MEE 4411/5411 Homework 1

Due: Thursday, December 7, 12:30pm

1. Consider the following rotation matrix:

$$R = \begin{bmatrix} 0.0000 & -0.7071 & -0.7071 \\ 0.5000 & 0.6124 & -0.6124 \\ 0.8660 & -0.3536 & 0.3536 \end{bmatrix}$$

(a) Find the XYZ Euler angles representing the rotation, meaning $R = R_{x,\phi}R_{y,\theta}R_{z,\psi}$. Hint:

$$R = \begin{bmatrix} \cos\psi\cos\theta & -\cos\theta\sin\psi & \sin\theta \\ \cos\phi\sin\psi + \cos\psi\sin\phi\sin\theta & \cos\phi\cos\psi - \sin\phi\sin\psi\sin\theta & -\cos\theta\sin\phi \\ \sin\phi\sin\psi - \cos\phi\cos\psi\sin\theta & \cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta & \cos\phi\cos\theta \end{bmatrix}$$

(b) Find the axis-angle representation of the rotation. Hint: use Rodrigues' formula.

(c) Find the quaternion representation of the rotation.

2. A planar (2D) quadrotor flies in the YZ plane. The state of the quadrotor is therefore $[y, z, \phi]^T$ and there are two inputs, u_1, u_2, which are the thrust and the moment about the x-axis, respectively. The equations of motion are:

$$\begin{aligned} \ddot{y} &= -\frac{1}{m}\sin\phi u_1 \\ \ddot{z} &= -g + \frac{1}{m}\cos\phi u_1 \\ \ddot{\phi} &= \frac{1}{I_x x}u_2 \end{aligned}$$

The hover state, like the 3D quadrotor, is $[y_0, z_0, 0]$.

(a) Find the inputs u_1, u_2 at the hover state.

(b) Linearize the equations of motion about the hover state. Hint: you need to linearize the equations with respect to both the inputs and the states of the robot.

(c) We want to use a PD controller for our robot. Write down the equations for the control inputs as a function of the desired and actual states of the robot.

3. In addition to optimal trajectory planning, the Euler-Lagrange equations can be used to solve a variety of other problems. One of these is to find the equations of motion of a dynamic system. Let T be the kinetic energy of a system and let U be the potential energy of the system. Then the Lagrangian of the system is $L = T - U$. The equations of motion with respect to any state variable can then be found by plugging this Lagrangian into the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = F$$

where x_i is the ith state and F is the external force (or moment) in the direction of that state. Note that gravity is *not* considered an external force since it is accounted for in the potential energy term.

(a) Write down the kinetic energy of the planar quadrotor from problem 2. Hint: consider both the translational and rotational velocities.

(b) Write down the potential energy of the planar quadrotor from problem 2.

(c) Write down the external force (or moment) in each direction.

(c) Write down the Lagrangian for the system and solve the Euler-Lagrange equations for the planar quadrotor. The result should be the same equations of motion as are given in problem 2.