

Homework 4

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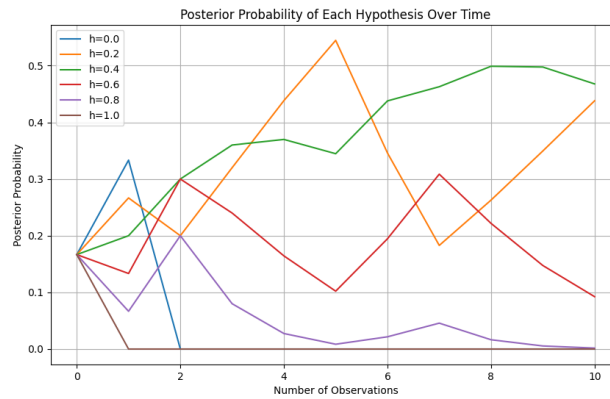
Problem 1: Bayesian Learning

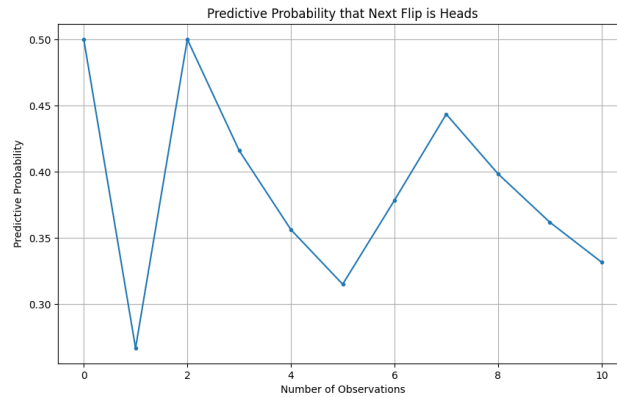
Bayesian Learning calculates the probability of each hypothesis given the data, and makes predictions by weighing all hypothesis by their probabilities

- Will assumed equal prior start for each hypothesis $P(h_i) = \frac{1}{6}$ for all i

Observations

Plots





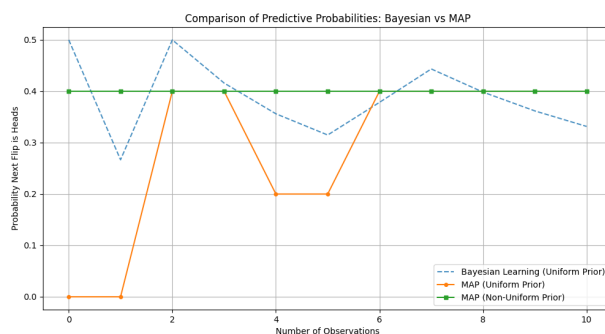
Most likely hypothesis after all observations: $h=0.4$

Insights

The first plot: Posterior Probability plot: the plot shows the evolution of the posterior probabilities over time for each hypothesis. At the start all hypotheses have the same prior however as the observation came in the posterior changes and having the likely of $h=0.4$.

The second plot: Predictive Probability plot: the second plot depicts the predictive probability that the next flip will be heads, calculated after each observation. The model fluctuates at first but it tends to stabilize around 0.4. The highest posterior probability influences the dominance of the hypothesis on predictions. The predictive probability tends to be stable around 0.4 since the posterior of h_3 becomes quite large.

Problem 2: Maximum a Posteriori (MAP) Estimation



Observation and Insights

MAP vs Bayesian

- Smoothness: Bayesian with the blue dotted line smoothly converges to the true bias by averaging all hypotheses; however, the MAP prediction jumps discretely as the most probable hypothesis changes.

- Prior sensitivity: With the uniform prior, MAP predictions start at 0 due to a limitation in tie-breaking, but it adjusts faster as the data comes in. While with a non-uniform prior, a biased MAP initially predicts 0.4 and is constant for that.

Generally, the MAP is computationally efficient; however, it is prone to abrupt changes and prior bias, especially with limited data. It trades off accuracy for simplicity while Bayesian provides robust predictions by combining uncertainty but requires more computation.

Problem 3: Maximum Likelihood (ML) Estimation