

CAS CS 330. Problem Set 1

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Problem 1.

$n = 2$   $2n = 4$ , four people will be A,B,C,D

preference for A,B,C,D:

A: B | C | D

B: C | A | D

C: A | B | D

In this way, whoever paired with D would like to swap and pair with anyone who propose to him.

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Problem 2.

Assuming that the output wires are going from left to the right, that is, the left side is the beginning point and the right side is the ending point. showed as following  
output ----->

Algorithm: assuming that there are  $n$  inputs and  $n$  outputs, we know that each input would be able to connect with any output directly or indirectly. The leftmost input has the most choice in selecting output. So we can start the matching from the rightmost input, the first input switches to the output at its first junction, and the second input switches to the output at its second junction, and so on. In this way, we would always find a valid matching.

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Problem 3.

$$\begin{aligned} (2^{2n+1}) &= \Omega(2^{2n}) & (2^{2n}) &= \Omega(n2^n) & (n2^n) &= \Omega(2^{\lg 1.001n}) \\ (2^{\lg 1.001n}) &= \Omega(2^{100 \lg n}) & (2^{100 \lg n}) &= \Omega(4^{\lg n}) & (4^{\lg n}) &= \Omega(\sqrt{2}^{\lg n}) \\ (\sqrt{2}^{\lg n}) &= \Omega(n^{100}) & (n^{100}) &= \Omega(n^3) & (n^3) &= \Omega(n^2) \\ (n^2) &= \Omega(n) & (n) &= \Omega(\sqrt{n}) & (\sqrt{n}) &= \Omega\left(\binom{n}{2} + n \lg n\right) \\ \left(\binom{n}{2} + n \lg n\right) &= \Omega(n \lg n) & (n \lg n) &= \Omega(\lg(n!)) & (\lg(n!)) &= \Omega(\lg^2 n) \\ (\lg^2 n) &= \Omega(\log_3(5n)) & (\log_3(5n)) &= \Omega(\lg n) & (\lg n) &= \Omega(n^{\lg \lg n}) \\ (n^{\lg \lg n}) &= \Omega\left(n^{\frac{1}{\lg n}}\right) \end{aligned}$$

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Problem 4.

(a)  $f(n) = o(g(n))$  and  $f(n) \neq \Theta(g(n))$ .

$$f(n) = \lg n \qquad g(n) = n$$

(b)  $f(n) = \Theta(g(n))$  and  $f(n) = o(g(n))$ .

$$f(n) = n^2 + n \qquad g(n) = n^2$$

(c)  $f(n) = \Theta(g(n))$  and  $f(n) \neq O(g(n))$ .

None

because  $f(n) = \Theta(g(n))$  means that  $f(n) = O(g(n))$  and  $f(n) = \Omega(g(n))$

(d)  $f(n) = \Omega(g(n))$  and  $f(n) \neq O(g(n))$ .

$$f(n) = n^2 + n \qquad g(n) = (n)$$

(e)  $f(n) = \Omega(g(n))$  and  $f(n) \neq o(g(n))$ .

$$f(n) = n^2 + n \qquad g(n) = (n)$$

(f)  $f(n) = \omega(g(n))$  and  $g(n) \neq o(f(n))$ .

$$f(n) = n^2 + n \qquad g(n) = (n)$$