FILTER DESIGN ASSIGNMENT

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1 INTRODUCTION

We are supposed to design the equivalent FIR and IIR filter realizations for given filter number. This is a bandpass filter whose specifications are available below.

2 Filter Specifications

The sampling rate for the filter has been specified as $F_s = 48$ kHz. If the un-normalized discrete-time (natural) frequency is F, the corresponding normalized digital filter (angular) frequency is given by $\omega = 2\pi\left(\frac{F}{F_s}\right)$.

2.1 The Digital Filter

1. *Passband:* The passband of filter number j, is from {4 + 0.6(j)}kHz to {4 + 0.6(j+2)}kHz where

$$j = (r - 11000) \mod \sigma \tag{1}$$

where σ is sum of digits of roll number and r is roll number.

$$r = 11011 \tag{2}$$
$$\sigma = 4 \tag{3}$$
$$j = 3 \tag{4}$$

Substituting $j = 3$ gives the passband range for our bandpass filter as 5.8 kHz - 7 kHz. Hence, the un-normalized discrete time filter passband frequencies are $F_{p1} = 7$ kHz and $F_{p2} = 5.8$ kHz. The corresponding normalized digital filter passband frequencies are

$$\omega_{p1} = 2\pi\frac{F_{p1}}{F_s} = 0.29\pi \tag{5}$$

$$\omega_{p2} = 2\pi\frac{F_{p2}}{F_s} = 0.24\pi \tag{6}$$

The centre frequency is then given by

$$\omega_c = \frac{\omega_{p1} + \omega_{p2}}{2} = 0.265\pi. \tag{7}$$

2. *Tolerances:* The passband (δ_1) and stopband (δ_2) tolerances are given to be equal, so we let $\delta_1 = \delta_2 = \delta = 0.15$.

3. *Stopband:* The *transition band* for bandpass filters is $\Delta F = 0.3$ kHz on either side of the passband. Hence, the un-normalized *stopband* frequencies are

$$F_{s1} = 7 + 0.3 = 7.3kHz \tag{8}$$

$$F_{s2} = 5.8 - 0.3 = 5.5kHz. \tag{9}$$

The corresponding normalized frequencies are

$$\omega_{s1} = 0.3041\pi \tag{10}$$

$$\omega_{s2} = 0.2292\pi. \tag{11}$$

2.2 The Analog filter

In the bilinear transform, the analog filter frequency (Ω) is related to the corresponding digital filter frequency (ω) as

$$\Omega = \tan\frac{\omega}{2}. \tag{12}$$

Using this relation, we obtain the analog passband and stopband frequencies as $\Omega_{p1} = 0.4899$, $\Omega_{p2} = 0.3959$ and $\Omega_{s1} = 0.5177$, $\Omega_{s2} = 0.3764$ respectively.

3 The IIR Filter Design

Filter Type: We are supposed to design filters whose stopband is monotonic and passband equiripple. Hence, we use the *Chebyschev approximation* to design our bandpass IIR filter.

3.1 The Analog filter

1. *Low Pass Filter Specifications:* If $H_{a,BP}(j\Omega)$ be the desired analog band pass filter, with the specifications provided in Section 2.2 , and $H_{a,LP}(j\Omega_L)$ be the equivalent low pass filter, then

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{13}$$

where $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.4404$ and $B = \Omega_{p1} - \Omega_{p2} = 0.094$.
The low pass filter has the passband edge at $\Omega_{Lp} = 1$ and stopband edges at

$$\Omega_{Ls_1} = 1.5219 \tag{14}$$
$$\Omega_{Ls_2} = -1.4775 \tag{15}$$

We choose the stopband edge of the analog low pass filter as

$$\Omega_{Ls} = \min(|\Omega_{Ls_1}|, |\Omega_{Ls_2}|) = 1.4775. \tag{16}$$

2. *The Low Pass Chebyschev Filter Paramters:* The magnitude squared of the Chebyschev low pass filter is given by

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 c_N^2(\Omega_L/\Omega_{Lp})} \tag{17}$$

Since $\Omega_{Lp} = 1$, (17) may be rewritten as

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 c_N^2(\Omega_L)} \tag{18}$$

where

$$c_N(x) = \cosh(N \cosh^{-1} x) \tag{19}$$
$$c_0(x) = 1 \tag{20}$$
$$c_1(x) = x \tag{21}$$

and the integer N, which is the order of the filter, and ϵ are design paramters. There exists a recurssive relation from which all the polynomials can be found out.

$$c_{N+2} = 2xc_{N+1} - c_N \tag{22}$$

Imposing the band restrictions on (17)

$$|H_{a,LP}(j\Omega_L)|^2 < \delta_2 \text{ for } \Omega_L = \Omega_{Ls} \tag{23}$$
$$1 - \delta_1 < |H_{a,LP}(j\Omega_L)|^2 < 1 \text{ for } \Omega_L = \Omega_{Lp} \tag{24}$$

we obtain :

$$\frac{\sqrt{D_2}}{c_N(\Omega_{Ls})} \leq \epsilon \leq \sqrt{D_1},$$
$$N \geq \left\lceil \frac{\cosh^{-1} \sqrt{D_2/D_1}}{\cosh^{-1} \Omega_{Ls}} \right\rceil, \tag{25}$$

where $D_1 = \frac{1}{(1-\delta)^2} - 1$ and $D_2 = \frac{1}{\delta^2} - 1$ and $\lceil . \rceil$ is known as the ceiling operator.

Parameter	Value
D_1	0.384
D_2	43.44
N	4
$c_4(x)$	$8x^4 + 8x^2 + 1$

Table 2: Parameter Table

The below code plots (17) for different values of ϵ .

https://github.com/BATCHUISHITHA/EE−1205/blob/main/filterdesign/codes
/1.py

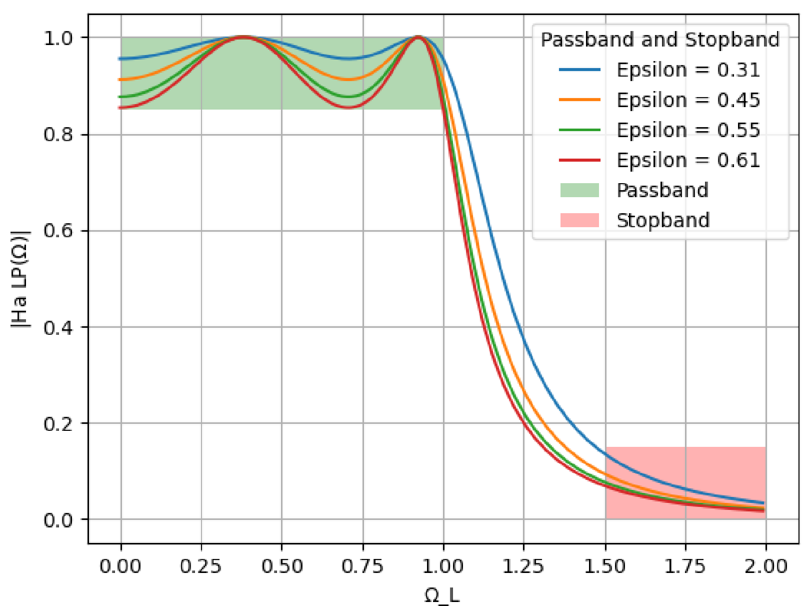

Figure 2: The Analog Low-Pass Frequency Response for $0.31 \leq \epsilon \leq 0.62$

In Fig. 2 we can observe the equiripple behaviour in passband and monotonic behaviour in stopband. As the value of ϵ increases the value of $|H_{a,LP}(j\Omega_L)|$ decreases.

3. *The Low Pass Chebyschev Filter:* Thus, we obtain

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + 0.16c_4^2(\Omega_L)} \tag{26}$$

where

$$c_4(x) = 8x^4 + 8x^2 + 1. \tag{27}$$

The poles of the frequency response in (17) lying in the left half plane are in general obtained as $r_1 \cos\phi_k + jr_2 \sin\phi_k$, where

$$\begin{aligned} \phi_k &= \frac{\pi}{2} + \frac{(2k+1)\pi}{2N}, k = 0, 1, \ldots, N-1 \\ r_1 &= \frac{\beta^2 - 1}{2\beta}, r_2 = \frac{\beta^2 + 1}{2\beta}, \beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{\frac{1}{N}} \end{aligned} \tag{28}$$

Thus, for N even, the low-pass stable Chebyschev filter, with a gain G has the form

$$H_{a,LP}(s_L) = \frac{G_{LP}}{\prod_{k=1}^{\frac{N}{2}-1}(s_L^2 - 2r_1 \cos\phi_k s_L + r_1^2 \cos^2\phi_k + r_2^2 \sin^2\phi_k)} \tag{29}$$

Substituting $N = 4$, $\epsilon = 0.5$ and $H_{a,LP}(j) = \frac{1}{\sqrt{1+\epsilon^2}}$, from (28) and (29), we obtain

$$H_{a,LP}(s_L) = \frac{0.3125}{s_L^4 + 1.1068s_L^3 + 1.6125s_L^2 + 0.9140s_L + 0.3366} \tag{30}$$

In Figure 2 we plot $|H(j\Omega)|$ using (26) and (30), thereby verifying that our low-pass Chebyschev filter design meets the specifications.