Final Exam
CS 1103 Computer Science I Honors
Fall 2016

KEY

Friday December 16, 2016

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Before reading further, please arrange to have an empty seat on either side of you. Now that you are seated, please write your name **on the back** of this exam.

This is a closed-notes and closed-book exam. Computers, calculators, and books are prohibited.

This is a 30 point exam.

- Partial credit will be given so be sure to show your work.
- Feel free to write helper functions if you need them.
- **Please write neatly.**

Problem	Points	Out Of
1 Snippets		**6**
2 Storage		**8**
3 Lists		**12**
3 SVM		**4**
Total		**30**

Section 1: Snippets (6 Points Total)

1. (1 Point) In a sentence or two, what is a *value*? Give an example of an OCaml expression that is a value, an example of an expression that is not a value but which has a value and give an example of an expression which has no value.

   Answer: A *value* is an expression that cannot be further simplified. For example, `(2, 3)` is value. The expression `(2, 1 2)+` is an expression which has a value but is not a value and `(2, 1 / 0)` is an expression that does not have a value.

2. (1 Point) In a sentence or two, what is a *variable*? Are there legal OCaml expressions containing more than one variable with the same name? If so, show one.

   Answer: A variable is a symbol that can be associated with a value.

   ```
   let x = 1 in (let x = 2 in x) + x
   ```

   has two different variables named `x`.

3. (1 Point) Is the following expression well formed? If so, simplify the expression, one step at a time.

   ```
   let x = match (1 + 1) = 3 with | true -> 4 | false -> 5 in x * 2
   ```

   Answer:

   ```
   let x = match (1 + 1) = 3 with | true -> 4 | false -> 5 in x * 2 ->
     let x = match 2 = 3 with | true -> 4 | false -> 5 in x * 2 ->
     let x = match false with | true -> 4 | false -> 5 in x * 2 ->
     let x = 5 in x * 2 ->
     5 * 2 ->
     10
   ```

4. (1 Point) Is the following function well-defined? If so, what is its type?

```
let f (x, y) = (y + 1, x)
```

Answer: Yes it is well typed with type `'a * int -> int * 'a`.

5. (1 Point) $200_4 = X_{16}$. Solve for X.

Answer: X = 20.

6. (1 Point) $123_5 = 212_X$. Solve for X.

Answer: $X = 4$.

Section 2: Storage Diagrams (8 Points)

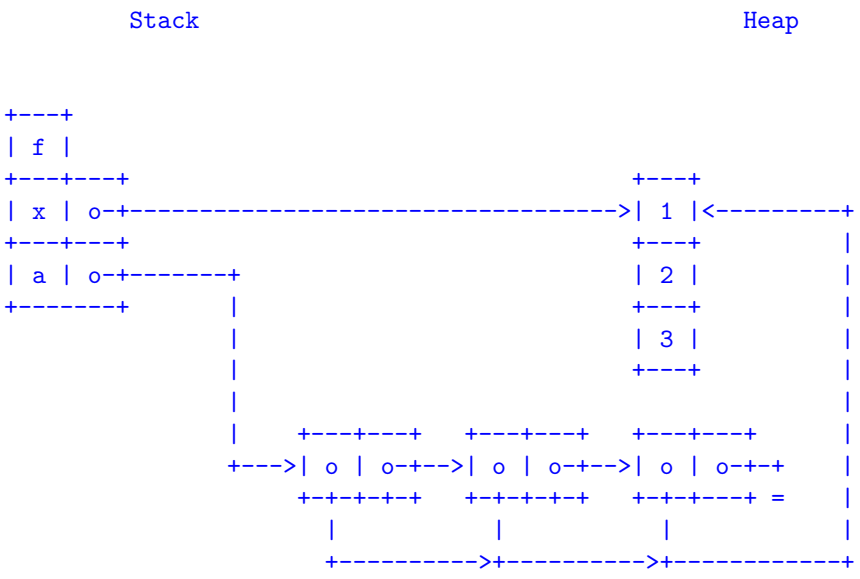
1. (2 Points) Consider the following code:

```
let f x =
  let a = [x; x; x]            (1)
  in
  a                            (2)

f (1, 2, 3)
```

Show the state of the stack and the heap after (1) has executed but before (2) has executed.

Answer:



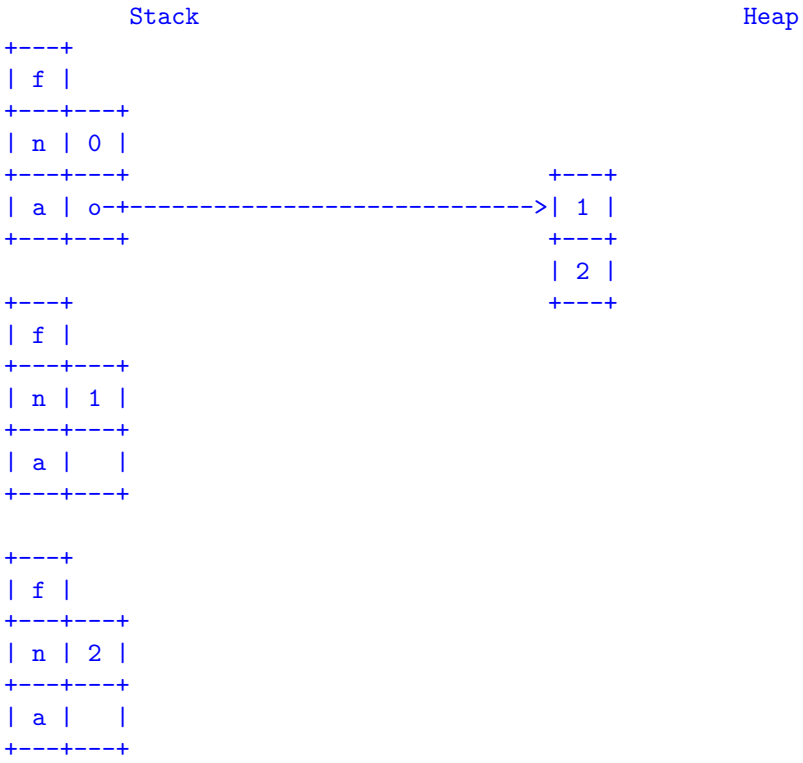
2. (2 Points) Consider the following code:

```
let rec f n =
  match n = 0 with
  | true  ->
    let a = (1, 2)                    (1)
    in
    a                                 (2)
  | false -> f (n - 1)

f (1 + 1)
```

Show the state of the stack and the heap after (1) has executed but before (2) has executed.

Answer:



3. (2 Points) Consider the following definitions.

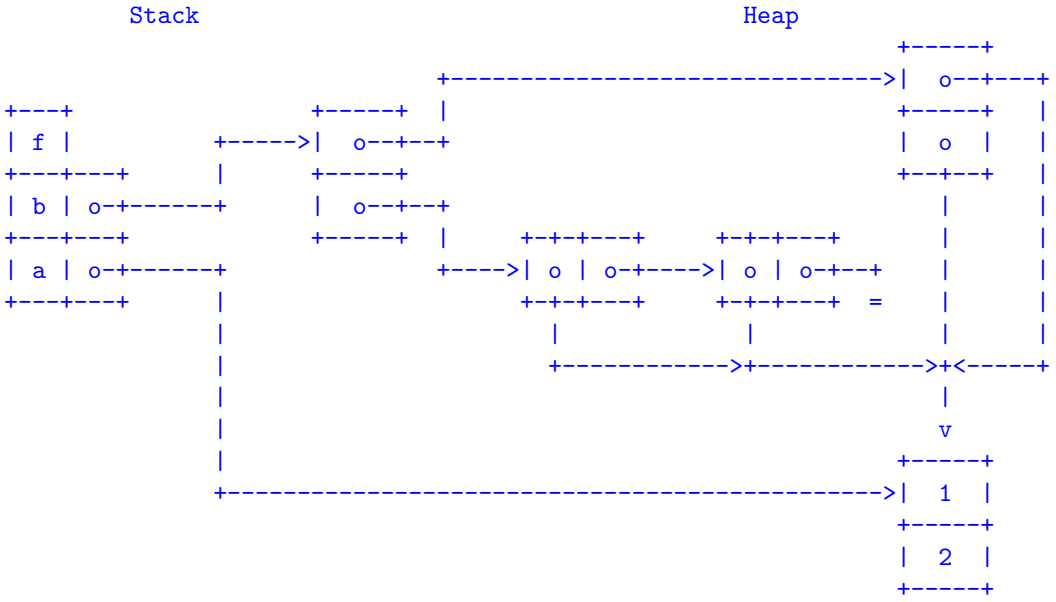
```
let g (b, c) =
  b.(1) <- (3, 4) ;                (3)
  b                                (4)

let f a =
  let b = ([| a; a |], [a; a])     (1)
  in
  g b                              (2)

f (1, 2)
```

Show the state of the stack and the heap after (1) has executed but before (2) has executed.

Answer:



4. (2 Points) Consider the following definition.

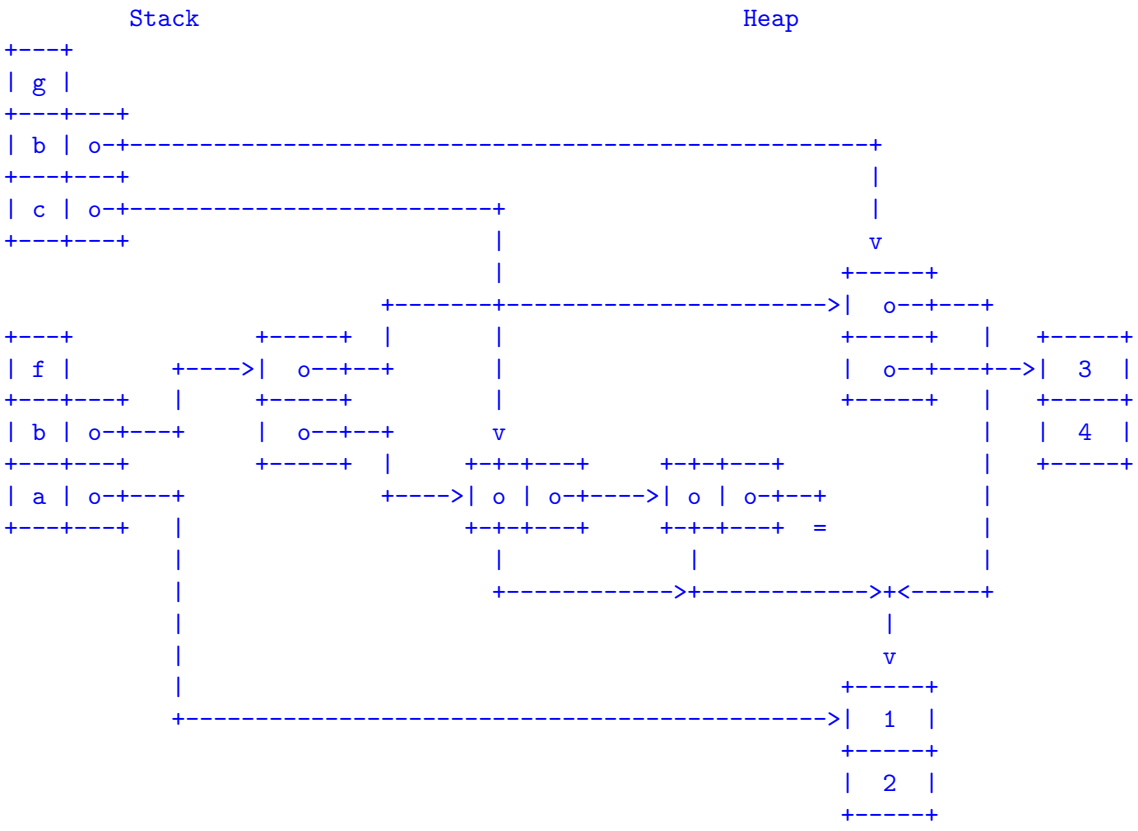
```
let g (b, c) =
  b.(1) <- (3, 4) ;                (3)
  b                                (4)

let f a =
  let b = ([| a; a |], [a; a])     (1)
  in
  g b                              (2)

f (1, 2)
```

Show the state of the stack and the heap after (3) has executed but before (4) has executed.

Answer:



Section 3: Lists, Trees and Arrays

1. (3 Points) Write an OCaml function `oddsToZero : int list -> int list` such that a call of the function `(oddsToZero ns)` returns a list like `ns` but in which all of the odd integers have been replace by zeros. For example, the call `(oddsToZero [1; 2; 3; 4; 5])` would return the list `[0; 2; 0; 4; 0]`.

   Answer:

```
let rec oddsToZero ns = List.map (fun n -> if n mod 2 = 0 then n else 0) ns
```

2. (3 Points) Let `a` and `b` be arrays of integers that are in ascending order. Write the function

   ```
   merge : int array -> int array -> int array
   ```

   such that a call `(merge a b)` returns a new array containing the elements of both `a` and `b` in ascending order. For example, the call `(merge [|1; 3|] [|2; 4; 5|])` should evaluate to the array `[|1; 2; 3; 4; 5|]`.

   Answer:

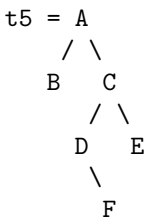
```
let merge a b =
  let m = Array.length a in
  let n = Array.length b in
  let c = Array.make (m + n) 0 in
  let i = ref 0 in
  let j = ref 0
  in
  while (!i < m || !j < n) do
    match !i = m with
    | true  -> c.(!i + !j) <- b.(!j) ; j := !j + 1
    | false ->
      (match !j = n with
       | true  -> c.(!i + !j) <- a.(!i) ; i := !i + 1
       | false ->
         (match a.(!i) < b.(!j) with
          | true  -> c.(!i + !j) <- a.(!i); i := !i + 1
          | false -> c.(!i + !j) <- b.(!j); j := !j + 1))
  done
  ; c
```

3. (3 Points) Consider the following representation of binary trees. This definition differs from those discussed in class in that here we use an explicit empty node `Empty` while the in-class version had a leaf `(Leaf t)`. In this representation a leaf would be `Node{info = t; left = Empty; right = Empty}`.

```
type t = A | B | C | D | E | F

type tree = Empty
          | Node of {info : t; left : tree; right : tree}

let t0 = Node {info = F; left = Empty; right = Empty}
let t1 = Node {info = B; left = Empty; right = Empty}
let t2 = Node {info = D; left = Empty; right = t1}
let t3 = Node {info = E; left = Empty; right = Empty}
let t4 = Node {info = C; left = t2; right = t3}
let t5 = Node {info = A; left = t1; right = t4}
```



The *root* of tree `t5` is `A`. The length of the path from `A` to `A` is 0. The length of the path from `A` to `C` is 1. The length of the path from `A` to `F` is 3. The *height* of a tree is the length of the longest path from the root. Write the function `height : tree -> int` such that a call `(height tree)` returns the height of the tree.

Answer:

```
let rec height tree =
  match tree with
  | Empty -> 0
  | Node{left = Empty; right = Empty} -> 0
  | Node{left; right} -> 1 + max (height left) (height right)
```

4. (3 Points) Early on we wrote the predicate `isPrime : int -> bool` which tested `n` for primality. Our definition looked something like this:

```
let sqrtInt n = int_of_float (sqrt (float n))
let isFactor m n = n mod m == 0

let isPrime n =
  let top = sqrtInt n in
  let rec repeat candidate =
    match candidate > top with
    | true  -> true
    | false -> match isFactor candidate n with
       | true  -> false
               | false -> repeat (candidate + 1)
  in
  repeat 2
```

Rewrite the `isPrime` function without using recursion.

Answer:

```
let isPrime n =
  let top = sqrtInt n in
  let i = ref 2
  in
  while (!i) <= top && not(isFactor !i n) do i := !i + 1 done;
  !i > top
```

Section 4: SVM (4 Points Total)

Assume that the data segment contains a list of non-zero numbers ending with a sentinal zero, something like `[3; 1; 2; 4; 8; 0]`. Write an SVM program that when called with a number n in `R1`, will `Hlt` with a `1` in register `R0` if n is in the data segment and a `0` in `R0` if it isn't.

Answer:

```
Li  R0, 1
Mov R2, Zero
Lod R3, 0(R2)
Cmp R3, Zero
Beq 4
Cmp R3, R1
Beq 3
Add R2, R2, R0
Jmp -7
Mov R0, Zero
Hlt
```

1 The Simple Virtual Machine

The instruction set of SVM is as follows.

- `Lod Rd, offset(Rs)`: Let `base` be the contents of register `Rs`. Then this instruction loads the contents of data segment location `offset + base` into register `Rd`.
- `Sto Rs, offset(Rd)`: Let `base` be the contents of register `Rd`. Then this instruction stores the contents of register `Rs` into data segment location `offset + base`.
- `Li Rd, number`: loads `number` into register Rd.
- `Mov Rd, Rs`: copies the contents of register Rs into register Rd.
- `Add Rd, Rs, Rt`: adds the contents of registers Rs and Rt and stores the sum in register Rd.
- `Sub Rd, Rs, Rt`: subtracts the contents of register Rt from Rs and stores the difference in register Rd.
- `Mul Rd, Rs, Rt`: multiplies the contents of register Rt by Rs and stores the product in register Rd.
- `Div Rd, Rs, Rt`: divides the contents of register Rs by Rt and stores the integer quotient in register Rd.
- `Cmp Rs, Rt`: sets PSW = Rs - Rt. Note that if Rs > Rt, then PSW will be positive, if Rs == Rt, then PSW will be 0 and if Rs < Rt, then PSW will be negative.
- `Blt disp`: if PSW is negative, causes the new value of PC to be the sum PC + disp. Note that if disp is negative, this will cause the program to jump backward in the sequence of instructions. If PSW $\geq$ 0, this instruction does nothing.
- `Beq disp`: if PSW == 0, causes the new value of PC to be the sum PC + disp. Note that if disp is negative, this will cause the program to jump backward in the sequence of instructions. If PSW $\neq$ 0, this instruction does nothing.
- `Bgt disp`: if PSW, is positive, causes the new value of PC to be the sum PC + disp. Note that if disp is negative, this will cause the program to jump backward in the sequence of instructions. If PSW $\leq$ 0, this instruction does nothing.
- `Jmp disp`: causes the new value of PC to be the sum PC + disp.
- `Jsr disp`: Jump subroutine: `RA := PC` then `PC := PC + disp`.
- `R`: Return from subroutine: `PC := RA`.
- `Hlt`: causes the svm machine to print the contents of registers PC, PSW, R0, R1, R2 and R3. It then halts.