

GATE IN-13 2022

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Q: A periodic function $f(x)$, with period 2, is defined as

$$f(x) = \begin{cases} -1 - x & -1 \leq x < 0 \\ 1 - x & 0 < x \leq 1 \end{cases} \quad (1)$$

The Fourier series of this function contains

- A. Both $\cos(n\pi x)$ and $\sin(n\pi x)$ where $n=1,2,3\dots$
- B. Only $\sin(n\pi x)$ where $n=1,2,3\dots$
- C. Only $\cos(n\pi x)$ where $n=1,2,3\dots$
- D. Only $\cos(2n\pi x)$ where $n=1,2,3\dots$

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Solution:

Parameter	Description
$f(x)$	Polynomial function
$2L$	Period of the Polynomial function
$c(n)$	Complex Fourier Coefficients

TABLE 4
INPUT PARAMETERS

The complex exponential Fourier Series of $f(x)$ is,

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega x} \quad (2)$$

$$\Rightarrow c(n) = \frac{1}{2L} \int_{-L}^L f(x) e^{-jn\omega x} dx \quad (3)$$

$$(4)$$

For $n \neq 0$;

$$c(n) = \frac{1}{2} \int_{-1}^1 f(x) e^{-jn\omega x} dx \quad (5)$$

$$= \frac{1}{2} \left(\int_{-1}^0 (-1 - x) e^{-jn\omega x} dx + \int_0^1 (1 - x) e^{-jn\omega x} dx \right) \quad (6)$$

$$= \frac{1}{2} \left(- \int_{-1}^0 e^{-jn\omega x} dx - \int_{-1}^1 x e^{-jn\omega x} dx + \int_0^1 e^{-jn\omega x} dx \right) \quad (7)$$

$$= \frac{1}{2} \left[\frac{-1}{jn\omega} \left[- (1 - e^{+jn\omega}) + (e^{-jn\omega} - 1) \right] - \int_{-1}^1 x e^{-jn\omega x} dx \right] \quad (8)$$

$$= \frac{1}{2} \left[\frac{-1}{jn\omega} \left[-2 + e^{+jn\omega} + e^{-jn\omega} \right] + \left(\frac{e^{-jn\omega x}}{jn\omega} \left[x + \frac{1}{jn\omega} \right] \right) \Big|_{-1}^1 \right] \quad (9)$$

$$= \frac{-1}{jn\omega} [-1 + \cos(n\omega)] + \frac{1}{2(jn\omega)^2} \left[(e^{-jn\omega})(1 + jn\omega) - (e^{jn\omega})(-jn\omega + 1) \right] \quad (10)$$

$$\Rightarrow c(n) = \frac{j}{(n\omega)^2} [-n\omega + \sin(n\omega)] \quad (11)$$

For $n = 0$;

$$c(0) = \frac{1}{2} \int_{-1}^1 f(x) dx \quad (12)$$

$$= \frac{1}{2} \left[\int_{-1}^0 (-1-x) dx + \int_0^1 (1-x) dx \right] \quad (13)$$

$$= \frac{1}{2} \left[\left(-x - \frac{x^2}{2} \right)_{-1}^0 + \left(x - \frac{x^2}{2} \right)_{0}^1 \right] \quad (14)$$

$$= \frac{1}{2} \left[0 - 1 + \frac{1}{2} + 1 - \frac{1}{2} - 0 \right] \quad (15)$$

$$= 0 \quad (16)$$

\therefore The Fourier series of this function contains only $\sin(n\pi x)$ where $n=1,2,3,\dots$

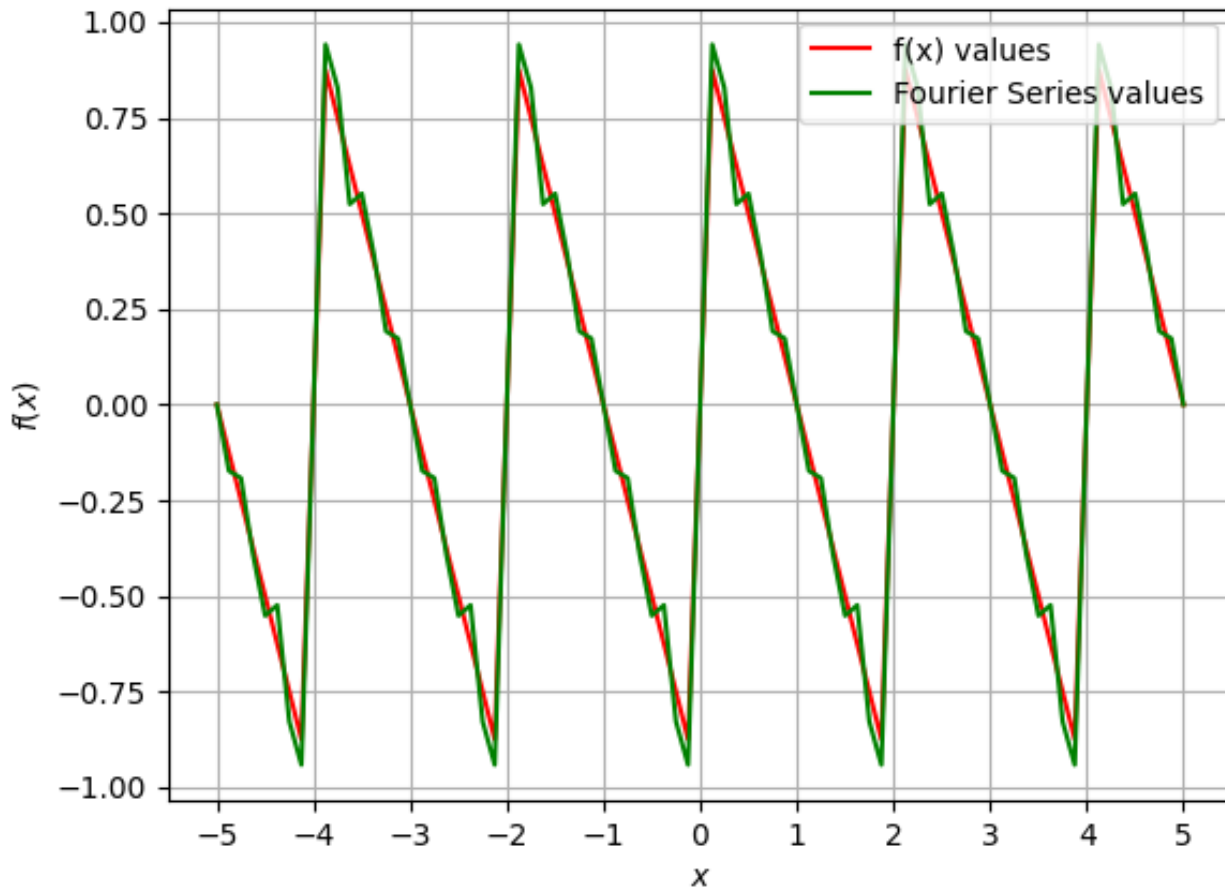


Fig. 4.