GATE NM-54 2022

EE23BTECH11011- Batchu Ishitha*

Q: A system with two degrees of freedom, as shown in the figure, has masses $m_1 = 200kg$ and $m_2 = 100kg$ and stiffness coefficients $k_1 = k_2 = 200N/m$. Then the lowest natural frequency of the system is _____ rad/s (rounded off to one decimal place).

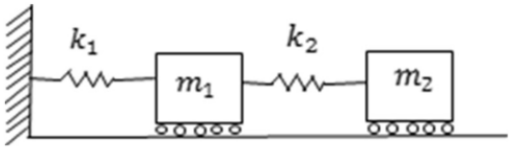

Fig. 0.

GATE NM 2022

Solution:

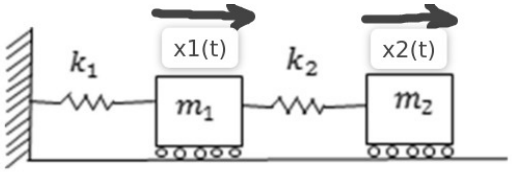

Fig. 0.

$$m_2\ddot{x_2}(t) + k_2\left(\ddot{x_2}(t) - \ddot{x_1}(t)\right) = 0 \quad (1)$$

$$m_1\ddot{x_1}(t) + k_2\left(\ddot{x_2}(t) - \ddot{x_1}(t)\right) + k_1\ddot{x_1}(t) = 0 \quad (2)$$

$$\ddot{x_1}(t) + x_2(t) = 0 \quad (3)$$

$$\ddot{x_2}(t) + 2\left(x_2(t) - x_1(t)\right) = 0 \quad (4)$$

Substituting (3) in (4)

$$\dddot{x_1}(t) + 2\ddot{x_1}(t) + x_1(t) = 0 \quad (5)$$

Applying Laplace transform on both sides of (5)

$$\mathcal{L}\left(\dddot{x_1}(t) + 2\ddot{x_1}(t) + x_1(t)\right) = 0 \quad (6)$$

$$X_1(s)\left(s^4 - s^3x_1(0) - s^2\dot{x_1}(0) - s\ddot{x_1}(0) - \dddot{x_1}(0)\right) + 2X_1(s)\left(s^2 - sx_1(0) - \dot{x_1}(0)\right) + X_1(s) = 0 \quad (7)$$

let $x_1(t)$ be constant at t=0

$$X_1(s)\left(s^4 - s^3x_1(0) - s^2\dot{x_1}(0)\right) + 2X_1(s)\left(s^2 - sx_1(0)\right) + X_1(s) = 0 \quad (8)$$

$$X_1(s)\left(s^4 - \left(s^3 + 2s\right)x_1(0) - s^2\left(\dot{x_1}(0) - 2\right) + 1\right) = 0 \quad (9)$$