
Modal_Example_Week7

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Computation of modal matrix using numeric codes in Matlab

start with diagonal form with roots $\{-1, -5, -2 \pm j3\}$

```
format rat
J = [-1      0      0      0;...
      0     -5      0      0;...
      0      0    -2+j*3    0;...
      0      0      0    -2-j*3]

% define QB as on pg 11 of Week 7 notes
QB = [1      0      0      0;...
      0      1      0      0;...
      0      0     1/2    -j*(1/2);...
      0      0     1/2     j*(1/2)]

J_in_M = inv(QB)*J*QB % "modal" form
```

note: if you put one complex number in an array, Matlab generally will display the array as shown below

$J =$

Columns 1 through 2

-1	+	0i	0	+	0i
0	+	0i	-5	+	0i
0	+	0i	0	+	0i
0	+	0i	0	+	0i

Columns 3 through 4

0	+	0i	0	+	0i
0	+	0i	0	+	0i
-2	+	3i	0	+	0i
0	+	0i	-2	-	3i

$QB =$

Columns 1 through 2

$$\begin{array}{rclclcl} 1 & + & 0i & 0 & + & 0i \\ 0 & + & 0i & 1 & + & 0i \\ 0 & + & 0i & 0 & + & 0i \\ 0 & + & 0i & 0 & + & 0i \end{array}$$

Columns 3 through 4

$$\begin{array}{rclclcl} 0 & + & 0i & 0 & + & 0i \\ 0 & + & 0i & 0 & + & 0i \\ 1/2 & + & 0i & 0 & - & 1/2i \\ 1/2 & + & 0i & 0 & + & 1/2i \end{array}$$

$J_{in_M} =$

$$\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & -3 & -2 \end{array}$$

Computation of modal matrix using symbolic variables and tools

this surpsres the +j0 entries in the JB martix for real entries remember these are symbolic variables see note at end of the cell

```
clear all;
clc
J = sym([-1      0      0      0;...
         0      -5      0      0;...
         0      0     -2+j*3  0;...
         0      0      0     -2-j*3])

% define QB as on pg 11 of Week 7 notes
QB = sym([1      0      0      0;...
          0      1      0      0;...
          0      0     1/2    -j*(1/2);...
          0      0     1/2     j*(1/2)])

J_in_M = inv(QB)*J*QB
% one way to to get back to numeric so you can use in numeric program
J_in_M_num= eval(J_in_M)

J =

[ -1,  0,      0,      0]
[  0, -5,      0,      0]
[  0,  0, - 2 + 3i,      0]
```

$[\quad 0, \quad 0, \quad \quad \quad 0, \quad -2 - 3i]$

$QB =$

$[\quad 1, \quad 0, \quad \quad 0, \quad \quad 0]$
 $[\quad 0, \quad 1, \quad \quad 0, \quad \quad 0]$
 $[\quad 0, \quad 0, \quad 1/2, \quad -1i/2]$
 $[\quad 0, \quad 0, \quad 1/2, \quad 1i/2]$

$J_{in_M} =$

$[\quad -1, \quad 0, \quad 0, \quad 0]$
 $[\quad 0, \quad -5, \quad 0, \quad 0]$
 $[\quad 0, \quad 0, \quad -2, \quad 3]$
 $[\quad 0, \quad 0, \quad -3, \quad -2]$

$J_{in_M_num} =$

-1	0	0	0
0	-5	0	0
0	0	-2	3
0	0	-3	-2

Finding $Q \cdot Q_bar = inv(P)$ see pg 11 notes bottom

this is direct transform to get modal form from A start with A in PVCV similar to above diagonal form with roots $\{-1, -5, -2 \pm j3\}$

```
den_coeff = conv([conv([1 1], [1,5])], [conv([1 2+j*3],[1 2-j*3])]) %
    coeff of CE
% validate by finding roots
roots(den_coeff)
coeffs= fliplr(den_coeff) % flip so we can put in PVCF matrix easily
A = [0 1 0 0; 0 0 1 0; 0 0 0 1; -coeffs(1:4)] % put in PVCF (only 1st
    4 coeffs)
[V, D] = eig(A) % eig works since no repeated roots and no GEVs
```

$den_coeff =$

1	10	42	98	65
---	----	----	----	----

$ans =$

-5	+	0i
-2	+	3i

$$\begin{array}{rcl} -2 & - & 3i \\ -1 & + & 0i \end{array}$$

coeffs =

$$\begin{array}{ccccc} 65 & & 98 & & 42 & & 10 & & 1 \end{array}$$

A =

$$\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -65 & -98 & -42 & -10 \end{array}$$

V =

Columns 1 through 2

$$\begin{array}{rclcl} 71/9058 & + & 0i & -176/8749 & + & 94/23883i \\ -97/2475 & + & 0i & 479/16851 & - & 430/6303i \\ 97/495 & + & 0i & 997/6745 & + & 3040/13711i \\ -97/99 & + & 0i & -13206/13745 & + & 0i \end{array}$$

Columns 3 through 4

$$\begin{array}{rclcl} -176/8749 & - & 94/23883i & 1/2 & + & 0i \\ 479/16851 & + & 430/6303i & -1/2 & + & 0i \\ 997/6745 & - & 3040/13711i & 1/2 & + & 0i \\ -13206/13745 & + & 0i & -1/2 & + & 0i \end{array}$$

D =

Columns 1 through 2

$$\begin{array}{rclcl} -5 & + & 0i & 0 & + & 0i \\ 0 & + & 0i & -2 & + & 3i \\ 0 & + & 0i & 0 & + & 0i \\ 0 & + & 0i & 0 & + & 0i \end{array}$$

Columns 3 through 4

$$\begin{array}{rclcl} 0 & + & 0i & 0 & + & 0i \\ 0 & + & 0i & 0 & + & 0i \\ -2 & - & 3i & 0 & + & 0i \\ 0 & + & 0i & -1 & + & 0i \end{array}$$

as seen the complex conjugate roots of *D* are in the two center locations corresponding to cols 2 and 3 of *V* this was a Matlab ordering - you have to check

notation from lecture pg 11

```

QQB = [V(:,1), real(V(:,2)), imag(V(:,3)), V(:,4)] % where A = QQB
Modal inv(QB)Inv(Q)
% starting with A we get modal for and bypass diagonal with complex
% roots
% on diagonal. the basic algorithm is to find the cols in V
% corresponding
% to the complex roots and make the first col real components the
% second
% imag components
format short
QQB

```

QQB =

71/9058	-176/8749	-94/23883	1/2
-97/2475	479/16851	430/6303	-1/2
97/495	997/6745	-3040/13711	1/2
-97/99	-13206/13745	0	-1/2

QQB =

0.0078	-0.0201	-0.0039	0.5000
-0.0392	0.0284	0.0682	-0.5000
0.1960	0.1478	-0.2217	0.5000
-0.9798	-0.9608	0	-0.5000

note no imaginary cols are in QQB

MM = inv(QQB)*A*QQB

MM =

-5.0000	0.0000	0.0000	-0.0000
-0.0000	-2.0000	-3.0000	-0.0000
0.0000	3.0000	-2.0000	0.0000
0.0000	0.0000	0	-1.0000

Modal form

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