

SEQUENCE AND SERIES

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Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n - 1)^2$?

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(2n - 1)^2$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$y(n) = \sum_{k=0}^n x(k) \quad (1)$$

$$= x(n) * u(n) \quad (2)$$

$$x(n) = (2n - 1)^2 u(n) \quad (3)$$

$$u(n) \xleftrightarrow{z} \frac{1}{(1 - z^{-1})} \quad |z| > 1 \quad (4)$$

$$nu(n) \xleftrightarrow{z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (5)$$

$$n^2 u(n) \xleftrightarrow{z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \quad |z| > 1 \quad (6)$$

$$\Rightarrow X(z) = \frac{4z^{-1}(z^{-1} + 1)}{(1 - z^{-1})^3} + \frac{1}{(1 - z^{-1})} - \frac{4z^{-1}}{(1 - z^{-1})^2} \quad (7)$$

$$Y(z) = X(z)U(z) \quad (8)$$

$$= \left(\frac{4z^{-1}(z^{-1} + 1)}{(1 - z^{-1})^3} + \frac{1}{(1 - z^{-1})} - \frac{4z^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (9)$$

$$= \frac{z^4 - 2z^3 + 9z^2}{(z - 1)^4} \quad |z| > 1 \quad (10)$$

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (11)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{z^4 - 2z^3 + 9z^2}{(z - 1)^4} \right) z^{n-1} dz \quad (12)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{z^{n+3} - 2z^{n+2} + 9z^{n+1}}{(z - 1)^4} \right) dz \quad (13)$$

$$= \frac{1}{(4 - 1)!} \frac{d^3}{dz^3} (z^{n+3} - 2z^{n+2} + 9z^{n+1}) \quad (14)$$

$$= \left(\frac{(n + 3)(n + 2)(n + 1) - 2(n + 2)(n + 1)(n) + 9(n + 1)(n)(n - 1)}{6} \right) u(n) \quad (15)$$

On solving, we get

$$y(n) = \left(\frac{4n^3 - n + 3}{3} \right) u(n) \quad (16)$$

On replacing n by $n - 1$,

$$y(n) = \left(\frac{4n^3 - 12n^2 + 11n}{3} \right) u(n) \quad (17)$$

$$y(n) = \sum_{n=0}^{n-1} x(n) \quad (18)$$

$$= \sum_{n=0}^{n-1} (2n - 1)^2 u(n) \quad (19)$$

$$= \sum_{n=0}^{n-1} (4n^2 + 1 - 4n) \quad (20)$$

$$= 4 \frac{(n-1)(n)(2n-1)}{6} + n - 4 \frac{(n-1)(n)}{2} \quad (21)$$

$$= \frac{4(n-1)n}{6} (2n-1-3) + n \quad (22)$$

$$= \frac{2(n-1)n}{3} (2(n-2)) + n \quad (23)$$

$$= \frac{4(n-1)n(n-2)}{3} + n \quad (24)$$

$$= \frac{4n^3 - 12n^2 + 11n}{3} \quad (25)$$

$$y(n) = \left(\frac{4n^3 - 12n^2 + 11n}{3} \right) u(n) \quad (26)$$

Therefore, sum of n terms of the series whose n^{th} term is given by $(2n - 1)^2$ is $\frac{n(4n^2 - 12n + 11)}{3}$.

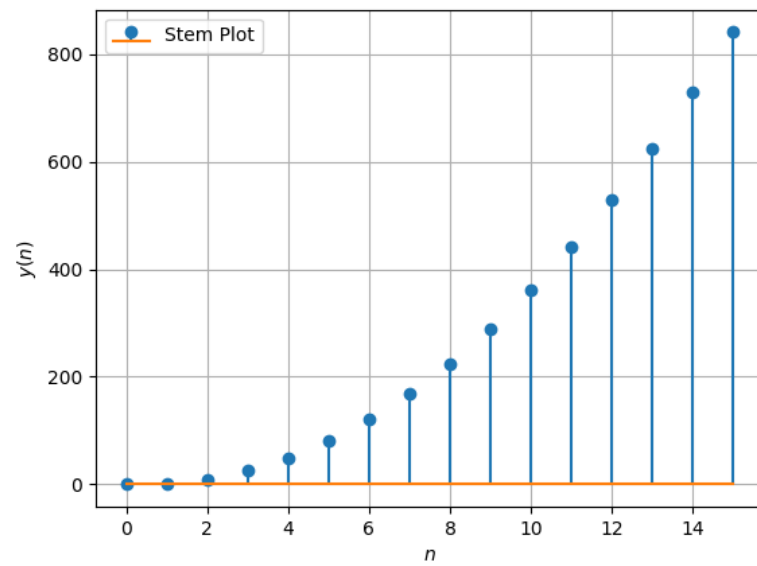


Fig. 0.