GATE NM-50 2022

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Q: Let $y(x)$ be the solution of the differential equation

$$y'' - 4y' - 12y = 3e^{5x}$$

satisfying $y(0) = \frac{18}{7}$ and $y'(0) = \frac{-1}{7}$.
Then $y(1)$ is _____ (rounded off to nearest integer). GATE NM 2022

Solution:

Parameter	Description	Value
$y'' - 4y' - 12y = 3e^{5x}$	Differential equation	none
$y(x)$	Solution of differential equation	$y(0) = \frac{18}{7}$
$y'(x)$	First order derivative of solution of differential equation	$y'(0) = \frac{-1}{7}$

TABLE 0
INPUT PARAMETERS

$$\mathcal{L}\left(y''(x)\right) = s^2Y(s) - sy(0) - y'(0) \tag{1}$$

$$\mathcal{L}\left(y'(x)\right) = sY(s) - y(0) \tag{2}$$

$$\mathcal{L}\left(y(x)\right) = Y(s) \tag{3}$$

$$\mathcal{L}\left(e^{ax}\right) = \frac{1}{s-a} \tag{4}$$

Applying Laplace transform on both sides of the given differential equation,

$$\mathcal{L}\left(y''(x) - 4y'(x) - 12y(x)\right) = \mathcal{L}\left(3e^{5x}\right) \tag{5}$$

$$\mathcal{L}\left(y''(x)\right) - \mathcal{L}\left(4y'(x)\right) - \mathcal{L}\left(12y(x)\right) = \mathcal{L}\left(3e^{5x}\right) \tag{6}$$

From (1), (2), (3), (4)

$$\left(s^2Y(s) - sy(0) - y'(0)\right) - 4\left(sY(s) - y(0)\right) - 12\left(Y(s)\right) = \frac{3}{s-5} \tag{7}$$

$$Y(s)\left(s^2 - 4s - 12\right) - y(0)\left(s-4\right) - y'(0) = \frac{3}{s-5} \tag{8}$$

$$Y(s)\left(s^2 - 4s - 12\right) - \frac{18\left(s-4\right)}{7} - \frac{-1}{7} = \frac{3}{(s-5)} \tag{9}$$

$$Y(s)\left(s^2 - 4s - 12\right) - \frac{(18s-73)}{7} = \frac{3}{(s-5)} \tag{10}$$

$$Y(s) = \frac{3}{(s-5)(s^2-4s-12)} + \frac{(18s-73)}{7(s^2-4s-12)} \tag{11}$$

$$= \frac{3}{8(s-6)} - \frac{3}{7(s-5)} + \frac{3}{56(s+2)} + \frac{5}{8(s-6)} + \frac{109}{56(s+2)} \tag{12}$$

$$\implies Y(s) = \frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{1}{(s+2)} \tag{13}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s-a)}\right) = e^{ax} \tag{14}$$

Now finding Inverse Laplace Transform of Y(s), From (14)

$$y(x) = \mathcal{L}^{-1}\left(\frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{2}{(s+2)}\right) \tag{15}$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s-6}\right) - \mathcal{L}^{-1}\left(\frac{3}{7(s-5)}\right) + \mathcal{L}^{-1}\left(\frac{2}{s+2}\right) \tag{16}$$

$$\implies y(x) = e^{6x} - \frac{3}{7}e^{5x} + 2e^{-2x} \tag{17}$$

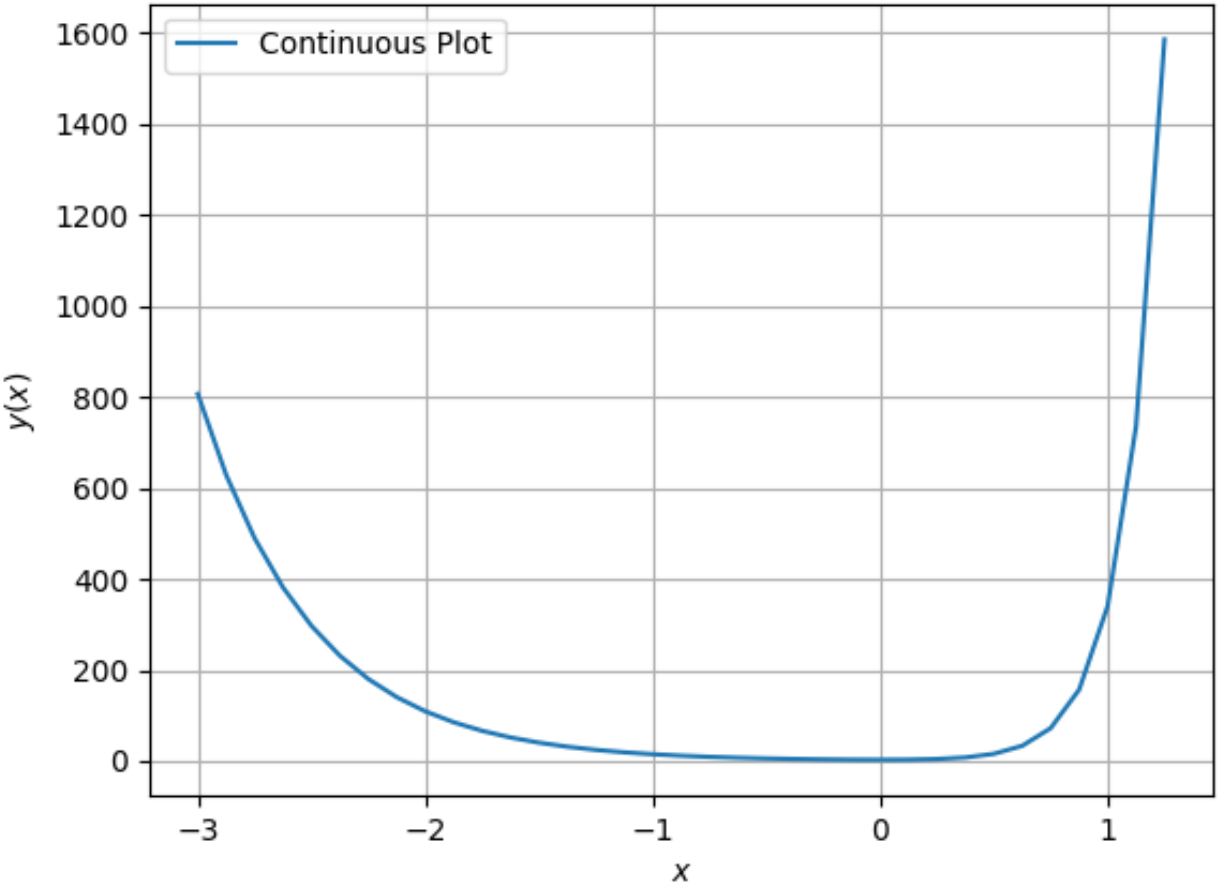
Continuous Plot
1600
1400
1200
1000
800
600
400
200
0
y(x)
−3
−2
−1
0
1
x

Fig. 0.