① 본과제에 작성된 내용은 기본적으로는 본인이 스스로 해결하고 작성한 것임을 서약합니다. 서약이 거짓임이 밝혀지면 F학점을 받는 것을 동의합니다.

2022/03/26

2018008559 신상원 신상원

⑥ 참고문헌 : 교재.

2.1 - #12.

(a) 실제 해 : $-2, -1, 0, 1, 2$

$a_1 = -1.5$, $a_2 = 2.5$ $(-1.5, 2.5)$

$a_3 = 0.5$ $(-1.5, 0.5)$

$a_4 = -0.5$ $(-0.5, 0.5)$

이후 0으로 수렴.

(b) $a_1 = -0.5$ $a_2 = 2.4$ $(-0.5, 2.4)$

$a_3 = 0.95$ $(-0.5, 0.95)$

이후 0으로 수렴.

(c) $a_1 = -0.5$, $a_2 = 3$ $(-0.5, 3)$

$a_3 = 1.25$ $(1.25, 3)$

이후 2로 수렴

(d) $a_1 = -3$, $a_2 = -0.5$ $(-3, -0.5)$

$a_3 = -1.75$ $(-3, -1.75)$

이후 -2로 수렴

수치해석I 과제 #2.

2.1 - #19

$p_n = \sum_{k=1}^{n} \frac{1}{k}$,

$p_n - p_{n-1} = \frac{1}{n}$

$\lim_{n\to\infty} p_n - p_{n-1} = \lim_{n\to\infty} \frac{1}{n} = 0$

$\lim_{n\to\infty} p_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

$> 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \cdots$

$= \frac{3}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$

$= \infty$

$p_n - p_{n-1}$이 0으로 수렴함에도 불구하고 p_n은 발산한다.

2.1 - #20.

$f(x) = (x-1)^{10}$, $p = 1$, $p_n = 1 + \frac{1}{n}$ $(n \geq 1)$

$|f(p_n)| = |(1 + \frac{1}{n} - 1)^{10}| = \frac{1}{n^{10}} < 10^{-3}$.

$10^3 < n^{10}$, $n > 1$.

$|p - p_n| = |\frac{1}{n}| < 10^{-3}$

$10^3 < n$, $1000 < n$.

2.2-#8

let $x = x - \frac{x^3-x-1}{3x^2-1}$

(뉴턴 메소드).

$P_0 = 1$, $P_1 = 1.5$, $P_2 = 1.34783$

$P_3 = 1.3252$, $P_4 = 1.32472$

실제 해 = 1.3247

P_3에서 오차 10^{-2} 이내 이다.

$\therefore$ 1.3252.

2.3-#12

(a) $x^2-4x+4-\ln x = 0$

실제 해: 1.41239, 3.0571

① newton's method

$x = x - \frac{x^2-4x+4-\ln x}{2x-4-\frac{1}{x}}$

$P_0 = 1.5$, $P_1 = 1.406720935351$

$P_2 = 1.41236995725119$

$P_3 = 1.41239117172501$

② Secant method

$P_0 = 1$, $P_1 = 2$, $P_2 = 1.59061610914$

$P_3 = 1.28454784959$

$P_4 = 1.42796611008566$

$P_5 = 1.41363463974$

$P_6 = 1.41237818603536$

$P_7 = 1.41239118215496$

③ False position

$P_0 = 1$, $P_1 = 2$, $P_2 = 1.590616109$

$P_3 = 1.45553732836782$

$P_4 = 1.42220996148$

$P_5 = 1.4145936098$

$P_6 = 1.41288359108$

$P_7 = 1.41250118679$

$P_8 = 1.41241574714$

$P_9 = 1.41239666142125$

$P_{10} = 1.412392398 19$

$P_{11} = 1.41239144591295$

$P_{12} = 1.41239123320243$

at (2, 4) 실제 = 3.057103549947

① newton

$P_0 = 3$, $P_1 = 3.0591673732$

$P_2 = 3.05710605469 16$

$P_3 = 3.05710354999 84$.

② secant

$P_0 = 2$, $P_1 = 4$

$P_2 = 2.4192186450889$,

$P_3 = 2.75603991220$

$P_4 = 3.3170225040112$

$P_5 = 3.00976895937$

$P_6 = 3.05067010972576$

$P_7 = 3.05728904165992$

$P_8 = 3.05710284392887$

$P_9 = 3.05710354991754$

③ False position

$P_2 = 2.41921864588$

$P_3 = 2.75603991220601$

$P_4 = 2.93604457930724$

$P_5 = 3.01198156637919$

$P_6 = 3.05126965792$

$P_7 =$

⋮

$P_{18} = 3.05710348330573$

12-(b) $f(x) = x+1-2\sin\pi x$
$f'(x) = 1-2\pi\cos\pi x$

at $0 \le x \le \frac{1}{2}$

① newton $p_0 = 0.25$, $p_3 = 0.2060351189$

② secant $p_0 = 0$, $p_1 = 0.5$

$p_7 = 0.20603512029448$

③ False Position

$p_0 = 0$, $p_1 = 0.5$, $p_n = 0.206035\,14924078$

at $\frac{1}{2} \le x \le 1$

① newton $p_0 = 0.75$, $p_3 = 0.6819748188$

② secant $p_0 = 0.5$, $p_1 = 1$

$p_7 = 0.68197481172852$

③ False Position

$p_0 = 0.5$, $p_1 = 1$, $p_{14} = 0.68197475045789$

2.4-#12

($\Rightarrow$) $f(x) = (x-p)^m q(x)$

$\lim_{x\to p} q(x) \ne 0$, $f(p) = 0$

$f'(x) = m(x-p)^{m-1}q(x) + (x-p)^m q'(x)$

$f'(p) = 0$

$f''(x) = m(m-1)(x-p)^{m-2}q(x)$
$+ m(x-p)^{m-1}q'(x) + m(x-p)^{m-1}q'(x)$
$+ (x-p)^m q''(x) = (x-p)\xi(x)$
$+ m(m-1)(x-p)^{m-2}q(x)$

$f''(p) = 0$

재귀적으로

$f^{(k)}(x) = {}_mP_k\, q(x) + (x-p)Q(x)$

$0 = f(p) = f'(p) = \cdots = f^{(m-1)}(p)$

$f^{(m)}(p) = m!\, q(x)$ 이고,

$\lim_{x\to p} q(x) \ne 0$ 이므로

$f^{(m)}(p) \ne 0$ 이다.

($\Leftarrow$) p에서 테일러 전개를 하면

$f(x) = f(p) + \frac{f'(p)}{1!}(x-p) + \frac{f''(p)}{2!}(x-p)^2$
$+ \cdots + \frac{f^{(m-1)}(p)}{(m-1)!}(x-p)^{m-1} + \frac{f^{(m)}(\xi(x))}{m!}(x-p)^m$

$f^{(k)}(p) = 0$ $(k<m)$ 이므로

$f(x) = \frac{f^{(m)}(\xi(x))}{m!}(x-p)^m$.

$\xi(x)$는 x와 p 사이의 값이므로

$x < \xi(x) < p$ $\lim_{x\to p} \xi(x) = p$
$\to p$

$\frac{f^{(m)}(\xi(x))}{m!} = g(x)$라 하면

$\lim_{x\to p} g(x) \ne 0$ 이다. ($f^{(m)}(p) \ne 0$)

$\therefore f(x) = (x-p)^m g(x)$ $\left(\underset{x\to p}{g(x)} \ne 0\right)$

으로 나타낼수 있으므로

f has a zero of multiplicity m at p.

2.4-#14

by definition $\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^\alpha} = \lambda$

$|p_{n+1}-p| \approx \lambda |p_n-p|^\alpha$

$|p_{n+1}-p| \approx C|p_n-p||p_{n-1}-p|$

$$|p_{n+1}-p| \approx \lambda |p_n - p|^{\alpha}$$

$$|p_n - p|^{\alpha} \approx \lambda^{\alpha} |p_{n-1}-p|^{\alpha^2}$$

$$|p_{n+1}-p| \approx \lambda^{\alpha+1} |p_{n-1}-p|^{\alpha^2}$$

$$|p_{n+1}-p| \approx C|p_n-p||p_{n-1}-p|$$

$$\lambda^{\alpha+1}|p_{n-1}-p|^{\alpha^2} \approx C|p_n-p||p_{n-1}-p|$$

$$\lambda^{\alpha+1}|p_{n-1}-p|^{\alpha^2} \approx C\lambda|p_{n-1}-p|^{\alpha+1}$$

$$|p_{n-1}-p|^{\alpha^2-\alpha-1} \approx C\lambda^{-\alpha} = \text{상수}$$

$\therefore \alpha^2-\alpha-1=0, \quad \alpha = \frac{1\pm\sqrt{5}}{2}$

$\alpha > 0, \quad \alpha = \frac{1+\sqrt{5}}{2}$

2.5 - #14

(a) if $p_n \to p$ of order $\alpha > 1$

$$\Rightarrow \lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^{\alpha}} = \lambda.$$

$$\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|} = \lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^{\alpha}} \cdot |p_n-p|^{\alpha-1}$$

$$= \lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^{\alpha}} \cdot \lim_{n\to\infty} |p_n-p|^{\alpha-1}$$

$$= \lambda \cdot \lim_{n\to\infty} |p_n-p|^{\alpha-1}$$

$\left(p_n \to p \ \ \alpha \ \ \alpha>1 \text{ 이므로} \quad \lim_{n\to\infty} |p_n-p|^{\alpha-1} = 0 \right)$

$$= \lambda \cdot 0 = 0$$

(b) $p_n = \frac{1}{n^n}, \ p = 0$

$$\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|} = \lim_{n\to\infty} \frac{\left|\frac{1}{(n+1)^{n+1}}\right|}{\left|\frac{1}{n^n}\right|}$$

$$= \lim_{n\to\infty} \frac{n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1}$$

$\left(\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e, \quad \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \frac{1}{e} \right)$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \lim_{n\to\infty} \frac{1}{n+1} = \frac{1}{e} \cdot 0 = 0$$

$\therefore p_n$ superlinearly convergent to 0

if p_n converge order of $\alpha > 1$

$$\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^{\alpha}} = \lambda.$$

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)^{n+1}}}{n^{n\alpha}} = \lim_{n\to\infty} \frac{n^{n\alpha}}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot n^{n\alpha-n} \cdot \frac{1}{n+1}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \underbrace{\lim_{n\to\infty} \frac{n^{n(\alpha-1)}}{n+1}}_{\text{(c)}} \quad \text{--- (c)}$$

$$= \frac{1}{e} \cdot \text{(c)}$$

$\Bigg($ (c) by L'Hopital's rule

$$\lim_{n\to\infty} \frac{n^{n(\alpha-1)}}{n+1} = \lim_{n\to\infty} \frac{n^{n(\alpha-1)} \cdot (\alpha-1)(\ln n+1)}{1}$$

$$= \lim_{n\to\infty} n^{n(\alpha-1)} (\alpha-1)(\ln n+1) = \infty \quad (\alpha>1) \Bigg)$$

$$= \frac{1}{e} \cdot \infty = \infty$$

따라서 발산한다.

$\therefore$ order α로 수렴 불가능.

2.5 - #16

theorem 2.14

$p_n \to p$, $\lim_{n\to\infty} \frac{p_{n+1}-p}{p_n-p} < 1$

then $\hat{p}_n$ converges faster then p_n.

$\Rightarrow \lim_{n\to\infty} \frac{\hat{p}_n - p}{p_n - p} = 0$

$\lim_{n\to\infty} \frac{p_{n+1}-p}{p_n-p} = \lambda$ 라 하자.

Let $x_n = \frac{p_{n+1}-p}{p_n-p} - \lambda$

$\lim_{n\to\infty} x_n = \lambda - \lambda = 0$

$$\lim_{n\to\infty} \frac{\hat{p}_n - p}{p_n - p} = \frac{p_n - \frac{(p_{n+1}-p_n)^2}{p_{n+2}-2p_{n+1}+p_n} - p}{p_n - p}$$

$$= \frac{p_n-p}{p_n-p} - \frac{(p_{n+1}-p_n)^2}{(p_n-p)(p_{n+2}-p_{n+1}\cdot 2+p_n)}$$

$$= 1 - \frac{p_{n+1}-p_n}{p_n-p} \times \frac{p_{n+1}-p_n}{(p_{n+2}-p_{n+1})-(p_{n+1}-p_n)}$$

$$\left(\frac{p_{n+1}-p_n}{p_n-p} = \frac{p_{n+1}-p-(p_n-p)}{p_n-p} = x_n+\lambda-1 \right)$$

$$\left(\frac{p_{n+1}-p_n}{(p_{n+2}-p_{n+1})-(p_{n+1}-p_n)} = \frac{1}{\frac{p_{n+2}-p_{n+1}}{p_{n+1}-p_n} - 1} \right.$$

$$\left. \frac{p_{n+2}-p_{n+1}}{p_{n+1}-p_n} \cdot \frac{p_{n+1}-p}{p_{n+1}-p} \; // \right)$$

$$\left(\frac{p_{n+2}-p_{n+1}}{p_{n+1}-p} = \frac{p_{n+2}-p-(p_{n+1}-p)}{p_{n+1}-p} = x_{n+1}+\lambda-1 \right)$$

$$\left(\frac{p_{n+1}-p}{p_{n+1}-p_n} = \frac{1}{\frac{p_{n+1}-p_n}{p_{n+1}-p}} = \frac{1}{\frac{p_{n+1}-p-(p_n-p)}{p_{n+1}-p}} = \frac{1}{1-\frac{p_n-p}{p_{n+1}-p}} = \frac{1}{1-\frac{1}{x_n+\lambda}} \right)$$

$$= \text{정리하면 } 1-(x_n+\lambda-1)\frac{1}{(x_{n+1}+\lambda-1)\cdot\frac{1}{1-\frac{1}{x_n+\lambda}}-1}$$

$$= 1-(\lambda-1)\frac{1}{(\lambda-1)\cdot\frac{1}{1-\frac{1}{\lambda}}-1} \quad \left(\lim_{n\to\infty} x_n = 0\right)$$

$$= \lim_{n\to\infty} 1-(\lambda-1)\frac{1}{\lambda-1} = \lim_{n\to\infty} 1-1 = 0$$

$$\therefore \lim_{n\to\infty} \frac{\hat{p}_n-p}{p_n-p} = 0 \; //$$

2.5 - #17

$f(x) = e^x = P_n(x) + R_n(x)$

$= \sum_{k=0}^{n} \frac{x^k}{k!} + \frac{x^{n+1}}{(n+1)!}\cdot e^{\xi(x)}$

at $n \to \infty$

$P_n(x) \to e^x$, $R_n(x) \to 0$

$$\lim_{n\to\infty} \frac{P_{n+1}(x)-e^x}{P_n(x)-e^x} = \lim_{n\to\infty} \frac{R_{n+1}(x)}{R_n(x)}$$

$$= \lim_{n\to\infty} \frac{\frac{x^{n+2}}{(n+2)!}e^{\xi(x)}}{\frac{x^{n+1}}{(n+1)!}\cdot e^{\xi(x)}} = \lim_{n\to\infty} \frac{x}{n+2}\cdot e^{\xi_2(x)}$$

$= 0$ (x, $e^{\xi_2(x)}$ 상수) < 1

(a) Theorem 2.14의 가정을 만족한다.

(b) $x=1$ 이므로 $P_n(1) = \sum_{k=0}^{n} \frac{1}{k!}$

$\hat{p}_0 = 3$, $\hat{p}_1 = 2.75$, $\hat{p}_2 = 2.722222$

$\hat{p}_3 = 2.71875$, $\hat{p}_4 = 2.7183333$

$\hat{p}_5 = 2.718281037$, $\hat{p}_6 = 2.718282313$

$\hat{p}_7 = 2.718281187$, $\hat{p}_8 = 2.718281832$

17-(c) Aitken's method 가 더 빨리 수렴하였다.

2.6-#1

(a) 실제근: $x = 2.69064744802861$

$P_0 = 2$, $P_4 = 2.69068$

(b) real $= -2.87938524157$
$= -0.65270364466$
$= 0.53208888$

$P_0 = -3$, $P_2 = -2.87945$
$P_0 = -1$, $P_2 = -0.65278$
$P_0 = 1$, $P_4 = 0.53209$

$P'(-2) = P'(0) = 0$ 임을 주의.

(c) real $= 1.324717957$

$P_0 = 3$, $P_5 = 1.32472$

(d) real $= -0.87605311$
$= 1.124123029$

$P_0 = 0$, $P_7 = -0.87606$
$P_0 = 3$, $P_6 = 1.12412$

(e) real $= -2.64561$
$= -0.88533$
$= -0.470064$

$P_0 = -4$ $P_5 = -2.64561$
$P_0 = -1$ $P_3 = -0.88533$
$P_0 = 1$ $P_6 = -0.47003$

(f) real $= 1.49818998847 25$
$P_0 = 5$, $P_9 = 1.49819$

2.3-#12

"C:₩Users₩kas12₩OneDrive₩바탕 화면₩도움!₩tnclgotjr₩bin₩Debug₩tnclgotjr.exe"

```
newton's method
0 1.50000000000000
1 1.40672093513510
2 1.41236995725119
3 1.41239117172501
secant method
0 1.00000000000000 2.00000000000000
1 2.00000000000000 1.59061610914964
2 1.59061610914964 1.28454784959203
3 1.28454784959203 1.42796611008566
4 1.42796611008566 1.41363463957469
5 1.41363463957469 1.41237818603536
6 1.41237818603536 1.41239118275496
False Position
0 1.00000000000000 2.00000000000000
1 1.00000000000000 1.59061610914964
2 1.00000000000000 1.45553732836782
3 1.00000000000000 1.42220996149836
4 1.00000000000000 1.41459360982987
5 1.00000000000000 1.41288359188089
6 1.00000000000000 1.41250118679691
7 1.00000000000000 1.41241574714899
8 1.00000000000000 1.41239666142125
9 1.00000000000000 1.41239239819215
10 1.00000000000000 1.41239144591295
11 1.00000000000000 1.41239123320243

Process returned 0 (0x0)   execution time : 0.414 s
Press any key to continue.
```

"C:₩Users₩kas12₩OneDrive₩바탕 화면₩도움!₩tnclgotjr₩bin₩Debug₩tnclgotjr.exe"

```
newton's method
0 3.00000000000000
1 3.05916737320087
2 3.05710605469160
3 3.05710354999844
secant method
0 2.00000000000000 4.00000000000000
1 4.00000000000000 2.41921864588900
2 2.41921864588900 2.75603971220601
3 2.75603971220601 3.31702250400112
4 3.31702250400112 3.00976895937103
5 3.00976895937103 3.05067070972576
6 3.05067070972576 3.05728904165992
7 3.05728904165992 3.05710284392887
8 3.05710284392887 3.05710354991754
False Position
0 2.00000000000000 4.00000000000000
1 2.41921864588900 4.00000000000000
2 2.75603971220601 4.00000000000000
3 2.93604457430724 4.00000000000000
4 3.01198156637919 4.00000000000000
5 3.04078742422821 4.00000000000000
6 3.05126965792156 4.00000000000000
7 3.05502607811050 4.00000000000000
8 3.05636482744996 4.00000000000000
9 3.05684100541793 4.00000000000000
10 3.05701025788230 4.00000000000000
11 3.05707040191052 4.00000000000000
12 3.05709177225753 4.00000000000000
13 3.05709936532003 4.00000000000000
14 3.05710206316830 4.00000000000000
15 3.05710302172176 4.00000000000000
16 3.05710336229816 4.00000000000000
17 3.05710348330573 4.00000000000000

Process returned 0 (0x0)   execution time : 0.365 s
```

"C:\Users\kas12\OneDrive\바탕 화면\도움!\tnclgotjr\bin\Debug\tnclgotjr.exe"

```
newton's method
0 0.75000000000000
1 0.68830724609695
2 0.68204806909390
3 0.68197481886042
secant method
0 0.50000000000000 1.00000000000000
1 1.00000000000000 0.60000000000000
2 0.60000000000000 0.65249317098047
3 0.65249317098047 0.68822269974871
4 0.68822269974871 0.68160555624992
5 0.68160555624992 0.68197052096669
6 0.68197052096669 0.68197481172852
False Position
0 0.50000000000000 1.00000000000000
1 0.60000000000000 1.00000000000000
2 0.65249317098047 1.00000000000000
3 0.67252691066148 1.00000000000000
4 0.67907709312339 1.00000000000000
5 0.68109873665095 1.00000000000000
6 0.68171111588159 1.00000000000000
7 0.68189554518608 1.00000000000000
8 0.68195099250922 1.00000000000000
9 0.68196765357362 1.00000000000000
10 0.68197265917439 1.00000000000000
11 0.68197416297077 1.00000000000000
12 0.68197461473897 1.00000000000000
13 0.68197475045789 1.00000000000000

Process returned 0 (0x0)   execution time : 0.780 s
Press any key to continue.
```

"C:\Users\kas12\OneDrive\바탕 화면\도움!\tnclgotjr\bin\Debug\tnclgotjr.exe"

```
newton's method
0 0.25000000000000
1 0.20230347202541
2 0.20601490020939
3 0.20603511896462
secant method
0 0.00000000000000 0.50000000000000
1 0.50000000000000 0.33333333333333
2 0.33333333333333 -0.32278095559281
3 -0.32278095559281 0.23902382930864
4 0.23902382930864 0.21080900654599
5 0.21080900654599 0.20577426954979
6 0.20577426954979 0.20603699084695
7 0.20603699084695 0.20603512029448
False Position
0 0.00000000000000 0.50000000000000
1 0.00000000000000 0.33333333333333
2 0.00000000000000 0.23831355471949
3 0.00000000000000 0.21220451672207
4 0.00000000000000 0.20712481220511
5 0.00000000000000 0.20622462812000
6 0.00000000000000 0.20606798634282
7 0.00000000000000 0.20604081697468
8 0.00000000000000 0.20603610712486
9 0.00000000000000 0.20603529074518
10 0.00000000000000 0.20603514924078

Process returned 0 (0x0)   execution time : 0.656 s
Press any key to continue.
```

C++ code

```
#include <iostream>
#include <cmath>
using namespace std;
typedef long double ld;
ld pi = 3.14159265358979;
ld f(ld x){
    return x+1-2*sin(pi*x);
}
ld fp(ld x){
    return 1-2*pi*cos(pi*x);
}
bool sign(ld x, ld y){
    if(x > 0 && y > 0) return true;
    if(x < 0 && y < 0) return true;
    else return false;
}
int main()
{
    ld x = 0.75;
    ld real = 0.68197480873864749907;
    cout<<fixed;
    cout.precision(14);
    cout << "newton's method\n";
    int cnt = 0;
    while(abs(real - x) > 1e-7){
        cout << cnt++ << " " << x << "\n";
        x = x - f(x)/fp(x);
    }
    cout << cnt++ << " " << x << "\n";
    cout << "secant method\n";
    cnt = 0;
    ld x1 = 0.5;
    ld x2 = 1;
    while(abs(real - x2) > 1e-7){
        cout << cnt++ << " " << x1 << " " << x2 << "\n";
        ld tmp = x2;
        x2 = x2 - f(x2)*(x2-x1)/(f(x2)-f(x1));
        x1 = tmp;
    }
```

```
    cout << cnt++ << " " << x1 << " " << x2 << "\n";
    cout << "False Position\n";
    cnt = 0;
    x1 = 0.5;
    x2 = 1;
    while(abs(real - x2) > 1e-7 && abs(real - x1) > 1e-7){
        cout << cnt++ << " " << x1 << " " << x2 << "\n";
        ld tmp = x2;
        x2 = x2 - f(x2)*(x2-x1)/(f(x2)-f(x1)); //a = x1, b = tmp, c = x2
        if(sign(f(x1),f(x2))){
            x1 = x2;
            x2 = tmp;
        }
    }
    cout << cnt++ << " " << x1 << " " << x2 << "\n";
}
```

2.5-#17 (b)

```
"C:\Users\kas12\OneDrive\바탕 화면\도움!\aitkens method\bin\Debug\aitkens method.exe"
0 1.000000000 3.000000000
1 2.000000000 2.750000000
2 2.500000000 2.722222222
3 2.666666667 2.718750000
4 2.708333333 2.718333333
5 2.716666667 2.718287037
6 2.718055556 2.718282313
7 2.718253968 2.718281870
8 2.718278770 2.718281832
9 2.718281526
10 2.718281801

Process returned 0 (0x0)   execution time : 0.410 s
Press any key to continue.
```

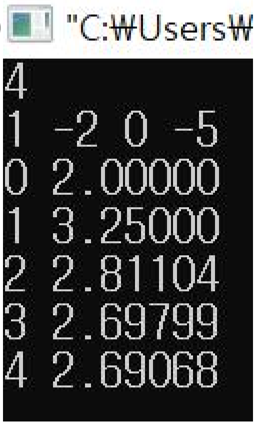
C++ code

```
#include <iostream>
#include <cmath>
using namespace std;
typedef long double ld;
ld pn(int x){
    ld ans = 1;
    ld pac = 1;
    for(int i=1; i<=x; i++){
```

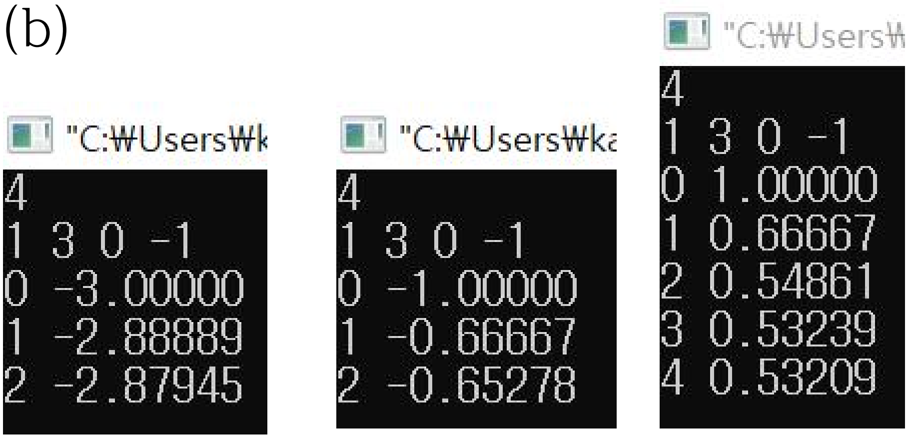
```
            pac *= i;
            ans += 1/pac;
        }
        return ans;
}
ld p[11];
ld pen(int n){
        return p[n] - (p[n+1]-p[n])*(p[n+1]-p[n])/(p[n+2]-2*p[n+1]+p[n]);
}
int main()
{
        cout << fixed;
        cout.precision(9);
        for(int i=0; i<11; i++){
            p[i] = pn(i);
        }
        for(int i=0; i<11; i++){
            cout << i << " " << p[i];
            if(i<9) cout << " " << pen(i);
            cout << "\n";
        }
}
```

2.6-#1

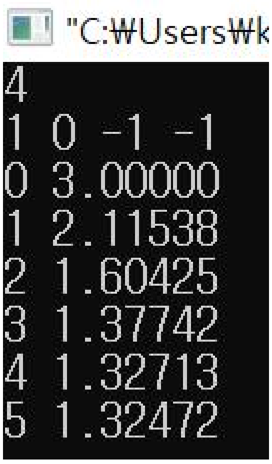
(a)


```
"C:₩Users₩
4
1 -2 0 -5
0 2.00000
1 3.25000
2 2.81104
3 2.69799
4 2.69068
```

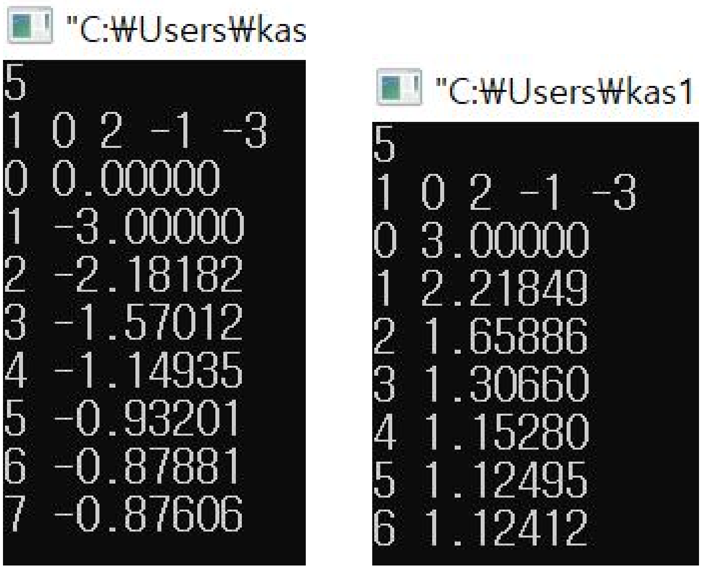
(b)


"C:₩Users₩k
4
1 3 0 -1
0 -3.00000
1 -2.88889
2 -2.87945
"C:₩Users₩ka
4
1 3 0 -1
0 -1.00000
1 -0.66667
2 -0.65278
"C:₩Users₩
4
1 3 0 -1
0 1.00000
1 0.66667
2 0.54861
3 0.53239
4 0.53209

(c)


"C:₩Users₩k
4
1 0 -1 -1
0 3.00000
1 2.11538
2 1.60425
3 1.37742
4 1.32713
5 1.32472

(d)


"C:₩Users₩kas
5
1 0 2 -1 -3
0 0.00000
1 -3.00000
2 -2.18182
3 -1.57012
4 -1.14935
5 -0.93201
6 -0.87881
7 -0.87606
"C:₩Users₩kas1
5
1 0 2 -1 -3
0 3.00000
1 2.21849
2 1.65886
3 1.30660
4 1.15280
5 1.12495
6 1.12412

(e)

```
"C:\Users\kas12\OneDri
4
1 4.001 4.002 1.101
0 -4.00000
1 -3.25523
2 -2.84224
3 -2.67608
4 -2.64652
5 -2.64561
```

```
"C:\Users\kas12\OneDr
4
1 4.001 4.002 1.101
0 1.00000
1 0.32658
2 -0.08718
3 -0.32217
4 -0.43368
5 -0.46685
6 -0.47003
```

```
"C:\Users\kas12\OneDr
4
1 4.001 4.002 1.101
0 -1.00000
1 -0.90000
2 -0.88570
3 -0.88533
```

(f)

```
"C:\Users\kas12\
6
1 -1 2 -3 1 -4
0 5.00000
1 4.02549
2 3.24728
3 2.63066
4 2.15296
5 1.80552
6 1.59295
7 1.50999
8 1.49840
9 1.49819
```

C++ code

```
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
typedef long double ld;
vector<ld> P,Q;
ld newton(ld x){
    Q.clear();
    ld tmp = P[0];
    Q.push_back(tmp);
    for(int i=1; i<P.size(); i++){
        tmp = P[i] + Q[i-1] * x;
        if(i == P.size()-1) break;
        Q.push_back(tmp);
    } //tmp = P(x)
    ld gap = tmp;
    tmp = Q[0];
    for(int i=1; i<Q.size(); i++){
        tmp = Q[i] + tmp * x;
    } //tmp = Q(x) = P'(x)
    return gap/tmp;
}
int main()
{
    int n;
    cin >> n;
    for(int i=0; i<n; i++){
        ld x;
        cin >> x;
        P.push_back(x);
    }
    ld real = 1.4981899847252999275;
    ld x0 = 5;
    cout << fixed;
    cout.precision(5);
    int cnt = 0;
    while(abs(real - x0) > 1e-4){
        cout << cnt++ << " " << x0 << "\n";
```

```
        x0 = x0 - newton(x0);
    }
    cout << cnt++ << " " << x0 << "\n";
}
```

theorem 2.6.

$\boxed{\text{Lemma}}$

Let $f \in C[a,b]$, $P \in (a,b)$

(a) $f(P) \neq 0$ 이라 가정. then $\delta > 0$ exists for all $x \in [P-\delta, P+\delta] \subset [a,b]$ with $f(x) \neq 0$

Pf) f는 P에서 연속이므로

$\forall \varepsilon > 0$, $\exists \delta > 0$ s.t "$0 < |x-P| < \delta \rightarrow |f(x) - f(P)| < \varepsilon$"

$0 < \varepsilon < |f(P)|$인 ε을 고른다면

$|f(P)| - |f(x)| \leq |f(P) - f(x)| < \varepsilon$

$|f(x)| \geq |f(P)| - \varepsilon > 0 \rightarrow f(x) \neq 0$ 인 $\delta > 0$ 존재, for $\forall x \in [P-\delta, P+\delta] \subset [a,b]$

(b) $f(P) = 0$ 이라 가정, $k > 0$ 이 주어질때 then $\delta > 0$ exists for all $x \in [P-\delta, P+\delta] \subset [a,b]$ with $|f(x)| \leq k$

Pf) f는 P에서 연속이므로

$\forall \varepsilon > 0$, $\exists \delta > 0$. s.t "$0 < |x-P| < \delta \rightarrow |f(x) - f(P)| < \varepsilon$"

$\varepsilon = k+1$ 로 고르자.

$|f(x) - f(P)| = |f(x)| < \varepsilon = k+1 \rightarrow |f(x)| \leq k$인 $\delta > 0$ 존재, for $\forall x \in [P-\delta, P+\delta] \subset [a,b]$

(증명)

$g(x) = x - \dfrac{f(x)}{f'(x)}$, $P_n = g(P_{n-1})$

이라 하자.

$f \in C^2[a,b]$ 이므로 $(P \in (a,b))$

f'도 $[a,b]$에서 연속, $f'(P) \neq 0$ 이므로

by Lemma-(a)

for all $x \in [P-\delta_1, P+\delta_1] \subset [a,b]$ 에서 $f'(x) \neq 0$인 $\delta_1 > 0$ 존재

$x \in [P-\delta_1, P+\delta_1]$ 에서 $f'(x) \neq 0$ 이므로

$g(x)$는 $[P-\delta_1, P+\delta_1]$에서 연속

$g'(x) = \dfrac{f(x) f''(x)}{(f'(x))^2}$ 이므로 $g'(x)$도 $[P-\delta_1, P+\delta_1]$ 에서 연속 $\therefore g \in C^1[P-\delta_1, P+\delta_1]$

문제 가정에 의해 $f(P) = 0$ 이므로

$g'(P) = 0$ 이다.

g' 연속, $0 < k < 1$ 이라 할때

by Lemma-(b)

for all $x \in [P-\delta, P+\delta] \subset [P-\delta_1, P+\delta_1]$에서

$|g'(x)| \leq k$ 인 $0 < \delta < \delta_1$ 존재.

by MVT $\dfrac{|g(x) - g(P)|}{|x-P|} = g'(\alpha)$ 인

α가 (x, P)에 존재. 다시 쓰면

$|g(x) - P| = |g(x) - g(P)| = g'(\alpha)|x-P|$

$\leq k|x-P| < |x-P| < \delta$, $(x \in [P-\delta, P+\delta])$

$\rightarrow |g(x) - P| < \delta$, $|x-P| < \delta$.

$\rightarrow g(x) \in [P-\delta, P+\delta]$, $x \in [P-\delta, P+\delta]$

g, g는 $[P-\delta, P+\delta] \subset [P-\delta_1, P+\delta_1]$ 에서 연속

$|g'(x)| \leq k$, for all $x \in [P-\delta, P+\delta]$

$0 < k < 1$ 이므로 fixed point theorem의 가정들을 모두 만족한다.

by fixed point theorem, for any number $P_0 \in [P-\delta, P+\delta]$, $P_n = g(P_{n-1})$, $n \geq 1$ 은 어떤 유일한 P에 수렴한다.