

9/24/2010

Finite Form Lagrangian

coupled $\begin{cases} \textcircled{A} Ml\ddot{\theta} = (M+m)g\theta - U \\ \textcircled{B} M\ddot{x} = U - mg\theta \end{cases} \quad \dot{x} = f(x, \theta)$

$$\begin{aligned} \textcircled{1} \quad (M+m)\ddot{x} + ml\ddot{\theta} &= U \\ \textcircled{2} \quad ml^2\ddot{\theta} + ml\ddot{x} &= mgl\theta \end{aligned} \quad \left. \begin{array}{l} \text{From Newtonian} \\ \text{model or} \\ \text{Lagrangian model} \end{array} \right\}$$

sub $l \times \textcircled{1}$ from 2

$$ml^2\ddot{\theta} + ml\ddot{x} - Ml\ddot{x} - ml\ddot{x} - ml\ddot{\theta} = mgl\theta - lU$$

$$Ml\ddot{x} = mgl\theta - lU$$

$$\textcircled{3} \quad \boxed{M\ddot{x} = U - mg\theta} \quad \text{from } \dot{x} = f(x, \theta) \text{ this is EQ B}$$

To get EQ \textcircled{A}

Substitute $\textcircled{3}$ into $\textcircled{2}$

$$ml^2\ddot{\theta} + ml \left[\frac{U}{M} - \frac{mg}{M}\theta \right] = mgl\theta$$

Multiply both sides by $\left(\frac{M}{m}\right)$

$$Ml^2\ddot{\theta} + Ml \left[\frac{U}{m} - \frac{mg}{M}\theta \right] = Mgl\theta$$

Resulting in

$$Ml^2\ddot{\theta} + lU - mgl\theta = Mgl\theta$$

dividing by l

$$\textcircled{4} \quad \boxed{\text{or } Ml\ddot{\theta} = (M+m)g\theta - U}$$

~ End ~