

# Combinatorial Optimization Project:

## The $p$ -Center Problem #1

Professor: Michaël Poss

Due date: 8<sup>th</sup> of May, 2017

April 12, 2017

The  $p$ -center problem is a well known combinatorial optimization problem that requires location of  $p$  centers on a given network and allocation of the nodes to the selected centers, so that the maximum of distances between the nodes and their assigned centers is minimum. If the centers can be placed anywhere on the network (vertices as well as edges), the problem is called the absolute  $p$ -center problem while; if they can only be located on the vertices, it is called the vertex restricted  $p$ -center problem.

The focus of this project will be on the vertex restricted  $p$ -center problem, which can be formally defined as follows: Let  $G = (N, E)$  be a given network with vertex set  $N = \{1, \dots, n\}$  and edge set  $E$ . Define  $d_{ij}$  as the length of a shortest path from vertex  $i \in N$  to vertex  $j \in N$  in the given network and  $f(X) = \max_{i \in N} \min_{x \in X} d_{xi}$  for any point set  $X \subset G$ . Then, the vertex restricted  $p$ -center problem is to find a set  $X^* \subseteq N$  with  $|X^*| = p$  so that  $f(X^*) \leq f(X)$  for any  $X \subseteq N$  with  $|X| = p$ .

Several integer programming (IP) formulations for solving this problem are available in the literature. In this project, you will study two IP formulations:

- The formulation proposed by Daskin (1995). You can find this formulation (P1) in page 2992 of Calik and Tansel (2013);
- The formulation proposed by Calik and Tansel (2013). You can find this formulation (P3) in page 2993 of Calik and Tansel (2013).

For this project:

- You must implement these formulations in Julia language combined with JuMP package.
- You must prepare a project report written in L<sup>A</sup>T<sub>E</sub>X. In this report you should describe the mathematical formulations referenced above (explaining the meaning of each variable and constraint set), describe the computational experiments, and discuss the results;
- You must send the report and code to guillaume.duvillie@ulb.ac.be and leave a physical copy at the *Secrétariat des Étudiants du Département d'Informatique* at the 8th floor of the NO building, by 8<sup>th</sup> of May.

Practical informations:

- the ease of use (read/write on the standard input, options, CLI, etc) is taken into consideration in the grading,
- the performance of the implementation is also taken into consideration in the grading.

## References

- Calik H, Tansel BC (2013) Double bound method for solving the  $p$ -center location problem. *Comput Oper Res* 40:2991–2999
- Daskin MS (1995) *Network and discrete location: models, algorithms, and applications*, Wiley, New York