

SEQUENCE AND SERIES

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Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n - 1)^2$?

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(2n + 1)^2 u(n)$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$y(n) = x(n) * u(n) \quad (1)$$

$$x(n) = (2n + 1)^2 u(n) \quad (2)$$

$$u(n) \xleftrightarrow{Z} \frac{1}{(1 - z^{-1})} \quad |z| > 1 \quad (3)$$

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (4)$$

$$n^2 u(n) \xleftrightarrow{Z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \quad |z| > 1 \quad (5)$$

$$n^3 u(n) \xleftrightarrow{Z} \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4} \quad |z| > 1 \quad (6)$$

$$\Rightarrow X(z) = \frac{4z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} + \frac{1}{(1 - z^{-1})} + \frac{4z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (7)$$

$$Y(z) = X(z)U(z) \quad (8)$$

$$= \left(\frac{4z^{-1}(z^{-1} + 1)}{(1 - z^{-1})^3} + \frac{1}{(1 - z^{-1})} + \frac{4z^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (9)$$

$$= \frac{4z^{-1}(z^{-1} + 1)}{(1 - z^{-1})^4} + \frac{1}{(1 - z^{-1})^2} + \frac{4z^{-1}}{(1 - z^{-1})^3} \quad (10)$$

$$\Rightarrow Y(Z) = \frac{1}{(1 - z^{-1})} + \frac{9z^{-1}}{(1 - z^{-1})} + \frac{25z^{-2}}{(1 - z^{-1})^2} + \frac{24z^{-3}}{(1 - z^{-1})^3} + \frac{8z^{-4}}{(1 - z^{-1})^4} \quad |z| > 1 \quad (11)$$

Now from (3), (4), (5), (6), (11)

By using inverse Z-transform pairs,

$$y(n) = u(n) + 9u(n - 1) + 25(n - 1)u(n - 2) + 24 \frac{(n - 1)(n - 2)}{2} u(n - 3) + 8 \frac{(n - 1)(n - 2)(n - 3)}{6} u(n - 4) \quad (12)$$

$$\Rightarrow y(n) = \left(\frac{4n^3 + 12n^2 + 11n + 3}{3} \right) u(n) \quad (13)$$

\therefore Sum of n terms of the series whose n^{th} term is given by $(2n + 1)^2$ is $\frac{4n^3 + 12n^2 + 11n + 3}{3}$.

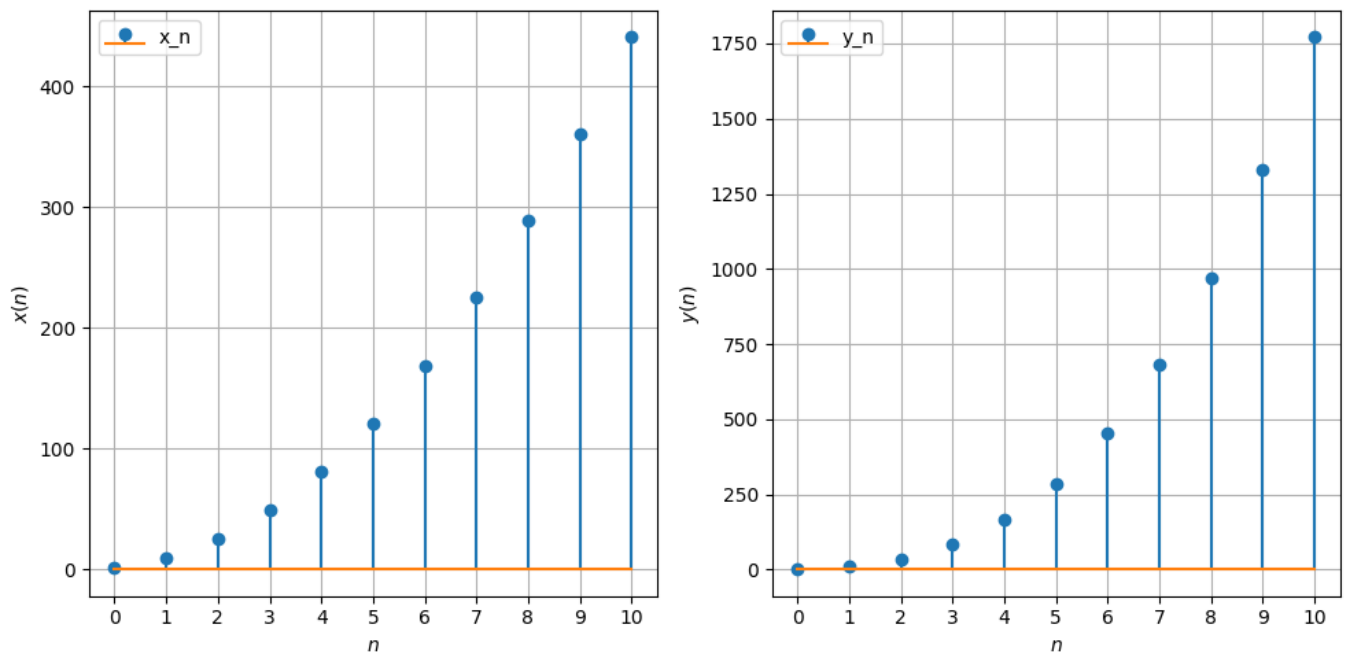


Fig. 0.