Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

Solution:

Let us assume given two APs(Arithmetic Progressions) as X and Y whose first terms are given by $x(0)$ and $y(0)$. Let d be the common difference of the APs.

We know that,

nth term of an AP is given by

$$x(n) = x(0) + [(n)d]u(n) \tag{1}$$
$$\implies x(99) - y(99) = 100 \tag{2}$$
$$\implies (x(0) + 99d) - (y(0) + 99d) = 100 \tag{3}$$
$$\implies x(0) - y(0) = 100 \tag{4}$$

Now to find the difference between the 1000th terms of APs;

$$\implies x(999) - y(999) = (x(0) + 999d) - (y(0) + 999d) \tag{5}$$
$$= x(0) - y(0) \tag{6}$$
$$= 100 \tag{7}$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,

The Z-transform of a discrete signal $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering x(n-1) and y(n-1) as n^{th} terms of the APs(Arithmetic Progressions), Z-transform for x(n-1) and y(n-1) can be given by

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{8}$$
$$= \sum_{n=0}^{n} x(n)z^{-n} \tag{9}$$
$$= x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + \ldots + x(n)z^{-(n)} \tag{10}$$
$$= x(0) + (x(0) + d)z^{-1} + (x(0) + 2d)z^{-2} + \ldots + (x(0) + (n)d)z^{-n} \tag{11}$$
$$= x(0)[1 + z^{-1} + z^{-2} + \ldots + z^{-n}] + d[1.z^{-1} + 2.z^{-2} + \ldots + n.z^{-n}] \tag{12}$$
$$= x(0)(U(z)) + d[-z \cdot \frac{d(U(z))}{dz}] \tag{13}$$
$$\implies X(z) = x(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz} \tag{14}$$
$$\implies Y(z) = y(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz} \tag{15}$$
$$\tag{16}$$

Variable	**Description**	**Value**
$x(n)$	n^{th} term of X	none
$y(n)$	n^{th} term of Y	none
n	position of the term in the AP starting from 0	none
d	common difference between the terms of AP	none
$X(z)$	z-transform of x(n)	$x(0) \cdot U(z) + d \cdot -z \cdot \frac{d(U(z))}{dz}$
$Y(z)$	z-transform of y(n)	$y(0) \cdot U(z) + d \cdot -z \cdot \frac{d(U(z))}{dz}$
$U(z)$	z-transform of u(n)	$\sum_{n=1}^{\infty} z^{-n}$
$\frac{d(U(z))}{dz}$	Derivative of U(z)	$-\sum_{n=1}^{\infty} nz^{-n-1}$

TABLE 0
INPUT PARAMETERS