Population demography_HW4

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Homework Q1: life table

```
#set data as data frame
barnacle <- data.frame(Age_x = 0:9,
                       lx = c(1.0000000, 0.0000620, 0.0000340, 0.0000200, 0.0000155, 0.000011
0, 0.0000065, 0.0000020, 0.0000020, 0),
                       mx = c(0, 4600, 8700, 11600, 12700, 12700, 12700, 12700, 12700, 0))
barnacle
```

```
##    Age_x       lx    mx
## 1      0 1.00e+00     0
## 2      1 6.20e-05  4600
## 3      2 3.40e-05  8700
## 4      3 2.00e-05 11600
## 5      4 1.55e-05 12700
## 6      5 1.10e-05 12700
## 7      6 6.50e-06 12700
## 8      7 2.00e-06 12700
## 9      8 2.00e-06 12700
## 10     9 0.00e+00     0
```

Calculation

```
library(tidyverse)
```

```
## ── Attaching packages ─────────────────────────────────────── tidyverse 1.3.2 ──
## ✓ ggplot2 3.3.6      ✓ purrr   0.3.4
## ✓ tibble  3.1.8      ✓ dplyr   1.0.10
## ✓ tidyr   1.2.1      ✓ stringr 1.4.1
## ✓ readr   2.1.2      ✓ forcats 0.5.2
## ── Conflicts ────────────────────────────────────────── tidyverse_conflicts() ──
## ✕ dplyr::filter() masks stats::filter()
## ✕ dplyr::lag()    masks stats::lag()
```

```
#lx * mx -> Σ (lx*mx) = Ro
a <- barnacle$lx*barnacle$mx
barnacle <- barnacle %>% data.frame(lx.mx = a)
```

```
#x*lx*mx -> Σx*lx*mx/ΣRo = Σx*lx*mx/Σ(lx*mx) = G
b <- barnacle$Age_x*barnacle$lx*barnacle$mx
barnacle <- barnacle %>% data.frame(x.lx.mx = b)
```

```r
#lx*mx*e^-rx
c <- paste(barnacle$lx*barnacle$mx, 'e^-', barnacle$Age_x, 'r')
barnacle <- barnacle %>% data.frame(lx.mx.e_rx = c)
```

```r
#Lx = (lx + lx+1)/2  每個年齡區間的平均存活率
Lx <- rep(0,9)
for (i in 1:9){
  Lx[i] <- (barnacle$lx[i] + barnacle$lx[i+1])/2
}
Lx
```

```
## [1] 0.50003100 0.00004800 0.00002700 0.00001775 0.00001325 0.00000875 0.00000425
## [8] 0.00000200 0.00000100
```

```r
#ex = (Lx + Lx+1 + Lx+2 + … + Lmax)/lx (life expectancy)
ex <- NULL
for (i in 1:length(Lx)){
  ex[i] <- sum(Lx[i:length(Lx)])/barnacle$lx[i]
}
ex
```

```
## [1] 0.500153 1.967742 2.176471 2.350000 1.887097 1.454545 1.115385 1.500000
## [9] 0.500000
```

```r
#Ro = Σ (lx*mx) (net reproductive rate)
Ro <- sum(a)
Ro
```

```
## [1] 1.2829
```

```r
#G = Σx*lx*mx/ΣRo = Σx*lx*mx/Σ(lx*mx)  (generation time)
G <- sum(b)/sum(a)
G
```

```
## [1] 3.067269
```

```r
#估計r (approximate r)
#r~lnRo/G
#r~(ln(Σ lx*mx))/(Σx*lx*mx/Σ(lx*mx))
r <- log(sum(a))/(sum(b)/sum(a))
r
```

```
## [1] 0.08121984
```

Approximate r ~ 0.08121984

```r
#true r
#function for r
lx.mx.e_rx <- NULL
true.r <- function(r){
  for (i in 1:9){
    lx.mx.e_rx[i] <- barnacle$lx[i]*barnacle$mx[i]*exp(-r*barnacle$Age_x[i])
  }
  return(sum(lx.mx.e_rx)-1)
}

#解方程式
Tr <- uniroot(true.r, lower = 0.08, upper = 0.09, tol = 0.0000000000001) #tol= accuracy
# true r = 0.08471204
Tr$root
```

```
## [1] 0.08471204
```

true r = 0.08471204

```r
#lx*mx*e^-rx
#plug approximate r in it
lme.r <- barnacle$lx*barnacle$mx*exp(-r*barnacle$Age_x)
barnacle <- barnacle %>% data.frame(lme.r = lme.r)
#plug true r in it
lme.tr <- barnacle$lx*barnacle$mx*exp(-Tr$root*barnacle$Age_x)
barnacle <- barnacle %>% data.frame(lme.tr = lme.tr)
```

```r
#Vx (reproductive value) (Σ ly.my.e^-ry)/(e^-rx*lx)
#plug approximate r in it
Vx <- NULL
for (i in 1:length(lme.r)){
  Vx[i] <- sum(lme.r[i:length(lme.r)])/(barnacle$lx[i]*exp(-r*barnacle$Age_x[i]))
}
Vx
```

```
##  [1]     1.00991 17667.06693 25844.26687 31611.21357 28005.61108 23391.76626
##  [7] 19624.65874 24409.28541 12700.00000         NaN
```

```r
#plug true r in it
Vx.tr <- NULL
for (i in 1:length(lme.tr)){
  Vx.tr[i] <- round(sum(lme.tr[i:length(lme.tr)])/(barnacle$lx[i]*exp(-Tr$root*barnacle$Age_x
[i])), digits = 4)
}
Vx.tr <- sprintf("%0.4f", Vx.tr)
Vx.tr
```

```
##  [1] "1.0000"     "17554.8969" "25712.0502" "31477.1605" "27915.3203"
##  [6] "23335.1225" "19588.9788" "24368.4655" "12700.0000" "NaN"
```

```
#年齡區間dataframe
barnacle_Age.range <- data.frame(Age_range = c('0-1', '1-2', '2-3', '3-4', '4-5', '5-6', '6-7', '7-8', '8-9'),
                                 Lx = Lx,
                                 ex = ex,
                                 vx = Vx[1:9],
                                 Vx.true.r = Vx.tr[1:9])
```

My dataframe

```
barnacle
```

```
##    Age_x       lx    mx   lx.mx x.lx.mx      lx.mx.e_rx      lme.r     lme.tr
## 1      0 1.00e+00     0 0.00000  0.0000       0 e^- 0 r 0.00000000 0.00000000
## 2      1 6.20e-05  4600 0.28520  0.2852  0.2852 e^- 1 r 0.26295183 0.26203515
## 3      2 3.40e-05  8700 0.29580  0.5916  0.2958 e^- 2 r 0.25144993 0.24969982
## 4      3 2.00e-05 11600 0.23200  0.6960   0.232 e^- 3 r 0.18183103 0.17993600
## 5      4 1.55e-05 12700 0.19685  0.7874 0.19685 e^- 4 r 0.14224667 0.14027347
## 6      5 1.10e-05 12700 0.13970  0.6985  0.1397 e^- 5 r 0.09307430 0.09146323
## 7      6 6.50e-06 12700 0.08255  0.4953 0.08255 e^- 6 r 0.05070807 0.04965663
## 8      7 2.00e-06 12700 0.02540  0.1778  0.0254 e^- 7 r 0.01438535 0.01403796
## 9      8 2.00e-06 12700 0.02540  0.2032  0.0254 e^- 8 r 0.01326316 0.01289775
## 10     9 0.00e+00     0 0.00000  0.0000       0 e^- 9 r 0.00000000 0.00000000
```

```
barnacle_Age.range
```

```
##   Age_range         Lx       ex          vx  Vx.true.r
## 1       0-1 0.50003100 0.500153     1.00991     1.0000
## 2       1-2 0.00004800 1.967742 17667.06693 17554.8969
## 3       2-3 0.00002700 2.176471 25844.26687 25712.0502
## 4       3-4 0.00001775 2.350000 31611.21357 31477.1605
## 5       4-5 0.00001325 1.887097 28005.61108 27915.3203
## 6       5-6 0.00000875 1.454545 23391.76626 23335.1225
## 7       6-7 0.00000425 1.115385 19624.65874 19588.9788
## 8       7-8 0.00000200 1.500000 24409.28541 24368.4655
## 9       8-9 0.00000100 0.500000 12700.00000 12700.0000
```