Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

Solution:

Let us assume given two APs(Arithmetic Progressions) as X and Y whose first terms are given by $x(1)$ and $y(1)$. Let d be the common difference of the APs.
We know that, nth term of an AP is given by $x(n) = x(1) + (n-1)d$.
Given that the difference between the 100th terms is 100.

$$x(100) - y(100) = 100 \tag{1}$$
$$(x(1) + 99d) - (y(1) + 99d) = 100(\text{since } 999d \text{ cancels out, the equation will be}) \tag{2}$$
$$x(1) - y(1) = 100 \tag{3}$$

Now to find the difference between the 1000th terms of APs;

$$x(1000) - y(1000) = (x(1) + 999d) - (y(1) + 999d)(\text{since } 999d \text{ cancels out, the equation will be}) \tag{4}$$
$$= x(1) - y(1) \tag{5}$$
$$= 100(\text{from the above equation}) \tag{6}$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,
The Z-transform of a discrete signal $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering x(n) and y(n) as n^{th} terms of the APs(Arithmetic Progressions), Z-transform for x(n) and y(n) can be given by

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{7}$$
$$= \sum_{n=1}^{n} x(n)z^{-n} \tag{8}$$
$$= x(1)z^{-1} + x(2)z^{-2} + \ldots + x(n)z^{-n} \tag{9}$$
$$= x(1)z^{-1} + (x(1) + d)z^{-2} + \ldots + (x(1) + (n-1)d)z^{-n} \tag{10}$$
$$= x(1)[z^{-1} + z^{-2} + \ldots + z^{-n}] + d[1.z^{-2} + \ldots + (n-1).z^{-n}] \tag{11}$$
$$= x(1) \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz} - U(z)] \tag{12}$$
$$\tag{13}$$

$X(z) = [x(1) - d] \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz}]$

similarly

$Y(z) = [y(1) - d] \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz}]$

Variable	Description	Value
X, Y	Two given APs	none
$x(n)$,$y(n)$	n^{th} term of X, n^{th} term of Y	none
n	position of the term in the AP	none
d	common difference between the terms of AP	none
$X(z)$	z-transform of x(n)	$[x(1) - d] \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz}]$
$Y(z)$	z-transform of y(n)	$[y(1) - d] \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz}]$
$U(z)$	z-transform of u(n)	$\sum_{n=1}^{\infty} z^{-n}$
$\frac{d(U(z))}{dz}$	Derivative of U(z)	$-\sum_{n=1}^{\infty} nz^{-n-1}$

TABLE 0

VARIABLES AND THEIR VALUES