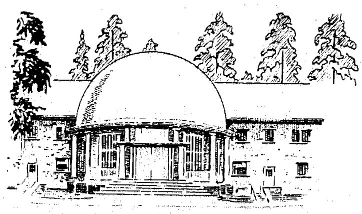

LOWELL OBSERVATORY

BULLETIN NO. 121

VOL. VI | FLAGSTAFF, ARIZONA | NO. 2

PHOTOELECTRIC INVESTIGATION OF MAGNETIC AND SPECTRUM VARIABLE STARS II

Karl D. Rakos*

I. INTRODUCTION

The first part of the photoelectric measurements of magnetic and spectrum-variable stars was published in LOWELL OBSERVATORY BULLETIN No. 117. The present paper contains the results of the observations of the stars HD 224801, Gamma Ari (S), 73 Dra and HD 204411. The following is a discussion of all stars of this type which have been observed up to the present time.

II. MEASURING PROCEDURE

The photoelectric observations described here were carried out with the 42-inch reflector in the autumn of 1960. About 50 measures were taken for each color (UBV system). Each measure represents the average of 4 to 7 rapid comparisons of the star HD 224801 with its comparison star. Table I gives pertinent data for both stars. Table II gives a survey of the observations taken on a single night. The correction for differential extinction was always very small, but not negligible. The comparison star, HD 3 = BD + 44°4450, was also used by Provin (3).

TABLE I List of Observed Stars

HD No.	BD No.	α1900	δ1900	m_{pv}	m_{pg}	Sp
224801	+44°4538	$23^h55^m.7$	+44°42'	6.25	6.25	A0p
003	44 4550	0 00.0	44 40	6.51	6.51	A0
11502	18 243	1 48.0	18 49	4.75	4.75	B9
11503	18 243	1 48.0	18 48	4.83	4.83	A0p
10845	16 196	1 41.2	16 55	6.46	6.74	F0
11326	17 276	1 46.3	17 48	6.73	7.51	G5
196502	74 872	20 32.8	74 37	5.18	5.24	A2p
192907	77 764	20 12.3	77 25	4.40	4.38	B9
204411	48 3390	21 23.3	48 24	5.31	5.39	A3
203245	48 3345	21 16.0	49 06	5.65	5.53	B5
203644	48 3357	21 18.5	48 58	5.87	6.87	K0

The observing technique, the photometer, and other instrumental details, such as the method of reduction, were the same as those used for the observations of other magnetic and spectrum-variable stars (5, 6). The accuracy attained for a single point is usually better than $\pm 0^m.002$ and corresponds therefore to the accuracy which was attained in measurements of other stars of this group. In order to attain even higher accuracy, one would have to shorten the duration of a single comparison measure in order to diminish the effect of the extinction fluctuations. The time constant of the measuring device would have to be shortened and the surface area of the deflection integrated with time and more accurate recording equipment must be used. (For example the incoming voltage could be linearly converted into frequency and digitized.) The greater accuracy of observations thus obtained would be particularily worthwhile for observing rapidly varying stars with periods of less than three hours or for the investigation of short period brightness fluctuations of magnetic and spectrum-variable stars. Such fluctuations as were found in three stars HD 71866, HD 32633 and HD 224801 might be found in others.

III. RESULTS

HD 224801

The star HD 224801 is classified both as a spectrum-variable and as a magnetic star. Twenty years ago it was established that the radial velocity of the star was variable because of the variability of the spectrum, (1). H. W. Babcock found a strongly variable magnetic field in the atmosphere of this star (2). S. S. Provin observed it photoelectrically and found small amplitude periodic brightness variations (3). Finally it was again spectroscopically observed by M. Hack and T. Tamburini (4).

The spectrum of the star is characterized in particular by the very strong Si II and Sr II lines. The hydrogen lines are in comparison strikingly weak. According to the H gamma lines the star should be classified as a B4V star. But since the helium lines are completely absent and a few metallic lines ap-

*Staff member, University Observatory at Graz, Austria. Presently a guest investigator on an NSF grant at Lowell Observatory.

TABLE II Observations of HD 224801

Date J.D.	Phase	Δm Yellow	Δm Blue	Δm UV
2437				
219.8046	0.416	-0.333	-0.443	-0.870
219.8400	.426	.338	.440	.864
220.8671	.700	.335	.447	.864
221.7199	.928	.364	.471	.934
226.8102	.290	.334	.443	.878
227.8220	.560	.341	.452	.872
228.8248	.829	.350	.461	.895
229.8234	.095	.369	.473	.917
232.8309	.900	.358	.471	.923
233.6809	.127	.347	.439	.888
233.7122	.135	.346	.441	.887
233.7670	.150	.342	.441	.878
233.7955	.157	.339	.442	.879
233.8344	.168	.331	.443	.871
234.8233	.432	.332	.457	.878
234.8427	.437	.332	.442	.872
235.6643	.657	.346	.452	.884
235.6816	.662	.342	.455	.889
235.7031	.668	.336	.454	.875
235.7226	.673	.335	.453	.874
235.7427	.678	.333	.447	.871
235.7649	.684	.337	.449	.869
235.7899	.691	.334	.451	.870
236.6358	.917	.372	.470	.935
237.6351	.184	.347	.456	.908
238.6475	.455	.334	.450	.875
239.6239	.716	.331	.440	.870
240.7385	.014	.374	.478	.943
254.6122	.723	.329	.442	.868
257.7898	.574	.339	.459	.880
258.7015	.817	.338	.457	.880
259.7515	.098	.361	.460	.913
260.5917	.323	.330	.444	.871
260.6119	.328	.331	.437	.870
260.6313	.333	.329	.433	.876
260.6528	.339	.332	.443	.880
260.6737	.345	.327	.436	.873
260.6952	.350	.332	.434	.876
260.7153	.356	.330	.431	.869
260.7396	.362	.325	.434	.873
260.7667	.370	.332	.436	.871
261.5965	.591	.342	.460	.878
261.6166	.597	.343	.460	.880
261.6395	.603	.339	.458	.875
262.6319	.868	.358	.470	.907
263.7276	.161	.356	.455	.899
264.7846	.444	.336	.432	-0.858
199.8606	0.083	-0.378	-0.470	---

TABLE II (cont'd)

Observations of HD 224801

Date J.D.	Phase	Δm Yellow	Δm Blue	Δm UV
199.8918	0.092	-0.372	-0.481	---
201.7961	.601	.336	---	---
203.6920	0.108	-0.367	-0.460	-0.938

pear, it should be classified as A0V. Its color index corresponds to that of a later B type star, which is altogether usual in a spectrum-variable star. The spectrum variation involves several lines. Because of the lack of sufficient observations, it is still not possible to state anything definite about the nature of these variations. It seems, however, that they are not simply correlated with the brightness variation as is usually the case. The same is true for the variations in radial velocity; because different lines change quite differently and so far it has not been possible to find any regularity (1, 4).

The star HD 224801 shows the greatest amplitude of light fluctuations in the ultraviolet. These measurements were also used to calculate the epochs of the light maxima. From the twelve dates on which the star was found to be brighter than $-0^m.890$ in the ultraviolet with respect to the comparison star, the time of maximum was determined, with the help of the known period length, to be as follows:

Maximum light = JD 2437240.686

This value was used to determine the phase of the single observations in Table II. Figure 1 shows the light-curves in all three colors and Figure 2 shows the change in color indices with phase. If these are compared with the observations of Provin, it can be seen that the shape of the light-curve has not changed in the last 9 years. This seems to be true in general for the stars in this group. In contrast to the internal accuracy of the single observations; the great scatter of measured points around an average curve is striking. This phenomenon is typical of many stars of this group. One must conclude therefore that there are in fact additional very short period brightness fluctuations. For example the star was observed continuously for several hours on some nights and it was not difficult to establish the presence of a short period of 2^h04^m from these additional fluctuations in brightness. Figure 3 shows the single comparison measurements in all three colors during the night of November 21 to 22, 1960. The length of the period and the amplitude in the blue spectral region proved to be constant within the error of measurement. The observations with the yellow filter show no regular fluctuations. In the ultraviolet the amplitude is variable and the light-curve is shifted in phase with respect to the blue curve. It was possible to pursue these regular fluctuations of the light in the blue region over a longer time, namely during the nights of September 21, October 11 and 25 and

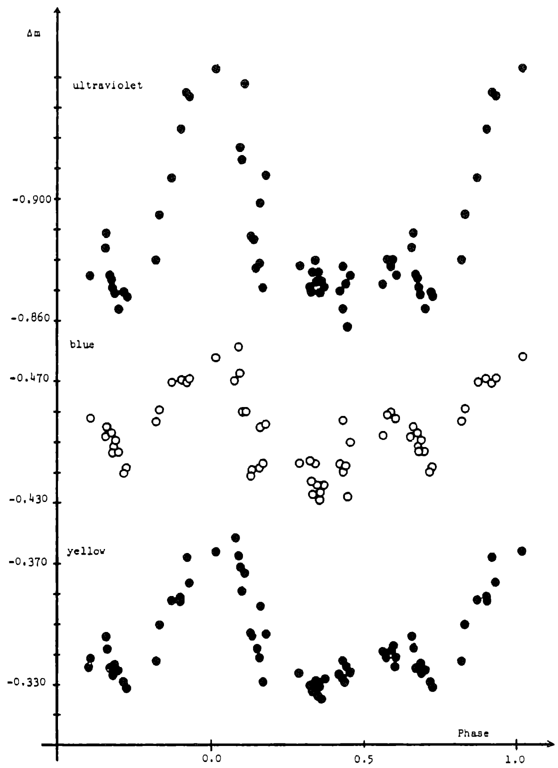

Figure 1. Mean Light Curves for HD 224801.

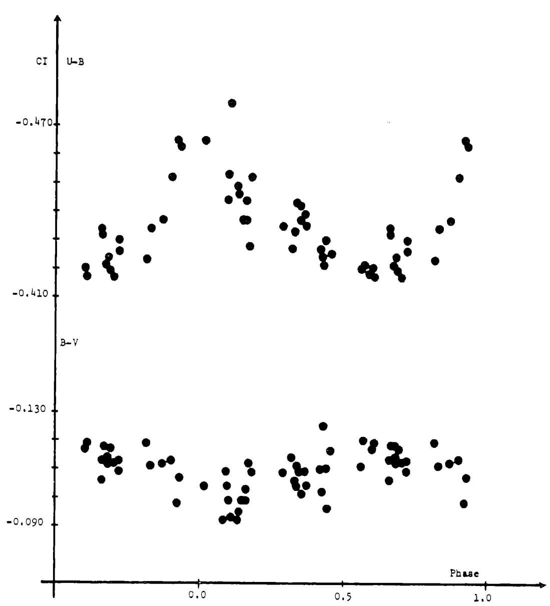

Figure 2. Color Index Curves for HD 224801.

November 21. If all these measurements are reduced according to the period:

$$P = 0.08608 \text{ days}$$
$$\pm 0.00001 \text{ p.e.}$$

and the corresponding position on the primary light-curve considered, the result is a rather well defined average curve (Figure 4). The epoch of the maximum is:

Maximum light = JD 2437199.8877

The amplitude of this short period fluctuation amounts to about $0^{m}_{.}011$ and corresponds to the values which were found for the stars HD 71866 and HD 32633.

It appears that the length of the period of the primary light-curve has changed since the measure-

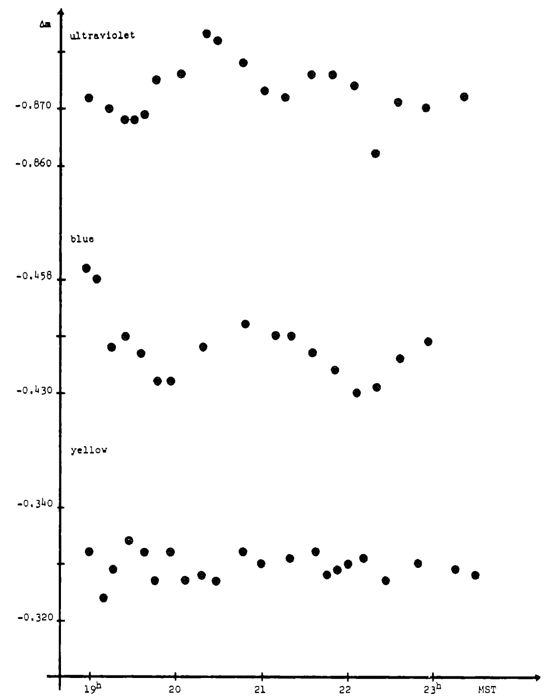

Figure 3. Short Period Variation of HD 224801 on November 21, 1960.

ments of Provin. Either 806 or 807 periods could have passed between the epoch of maximum light observed by Provin and the same epoch observed by the author. Accordingly it is not possible to determine whether the period has lengthened or shortened on the basis of the observations. In view of the fact that the stars HD 71866, HD 32633 and 73 Draconis show a shortening of their periods of the same order of magnitude, it is possible that HD 224801 also shows a shortening of period. Furthermore, observations taken in 1963 support this conclusion. The average length of the period for the time between the observations of Provin and those of the author amounts to:

$$P = 3.7397 \text{ days}$$

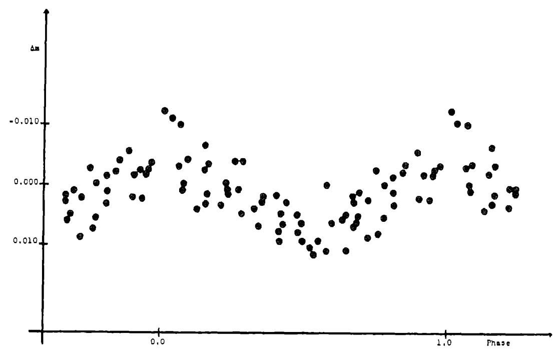

Figure 4. Mean Light Curve of Short Period Variation for HD 224801.

Therefore, the shortening of the period would amount to about 0.000006 days per period.

In summary, the following can be established:
1) The average light-curve shape has remained constant over nine years.
2) The period shows a noticeable change with time.
3) Besides the main brightness variation, the star shows also a very constant short period brightness fluctuation with a period of about two hours.
4) The brightness variation corresponds to the brightness variation of almost all magnetic and spectrum-variable stars and fits nicely in the period-amplitude diagram.

As previously mentioned, it has not been possible to present the changes in radial velocity as a function of the period of the light-curve. However, a rather impressive radial velocity curve can easily be derived from the existing measurements (Figure 5). In order to do this the author averaged the radial velocity of single lines measured from the

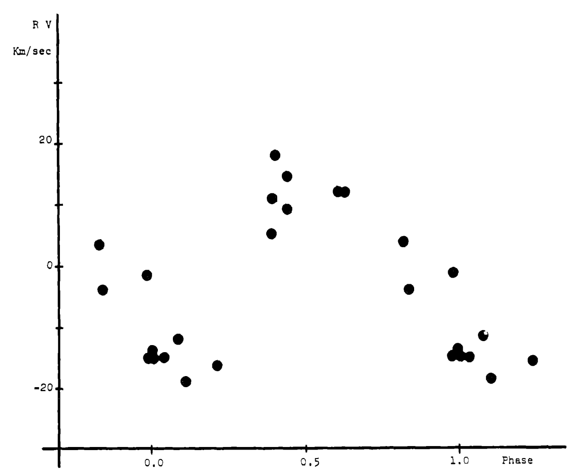

Figure 5. Mean Radial Velocity Curve for HD 224801.

same spectrum taken by Hack and Tamburini and subtracted throughout a constant of −10 Km/sec in order to force agreement with the measures of R. K. Young. The epoch of phase zero was chosen as:

$$\text{JD } 2428084.717$$

and the period of the radial velocity curve amounted to:

$$P = 4.8833 \text{ days}$$

The assumption that the star HD 224801 is a spectroscopic binary seems plausible. However, the radial velocities of single spectral lines are scattered strikingly with respect to their average value. This suggests possible gas turbulence in the tight double star system. Were the assumption to be made that because of the proximity of the components, the rotation of the stars is coupled with their revolution, then the hypothesis of the oblique rotators of A. J. Deutsch is not applicable to these stars since the period of the light-curve is about one day shorter than the period of rotation. It must be mentioned however, that the measurements of the strength of the magnetic field by Babcock show an alternating magnetic field. Unfortunately because of incompleteness of observation, the question of regularity and periodicity is still open.

The star HD 224801 is in every respect a very interesting star. The possibility that it is a spectroscopic binary in which the period of rotation is in no relation to the period of the brightness fluctuations, is enough reason to put it on a well prepared program of spectroscopic and photoelectric observation. In particular, the magnetic variations should be investigated for periodicity.

Gamma Arietis (S)

Gamma Ari is to all appearances a physical double star (HD 11503 and HD 11502) with components separated by about eight seconds. The first star has an A0p spectrum and the second is about $0^{m}.08$ brighter and according to Deutsch has the spectral characteristics of a B9V star (7). First Gutnick and later W. W. Morgan pointed out the peculiarities in the spectrum of Gamma Ari (S) = HD 11503. Deutsch has reported on the strongly variable lines Ca II and Cr II and has found the periodicity of these changes. According to his measurements:

$$\text{Ca II minimum} = \text{Cr II minimum} = \text{JD } 2431782.5 + 2.607 \text{ E}$$

In this respect the star resembles the well known star BD−18°3789. The question of whether a strong magnetic field exists in this star cannot be answered because the lines in the spectrum are too broad.

The present observations by the author are the first photoelectric measures of Gamma Ari. The observations were taken at Lowell Observatory in the fall of 1960 during the course of seven nights. Both components were always measured together and the assumption is made that the second component is of constant brightness. The measures were made in two colors, blue and yellow. The comparison stars were HD 10845 and HD 11326. Table I shows a summary of the general data on all four stars. The brightness of both comparison stars was constant within the range of observational error. In order to avoid too great a difference in brightness, the light from Gamma Ari was cut down by a neutral density filter (NG3 1mm). Table III shows the results of the measurements in the form Gamma Ari+NG3−HD 11326. The average error and the number of single observations from which the brightness differences were calculated are included as well as the time, phase and brightness differences. The value of the period was taken from Deutsch and the elements of the light-curve are:

Minimum light=JD 2437207.85+2.607 E

The light-curves are shown in Figure 6. If it is assumed that the brightness fluctuations are only at-

TABLE III

Observations of Gamma Arietis

Yellow Date J.D.	Phase	Δm	p.e.	N
2437				
207.8505	0.000	-0.080	0.003	5
207.8671	.005	.080	.002	4
211.7839	.509	.082	.003	5
211.8034	.516	.092	.001	5
211.8158	.521	.094	.003	5
211.8325	.527	.100	.001	6
211.8499	.534	.096	.001	6
212.7985	.897	.072	.002	7
247.6818	.279	.080	.002	6
248.6506	.650	.080	.003	6
249.6776	.044	.059	.002	5
250.6865	0.431	-0.077	0.004	5
Blue				
207.8498	0.000	-0.766	0.002	5
207.8664	.005	.766	.002	4
211.7832	.509	.771	.002	5
211.8027	.516	.783	.003	5
211.8152	.521	.781	.001	5
211.8318	.527	.779	.001	6
211.8492	.534	.780	.002	6
212.7978	.897	.760	.001	7
247.6811	.279	.775	.002	6
248.6499	.650	.775	.004	6
249.6783	.044	.753	.003	5
250.6872	0.431	-0.769	0.004	5

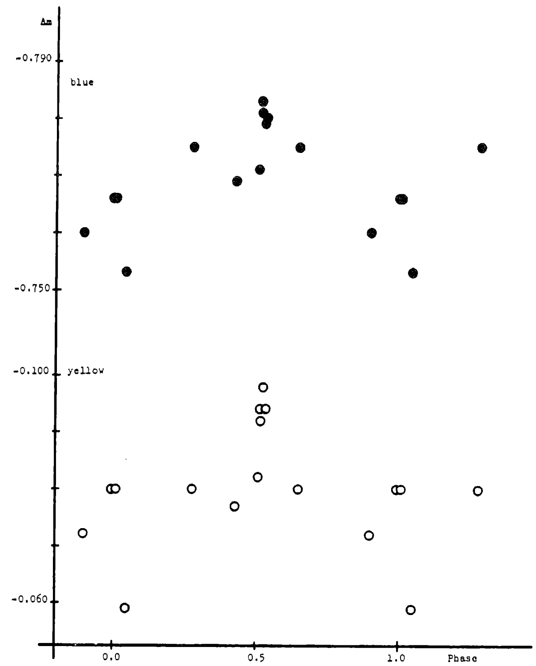

Figure 6. Mean Light Curves for Gamma Ari.

tributed to the Gamma Ari (S), then the true amplitude of brightness change is almost twice as great for the blue and yellow, nearly $0^{m}03$. Unfortunately there are too few observations to enable a better determination of the light variation. On the other hand, the poorly determined period does not permit a comparison between the spectral and photometric observations.

During only one night was the star observed for a longer time, (about two hours). Apparently the short period fluctuations in brightness also exist in this star (Figure 7). It would be very worthwhile to make more extended photoelectric observations of this object in the ultraviolet and if possible the observations should be limited to one of the components. Should Gamma Ari show double star characteristics, the knowledge of the orbit would be of great interest.

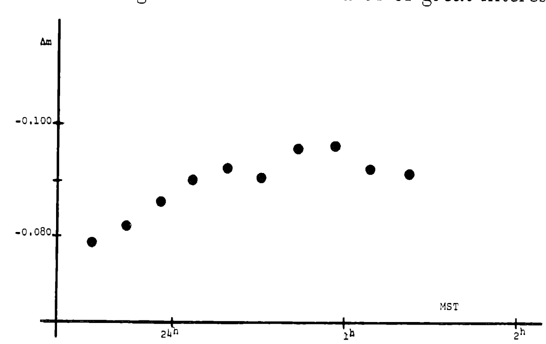

Figure 7. Short Period Variation of Gamma Ari on October 30, 1960.

One could in all probability determine from it the positions of the axes of rotation of both stars and therefore prove the applicability of theory of the oblique rotators in this case.

73 Draconis

The first extensive study of the spectral characteristics of this star were published by Morgan (8). Especially noticeable in the spectrum of the star is the existence of many Cr I lines and the particularly weak Ca II K line. The lines of Eu II, Fe II and Ti II change their intensity with a period of 20.7 days. The star was again spectroscopically observed by G. E. Durham ten years after Morgan's work (9). She confirmed the previously discovered peculiarities in the spectrum and improved the determination of the spectral period to 20.27 days. In addition, it was shown that apparently no innate correlation exists between the changes in line intensity and the small radial velocity changes. It seems that this is true in general. It was possible to show an obvious regularity in the changes in radial velocity for HD 224801. (Figure 5). However, this is not true for 73 Dra. The radial velocity changes have, however, quite a different period than the brightness fluctuations. On the other hand, the period of the brightnesss variation has always been identical to the period of spectral change in all regular spectrum variable stars. The first photoelectric observation of 73 Dra were made by Provin (3). His measures yielded a periodic brightness change with an amplitude of about $0^m.04$, which was synchronized with the changes in the spectrum. The author carried out his photoelectric observations almost 10 years later, in the fall of 1960. Unfortunately due to lack of time, the observations were not extensive enough for a complete light-curve. Luckily the epoch of minimum light can be determined accurately from the observations. There are therefore two epochs of photoelectric measures available:

Provin minimum light = JD 2433908.5
Rakos minimum light = JD 2437210.55

The interval includes 163 cycles and yields a period of:

$$P = 20.2577 \text{ days}$$

The spectroscopic observations of Morgan and Durham yield two epochs for the maxima of the Eu II lines. These were calculated anew from the above observational material.

Morgan maximum Eu II = JD 2426906.8
Durham maximum Eu II = JD 2430678.07

The interval includes 186 cycles and yields a period of:

$$P = 20.2756 \text{ days}$$

It seems that the period of this star has also been shortened. Assuming a linear decrease, it would be 0.000054 days or about 4 seconds per period.

The comparison star used by the author was HD 192907. The general data on both stars can be found in Table I and Table IV contains the brightness differences 73 Dra—HD 192907 for all three colors. These values plotted against time are shown in Figure 8. The existence of short period brightness fluctuations can be determined from the observations of October 3, 1960. In order to really prove the

TABLE IV

Observations of 73 Draconis

Date J.D.	Δm Yellow	Δm Blue	Δm UV
2437			
207.650	0.792	0.902	1.103
208.657	.822	.924	1.124
209.618	.813	.927	1.122
211.730	.838	.926	1.120
211.747	.834	.926	1.124
212.710	.826	.919	1.106
212.730	0.826	0.920	1.108

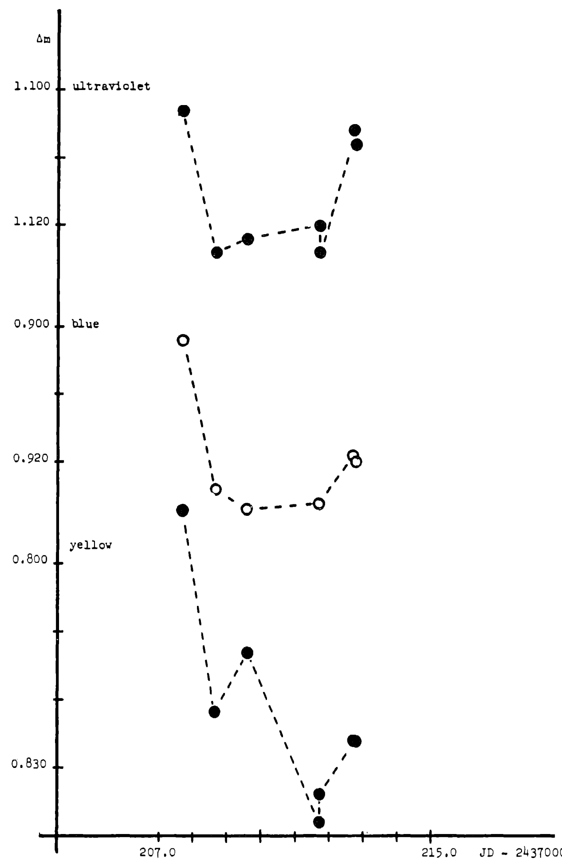

Figure 8. Light Variation for 73 Dra from September 29 to October 5, 1960.

existence of these fluctuations as well as the change in the period length, more extensive observations are needed.

HD 204411

Babcock conjectured that this star has a magnetic field. He classified it as an F0 star (2). Provin made photoelectric observations on eight nights and found an indication of variability (3). The peculiarities of the spectrum involve the lines 4128, 4131, 4171A.

I observed this star on nine nights at the 21-inch reflector of the Lowell Observatory in the fall of 1960 along with the magnetic and spectrum-variable stars. The general data on HD 204411 and on two further stars which were used as comparison stars is shown in Table I. The observations do not suffice to tell whether the brightness changes are periodic. These changes are usually very small and are most pronounced in the ultraviolet. The period is probably shorter than a day. Figure 9 shows the

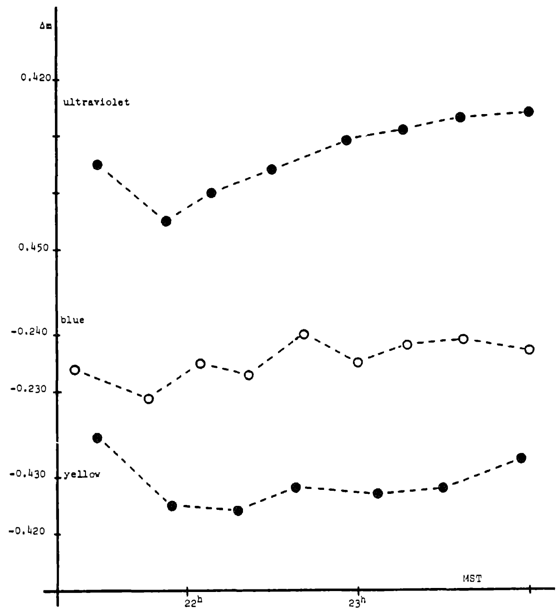

Figure 9. Light Variation for HD 204411 on September 30, 1960.

brightness variations in all three colors on the night of September 30, 1960. Spectroscopic as well as more photometric observations are needed.

IV. DISCUSSION OF THE RESULTS

Of the 6,300 A0 stars given in the Henry Draper Catalog, more than 17 percent are listed as peculiar. In the spectra of most of these stars the lines of the elements Si, Sr, Eu, Cr and others are unusually strong. This is true not only of the A0p stars but also of the less numerous stars between the spectral types B8p to F0p. The spatial distribution of these stars, that lie along main sequence in the Hertzsprung-Russell diagram, is identical to the distribution of normal stars of the same spectral class, or of Baade's Population I. They are found in some open clusters and even as components in double star systems as A. Slettebak recently proved (10). According to color alone in relation to spectral type they are a few tenths of a spectral class earlier than they are classified in the Henry Draper Catalog (11). It is therefore not to be expected that they are significantly different in origin, chemical composition or inner structure from other normal A stars within a distance of 200 pcs from the local star system. Therefore it was convenient to propose the hypothesis that they are normal, rapidly-rotating A stars in which the axis of rotation is pointing towards the observer. As early as the beginning of the century, certain specific periodic changes in the intensity of single spectral lines were found. Since 1947 Babcock has shown the existence of strong magnetic fields in many of these stars and Gutnick, Provin, Stibbs and Jarzebowski, found brightness fluctuations of small amplitude in others. The present status of the research on these stars is best illustrated by a diagram such as that shown in Figure 10. The periodic brightness fluctuations could be established in 25 observed stars. This group also contains 17 stars with periodic changes in their spectra. The number of known spectrum-variables would surely be much larger if more observations were available. Of the stars with known magnetic fields, 10 have been determined as periodic variables. Wherever periodic variations exist, the variations in spectrum, in magnetic field strength and in brightness are synchronized. The light-curves show no double waves as would at first be expected.

The results of the photoelectric light-curves for the 25 stars mentioned are shown in Table V. Periodic brightness changes seem to be a general characteristic of these stars. Up to the present time all observed periodic magnetic or spectroscopic variables

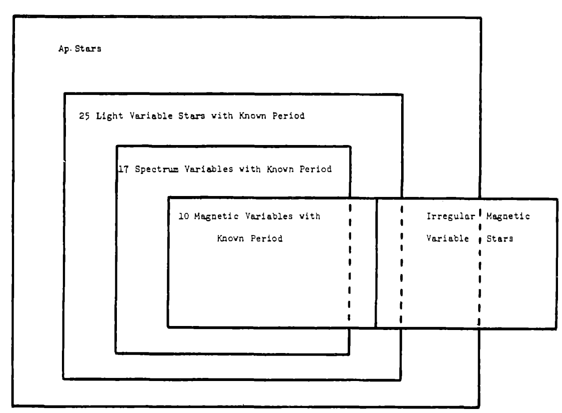

Figure 10. The Distribution of Ap Stars.

TABLE V
Summary of Photometric Results for Known
Spectrum and Magnetic Variables

Name HD No.	Sp. Type	Period Days	Ampl. Yellow Blue UV	Short Period	Magnetic Field Gauss	Spectrum Variable	Line Widths	Observer
γ Ari 11503	A0p var.	2.607	$0^{m}.030$ 0.030 --	present	broad lines	CaII,SrII	1.2	Rakos
ι Cas 15089	A5p var.	1.741551	0.021 0.019 --		broad lines	CaII,SrII	1.3	Rakos
18296	A0p --	1.729	0.016 0.012 --		periodic ? +1000 -1000	----	0.6	Rakos
56 Ari 19832	A0p var.	0.7279019	0.037 0.055 0.070		broad lines	HeI,SiII	3.0	Rakos
25354	A0p var.	3.901	0.032 0.022 0.044		variable	Eu, Cr	0.15- 0.40	Rakos
25823	A0p --	11.94	0.035 0.033 --	present	variable	----	0.43	Rakos
32633	B8p --	6.429	0.024 0.028 0.042	present $1^{h}46^{m}$	periodic +2000 -4000	----	0.3- 0.5	Rakos
34452	A0p var.	2.4660	0.020 0.026 --		present ?	HeI,λ4201	1.2	Rakos
53 Cam 65339	A2p var.	8.00552	0.006 0.010 0.023		periodic +3700 -4350	TiII,MgII	0.15- 1.0	Rakos
71866	A0p var.	6.79760	0.020 0.038 0.025	present $1^{h}34^{m}$	periodic +2000 -2000	EuII	0.26	Rakos
21 Com 108945	A3p var.	1.10	0.013 -- --	present $0^{h}32^{m}$	broad lines	CaII,SrII	1.0	Bahner
ε U Ma 112185	A0p var.	5.0887	0.024 0.024 --		present ?	CaII,CrII	0.6	Provin
112413	A0p var.	5.469	0.045 0.030 --		periodic +1600 -1400	CrII,EuII	0.36	Provin
124224	A0p var.	0.52067	0.070 0.080 0.150		broad lines	HeI,SiII	4.0	Hardie
125248	A0p var.	9.225	-- 0.053 --		periodic +2100 -1900	EuII,Cr	0.18	Stibbs
χ Ser 140160	A0p var.	1.59584	0.018 0.018 --		broad lines	CaII,SrII	1.2	Provin
140728	A0p --	1.30508	0.010 0.014 --		broad lines	----	2.0	Wehlau
153882	A0p --	6.007	0.019 $0^{m}.019$ --		periodic +4000 -4000	----	0.4	Jarzebowski

TABLE V (cont'd)
Summary of Photometric Results for Known Spectrum and Magnetic Variables

Name HD No.	Sp. Type	Period Days	Ampl. Yellow Blue UV	Short Period	Magnetic Field Gauss	Spectrum Variable	Line Widths	Observer
173650	A0p var.	10.1	$0^m.045$ 0.039 --		periodic +600 -400	SrII,EuII	0.2- 0.4	Wehlau
73 Dra 196502	A2p var.	20.2577	0.040 0.024 --	present	variable	CaII,EuII	0.13	Rakos
215038	A0p --	2.0357	0.070 0.070 --		variable	----	0.6	Jarzebowski
215441	A0p var.	9.49	-- 0.140 --		variable	small var.	0.3- 1.0	Jarzebowski
219749	B9p --	2.604	0.018 0.023 0.044		36K Gauss broad lines	----	5.0	Rakos
κ Pis 220825	A2p var.	0.5805	0.011 0.007 --		present ?	SrII,EuII	0.3- 1.2	Rakos
224801	A0p var	3.7397	0.037 0.027 $0^m.065$	present 2^h04^m	variable	EuII	0.8	Rakos

show periodic brightness fluctuations. On the other hand there are stars with periodically varying magnetic fields which are not at the same time spectrum variables.

The above material does not give much information about the distribution of the period lengths, as there are surely selection effects which are not negligible. In any case, there is among these 25 stars an almost exponential decrease in the number of stars with increase in period; only one star has a period of more than 12 days. It is natural to want to correlate this distribution with the shortening of period which occurs in the four stars discussed here and to ascribe it to a time development of these stars. It seems in fact, that the period of those stars having the longer periods decrease much more rapidly than do those with shorter periods (Table VI). The first task now seems to be to prove the reality of the changes in period length and to support their secular character. One could easily imagine a forced change in period length due to a change in amplitude of oscillation, without immediately ascribing it to the lifetime development of the star.

TABLE VI

The Observed Change of Period Length

Star	Period	ΔP/day	Change
HD 15089	1.741551	0	no
56 Ari	0.7279019	0	no
HD 224801	3.7397	-0.13 s	no
HD 32633	6.429	-0.1 s	no
HD 71866	6.79760	-0.06 s	no
73 Dra	20.2577	-0.20 s	--

Besides the normal brightness change of a few hundredths of a magnitude, most of the stars show additional brightness fluctuations over very short intervals. It was possible to show the periodicity of these fluctuations in the case of four stars (fourth column, Table V). It seems reasonable to attribute these short period fluctuations to a free radial oscillation of the star. This hypothesis permits some thought to be given to the average density and absolute magnitude of this star.

It is known that a simple relationship between period length, central density and polytropic index can be deduced for adiabatically constructed stellar models and pulsating stars. Therefore according to A. S. Eddington:

$$P\sqrt{\rho_c} = \frac{0.290}{\sqrt{3\gamma - 4}}$$

This relation was empirically proven to be true over a large range. Despite the fact that the period as well as the density is still very dependent on γ, and that γ, because of the radiation pressure, cannot be considered equal to the ratio of specific heats, Cp/Cv, this relationship is in very good agreement with the measurements of almost all pulsating stars known to us even though they may have very different characteristics. The relation could be applied equally well to the Cepheids, RR Lyrae stars and to low-luminosity intrinsic variables with periods of less than 0.2 day. If the ratio of the pulsation periods of a gas mass of corresponding average density is calculated for the stars of the main sequence, it can immediately be seen that the period length of the stars on the main sequence changes slowly with spectral class (Table VII). This is in definite contrast to the case of the stars discussed here whose known period lengths deviate rather considerably from one another. This is also true even if the star 21 Comae is excluded from consideration as it is probably oscillating in a high harmonic.

TABLE VII

The Calculated Period Length of Radial Pulsation for the Stars on the Main Sequence

Sp	M_{bol}	$M/M_{\odot}$	$R/R_{\odot}$	$P/P_{\odot}$	P
B8	-1.0	4.0	2.9	2.43	2^h45^m
A0	+0.3	2.84	2.27	2.04	2 19
A2	+1.1	2.29	1.98	1.86	2 06
A3	+1.5	2.06	1.83	1.73	1 58
A5	+2.0	1.78	1.64	1.56	1 46

If, with reference to the polytropic index, a similar internal constitution is assumed for these variables as for RR Lyrae stars (since the existence of magnetic fields in RR Lyrae is established), the following empirically-found relationship for RR Lyrae stars is used:

$$P\sqrt{\rho} = 0.056$$

it is possible to derive immediately the period in hours (last column, Table VII).

Ap stars seem to be full of paradoxical surprises. It seems as if in all regular magnetic variables the light minimum falls at the same time as the maximum of positive magnetic field strength. A probable exception is 53 Cam (12) where it is still to be clarified whether the period of this star is not significantly smaller than has so far been assumed, or whether it does not also have a variable period. Determination of periods from measurements of the magnetic fields was made several years before photoelectric observations were made and the values determined at the time differ substantially from those later obtained from photometric measurements.

Up to the present time it has not been possible to find any correlation between the amplitude of the variation in the magnetic field strength and other characteristics of these stars. The same is true for the degree of spectrum variability. However, the amplitude of the light variation shows a dependence upon the length of the period (Figure 11). These amplitudes were entered in the diagram as the arithmetic mean of the amplitudes measured with the blue and yellow filters. The scatter is very small considering the accuracy of measurement. Four stars in the diagram have significantly larger amplitudes than the others. It is not possible to cite a particular difference between these stars and others to which one could attribute their deviation, except perhaps for the star HD 215441 which has an unusually strong magnetic field of 36K Gauss. However, most of the stars appear to form such a close group that the deviation cannot merely be considered as scatter.

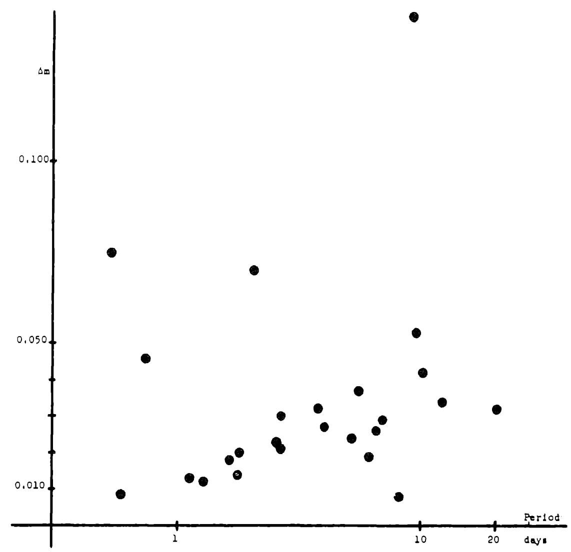

Figure 11. Period-Amplitude Relationship for Stars Given in Table V.

From the existence of a period-amplitude relation and the assumption generally made that the rotational axes of these stars are randomly oriented, it is possible to say, without knowing more about the events on these stars, that the brightness variations must involve a great part of the upper surface of the star. It must be mentioned here that G. Thiessen has found an indication of a preferential direction of the rotational axes of these stars which is perpendicular to the plane of the galaxy (13). An outstanding fact is that the controversial star, 53 Cam, in which it seems that the light minimum corresponds to the maximum of negative magnetic field strength, (opposite to that of all other regular magnetic variables) has a light-curve with a very small amplitude. Should new observations support this behavior, then there is some possibility that the period-amplitude relation is only an effect of the chance selection of stars with their rotational axes pointed towards the observer. It can be seen from Figure 12 that the likelihood of such a selection is small, even if the rotational axes take a preferential direction perpendicular to the galactic plane as suggested by Thiessen. In this diagram all 25 stars are entered according to their galactic coordinates. All the stars which stand out in the period-am-

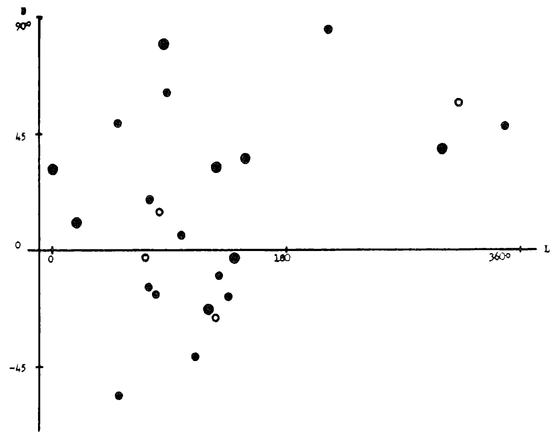

Figure 12. The Galactic Distribution of Stars Given in Table V.

plitude diagram because of too large a light amplitude are marked with open circles. The regular magnetic variables are marked with larger filled circles. The impression of complete randomness of distribution is inescapable. There is therefore good reason to believe that the changes in brightness are produced by most of the upper surface of the star. Thus, one is led to argue in favor of the oscillator theory. The amplitude of light variation according to the oblique rotator hypothesis would be very dependent on the angle of inclination of the axis with respect to the observer.

Without repeating the arguments which other authors have used in favor of one or other models, the star HD 219749 should be mentioned. Judging from the breadth of the lines in its spectrum, one is led to believe that HD 219749 has a significantly shorter period of rotation than the 2.6 days indicated by its light-curve.

September 15, 1963

1. Young, R. K., *Publ. of the David Dunlap Obs., I,* No. 3, 1939.
2. Babcock, H. W., *Ap.J., Suppl. 3,* 1958.
3. Provin, S. S., *Ap.J., 117,* 21, 1953; *Ap.J., 118,* 489, 1953.
4. Hack, M. and Tamburini, T., Contr.dell, *Osservatorio Astr. di Milano Merate, Nuova Serie, 134,* 1958.
5. Rakos, K. D., *Lowell Obs. Bull., V, No.* 117, 1962; *Zs.f.Ap., 56,* 153, 1962.
6. Rakos, K. D., *Acta Physica Austriaca, XVI,* 70, 1963.
7. Deutsch, A. J., *Ap.J., 105,* 283, 1947.
8. Morgan, W. W., *Ap.J., 77,* 77, 1933.
9. Durham, G. E. *Ap.J., 98,* 504, 1943.
10. Slettebak, A., *A.J., 68,* 292, 1963.
11. Abt, H. A. and Golson, J. C., *Ap.J., 136,* 35, 1962.
12. Babcock, H. W., *P.A.S.P., 75,* 74, 1963.
13. Thiessen, G., *Astr. Abhandlungen der Hamburger Sternwarte, V,* Nr. 9, 1961.