

GATE NM-54 2022

EE23BTECH11011- Batchu Ishitha*

Q: A system with two degrees of freedom, as shown in the figure, has masses $m_1 = 200\text{kg}$ and $m_2 = 100\text{kg}$ and stiffness coefficients $k_1 = k_2 = 200\text{N/m}$. Then the lowest natural frequency of the system is _____ rad/s (rounded off to one decimal place).

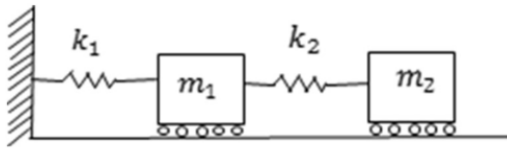


Fig. 0.

GATE NM 2022

Solution:

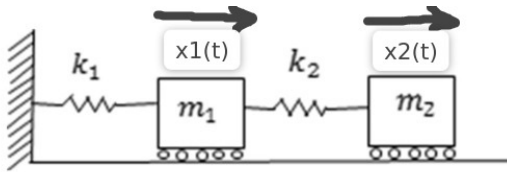


Fig. 0.

$$m_2 \ddot{x}_2(t) + k_2 (x_2(t) - x_1(t)) = 0 \quad (1)$$

$$m_1 \ddot{x}_1(t) - k_2 (x_2(t) - x_1(t)) + k_1 x_1(t) = 0 \quad (2)$$

$$\ddot{x}_2(t) + 2(x_2(t) - x_1(t)) = 0 \quad (3)$$

$$\ddot{x}_1(t) + 2x_1(t) - x_2(t) = 0 \quad (4)$$

Substituting (3) in (4)

$$\ddot{x}_2(t) + 4\ddot{x}_2(t) + 2x_2(t) = 0 \quad (5)$$

Applying Laplace transform on both sides of (5)

$$\mathcal{L}(\ddot{x}_2(t) + 4\ddot{x}_2(t) + 2x_2(t)) = 0 \quad (6)$$

$$X_2(s) \left(s^4 - s^3 x_2(0) - s^2 \dot{x}_2(0) - s \ddot{x}_2(0) - \ddot{x}_2(0) \right) + 4X_2(s) \left(s^2 - s x_2(0) - \dot{x}_2(0) \right) + 2X_2(s) = 0 \quad (7)$$

$$X_2(s) \left(s^4 - s^3 x_2(0) - s^2 \dot{x}_2(0) \right) +$$

$$4X_2(s) \left(s^2 - s x_1(0) \right) + 2X_2(s) = 0 \quad (8)$$

$$X_2(s) \left(s^4 - (s^3 + 4s) x_2(0) - s^2 (\dot{x}_2(0) - 4) + 2 \right) = 0 \quad (9)$$

let $x_2(t)$ be constant at $t=0$

$$\Rightarrow X_2(s)(s^4 + 2) = 0 \quad (10)$$