Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

Solution:

Let us assume given two APs(Arithmetic Progressions) as X and Y whose first terms are given by $x(0)$ and $y(0)$. Let d be the common difference of the APs.
We know that,
nth term of an AP is given by

$$x(n) = x(0) + [(n)d]u(n) \tag{1}$$

Given that the difference between the 100th terms is 100 (2)

$$\implies x(99) - y(99) = 100 \tag{3}$$

$$\implies (x(0) + 99d) - (y(0) + 99d) = 100(\text{since } 999d \text{ cancels out, the equation will be}) \tag{4}$$

$$\implies x(0) - y(0) = 100 \tag{5}$$

Now to find the difference between the 1000th terms of APs;

$$\begin{aligned}
\implies x(999) - y(999) &= (x(0) + 999d) - (y(0) + 999d)(\text{since } 999d \text{ cancels out, the equation will be}) && (6)\\
&= x(0) - y(0) && (7)\\
&= 100(\text{from the above equation}) && (8)
\end{aligned}$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,
The Z-transform of a discrete signal $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering x(n-1) and y(n-1) as n^{th} terms of the APs(Arithmetic Progressions), Z-transform for x(n-1) and y(n-1) can be given by

$$\begin{aligned}
X(z) = \mathcal{Z}\{x(n)\} &= \sum_{n=-\infty}^{\infty} x(n)z^{-n} && (9)\\
&= \sum_{n=0}^{n-1} x(n)z^{-n} && (10)\\
&= x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + \ldots + x(n-1)z^{-(n-1)} && (11)\\
&= x(0) + (x(0) + d)z^{-1} + (x(0) + 2d)z^{-2} + \ldots + (x(0) + (n-1)d)z^{-(n-1)} && (12)\\
&= x(0)[1 + z^{-1} + z^{-2} + \ldots + z^{-(n-1)}] + d[1.z^{-1} + 2.z^{-2} + \ldots + n.z^{-(n-1)}] && (13)\\
&= x(0)(U(z)) + d[-z \cdot \frac{d(U(z))}{dz}] && (14)\\
\implies X(z) &= x(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz} && (15)\\
\implies Y(z) &= y(0) \cdot U(z) + d \cdot (-z) \cdot \frac{d(U(z))}{dz} && (16)\\
& && (17)
\end{aligned}$$