For return on 27 January 2017 (late submission: 10 February 2017)

Submit a hard copy to MAL263 or MAL161
or send a single pdf file to `michael@dcs.bbk.ac.uk`

1. **(5%)** Construct the truth-table for the Boolean function given by the formula

$$F \;=\; \neg(A_1 \to A_2) \vee (A_1 \wedge A_2) \vee (A_2 \wedge A_3).$$

Find a Boolean circuit with AND, OR and NOT gates only that computes that Boolean function and contains as few gates as possible. Determine whether the formula F is equivalent to the formula

$$A_1 \vee (A_2 \wedge (A_2 \to A_3)).$$

Show your working.

2. **(3%)** A parity function is a Boolean function whose value is 1 if the input has an odd number of ones. Design a Boolean circuit for the 2-bit parity function. Show your working. (Hint: you may find XOR gates useful.)

3. **(6%)** Given the machine 32-bit word

$$1100\ 0001\ 1011\ 0000\ 0000\ 0000\ 0000\ 0000$$

find the decimal number represented by this word assuming that it is

(a) a two's complement integer;

(b) an unsigned integer;

(c) a single precision IEEE 754 floating-point number.

4. **(6%)** Find computer representations of the following numbers:

(a) -44 as a two's complement 32-bit binary number;

(b) -44 as an IEEE 754 32-bit floating-point number;

(c) -15.375 as an IEEE 754 32-bit floating-point number.

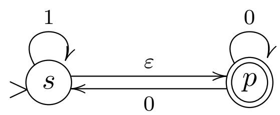
5. **(6%)** For each of the following relations, determine whether or not it is a function from $\mathbb{Z}$ to $\mathbb{Z}$ (where $\mathbb{Z}$ is the set of integer numbers). Explain your answer.

(a) $\{(x, y) \mid x, y \in \mathbb{Z},\ y = 3x\}$;

(b) $\{(x, y) \mid x, y \in \mathbb{Z},\ x = 3y\}$;

(c) $\{(x,y) \mid x,y \in \mathbb{Z},\, x \geq y\}$.

6. **(4%)** Consider the following NFA:



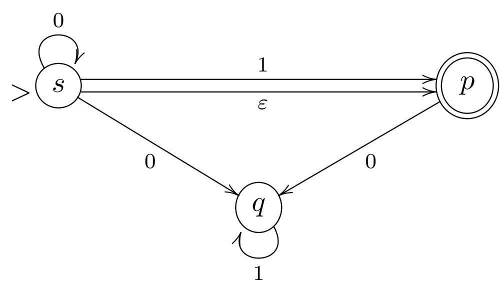
(a) Give all the computations of the automaton on the input strings 001, 010, and ε, and determine if the strings are accepted.

(b) Describe the language of the automaton in English.

(c) Describe the language of the automaton by means of a regular expression.

(d) Describe the language of the automaton by means of a context-free grammar.

7. **(10%)** Transform, using the subset construction, the following nondeterministic finite automaton into an equivalent deterministic finite automaton. Show your working.



What is the language of this automaton?

8. **(8%)** Design a (deterministic or nondeterministic) finite automaton A such that $L(A)$ consists of all words over the alphabet $\{0,1\}$ that contain at least two 0's and do not end with 11. Find a regular expression representing the language $L(A)$.

9. **(8%)** Convert the regular language $L\big[\,01\big((0\cup 11)11^*\big)^*0\,\big]$ to a finite automaton accepting it.

10. **(4%)** Give a context-free grammar for the language over the alphabet $\{a,b\}$ containing all words with at most three a's. Show the derivation of $abba$ in your grammar.

11. **(4%)** What is the language over $\{0,1\}$ defined by the following context-free grammar with start variable S?

$$\begin{array}{llll} S \to TS, & S \to 1T, & S \to 1S & \\ T \to TT, & T \to 0T1, & T \to 1T0 & T \to \varepsilon \end{array}$$

Is this language regular? Give an informal explanation of your answer.

12. **(13%)** Construct a context free grammar and a pushdown automaton for the language that consists of all strings over the alphabet $\{a,), (\}$ with balanced parentheses.

13. **(5%)** Consider the following transition table of a Turing machine (with s being its initial state):

s	0	h	0
s	1	q	$\rightarrow$
s	$\sqcup$	s	$\sqcup$
s	$\triangleright$	s	$\rightarrow$
q	0	q	$\rightarrow$
q	1	q	$\rightarrow$
q	$\sqcup$	p	$\leftarrow$
q	$\triangleright$	q	$\rightarrow$
p	0	p	$\rightarrow$
p	1	h	0
p	$\sqcup$	h	0
p	$\triangleright$	p	$\rightarrow$

(i) Give the computations of the machine on inputs 10, 111 and 110.

(ii) Describe in English what this Turing machine does.

14. **(10%)** Consider the following $\mathbb{N} \rightarrow \mathbb{N}$ function f:

$$f(n) = \begin{cases} 2n+1 & \text{if } n \text{ is even,} \\ 2n-2 & \text{if } n \text{ is odd.} \end{cases}$$

(Do not forget that all numbers are represented in binary.)

(i) Explain what it means to say that a Turing machine *computes* this function f.

(ii) Give an implementation level description in English of a Turing machine that computes this f.

(iii) Give the complete transition table of this Turing machine.

(iv) Give the computations of your Turing machine on inputs 0, 11 and 100.

15. **(8%)** Are the following problems decidable:

– a given CFG generates a given word;

– a given regular expression generates an infinite language.

Explain your answer.