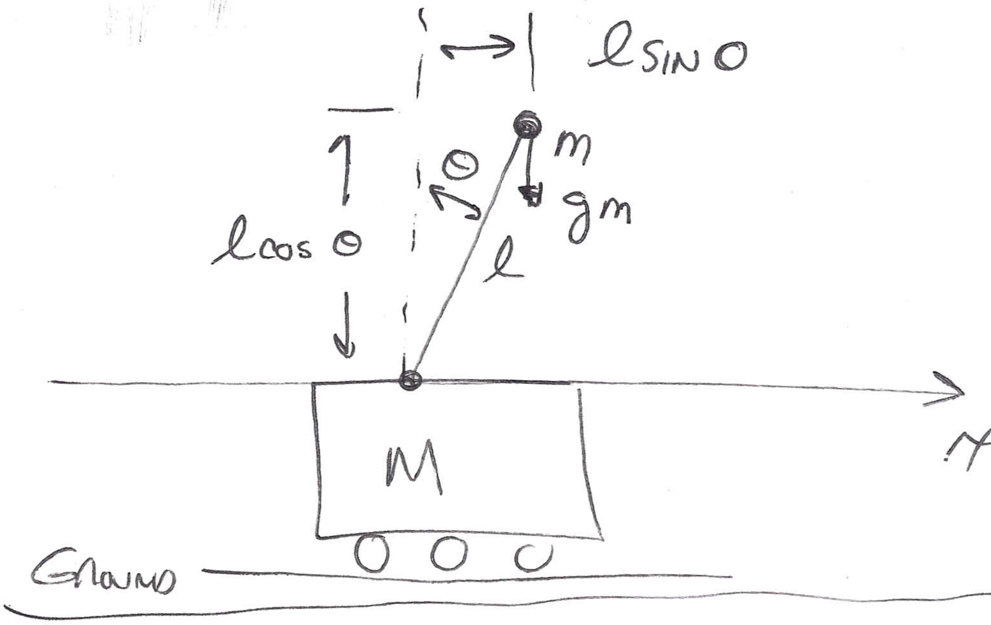
21 NOVEMBER 2005

INVERTED PENDULUM



Position of cart $x_c = x \qquad y_c = 0$

Position of mass m

$$x_m = x + l\sin\theta$$
$$y_m = l\cos\theta$$

two generalized coordinates

$x = x_c$

θ

Cart mass M

$$K_E = \tfrac{1}{2} M \dot{x}^2 \qquad P_E = 0$$

Always same height

Mass at end of pendulum m

$$P_E = mg\,l\cos\theta$$

$$\dot{x}_m = \dot{x} + l\cos\theta\,\dot{\theta} \qquad \dot{y}_m = -l\sin\theta\,\dot{\theta}$$

$$v^2 = \dot{x}_m^2 + \dot{y}_m^2$$

$$v^2 = (\dot{x} + l\cos\theta\,\dot{\theta})(\dot{x} + l\cos\theta\,\dot{\theta}) + l^2\sin^2\theta\,\dot{\theta}^2$$

$$v^2 = \dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2\cos^2\theta + l^2\dot{\theta}^2\sin^2\theta$$

$$v^2 = \dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2$$

$$\therefore\ KE_m = \frac{1}{2}m\left(\dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2\right)$$

LAGRANGIAN $L = E_k - E_p$

$$\boxed{L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{x}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta}$$

GENERALIZED COORDINATE x

$$F_x = \frac{\partial}{\partial t}\left[\frac{\partial L}{\partial \dot{x}}\right] - \frac{\partial L}{\partial x} \qquad \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = M\dot{x} + m\dot{x} + ml\dot{\theta}\cos\theta$$

$$\frac{\partial}{\partial t}\left[\frac{\partial L}{\partial \dot{x}}\right] = M\ddot{x} + m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$$

$$\boxed{\therefore\ F_x = (M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta}$$

IF WE ASSUME $\theta \approx 0$ $\quad \sin\theta = \theta \quad \cos\theta = 1$

$$F_x = (M+m)\ddot{x} + ml\ddot{\theta} - ml\dot{\theta}^2\theta$$

FURTHERMORE ALSO IF $\theta\dot{\theta}^2 \approx 0$

$$\boxed{F_x = (M+m)\ddot{x} + ml\ddot{\theta}}$$ LINEARIZED EQUATION

GENERALIZED COORDINATE θ

$$T_\theta = \frac{\partial}{\partial t}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta}$$

$$\frac{\partial L}{\partial \theta} = 0 + 0 - \sin\theta\, m l \dot{x} \dot{\theta} + 0 + mgl\sin\theta$$

$$= \left[-\sin\theta\, m l \dot{x} \dot{\theta} + mgl\sin\theta\right]$$

$$\frac{\partial L}{\partial \dot{\theta}} = 0 + 0 + m l \dot{x} \cos\theta + m l^2 \dot{\theta}$$

$$\frac{\partial}{\partial t}\left[\frac{\partial L}{\partial \dot{\theta}}\right] = m l \ddot{x} \cos\theta - \sin\theta\, m l \dot{x} \dot{\theta} + m l^2 \ddot{\theta}$$

$$T_\theta = m l \ddot{x} \cos\theta - \sin\theta\, m l \dot{x} \dot{\theta} + m l^2 \ddot{\theta} + \sin\theta\, m l \dot{x} \dot{\theta} - mgl\sin\theta$$

$$\boxed{T_\theta = m l \ddot{x} \cos\theta + m l^2 \ddot{\theta} - mgl\sin\theta}$$

Assuming $\theta \approx 0$ $\quad \sin\theta = \theta \quad \cos\theta = 1$

$$T_\theta = m l \ddot{x} + m l^2 \ddot{\theta} - mgl\theta$$ LINEARIZED EQUATION

SINCE $T_\theta = 0$ NO ACTUATOR

$$\boxed{m l \ddot{x} + m l^2 \ddot{\theta} = mgl\theta}$$