Name: Brandon Botzer

Date: 1/11/2024

Class: Penn State – AI 879

**Homework 3: Probability**

**Problem 2.1** Give a real-world example of a joint distribution P r(x, y) where x is discrete  
and y is continuous.

Answer:

The discrete value can be the number of sunny days in a year.

The continuous value can be the amount of rainfall measured in cm.

Thus, the join probability can be constructed as:

the probability of x sunny days AND the probability of y cm of rainfall in a year.

This can be represented as:

**Problem 2.4** In my pocket there are two coins. Coin 1 is unbiased, so the likelihood  
P r(h = 1|c = 1) of getting heads is 0.5 and the likelihood P r(h = 0|c = 1) of getting  
tails is also 0.5. Coin 2 is biased, so the likelihood P r(h = 1|c = 2) of getting heads is  
0.8 and the likelihood P r(h = 0|c = 2) of getting tails is 0.2. I reach into my pocket and  
draw one of the coins at random. There is an equal prior probability I might have picked  
either coin. I flip the coin and observe a head. Use Bayes’ rule to compute the posterior  
probability that I chose coin 2.

Answer:

Probability of drawing the Fair or Bias coin:

Probabilities of Heads and Tails for the Fair coin (coin 1): Probabilities of Heads and Tails for the Bias coin (coin 2):

Applying Bayes theorem and given the coin flip is heads:

**Problem 2.5** If variables x and y are independent and variables x and z are independent,  
does it follow that variables y and z are independent?

Answer:

It is easiest to show this proof by example. Imagine that I have in possession both a fair coin and a bag of marbles, the color of which are either red or blue. Say *x* is the flipping of a fair coin while *y* and *z* are drawing red or blue marbles respectively from a bag. The coin flip is independent of both drawing a red or of drawing a blue marble. However, if a red marble is drawn from the bag (without replacement), this would influence the probability of what could be drawn the next time. Thus, *y* and *z* are not independent.