
file ccf_state

Table of Contents

original system	1
find transfer function	2
write in controllable canonical form	2
get characteristic equation same for both representations	2
Compute feedback gain based on characteristic equation coefficients	3
check eigenvalues using kb	3
find $P = Cbar * inv(C)$ and k for original system	3
verify k with original system	4
Verify matrices using transform, P are same as by inspection of $G(s)$	4
compare step responses of both systems without state feedback	5
compare step responses systems of both systems with state feedback	6
show transfer functions	6
look at components of transfer function with state feedback	7
Phase plane analysis for system without and with state feedback	8

revised 28 JAn 2017 Dr. Tom Chmielewski

original system

this is given in diagonal form and controllable and completely observable

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$D = 0$$

$$A =$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$B =$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C =$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$D =$$

$$0$$

find transfer function

```
tf(ss(A,B,C,D))
```

```
ans =
```

$$\frac{2s + 3}{s^2 + 3s + 2}$$

Continuous-time transfer function.

write in controllable canonical form

this is done by finding the transfer function and doing a coefficient match to the model of Eq. 8.7

```
Ab = [-3 -2; 1 0]
```

```
Bb = [1; 0]
```

```
Cb = [2 3]
```

```
Ab =
```

$$\begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$$

```
Bb =
```

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

```
Cb =
```

$$\begin{bmatrix} 2 & 3 \end{bmatrix}$$

get characteristic equation same for both representations

```
CEA = poly(A)
```

```
CEA =
```

$$\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$$

Compute feedback gain based on characteristic equation coefficients

```
% desired closed loop ce = s^2 + 11s + 30 = (s+5)(s+6)
% desired - actual high-1 ro low
CED = [1 11 30]

DD = CED-CEA
% strip off first element s^2 coefficient

kb = DD(2:3)

kb = [8 28] % state feedback in CCF

CED =

    1    11    30

DD =

    0     8    28

kb =

     8    28

kb =

     8    28
```

check eigenvalues using kb

```
eig((Ab-Bb*kb)) % verify correct

ans =

   -6.0000
   -5.0000
```

find $P = Cbar * inv(C)$ and k for original system

```
Cbar = [Bb, Ab*Bb]
CT = [B, A*B]
```

```
P = Cbar*inv(CT) % this is transformation
```

```
k= kb*P
```

```
Cbar =
```

$$\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

```
CT =
```

$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$$

```
P =
```

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

```
k =
```

$$\begin{bmatrix} 20 & -12 \end{bmatrix}$$

verify k with original system

```
eig((A-B*k))
```

```
ans =
```

$$\begin{bmatrix} -6.0000 \\ -5.0000 \end{bmatrix}$$

Verify matrices using transform, P are same as by inspection of G(s)

```
AA = P*A*inv(P)
```

```
BB = P*B
```

```
CC = C*inv(P)
```

```
AA =
```

$$\begin{bmatrix} -3 & -2 \end{bmatrix}$$

$$\begin{matrix} 1 & 0 \end{matrix}$$

$$BB =$$

$$\begin{matrix} 1 \\ 0 \end{matrix}$$

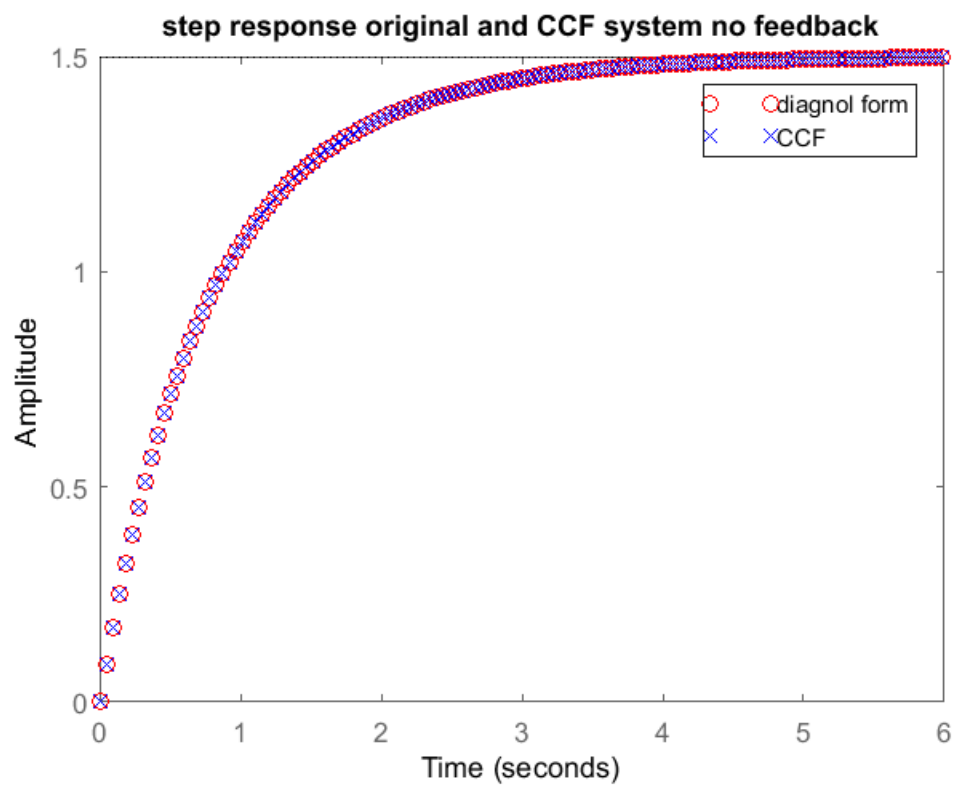
$$CC =$$

$$\begin{matrix} 2 & 3 \end{matrix}$$

compare step responses of both systems without state feedback

same y so they should yield same results even though states are different

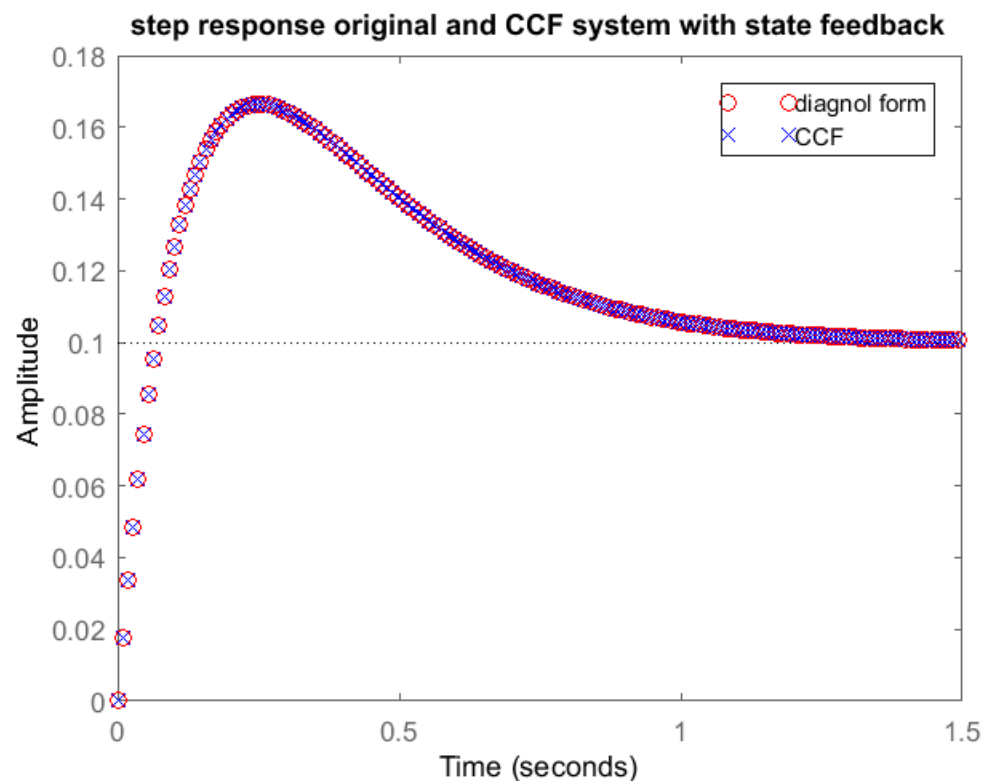
```
sys_orig = ss(A, B, C, 0);
sys_CCF = ss(Ab, Bb, Cb, 0);
figure
step(sys_orig, 'ro', sys_CCF, 'bx')
title('step response original and CCF system no feedback')
legend('diagnol form', 'CCF')
```



compare step responses systems of both systems with state feedback

same y so they should yield same results even though states are different

```
sys_orig_f = ss((A-B*k), B, C, 0);
sys_CCF_f = ss((Ab-Bb*kb), Bb, Cb, 0);
figure
step(sys_orig_f, 'ro', sys_CCF_f, 'bx')
title('step response original and CCF system with state feedback')
legend('diagnol form', 'CCF')
```



show transfer functions

original transfer function

```
gg1= tf(sys_orig_f)
% transfer function with state feedback
gg2 = tf(sys_CCF_f)
```

gg1 =

$2s + 3$

$$\frac{\quad}{s^2 + 11 s + 30}$$

Continuous-time transfer function.

gg2 =

$$\frac{2 s + 3}{s^2 + 11 s + 30}$$

Continuous-time transfer function.

look at components of transfer function with state feedback

```
s = tf('s')

g1 = -7/(s+5)
g2 = 9/(s+6)

[y1, t1] = step(g1);
[y2, t2] = step(g2, t1);
figure
plot(t1, y1, t2, y2, t1, (y1+y2))

hold on
step((g1+g2), 'r.')
legend('-7/(s+5)', '9/(s+6)', 'sum')
title('components contributing to step response')
```

s =

s

Continuous-time transfer function.

g1 =

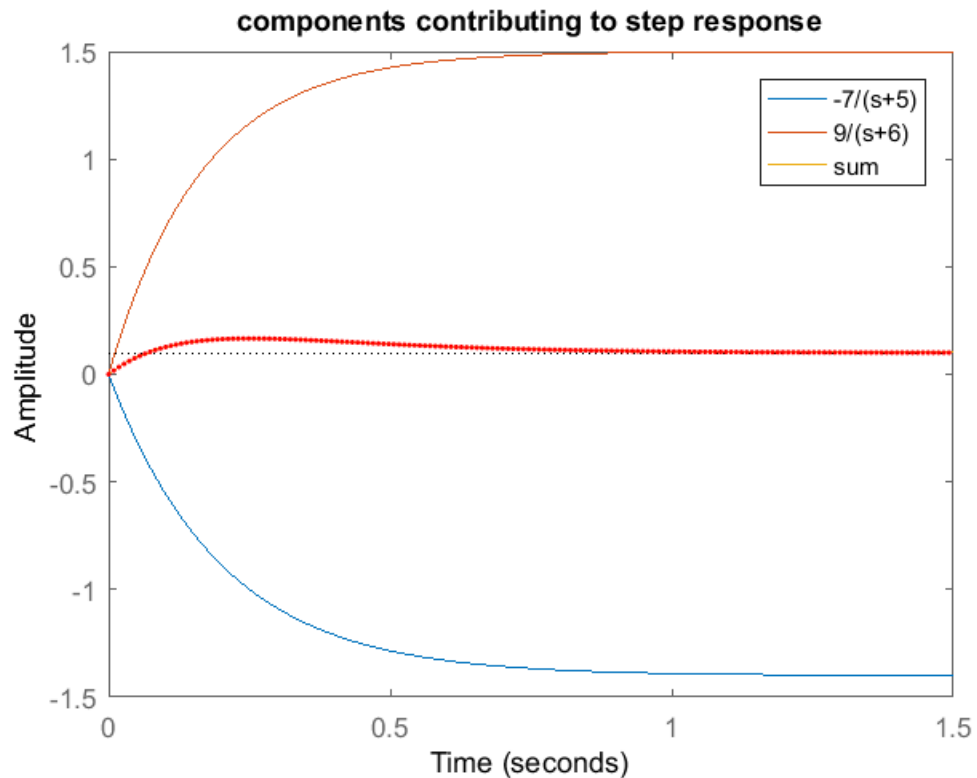
$$\frac{-7}{s + 5}$$

Continuous-time transfer function.

g2 =

$$\frac{9}{s+6}$$

Continuous-time transfer function.



Phase plane analysis for system without and with state feedback

this is solution to homogeneous system $\dot{x} = Ax$ with $x(0)$ or $\dot{z} = (A-Bk)z$ note with state feedback we saw overshoot in $y(t)$

```
t = 0:.01:10; % create time vector
u = zeros(size(t)); % input u = 0;
x0 = [10;10] % initial condition;
C = eye(2) ; % to see both states
sys_CCF = ss(Ab, Bb, C, 0); % modified to see both states
sys_CCF_f = ss((Ab-Bb*kb), Bb, C, 0); % modified to see both states
X = lsim(sys_CCF,u,t, x0); % CCF
Xf = lsim(sys_CCF_f,u,t, x0); % CCF with state feedback
figure
plot(X(:,1), X(:,2), 'b', 'LineWidth', 2)
hold on
```



```

plot(Xf(:,1), Xf(:,2), 'g', 'LineWidth', 2)

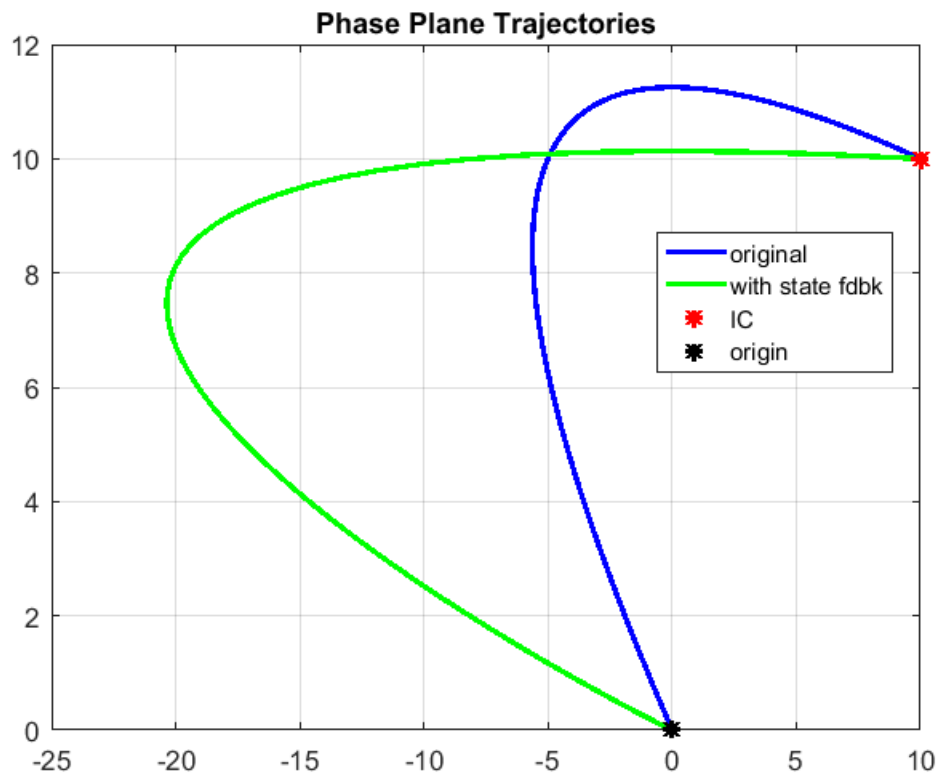
plot(10, 10, 'r*', 'LineWidth', 2)
plot(0, 0, 'k*', 'LineWidth', 2)
legend('original', 'with state
      fdbk', 'IC', 'origin', 'Location', 'Best')
title('Phase Plane Trajectories')
grid on

```

```
x0 =
```

```
10
```

```
10
```



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