Reduced Order Observer Example

See ECE 5512 Week 4 pgs 6 thru 9

2/5/2017

Particle Dynamics $\dot{x} = \left[\begin{array}{c|c} 0 & 1 \\ \hline 0 & 0 \end{array}\right] x + \left[\begin{array}{c} 0 \\ \hline 1/M \end{array}\right] u \qquad y = [1\ 0]\, x$

$x = \begin{bmatrix} pos \\ vel \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ We want to estimate x_2

$M = 1$ for simulations

Slow System $F = -1 \quad l = 1 \quad T_r = [\tau_1, \tau_2] = [1\ -1]$

$$T = \begin{bmatrix} C \\ T_r \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \qquad inv(T) = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \tau_{21} & \tau_{22} \end{bmatrix}$$

In z coords $\begin{bmatrix} x_1 = y \\ \hat{x}_2 \end{bmatrix} = T^{-1} \begin{bmatrix} y \\ z \end{bmatrix}$

$$\dot{z} = Fz - T_r b u + l y \Rightarrow \dot{z} = (-1) z - \left(\tfrac{1}{M}\right) u + (1) y \qquad M = 1$$

Assuming we want IC of $\hat{x}_2 = 1$

$$\Rightarrow \begin{pmatrix} y \\ z \end{pmatrix} = T \begin{bmatrix} y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad z(0) = -1$$

Combining equation $\hat{x}_2 = \tau_{21} y + \tau_{22} z = 1 y - z$

Fast System $F = -10 \quad l = 1 \quad T_r = \left[\frac{1}{10} \ -\frac{1}{100}\right]$

$$T = \begin{bmatrix} 1 & 0 \\ 0.1 & -0.01 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 10 & -100 \end{bmatrix}$$

$$\dot{z} = -10 z - \frac{1}{100} u + (1) y$$

$$IC \Rightarrow T \begin{pmatrix} y \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 10 & -0.01 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \hat{z}(0) = -0.01$$

Combining $\hat{x}_2 = 10 y - 100 z$