Amirkabir University of Technology
(Tehran Polytechnic)

Department of Electrical Engineering

IoT-Based Electric Vehicle State Estimation and Control Algorithms Under Cyber Attacks

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Table of Contents

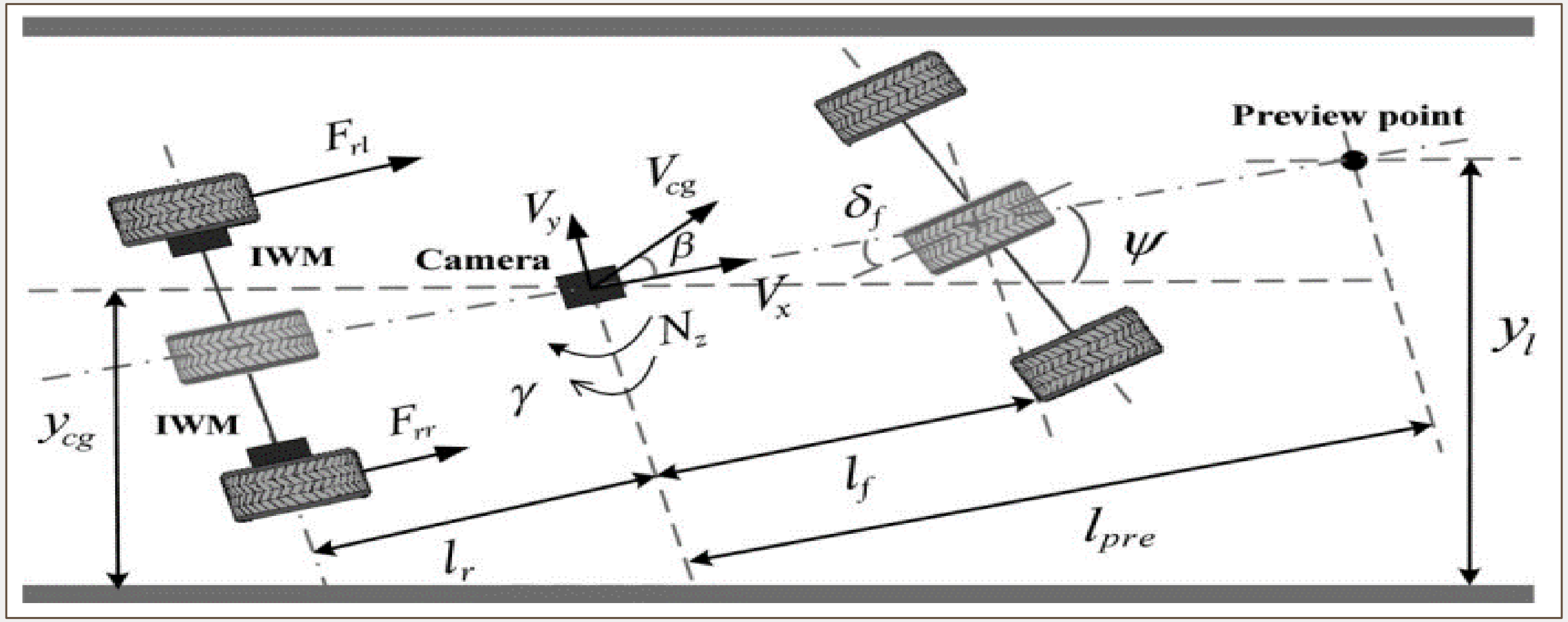
Misson Statement

In this presentation, we are trying to study Implementations of Estimation and different optimization approaches on IoT-based electric vehicle.

01

System Model

Model Schematic



Dynamic Equations

$$\dot{\beta} = 2\frac{C_f}{mV_x}\left(\delta_f - \gamma\frac{l_f}{V_x} - \beta\right) - \frac{\gamma}{mV_x} + \frac{2C_r}{mV_x}\left(\gamma\frac{l_r}{V_x} - \beta\right)$$

$$\dot{\gamma} = 2\frac{l_f C_f}{I}\left(\delta_f - \gamma\frac{l_f}{V_x} - \beta\right) + \frac{N_z}{I} + \frac{2 l_r C_r}{I}\left(\gamma\frac{l_r}{V_x} - \beta\right)$$

$$T_l = F_{rl} r = \frac{m r a_x}{2} + \frac{r N_z}{d_r}, \qquad T_r = F_{rr} r = \frac{m r a_x}{2} - \frac{r N_z}{d_r}$$

$$\dot{\psi} = \gamma, \quad y_l \approx y_{cg} + \psi l_{pev}, \quad \dot{y}_{cg} \approx V_x(\beta + \psi), \quad \dot{y}_l = V_x(\beta + \psi) + \gamma l_{pev}$$

State Space

$$A_c = \begin{bmatrix} -2\dfrac{C_r + C_f}{mV_x} & 2\dfrac{C_f l_f - C_f l_f}{mV_x^2} - 1 & 0 & 0 \\ 2\dfrac{C_r l_r - C_f l_f}{I} & 2\dfrac{-C_r l_r^2 - C_f l_f^2}{IV_x} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ V_x & l_{pre} & V_x & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 2\dfrac{C_f}{mV_x} & 2\dfrac{C_f l_f}{I} & 0 & 0 \\ 0 & \dfrac{1}{I} & 0 & 0 \end{bmatrix}'$$

Eigen values of the system: [0 0 -7.27 1.22]

Symbols	Values	Symbols	Values
m	380 kg	l_f	0.8 m
l_r	0.6 m	d_r	0.82 m
r	0.22 m	C_f	6000 N/rad
C_r	6000 N/rad	**Q**	0.0005***I**
T	0.001 sec	**R**	0.05***I**

Discrete Time:

$$X_{k+1} = A_d X_k + B_d u_k + n_k$$

$$y_k = C X_k + v_k$$

Eigen Values

$$[1 \quad 1 \quad 0.9927 \quad 1.0012]$$

$$A_d = \begin{bmatrix} 0.9975 & -0.0010 & 0 & 0 \\ -0.0176 & 0.9965 & 0 & 0 \\ 0 & 0.0010 & 1 & 0 \\ 0.025 & 0.0015 & 0.025 & 1 \end{bmatrix}$$

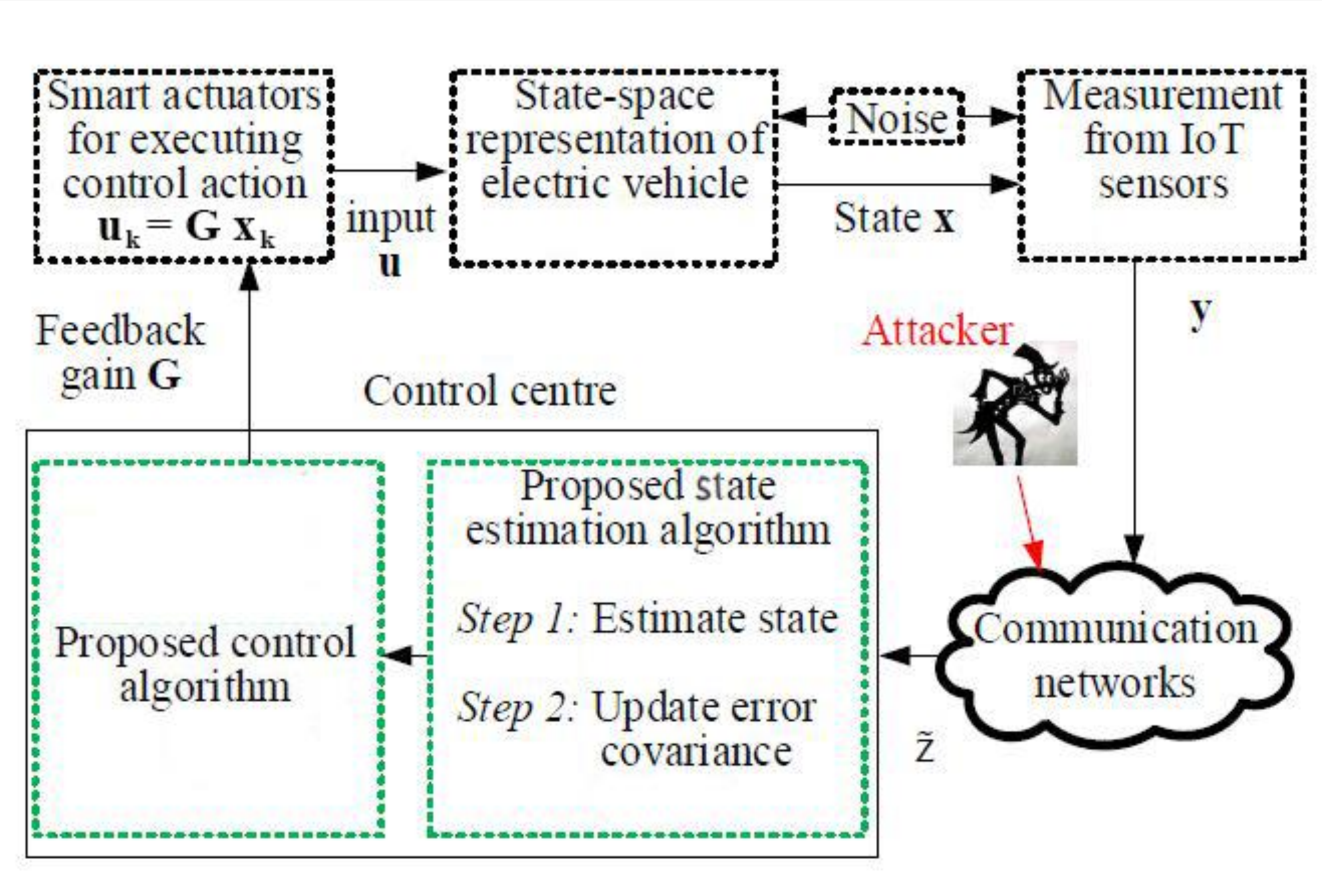
$$B_d = \begin{bmatrix} 0.0012 & 0 \\ 0.0704 & 0 \\ 0 & 0 \\ 0.0001 & 0 \end{bmatrix}$$

02

State Estimation

Based on the mean square error principle, the optimal state estimation algorithm is derived

State Estimation:



State Estimation:

- Information that is sent through the communication channel:

$$z_k = y_k - C\hat{x}_k^-$$

- Manipulated information received at the center:

$$\tilde{\mathbf{z}}_k = \underbrace{\boldsymbol{T}_k}_{\text{Attacker Matrix}} \mathbf{z}_k + \underbrace{\boldsymbol{a}_k}_{\text{Channel noise}}$$

State Estimation:

$$\tilde{x}_k = \hat{x}_k^- + K\,\tilde{z}_k$$

Prediction

Correction

$$K_k = \tilde{P} C^T (C \tilde{P} C^T + R)^{-1}$$

$$\tilde{P}_k = \tilde{P}_k^- + \bar{P} C^T \left(\breve{P} - T_k^T \breve{P} - \breve{P} T_k\right) C \bar{P}$$

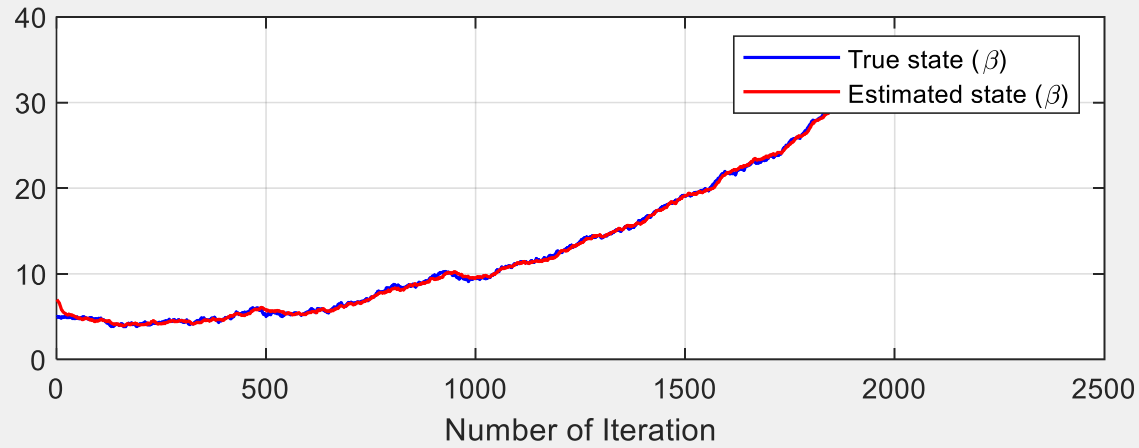
$$\tilde{P}_k^- = A_d \tilde{P}_{k-1} A_d^T + Q$$

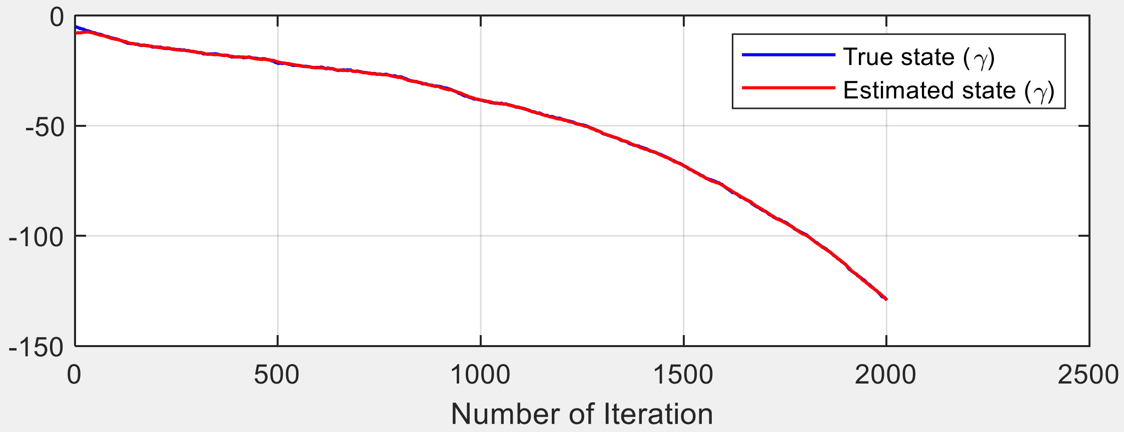
$$\breve{P} = (C \bar{P} C^T + R)^{-1}$$

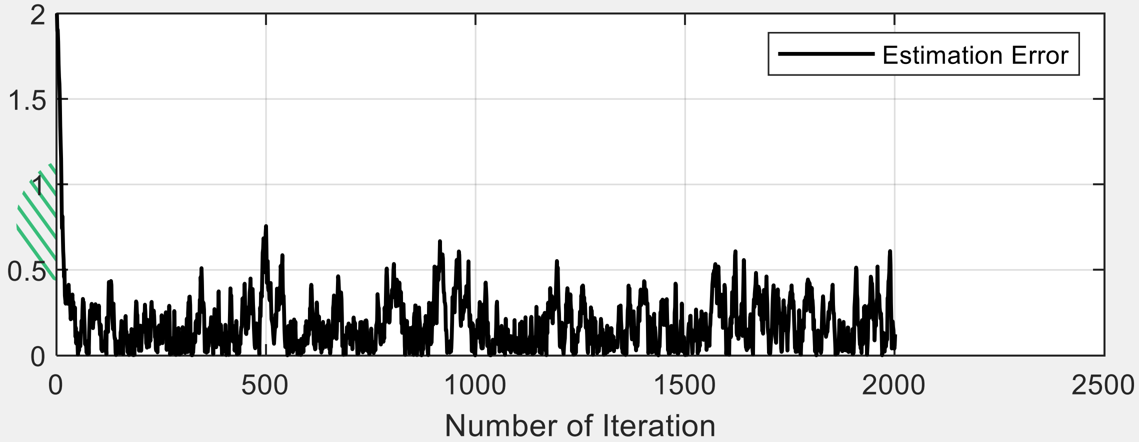
$$\bar{P} = \tilde{P}_0$$

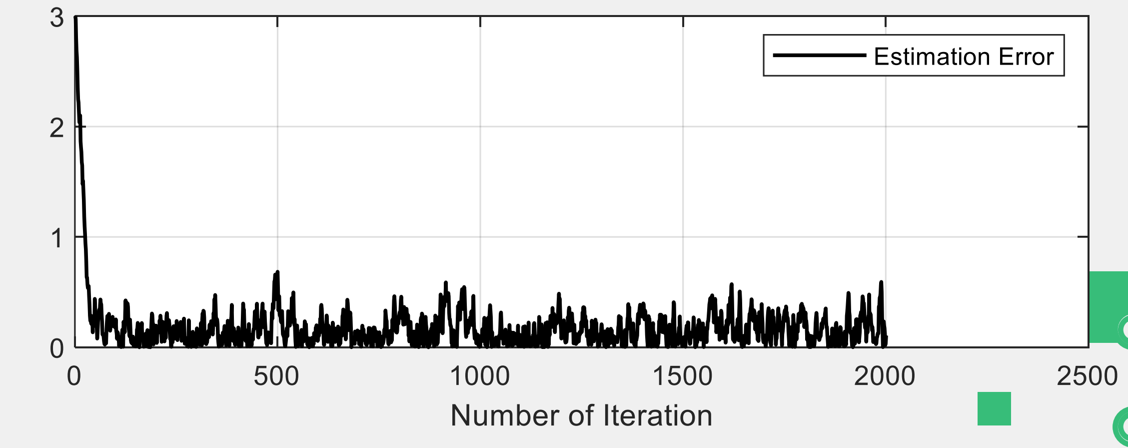
Simulation:

$$\bar{P} = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0.02 \end{bmatrix} \quad u_k = \begin{bmatrix} \exp(-10k) \\ \sin\left(\frac{k}{2}\right) \end{bmatrix}$$

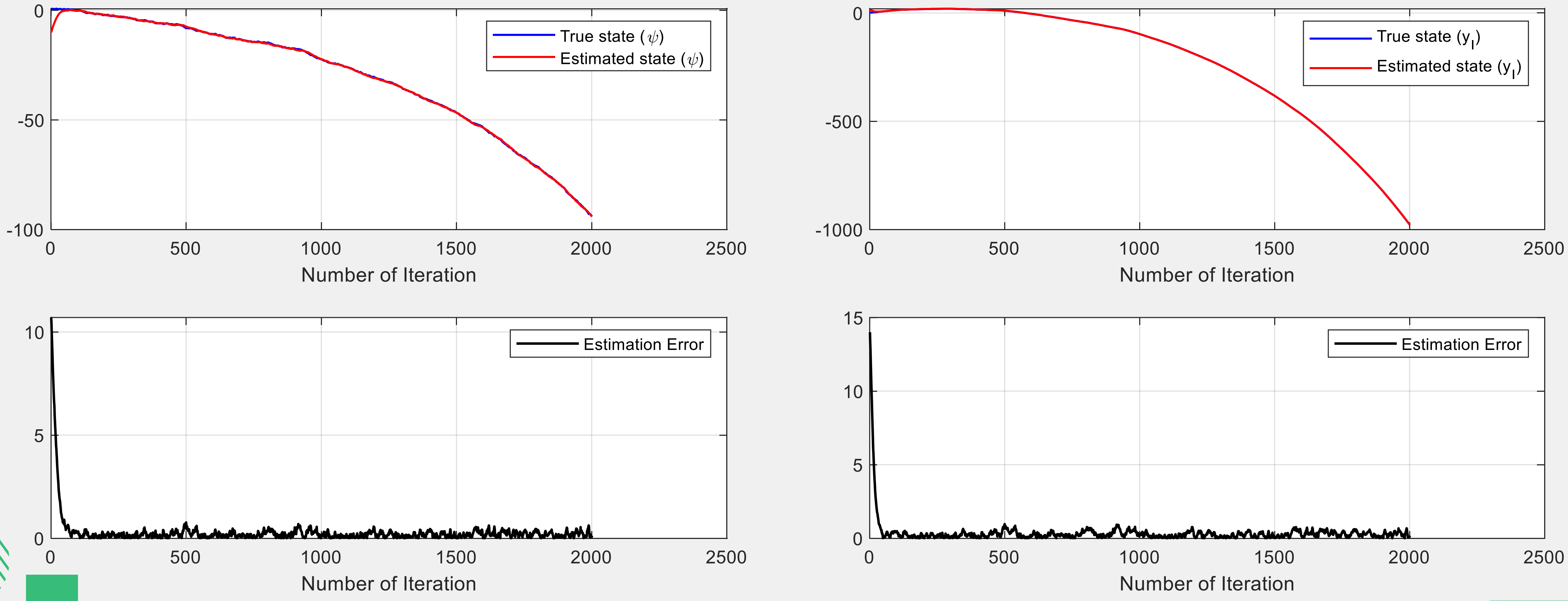








Simulation:


True state (ψ)
Estimated state (ψ)
True state (y_l)
Estimated state (y_l)
Estimation Error
Estimation Error
Number of Iteration
Number of Iteration
Number of Iteration
Number of Iteration
0
500
1000
1500
2000
2500
0
-50
-100
-500
-1000
10
5
15

03

Proposed Control Algorithm

Based on the mean square error principle, the optimal state estimation algorithm is derived

Control Signal:

- the optimal control algorithm is designed based on the semidefinite programming approach. According to the separation principle, the feedback control law is defined:

$$\boldsymbol{u}_k = \boldsymbol{G}\boldsymbol{x}_k$$

- Inspired by Bounded Real Lemma:

$$\begin{aligned} &\text{minimise} \quad \boxed{\xi} \\ &\text{subject to} \quad \mathbf{A}'_{cl}\mathbf{P}\mathbf{A}_{cl} - \mathbf{P} + \xi < \mathbf{0}, \mathbf{P} > \mathbf{0}. \end{aligned}$$

Control Signal:

$$X = P^{-1}$$

$$(\mathbf{A}_d + \mathbf{B}_d\mathbf{G})'\mathbf{X}^{-1}(\mathbf{A}_d + \mathbf{B}_d\mathbf{G}) - \mathbf{X}^{-1} + \xi < \mathbf{0}.$$

$$\mathbf{X}(\mathbf{A}_d + \mathbf{B}_d\mathbf{G})'\mathbf{X}^{-1}(\mathbf{A}_d + \mathbf{B}_d\mathbf{G})\mathbf{X} - \mathbf{X} + \xi\mathbf{X}\mathbf{X} < \mathbf{0}.$$

- Applying the Schur's Complement:

$$\begin{bmatrix} -\mathbf{X} & \mathbf{X}(\mathbf{A}_d' + \mathbf{B}_d'\mathbf{G}') & \mathbf{X} \\ \mathbf{X}(\mathbf{A}_d' + \mathbf{B}_d'\mathbf{G}')' & -\mathbf{X} & \mathbf{0} \\ \mathbf{X} & \mathbf{0} & -\xi\mathbf{I} \end{bmatrix} < \mathbf{0}$$

Control Signal:

$$S = GX$$

YALMIP

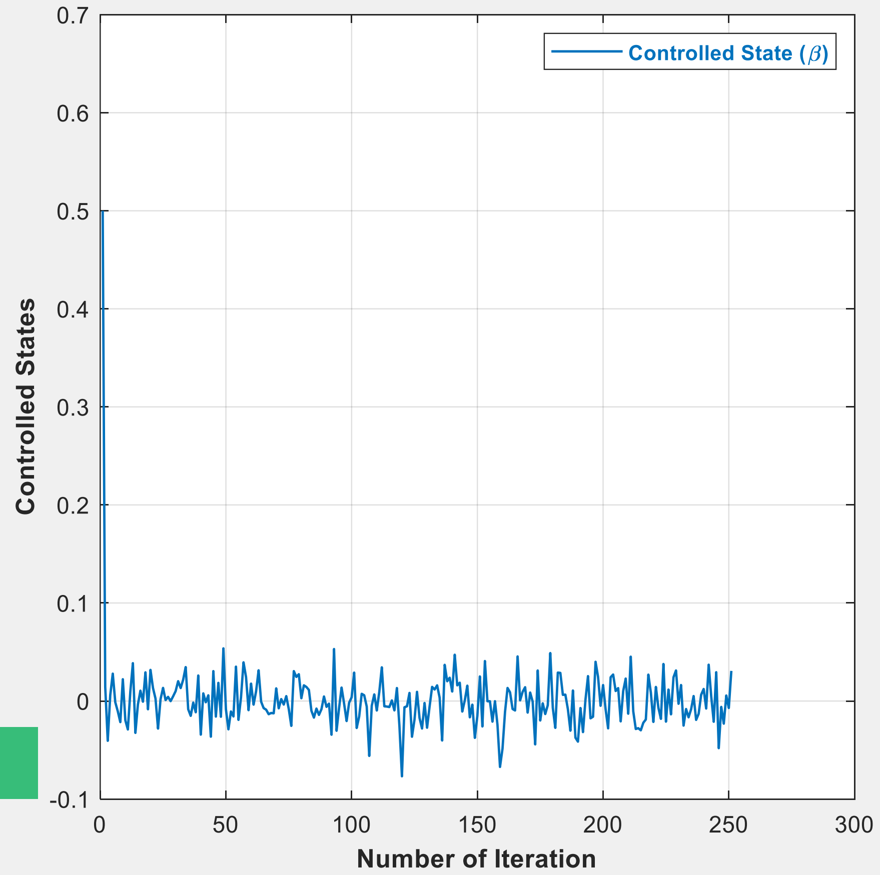
$$\begin{bmatrix} -\mathbf{X} & \mathbf{X}\mathbf{A}_d' + \mathbf{S}'\mathbf{B}_d' & \mathbf{X} \\ (\mathbf{X}\mathbf{A}_d' + \mathbf{S}'\mathbf{B}_d')' & -\mathbf{X} & \mathbf{0} \\ \mathbf{X} & \mathbf{0} & -\xi\mathbf{I} \end{bmatrix} < \mathbf{0}$$

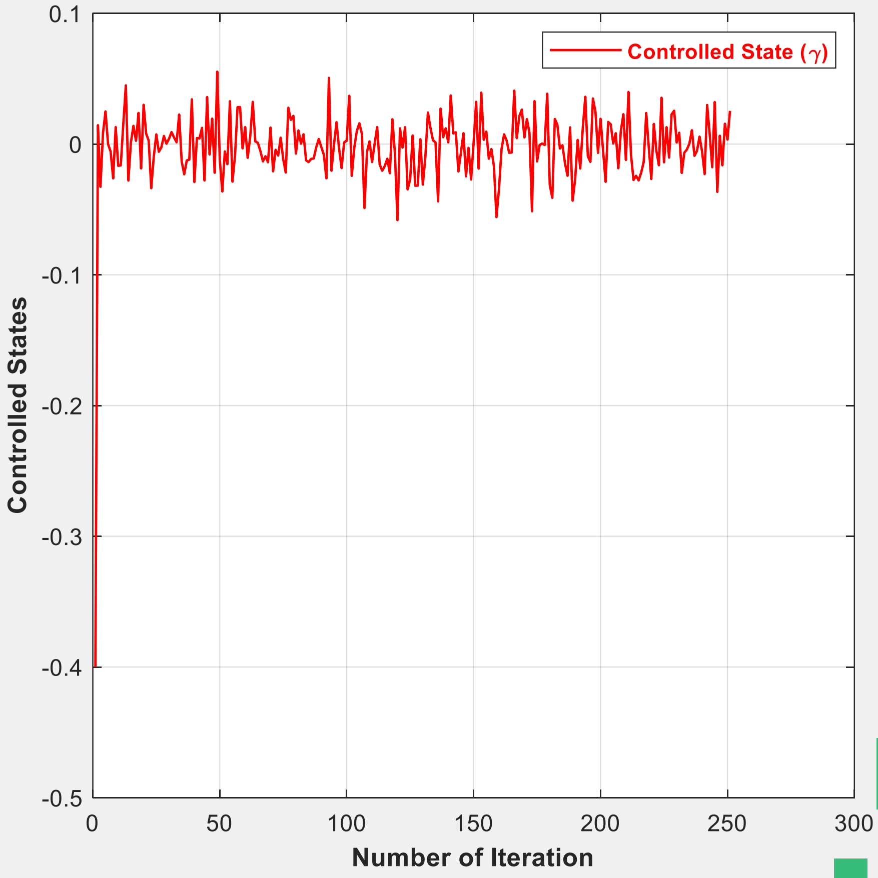
$$G = SX^{-1}$$

Control Signal:

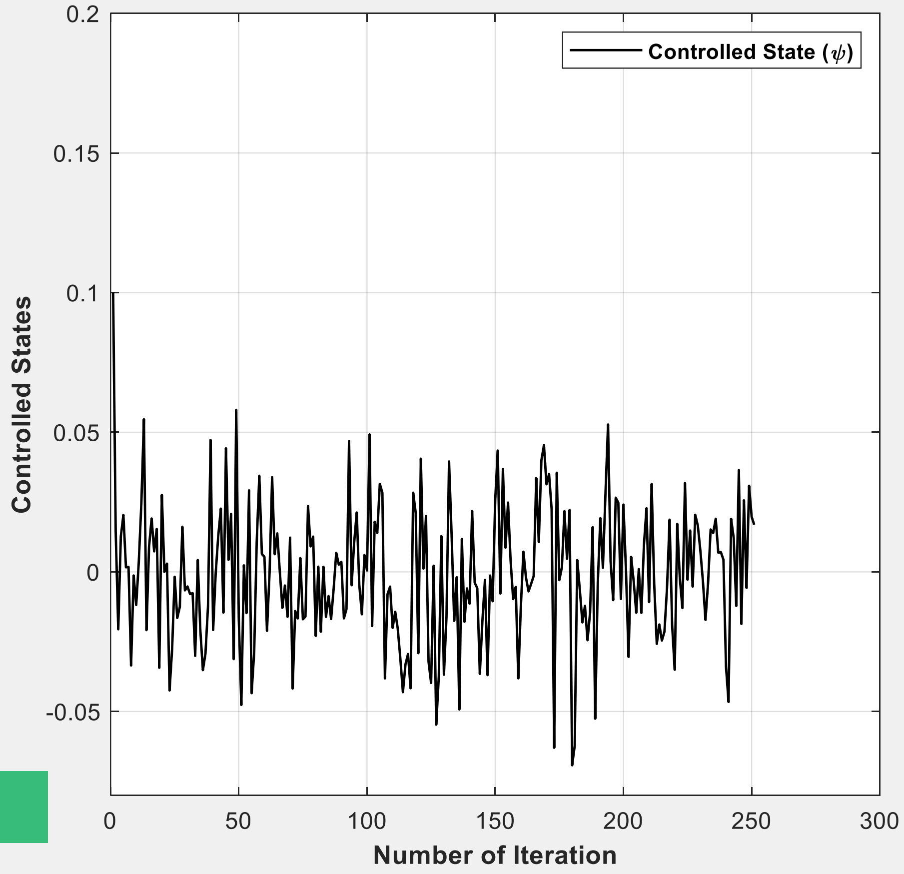
```
The optimal State feedback gain is:
   1.0e+04 *

   -0.0001    0.0000   -0.0000   -0.0000
    1.2405   -0.0229    0.0008    0.0148

The Closed-Loop system eigenvalues:
  -0.6326 + 0.0000i
   0.0000 + 0.0000i
   0.9998 + 0.0001i
   0.9998 - 0.0001i

The Optimal Value of Zeta is:
    0.0790

The Ellapsed Time to Calculate the Optimal StateFeedback gain is:2.5248
```

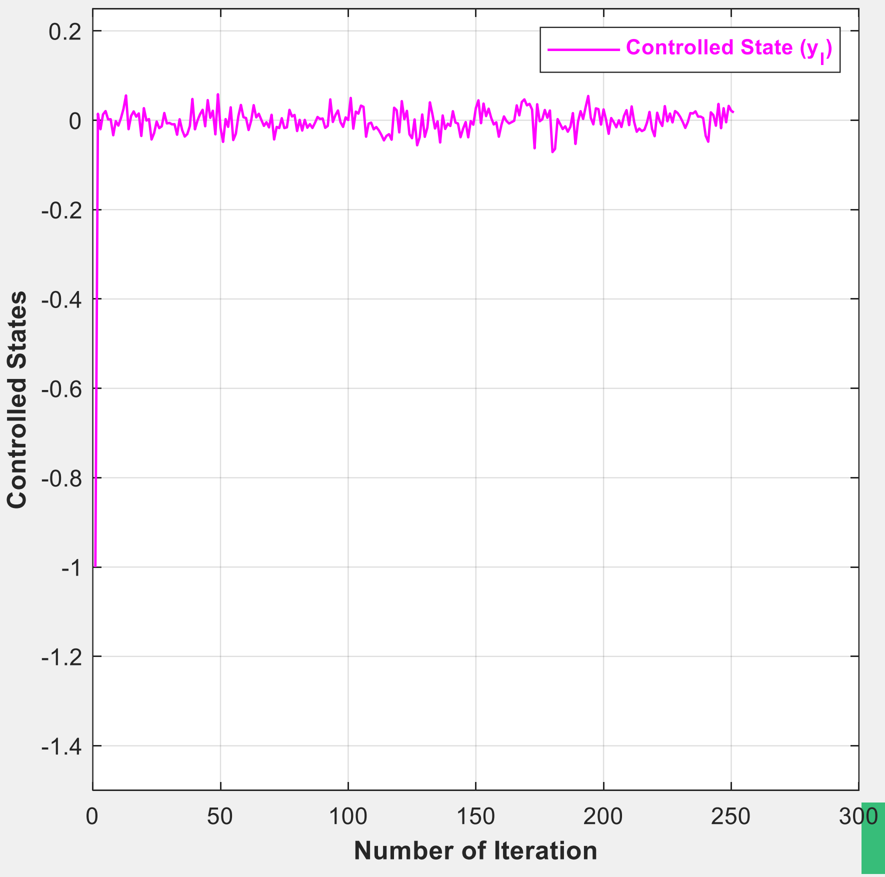
Simulation Result:





Simulation Result:





04

Solve SDLQR usnig RL

SDLQR:

$$V_h(x(k)) = r\left(x(k), h(x(k))\right) + \boxed{\gamma}\, V_h(x(k+1))$$

$$r(x(k), u(k)) = x(k)^T Q x(k) + u(k)^T R u(k)$$

$$u(k) = h(x(k)) = -K.x(k)$$

$$V_h(x(k)) = E\left(\sum_{i=k}^{k+T} \gamma^{i-k} r_i\right)$$

SDLQR:

- Discrete System with Additive and Multiplicative Noise:

$$x_{k+1} = Ax_k + Bu_k + (Cx_k + Du_k)d_k + w_k$$

$$E(x_0\, d_i) = E(x_0\, w_i) = E(w_i\, d_j) = 0 \; for \; all \; i, j$$

SDLQR:

- The ASS System (Unforced):

$$\rho(A \otimes A + C \otimes C) < 1$$

- Admissible Control Policy (u = Lx):

$$P = (A+BL)^\top P(A+BL)+(C+DL)^\top P(C+DL)+F$$

SDLQR:

- The ASS System (Unforced):

$$\rho(A \otimes A + C \otimes C) < 1$$

- Admissible Control Policy (u = Lx):

$$P = (A+BL)^\top P(A+BL)+(C+DL)^\top P(C+DL)+F$$

SDLQR:

- Cost Function and Optimal Control Policy:

$$V_h\big(x(k)\big) = E\Big(\sum_{i=k}^{k+T} \gamma^{i-k} c_i(x_i, u_i)\Big)$$

- Defining the SDLQR Problem as:

$$V^*(x_0) = \min_{u \in U_{ad}} V(x_0)$$

SDLQR:

- Cost Function and Optimal Control Policy:

$$V_h(x(k)) = E(\sum_{i=k}^{k+T} \gamma^{i-k} c_i(x_i, u_i))$$

- Defining the SDLQR Problem as:

$$V^*(x_0) = \min_{u \in U_{ad}} V(x_0)$$

SDLQR:

- Well-Posed SDLQR:

$$-\infty < V^*(x_k) < +\infty$$

- It's proven that For an Admissible u = Lx:

$$V(x_k) = \mathrm{E}(x_k^\top P x_k) + \boxed{\frac{\gamma}{1-\gamma}\mathrm{tr}(PW)}$$

SLE

$$P = \gamma(A+BL)^\top P(A+BL) + \gamma(C+DL)^\top P \times (C+DL) + L^\top RL + Q.$$

SDLQR:

- Back to the Cost Function:

$$V(x_k) = \mathrm{E}\big(c(x_k, u_k)\big) + \gamma \mathrm{E}\Big(\sum_{i=k+1}^{\infty} \gamma^{i-k-1} c(x_i, u_i)\Big)$$

Bellman's Equation

Stochastic form

$$V(x_k) = \mathrm{E}\big(c(x_k, u_k)\big) + \gamma V(x_{k+1})$$

$$\mathrm{E}(x_k^\top P x_k) = \mathrm{E}(x_k^\top Q x_k + u_k^\top R u_k)$$

$$+ \gamma \mathrm{E}(x_{k+1}^\top P x_{k+1}) - \gamma \mathrm{tr}(PW)$$

SDLQR:

- Hamiltonian for u = Lx:

$$H(x_k, L) = \mathrm{E}\big(x_k^\top (Q + L^\top RL)x_k\big) + \gamma \mathrm{E}(x_{k+1}^\top P x_{k+1}) \\ - \mathrm{E}(x_k^\top P x_k) - \gamma \mathrm{tr}(PW).$$

$$\frac{\partial H(x_k, L)}{\partial L}$$

$$L^* = -(R+\gamma B^\top P^* B+\gamma D^\top P^* D)^{-1}(\gamma B^\top P^* A+\gamma D^\top P^* C)$$

SARE

$$P^* = Q + \gamma A^\top P^* A + \gamma C^\top P^* C - (\gamma A^\top P^* B + \gamma C^\top P^* D) \\ \times (R + \gamma B^\top P^* B + \gamma D^\top P^* D)^{-1}(\gamma B^\top P^* A + \gamma D^\top P^* C)$$

SDLQR:

- Fixed Point Algorithm:

$$f(x) = 0$$

$$x = g(x)$$

- Conditions:

$$|\acute{g}(x)| < 1$$

$$if\ x \in [a,b] \rightarrow g(x) \in [a,b]$$

$$x(j+1) = g(x(j))$$

SDLQR:

Input: Admissible control gain $L^{(0)}$, discount factor γ, maximum number of iterations i_{max}, convergence tolerance ε

Output: The estimated optimal control gain $\hat{L}$

for $i = 0 : i_{max}$ **do**

  Policy Evaluation:

$$P^{(i)} = \gamma(A + BL^{(i)})^\top P^{(i)}(A + BL^{(i)}) + \gamma(C + DL^{(i)})^\top \times P^{(i)}(C + DL^{(i)}) + (L^{(i)})^\top RL^{(i)} + Q$$

  Policy Improvement:

$$L^{(i+1)} = -(R + \gamma B^\top P^{(i)} B + \gamma D^\top P^{(i)} D)^{-1} \times (\gamma B^\top P^{(i)} A + \gamma D^\top P^{(i)} C)$$

  if $\|L^{(i+1)} - L^{(i)}\| < \varepsilon$ **then**

    Break

  endif

endfor

$\hat{L} = L^{(i+1)}$

SDLQR:

- :(

$$x_{k+1} = \begin{bmatrix} 0.8 & 1 \\ 1.1 & 2 \end{bmatrix} x_k + \begin{bmatrix} 0.2 \\ 1.4 \end{bmatrix} u_k + \left(\begin{bmatrix} 0.7 & 0 \\ -1 & -0.5 \end{bmatrix} x_k + \begin{bmatrix} -1 \\ 0.8 \end{bmatrix} u_k \right) d_k + w_k$$

$$P^* = \begin{bmatrix} 8.2254 & 8.0704 \\ 8.0704 & 10.3873 \end{bmatrix}$$

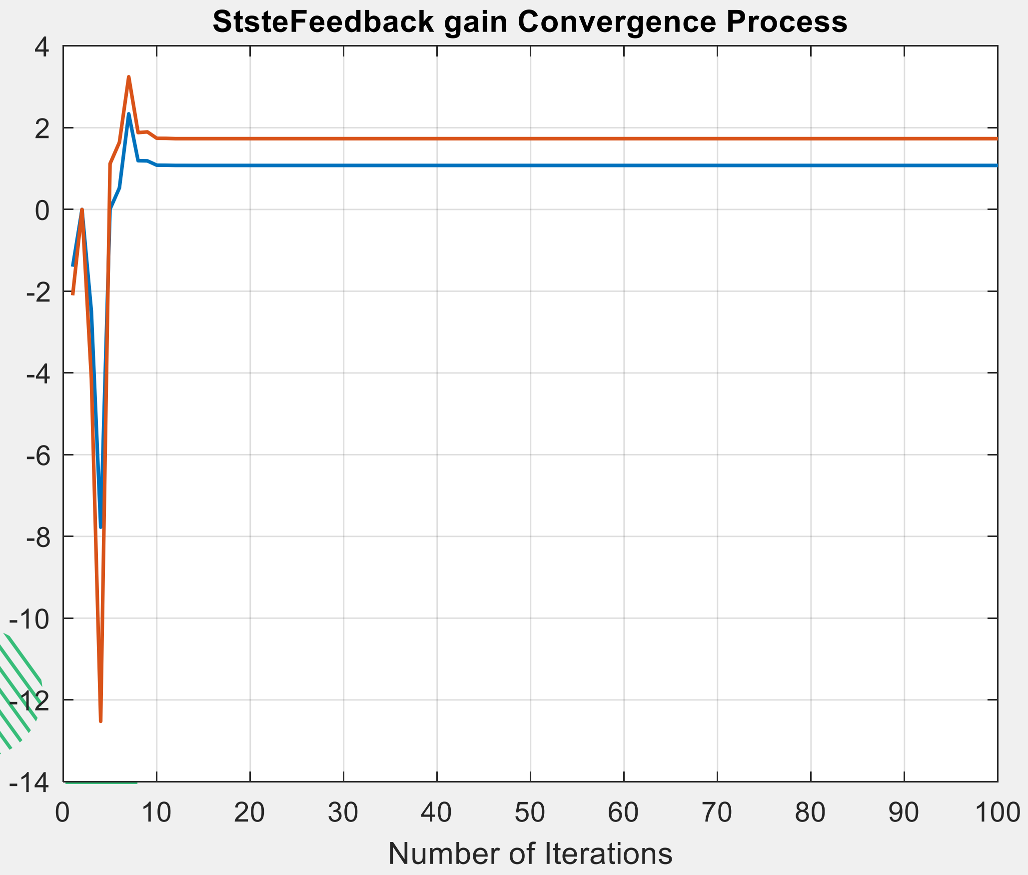
$$L^* = [-0.9319 \ \ -1.5784]$$

$$V^*(x_0) \ = \ 62.0422$$

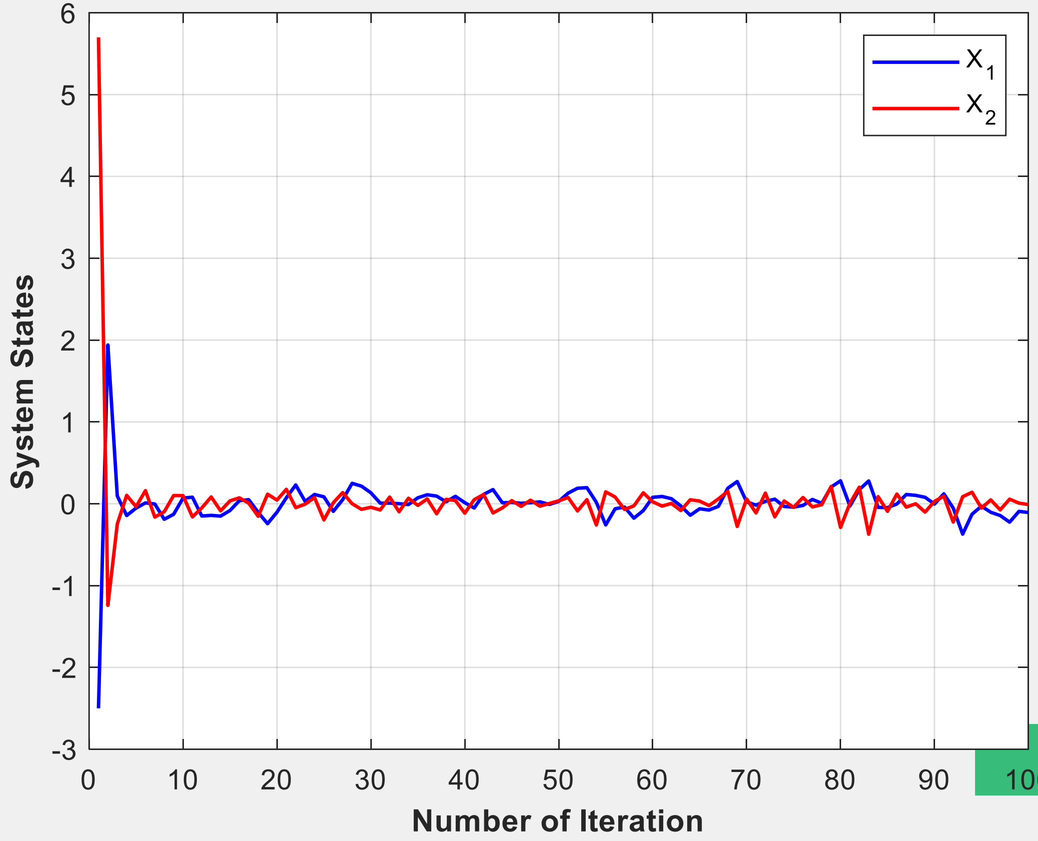
```
Iteration(17)
Iteration(18)
Iteration(19)
Iteration(20)
Elapsed Time = 2.6345 Seconds
The Policy Iteration P Matrix is:
    8.6754    8.5527
    8.5527   10.9058

The Policy Iteration Method Gain is:
   -0.9424   -1.5907
```

$$\hat{V}(x_0) = 62.1118$$

SDLQR:



StsteFeedback gain Convergence Process
4
2
0
-2
-4
-6
-8
-10
-12
-14
0
10
20
30
40
50
60
70
80
90
100
Number of Iterations



System States
6
5
4
3
2
1
0
-1
-2
-3
0
10
20
30
40
50
60
70
80
90
100
Number of Iteration
X_1
X_2

SDLQR:

$$Q_h(x_k, u_k) = r(x_k, u_k) + \gamma V_h(x_{k+1})$$

- Q-Learning:

$$\begin{aligned}
&Q(x_k, u_k) \\
&= \gamma\Big(\mathrm{E}(x_{k+1}^\top P x_{k+1}) + \frac{\gamma}{1-\gamma}\mathrm{tr}(PW)\Big) + \mathrm{E}\big(c(x_k, u_k)\big) \\
&= \gamma \mathrm{E}\Big(\big(Ax_k + Bu_k + (Cx_k + Du_k)d_k + w_k\big)^\top P \\
&\quad \times \big(Ax_k + Bu_k + (Cx_k + Du_k)d_k + w_k\big) \\
&\quad + \frac{\gamma}{1-\gamma}\mathrm{tr}(PW)\Big) + \mathrm{E}(x_k^\top Q x_k + u_k^\top R u_k) \\
&= \mathrm{E}\big(x_k^\top (Q + \gamma A^\top PA + \gamma C^\top PC)x_k + 2\gamma x_k^\top (A^\top PB \\
&\quad + C^\top PD)u_k + u_k^\top (R + \gamma B^\top PB + \gamma D^\top PD)u_k\big) \\
&\quad + \frac{\gamma}{1-\gamma}\mathrm{tr}(PW) \\
&= \mathrm{E}\Big(\begin{bmatrix} x_k \\ u_k \end{bmatrix}^\top H \begin{bmatrix} x_k \\ u_k \end{bmatrix}\Big) + \frac{\gamma}{1-\gamma}\mathrm{tr}(PW), \qquad (28)
\end{aligned}$$

SDLQR:

- Q-Learning:

$$H = \begin{bmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{bmatrix} \in \mathcal{S}_{+}^{n+m},$$

$$\begin{aligned} H_{xx} &= Q + \gamma A^\top P A + \gamma C^\top P C \\ H_{xu} &= \gamma A^\top P B + \gamma C^\top P D = H_{ux}^\top \\ H_{uu} &= R + \gamma B^\top P B + \gamma D^\top P D. \end{aligned}$$

SDLQR:

- Optimal Control Policy:

$$\boldsymbol{Q}^*(\boldsymbol{x}_k, \boldsymbol{u}_k) = \boldsymbol{r}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{\gamma} \boldsymbol{V}^*(\boldsymbol{x}_{k+1})$$

$$\frac{\partial Q^*(x_k, u_k)}{\partial u_k} = 0$$

$$L^* = -(H^*_{uu})^{-1} H^*_{ux}$$

$$Q(x_k, u_k) = \mathrm{E}\big(c(x_k, u_k)\big) + \gamma Q(x_{k+1}, u_{k+1})$$

SDLQR:

Input: Admissible control gain $L^{(0)}$, initial state covariance matrix X_0, additive noise covariance matrix W, discount factor γ, maximum number of iterations i_{max}, convergence tolerance ε

Output: The estimated optimal control gain $\hat{L}$

for $i = 0 : i_{max}$ **do**

  Policy Evaluation:

$$\mathrm{E}\left(\begin{bmatrix} x_k \\ u_k^{(i)} \end{bmatrix}^\top H^{(i)} \begin{bmatrix} x_k \\ u_k^{(i)} \end{bmatrix} \right) = \mathrm{E}\big(c(x_k, u_k^{(i)})\big) + \gamma \mathrm{E}\left(\begin{bmatrix} x_{k+1} \\ u_{k+1}^{(i)} \end{bmatrix}^\top H^{(i)} \begin{bmatrix} x_{k+1} \\ u_{k+1}^{(i)} \end{bmatrix} \right) - \gamma \mathrm{tr}\left(H^{(i)} \begin{bmatrix} I \\ L^{(i)} \end{bmatrix} W \begin{bmatrix} I \\ L^{(i)} \end{bmatrix}^\top \right) \tag{34}$$

  Policy Improvement:

$$L^{(i+1)} = -(H_{uu}^{(i)})^{-1} H_{ux}^{(i)} \tag{35}$$

  if $\|L^{(i+1)} - L^{(i)}\| < \varepsilon$ **then**

    Break

  endif

endfor

$\hat{L} = L^{(i+1)}$

SDLQR:

$$P^* = \begin{bmatrix} 8.2254 & 8.0704 \\ 8.0704 & 10.3873 \end{bmatrix}$$

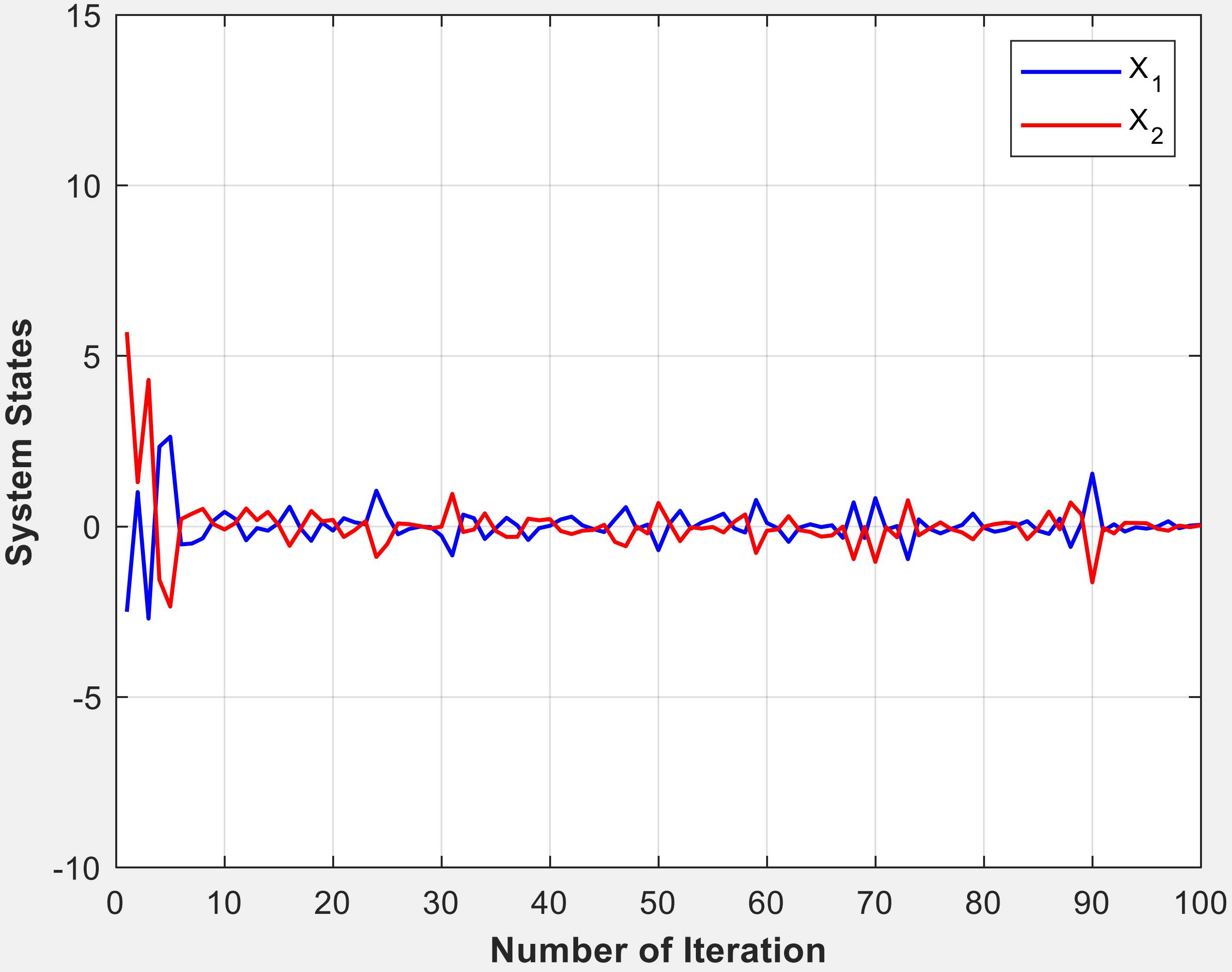
$$L^* = [-0.9319 \quad -1.5784]$$

$$V^*(x_0) = 62.0422$$

$$\hat{V}(x_0) = 62.2118$$

```
Iteration(1)
Iteration(2)
Iteration(3)
Iteration(4)
Iteration(5)
Iteration(6)
Iteration(7)
Elapsed Time = 0.033295 Seconds
Optimal Control Policy obtained by Qlearning =
   -0.9211   -1.5259
```

SDLQR:



15
10
5
0
-5
-10
0
10
20
30
40
50
60
70
80
90
100
System States
Number of Iteration
X_1
X_2

Thanks for your Attention

References:

[1] M. M. Rana, “IoT-based electric vehicle state estimation and control algorithms under cyber attacks,” IEEE Internet Things J., vol. 7, no. 2, pp. 874–881, 2020.

[2] Y. Wang, B. M. Nguyen, H. Fujimoto, and Y. Hori, “Multirate estimation and control of body slip angle for electric vehicles based on onboard vision system,” IEEE Trans. Ind. Electron., vol. 61, no. 2, pp. 1133–1143, 2014.

[3] K. Nam, S. Oh, H. Fujimoto, and Y. Hori, “Estimation of sideslip and roll angles of electric vehicles using lateral tire force sensors through RLS and Kalman filter approaches,” IEEE Trans. Ind. Electron., vol. 60, no. 3, pp. 988–1000, 2013.

[4] H. Zhang, G. Zhang, and J. Wang, “Sideslip angle estimation of an electric ground vehicle via finite-frequency H approach,” IEEE trans. transp. electrif., vol. 2, no. 2, pp. 200–209, 2016.

[5] W. Cheng, S. Chuanxue, and L. Jianhua, “Research on key state parameters estimation of electric vehicle ESP based on multi-sensor,” in 2015 Fifth International Conference on Instrumentation and Measurement, Computer, Communication and Control (IMCCC), 2015.

[6] L. Wang, L. Wang, C. Liao, and W. Zhang, “Research on multiple states joint estimation algorithm for electric vehicles under charge mode,” IEEE Access, vol. 6, pp. 40143–40153, 2018.

[7] M. M. Rana, “Attack resilient wireless sensor networks for smart electric vehicles,” IEEE Sens. Lett., vol. 1, no. 2, pp. 1–4, 2017.

[8] Arif Ahmed Ubaid M. Al-Saggaf Muhammad Moinuddin, “State Space Least Mean Fourth Algorithm for State Estimation of Synchronous Motor,” ASIAN JOURNAL OF ENGINEERING, SCIENCES & TECHNOLOGY, vol. 4, no. 1, 2014.

[9] M. B. Malik and M. Salman, “State-space least mean square,” Digit. Signal Process., vol. 18, no. 3, pp. 334–345, 2008.

References:

[10] Q. Huang, Z. Huang, and H. Zhou, “Nonlinear optimal and robust speed control for a light-weighted all-electric vehicle,” IET Control Theory Appl., vol. 3, no. 4, pp. 437–444, 2009.

[11] M. M. Rana and R. Bo, “IoT-based improved human motion estimations method under cyber attacks,” IEEE Internet Things J., vol. 6, no. 6, pp. 10934–10935, 2019.

[12] Z. Guo, D. Shi, K. H. Johansson, and L. Shi, “Optimal linear cyber-attack on remote state estimation,” IEEE Trans. Control Netw. Syst., vol. 4, no. 1, pp. 4–13, 2017.

[13] J. Löfberg, “YALMIP.” [Online]. Available: https://yalmip.github.io/.

[14] S. J. Bradtke, B. E. Ydstie, and A. G. Barto, “Adaptive linear quadratic control using policy iteration,” in Proceedings of 1994 American Control Conference - ACC ’94, 2005.

[15] H. Xu, S. Jagannathan, and F. L. Lewis, “Stochastic optimal control of unknown linear networked control system in the presence of random delays and packet losses,” Automatica (Oxf.), vol. 48, no. 6, pp. 1017–1030, 2012.

[16] A. S. Leong, A. Ramaswamy, D. E. Quevedo, H. Karl, and L. Shi, “Deep reinforcement learning for wireless sensor scheduling in cyber–physical systems,” Automatica (Oxf.), vol. 113, no. 108759, p. 108759, 2020.

[17] Vaishaal Shankar Karl Krauth Kailas Vodrahalli Qifan Pu Benjamin Recht Ion Stoica Jonathan Ragan-Kelley Eric Jonas Shivaram Venkataraman, Finite-time analysis of approximate policy iteration for the linear quadratic regulator. University of California at Berkeley, 2021.

[18] B. Kiumarsi, F. L. Lewis, H. Modares, A. Karimpour, and M.-B. Naghibi-Sistani, “Reinforcement -learning for optimal tracking control of linear discrete-time systems with unknown dynamics,” Automatica (Oxf.), vol. 50, no. 4, pp. 1167–1175, 2014.