Big Data Analytics

Session 2

Basic Statistics

Where were we last week

- Introduction to Big Data Analysis
  - Big: 4V dimension of Data
  - Data: Turning data to data products
  - Analysis: Statistical learning
    - Why estimate f ?
    - How do we estimate f ?
    - The trade-off between prediction accuracy and model interpretability
    - Supervised vs. unsupervised learning
    - Regression vs. classification problems

What is Statistics?

Main purpose of statistics, among others, is to

develop and apply methodology for

extracting useful knowledge from data.

Statistical data analysis

- Data
  - Nominal, Ordinal, Interval, and Ratio
- Descriptive statistics
  - Exploring, visualising, and summarising data without fitting the data to any models
- Inferential statistics
  - Identification of a suitable model
  - Testing either predictions or hypotheses of the model

  → Will be covered in the following sessions

Statistical data analysis

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Data

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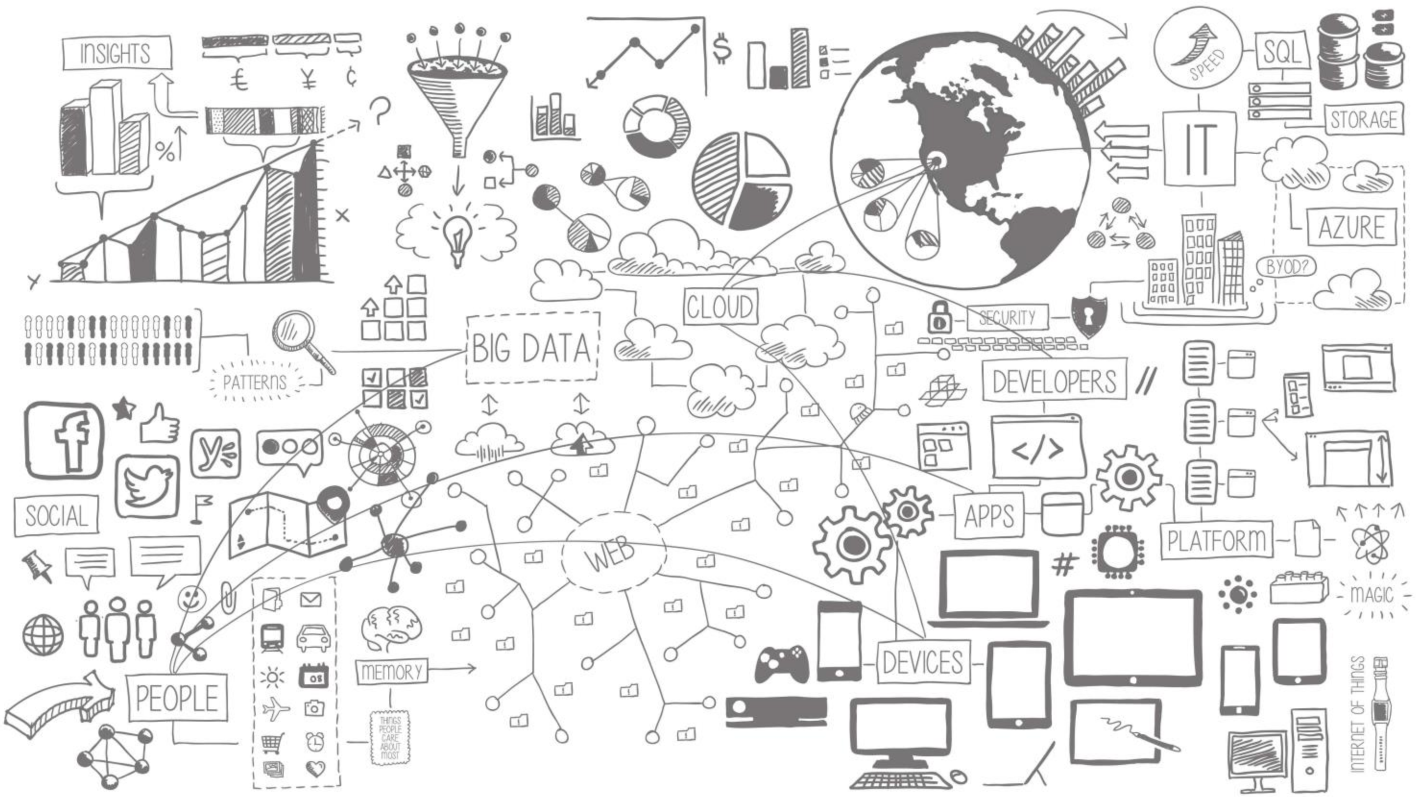
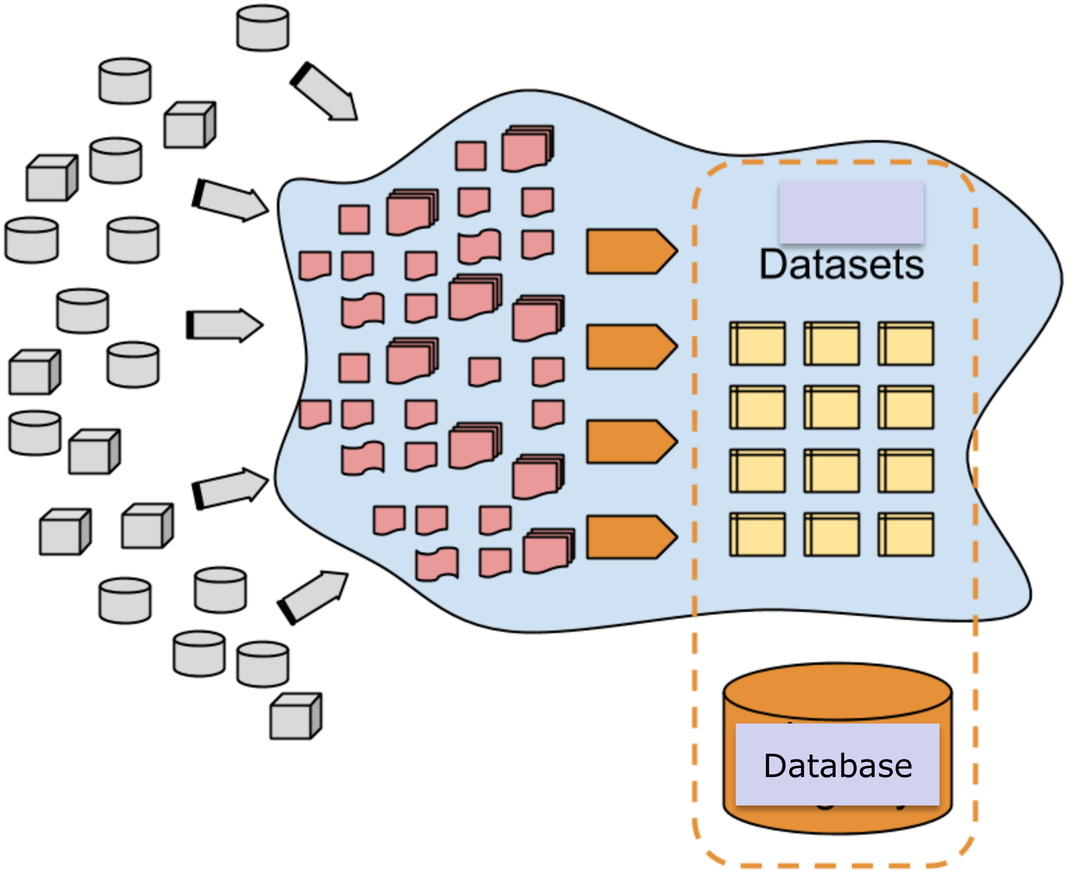


- Data are the facts and figures collected, summarised, analysed, and interpreted.

Data

- Data are the results of measurements and can be the basis of graphs, images, or observations of a set of variables.

Data Sets



- The data collected in a particular study are referred to as the data set.

Data Sets

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Observation

Variables

Element Names

Company	Stock Exchange	Annual Sales($M)	Earn/ Share($)
Dataram	AMEX	73.10	0.86
EnergySouth	OTC	74.00	1.67
Keystone	NYSE	365.70	0.86
LandCare	NYSE	111.40	0.33
Psychemedics	AMEX	17.60	0.13

Data Set

Scales of Measurement

- Scales of measurement include:
  - Nominal
  - Ordinal
  - Interval
  - Ratio

- The scale determines the amount of information contained in the data.

- The scale indicates the data summarisation and statistical analysis that are most appropriate.

Scales of Measurement

- Nominal: You cannot count them.
  - Data are labels or names to identify an attribute of the element
  - A non-numeric label or numeric code may be used
  - Example:
    - Naming Schools in a University: Education, Business, Humanities, etc
    - Alternatively, using numeric code: 1 for Education, 2 for Business, etc

- Ordinal: You can count and order, but not add or subtract them.
  - The ordinal type allows for rank order by which data can be sorted
  - But it still does not allow for relative degree of difference between them.
  - Example:
    - Measuring opinion: completely agree, mostly agree, mostly disagree, etc.
    - Alternatively, using numeric code: 1 for completely agree, 2 mostly agree, etc

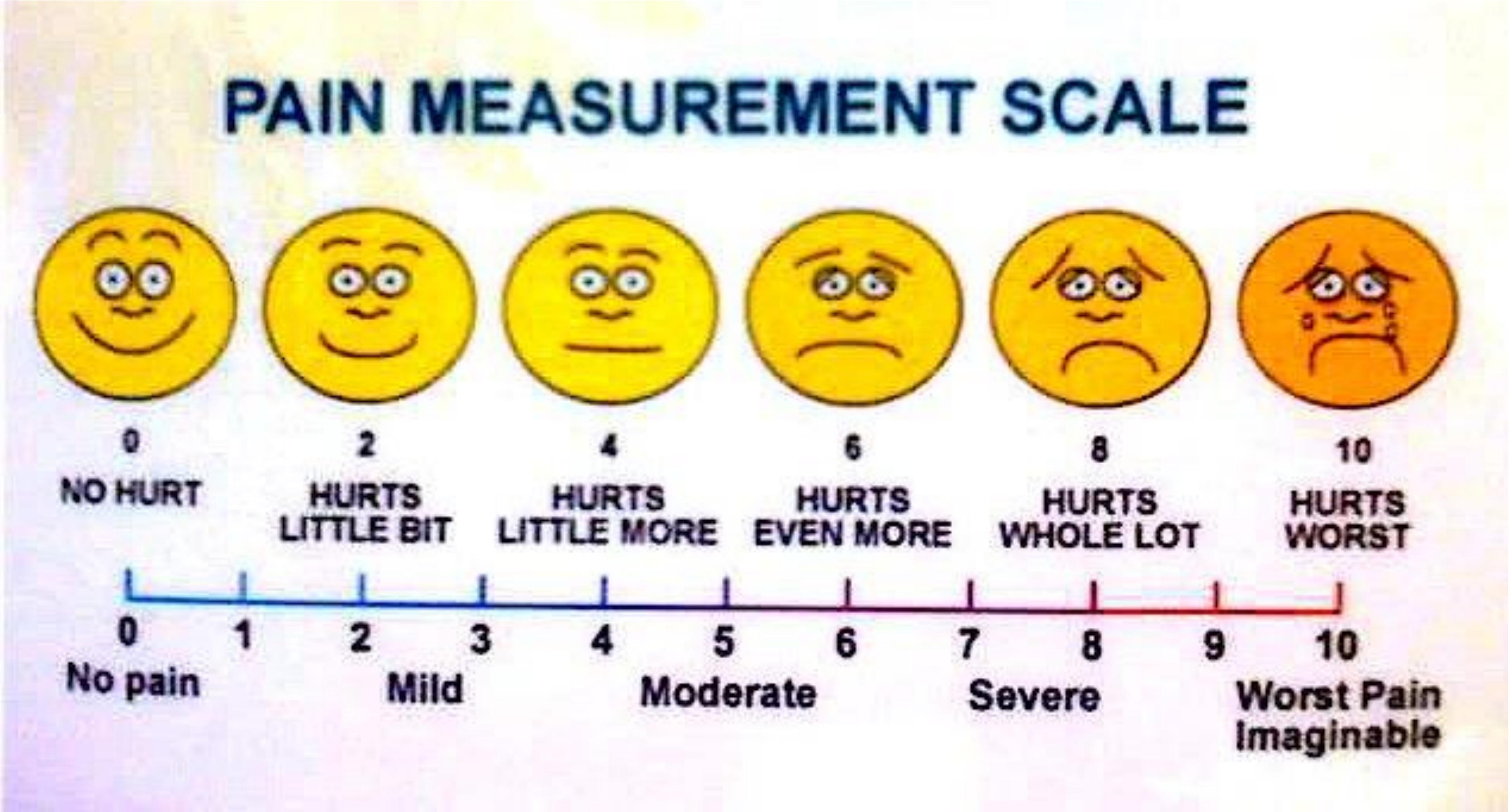
Scales of Measurement

- Interval: can be added or subtracted, but not multiplied or divided
  - The interval type allows for the degree of difference between items, but not the ratio between them.
  - Interval data are always numeric.
  - Example:
    - Measuring temp.: 20˚C, 10˚C. We cannot say 20˚C is twice as hot as 10˚C.

- Ratio: can be multiplied or divided, has zero value
  - The ratio of two values is meaningful.
  - Variables such as distance, height, weight and time use the ratio scale.
  - A ratio scale possesses a unique and non-arbitrary zero value.
  - Example:
    - Measuring length: 10cm is twice as long as 5cm.

Scales of Measurement

- Examples: pain measurement



Scales of Measurement

- Examples: date

In-class Exercise

- Give your own examples of the following levels of measurements
  - Nominal
  - Ordinal
  - Interval
  - Ratio

Qualitative and Quantitative Data

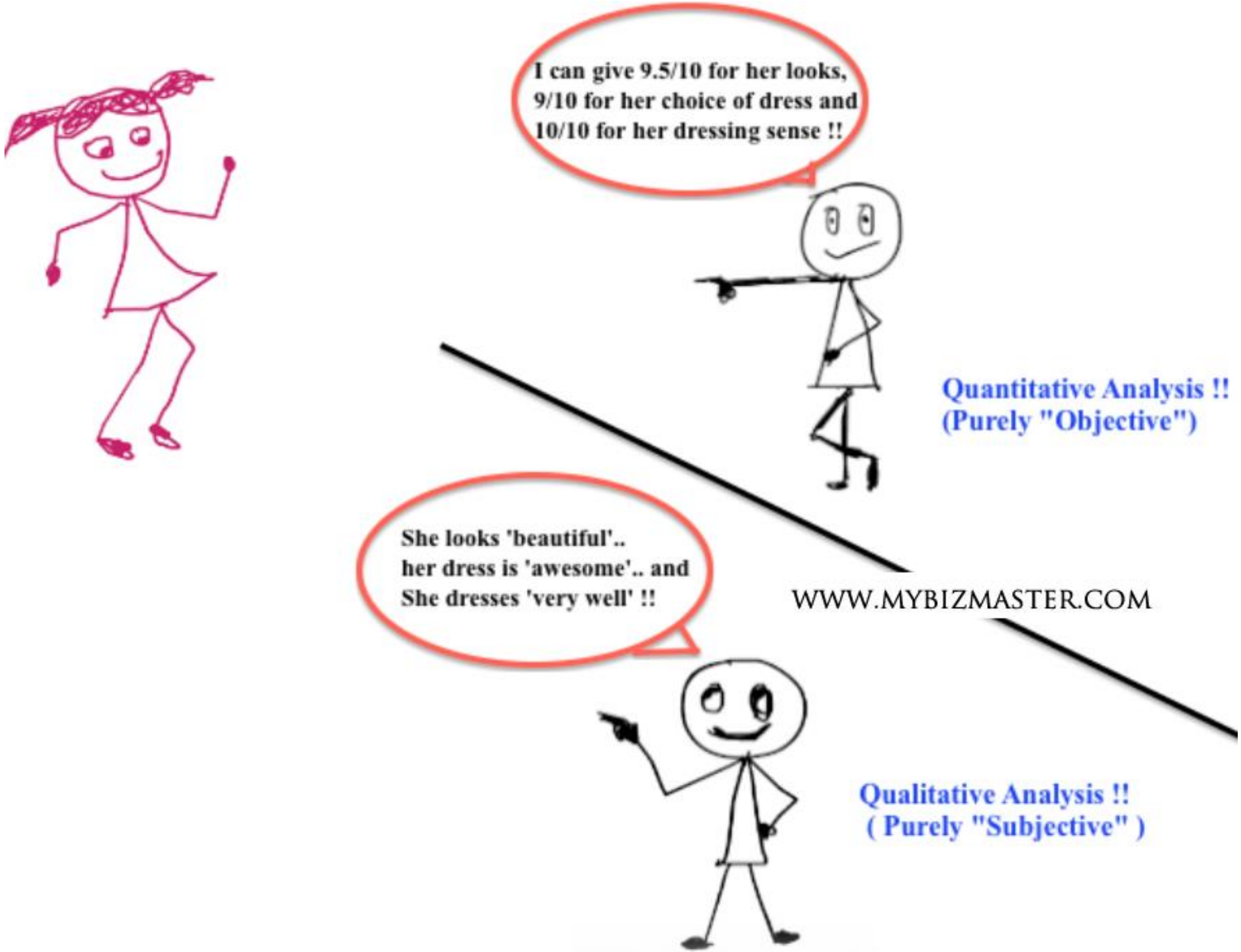
- Data can be further classified as being

  qualitative and quantitative.

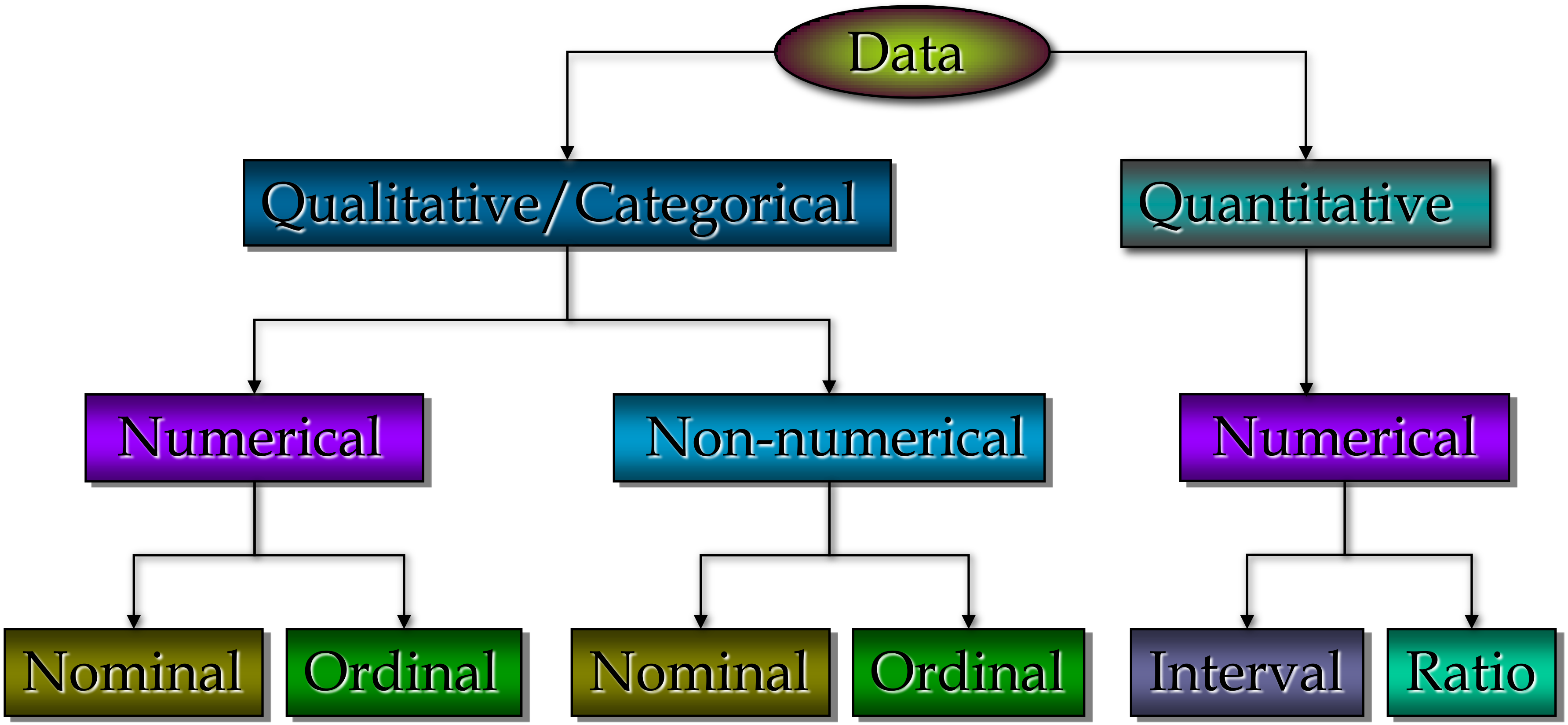
- The statistical analysis that is appropriate depends on whether the data for the variable are qualitative or quantitative.
  - Qualitative data ➔ qualitative analysis
  - Quantitative data ➔ quantitative analysis

- In general, there are more alternatives for statistical analysis when the data are quantitative.

Qualitative vs. Quantitative Data



Data



1 (Education), 2 (Business), …

1 (Completely agree), 2 (Mostly agree), …

Education, Business, …

Completely agree, Mostly agree, …

10°C, 20°C …

10cm, 20cm …

Statistical data analysis

- Data
  - Nominal, Ordinal, Interval, and Ratio
- Descriptive statistics
  - Exploring, visualising, and summarising data without fitting the data to any models
- Inferential statistics
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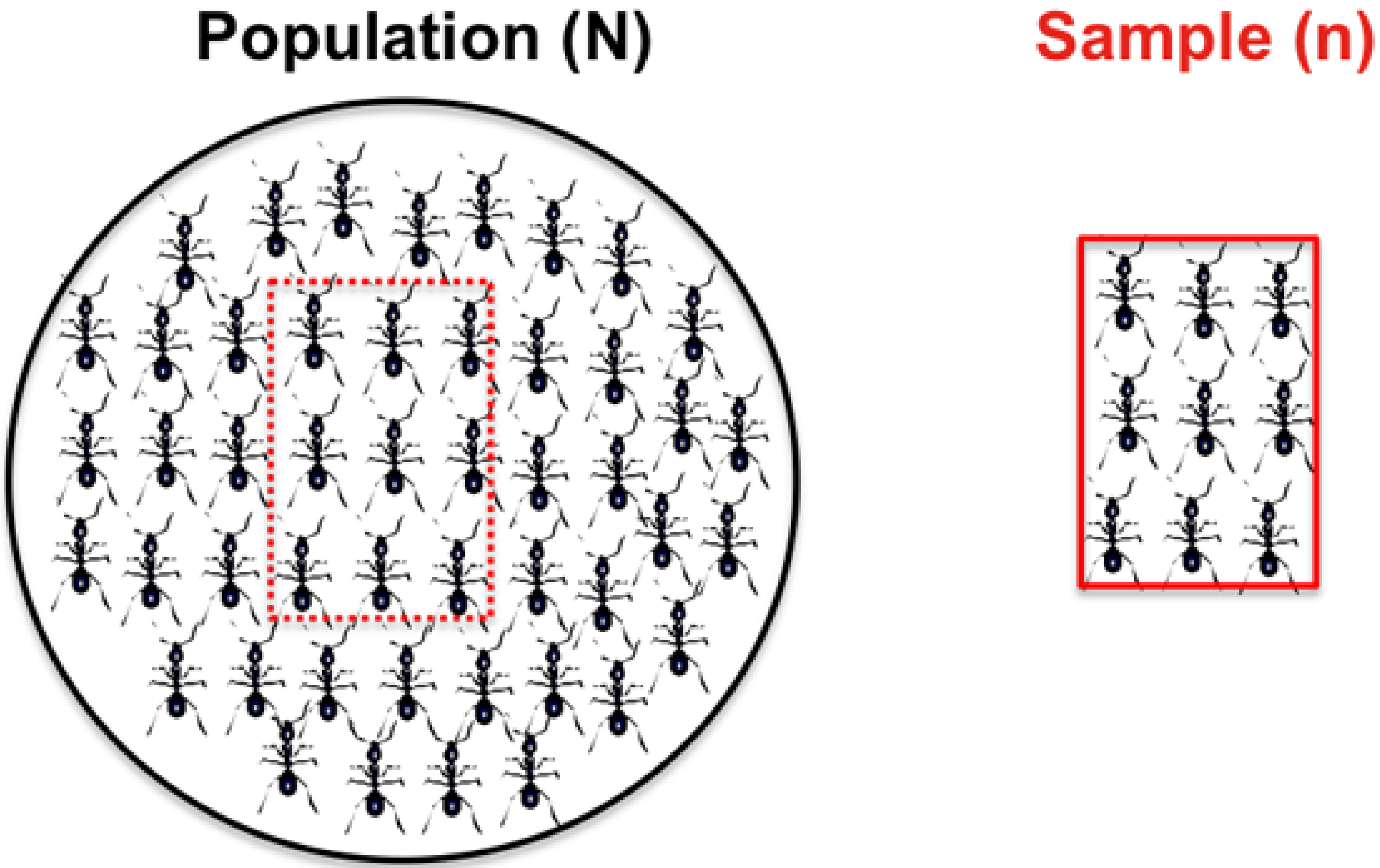
  → Will be covered in the following sessions

Descriptive Statistics

- Numerical measures

- Tabular and graphical presentation
  - Frequency distribution table
  - Histogram
  - Box plot
  - Scatter diagram

Sample and Population



Numerical Measures

- If the measures are computed for data from a sample, they are called sample statistics.
- If the measures are computed for data from a population, they are called population parameters.
- A sample statistic is referred to as the **point estimator** of the corresponding population parameter.

Descriptive Analysis

- Univariate analysis: describing the distribution of a single variable
  - Measures of central tendency
    - Mean, Median, Mode
  - Measures of spread
    - Variance, Standard Deviation
  - Measures of dispersion
    - Range, Quartiles, Interquartile Range

- Bivariate analysis: describing the relationship between pairs of variables
  - Quantitative measures of dependence
    - Correlation, Covariance

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Measures of Central Tendency

- It identifies the central position within a set of data.
- As such, measures of central tendency are sometimes called measures of central location.
  - Mean
  - Median
  - Mode

Mean

- The **mean** of a data set is the arithmetic average of all the data values.

- The sample mean $\bar{x}$ is the point estimator of the population mean μ
- The sample mean is a statistic and the population mean is a parameter

Sum of the values of the *n* observations

$$\bar{x} = \frac{\sum x_i}{n}$$

Number of observations in the sample

Sum of the values of the *N* observations

$$\mu = \frac{\sum x_i}{N}$$

Number of observations in the population

Median

- The **median** is the middle observation in a group of data when the data are ranked in order of magnitude
  - Odd number of observations: the middle one

(11)

65	55	89	56	35	14	56	55	87	45	92

14	35	45	55	55	**56**	56	65	87	89	92

Median = 56

  - Even number of observations: the average of the middle two

(10)

65	55	89	56	35	14	56	55	87	45

14	35	45	55	**55**	**56**	56	65	87	89

Median = (55+56)/2 = 55.5

Mean or Median?

- Consider data set with an outlier (extreme value)
  - Example: graduate salary

27K, 29K, 33K, 34K, 35K, 39K, 500K (an outlier)

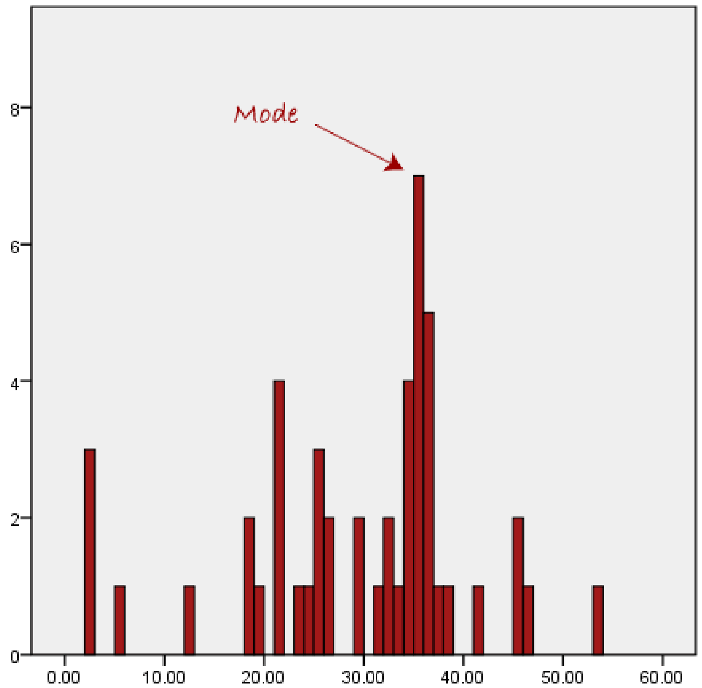
Mean = 99.6K, Median = 34K

Which is better as a representative of the central location?

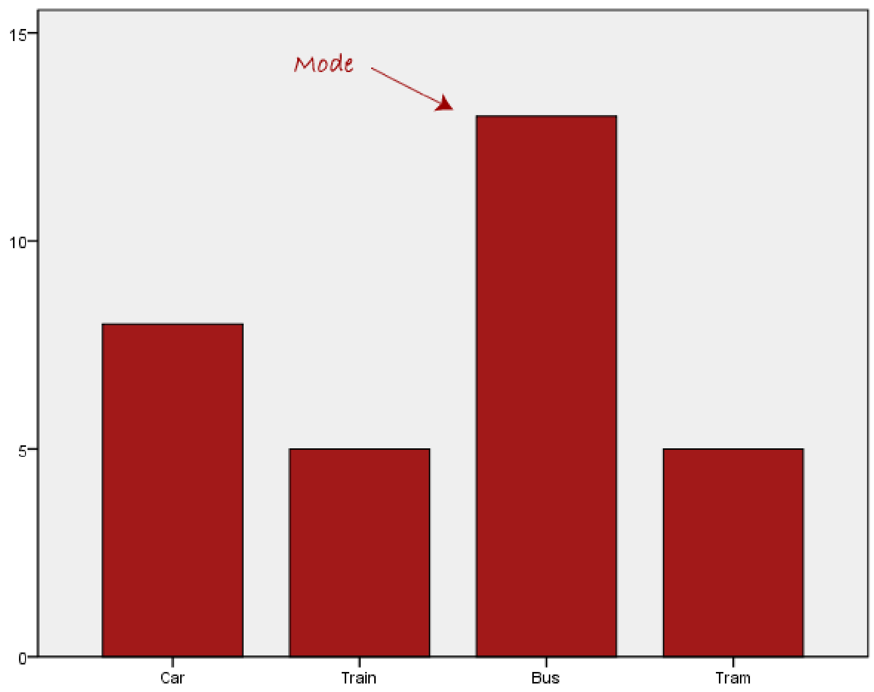
- Mean is highly influenced by one or two oddly high or low values.
- Whenever a data set has extreme values, the median is the preferred measure of central location.

Mode

- The Mode is the most frequent value in a data set
  - The mode describes the most popular option (categorical data)



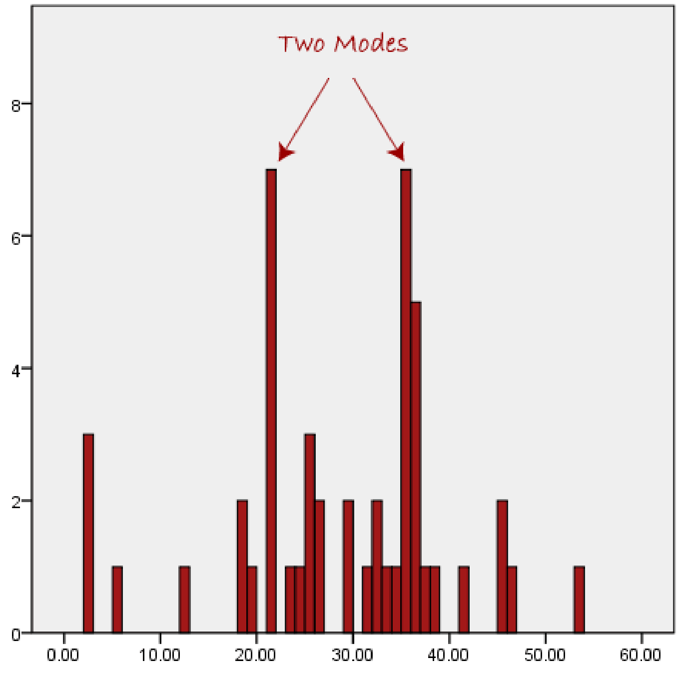
Highest bar in a Histogram



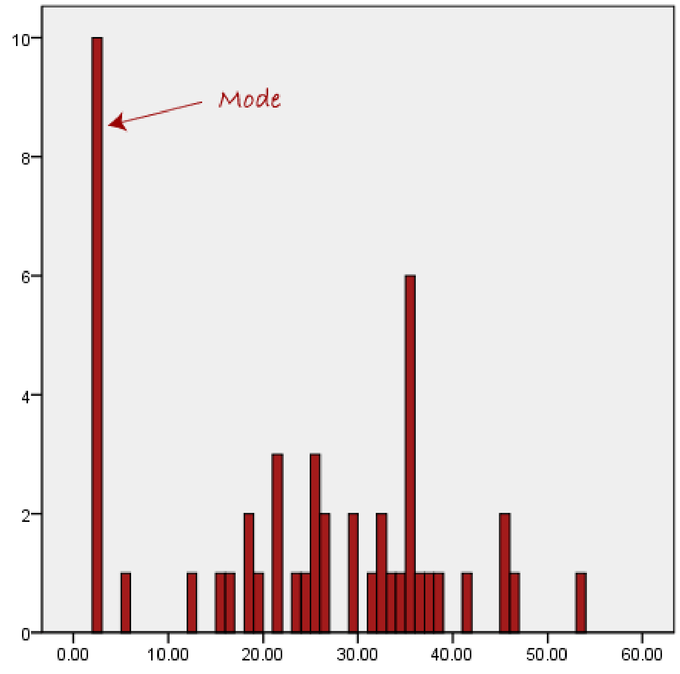
Bus being the most popular means of transportation

Mode

- The mode is rare in the continuous data
- A data set might have more than one mode
- The limitation of using the mode



Two modes, which is more representative?



The mode that is far away from the rest of the data

Mean, Median and Mode - Exercise

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- What is the mean, median and mode for weight and height?

variables

observations

	weight	height
student1	145	170
student2	170	190
student3	155	172
student4	122	180
student5	167	187
student6	160	174
student7	143	174
student8	142	166
studnet9	139	164
Student10	165	182

- Consider height
  - Mean

(170+190+172+…+182)/10 = 175.9

  - Median

164 166 170 172 174 174 180 182 187 190

  - Mode

174

Descriptive Analysis

- Univariate analysis: describing the distribution of a single variable
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- Bivariate analysis: describing the relationship between pairs of variables
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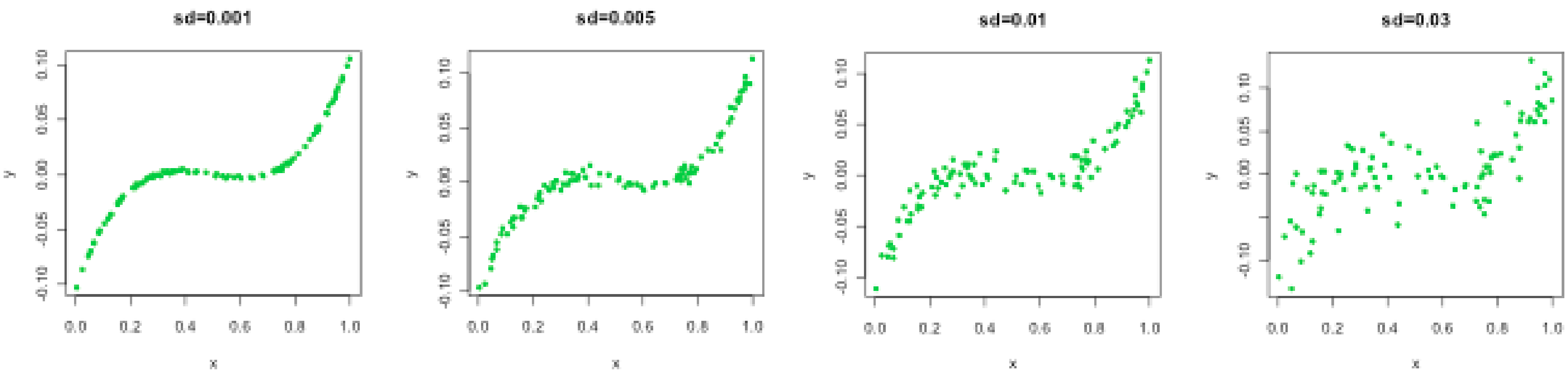
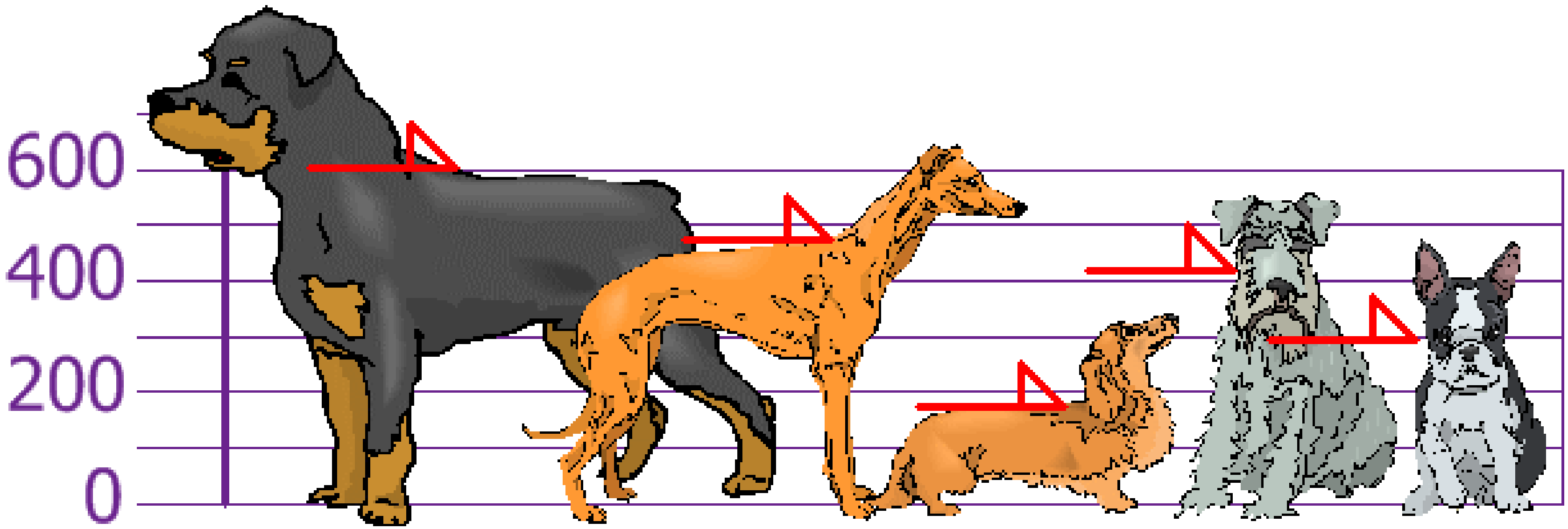
Measures of spread

- It tells us how spread out numbers are.
  - Variance (s^2(sample); σ^2(population))
    - The average of the squared differences from the mean
  - Standard Deviation (s(sample); σ(population))
    - The square root of variance



Example – variance & standard deviation

You and your friends have just measured the heights of your dogs (in millimeters):



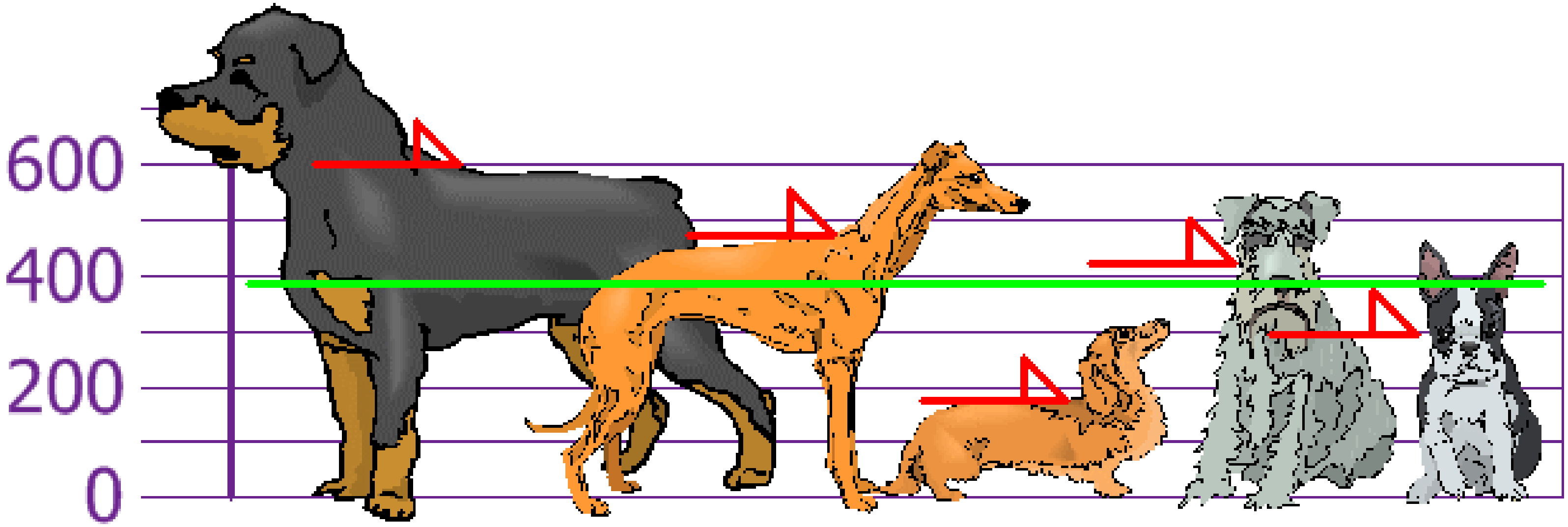
The heights (at the shoulders) are: 600mm, 470mm, 170mm, 430mm and 300mm.

Find out the Mean, the Variance, and the Standard Deviation.

Example – variance & standard deviation

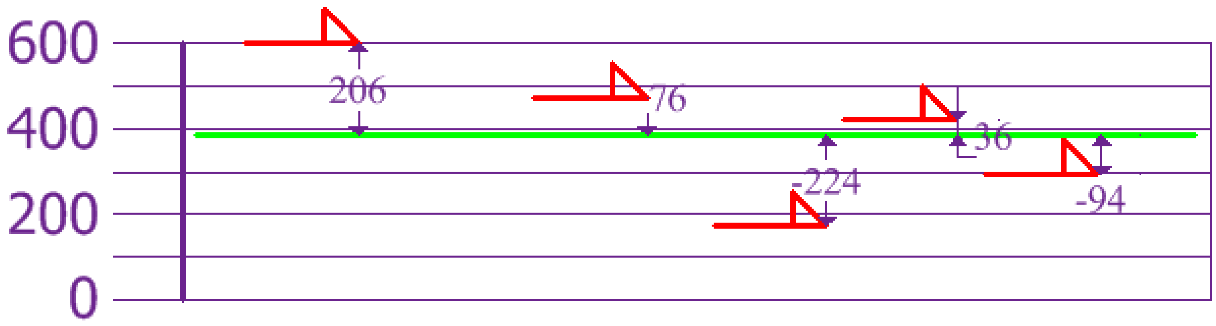
$$\text{Mean} = \frac{600 + 470 + 170 + 430 + 300}{5} = \frac{1970}{5} = 394$$

so the mean (average) height is 394 mm. Let's plot this on the chart:



Example – variance & standard deviation

Now, we calculate each dogs difference from the Mean:



To calculate the Variance, take each difference, square it, and then average the result:

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5}$$

$$= \frac{42,436 + 5,776 + 50,176 + 1,296 + 8,836}{5}$$

$$= \frac{108,520}{5} = 21,704$$

So, the Variance is **21,704**.

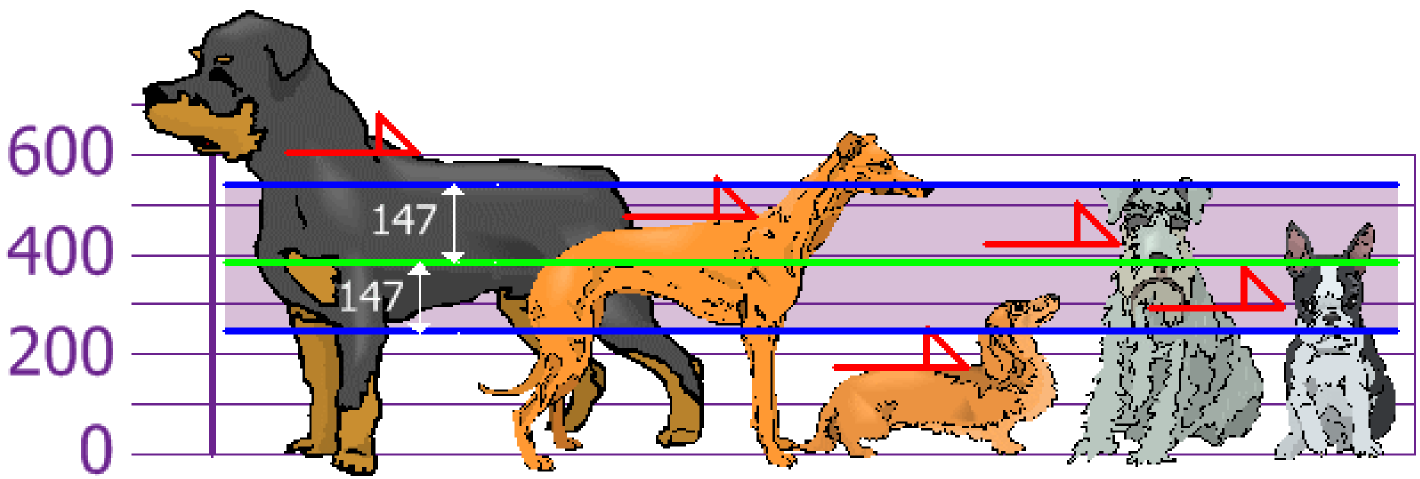
Note that SD has the same unit as mean

And the Standard Deviation is just the square root of Variance, so:

Standard Deviation: **σ = √21,704 = 147.32...** = **147** (to the nearest mm)

Standard Deviation

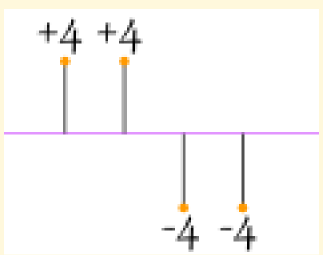
- Why study standard deviation (SD)?
  - From the point of view of one data:
    - A ‘standard’ way of knowing what is normal, and what is extra large or extra small.



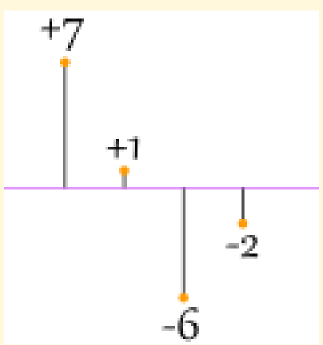
Rottweilers **are** tall dogs. And Dachshunds **are** a bit short ... but don't tell them!

Standard Deviation

- Why study standard deviation (SD)?
  - From the point of view of the data set:
    - A low SD indicates that the data points tend to be very close to the mean;
    - A high SD indicates that the data points are spread out over a large range of values.



$$\sqrt{\frac{4^2 + 4^2 + 4^2 + 4^2}{4}} = \sqrt{\frac{64}{4}} = \boxed{4}$$



$$\sqrt{\frac{7^2 + 1^2 + 6^2 + 2^2}{4}} = \sqrt{\frac{90}{4}} = \boxed{4.74...}$$

SD for Sample & Population

The "**Population** Standard Deviation": $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$

The "**Sample** Standard Deviation": $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}$

- Example:

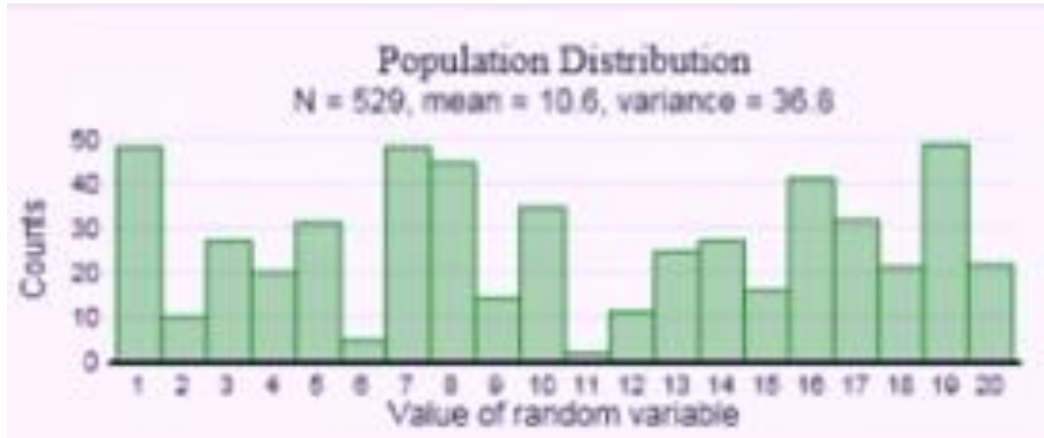
  If our 5 dogs were just a **sample** of a bigger population of dogs, we would divide by **4 instead of 5** like this:

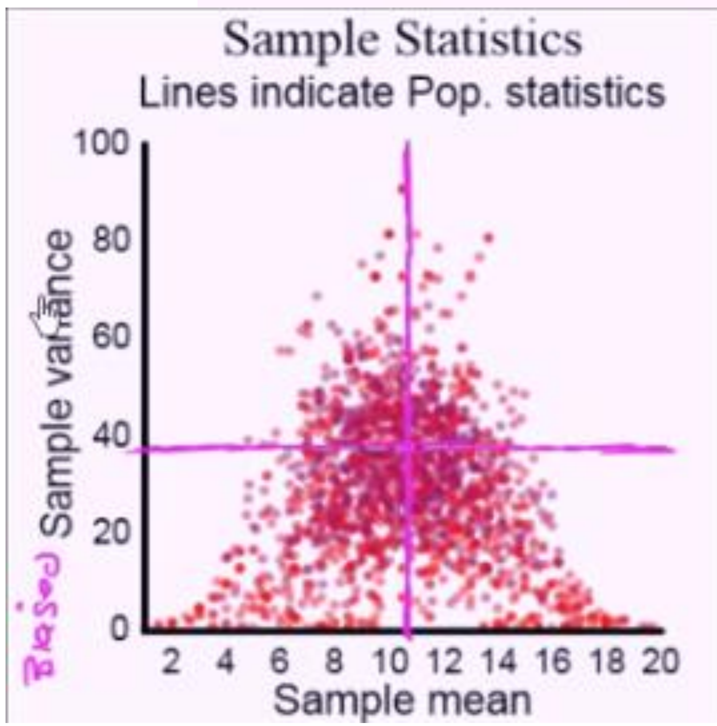
  - Sample Variance = 108,520 / **4** = **27,130**
  - Sample Standard Deviation = √27,130 ≈ **164**

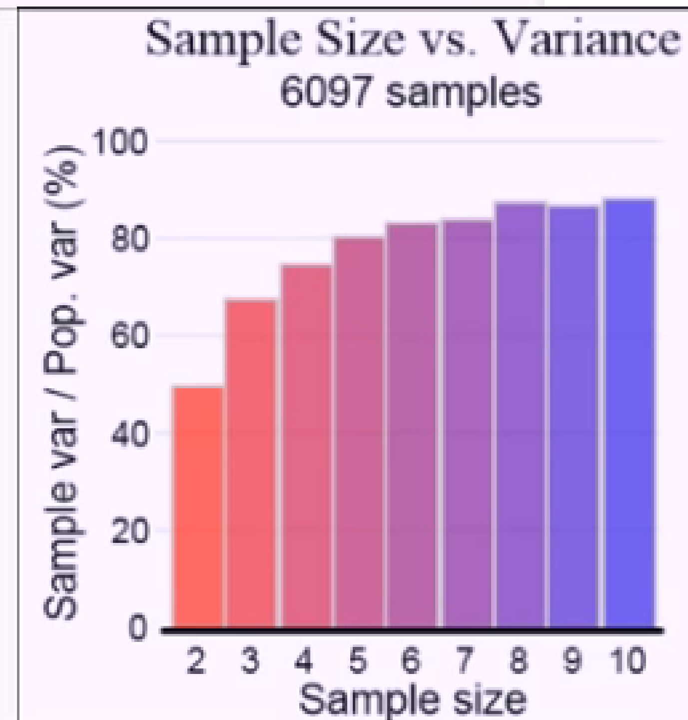
- Why divided by n-1?

Why divided by n-1?

Simulation showing
bias in sample variance







Descriptive Analysis - Exercise

- Compute the variance and standard deviation of height and weight

variables

observations

	weight	height
student1	145	170
student2	170	190
student3	155	172
student4	122	180
student5	167	187
student6	160	174
student7	143	174
student8	142	166
studnet9	139	164
Student10	165	182

- Height
  - Mean

(170+190+172+…+182)/10 = 175.9

  - Variance

$[(170-175.9)^2 + (190-175.9)^2 + \ldots + (182-175.9)^2\,]/10 = 67.29$

  - Standard deviation

$\sqrt{67.29} \approx 8.20$

Descriptive Analysis

- Univariate analysis: describing the distribution of a single variable
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    - Mean, Median, Mode
  - Measures of spread
    - Variance, Standard Deviation
  - Measures of dispersion (variability)
    - Range, Quartiles, Interquartile Range

- Bivariate analysis: describing the relationship between pairs of variables
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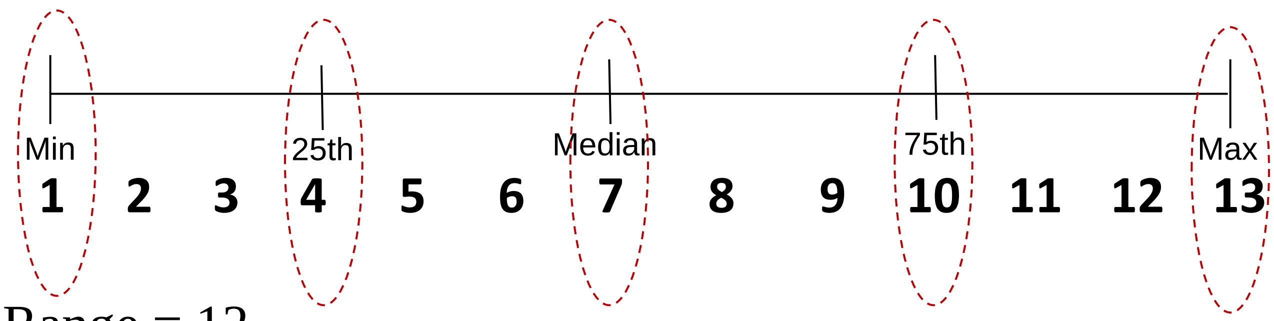
Measures of dispersion

- Range: difference between maximum and minimum value
  - Min: the lowest, or minimum value in variable
  - Max: the highest, or maximum value in variable
- Q1: the first (or 25^{th}) quartile
- Q2: the second (or 50^{th}) quartile – the Median
- Q3: the third (or 75^{th}) quartile

- Box Plot



  - Range = 12

In-class Exercises

Use the following observations of variable x to find the values below.

10, 2, 15, 6, 4, 9, 12, 11, 3, 0, 12, 10, 9, 7, 11, 10, 8, 5, 10, 6

- Q1: n (number of observations)
- Q2: sum of x's
- Q3: X (mean)
- Q4: Median and Mode
- Q5: Five number summary
  - Min, Q1, M, Q3, Max
- Q6: s^2 (variance)
- Q7: s (standard deviation)

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- Bivariate analysis: describing the relationship between pairs of variables
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Covariance and Correlation

- Variables may change in relation to each other.

- Both quantify relationship.

- Difference:
  - Covariance is a dimensional quantity
    - The value depends on the units of the data

    ➔ difficult to compare covariances among data sets that have different scales.

  - Correlation is a dimensionless quantity
    - Always between -1 and 1

    ➔ facilitates the comparison of different data sets

Variance

- First recall: variance of one variable

Case	X	X - Avg	(X - Avg)^2
A	3	-1	1
B	1	-3	9
C	3	-1	1
D	9	5	25
Sum:	16	Sum:	36
Avg:	4	Variance:	9

Variance = Σ (x-Avg)^2 / N =36/4 =9

- X: 4, 4, 4, 4; variance = 0
- X: 1, 1, 1, 13; variance = [(-3)^2+(-3)^2+(-3)^2+9^2]/4=108/4=27

Covariance

- Variance of one variable
- Covariance of two variables

Case	X	Y	(X - Xavg)	(Y - YAvg)	Multiplied
A	3	4	-1	-2	2
B	1	4	-3	-2	6
C	3	8	-1	2	-2
D	9	8	5	2	10
Sum:	16	24		Sum:	16
Avg.	4	6		Avg:	4

covariance coefficient

Covariance = $\Sigma (X_i - Xavg)(Y_i - Yavg) / N = (2+6-2+10)/4 = 4$

- X: 4, 4, 4, 4; covariance = 0
- X: 1, 1, 1, 13; covariance = 6
- X: 13, 1, 1, 1; covariance = -6

We write covariance of X and Y as σ_{XY}

σ_{XX} is the covariance of X with itself
→ Variance of X: σ_X^2

Covariance

- Formally, covariance coefficient can be calculated as:

$$s_{xy} = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

for samples

$$\sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N}$$

for populations

Covariance

- Covariance measures how much the movement in one variable predicts the movement in a corresponding variable
  - Positive covariance indicates that higher than average values of one variable tend to be paired with higher than average values of the other variable.
  - Negative covariance indicates that higher than average values of one variable tend to be paired with lower than average values of the other variable.
- In other words, it measures the degree of linkage between two variables that covary.

From Covariance to Correlation

- Covariance is a dimensional quantity
  - The value depends on the units of the data

  ➔ difficult to compare covariances among data sets that have different scales.

- We need a dimensionless quantity to facilitate comparison → correlation
  - Always between -1 and 1

- The correlation of X and Y, denoted ρ_{XY}, is simply calculated as:

$$\text{correlation of X and Y} = \frac{\text{covariance of X and Y}}{\text{standard deviation of X * standard deviation of Y}}$$

$$\rho_{XY} = \sigma_{XY} / \ (\sigma_{X*}\sigma_{Y})$$

Correlation

$$\text{correlation of X and Y} = \frac{\text{covariance of X and Y}}{\text{standard deviation of X * standard deviation of Y}}$$

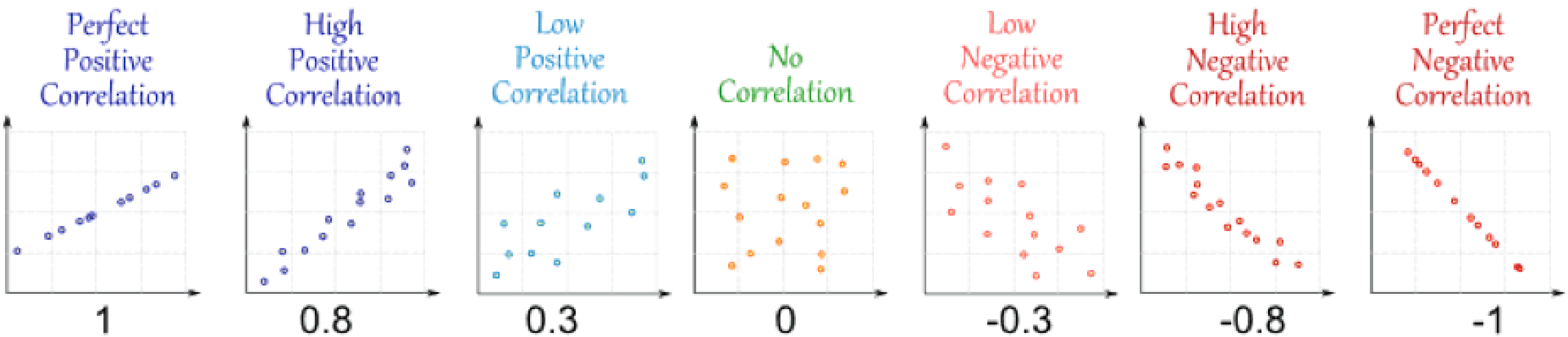
- Example:

Case	X	Y	(X - Xavg)	(Y - YAvg)	Multiplied
A	3	4	-1	-2	2
B	1	4	-3	-2	6
C	3	8	-1	2	-2
D	9	8	5	2	10
Sum:	16	24		Sum:	16
Avg.	4	6		Avg:	4

- SD of X: $\sigma_X = 3$
- SD of Y: $\sigma_Y = 2$
- Covariance of X and Y: $\sigma_{XY} = 4$
- Correlation of X and Y: $\rho_{XY} = 4/(2*3) = 0.67$

Correlation Coefficient

- When the two sets of data are strongly linked together, we say they have a high correlation.
- Correlation is positive when the values increase together, and
- Correlation is negative when one value decreases as the other increase.
- The coefficient can take on values between -1 and +1.
- Values near +1 indicate a strong positive linear relationship.
- Values near -1 indicate a strong negative linear relationship.



Correlation Coefficient

- The correlation coefficient is computed as follows:

$$r_{xy} = \frac{s_{xy}}{s_x s_y}$$

for samples

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

for populations

- It is obtained by dividing the covariance of the two variables by the product of their standard deviations.

In R

- The commands are most straightforward

```
x=c(3,1,3,9)
mean(x)
median(x)
var(x)
sd(x)

y=c(4,4,8,8)
cov(x,y)
cor(x,y)
```

- Question: what is the sd(x) function computing? Unbiased or biased SD? How to calculate the other one?

Correlation and Causation

- Correlation is a measure of linear association and not necessarily causation.
- Just because two variables are highly correlated, it does not mean that one variable is the cause of the other.

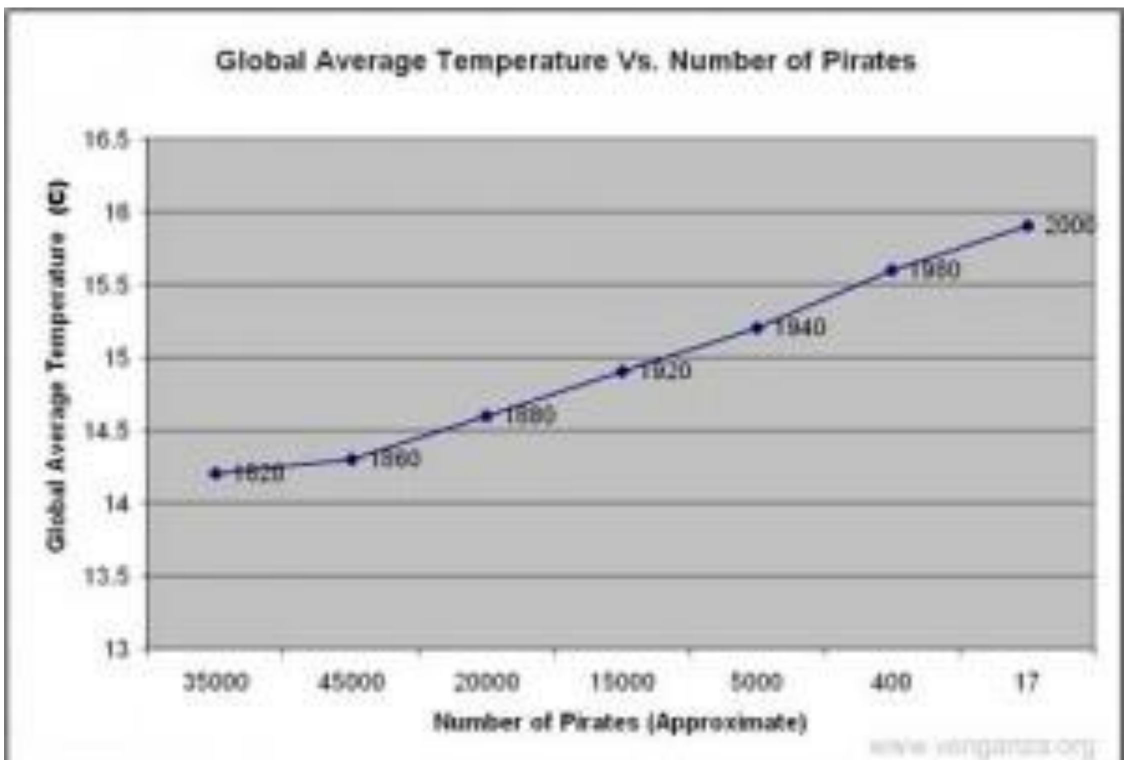
Example

- Ice cream and homicide rates are positively correlated

- Do they have a causal relationship?
  – Does ice cream consumption turn harmless Joe into a murderous monster?

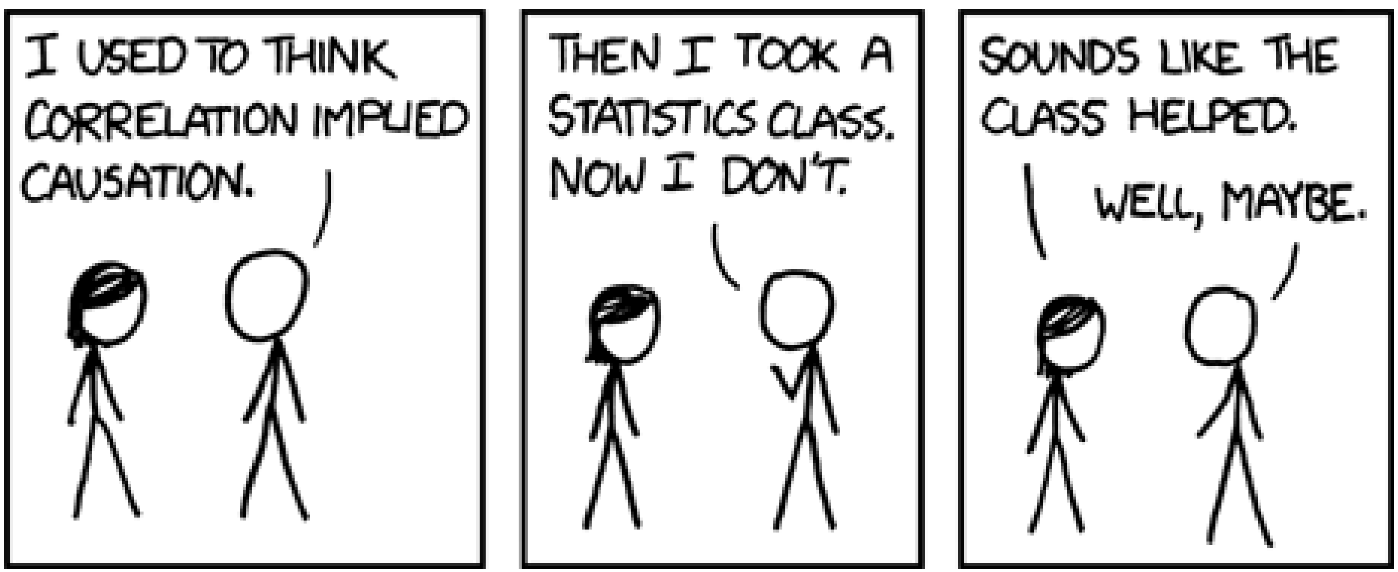
Example

- An increase in both global temperature and number of pirates
  - That’s a positive correlative relationship
- Do the pirates cause the global warming?



Reflection

- Every correlation you have ever heard of can be questioned in your own mind.
  - Is there a cause and effect here, or
  - Is it just coincidence?
  - How are the two factors really related?



Summary

	Population (parameter)	Sample (statistic)
mean	$\mu = \frac{\sum x_i}{N}$	$\overline{x} = \frac{\sum x_i}{n}$
Variance	$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$	$s^2 = \frac{\sum (x_i - \overline{x})^2}{n-1}$
Standard deviation	$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$
Covariance	$\sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N}$	$s_{xy} = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{n-1}$
Correlation	$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$	$r_{xy} = \frac{s_{xy}}{s_x s_y}$

Scale of Measurement

- Nominal
  - Mode
- Ordinal
  - Median, Mode
- Interval
  - Mode, Median, Mean
  - Range, Variance, Standard deviation
- Ratio
  - Mode, Median, Mean
  - Range, Variance, Standard deviation
  - And many more: geometric mean, harmonic mean, coefficient of variation, and all the other statistical measures

Exercises

- Investigate relationship between cigarette smoking and lung capacity
- Data: sample group response data on
  - smoking habits (number of years)
  - measured lung capacities (Spirometer)

N	Cigarettes (X)	Lung Capacity (Y)
1	0	45
2	5	42
3	10	33
4	15	31
5	20	29

Questions:

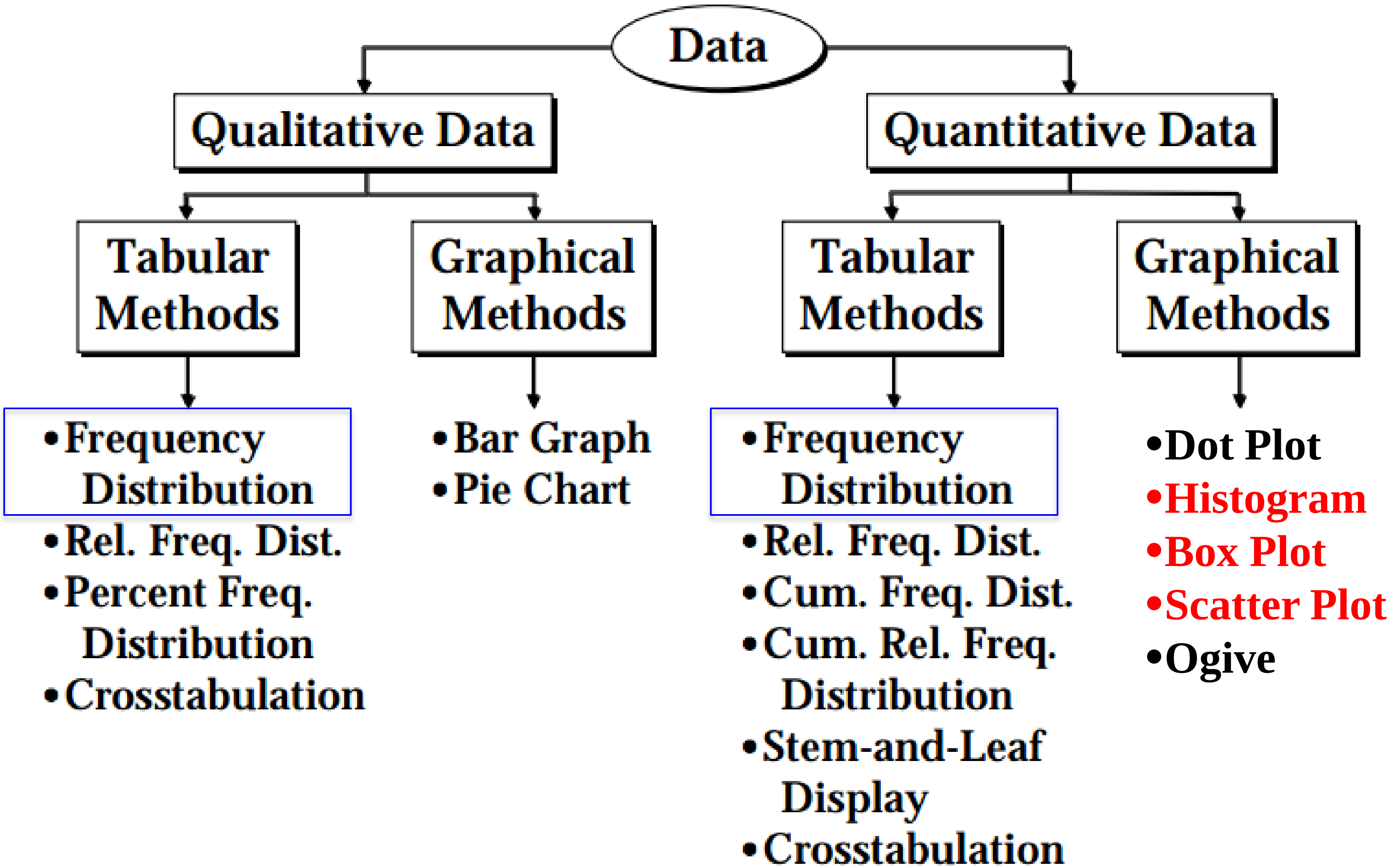
1. Try to plot the data
2. Calculate the sample covariance
3. Calculate the sample correlation
4. How are X and Y correlated?
5. Is there a causation between X and Y?

Descriptive Statistics

- Numerical measures

- Tabular and graphical presentation
  - Frequency distribution
  - Histogram
  - Box plot
  - Scatter plot

Tabular & Graphical Presentation



Frequency distribution

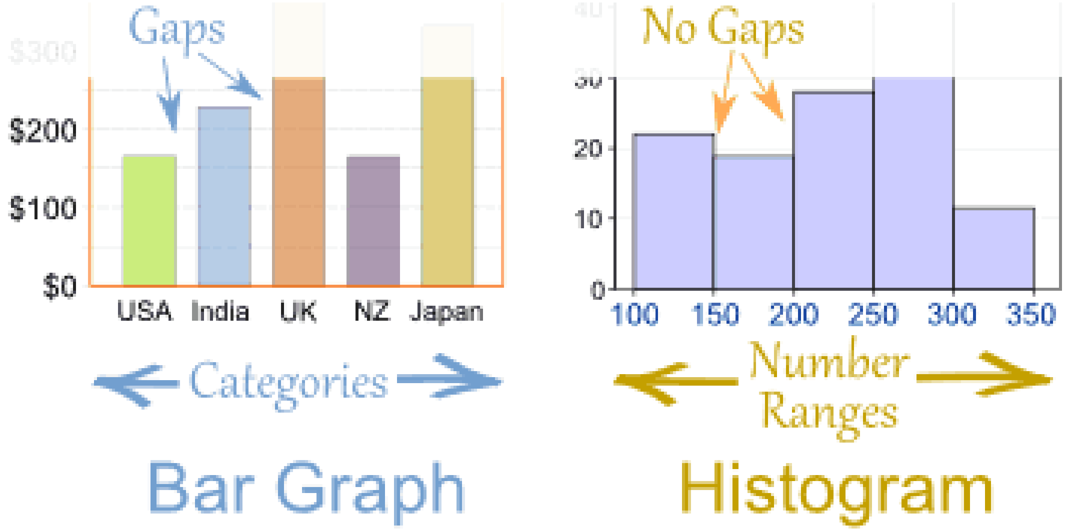
- A table that displays the frequency of various outcomes in a data set
  - Example:

Frequency Distribution for a Class of 25 M.B.A. Students

Grade Scale	Student/Grade Frequency	Relative Frequency
A	5	20%
B	12	48%
C	4	16%
D	2	8%
F	1	4%
I (Incomplete)	1	4%
TOTAL	25	100%

Histogram

- A Histogram is a graphical display of data using bars of different heights.
- It is similar to a Bar Chart, but a histogram groups numbers into **ranges.** And you decide what ranges to use!
- Histograms are a great way to show results of continuous data, such as weight, height, how much time, etc.
- When the data is in **categories** (such as Country or Favorite Movie), we should use a Bar Chart.



Histogram

- Every month you measure how much weight your puppy has gained and get these results:

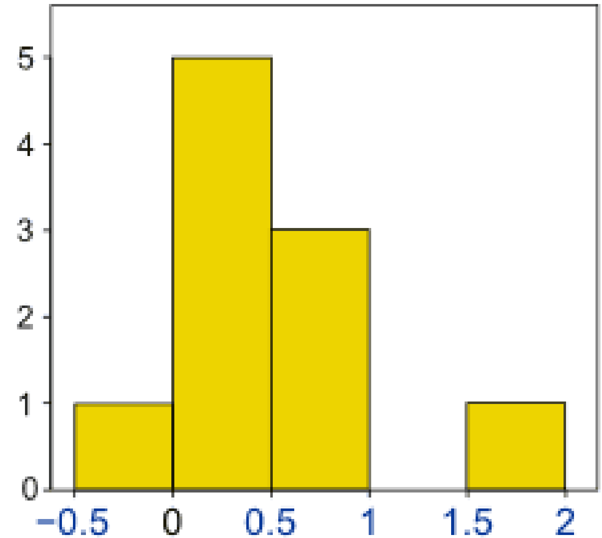
  0.5, 0.5, 0.3, -0.2, 1.6, 0, 0.1, 0.1, 0.6, 0.4

- They vary from -0.2 (the puppy lost weight that month) to 1.6

- Put in order from lowest to highest weight gain:

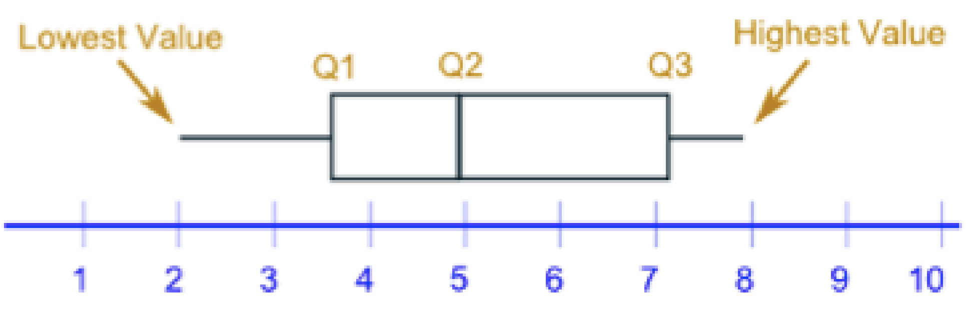
  -0.2, 0, 0.1, 0.1, 0.3, 0.4, 0.5, 0.5, 0.6, 1.6

- You decide to put the results into groups of 0.5:
  - The -0.5 to just below 0 range,
  - The 0 to just below 0.5 range,
  - etc …



Box Plot

- You can show all the important values in a “Box and Whisker Plot”, like this:



- **Example: Box Plot and Interquartile Range for**

  4, 17, 7, 14, 18, 12, 3, 16, 10, 4, 4, 11

- Put them in order:

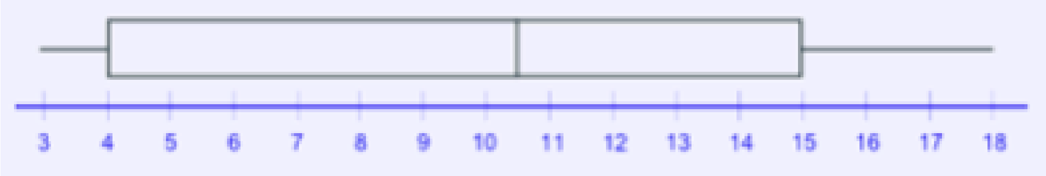
  3, 4, 4, 4, 7, 10, 11, 12, 14, 16, 17, 18

- Cut it into quarters:

  3, 4, 4 | 4, 7, 10 | 11, 12, 14 | 16, 17, 18

Box Plot

- 3, 4, 4 | 4, 7, 10 | 11, 12, 14 | 16, 17, 18
- In this case all the quartiles are between numbers:
  - Quartile 1 (Q1) = (4+4)/2 = 4
  - Quartile 2 (Q2) = (10+11)/2 = 10.5
  - Quartile 3 (Q3) = (14+16)/2 = 15
- Also:
  - The lowest value is 3,
  - The highest value is 18
- So now we have enough data for the Box and Whisker Plot:



- And the Interquartile Range is:

Q3 – Q1 = 15 – 4 = 11

Scatter Plots

- A graph of plotted points that show the relationship between two sets of data.
- In this example, each dot represents one person's weight versus their height.
- (The data is plotted on the graph as "Cartesian (x, y) Coordinates")