

Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

Solution:

Let us assume given two APs(Arithmetic Progressions) as X and Y whose first terms are given by $x(0)$ and $y(0)$. Let d be the common difference of the APs.

We know that, n th term of an AP is given by $x(n) = x(0) + (n)d$.

Given that the difference between the 100th terms is 100.

$$x(99) - y(99) = 100 \quad (1)$$

$$(x(0) + 99d) - (y(0) + 99d) = 100 \text{ (since } 99d \text{ cancels out, the equation will be)} \quad (2)$$

$$x(0) - y(0) = 100 \quad (3)$$

Now to find the difference between the 1000th terms of APs;

$$x(999) - y(999) = (x(0) + 999d) - (y(0) + 999d) \text{ (since } 999d \text{ cancels out, the equation will be)} \quad (4)$$

$$= x(0) - y(0) \quad (5)$$

$$= 100 \text{ (from the above equation)} \quad (6)$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,

The Z-transform of a discrete signal $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering $x(n)$ and $y(n)$ as n^{th} terms of the APs(Arithmetic Progressions), Z-transform for $x(n)$ and $y(n)$ can be given by

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (7)$$

$$= \sum_{n=0}^{n-1} x(n)z^{-n} \quad (8)$$

$$= x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + \dots + x(n-1)z^{-(n-1)} \quad (9)$$

$$= x(0) + (x(0) + d)z^{-1} + (x(0) + 2d)z^{-2} + \dots + (x(0) + (n-1)d)z^{-(n-1)} \quad (10)$$

$$= x(0)[1 + z^{-1} + z^{-2} + \dots + z^{-(n-1)}] + d[1.z^{-1} + 2.z^{-2} + \dots + n.z^{-(n-1)}] \quad (11)$$

$$= x(0)(U(z)) + d[-z \cdot \frac{d(U(z))}{dz}] \quad (12)$$

$$X(z) = x(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz}$$

similarly

$$Y(z) = y(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz}$$