Question 2

Part 1

a) $E[K_{100}] = p.n = 0.4 \times 100$

$= 40$ calls

b) $\sigma_{K100} = \sqrt{Var(K_{100})} = \sqrt{np(1-p)}$

$= \sqrt{40 \times 0.6} = \sqrt{24} = 4.9$

c) $P[K_{100} \geq 18]$

CLT $\Rightarrow$ $Z = \frac{\bar{x} - \mu}{\sigma} = \frac{18 - 40}{4.9} = -4.49$

$P[Z \geq -4.449]$

d) $P[16 \leq K_{100} \leq 24]$

i) $P[K_{100} \geq 16]$

$\Rightarrow Z = \frac{16 - 40}{4.9} = -4.90$

$P[K_{100} \leq 24]$

$\Rightarrow Z = \frac{24 - 40}{4.9} = -3.27$

$P(-4.90 \leq Z \leq -3.27)$

e) Infer

- From the above the probability of having data calls between 16 to 24 is so little the indicates that very high calls are around 40 calls

Telecom Operator

- ✓ This will help the operator to allocate enough bandwidth for data call traffic.
- ✓ Resource planning to avoid congestion / under utilization.

Part 2 : Chernoff & Gaussian RV

Chernoff bound

$$\Rightarrow P[X \geq c] \leq e^{-tc} M_X(t)$$

Generative Moment function $[M_X(t)]$ for Gaussian.

$$\Rightarrow M_X(t) = e^{ut + \frac{\sigma^2 t^2}{2}}$$

$$t = \frac{c-u}{\sigma^2}$$

Substitute to Chernoff

$$\therefore \Rightarrow P[X \geq c] \leq e^{\frac{-c(c-u)}{\sigma^2}} \cdot e^{\frac{u(c-u)}{\sigma^2} + \frac{\sigma^2\left(\frac{c-u}{\sigma^2}\right)^2}{2}}$$

$$\leq e^{\frac{-c(c-u)}{\sigma^2}} \cdot e^{\frac{u(c-u)}{\sigma^2} + \frac{(c-u)^2}{2\sigma^2}}$$

$\Rightarrow$

This

$$= -\frac{c(c-u)}{\sigma^2} + \frac{u(c-u)}{\sigma^2} + \frac{(c-u)^2}{2\sigma^2} = -\frac{(c-u)^2}{2\sigma^2}$$

Substute it

$$\therefore P[X \geq c] \leq e^{\frac{-(c-u)^2}{2\sigma^2}}$$

Applications

1) Stock prices surges
   → Setting c as the price level that is considered a surge, then use available data / historical data to find μ and σ and find the probability of surge this will guide in risk management plans

2) Network family spikes:
   → Set c as the critical level and use find probability for the surge, Helps in bandwidth planning and better prevention measures to avoid the surges

Part 3

win (s) = 3 | tie (s) = 1 | loss (s) = 0
P(w) = 1/3 | P(t) = 1/3 | P(L) = 1/3

a) $\phi_Y(s) \Rightarrow E[e^{ts}] \Rightarrow P(w)\cdot e^{3s} + P(t)\cdot e^{s} + P(L)\, e^{0}$

$$\Rightarrow \frac{1}{3}e^{3t} + \frac{1}{3}e^{t} + \frac{1}{3}e^{0t} = \frac{1}{3}e^{3t} + \frac{1}{3}e^{t} + \frac{1}{3}$$

$$\phi_{\dot{Y}}(s) = [E(e^{ts})]^n = \left(\frac{1}{3}e^{3t} + \frac{1}{3}e^{t} + \frac{1}{3}\right)^n$$

b) $E[Y] = n\,E[X_i]$

$E[X_i] = \frac{1}{3}\times 3 + \frac{1}{3}\times 1 + \frac{1}{3}(0)$

$= 4/3$

$\therefore E[Y] = \frac{4n}{3}$

$Var[Y] = n[Var[X_i]]$

$Var[X_i] = E[X^2] - [E(X)]^2$

$\therefore E[X^2] = 3^2\times\frac{1}{3} + 1^2\times\frac{1}{3} + 0^2\times\frac{1}{3}$

$= 10/3$

$(E[X])^2 = (4/3)^2 = \frac{16}{9}$

$Var[Y] = \left[\frac{10}{3} - \frac{16}{9}\right]n$

$= \frac{14n}{9}$

c) **Infer**

↳ For multiple tournaments Man-U is expected to have points of $\frac{4n}{3}$

↳ The variance shows how the points/performance vary overaway from the mean after each game

Impact

↳ Consistent achievement of points close to the expected points would improve the rank of the team.

↳ A low covariance will indicate consistency of the teams performance.

Part 4

$\mu = 100 \quad \sigma = 10$

a) $P[X \geq 120]$

Poisson (100) $\sim N(100, 10^2)$

$\therefore Z = \frac{X - \mu}{\sigma} = \frac{120 - 10}{10} = 2$

$P(Z \geq 2) = 1 - P(Z \leq 2)$

$= 0.0228$

$= 2.28\%$

b) The probability to have two sections is 2.28%.

1) The prof should primarily prepare to have one section since the percentage is quit low of 2.28%.
2) Regardless of it being low the prof should ~~Continue Prepare~~ Plan for two section.
3) Prof should be flexible with the resource that can be scaled up incase the 2.28% chance occurs