

# GATE NM-54 2022

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Q: A system with two degrees of freedom, as shown in the figure, has masses  $m_1 = 200\text{kg}$  and  $m_2 = 100\text{kg}$  and stiffness coefficients  $k_1 = k_2 = 200\text{N/m}$ . Then the lowest natural frequency of the system is \_\_\_\_\_ rad/s (rounded off to one decimal place).

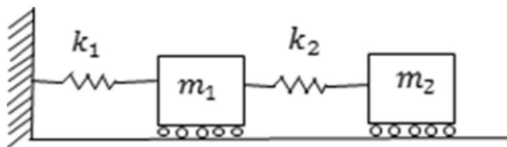


Fig. 0.

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**Solution:**

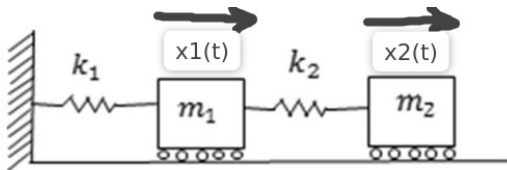


Fig. 0.

Variable	Description	Value
$m_1$	Mass of block 1	$200\text{kg}$
$m_2$	Mass of block 2	$100\text{kg}$
$k_1$	Stiffness coefficient of spring1	$200\text{N/m}$
$k_2$	Stiffness coefficient of spring2	$200\text{N/m}$
$x_i(t)$	Displacement of $i^{\text{th}}$ block	$x_i(t) = \cos(\omega t + \phi_i), x_i \geq 0$

TABLE 0  
INPUT PARAMETERS

$$m_1 \ddot{x}_1(t) - k_2 (x_2(t) - x_1(t)) + k_1 x_1(t) = 0 \quad (1)$$

$$m_2 \ddot{x}_2(t) + k_2 (x_2(t) - x_1(t)) = 0 \quad (2)$$

Writing (1) and (2) in matrix form:

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{pmatrix} + \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3)$$

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \quad (4)$$

$$x''(t) \xleftrightarrow{\mathcal{L}} s^2 X(s) - sx(0) - x'(0) \quad (5)$$

Applying Laplace transform for (1) and (2) assuming they are at their respective maximum displacement at  $t=0$

$$m_1 (s^2 X_1(s) - sx_1(0) - 0) - k_2 (X_2(s) - X_1(s)) + k_1 X_1(s) = 0 \quad (6)$$

$$m_2 (s^2 X_2(s) - sx_2(0) - 0) + k_2 (X_2(s) - X_1(s)) = 0 \quad (7)$$

Writing (6) and (7) in matrix form:

$$\begin{pmatrix} m_1 s^2 + (k_1 + k_2) & -k_2 \\ -k_2 & m_2 s^2 + k_2 \end{pmatrix} \begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = \begin{pmatrix} m_1 s x_1(0) \\ m_2 s x_2(0) \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = \frac{1}{m_1 m_2 s^4 + (k_1 + k_2) m_2 s^2 + k_2 m_1 s^2 + k_1 k_2} \quad (9)$$

$$\begin{pmatrix} m_2 s^2 + k_2 & k_2 \\ k_2 & m_1 s^2 + (k_1 + k_2) \end{pmatrix} \begin{pmatrix} m_1 s x_1(0) \\ m_2 s x_2(0) \end{pmatrix} \quad (10)$$

$$X_1(s) = \frac{m_1 m_2 s^3 x_1(0) + k_2 m_1 s x_1(0) + m_2 k_2 s x_2(0)}{m_1 m_2 s^4 + (k_1 + k_2) m_2 s^2 + k_2 m_1 s^2 + k_1 k_2} \quad (11)$$

$$X_2(s) = \frac{m_1 k_2 s x_1(0) + m_1 m_2 s^3 x_2(0) + (k_1 + k_2) m_2 s x_2(0)}{m_1 m_2 s^4 + (k_1 + k_2) m_2 s^2 + k_2 m_1 s^2 + k_1 k_2} \quad (12)$$

Considering denominator of  $X_i(s)$  for  $i = 1, 2$  (ie: characteristic equation of the system):

$$m_1 m_2 s^4 + (k_1 + k_2) m_2 s^2 + k_2 m_1 s^2 + k_1 k_2 = 0 \quad (13)$$

$$s^4 + 4s^2 + 2 = 0 \quad (14)$$

$$\Rightarrow s^2 = \frac{-4 \pm \sqrt{16 - 8}}{2} \quad (15)$$

$$= -2 \pm \sqrt{2} \quad (16)$$

$$\omega = \pm \sqrt{2 \mp \sqrt{2}} \quad (17)$$

$$\Rightarrow \omega_{least} = 0.765 rad/s \quad (18)$$