# SEQUENCE AND SERIES

EE23BTECH11011- Batchu Ishitha*

Q: Find the sum to $n$ terms of the series whose $n^{th}$ term is given by $(2n-1)^2$ ?

**Solution:**

| Variable | Description | Value |
|---|---|---|
| $x(n)$ | $n^{th}$ term of sequence | $(2n-1)^2$ |

TABLE 0
INPUT PARAMETERS

$$S_n = \sum_{n=0}^{n-1} x(n) \tag{1}$$

$$= \sum_{n=0}^{n-1} (2n-1)^2 \tag{2}$$

$$= \sum_{n=0}^{n-1} 4n^2 + 1 - 4n \tag{3}$$

$$= \frac{4(n-1)n(2n-1)}{6} + n - \frac{4(n-1)n}{2} \tag{4}$$

$$\because 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6} \tag{5}$$

$$\because 1 + 2 + 3 + \ldots + n = \frac{n \cdot (n+1)}{2} \tag{6}$$

$$= \frac{2n(n-1)(2n-1)}{3} + n - 2(n)(n-1) \tag{7}$$

$$\text{(taking} \frac{2n(n-1)}{3} \text{ common from first and third term;)} \tag{8}$$

$$= \frac{2n(n-1)}{3}(2n-1-3) + n \tag{9}$$

$$= \frac{2n(n-1)}{3}2(n-2) + n \tag{10}$$

$$= \frac{4n(n-1)(n-2)}{3} + n \tag{11}$$

$$\text{( taking } \frac{n}{3} \text{ common from both terms; )} \tag{12}$$

$$= \frac{n(4(n^2-3n+2)+3)}{3} \tag{13}$$

$$= \frac{n(4n^2-12n+11)}{3} \tag{14}$$

Therefore, sum of $n$ terms of the series whose $n^{th}$ term is given by $(2n-1)^2$ is $\frac{n(4n^2-12n+11)}{3}$ .

$$x(n) = (2n-1)^2 u(n) \tag{15}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{16}$$

$$= \sum_{n=0}^{\infty} (2n-1)^2 z^{-n} \tag{17}$$

$$= U(z) + 8z\frac{d(U(z))}{dz} + 4z^2\frac{d^2(U(z))}{dz^2} \tag{18}$$

$$= \frac{(1-z^{-1})^2 - 8z^{-1}(2-z^{-1})}{(1-z^{-1})^3} \tag{19}$$

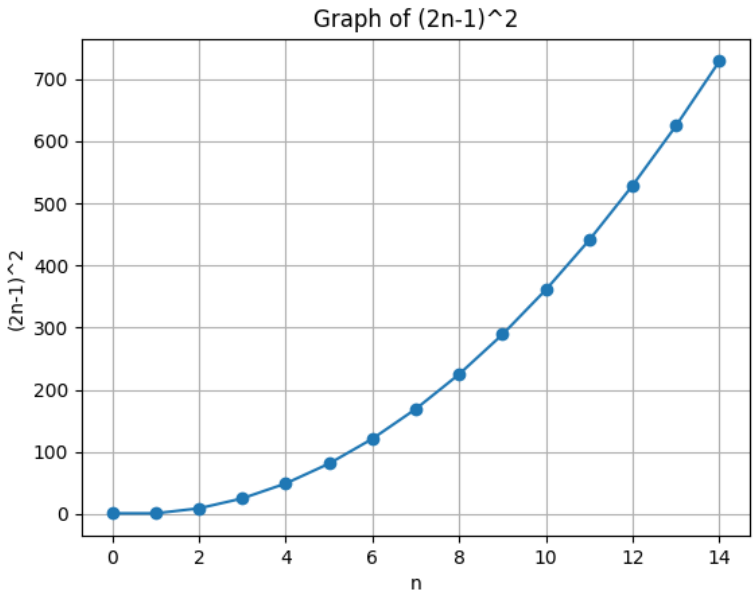


Fig. 0.