EE1204 ASSIGNMENT-1

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SOFTWARE USED:
MATHEMATICA FOR QUESTION 1 AND 2; SIM SCALE FOR QUESTION 3.

1: A field is given in spherical co-ordinates: $[5 points]$

$$\overrightarrow{F} = \frac{1}{r^2}\cos\phi \hat{\mathrm{r}} + \frac{\sin\phi}{r^2\sin\theta}\hat{\phi}$$

(1.1) Plot (a)$\left|\overrightarrow{F}\right|$ vs ϕ for $r = 0.8$ (b) $\left|\overrightarrow{F}\right|$ vs r for $\phi = 45°$

(1.2) Calculate with computer , verify with hand calculations (a)$\overrightarrow{\nabla}.\overrightarrow{F}$ (b)$\overrightarrow{\nabla} X \overrightarrow{F}$ (c)$\overrightarrow{\nabla}\left(\overrightarrow{\nabla}.\overrightarrow{F}\right)$

(1.3) Check if $\overrightarrow{F}$ is a valid electrostatic field using two methods.

Solution:
(1.1)(a)

```
(*Define the field components*)
f[r_, θ_, ϕ_] := {1 / r^2 Cos[ϕ], 0, Sin[ϕ] / (r^2 Sin[θ])};

(*Compute the norm of f*)
normF[r_, θ_, ϕ_] := Sqrt[f[r, θ, ϕ].f[r, θ, ϕ]];

(*Set the constant value for r*)
rValue = 0.8;

(*Create the contour plot*)
ContourPlot[normF[rValue, θ, ϕ], {θ, 0, π}, {ϕ, 0, 2 π}, Contours → 20,
 (*Number of contour levels*)ColorFunction → "Rainbow", (*Choose a color scheme*)
 FrameLabel → {"θ", "ϕ"}, (*Axis labels*)PlotLegends → Automatic (*Show a legend*)]
```

Fig. 0.

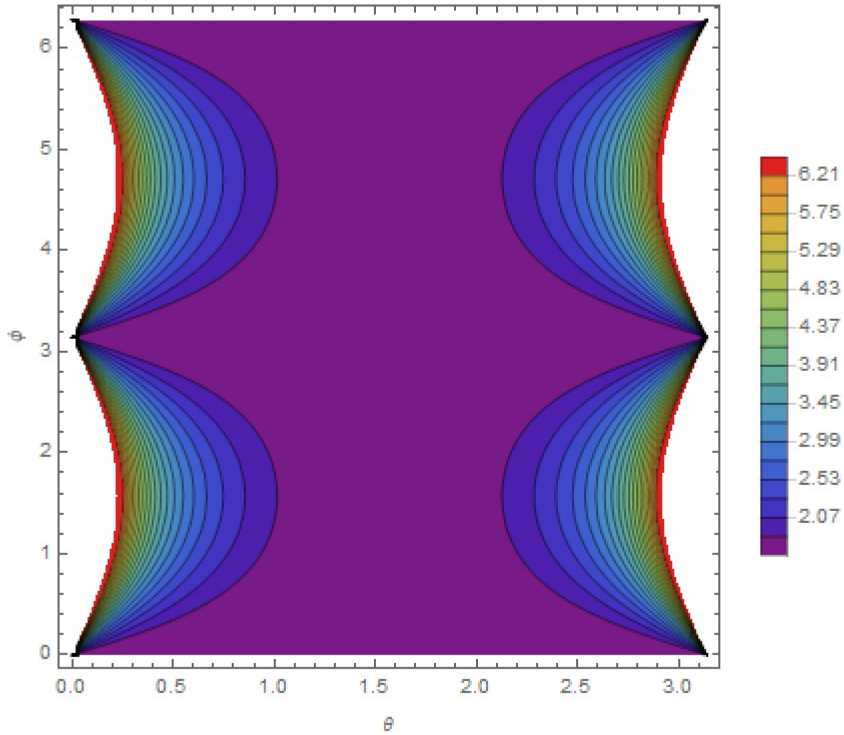


Fig. 0.

(b)

```
(*Define the field components*)f[r_, θ_, ϕ_] := {1 / r^2 Cos[ϕ], 0, Sin[ϕ] / (r^2 Sin[θ])};

(*Compute the norm of f*)
normF[r_, θ_, ϕ_] := Sqrt[f[r, θ, ϕ].f[r, θ, ϕ]];

(*Create the contour plot*)
ContourPlot[normF[r, θ, 45 Degree], {θ, 0, π}, {r, 0.1, 2}, Contours → 20,
 (*Number of contour levels*)ColorFunction → "Rainbow", (*Choose a color scheme*)
 FrameLabel → {"θ", "r"}, (*Axis labels*)PlotLegends → Automatic (*Show a legend*)]
```

Fig. 0.

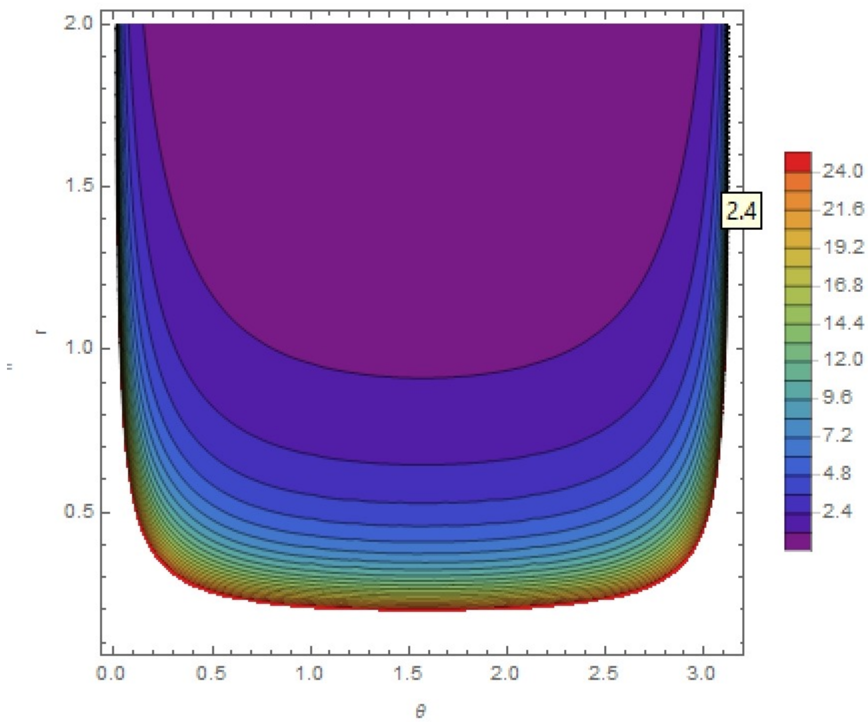


Fig. 0.

(1.2) (a)

```
F = {1 / r^2 Cos[ϕ], 0, Sin[ϕ] / (r^2 Sin[θ])};

divergence = Div[F, {r, θ, ϕ}, "Spherical"];

Simplify[divergence]
```

$$\frac{\mathrm{Cos}[\phi]\,\mathrm{Csc}[\theta]^2}{r^3}$$

Fig. 0.

(a) HAND CALCULATION:

$$\vec{\nabla}.\vec{F} = \left(\frac{1}{r^2}\frac{\partial r^2}{\partial r}\hat{r} + \frac{1}{r\sin\theta}\frac{\partial \sin\theta}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}\right).\left(\frac{1}{r^2}\cos\phi\hat{r} + \frac{\sin\phi}{r^2\sin\theta}\hat{\phi}\right) \tag{1}$$

$$= \frac{1}{r^2}\frac{\partial\left(r^2\frac{\cos\phi}{r^2}\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\sin\theta(0)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\left(\frac{\sin\phi}{r^2\sin\theta}\right)}{\partial\phi} \tag{2}$$

$$= 0 + 0 + \frac{\cos\phi}{r^3\sin^2\theta} \tag{3}$$

$$\implies \vec{\nabla}.\vec{F} = \frac{\cos\phi}{r^3\sin^2\theta} \tag{4}$$

(b)

```
F = {1 / r^2 Cos[ϕ], 0, Sin[ϕ] / (r^2 Sin[θ])};

curl = Curl[F, {r, θ, ϕ}, "Spherical"];

Simplify[curl]

= {0, 0, 0}
```

Fig. 0.

HAND CALCULATION:

$$\overrightarrow{\nabla} X \overrightarrow{F} = \frac{1}{r^2 \sin\theta} det \begin{pmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{1}{r^2}\cos\phi & r.0 & r\sin\theta\frac{\sin\phi}{r^2\sin\theta} \end{pmatrix} \tag{5}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{r}\left(\frac{\partial\left(\frac{\sin\phi}{r}\right)}{\partial\theta} - \frac{\partial(0)}{\partial\phi}\right) - r\hat{\theta}\left(\frac{\partial\left(\frac{\sin\phi}{r}\right)}{\partial r} - \frac{\partial\left(\frac{\cos\phi}{r^2}\right)}{\partial\phi}\right) + r\sin\theta\hat{\phi}\left(\frac{\partial(0)}{\partial r} - \frac{\partial\left(\frac{\cos\phi}{r^2}\right)}{\partial\theta}\right)\right] \tag{6}$$

$$= r^2\sin\theta\left[(0-0)\hat{r} - \left(\frac{-\sin\phi}{r^2} - \frac{-\sin\phi}{r^2}\right)r\hat{\theta} + (0-0)r\sin\theta\hat{\phi}\right] \tag{7}$$

$$= 0\hat{r} + 0\hat{\theta} + 0\hat{\phi} \tag{8}$$

(c)

```
F = {1 / r^2 Cos[ϕ], 0, Sin[ϕ] / (r^2 Sin[θ])};

divergence = Div[F, {r, θ, ϕ}, "Spherical"];

gradient = Grad[divergence, {r, θ, ϕ}, "Spherical"];

Simplify[gradient]

{-(3 Cos[ϕ] Csc[θ]^2)/r^4, -(2 Cos[ϕ] Cot[θ] Csc[θ]^2)/r^4, -(Csc[θ]^3 Sin[ϕ])/r^4}
```

Fig. 0.

HAND CALCULATION:

$$\overrightarrow{\nabla}\left(\overrightarrow{\nabla}.\overrightarrow{F}\right) = \overrightarrow{\nabla}\left(\frac{\cos\phi}{r^3\sin^2\theta}\right) \tag{9}$$

$$= \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}\right)\left(\frac{\cos\phi}{r^3\sin^2\theta}\right) \tag{10}$$

$$= \left(\frac{-3\cos\phi}{r^4\sin^2\theta}\right)\hat{r} - \left(\frac{2\cos\phi\cot\theta}{r^4\sin^2\theta}\right)\hat{\theta} + \left(-\frac{\sin\phi}{r^4\sin^3\theta}\right)\hat{\phi} \tag{11}$$

(1.3)
METHOD-1: CURL OF $\overrightarrow{F}$:
If the curl of the electrostatic field is zero, it indicates that electrostatic field is conservative, ie: $\overrightarrow{F}$ is a valid electrostatic field.

$$\overrightarrow{\nabla}X\overrightarrow{F} = \frac{1}{r^2\sin\theta}det\begin{pmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{1}{r^2}\cos\phi & r.0 & r\sin\theta\frac{\sin\phi}{r^2\sin\theta} \end{pmatrix} \tag{12}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{r}\left(\frac{\partial\left(\frac{\sin\phi}{r}\right)}{\partial\theta} - \frac{\partial(0)}{\partial\phi}\right) - r\hat{\theta}\left(\frac{\partial\left(\frac{\sin\phi}{r}\right)}{\partial r} - \frac{\partial\left(\frac{\cos\phi}{r^2}\right)}{\partial\phi}\right) + r\sin\theta\hat{\phi}\left(\frac{\partial(0)}{\partial r} - \frac{\partial\left(\frac{\cos\phi}{r^2}\right)}{\partial\theta}\right)\right] \tag{13}$$

$$= r^2\sin\theta\left[(0-0)\hat{r} - \left(\frac{-\sin\phi}{r^2} - \frac{-\sin\phi}{r^2}\right)r\hat{\theta} + (0-0)r\sin\theta\hat{\phi}\right] \tag{14}$$

$$= 0\hat{r} + 0\hat{\theta} + 0\hat{\phi} \tag{15}$$

Therefore, $\overrightarrow{F}$ is a valid electrostatic field.
METHOD-2:STOKE'S THEOREM:

$$\iint_S (\nabla X\overrightarrow{F})\cdot d\mathbf{a} = \oint_C \overrightarrow{F}\cdot d\mathbf{l}$$

From the above equation , since curl of $\overrightarrow{F}$ is zero,it is conservative in nature,ie: it is a valid electrostatic field.

2: Consider the charge configuration :[$5points$]

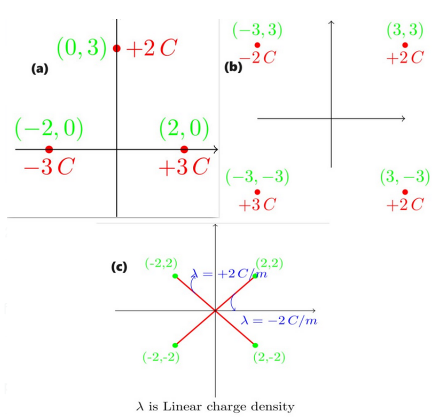


Fig. 0.

(2.1) Plot for all the charge configurations for $(x, y) = (\pm 6, \pm 6)$ (a) $\overrightarrow{E}$ (b) V (c) U(Electrostatic Energy)
(2.2) Verify through graphical visualisation/representation (a) GAUSS'S LAW (b)$\overrightarrow{E} = \nabla V$
(c) $\overrightarrow{E}$ is conservative
(2.3) Calculate the force acting on unit charge sitting in origin due to charge configurations.

Solution:
(2.1.(a))
(a)

```
(*Define Coulomb's constant*)k = 8.99 * 10^9;

(*Define the electric field due to the ensemble of charges*)
parametricField[q1_, q2_, q3_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}][x_, y_] := Module[{r1, r2, r3}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  {(k * q1 (x - x1)) / r1^(3 / 2) + k * (q2 (x - x2)) / r2^(3 / 2) + k * (q3 (x - x3)) / r3^(3 / 2), (k * q1 (y - y1)) / r1^(3 / 2) + (k * q2 (y - y2)) / r2^(3 / 2) + (k * q3 (y - y3)) / r3^(3 / 2)}]

(*Specify parameters for the charges*)
specificField[x_, y_] = parametricField[2, -3, 3, {0, 3}, {-2, 0}, {2, 0}][x, y];

(*Create the stream plot*)
StreamPlot[specificField[x, y], {x, -6, 6}, {y, -6, 6}, VectorStyle → "Arrow", VectorScale → {0.03, Automatic}, AspectRatio → Automatic, FrameLabel → {"x", "y"}]
```

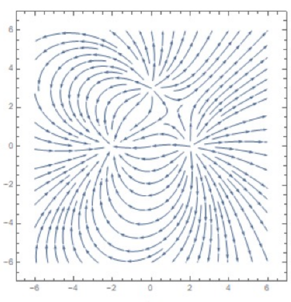


Fig. 0.

(b)

```
(*Define Coulomb's constant*)k = 8.99 * 10^9;

(*Define the electric field due to the ensemble of charges*)
parametricField[q1_, q2_, q3_, q4_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}, {x4_, y4_}][x_, y_] := Module[{r1, r2, r3, r4}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  r4 = (x - x4)^2 + (y - y4)^2;
  {(k * q1 (x - x1)) / r1^(3 / 2) + k * (q2 (x - x2)) / r2^(3 / 2) + k * (q3 (x - x3)) / r3^(3 / 2) + k * (q4 (x - x4)) / r4^(3 / 2),
   (k * q1 (y - y1)) / r1^(3 / 2) + k * (q2 (y - y2)) / r2^(3 / 2) + k * (q3 (y - y3)) / r3^(3 / 2) + k * (q4 (y - y4)) / r4^(3 / 2)}]

(*Specify parameters for the charges*)
specificField[x_, y_] = parametricField[-2, 2, 2, 3, {-3, 3}, {3, 3}, {3, -3}, {-3, -3}][x, y];

(*Create the stream plot*)
StreamPlot[specificField[x, y], {x, -6, 6}, {y, -6, 6}, VectorStyle → "Arrow", VectorScale → {0.03, Automatic}, AspectRatio → Automatic, FrameLabel → {"x", "y"}]
```

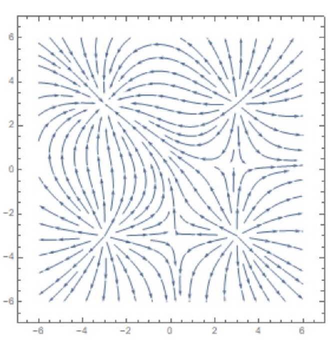


Fig. 0.

(c)

```
k = 8.99 * 10 ^9;

λ1 = 2;
λ2 = -2;

potential1[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2) ^2 + (y + 2) ^2];
   r2 = Sqrt[(x + 2) ^2 + (y - 2) ^2];
   d = (x + y) / Sqrt[2];
   k * λ1 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential2[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2) ^2 + (y - 2) ^2];
   r2 = Sqrt[(x + 2) ^2 + (y + 2) ^2];
   d = (x - y) / Sqrt[2];
   k * λ2 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential[x_, y_] := potential1[x, y] + potential2[x, y];

negGrad[x_, y_] := Evaluate[-D[potential[x, y], {{x, y}}]];

VectorPlot[negGrad[x, y], {x, -6, 6}, {y, -6, 6}, VectorStyle → "Arrow", VectorPoints → Fine, VectorScale → Automatic, PlotRange → All, AspectRatio → Automatic, FrameLabel → {"x", "y"}, AxesLabel → Automatic, ImageSize → Large, Epilog → {Red, PointSize[Large]}]
```

Fig. 0.

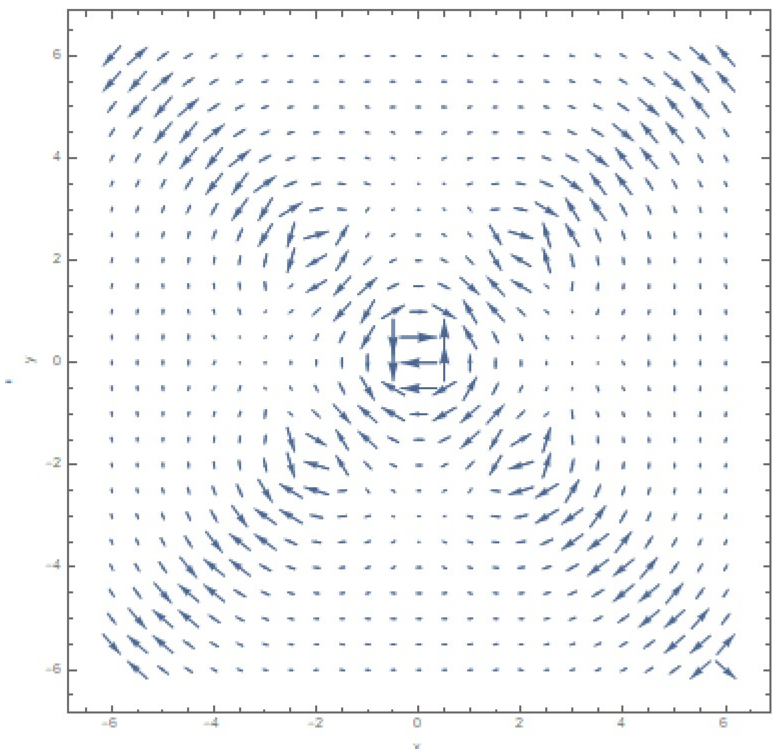


Fig. 0.

(2.1.(b))
(a)

```
(*Define Coulomb's constant*)k = 8.99*10^9;

(*Define the electric potential due to the ensemble of charges*)
parametricPotential[q1_, q2_, q3_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}][x_, y_] := Module[{r1, r2, r3}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  {k*q1/Sqrt[r1] + k*q2/Sqrt[r2] + k*q3/Sqrt[r3], k*q1/Sqrt[r1] + k*q2/Sqrt[r2] + k*q3/Sqrt[r3]}]

(*Specify parameters for the charges*)
specificPotential[x_, y_] = parametricPotential[2, -3, 3, {0, 3}, {-2, 0}, {2, 0}][x, y];

(*Create the contour plot*)
ContourPlot[specificPotential[x, y], {x, -6, 6}, {y, -6, 6}, ColorFunction → "Rainbow", Contours → 20, ContourLabels → False, PlotLegends → Automatic,
 AspectRatio → Automatic, FrameLabel → {"x", "y"}, (*Add frame labels*)AxesLabel → Automatic (*Add axes labels*)]
```

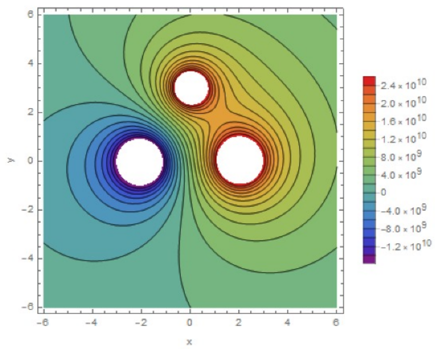


Fig. 0.

(b)

```
(*Define Coulomb's constant*)k = 8.99*10^9;

(*Define the electric potential due to the ensemble of charges*)
parametricPotential[q1_, q2_, q3_, q4_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}, {x4_, y4_}][x_, y_] := Module[{r1, r2, r3, r4}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  r4 = (x - x4)^2 + (y - y4)^2;
  {k*q1/Sqrt[r1] + k*q2/Sqrt[r2] + k*q3/Sqrt[r3] + k*q4/Sqrt[r4], k*q1/Sqrt[r1] + k*q2/Sqrt[r2] + k*q3/Sqrt[r3] + k*q4/Sqrt[r4]}]

(*Specify parameters for the charges*)
specificPotential[x_, y_] = parametricPotential[-2, 2, 2, 3, {-3, 3}, {3, 3}, {3, -3}, {-3, -3}][x, y];

(*Create the contour plot*)
ContourPlot[specificPotential[x, y], {x, -6, 6}, {y, -6, 6}, ColorFunction → "Rainbow", Contours → 20, ContourLabels → False, PlotLegends → Automatic,
 AspectRatio → Automatic, FrameLabel → {"x", "y"}, (*Add frame labels*)AxesLabel → Automatic (*Add axes labels*)]
```

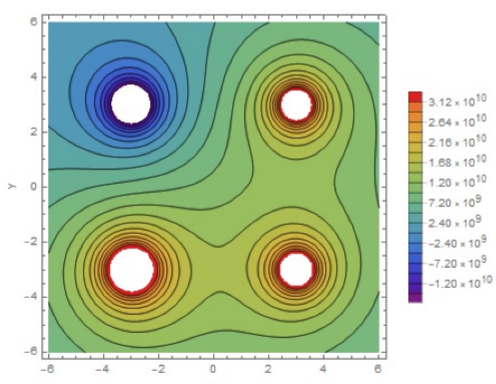


Fig. 0.

(c)

```
k = 9 * 10^9;
λ1 = 2;
λ2 = -2;

potential1[x_, y_] := Module[{r1, r2, d},
   r1 = Sqrt[(x - 2)^2 + (y + 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y - 2)^2];
   d = (x + y) / Sqrt[2];
   k * λ1 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential2[x_, y_] := Module[{r1, r2, d},
   r1 = Sqrt[(x - 2)^2 + (y - 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y + 2)^2];
   d = (x - y) / Sqrt[2];
   k * λ2 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential[x_, y_] := potential1[x, y] + potential2[x, y];

ContourPlot[potential[x, y], {x, -6, 6}, {y, -6, 6}, ColorFunction → "Rainbow", Contours → 20, ContourLabels → False, PlotRange → All,
 PlotLegends → Automatic, AspectRatio → Automatic, FrameLabel → {"x", "y"}, AxesLabel → Automatic]
```

Fig. 0.

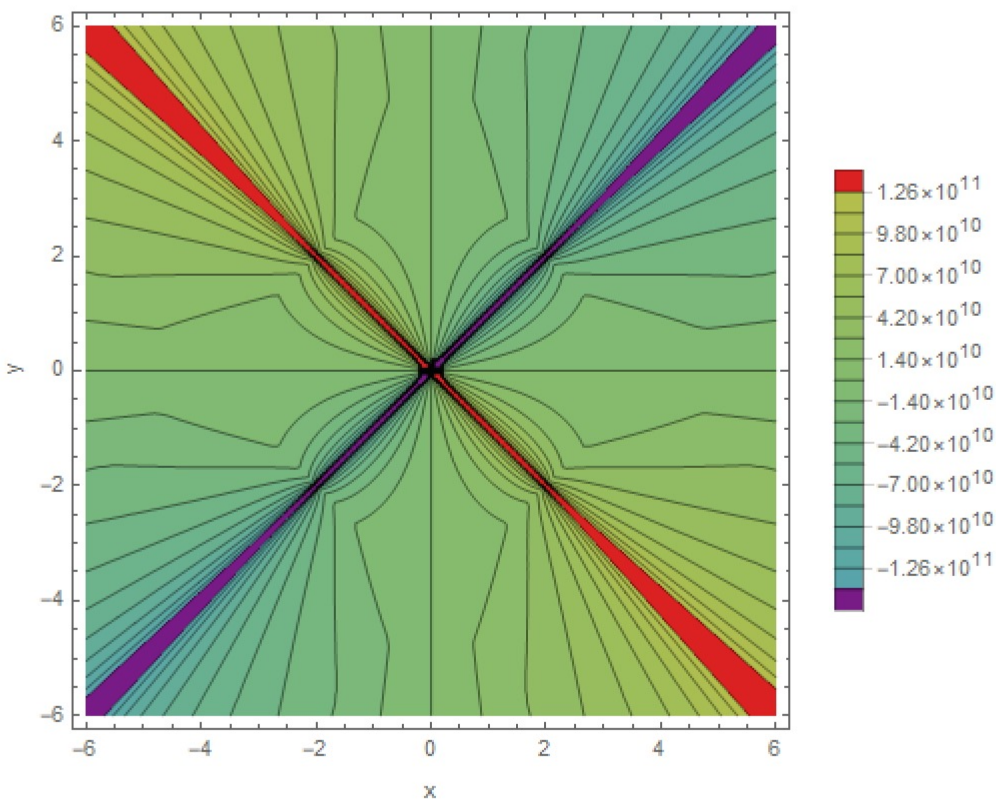


Fig. 0.

(2.1.(c))
(a)

```
(*Define the positions and charges*)r = {{0, 3}, {2, 0}, {-2, 0}};
q = {2, 3, -3};
k = 8.99*10^9;
(*Coulomb's constant in N m^2/C^2*)(*Function to calculate distance between two points*)
distance[c1_, c2_] := Sqrt[(c1[[1]] - c2[[1]])^2 + (c1[[2]] - c2[[2]])^2];

(*Function to calculate electrostatic energy*)
calculateElectrostaticEnergy[r_, q_, k_] :=
  Module[{U}, U = k Sum[(q[[i]] q[[j]])/distance[r[[i]], r[[j]]], {i, 1, Length[r]}, {j, i+1, Length[r]}];
   U];

(*Create a grid of (x,y) points within the boundary*)
gridPoints = Flatten[Table[{x, y}, {x, -6, 6, 0.1}, {y, -6, 6, 0.1}], 1];

(*Calculate electrostatic energy for each grid point*)
energies = Map[{#[[1]], #[[2]], calculateElectrostaticEnergy[r, q, k]} &, gridPoints];

(*Plot the electrostatic energy in 3D with the "Rainbow" color scheme*)
plot = ListPlot3D[energies, PlotRange → All, Mesh → None, ColorFunction → (ColorData["Rainbow"][Rescale[#, {-1*10^11, 1*10^11}]] &),
   AxesLabel → {"x", "y", "Energy"}, PlotLegends → BarLegend[{"Rainbow", {-1*10^11, 1*10^11}}, LegendLabel → "Energy"]];

(*Show the plot with the scale bar*)
plot
```

Fig. 0.

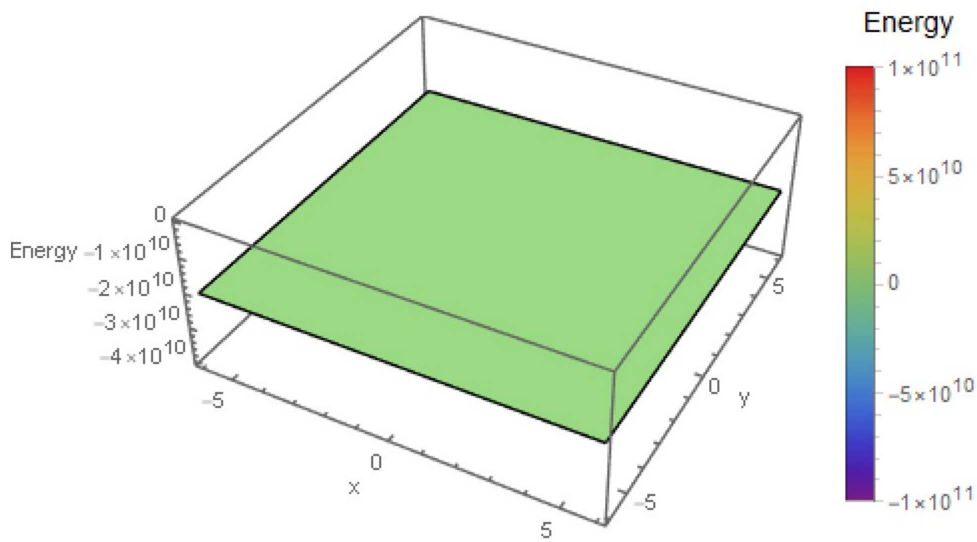


Fig. 0.

(b)

```
(*Define the positions and charges for the four-charge system*)r = {{-3, 3}, {3, 3}, {-3, -3}, {3, -3}};
q = {-2, 2, 3, 2};
(*Charges in Coulombs*)k = 8.99*10^9;
(*Coulomb's constant in N m^2/C^2*)(*Function to calculate distance between two points*)
distance[c1_, c2_] := Sqrt[(c1[[1]] - c2[[1]])^2 + (c1[[2]] - c2[[2]])^2];

(*Function to calculate electrostatic energy*)
calculateElectrostaticEnergy[r_, q_, k_] :=
  Module[{U}, U = k Sum[(q[[i]] q[[j]]) / distance[r[[i]], r[[j]]], {i, 1, Length[r]}, {j, i + 1, Length[r]}];
   U];

(*Create a grid of (x,y) points within the boundary*)
gridPoints = Flatten[Table[{x, y}, {x, -6, 6, 0.1}, {y, -6, 6, 0.1}], 1];

(*Calculate electrostatic energy for each grid point*)
energies = Map[{#[[1]], #[[2]], calculateElectrostaticEnergy[r, q, k]} &, gridPoints];

(*Plot the electrostatic energy in 3D with the "Rainbow" color scheme*)
plot = ListPlot3D[energies, PlotRange → All, Mesh → None, ColorFunction → (ColorData["Rainbow"][Rescale[#, {-1*10^11, 1*10^11}]] &),
   AxesLabel → {"x", "y", "Energy"}, PlotLegends → BarLegend[{"Rainbow", {-1*10^11, 1*10^11}}, LegendLabel → "Energy"]];

(*Show the plot with the scale bar*)
plot
```

Fig. 0.

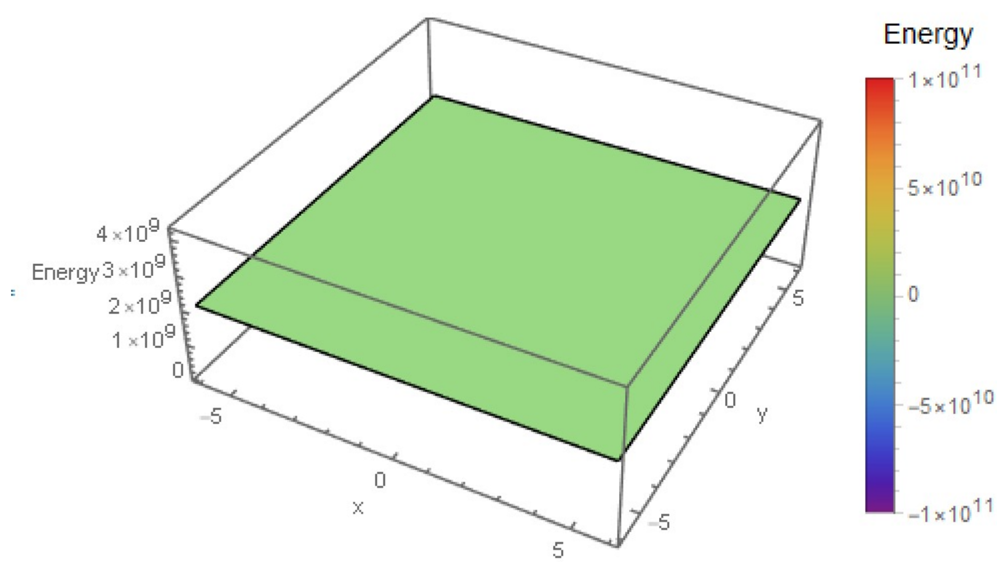
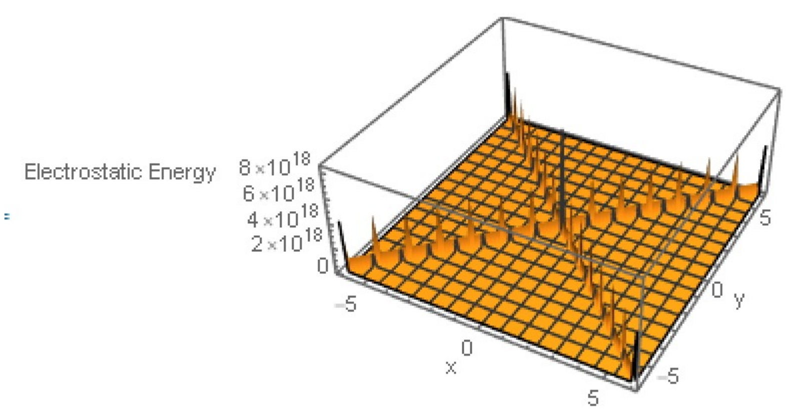


Fig. 0.

(c)

```
k = 8.99 * 10^9;
ϵ0 = 8.85 * 10^(-12);
λ1 = 2;
λ2 = -2;

potential1[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y + 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y - 2)^2];
   d = (x + y) / Sqrt[2];
   k * λ1 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential2[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y - 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y + 2)^2];
   d = (x - y) / Sqrt[2];
   k * λ2 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential[x_, y_] := potential1[x, y] + potential2[x, y];
negGrad[x_, y_] := Evaluate[-D[potential[x, y], {{x, y}}]];

Fx[x_, y_] := Evaluate[negGrad[x, y][[1]]];
Fy[x_, y_] := Evaluate[negGrad[x, y][[2]]];

(*Define the integrand for the electrostatic energy*)
integrand[x_, y_] := (ϵ0 / 2) * (Fx[x, y]^2 + Fy[x, y]^2);

(*Define the range for plotting*)
xmin = -6;
xmax = 6;
ymin = -6;
ymax = 6;

(*Generate the 3D plot of the electrostatic energy*)
Plot3D[integrand[x, y], {x, xmin, xmax}, {y, ymin, ymax}, PlotRange → All, AxesLabel → {"x", "y", "Electrostatic Energy"}]

(*Print the result*)
Print["Electrostatic energy:", energy]
```

Fig. 0.



Electrostatic energy:4.7856×10^{17}

Fig. 0.

(2.2.(a))
(2.2.(b))
(a)

```
(*Define Coulomb's constant*)k = 8.99 * 10 ^9;
(*Define the electric potential due to the ensemble of charges*)
parametricPotential[q1_, q2_, q3_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}][x_, y_] := Module[{r1, r2, r3}, r1 = (x - x1) ^2 + (y - y1) ^2;
  r2 = (x - x2) ^2 + (y - y2) ^2;
  r3 = (x - x3) ^2 + (y - y3) ^2;
  {k * q1 / Sqrt[r1] + k * q2 / Sqrt[r2] + k * q3 / Sqrt[r3], k * q1 / Sqrt[r1] + k * q2 / Sqrt[r2] + k * q3 / Sqrt[r3]}]
(*Specify parameters for the charges*)
specificPotential[x_, y_] := parametricPotential[2, -3, 3, {0, 3}, {-2, 0}, {2, 0}][x, y][[1]]
(*Define the gradient function*)
grad[x_, y_] := Evaluate[D[specificPotential[x, y], {{x, y}}]]
(*Plot the streamlines using StreamPlot with arrows*)
StreamPlot[grad[x, y], {x, -6, 6}, {y, -6, 6}, StreamStyle → "Arrow", StreamPoints → Fine, StreamScale → Automatic, PlotRange → All, AspectRatio → Automatic, FrameLabel → {"x", "y"}, AxesLabel → Automatic, ImageSize → Large]
```

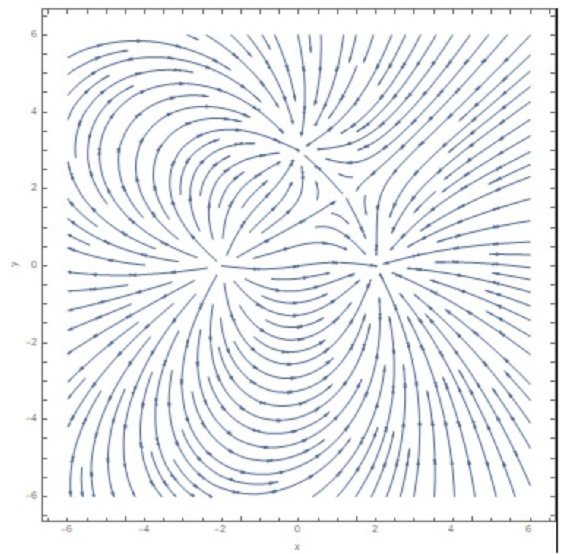


Fig. 0.

hence, $\overrightarrow{E} = -\nabla\overrightarrow{V}$

(b)

```
(*Define Coulomb's constant*)k = 8.99*10^9;
(*Define the electric potential due to the ensemble of charges*)
parametricPotential[q1_, q2_, q3_, q4_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}, {x4_, y4_}][x_, y_] := Module[{r1, r2, r3, r4}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  r4 = (x - x4)^2 + (y - y4)^2;
  {k*q1/Sqrt[r1] + k*q2/Sqrt[r2] + k*q3/Sqrt[r3] + k*q4/Sqrt[r4], k*q1/Sqrt[r1] + k*q2/Sqrt[r2] + k*q3/Sqrt[r3] + k*q4/Sqrt[r4]}]
(*Specify parameters for the charges*)
specificPotential[x_, y_] := parametricPotential[-2, 2, 2, 3, {-3, 3}, {3, 3}, {3, -3}, {-3, -3}][x, y][[1]]
(*Define the gradient function*)
grad[x_, y_] := Evaluate[D[specificPotential[x, y], {{x, y}}]]
(*Plot the streamlines using StreamPlot with arrows*)
StreamPlot[grad[x, y], {x, -6, 6}, {y, -6, 6}, StreamStyle → "Arrow", StreamPoints → Fine, StreamScale → Automatic, PlotRange → All, AspectRatio → Automatic, FrameLabel → {"x", "y"}, AxesLabel → Automatic, ImageSize → Large]
```

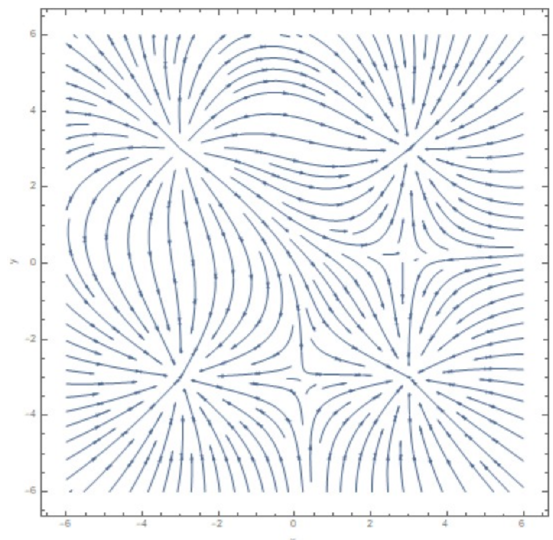


Fig. 0.

hence, $\overrightarrow{E} = -\nabla\overrightarrow{V}$

(c)

```
k = 8.99*10^9;

λ1 = 2;
λ2 = -2;

potential1[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y + 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y - 2)^2];
   d = (x + y)/Sqrt[2];
   k*λ1*Log[(r1 + Sqrt[r1^2 - d^2])/(r2 - Sqrt[r2^2 - d^2])]];

potential2[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y - 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y + 2)^2];
   d = (x - y)/Sqrt[2];
   k*λ2*Log[(r1 + Sqrt[r1^2 - d^2])/(r2 - Sqrt[r2^2 - d^2])]];

potential[x_, y_] := potential1[x, y] + potential2[x, y];

negGrad[x_, y_] := Evaluate[-D[potential[x, y], {{x, y}}]];

VectorPlot[negGrad[x, y], {x, -6, 6}, {y, -6, 6}, VectorStyle → "Arrow", VectorPoints → Fine, VectorScale → Automatic, PlotRange → All, AspectRatio → Automatic, FrameLabel → {"x", "y"}, AxesLabel → Automatic, ImageSize → Large, Epilog → {Red, PointSize[Large]}]
```

Fig. 0.

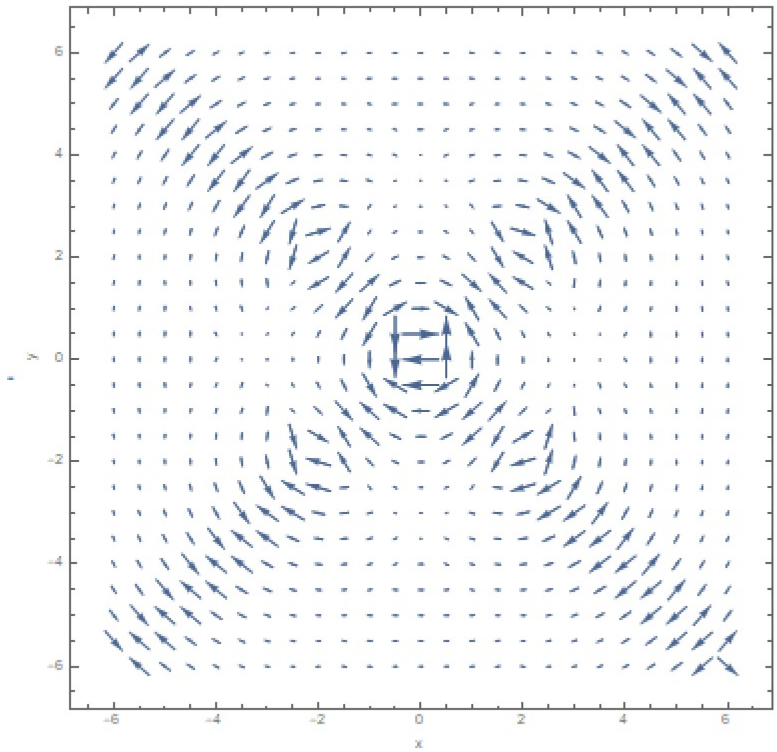


Fig. 0.

hence, $\overrightarrow{E} = -\nabla\overrightarrow{V}$

(2.2.(c))

(a)

```
(*Define Coulomb's constant*)k = 8.99 * 10^9;

(*Define the electric field due to the ensemble of charges*)
parametricField[q1_, q2_, q3_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}][x_, y_] := Module[{r1, r2, r3}, r1 = Sqrt[(x - x1)^2 + (y - y1)^2];
  r2 = Sqrt[(x - x2)^2 + (y - y2)^2];
  r3 = Sqrt[(x - x3)^2 + (y - y3)^2];
  {(k * q1 (x - x1)) / r1^3 + k * (q2 (x - x2)) / r2^3 + k * (q3 (x - x3)) / r3^3, (k * q1 (y - y1)) / r1^3 + (k * q2 (y - y2)) / r2^3 + (k * q3 (y - y3)) / r3^3}]

(*Specify parameters for the charges*)
specificField[x_, y_] = parametricField[2, -3, 3, {0, 3}, {-2, 0}, {2, 0}][x, y];

(*Calculate the curl of the electric field*)
curl = Curl[specificField[x, y], {x, y}];

(*Create a 3D plot of the curl components*)
Plot3D[Evaluate[curl], {x, -6, 6}, {y, -6, 6}, PlotLegends → {"Curl_x", "Curl_y", "Curl_z"}, AxesLabel → {"x", "y", "Curl"}]
```

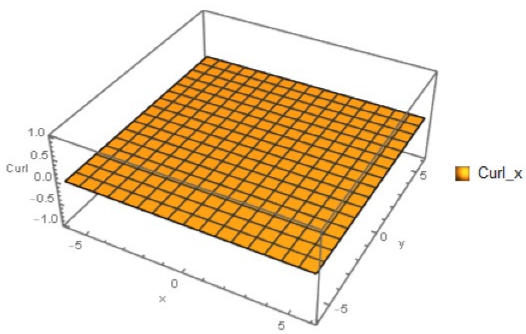


Fig. 0.

(b)

```
(*Define Coulomb's constant*)k = 8.99 * 10^9;
(*Define the electric field due to the ensemble of charges*)
parametricField[q1_, q2_, q3_, q4_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}, {x4_, y4_}][x_, y_] := Module[{r1, r2, r3, r4}, r1 = (x - x1)^2 + (y - y1)^2;
   r2 = (x - x2)^2 + (y - y2)^2;
   r3 = (x - x3)^2 + (y - y3)^2;
   r4 = (x - x4)^2 + (y - y4)^2;
   {(k * q1 (x - x1)) / r1^(3 / 2) + k * (q2 (x - x2)) / r2^(3 / 2) + k * (q3 (x - x3)) / r3^(3 / 2) + k * (q4 (x - x4)) / r4^(3 / 2),
    (k * q1 (y - y1)) / r1^(3 / 2) + k * (q2 (y - y2)) / r2^(3 / 2) + k * (q3 (y - y3)) / r3^(3 / 2) + k * (q4 (y - y4)) / r4^(3 / 2)}];
(*Specify parameters for the charges*)
specificField[x_, y_] = parametricField[-2, 2, 2, 3, {-3, 3}, {3, 3}, {3, -3}, {-3, -3}][x, y];
(*Compute the curl of the parametric field*)
curl[x_, y_] := Evaluate[D[specificField[x, y][[2]], x] - D[specificField[x, y][[1]], y]];
(*Create a 3D plot of the curl field*)
plotCurl = Plot3D[curl[x, y], {x, -6, 6}, {y, -6, 6}, PlotRange → All, AxesLabel → {"x", "y", "curlZ"}, ColorFunction → "Rainbow", PlotPoints → 100, MaxRecursion → 5];
(*Show the 3D plot of the curl field*)
plotCurl
```

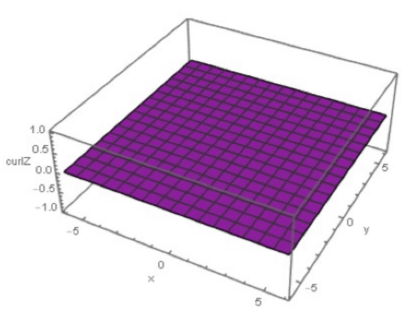


Fig. 0.

(c)

```
k = 8.99 * 10^9;
λ1 = 2;
λ2 = -2;

potential1[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y + 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y - 2)^2];
   d = (x + y) / Sqrt[2];
   k * λ1 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential2[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y - 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y + 2)^2];
   d = (x - y) / Sqrt[2];
   k * λ2 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential[x_, y_] := potential1[x, y] + potential2[x, y];
negGrad[x_, y_] := Evaluate[-D[potential[x, y], {{x, y}}]];

Fx[x_, y_] := Evaluate[negGrad[x, y][[1]]];
Fy[x_, y_] := Evaluate[negGrad[x, y][[2]]];

curl = D[Fy[x, y], x] - D[Fx[x, y], y];

(*3D plot of curl*)
Plot3D[curl, {x, -6, 6}, {y, -6, 6}, PlotRange → All, AxesLabel → {"x", "y", "curl"}, PlotLabel → "3D Plot of Curl of Vector Field", ImageSize → Large]
```

Fig. 0.

Out[11]=

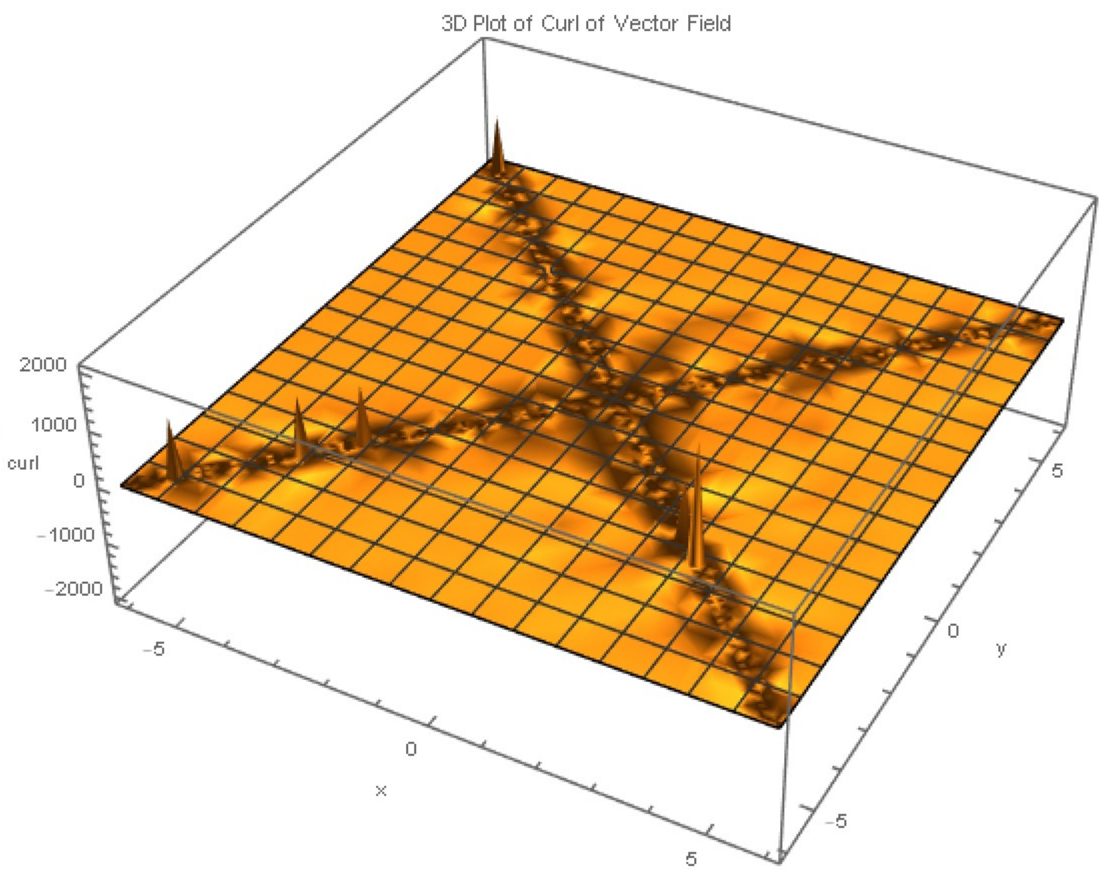


Fig. 0.

(2.3)
(a)

```
k = 8.99 * 10^9; (*Coulomb's constant*)
(*Define the electric force due to the ensemble of charges*)
parametricForce[q1_, q2_, q3_, q4_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}, {x4_, y4_}][x_, y_] := Module[{r1, r2, r3, r4}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  r4 = (x - x4)^2 + (y - y4)^2;
  {(k * q1 (x - x1)) / r1^(3/2) + (k * q2 (x - x2)) / r2^(3/2) + (k * q3 (x - x3)) / r3^(3/2) + (k * q4 (x - x4)) / r4^(3/2),
   (k * q1 (y - y1)) / r1^(3/2) + (k * q2 (y - y2)) / r2^(3/2) + (k * q3 (y - y3)) / r3^(3/2) + (k * q4 (y - y4)) / r4^(3/2)}]

(*Specify parameters for the charges*)
specificForce[x_, y_] = parametricForce[-2, 2, 2, 3, {-3, 3}, {3, 3}, {3, -3}, {-3, -3}][x, y];

(*Calculate the force on a unit positive charge at the origin*)
unitChargeForce = specificForce[0, 0];

netForceMagnitude = Norm[unitChargeForce];


(*Print the net force vector*)
Print["Net Force (Vector) experienced by unit charge at origin due to configuration of charges: ", unitChargeForce];

(*Print the net force magnitude*)
Print["Net Force experienced by unit charge at origin due to configuration of charges:", netForceMagnitude];
Net Force (Vector) experienced by unit charge at origin due to configuration of charges: {-1.05948×10^9, 1.7658×10^9}

Net Force experienced by unit charge at origin due to configuration of charges:2.05926×10^9
```

Fig. 0.

(b)

```
k = 8.99 * 10^9; (*Coulomb's constant*)

(*Define the electric force due to the ensemble of charges*)
parametricForce[q1_, q2_, q3_, {x1_, y1_}, {x2_, y2_}, {x3_, y3_}][x_, y_] := Module[{r1, r2, r3}, r1 = (x - x1)^2 + (y - y1)^2;
  r2 = (x - x2)^2 + (y - y2)^2;
  r3 = (x - x3)^2 + (y - y3)^2;
  {(k * q1 (x - x1)) / r1^(3/2) + (k * q2 (x - x2)) / r2^(3/2) + (k * q3 (x - x3)) / r3^(3/2),
   (k * q1 (y - y1)) / r1^(3/2) + (k * q2 (y - y2)) / r2^(3/2) + (k * q3 (y - y3)) / r3^(3/2)}]

(*Specify parameters for the charges*)
specificForce[x_, y_] = parametricForce[2, -3, 3, {0, 3}, {-2, 0}, {2, 0}][x, y];

(*Calculate the force on a unit positive charge at the origin*)
unitChargeForce = specificForce[0, 0];
netForceMagnitude = Norm[unitChargeForce];
(*Print the net force vector*)
Print["Net Force (Vector) experienced by unit charge at origin due to configuration of charges: ", unitChargeForce];

Print["Net Force experienced by unit charge at origin due to configuration of charges:", netForceMagnitude];


Net Force experienced by unit charge at origin due to configuration of charges:1.36322×10^10

Net Force (Vector) experienced by unit charge at origin due to configuration of charges: {-1.3485×10^10, -1.99778×10^9}
```

Fig. 0.

(c)

```
k = 8.99 * 10^9;
λ1 = 2;
λ2 = -2;

potential1[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y + 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y - 2)^2];
   d = (x + y) / Sqrt[2];
   k * λ1 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential2[x_, y_] := Module[{r1, r2, d}, r1 = Sqrt[(x - 2)^2 + (y - 2)^2];
   r2 = Sqrt[(x + 2)^2 + (y + 2)^2];
   d = (x - y) / Sqrt[2];
   k * λ2 * Log[(r1 + Sqrt[r1^2 - d^2]) / (r2 - Sqrt[r2^2 - d^2])]];

potential[x_, y_] := potential1[x, y] + potential2[x, y];

negGrad[x_, y_] := Evaluate[-D[potential[x, y], {{x, y}}]];

(*Evaluate the vector field at the origin*)
forceAtOrigin = negGrad[0, 0];

(*Force experienced by a unit charge at the origin*)
unitChargeForce = forceAtOrigin;

(*Print the force experienced by a unit charge in vector form*)
Print["Force experienced by a unit charge in vector form: ", unitChargeForce];

(*Calculate and print the magnitude of the force*)
forceMagnitude = Norm[unitChargeForce];
Print["Magnitude of the force: ", forceMagnitude];
```

Fig. 0.

```
Force experienced by a unit charge in vector form: {Indeterminate, Indeterminate}
Magnitude of the force: Indeterminate
```

Fig. 0.

3: Simulating fields and potential on a simulator
(a) Select any open source/institute licensed electromagnetics simulator.
(b) Draw a 2-D geometry comprising metal/insulator , semiconductor is optional. Explain the significance/ why you chose this geometry.[3*points*]
(c) Select/choose the correct equations & boundary conditions and justify them.[2*points*]
(d) Plot/Represent the $\overrightarrow{E}$ & V fields,what svientific sights you gain out of this?[5*points*]

Solution: The electromagnetic simulator we are using is *sim scale*, which is open source.

The geometry we are using is **circular plate**.

We are choosing a circular plate for the following reasons:

- The metal plate acts as a conductor and the surrounding medium (air) acts as an insulator. The geometry is very simple and allows the study of the behavior of electric fields and electric potential.
- The circular plate has more symmetry and a uniform field distribution.
- The circular geometry provides simple analytical solutions compared to other complex geometries.
- In our regular life, the circular structures are used in many practical appliances. so studying this geometry gives insights to other real world devices.

The boundary conditions for the circular plate are:

1) The boundary condition for the electric field is that the tangential part of the electric field must be zero so that the electric field lines will be perpendicular to the surface of the conductor, making it an ideal conductor:

$$E_t = 0$$

2) The boundary condition for the electrostatic potential is that the gradient of the electrostatic potential normal to the surface should be zero, ensuring that the electrostatic potential is constant throughout:

$$\frac{\partial V}{\partial n} = 0$$

where, n is the normal vector pointing outwards from the surface.