# SEQUENCE AND SERIES

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Q: Find the sum to $n$ terms of the series whose $n^{th}$ term is given by $(2n-1)^2$ ?

**Solution:**

| Variable | Description | Value |
|---|---|---|
| $x(n)$ | $n^{th}$ term of sequence | $(2n-1)^2$ |

TABLE 0
INPUT PARAMETERS

Sum of $n$ terms of AP is given by

$$y(n) = \sum_{k=0}^{n} x(k) \tag{1}$$

$$= x(n) * u(n) \tag{2}$$

$$x(n) = (2n-1)^2 u(n) \tag{3}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4}$$

$$= \sum_{n=0}^{\infty} (2n-1)^2 z^{-n} \tag{5}$$

$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} - \frac{4z^{-1}}{(1-z^{-1})^2} \tag{6}$$

$$Y(z) = X(z)U(z) \tag{7}$$

$$= \left(\frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} - \frac{4z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{8}$$

$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{1}{(1-z^{-1})^2} - \frac{4z^{-1}}{(1-z^{-1})^3} \tag{9}$$

$$= \frac{1+z^{-1}}{(1-z^{-1})} + \frac{9z^{-2}}{(1-z^{-1})^2} + \frac{16z^{-3}}{(1-z^{-1})^3} + \frac{8z^{-4}}{(1-z^{-1})^4} \tag{10}$$

$$= \frac{9z^{-2} - 2z^{-1} + 1}{(1-z^{-1})^4} \tag{11}$$

$$= \frac{z^4 - 2z^3 + 9z^2}{(z-1)^4} \quad |z| > 1 \tag{12}$$

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1}\, dz \tag{13}$$
$$= \frac{1}{2\pi j} \oint_C \left( \frac{z^4 - 2z^3 + 9z^2}{(z-1)^4} \right) z^{n-1}\, dz \tag{14}$$
$$= \frac{1}{2\pi j} \oint_C \left( \frac{z^{n+3} - 2z^{n+2} + 9z^{n+1}}{(z-1)^4} \right) dz \tag{15}$$
$$= \frac{1}{(4-1)!} \frac{d^3}{dz^3} (z^{n+3} - 2z^{n+2} + 9z^{n+1}) \tag{16}$$
$$= \frac{(n+3)(n+2)(n+1) - 2(n+2)(n+1)(n) + 9(n+1)(n)(n-1)}{6} \tag{17}$$

On solving, we get

$$y(n) = \frac{4n^3 - n + 3}{3} \tag{18}$$

Therefore, sum of $n$ terms of the series whose $n^{th}$ term is given by $(2n-1)^2$ is $\frac{4n^3 - n + 3}{3}$ .

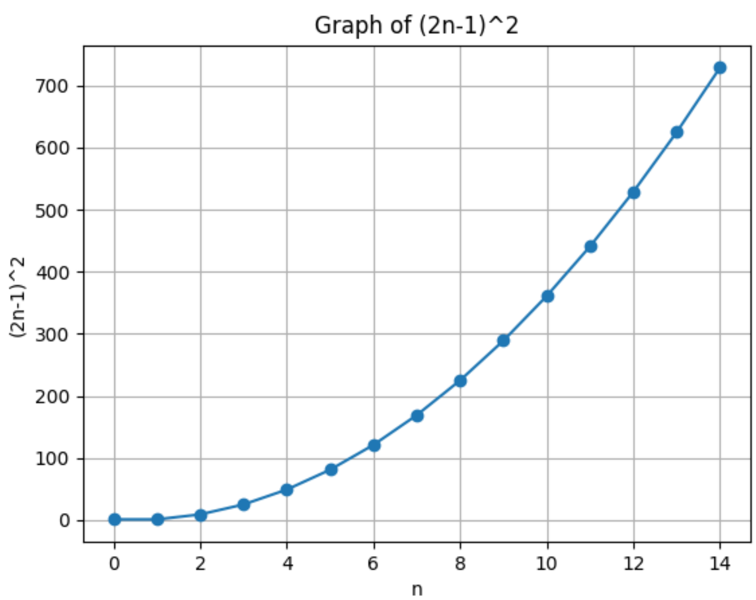


Fig. 0.