GATE NM-50 2022

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Q: Let $y(x)$ be the solution of the differential equation

$$y'' - 4y' - 12y = 3e^{5x}$$

satisfying $y(0) = \frac{18}{7}$ and $y'(0) = \frac{-1}{7}$.
Then $y(1)$ is ____________ (rounded off to nearest integer). GATE NM 2022

Solution:

Parameter	Description	Value
$y'' - 4y' - 12y = 3e^{5x}$	Differential equation	none
$y(x)$	Solution of differential equation	$y(0) = \frac{18}{7}$
$y'(x)$	First order derivative of solution of differential equation	$y'(0) = \frac{-1}{7}$

TABLE 0
INPUT PARAMETERS

$$y''(x) \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) \tag{1}$$

$$y'(x) \overset{\mathcal{L}}{\longleftrightarrow} sY(s) - y(0) \tag{2}$$

$$y(x) \overset{\mathcal{L}}{\longleftrightarrow} Y(s) \tag{3}$$

$$e^{ax} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-a} \tag{4}$$

Applying Laplace transform on both sides of the given differential equation,

$$\mathcal{L}\left(y''(x) - 4y'(x) - 12y(x)\right) = \mathcal{L}\left(3e^{5x}\right) \tag{5}$$

$$\mathcal{L}\left(y''(x)\right) - \mathcal{L}\left(4y'(x)\right) - \mathcal{L}\left(12y(x)\right) = \mathcal{L}\left(3e^{5x}\right) \tag{6}$$

From (1), (2), (3), (4)

$$Y(s)\left(s^2 - 4s - 12\right) - y(0)(s-4) - y'(0) = \frac{3}{s-5} \tag{7}$$

$$Y(s)\left(s^2 - 4s - 12\right) - \frac{(18s-73)}{7} = \frac{3}{(s-5)} \tag{8}$$

$$Y(s) = \frac{3}{(s-5)\left(s^2-4s-12\right)} + \frac{(18s-73)}{7\left(s^2-4s-12\right)} \tag{9}$$

$$\implies Y(s) = \frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{1}{(s+2)} \tag{10}$$

$$\frac{1}{s-a} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} e^{ax} \tag{11}$$

Now finding Inverse Laplace Transform on both sides of (10) ,

$$y(x) = \mathcal{L}^{-1}\left(\frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{2}{(s+2)}\right) \tag{12}$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s-6}\right) - \mathcal{L}^{-1}\left(\frac{3}{7(s-5)}\right) + \mathcal{L}^{-1}\left(\frac{2}{s+2}\right) \tag{13}$$

From (11)

$$\implies y(x) = \left(e^{6x} - \frac{3}{7}e^{5x} + 2e^{-2x}\right)u(x) \tag{14}$$

$$\implies y(1) = e^6 - \frac{3}{7}e^5 + 2e^{-2} \tag{15}$$

$$\therefore y(1) = 340 \tag{16}$$

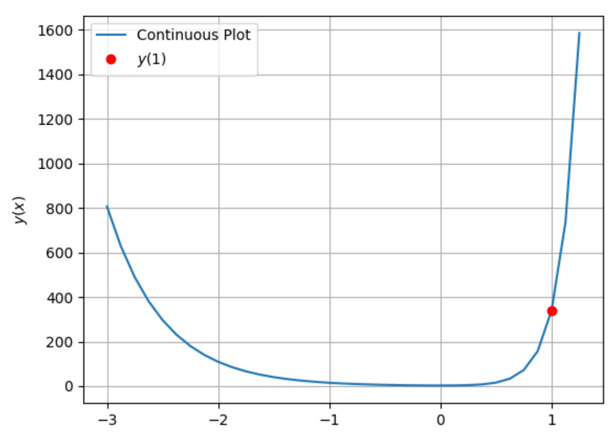

Fig. 0.