GATE NM-54 2022

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Q: A system with two degrees of freedom, as shown in the figure, has masses $m_1 = 200kg$ and $m_2 = 100kg$ and stiffness coefficients $k_1 = k_2 = 200N/m$. Then the lowest natural frequency of the system is ___________ rad/s (rounded off to one decimal place).

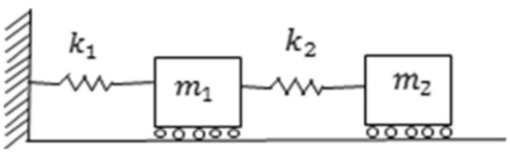

Fig. 0.

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Solution:

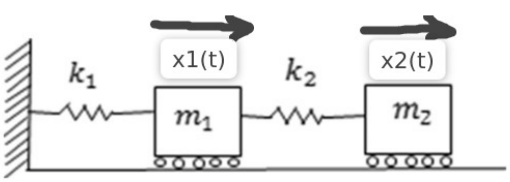

Fig. 0.

$$m_2\ddot{x}_2(t) + k_2\left(x_2(t) - x_1(t)\right) = 0 \quad (1)$$
$$m_1\ddot{x}_1(t) - k_2\left(x_2(t) - x_1(t)\right) + k_1x_1(t) = 0 \quad (2)$$

$$\ddot{x}_2(t) + 2\left(x_2(t) - x_1(t)\right) = 0 \quad (3)$$
$$\ddot{x}_1(t) + 2x_1(t) - x_2(t) = 0 \quad (4)$$

Substituting (3) in (4)

$$\ddddot{x}_2(t) + 4\ddot{x}_2(t) + 2x_2(t) = 0 \quad (5)$$

Applying Laplace transform on both sides of (5)

$$\mathcal{L}\left(\ddddot{x}_2(t) + 4\ddot{x}_2(t) + 2x_2(t)\right) = 0 \quad (6)$$

$$X_2(s)s^4 - s^3x_2(0) - s^2\dot{x}_2(0) - s\ddot{x}_2(0) - \dddot{x}_2(0) + 4\left(X_2(s)s^2 - sx_2(0) - \dot{x}_2(0)\right) + 2X_2(s) = 0 \quad (7)$$

$$X_2(s)s^4 - s^3x_2(0) - s^2\dot{x}_2(0) - s\ddot{x}_2(0) + 4\left(X_2(s)s^2 - sx_2(0) - \dot{x}_2(0)\right) + 2X_2(s) = 0 \quad (8)$$

$$X_2(s)\left(s^4 + 4s^2 + 2\right) - s^3x_2(0) - s^2\dot{x}_2(0) - s\ddot{x}_2(0) - 4sx_2(0) - 4\dot{x}_2(0) = 0 \quad (9)$$

Assuming $x_2(t) = 0$ and $\ddot{x}_2(t) = 0$ at t=0

$$X_2(s)\left(s^4 + 4s^2 + 2\right) - s^2\dot{x}_2(0) - 4\dot{x}_2(0) = 0 \quad (10)$$

$$X_2(s) = \dot{x}_2(0)\frac{\left(s^2 + 4\right)}{\left(s^4 + 4s^2 + 2\right)} \quad (11)$$

$$X_2(s) = \dot{x}_2(0)\frac{\left(s^2 + 4\right)}{\left(s^2 + (2 + \sqrt{2})\right)\left(s^2 + (2 - \sqrt{2})\right)} \quad (12)$$

$$\implies X_2(s) = \dot{x}_2(0)\left(\left(\frac{1 - \sqrt{2}}{2\left(s^2 + (2 + \sqrt{2})\right)}\right) + \left(\frac{1 + \sqrt{2}}{2\left(s^2 + (2 - \sqrt{2})\right)}\right) \quad (13)$$

$$\frac{a}{s^2 + a^2} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} sinat \quad (14)$$

Applying Inverse Laplace Transform on both sides of (13)

$$x_2(t) = \dot{x}_2(0)\left(\frac{\left(1 - \sqrt{2}\right)\sin(2 + \sqrt{2})t}{2\left((2 + \sqrt{2})\right)} + \frac{\left(1 + \sqrt{2}\right)\sin(2 - \sqrt{2})t}{2\left((2 - \sqrt{2})\right)} \quad (15)$$

$$= \frac{3}{\sqrt{2}}\left(-\cos 2t \sin \sqrt{2}t + \sqrt{2}\sin 2t cos \sqrt{2}t\right) \quad (16)$$