"본 과제에 작성된 내용은 (아래에 작성한 바와 같이 일부 참고문헌의 도움을 받거나 동료들과 상의는 하였지만) 기본적으로는 본인 스스로 해결하고 작성한 것임을 서약합니다. 서약이 거짓임이 밝혀지면 F학점을 받는 것에 동의 합니다."

2022/6/3 2018008559 신상윤 신상윤

참고 내용 : 교재.

6.2-#1-(a)

$$\begin{bmatrix}1&-5&1\\10&0&20\\5&0&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}7\\6\\4\end{bmatrix}\to\begin{bmatrix}1&-5&1\\0&50&10\\0&25&-6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}7\\-64\\-31\end{bmatrix}$$

$E_2 := E_2 - 10E_1$

$E_3 := E_3 - 5E_1$

$E_3 := E_3 - \frac{1}{2}E_2$

$$\to\begin{bmatrix}1&-5&1\\0&50&10\\0&0&-11\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}7\\-64\\1\end{bmatrix}$$

∴ no row change

#3-(a)

$$\begin{bmatrix}1&-5&1\\10&0&20\\5&0&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}7\\6\\4\end{bmatrix}\xrightarrow[E_1\leftrightarrow E_2]{\text{row exchange. } 10>1.5}\begin{bmatrix}10&0&20\\1&-5&1\\5&0&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}6\\7\\4\end{bmatrix}$$

$\to$ $E_2 := E_2 - \frac{1}{10}E_1$, $E_3 := E_3 - \frac{1}{2}E_1$

$$\begin{bmatrix}10&0&20\\0&-5&-1\\0&0&-11\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}6\\6.4\\1\end{bmatrix}$$

∴ row 1 ⟷ row 2 , 1 row exchange

#5-(a)

$s_1=5,\ s_2=20,\ s_3=5$

$\max\left(\frac{1}{5}, \frac{10}{20}, \frac{5}{5}\right)=1$ → row exchange $E_1\leftrightarrow E_3$

$$\begin{bmatrix}5&0&-1\\10&0&20\\1&-5&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\6\\7\end{bmatrix}\xrightarrow[E_3:=E_3-\frac{1}{5}E_1]{E_2:=E_2-2E_1}\begin{bmatrix}5&0&-1\\0&0&22\\0&-5&\frac{6}{5}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\-2\\\frac{31}{5}\end{bmatrix}$$

0 pivot row exchange $E_2\leftrightarrow E_3$

$$\to\begin{bmatrix}5&0&-1\\0&-5&\frac{6}{5}\\0&0&22\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\\frac{31}{5}\\-2\end{bmatrix}$$

$E_1\leftrightarrow E_3$

$E_2\leftrightarrow E_3$

2 row exchange.

7-(a)

$\max|a_{ij}|=20$ → row exchange $E_1\leftrightarrow E_2$, column exchange $C_1\leftrightarrow C_3$

$$\begin{bmatrix}20&0&10\\1&-5&1\\-1&0&5\end{bmatrix}\begin{bmatrix}x_3\\x_2\\x_1\end{bmatrix}=\begin{bmatrix}6\\7\\4\end{bmatrix}\to$$

$E_2 := E_2 - \frac{1}{20}E_1$

$E_3 := E_3 + \frac{1}{20}E_1$

$$\begin{bmatrix}20&0&10\\0&-5&\frac{1}{2}\\0&0&\frac{11}{2}\end{bmatrix}\begin{bmatrix}x_3\\x_2\\x_1\end{bmatrix}=\begin{bmatrix}6\\6.7\\4.3\end{bmatrix}\quad \max|a_{ij}|=\frac{11}{2}$$

→ $E_2\leftrightarrow E_3$, $C_2\leftrightarrow C_3$

$$\begin{bmatrix}20&10&0\\0&\frac{11}{2}&0\\0&\frac{1}{2}&-5\end{bmatrix}\begin{bmatrix}x_3\\x_1\\x_2\end{bmatrix}=\begin{bmatrix}6\\4.3\\6.7\end{bmatrix}$$

$E_3 := E_3 - \frac{1}{11}E_2$

$$\to\begin{bmatrix}20&10&0\\0&\frac{11}{2}&0\\0&0&-5\end{bmatrix}\begin{bmatrix}x_3\\x_1\\x_2\end{bmatrix}=\begin{bmatrix}6\\4.3\\\frac{347}{55}\end{bmatrix}$$

$E_1\leftrightarrow E_2$ & $E_2\leftrightarrow E_3$

$C_1\leftrightarrow C_3$ $C_2\leftrightarrow C_3$

2 row exchange.

#26

if $\alpha=6$. $s_1=3,\ s_2=8,\ s_3=10$

$\text{Max}\left|\frac{a_{i1}}{s_i}\right|=\max\left(\frac{2}{3},\frac{4}{8},\frac{6}{10}\right)=\frac{2}{3}$

no row exchange at first

$$\begin{bmatrix}2&1&3&:&1\\0&4&2&:&3\\0&3&1&:&2\end{bmatrix}\quad \max\left(\frac{4}{8},\frac{3}{10}\right)=\frac{1}{2}$$

no row exchang at second

$$\begin{bmatrix}2&1&3&:&1\\0&4&2&:&3\\0&0&-\frac{1}{2}&:&-\frac{1}{4}\end{bmatrix}$$

if $\alpha=9$ $s_1=3,\ s_2=8,\ s_3=10$.

$\max\left(\frac{2}{3},\frac{4}{8},\frac{6}{10}\right)=\frac{2}{3}$

no row exchange at first

$$\begin{bmatrix}2&1&3&:&1\\0&4&2&:&3\\0&6&1&:&2\end{bmatrix}\quad \max\left(\frac{4}{8},\frac{6}{10}\right)=\frac{6}{10}$$

row exchange needed $E_2\leftrightarrow E_3$

if $\alpha = -3$

$$\left[\begin{array}{ccc:c} 2 & 1 & 3 & 1 \\ 4 & 6 & 8 & 5 \\ 6 & -3 & 10 & 5 \end{array}\right]$$

$S_1 = 3$, $S_2 = 8$, $S_3 = 10$

$$\max\left(\frac{2}{3}, \frac{4}{8}, \frac{6}{10}\right) = \frac{2}{3}$$

no rowexchange at first

$$\left[\begin{array}{ccc:c} 2 & 1 & 3 & 1 \\ 0 & 4 & 2 & 3 \\ 0 & -6 & 1 & 2 \end{array}\right] \quad \max\left(\frac{4}{8}, \frac{6}{10}\right) = \frac{6}{10}$$

row exchange needed. $E_2 \leftrightarrow E_3$

$\therefore$ if $\alpha = 6$ there will be no row interchange required.

6.5-#8-(a)

LU Factorization Algorithm give us

$LU = A$, to solve $Ax = b$

$LUx = b$, $Ux = C$

First solve $Lc = b$ by front substitution and then solve $Ux = C$ by back substitution.

$$\begin{bmatrix} 1 & & \\ l_{21} & 1 & \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ & U_{22} & U_{23} \\ & & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

solve $Lc = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ \frac{17}{2} \end{bmatrix}$$

solve $U_x = C$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ \frac{17}{2} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -12 \\ -14 \\ 17 \end{bmatrix}$$

#9-(a)

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{P_{12}} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$E_3 : E_3 + \frac{1}{2}E_2$ $\rightarrow$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & \frac{1}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & & \\ & 1 & \\ & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$A = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{= P^T} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}}_{= L} \underbrace{\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}}_{= U}$$

6.6-#14-(a)

A : 정사각 행렬 일때 $A=LU$ 항상 가능.

$A=LU=A^T=U^TL^T$ (symmetric)

L은 대각선성분이 1인 하반 행렬

$\det(L)=1 \rightarrow L^{-1}$ 존재.

$LU=U^TL^T \rightarrow U=L^{-1}U^TL^T$

L^T도 역행렬존재 $\rightarrow U(L^T)^{-1}=L^{-1}U^T$

좌변은 상반 행렬이고, 우변은 하반 행렬이므로

양변은 대각행렬 $D=U(L^T)^{-1}=L^{-1}U^T$

$\therefore U=DL^T$

$A=LU=LDL^T$ 로 항상 분해 가능.

$$A=\begin{bmatrix}3&-3&6\\-3&2&-7\\6&-7&13\end{bmatrix}$$

$$\begin{bmatrix}1&&\\l_{21}&1&\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}D_1&&\\&D_2&\\&&D_3\end{bmatrix}\begin{bmatrix}1&l_{21}&l_{31}\\&1&l_{32}\\&&1\end{bmatrix}=A$$

$$\begin{bmatrix}D_1&0&0\\l_{21}D_1&D_2&0\\l_{31}D_1&D_2l_{32}&D_3\end{bmatrix}\begin{bmatrix}1&l_{21}&l_{31}\\&1&l_{32}\\&&1\end{bmatrix}=A$$

$D_1=3,\ l_{21}=-1,\ l_{31}=2$

$l_{21}^2D_1+D_2=2,\ D_2=-1$

$l_{31}l_{21}D_1+D_2l_{32}=-7$

$-6-l_{32}=-7,\ l_{32}=1$

$D_3+12+(-1)=13,\ D_3=2$

$$\therefore L=\begin{bmatrix}1&0&0\\-1&1&0\\2&1&1\end{bmatrix},\ D=\begin{bmatrix}3&0&0\\0&-1&0\\0&0&2\end{bmatrix}$$

6.6-#16.

$\det|\alpha|=\alpha>0$

$\det\begin{vmatrix}\alpha&1\\1&2\end{vmatrix}=2\alpha-1>0$

$\det\begin{vmatrix}\alpha&1&-1\\1&2&1\\-1&1&4\end{vmatrix}=\det\begin{vmatrix}0&1-2\alpha&-1-\alpha\\1&2&1\\0&3&5\end{vmatrix}$

$=-\det\begin{vmatrix}1-2\alpha&-1-\alpha\\3&5\end{vmatrix}$

$=-(5-10\alpha+3+3\alpha)$

$=-5+10\alpha-3-3\alpha=7\alpha-8>0$

$\alpha>0,\ \alpha>\frac{1}{2},\ \alpha>\frac{8}{7}$

$\therefore \alpha>\frac{8}{7}.$

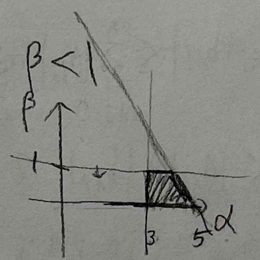
#19.

$3>2+\beta,\ \beta<1$

$5>\alpha+\beta$

$\alpha>3$

$\alpha>0,\beta>0$



$\therefore 0<\beta<1$ 에서 $3<\alpha<5-\beta$

#24.

A, B are strictly diagonally dominant $n \times n$

(a) $-A$ $\sum_{\substack{j=1 \\ i \neq j}}^{n} |-a_{ij}| = \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}| < |a_{ii}|$

$\therefore -A$도 strictly diagonally dominant

(b) A^T $\sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ji}| \neq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$

반례

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$은 맞지만 $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$는 아님.

(c) $A+B$

$\sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}+b_{ij}| < \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}| + \sum_{\substack{j=1 \\ i \neq j}}^{n} |b_{ij}|$

$< |a_{ii}| + |b_{ii}| \ ? \ |a_{ii}+b_{ii}|$

반례 $\begin{bmatrix} 5 & 2 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} -5 & 3 \\ 2 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$

(d) A^2

반례 $\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 12 & 7 \end{bmatrix}$

(e) $A-B$

반례 $\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$