GATE IN-13 2022

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Q: A periodic function $f(x)$, with period 2, is defined as

$$f(x) = \begin{cases} -1 - x & -1 \le x < 0 \\ 1 - x & 0 < x \le 1 \end{cases} \tag{1}$$

The Fourier series of this function contains

A. Both $cos(n\pi x)$ and $sin(n\pi x)$ where n=1,2,3...
B. Only $sin(n\pi x)$ where n=1,2,3...
C. Only $cos(n\pi x)$ where n=1,2,3...
D. Only $cos(2n\pi x)$ where n=1,2,3... GATE IN 2022

Solution:

$$C_n = \frac{1}{2}\int_{-1}^{1} f(x)e^{-jn\omega_0 x}\,dx \tag{2}$$

$$= \frac{1}{2}\left(\int_{-1}^{0}(-1-x)\,e^{-jn\omega_0 x}\,dx + \int_{0}^{1}(+1-x)\,e^{-jn\omega_0 x}\,dx\right) \tag{3}$$

$$= \frac{1}{2}\left(-\int_{-1}^{0} e^{-jn\omega_0 x}\,dx - \int_{-1}^{1} xe^{-jn\omega_0 x}\,dx + \int_{0}^{1} e^{-jn\omega_0 x}\,dx\right) \tag{4}$$

$$= \frac{1}{2}\left[\frac{-1}{jn\omega_0}\left[-\left(1 - e^{+jn\omega_0}\right) + \left(e^{-jn\omega_0} - 1\right)\right] - \int_{-1}^{1} xe^{-jn\omega_0 x}\,dx\right] \tag{5}$$

$$= \frac{1}{2}\left[\frac{-1}{jn\omega_0}\left[-2 + e^{+jn\omega_0} + e^{-jn\omega_0}\right] + \left(\frac{e^{-jn\omega_0 x}}{jn\omega_0}\left[x + \frac{1}{jn\omega_0}\right]\right)_{-1}^{1}\right] \tag{6}$$

$$= \frac{-1}{jn\omega_0}\left[-1 + cos(n\omega_0)\right] + \frac{1}{2(jn\omega_0)^2}\left[\left(e^{-jn\omega_0}\right)(1 + jn\omega_0) - \left(e^{jn\omega_0}\right)(-jn\omega_0 + 1)\right] \tag{7}$$

$$\implies C_n = \frac{-1}{(jn\omega_0)^2}\left[-jn\omega_0 + jsin(n\omega_0)\right] \tag{8}$$

$$a_n = C_n + C_{-n} \tag{9}$$

$$= 0 \tag{10}$$

$$b_n = C_n - C_{-n} \tag{11}$$

$$= \frac{-2}{(jn\omega_0)^2}\left[-jn\omega_0 + jsin(n\omega_0)\right] \tag{12}$$

$\therefore$ The Fourier series of this function contains only $sin(n\pi x)$ where n=1,2,3...