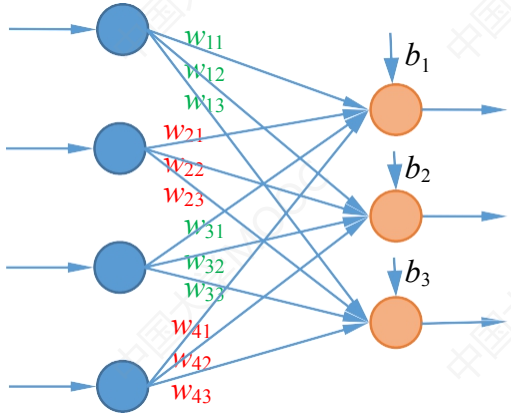
12.4 误差反向传播算法

感知机 / 单层神经网络：线性分类

多层神经网络：非线性分类

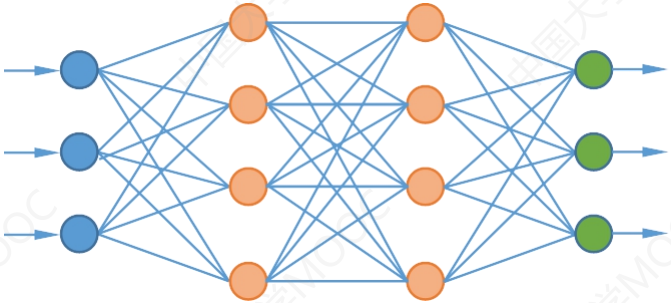


$$W = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{31} & w_{33} \\ w_{41} & w_{42} & w_{43} \end{bmatrix}$$

$$B = [\, b_1, b_2, b_3 \,]$$

$$W^{(k+1)} = W^{(k)} - \eta \frac{\partial Loss(W,B)}{\partial W}$$

$$B^{(k+1)} = B^{(k)} - \eta \frac{\partial Loss(W,B)}{\partial B}$$



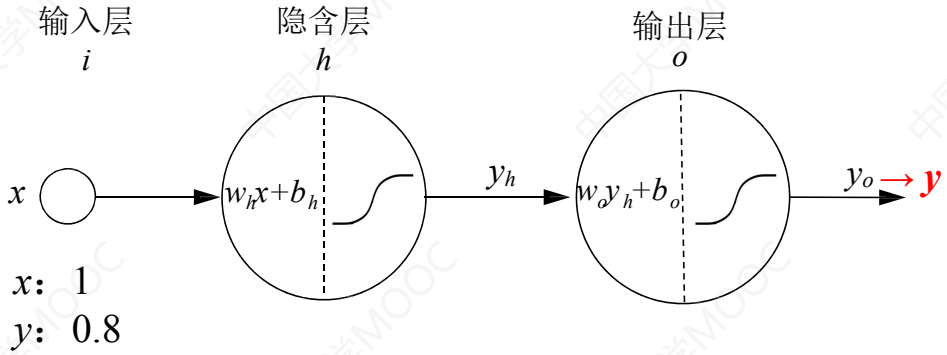
误差反向传播算法（Backpropagation，BP）

利用链式法则，反向传播损失函数的梯度信息，计算出损失函数对网络中所有模型参数的梯度

神经网络的训练

- 使用误差反向传播算法计算梯度
- 使用梯度下降法学习模型参数

1-1-1神经网络的误差反向传播

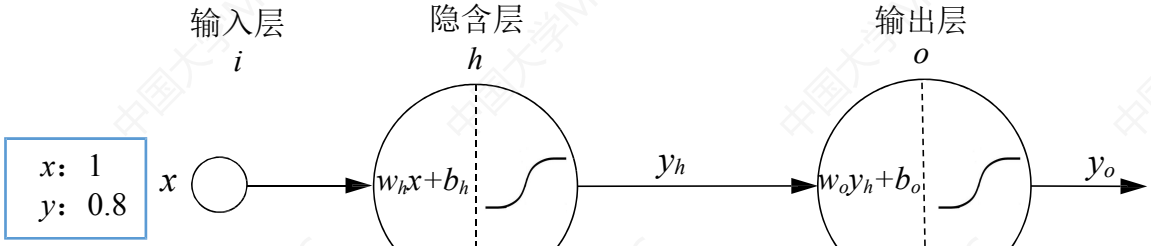


$$z_h = w_h x + b_h \qquad y_h = \frac{1}{1+e^{-z_h}}$$

$$z_o = w_o y_h + b_o \qquad y_o = \frac{1}{1+e^{-z_o}}$$

Step1: 设置模型参数初始值

w_h=0.2, b_h=0.1, w_o=0.3, b_o=0.2



Step2: 正向计算预测值

$$y_h=\frac{1}{1+e^{-(0.2\times1+0.1)}}=0.57$$

$$y_o=\frac{1}{1+e^{-(0.3\times0.57+0.2)}}=0.59$$

Step3: 计算误差

$$Loss=\frac{1}{2}(y-y_o)^2=0.02205$$

Step4: 误差反向传播

$$w_o^{(k+1)}=w_o^{(k)}-\eta\frac{\partial Loss}{\partial w_o}$$

$$b_o^{(k+1)}=b_o^{(k)}-\eta\frac{\partial Loss}{\partial b_o}$$

链式求导法则

$$\frac{\partial Loss}{\partial w_o} = \frac{\partial Loss}{\partial y_o} \cdot \frac{\partial y_o}{\partial z_o} \cdot \frac{\partial z_o}{\partial w_o} = -0.21 \times 0.2419 \times 0.57 = -0.02895543$$

$$Loss = \frac{1}{2}(y - y_o)^2 \qquad \frac{\partial Loss}{\partial y_o} = 2 \times \frac{1}{2} \times -(y - y_o) = -(0.8 - 0.59) = -0.21$$

$$y_o = \frac{1}{1 + e^{-z_o}} \qquad \frac{\partial y_o}{\partial z_o} = \frac{e^{-z_o}}{(1 + e^{-z_o})^2} = y_o(1 - y_o)$$

$$= 0.59 \times (1 - 0.59) = 0.2419$$

$$z_o = w_o y_h + b_o \qquad \frac{\partial z_o}{\partial w_o} = y_h = 0.57$$

$$\eta = 0.5$$

$$w_o^{(1)} = w_o^{(0)} - \eta \frac{\partial Loss}{\partial w_o}$$

$$= 0.3 - 0.5 \times (-0.02895543)$$

$$= 0.314477715$$

链式求导法则

$$\eta = 0.5$$

$$\frac{\partial Loss}{\partial w_o} = \frac{\partial Loss}{\partial y_o} \cdot \frac{\partial y_o}{\partial z_o} \cdot \frac{\partial z_o}{\partial w_o} = -0.21 \times 0.2419 \times 0.57 = -0.02895543$$

$$w_o^{(1)} = w_o^{(0)} - \eta \frac{\partial Loss}{\partial w_o} = 0.314477715$$

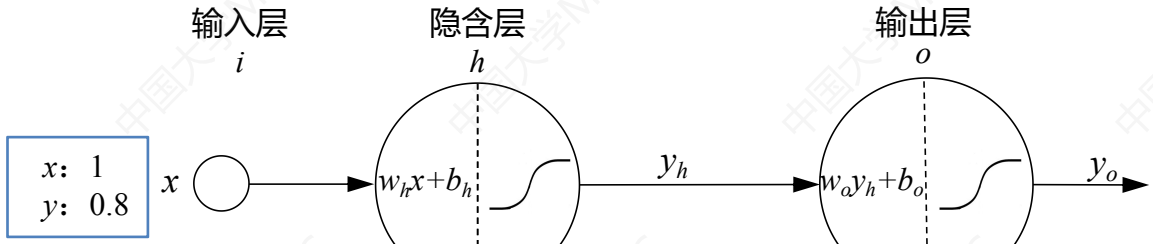
$$\frac{\partial Loss}{\partial b_o} = \frac{\partial Loss}{\partial y_o} \cdot \frac{\partial y_o}{\partial z_o} \cdot \frac{\partial z_o}{\partial b_o} = -0.21 \times 0.2419 = -0.050799$$

$$b_o^{(1)} = b_o^{(0)} - \eta \frac{\partial Loss}{\partial b_o} = 0.2253995$$

$$z_o = w_o y_h + b_o \qquad \frac{\partial z_o}{\partial b_o} = 1$$

Step1: 设置模型参数初始值

$w_h=0.2,\ b_h=0.1,\ w_o=0.3,\ b_o=0.2$



Step2: 正向计算预测值

$$y_h=\frac{1}{1+e^{-(0.2\times1+0.1)}}=0.57 \qquad y_o=\frac{1}{1+e^{-(0.3\times0.57+0.2)}}=0.59$$

Step3: 计算误差

$$Loss=\frac{1}{2}(y-y_o)^2=0.02205$$

Step4: 误差反向传播

$$w_h^{(k+1)}=w_h^{(k)}-\eta\frac{\partial Loss}{\partial w_h} \qquad b_h^{(k+1)}=b_h^{(k)}-\eta\frac{\partial Loss}{\partial b_h}$$

$$w_o^{(1)}=w_o^{(0)}-\eta\frac{\partial Loss}{\partial w_o}=0.314477715 \qquad b_o^{(1)}=b_o^{(0)}-\eta\frac{\partial Loss}{\partial b_o}=0.2253995$$

链式求导法则

$$\frac{\partial Loss}{\partial w_h}=\frac{\partial Loss}{\partial y_o}\cdot\frac{\partial y_o}{\partial z_o}\cdot\frac{\partial z_o}{\partial y_h}\cdot\frac{\partial y_h}{\partial z_h}\cdot\frac{\partial z_h}{\partial w_h}=-0.21\times0.2419\times0.3\times0.2451\times1=-0.00373525$$

$$Loss=\frac{1}{2}(y-y_o)^2 \qquad \frac{\partial Loss}{\partial y_o}=2\times\frac{1}{2}\times-(y-y_o)=-(0.8-0.59)=\boxed{-0.21}$$

$$y_o=\frac{1}{1+e^{-z_o}} \qquad \frac{\partial y_o}{\partial z_o}=\frac{e^{-z_o}}{(1+e^{-z_o})^2}=y_o(1-y_o)=\boxed{0.2419}$$

$$z_o=w_o y_h+b_o \qquad \frac{\partial z_o}{\partial y_h}=w_o=\boxed{0.3}$$

$$y_h=\frac{1}{1+e^{-z_h}} \qquad \frac{\partial y_h}{\partial z_h}=y_h(1-y_h)=0.57\times(1-0.57)=\boxed{0.2451}$$

$$z_h=w_h x+b_h \qquad \frac{\partial z_h}{\partial w_h}=x=\boxed{1}$$

链式求导法则

$$\frac{\partial Loss}{\partial b_h}=\frac{\partial Loss}{\partial y_o}\cdot\frac{\partial y_o}{\partial z_o}\cdot\frac{\partial z_o}{\partial y_h}\cdot\frac{\partial y_h}{\partial z_h}\cdot\frac{\partial z_h}{\partial b_h}=-0.21\times0.2419\times0.3\times0.2451\times1=-0.00373525$$

$$Loss=\frac{1}{2}(y-y_o)^2 \qquad \frac{\partial Loss}{\partial y_o}=2\times\frac{1}{2}\times-(y-y_o)=-(0.8-0.59)=-0.21$$

$$y_o=\frac{1}{1+e^{-z_o}} \qquad \frac{\partial y_o}{\partial z_o}=\frac{e^{-z_o}}{(1+e^{-z_o})^2}=y_o(1-y_o)=0.2419$$

$$z_o=w_o y_h+b_o \qquad \frac{\partial z_o}{\partial y_h}=w_o=0.3$$

$$y_h=\frac{1}{1+e^{-z_h}} \qquad \frac{\partial y_h}{\partial z_h}=y_h(1-y_h)=0.57\times(1-0.57)=0.2451$$

$$z_h=w_h x+b_h \qquad \frac{\partial z_h}{\partial b_h}=1$$

链式求导法则

$$\frac{\partial Loss}{\partial w_h}=\frac{\partial Loss}{\partial y_o}\cdot\frac{\partial y_o}{\partial z_o}\cdot\frac{\partial z_o}{\partial y_h}\cdot\frac{\partial y_h}{\partial z_h}\cdot\frac{\partial z_h}{\partial w_h}=-0.21\times0.2419\times0.3\times0.2451\times1=-0.00373525$$

$$\frac{\partial Loss}{\partial b_h}=\frac{\partial Loss}{\partial y_o}\cdot\frac{\partial y_o}{\partial z_o}\cdot\frac{\partial z_o}{\partial y_h}\cdot\frac{\partial y_h}{\partial z_h}\cdot\frac{\partial z_h}{\partial b_h}=-0.21\times0.2419\times0.3\times0.2451\times1=-0.00373525$$

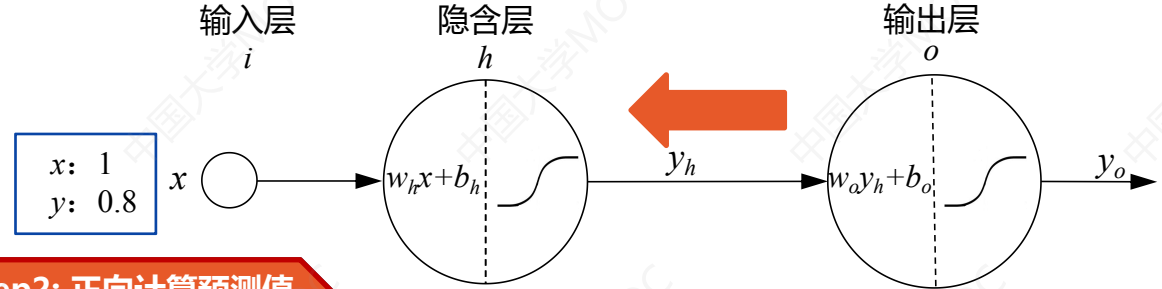
迭代公式

$$w_h^{(1)}=w_h^{(0)}-\eta\frac{\partial Loss}{\partial w_h}=0.2-0.5\times(-0.00373525)=0.201867625$$

$$b_h^{(1)}=b_h^{(0)}-\eta\frac{\partial Loss}{\partial b_h}=0.1-0.5\times(-0.00373525)=0.101867625$$

Step1: 设置模型参数初始值

$w_h=0.2,\ b_h=0.1,\ w_o=0.3,\ b_o=0.2$



Step2: 正向计算预测值

$$y_h=\frac{1}{1+e^{-(0.2\times1+0.1)}}=0.57 \qquad y_o=\frac{1}{1+e^{-(0.3\times0.57+0.2)}}=0.59$$

Step3: 计算误差

$$Loss=\frac{1}{2}(y-y_o)^2=0.02205$$

Step4: 误差反向传播

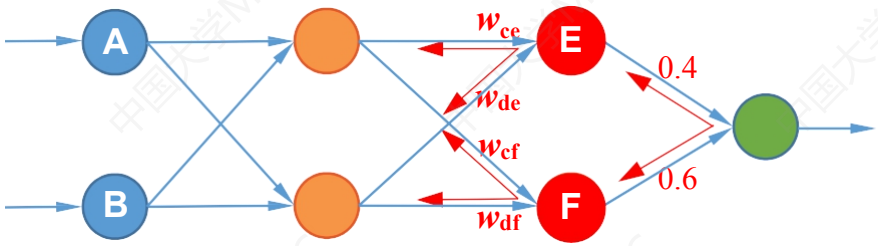
$$w_h^{(1)}=w_h^{(0)}-\eta\frac{\partial Loss}{\partial w_h}=0.201867625$$

$$b_h^{(1)}=b_h^{(0)}-\eta\frac{\partial Loss}{\partial b_h}=0.101867625$$

$$w_o^{(1)}=w_o^{(0)}-\eta\frac{\partial Loss}{\partial w_o}=0.314477715$$

$$b_o^{(1)}=b_o^{(0)}-\eta\frac{\partial Loss}{\partial b_o}=0.2253995$$

隐含层有多个神经元的误差反向传播



$$LossE = 0.4LossG$$
$$LossF = 0.6LossG$$

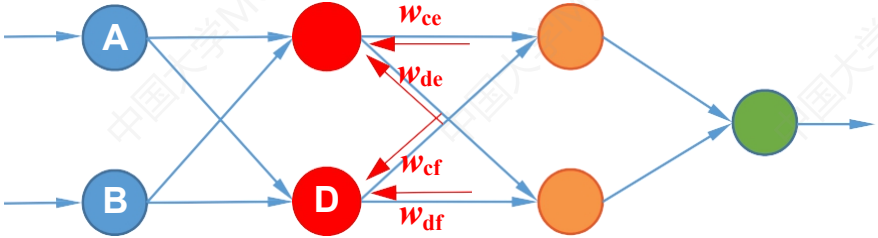
$$LossC_E = \frac{w_{ce}}{w_{ce}+w_{de}} LossE$$

$$LossD_E = \frac{w_{de}}{w_{ce}+w_{de}} LossE$$

$$LossC_F = \frac{w_{cf}}{w_{cf}+w_{df}} LossF$$

$$LossD_F = \frac{w_{df}}{w_{cf}+w_{df}} LossF$$

隐含层有多个神经元的误差反向传播



$$LossE = 0.4LossG$$
$$LossF = 0.6LossG$$

$$LossC = LossC_E + LossC_F = \frac{w_{ce}}{w_{ce}+w_{de}}LossE + \frac{w_{cf}}{wcf+wdf}LossF$$

$$LossD = LossD_E + LossD_F = \frac{w_{de}}{w_{ce}+w_{de}}LossE + \frac{w_{df}}{w_{cf}+w_{df}}LossF$$

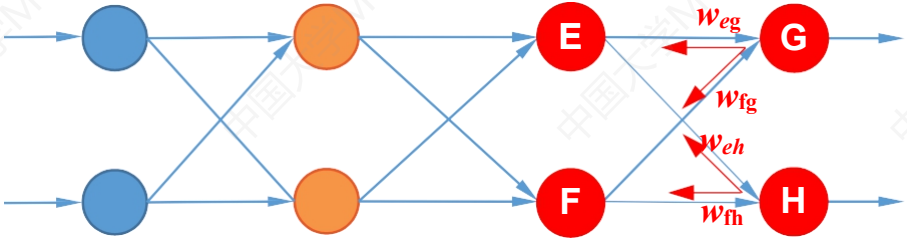
$$LossC_E = \frac{w_{ce}}{w_{ce}+w_{de}}LossE$$

$$LossD_E = \frac{w_{de}}{w_{ce}+w_{de}}LossE$$

$$LossC_F = \frac{w_{cf}}{w_{cf}+w_{df}}LossF$$

$$LossD_F = \frac{w_{df}}{w_{cf}+w_{df}}LossF$$

隐含层有多个神经元的误差反向传播



$$LossE = \frac{w_{eg}}{w_{eg} + w_{fg}} LossG + \frac{w_{eh}}{w_{eh} + w_{fh}} LossH$$

$$LossF = \frac{w_{fg}}{w_{eg} + w_{fg}} LossG + \frac{w_{fh}}{w_{eh} + w_{fh}} LossH$$

多层神经网络的训练

通过梯度下降法训练模型参数

通过误差反向传播法计算梯度