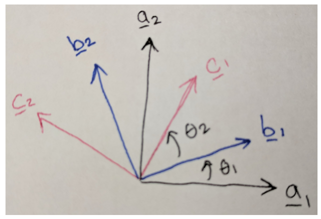
MEE 4411/5411 Homework 1

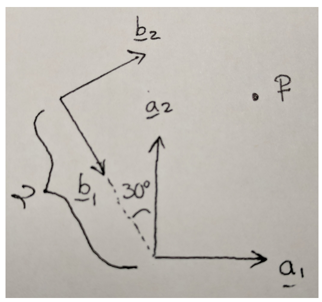
Due: Thursday, September 14, 12:30pm

1. Consider the three coordinate frames A, B, and C in the figure below and let $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.



We know that ${}^{A}R_{B} = R(\theta_1)$ and ${}^{B}R_{C} = R(\theta_2)$. Prove that ${}^{A}R_{C} = R(\theta_1 + \theta_2)$. (This is obvious from the diagram, the challenge here is to mathematically prove it. You must show your work.)

2. Consider the two coordinate frames A and B in the figure below.

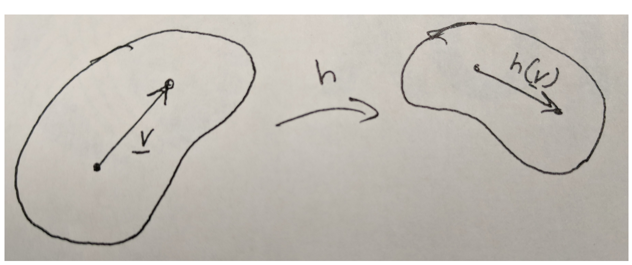


(a) Find the homogeneous transformation that represents the position and orientation of frame B with respect to frame A.

(b) Find the homogeneous transformation that takes a point in frame B and puts it into frame A.

(c) Consider the point ${}^{A}\mathbf{p} = [1, \sqrt{3}]^T$ in frame A. What is ${}^{B}\mathbf{p}$?

3. Let $\mathbf{v}$ be a vector connecting two points on a rigid body and let $h(\mathbf{v})$ be that same vector after the rigid body has undergone transformation h, as in the diagram below.



In lecture you saw that rigid body transformations preserve both lengths and angles. Mathematically, we can state these constraints as $\|h(\mathbf{u})\| = \|\mathbf{u}\|$ for any vector $\mathbf{u}$ and $h(\mathbf{u}) \times h(\mathbf{v}) = h(\mathbf{u} \times \mathbf{v})$ for any vectors $\mathbf{u}, \mathbf{v}$, respectively. Using these facts, prove that:

(a) orthogonal vectors are mapped to orthogonal vectors, *i.e.*, for any vectors $\mathbf{u}, \mathbf{v}$ such that $\mathbf{u} \perp \mathbf{v}$ then $h(\mathbf{u}) \perp h(\mathbf{v})$

(b) **(MEE 5411 only)** rigid body transformation preserve inner products, *i.e.*, $h(\mathbf{u}) \cdot h(\mathbf{v}) = \mathbf{u} \cdot \mathbf{v}$ for any vectors $\mathbf{u}, \mathbf{v}$.

4. (MEE 5411 only) Rotation matrices have 4 elements but are completely determined by a single parameter θ. This is due to the fact that there are 3 constraints on the elements of a rotation matrix, which is an orthogonal matrix with determinant 1 (also called a Special Orthogonal matrix). Letting

$$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

one set of three constraints is $r_{11}^2 + r_{21}^2 = 1$, $r_{12}^2 + r_{22}^2 = 1$, and $r_{11}r_{22} - r_{12}r_{21} = 1$, which ensure that each column is a unit vector and that the determinant is 1, respectively. Show that the *only* solution to these 3 equations is the standard rotation matrix (shown in problem 1). (Note: It is *not* sufficient to simply plug in the elements and show that they satisfy the constraints, this does not prove that no other solutions exist.) (Hint: Let r_{11} be the independent variable and try to solve for the other three elements as a function of r_{11}.)