

Practical Work : Dice trading

Vincent Leclère

April 23, 2020

1. We consider a game of dices.

At the beginning of a turn the player have a certain amount of points (starting with 0). He can decide to buy one new dice (one per turn maximum, to be kept until the end) for $p = 5$ points, only if he have at least $p + 1$ points. The player then throw his dices (6 faces, independent, equilibrated), and add the maximum of all dices to his points. The game play for $T = 10$ turns. The player want to maximize the expected number of points at the end.

Example (9 points, 5 turn game) :

turn	dice roll	action	total points
1	3	can't buy	3
2	4	do not buy	7
3	5	buy	6
4	$\max(3,2)$	buy	3
5	$\max(1,2,6)$	do not buy	9

- (a) Determine the dynamical system considered in this problem. (specify state, control, dynamics)
 - (b) A strategy is a function taking as argument the time-step t and the current state x and returning a control. Implement a very simple strategy (a heuristic).
 - (c) Write a simulator taking as argument a strategy and an integer returning the estimated expected cost associated to the strategy with 95% confidence interval.
 - (d) Compute the law of the maximum of 1,2 or 3 dices
 - (e) Find the optimal value V_0 and strategy π^* by Dynamic Programming. Describe the optimal strategy in simple term.
 - (f) Check by simulation that the optimal value V_0 is indeed obtained when using the strategy π^*
 - (g) For which horizon will it never be interesting to buy (at price 5 ?)
2. We now consider an extension of the previous game. At any turn, once the dices are thrown, the player can loose a dice (that can be rebought afterwards) to double the gain of the throw. Compute the optimal value and describe the optimal strategy.