Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n-1)^2$?

Solution:

Let nth term of the series be given by T_n.

Given, n^{th} of the series is given by $(2n-1)^2$.

$x(n)$= $(2n-1)^2$.

We know that summation of the above term from r=1 to r=n gives the sum of the series.

$$= S_n = \sum_{n=1}^{n} x(n) \tag{1}$$

$$= \sum (2n-1)^2 \tag{2}$$

$$= \sum 4n^2 + 1 - 4n \tag{3}$$

$$= \frac{4 \cdot n \cdot (n+1) \cdot (2n+1)}{6} + n - \frac{4 \cdot n \cdot (n+1)}{2} \tag{4}$$

$$\because 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6} \tag{5}$$

$$\because 1 + 2 + 3 + \ldots + n = \frac{n \cdot (n+1)}{2} \tag{6}$$

$$= \frac{2 \cdot n \cdot (n+1) \cdot (2n+1)}{3} + n - 2(n)(n+1) \tag{7}$$

$$\text{(taking} \frac{2 \cdot n \cdot (n+1)}{3} \text{ common from first and third term;)} \tag{8}$$

$$= \frac{2 \cdot n \cdot (n+1) \cdot (2n+1-3)}{3} + n \tag{9}$$

$$= \frac{4 \cdot n \cdot (n+1) \cdot (n-1)}{3} + n \tag{10}$$

$$= \frac{4 \cdot n \cdot (n^2-1)}{3} + n \tag{11}$$

$$\text{(taking } \frac{n}{3} \text{ common from both terms;)} \tag{12}$$

$$= \frac{n \cdot (4n^2 - 4 + 3)}{3} \tag{13}$$

$$= \frac{n \cdot (4n^2 - 1)}{3} \tag{14}$$

Therefore, sum of n terms of the series whose n^{th} term is given by $(2n-1)^2$ is $\frac{n \cdot (4n^2-1)}{3}$.

$x(n)$= $(2n-1)^2 \cdot u(n)$.

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{15}$$

$$= 1^2 \cdot z^{-1} + 3^2 \cdot z^{-2} + \ldots + (2n-1)^2 \cdot z^{-n} \tag{16}$$

$$= U(z) + 8 \cdot z \cdot \frac{d(U(z))}{dz} + 4 \cdot z^2 \cdot \frac{d^2(U(z))}{dz^2} \tag{17}$$