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Homework 3: Probability

Problem 2.1 Give a real-world example of a joint distribution P r(x, y) where x is discrete and y is continuous.

Answer:

The discrete value can be the number of sunny days in a year.

The continuous value can be the amount of rainfall measured in cm.

Thus, the join probability can be constructed as:

the probability of x sunny days AND the probability of y cm of rainfall in a year.

This can be represented as:

$$P(x \cap y) = P(x) * P(y)$$

Problem 2.4 In my pocket there are two coins. Coin 1 is unbiased, so the likelihood P r(h = 1|c = 1) of getting heads is 0.5 and the likelihood P r(h = 0|c = 1) of getting tails is also 0.5. Coin 2 is biased, so the likelihood P r(h = 1|c = 2) of getting heads is 0.8 and the likelihood P r(h = 0|c = 2) of getting tails is 0.2. I reach into my pocket and draw one of the coins at random. There is an equal prior probability I might have picked either coin. I flip the coin and observe a head. Use Bayes' rule to compute the posterior probability that I chose coin 2.

Answer:

Probability of drawing the Fair or Bias coin: $P(F) = 0.5,\ P(B) = 0.5$

Probabilities of Heads and Tails for the Fair coin (coin 1): $P(H|F) = 0.5,\ P(T|F) = 0.5$
Probabilities of Heads and Tails for the Bias coin (coin 2): $P(H|B) = 0.8,\ P(T|B) = 0.2$

Applying Bayes theorem and given the coin flip is heads:

$$P(B|H) = \frac{P(H|B)P(B)}{P(H)} = \frac{P(H|B)P(B)}{P(H|B)P(B) + P(H|F)P(F)}$$

$$= \frac{(0.8)(0.5)}{\big((0.8)(0.5)\big) + ((0.5)(0.5))} = \frac{0.4}{0.4 + 0.25} = \frac{0.4}{0.65} = 0.615 = 61.5\%$$

Problem 2.5 If variables x and y are independent and variables x and z are independent, does it follow that variables y and z are independent?

Answer:

It is easiest to show this proof by example. Imagine that I have in possession both a fair coin and a bag of marbles, the color of which are either red or blue. Say *x* is the flipping of a fair coin while *y* and *z* are drawing red or blue marbles respectively from a bag. The coin flip is independent of both drawing a red or of drawing a blue marble. However, if a red marble is drawn from the bag (without replacement), this would influence the probability of what could be drawn the next time. Thus, *y* and *z* are not independent.