Q:Two APs have the same common difference.The difference between their 100th terms is 100,what is the difference between their 1000th terms?

Solution:

Let us assume given two APs(Arithmetic Progressions) as A and B whose first terms are given by $a(1)$ and $b(1)$. Let d be the common difference of the APs.
We know that, nth term of an AP is given by $a(n) = a(1) + (n-1)d$.
Given that the difference between the 100th terms is 100.

$$a(100) - b(100) = 100 \tag{1}$$
$$(a(1) + 99d) - (b(1) + 99d) = 100(\text{since } 999d \text{ cancels out, the equation will be}) \tag{2}$$
$$a(1) - b(1) = 100 \tag{3}$$

Now to find the difference between the 1000th terms of APs;

$$a(1000) - b(1000) = (a(1) + 999d) - (b(1) + 999d)(\text{since } 999d \text{ cancels out, the equation will be}) \tag{4}$$
$$= a(1) - b(1) \tag{5}$$
$$= 100(\text{from the above equation}) \tag{6}$$

Therefore,the difference between the 1000th terms of two given APs is 100.

We know that,
The Z-transform of a discrete signal $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Considering a(n) and b(n) as n^{th} terms of the APs(Arithmetic Progressions), Z-transform for a(n) and b(n) can be given by

$$A(z) = \mathcal{Z}\{a(n)\} = \sum_{n=-\infty}^{\infty} a(n)z^{-n} \tag{7}$$
$$= \sum_{n=1}^{n} a(n)z^{-n} \tag{8}$$
$$= a(1)z^{-1} + a(2)z^{-2} + \ldots + a(n)z^{-n} \tag{9}$$
$$= a(1)z^{-1} + (a(1) + d)z^{-2} + \ldots + (a(1) + (n-1)d)z^{-n} \tag{10}$$
$$= a(1)[z^{-1} + z^{-2} + \ldots + z^{-n}] + d[1.z^{-2} + \ldots + (n-1).z^{-n}] \tag{11}$$
$$= a(1) \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz} - U(z)] \tag{12}$$
$$\tag{13}$$

$A(z) = [a(1) - d] \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz}]$

similarly

$B(z) = [b(1) - d] \cdot U(z) + d[-z \cdot \frac{d(U(z))}{dz}]$