
finding square root of matrix

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Revised 23 April 2013 Dr. Tom Chmielewski Matlab has a built in function to find the square root of a matrix - however we need to be a bit careful to understand how it works in regard to pg 70 of Lewis (Ed III) C in $Q = C'C$ could be a matrix, or vector - we want to use the vector solution. Matlab has a function `sqrtm()`

given a symmetric matrix - this one is also PD with unique eigenvalues

```
Q = [100 5; 5,50]
% note if we start with a symmetric matrix then the square root is also
% symmetric and hence we can use the transpose
```

$Q =$

$$\begin{bmatrix} 100 & 5 \\ 5 & 50 \end{bmatrix}$$

here is the square root note it is symmetric

```
Sq = sqrtm(Q)
```

$Sq =$

$$\begin{bmatrix} 9.9957 & 0.2931 \\ 0.2931 & 7.0650 \end{bmatrix}$$

one form of recovering Q

```
Sq*Sq'
```

$ans =$

$$\begin{bmatrix} 100.0000 & 5.0000 \\ 5.0000 & 50.0000 \end{bmatrix}$$

another form for recovering Q

```
Sq'*Sq
```

ans =

```
100.0000    5.0000
 5.0000    50.0000
```

a final form for recovering Q

Sq*Sq

ans =

```
100.0000    5.0000
 5.0000    50.0000
```

now consider a nonsymmetric matrix that is still PD

QQ = [100, 7; 5, 50]

QQ =

```
100    7
  5   50
```

this is PD since eigenvalues are >0

eig(QQ)

ans =

```
100.6905
 49.3095
```

the square root is not symmetric

Sq1 = sqrtm(QQ)

Sq1 =

```
9.9940    0.4104
 0.2931    7.0626
```

this form gives the wrong answer

Sq1*Sq1 '

ans =

```
100.0481    5.8281
    5.8281    49.9656
```

this form also gives wrong answer

Sq1'*Sq1

ans =

```
99.9656    6.1719
    6.1719    50.0481
```

this form gives right answer

Sq1*Sq1

ans =

```
100.0000    7.0000
    5.0000    50.0000
```

The diagonalization algorithm for square root

```
% digaonalize the matrix finding eigenvalues and eigenvectors
[E D] = eig(Q)
% define the square root as
Sq = E * sqrt(D) * E' % where D is a diagonal matrix and sqrt(D) is formed ...
%                       by taking the square root of the diagonal entries in
%                       D.

% Q = Sq'*Sq
Sq'*Sq
Q
```

E =

```
0.0985    -0.9951
-0.9951    -0.0985
```

D =

```
49.5049    0
    0    100.4951
```

Sq =

| | |
|--------|--------|
| 9.9957 | 0.2931 |
| 0.2931 | 7.0650 |

ans =

| | |
|----------|---------|
| 100.0000 | 5.0000 |
| 5.0000 | 50.0000 |

Q =

| | |
|-----|----|
| 100 | 5 |
| 5 | 50 |

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