

Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

**Solution:**

Let us assume given two APs(Arithmetic Progressions) as X and Y whose first terms are given by  $x(0)$  and  $y(0)$ . Let  $d$  be the common difference of the APs.

We know that,

$n$ th term of an AP is given by

$$x(n) = [x(0) + (n)d]u(n) \quad (1)$$

$$\implies x(99) - y(99) = 100 \quad (2)$$

$$\implies (x(0) + 99d) - (y(0) + 99d) = 100 \quad (3)$$

$$\implies x(0) - y(0) = 100 \quad (4)$$

Now to find the difference between the 1000th terms of APs;

$$\implies x(999) - y(999) = (x(0) + 999d) - (y(0) + 999d) \quad (5)$$

$$= x(0) - y(0) \quad (6)$$

$$= 100 \quad (7)$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,

The Z-transform of a discrete signal  $x(n)$  is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering  $x(n-1)$  and  $y(n-1)$  as  $n^{th}$  terms of the APs(Arithmetic Progressions), Z-transform for  $x(n-1)$  and  $y(n-1)$  can be given by

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (8)$$

$$= \sum_{n=0}^n x(n)z^{-n} \quad (9)$$

$$= x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + \dots + x(n)z^{-n} \quad (10)$$

$$= x(0) + (x(0) + d)z^{-1} + (x(0) + 2d)z^{-2} + \dots + (x(0) + nd)z^{-n} \quad (11)$$

$$= x(0)[1 + z^{-1} + z^{-2} + \dots + z^{-n}] + d[1 \cdot z^{-1} + 2 \cdot z^{-2} + \dots + n \cdot z^{-n}] \quad (12)$$

$$= x(0)\left[\frac{1 \cdot (z^{-(n+1)} - 1)}{z^{-1} - 1}\right] + d\left[\frac{1}{1 - z^{-1}} + 1 \cdot \frac{z^{-1} \cdot (1 - z^{n-1})}{1 - z^{-1}} - n \cdot z^n\right] \quad (13)$$

$$= x(0)\left[\frac{1 \cdot (z^{-(n+1)} - 1)}{z^{-1} - 1}\right] + d\left[\frac{1 + z^{-1} \cdot (1 - z^{n-1})}{1 - z^{-1}} - n \cdot z^n\right] \quad (14)$$

$$= \frac{x(0) \cdot (z^{-(n+1)} - 1) - d \cdot (1 + z^{-1} \cdot (1 - z^{n-1}))}{z^{-1} - 1} - n \cdot d \cdot z^n \quad (15)$$

$$= x(0)(U(z)) + d\left[-z \cdot \frac{d(U(z))}{dz}\right] \quad (16)$$

$$Y(z) = \frac{y(0) \cdot (z^{-(n+1)} - 1) - d \cdot (1 + z^{-1} \cdot (1 - z^{n-1}))}{z^{-1} - 1} - n \cdot d \cdot z^n \quad (17)$$

$$(18)$$

$$\Rightarrow X(z) = x(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz} \quad (19)$$

$$\Rightarrow Y(z) = y(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz} \quad (20)$$

$$(21)$$

Variable	Description	Value
$x(n)$	$n^{th}$ term of X	none
$y(n)$	$n^{th}$ term of Y	none
$d$	common difference between the terms of AP	none
$X(z)$	z-transform of x(n)	none
$Y(z)$	z-transform of y(n)	none
$U(z)$	z-transform of u(n)	$\frac{1}{1-z^{-1}},  z  > 1$

TABLE 0  
INPUT PARAMETERS