

### Problem 6.3

Describe a regression model that relates univariate binary discrete data  $x \in \{0, 1\}$  to a univariate continuous world state  $w \in [-\infty, \infty]$ . Use a generative formulation in which  $P(x|w)$  and  $P(w)$  are modeled.

A risk formula (ie. credit score) could be a similar use case to this. A good or bad score could map to any given range of values  $[-\infty, \infty]$  based on how the risk score is calculated.

The problem is structurally similar to the last but with a change in the denominator.

In total, the probability distribution over the data  $x$  could use the Bernoulli distribution where:  $P(x|w, \theta) = \text{Bern}[\lambda]$ .

A prior distribution would need to be chosen. One such could be a univariate normal with variance  $\sigma^2$  and mean  $\mu$ :  $P(w) = \text{Norm}_w[\sigma^2, \mu]$ .

The parameters  $\theta$  would need to be fit from training data examples for each class  $\{x_i, w_i\}_{i=1}^l$ . It may be best to use a sort of logit function ( $\log\left(\frac{p}{1-p}\right)$ ) for this task since we would be mapping the predictor variables back into some linear relationship.

The large change in Baye's rule is in the denominator as  $w \in [-\infty, \infty]$ :

$$P(w|x) = \frac{P(x|w)P(w)}{\int_{-\infty}^{\infty} P(x|w)P(w)dw}$$