SEQUENCE AND SERIES

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Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n-1)^2$?

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(2n+1)^2u(n)$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$y(n) = x(n) * u(n) \tag{1}$$
$$x(n) = (2n+1)^2u(n) \tag{2}$$
$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})} \quad |z| > 1 \tag{3}$$
$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{4}$$
$$n^2u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \quad |z| > 1 \tag{5}$$
$$\implies X(z) = \frac{4z^{-1}(1+z^{-1})}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} + \frac{4z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{6}$$

$$Y(z) = X(z)U(z) \tag{7}$$
$$= \left(\frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} + \frac{4z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{8}$$
$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{1}{(1-z^{-1})^2} + \frac{4z^{-1}}{(1-z^{-1})^3} \tag{9}$$

$$Y(Z) = \frac{1}{(1-z^{-1})} + \frac{9z^{-1}}{(1-z^{-1})} + \frac{25z^{-2}}{(1-z^{-1})^2} + \frac{24z^{-3}}{(1-z^{-1})^3} + \frac{8z^{-4}}{(1-z^{-1})^4} \quad |z| > 1 \tag{10}$$

By using inverse Z-transform pairs,

$$Z^{-1}\left[\frac{z^{-1}}{1-z^{-1}}\right] \to u(n-1) \tag{11}$$
$$Z^{-1}\left[\frac{z^{-2}}{(1-z^{-1})^2}\right] \to (n-1)u(n-2) \tag{12}$$
$$Z^{-1}\left[\frac{z^{-3}}{(1-z^{-1})^3}\right] \to \frac{(n-1)(n-2)}{2}u(n-3) \tag{13}$$
$$Z^{-1}\left[\frac{z^{-4}}{(1-z^{-1})^4}\right] \to \frac{(n-1)(n-2)(n-3)}{6}u(n-4) \tag{14}$$

Now from (10)

$$y(n) = u(n) + 9u(n-1) + 25(n-1)u(n-2) + 24\frac{(n-1)(n-2)}{2}u(n-3) + 8\frac{(n-1)(n-2)(n-3)}{6}u(n-4) \tag{15}$$

$$\implies y(n) = \left(\frac{4n^3 + 12n^2 + 11n + 3}{3}\right)u(n) \tag{16}$$

$\therefore$ Sum of n terms of the series whose n^{th} term is given by $(2n+1)^2$ is $\frac{4n^3+12n^2+11n+3}{3}$.

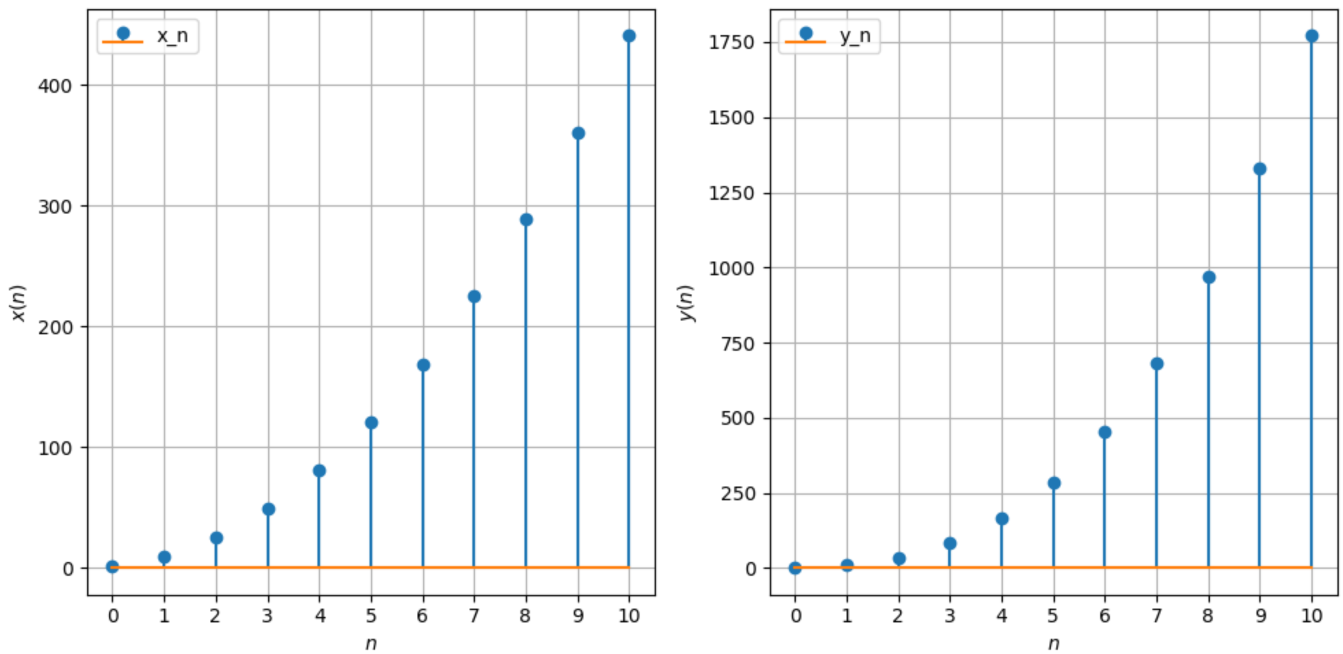

Fig. 0.