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ASSIGNMENT

ANALYTICAL QUESTIONS

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1. Solve the following recurrence relations

a.) $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

$x(2) = x(2-1)+5 = x(1)+5 = 0+5 = 5$
$x(3) = x(3-1)+5 = x(2)+5 = 5+5 = 10$
$x(4) = x(4-1)+5 = x(3)+5 = 10+5 = 15$

So from above recurrence relation,

$x(n) = 5n$ for $n>1$

b.) $x(n) = 3x(n-1)$ for $n>1$ $x(1)=4$

$x(2) = 3x(2-1) = 3.x(1) = 3\times4 = 12$
$x(3) = 3.x(3-1) = 3x(2) = 3\times12 = 36$
$x(4) = 3x(4-1) = 3.x(3) = 3\times36 = 108$

From the above recurrence relation, we can state that

$x(n) = 4\times3^{n-1}$ for. $n>1$

c.) $x(n) = x(n/2)+n$. for $n>1$ $x(1)=1$ [solve for $n=2k$]

$n=2$: $x(2) \Rightarrow x(2/2)+2 = x(1)+2 = 1+2 = 3$

$n=3$: $x(3) = x(3/2)+2$

$n=4$: $x(4) \Rightarrow x(4/2)+2 = x(2)+2 = 3+2 = 5$

$n=6$: $x(6) \Rightarrow x(6/2)+2 = x(3)+2 = 5+2 = 7$

$x(2^k) = 1+(2^1+2^2+\dots+2^k)$.

$= 1+(2(2^k-1)) = 1+2^{k+1}-2$

$= 2^{k+1}-1$

$\Rightarrow x(2^k) = 2^{k+1}-1$

d.) $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$ [Solve for $n = 3k$]

For $n = 3$: $x(3) \Rightarrow x(3/3) + 1 = x(1) + 1 = 1 + 1 = 2$

$x(6) \Rightarrow x(6/3) + 1 = x(2) + 1 = 2 + 1 = 3$

$x(9) \Rightarrow x(9/3) + 1 = x(3) + 1 = 3 + 1 = 4$

$\therefore$ The recurrence relation is $x(3^k) = k + 1$.

2.) Evaluate the following recurrence completely.

i) $T(n) = T(n/2) + 1$, where $n = 2k$ for all $k \geq 0$.

here $T(1) = 0$

$n = 2$, $T(2) = T(2/2) + 1 = T(1) + 1 = 1$

$n = 4$, $T(4) = T(4/2) + 1 = T(2) + 1 = 1 + 1 = 2$

$n = 8$, $T(8) = T(8/2) + 1 = T(4) + 1 = 2 + 1 = 3$

$\therefore$ For $n = 2^k$,

$T(2^k) = T(2^{k-1}) + 1$.

$= T(2^{k-2}) + 2 \ \ldots \ T(2^0) + k = k$

Case 2:

$T(n) = \Theta(n^k \log_2(1) \log n)$

$= \Theta(\log n)$

Since $n = 2^k$, $k = \log_2 n$

$T(n) = \log_2 n$

$\Rightarrow O(\log n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$, c - constant & n - input size.

$\Rightarrow T(n) = aT(n/b) + f(n)$

where $a = 2$, $b = 3$; $f(n) = cn$

$\log_a b = \log_2 3 = 1.585$

Case - 3: $\log_a b < k$.

$T(n) = \Theta(n^k \log_3 2) = \Theta(n^k 0.63)$

$O(\log n)$

3.) Consider the following recursion algorithm

```
Min 1[A[0 .......n-1])
  if n=1 return A[0]
  Else temp = Min1[A[0.......n-2])
      if temp <= A[n-1] return temp
  Else
      Return A[n-1]
```

a.) What does this algorithm compute?

This algorithm computes the minimum value in an array A of size n.

b.) Setup a recurrence relation for the algorithms basic operation count and solve it.

$T(n)$ be the basic operation count for an array of size n

$T(n) = T(n-1)+1$, when $n>1$

$T(1) = 1$

$T(n) = T(n-1)+1 = T(n-2)+2 \ldots\ldots T(1)+(n-1) = n$

$\therefore$ Basic operation count for this algorithm is $O(n)$.

4) Analyze the order of growth

(i) $F(n) = 2n^2+5$ & $g(n) = 7n$. Use the $\Omega(g(n))$ notation

$$2n^2+5 \geq c.7n$$

$\Rightarrow F(n) \geq c * g(n)$

$$2n + 5/n \geq 7C/n$$

$F(n)$ is asymptotically greater than $g(n)$ for all $n \geq n_0$.

Thus, $F(n)$ is not in the $\Omega(g(n))$, notation with $g(n) = 7n$