

Roll No: 0039-BSCS-17

ML Syed M. Tafseer

ul Hassan

Q # 1

i)

Stolen

Yes
(3)

No
(5)

$$P(\text{Yes}) = \frac{5}{10} = \frac{1}{2}$$

$$P(\text{No}) = \frac{5}{10} = \frac{1}{2}$$

ii)

Origin

Domestic

Yes
3

No
3

Imported

Yes
2

No
2

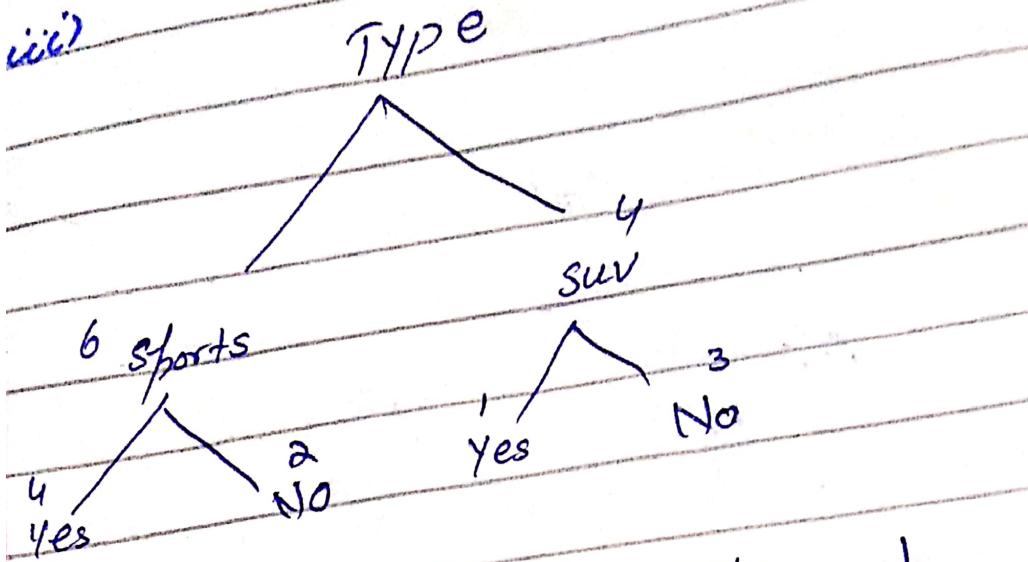
$$P\left(\frac{\text{Domestic}}{\text{Yes}}\right) = \frac{3}{5}$$

$$P\left(\frac{\text{Imported}}{\text{Yes}}\right) = \frac{2}{5}$$

$$P\left(\frac{\text{Domestic}}{\text{No}}\right) = \frac{3}{5}$$

$$P\left(\frac{\text{Imported}}{\text{No}}\right) = \frac{2}{3}$$

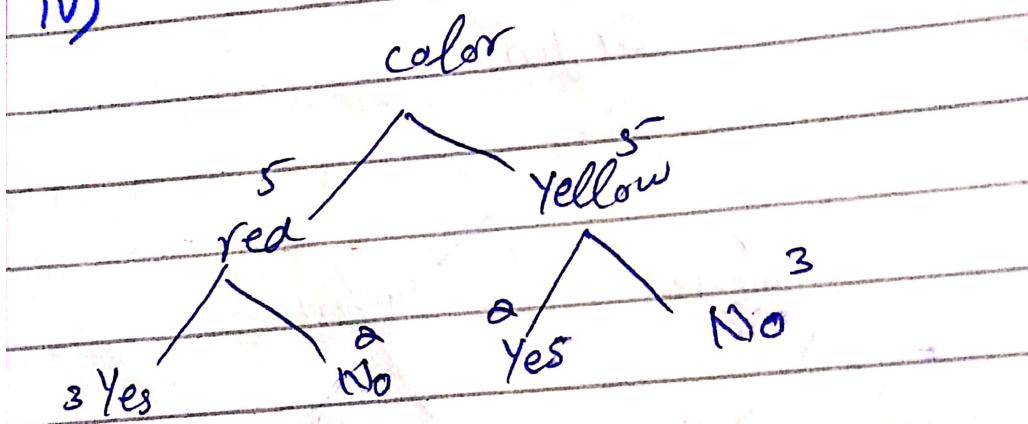
iii)



$$P\left(\frac{\text{sports}}{\text{Yes}}\right) = \frac{4}{5} \quad P\left(\frac{\text{suv}}{\text{Yes}}\right) = \frac{1}{5}$$

$$P\left(\frac{\text{sports}}{\text{No}}\right) = \frac{2}{5} \quad P\left(\frac{\text{suv}}{\text{No}}\right) = \frac{3}{5}$$

iv)



$$P\left(\frac{\text{red}}{\text{Yes}}\right) = \frac{3}{5} \quad P\left(\frac{\text{yellow}}{\text{Yes}}\right) = \frac{2}{5}$$

$$P\left(\frac{\text{red}}{\text{No}}\right) = \frac{2}{5} \quad P\left(\frac{\text{yellow}}{\text{No}}\right) = \frac{3}{5}$$

$$P\left(\frac{x}{\text{Yes}}\right) = \{P(\text{Yes}), P\left(\frac{\text{red}}{\text{Yes}}\right), P\left(\frac{\text{suv}}{\text{Yes}}\right), P\left(\frac{\text{clones}}{\text{Yes}}\right)\}$$

$$= \frac{1}{2} \times \frac{3}{5} \times \frac{1}{5} \times \frac{3}{5}$$

$$= 0.2 \times 0.6 \times 0.2 \times 0.6$$
$$= 0.036$$

$$P\left(\frac{x}{\text{No}}\right) = \{P(\text{No}), P\left(\frac{\text{red}}{\text{No}}\right), P\left(\frac{\text{suv}}{\text{No}}\right), P\left(\frac{\text{clones}}{\text{No}}\right)\}$$

$$= \frac{1}{2} \times \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$$

$$= 0.072$$

~~P~~b. will Not be stolen.

Q#3

a) Draw clusters using given dataset

(2, 3, 4, 10, 11, 12, 20, 25 & 30)

$$k = 2$$

$$c_1 = 4 \quad \& \quad c_2 = 12$$

By finding the distance of dataset with c_1 & c_2 :

$$K_1 = \{2, 3, 4\}$$

$$K_2 = \{10, 11, 12, 20, 25, 30\}$$

$$c_1 = \frac{(2+3+4)}{3} = 3$$

$$c_2 = \frac{(10+11+12+20+25+30)}{6} = 18$$

Now for iteration #12:

$$c_1 = 3 \quad \& \quad c_2 = 18$$

By finding distance of dataset with c_1 & c_2 :

$$K_1 = \{2, 3, 4, 10\}$$

$$K_2 = \{11, 12, 20, 25, 30\}$$

$$c_1 = \frac{(2+3+4+10)}{4} = 4.75 \approx 5$$

$$c_2 = \frac{(11+12+20+25+30)}{5} = 19.6 \approx$$

Now for iteration #3

$$c_1 = 4.75 \approx 5 \quad \& \quad c_2 = 19.6 \approx 20$$

By finding distance from CR (c_2):

$$K_1 = \{2, 3, 4, 10, 11, 12\}$$

$$K_2 = \{20, 25, 30\}$$

$$c_1 = \frac{2+3+4+10+11+12}{6} = 7$$

$$c_2 = \frac{20+25+30}{3} = 25$$

Now for iteration #4:

$$c_1 = 7 \quad \& \quad c_2 = 25$$

$$K_1 = \{2, 3, 4, 10, 11, 12\}$$

$$K_2 = \{20, 25, 30\}$$

As, K_1 & K_2 of this iteration are same iteration #3, so,

$$c_1 = 7 \quad \& \quad c_2 = 25$$

so the clusters of this dataset are:

$$K_1 = \{2, 3, 4, 10, 11, 12\}$$

$$K_2 = \{20, 25, 30\}$$

b)

draw the clusters for given dataset

(2,3) (5,6) (8,7) (1,4) (2,2)
(6,7) (3,4) (8,6)

where $K=2$

① Assume two points as mean

$$c_1 = (2,3) \quad c_2 = (5,6)$$

② Assign values in two clusters

$$\text{distance formula} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

x	y	distance $c_1(2,3)$	distance $c_2(5,6)$	selected cluster
2	3	0	4.24	c_1
5	6	4.24	0	c_2
8	7	7.21	3.16	c_2
1	4	1.41	4.47	c_1
2	2	1	5	c_1
6	7	5.65	1.41	c_2
3	4	1.41	2.82	c_1
8	6	6.70	3	c_2

③ check which one in $c_1 \cup c_2$

$$c_1 = (2,3) (1,4) (2,2) (3,4)$$

$$\text{mean}(x,y) = \frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}$$

$$= \frac{2+1+2+3}{4}, \frac{3+4+2+4}{4}$$

$$c_1 = (2, 3.25)$$

$$c_2 = (5, 6)(8, 7)(6, 7)(8, 6)$$

$$\text{mean}(x, y) = \frac{5+8+6+3}{4}, \frac{6+7+7+6}{4}$$

$$c_2 = (6.75, 6.8)$$

mean is not same repeat the steps.
with new men.

x	y	distance $c_1(2, 3.25)$	distance $c_2(6.75, 6.5)$	rek
2	3	0.25	5.90	c1
5	6	4.069	1.820	c2
8	7	7.075	1.346	c2
1	4	1.25	6.269	c1
2	2	1.25	6.54	c2
6	7	5.482	0.90	c2
3	4	1.25	4.80	c1
8	6	6.005	1.346	c2

$$c_1 = (2, 3)(1, 4)(2, 2)(3, 4)$$

$$\text{mean}(x, y) = \frac{2+1+2+3}{4}, y = \frac{3+4+2+4}{4}$$

$$c_1 = (2, 3.25)$$

$$c_2 = (5, 6)(8, 7)(6, 7)(8, 6)$$

$$\text{mean}(x, y) x = \frac{5+8+6+3}{4}, y = \frac{6+7+7}{4}$$

$$c_2 = (6.75, 6.5)$$

Now mean is same so stop it.

Q#2

$$P(\text{Yes}) = 9, P(\text{No}) = 5$$

$$\text{total} = 14$$

calculating Entropy

$$\text{Entropy}(S) = -\frac{9}{9+5} \log_2 \left(\frac{9}{9+5} \right) = \frac{5}{9+5} \log_2$$

$$\frac{5}{9+5} = 0.940$$

Now, calculating entropy for each attribute

outlook	yes	no	Entropy
sunny	2	3	0.971
Rain	3	2	0.971
overcast	4	0	0

using formula

$$E(\text{outlook} = \text{sunny}) = -\frac{2}{5} \log_2 \left(\frac{2}{5} \right) - \frac{3}{5} \log_2 \left(\frac{3}{5} \right) = 0.971$$

Similarly calculate others.

now, calculating average information

entropy

$$I(\text{outlook}) = \frac{3+2}{9+5} \times 0.971 + \frac{2+3}{9+5} \times 0.971 + 0 \times 0 = 0.971$$

Hence further splitting is required, we will have to calculate entropy for sunny & rainy values of overcast

For these two values

$$P(\text{yes}) = 2, P(\text{no}) = 3$$

$$\text{total} = 5$$

$$\text{Entropy } (S) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5}$$

$$E(\text{sunny}) = 0.971$$

For Humidity,

Humidity	yes	no	Entropy
high	0	3	0
normal	0	0	0

Avg info Entropy

$$I(\text{humidity}) = 0$$

$$\text{rain } (0.971)$$

For wind,

wind	yes	no	Entropy
strong	1	1	1
weak	1	2	0.918

$$I(\text{windy}) = 0.95$$

$$\text{Gain} = 0.020$$

For Temperature

Temp	Yes	No	Entropy
Cool	Yes!	0	0
Hot	0	0	0
mild	0	1	1

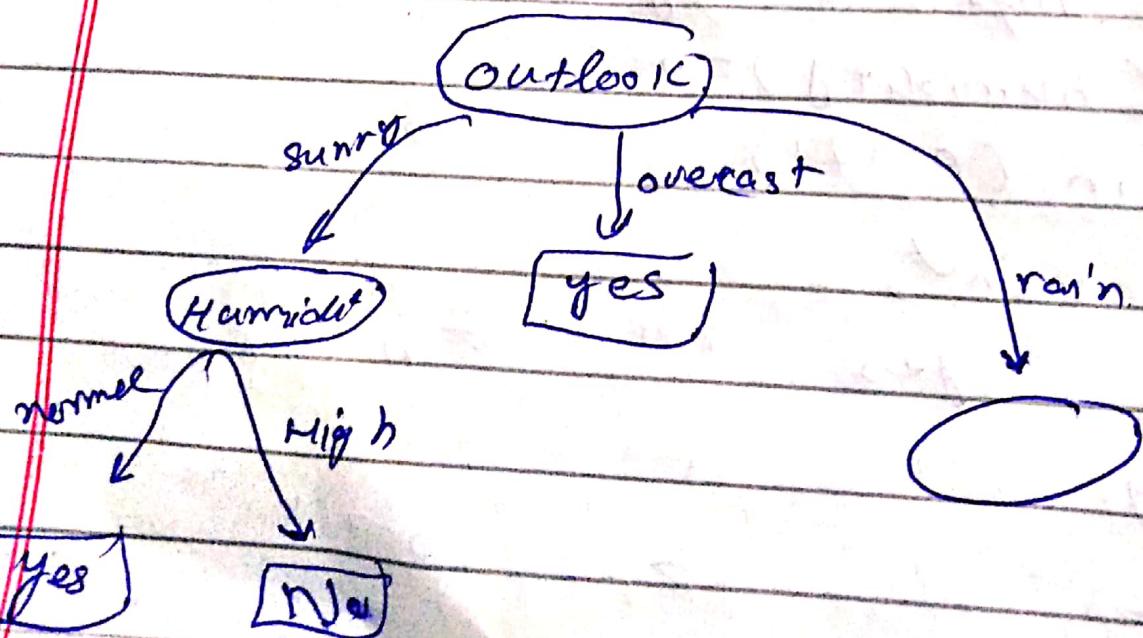
$$I(\text{Temp}) = 0.4$$

$$\text{Gain} = 0.571$$

now,

Attributes	Gain
temperature	0.571
Humidity	0.971
windy	0.02

next node humidity.



now. Calculating gain

$$\text{Gain} = \text{Entropy}(S) - I(\text{Attribute})$$

$$\text{Entropy}(S) = 0.940$$

$$\text{Gain}(\text{outlook}) = 0.940 - 0.693 = 0.247$$

For temperature,

Temp	yes	No	Entropy
Hot	2	2	1
mild	4	2	0.918
cool	3	1	0.311

Avg information Entropy

$$I(\text{Temp}) = \frac{2+2}{9+5} \times 1 + \frac{4+2}{9+5} \times 0.918 + \frac{3+1}{9+5} \times 0.311 \\ = 0.911$$

$$\text{Entropy}(S) = 0.940$$

$$\text{Gain} = 0.940 - 0.911 = 0.029$$

For humidity

Humidity	yes	No	Entropy
High	3	4	0.985
Normal	6	1	0.591

Avg information Entropy

$$I(\text{Humidity}) = \frac{3+4}{9+5} \times 0.985 + \frac{6+1}{9+5} \times 0.591 \\ = 0.788$$

$$E(S) = 0.940$$

$$\text{Gain(Humidity)} = 0.940 - 0.788 = 0.152$$

For wind,

wind	yes	No	Entropy
strong	3	3	1
weak	6	2	
			0.811

Avg Info Entropy

$$I(\text{windy}) = \frac{3+3}{9+5} \times \frac{1+6+2}{9+5} \times 0.811 = 0.892$$

$$E(S) = 0.940$$

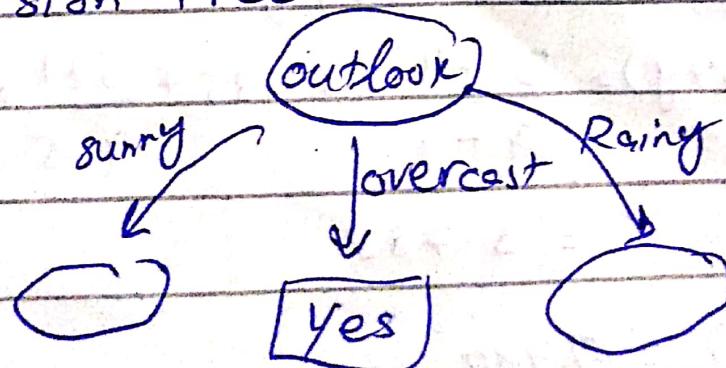
$$\text{Gain(wind)} = 0.940 - 0.892 = 0.048$$

Since, outlook has biggest gain.

it will be our root node.

Attributes	Gain
outlook	0.242
Temp	0.029
Humidity	0.152
wind	0.048

Decision Tree



now for rain, further splitting is necessary

$$P = 3 \rightarrow N = 2$$

$$\text{Total} = 5$$

$$\text{Entropy}(S_{\text{rainy}}) = -\frac{3}{3+2} \log_2 \frac{3}{3+2} - \frac{2}{3+2} \log_2 \left(\frac{2}{3+2} \right)$$
$$= 0.971$$

now, calculating entropy for each attribute

for Humidity

Avg info Entropy

$$I(\text{Humidity}) = 0.951$$

$$\text{Gain} = 0.020$$

For wind,

Avg info Entropy

$$I(\text{windy}) = 0$$

$$\text{Gain} = 0.971$$

For Temp,

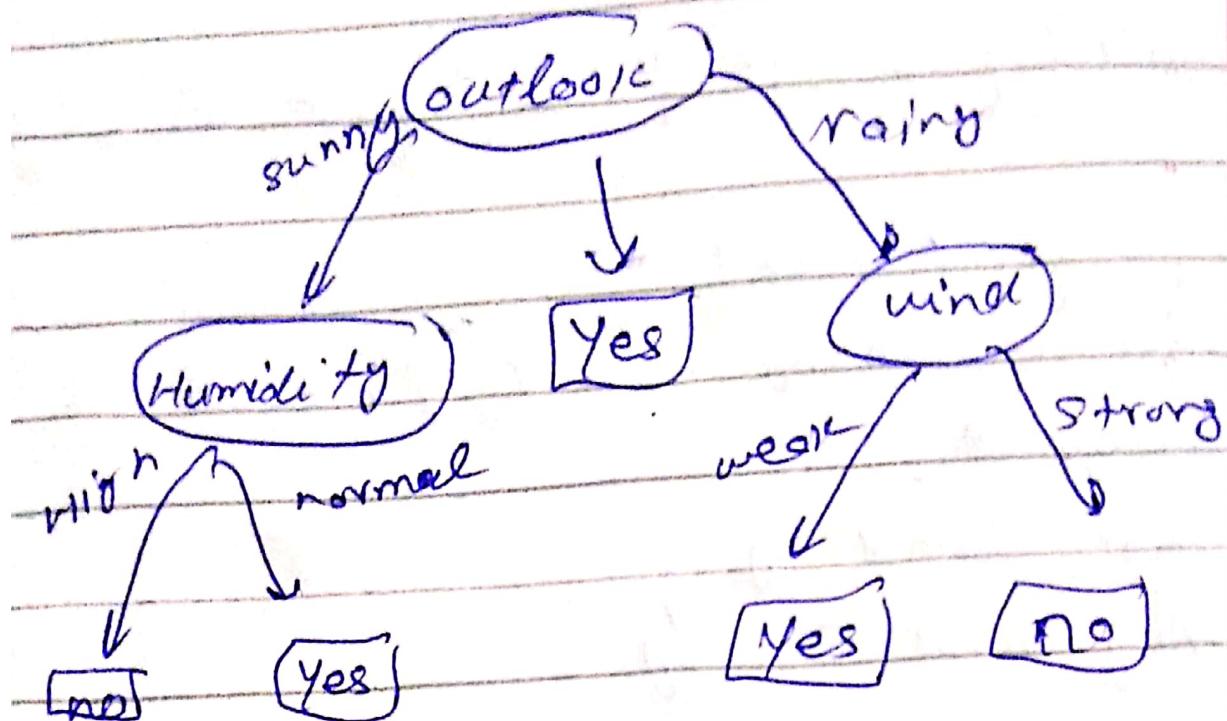
$$I(\text{Temp}) = 0.951$$

$$\text{Gain} = 0.020$$

Attributes	Gain
Humidity	0.020
wind	0.971
Temperature	0.020

next node : wind

Final decision tree :



Q#4 B/\bar{X} B/\bar{N} core core core B/\bar{X} B/\bar{X}
a 0.00 2.83 4.24 4.12 2.24 6.40 4.12
b 2.83 0.00 8.10 3.61 3.00 6.71 4.00
c 4.24 5.10 0.00 2.24 2.24 2.24 2.24
d 4.12 3.61 2.24 0.00 2.00 3.16 2.91
e 2.24 3.00 2.24 2.00 0.00 4.24 3.16
f 6.40 6.71 2.24 3.16 4.24 0.00 4.00
g 4.12 6.08 2.24 4.24 3.16 4.00 0.00

$2\text{-neighborhood} = \{a, b, c\}$

Core points: - c, d, e

Border points: a, f, g

Noise points: b