SEQUENCE AND SERIES

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Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n-1)^2$?

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(2n+1)^2$

TABLE 0
INPUT PARAMETERS

Sum of n terms of AP is given by

$$y(n) = \sum_{k=0}^{n} x(k) \tag{1}$$

$$= x(n) * u(n) \tag{2}$$

$$x(n) = (2n+1)^2 u(n) \tag{3}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4}$$

$$= \sum_{n=0}^{\infty} (2n+1)^2 z^{-n} \tag{5}$$

$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} + \frac{4z^{-1}}{(1-z^{-1})^2} \tag{6}$$

$$Y(z) = X(z)U(z) \tag{7}$$

$$= \left(\frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} + \frac{4z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \quad |z| > 1 \tag{8}$$

$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{1}{(1-z^{-1})^2} + \frac{4z^{-1}}{(1-z^{-1})^3} \quad |z| > 1 \tag{9}$$

$$= \frac{1+9z^{-1}}{(1-z^{-1})} + \frac{25z^{-2}}{(1-z^{-1})^2} + \frac{24z^{-3}}{(1-z^{-1})^3} + \frac{8z^{-4}}{(1-z^{-1})^4} \quad |z| > 1 \tag{10}$$

On solving, we get

$$y(n) = \left(\frac{4n^3 + 12n^2 + 11n + 3}{3}\right) u(n) \tag{11}$$

Therefore, sum of n terms of the series whose n^{th} term is given by $(2n+1)^2$ is $\frac{n(4n^2-1)}{3}$.

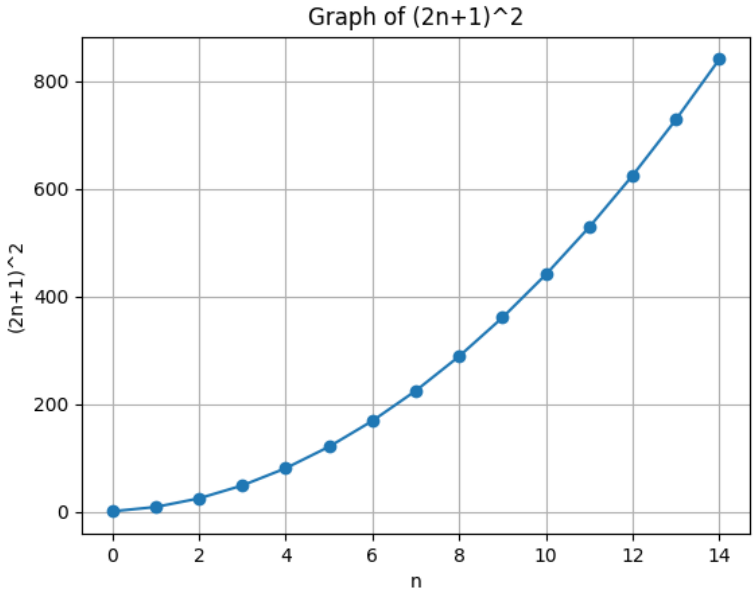

Fig. 0.