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CSC 421 Artificial Intelligence
Assignment #3

1. *Using propositional resolution, show the following propositional sentence is unsatisfiable.*

$$(p \mid q \mid -r) \mathbin{\&} ((-r \mid q \mid p) \;{->}\; ((r \mid q) \mathbin{\&} -q \mathbin{\&} -p)$$

a) Convert this sentence into clausal form

$(p \mid q \mid -r) \mathbin{\&} ((-r \mid q \mid p) \;{->}\; ((r \mid q) \mathbin{\&} -q \mathbin{\&} -p)$
$(p \vee q \vee \neg r) \wedge ((\neg r \vee q \vee p) \Rightarrow ((r \vee q) \wedge \neg q \wedge \neg p)$

I $(p \vee q \vee \neg r) \wedge (\neg(\neg r \vee q \vee p) \vee ((r \vee q) \wedge \neg q \wedge \neg p)$

N $(p \vee q \vee \neg r) \wedge (\neg\neg r \wedge \neg q \wedge \neg p) \vee ((r \vee q) \wedge \neg q \wedge \neg p)$
$(p \vee q \vee \neg r) \wedge (r \wedge \neg q \wedge \neg p) \vee ((r \vee q) \wedge \neg q \wedge \neg p)$

D $(p \vee q \vee \neg r) \wedge$
$(r \vee \neg q \vee \neg p) \vee (r \vee q))$
$((r \vee \neg q \vee \neg p) \vee \neg q))$
$((r \vee \neg q \vee \neg p) \vee \neg p))$

$(p \vee q \vee \neg r) \wedge$
$(r \vee q)) \wedge$
$(\neg q \vee (r \vee q)) \wedge$
$(\neg p \vee (r \vee q)) \wedge$
$(r \vee \neg q) \wedge$
$(\neg q) \wedge$
$(\neg p \vee \neg q) \wedge$
$(r \vee \neg p) \wedge$
$(\neg q \vee \neg p) \wedge$
$(\neg p)$

O $\{p, q, \neg r\}$
$\{r, q\}$
$\{\neg q, r, q\}$
$\{\neg p, r, q\}$
$\{r, \neg q\}$
$\{\neg q\}$
$\{\neg p, \neg q\}$
$\{r, \neg p\}$
$\{\neg q, \neg p\}$
$\{\neg p\}$

b) Resolution

1. $\{p, q, \neg r\}$
2. $\{r, q\}$
3. $\{\neg q, r, q\}$
4. $\{\neg p, r, q\}$
5. $\{r, \neg q\}$
6. $\{\neg q\}$
7. $\{\neg p, \neg q\}$
8. $\{r, \neg p\}$
9. $\{\neg q, \neg p\}$
10. $\{p\}$ Negated Conclusion
11. $\{q\}$ 9, 10
12. $\{\}$ 6, 11

2. Every horse can outrun every dog.
Some greyhounds can outrun every rabbit.
Show that every horse can outrun every rabbit.

a) FOL Formulation

$\forall x.\ \forall y.\ (Horse(x) \wedge Dog(y) \Rightarrow Faster(x, y)$
$\exists y.\ (Greyhound(y) \wedge \forall z.(Rabbit(z) \Rightarrow Faster(y, z)))$
$\forall y.\ (Greyhound(y) \Rightarrow Dog(y))$ $(Background\ knowledge)$
$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge Faster(y, z) \Rightarrow Faster(x, z))$ $(Background\ knowledge)$
$\neg \forall x.\ \forall y.\ (Horse(x) \wedge Rabbit(y) \Rightarrow Faster(x, y))$ $(Negated\ conclusion)$

b) Convert to clausal form.

I $\forall x.\ \forall y.\ (Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$\exists y.\ (Greyhound(y) \wedge \forall z.(\neg Rabbit(z) \vee Faster(y, z)))$
$\forall y.\ (\neg Greyhound(y) \vee Dog(y))$

$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$\neg \forall x.\ \forall y.\ (Horse(x) \wedge (\neg Rabbit(y) \vee Faster(x, y)))$

N $\forall x.\ \forall y.\ (Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$\exists y.\ (Greyhound(y) \wedge \forall z.(\neg Rabbit(z) \vee Faster(y, z)))$
$\forall y.\ (\neg Greyhound(y) \vee Dog(y))$
$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$\forall x.\ \forall y.\ (\neg Horse(x) \wedge \neg(\neg Rabbit(y) \vee Faster(x, y)))$

$\forall x.\ \forall y.\ (Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$\exists y.\ (Greyhound(y) \wedge \forall z.(\neg Rabbit(z) \vee Faster(y, z)))$
$\forall y.\ (\neg Greyhound(y) \vee Dog(y))$
$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$\forall x.\ \forall y.\ (\neg Horse(x) \vee (\neg\neg Rabbit(y) \wedge \neg Faster(x, y)))$

$\forall x.\ \forall y.\ (Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$\exists y.\ (Greyhound(y) \wedge \forall z.(\neg Rabbit(z) \vee Faster(y, z)))$
$\forall y.\ (\neg Greyhound(y) \vee Dog(y))$
$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$\forall x.\ \forall y.\ (\neg Horse(x) \vee (Rabbit(y) \wedge \neg Faster(x, y)))$

S $\forall x.\ \forall y.\ (Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$\exists y.\ (Greyhound(y) \wedge \forall z.(\neg Rabbit(z) \vee Faster(y, z)))$
$\forall y.\ (\neg Greyhound(y) \vee Dog(y))$
$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$\forall x.\ \forall z.\ (\neg Horse(x) \vee (Rabbit(z) \wedge \neg Faster(x, z)))$

E $\forall x.\ \forall y.\ (Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$(Greyhound(Rocky) \wedge \forall z.(\neg Rabbit(z) \vee Faster(Rocky, z)))$
$\forall y.\ (\neg Greyhound(y) \vee Dog(y))$
$\forall x.\ \forall y.\ \forall z.\ (Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$\forall x.\ \forall z.\ (\neg Horse(x) \vee (Rabbit(z) \wedge \neg Faster(x, z)))$

A $(Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$
$(Greyhound(Rocky) \wedge (\neg Rabbit(z) \vee Faster(Rocky, z)))$
$(\neg Greyhound(y) \vee Dog(y))$
$(Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$(\neg Horse(x) \vee (Rabbit(z) \wedge \neg Faster(x, z)))$

D $(Horse(x) \wedge (\neg Dog(y) \vee Faster(x, y)))$

$(Greyhound(Rocky) \wedge (\neg Rabbit(z) \vee Faster(Rocky, z)))$
$(\neg Greyhound(y) \vee Dog(y))$
$(Faster(x, y) \wedge (\neg Faster(y, z) \vee Faster(x, z)))$
$((\neg Horse(x) \vee Rabbit(z)) \wedge (\neg Horse(x) \vee \neg Faster(x, z))$

O $\{Horse(x)\}$
$\{\neg Dog(y), Faster(x, y)\}$
$\{Greyhound(Rocky)\}$
$\{\neg Rabbit(z), Faster(Rocky, z)\}$
$\{\neg Greyhound(y), Dog(y)\}$
$\{Faster(x, y)\}$
$\{\neg Faster(y, z), Faster(x, z)\}$
$\{\neg Horse(x), Rabbit(z)\}$
$\{\neg Horse(x), \neg Faster(x, z)\}$

c) Resolution

1. $\{Horse(x)\}$
2. $\{\neg Dog(y), Faster(x, y)\}$
3. $\{Greyhound(Rocky)\}$
4. $\{\neg Rabbit(z), Faster(Rocky, z)\}$
5. $\{\neg Greyhound(y), Dog(y)\}$
6. $\{Faster(x, y)\}$
7. $\{\neg Faster(y, z), Faster(x, z)\}$
8. $\{\neg Horse(x), Rabbit(z)\}$
9. $\{\neg Horse(x), \neg Faster(x, z)\}$
10. $\{Rabbit(z)\}$ 1, 8
11. $\{\neg Faster(x, z)\}$ 1, 9
12. $\{\neg Greyhound(y), Faster(x, y)\}$ 2, 5
13. $\{\neg Horse(x), \neg Faster(y, z)\}$ 7, 9
14. $\{\neg Rabbit(z), Faster(x, z)\}$ 4, 7 using unifier $y \leftarrow Rocky$ and transitivity

Want to show that all horses are faster than all rabbits:

$\forall x. \forall y. (Horse(x) \wedge Rabbit(z) \Rightarrow Faster(x, z)$

$\{Horse(x)\}$
$\{\neg Rabbit(z), Faster(x, z)\}$

This is shown from lines 1, 14 of our resolution.