Import necessary libraries

```
import numpy as np
import matplotlib.pyplot as plt
```

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Step 1: Generate Synthetic Data

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```
np.random.seed(42)
N = 500  # Total data points
d = 12   # Number of features
train_ratio = 0.7
val_ratio = 0.15

# Generate feature matrix and true weights
X = np.random.normal(0, 1, (N, d))
true_weights = np.linspace(1, 5, d)  # Linearly spaced true weights
epsilon = np.random.normal(0, 0.5, N)  # Noise
y = X @ true_weights + epsilon  # Generate target values

# Split data into train, validation, and test sets
train_size = int(N * train_ratio)
val_size = int(N * val_ratio)
test_size = N - train_size - val_size

X_train, X_val, X_test = X[:train_size],
X[train_size:train_size+val_size], X[train_size+val_size:]
y_train, y_val, y_test = y[:train_size],
y[train_size:train_size+val_size], y[train_size+val_size:]
```

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Step 2: Ridge Regression Functions

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```
def ridge_loss(w, X, y, lam):
    """Calculate the ridge regression loss."""
    residuals = y - X @ w
    return np.sum(residuals**2) + lam * np.sum(w**2)

def ridge_gradient(w, X, y, lam):
    """Calculate the gradient of the ridge regression loss."""
    residuals = y - X @ w
    grad = - 2 * X.T @ residuals + 2* lam * w
    grad = grad/len(y)
```

```
    return grad

def gradient_descent(loss_fn, grad_fn, w_init, X, y, lam, lr=0.01,
tol=1e-6, max_iters=1000):
    """Perform gradient descent to minimize the ridge regression
loss."""
    w = w_init
    for i in range(max_iters):
        grad = grad_fn(w, X, y, lam)
        w_new = w - lr * grad
        if np.linalg.norm(w_new - w, ord=2) < tol:
            break
        w = w_new
    return w
```

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Step 3: Variance and Bias Calculation

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```
def calculate_bias_variance(X_train, y_train, X_val, y_val, lambdas,
num_datasets=20,
                            sub_sample_size=50):
    """
    Calculate the bias and variance for ridge regression models
trained on multiple datasets.
    """
    biases, variances = [], []
    for lam in lambdas:
        predictions = []
        for _ in range(num_datasets):
            # Sample with replacement
            indices = np.random.choice(len(X_train),
size=sub_sample_size, replace=True)
            X_sample, y_sample = X_train[indices], y_train[indices]

            # Train ridge regression
            w_init = np.zeros(d)
            w = gradient_descent(ridge_loss, ridge_gradient, w_init,
X_sample, y_sample, lam)

            # Predict on validation data
            predictions.append(X_val @ w)

        # Average predictions
        predictions = np.array(predictions)
        mean_prediction = np.mean(predictions, axis=0)
        bias = np.mean((mean_prediction - y_val)**2)
```

```
        variance = np.mean(np.var(predictions, axis=0))

        biases.append(bias)
        variances.append(variance)

    return biases, variances

lambdas = [a * 10**b for a in range(1, 10) for b in range(-5, 3)]  #
For λ values
lambdas.sort()
```

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Step 4: Plotting Functions

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```
# Empty sections for students to complete
def plot_coefficients_vs_lambda():
    """Plot the learned coefficients for different lambda values."""
    coeffs = []
    for lam in lambdas:
        w_init = np.zeros(d)
        w_opt = gradient_descent(ridge_loss, ridge_gradient, w_init,
X_train, y_train, lam)
        coeffs.append(w_opt)

    coeffs = np.array(coeffs)  # Convert to numpy array for plotting
    plt.figure(figsize=(8, 5))
    plt.plot(lambdas, coeffs)
    plt.xscale("log")  # Log scale for lambda
    plt.xlabel("Lambda (log scale)")
    plt.ylabel("Coefficient Values")
    plt.title("Coefficient Values vs. Lambda")
    plt.grid()
    plt.savefig("coefficients_vs_lambda.png")
    plt.show()


def plot_rmse_vs_lambda():
    """Plot RMSE on validation data vs lambda."""
    rmse_values = []
    for lam in lambdas:
        w_init = np.zeros(d)
        w_opt = gradient_descent(ridge_loss, ridge_gradient, w_init,
X_train, y_train, lam)
        y_pred = X_val @ w_opt
        rmse = np.sqrt(np.mean((y_val - y_pred) ** 2))
        rmse_values.append(rmse)
```

```
    optimal_lambda = lambdas[np.argmin(rmse_values)]  # Find lambda
that minimizes RMSE


    plt.figure(figsize=(8, 5))
    plt.plot(lambdas, rmse_values)
    plt.plot(optimal_lambda, min(rmse_values), 'ro')  # Highlight
optimal lambda
    plt.xscale("log")
    plt.xlabel("Lambda (log scale)")
    plt.ylabel("Validation RMSE")
    plt.title(f"Validation RMSE vs. Lambda (Best λ =
{optimal_lambda:.2e})")
    plt.grid()
    plt.savefig("rmse_vs_lambda.png")
    plt.show()

    return optimal_lambda  # Return best lambda

# def plot_predicted_vs_true():
def plot_predicted_vs_true(lambda_opt):
    """Plot predicted vs true values using optimal lambda."""
    w_init = np.zeros(d)
    w_opt = gradient_descent(ridge_loss, ridge_gradient, w_init,
np.vstack((X_train, X_val)), np.hstack((y_train, y_val)), lambda_opt)

    y_pred = X_test @ w_opt  # Predict test values

    plt.figure(figsize=(6, 6))
    plt.scatter(y_test, y_pred, alpha=0.7)
    plt.plot([min(y_test), max(y_test)], [min(y_test), max(y_test)],
'r', linestyle="--")  # 45-degree reference line
    plt.xlabel("True Values")
    plt.ylabel("Predicted Values")
    plt.title("Predicted vs True Values")
    plt.grid()
    plt.savefig("predicted_vs_true.png")
    plt.show()



# def plot_bias_variance_tradeoff():
def plot_bias_variance_tradeoff():
    """Plot bias and variance vs lambda."""
    biases, variances = calculate_bias_variance(X_train, y_train,
X_val, y_val, lambdas)

    plt.figure(figsize=(8, 5))
    plt.plot(lambdas, biases, label="Bias")
    plt.plot(lambdas, variances, label="Variance")
```

```
    plt.xscale("log")
    plt.xlabel("Lambda (log scale)")
    plt.ylabel("Variance")
    plt.title("Variance Tradeoff")
    plt.legend()
    plt.grid()
    plt.savefig("bias_variance_tradeoff.png")
    plt.show()
```

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Step 5: Main Execution

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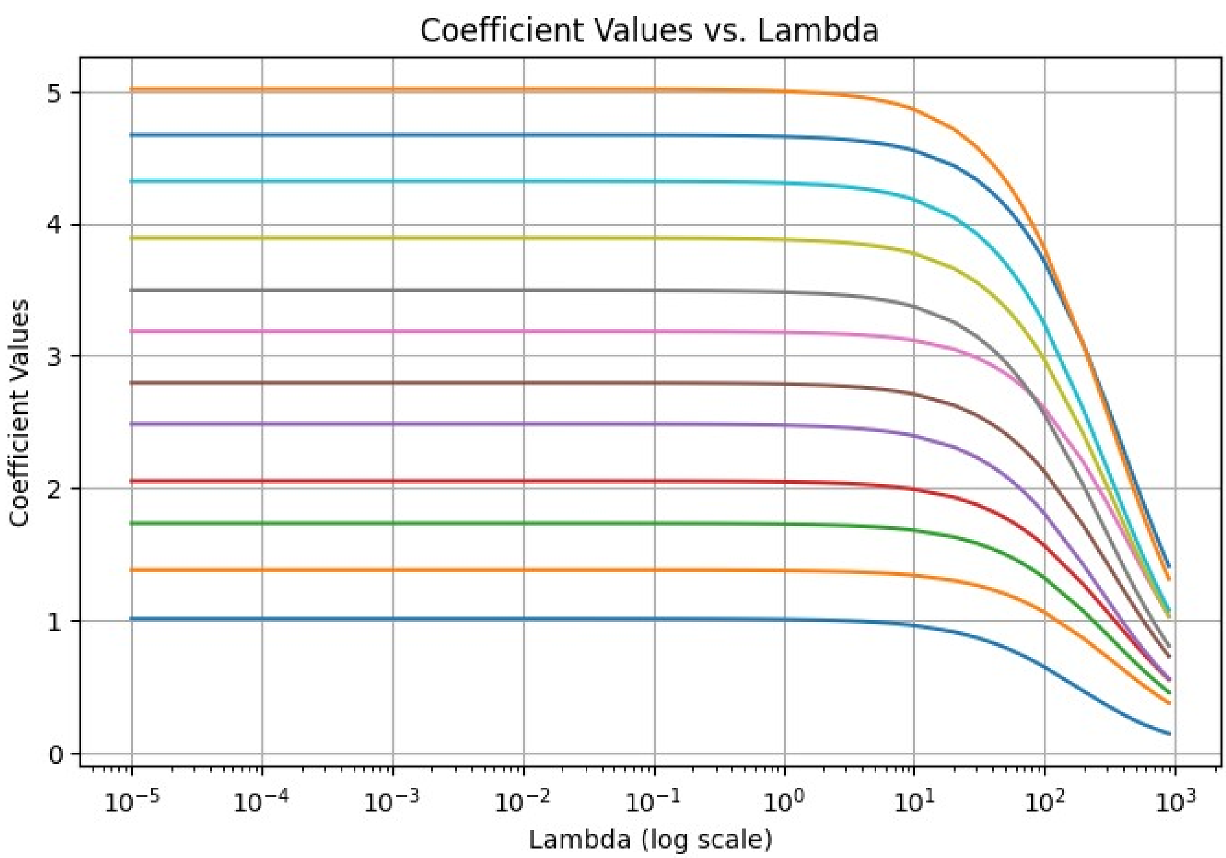
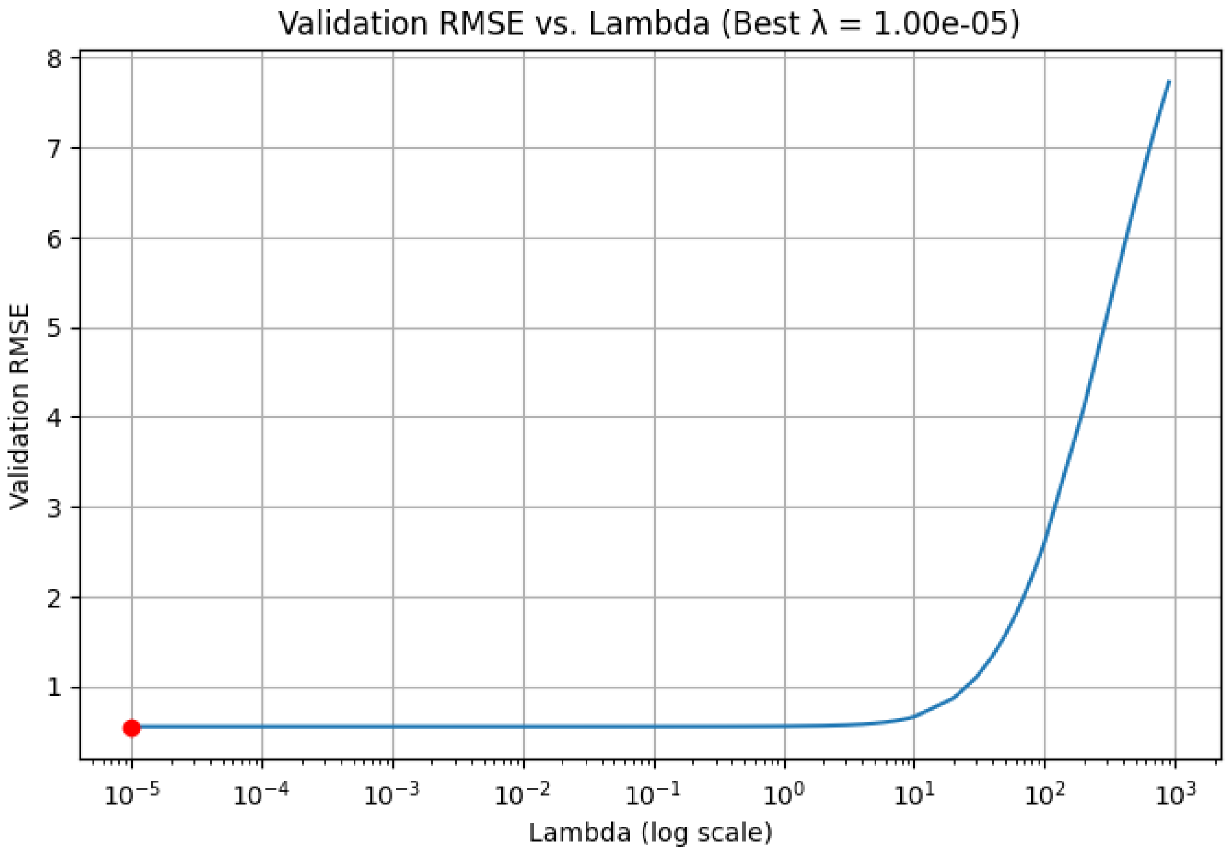
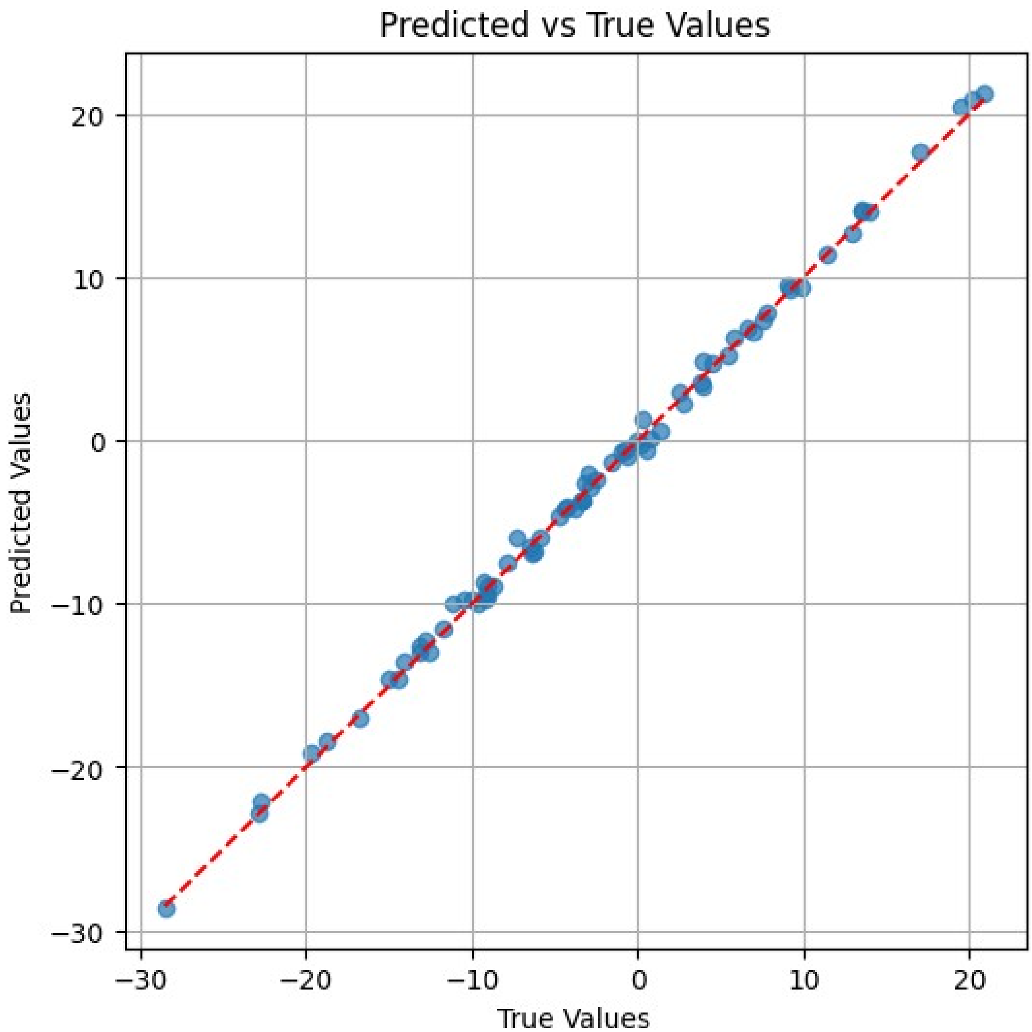
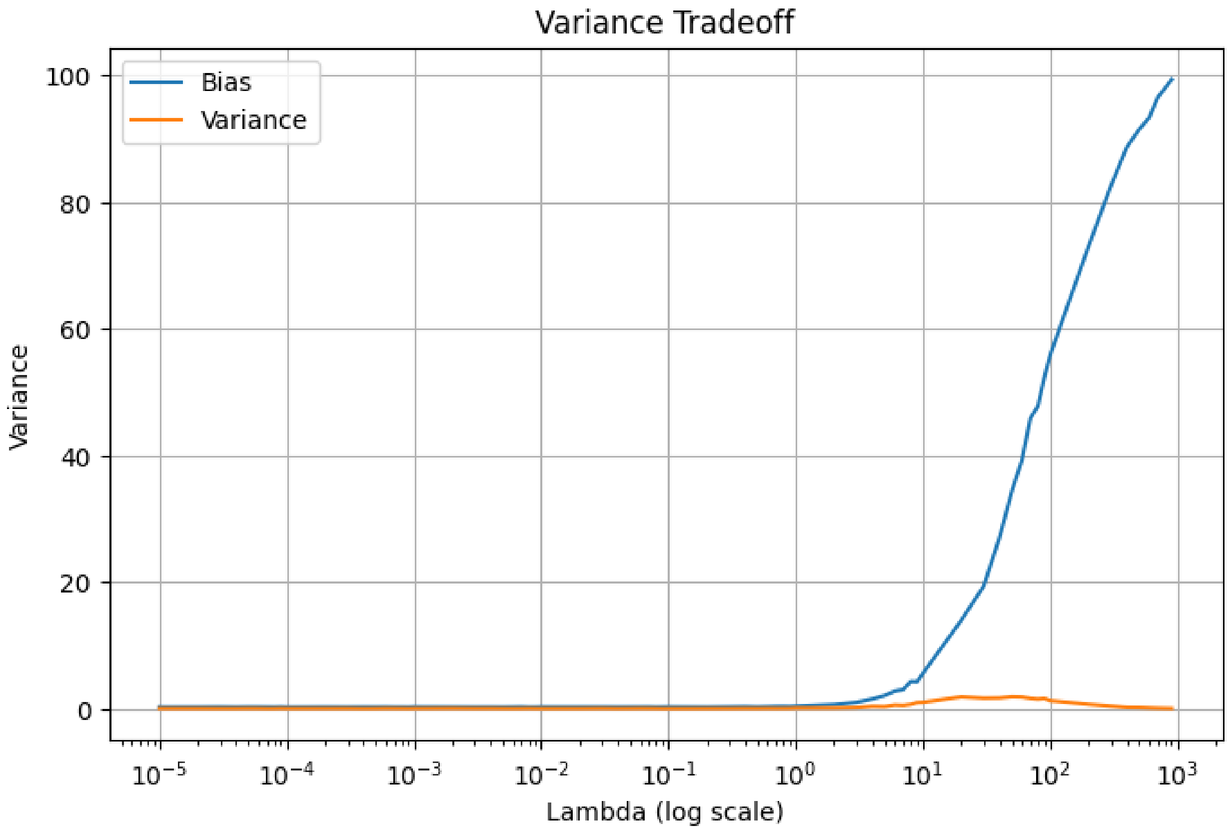
```
plot_coefficients_vs_lambda()
best_lambda = plot_rmse_vs_lambda()
plot_predicted_vs_true(best_lambda)
plot_bias_variance_tradeoff()
```





Validation RMSE vs. Lambda (Best λ = 1.00e-05)
Validation RMSE
Lambda (log scale)
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
1
2
3
4
5
6
7
8


Predicted vs True Values
20
10
0
−10
−20
−30
Predicted Values
−30
−20
−10
0
10
20
True Values


Variance Tradeoff
Bias
Variance
100
80
60
40
20
0
Variance
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Lambda (log scale)

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Analysis

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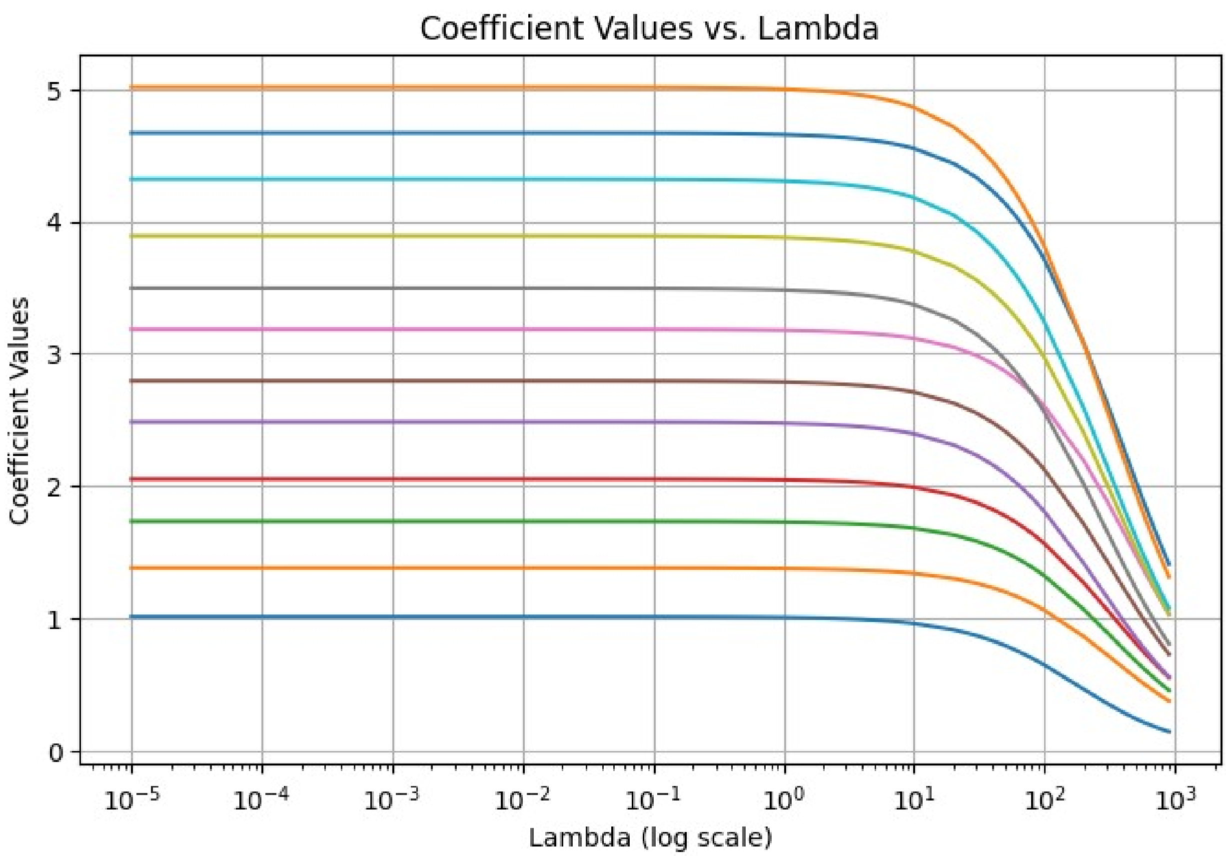
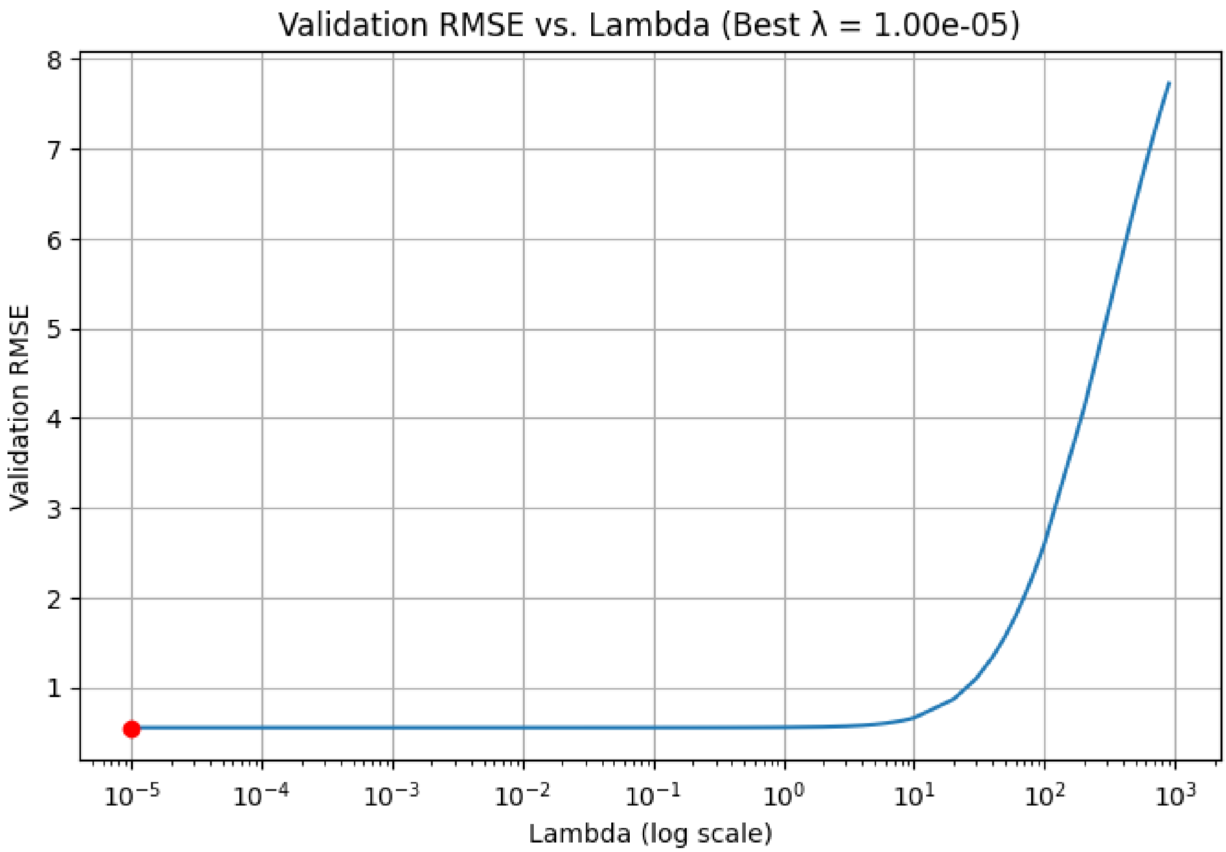
1. **How coefficients behave as λ increases.**



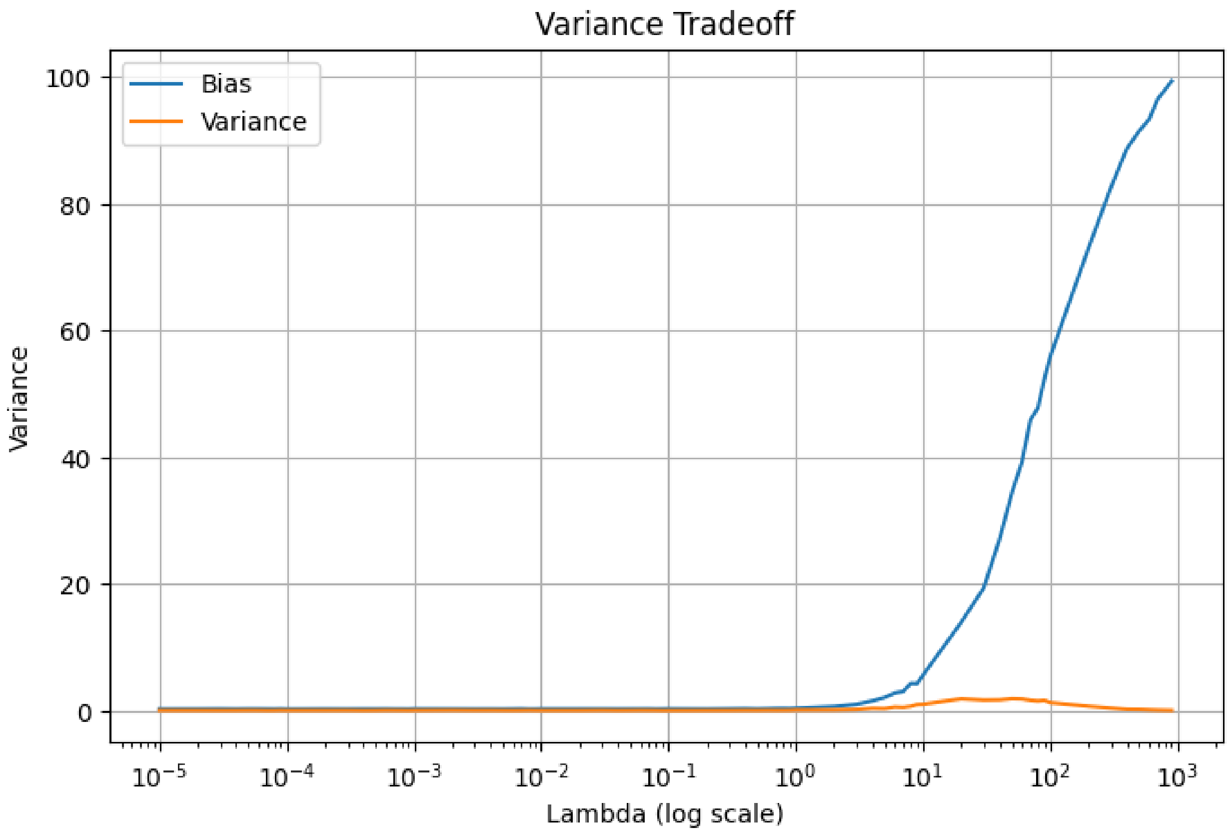
- The small values of λ the coefficients take on their unrestricted values, this indicates that the model behaves like ordinary least squares (OLS), while as λ increases, the coefficients tends to go towards zero. This is an effect of the ridge regression penalty that discourages large coeeficint values to prevent overfitting.
- For the high values of λ, all coefficients approach zero, this indicates that the model is being constrainde which leads to underfitting.

2. **The trade-off between RMSE and λ.**



- At very small λ, RMSE is low, ths indicates that the model fits the data well. As it increases, RMSE remains low for a while, this shows the region where the model generalizes well.However, as λ increases, RMSE starts increasing sharply, indicating underfitting since the model becomes too simple and fails to capture the data complexity.
- The red dot marks indicates an optimal λ, that achieves the lowest RMSE on the validation set. This represents the best balance between bias and variance.

3. **Observations from the bias-variance trade-off plot.**



- For very small values of λ, variance is high, as bias remains low. This could indicate to a very flexible model that might lead to overfitting the training data. However, as λ increases, variance decreases because the model becomes more constrained, reducing sensitivity to noise.
- Bias also starts increasing at higher λ values as the model becomes too simple to capture the underlying pattern.
- The most ideal λ is in the region where variance is controlled without significantly increasing bias.

```
# Please complete this field.
```