CAS CS 330. Problem Set 4 (snow-themed)
Due by 11:59pm on Wednesday, February 25

Problem 1
(a)

Algorithm: Sort the list of input by increasing order. Each time value corresponding to a person, put that person's number into the list one by one, this would be the output.

Proof: If there is a person S in the middle of the schedule, then every customer who has schedule after S need to wait for S to complete the lesson, which will add ts to their waiting time. We want to minimize S. Therefore, we want to put customers who have shorter value t in the beginning.

Analyze: We only need to sort the list and then take every element out from the list one by one, so the running time will be the time for sorting, which is Theta(nlogn)

(b)

Algorithm: First we need to calculate a new weighted list W. For an element wi in W, wi equals to vi / t1, which is the ratio of the customer value to the customer time. Then we sort the list W with a decreasing order. The order of the corresponding customer for ratio is the output.

Proof: The ratio of customer value to customer time representing the customer value per minute. If the customer value per minute is larger, that means this customer is more valuable and we need to schedule his first.

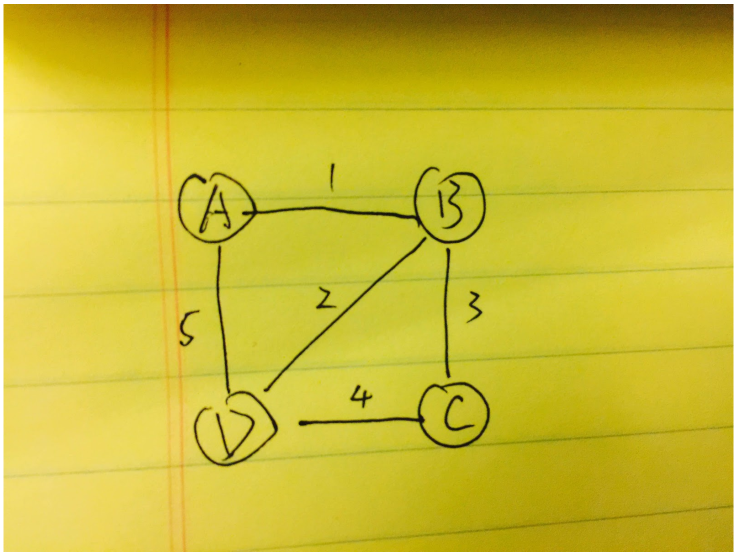
Analyze: we only need to do the sorting for the list and this will be in Theta(nlogn)

Problem 2

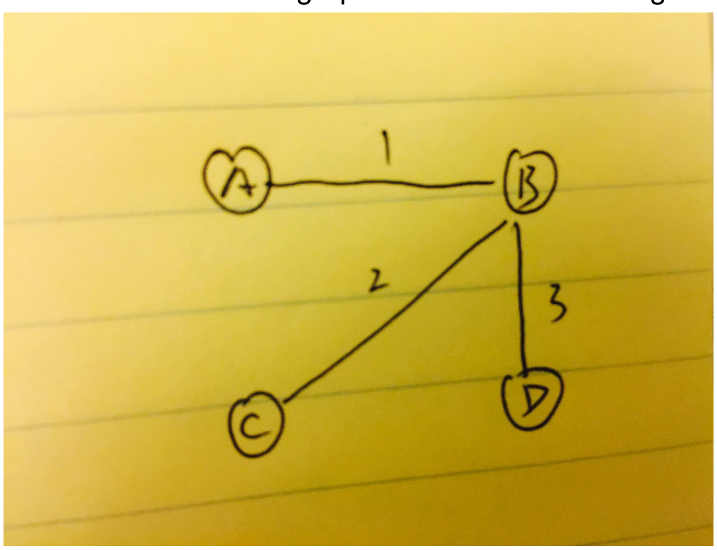
Conjecture 2:

The definition for a Minimum Spanning Tree is that it contains the minimum edges across every cut. The definition for a Minimum-Altitude Connected Subgraph is that: For a subgraph (V,E'), for every pair of towns i and j, the height of the winter-optimal path in (V,E') should be no greater than it is in the full graph G = (V,E). Therefore, if a subgraph contains all the minimum spanning tree, it will have a optimal path that it no greater than it is in the full graph, which implies that this subgraph is a minimum-altitude connected subgraph. So the second conjecture is True.

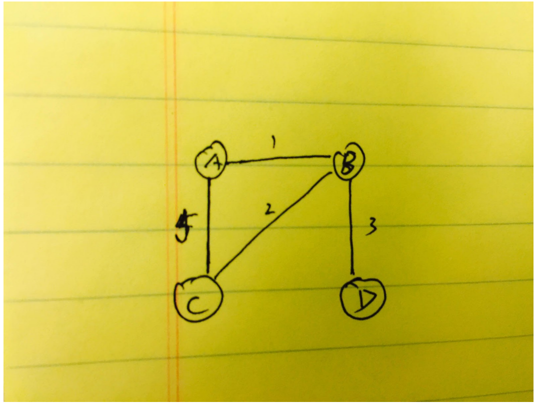
For example, we have a graph with altitude-weight as following:



Then the MST for this graph would be as following:



A subgraph contains all the edges of the MST could look like as following:



From this we can see clearly that the optimal path will the the edges of the MST, and it is no greater than it is in the whole graph, so it is a minimum-altitude connected subgraph.

According to the prompt in the question that “the second conjecture would immediately imply the first one”. I have proofed that the second conjecture is true, therefore, the first conjecture is true as well.

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