GATE NM-54 2022

EE23BTECH11011- Batchu Ishitha*

Q: A system with two degrees of freedom, as shown in the figure, has masses $m_1 = 200kg$ and $m_2 = 100kg$ and stiffness coefficients $k_1 = k_2 = 200N/m$. Then the lowest natural frequency of the system is ____________ rad/s (rounded off to one decimal place).

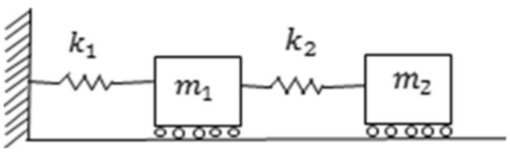

Fig. 0.

GATE NM 2022

Solution:

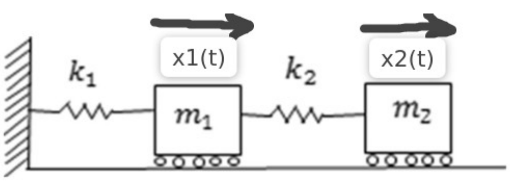

Fig. 0.

$$m_2\ddot{x}_2(t) + k_2\left(x_2(t) - x_1(t)\right) = 0 \quad (1)$$
$$m_1\ddot{x}_1(t) - k_2\left(x_2(t) - x_1(t)\right) + k_1x_1(t) = 0 \quad (2)$$

$$\ddot{x}_2(t) + 2\left(x_2(t) - x_1(t)\right) = 0 \quad (3)$$
$$\ddot{x}_1(t) + 2x_1(t) - x_2(t) = 0 \quad (4)$$

Substituting (3) in (4)

$$\ddddot{x}_2(t) + 4\ddot{x}_2(t) + 2x_2(t) = 0 \quad (5)$$

Applying Laplace transform on both sides of (5)

$$\mathcal{L}\left(\ddddot{x}_2(t) + 4\ddot{x}_2(t) + 2x_2(t)\right) = 0 \quad (6)$$

$$X_2(s)\left(s^4 - s^3x_2(0) - s^2\dot{x}_2(0) - s\ddot{x}_2(0) - \dddot{x}_2(0)\right) + 4X_2(s)\left(s^2 - sx_2(0) - \dot{x}_2(0)\right) + 2X_2(s) = 0 \quad (7)$$

$$X_2(s)\left(s^4 - s^3x_2(0) - s^2\dot{x}_2(0)\right) + 4X_2(s)\left(s^2 - sx_2(0)\right) + 2X_2(s) = 0 \quad (8)$$

$$X_2(s)\left(s^4 - \left(s^3 + 4s\right)x_2(0) - s^2\left(\dot{x}_2(0) - 4\right) + 2\right) = 0 \quad (9)$$

let $x_2(t)$ be constant at t=0

$$\implies X_2(s)(s^4 + 2) = 0 \quad (10)$$