SEQUENCE AND SERIES

EE23BTECH11011- Batchu Ishitha*

Q: Find the sum to n terms of the series whose n^{th} term is given by $(2n-1)^2$?

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(2n-1)^2$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$y(n) = \sum_{k=0}^{n} x(k) \tag{1}$$

$$= x(n) * u(n) \tag{2}$$

$$x(n) = (2n-1)^2 u(n) \tag{3}$$

$$X(z) = \sum_{n=0}^{\infty} \left(4n^2 z^{-n} + z^{-n} - 4nz^{-n}\right) \tag{4}$$

$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} - \frac{4z^{-1}}{(1-z^{-1})^2} \tag{5}$$

$$Y(z) = X(z)U(z) \tag{6}$$

$$= \left(\frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{1}{(1-z^{-1})} - \frac{4z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{7}$$

$$= \frac{4z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{1}{(1-z^{-1})^2} - \frac{4z^{-1}}{(1-z^{-1})^3} \tag{8}$$

$$= \frac{1+z^{-1}}{(1-z^{-1})} + \frac{9z^{-2}}{(1-z^{-1})^2} + \frac{16z^{-3}}{(1-z^{-1})^3} + \frac{8z^{-4}}{(1-z^{-1})^4} \tag{9}$$

$$= \frac{9z^{-2} - 2z^{-1} + 1}{(1-z^{-1})^4} \tag{10}$$

$$= \frac{z^4 - 2z^3 + 9z^2}{(z-1)^4} \quad |z| > 1 \tag{11}$$

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1}\, dz \tag{12}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{z^4 - 2z^3 + 9z^2}{(z-1)^4}\right) z^{n-1}\, dz \tag{13}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{z^{n+3} - 2z^{n+2} + 9z^{n+1}}{(z-1)^4}\right) dz \tag{14}$$

$$= \frac{1}{(4-1)!} \frac{d^3}{dz^3}(z^{n+3} - 2z^{n+2} + 9z^{n+1}) \tag{15}$$

$$= \left(\frac{(n+3)(n+2)(n+1) - 2(n+2)(n+1)(n) + 9(n+1)(n)(n-1)}{6}\right) u(n) \tag{16}$$

On solving, we get

$$y(n) = \frac{4n^3 - n + 3}{3} \tag{17}$$

Therefore, sum of n terms of the series whose n^{th} term is given by $(2n-1)^2$ is $\frac{4n^3-n+3}{3}$.

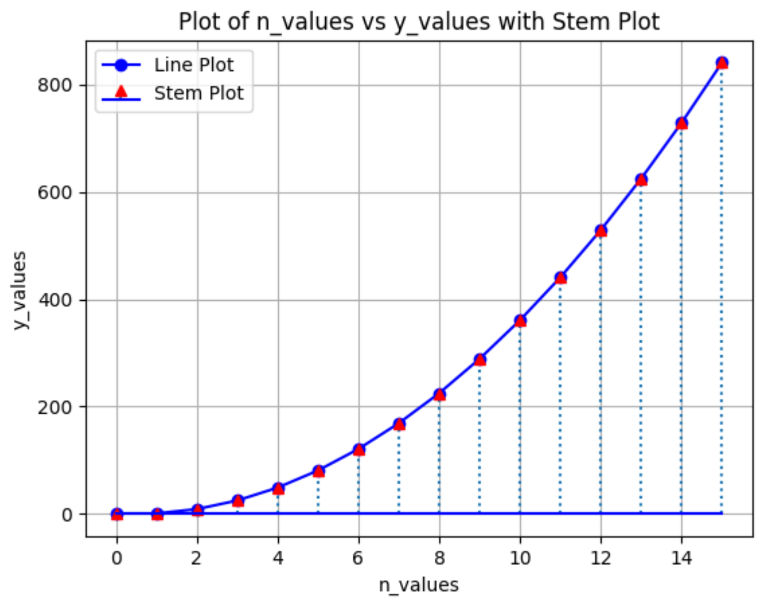

Fig. 0.