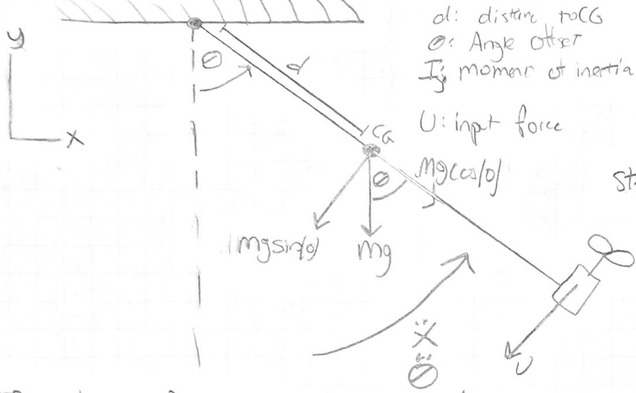
Aeropendulum Model

Assume force out is proportional to some input voltage

$U = K \cdot V$



M: mass motor

m: mass arm link

d: distance to CG

θ: Angle offset

I_y: moment of inertia

U: input force

System reaches stability when $\dot{\theta}, \ddot{\theta} \Rightarrow 0$ and θ is constant ($\theta_{desired}$)

Steady state eqn

$U = T = mg\sin(\theta)$

$mg\sin(\theta) = KV$

$\frac{mg\sin(\theta)}{V} = K$

State space

$x_1 = \theta$

$x_2 = \dot{x}_1 = \dot{\theta}$

$\dot{x}_2 = \ddot{\theta}$

$\ddot{x}$

$\ddot{\theta}$

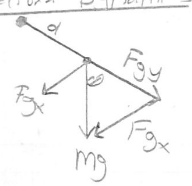
$\dot{x}$ $\dot{\theta}$

non linear ss rep:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ f_1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\dot{x}_2 = \frac{g\sin(x_1)}{l} + \frac{Ul}{m}$$

Translational Dynamics



$F_{gx} = mg\sin(\theta)$

$F_{gy} = mg\cos(\theta)$

Linearization $\sin(\theta) \approx \theta$ (small angle theorem)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ g/l & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{l}{m} \end{bmatrix} U \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

Tangential Accel

Centripetal Accel

$F_x = ma$

$a = \ddot{\theta} \times r + \dot{\theta} \times V$

$m\ddot{x} = F_{gx} + U$

$ma = mg\sin(\theta) + U$

$a = g\sin(\theta) + U/m$

or use arc length formula:

$s = l\theta$

$V = \dot{s} = l\dot{\theta}$

$a = \ddot{s} = l\ddot{\theta}$

s: arc length

l: length arm

θ: angle

$$g\sin(\theta) + \frac{U}{m} = l\ddot{\theta}$$

$$\left[\ddot{\theta} = \frac{g\sin(\theta)}{l} + \frac{Ul}{m}\right]$$