CAS CS 330. Problem Set 9 (NP-completeness)
Due by 11:59pm on Thursday, April 23

Problem 1

1 . Diverse Subset problem is in NP:

For any given chart and data, we can simply compare two customers in the data and see if they have bought anything in common.

2 . Diverse Subset problem is in NP-complete

proof Independent Set $\leq_p$ Diverse Subset

Creating a graph G = (V,E). Let customers be the vertices, and every vertex can have many edges. Edges between vertices represent the product that two customers both bought, which can be liquid detergent, beer, diapers and cat litter. For example, if four customers all bought beer, then we need to connect all four of them with edges.

If we are able to process Independent Set algorithm and apply this to this graph, the algorithm will give us a subset of the graph that contains the vertices that are not connecting with each other. This means that the customers in that subset never bought anything in common, and this is what Diverse Subset wants.

Therefore, Diverse Subset is reducing from Independent Set, and Independent Set in NP-complete, therefore, Diverse Subset is NP-complete

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Problem 2

1 .Resource Reservation Problem is in NP

To check whether two processes are both active, we just need to check whether they share the same resources, and this can be done in polynomial time.

2. Resource Reservation Problem is NP-complete

Proof Vertex Cover $\leq_p$ Resource Reservation

We can create a graph G = (V,E), and let processes be the vertices. make resources be the edges and connect each two processes that share the same resource. If a resource is only required by one process, then we don’t need to label that as a edge.

if we are able to process Vertex Cover on this G, we can get a subset S. The Vertex Cover will give us the subset S that cover the whole graph, which means that all the edges/resources have been allocated.

Therefore, we know that Resource Reservation is a reducing from Vertex cover, and Vertex cover is NP-complete, then Resource Reservation must be in NP-complete.

Problem 3

1 . Low-Diameter Clustering problem is in NP

Given a solution S, we can easily compute the distance between objects in the same cluster, if all the distances are less than B, then the solution is valid. This can be done in polynomial time.

2 . Low-Diameter Clustering problem is in NP-complete

Proof k-coloring $\leq_p$ Low-Diameter Clustering

Create a graph G = (G,E), and let Objects P be the vertices. For every edge (u,v), u $\neq$ v and u,v $\in$ P, edge (u,v) = d (u,v).

If we can use k-coloring, we can get a solution S. For u and v that are representing the same color in S, d(P_u , P_v) must less than the bound B, otherwise, P_u and P_v should be marked as different color. Therefore, the k colors in S represent the k different sets we want in Low-Diameter Clustering problem.

Low-Diameter Clustering problem can be reducing from K-coloring problem, and K-Coloring problem is in NP-complete, therefore, Low-Diameter Clustering problem is in NP-complete