Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

Solution:

$$x(n) = \{x(0) + nd\}u(n) \tag{1}$$
$$x(99) - y(99) = 100 \tag{2}$$
$$\implies (x(0) + 99d) - (y(0) + 99d) = 100 \tag{3}$$
$$\implies x(0) - y(0) = 100 \tag{4}$$

$$x(n) - y(n) = (x(0) + nd) - (y(0) + nd) \tag{5}$$
$$= x(0) - y(0) \tag{6}$$
$$= 100 \tag{7}$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,

The Z-transform of a discrete signal $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering x(n-1) and y(n-1) as n^{th} terms of the APs(Arithmetic Progressions), Z-transform for x(n) and y(n) can be given by

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{8}$$
$$= \sum_{n=0}^{\infty} x(n)z^{-n} \tag{9}$$
$$= x(0)U(z) + d[-z\frac{d(U(z))}{dz}] \tag{10}$$
$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad ROC : |z| > 1 \tag{11}$$
$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad ROC : |z| > 1 \tag{12}$$
$$Y(z) = \frac{y(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad ROC : |z| > 1 \tag{13}$$

$$x(n) = \{101, 106, 111, ...\} \tag{14}$$
$$y(n) = \{1, 6, 11, ...\} \tag{15}$$

Variable	Description	Value
$x(n)$	n^{th} term of X	none
$y(n)$	n^{th} term of Y	none
d	common difference between the terms of AP	none
$X(z)$	z-transform of x(n)	none
$Y(z)$	z-transform of y(n)	none
$U(z)$	z-transform of u(n)	$\frac{1}{1-z^{-1}}, \lvert z\rvert > 1$
$x(n) - y(n)$	difference of n^{th} terms of X and Y	$x(99) - y(99) = 100$

TABLE 0
INPUT PARAMETERS

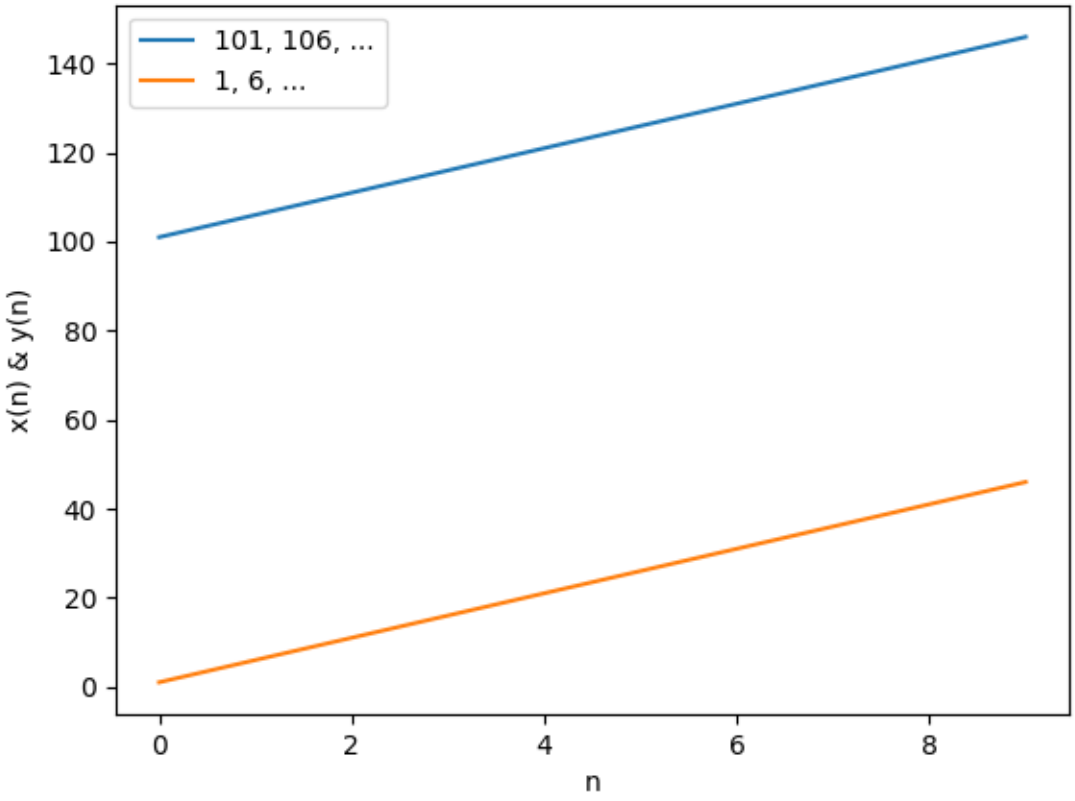

Fig. 0.

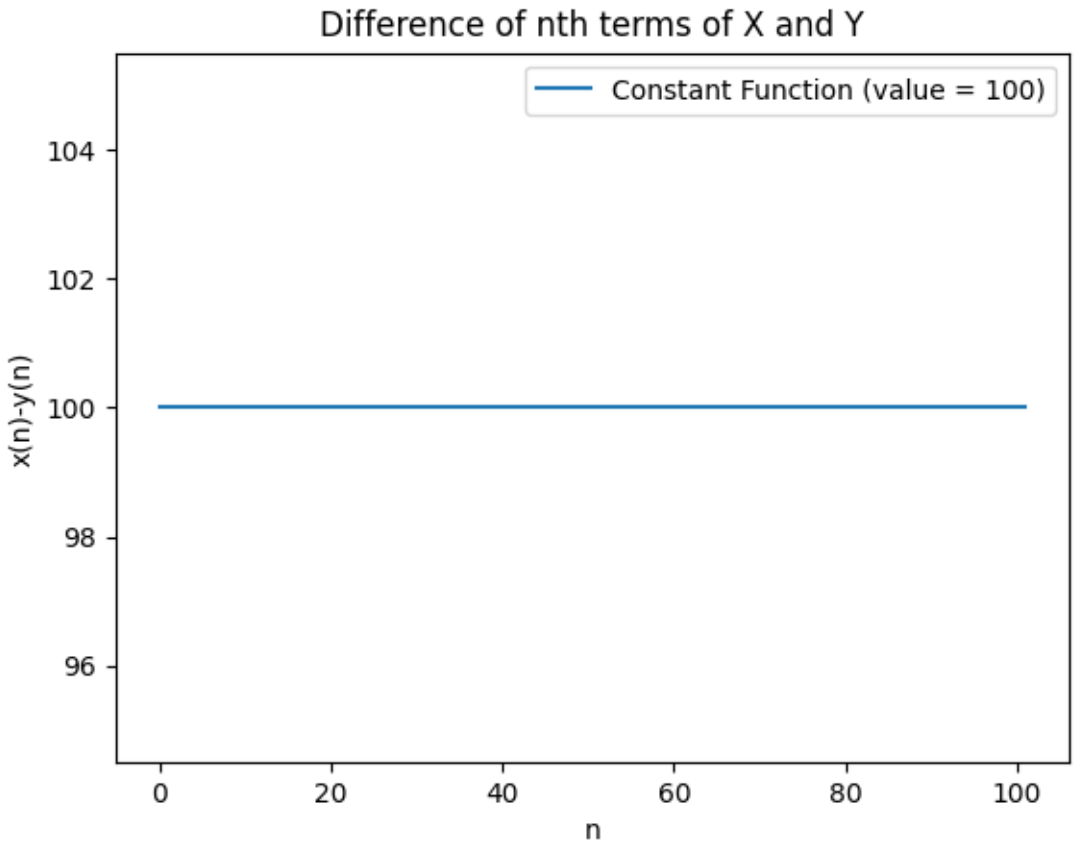
Difference of nth terms of X and Y
Constant Function (value = 100)
104
102
100
98
96
x(n)-y(n)
0
20
40
60
80
100
n

Fig. 0.