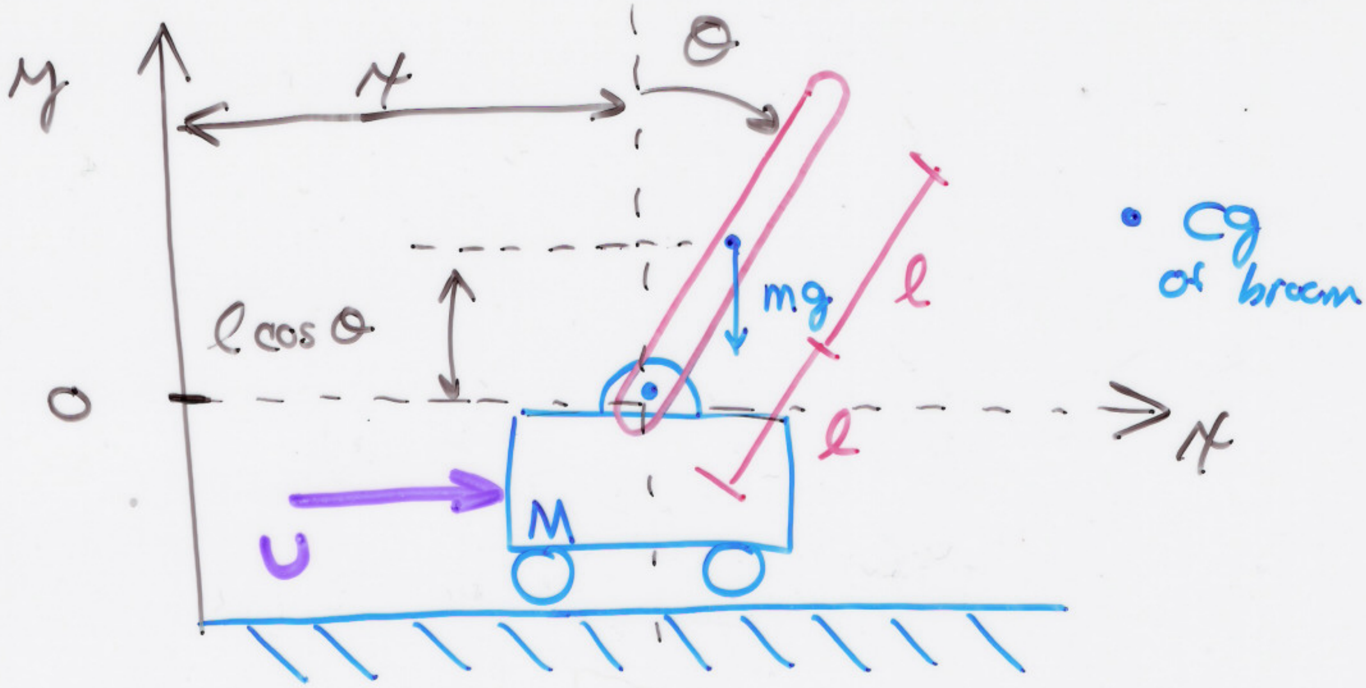
INVERTED PENDULUM / BROOM BALANCER



OBJECTIVE IS TO KEEP PENDULUM VERTICAL

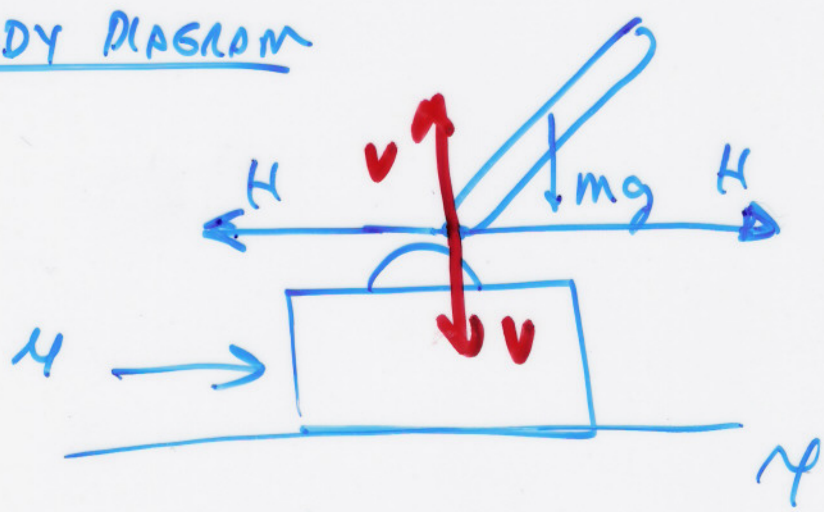
$$X_{\bar{g}} = x + l \sin\theta$$
$$Y_{\bar{g}} = l \cos\theta$$

] LOCATION OF CG OF BROOM

CART AND PENDULUM MOVE IN PLANE OF PAPER

NATURALLY UNSTABLE SYSTEM (FALL DOWN)

FREE BODY DIAGRAM



I PENDULUM ROTATION

$$J\ddot{\theta} = Vl\sin\theta - Hl\cos\theta$$

II HORIZONTAL MOTION OF CG

$$m\frac{d^2}{dt}(x + l\sin\theta) = H$$

III VERTICAL MOTION OF CG

$$m\frac{d^2}{dt}(l\cos\theta) = V - mg$$

IV HORIZONTAL CART MOTION

$$M\frac{d^2}{dt}x = U - H$$

θ SMALL NOMINALLY SET VERTICALLY

$$\theta \approx 0$$

$\sin\theta \approx \theta$ SMALL ANGLE THEORY

$\cos\theta \approx 1$

Ia $J\ddot{\theta} = Vl\theta - Hl$

IIa $m(\ddot{x} + l\ddot{\theta}) = H$

IIIa $0 = V - mg$

IVa $M\ddot{x} = U - H$

LINEARIZATION OF N-L DIFF EQS USING SMALL ANGLE THEORY

EQS IVa AND IIIa

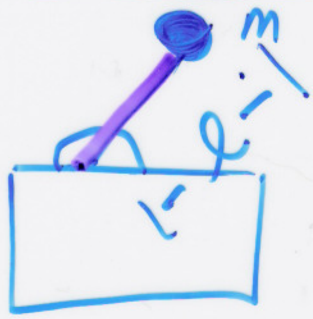
$$\boxed{(M+m)\ddot{x} + ml\ddot{\theta} = u}$$

EQS Ia IIa IIIa

$$\boxed{\ddot{\theta}(J+ml^2) + ml\ddot{x} = mgl\theta}$$

two coupled 2nd order diff EQ's

MASS TO TOP
Length = l



cg of BALL

If we ignore J broom $J \approx 0$
Compared to mass m

$$\boxed{\begin{aligned} &(M+m)\ddot{x} + ml\ddot{\theta} = u && \text{Same} \\ &ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta && \text{Removed } J \end{aligned}}$$

$x_1 = \theta$
$x_2 = \dot{\theta}$
$x_3 = x$
$x_4 = \dot{x}$

Choose SV's
CONVENTIONAL SENSE

Reformulate EQs to be slightly simpler for finding S.V. This would happen if you started with previous EQs and solved for $\frac{d\,sv}{dt} = f(sv, u)$

$$M\ell\ddot{\theta} = (M+m)g\theta - u$$

$$M\ddot{x} = u - mg\theta$$

$$\Rightarrow \dot{x}_2 = \ddot{\theta} = \left[\frac{M+m}{M\ell}g\right]\theta - \left[\frac{1}{M\ell}\right]u$$

$$\dot{x}_4 = \ddot{x} = -\left[\frac{mg}{M}\right]\theta + \left[\frac{1}{M}\right]u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{M+m}{M\ell}g & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ -\frac{mg}{M} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{M\ell}\\ 0\\ \frac{1}{M}\end{bmatrix}u$$

$$\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix} = \vec{sv}$$

Output is position of "broom"

$$y = \theta = [1\ 0\ 0\ 0]\,\vec{sv}$$