Essentials of Traffic Flow

Anush Badii

anushbadii@gmail.com

Staff of

Ramp Metering and Census District 11

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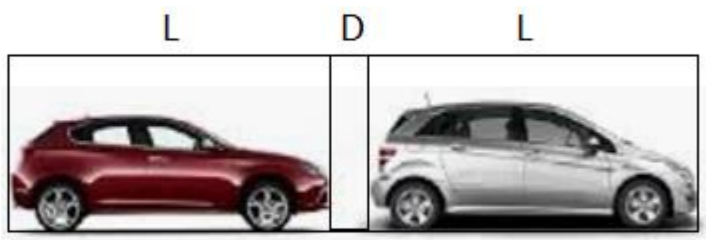
Consider two vehicles with lengths L and the distance between them D



We calculate the number of vehicles with the above configuration (L+D) that fit in one mile as density, *n.*

$$n = \frac{5280}{L + D}$$

We solve for the distance D

$$D = \frac{5280 - nL}{n}$$

Divide both sides of equation by the speed *v* to obtain the time of impact or headway, τ

$$\tau = \frac{D}{v} = \frac{5280 - nL}{nv}$$

We contend that there is a minimum τ that the drivers are comfortable with. This is usually geographical. In southern California τ is measured at 1.75 seconds and in Korea is measured at 2.5 seconds and the Midwest US values $\tau < 3.0$ seconds is considered reckless driving.

We solve for v from the equation above to obtain

$$v = \frac{5280 - nL}{n\tau}$$

The units of speed in the above equation is in ft/sec. To convert this to MPH, we divide it by a factor $\bar{c}$

$$\bar{c} = \frac{5280}{3600}$$

To obtain speed in MPH

$$V = \frac{v}{\bar{c}} = \frac{5280 - nL}{n\tau\bar{c}}$$

The above equation does not fulfil traffic requirements of speed limit as such we introduce V_f as the speed limit and rewrite the above as

$$V = \min\{V_f, \frac{5280 - nL}{n\tau\bar{c}}]$$

The traffic flow measured in vehicles/hour is then

$$Q = nV = n\,\frac{5280 - nL}{n\tau\bar{c}} = \frac{5280 - nL}{\tau\bar{c}}$$

We rewrite the above as

$$\mathrm{Q} = \frac{5280}{\tau\bar{c}} - \frac{nL}{\tau\bar{c}}$$

Examination of the above defines the maximum capacity of a roadway ($\frac{5280}{\tau\bar{c}}$) and any subsequent reduction in capacity, represented by ($\frac{nL}{\tau\bar{c}}$) . Note that the maximum capacity is dependent on τ, as such the capacity of a roadway depends on the driver's aggressiveness. So the same roadway with NASCAR drivers can exhibit a capacity of 14400 vehicles/hour and headways of .25 seconds.

We know that drivers mostly drive at a speed limit of V_f =65 MP, therefore the flow

$$Q = nV_f = 65n$$

Plotting the two equations together will have a point of the intersection of the two lines, called the critical density and is 29 vehicles/mile for a 65 MPH speed. At this point any further increase in the density would diminish the traffic flow, rendering reduced traffic speed.

The above diagram was first proposed by Newell at UC Berkeley. For further reading refer to the link below:

http://www.dot.ca.gov/dist11/d11tmc/tmc_docs/Newell.pdf

For practical traffic flow engineering Two questions remain

1. Lane drop and lane addition.
2. Merging ramp traffic.

The convergence of two vehicular traffic streams, the on-ramp with the main lanes is only possible when the two traffic streams merge at the same speed and the densities are combined, resulting in a flow at the increased density n_{merge}

$$n_{merge} = n_1 + n_2$$

The speed is governed by

$$V = min\left[V_f \quad , \frac{5280 - n_{merge}L}{n_{merge}\tau}\right]$$

Where V_f is the posted speed limit.

Applying the above equation to the ramp with known vehicle release rate q_{ramp} and the knowledge that the speed of the ramp would need to match the speed of the main lanes at the merge.

We calculate the density of a ramp as

$$n_{ramp} = \frac{q_{ramp}}{v_{main\ lane}}$$

$$n_{merge} = n_{ramp} + n_{mainlane}$$

Resulting in merge speed of

$$V_{merge} = min\left[V_f \quad , \frac{5280 - n_{merge}L}{n_{merge}\tau}\right]$$

For a multi lane facility we can distribute the added density from the ramp over all main lanes to arrive at

$$n_{merge} = \frac{n_{ramp}}{\aleph} + n_{mainlane}$$

Resulting in merge flow rate of

$$Q_{merge} = n_{merge}V_{merge}$$

In ramp metering we only measure the flow Q and percentage Occupancy $\% \ OCC_c$. Therefore, we would need to convert our measurements of occupancy in to density.

A VDS (vehicle detection station) is a conductive loop of wires with dimension l installed in the pavement. The loop is activated when it detects a vehicle entering it and deactivates when the vehicle leaves.

The time of occupancy of the loop t_{occ}

$$t_{occ} = \frac{L + l}{V}$$

Similarly the time of vacancy of the loop t_{vac}

$$t_{vac} = \frac{D}{V}$$

The percentage Occupancy is then

$$\%O_{cc} = \frac{\frac{L + l}{V}}{\frac{L + l}{V} + \frac{D}{V}} = \frac{L + l}{L + D + l}$$

From the expression for density

$$n = \frac{5280}{L + D}$$

May be written as

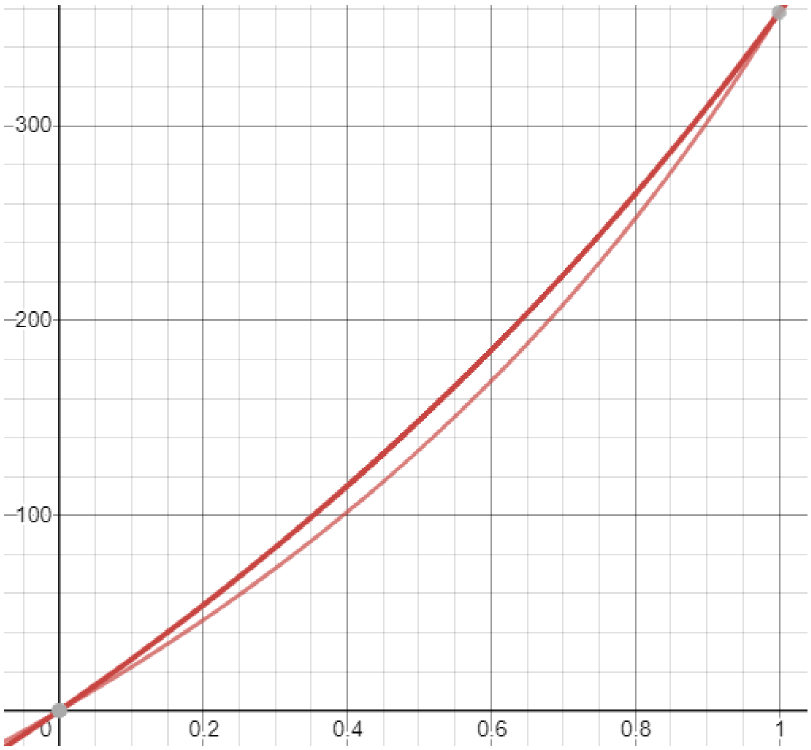
$$L + D = \frac{5280}{n}$$

And substitute in the percentage Occupancy, $\%OCC$ to obtain a relation between occupancy and density

$$\%OCC \quad = \frac{L + l}{L + D + l} = \frac{L + l}{\frac{5280}{n} + l} = \frac{n(L + l)}{5280 + nl}$$

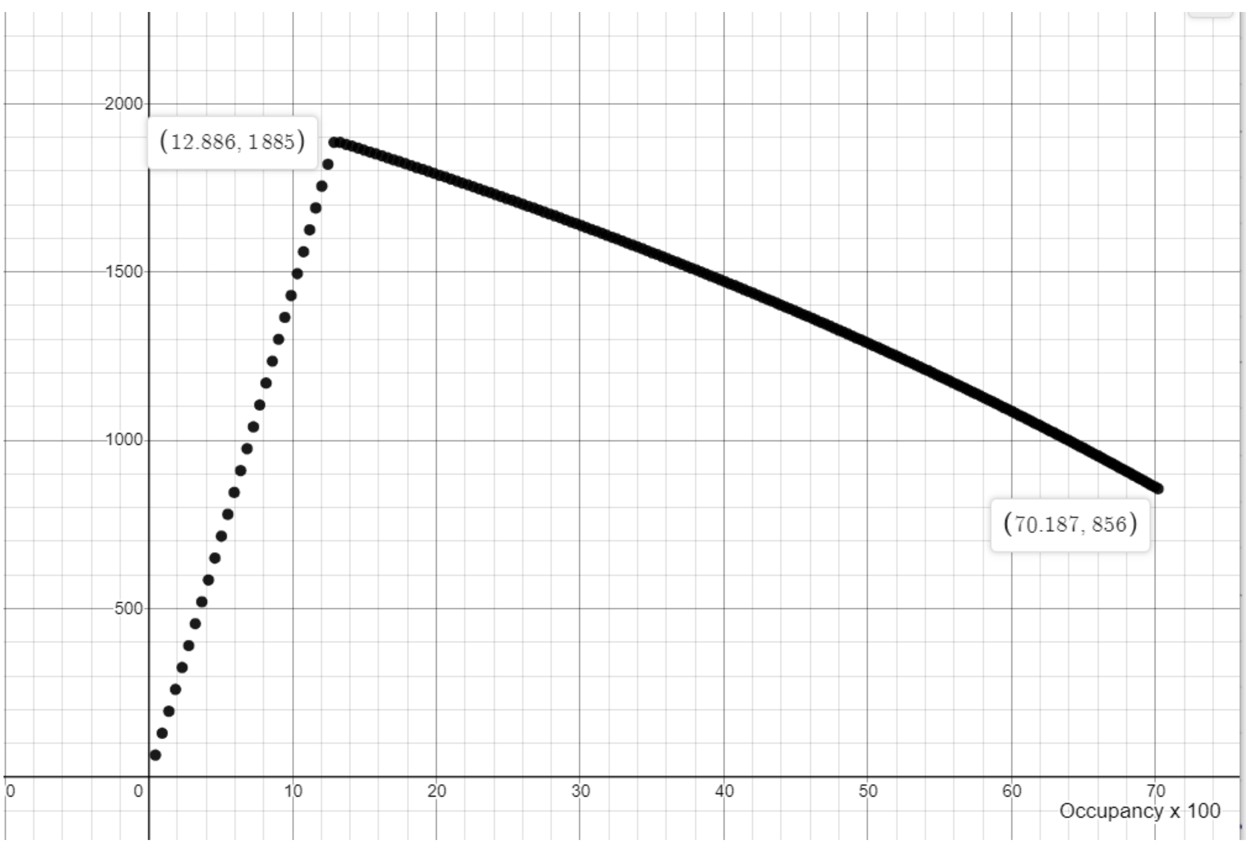
Which may be inverted to result in equation below for :

$$n = \frac{-5280 \ \%OCC}{l \ \%OCC \quad - L - l}$$

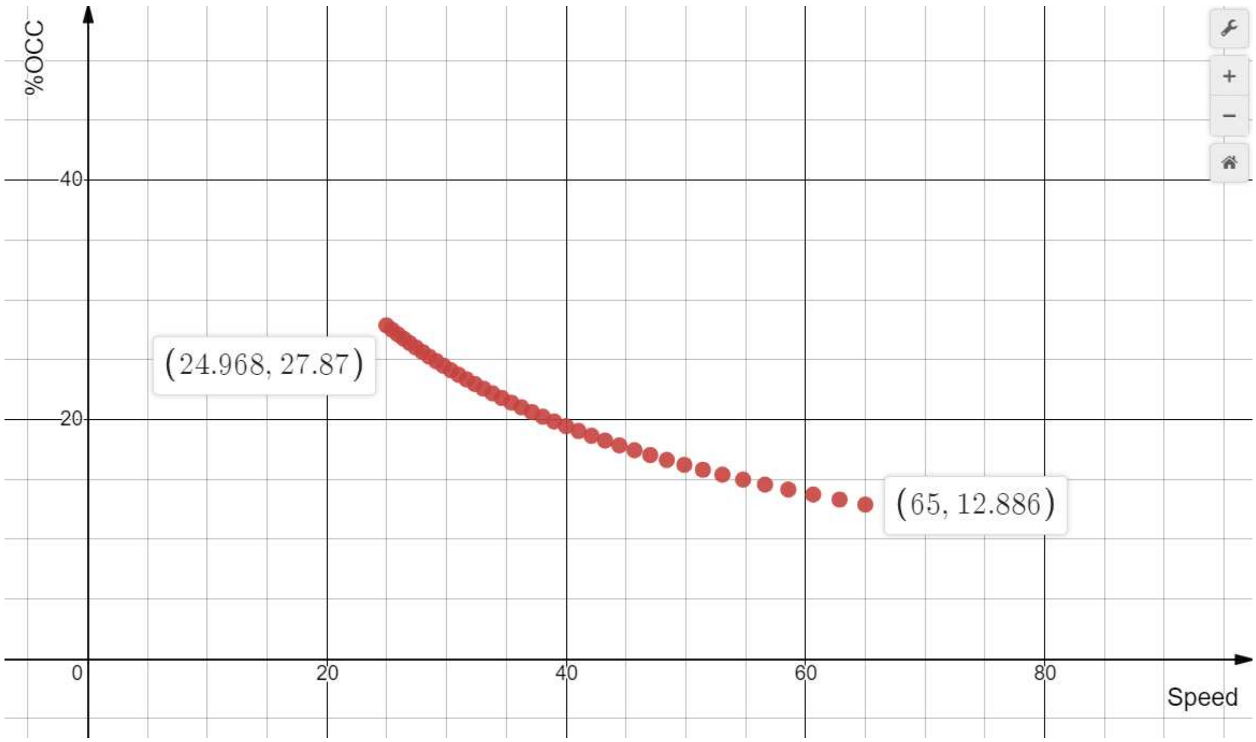


Density vs % Occupancy for 6ft and 10 ft loops

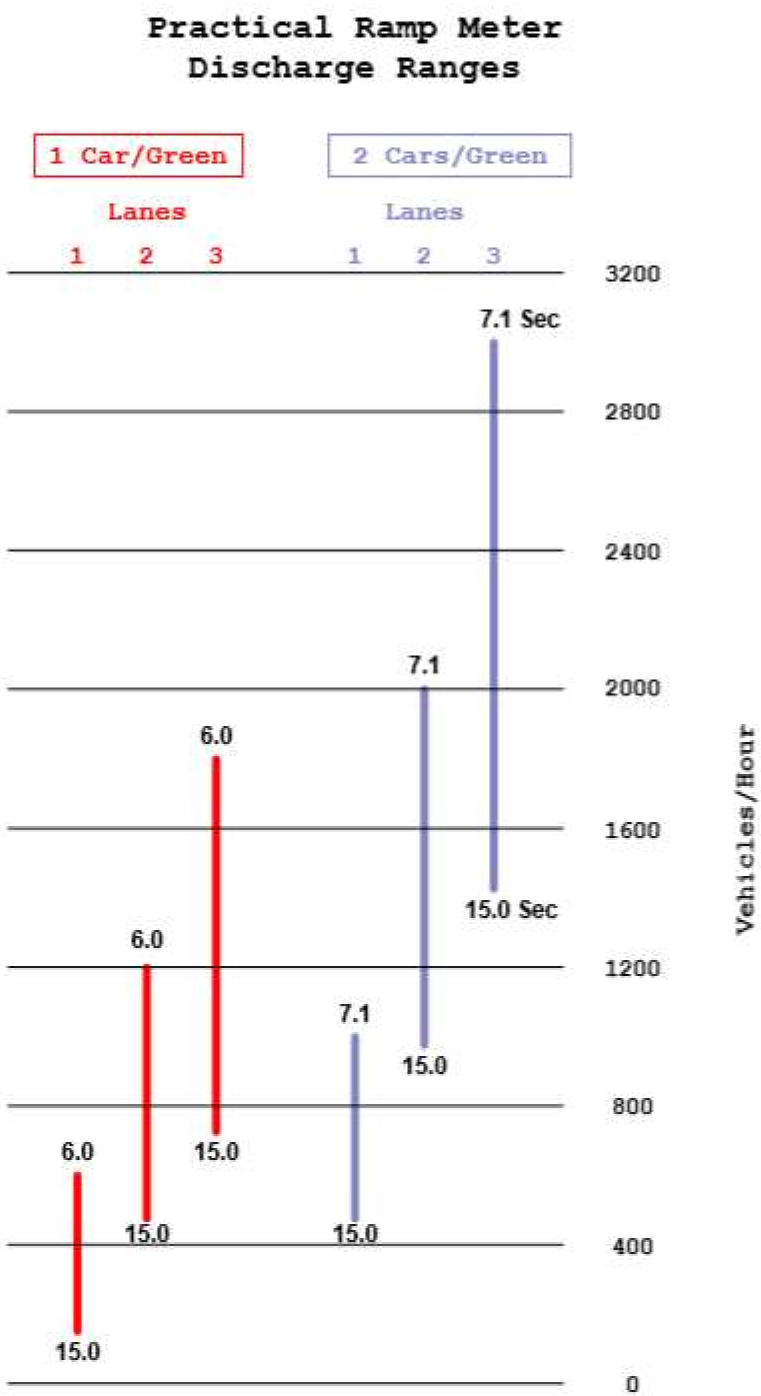
Now we plot the flow diagram versus Occupancy



From the diagram above we should be able to decipher the Occupancy and corresponding flow volumes for starting the ramp meter. The practical Range for ramp metering should be between free flow occupancy of 12.86% and 27.87% for the speed of 25 MPH. The occupancy differential can be divided evenly between rate 1 and rate 15 (Hex, F), with 1 percentage point tick in occupancy.



Ramp queue backing on to surface street consideration has influenced the ramp metering operations. In the diagram below, the maximum and minimum signal cycle times was established for different ramp configuration and discharge rates.



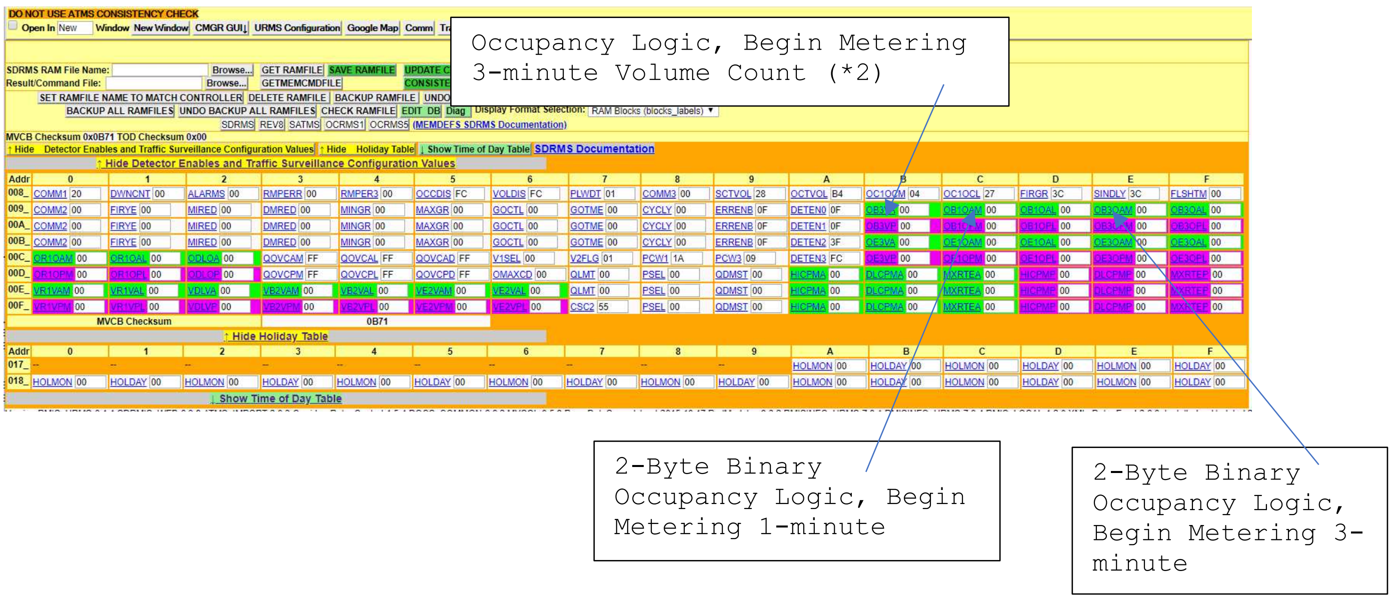
These quantities are used in queue analysis spread sheet

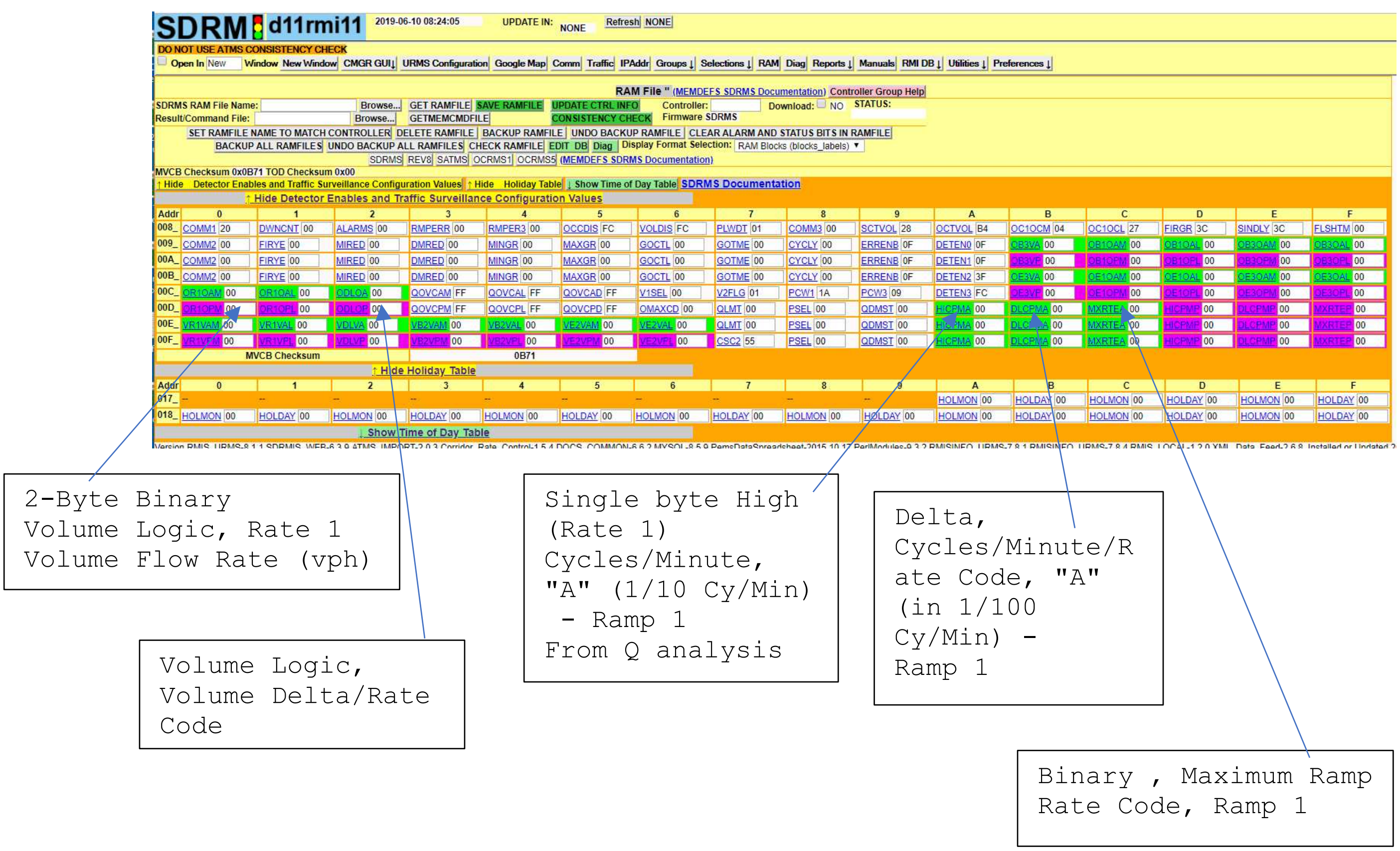
http://www.dot.ca.gov/dist11/d11tmc/academy/QueueAnalysis.xlsm

Once the cycle timing is established by Queue Analysis, the Occupancy and flow thresholds need to be programmed into the controller to effectuate a traffic responsive ramp metering.

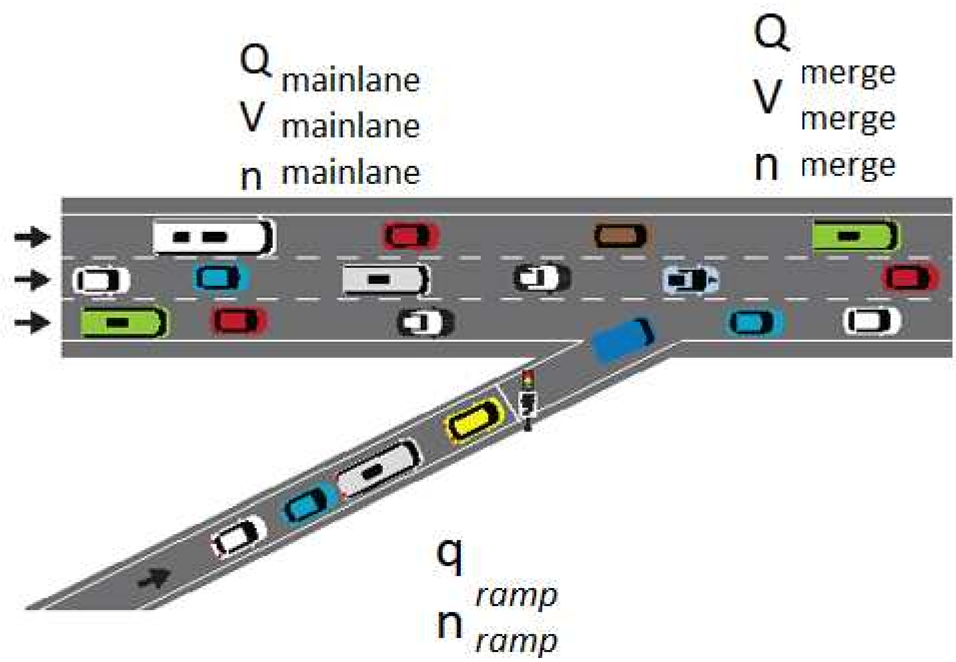
The start of ramp metering should be based on judicious choice of the flow parameters. Because the flow is a dual valued function we cannot rely on the measured values of the flow to take action. As such the authors of the ramp metering algorithm, also used the %OCC values to implement the algorithm.

To do this they relied on three-minute exponentially weighted average %OCC and one-minute average %OCC and Flow Volumes. These values need to be encoded in hexadecimal and put into specific locations in the ram sheet.





Ramp metering



$$n_{merge} = n_{ramp} + n_{mainlane}$$

$$n_{ramp} = \frac{1}{\aleph}\frac{q_{ramp}}{V_{mainlane}}$$

$$V_{merge} = \min[V_f\,, \frac{5280 - n_{merge}L}{n_{merge}\tau\,\bar{c}}]$$

$$Q_{merge} = n_{merge}\,V_{merge}$$

$$n_{merge} = \frac{1}{\aleph} \frac{q_{ramp}}{V_{mainlane}} + \frac{Q_{mainlane}}{V_{mainlane}}$$

Above equation can be used as a ramp metering equation to control the discharge from the ramp to achieve a desired merge density n_{merge}, or merge speed V_{merge}. Thus, we can solve for q_{ramp}

$$\frac{1}{\aleph} q_{ramp} = n_{merge} V_{mainlane} - Q_{mainlane}$$

$$q_{ramp} = \aleph \left[n_{merge} V_{mainlane} - Q_{mainlane} \right]$$

$$q_{ramp} = \aleph \left[n_{merge} V_{mainlane} - n_{mainlane} V_{mainlane} \right]$$

$$q_{ramp} = \aleph V_{mainlane} [n_{merge} - n_{mainlane}]$$

Since n_{merge} is always larger than $n_{mainlane}$ we may express

$$n_{merge} = \delta n_{mainlane}$$

Where delta δ is an arbitrary number greater than one $\delta \geq 1$

Now we may write

$$q_{ramp} = \aleph V_{mainlane} [\delta n_{mainlane} - n_{mainlane}]$$

Factoring $n_{mainlane}$ we may write

$$q_{ramp} = \aleph V_{mainlane} [n_{mainlane} (\delta - 1)]$$

We may use δ as a control variable for determining the appropriate ramp discharge rate.

By similar reasoning we can control the flow volume on the main lanes by noting that

$Q_{mainlane=} V_{mainlane} n_{mainlane}$ then, the ramp discharge rate may be written as

$$q_{ramp} = \aleph Q_{mainlane} (\delta - 1)$$

Where δ remains the controlling factor in determining the flow rate at the merge.

To control the speed of the merge, Starting from

$$V_{merge} = \min[V_f, \frac{5280}{n_{merge} \tau \bar{c}} - \frac{L}{\tau \bar{c}}]$$

Substitute $n_{ramp} + n_{mainlane}$ for n_{merge}

$$V_{merge} = \min[V_f\ , \frac{5280}{(n_{ramp} + n_{mainlane})\tau\ \bar{c}} - \frac{L}{\tau\ \bar{c}}]$$

Followed by another substitution for

$$n_{ramp} = \frac{1}{\aleph} \frac{q_{ramp}}{V_{mainlane}}$$

$$V_{merge} = \min[V_f\ , \frac{5280}{(\frac{1}{\aleph}\frac{q_{ramp}}{V_{mainlane}} + n_{mainlane})\tau\ \bar{c}} - \frac{L}{\tau\ \bar{c}}]$$

$$V_{merge} = \min[V_f\ , \frac{5280}{(\frac{1}{\aleph}\frac{q_{ramp} + \aleph Q_{mainlane}}{V_{mainlane}})\tau\ \bar{c}} - \frac{L}{\tau\ \bar{c}}]$$

$$V_{merge} = \min[V_f\ , \frac{5280\aleph V_{mainlane}}{(\ q_{ramp} + \aleph Q_{mainlane}\ )\tau\ \bar{c}} - \frac{L}{\tau\ \bar{c}}]$$

Solving for q_{ramp}

We arrive at the following equation

as

$$q_{ramp} = \frac{\aleph\left(5280\ \ V_{mainlane} - L n_{mainlane} - \ \ V_{merge} Q_{mainlane} \tau c\ \right)}{V_{merge}\tau\ \bar{c} + L}$$

If we were too write as before

$$V_{merge} = \alpha\ \ V_{mainlane} \quad \alpha \leq 1$$

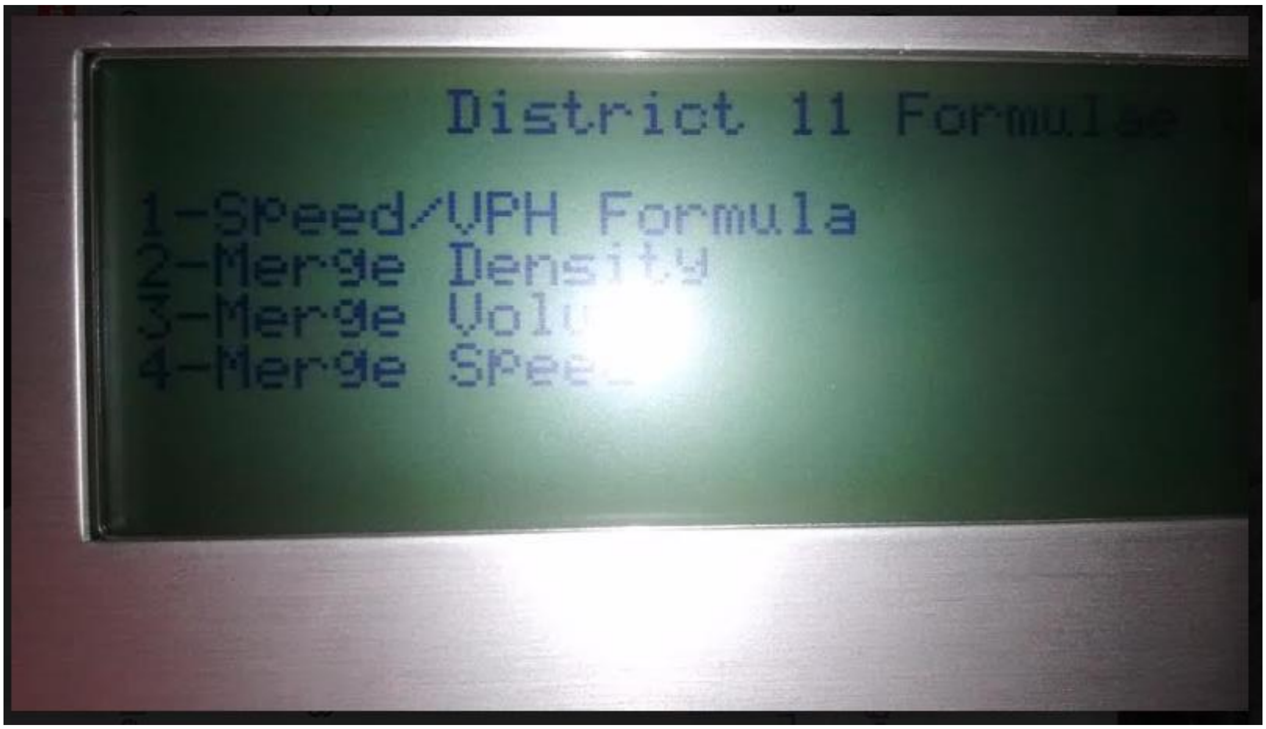
Then we may write

$$q_{ramp} = \frac{\aleph(5280\ \ V_{mainlane} - L n_{mainlane} - \alpha\ \ V_{mainlane} Q_{mainlane} \tau c\ )}{\alpha\ \ V_{mainlane}\tau\ \bar{c} + L}$$

$$q_{ramp} = \frac{\aleph(\ V_{mainlane}(5280 - Q_{mainlane}\tau c\alpha)\ - L n_{mainlane}\ )}{\alpha\ \ V_{mainlane}\tau\ \bar{c} + L}$$

Now α may become a controlling factor for ramp metering, if we put say $\alpha = .9$ the merge speed will be pegged at 90% of the mainlane speed.

These equations are already programmed into the 2070 controller and may be used at the traffic engineers discretion on any ramp. To access the algorithms through the front the front panel under option 4 traffic responsive plans select C where one would encounter District 11 Formulas.



Option 1 is Alinea algorithm

Options 2 through 4 are merge density, Volume and speed options may be expressed as absolute value or a percentage of mainlane values.