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Intelligent robust PI adaptive control strategy for speed control of EV(s)

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Abstract

- The intensive non-linear system and plants of modern industry highly motivating researcher to extend and evolve non-linear control systems. In **this** study, in order to control a class of non-linear uncertain power systems in the presence of large and fast disturbances, a new simple indirect adaptive proportional-integral is proposed.
- For handling dynamic uncertainties, the proposed controller utilities the advantages of least squares support vectors regression (LS-SVR) to approximate unknown non-linear actions and noisy data.

Problem Statement

- Consider a class of single-input single-output nth order non-linear system in the following form

$$x^{(n)} = f(x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}) + g(x, \dot{x}, \ddot{x}, \dots, x^{(n-1)})u(t) + d(x, t)$$
$$y = x$$

- Assumptions:

- $d(x, t) \leq D$

- $X_d^T = [x_d \quad \dot{x}_d \quad \dots \quad x_d^{(n-1)}]$

- $\|x_d\| \leq \psi$

$$E = x - x_d = [e \quad \dot{e} \quad \dots \quad e^{(n-1)}]^T$$

LS - SVR

- The goal is to Estimate Unknown Nonlinear Functions.

$$\{(x_k, y_k) | k = 1, 2, \dots, N\}, x_k \in R^n, y_k \in R$$

$$y(x) = w^T \varphi(x) + b$$

Defining Cos Function and Optimality Condition

$$\min J(w, e) = \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^N e_k^2$$

$$y(k) = w^T \varphi(x_k) + b + e_k, k = 1, 2, \dots, N$$

- The Lagrangian is defined as follows:

$$L(w, b, e; \alpha) = J(w, e) - \sum_{k=1}^N \alpha_k \{w^T \varphi(x_k) + b + e_k - y_k\}$$

Defining Cos Function and Optimality Condition

- Karush – Kuhn – Tucker Optimality Condition:

$$\left\{ \begin{array}{l} \frac{\partial L}{\partial w} = 0 \rightarrow w = \sum_{k=1}^N \alpha_k \varphi(x_k) \\ \frac{\partial L}{\partial b} = 0 \rightarrow \sum_{k=1}^N \alpha_k = 0 \\ \frac{\partial L}{\partial e_k} = 0 \rightarrow \alpha_k = \gamma e_k, k = 1, 2, \dots, N \\ \frac{\partial L}{\partial \alpha_k} = 0 \rightarrow w^T \varphi(x_k) + b + e_k - y_k = 0 \end{array} \right.$$

Defining Cos Function and Optimality Condition

- After elimination of the variable w and e_k :

$$\begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{1}_N^T \\ \mathbf{1}_N & \Omega + \gamma^{-1} I_N \end{bmatrix} \begin{bmatrix} 0 \\ y \end{bmatrix}$$

$$b = \frac{\mathbf{1}_N^T (\Omega + \gamma^{-1} I_N)^{-1} y}{\mathbf{1}_N^T (\Omega + \gamma^{-1} I_N)^{-1} \mathbf{1}_N}$$

$$\alpha = (\Omega + \gamma^{-1} I_N)^{-1} (y - \mathbf{1}_N^T b)$$



$$y(x) = \sum_{k=1}^N \alpha_k \mathbf{K}(x_k, x_l) + b$$

$$\alpha = [\alpha_1, \dots, \alpha_N]$$

$$\mathbf{1}_N = [1; \dots; 1]$$

$$y = [y_1; \dots; y_N]$$

$$\Omega_{kl} = \varphi(x_k)^T \varphi(x_l)$$

$$= \mathbf{K}(x_k, x_l), \quad k, l = 1, \dots, N$$

Defining Cos Function and Optimality Condition

- Kernel Functions

Name	Kernel function expression
linear kernel	$k(x, x_i) = x^T x_i$
polynomial kernel	$k(x, x_i) = (t + x^T x_i)^d$
RBF kernel	$k(x, x_i) = \exp(-\ x - x_i\ ^2 / \sigma^2)$
MLP kernel	$k(x, x_i) = \tanh(\beta_0 x^T x_i + \beta_1)$



Defining The Controller

- The non-linear system can be rewritten in the following form

$$\begin{aligned}\dot{X} &= AX + B(f(X, t) + g(X, t)u + d(X, t)) \\ y &= CX\end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 \end{bmatrix}_{n \times n}$$

$$B^T = [0 \dots 0 \ 1]_{1 \times n}, \quad C = [1 \dots 0 \ 0]_{1 \times n}$$

Defining The Controller

- Algebraic Part

$$x^{(n)} = f(x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}) + g(x, \dot{x}, \ddot{x}, \dots, x^{(n-1)})u(t) + d(x, t)$$
$$y = x$$

$$u = \frac{1}{g(X, t)} [-f(X, t) - d(X, t) + \ddot{x}_d^{(n)} - K^T E]$$

$$\pm AX_d$$

$$\dot{X} - \dot{X}_d = (A - BK^T)E \rightarrow \dot{E} = (A - BK^T)E$$

Hurwitz



Defining The Controller

- Analytic Part

$$u = \frac{1}{\hat{g}(X|W_g)} [-\hat{f}(X|W_f) - S(E^T \mathbf{P} B | W_s) + x_d^{(n)} - \mathbf{K}^T E]$$

$$\hat{f}(X|W_f) = W_f \beta(X)$$

$$\hat{g}(X|W_g) = W_g \beta(X)$$

$$\beta = [1 \ K(x, x_1) \ \cdots \ K(x, x_i) \ \cdots \ K(x, x_N)]$$

Defining The Controller

- Analytic Part - Assumptions

$$W_f^* = \arg \min_{W_f \in R^M} [\sup_{X \in R^n} |\hat{f}(X|W_f) - f(X, t)|]$$

$$W_g^* = \arg \min_{W_g \in R^M} [\sup_{X \in R^n} |\hat{g}(X|W_g) - g(X, t)|]$$

- The minimum Approximation Error:

$$\omega = f(X, t) - \hat{f}(X|W_f^*) + (g(X, t) - \hat{g}(X|W_g^*))u$$

- Since f and g are bounded:

$$|\omega| < \Phi$$

Defining The Controller

- Proposed PI Controller:

$$(E^T \mathbf{P} \mathbf{B} | W_s) = \begin{cases} W_s^T \psi(E^T \mathbf{P} \mathbf{B}) = K_p E^T \mathbf{P} \mathbf{B} + \\ K_I \int (E^T \mathbf{P} \mathbf{B}) dt, & |E^T \mathbf{P} \mathbf{B}| \leq \Omega \\ \hat{D}_\omega \operatorname{sgn}(E^T \mathbf{P} \mathbf{B}), & |E^T \mathbf{P} \mathbf{B}| > \Omega \end{cases}$$

$k_1 = 2$ and $k_2 = 1$.

$$(A - B\mathbf{K}^T)^T \mathbf{P} + \mathbf{P}(A - B\mathbf{K})^T = -\mathbf{Q}$$

$$\mathbf{Q} = \operatorname{diag}(10, 10) \quad \mathbf{P} = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

$$\psi(E^T \mathbf{P} \mathbf{B}) = [E^T \mathbf{P} \mathbf{B}, \int (E^T \mathbf{P} \mathbf{B}) dt]^T$$

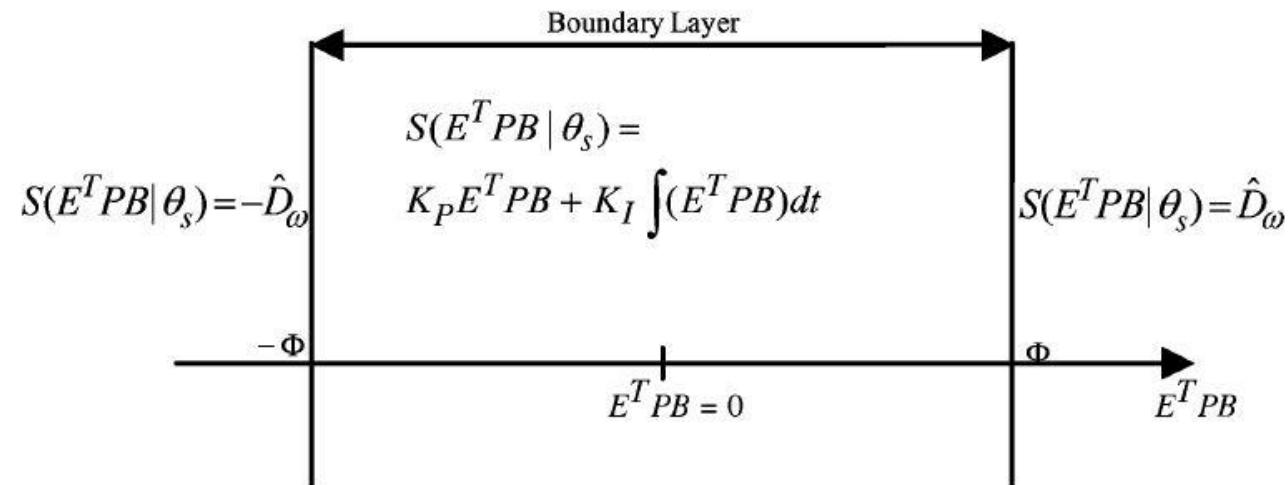
$$W_s^T = [K_p, K_I]$$

$$\hat{D}_\omega = \hat{D} + \hat{\Phi}$$

Defining The Controller

- Proposed PI Controller:

$$(E^T \mathbf{P}B | W_s) = \begin{cases} W_s^T \psi(E^T \mathbf{P}B) = K_p E^T \mathbf{P}B + \\ K_I \int (E^T \mathbf{P}B) dt, & |E^T \mathbf{P}B| \leq \Omega \\ \hat{D}_\omega \operatorname{sgn}(E^T \mathbf{P}B), & |E^T \mathbf{P}B| > \Omega \end{cases}$$



Lyapunov Function

- The candidate:

$$V = \frac{1}{2} E^T \mathbf{P} E + \frac{1}{2\Gamma_1} \tilde{W}_f^T \tilde{W}_f + \frac{1}{2\Gamma_2} \tilde{W}_g^T \tilde{W}_g + \frac{1}{2\Gamma_3} \tilde{W}_s^T \tilde{W}_s + \frac{1}{2\Gamma_4} \tilde{D}_\omega^2$$

$$\tilde{W}_s = W_s^* - W_s$$

$$\tilde{D}_\omega = D_\omega - \hat{D}_\omega$$

Lyapunov Function

- Differentiating with respect to time, we have:

$$\begin{aligned}\dot{V} = & -\frac{1}{2}E^T \mathbf{Q}E + E^T \mathbf{P}B\omega + \tilde{W}_f^T \left(E^T \mathbf{P}B\beta(X) + \frac{1}{\Gamma_1} \dot{\tilde{W}}_f \right) \\ & + \tilde{W}_g^T \left(E^T \mathbf{P}B\beta(X)u + \frac{1}{\Gamma_2} \dot{\tilde{W}}_g \right) \\ & + \tilde{W}_s^T \left(E^T \mathbf{P}B\psi(E^T \mathbf{P}B) + \frac{1}{\Gamma_3} \dot{\tilde{W}}_s \right) + \frac{1}{\Gamma_4} \tilde{D}_\omega \dot{\tilde{D}}_\omega \\ & + E^T \mathbf{P}Bd(X, t) - E^T \mathbf{P}BS(E^T \mathbf{P}B|W_s^*).\end{aligned}$$

Lyapunov Function

- Differentiating with respect to time, we have:

$$\begin{aligned}\dot{V} \leq & -\frac{1}{2}E^TQE + \tilde{W}_f^T \left(E^T PB\beta(X) + \frac{1}{\Gamma_1} \dot{\tilde{W}}_f \right) \\ & + \tilde{W}_g^T \left(E^T PB\beta(X)u + \frac{1}{\Gamma_2} \dot{\tilde{W}}_g \right) \\ & + \tilde{W}_s^T \left(E^T PB\psi(E^T PB) + \frac{1}{\Gamma_3} \dot{\tilde{W}}_s \right) + \frac{1}{\Gamma_4} \tilde{D}_\omega \dot{\tilde{D}}_\omega + |E^T PB||\omega| \\ & + |E^T PB||d(X, t)| - |E^T PB||S(E^T PB|W_s^*)|\end{aligned}$$



$$\begin{aligned}\dot{V} \leq & -\frac{1}{2}E^TQE + \tilde{W}_f^T \left(E^T PB\beta(X) - \frac{1}{\Gamma_1} \dot{\tilde{W}}_f \right) \\ & + \tilde{W}_g^T \left(E^T PB\beta(X)u - \frac{1}{\Gamma_2} \dot{\tilde{W}}_g \right) \\ & + \tilde{W}_s^T \left(E^T PB\psi(E^T PB) - \frac{1}{\Gamma_3} \dot{\tilde{W}}_s \right) \\ & - \frac{1}{\Gamma_4} \tilde{D}_\omega \dot{\tilde{D}}_\omega + |E^T PB|D_\omega - |E^T PB|\hat{D}_\omega.\end{aligned}$$

Lyapunov Function

- As a result, Adaptation Laws are obtained as follows

$$\dot{W}_f = \Gamma_1 E^T P B \beta(X)$$

$$\dot{W}_g = \Gamma_2 E^T P B \beta(X) u$$

$$\dot{W}_s = \Gamma_3 E^T P B \psi(E^T P B)$$

$$\dot{\hat{D}}_\omega = \Gamma_4 |E^T P B|$$

Lyapunov Function

- With respect to the Adaptation Laws:

$$\dot{V} \leq -\frac{1}{2}E^T \mathbf{Q}E \leq 0$$

$$E(t), \tilde{W}_f(t), \tilde{W}_g(t), \tilde{W}_s(t), \tilde{D}_\omega(t) \in L^\infty$$

Closed Loop Stability

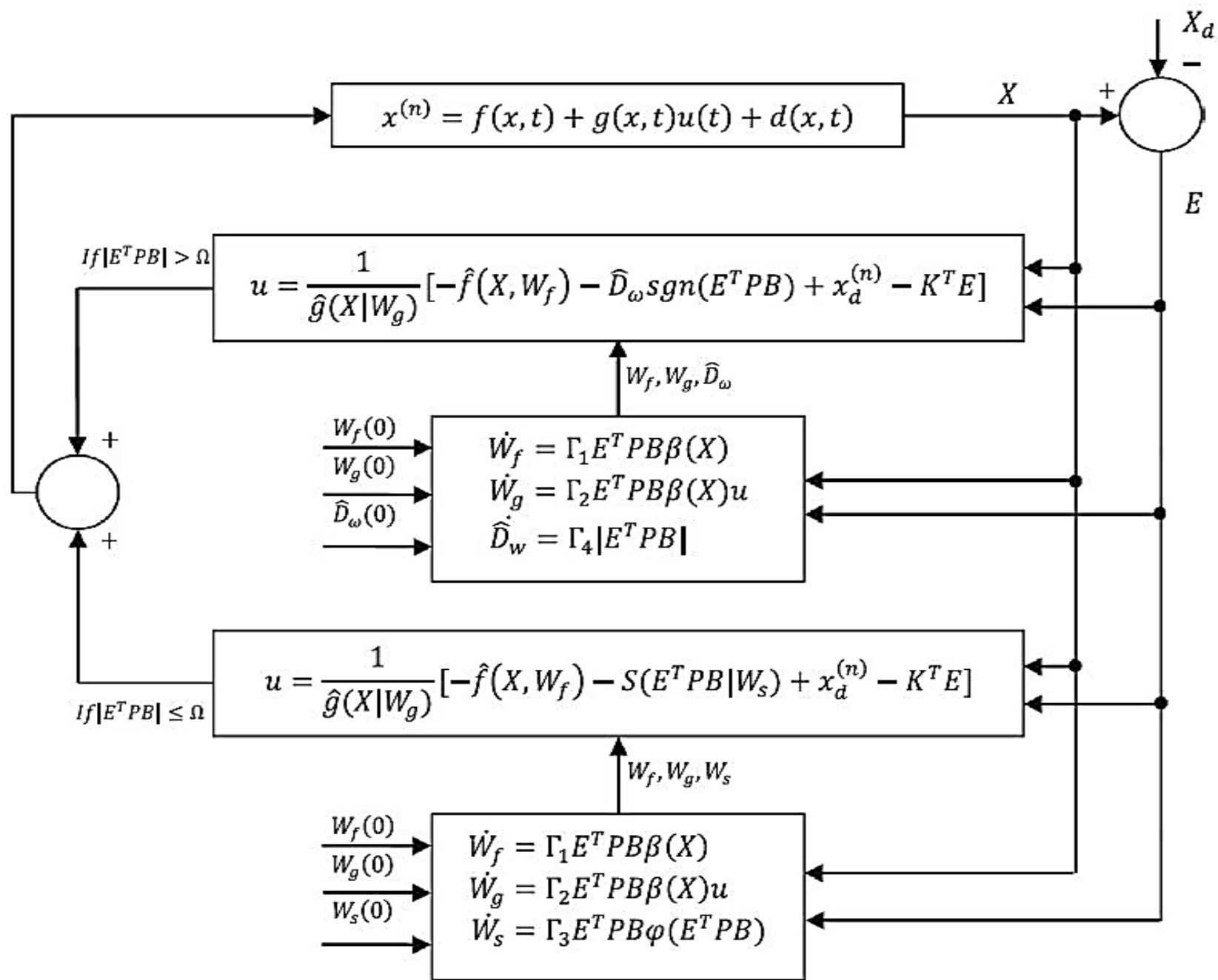
- Barbalat's Lemma:

$$W(t) = \frac{1}{2} E^T Q E \leq -V$$

$$V\left(E(t), \tilde{W}_f(t), \tilde{W}_g(t), \tilde{W}_s(t), \tilde{D}_\omega(t)\right) \leq V\left(E(0), \tilde{W}_f(0), \tilde{W}_g(0), \tilde{W}_s(0), \tilde{D}_\omega(0)\right)$$

$$\begin{aligned} \int_0^t \mathcal{W}(\tau) \, d\tau &\leq V\left(E(0), \tilde{W}_f(0), \tilde{W}_g(0), \tilde{W}_s(0), \tilde{D}_\omega(0)\right) \\ &\quad - V\left(E(t), \tilde{W}_f(t), \tilde{W}_g(t), \tilde{W}_s(t), \tilde{D}_\omega(t)\right) \end{aligned}$$

$$\lim_{t \rightarrow \infty} \int_0^t \mathcal{W}(\tau) \, d\tau < \infty \quad \longrightarrow \quad \lim_{t \rightarrow \infty} \dot{V}(t) \text{ and } E(t) = 0$$



Simulation

- Motor Model:

Motor		Vehicle	
$L_a + L_f$ (mH)	6.008	m (kg)	800
$R_a + R_f$ (Ω)	0.12	A (m^2)	1.8
B (N.M.s)	0.0002	ρ (kg/ m^3)	1.25
J (kg m^2)	0.05	C_d	0.3
L_{af} (mH)	1.766	ϕ ($^\circ$)	0
V (V)	0 ~ 48	μ_{rr}	0.015
i (A)	78A (250 max)	r (m)	0.25
ω_{nom} (r/min)	2800 ($v = 25$ km/h)	G	11

$$(L_a + L_{\text{field}}) \frac{di}{dt} = V - (R_a + R_f)i - L_{af}i \cdot \omega$$

$$(J + m \frac{r^2}{G^2}) \frac{d\omega}{dt} = L_{af}i^2 - B\omega - \frac{r}{G}(\mu_{rr}mg + \frac{1}{2}\rho AC_d v^2 + mg \sin \phi)$$

Simulation

- Motor Model:

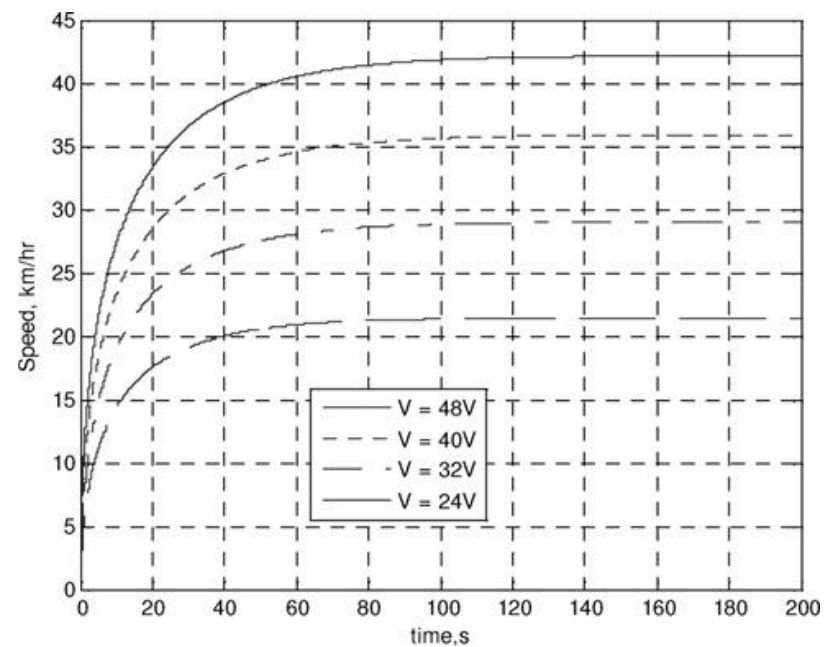
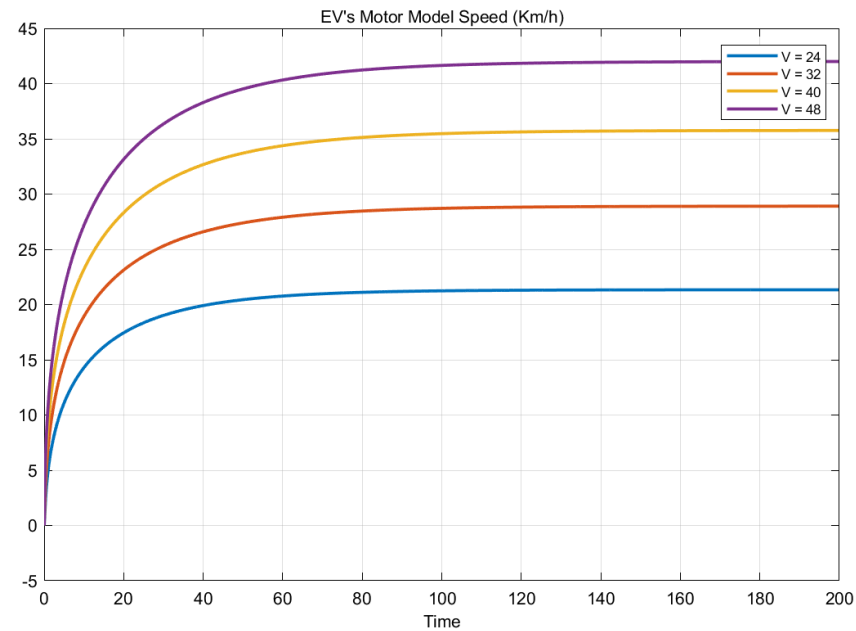
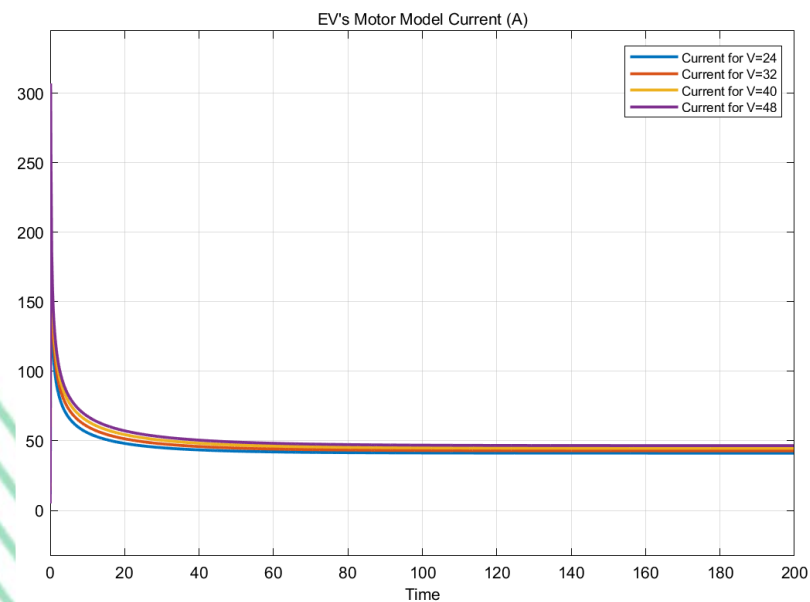
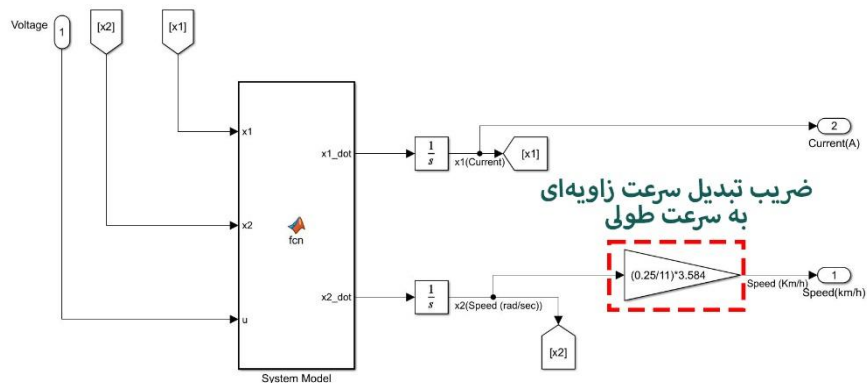
$$X = \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} \left(\frac{1}{L_a + L_{\text{field}}} \right) \times \left\{ -\left(R_a + R_f \right) \cdot x_1 - L_{af} \cdot x_1 \cdot x_2 \right\} \\ \left(\frac{1}{J + m(r/G)^2} \right) \times \left\{ L_{af} \cdot x_1^2 - Bx_2 - \left(\frac{r}{G} \right) \times \left(\mu_{rr}mg + \frac{1}{2}\rho AC_d \left(\frac{r}{G} \right)^2 x_2^2 + mgsin\varphi \right) \right\} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} \left(\frac{1}{L_a + L_{\text{field}}} \right) \\ 0 \end{bmatrix}$$

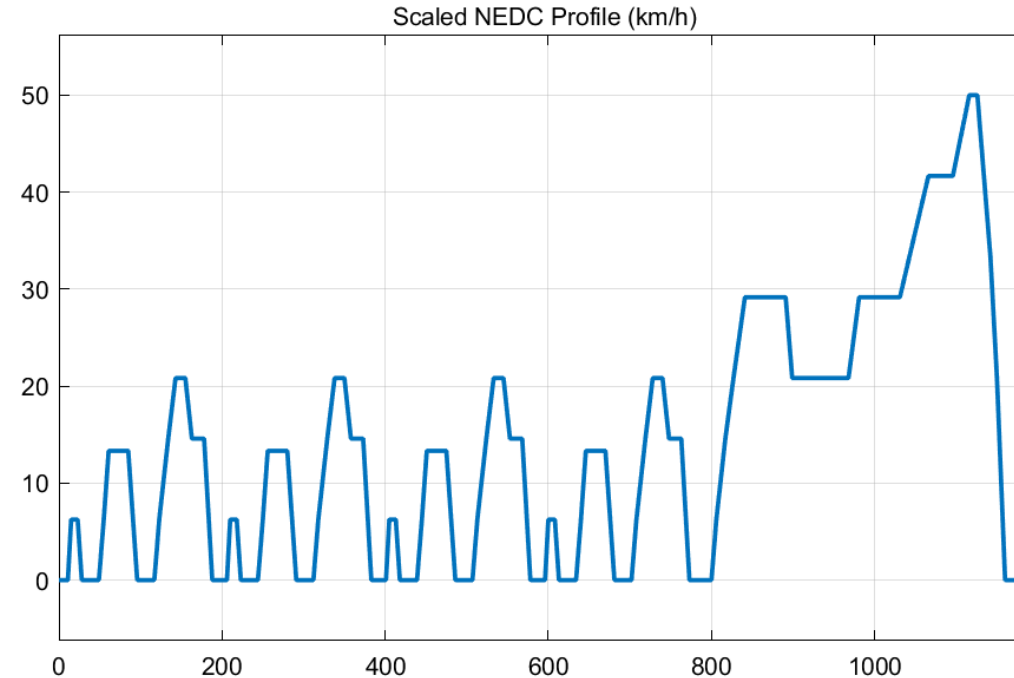
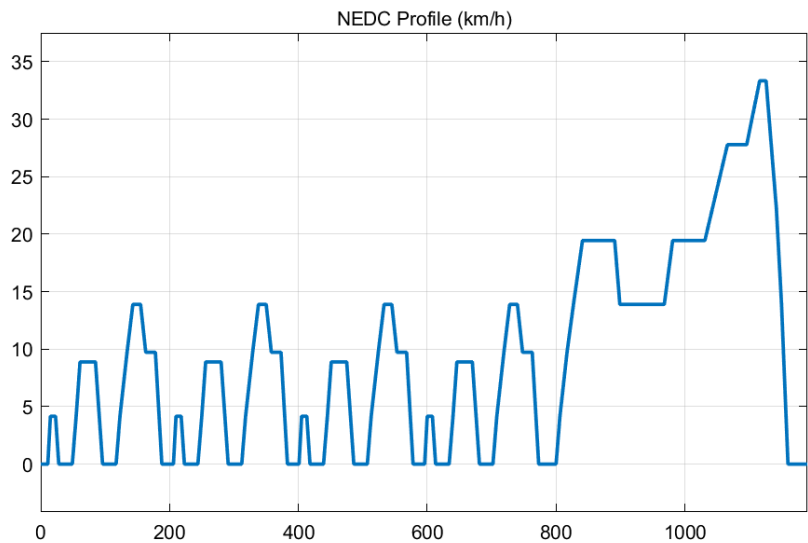
Simulation

- Open Loop Response:



Simulation

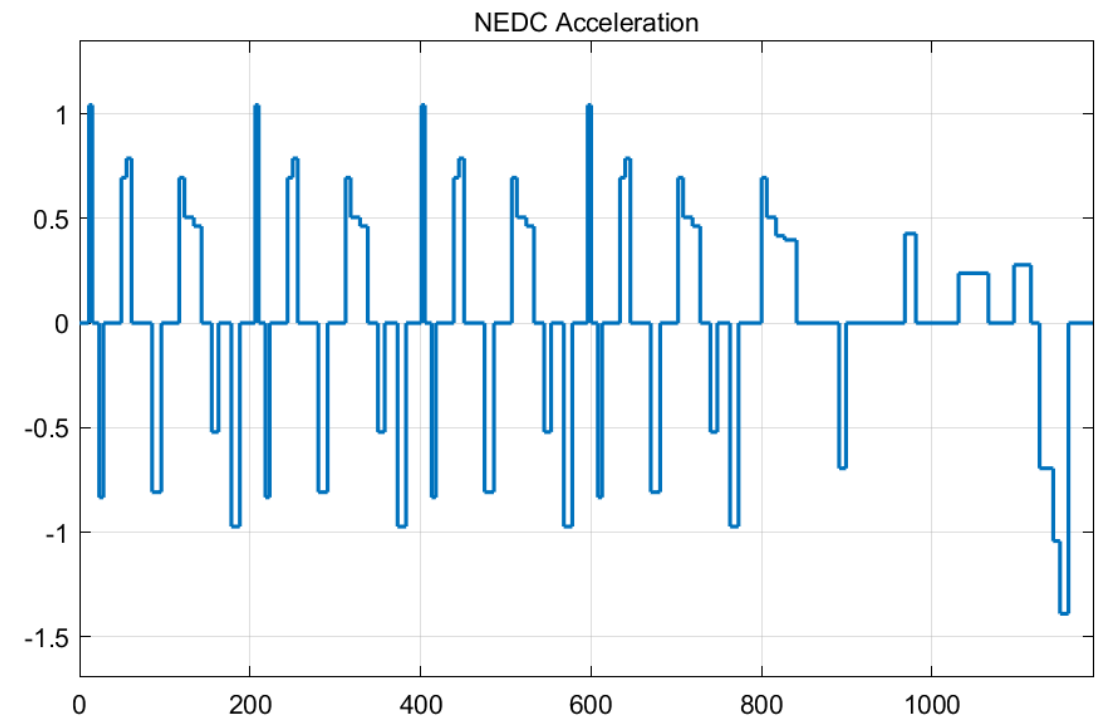
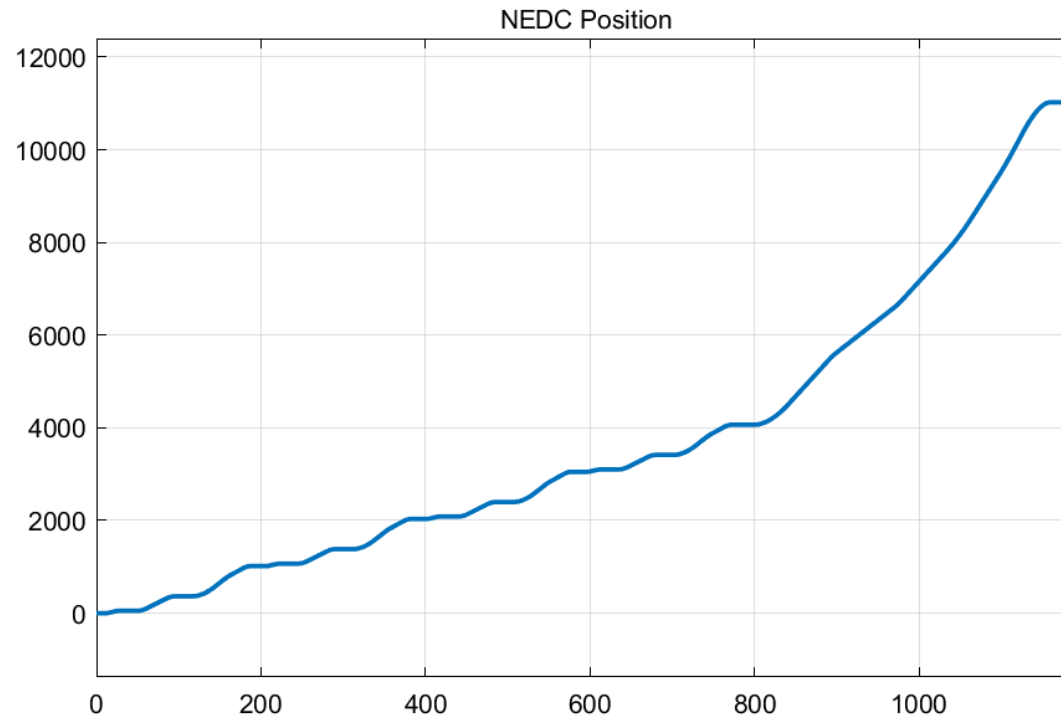
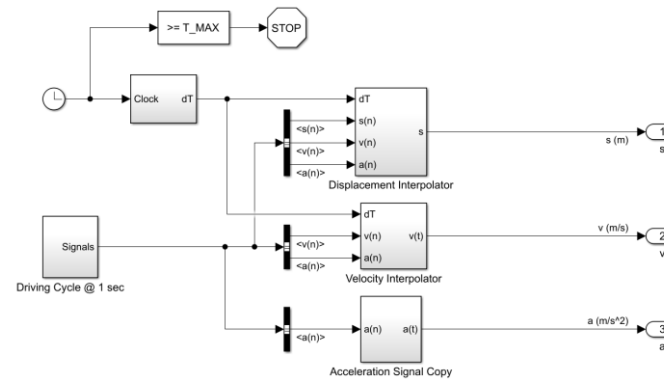
- Desired Speed Values (NEDC):



$$\omega = G \frac{v}{r} \text{ rad.s}^{-1}$$

Simulation

- NEDC Profiles:

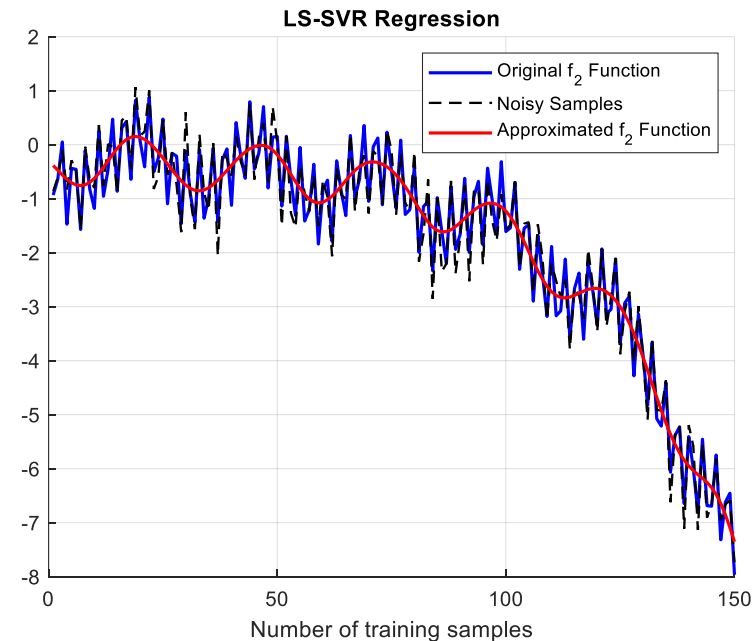
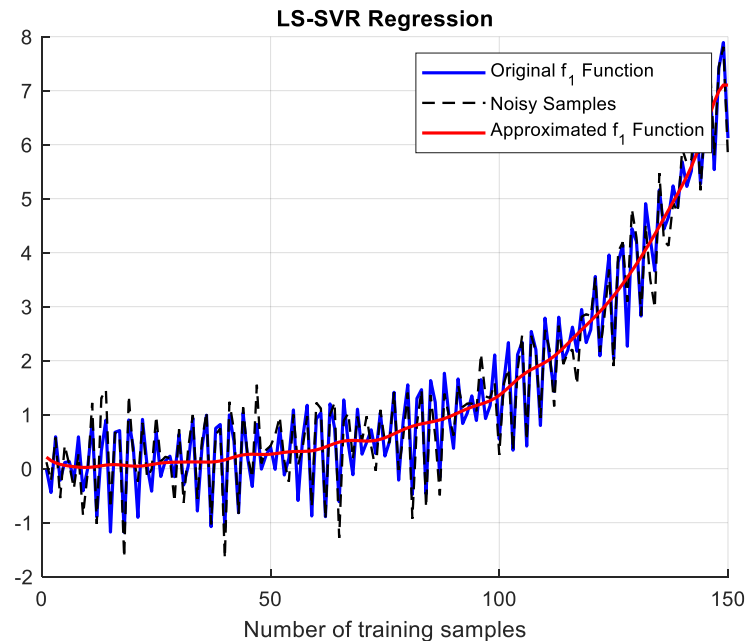


Simulation

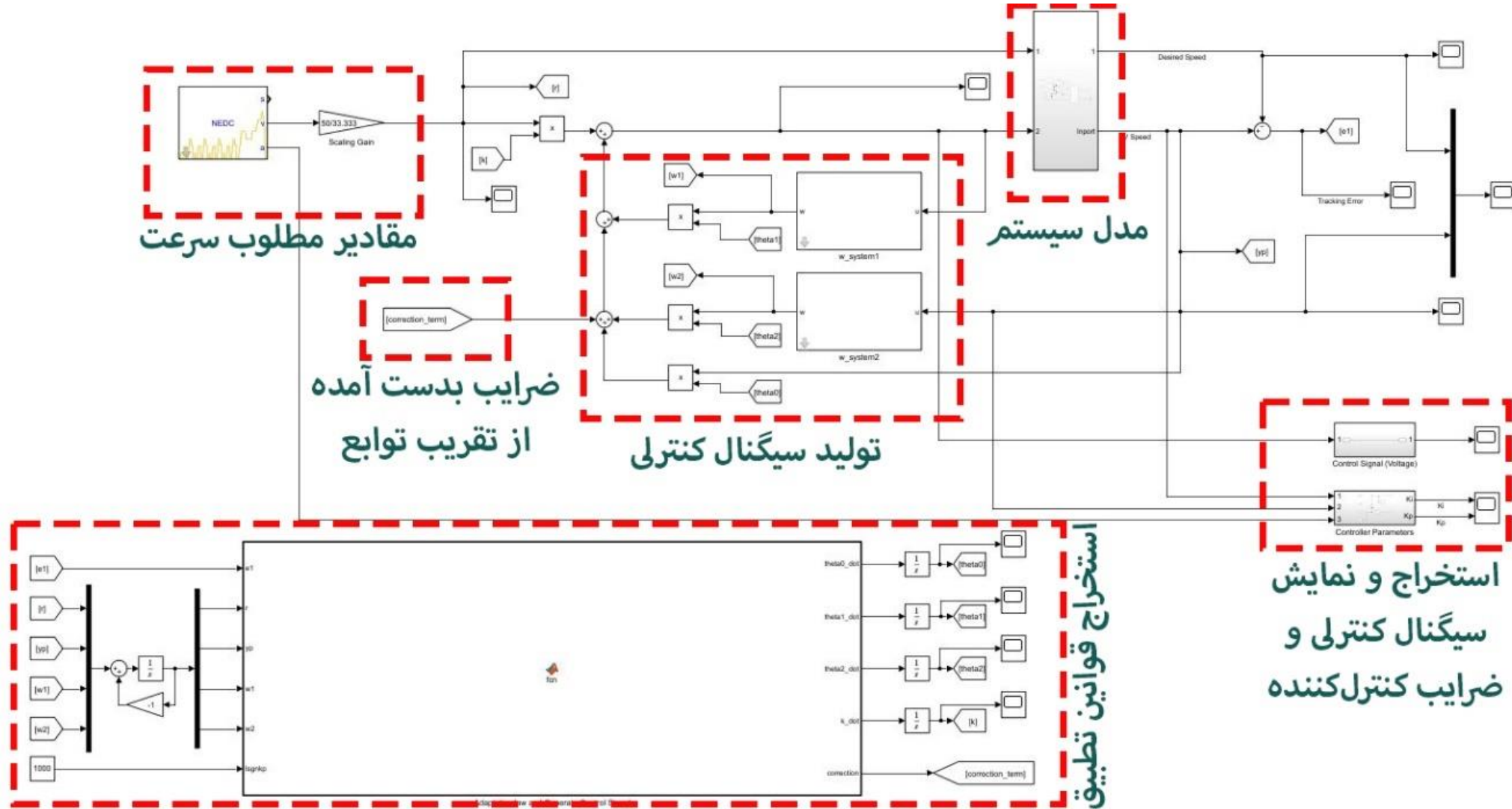
- Approximate f_1 and f_2 using LS-SVR:

$C = 100$; %Parameter defined to avoid overfitting

$g = 0.01$; %Radial Basis Function learning parameter, is equal to $1/2\sigma^2$

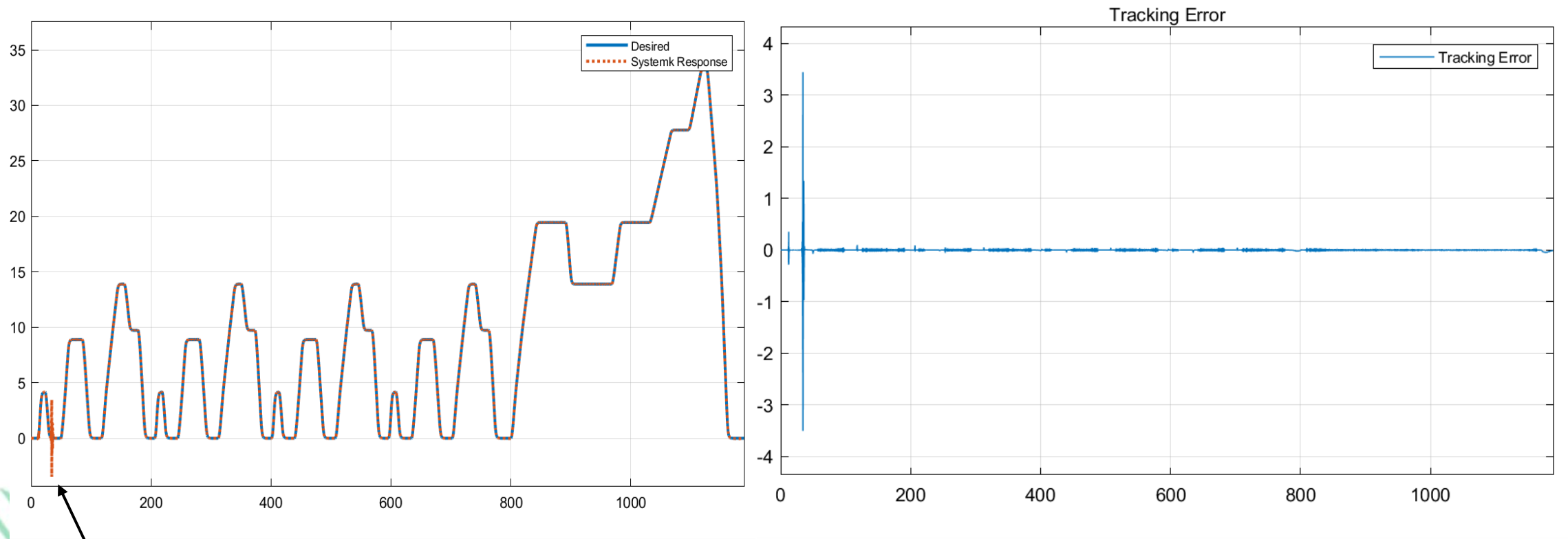


Proposed Control Algorithm



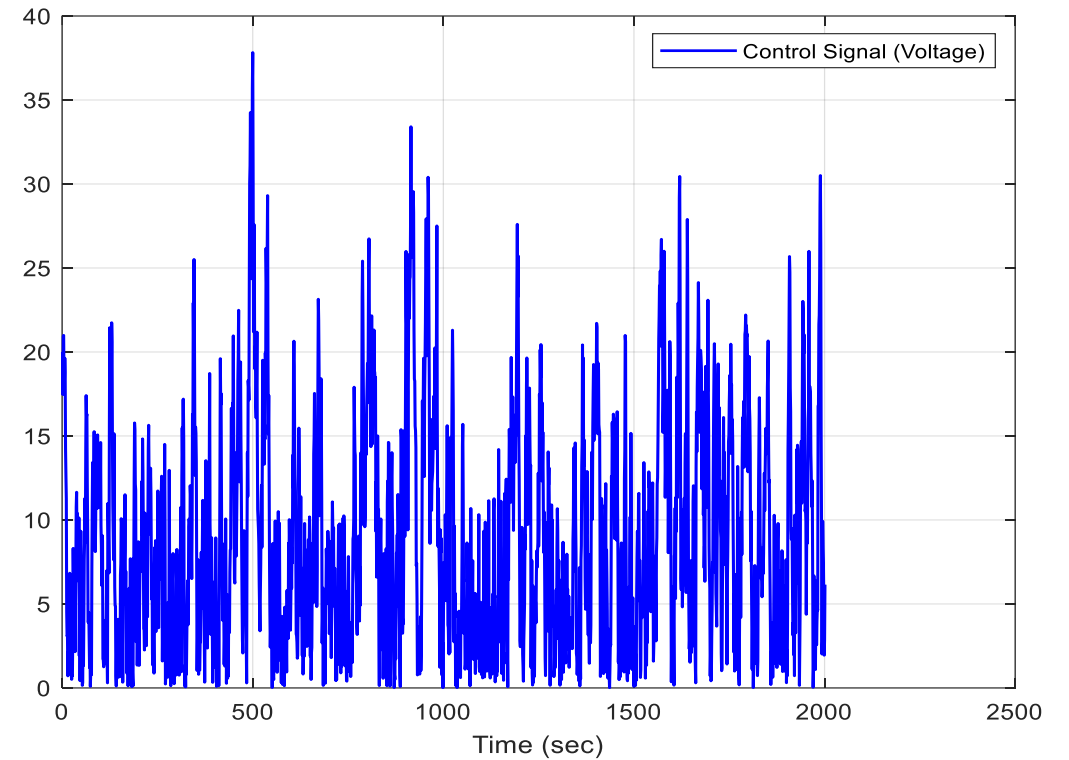
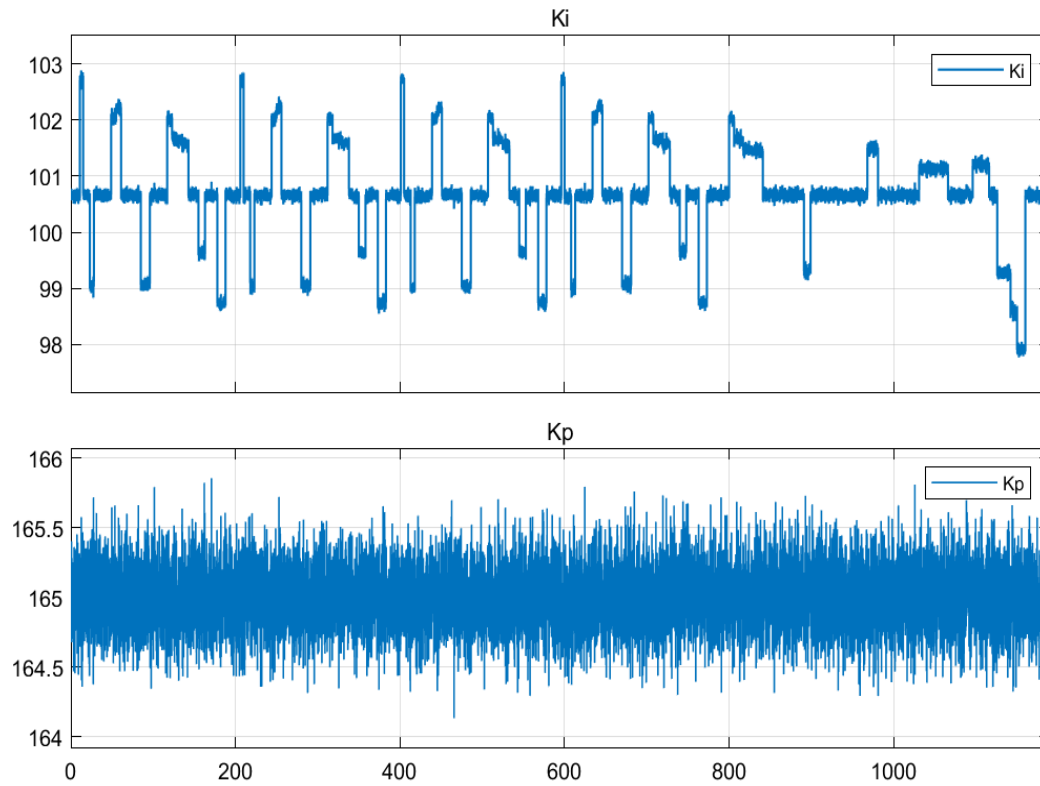
Proposed Control Algorithm

- By applying the NEDC Profiles as Desired input, The Response is:



$5 \cos(2\pi t)$ @ $t=35$

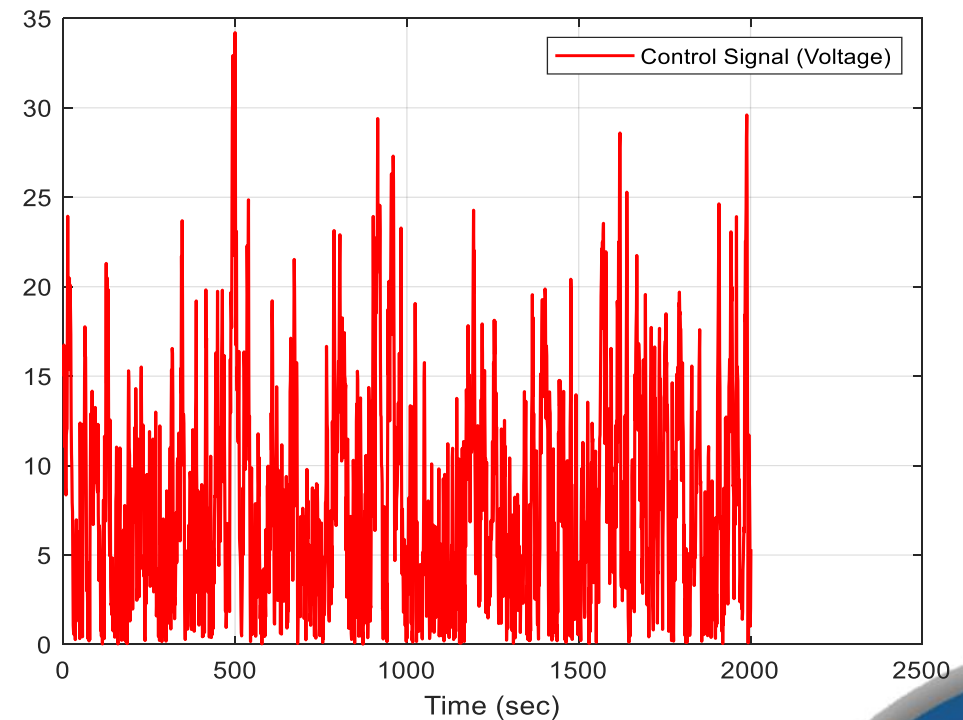
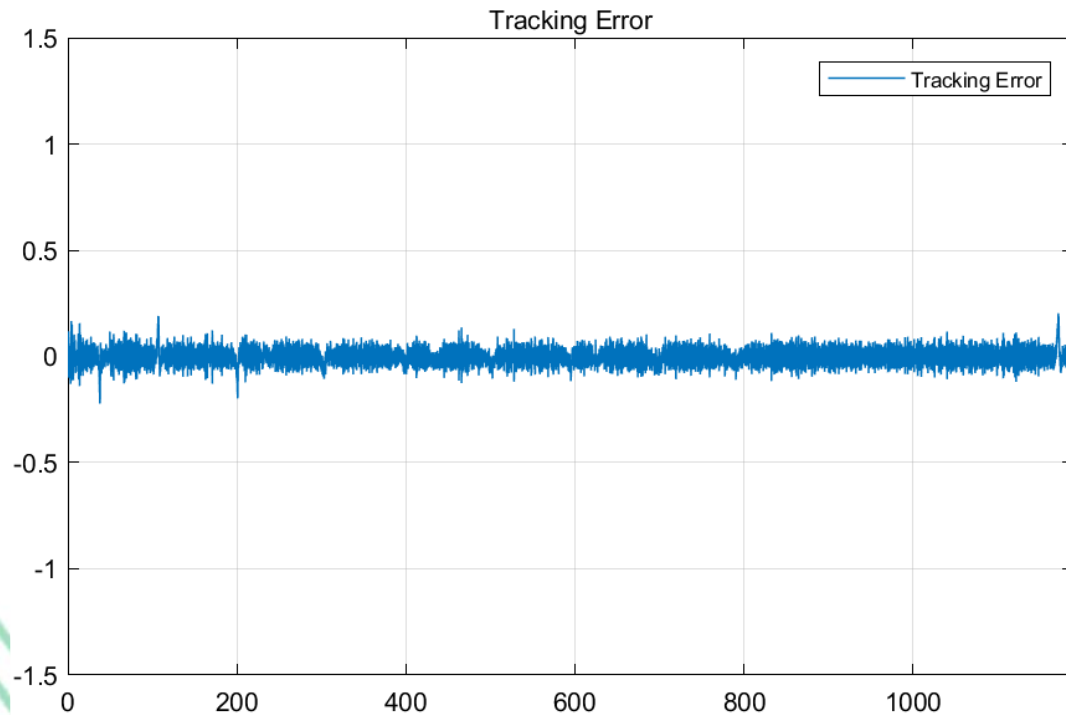
Proposed Control Algorithm



Proposed Control Algorithm

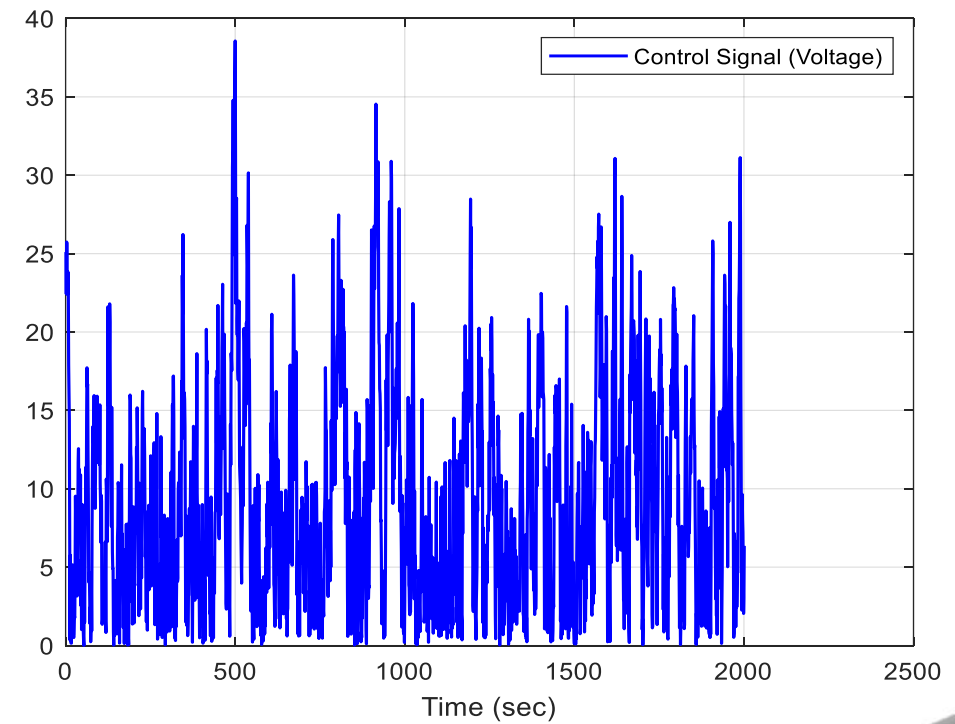
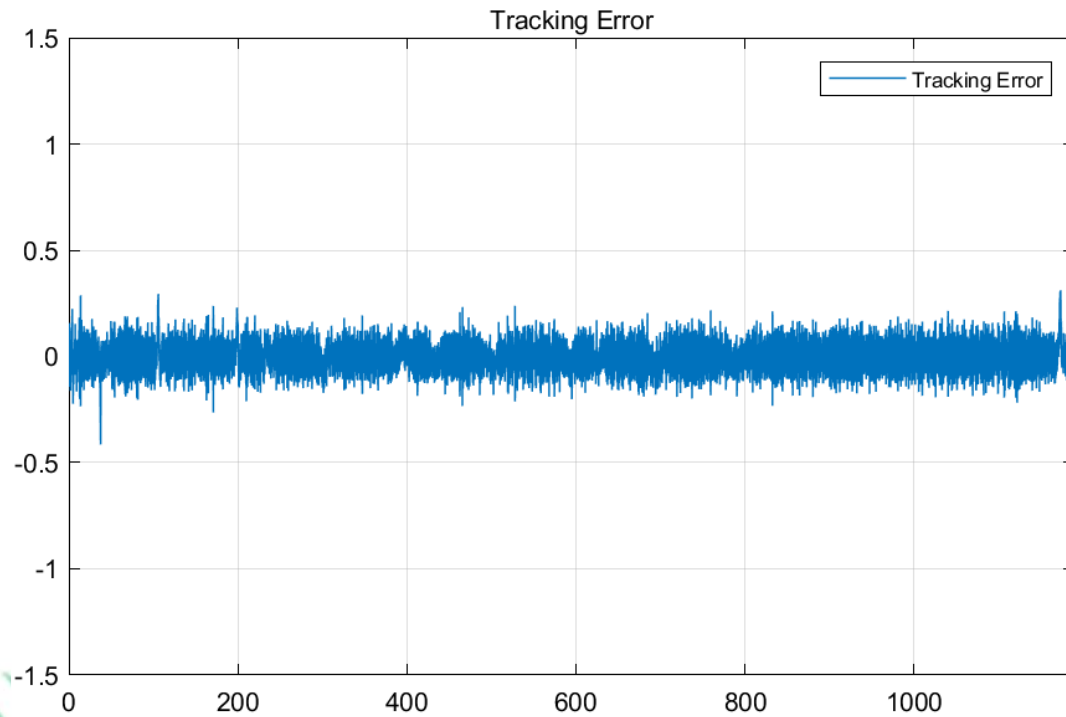
- Learning Rate's Effect
 - $\Gamma_i = 100$

$$\Gamma_i = [100 \quad 150 \quad 200]$$



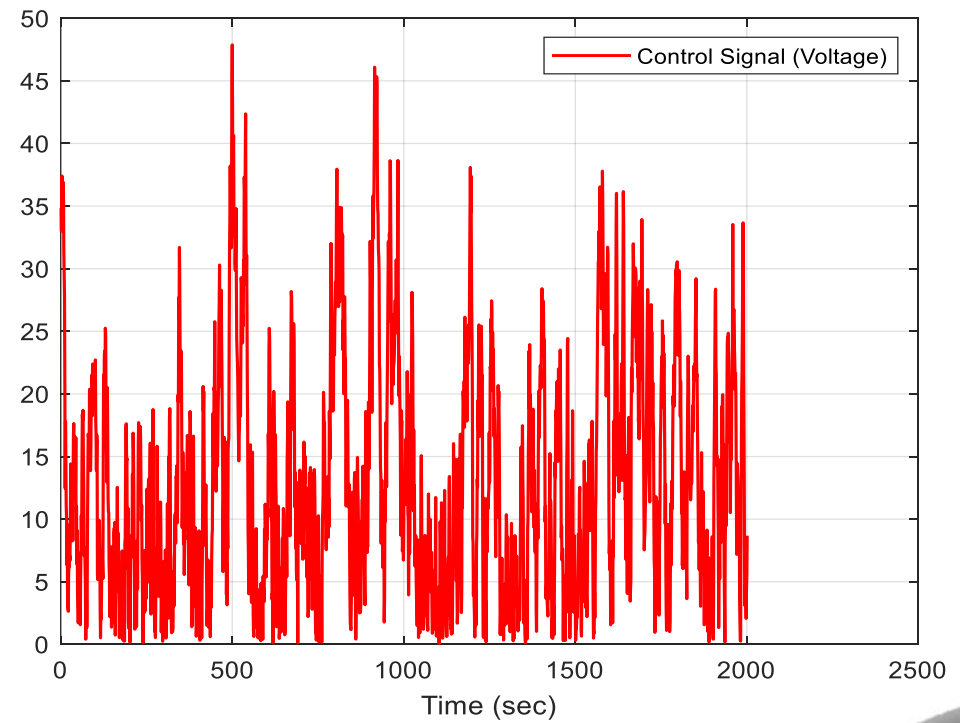
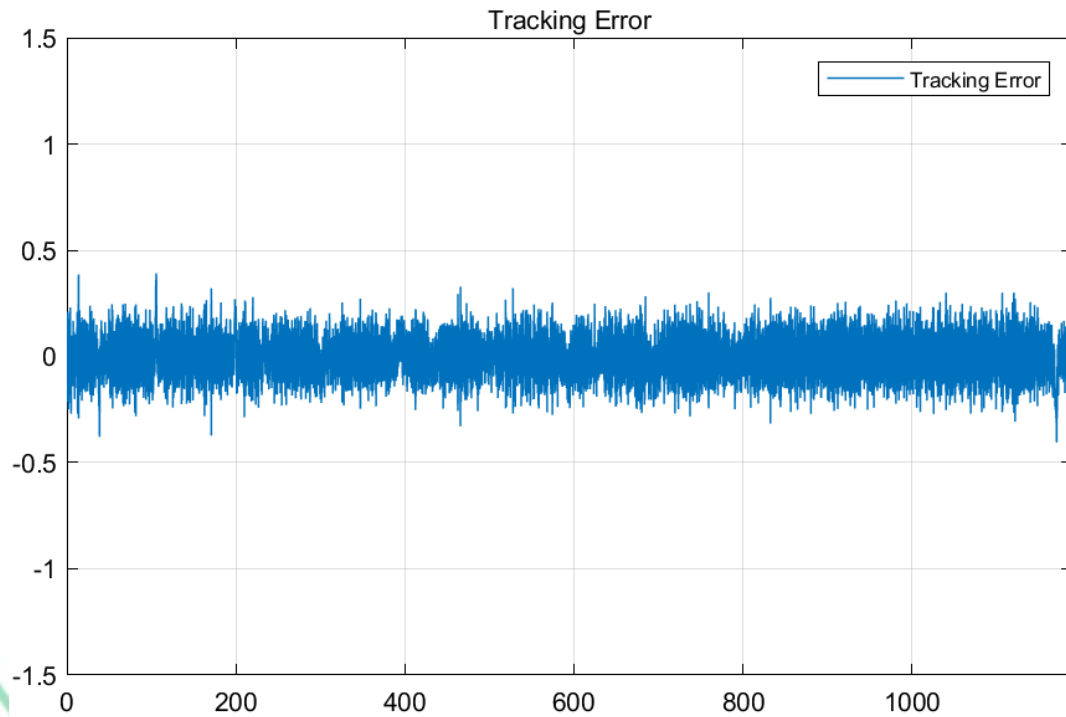
Proposed Control Algorithm

- $\Gamma_i = 150$



Proposed Control Algorithm

- $\Gamma_i = 200$



Thanks For Your Attention

References

- [1] M. H. Khooban, O. Naghash-Almasi, T. Niknam, and M. Sha-Sadeghi, "Intelligent robust PI adaptive control strategy for speed control of EV(s)," IET Sci. Meas. Technol., vol. 10, no. 5, pp. 433–441, 2016.
- [2] Q. Huang, Z. Huang, and H. Zhou, "Nonlinear optimal and robust speed control for a light-weighted all-electric vehicle," IET Control Theory Appl., vol. 3, no. 4, pp. 437–444, 2009.
- [3] W. Wei, "On disturbance rejection for a class of nonlinear systems," Complexity, vol. 2018, pp. 1–14, 2018.
- [4] E. Wei, T. Li, J. Li, Y. Hu, and Q. Li, "Neural network-based adaptive dynamic surface control for inverted pendulum system," in Advances in Intelligent Systems and Computing, Berlin, Heidelberg: Springer Berlin Heidelberg, 2014, pp. 695–704.
- [5] B. Hammer and K. Gersmann, "A note on the universal approximation capability of support vector machines," Neural Process. Lett., vol. 17, no. 1, pp. 43–53, 2003.

References

- [6] J. L. J. Lowry, *Electric Vehicle Technology Explained*. London, England: ISTE Ltd and John Wiley & Sons, 2003.

- [7] S. S. Keerthi and C.-J. Lin, "Asymptotic behaviors of support vector machines with Gaussian kernel," *Neural Comput.*, vol. 15, no. 7, pp. 1667–1689, 2003.

- [8] S. An, W. Liu, and S. Venkatesh, "Fast cross-validation algorithms for least squares support vector machine and kernel ridge regression," *Pattern Recognit.*, vol. 40, no. 8, pp. 2154–2162, 2007.

- [9] S. Wang and B. Meng, "Parameter selection algorithm for support vector machine," *Procedia Environ. Sci.*, vol. 11, pp. 538–544, 2011.

- [10] S. Lin, X. Liu, Y. Rong, and Z. Xu, "Almost optimal estimates for approximation and learning by radial basis function networks," *Mach. Learn.*, vol. 95, no. 2, pp. 147–164, 2014.

References

- [11] C.-N. Ko and C.-M. Lee, "Short-term load forecasting using SVR (support vector regression)-based radial basis function neural network with dual extended Kalman filter," *Energy (Oxf.)*, vol. 49, pp. 413–422, 2013.
- [12] C.-C. Chuang, "Fuzzy weighted support vector regression with a fuzzy partition," *IEEE Trans. Syst. Man Cybern. B Cybern.*, vol. 37, no. 3, pp. 630–640, 2007.
- [13] B. De Moor, J. P. Vandewalle, J. A. Suykens, T. Van Gestel, and J. De Brabanter, *Least Squares Support Vector Machines*. World Scientific Publishing Company, 2002.
- [14] H. Yang, F. Luo, and Y. Xu, "Robust control based on LS-SVM for uncertain nonlinear system," in *2008 IEEE Conference on Cybernetics and Intelligent Systems*, 2008.
- [15] C. Xie, C. Shao, and D. Zhao, "Indirect adaptive control for a class of unknown nonlinear systems based on LS-SVM," in *The 2010 IEEE International Conference on Information and Automation*, 2010.

References

- [16] Y. Gao, Y. Liu, H. Wang, and P. Li, "Adaptive control of a class of nonlinear discrete-time systems with online kernel learning," IFAC proc. vol., vol. 41, no. 2, pp. 7937–7942, 2008.
- [17] H. Zargarzadeh, T. Dierks, and S. Jagannathan, "Adaptive neural network-based optimal control of nonlinear continuous-time systems in strict-feedback form: ADAPTIVE NN-BASED OPTIMAL CONTROL," Int. J. Adapt. Control Signal Process., vol. 28, no. 3–5, pp. 305–324, 2014.
- [18] H.-J. Rong and G.-S. Zhao, "Stable indirect adaptive fuzzy-neuro control for a class of nonlinear systems," in International Conference on Fuzzy Systems, 2010.
- [19] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," IEEE Trans. Fuzzy Syst., vol. 14, no. 5, pp. 676–697, 2006.
- [20] M. Shamsadeghi, M. H. Khooban, and T. Niknam, "A robust and simple optimal type II fuzzy sliding mode control strategy for a class of nonlinear chaotic systems," J. Intell. Fuzzy Syst., vol. 27, no. 4, pp. 1849–1859, 2014.
- [21] M. R. Soltanpour, P. Otadolajam, and M. H. Khooban, "Robust control strategy for electrically driven robot manipulators: adaptive fuzzy sliding mode," IET Sci. Meas. Technol., vol. 9, no. 3, pp. 322–334, 2015.

References

- [22] T. Niknam and M. H. Khooban, "Fuzzy sliding mode control scheme for a class of non-linear uncertain chaotic systems," IET Sci. Meas. Technol., vol. 7, no. 5, pp. 249–255, 2013.
- [23] L.-X. Wang, "Stable adaptive fuzzy controllers with application to inverted pendulum tracking," IEEE Trans. Syst. Man Cybern. B Cybern., vol. 26, no. 5, pp. 677–691, 1996.
- [24] G. F. Montúfar, "Universal approximation depth and errors of narrow belief networks with discrete units," 2013.
- [25] M. Ghaemi, S. K. Hosseini-Sani, and M. H. Khooban, "Direct adaptive general type-2 fuzzy control for a class of uncertain non-linear systems," IET Sci. Meas. Technol., vol. 8, no. 6, pp. 518–527, 2014.
- [26] C.-H. Wang, T.-C. Lin, T.-T. Lee, and H.-L. Liu, "Adaptive hybrid intelligent control for uncertain nonlinear dynamical systems," IEEE Trans. Syst. Man Cybern. B Cybern., vol. 32, no. 5, pp. 583–597, 2002.