

D3 : Tímaflækja og röðun/Time complexity and Sorting

1. (12%) Gefðu einföldustu mögulega tildu-nálgun (\sim) fyrir eftirfarandi stærðir / Give the simplest possible tilde approximations for the following quantities:

a. $(1 - 1/N)(1 - 2/N)$

b. $\lg N^2 / \lg N$

c. $2^{\lg N}$

Munið að $\log ab = \log a + \log b$.

2. (15%) Hver er tilda tímaflækja eftirfarandi kóðabúta? / Give the tilde time complexity of each of the following code fragments:

a.

```
int sum = 0;
for (i=0; i < n*n; i++)
    for (j=0; j < 2*n; j++)
        sum++;
```

b.

```
int sum = 0;
for (int t = n; t > 0; t /= 2)
    for (int i = 0; i < t; i++)
        sum++;
```

c.

```
int sum = 0;
for (i=0; i < n; i++)
    for (j=0; j < i; j+=2)
        sum++;
```

4. (15%) Ákveðið forrit tekur 2.1 s fyrir inntök af stærð 1000 en 16.8 s fyrir inntök af stærð 4000. Leiddu út tímaflækju reikniritsins á forminu $T(n) = a n^b$. / A given program runs in 2.1s on inputs of size 1000 but 16.8s on inputs of size 4000. Derive the time complexity of the program of the form $T(n) = a n^b$.

5. (15%) We are given a randomly ordered array where elements have only three values (small, medium, big), equally many of each type. What is the tilde running time of Insertion sort on this array?

6. (13%) A clerk at a shipping company is charged with the task of rearranging a number of large crates in order of the time they are to be shipped out. Thus, the cost of compares is very low (just look at the labels) relative to the cost of exchanges (move the crates). The warehouse is nearly full – there is extra space to hold any one of the crates, but not two. What sorting method should the clerk use and why (briefly)?

7. (15%) Suppose Mergesort is modified to divide the input into three parts, each with one

third of the input. It then sorts each part recursively and combines using three-way merge. Explain why the order of growth of the overall running time is $\Theta(n \log n)$.

8. (15%) Explain succinctly how to solve the following problem efficiently: Given two array, determine which elements appear in both arrays.

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Class problems

a. How efficiently can we solve this problem: Given an array of numbers, determine if they are all different.

b. Ken and Barbie want to check how good a fit they are by comparing their movie preferences. They have ordered all the Netflix horror films in order of preference. They have come up with the following measure: they count how many pairs of films are pairwise in the same order in their preference lists. (E.g., if Ken prefers the films A B C D in the order B A D C, while Barbie orders them as A D B C, their score is 4 (AD, AC, BC, DC)). How can they compute this measure?

c. What is the tilde time complexity:

```
int sum = 0;
for (i=n; i > 0; i /= 2)
    for (j=0; j < i; j++)
        sum++;
```