GATE NM-50 2022

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Q: Let $y(x)$ be the solution of the differential equation

$$y'' - 4y' - 12y = 3e^{5x} \tag{1}$$

satisfying $y(0) = \frac{18}{7}$ and $y'(0) = \frac{-1}{7}$.
Then $y(1)$ is _____ (rounded off to nearest integer). GATE NM 2022

Solution:

Parameter	Description	Value
$y'' - 4y' - 12y = 3e^{5x}$	Differential equation	none
$y(x)$	Solution of differential equation	$y(0) = \frac{18}{7}$
$y'(x)$	First order derivative of solution of differential equation	$y'(0) = \frac{-1}{7}$

TABLE 0
INPUT PARAMETERS

$$y''(t) \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) \tag{2}$$

$$y'(t) \overset{\mathcal{L}}{\longleftrightarrow} sY(s) - y(0) \tag{3}$$

$$y(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s) \tag{4}$$

$$e^{at} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-a} \tag{5}$$

Applying Laplace transform on both sides of the given differential equation,

$$\mathcal{L}\left(y''(t) - 4y'(t) - 12y(t)\right) = \mathcal{L}\left(3e^{5x}\right) \tag{6}$$

From (2), (3), (4), (5)

$$Y(s)\left(s^2 - 4s - 12\right) - y(0)(s-4) - y'(0) = \frac{3}{s-5} \tag{7}$$

$$Y(s)\left(s^2 - 4s - 12\right) - \frac{(18s - 73)}{7} = \frac{3}{(s-5)} \tag{8}$$

$$Y(s) = \frac{3}{(s-5)(s^2 - 4s - 12)} + \frac{(18s - 73)}{7(s^2 - 4s - 12)} \tag{9}$$

$$\implies Y(s) = \frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{1}{(s+2)} \tag{10}$$

$$\frac{1}{s-a} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} e^{at} \tag{11}$$

Now finding Inverse Laplace Transform on both sides of (10) ,
From (11)

$$\implies y(t) = \left(e^{6t} - \frac{3}{7}e^{5t} + 2e^{-2t}\right)u(t) \tag{12}$$

$$\implies y(1) = e^6 - \frac{3}{7}e^5 + 2e^{-2} \tag{13}$$

$$\therefore y(1) = 340 \tag{14}$$

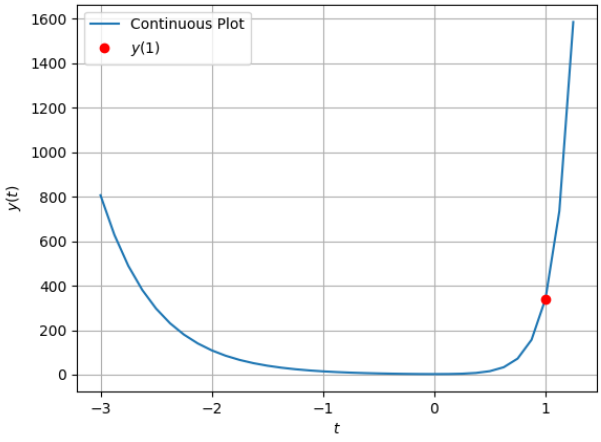

Fig. 0.