SEQUENCE AND SERIES

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Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n-1)^2$?

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(2n-1)^2$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$y(n) = \sum_{k=0}^{n} x(k) \tag{1}$$

$$= x(n) * u(n) \tag{2}$$

$$x(n) = (2n-1)^2 u(n) \tag{3}$$

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})} \quad |z| > 1 \tag{4}$$

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{5}$$

$$n^2 u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \quad |z| > 1 \tag{6}$$

$$\implies X(z) = \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} - \frac{4z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{7}$$

$$Y(z) = X(z)U(z) \tag{8}$$

$$= \left(\frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} - \frac{4z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{9}$$

$$= \frac{z^4 - 2z^3 + 9z^2}{(z-1)^4} \quad |z| > 1 \tag{10}$$

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1}\, dz \tag{11}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{z^4 - 2z^3 + 9z^2}{(z-1)^4}\right) z^{n-1}\, dz \tag{12}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{z^{n+3} - 2z^{n+2} + 9z^{n+1}}{(z-1)^4}\right) dz \tag{13}$$

$$= \frac{1}{(4-1)!} \frac{d^3}{dz^3} (z^{n+3} - 2z^{n+2} + 9z^{n+1}) \tag{14}$$

$$= \left(\frac{(n+3)(n+2)(n+1) - 2(n+2)(n+1)(n) + 9(n+1)(n)(n-1)}{6}\right) u(n) \tag{15}$$

On solving, we get

$$y(n) = \left(\frac{4n^3 - n + 3}{3}\right) u(n) \tag{16}$$

Therefore, sum of n terms of the series whose n^{th} term is given by $(2n-1)^2$ is $\frac{4n^3-n+3}{3}$.

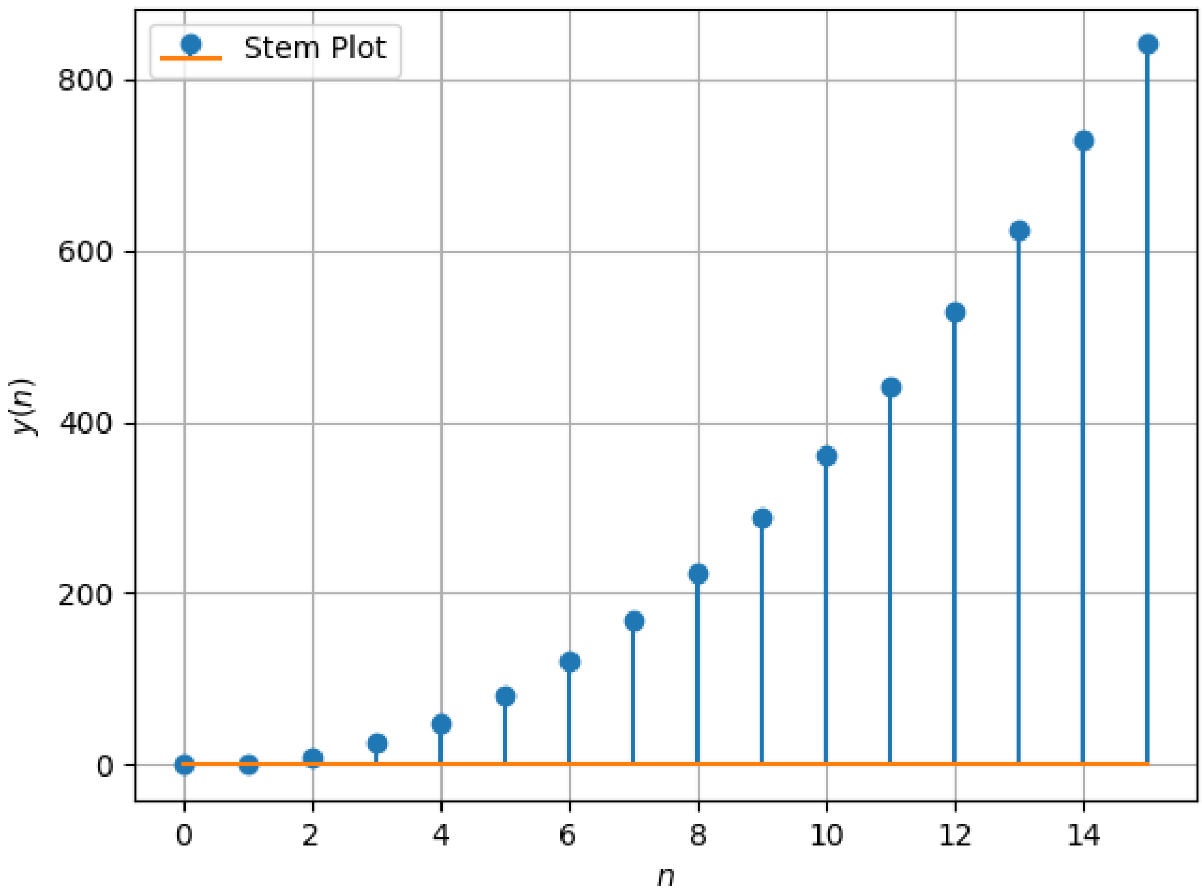

Fig. 0.