GATE NM-54 2022

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Q: A system with two degrees of freedom, as shown in the figure, has masses $m_1 = 200kg$ and $m_2 = 100kg$ and stiffness coefficients $k_1 = k_2 = 200N/m$. Then the lowest natural frequency of the system is _____ rad/s (rounded off to one decimal place).

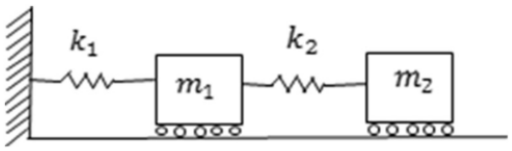

Fig. 0.

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Solution:

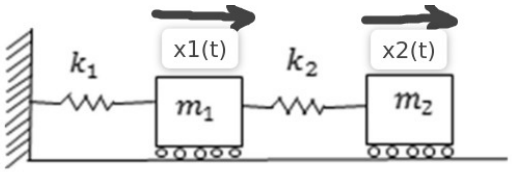

Fig. 0.

Variable	Description	Value
m_1	Mass of block 1	$200kg$
m_2	Mass of block 2	$100kg$
k_1	Stiffness coefficient of spring1	$200N/m$
k_2	Stiffness coefficient of spring2	$200N/m$
$x_i(t)$	Displacement of i^{th} block	$x_i(0) = 0$
$\dot{x}_i(t)$	Velocity of i^{th} block	$\dot{x}_i(0) = 0$
$\ddot{x}_i(t)$	Acceleration of i^{th} block	$\ddot{x}_i(0) = 0$

TABLE 0
INPUT PARAMETERS

$$m_2\ddot{x}_2(t) + k_2\left(x_2(t) - x_1(t)\right) = 0 \quad (1)$$

$$m_1\ddot{x}_1(t) - k_2\left(x_2(t) - x_1(t)\right) + k_1x_1(t) = 0 \quad (2)$$

Applying Laplace transform for (1) and (2)

$$m_2s^2X_2(s) + k_2\left(X_2(s) - X_1(s)\right) = 0 \quad (3)$$

$$m_1s^2X_1(s) - k_2\left(X_2(s) - X_1(s)\right) + k_1X_1(s) = 0 \quad (4)$$

Equating Transfer function from (3) and (4)

$$\frac{m_2s^2 + k_2}{k_2} = \frac{k_2}{m_1s^2 + k_1 + k_2} \quad (5)$$

$$\frac{100s^2 + 200}{200} = \frac{200}{200s^2 + 200 + 200} \quad (6)$$

$$\implies \left(s^2 + 2\right)^2 = 2 \quad (7)$$

Substituting s as $j\omega$ in (7)

$$\left(2 - \omega^2\right)^2 = 2 \quad (8)$$

$$\omega = \pm\sqrt{2 \mp \sqrt{2}} \quad (9)$$

$$\implies \omega_{min} = 0.766rad/sec \quad (10)$$