A Brief Introduction to Active Disturbance Rejection Control (ADRC)

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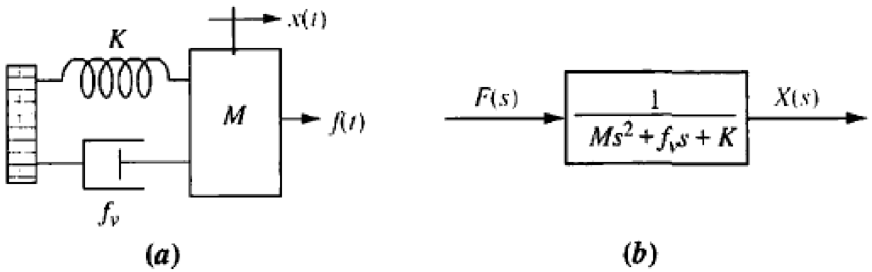
Most copied figures and equations from [4, others referenced appropriately

Motivation

Conventional and modern Closed Loop Control System design relies on having a model of the system prior to applying a control strategy.



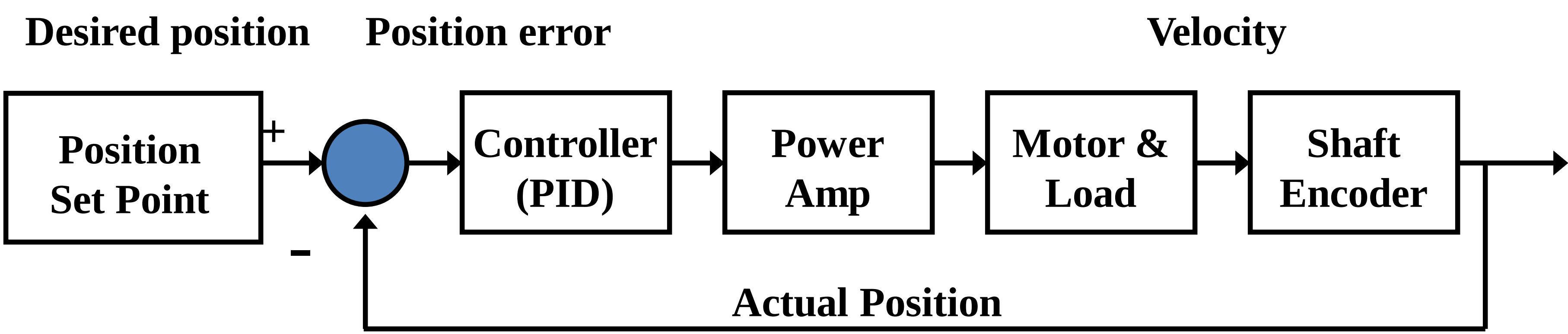
Simple model of a mass, spring, and damper system with its corresponding block diagram [1]

- Writing model equations for a physical system can become complicated
- Unmodeled high frequency dynamics – would make the model's order large – many times ignored until they become an issut
- Parameter values may know be known exactly or difficult to obtain
- Validation may be difficult

ADRC Converts the problem from modeling to controls.

Classical Approach

- Closed loop control using model of system with cascade compensators, multiple loops etc. Requires analysis based on model to obtain correct phase margin, gain margin, steady state performance.
  - Uncertainty of models/ parameters
- Many engineers still use PID due to its simplicity and non reliance on specific model parameters (relatively forgiving)
  - May be difficult to tune under many circumstances
  - Ziegler Nichols is a starting point



Modern Control

Observer based state variable feedback (pole placement)

- Reliant on state equations of system
- Observer Estimates the states of the system
- Observers model is assumed to be "close" to that of the system
  - Modeling error small
  - Noise and Disturbances are handled by feedback

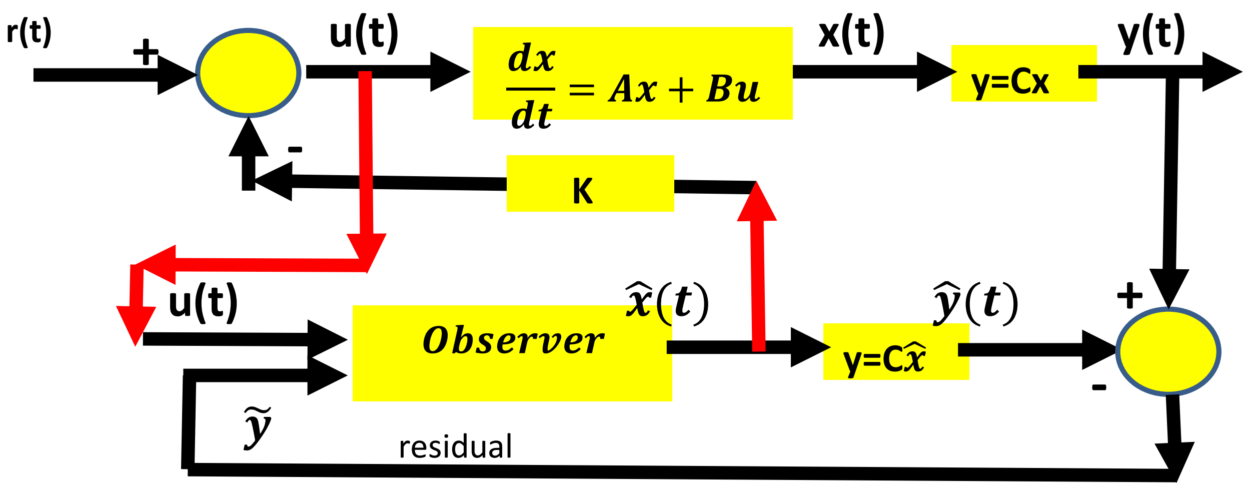


Figure of observer based state feedback [2]

Other Techniques and a Solution

Non-linear Control – still need model

- Feedback linearization
- Sliding mode
- Model Reference Adaptive Control
- Robust Control approaches
- Etc.

Solution

Active Disturbance Rejection Control (ADRC) introduced by GAO [3], [5]

- Reduces dependency on accurate mathematical model
- Converts the problem from modeling to controls
- Detailed Stability analysis shown in [5] Lyapounov based

Linear approach and Nonlinear ADRC
Briefing focus on Linear ADRC

Briefing Material

Implement 2nd order control system with disturbances using ADRC

- Model 2nd order systems using ADRC formulation
- Simulate using MATLAB/Simulink
- Try and see if a NL system will also work – just slip into controller – no changes

Linear Active Disturbance Rejection Control (ADRC)

Characteristics:

- Reduces/removes dependency on accurate mathematical model
- All input disturbances and unknown model dynamics are considered a **general disturbance** (represented as an additional state)
- The plant is then **simply a series of integrators depending** on its order i.e., a double integrator for a 2nd order system
- An **Extended state observer** is use to estimate the **states** and the **generalized disturbance** (for a 2nd order system the ESO is 3rd order)
- Converted the problem from modeling to controls since we are controlling a nth order plant consisting of all integrators. – class of problems

ADRC formulation for 2nd Order system

From Herbst [4]:

Given a 2nd order system, *P(s)*, with DC gain, *K*, damping factor, *D*, and time constant, T

$$P(s) = \frac{y(s)}{u(s)} = \frac{K}{T^2 s^2 + 2DTs + 1} \quad \bullet\!\!-\!\!\circ \quad T^2 \cdot \ddot{y}(t) + 2DT \cdot \ddot{y}(t) + y(t) = K \cdot u(t)$$

Add an input disturbance, *d(t) to input*, let $b = K/T^2$ and split *b* into a known and unknown part $b = b_0 + \Delta b$:

$$\ddot{y}(t) = \underbrace{\left(-\frac{2D}{T} \cdot \dot{y}(t) - \frac{1}{T^2} \cdot y(t) + \frac{1}{T^2} \cdot d(t) + \Delta b \cdot u(t)\right)}_{\text{generalized disturbance } f(t)} + b_0 \cdot u(t) = \underline{f(t) + b_0 \cdot u(t)}$$

ADRC formulation for 2nd Order system

From Herbst [4]:

System State Space Formulation:

$$\ddot{y} = f(t) + b_0 u(t)$$
$$y = x_1$$
$$\dot{y} = x_2$$

$$- - - - - - - -$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 + b_0 u(t)$$
$$\dot{x}_3 = \dot{f}$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{A} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ b_0 \\ 0 \end{pmatrix}}_{B} \cdot u(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \dot{f}(t)$$

Virtual input

$$y(t) = \underbrace{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}}_{C} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

ADRC formulation for 2nd Order system

From Herbst [4]:

Extended State Observer (ESO)

States are estimates of y, y_dot and d(t)

$$
\begin{aligned}
\begin{pmatrix} \dot{\hat{x}}_1(t) \\ \dot{\hat{x}}_2(t) \\ \dot{\hat{x}}_3(t) \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{x}_3(t) \end{pmatrix} + \begin{pmatrix} 0 \\ b_0 \\ 0 \end{pmatrix} \cdot u(t) + \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} \cdot (y(t) - \hat{x}_1(t)) \\
&= \underbrace{\begin{pmatrix} -l_1 & 1 & 0 \\ -l_2 & 0 & 1 \\ -l_3 & 0 & 0 \end{pmatrix}}_{A-LC} \cdot \begin{pmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{x}_3(t) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ b_0 \\ 0 \end{pmatrix}}_{B} \cdot u(t) + \underbrace{\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}}_{L} \cdot y(t)
\end{aligned} \tag{12}
$$

Control Implementation

Disturbance rejection and linear control (state feedback)

$$\ddot{y} = f(t) + b_0 u(t)$$

$$u(t) = \frac{u_0(t) - \hat{f}(t)}{b_0}$$

Remove disturbances and unknown dynamics by using Estimate of d(t)

$$u_0(t) = K_P \cdot (r(t) - \hat{y}(t)) - K_D \cdot \dot{\hat{y}}(t)$$

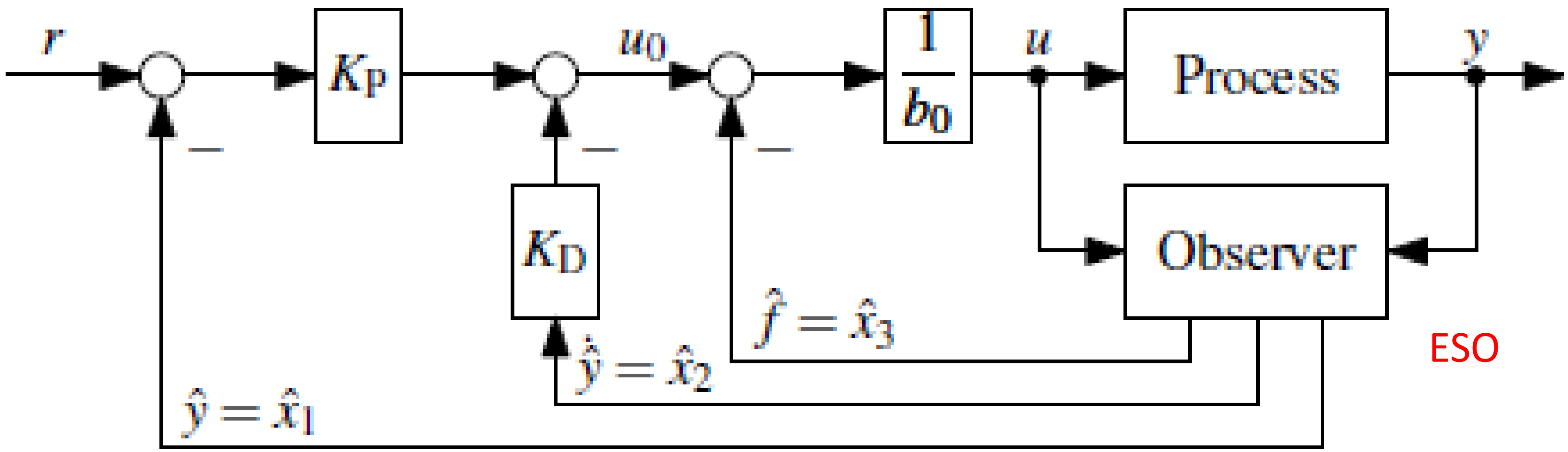
Observer based state feedback with input r(t)

ADRC formulation for 2^{nd} Order system

From Herbst [4]:

ADRC Control with ESO

Only know system order and b_0



ESO

Control Result

If the state estimates are close to actual states

$$\ddot{y}(t) = \left(f(t) - \hat{f}(t)\right) + u_0(t) \approx u_0(t) \approx K_P \cdot (r(t) - y(t)) - K_D \cdot \dot{y}(t)$$

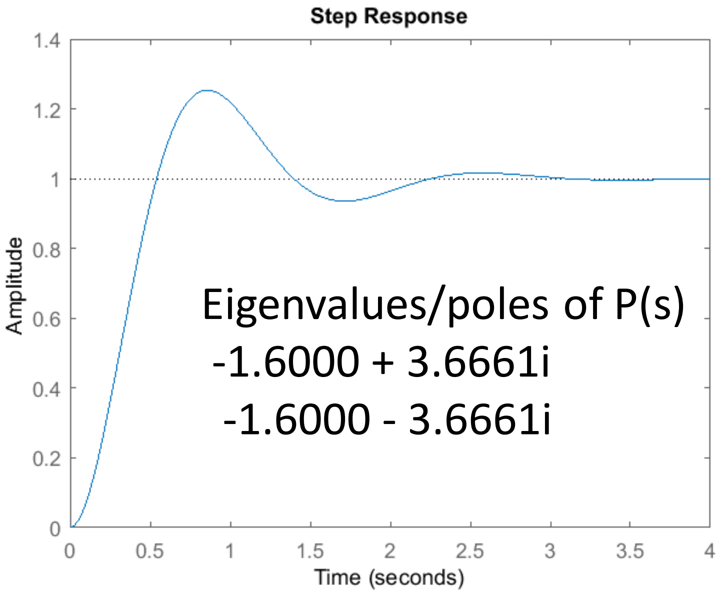
$$\frac{1}{K_P} \cdot \ddot{y}(t) + \frac{K_D}{K_P} \cdot \dot{y}(t) + y(t) = r(t)$$

Bandwidth parameterization:

- Choose {Kp, Kd} to get critically damped response
- Choose observer eigenvalues (3-10) times faster then critically damped “pole”
- Only “one” eigenvalue/pole value to worry about basically only one “knob” to turn

Simulation
nominal system P(s)

System Step Response



Eigenvalues/poles of P(s)
-1.6000 + 3.6661i
-1.6000 - 3.6661i

Key Parameters

P =

$$P = \frac{1}{0.0625\,s^2 + 0.2\,s + 1}$$

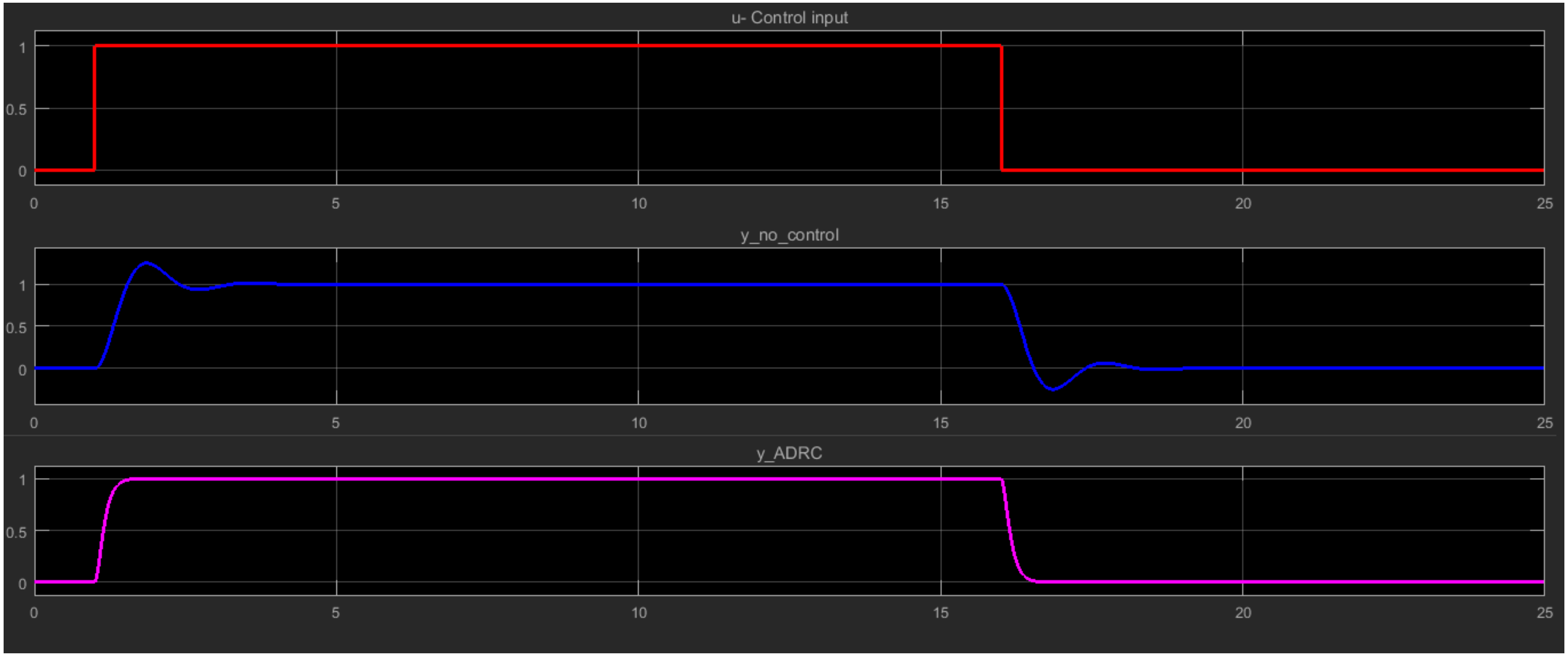
Set critically damped poles at -12

Kp =144

Kd = 24

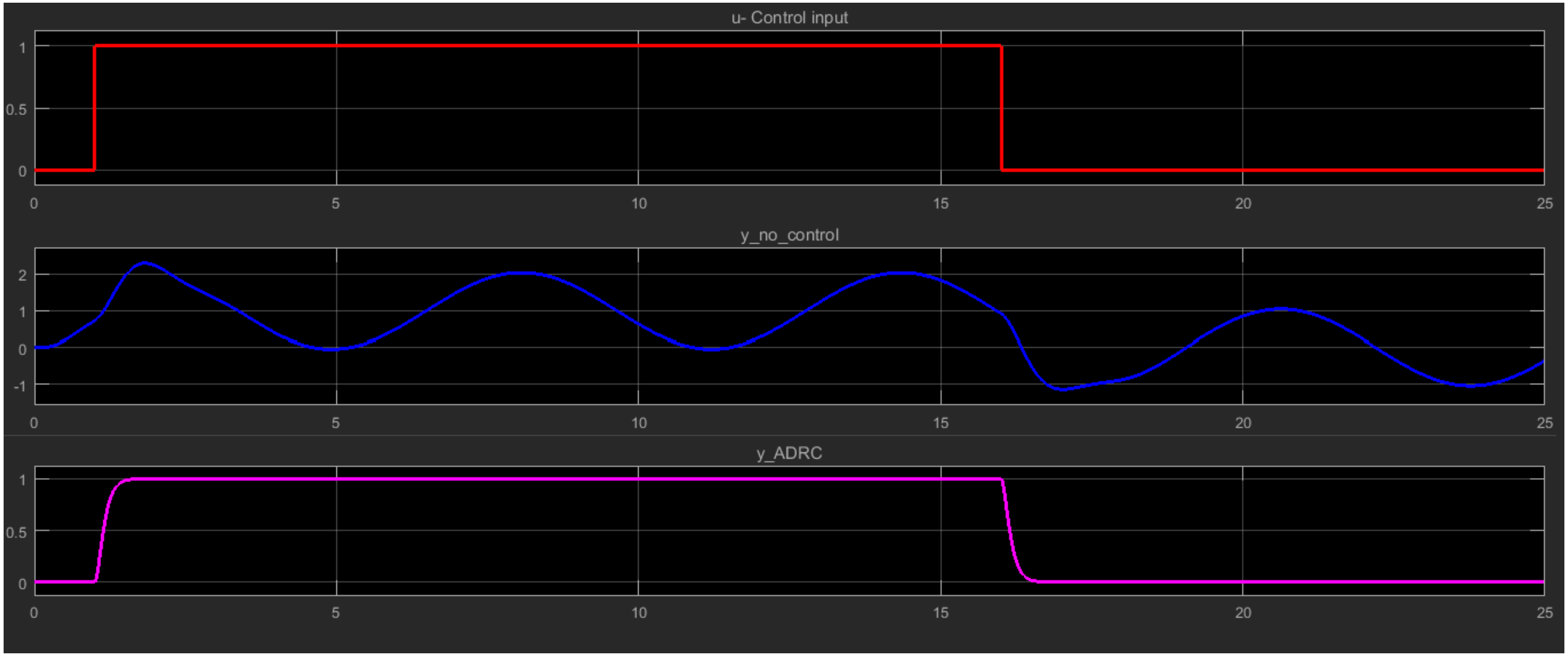
p_obs = -300 (25*faster) for better external disturbance rejection (three eigenvalues)

Response – no disturbance input



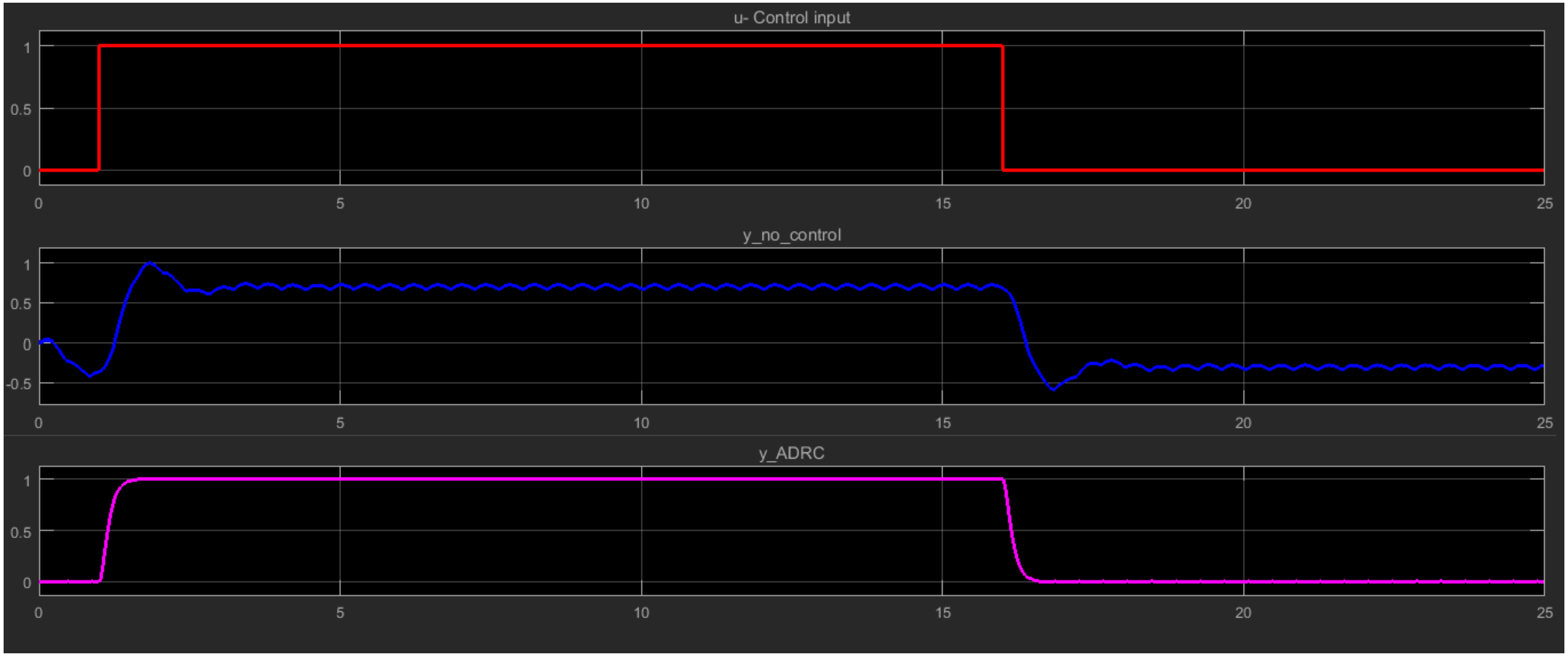
Input step on- step off, response of P(s) no control, ADRC control

Response – sine disturbance input added to input u(s)



Input step on- step off, response of P(s) no control, ADRC control

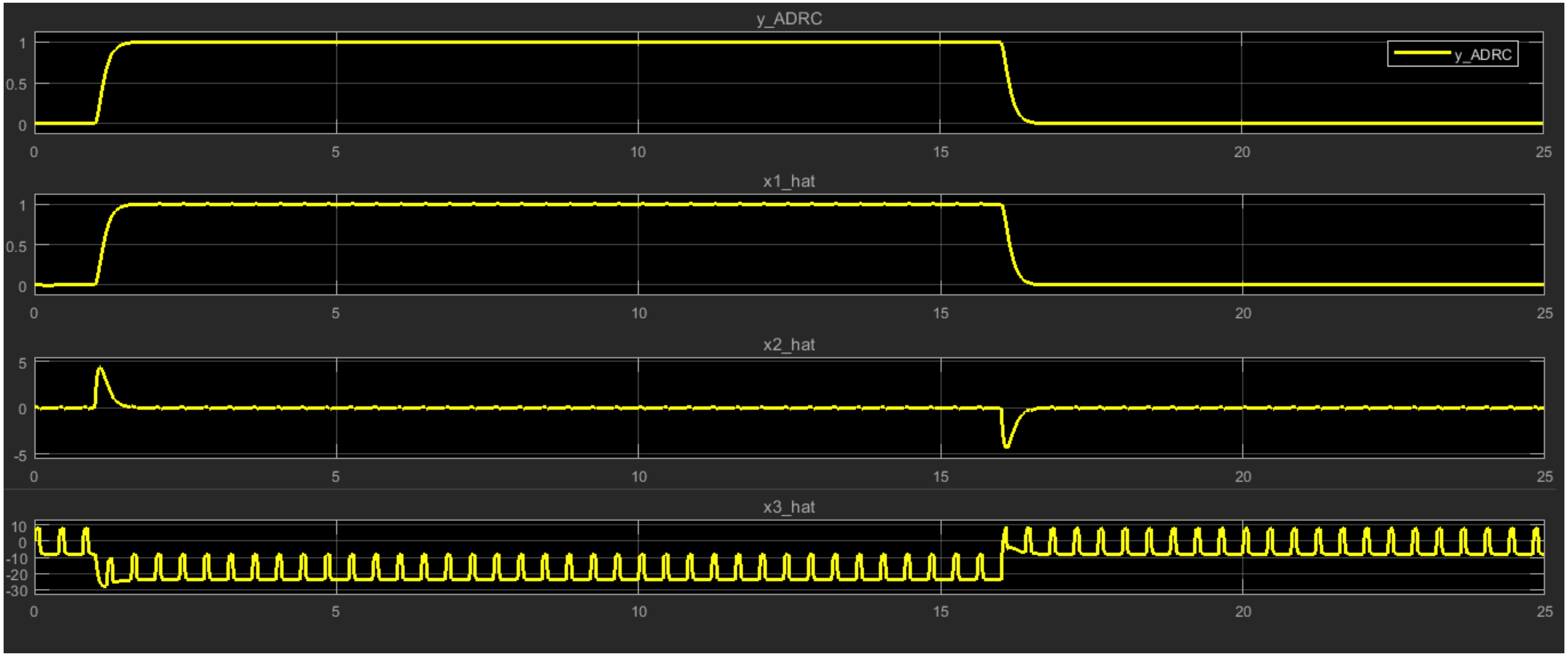
Response – pulse disturbance input



Input step on- step off, response of P(s) no control, ADRC control

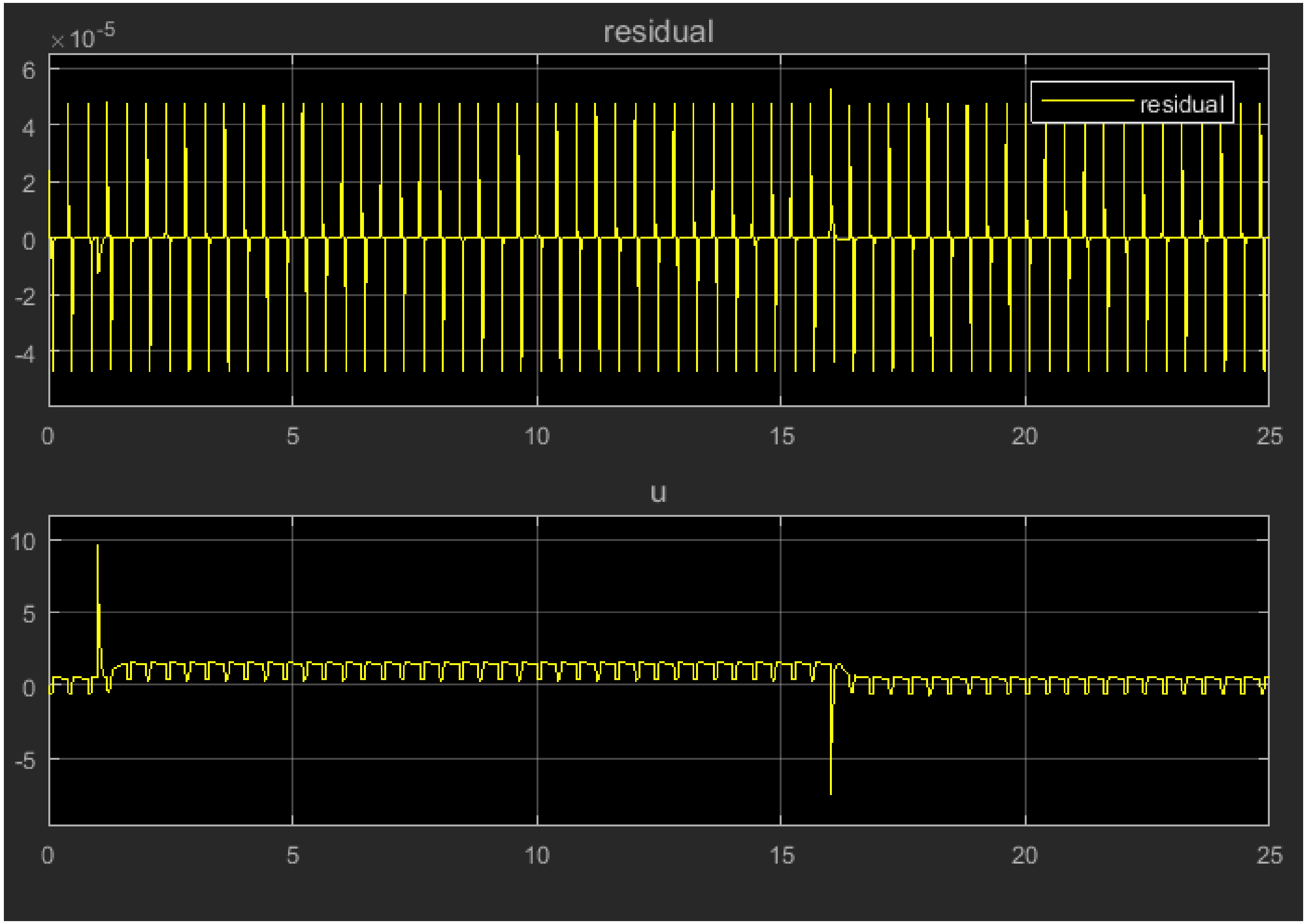
ESO States – pulse disturbance input

states show control to compensate for disturbance

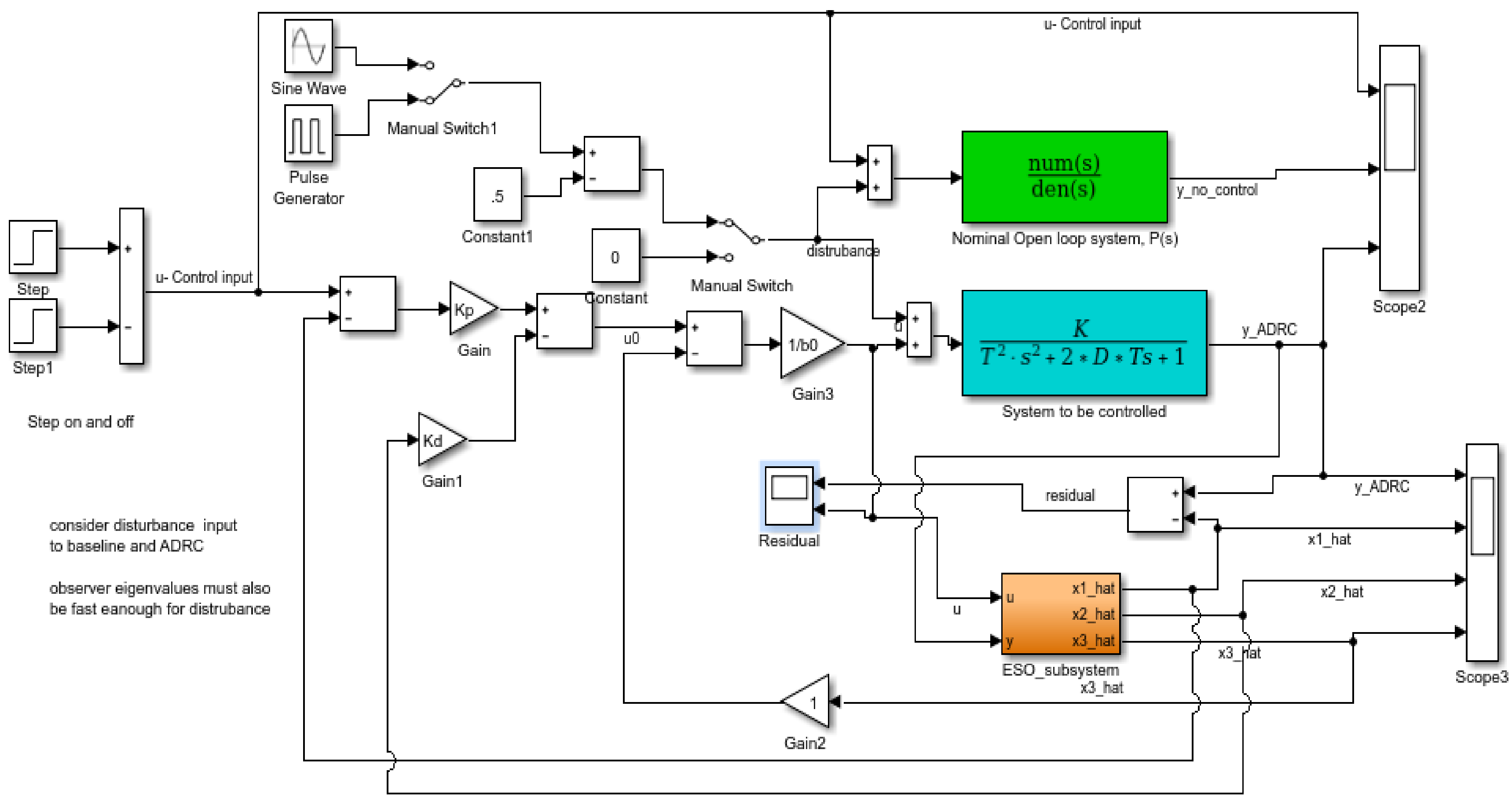


Output of ADRC, estimated states x1, x2, x3

Response – pulse disturbance input residual and u(t)



Simulation Diagram



Simulation

nominal system P(s) replaced with unstable system no changes to tuning parameters

New Unstable plant inserted

Pnew =

$$\frac{16}{s^2 - 3.2s + 16}$$

poles

1.6000 + 3.6661i

1.6000 - 3.6661i

Key Parameters – nominal design

P =

$$\frac{1}{0.0625\, s^2 + 0.2\, s + 1}$$

Eigenvalues

-1.6000 + 3.6661i

-1.6000 - 3.6661i

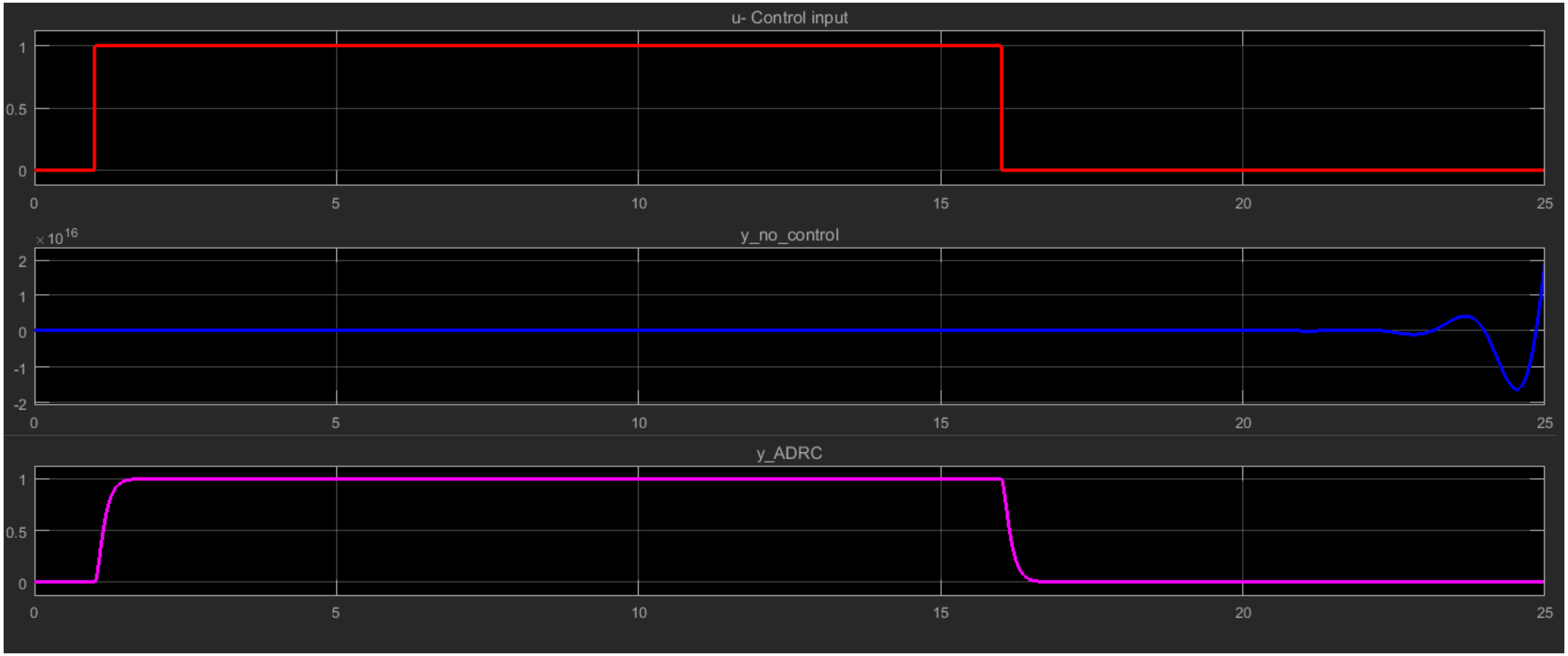
Set critically damped poles at -12

Kp =144

Kd = 24

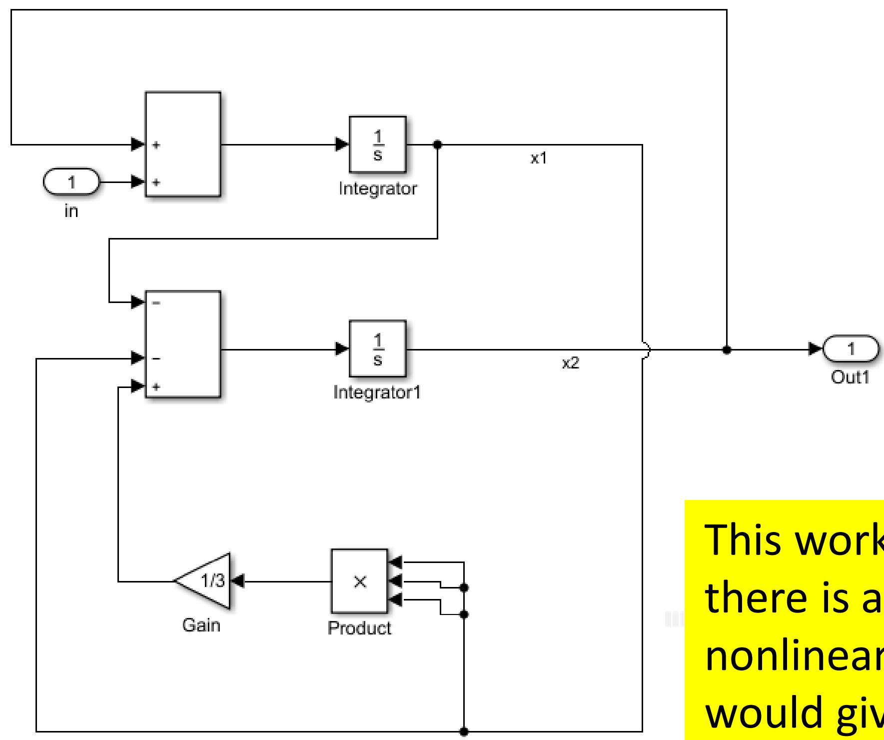
p_obs = -300 (25*faster) for better external disturbance rejection (three eigenvalues)

Response – with unstable plant
all tuning the same and sine wave disturbance



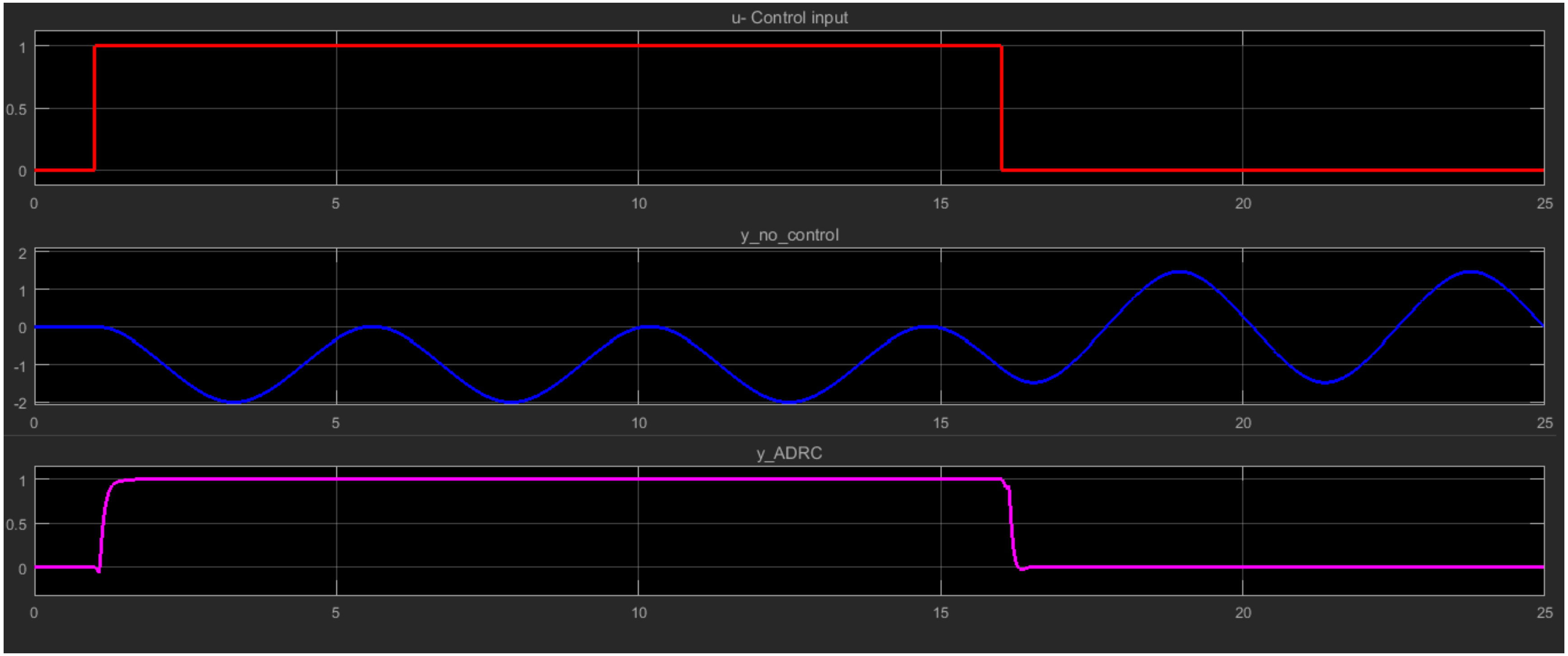
Input step on- step off, response of P(s) no control, ADRC control

Experiment
replace system with 2nd order NL System



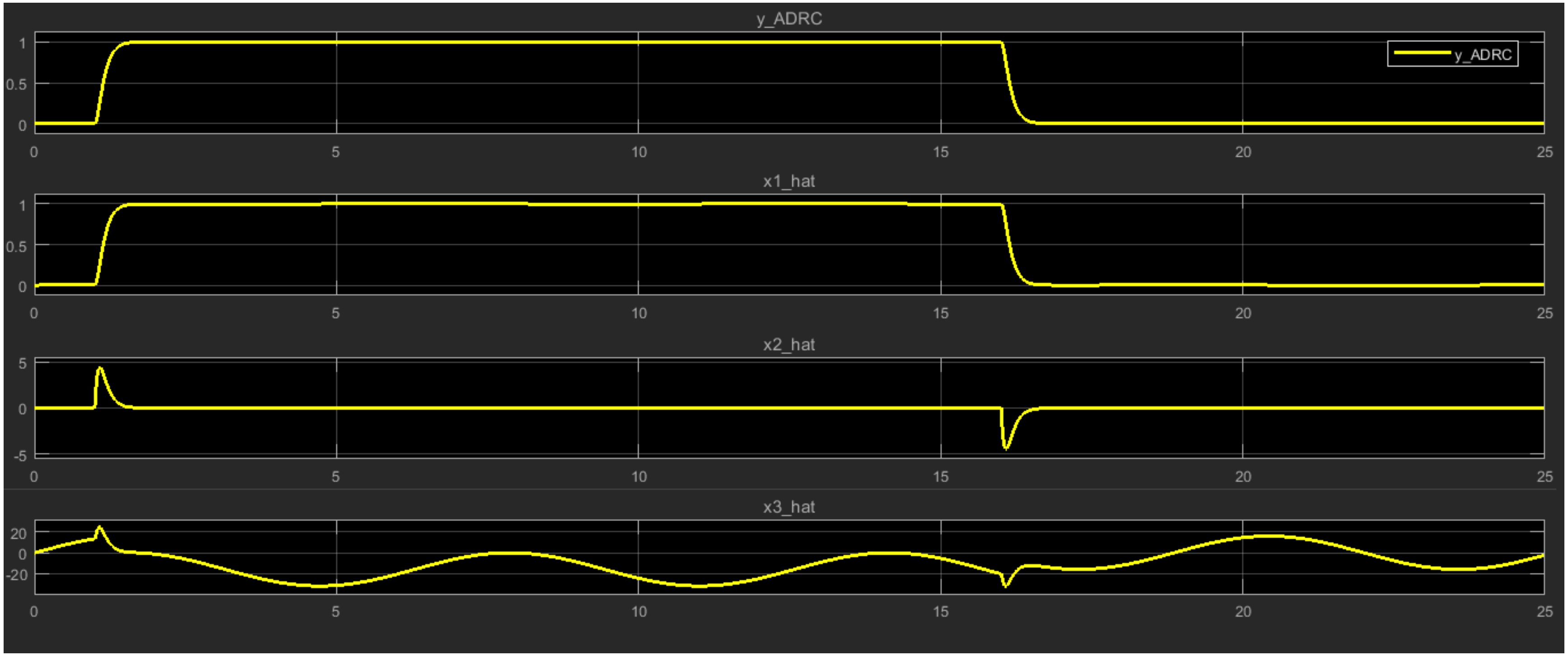
This worked – however there is an ADRC format for nonlinear systems which would give improved results

Response – ADRC with NL Plant and Linear ADRC – no tuning changes



Input step on- step off, response of NL system no control, ADRC control

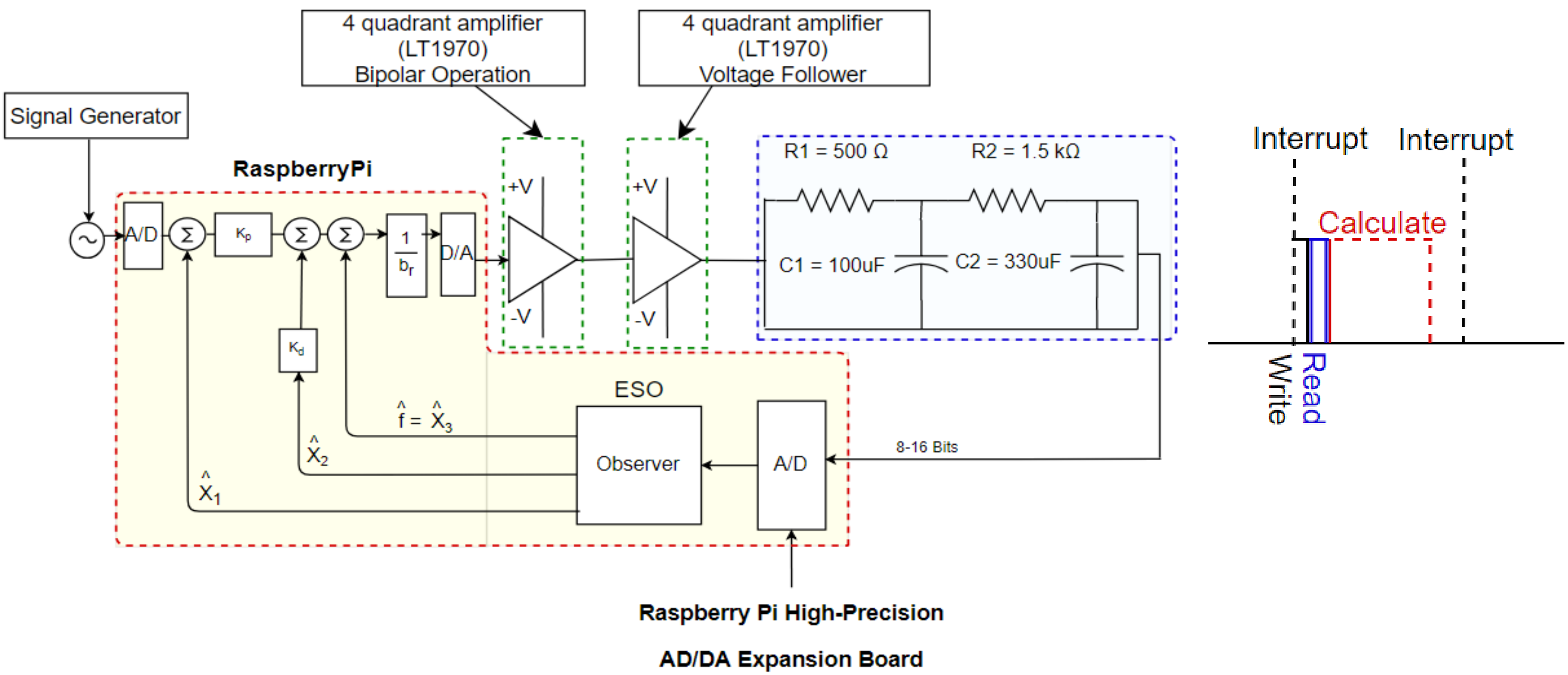
ESO states – ADRC with NL Plant and Linear ADRC – tuning no changes



States

Senior Design Implementation

Hybrid Implementation of ADRC



References

[1] N. S. Nise, Control Systems Engineering. Wiley, 2010, p. 682-689.

[2] T. Chimelewski, “Dr. Chmielewski’s Observer Presentation to Siemen’s Medical Systems Oct 2017”, 2017

[3] Zhiqiang Gao, Yi Huang and Jingqing Han, "An alternative paradigm for control system design," Proceedings of the 40th IEEE Conference on Decision and Control (Cat. No.01CH37228), Orlando, FL, 2001, pp. 4578-4585 vol.5.doi: 10.1109/.2001.980926

[4] G. Herbst, “A Simulative Study on Active Disturbance Rejection Control (ADRC) as a Control Tool for Practitioners,” Electronics, Vol. 2,No. 3, 2013, pp. 246–279. doi:10.3390/electronics2030246

[5] Zhiqiang Gao, "Active disturbance rejection control: a paradigm shift in feedback control system design," 2006 American Control Conference, Minneapolis, MN, 2006, pp. 7 pp.-.doi: 10.1109/ACC.2006.1656579

