수치해석 HW#6

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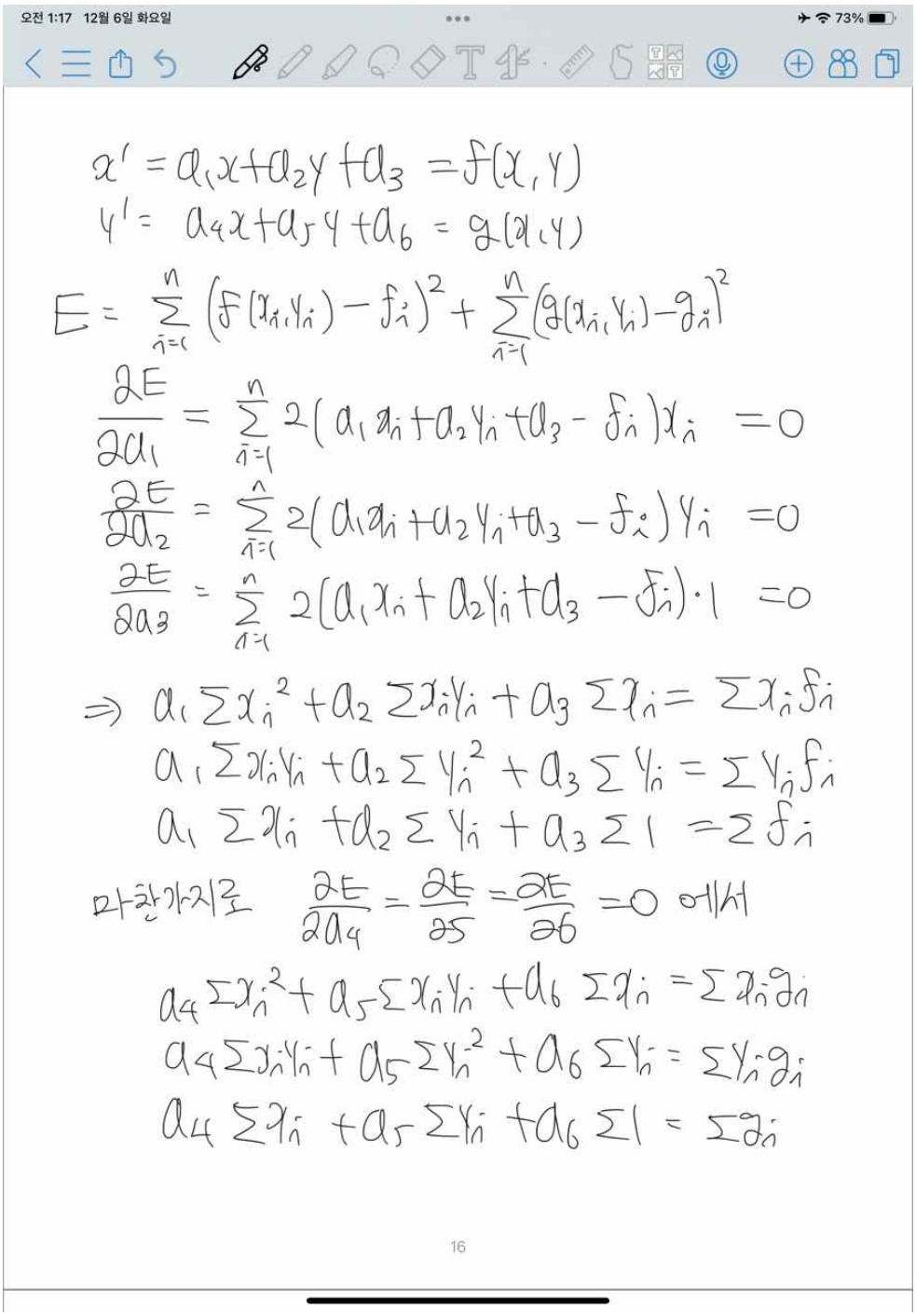
1. Introduction

1. 목적

최소제곱법을 통해 data를 fitting 시키는 함수의 parameter를 최적화한다.

2. 최소제곱법을 통한 parameter 최적화 공식



2. Process

```
tmp = []
with open("fitdata1.dat", "r") as f:
    while True:
        s = f.readline()
        if not s:
            break
        tmp.append(s.split())
n = len(tmp)
print("n = ",n)
x,y,f,g,xy,xx,yy,xf,yf,xg,yg = 0,0,0,0,0,0,0,0,0,0,0
for s in tmp:
    x_tmp, y_tmp, f_tmp, g_tmp = float(s[0]), float(s[1]),float(s[2]),float(s[3])
    x += x_tmp
    y += y_tmp
    f += f_tmp
    g += g_tmp
    xy += x_tmp * y_tmp
    xx += x_tmp * x_tmp
    yy += y_tmp * y_tmp
    xf += x_tmp * f_tmp
    yf += y_tmp * f_tmp
    xg += x_tmp * g_tmp
    yg += y_tmp * g_tmp
print(" x: ",x,"₩n","y: ",y,"₩n","f: ",f,"₩n","g: ",g,"₩n","xy: ",xy,"₩n","xx: ",xx,"₩n","yy: ",yy,"₩n","xf: ",xf,"₩n","yf: ",yf,"₩n",
```

data를 가져와서 x_i, y_i, f_i, g_i 값을 tmp에 저장한다. 이후에 parameter를 구하기위해 필요한 값인 ∑x, ∑y, ∑f, ∑g, ∑xy, ∑x^2, ∑y^2, ∑xf, ∑yf, ∑xg, ∑yg를 구한 후 식을 세우고 연립방정식을 풀면 된다.

3. Result

3.1 fitdata1.dat

```
n =  77
 x:  -592.8399999999998
 y:  -1971.5700000000045
 f:  -616.0
 g:  -1848.0
 xy:  7667.740399999995
 xx:  3274242.0570000014
 yy:  1355803.0851000005
 xf:  3215182.400000001
 yf:  11713.04
 xg:  10938.560000000063
 yg:  1329346.4799999995
```

연립 방정식:

$$\begin{cases} 3274242.05700000x_1 + 7667.74039999999x_2 - 592.839999999999x_3 + 0x_4 + 0 + 0 = 3215182.40000000 \\ 7667.74039999999x_1 + 1355803.08510000x_2 - 1971.57000000000x_3 + 0x_4 + 0x_5 + 0x_6 = 11713.04 \\ -592.839999999999x_1 - 1971.57000000000x_2 + 77x_3 + 0x_4 + 0x_5 + 0x_6 = -616.0 \\ 0x_1 + 0x_2 + 0x_3 + 3274242.05700000x_4 + 7667.74039999999x_5 - 592.839999999999x_6 = 10938.5600000000 \\ 0x_1 + 0x_2 + 0x_3 + 7667.74039999999x_4 + 1355803.08510000x_5 - 1971.57000000000x_6 = 1329346.47999999 \\ 0x_1 + 0x_2 + 0x_3 - 592.839999999999x_4 - 1971.57000000000x_5 + 77x_6 = -1848.0 \end{cases}$$

best a =

일반해 : $X = \begin{pmatrix} 0.9819 \\ 0.0025 \\ -0.3752 \\ 0.0013 \\ 0.9822 \\ 1.1577 \end{pmatrix}$

3.2 fitdata2.dat

```
n =  77
 x:  -534.0599999999993
 y:  -1924.2400000000002
 f:  -616.0
 g:  -1848.0
 xy:  16324.710100000011
 xx:  3288267.2390000005
 yy:  1360720.7228
 xf:  3222840.159999999
 yf:  18905.600000000057
 xg:  12224.799999999952
 yg:  1333014.4
```

연립 방정식:

$$\begin{cases}
3288267.23900000 x_1 + 16324.7101000000 x_2 + -534.059999999995 x_3 + 0 x_4 + 0 + 0 = 3222840.15999999 \\
16324.7101000000 x_1 + 1360720.7228 x_2 + -1924.24000000000 x_3 + 0 x_4 + 0 x_5 + 0 x_6 = 18905.6000000000 \\
-534.059999999995 x_1 + -1924.24000000000 x_2 + 77 x_3 + 0 x_4 + 0 x_5 + 0 x_6 = -616.0 \\
0 x_1 + 0 x_2 + 0 x_3 + 3288267.23900000 x_4 + 16324.7101000000 x_5 + -534.059999999995 x_6 = 12224.7999999999 \\
0 x_1 + 0 x_2 + 0 x_3 + 16324.7101000000 x_4 + 1360720.7228 x_5 + -1924.24000000000 x_6 = 1333014.4 \\
0 x_1 + 0 x_2 + 0 x_3 + -534.059999999995 x_4 + -1924.24000000000 x_5 + 77 x_6 = -1848.0
\end{cases}$$

best a =

일반해 : $X = \begin{pmatrix} 0.9799 \\ 0.0005 \\ -1.1922 \\ -0.0011 \\ 0.9803 \\ 0.4916 \end{pmatrix}$

3.3 fitdata3.dat

```
n =  77
 x:  -552.8399999999998
 y:  -1921.5700000000045
 f:  -616.0
 g:  -1848.0
 xy:  15451.140400000018
 xx:  3282454.0570000014
 yy:  1363876.0851000005
 xf:  3219982.400000001
 yf:  17713.04
 xg:  12538.560000000005
 yg:  1334546.4799999995
```

연립 방정식:

$$\begin{cases} 3282454.05700000x_1 + 15451.1404000000x_2 - 552.839999999995x_3 + 0x_4 + 0 + 0 = 3219982.40000000 \\ 15451.1404000000x_1 + 1363876.08510000x_2 - 1921.57000000000x_3 + 0x_4 + 0x_5 + 0x_6 = 17713.04 \\ -552.839999999995x_1 - 1921.57000000000x_2 + 77x_3 + 0x_4 + 0x_5 + 0x_6 = -616.0 \\ 0x_1 + 0x_2 + 0x_3 + 3282454.05700000x_4 + 15451.1404000000x_5 - 552.839999999995x_6 = 12538.5600000000 \\ 0x_1 + 0x_2 + 0x_3 + 15451.1404000000x_4 + 1363876.08510000x_5 - 1921.57000000000x_6 = 1334546.47999999 \\ 0x_1 + 0x_2 + 0x_3 - 552.839999999995x_4 - 1921.57000000000x_5 + 77x_6 = -1848.0 \end{cases}$$

$$\text{best a} = \text{일반해} : X = \begin{pmatrix} 0.9808 \\ 0.0005 \\ -0.9445 \\ -0.0007 \\ 0.9791 \\ 0.4289 \end{pmatrix}$$

4. Discussion

4.1 independent(독립성)

에러함수를 미분했을 때 a1, a2, a3와 y'은 독립이므로 a1, a2, a3에 대한 연립방정식이 3개 a4, a5, a6에 대한 연립방정식 3개 각각 총 6개의 연립방정식을 풀면 된다. 또한 x', y'식이 서로 관계가 없어 x, y에 대해서 x'을 fitting 해주고, x, y에 대해서 y'을 fitting 해도 문제가 없다는 것을 의미한다.

5. Reference

[1] http://numerical.recipes/book/book.html