Kalman Filter for Temperature Smoothing

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1 Introduction

The Kalman Filter is a recursive algorithm used to estimate the state of a system in the presence of noise and uncertainty. In this case, we apply the Kalman Filter to estimate the true average daily temperature, smoothing out the noise in the observed data. The filter operates in two steps: *prediction* and *update*.

2 Kalman Filter Equations

Let:

- μ_t: The estimated state at time t, representing the average temperature.
- α: The state transition coefficient, assumed constant.
- q: The standard deviation of the process noise, representing uncertainty in the state transition.
- r: The standard deviation of the measurement noise, representing uncertainty in the observed data.
- σ_t: The error covariance at time t, representing the uncertainty in our estimate of the state.

2.1 Prediction Step

In the prediction step, we predict the next state and error covariance based on the previous state and process noise.

$$\mu_{\text{prior},t} = \alpha \cdot \mu_{t-1} \tag{1}$$

$$\sigma_{\text{prior},t} = \alpha^2 \cdot \sigma_{t-1} + q^2 \tag{2}$$

Here:

- $\mu_{\text{prior},t}$ is the predicted state at time t.
- $\sigma_{\text{prior},t}$ is the predicted error covariance.

2.2 Update Step

In the update step, we refine our prediction using the observed measurement.

$$K_t = \frac{\sigma_{\text{prior},t}}{\sigma_{\text{prior},t} + r^2} \tag{3}$$

$$\mu_t = \mu_{\text{prior},t} + K_t \cdot (z_t - \mu_{\text{prior},t}) \tag{4}$$

$$\sigma_t = (1 - K_t) \cdot \sigma_{\text{prior},t} \tag{5}$$

Here:

- K_t is the Kalman gain, which determines the weight given to the observed measurement versus the prediction.
- μ_t is the updated state estimate.
- σ_t is the updated error covariance.

3 Application to Temperature Data

In our temperature smoothing problem, we use the following values:

- μ_t: The estimated average daily temperature at time t.
- α: The state transition coefficient, set to 1 (assuming that today's temperature is close to yesterday's).
- q: The process noise standard deviation, controlling the smoothness of the filter. Smaller values of q lead to a smoother estimate.
- r: The measurement noise standard deviation, based on the expected noise in the temperature data.

The observed temperature data, z_t, is calculated as the daily average:

$$\text{temp\_avg} = \frac{\text{temp\_max} + \text{temp\_min}}{2} \tag{6}$$

We initialize the filter with an initial state, μ_0, set to the first observed average temperature, and an initial variance, σ_0, set to 1.