9 Soluzioni esercizi Integrali

- Esercizio (65)

$$\int (nx)^{\frac{1-n}{n}} dx = \frac{1}{n}\int n(nx)^{\frac{1-n}{n}} dx = \frac{1}{n}\frac{(nx)^{\frac{1-n}{n}+1}}{\frac{1-n}{n}+1} + c = \sqrt[n]{nx} + c$$

- Esercizio (66)

$$\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}} dx = \int \frac{x^4-x^2-2}{x^{\frac{2}{3}}} dx = \int \left[x^{\frac{10}{3}} - x^{\frac{4}{3}} - 2x^{\frac{-2}{3}}\right] dx = x^{\frac{13}{3}} - x^{\frac{7}{3}} - 2x^{\frac{1}{3}} + c$$

- Esercizio (67)

$$\int \frac{2x+3}{2x+1} dx = \int \frac{2x+1}{2x+1} dx + \int \frac{2}{2x+1} dx = x + \log|2x+1| + c$$

- Esercizio (68)

$$\int \frac{x}{(x+1)^2} dx = \frac{1}{2}\int \frac{2}{x^2+2x+1} dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+1} dx - \frac{1}{2}\int \frac{2}{x^2+2x+1} dx =$$

$$= \frac{1}{2}\int \frac{2x+2}{x^2+2x+1} dx - \int (x+1)^{-2} dx = \frac{1}{2}\log|x^2+2x+1| + (x+1)^{-1} + c$$

- Esercizio (69)

$$\int \frac{e^x}{e^x-1} dx = \int \frac{D[e^x]}{e^x-1} = \left(\int \frac{1}{y-1} dy\right)_{y=e^x} = \log|e^x-1| + c$$

- Esercizio (70)

$$\int x7^{x^2} dx = \frac{1}{2}\int 2x7^{x^2} dx = \frac{1}{2\log 7}\int 2x7^{x^2} dx = \frac{1}{2\log 7}7^{x^2} + c$$

- Esercizio (71)

$$\int \frac{\sin(\log x)}{x} dx = \int \sin(\log x) D[\log x] dx = \left(\int \sin y dy\right)_{y=\log x} = -\cos(\log x) + c$$

- Esercizio (72)

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = \int \frac{1}{(\sin x \cos x)^2} dx = \int \frac{4}{(2\sin x\cos x)^2} dx = 4\int \frac{1}{\sin^2 2x} dx = -2\cot 2x + c$$

- Esercizio (73)

$$\int \frac{1}{x\sqrt{1+x^2}}dx = \left(\int \frac{1}{\sqrt{t^2-1}t}\frac{2t}{2\sqrt{t^2-1}}\right)_{t=\sqrt{1+x^2}} dt = \left(\int \frac{1}{t^2-1}dt\right)_{t=\sqrt{1+x^2}} = \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$

Ricordando che per risolvere l'ultimo integrale è stato utilizzato il seguente metodo

$$\int \frac{1}{t^2-1}dt = \int \frac{1}{(t-1)(t+1)}dt = \int \left(\frac{A}{t-1} + \frac{B}{t+1}\right)dt$$

Scelti A e B tali che $A(t+1)+B(t-1)=1$, cioè:

$$\begin{cases} A+B=0 \\ A-B=1 \end{cases}$$

cioè:

$$\begin{cases} A=\frac{1}{2} \\ B=-\frac{1}{2} \end{cases}$$

l'integrale sarà quindi:

$$\frac{1}{2}\int \left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt = \frac{1}{2}\log\left|\frac{t-1}{t+1}\right| + c$$

- Esercizio (74)

$$\int \sin x \cos x dx = \frac{1}{2}\int x \sin 2x dx = \frac{1}{2}\int x D[-\cos 2x] dx = -\frac{1}{2}x\cos 2x - \frac{1}{2}\int -\cos 2x dx =$$

$$= -\frac{1}{2}x\cos 2x + \frac{1}{2}\sin 2x + c$$

- Esercizio (75)

$$\int \log^2 x dx = \int \log^2 x D[x] dx = x\log^2 x - \int 2x \log x \frac{1}{x} dx = x\log^2 x - 2\int \log x dx =$$

$$= x\log^2 x - 2\int D[x]\log x dx = x\log^2 x - 2x\log x + 2x + c$$

- Esercizio (76)

$$\int \frac{1}{\sin x}dx = \int \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx = \frac{1}{2}\int \frac{1}{\tan\frac{x}{2}\cos^2 x\frac{x}{2}}dx =$$

$$= \int \frac{D[\tan\frac{x}{2}]}{\tan\frac{x}{2}}dx = \log\left|\tan\frac{x}{2}\right| + c$$

- Esericizio (77)

$$\int x^2 \sin x dx = -\int x^2 D[\cos x] dx = -x^2 \cos x + \int 2x \cos x dx = -x^2 \cos x + 2\int x D[\sin x] dx =$$

$$= \int -x^2 \cos x + 2(x \sin x - \int \sin x dx) = (2 - x^2)\cos x + 2x \sin x + c$$

- Esercizio (78)

$$\int \frac{log(1+x)}{x^2} dx = \int \log(1+x) D\left[-\frac{1}{x}\right] dx = -\frac{\log(1+x)}{x} + \int \frac{1}{x}\frac{1}{1+x} dx = -\frac{\log(1+x)}{x} + \int \frac{1+x-x}{x(1+x)} dx =$$

$$= -\frac{\log(1+x)}{x} + \int \frac{1}{x} - \frac{1}{1+x} dx = -\frac{\log(1+x)}{x} + \log|x| - \log|1+x| + c$$

- Esercizio (79)

$$\int \log(1+x^3)^{x^2} dx = \int x^2 \log(1+x^3) dx = \int D[1+x^3] \log(1+x^3) dx = \frac{1}{3}\left(\int \log(y) dy\right)_{y=1+x^3} =$$

$$= \frac{1}{3}(1+x^3)(\log(1+x^3) - 1) + c$$

Ricordando che

$$\int \log y dy = \int \log y D[y] dy = y \log y - \int y \frac{1}{y} dy = y(\log y - 1) + c$$

- Esercizio (80)

$$\int \frac{3x+1}{x^2-6x+5} dx = \int \frac{3x+1}{(x-5)(x+1)} dx = \int \left[\frac{A}{x-5} + \frac{B}{x+1}\right] dx$$

scelti A e B tali che $3x + 1 = (A + B)x - (A + 5B)$, cioè:

$$\begin{cases} A + B = 3 \\ A + 5B = 1 \end{cases}$$

cioè:

$$\begin{cases} A = 4 \\ B = -1 \end{cases}$$

$$\int \frac{3x+1}{x^2-6x+5} dx = \int \left[\frac{4}{x-5} - \frac{1}{x+1}\right] dx = 4\log|x-5| - \log|x-1| + c$$

- Esercizio (81)

$$\int \frac{x+2}{x^2+2} dx = \int \frac{x}{x^2+2} dx + \int \frac{2}{x^2+2} dx = \frac{1}{2}\int \frac{D[x^2+2]}{x^2+2} dx + \int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2 + 1} dx =$$

$$= \frac{1}{2}\log(x^2+2) + \sqrt{2}\int \frac{D\left[\frac{x}{\sqrt{2}}\right]}{\left(\frac{x}{\sqrt{2}}\right)^2 + 1} dx = \frac{1}{2}\log(x^2+2)\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + c$$

- Esercizio (82)

$$\int \frac{\sqrt{x}}{2+\sqrt{x}}dx = \left(\int \frac{t}{2+t}2tdt\right)_{t=\sqrt{x}} = 2\left(\int \frac{t^2}{2+t}dt\right)_{t=\sqrt{x}}$$

Risolviamo ora

$$\int \frac{t^2}{t+2}dt = \int \frac{t^2-4+4}{t+2}dt = \int \frac{t^2-4}{t+2}dt + 4\int \frac{1}{t+2}dt = \int (t-2)dt + 4\log|t+2| = \frac{t^2}{2} - 2t + 4\log|t+2| + c$$

La soluzione del nostro integrale quindi sarà:

$$\int \frac{\sqrt{x}}{2+\sqrt{x}}dx = 2\left(\frac{t^2}{2} - 2t + 4\log|t+2|\right)_{t=\sqrt{x}} + c = x - 4\sqrt{x} + 8\log(\sqrt{x}+2) + c$$

- Esercizio (83)

$$\int \sqrt{2^x-1}dx$$

Utilizzando la sostituzione di variabile

$$t = \sqrt{2^x-1} \iff t^2 = 2^x - 1 \iff 2^x = 1 + t^2 \iff x = \log_2(1+t^2)$$

avremo quindi

$$\varphi'(t) = \frac{1}{\log 2}\frac{2t}{1+t^2}$$

e il nostro integrale sarà quindi

$$\int \sqrt{2^x-1}dx = \frac{2}{\log 2}\left(\int \frac{t^2}{1+t^2}dt\right)_{t=\sqrt{2^x-1}} = \frac{2}{\log 2}\left(\int \frac{t^2-1+1}{1+t^2}dt\right)_{t=\sqrt{2^x-1}} =$$

$$= \frac{2}{\log 2}\left(\int 1 - \frac{1}{1+t^2}dt\right)_{t=\sqrt{2^x-1}} = \frac{2}{\log 2}\left(\sqrt{2^x-1} - \arctan\sqrt{2^x-1}\right) + c$$

- Esercizio (84)

$$\int \frac{x^2+1}{x-1}dx = \int \frac{x^2-1}{x-1}dx + \int \frac{2}{x-1}dx = \int (x+1)dx + \int \frac{2}{x-1}dx = \frac{x^2}{2} + x + 2\log|x-1| + c$$

- Esercizio (85)

$$\int 2e^{\sin^2 x}\frac{\sin x}{\sec x}dx = \int e^{\sin^2 x}D[\sin^2 x]dx = e^{\sin^2 x} + c$$