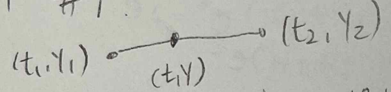
"본과제에 작성된 내용은 (아래에 작성한 바와 같이 일부 참고문헌의 도움을 받거나 동료들과 상의는 하였지만) 기본적으로는 본인 스스로 해결하고 작성한 것임을 서약합니다. 서약이 거짓임을 밝혀지면 F학점을 받는것에 동의합니다."

2022/6/1 2018008559 신상윤 신상윤

참고내용: 교재.

5.1 - #7



선 위에 있는 점 (t, y)를 생각. 기울기는 모두 같으므로 $\dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{y - y_1}{t - t_1}$

$\rightarrow \dfrac{y - y_1}{y_2 - y_1} = \dfrac{t - t_1}{t_2 - t_1} = \lambda$ 라 하자.

$t = (1-\lambda)t_1 + \lambda t_2$

$y = (1-\lambda)y_1 + \lambda y_2$ $\therefore ((1-\lambda)t_1 + \lambda t_2, (1-\lambda)y_1 + \lambda y_2)$ 에 대응됨.

특히 $\begin{pmatrix} t_1 \le t \le t_2 \\ y_1 \le y \le y_2 \end{pmatrix}$ 일때. $0 \le \lambda \le 1$ 이다.

5.1 - #9

Let $(t_1, y_1) \in D$, $(t_2, y_2) \in D$.

need to show $((1-\lambda)t_1 + \lambda t_2, (1-\lambda)y_1 + \lambda y_2) \in D$ at all $0 \le \lambda \le 1$

정의에 의해

$a \le t_1 \le b$, $a \le t_2 \le b$

$-\infty \le y_1 \le \infty$, $-\infty \le y_2 \le \infty$

$0 \le 1-\lambda \le 1 \rightarrow (1-\lambda)a \le (1-\lambda)t_1 \le (1-\lambda)b$

$\lambda a \le \lambda t_2 \le \lambda b$

$\therefore a = (1-\lambda)a + \lambda a \le (1-\lambda)t_1 + \lambda t_2 \le b$.

마찬가지로

$-\infty \le (1-\lambda)y_1 \le \infty$, $-\infty \le \lambda y_2 \le \infty$

$-\infty \le (1-\lambda)y_1 + \lambda y_2 \le \infty$

$\therefore ((1-\lambda)t_1 + \lambda t_2, (1-\lambda)y_1 + \lambda y_2) \in D$ 이므로

D는 convex.

5.2 - #1

(a) $y' = te^{3t} - 2y$, $0 \le t \le 1$, $y(0) = 0$, $h = 0.5$

$y(0) = 0$

$y(0.5) = y(0) + 0.5 \times f(0, y(0))$

$= 0 + 0.5 \times 0 = 0$

$y(1) = y(0.5) + 0.5 \times f(0.5, y(0.5))$

$= 0.5 \times 0.5 \times e^{1.5} = 1.12042$

(b) $y' = 1 + (t-y)^2$, $2 \le t \le 3$, $y(2) = 1$, $h = 0.5$

$y(2) = 1$

$y(2.5) = y(2) + 0.5 \times f(2, y(2))$

$= 1 + 0.5 \times 2 = 2$

$y(3) = y(2.5) + 0.5 \times f(2.5, y(2.5))$

$= 2 + 0.5 \times (1 + 0.5^2) = 2.625$

(c) $y' = 1 + \dfrac{y}{t}$, $1 \le t \le 2$, $y(1) = 2$, $h = 0.25$

$y(1) = 2$

$y(1.25) = y(1) + 0.25 \times f(1, y(1))$

$= 2 + 0.25 \times 3 = 2.75$

$y(1.5) = y(1.25) + 0.25 \times f(1.25, y(1.25))$

$= 2.75 + 0.25 \times (1 + \frac{2.75}{1.25})$

$= 3.55$

$y(1.75) = y(1.5) + 0.25 \times f(1.5, y(1.5))$

$= 3.55 + 0.25 \times (1 + \frac{3.55}{1.5}) = 4.39166$

$y(2) = y(1.75) + 0.25 \times f(1.75, y(1.75))$

$= 4.39166 + 0.25 \times (1 + \frac{4.39166}{1.75}) = 5.269$

(d) $y' = \cos 2t + \sin 3t$, $0 \le t \le 1$, $y(0) = 1$, $h = \frac{1}{4}$

$y(0) = 1$

$y(0.25) = y(0) + 0.25 \times f(0, y(0))$
$= 1 + 0.25 \times 1 = 1.25$

$y(0.5) = y(0.25) + 0.25 \times f(0.25, y(0.25))$
$= 1.25 + 0.25 \times (\cos 0.5 + \sin 0.75)$
$= 1.6398$

$y(0.75) = y(0.5) + 0.25 \times f(0.5, y(0.5))$
$= 1.6398 + 0.25 \times (\cos 1 + \sin 1.5)$
$= 2.0242546$

$y(1) = y(0.75) + 0.25 \times f(0.75, y(0.75))$
$= 2.02425 + 0.25 \times (\cos 1.5 + \sin 2.25)$
$= 2.23645$

5.2-#9

$y' = \frac{2}{t} y + t^2 e^t$, $1 \le t \le 2$, $y(1) = 0$

$y(t) = t^2(e^t - e)$

(a)

오일러	exact
0	0
0.271828	0.3459
0.684755	0.86664
1.276978	1.60721508
2.09354	2.62035955
3.18744	3.96766
4.620817	5.7209
6.466396	7.96387
8.809119	10.793624
11.74799	14.323081
15.3982	18.683097

(b) $1 < 1.04 < 1.1$

$y = \frac{f(1.1) - f(1)}{1.1 - 1}(x - 1) + f(1)$

추정 $f(1.04) = 0.108731273$

정확 $y(1.04) = 0.11998749$

$1.5 < 1.55 < 1.6$

추정: 3.90413148

정확: 4.788635

$1.9 < 1.97 < 2$

추정: 14.30316392

정확: 17.27929 8

(c) $y' = 2t(e^t - e) + t^2(e^t)$
$= (t^2 + 2t)e^t - 2et$

$y'' = (t^2 + 4t + 2)e^t - 2e$

$y''' = (t^2 + 6t + 6)e^t$

$1 \le t \le 2$ 에서

$|y''| \le 14e^2 - 2e = 98.01$

$M = 98.01$

convex 일때 L

$\left|\frac{\partial f}{\partial y}(t, y)\right| = \left|\frac{2}{t}\right| \le 2$ $(1 \le t \le 2)$

$L = 2$.

$\therefore \frac{h \times 98.01}{2 \times 2} \times (e^{2 \times 1} - 1) \le 0.1$

$h = 0.000638782$

5.3-#1 order 2

(a) $y' = te^{3t} - 2y$, $0 \le t \le 1$, $y(0)=0$ $h=0.5$

$$T^{(2)} = f(t,y) + \frac{h}{2} f'(t,y)$$
$$= te^{3t} - 2y + \frac{h}{2}\left(e^{3t} + 3te^{3t} - 2te^{3t} + 4y\right)$$
$$= te^{3t} - 2y + \frac{h}{2}\left((t+1)e^{3t} + 4y\right)$$

$y(0)=0$

$y(0.5) = y(0) + 0.5 \times T(0, y(0)) = \frac{0.5^2}{2} = 0.125$

$y(1) = y(0.5) + 0.5 \times T(0.5, y(0.5)) = 2.02324$

(b) $y' = 1 + (t-y)^2$, $2 \le t \le 3$, $y(2)=1$, $h=0.5$

$$T^{(2)} = f(t,y) + \frac{h}{2} f'(t,y)$$
$$= 1 + (t-y)^2 + \frac{h}{2}\left(2(t-y) - 2(t-y)(1+(t-y)^2)\right)$$
$$= 1 + (t-y)^2 + h(t-y)(-(t-y)^2)$$
$$= 1 + (t-y)^2(1 - h(t-y))$$

$y(2)=1$

$y(2.5) = y(2) + 0.5 \times T(2, y(2))$
$= 1 + 0.5 \times (1 + 1 - 0.5(1)) = 1.75$

$y(3) = y(2.5) + 0.5 \times T(2.5, 1.75)$
$= 1.75 + 0.5 \times (1.3515625) = 2.42578$

(c) $y' = 1 + \frac{y}{t}$, $1 \le t \le 2$, $y(1)=2$ $h=0.25$

$$T^{(2)} = f(t,y) + \frac{h}{2} f'(t,y)$$
$$= 1 + \frac{y}{t} + \frac{h}{2} \cdot \frac{y't - y}{t^2} = 1 + \frac{y}{t} + \frac{h}{2}\frac{1}{t}$$

$y(1)=2$

$y(1.25) = y(1) + 0.25 \times T(1, y(1))$
$= 2 + 0.25 \times 3.125 = 2.78125$

$y(1.5) = y(1.25) + 0.25 \times T(1.25, 2.78125)$
$= 3.6125$

$y(1.75) = y(1.5) + 0.25 \times T(1.5, 3.6125)$
$= 4.4854166$

$y(2) = y(1.75) + 0.25 \times T(1.75, 4.4854)$
$= 5.3940476$

(d) $y' = \cos 2t + \sin 3t$, $0 \le t \le 1$ $y(0)=1$ $h=0.25$

$$T^{(2)} = f(t,y) + \frac{h}{2} f'(t,y) = \cos 2t + \sin 3t + \frac{h}{2}\left(3\cos 3t - 2\sin 2t\right)$$
$$= \cos 2t + \sin 3t + \frac{3}{2} h \cos 3t - h \sin 2t$$

$y(0)=1$

$y(0.25) = y(0) + h \cdot T(0, y(0))$
$= 1.34375$

$y(0.5) = y(0.25) + h \cdot T(0.25, y(0.25))$
$= 1.772187$

$y(0.75) = y(0.5) + h \times T(0.5, y(0.5))$
$= 2.110676$

$y(1) = y(0.75) + h \times T(0.75, y(0.75))$
$= 2.2016439$

5.3-#10

$y' = \frac{1}{t^2} - \frac{y}{t} - y^2$, $1 \le t \le 2$ $y(1) = -1$

$$f(t,y) = \frac{1}{t^2} - \frac{y}{t} - y^2$$

$$f'(t,y) = -2 \cdot \frac{1}{t^3} - 2y \cdot y' - \left(\frac{y't - y}{t^2}\right)$$
$$= \frac{-2}{t^3} - \cancel{\frac{2y}{t^2}} + \cancel{\frac{2y^2}{t}} + 2y^3 + \cancel{\frac{y}{t^2}} - \left(\frac{1}{t^3} - \cancel{\frac{y}{t^2}} - \cancel{\frac{y^2}{t}}\right)$$
$$= \frac{-3}{t^3} + \frac{3y^2}{t} + 2y^3$$

$$f''(t,y) = 9t^{-4} + 6y^2 y' + 3\left(\frac{2yy't - y^2}{t^2}\right)$$
$$= \frac{9}{t^4} + \frac{6y^2}{t^2} - \frac{6y^3}{t} - 6y^4 - \frac{3y^2}{t^2} + \frac{6y}{t^3} - \frac{6y^2}{t^2} - \frac{6y^3}{t}$$
$$= \frac{9}{t^4} + \frac{6y}{t^3} - \frac{3y^2}{t^2} - \frac{12y^3}{t} - 6y^4$$

$$f^{(3)}(t,y) = \frac{-36}{t^5} + 6 \cdot \left(\frac{y't^3 - y3t^2}{t^6}\right) - 3\left(\frac{2yy't^2 - 2y^2t}{t^4}\right)$$
$$-12\left(\frac{3y^2y't - y^3}{t^2}\right) - 24y^3 \cdot y'$$

$$= \frac{-36}{t^5} + \frac{6}{t^5} - \frac{6y}{t^4} - \frac{6y^2}{t^3} - \frac{18y}{t^4} - \frac{6y}{t^4} + \frac{6y^2}{t^3} + \frac{6y^3}{t^2}$$
$$\frac{12y^3}{t^2} + \frac{6y^2}{t^3} - \frac{36y^2}{t^3} + \frac{36y^3}{t^2} + \frac{36y^4}{t} - \frac{24y^3}{t^2} + \frac{24y^4}{t} + 24y^5$$

$$= \frac{-30}{t^5} + \frac{-30y}{t^4} + \frac{-30y^2}{t^3} + \frac{30y^3}{t^2} + \frac{60y^4}{t} + 24y^5$$

(a), (c)

actual value	order2	order 4
-1	-1	-1
-0.95238	-0.9525	-0.95238125
-0.90909	-0.90931378	-0.90909144
-0.86956521	-0.8698798	-0.86956559
-0.83333	-0.833373	-0.833333422
-0.799999	-0.80047127	-0.800010259
-0.76923	-0.76977	-0.769231915
-0.74074074	-0.74143010	-0.7407419929
-0.7142857	-0.7149465	-0.714287
-0.6896551724	-0.6903711	-0.6896566
-0.666666	-0.6674346	-0.6666668
-0.64516129	-0.645978	-0.645162886
-0.6249999	-0.62586488	-0.625000166
-0.6060606	-0.6069410	-0.606062345
-0.58823529	-0.5891894	-0.588237
-0.571428571	-0.57242568	-0.57143
-0.5555--	-0.55659	-0.55555749
-0.54054054	-0.5416196	-0.5405425
-0.526315789	-0.5274345	-0.52631785
-0.51820512	-0.5139778	-0.51282263
-0.49999	-0.5011957	-0.500002189

(b), (d)

exact value	linear-app	cubic-app
-0.9505703422	-0.9507725515	-0.95057063
-0.643086816	-0.6439694	-0.6430884
-0.50556117	-0.5068199	-0.5055633

5.4-#29

$y' = -y + t + 1, \quad 0 \le t \le 1, \quad y(0) = 1$

by mid point method

$$w_{i+1} = w_i + h f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\right)$$
$$= w_i + h f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}(-w_i + t_i + 1)\right)$$
$$= \left(1 - h + \frac{h^2}{2}\right) w_i - \frac{h^2}{2} t_i + h(t_i + 1) \cdots ①$$

by Modified Euler method

$$w_{i+1} = w_i + \frac{h}{2}\left(f(t_i, w_i) + f(t_{i+1}, w_i + h f(t_i, w_i))\right)$$
$$= w_i + \frac{h}{2}\left(-w_i + t_i + 1 - w_i - h(-w_i + t_i + 1) + t_{i+1} + 1\right)$$
$$= \left(1 - h + \frac{h^2}{2}\right) w_i - \frac{h^2}{2}(t_i + 1) + \frac{h}{2}(t_i + t_{i+1}) + h \cdots ②$$

$$① - ② = h t_i + \frac{h^2}{2} - \frac{h}{2} t_i - \frac{h}{2} t_{i+1}$$
$$= h t_i + \frac{h^2}{2} - \frac{h}{2} t_i - \frac{h}{2}(t_i + h)$$
$$= 0$$

∴ h와 관계없이 같은 근사결과가 나온다.