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## CASE I

```
close all
%setting initial condition
A = [-5.0];
B = [2.0];
C = [1.0];
D = [0];
t = 0.1;
IC = 1;
U = 0;
sys = ss(A,B,C,D);
[Yc, Tc] = initial(sys, IC, 0:t:2);

% State Discretition
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);
yd = [];
yd(1) = 1;
Td = [];
Td(1) = 0;
% Exact Discrete
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
```

---

## Part 1

```
fprintf("After discretization Ad = expm(-5*t) Bd = ((-2/5)*expm(-5*t)) + (2/5)")  
fprintf("State Equation: X[k+1] = Ad*X[k] + Bd*U[k]")  
fprintf("Y[k] = X[k]")  
  
After discretization Ad = expm(-5*t) Bd = ((-2/5)*expm(-5*t)) + (2/5)  
State Equation: X[k+1] = Ad*X[k] + Bd*U[k]Y[k] = X[k]
```

## Part 2

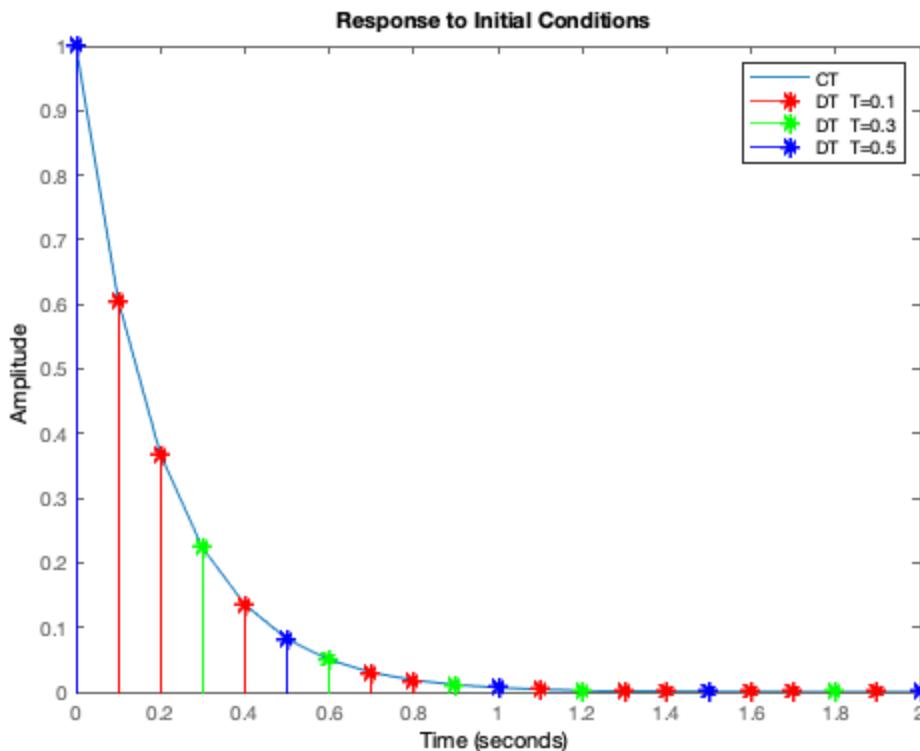
```
figure  
hold on  
initial(sys, IC, 0:0.1:2)  
  
yd = [];  
yd(1) = 1;  
Td = [];  
Td(1) = 0;  
% Exact Discrete  
t = 0.1;  
Ad = expm(-5*t);  
Bd = ((-2/5)*expm(-5*t)) + (2/5);  
for i = 1:length(Tc)  
    yd(i+1) = Ad*yd(i) +(Bd*U);  
    Td(i+1) = i; % save corresponding discrete time integer index  
end  
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');  
ed.Color = 'red';  
t = 0.3;  
Ad = expm(-5*t);  
Bd = ((-2/5)*expm(-5*t)) + (2/5);  
for i = 1:length(Tc)  
    yd(i+1) = Ad*yd(i) +(Bd*U);  
    Td(i+1) = i; % save corresponding discrete time integer index  
end  
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');  
ed.Color = 'green';  
t = 0.5;  
Ad = expm(-5*t);  
Bd = ((-2/5)*expm(-5*t)) + (2/5);  
for i = 1:length(Tc)  
    yd(i+1) = Ad*yd(i) +(Bd*U);  
    Td(i+1) = i; % save corresponding discrete time integer index  
end  
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');  
ed.Color = 'blue';  
legend('CT','DT T=0.1', 'DT T=0.3', 'DT T=0.5')  
  
hold off  
fprintf("\nThe continuous time signal is stable because as t approaches infinity, the amplitude approaches 0")
```

---

```
fprintf("\nThe discrete time signal is stable for any time T > 0 can  
be seen from the graph as well for T = 0.1, 0.3, 0.5, the amplitude  
also approaches 0")
```

The continuous time signal is stable because as  $t$  approaches infinity, the amplitude approaches 0

The discrete time signal is stable for any time  $T > 0$  can be seen from the graph as well for  $T = 0.1, 0.3, 0.5$ , the amplitude also approaches 0

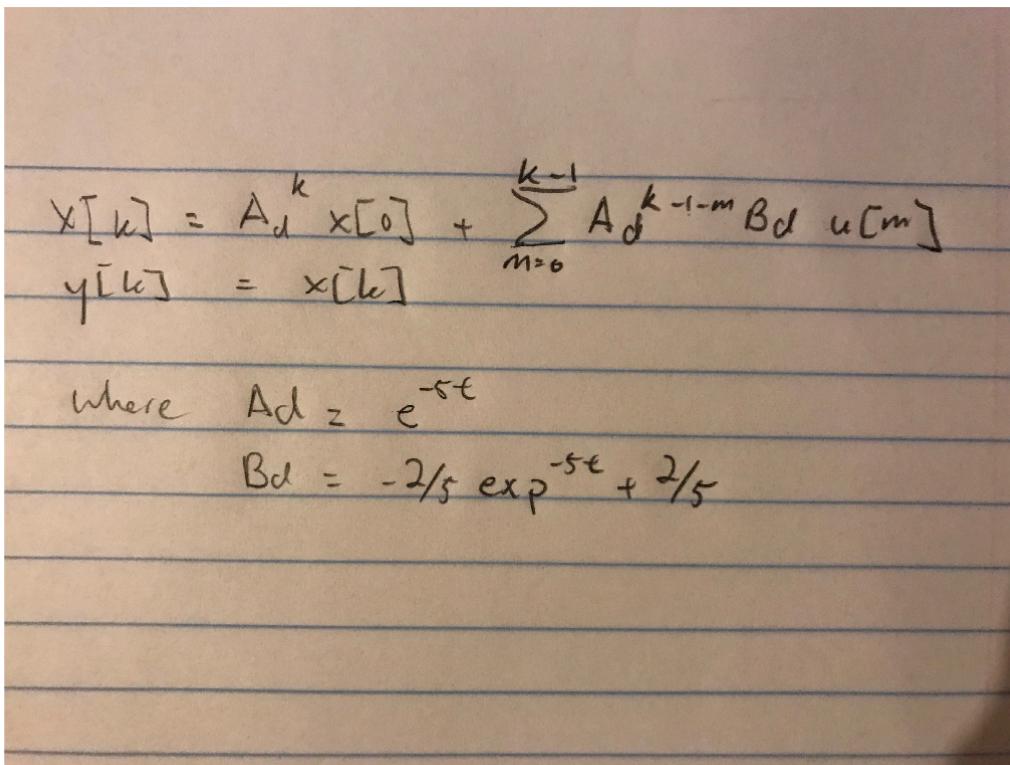


## Part 3a

```
figure  
fprintf("\nClose form continuous signal is x(t) = e^(A*t) * x(0) \n  
y(t) = x(t)")  
I = imread('d.jpg');  
imshow(I)
```

Close form continuous signal is  $x(t) = e^{(A*t)} * x(0)$   
 $y(t) = x(t)$ Warning: Image is too big to fit on screen; displaying at 50%

---



$$x[k] = A_d^k x[0] + \sum_{m=0}^{k-1} A_d^{k-1-m} B_d u[m]$$

$$y[k] = x[k]$$

where  $A_d = e^{-5t}$

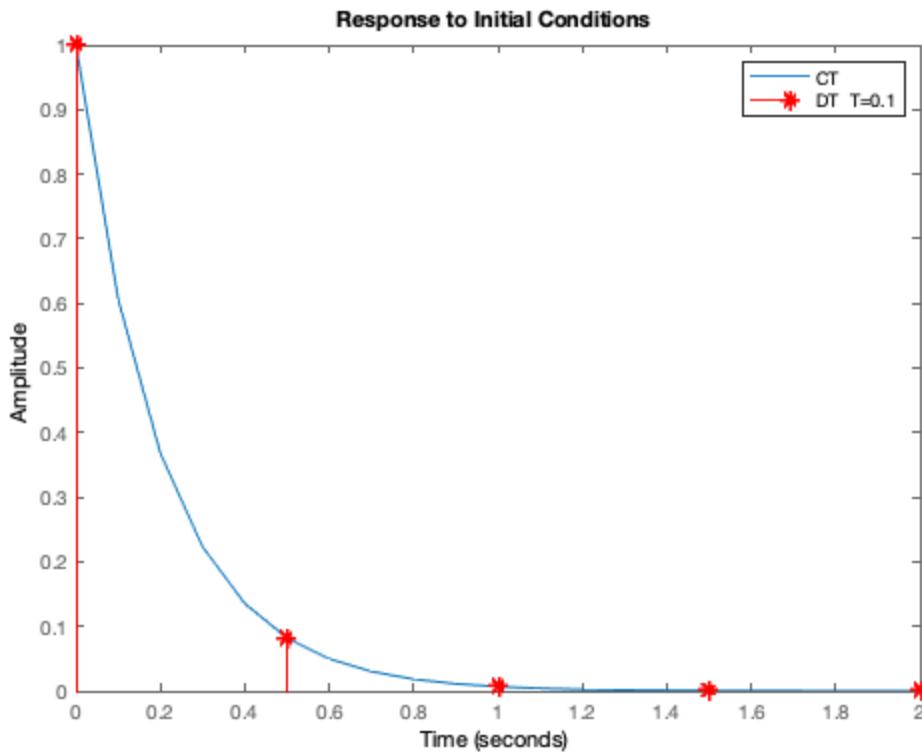
$$B_d = -\frac{2}{5} \exp^{-5t} + \frac{2}{5}$$

## Part 3b

```

figure
hold on
initial(sys, IC, 0:0.1:2)
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';
legend('CT','DT T=0.1')

```



## Part 3c

```

figure
hold on
initial(sys, IC, 0:0.1:2)

yd = [];
yd(1) = 1;
Td = [];
Td(1) = 0;
% Exact Discrete
t = 0.1;
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';
t = 0.3;
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);

```

---

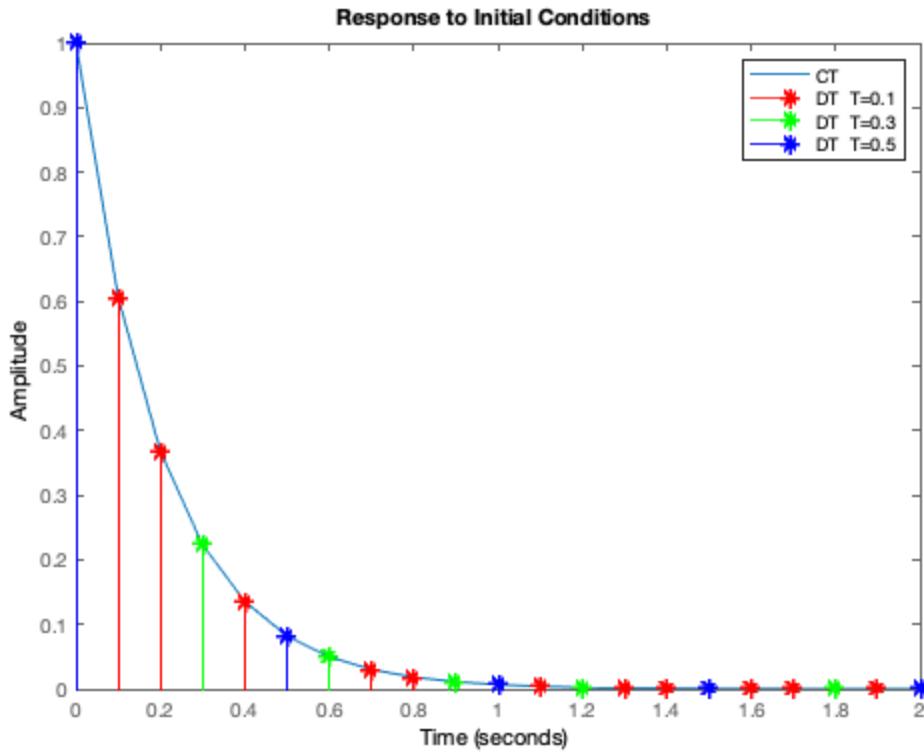
```

Td(i+1) = i; % save corresponding discrete time integer index
end
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'green';
t = 0.5;
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
ed = stem(Td*t,yd, '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'blue';
legend('CT','DT T=0.1', 'DT T=0.3', 'DT T=0.5')

hold off
fprintf("\nWhen T increases the discrete time signal matches better
with continous time. When T decreases the DT signal loses alot of
information but in general the discrete time signal is stable for all
T")

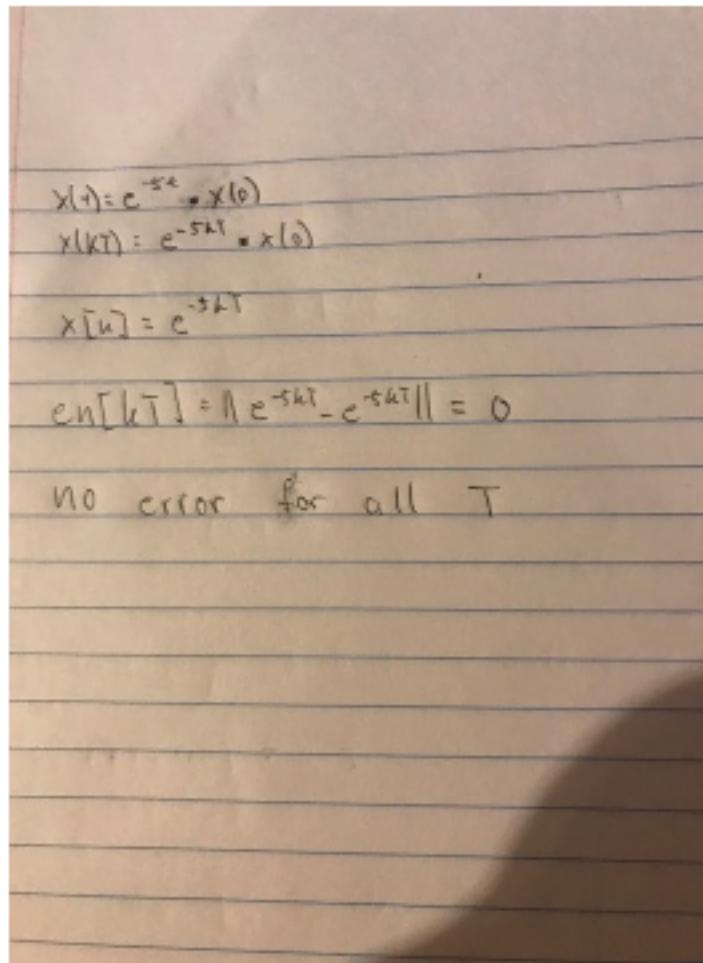
```

*When  $T$  increases the discrete time signal matches better with continuous time. When  $T$  decreases the DT signal loses alot of information but in general the discrete time signal is stable for all  $T$*



## Part 3d

```
figure  
I = imread('caselerr.jpg');  
imshow(I)
```



## Part 4a

```
figure  
I = imread('caselpart4ct.jpg');  
imshow(I)  
figure  
I = imread('caselpart4dt.jpg');  
imshow(I)
```

CT:  $\dot{x} = -5x + 2u$      $y = x$      $w(s) = 1$

$A = -5$     $B = 2$     $C = 1$     $D = 0$

$$H(s) = C(sI - A)^{-1}B + D$$

$$1 \cdot \frac{1}{s+5} \cdot 2 = \left[ \frac{2}{s+5} = H(s) \right]$$

DT:  $A = -5$     $B = 2$     $C = 1$     $D = 0$

$$H(z) = C \cdot (zI - A)^{-1} \cdot B$$

$$1 \cdot \frac{1}{z+5} \cdot 2 = \left[ \frac{2}{z+5} \right] = H(z)$$

## Part 4b

```

figure
step(sys, 0:0.01:2);
hold on;
U = 1;
yd(1) = 0;
t = 0.1;
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
ed = stem(Td*t,yd, 'filled', 'DisplayName', 'Exact Discrete');
ed.Color = 'blue';
t = 0.3;
Ad = expm(-5*t);

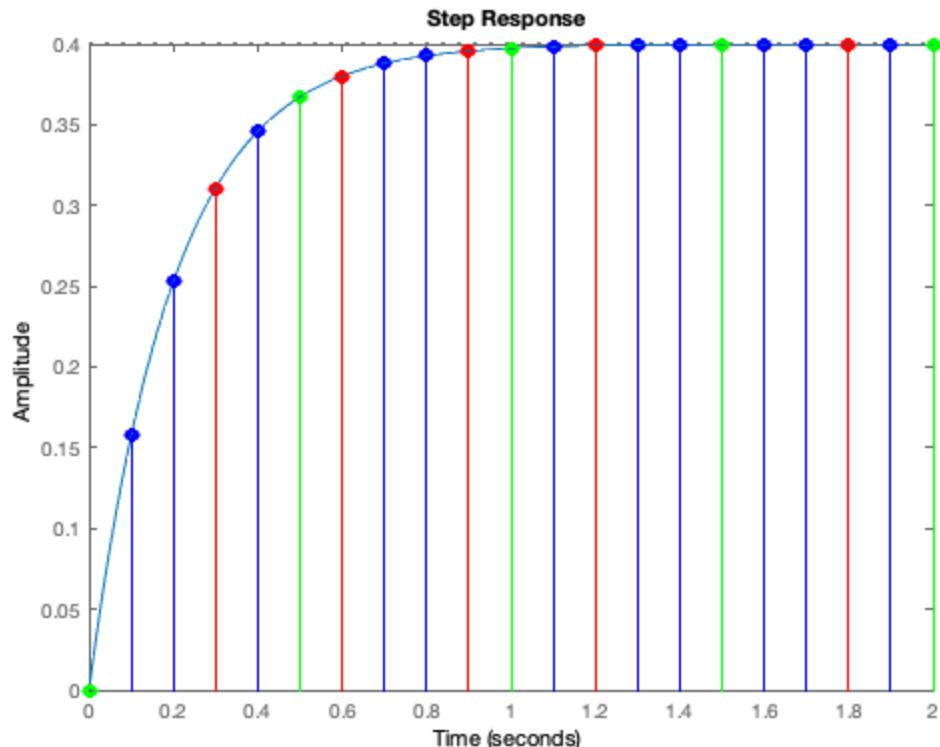
```

---

```

Bd = ((-2/5)*expm(-5*t)) + (2/5);
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
ed = stem(Td*t,yd, 'filled', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';
t = 0.5;
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
ed = stem(Td*t,yd, 'filled', 'DisplayName', 'Exact Discrete');
ed.Color = 'green';

```



## Case II

### Part 1

```

figure
I = imread('case2part1.jpg');
imshow(I)

```

---

Warning: Image is too big to fit on screen; displaying at 33%

Forward Difference

$$\begin{aligned} x[k+1] &= (I + AT)x[k] + BTu[k] \\ y[k] &= x[k] \end{aligned}$$

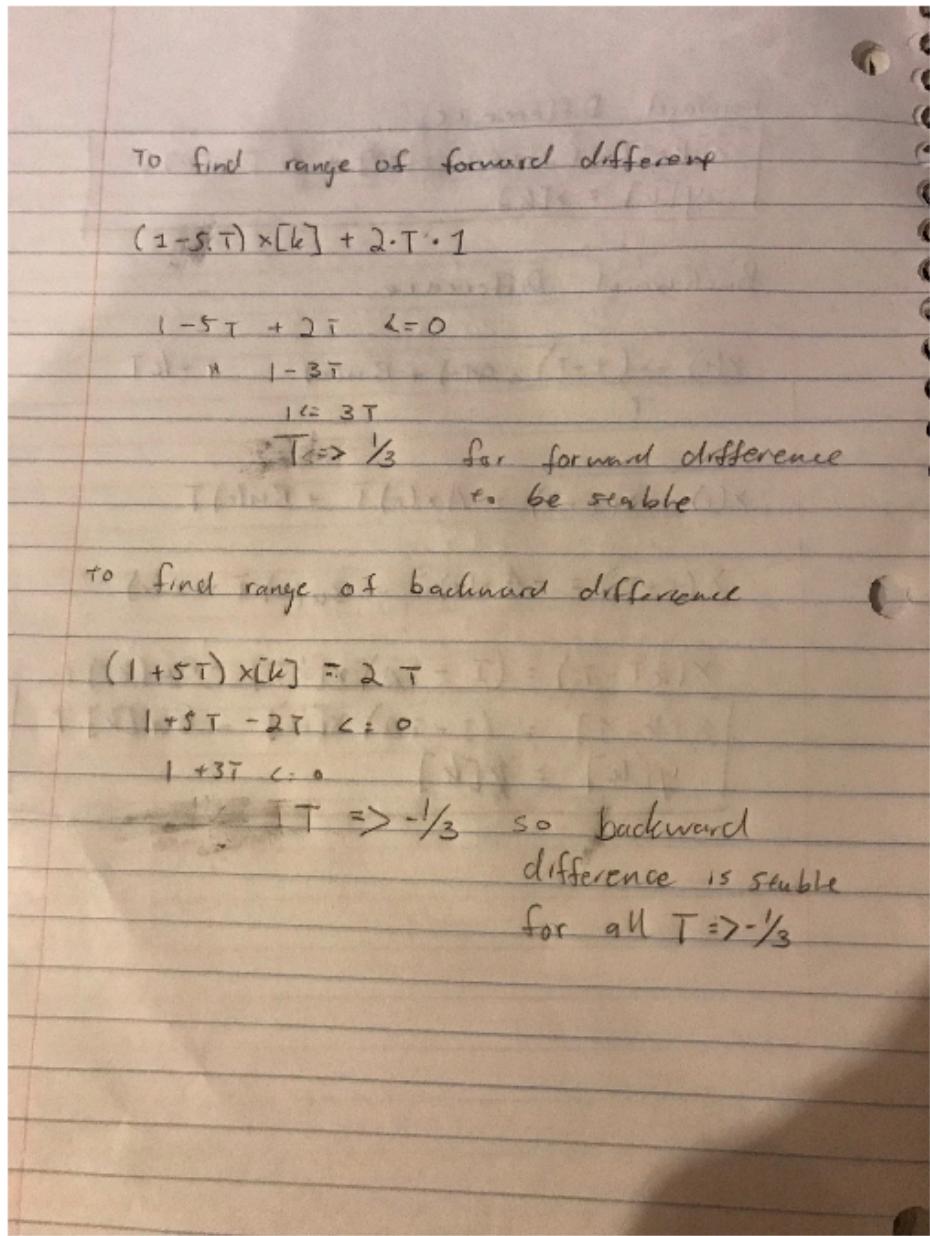
Backward Difference

$$\frac{x(t) - x(t - T)}{T} = Ax(t) + Bu(t) \quad t = kT$$
$$x(t) - x(t - T) = Ax(t)T + Bu(t)T$$
$$x(t - T) = -Ax(t)T - Bu(t)T + x(t)$$
$$x(kT - i) = (I - iTA)x[kT] - Bu(kT)T$$
$$\begin{aligned} x[k-1] &= (I - iTA)x[k] - Bu[k]T \\ y[k] &= x[k] \end{aligned}$$

## Part 2

```
figure  
I = imread('case2part2.jpg');  
imshow(I)
```

Warning: Image is too big to fit on screen; displaying at 33%



To find range of backward difference

$$(1+5T) \times [k] = 2 \cdot T$$

$$1+5T - 2T < 0$$

$$1+3T < 0$$

$T \Rightarrow -\frac{1}{3}$  so backward difference is stable  
 for all  $T \Rightarrow -\frac{1}{3}$

## Part 3a

```
figure
I = imread('case2part3a.jpg');
imshow(I)
```

Warning: Image is too big to fit on screen; displaying at 33%

forward difference

$$x[1] = (1+AT)x[0] + B \cdot T \cdot u[0]$$

$$x[2] = (1+AT)x[1] + B \cdot T \cdot u[1]$$

$$= (1+AT)^2 x[0] + (1+AT)B \cdot T \cdot u[0] + B \cdot T \cdot u[1]$$

$$x[3] = (1+AT)x[2] + B \cdot T \cdot u[2]$$
  

$$x[k] = (1+AT)^k x[0] + \sum_{m=1}^k (1+AT)^{k-m} B \cdot T \cdot u[m]$$

$$y[k] = x[k]$$

Backward difference

$$x[2] = x[0] \frac{1}{(1-AT)} - \frac{Bu[0]T}{(1-AT)}$$

$$x[3] = x[1] \frac{1}{(1-AT)} - \frac{Bu[1]T}{(1-AT)}$$

$$= x[0] \frac{1}{(1-AT)^2} - x[0] \frac{Bu[0]T}{(1-AT)} - \frac{Bu[1]T}{(1-AT)}$$

$$x[4] = x[0] \frac{1}{(1-AT)^3} + \sum_{m=1}^k \frac{Bu[m]T}{(1-AT)^{k-m+1}}$$

$$y[k] = x[k]$$

## Part 3b

```

figure
I = imread('case2part3b.jpg');
imshow(I)
fprintf("within the range of stability, smaller the period, the higher
        the lower the error between the estimates\n")
fprintf("this is because as you sample more frequently the estimates
        gets better matched to the original signal\n")

```

---

```
fprintf("Larger the K value, more time for the method to converge  
hence lowering the error")  
fprintf("The result for this is different compared to Case 1 part 3d  
because 3d was an exact solution, while this is an estimate, so 3d  
has no error component, while forward and backward methods do")  
fprintf("\n see next part for graph")  
  
Warning: Image is too big to fit on screen; displaying at 33%  
within the range of stability, smaller the period, the higher the  
lower the error between the estimates  
this is because as you sample more frequently the estimates gets  
better matched to the original signal  
Larger the K value, more time for the method to converge hence  
lowering the errorThe result for this is different compared to Case 1  
part 3d because 3d was an exact solution, while this is an estimate,  
so 3d has no error component, while forward and backward methods do  
see next part for graph
```

forward error norm zero input

$$x(kT) = e^{-5kT}$$

$$x[k] = (1 - 5T)^k$$

$$\text{err}[kT] = \|e^{-5kT} - (1 + 5T)^k\|$$

Backward err norm zero input

$$x(kT) = e^{-5kT}$$

$$x[k] = \frac{1}{(1 + 5T)^k}$$

$$\text{err}[kT] = \left\| e^{-5kT} - \frac{1}{(1 + 5T)^k} \right\|$$

## Part 3c

```
close all
A = [-5.0];
B = [2.0];
C = [1.0];
D = [];
t = .1;
IC = 1;
U = 0;
```

---

```

% Forward Difference
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);

sys = ss(A,B,C,D);
[Yc, Tc] = initial(sys, IC, 0:0.01:2);
hold on;
initial(sys, IC, 0:0.01:2)
ybd = [];
yd = [];
yd(1) = 1;
Td = [];
Td(1) = 0;
% Exact Discrete
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
hold on;
ed = stem(Td(1:11)*t,yd(1:11), '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';

yfd = [];
yfd(1) = 1;
Td = [];
Td(1) = 0;
% Forward Difference
for i = 1:length(Tc)
    yfd(i+1) = (1+(t*A))*yfd(i) +(B*t*U);
    Td(i+1) = i;
end
hold on;
fd = stem(Td(1:11)*t,yfd(1:11), 'filled','DisplayName', 'Forward
Difference');
fd.Color = 'green';
ybd = zeros(1, length(Td));
ybd(1) = 1;
% Backward Difference
for k = 2:length(Td)
    z = 0;
    for m = 1:k
        z = z + 1/(1-A*t)^(k-m+1) * (B*t*U);
    end
    ybd(k) = ybd(1)*(1/((1-A*t)^k)) + z;
end
bd = stem(Td(1:11)*t,ybd(1:11), 'filled','DisplayName', 'Backward
Difference');
bd.Color = 'blue';
legend('CT', 'CT(kT)', 'Fwd', 'Back')

en_forward = [];
en_backward = [];
t = 0.1;

```

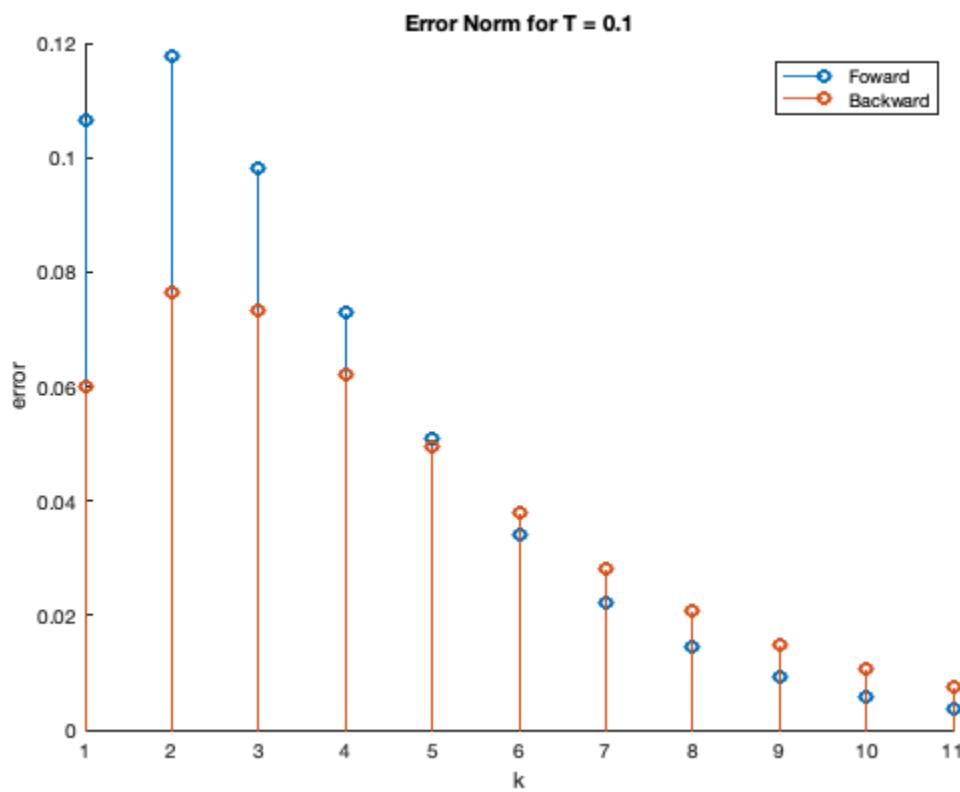
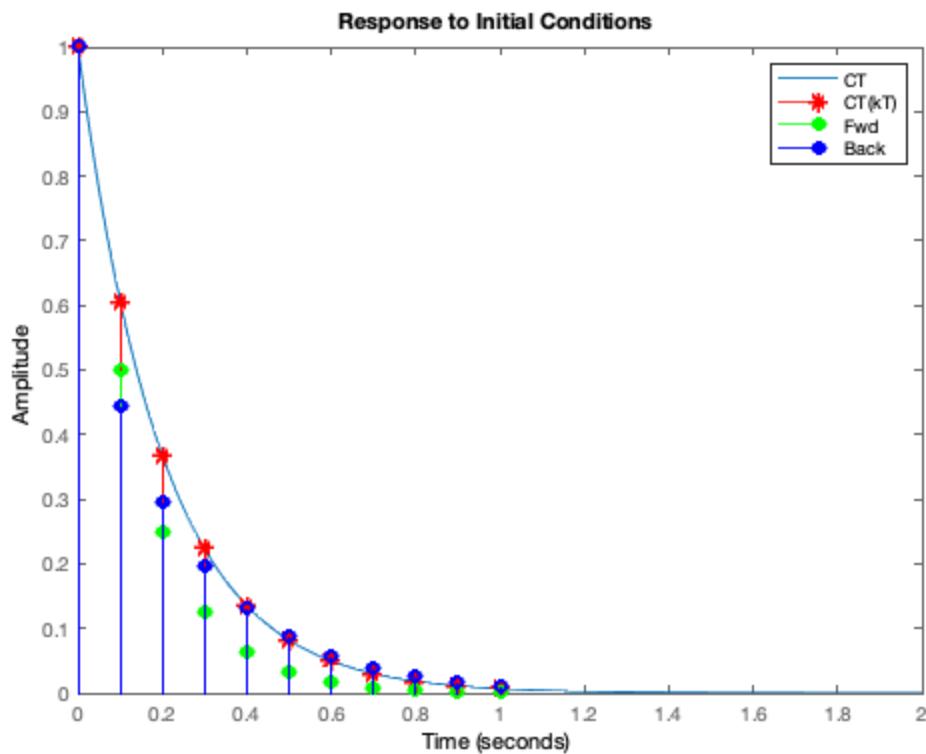
---

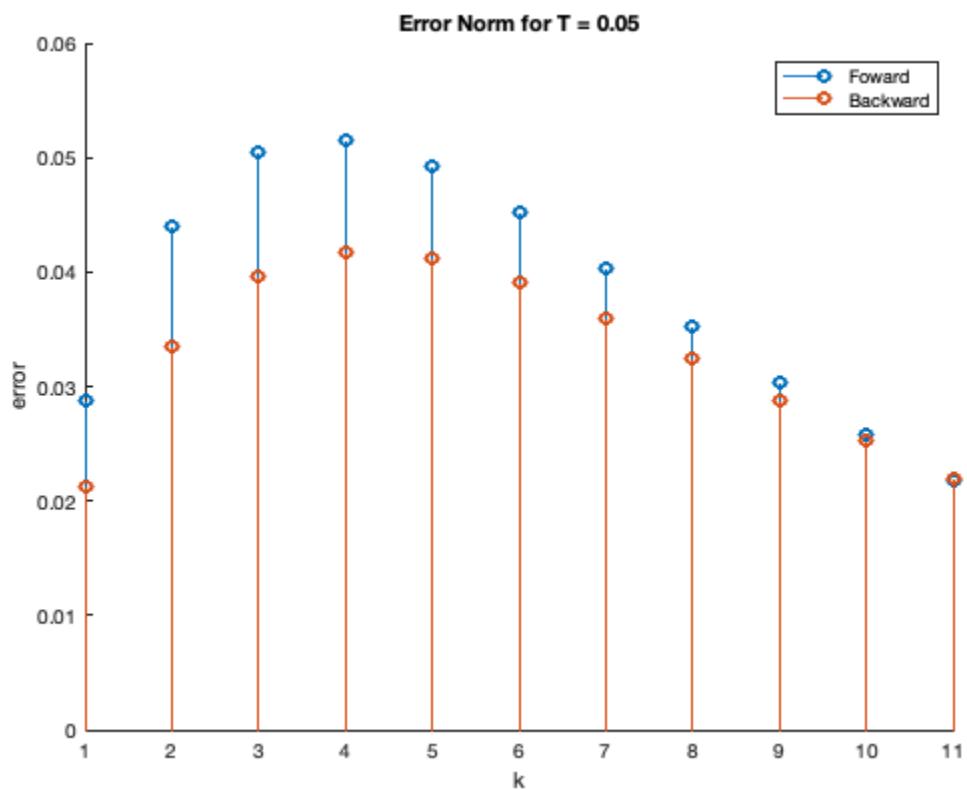
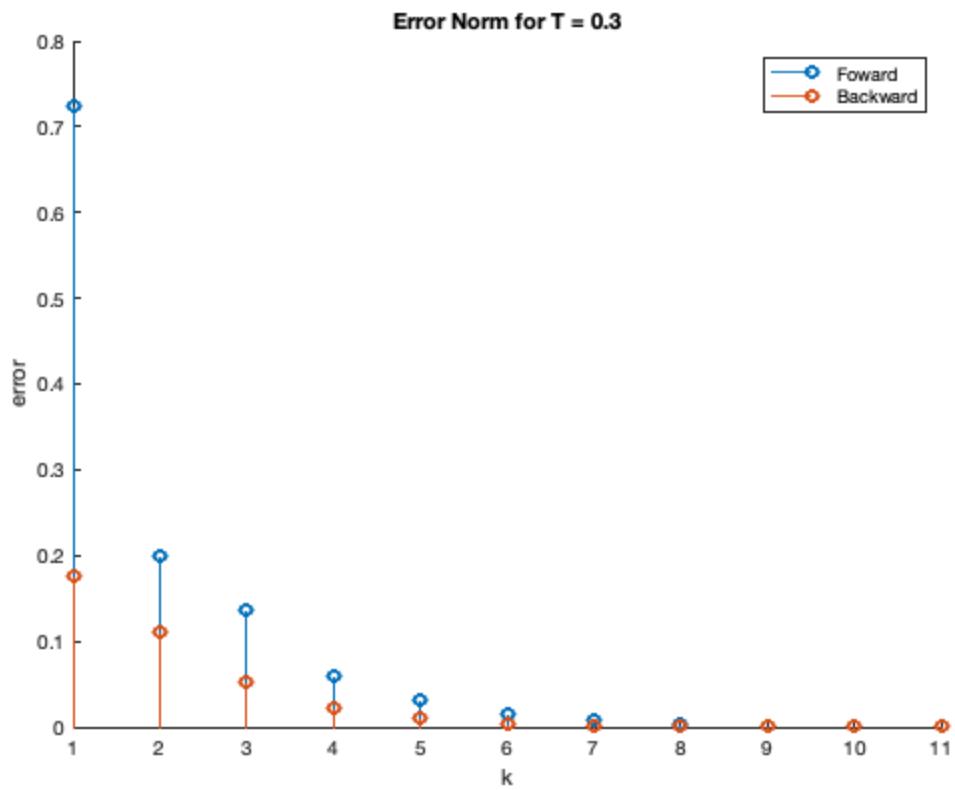
---

```

for k = 1:11
    en_forward(k) = abs(exp(-5*k*t) - (1-5*t)^k);
    en_backward(k) = abs(exp(-5*k*t) - (1/(1+5*t))^k));
end
k = 1:11;
figure
hold on
stem(k, en_forward)
stem(k, en_backward)
title("Error Norm for T = 0.1")
xlabel("k")
ylabel("error")
legend("Foward", "Backward")
t = 0.3;
for k = 1:11
    en_forward(k) = abs(exp(-5*k*t) - (1-5*t)^k);
    en_backward(k) = abs(exp(-5*k*t) - (1/(1+5*t))^k));
end
k = 1:11;
figure
hold on
stem(k, en_forward)
stem(k, en_backward)
title("Error Norm for T = 0.3")
xlabel("k")
ylabel("error")
legend("Foward", "Backward")
t = 0.05;
for k = 1:11
    en_forward(k) = abs(exp(-5*k*t) - (1-5*t)^k);
    en_backward(k) = abs(exp(-5*k*t) - (1/(1+5*t))^k));
end
k = 1:11;
figure
hold on
stem(k, en_forward)
stem(k, en_backward)
title("Error Norm for T = 0.05")
xlabel("k")
ylabel("error")
legend("Foward", "Backward")

```





---

## Part 4a

```
figure
I = imread('case2part4af.jpg');
imshow(I)
figure
I = imread('case2part4ab.jpg');
imshow(I)
```

BRIAN

Background:

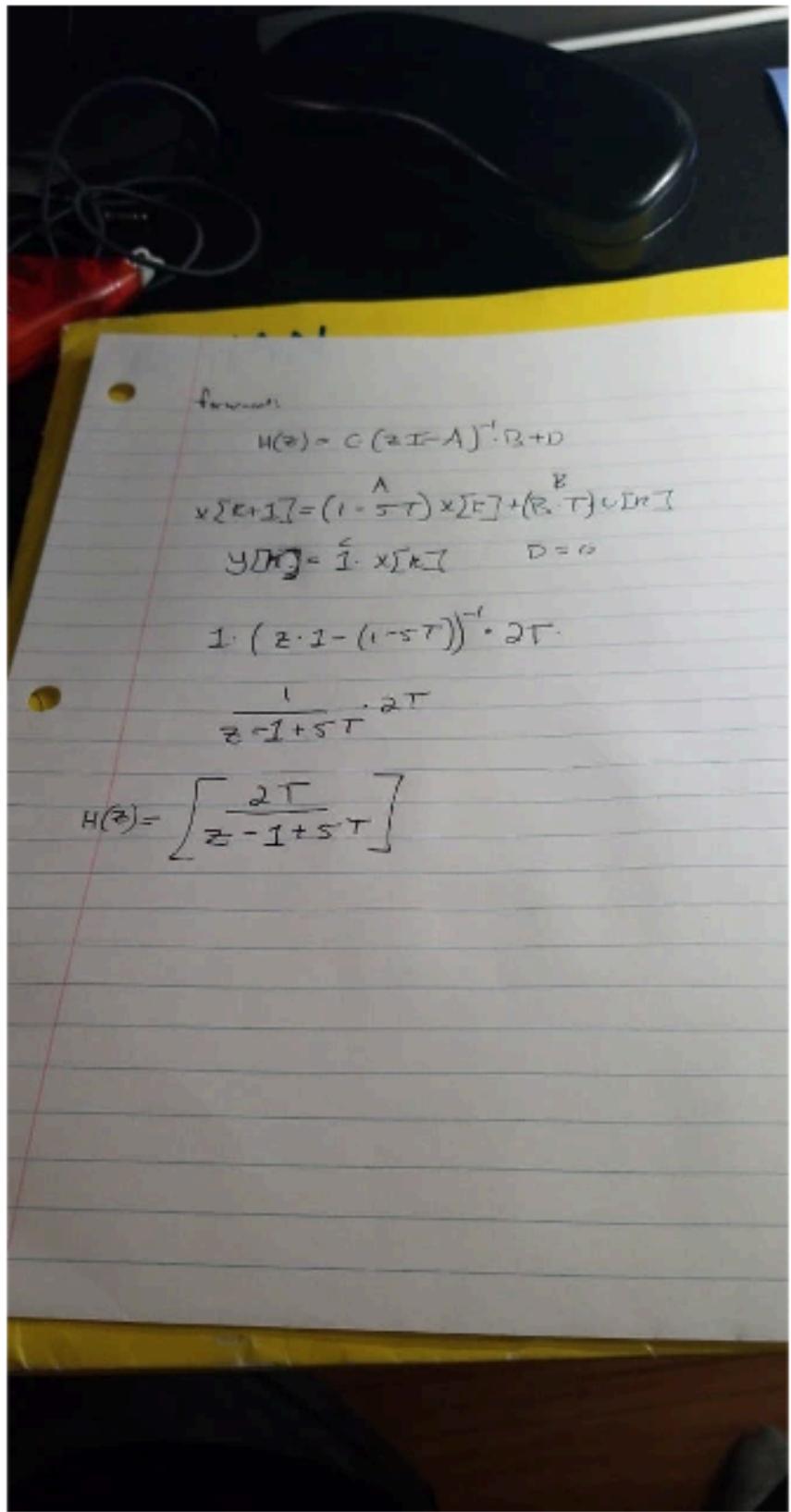
$$x[k-I] = (1 - \frac{1}{T}(-\tau))x[k] - \frac{\beta}{T}u[k-T]$$

$$y[k] = 1 \cdot x[k]$$

$$H(z) = (zI - (1 + \sigma T))^{-1}$$

$$H(z) = \frac{-2T}{z - 1 - \sigma T}$$

$$\frac{T_0}{T_0 + 1 - \frac{\sigma}{T}}$$



---

## Part 4b1

```
close all
A = [-5.0];
B = [2.0];
C = [1.0];
D = [];
IC = 1;
U = 0;
figure
% Exact Difference
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);

sys = ss(A,B,C,D);
[Yc, Tc] = initial(sys, IC, 0:0.01:2);
hold on;
initial(sys, IC, 0:0.01:2)
ybd = [];
yd = [];
yd(1) = 1;
Td = [];
Td(1) = 0;
% Exact Discrete
t = .1;
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
hold on;
ed = stem(Td(1:11)*t,yd(1:11), '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';

yfd = [];
yfd(1) = 1;
Td = [];
Td(1) = 0;
% Forward Difference
for i = 1:length(Tc)
    yfd(i+1) = (1+(t*A))*yfd(i) +(B*t*U);
    Td(i+1) = i;
end
hold on;
fd = stem(Td(1:11)*t,yfd(1:11), 'filled','DisplayName', 'Forward
Difference');
fd.Color = 'green';
ybd = zeros(1, length(Td));
ybd(1) = 1;
% Backward Difference
for k = 2:length(Td)
    z = 0;
    for m = 1:k
        z = z + 1/(1-A*t)^(k-m+1) * (B*t*U);
```

---

```

    end
    ybd(k) = ybd(1)*(1/((1-A*t)^k)) + z;
end
bd = stem(Td(1:11)*t,ybd(1:11), 'filled','DisplayName', 'Backward
Difference');
bd.Color = 'blue';
legend('CT','CT(kT)','Fwd','Back')
title('Discrete methods for T = 0.1')
figure
initial(sys, IC, 0:0.01:2)
t = .3;
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
hold on;
ed = stem(Td(1:11)*t,yd(1:11), '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';

yfd = [];
yfd(1) = 1;
Td = [];
Td(1) = 0;
% Forward Difference
for i = 1:length(Tc)
    yfd(i+1) = (1+(t*A))*yfd(i) +(B*t*U);
    Td(i+1) = i;
end
hold on;
fd = stem(Td(1:11)*t,yfd(1:11), 'filled','DisplayName', 'Forward
Difference');
fd.Color = 'green';
ybd = zeros(1, length(Td));
ybd(1) = 1;
% Backward Difference
for k = 2:length(Td)
    z = 0;
    for m = 1:k
        z = z + 1/(1-A*t)^(k-m+1) * (B*t*U);
    end
    ybd(k) = ybd(1)*(1/((1-A*t)^k)) + z;
end
bd = stem(Td(1:11)*t,ybd(1:11), 'filled','DisplayName', 'Backward
Difference');
bd.Color = 'blue';
legend('CT','CT(kT)','Fwd','Back')
title('Discrete methods for T = 0.3')
figure
t = .05;
initial(sys, IC, 0:0.01:2)
for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end

```

---

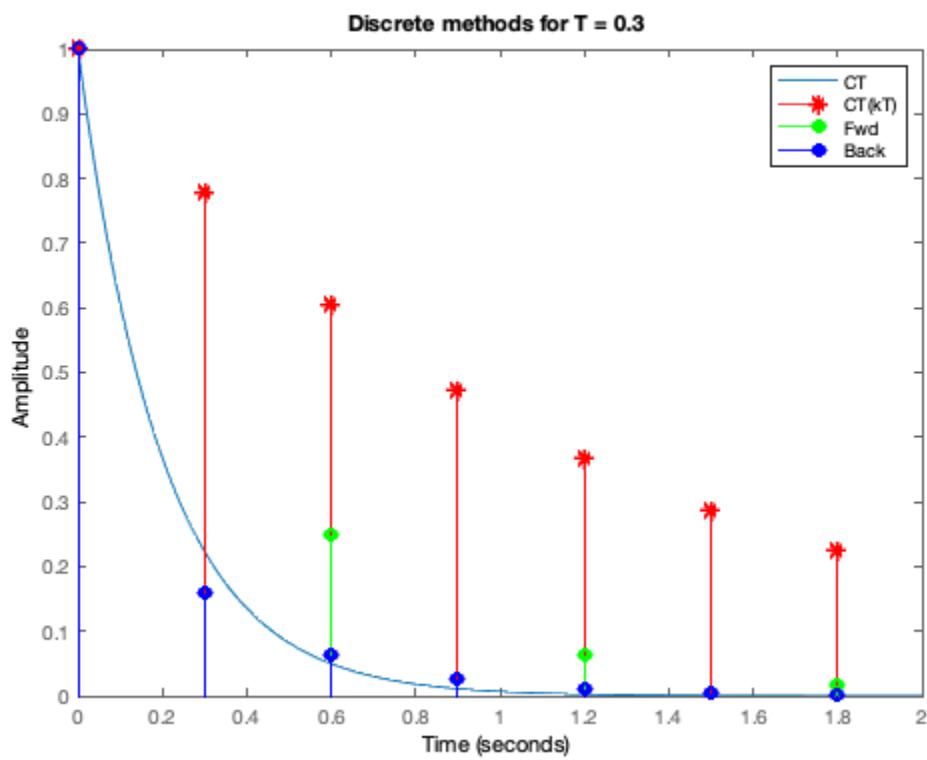
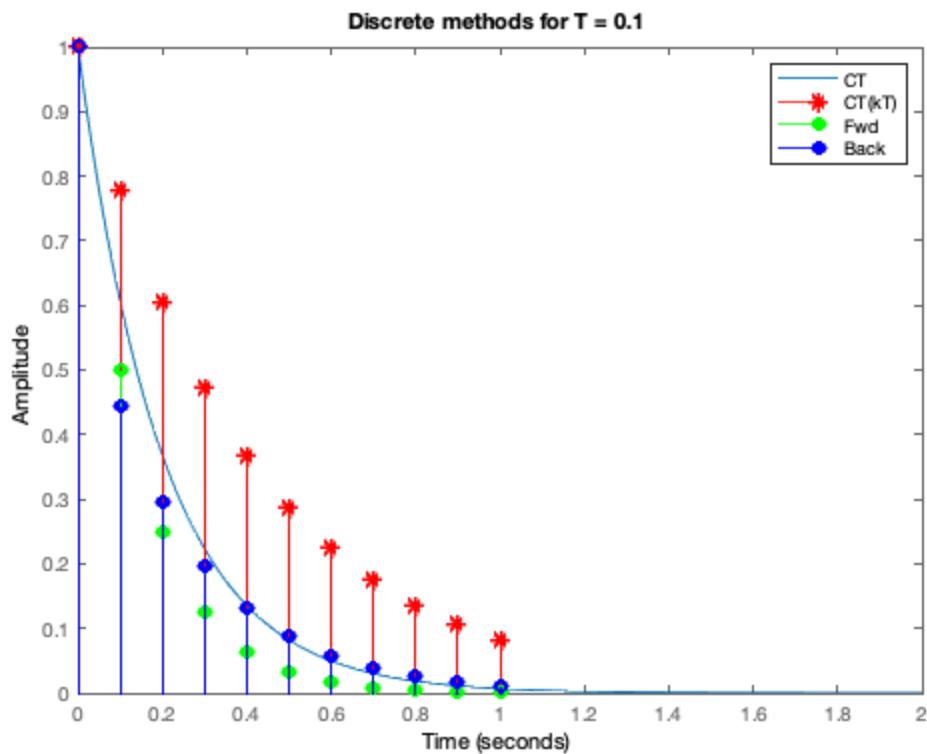
---

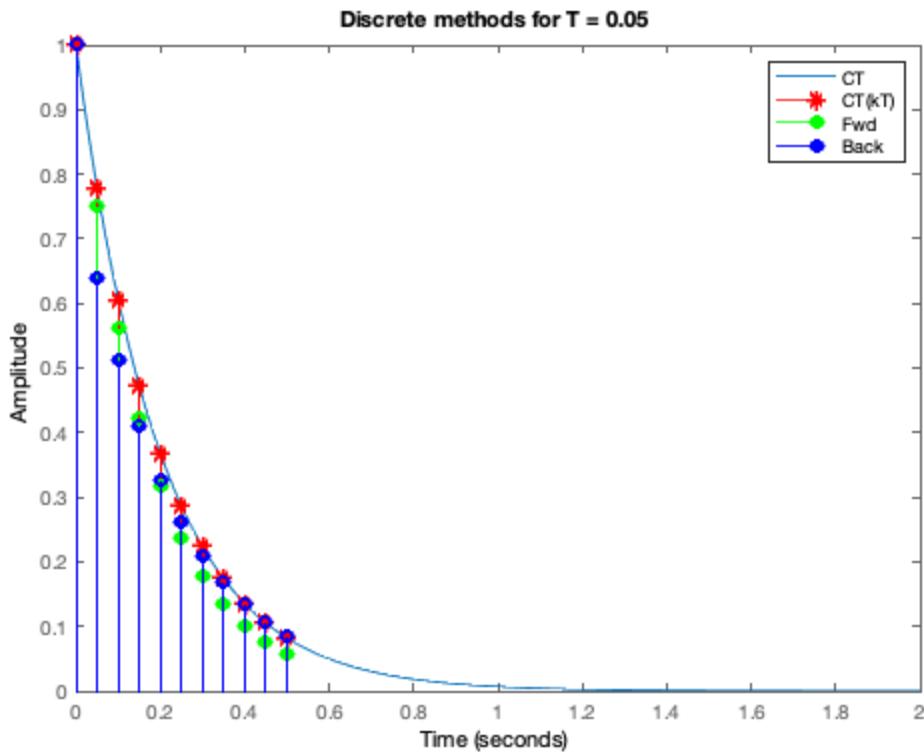
```

hold on;
ed = stem(Td(1:11)*t,yd(1:11), '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';

yfd = [];
yfd(1) = 1;
Td = [];
Td(1) = 0;
% Forward Difference
for i = 1:length(Tc)
    yfd(i+1) = (1+(t*A))*yfd(i) +(B*t*U);
    Td(i+1) = i;
end
hold on;
fd = stem(Td(1:11)*t,yfd(1:11), 'filled','DisplayName', 'Forward
Difference');
fd.Color = 'green';
ybd = zeros(1, length(Td));
ybd(1) = 1;
% Backward Difference
for k = 2:length(Td)
    z = 0;
    for m = 1:k
        z = z + 1/(1-A*t)^(k-m+1) * (B*t*U);
    end
    ybd(k) = ybd(1)*(1/((1-A*t)^k)) + z;
end
bd = stem(Td(1:11)*t,ybd(1:11), 'filled','DisplayName', 'Backward
Difference');
bd.Color = 'blue';
legend('CT','CT(kT)', 'Fwd', 'Back')
title('Discrete methods for T = 0.05')

```





## Part 4b2

```

close all
A = [-5.0];
B = [2.0];
C = [1.0];
D = [];
IC = 1;
U = 0;
figure
% Exact Difference
Ad = expm(-5*t);
Bd = ((-2/5)*expm(-5*t)) + (2/5);

sys = ss(A,B,C,D);
[Yc, Tc] = initial(sys, IC, 0:0.01:2);
hold on;
initial(sys, IC, 0:0.01:2)
ybd = [];
yd = [];
yd(1) = 1;
Td = [];
Td(1) = 0;
% Exact Discrete
t = .1;

```

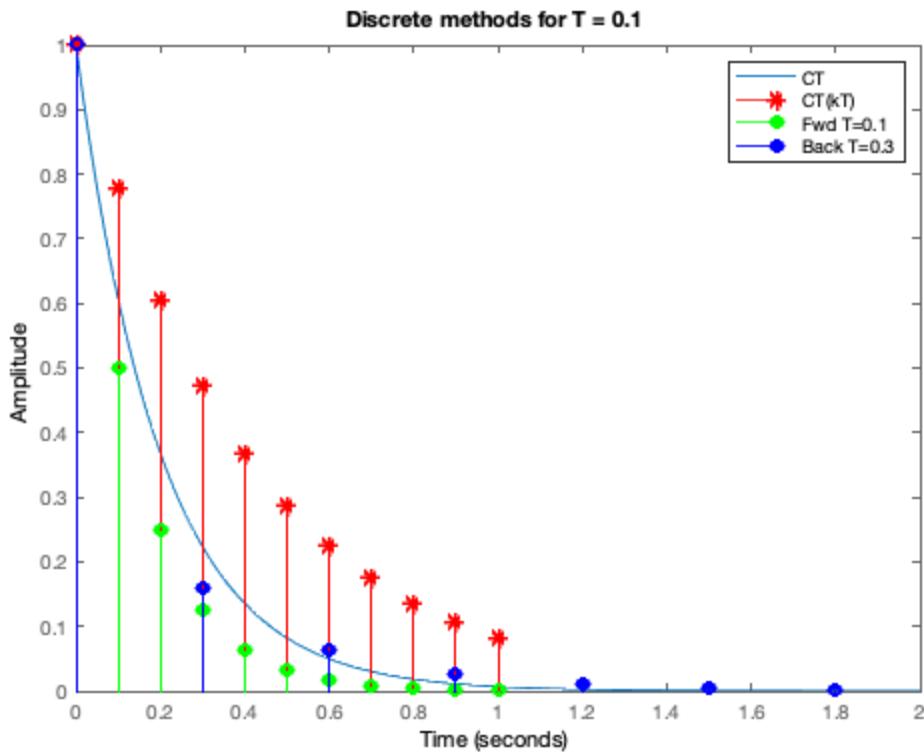
---

```

for i = 1:length(Tc)
    yd(i+1) = Ad*yd(i) +(Bd*U);
    Td(i+1) = i; % save corresponding discrete time integer index
end
hold on;
ed = stem(Td(1:11)*t,yd(1:11), '*', 'DisplayName', 'Exact Discrete');
ed.Color = 'red';

yfd = [ ];
yfd(1) = 1;
Td = [ ];
Td(1) = 0;
% Forward Difference
for i = 1:length(Tc)
    yfd(i+1) = (1+(t*A))*yfd(i) +(B*t*U);
    Td(i+1) = i;
end
hold on;
fd = stem(Td(1:11)*t,yfd(1:11), 'filled','DisplayName', 'Forward
Difference');
fd.Color = 'green';
ybd = zeros(1, length(Td));
ybd(1) = 1;
% Backward Difference
t = 0.3;
for k = 2:length(Td)
    z = 0;
    for m = 1:k
        z = z + 1/(1-A*t)^(k-m+1) * (B*t*U);
    end
    ybd(k) = ybd(1)*(1/((1-A*t)^k)) + z;
end
bd = stem(Td(1:11)*t,ybd(1:11), 'filled','DisplayName', 'Backward
Difference');
bd.Color = 'blue';
legend('CT','CT(kT)', 'Fwd T=0.1', 'Back T=0.3')
title('Discrete methods for T = 0.1')

```



## Part 4b3

```
% the results shows that backwards difference will work for all T >
-1/3
% and forward difference will only work for T = 0<T<1/3
% inorder to have the same negative poles for the transfer function,
the T
% for forward has to be less than 0.2 and the T for backward have to
be
% greater than 0.2 so in the graph, T for forward was 0.1 and backward
was
% 0.3 and this will have the same poles for the transfer function.

% also for forward difference the it grows very instable as T
increases,
% and the threshhold is 0.3333 before it becomes unstable and this can
be
% seen in the graph.
```