UNITS

This handbook uses the metric system of units. Ultimately, the FE examination will be entirely metric. However, currently some of the problems use both metric and U.S. Customary System (USCS). In the USCS system of units, both force and mass are called pounds. Therefore, one must distinguish the pound-force (lbf) from the pound-mass (lbm).

The pound-force is that force which accelerates one pound-mass at 32.174 ft/s². Thus, 1 lbf = 32.174 lbm-ft/s². The expression 32.174 lbm-ft/(lbf-s²) is designated as g_c and is used to resolve expressions involving both mass and force expressed as pounds. For instance, in writing Newton's second law, the equation would be written as $F = ma/g_c$, where F is in lbf, m in lbm, and a is in ft/s².

Similar expressions exist for other quantities. Kinetic Energy: $KE = mv^2/2g_c$, with KE in (ft-lbf); Potential Energy: $PE = mgh/g_c$, with PE in (ft-lbf); Fluid Pressure: $p = \rho gh/g_c$, with p in (lbf/ft²); Specific Weight: $SW = \rho g/g_c$, in (lbf/ft³); Shear Stress: $\tau = (\mu/g_c)(dv/dy)$, with shear stress in (lbf/ft²). In all these examples, g_c should be regarded as a unit conversion factor. It is frequently not written explicitly in engineering equations. However, its use is required to produce a consistent set of units.

Note that the conversion factor g_c [lbm-ft/(lbf-s²)] should not be confused with the local acceleration of gravity g, which has different units (m/s²) and may be either its standard value (9.807 m/s²) or some other local value.

If the problem is presented in USCS units, it may be necessary to use the constant g_c in the equation to have a consistent set of units.

METRIC PREFIXES			COMMONLY USED EQUIVALENTS	
Multiple	Prefix	Symbol		
10^{-18}	atto	a	1 gallon of water weighs	8.34 lbf
10^{-15}	femto	f	1 cubic foot of water weighs	62.4 lbf
10^{-12}	pico	p	1 cubic inch of mercury weighs	0.491 lbf
10^{-9}	nano	n		
10^{-6}	micro	μ	The mass of one cubic meter of water is 1,000 kilograms	
10^{-3}	milli	m		
10^{-2}	centi	c	TEMPERATURE CONVERSIONS	
10^{-1}	deci	d	$°F = 1.8 (°C) + 32$	
10^{1}	deka	da	$°C = (°F - 32)/1.8$	
10^{2}	hecto	h	$°R = °F + 459.69$	
10^{3}	kilo	k	$K = °C + 273.15$	
10^{6}	mega	M		
10^{9}	giga	G		
10^{12}	tera	T		
10^{15}	peta	P		
10^{18}	exa	E		

FUNDAMENTAL CONSTANTS

Quantity		Symbol	Value	Units
electron charge		e	1.6022×10^{-19}	C (coulombs)
Faraday constant		$\mathcal{F}$	96,485	coulombs/(mol)
gas constant	metric	$\overline{R}$	8,314	J/(kmol·K)
gas constant	metric	$\overline{R}$	8.314	kPa·m³/(kmol·K)
gas constant	USCS	$\overline{R}$	1,545	ft-lbf/(lb mole-°R)
		$\overline{R}$	0.08206	L-atm/mole-K
gravitation - newtonian constant		G	6.673×10^{-11}	m³/(kg·s²)
gravitation - newtonian constant		G	6.673×10^{-11}	N·m²/kg²
gravity acceleration (standard)	metric	g	9.807	m/s²
gravity acceleration (standard)	USCS	g	32.174	ft/s²
molar volume (ideal gas), $T = 273.15K$, $p = 101.3$ kPa		V_m	22,414	L/kmol
speed of light in vacuum		c	299,792,000	m/s

CONVERSION FACTORS

Multiply	By	To Obtain	Multiply	By	To Obtain
acre	43,560	square feet (ft²)	joule (J)	9.478×10^{-4}	Btu
ampere-hr (A-hr)	3,600	coulomb (C)	J	0.7376	ft-lbf
ångström (Å)	1×10^{-10}	meter (m)	J	1	newton·m (N·m)
atmosphere (atm)	76.0	cm, mercury (Hg)	J/s	1	watt (W)
atm, std	29.92	in, mercury (Hg)			
atm, std	14.70	lbf/in² abs (psia)	kilogram (kg)	2.205	pound (lbm)
atm, std	33.90	ft, water	kgf	9.8066	newton (N)
atm, std	1.013×10^5	pascal (Pa)	kilometer (km)	3,281	feet (ft)
			km/hr	0.621	mph
bar	1×10^5	Pa	kilopascal (kPa)	0.145	lbf/in² (psi)
barrels-oil	42	gallons-oil	kilowatt (kW)	1.341	horsepower (hp)
Btu	1,055	joule (J)	kW	3,413	Btu/hr
Btu	2.928×10^{-4}	kilowatt-hr (kWh)	kW	737.6	(ft-lbf)/sec
Btu	778	ft-lbf	kW-hour (kWh)	3,413	Btu
Btu/hr	3.930×10^{-4}	horsepower (hp)	kWh	1.341	hp-hr
Btu/hr	0.293	watt (W)	kWh	3.6×10^6	joule (J)
Btu/hr	0.216	ft-lbf/sec	kip (K)	1,000	lbf
			K	4,448	newton (N)
calorie (g-cal)	3.968×10^{-3}	Btu			
cal	1.560×10^{-6}	hp-hr	liter (L)	61.02	in³
cal	4.186	joule (J)	L	0.264	gal (US Liq)
cal/sec	4.186	watt (W)	L	10^{-3}	m³
centimeter (cm)	3.281×10^{-2}	foot (ft)	L/second (L/s)	2.119	ft³/min (cfm)
cm	0.394	inch (in)	L/s	15.85	gal (US)/min (gpm)
centipoise (cP)	0.001	pascal·sec (Pa·s)			
centistokes (cSt)	1×10^{-6}	m²/sec (m²/s)	meter (m)	3.281	feet (ft)
cubic feet/second (cfs)	0.646317	million gallons/day (mgd)	m	1.094	yard
cubic foot (ft³)	7.481	gallon	m/second (m/s)	196.8	feet/min (ft/min)
cubic meters (m³)	1,000	Liters	mile (statute)	5,280	feet (ft)
electronvolt (eV)	1.602×10^{-19}	joule (J)	mile (statute)	1.609	kilometer (km)
			mile/hour (mph)	88.0	ft/min (fpm)
foot (ft)	30.48	cm	mph	1.609	km/h
ft	0.3048	meter (m)	mm of Hg	1.316×10^{-3}	atm
ft-pound (ft-lbf)	1.285×10^{-3}	Btu	mm of H₂O	9.678×10^{-5}	atm
ft-lbf	3.766×10^{-7}	kilowatt-hr (kWh)			
ft-lbf	0.324	calorie (g-cal)	newton (N)	0.225	lbf
ft-lbf	1.356	joule (J)	N·m	0.7376	ft-lbf
ft-lbf/sec	1.818×10^{-3}	horsepower (hp)	N·m	1	joule (J)
gallon (US Liq)	3.785	liter (L)	pascal (Pa)	9.869×10^{-6}	atmosphere (atm)
gallon (US Liq)	0.134	ft³	Pa	1	newton/m² (N/m²)
gallons of water	8.3453	pounds of water	Pa·sec (Pa·s)	10	poise (P)
gamma (γ, Γ)	1×10^{-9}	tesla (T)	pound (lbm,avdp)	0.454	kilogram (kg)
gauss	1×10^{-4}	T	lbf	4.448	N
gram (g)	2.205×10^{-3}	pound (lbm)	lbf-ft	1.356	N·m
			lbf/in² (psi)	0.068	atm
hectare	1×10^4	square meters (m²)	psi	2.307	ft of H₂O
hectare	2.47104	acres	psi	2.036	in of Hg
horsepower (hp)	42.4	Btu/min	psi	6,895	Pa
hp	745.7	watt (W)			
hp	33,000	(ft-lbf)/min	radian	180/π	degree
hp	550	(ft-lbf)/sec			
hp-hr	2,544	Btu	stokes	1×10^{-4}	m²/s
hp-hr	1.98×10^6	ft-lbf			
hp-hr	2.68×10^6	joule (J)	therm	1×10^5	Btu
hp-hr	0.746	kWh			
			watt (W)	3.413	Btu/hr
inch (in)	2.540	centimeter (cm)	W	1.341×10^{-3}	horsepower (hp)
in of Hg	0.0334	atm	W	1	joule/sec (J/s)
in of Hg	13.60	in of H₂O	weber/m² (Wb/m²)	10,000	gauss
in of H₂O	0.0361	lbf/in² (psi)			
in of H₂O	0.002458	atm			

FLUID MECHANICS

DENSITY, SPECIFIC VOLUME, SPECIFIC WEIGHT, AND SPECIFIC GRAVITY

The definitions of density, specific volume, specific weight, and specific gravity follow:

$$\rho = \lim_{\Delta V \to 0} \Delta m / \Delta V$$

$$\gamma = \lim_{\Delta V \to 0} \Delta W / \Delta V$$

$$\gamma = \lim_{\Delta V \to 0} g \cdot \Delta m / \Delta V = \rho g$$

also $SG = \gamma / \gamma_w = \rho / \rho_w$, where

ρ = *density* (also *mass density*),

Δm = mass of infinitesimal volume,

ΔV = volume of infinitesimal object considered,

γ = *specific weight,*

ΔW = weight of an infinitesimal volume,

SG = *specific gravity*, and

ρ_w = mass density of water at standard conditions = 1,000 kg/m^3 (62.43 lbm/ft^3).

STRESS, PRESSURE, AND VISCOSITY

Stress is defined as

$$\tau(P) = \lim_{\Delta A \to 0} \Delta F / \Delta A \text{, where}$$

$\tau(P)$ = surface stress vector at point P,

ΔF = force acting on infinitesimal area ΔA, and

ΔA = infinitesimal area at point P.

$$\tau_n = -p$$

$$\tau_t = \mu \, (dv/dy) \quad \text{(one-dimensional; i.e., } y\text{), where}$$

τ_n and τ_t = the normal and tangential stress components at point P,

p = the pressure at point P,

μ = *absolute dynamic viscosity* of the fluid N·s/m^2 [lbm/(ft-sec)],

dv = velocity at boundary condition, and

dy = normal distance, measured from boundary.

$$v = \mu/\rho \text{, where}$$

v = *kinematic viscosity*; m^2/s (ft^2/sec).

For a thin Newtonian fluid film and a linear velocity profile,

$$v(y) = Vy/\delta; \; dv/dy = V/\delta \text{, where}$$

V = velocity of plate on film and

δ = thickness of fluid film.

For a power law (non-Newtonian) fluid

$$\tau_t = K \, (dv/dy)^n \text{, where}$$

K = consistency index, and

n = power law index.

$n < 1 \equiv$ pseudo plastic

$n > 1 \equiv$ dilatant

SURFACE TENSION AND CAPILLARITY

Surface tension σ is the force per unit contact length

$$\sigma = F/L \text{, where}$$

σ = surface tension, force/length,

F = surface force at the interface, and

L = length of interface.

The *capillary rise h* is approximated by

$$h = 4\sigma \cos \beta / (\gamma d) \text{, where}$$

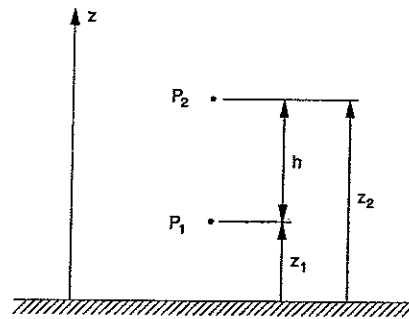h = the height of the liquid in the vertical tube,

σ = the surface tension,

β = the angle made by the liquid with the wetted tube wall,

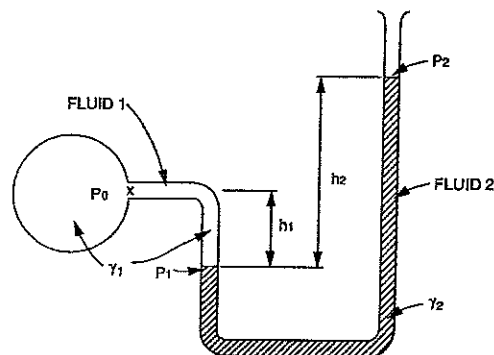γ = specific weight of the liquid, and

d = the diameter or the capillary tube.

THE PRESSURE FIELD IN A STATIC LIQUID AND MANOMETRY

The difference in pressure between two different points is

$$p_2 - p_1 = -\gamma \, (z_2 - z_1) = -\gamma h$$
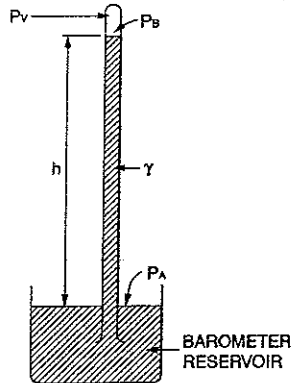
For a simple manometer,

$$p_o = p_2 + \gamma_2 h_2 - \gamma_1 h_1$$

Absolute pressure = atmospheric pressure + gage pressure reading

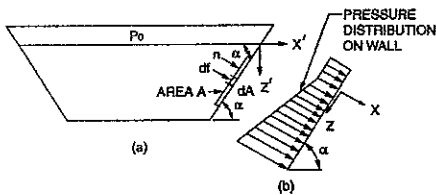Absolute pressure = atmospheric pressure − vacuum gage pressure reading

Another device that works on the same principle as the manometer is the simple barometer.

$$p_{atm} = p_A = p_v + \gamma h = p_B + \gamma h$$

◆

p_v = vapor pressure of the barometer fluid

FORCES ON SUBMERGED SURFACES AND THE CENTER OF PRESSURE

◆

Forces on a submerged plane wall. (a) Submerged plane surface. (b) Pressure distribution.

The pressure on a point at a distance Z' below the surface is

$$p = p_o + \gamma Z', \text{ for } Z' \geq 0$$

If the tank were open to the atmosphere, the effects of p_o could be ignored.

The coordinates of the *center of pressure CP* are

$$y^* = \left(\gamma I_{y_c z_c} \sin\alpha\right) / (p_c A) \quad \text{and}$$

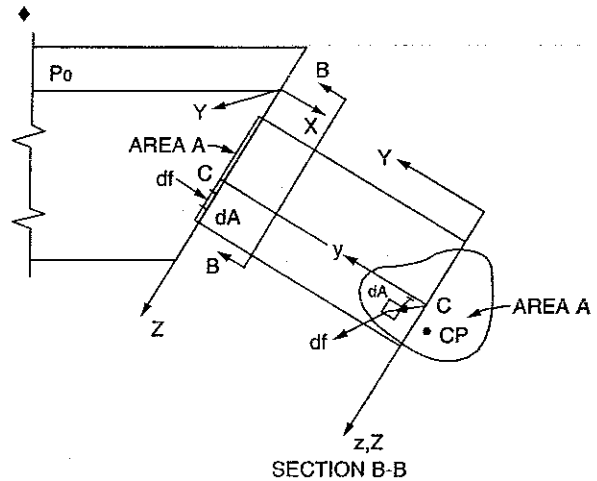$$z^* = \left(\gamma I_{y_c} \sin\alpha\right) / (p_c A), \text{ where}$$

y^* = the y-distance from the centroid (C) of area (A) to the center of pressure,

z^* = the z-distance from the centroid (C) of area (A) to the center of pressure,

I_{y_c} and $I_{y_c z_c}$ = the moment and product of inertia of the area,

p_c = the pressure at the centroid of area (A), and

Z_c = the slant distance from the water surface to the centroid (C) of area (A).

◆

SECTION B-B

If the free surface is open to the atmosphere, then

$p_o = 0$ and $p_c = \gamma Z_c \sin \alpha$.

$$y^* = I_{y_c z_c} / (A Z_c) \quad \text{and} \quad z^* = I_{y_c} / (A Z_c)$$

The force on the plate can be computed as

$$F = [p_1 A_v + (p_2 - p_1) A_v / 2]\mathbf{i} + V_f \gamma_f \mathbf{j}, \text{ where}$$

F = force on the plate,

p_1 = pressure at the top edge of the plate area,

p_2 = pressure at the bottom edge of the plate area,

A_v = vertical projection of the plate area,

V_f = volume of column of fluid above plate, and

γ_f = specific weight of the fluid.

ARCHIMEDES PRINCIPLE AND BUOYANCY

1. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.

2. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The *center of buoyancy* is located at the centroid of the submerged portion of the body.

In the case of a body lying at the *interface of two immiscible fluids*, the buoyant force equals the sum of the weights of the fluids displaced by the body.

ONE-DIMENSIONAL FLOWS

The Continuity Equation So long as the flow Q is continuous, the *continuity equation*, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point, $A_1 V_1 = A_2 V_2$.

$$Q = AV$$

$$\dot{m} = \rho Q = \rho AV, \text{ where}$$

Q = volumetric flow rate,

$\dot{m}$ = mass flow rate,

◆ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Copyright © 1980 by John Wiley & Sons, Inc. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

A = cross section of area of flow,

V = average flow velocity, and

ρ = the fluid density.

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then Q is constant.

The Field Equation is derived when the energy equation is applied to one-dimensional flows.

Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \text{, where}$$

p_1, p_2 = pressure at sections 1 and 2,

V_1, V_2 = average velocity of the fluid at the sections,

z_1, z_2 = the vertical distance from a datum to the sections (the potential energy),

γ = the specific weight of the fluid, and

g = the acceleration of gravity.

FLOW OF A REAL FLUID

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f$$

The pressure drop as fluid flows through a pipe of constant cross-section and which is held at a fixed elevation is

$$h_f = (p_1 - p_2)/\gamma \text{, where}$$

h_f = the head loss, considered a friction effect, and all remaining terms are defined above.

Fluid Flow

The velocity distribution for *laminar flow* in circular tubes or between planes is

$$v = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \text{, where}$$

r = the distance (m) from the centerline,

R = the radius (m) of the tube or half the distance between the parallel planes,

v = the local velocity (m/s) at r, and

v_{max} = the velocity (m/s) at the centerline of the duct.

v_{max} = 1.18V, for fully turbulent flow
    (Re > 10,000)

v_{max} = 2V, for circular tubes in laminar flow and

v_{max} = 1.5V, for parallel planes in laminar flow, where

V = the average velocity (m/s) in the duct.

The shear stress distribution is

$$\frac{\tau}{\tau_w} = \frac{r}{R} \text{, where}$$

τ and τ_w are the shear stresses at radii r and R respectively.

The *drag force F_D* on objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid is

$$F_D = \frac{C_D \rho V^2 A}{2} \text{, where}$$

C_D = the *drag coefficient* (see page 46),

V = the velocity (m/s) of the undisturbed fluid, and

A = the *projected area* (m²) of bluff objects such as spheres, ellipsoids, and disks and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow.

For flat plates placed parallel with the flow

$$C_D = 1.33/Re^{0.5} \ (10^4 < Re < 5 \times 10^5)$$

$$C_D = 0.031/Re^{1/7} \ (10^6 < Re < 10^9)$$

The characteristic length in the Reynolds Number (Re) is the length of the plate parallel with the flow. For bluff objects, the characteristic length is the largest linear dimension (diameter of cylinder, sphere, disk, etc.) which is perpendicular to the flow.

Reynolds Number

$$Re = VD\rho/\mu = VD/\nu$$

$$Re' = \frac{V^{(2-n)}D^n \rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{(n-1)}} \text{, where}$$

ρ = the mass density,

D = the diameter of the pipe or dimension of the fluid streamline,

μ = the dynamic viscosity,

ν = the kinematic viscosity,

Re = the Reynolds number (Newtonian fluid),

Re' = the Reynolds number (Power law fluid), and

K and n are defined on page 38.

The critical Reynolds number $(Re)_c$ is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Hydraulic Gradient (Grade Line)

The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the *pressure head* at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

Energy Line (Bernoulli Equation)

The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the "total head line" above a horizontal datum.

The difference between the hydraulic grade line and the energy line is the $V^2/2g$ term.

44

STEADY, INCOMPRESSIBLE FLOW IN CONDUITS AND PIPES

The energy equation for incompressible flow is

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f$$

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $V_1 = V_2$. The pressure drop $p_1 - p_2$ is given by the following:

$$p_1 - p_2 = \gamma h_f$$

The *Darcy equation* is

$$h_f = f \frac{L}{D} \frac{V^2}{2g}, \text{ where}$$

f = f(Re, e/D), the friction factor,

D = diameter of the pipe,

L = length over which the pressure drop occurs,

e = roughness factor for the pipe, and all other symbols are defined as before.

A chart that gives f versus Re for various values of e/D, known as a *Moody* or *Stanton diagram*, is available at the end of this section on page 45.

Friction Factor for Laminar Flow

The equation for Q in terms of the pressure drop Δp_f is the Hagen-Poiseuille equation. This relation is valid only for flow in the laminar region.

$$Q = \frac{\pi R^4 \Delta p_f}{8\mu L} = \frac{\pi D^4 \Delta p_f}{128\mu L}$$

Flow in Noncircular Conduits

Analysis of flow in conduits having a noncircular cross section uses the *hydraulic diameter* D_H, or the *hydraulic radius R_H*, as follows

$$R_H = \frac{\text{cross - sectional area}}{\text{wetted perimeter}} = \frac{D_H}{4}$$

Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.
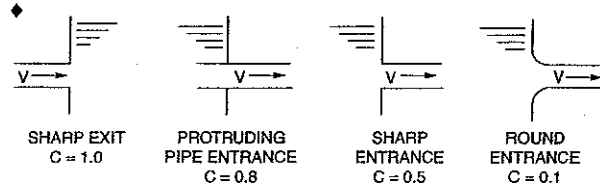
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f + h_{f,\text{ fitting}}, \text{ where}$$

$$h_{f,\text{ fitting}} = C \frac{V^2}{2g}$$

Specific fittings have characteristic values of C, which will be provided in the problem statement. A generally accepted *nominal value* for head loss in *well-streamlined gradual contractions* is

$$h_{f,\text{ fitting}} = 0.04 \, V^2 / 2g$$

The *head loss* at either an *entrance* or *exit* of a pipe from or to a reservoir is also given by the $h_{f,\text{ fitting}}$ equation. Values for C for various cases are shown as follows.

SHARP EXIT	PROTRUDING PIPE ENTRANCE	SHARP ENTRANCE	ROUND ENTRANCE
C = 1.0	C = 0.8	C = 0.5	C = 0.1

PUMP POWER EQUATION

$$\dot{W} = Q\gamma h / \eta, \text{ where}$$

Q = quantity of flow (m^3/s or cfs),

h = head (m or ft) the fluid has to be lifted,

η = efficiency, and

$\dot{W}$ = power (watts or ft-lbf/sec).

THE IMPULSE-MOMENTUM PRINCIPLE

The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

$$\Sigma F = Q_2 \rho_2 V_2 - Q_1 \rho_1 V_1, \text{ where}$$

ΣF = the resultant of all external forces acting on the control volume,

$Q_1 \rho_1 V_1$ = the rate of momentum of the fluid flow entering the control volume in the same direction of the force, and

$Q_2 \rho_2 V_2$ = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force.

Pipe Bends, Enlargements, and Contractions

The force exerted by a flowing fluid on a bend, enlargement, or contraction in a pipe line may be computed using the impulse-momentum principle.
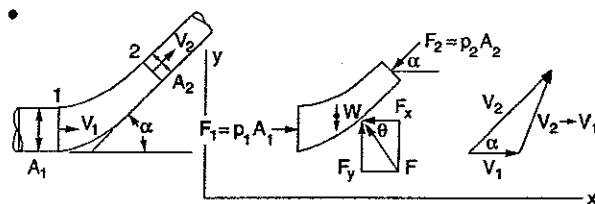
$$p_1 A_1 - p_2 A_2 \cos \alpha - F_x = Q\rho (V_2 \cos \alpha - V_1)$$

$$F_y - W - p_2 A_2 \sin \alpha = Q\rho (V_2 \sin \alpha - 0), \text{ where}$$

F = the force exerted by the bend on the fluid (the force exerted by the fluid on the bend is equal in magnitude and opposite in sign), F_x and F_y are the x-component and y-component of the force,

p = the internal pressure in the pipe line,

A = the cross-sectional area of the pipe line,

W = the weight of the fluid,

V = the velocity of the fluid flow,

α = the angle the pipe bend makes with the horizontal,

ρ = the density of the fluid, and

Q = the quantity of fluid flow.
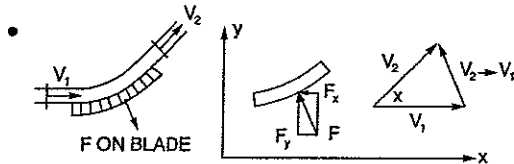
Jet Propulsion
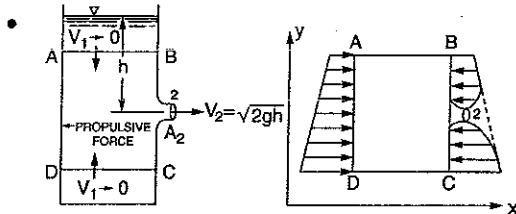
$$F = Q\rho(V_2 - 0)$$

$$F = 2\gamma h A_2, \text{ where}$$

F = the propulsive force,

γ = the specific weight of the fluid,

h = the height of the fluid above the outlet,

A_2 = the area of the nozzle tip,

Q = $A_2\sqrt{2gh}$, and

V_2 = $\sqrt{2gh}$.
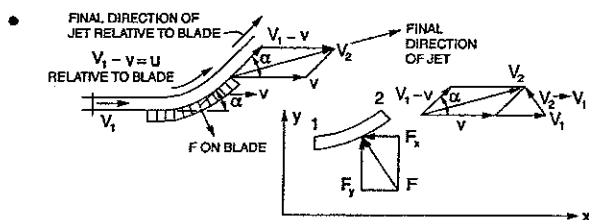
Deflectors and Blades

Fixed Blade

$$-F_x = Q\rho(V_2\cos\alpha - V_1)$$

$$F_y = Q\rho(V_2\sin\alpha - 0)$$
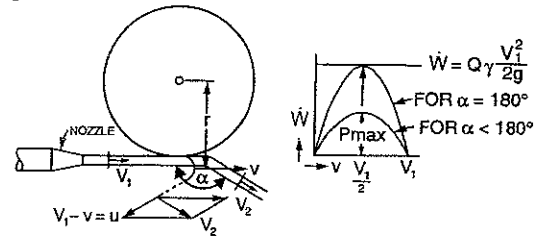
Moving Blade

$$-F_x = Q\rho(V_{2x} - V_{1x})$$
$$= -Q\rho(V_1 - v)(1 - \cos\alpha)$$
$$F_y = Q\rho(V_{2y} - V_{1y})$$
$$= +Q\rho(V_1 - v)\sin\alpha, \text{ where}$$

v = the velocity of the blade.

Impulse Turbine

$$\dot{W} = Q\rho(V_1 - v)(1 - \cos\alpha)v, \text{ where}$$

$\dot{W}$ = power of the turbine.

$$\dot{W}_{max} = Q\rho(V_1^2/4)(1 - \cos\alpha)$$

When $\alpha = 180°$,

$$\dot{W}_{max} = (Q\rho V_1^2)/2 = (Q\gamma V_1^2)/2g$$

MULTIPATH PIPELINE PROBLEMS
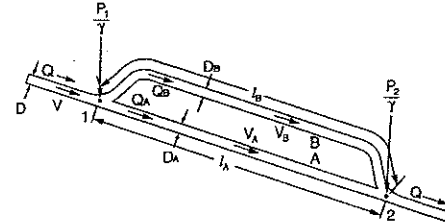
The same head loss occurs in each branch as in the combination of the two. The following equations may be solved simultaneously for V_A and V_B:

$$h_L = f_A\frac{l_A}{D_A}\frac{V_A^2}{2g} = f_B\frac{l_B}{D_B}\frac{V_B^2}{2g}$$
$$(\pi D^2/4)V = (\pi D_A^2/4)V_A + (\pi D_B^2/4)V_B$$

The flow Q can be divided into Q_A and Q_B when the pipe characteristics are known.

OPEN-CHANNEL FLOW AND/OR PIPE FLOW

Manning's Equation
$$V = (k/n)R^{2/3}S^{1/2}, \text{ where}$$

k = 1 for SI units,

k = 1.486 for USCS units,

V = velocity (m/s, ft/sec),

n = roughness coefficient,

R = hydraulic radius (m, ft), and

S = slope of energy grade line (m/m, ft/ft).

Hazen-Williams Equation
$$V = k_1CR^{0.63}S^{0.54}, \text{ where}$$

C = roughness coefficient,

k_1 = 0.849 for SI units, and

k_1 = 1.318 for USCS units.

Other terms defined as above.

MACH NUMBER

The *speed of sound* c in an ideal gas is given by

$$c = \sqrt{kRT}, \text{ where}$$

$k = c_P/c_v.$

This shows that the acoustic velocity in an ideal gas depends only on its temperature.

The *mach number* Ma is a ratio of the fluid velocity V to the speed of sound:

$$\text{Ma} = V/c$$

FLUID MEASUREMENTS

The Pitot Tube – From the stagnation pressure equation for an *incompressible fluid,*

$$V = \sqrt{(2/\rho)(p_o - p_s)} = \sqrt{2g(p_o - p_s)/\gamma}, \text{ where}$$

V = the velocity of the fluid,

p_o = the stagnation pressure, and

p_s = the static pressure of the fluid at the elevation where the measurement is taken.
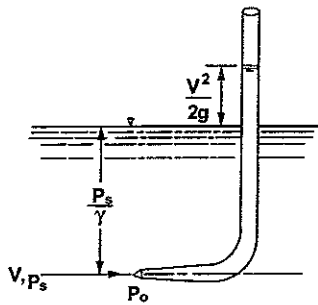
For a *compressible fluid,* use the above incompressible fluid equation if the mach number ≤ 0.3.

Venturi Meters

$$Q = \frac{C_v A_2}{\sqrt{1-(A_2/A_1)^2}} \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}, \text{ where}$$

C_v = the coefficient of velocity.

The above equation is for *incompressible fluids.*
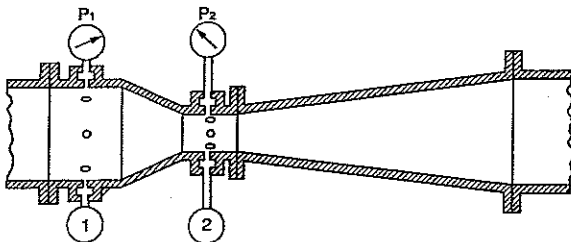
Orifices The cross-sectional area at the vena contracta A_2 is characterized by a *coefficient of contraction* C_c and given by $C_c A.$
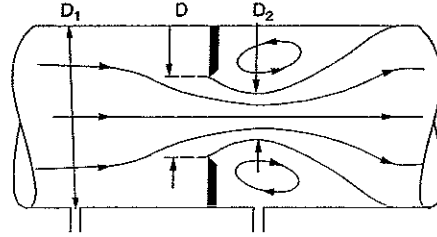
$$Q = CA\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

where C, the *coefficient of the meter,* is given by

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2(A/A_1)^2}}$$

ORIFICES AND THEIR NOMINAL COEFFICIENTS			
SHARP EDGED	ROUNDED	SHORT TUBE	BORDA
C 0.61	0.98	0.80	0.51
C_c 0.62	1.00	1.00	0.52
C_v 0.98	0.98	0.80	0.98

Submerged Orifice operating under steady-flow conditions:
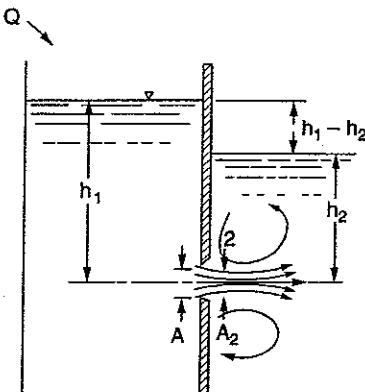
$$Q = A_2 V_2 = C_c C_v A \sqrt{2g(h_1 - h_2)}$$
$$= CA\sqrt{2g(h_1 - h_2)}$$

in which the product of C_c and C_v is defined as the *coefficient of discharge* of the orifice.

Orifice Discharging Freely Into Atmosphere
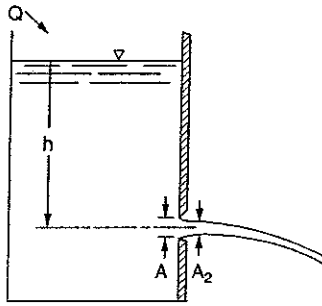
$$Q = CA\sqrt{2gh}$$

in which h is measured from the liquid surface to the centroid of the orifice opening.

DIMENSIONAL HOMOGENEITY AND DIMENSIONAL ANALYSIS

Equations that are in a form that do not depend on the fundamental units of measurement are called *dimensionally homogeneous* equations. A special form of the dimensionally homogeneous equation is one that involves only *dimensionless groups* of terms.

Buckingham's Theorem: The *number of independent dimensionless groups* that may be employed to describe a phenomenon known to involve n variables is equal to the number $(n - \bar{r})$, where $\bar{r}$ is the number of basic dimensions (i.e., M, L, T) needed to express the variables dimensionally.

SIMILITUDE

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically, kinematically,* and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_P}\right]_p = \left[\frac{F_I}{F_P}\right]_m = \left[\frac{\rho V^2}{p}\right]_p = \left[\frac{\rho V^2}{p}\right]_m$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{Vl\rho}{\mu}\right]_p = \left[\frac{Vl\rho}{\mu}\right]_m = [\text{Re}]_p = [\text{Re}]_m$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{V^2}{lg}\right]_p = \left[\frac{V^2}{lg}\right]_m = [\text{Fr}]_p = [\text{Fr}]_m$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m = \left[\frac{\rho V^2}{E_v}\right]_p = \left[\frac{\rho V^2}{E_v}\right]_m = [\text{Ca}]_p = [\text{Ca}]_m$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m = \left[\frac{\rho l V^2}{\sigma}\right]_p = \left[\frac{\rho l V^2}{\sigma}\right]_m = [\text{We}]_p = [\text{We}]_m$$

where

the subscripts p and m stand for *prototype* and *model* respectively, and

F_I = inertia force,

F_P = pressure force,

F_V = viscous force,

F_G = gravity force,

F_E = elastic force,

F_T = surface tension force,

Re = Reynolds number,

We = Weber number,

Ca = Cauchy number,

Fr = Froude number,

l = characteristic length,

V = velocity,

ρ = density,

σ = surface tension,

E_v = bulk modulus,

μ = dynamic viscosity,

p = pressure, and

g = acceleration of gravity.

$$\text{Re} = \frac{VD\rho}{\mu} = \frac{VD}{v}$$

PROPERTIES OF WATER[f]

Temperature °C	Specific Weight[a], γ, kN/m^3	Density[a], ρ, kg/m^3	Viscosity[a], $\mu \times 10^3$, Pa·s	Kinematic Viscosity[a], $v \times 10^6$, m^2/s	Vapor Pressure[e], p_v, kPa
0	9.805	999.8	1.781	1.785	0.61
5	9.807	1000.0	1.518	1.518	0.87
10	9.804	999.7	1.307	1.306	1.23
15	9.798	999.1	1.139	1.139	1.70
20	9.789	998.2	1.002	1.003	2.34
25	9.777	997.0	0.890	0.893	3.17
30	9.764	995.7	0.798	0.800	4.24
40	9.730	992.2	0.653	0.658	7.38
50	9.689	988.0	0.547	0.553	12.33
60	9.642	983.2	0.466	0.474	19.92
70	9.589	977.8	0.404	0.413	31.16
80	9.530	971.8	0.354	0.364	47.34
90	9.466	965.3	0.315	0.326	70.10
100	9.399	958.4	0.282	0.294	101.33

[a]From "Hydraulic Models," *A.S.C.E. Manual of Engineering Practice*, No. 25, A.S.C.E., 1942. See footnote 2.

[e]From J.H. Keenan and F.G. Keyes, *Thermodynamic Properties of Steam*, John Wiley & Sons, 1936.

[f]Compiled from many sources including those indicated, *Handbook of Chemistry and Physics*, 54th Ed., The CRC Press, 1973, and *Handbook of Tables for Applied Engineering Science*, The Chemical Rubber Co., 1970.

[2]Here, if E/10^6 = 1.98 then E = 1.98 × 10^6 kPa, while if μ × 10^3 = 1.781, then μ = 1.781 × 10^{-3} Pa's, and so on. Vennard, J.K. and Robert L. Street, *Elementary Fluid Mechanics*, Copyright 1954, John Wiley & Sons, Inc.

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MOODY (STANTON) DIAGRAM

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	e, (ft)	e, (mm)
Riveted steel	0.003-0.03	0.9-9.0
Concrete	0.001-0.01	0.3-3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015
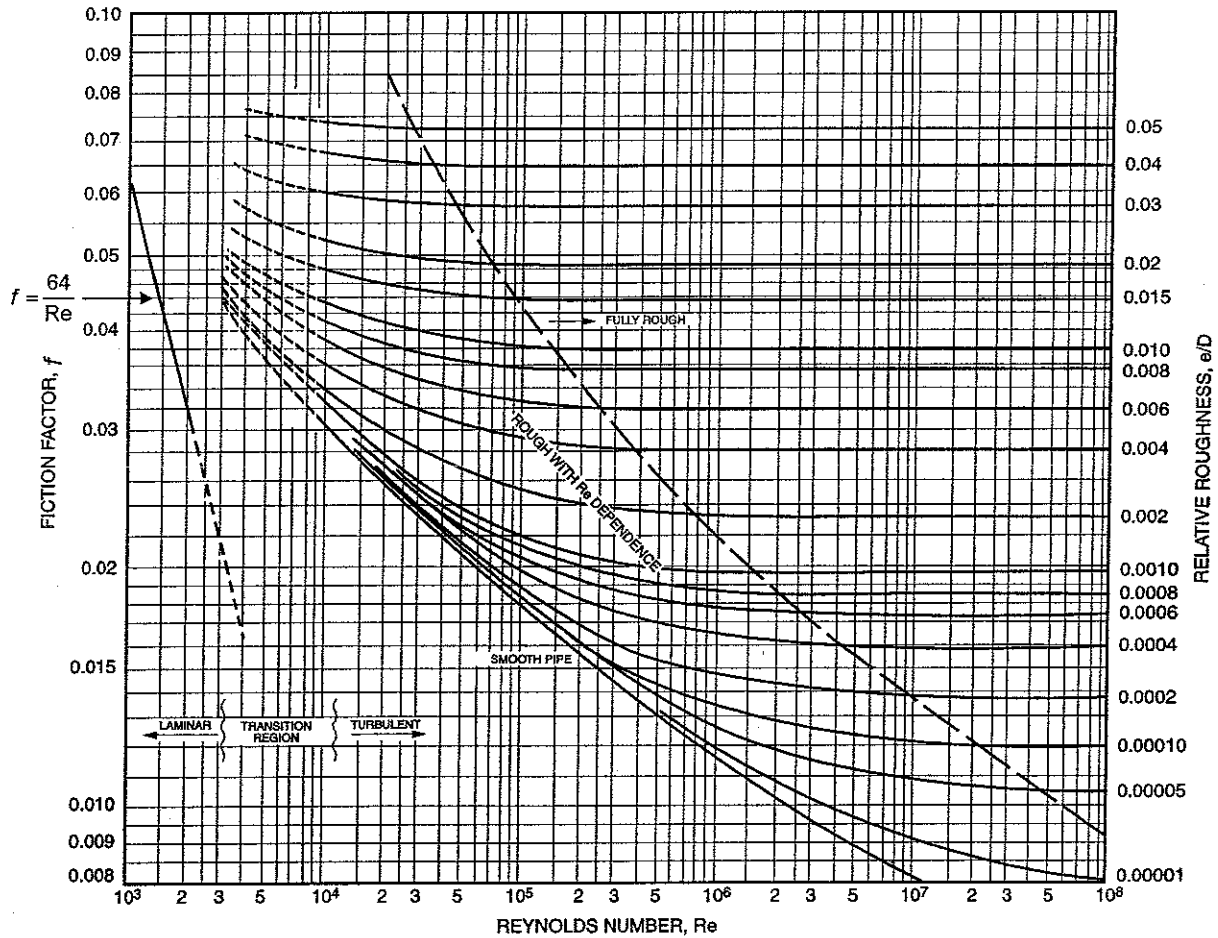
DRAG COEFFICIENTS FOR SPHERES, DISKS, AND CYLINDERS

$$C_D = \frac{24}{Re}, Re < 10$$

SPHERES

DISKS

CYLINDERS

REYNOLDS NUMBER $Re = \dfrac{DV\rho}{\mu}$

DRAG COEFFICIENT $C_D = \dfrac{2F_D}{\rho V^2 A}$

46

47
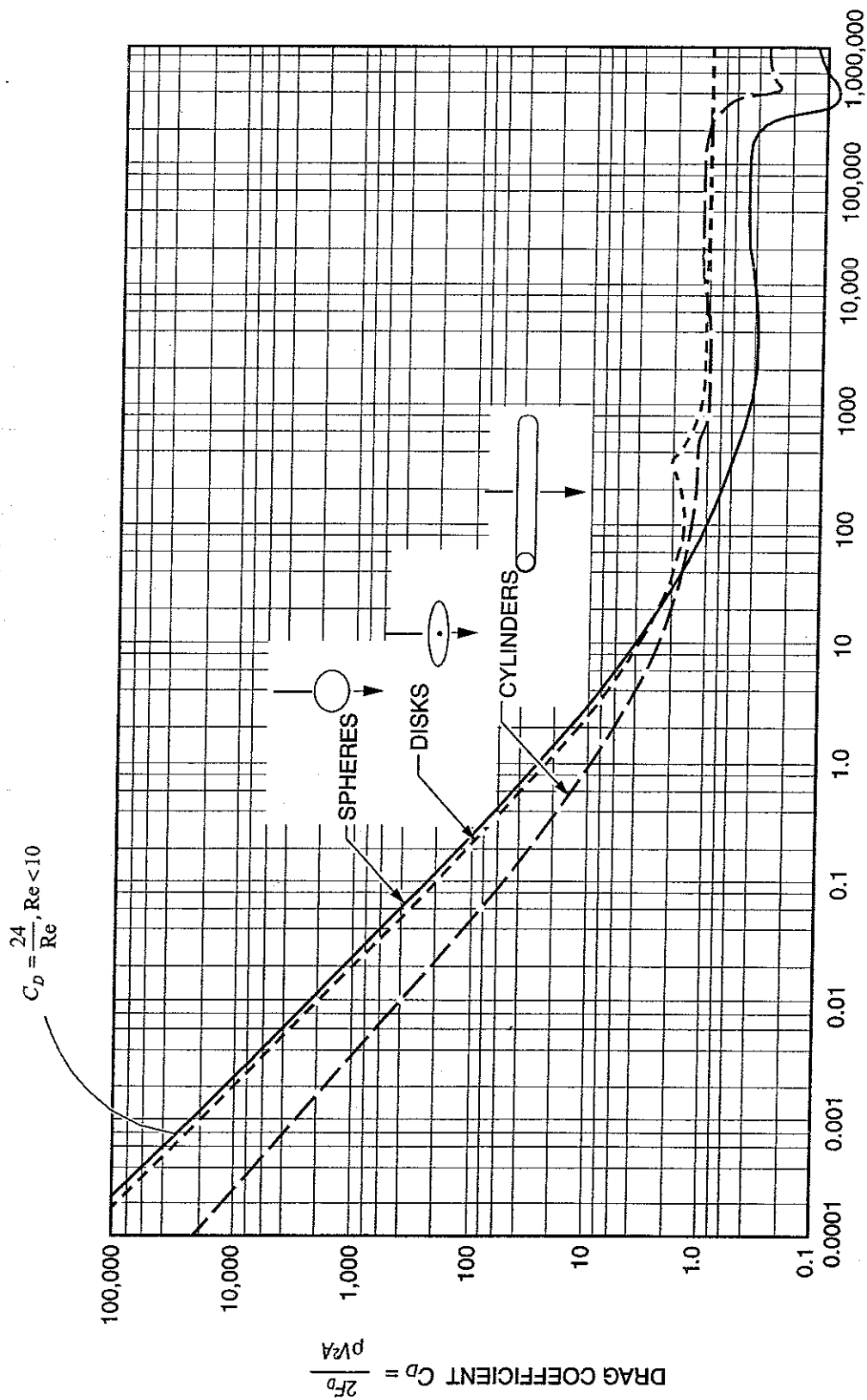
THERMODYNAMICS

PROPERTIES OF SINGLE-COMPONENT SYSTEMS

Nomenclature

1. Intensive properties are independent of mass.
2. Extensive properties are proportional to mass.
3. Specific properties are lower case (extensive/mass).

State Functions (properties)

Absolute Pressure, p (lbf/in² or Pa)

Absolute Temperature, T (°R or K)

Specific Volume, v (ft³/lbm or m³/kg)

Internal Energy, u (usually in Btu/lbm or kJ/kg)

Enthalpy, $h = u + Pv$ (same units as u)

Entropy, s [in Btu/(lbm-°R) or kJ/(kg·K)]

Gibbs Free Energy, $g = h - Ts$ (same units as u)

Helmholz Free Energy, $a = u - Ts$ (same units as u)

Heat Capacity at Constant Pressure, $c_p = \left(\dfrac{\partial h}{\partial T}\right)_P$

Heat Capacity at Constant Volume, $c_v = \left(\dfrac{\partial u}{\partial T}\right)_v$

Quality x (applies to liquid-vapor systems at saturation) is defined as the mass fraction of the vapor phase:

$x = m_g/(m_g + m_f)$, where

m_g = mass of vapor, and

m_f = mass of liquid.

Specific volume of a two-phase system can be written:

$v = xv_g + (1-x)v_f$ or $v = xv_{fg} + v_f$, where

v_f = specific volume of saturated liquid,

v_g = specific volume of saturated vapor, and

v_{fg} = specific volume change upon vaporization.

     $= v_g - v_f$.

Similar expressions exist for u, h, and s:

$u = xu_g + (1-x)\,u_f$

$h = xh_g + (1-x)\,h_f$

$s = xs_g + (1-x)\,s_f$

For a simple substance, *specification of any two intensive, independent properties is sufficient* to fix all the rest.

For an ideal gas, $Pv = RT$ or $PV = mRT$, and

     $P_1v_1/T_1 = P_2v_2/T_2$, where

p = pressure,

v = specific volume,

m = mass of gas,

R = gas constant, and

T = temperature.

R is *specific to each gas* but can be found from

$$R = \frac{\overline{R}}{(mol.\ wt.)}, \text{ where}$$

$\overline{R}$ = the universal gas constant

    = 1,545 ft-lbf/(lbmol-°R) = 8,314 J/(kmol·K).

For *Ideal Gases*, $c_P - c_v = R$

Also, for *Ideal Gases*:

$$\left(\frac{\partial h}{\partial v}\right)_T = 0 \qquad \left(\frac{\partial u}{\partial v}\right)_T = 0$$

For cold air standard, *heat capacities are assumed to be constant* at their room temperature values. In that case, the following are true:

$\Delta u = c_v\Delta T; \qquad\qquad \Delta h = c_P\,\Delta T$

$\Delta s = c_P \ln(T_2/T_1) - R \ln(P_2/P_1);$ and

$\Delta s = c_v \ln(T_2/T_1) + R \ln(v_2/v_1).$

For heat capacities that are temperature dependent, the value to be used in the above equations for Δh is known as the mean heat capacity $(\overline{c}_p)$ and is given by

$$\overline{c}_p = \frac{\int_{T_1}^{T_2} c_p\,dT}{T_2 - T_1}$$

Also, for *constant entropy* processes:

$P_1v_1{}^k = P_2v_2{}^k; \qquad T_1P_1{}^{(1-k)/k} = T_2P_2{}^{(1-k)/k}$

$T_1v_1{}^{(k-1)} = T_2v_2{}^{(k-1)}$, where $k = c_p/c_v$

FIRST LAW OF THERMODYNAMICS

The *First Law of Thermodynamics* is a statement of conservation of energy in a thermodynamic system. The net energy crossing the system boundary is equal to the change in energy inside the system.

Heat Q is *energy transferred* due to temperature difference and is considered positive if it is inward or added to the system.

Closed Thermodynamic System

(no mass crosses boundary)

$$Q - w = \Delta U + \Delta KE + \Delta PE$$

where

ΔKE = change in kinetic energy, and

ΔPE = change in potential energy.

Energy can cross the boundary only in the form of heat or work. Work can be boundary work, w_b, or other work forms (electrical work, etc.)

Work w is considered *positive if it is outward* or *work done by the system.*

Reversible boundary work is given by $w_b = \int P\,dv$.

Special Cases of Closed Systems

Constant Pressure (*Charles' Law*): $\qquad w_b = P\Delta v$

(ideal gas) $T/v = $ constant

Constant Volume: $\qquad w_b = 0$

(ideal gas) $T/P = $ constant

Isentropic (ideal gas), $\qquad Pv^k = $ constant:

$$w = (P_2v_2 - P_1v_1)/(1-k)$$
$$= R\,(T_2 - T_1)/(1-k)$$

Constant Temperature (*Boyle's Law*):

(ideal gas) $Pv = $ constant

$$w_b = RT\ln(v_2/v_1) = RT\ln(P_1/P_2)$$

Polytropic (ideal gas), $\qquad Pv^n = $ constant:

$$w = (P_2v_2 - P_1v_1)/(1-n)$$

Open Thermodynamic System

(allowing mass to cross the boundary)

There is flow work (PV) done by mass entering the system. The reversible flow work is given by:

$$w_{rev} = -\int v\,dP + \Delta KE + \Delta PE$$

First Law applies whether or not processes are reversible.

FIRST LAW (energy balance)

$$\Sigma \dot{m}[h_i + V_i^2/2 + gZ_i] - \Sigma \dot{m}[h_e + V_e^2/2 + gZ_e]$$
$$+ \dot{Q}_{in} - \dot{W}_{net} = d(m_s u_s)/dt, \text{ where}$$

$\dot{W}_{net}$ = rate of net or shaft work transfer,

m_s = mass of fluid within the system,

u_s = specific internal energy of system, and

$\dot{Q}$ = rate of heat transfer (neglecting kinetic and potential energy).

Special Cases of Open Systems

Constant Volume: $\qquad w_{rev} = -v\,(P_2 - P_1)$

Constant Pressure: $\qquad w_{rev} = 0$

Constant Temperature:

(ideal gas) $Pv = $ constant

$$w_{rev} = RT\ln(v_2/v_1) = RT\ln(P_1/P_2)$$

Isentropic (ideal gas): $\qquad Pv^k = $ constant:

$$w_{rev} = k\,(P_2v_2 - P_1v_1)/(1-k)$$
$$= kR\,(T_2 - T_1)/(1-k)$$
$$w_{rev} = \frac{k}{k-1}RT_i\left[1 - \left(\frac{P_2}{P_1}\right)^{(k-1)/k}\right]$$

Polytropic: $\qquad Pv^n = $ constant

$$w_{rev} = n\,(P_2v_2 - P_1v_1)/(1-n)$$

Steady-State Systems

The system does not change state with time. This assumption is valid for steady operation of turbines, pumps, compressors, throttling valves, nozzles, and heat exchangers, including boilers and condensers.

$$\Sigma \dot{m}_i\left(h_i + V_i^2/2 + gZ_i\right) - \Sigma \dot{m}_e\left(h_e + V_e^2/2 + gZ_e\right)$$
$$+ \dot{Q}_{in} - \dot{W}_{out} = 0 \quad \text{and}$$
$$\Sigma \dot{m}_i = \Sigma \dot{m}_e$$

where

$\dot{m}$ = mass flow rate (subscripts i and e refer to inlet and exit states of system),

g = acceleration of gravity,

Z = elevation,

V = velocity, and

$\dot{w}$ = rate of work.

Special Cases of Steady-Flow Energy Equation

Nozzles, Diffusers: Velocity terms are significant. No elevation change, no heat transfer, and no work. Single mass stream.

$$h_i + V_i^2/2 = h_e + V_e^2/2$$

Efficiency (nozzle) $= \dfrac{V_e^2 - V_i^2}{2(h_i - h_{es})}$, where

h_{es} = enthalpy at isentropic exit state.

Turbines, Pumps, Compressors: Often considered adiabatic (no heat transfer). Velocity terms usually can be ignored. There are significant work terms and a single mass stream.

$$h_i = h_e + w$$

Efficiency (turbine) $= \dfrac{h_i - h_e}{h_i - h_{es}}$

Efficiency (compressor, pump) $= \dfrac{h_{es} - h_i}{h_e - h_i}$

Throttling Valves and Throttling Processes: No work, no heat transfer, and single-mass stream. Velocity terms often insignificant.

$$h_i = h_e$$

Boilers, Condensers, Evaporators, One Side in a Heat Exchanger: Heat transfer terms are significant. For a single-mass stream, the following applies:

$$h_i + q = h_e$$

Heat Exchangers: No heat or work. Two separate flow rates $\dot{m}_1$ and $\dot{m}_2$:

$$\dot{m}_1\left(h_{1i} - h_{1e}\right) = \dot{m}_2\left(h_{2e} - h_{2i}\right)$$

Mixers, Separators, Open or Closed Feedwater Heaters:

$$\Sigma \dot{m}_i h_i = \Sigma \dot{m}_e h_e \quad \text{and}$$
$$\Sigma \dot{m}_i = \Sigma \dot{m}_e$$

BASIC CYCLES

Heat engines take in heat Q_H at a high temperature T_H, produce a net amount of work w, and reject heat Q_L at a low temperature T_L. The efficiency η of a heat engine is given by:

$$\eta = w/Q_H = (Q_H - Q_L)/Q_H$$

The most efficient engine possible is the *Carnot Cycle*. Its efficiency is given by:

$$\eta_c = (T_H - T_L)/T_H, \text{ where}$$

T_H and T_L = absolute temperatures (Kelvin or Rankine).

The following heat-engine cycles are plotted on P-v and T-s diagrams (see page 52):

Carnot, Otto, Rankine

Refrigeration Cycles are the reverse of heat-engine cycles. Heat is moved from low to high temperature requiring work W. Cycles can be used either for refrigeration or as heat pumps.

Coefficient of Performance (COP) is defined as:

$$\text{COP} = Q_H/W \text{ for heat pump, and as}$$

$$\text{COP} = Q_L/W \text{ for refrigerators and air conditioners.}$$

Upper limit of COP is based on reversed Carnot Cycle:

$$\text{COP}_c = T_H/(T_H - T_L) \text{ for heat pump and}$$

$$\text{COP}_c = T_L/(T_H - T_L) \text{ for refrigeration.}$$

1 ton refrigeration = 12,000 Btu/hr = 3,516 W

IDEAL GAS MIXTURES

$i = 1, 2, \ldots, n$ constituents. Each constituent is an ideal gas.

Mole Fraction: N_i = number of moles of component i.

$$x_i = N_i/N; N = \Sigma N_i; \Sigma x_i = 1$$

Mass Fraction: $y_i = m_i/m; m = \Sigma m_i; \Sigma y_i = 1$

Molecular Weight: $M = m/N = \Sigma x_i M_i$

Gas Constant: $R = \overline{R}/M$

To convert *mole fractions to mass fractions*:

$$y_i = \frac{x_i M_i}{\Sigma (x_i M_i)}$$

To convert *mass fractions to mole fractions*:

$$x_i = \frac{y_i/M_i}{\Sigma (y_i/M_i)}$$

Partial Pressures $\quad p = \Sigma p_i; p_i = \dfrac{m_i R_i T}{V}$

Partial Volumes $\quad V = \Sigma V_i; V_i = \dfrac{m_i R_i T}{p}$, where

p, V, T = the pressure, volume, and temperature of the mixture.

$$x_i = p_i/p = V_i/V$$

Other Properties

$u = \Sigma (y_i u_i); h = \Sigma (y_i h_i); s = \Sigma (y_i s_i)$

u_i and h_i are evaluated at T, and

s_i is evaluated at T and p_i.

PSYCHROMETRICS

We deal here with a mixture of dry air (subscript a) and water vapor (subscript v):

$$p = p_a + p_v$$

Specific Humidity (absolute humidity) ω:

$$\omega = m_v/m_a, \text{ where}$$

m_v = mass of water vapor and

m_a = mass of dry air.

$$\omega = 0.622 p_v/p_a = 0.622 p_v/(p - p_v)$$

Relative Humidity ϕ:

$$\phi = m_v/m_g = p_v/p_g, \text{ where}$$

m_g = mass of vapor at saturation, and

p_g = saturation pressure at T.

Enthalpy h: $h = h_a + \omega h_v$

Dew-Point Temperature T_{dp}:

$$T_{dp} = T_{sat} \text{ at } p_g = p_v$$

Wet-bulb temperature T_{wb} is the temperature indicated by a thermometer covered by a wick saturated with liquid water and in contact with moving air.

Humidity Volume: Volume of moist air/mass of dry air.

Psychrometric Chart

A plot of specific humidity as a function of dry-bulb temperature plotted for a value of atmospheric pressure. (See chart at end of section.)

PHASE RELATIONS

Clapeyron Equation for Phase Transitions:

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}} = \frac{s_{fg}}{v_{fg}}, \text{ where}$$

h_{fg} = enthalpy change for phase transitions,

v_{fg} = volume change,

s_{fg} = entropy change,

T = absolute temperature, and

$(dP/dT)_{sat}$ = slope of vapor-liquid saturation line.

Gibbs Phase Rule

$$P + F = C + 2, \text{ where}$$

P = number of phases making up a system,

F = degrees of freedom, and

C = number of components in a system.

Gibbs Free Energy

Energy released or absorbed in a reaction occurring reversibly at constant pressure and temperature ΔG.

Helmholtz Free Energy

Energy released or absorbed in a reaction occurring reversibly at constant volume and temperature ΔA.

COMBUSTION PROCESSES

First, the combustion equation should be written and balanced. For example, for the stoichiometric combustion of methane in oxygen:

$$CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$$
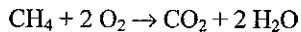
Combustion in Air

For each mole of oxygen, there will be 3.76 moles of nitrogen. For stoichiometric combustion of methane in air:

$$CH_4 + 2\,O_2 + 2(3.76)\,N_2 \rightarrow CO_2 + 2\,H_2O + 7.52\,N_2$$
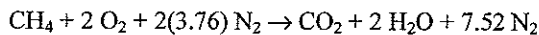
Combustion in Excess Air

The excess oxygen appears as oxygen on the right side of the combustion equation.

Incomplete Combustion

Some carbon is burned to create carbon monoxide (CO).

Air-Fuel Ratio (A/F): $A/F = \dfrac{\text{mass of air}}{\text{mass of fuel}}$

Stoichiometric (theoretical) air-fuel ratio is the air-fuel ratio calculated from the stoichiometric combustion equation.

$$\text{Percent Theoretical Air} = \frac{(A/F)_{\text{actual}}}{(A/F)_{\text{stoichiometric}}} \times 100$$

$$\text{Percent Excess Air} = \frac{(A/F)_{\text{actual}} - (A/F)_{\text{stoichiometric}}}{(A/F)_{\text{stoichiometric}}} \times 100$$

SECOND LAW OF THERMODYNAMICS

Thermal Energy Reservoirs

$$\Delta S_{\text{reservoir}} = Q/T_{\text{reservoir}} \text{ , where}$$

Q is measured with respect to the reservoir.

Kelvin-Planck Statement of Second Law

No heat engine can operate in a cycle while transferring heat with a single heat reservoir.

COROLLARY to Kelvin-Planck: No heat engine can have a higher efficiency than a Carnot cycle operating between the same reservoirs.

Clausius' Statement of Second Law

No refrigeration or heat pump cycle can operate without a net work input.

COROLLARY: No refrigerator or heat pump can have a higher COP than a Carnot cycle refrigerator or heat pump.

VAPOR-LIQUID MIXTURES

Henry's Law at Constant Temperature

At equilibrium, the partial pressure of a gas is proportional to its concentration in a liquid. Henry's Law is valid for low concentrations; i.e., $x \approx 0$.

$$p_i = p y_i = h x_i, \text{ where}$$

h = Henry's Law constant,

p_i = partial pressure of a gas in contact with a liquid,

x_i = mol fraction of the gas in the liquid,

y_i = mol fraction of the gas in the vapor, and

p = total pressure.

Raoult's Law for Vapor-Liquid Equilibrium

Valid for concentrations near 1; i.e., $x_i \approx 1$.

$$p_i = x_i p_i^{\,*}, \text{ where}$$

p_i = partial pressure of component i,

x_i = mol fraction of component i in the liquid, and

$p_i^{\,*}$ = vapor pressure of pure component i at the temperature of the mixture.

ENTROPY

$$ds = (1/T)\,\delta Q_{\text{rev}}$$

$$s_2 - s_1 = \int_1^2 (1/T)\,\delta Q_{\text{rev}}$$

Inequality of Clausius

$$\oint (1/T)\,\delta Q_{\text{rev}} \le 0$$

$$\int_1^2 (1/T)\,\delta Q \le s_2 - s_1$$

Isothermal, Reversible Process

$$\Delta s = s_2 - s_1 = Q/T$$

Isentropic process

$$\Delta s = 0;\ ds = 0$$

A reversible adiabatic process is isentropic.

Adiabatic Process

$$\delta Q = 0;\ \Delta s \ge 0$$

Increase of Entropy Principle

$$\Delta s_{\text{total}} = \Delta s_{\text{system}} + \Delta s_{\text{surroundings}} \ge 0$$

$$\Delta \dot{s}_{\text{total}} = \sum \dot{m}_{\text{out}} s_{\text{out}} - \sum \dot{m}_{\text{in}} s_{\text{in}}$$

$$- \sum \left(\dot{Q}_{\text{external}} / T_{\text{external}} \right) \ge 0$$

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Temperature-Entropy (*T-s*) Diagram

$$Q_{rev} = \int_1^2 T\, ds$$
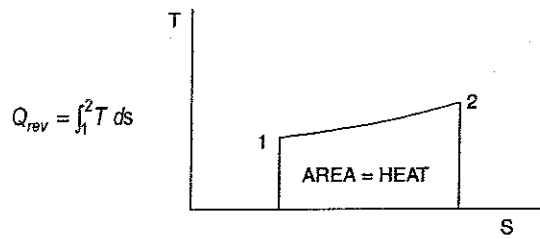
Entropy Change for Solids and Liquids

$$ds = c\,(dT/T)$$

$$s_2 - s_1 = \int c\,(dT/T) = c_{mean}\ln\,(T_2/T_1),$$

where c equals the heat capacity of the solid or liquid.

Irreversibility

$$I = w_{rev} - w_{actual}$$

Closed-System Availability

(no chemical reactions)

$$\phi = (u - u_o) - T_o\,(s - s_o) + p_o\,(v - v_o)$$

$$w_{reversible} = \phi_1 - \phi_2$$

Open-System Availability

$$\psi = (h - h_o) - T_o\,(s - s_o) + V^2/2 + gz$$

$$w_{reversible} = \psi_1 - \psi_2$$

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COMMON THERMODYNAMIC CYCLES

Carnot

$$\eta = 1 - \frac{T_L}{T_H}$$
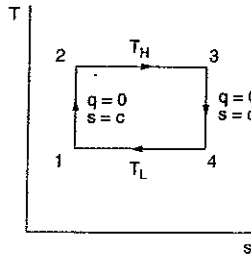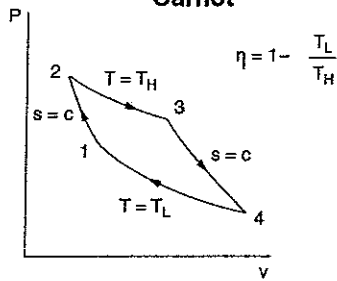
Reversed Carnot
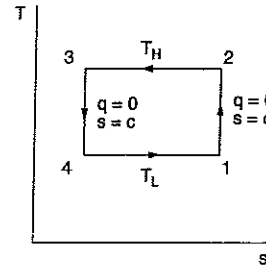
Otto

(gasoline engine)

$$\eta = 1 - r^{1-k}$$

$$r = v_1/v_2$$

Rankine

$$\eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

Refrigeration

(Reversed Rankine Cycle)

$$COP_{ref} = \frac{h_1 - h_4}{h_2 - h_1} \qquad COP_{HP} = \frac{h_2 - h_3}{h_2 - h_1}$$

Saturated Water - Temperature Table

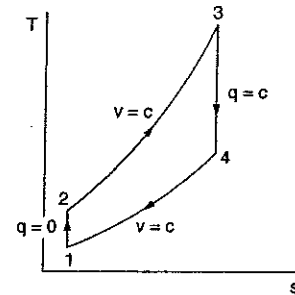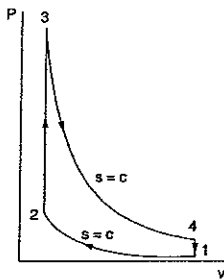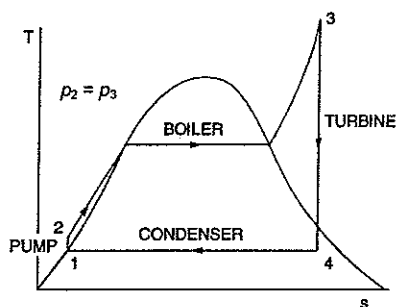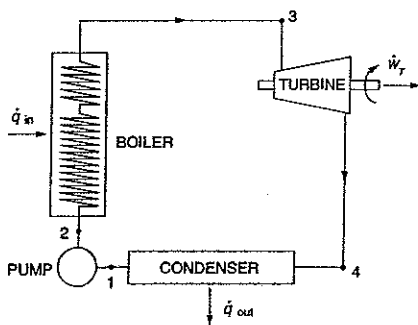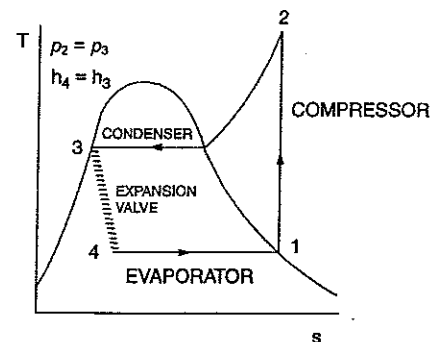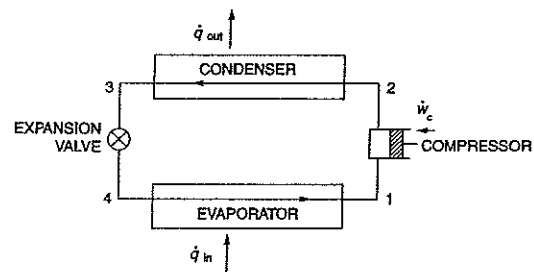| Temp. °C T	Sat. Press. kPa p_{sat}	Specific Volume m³/kg		Internal Energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/(kg·K)		
		Sat. liquid v_f	Sat. vapor v_g	Sat. liquid u_f	Evap. u_{fg}	Sat. vapor u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapor h_g	Sat. liquid s_f	Evap. s_{fg}	Sat. vapor s_g
0.01	0.6113	0.001 000	206.14	0.00	2375.3	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	9.1562
5	0.8721	0.001 000	147.12	20.97	2361.3	2382.3	20.98	2489.6	2510.6	0.0761	8.9496	9.0257
10	1.2276	0.001 000	106.38	42.00	2347.2	2389.2	42.01	2477.7	2519.8	0.1510	8.7498	8.9008
15	1.7051	0.001 001	77.93	62.99	2333.1	2396.1	62.99	2465.9	2528.9	0.2245	8.5569	8.7814
20	2.339	0.001 002	57.79	83.95	2319.0	2402.9	83.96	2454.1	2538.1	0.2966	8.3706	8.6672
25	3.169	0.001 003	43.36	104.88	2304.9	2409.8	104.89	2442.3	2547.2	0.3674	8.1905	8.5580
30	4.246	0.001 004	32.89	125.78	2290.8	2416.6	125.79	2430.5	2556.3	0.4369	8.0164	8.4533
35	5.628	0.001 006	25.22	146.67	2276.7	2423.4	146.68	2418.6	2565.3	0.5053	7.8478	8.3531
40	7.384	0.001 008	19.52	167.56	2262.6	2430.1	167.57	2406.7	2574.3	0.5725	7.6845	8.2570
45	9.593	0.001 010	15.26	188.44	2248.4	2436.8	188.45	2394.8	2583.2	0.6387	7.5261	8.1648
50	12.349	0.001 012	12.03	209.32	2234.2	2443.5	209.33	2382.7	2592.1	0.7038	7.3725	8.0763
55	15.758	0.001 015	9.568	230.21	2219.9	2450.1	230.23	2370.7	2600.9	0.7679	7.2234	7.9913
60	19.940	0.001 017	7.671	251.11	2205.5	2456.6	251.13	2358.5	2609.6	0.8312	7.0784	7.9096
65	25.03	0.001 020	6.197	272.02	2191.1	2463.1	272.06	2346.2	2618.3	0.8935	6.9375	7.8310
70	31.19	0.001 023	5.042	292.95	2176.6	2569.6	292.98	2333.8	2626.8	0.9549	6.8004	7.7553
75	38.58	0.001 026	4.131	313.90	2162.0	2475.9	313.93	2321.4	2635.3	1.0155	6.6669	7.6824
80	47.39	0.001 029	3.407	334.86	2147.4	2482.2	334.91	2308.8	2643.7	1.0753	6.5369	7.6122
85	57.83	0.001 033	2.828	355.84	2132.6	2488.4	355.90	2296.0	2651.9	1.1343	6.4102	7.5445
90	70.14	0.001 036	2.361	376.85	2117.7	2494.5	376.92	2283.2	2660.1	1.1925	6.2866	7.4791
95	84.55	0.001 040	1.982	397.88	2102.7	2500.6	397.96	2270.2	2668.1	1.2500	6.1659	7.4159
	MPa											
100	0.101 35	0.001 044	1.6729	418.94	2087.6	2506.5	419.04	2257.0	2676.1	1.3069	6.0480	7.3549
105	0.120 82	0.001 048	1.4194	440.02	2072.3	2512.4	440.15	2243.7	2683.8	1.3630	5.9328	7.2958
110	0.143 27	0.001 052	1.2102	461.14	2057.0	2518.1	461.30	2230.2	2691.5	1.4185	5.8202	7.2387
115	0.169 06	0.001 056	1.0366	482.30	2041.4	2523.7	482.48	2216.5	2699.0	1.4734	5.7100	7.1833
120	0.198 53	0.001 060	0.8919	503.50	2025.8	2529.3	503.71	2202.6	2706.3	1.5276	5.6020	7.1296
125	0.2321	0.001 065	0.7706	524.74	2009.9	2534.6	524.99	2188.5	2713.5	1.5813	5.4962	7.0775
130	0.2701	0.001 070	0.6685	546.02	1993.9	2539.9	546.31	2174.2	2720.5	1.6344	5.3925	7.0269
135	0.3130	0.001 075	0.5822	567.35	1977.7	2545.0	567.69	2159.6	2727.3	1.6870	5.2907	6.9777
140	0.3613	0.001 080	0.5089	588.74	1961.3	2550.0	589.13	2144.7	2733.9	1.7391	5.1908	6.9299
145	0.4154	0.001 085	0.4463	610.18	1944.7	2554.9	610.63	2129.6	2740.3	1.7907	5.0926	6.8833
150	0.4758	0.001 091	0.3928	631.68	1927.9	2559.5	632.20	2114.3	2746.5	1.8418	4.9960	6.8379
155	0.5431	0.001 096	0.3468	653.24	1910.8	2564.1	653.84	2098.6	2752.4	1.8925	4.9010	6.7935
160	0.6178	0.001 102	0.3071	674.87	1893.5	2568.4	675.55	2082.6	2758.1	1.9427	4.8075	6.7502
165	0.7005	0.001 108	0.2727	696.56	1876.0	2572.5	697.34	2066.2	2763.5	1.9925	4.7153	6.7078
170	0.7917	0.001 114	0.2428	718.33	1858.1	2576.5	719.21	2049.5	2768.7	2.0419	4.6244	6.6663
175	0.8920	0.001 121	0.2168	740.17	1840.0	2580.2	741.17	2032.4	2773.6	2.0909	4.5347	6.6256
180	1.0021	0.001 127	0.194 05	762.09	1821.6	2583.7	763.22	2015.0	2778.2	2.1396	4.4461	6.5857
185	1.1227	0.001 134	0.174 09	784.10	1802.9	2587.0	785.37	1997.1	2782.4	2.1879	4.3586	6.5465
190	1.2544	0.001 141	0.156 54	806.19	1783.8	2590.0	807.62	1978.8	2786.4	2.2359	4.2720	6.5079
195	1.3978	0.001 149	0.141 05	828.37	1764.4	2592.8	829.98	1960.0	2790.0	2.2835	4.1863	6.4698
200	1.5538	0.001 157	0.127 36	850.65	1744.7	2595.3	852.45	1940.7	2793.2	2.3309	4.1014	6.4323
205	1.7230	0.001 164	0.115 21	873.04	1724.5	2597.5	875.04	1921.0	2796.0	2.3780	4.0172	6.3952
210	1.9062	0.001 173	0.104 41	895.53	1703.9	2599.5	897.76	1900.7	2798.5	2.4248	3.9337	6.3585
215	2.104	0.001 181	0.094 79	918.14	1682.9	2601.1	920.62	1879.9	2800.5	2.4714	3.8507	6.3221
220	2.318	0.001 190	0.086 19	940.87	1661.5	2602.4	943.62	1858.5	2802.1	2.5178	3.7683	6.2861
225	2.548	0.001 199	0.078 49	963.73	1639.6	2603.3	966.78	1836.5	2803.3	2.5639	3.6863	6.2503
230	2.795	0.001 209	0.071 58	986.74	1617.2	2603.9	990.12	1813.8	2804.0	2.6099	3.6047	6.2146
235	3.060	0.001 219	0.065 37	1009.89	1594.2	2604.1	1013.62	1790.5	2804.2	2.6558	3.5233	6.1791
240	3.344	0.001 229	0.059 76	1033.21	1570.8	2604.0	1037.32	1766.5	2803.8	2.7015	3.4422	6.1437
245	3.648	0.001 240	0.054 71	1056.71	1546.7	2603.4	1061.23	1741.7	2803.0	2.7472	3.3612	6.1083
250	3.973	0.001 251	0.050 13	1080.39	1522.0	2602.4	1085.36	1716.2	2801.5	2.7927	3.2802	6.0730
255	4.319	0.001 263	0.045 98	1104.28	1596.7	2600.9	1109.73	1689.8	2799.5	2.8383	3.1992	6.0375
260	4.688	0.001 276	0.042 21	1128.39	1470.6	2599.0	1134.37	1662.5	2796.9	2.8838	3.1181	6.0019
265	5.081	0.001 289	0.038 77	1152.74	1443.9	2596.6	1159.28	1634.4	2793.6	2.9294	3.0368	5.9662
270	5.499	0.001 302	0.035 64	1177.36	1416.3	2593.7	1184.51	1605.2	2789.7	2.9751	2.9551	5.9301
275	5.942	0.001 317	0.032 79	1202.25	1387.9	2590.2	1210.07	1574.9	2785.0	3.0208	2.8730	5.8938
280	6.412	0.001 332	0.030 17	1227.46	1358.7	2586.1	1235.99	1543.6	2779.6	3.0668	2.7903	5.8571
285	6.909	0.001 348	0.027 77	1253.00	1328.4	2581.4	1262.31	1511.0	2773.3	3.1130	2.7070	5.8199
290	7.436	0.001 366	0.025 57	1278.92	1297.1	2576.0	1289.07	1477.1	2766.2	3.1594	2.6227	5.7821
295	7.993	0.001 384	0.023 54	1305.2	1264.7	2569.9	1316.3	1441.8	2758.1	3.2062	2.5375	5.7437
300	8.581	0.001 404	0.021 67	1332.0	1231.0	2563.0	1344.0	1404.9	2749.0	3.2534	2.4511	5.7045
305	9.202	0.001 425	0.019 948	1359.3	1195.9	2555.2	1372.4	1366.4	2738.7	3.3010	2.3633	5.6643
310	9.856	0.001 447	0.018 350	1387.1	1159.4	2546.4	1401.3	1326.0	2727.3	3.3493	2.2737	5.6230
315	10.547	0.001 472	0.016 867	1415.5	1121.1	2536.6	1431.0	1283.5	2714.5	3.3982	2.1821	5.5804
320	11.274	0.001 499	0.015 488	1444.6	1080.9	2525.5	1461.5	1238.6	2700.1	3.4480	2.0882	5.5362
330	12.845	0.001 561	0.012 996	1505.3	993.7	2498.9	1525.3	1140.6	2665.9	3.5507	1.8909	5.4417
340	14.586	0.001 638	0.010 797	1570.3	894.3	2464.6	1594.2	1027.9	2622.0	3.6594	1.6763	5.3357
350	16.513	0.001 740	0.008 813	1641.9	776.6	2418.4	1670.6	893.4	2563.9	3.7777	1.4335	5.2112
360	18.651	0.001 893	0.006 945	1725.2	626.3	2351.5	1760.5	720.3	2481.0	3.9147	1.1379	5.0526
370	21.03	0.002 213	0.004 925	1844.0	384.5	2228.5	1890.5	441.6	2332.1	4.1106	0.6865	4.7971
374.14	22.09	0.003 155	0.003 155	2029.6	0	2029.6	2099.3	0	2099.3	4.4298	0	4.4298

Superheated Water Tables

T Temp. °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m³/kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)
	p = 0.01 MPa (45.81°C)				p = 0.05 MPa (81.33°C)			
Sat.	14.674	2437.9	2584.7	8.1502	3.240	2483.9	2645.9	7.5939
50	14.869	2443.9	2592.6	8.1749				
100	17.196	2515.5	2687.5	8.4479	3.418	2511.6	2682.5	7.6947
150	19.512	2587.9	2783.0	8.6882	3.889	2585.6	2780.1	7.9401
200	21.825	2661.3	2879.5	8.9038	4.356	2659.9	2877.7	8.1580
250	24.136	2736.0	2977.3	9.1002	4.820	2735.0	2976.0	8.3556
300	26.445	2812.1	3076.5	9.2813	5.284	2811.3	3075.5	8.5373
400	31.063	2968.9	3279.6	9.6077	6.209	2968.5	3278.9	8.8642
500	35.679	3132.3	3489.1	9.8978	7.134	3132.0	3488.7	9.1546
600	40.295	3302.5	3705.4	10.1608	8.057	3302.2	3705.1	9.4178
700	44.911	3479.6	3928.7	10.4028	8.981	3479.4	3928.5	9.6599
800	49.526	3663.8	4159.0	10.6281	9.904	3663.6	4158.9	9.8852
900	54.141	3855.0	4396.4	10.8396	10.828	3854.9	4396.3	10.0967
1000	58.757	4053.0	4640.6	11.0393	11.751	4052.9	4640.5	10.2964
1100	63.372	4257.5	4891.2	11.2287	12.674	4257.4	4891.1	10.4859
1200	67.987	4467.9	5147.8	11.4091	13.597	4467.8	5147.7	10.6662
1300	72.602	4683.7	5409.7	11.5811	14.521	4683.6	5409.6	10.8382
	p = 0.10 MPa (99.63°C)				p = 0.20 MPa (120.23°C)			
Sat.	1.6940	2506.1	2675.5	7.3594	0.8857	2529.5	2706.7	7.1272
100	1.6958	2506.7	2676.2	7.3614				
150	1.9364	2582.8	2776.4	7.6134	0.9596	2576.9	2768.8	7.2795
200	2.172	2658.1	2875.3	7.8343	1.0803	2654.4	2870.5	7.5066
250	2.406	2733.7	2974.3	8.0333	1.1988	2731.2	2971.0	7.7086
300	2.639	2810.4	3074.3	8.2158	1.3162	2808.6	3071.8	7.8926
400	3.103	2967.9	3278.2	8.5435	1.5493	2966.7	3276.6	8.2218
500	3.565	3131.6	3488.1	8.8342	1.7814	3130.8	3487.1	8.5133
600	4.028	3301.9	3704.4	9.0976	2.013	3301.4	3704.0	8.7770
700	4.490	3479.2	3928.2	9.3398	2.244	3478.8	3927.6	9.0194
800	4.952	3663.5	4158.6	9.5652	2.475	3663.1	4158.2	9.2449
900	5.414	3854.8	4396.1	9.7767	2.705	3854.5	4395.8	9.4566
1000	5.875	4052.8	4640.3	9.9764	2.937	4052.5	4640.0	9.6563
1100	6.337	4257.3	4891.0	10.1659	3.168	4257.0	4890.7	9.8458
1200	6.799	4467.7	5147.6	10.3463	3.399	4467.5	5147.5	10.0262
1300	7.260	4683.5	5409.5	10.5183	3.630	4683.2	5409.3	10.1982
	p = 0.40 MPa (143.63°C)				p = 0.60 MPa (158.85°C)			
Sat.	0.4625	2553.6	2738.6	6.8959	0.3157	2567.4	2756.8	6.7600
150	0.4708	2564.5	2752.8	6.9299				
200	0.5342	2646.8	2860.5	7.1706	0.3520	2638.9	2850.1	6.9665
250	0.5951	2726.1	2964.2	7.3789	0.3938	2720.9	2957.2	7.1816
300	0.6548	2804.8	3066.8	7.5662	0.4344	2801.0	3061.6	7.3724
350					0.4742	2881.2	3165.7	7.5464
400	0.7726	2964.4	3273.4	7.8985	0.5137	2962.1	3270.3	7.7079
500	0.8893	3129.2	3484.9	8.1913	0.5920	3127.6	3482.8	8.0021
600	1.0055	3300.2	3702.4	8.4558	0.6697	3299.1	3700.9	8.2674
700	1.1215	3477.9	3926.5	8.6987	0.7472	3477.0	3925.3	8.5107
800	1.2372	3662.4	4157.3	8.9244	0.8245	3661.8	4156.5	8.7367
900	1.3529	3853.9	4395.1	9.1362	0.9017	3853.4	4394.4	8.9486
1000	1.4685	4052.0	4639.4	9.3360	0.9788	4051.5	4638.8	9.1485
1100	1.5840	4256.5	4890.2	9.5256	1.0559	4256.1	4889.6	9.3381
1200	1.6996	4467.0	5146.8	9.7060	1.1330	4466.5	5146.3	9.5185
1300	1.8151	4682.8	5408.8	9.8780	1.2101	4682.3	5408.3	9.6906
	p = 0.80 MPa (170.43°C)				p = 1.00 MPa (179.91°C)			
Sat.	0.2404	2576.8	2769.1	6.6628	0.194 44	2583.6	2778.1	6.5865
200	0.2608	2630.6	2839.3	6.8158	0.2060	2621.9	2827.9	6.6940
250	0.2931	2715.5	2950.0	7.0384	0.2327	2709.9	2942.6	6.9247
300	0.3241	2797.2	3056.5	7.2328	0.2579	2793.2	3051.2	7.1229
350	0.3544	2878.2	3161.7	7.4089	0.2825	2875.2	3157.7	7.3011
400	0.3843	2959.7	3267.1	7.5716	0.3066	2957.3	3263.9	7.4651
500	0.4433	3126.0	3480.6	7.8673	0.3541	3124.4	3478.5	7.7622
600	0.5018	3297.9	3699.4	8.1333	0.4011	3296.8	3697.9	8.0290
700	0.5601	3476.2	3924.2	8.3770	0.4478	3475.3	3923.1	8.2731
800	0.6181	3661.1	4155.6	8.6033	0.4943	3660.4	4154.7	8.4996
900	0.6761	3852.8	4393.7	8.8153	0.5407	3852.2	4392.9	8.7118
1000	0.7340	4051.0	4638.2	9.0153	0.5871	4050.5	4637.6	8.9119
1100	0.7919	4255.6	4889.1	9.2050	0.6335	4255.1	4888.6	9.1017
1200	0.8497	4466.1	5145.9	9.3855	0.6798	4465.6	5145.4	9.2822
1300	0.9076	4681.8	5407.9	9.5575	0.7261	4681.3	5407.4	9.4543

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P-h DIAGRAM FOR REFRIGERANT HFC-134a

(metric units)

(Reproduced by permission of the DuPont Company)

ASHRAE PSYCHROMETRIC CHART NO. 1

(metric units)

Reproduced by permission of ASHRAE

ASHRAE PSYCHROMETRIC CHART NO. 1
NORMAL TEMPERATURE   SEA LEVEL
BAROMETRIC PRESSURE   101.325 kPa

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AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.
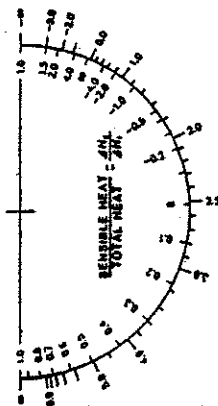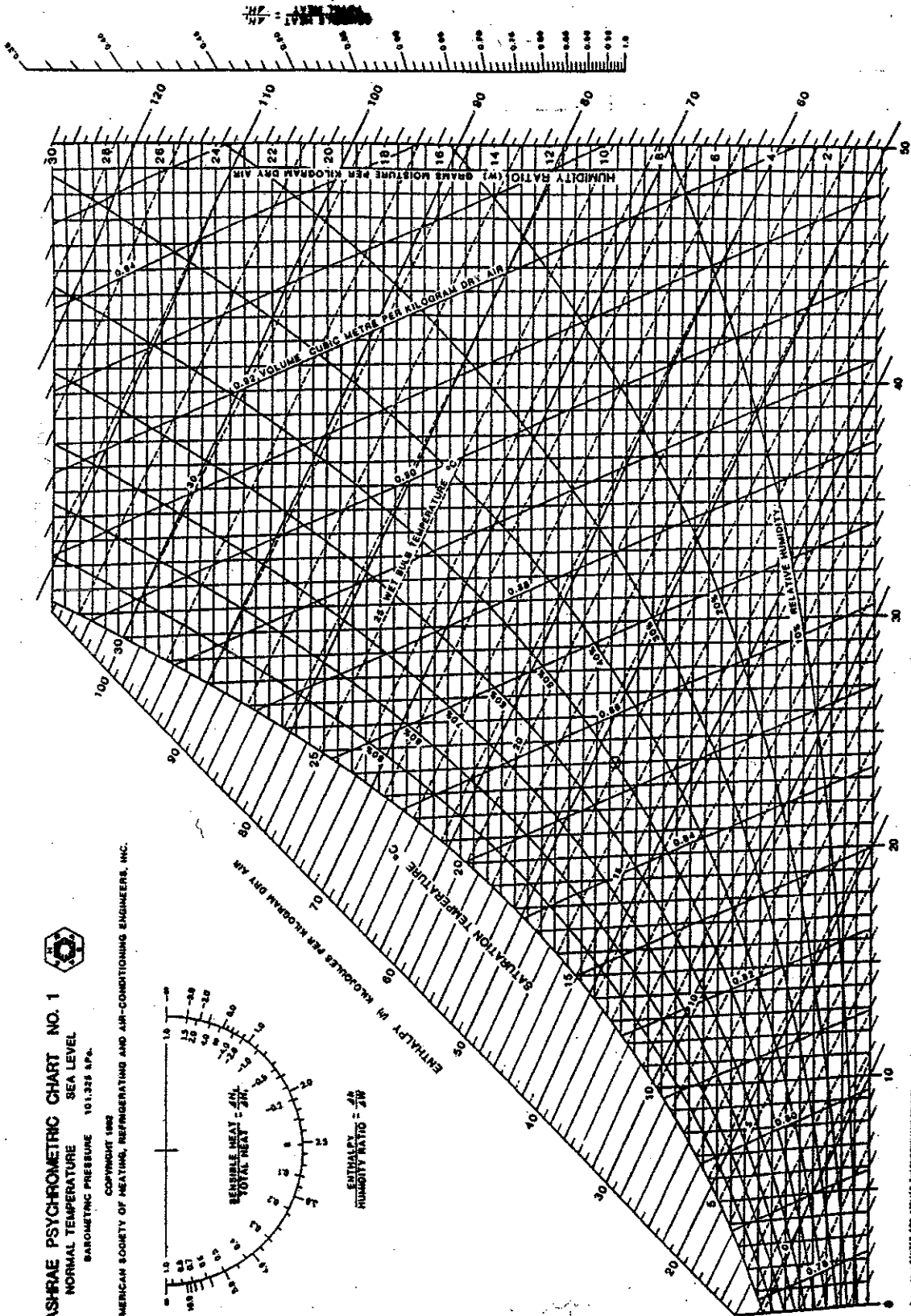
HEAT CAPACITY

(at Room Temperature)

Substance	Mol wt	c_p		c_v		k
		kJ/(kg·K)	Btu/(lbm-°R)	kJ/(kg·K)	Btu/(lbm-°R)	
Gases						
Air	29	1.00	0.240	0.718	0.171	1.40
Argon	40	0.520	0.125	0.312	0.0756	1.67
Butane	58	1.72	0.415	1.57	0.381	1.09
Carbon dioxide	44	0.846	0.203	0.657	0.158	1.29
Carbon monoxide	28	1.04	0.249	0.744	0.178	1.40
Ethane	30	1.77	0.427	1.49	0.361	1.18
Helium	4	5.19	1.25	3.12	0.753	1.67
Hydrogen	2	14.3	3.43	10.2	2.44	1.40
Methane	16	2.25	0.532	1.74	0.403	1.30
Neon	20	1.03	0.246	0.618	0.148	1.67
Nitrogen	28	1.04	0.248	0.743	0.177	1.40
Octane vapor	114	1.71	0.409	1.64	0.392	1.04
Oxygen	32	0.918	0.219	0.658	0.157	1.40
Propane	44	1.68	0.407	1.49	0.362	1.12
Steam	18	1.87	0.445	1.41	0.335	1.33

Substance	c_p		Density	
	kJ/(kg·K)	Btu/(lbm-°R)	kg/m³	lbm/ft³
Liquids				
Ammonia	4.80	1.146	602	38
Mercury	0.139	0.033	13,560	847
Water	4.18	1.000	997	62.4
Solids				
Aluminum	0.900	0.215	2,700	170
Copper	0.386	0.092	8,900	555
Ice (0°C; 32°F)	2.11	0.502	917	57.2
Iron	0.450	0.107	7,840	490
Lead	0.128	0.030	11,310	705

HEAT TRANSFER

There are three modes of heat transfer: conduction, convection, and radiation. Boiling and condensation are classified as convection.

Conduction

Fourier's Law of Conduction

$$\dot{Q} = -kA(dT/dx), \text{ where}$$

$\dot{Q}$ = rate of heat transfer.

Conduction Through a Plane Wall:
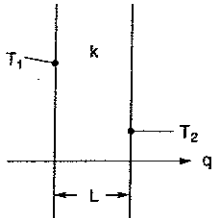
$$\dot{Q} = -kA(T_2 - T_1)/L, \text{ where}$$

k = the thermal conductivity of the wall,

A = the wall surface area,

L = the wall thickness, and

T_1, T_2 = the temperature on the near side and far side of the wall respectively.

Thermal resistance of the wall is given by

$$R = L/(kA)$$

Resistances in series are added.

Composite Walls:
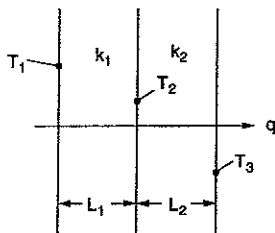
$R_{total} = R_1 + R_2$, where

R_1 = $L_1/(k_1A)$, and

R_2 = $L_2/(k_2A)$.

To Evaluate Surface or Intermediate Temperatures:

$$T_2 = T_1 - \dot{Q}R_1; T_3 = T_2 - \dot{Q}R_2$$

Conduction through a cylindrical wall is given by

CYLINDER (LENGTH = L)
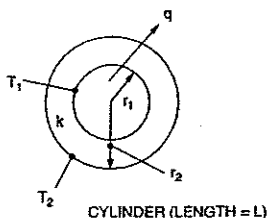
$$\dot{Q} = \frac{2\pi kL(T_1 - T_2)}{\ln(r_2/r_1)}$$

$$R = \frac{\ln(r_2/r_1)}{2\pi kL}$$

Convection

Convection is determined using a convection coefficient (heat transfer coefficient) h.

$$\dot{Q} = hA(T_w - T_\infty), \text{ where}$$

A = the heat transfer area,

T_w = work temperature, and

T_∞ = bulk fluid temperature.

Resistance due to convection is given by

$$R = 1/(hA)$$

FINS: For a straight fin,

$$\dot{Q} = \sqrt{hpkA_c}(T_b - T_\infty)\tanh mL_c, \text{ where}$$

h = heat transfer coefficient,

p = exposed perimeter,

k = thermal conductivity,

A_c = cross-sectional area,

T_b = temperature at base of fin,

T_∞ = fluid temperature,

m = $\sqrt{hp/(kA_c)}$, and

L_c = $L + A_c/p$, corrected length.
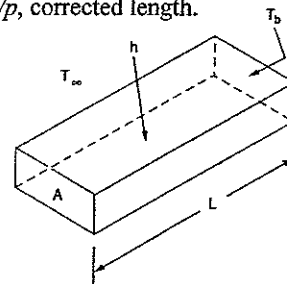
Radiation

The radiation emitted by a body is given by

$$\dot{Q} = \varepsilon\sigma AT^4, \text{ where}$$

T = the absolute temperature (K or °R),

σ = 5.67×10^{-8} W/(m²·K⁴)

    [0.173×10^{-8} Btu/(hr-ft²-°R⁴)],

ε = the emissivity of the body, and

A = the body surface area.

For a body (1) which is small compared to its surroundings (2)

$$\dot{Q}_{12} = \varepsilon\sigma A(T_1^4 - T_2^4), \text{ where}$$

$\dot{Q}_{12}$ = the net heat transfer rate from the body.

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A *black body* is defined as one which absorbs all energy incident upon it. It also emits radiation at the maximum rate for a body of a particular size at a particular temperature. For such a body

$$\alpha = \varepsilon = 1, \text{ where}$$

α = the absorptivity (energy absorbed/incident energy).

A *gray body* is one for which $\alpha = \varepsilon$, where

$$0 < \alpha < 1; \; 0 < \varepsilon < 1$$

Real bodies are frequently approximated as gray bodies.

The net energy exchange by radiation between two black bodies, which see each other, is given by

$$\dot{Q}_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4 \right), \text{ where}$$

F_{12} = the shape factor (view factor, configuration factor); $0 \leq F_{12} \leq 1$.

For any body, $\alpha + \rho + \tau = 1$, where

α = absorptivity,

ρ = reflectivity (ratio of energy reflected to incident energy), and

τ = transmissivity (ratio of energy transmitted to incident energy).

For an opaque body, $\alpha + \rho = 1$

For a gray body, $\varepsilon + \rho = 1$

The following is applicable to the PM examination for mechanical and chemical engineers.

The overall *heat-transfer coefficient for a shell-and-tube heat exchanger* is

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{t}{k A_{avg}} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}, \text{ where}$$

A = any convenient reference area (m^2),

A_{avg} = average of inside and outside area (for thin-walled tubes) (m^2),

A_i = inside area of tubes (m^2),

A_o = outside area of tubes (m^2),

h_i = *heat-transfer coefficient* for inside of tubes [W/(m^2·K)],

h_o = *heat-transfer coefficient* for outside of tubes [W/(m^2·K)],

k = *thermal conductivity* of tube material [W/(m·K)],

R_{fi} = *fouling factor* for inside of tube (m^2·K/W),

R_{fo} = *fouling factor* for outside of tube (m^2·K/W),

t = tube-wall thickness (m), and

U = *overall heat-transfer coefficient* based on area A and the log mean temperature difference [W/(m^2·K)].

The *log mean temperature difference* (LMTD) *for countercurrent flow in tubular heat exchangers* is

$$\Delta T_{lm} = \frac{\left(T_{Ho} - T_{Ci} \right) - \left(T_{Hi} - T_{Co} \right)}{\ln\left(\dfrac{T_{Ho} - T_{Ci}}{T_{Hi} - T_{Co}} \right)}$$

The *log mean temperature difference for concurrent* (parallel) *flow in tubular heat exchangers* is

$$\Delta T_{lm} = \frac{\left(T_{Ho} - T_{Co} \right) - \left(T_{Hi} - T_{Ci} \right)}{\ln\left(\dfrac{T_{Ho} - T_{Co}}{T_{Hi} - T_{Ci}} \right)}, \text{ where}$$

ΔT_{lm} = log mean temperature difference (K),

T_{Hi} = inlet temperature of the hot fluid (K),

T_{Ho} = outlet temperature of the hot fluid (K),

T_{Ci} = inlet temperature of the cold fluid (K), and

T_{Co} = outlet temperature of the cold fluid (K).

For individual heat-transfer coefficients of a fluid being heated or cooled in a tube, one pair of temperatures (either the hot or the cold) are the surface temperatures at the inlet and outlet of the tube.

Heat exchanger effectiveness =

$$\frac{\text{actual heat transfer}}{\text{max possible heat transfer}} = \frac{q}{q_{max}}$$

$$\varepsilon = \frac{C_H \left(T_{Hi} - T_{Ho} \right)}{C_{min} \left(T_{Hi} - T_{Ci} \right)}$$

or

$$\varepsilon = \frac{C_C \left(T_{Co} - T_{Ci} \right)}{C_{min} \left(T_{Hi} - T_{Ci} \right)}$$

Where C_{min} = smaller of C_c or C_H and $C = \dot{m} c_p$

Number of transfer units, $NTU = \dfrac{UA}{C_{min}}$

At a cross-section in a tube where heat is being transferred

$$\frac{\dot{Q}}{A} = h\left(T_w - T_b \right) = \left[k_f \left(\frac{dt}{dr} \right)_w \right]_{fluid}$$

$$= \left[k_m \left(\frac{dt}{dr} \right)_w \right]_{metal}, \text{ where}$$

$\dot{Q}/A$ = local inward radial heat flux (W/m^2),

h = local heat-transfer coefficient [W/(m^2·K)],

k_f = thermal conductivity of the fluid [W/(m·K)],

k_m = thermal conductivity of the tube metal [W/(m·K)],

$(dt/dr)_w$ = radial temperature gradient at the tube surface (K/m),

T_b = local bulk temperature of the fluid (K), and

T_w = local inside surface temperature of the tube (K).

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Rate of Heat Transfer in a Tubular Heat Exchanger

For the equations below, the following definitions along with definitions previously supplied are required.

D = inside diameter

Gz = Graetz number [RePr (D/L)],

Nu = Nusselt number (hD/k),

Pr = Prandtl number $(c_p\mu/k)$,

A = area upon which U is based (m^2),

F = configuration correction factor,

g = acceleration of gravity (9.81 m/s^2),

L = heated (or cooled) length of conduit or surface (m),

$\dot{Q}$ = inward rate of heat transfer (W),

T_s = temperature of the surface (K),

T_{sv} = temperature of saturated vapor (K), and

λ = heat of vaporization (J/kg).

$$\dot{Q} = UAF\Delta T_{lm}$$

Heat-transfer for laminar flow (Re < 2,000) in a closed conduit.

$$Nu = 3.66 + \frac{0.19Gz^{0.8}}{1 + 0.117Gz^{0.467}}$$

Heat-transfer for turbulent flow (Re > 10^4, Pr > 0.7) in a closed conduit (Sieder-Tate equation).

$$Nu = \frac{h_i D}{k_f} = 0.023Re^{0.8}Pr^{1/3}(\mu_b/\mu_w)^{0.14}, \text{ where}$$

μ_b = μ (T_b), and

μ_w = μ (T_w), and Re and Pr are evaluated at T_b.

For non-circular ducts, use the equivalent diameter.

The equivalent diameter is defined as

$$D_H = \frac{4\,(\text{cross - sectional area})}{\text{wetted perimeter}}$$

For a circular annulus ($D_o > D_i$) the equivalent diameter is

$$D_H = D_o - D_i$$

For liquid metals (0.003 < Pr < 0.05) flowing in closed conduits.

$Nu = 6.3 + 0.0167Re^{0.85}Pr^{0.93}$ (constant heat flux)

$Nu = 7.0 + 0.025Re^{0.8}Pr^{0.8}$ (constant wall temperature)

Heat-transfer coefficient for condensation of a pure vapor on a vertical surface.

$$\frac{hL}{k} = 0.943\left(\frac{L^3\rho^2 g\lambda}{k\mu(T_{sv} - T_s)}\right)^{0.25}$$

Properties other than λ are for the liquid and are evaluated at the average between T_{sv} and T_s.

For condensation outside horizontal tubes, change 0.943 to 0.73 and replace L with the tube outside diameter.

Heat Transfer to/from Bodies Immersed in a Large Body of Flowing Fluid

In all cases, evaluate fluid properties at average temperature between that of the body and that of the flowing fluid.

For flow parallel to a constant-temperature flat plate of length L (m)

$Nu = 0.648Re^{0.5}Pr^{1/3}$ (Re < 10^5)

$Nu = 0.0366Re^{0.8}Pr^{1/3}$ (Re > 10^5)

Use the plate length in the evaluation of the Nusselt and Reynolds numbers.

For flow perpendicular to the axis of a constant-temperature circular cylinder

$Nu = cRe^n Pr^{1/3}$ (values of c and n follow)

Use the cylinder diameter in the evaluation of the Nusselt and Reynolds numbers.

Re	n	c
1 – 4	0.330	0.989
4 – 40	0.385	0.911
40 – 4,000	0.466	0.683
4,000 – 40,000	0.618	0.193
40,000 – 250,000	0.805	0.0266

For flow past a constant-temperature sphere, $Nu = 2.0 + 0.60Re^{0.5}Pr^{1/3}$

(1 < Re < 70,000, 0.6 < Pr < 400)

Use the sphere diameter in the evaluation of the Nusselt and Reynolds numbers.

Conductive Heat Transfer

Steady Conduction with Internal Energy Generation

For one-dimensional steady conduction, the equation is

$$d^2T/dx^2 + \dot{Q}_{gen}/k = 0, \text{ where}$$

$\dot{Q}_{gen}$ = the heat generation rate per unit volume, and

k = the thermal conductivity.

For a plane wall:
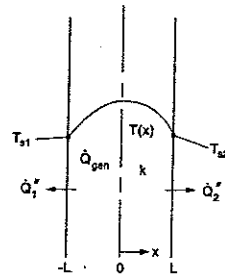
$$T(x) = \frac{\dot{Q}_{gen}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_{s2} - T_{s1}}{2}\right)\left(\frac{x}{L}\right) + \left(\frac{T_{s1} + T_{s2}}{2}\right)$$

$$\dot{Q}_1'' + \dot{Q}_2'' = 2\dot{Q}_{gen}L, \text{ where}$$

$$\dot{Q}_1'' = k(dT/dx)_{-L}$$
$$\dot{Q}_2'' = -k(dT/dx)_L$$

For a long circular cylinder:
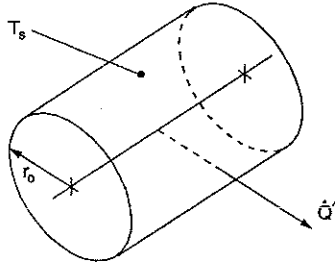
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{Q}_{gen}}{k}=0$$

$$T(r)=\frac{\dot{Q}_{gen}r_0^2}{4k}\left(1-\frac{r^2}{r_0^2}\right)+T_s$$

$$\dot{Q}'=\pi r_0^2\dot{Q}_{gen}\text{, where}$$

$\dot{Q}'$ = the heat-transfer rate from the cylinder per unit length.

Transient Conduction Using the Lumped Capacitance Method
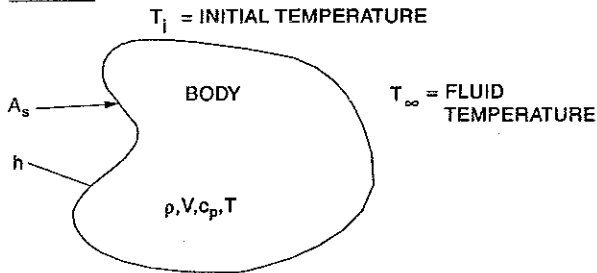
If the temperature may be considered uniform within the body at any time, the change of body temperature is given by

$$\dot{Q}=hA_s(T-T_\infty)=-\rho c_p V(dT/dt)$$

The temperature variation with time is

$$T-T_\infty=(T_i-T_\infty)e^{-(hA_s/\rho c_p V)t}$$

The total heat transferred up to time t is

$$Q_{total}=\rho c_P V(T_i-T)\text{, where}$$

ρ = density,

V = volume,

c_P = heat capacity,

t = time,

A_s = surface area of the body,

T = temperature, and

h = the heat-transfer coefficient.

The lumped capacitance method is valid if

$$\text{Biot number}=\text{Bi}=hV/kA_s\ll1$$

Natural (Free) Convection

For free convection between a vertical flat plate (or a vertical cylinder of sufficiently large diameter) and a large body of stationary fluid,

$$h=C\,(k/L)\,\text{Ra}_L{}^n\text{, where}$$

L = the length of the plate in the vertical direction,

Ra_L = Rayleigh Number = $\dfrac{g\beta(T_s-T_\infty)L^3}{v^2}\text{Pr}$,

T_s = surface temperature,

T_∞ = fluid temperature,

β = coefficient of thermal expansion ($\dfrac{2}{T_s+T_\infty}$ for an ideal gas where T is absolute temperature), and

v = kinematic viscosity.

Range of Ra_L	C	n
10^4-10^9	0.59	1/4
10^9-10^{13}	0.10	1/3

For free convection between a long horizontal cylinder and a large body of stationary fluid

$$h=C(k/D)\text{Ra}_D^n\text{, where}$$

$$\text{Ra}_D=\frac{g\beta(T_s-T_\infty)D^3}{v^2}\text{Pr}$$

Range of Ra_D	C	n
$10^{-3}-10^2$	1.02	0.148
10^2-10^4	0.850	0.188
10^4-10^7	0.480	0.250
10^7-10^{12}	0.125	0.333

Radiation

Two-Body Problem

Applicable to any two diffuse-gray surfaces that form an enclosure.

$$\dot{Q}_{12}=\frac{\sigma(T_1^4-T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1A_1}+\dfrac{1}{A_1F_{12}}+\dfrac{1-\varepsilon_2}{\varepsilon_2A_2}}$$

Generalized Cases
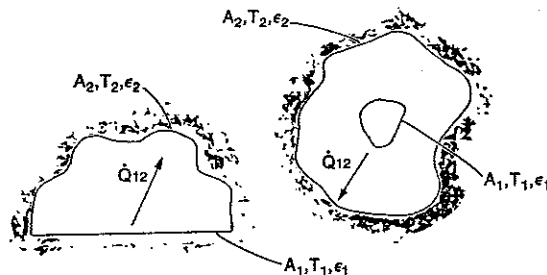
Radiation Shields

One-dimensional geometry with low-emissivity shield inserted between two parallel plates.
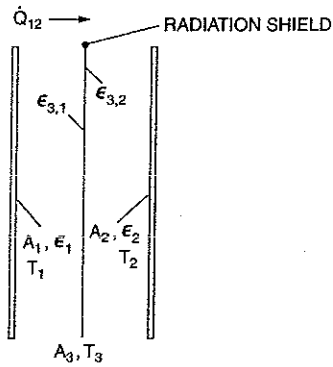
$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{13}} + \dfrac{1-\varepsilon_{3,1}}{\varepsilon_{3,1} A_3} + \dfrac{1-\varepsilon_{3,2}}{\varepsilon_{3,2} A_3} + \dfrac{1}{A_3 F_{32}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Shape Factor Relations

Reciprocity relations:

$$A_i F_{ij} = A_j F_{ji}$$

Summation rule:

$$\sum_{j=1}^{N} F_{ij} = 1$$

Reradiating Surface

Reradiating surfaces are considered to be insulated, or adiabatic $(\dot{Q}_R = 0)$.
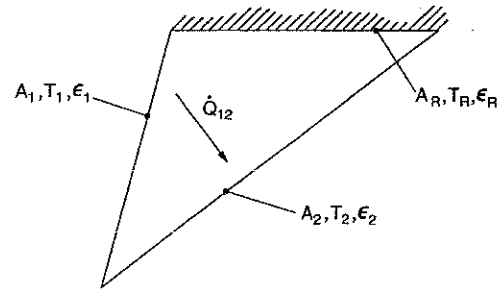
$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left[\left(\dfrac{1}{A_1 F_{1R}}\right) + \left(\dfrac{1}{A_2 F_{2R}}\right)\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

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TRANSPORT PHENOMENA

MOMENTUM, HEAT, AND MASS TRANSFER ANALOGY

For the equations which apply to **turbulent flow in circular tubes**, the following definitions apply:

Nu = Nusselt Number $\left[\dfrac{hD}{k}\right]$

Pr = Prandtl Number $(c_P\mu/k)$,

Re = Reynolds Number $(DV\rho/\mu)$,

Sc = Schmidt Number $[\mu/(\rho D_m)]$,

Sh = Sherwood Number $(k_m D/D_m)$,

St = Stanton Number $[h/(c_P G)]$,

c_m = concentration (mol/m^3),

c_P = heat capacity of fluid [J/(kg·K)],

D = tube inside diameter (m),

D_m = diffusion coefficient (m^2/s),

$(dc_m/dy)_w$ = concentration gradient at the wall (mol/m^4),

$(dT/dy)_w$ = temperature gradient at the wall (K/m),

$(dv/dy)_w$ = velocity gradient at the wall (s^{-1}),

f = Moody friction factor,

G = mass velocity [kg/(m^2·s)],

h = heat-transfer coefficient at the wall [W/(m^2·K)],

k = thermal conductivity of fluid [W/(m·K)],

k_m = mass-transfer coefficient (m/s),

L = length over which pressure drop occurs (m),

$(N/A)_w$ = inward mass-transfer flux at the wall [mol/(m^2·s)],

$\left(\dot{Q}/A\right)_w$ = inward heat-transfer flux at the wall (W/m^2),

y = distance measured from inner wall toward centerline (m),

Δc_m = concentration difference between wall and bulk fluid (mol/m^3),

ΔT = temperature difference between wall and bulk fluid (K),

μ = absolute dynamic viscosity (N·s/m^2), and

τ_w = shear stress (momentum flux) at the tube wall (N/m^2).

Definitions already introduced also apply.

Rate of transfer as a function of gradients at the wall

Momentum Transfer:

$$\tau_w = -\mu\left(\frac{dv}{dy}\right)_w = -\frac{f\rho V^2}{8} = \left(\frac{D}{4}\right)\left(-\frac{\Delta p}{L}\right)_f$$

Heat Transfer:

$$\left(\frac{\dot{Q}}{A}\right)_w = -k\left(\frac{dT}{dy}\right)_w$$

Mass Transfer in Dilute Solutions:

$$\left(\frac{N}{A}\right)_w = -D_m\left(\frac{dc_m}{dy}\right)_w$$

Rate of transfer in terms of coefficients

Momentum Transfer:

$$\tau_w = \frac{f\rho V^2}{8}$$

Heat Transfer:

$$\left(\frac{\dot{Q}}{A}\right)_w = h\Delta T$$

Mass Transfer:

$$\left(\frac{N}{A}\right)_w = k_m\Delta c_m$$

Use of friction factor (f) to predict heat-transfer and mass-transfer coefficients (turbulent flow)

Heat Transfer:

$$j_H = \left(\frac{\text{Nu}}{\text{Re Pr}}\right)\text{Pr}^{2/3} = \frac{f}{8}$$

Mass Transfer:

$$j_M = \left(\frac{\text{Sh}}{\text{Re Sc}}\right)\text{Sc}^{2/3} = \frac{f}{8}$$

CHEMISTRY

Avogadro's Number: The number of elementary particles in a mol of a substance.

$$1 \text{ mol} = 1 \text{ gram-mole}$$

$$1 \text{ mol} = 6.02 \times 10^{23} \text{ particles}$$

A *mol* is defined as an amount of a substance that contains as many particles as 12 grams of ^{12}C (carbon 12). The elementary particles may be atoms, molecules, ions, or electrons.

ACIDS AND BASES (aqueous solutions)

$$pH = \log_{10}\left(\frac{1}{[H^+]}\right), \text{ where}$$

$[H^+]$ = molar concentration of hydrogen ion,

Acids have pH < 7.

Bases have pH > 7.

ELECTROCHEMISTRY

Cathode – The electrode at which reduction occurs.

Anode – The electrode at which oxidation occurs.

Oxidation – The loss of electrons.

Reduction – The gaining of electrons.

Oxidizing Agent – A species that causes others to become oxidized.

Reducing Agent – A species that causes others to be reduced.

Cation – Positive ion

Anion – Negative ion

DEFINITIONS

Molarity of Solutions – The number of gram moles of a substance dissolved in a liter of solution.

Molality of Solutions – The number of gram moles of a substance per 1,000 grams of solvent.

Normality of Solutions – The product of the molarity of a solution and the number of valences taking place in a reaction.

Equivalent Mass – The number of parts by mass of an element or compound which will combine with or replace directly or indirectly 1.008 parts by mass of hydrogen, 8.000 parts of oxygen, or the equivalent mass of any other element or compound. For all elements, the atomic mass is the product of the equivalent mass and the valence.

Molar Volume of an Ideal Gas [at 0°C (32°F) and 1 atm (14.7 psia)]; 22.4 L/(g mole) [359 ft³/(lb mole)].

Mole Fraction of a Substance – The ratio of the number of moles of a substance to the total moles present in a mixture of substances. Mixture may be a solid, a liquid solution, or a gas.

Equilibrium Constant of a Chemical Reaction

$$aA + bB \rightleftharpoons cC + dD$$
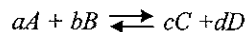
$$K_{eq} = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

Le Chatelier's Principle for Chemical Equilibrium – When a stress (such as a change in concentration, pressure, or temperature) is applied to a system in equilibrium, the equilibrium shifts in such a way that tends to relieve the stress.

Heats of Reaction, Solution, Formation, and Combustion – Chemical processes generally involve the absorption or evolution of heat. In an endothermic process, heat is absorbed (enthalpy change is positive). In an exothermic process, heat is evolved (enthalpy change is negative).

Solubility Product of a slightly soluble substance *AB*:

$$A_m B_n \rightarrow mA^{n+} + nB^{m-}$$

Solubility Product Constant = $K_{SP} = [A^+]^m [B^-]^n$

Metallic Elements – In general, metallic elements are distinguished from non-metallic elements by their luster, malleability, conductivity, and usual ability to form positive ions.

Non-Metallic Elements – In general, non-metallic elements are not malleable, have low electrical conductivity, and rarely form positive ions.

Faraday's Law – In the process of electrolytic changes, equal quantities of electricity charge or discharge equivalent quantities of ions at each electrode. One gram equivalent weight of matter is chemically altered at each electrode for 96,485 coulombs, or one Faraday, of electricity passed through the electrolyte.

A *catalyst* is a substance that alters the rate of a chemical reaction and may be recovered unaltered in nature and amount at the end of the reaction. The catalyst does not affect the position of equilibrium of a reversible reaction.
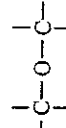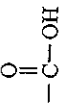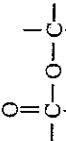
The *atomic number* is the number of protons in the atomic nucleus. The atomic number is the essential feature which distinguishes one element from another and determines the position of the element in the periodic table.

Boiling Point Elevation – The presence of a non-volatile solute in a solvent raises the boiling point of the resulting solution compared to the pure solvent; i.e., to achieve a given vapor pressure, the temperature of the solution must be higher than that of the pure substance.

Freezing Point Depression – The presence of a non-volatile solute in a solvent lowers the freezing point of the resulting solution compared to the pure solvent.

PERIODIC TABLE OF ELEMENTS

Legend:
- Atomic Number
- Symbol
- Atomic Weight

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H 1.0079																	2 He 4.0026
3 Li 6.941	4 Be 9.0122											5 B 10.811	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.179
11 Na 22.990	12 Mg 24.305											13 Al 26.981	14 Si 28.086	15 P 30.974	16 S 32.066	17 Cl 35.453	18 Ar 39.948
19 K 39.098	20 Ca 40.078	21 Sc 44.956	22 Ti 47.88	23 V 50.941	24 Cr 51.996	25 Mn 54.938	26 Fe 55.847	27 Co 58.933	28 Ni 58.69	29 Cu 63.546	30 Zn 65.39	31 Ga 69.723	32 Ge 72.61	33 As 74.921	34 Se 78.96	35 Br 79.904	36 Kr 83.80
37 Rb 85.468	38 Sr 87.62	39 Y 88.906	40 Zr 91.224	41 Nb 92.906	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.42	47 Ag 107.87	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.75	52 Te 127.60	53 I 126.90	54 Xe 131.29
55 Cs 132.91	56 Ba 137.33	57* La 138.91	72 Hf 178.49	73 Ta 180.95	74 W 183.85	75 Re 186.21	76 Os 190.2	77 Ir 192.22	78 Pt 195.08	79 Au 196.97	80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra 226.02	89** Ac 227.03	104 Rf (261)	105 Ha (262)													

*Lanthanide Series

58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.92	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.04	71 Lu 174.97

**Actinide Series

90 Th 232.04	91 Pa 231.04	92 U 238.03	93 Np 237.05	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)

IMPORTANT FAMILIES OF ORGANIC COMPOUNDS

FAMILY

	Alkane	Alkene	Alkyne	Arene	Haloalkane	Alcohol	Ether	Amine	Aldehyde	Carboxylic Acid	Ester
Specific Example	CH_3CH_3	$H_2C=CH_2$	$HC\equiv CH$		CH_3CH_2Cl	CH_3CH_2OH	CH_3OCH_3	CH_3NH_2	$CH_3\overset{O}{\overset{\|}{C}}H$	$CH_3\overset{O}{\overset{\|}{C}}OH$	$CH_3\overset{O}{\overset{\|}{C}}OCH$
IUPAC Name	Ethane	Ethene or Ethylene	Ethyne or Acetylene	Benzene	Chloroethane	Ethanol	Methoxy-methane	Methan-amine	Ethanal	Ethanoic Acid	Methyl ethanoate
Common Name	Ethane	Ethylene	Acetylene	Benzene	Ethyl chloride	Ethyl alcohol	Dimethyl ether	Methyl-amine	Acetal-dehyde	Acetic Acid	Methyl acetate
General Formula	RH	$RCH=CH_2$ $RCH=CHR$ $R_2C=CHR$ $R_2C=CR_2$	$RC\equiv CH$ $RC\equiv CR$	ArH	RX	ROH	ROR	RNH_2 R_2NH R_3N	$\overset{O}{\overset{\|}{R}}CH$	$\overset{O}{\overset{\|}{R}}COH$	$\overset{O}{\overset{\|}{R}}COR$
Functional Group	C–H and C–C bonds	$\,\diagup\!C\!=\!C\diagdown$	$-C\equiv C-$	Aromatic Ring	$-\overset{\|}{\underset{\|}{C}}-X$	$-\overset{\|}{\underset{\|}{C}}-OH$	$-\overset{\|}{\underset{\|}{C}}-O-\overset{\|}{\underset{\|}{C}}-$	$-\overset{\|}{\underset{\|}{C}}-\overset{\|}{N}-$	$-\overset{O}{\overset{\|}{C}}-H$	$-\overset{O}{\overset{\|}{C}}-OH$	$-\overset{O}{\overset{\|}{C}}-O-\overset{\|}{\underset{\|}{C}}-$

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Standard Oxidation Potentials for Corrosion Reactions*	
Corrosion Reaction	Potential, E_o, Volts vs. Normal Hydrogen Electrode
$Au \rightarrow Au^{3+} + 3e$	−1.498
$2H_2O \rightarrow O_2 + 4H^+ + 4e$	−1.229
$Pt \rightarrow Pt^{2+} + 2e$	−1.200
$Pd \rightarrow Pd^{2+} + 2e$	−0.987
$Ag \rightarrow Ag^+ + e$	−0.799
$2Hg \rightarrow Hg_2^{2+} + 2e$	−0.788
$Fe^{2+} \rightarrow Fe^{3+} + e$	−0.771
$4(OH)^- \rightarrow O_2 + 2H_2O + 4e$	−0.401
$Cu \rightarrow Cu^{2+} + 2e$	−0.337
$Sn^{2+} \rightarrow n^{4+} + 2e$	−0.150
$H_2 \rightarrow 2H^+ + 2e$	0.000
$Pb \rightarrow Pb^{2+} + 2e$	+0.126
$Sn \rightarrow Sn^{2+} + 2e$	+0.136
$Ni \rightarrow Ni^{2+} + 2e$	+0.250
$Co \rightarrow Co^{2+} + 2e$	+0.277
$Cd \rightarrow Cd^{2+} + 2e$	+0.403
$Fe \rightarrow Fe^{2+} + 2e$	+0.440
$Cr \rightarrow Cr^{3+} + 3e$	+0.744
$Zn \rightarrow Zn^{2+} + 2e$	+0.763
$Al \rightarrow Al^{3+} + 3e$	+1.662
$Mg \rightarrow Mg^{2+} + 2e$	+2.363
$Na \rightarrow Na^+ + e$	+2.714
$K \rightarrow K^+ + e$	+2.925

* Measured at 25°C. Reactions are written as anode half-cells. Arrows are reversed for cathode half-cells.

Flinn, Richard A. and Paul K. Trojan, *Engineering Materials and Their Applications*, 4th Edition. Copyright © 1990 by Houghton Mifflin Company. Table used with permission.

NOTE: In some chemistry texts, the reactions and the signs of the values (in this table) are reversed; for example, the half-cell potential of zinc is given as −0.763 volt for the reaction $Zn^{2+} + 2e \rightarrow Zn$. When the potential E_o is positive, the reaction proceeds spontaneously as written.

COMPUTERS, MEASUREMENT, AND CONTROLS

COMPUTER KNOWLEDGE

Examinees are expected to possess a level of computer expertise required to perform in a typical undergraduate environment. Thus only generic problems that do not require a knowledge of a specific language or computer type will be required. Examinees are expected to be familiar with flow charts, pseudo code, and spread sheets (Lotus, Quattro-Pro, Excel, etc.).

INSTRUMENTATION

General Considerations

In making any measurement, the response of the total measurement system, including the behavior of the sensors and any signal processors, is best addressed using the methods of control systems. Response time and the effect of the sensor on the parameter being measured may affect accuracy of a measurement. Moreover, many transducers exhibit some sensitivity to phenomena other than the primary parameter being measured. All of these considerations affect accuracy, stability, noise sensitivity, and precision of any measurement. In the case of digital measurement systems, the limit of resolution corresponds to one bit.

Examples of Types of Sensors

Fluid-based sensors such as manometers, orifice and venturi flow meters, and pitot tubes are discussed in the **FLUID MECHANICS** section.

Resistance-based sensors include resistance temperature detectors (RTDs), which are metal resistors, and thermistors, which are semiconductors. Both have electrical resistivities that are temperature dependent.

Electrical-resistance strain gages are metallic or semiconducting foils whose resistance changes with dimensional change (strain). They are widely used in load cells. The gage is attached to the surface whose strain is to be measured. The gage factor (G.F.) of these devices is defined by

$$\text{G.F.} = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon}, \quad \text{where}$$

R = electrical resistance,

L = the length of the gage section, and

ε = the normal strain sensed by the gage.

Strain gages sense normal strain along their principal axis. They do not respond to shear strain. Therefore, multiple gages must be used along with Mohr's circle techniques to determine the complete plane strain state.

Resistance-based sensors are generally used in a bridge circuit that detects small changes in resistance. The output of a bridge circuit with only one variable resistor (quarter bridge configuration) is given by

$$V_{\text{out}} = V_{\text{input}} \times [\Delta R/(4R)]$$
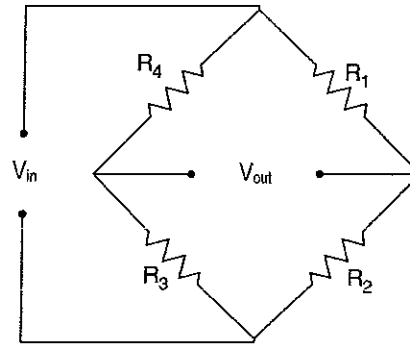
Half-bridge and full-bridge configurations use two or four variable resistors, respectively. A full-bridge strain gage circuit gives a voltage output of

$$V_{\text{out}} = V_{\text{input}} \times \text{G.F.} \times (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4)/4$$

Half- or full-strain gage bridge configurations can be developed that are sensitive to only some types of loading (axial, bending, shear) while being insensitive to others.

Piezoelectric sensors produce a voltage in response to a mechanical load. These transducers are widely used as force or pressure transducers. With the addition of an inertial mass, they are used as accelerometers.

Thermocouples are junctions of dissimilar metals which produce a voltage whose magnitude is temperature dependent.

Capacitance-based transducers are used as position sensors. The capacitance of two flat plates depends on their separation or on the area of overlap.

Inductance-based transducers or differential transformers also function as displacement transducers. The inductive coupling between a primary and secondary coil depends on the position of a soft magnetic core. This is the basis for the Linear Variable Differential Transformer (LVDT).

MEASUREMENT UNCERTAINTY

Suppose that a calculated result R depends on measurements whose values are $x_1 \pm w_1$, $x_2 \pm w_2$, $x_3 \pm w_3$, etc., where $R = f(x_1, x_2, x_3, \dots x_n)$, x_i is the measured value, and w_i is the uncertainty in that value. The uncertainty in R, w_R, can be estimated using the Kline-McClintock equation:

$$w_R = \sqrt{\left(w_1 \frac{\partial f}{\partial x_1}\right)^2 + \left(w_2 \frac{\partial f}{\partial x_2}\right)^2 + \dots + \left(w_n \frac{\partial f}{\partial x_n}\right)^2}$$

CONTROL SYSTEMS

The linear time-invariant transfer function model represented by the block diagram
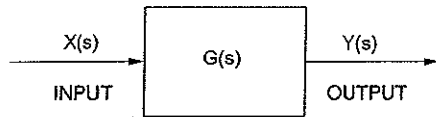
can be expressed as the ratio of two polynomials in the form

$$\frac{X(s)}{Y(s)} = G(s) = \frac{N(s)}{D(s)} = K \frac{\prod_{m=1}^{M}(s - z_m)}{\prod_{n=1}^{N}(s - p_n)}$$

where the M zeros, z_m, and the N poles, p_n, are the roots of the numerator polynomial, $N(s)$, and the denominator polynomial, $D(s)$, respectively.

One classical negative feedback control system model block diagram is
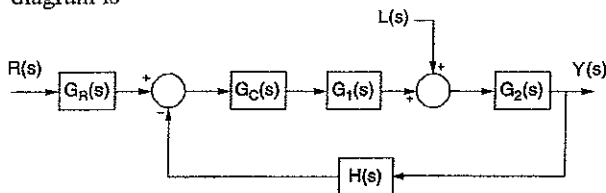
where $G_R(s)$ describes an input processor, $G_C(s)$ a controller or compensator, $G_1(s)$ and $G_2(s)$ represent a partitioned plant model, and $H(s)$ a feedback function. $Y(s)$ represents the controlled variable, $R(s)$ represents the reference input, and $L(s)$ represents a load disturbance. $Y(s)$ is related to $R(s)$ and $L(s)$ by

$$Y(s) = \frac{G_c(s)G_1(s)G_2(s)G_R(s)}{1 + G_c(s)G_1(s)G_2(s)H(s)} R(s)$$

$$+ \frac{G_2(s)}{1 + G_c(s)G_1(s)G_2(s)H(s)} L(s)$$

$G_C(s)$ $G_1(s)$ $G_2(s)$ $H(s)$ is the open-loop transfer function. The closed-loop characteristic equation is

$$.1 + G_c(s)\, G_1(s)\, G_2(s)\, H(s) = 0$$

System performance studies normally include:

1. Steady-state analysis using constant inputs is based on the Final Value Theorem. If all poles of a $G(s)$ function have negative real parts, then

$$\text{Steady State Gain} = \lim_{s \to 0} G(s)$$

For the unity feedback control system model
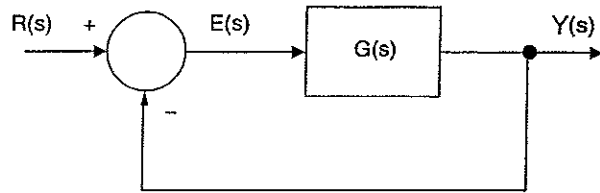
with the open-loop transfer function defined by

$$G(s) = \frac{K_B}{s^T} \times \frac{\prod_{m=1}^{M}(1 + s/\omega_m)}{\prod_{n=1}^{N}(1 + s/\omega_n)}$$

The following steady-state error analysis table can be constructed where T denotes the type of system; i.e., type 0, type 1, etc.

Steady-State Error $e_{ss}(t)$			
Input　　　Type	$T = 0$	$T = 1$	$T = 2$
Unit Step	$1/(K_B + 1)$	0	0
Ramp	∞	$1/K_B$	0
Acceleration	∞	∞	$1/K_B$

2. Frequency response evaluations to determine dynamic performance and stability. For example, relative stability can be quantified in terms of

   a. Gain margin (GM) which is the additional gain required to produce instability in the unity gain feedback control system. If at $\omega = \omega_{180}$,

   $$\angle G(j\omega_{180}) = -180°; \text{ then}$$

   $$GM = -20\log_{10}(\,|G(j\omega_{180})|\,)$$

   b. Phase margin (PM) which is the additional phase required to produce instability. Thus,

   $$PM = 180° + \angle G(j\omega_{0dB})$$

   where ω_{0dB} is the ω that satisfies $|G(j\omega)| = 1$.

3. Transient responses are obtained by using Laplace Transforms or computer solutions with numerical integration.

Common Compensator/Controller forms are

PID Controller $G_C(s) = K\left(1 + \dfrac{1}{T_I s} + T_D s\right)$

Lag or Lead Compensator $G_C(s) = K\left(\dfrac{1 + sT_1}{1 + sT_2}\right)$

depending on the ratio of T_1/T_2.

Routh Test

For the characteristic equation

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0 = 0$$

the coefficients are arranged into the first two rows of an array. Additional rows are computed. The array and coefficient computations are defined by:

a_n	a_{n-2}	a_{n-4}	...	...	...
a_{n-1}	a_{n-3}	a_{n-5}	...	...	...
b_1	b_2	b_3	...	...	...
c_1	c_2	c_3	...	...	...

where

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad c_1 = \frac{a_{n-3}b_1 - a_{n-1}b_2}{b_1}$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}} \qquad c_2 = \frac{a_{n-5}b_1 - a_{n-1}b_3}{b_1}$$

...

The necessary and sufficient conditions for all the roots of the equation to have negative real parts is that all the elements in the first column be of the same sign and nonzero.

Second-Order Control-System Models

One standard second-order control-system model is

$$\frac{Y(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \text{ where}$$

K = steady state gain,

ζ = the damping ratio,

ω_n = the undamped natural ($\zeta = 0$) frequency,

$\omega_d = \omega_n\sqrt{1 - \zeta^2}$, the damped natural frequency,

and

$\omega_p = \omega_n\sqrt{1 - 2\zeta^2}$, the damped resonant frequency.

If the damping ratio ζ is less than unity, the system is said to be underdamped; if ζ is equal to unity, it is said to be critically damped; and if ζ is greater than unity, the system is said to be overdamped.

For a unit step input to a normalized underdamped second-order control system, the time required to reach a peak value t_p and the value of that peak M_p are given by

$$t_p = \pi \big/ \left(\omega_n \sqrt{1 - \zeta^2} \right)$$

$$M_p = 1 + e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

For an underdamped second-order system, the logarithmic decrement is

$$\delta = \frac{1}{m}\ln\left(\frac{x_k}{x_{k+m}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

where x_k and x_{k+m} are the amplitudes of oscillation at cycles k and $k + m$, respectively. The period of oscillation τ is related to ω_d by

$$\omega_d \tau = 2\pi$$

The time required for the output of a second-order system to settle to within 2% of its final value is defined to be

$$T_s = \frac{4}{\zeta\omega_n}$$

State-Variable Control-System Models

One common state-variable model for dynamic systems has the form

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) \qquad \text{(state equation)}$$

$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t) \qquad \text{(output equation)}$$

where

$x(t)$ = N by 1 state vector (N state variables),

$u(t)$ = R by 1 input vector (R inputs),

$y(t)$ = M by 1 output vector (M outputs),

$\mathbf{A}$ = system matrix,

$\mathbf{B}$ = input distribution matrix,

$\mathbf{C}$ = output matrix, and

$\mathbf{D}$ = feed-through matrix.

The orders of the matrices are defined via variable definitions.

State-variable models automatically handle multiple inputs and multiple outputs. Furthermore, state-variable models can be formulated for open-loop system components or the complete closed-loop system.

The Laplace transform of the time-invariant state equation is

$$s\mathbf{X}(s) - x(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)$$

from which

$$\mathbf{X}(s) = \mathbf{\Phi}(s)\,x(0) + \mathbf{\Phi}(s)\,\mathbf{B}\mathbf{U}(s)$$

where the Laplace transform of the state transition matrix is

$$\mathbf{\Phi}(s) = [s\mathbf{I} - \mathbf{A}]^{-1}.$$

The state-transition matrix

$$\mathbf{\Phi}(t) = L^{-1}\{\mathbf{\Phi}(s)\}$$

(also defined as $e^{\mathbf{A}t}$) can be used to write

$$x(t) = \mathbf{\Phi}(t)\,x(0) + \int_0^t \mathbf{\Phi}(t - \tau)\,\mathbf{B}u(\tau)\,d\tau$$

The output can be obtained with the output equation; e.g., the Laplace transform output is

$$\mathbf{Y}(s) = \{\mathbf{C}\mathbf{\Phi}(s)\,\mathbf{B} + \mathbf{D}\}\mathbf{U}(s) + \mathbf{C}\mathbf{\Phi}(s)\,x(0)$$

The latter term represents the output(s) due to initial conditions whereas the former term represents the output(s) due to the $\mathbf{U}(s)$ inputs and gives rise to transfer function definitions.

ENGINEERING ECONOMICS

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\dfrac{i}{(1+i)^n - 1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\dfrac{i(1+i)^n}{(1+i)^n - 1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\dfrac{(1+i)^n - 1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\dfrac{(1+i)^n - 1}{i(1+i)^n}$
Uniform Gradient ** Present Worth	to P given G	$(P/G, i\%, n)$	$\dfrac{(1+i)^n - 1}{i^2(1+i)^n} - \dfrac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\dfrac{(1+i)^n - 1}{i^2} - \dfrac{n}{i}$
Uniform Gradient ‡ Uniform Series	to A given G	$(A/G, i\%, n)$	$\dfrac{1}{i} - \dfrac{n}{(1+i)^n - 1}$

NOMENCLATURE AND DEFINITIONS

A.......... Uniform amount per interest period
B.......... Benefit
BV....... Book Value
C.......... Cost
d.......... Combined interest rate per interest period
D_j........ Depreciation in year j
F.......... Future worth, value, or amount
f.......... General inflation rate per interest period
G......... Uniform gradient amount per interest period
i.......... Interest rate per interest period
i_e......... Annual effective interest rate
m......... Number of compounding periods per year
n.......... Number of compounding periods; or the expected life of an asset
P.......... Present worth, value, or amount
r.......... Nominal annual interest rate
S_n........ Expected salvage value in year n

Subscripts
j.......... at time j
n.......... at time n
** .... $P/G = (F/G)/(F/P) = (P/A) \times (A/G)$
† .... $F/G = (F/A - n)/i = (F/A) \times (A/G)$
‡ .... $A/G = [1 - n(A/F)]/i$

NON-ANNUAL COMPOUNDING

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Discount Factors for <u>Continuous Compounding</u>
(*n* is the number of years)

$(F/P, r\%, n) = e^{rn}$

$(P/F, r\%, n) = e^{-rn}$

$(A/F, r\%, n) = \dfrac{e^r - 1}{e^{rn} - 1}$

$(F/A, r\%, n) = \dfrac{e^{rn} - 1}{e^r - 1}$

$(A/P, r\%, n) = \dfrac{e^r - 1}{1 - e^{-rn}}$

$(P/A, r\%, n) = \dfrac{1 - e^{-rn}}{e^r - 1}$

BOOK VALUE

$$BV = \text{initial cost} - \Sigma\, D_j$$

DEPRECIATION

Straight Line $\qquad D_j = \dfrac{C - S_n}{n}$

Accelerated Cost Recovery System (ACRS)

$\qquad D_j = (factor)\, C$

A table of modified factors is provided below.

CAPITALIZED COSTS

Capitalized costs are present worth values using an assumed perpetual period of time.

$$\text{Capitalized Costs} = P = \frac{A}{i}$$

BONDS

Bond Value equals the present worth of the payments the purchaser (or holder of the bond) receives during the life of the bond at some interest rate i.

Bond Yield equals the computed interest rate of the bond value when compared with the bond cost.

RATE-OF-RETURN

The minimum acceptable rate-of-return is that interest rate that one is willing to accept, or the rate one desires to earn on investments. The rate-of-return on an investment is the interest rate that makes the benefits and costs equal.

BREAK-EVEN ANALYSIS

By altering the value of any one of the variables in a situation, holding all of the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical. This value is the break-even point.

Break-even analysis is used to describe the percentage of capacity of operation for a manufacturing plant at which income will just cover expenses.

The payback period is the period of time required for the profit or other benefits of an investment to equal the cost of the investment.

INFLATION

To account for inflation, the dollars are deflated by the general inflation rate per interest period f, and then they are shifted over the time scale using the interest rate per interest period i. Use a combined interest rate per interest period d for computing present worth values P and Net P. The formula for d is

$$d = i + f + (i \times f)$$

BENEFIT-COST ANALYSIS

In a benefit-cost analysis, the benefits B of a project should exceed the estimated costs C.

$$B - C \geq 0, \text{ or} \quad B/C \geq 1$$

MODIFIED ACRS FACTORS			
Recovery Period (Years)			
3	5	7	10
Year	Recovery Rate (Percent)		
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		11.5	8.9	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

Factor Table - $i = 0.50\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9950	0.9950	0.0000	1.0050	1.0000	1.0050	1.0000	0.0000
2	0.9901	1.9851	0.9901	1.0100	2.0050	0.5038	0.4988	0.4988
3	0.9851	2.9702	2.9604	1.0151	3.0150	0.3367	0.3317	0.9967
4	0.9802	3.9505	5.9011	1.0202	4.0301	0.2531	0.2481	1.4938
5	0.9754	4.9259	9.8026	1.0253	5.0503	0.2030	0.1980	1.9900
6	0.9705	5.8964	14.6552	1.0304	6.0755	0.1696	0.1646	2.4855
7	0.9657	6.8621	20.4493	1.0355	7.1059	0.1457	0.1407	2.9801
8	0.9609	7.8230	27.1755	1.0407	8.1414	0.1278	0.1228	3.4738
9	0.9561	8.7791	34.8244	1.0459	9.1821	0.1139	0.1089	3.9668
10	0.9513	9.7304	43.3865	1.0511	10.2280	0.1028	0.0978	4.4589
11	0.9466	10.6770	52.8526	1.0564	11.2792	0.0937	0.0887	4.9501
12	0.9419	11.6189	63.2136	1.0617	12.3356	0.0861	0.0811	5.4406
13	0.9372	12.5562	74.4602	1.0670	13.3972	0.0796	0.0746	5.9302
14	0.9326	13.4887	86.5835	1.0723	14.4642	0.0741	0.0691	6.4190
15	0.9279	14.4166	99.5743	1.0777	15.5365	0.0694	0.0644	6.9069
16	0.9233	15.3399	113.4238	1.0831	16.6142	0.0652	0.0602	7.3940
17	0.9187	16.2586	128.1231	1.0885	17.6973	0.0615	0.0565	7.8803
18	0.9141	17.1728	143.6634	1.0939	18.7858	0.0582	0.0532	8.3658
19	0.9096	18.0824	160.0360	1.0994	19.8797	0.0553	0.0503	8.8504
20	0.9051	18.9874	177.2322	1.1049	20.9791	0.0527	0.0477	9.3342
21	0.9006	19.8880	195.2434	1.1104	22.0840	0.0503	0.0453	9.8172
22	0.8961	20.7841	214.0611	1.1160	23.1944	0.0481	0.0431	10.2993
23	0.8916	21.6757	233.6768	1.1216	24.3104	0.0461	0.0411	10.7806
24	0.8872	22.5629	254.0820	1.1272	25.4320	0.0443	0.0393	11.2611
25	0.8828	23.4456	275.2686	1.1328	26.5591	0.0427	0.0377	11.7407
30	0.8610	27.7941	392.6324	1.1614	32.2800	0.0360	0.0310	14.1265
40	0.8191	36.1722	681.3347	1.2208	44.1588	0.0276	0.0226	18.8359
50	0.7793	44.1428	1,035.6966	1.2832	56.6452	0.0227	0.0177	23.4624
60	0.7414	51.7256	1,448.6458	1.3489	69.7700	0.0193	0.0143	28.0064
100	0.6073	78.5426	3,562.7934	1.6467	129.3337	0.0127	0.0077	45.3613

Factor Table - $i = 1.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9901	0.9901	0.0000	1.0100	1.0000	1.0100	1.0000	0.0000
2	0.9803	1.9704	0.9803	1.0201	2.0100	0.5075	0.4975	0.4975
3	0.9706	2.9410	2.9215	1.0303	3.0301	0.3400	0.3300	0.9934
4	0.9610	3.9020	5.8044	1.0406	4.0604	0.2563	0.2463	1.4876
5	0.9515	4.8534	9.6103	1.0510	5.1010	0.2060	0.1960	1.9801
6	0.9420	5.7955	14.3205	1.0615	6.1520	0.1725	0.1625	2.4710
7	0.9327	6.7282	19.9168	1.0721	7.2135	0.1486	0.1386	2.9602
8	0.9235	7.6517	26.3812	1.0829	8.2857	0.1307	0.1207	3.4478
9	0.9143	8.5650	33.6959	1.0937	9.3685	0.1167	0.1067	3.9337
10	0.9053	9.4713	41.8435	1.1046	10.4622	0.1056	0.0956	4.4179
11	0.8963	10.3676	50.8067	1.1157	11.5668	0.0965	0.0865	4.9005
12	0.8874	11.2551	60.5687	1.1268	12.6825	0.0888	0.0788	5.3815
13	0.8787	12.1337	71.1126	1.1381	13.8093	0.0824	0.0724	5.8607
14	0.8700	13.0037	82.4221	1.1495	14.9474	0.0769	0.0669	6.3384
15	0.8613	13.8651	94.4810	1.1610	16.0969	0.0721	0.0621	6.8143
16	0.8528	14.7179	107.2734	1.1726	17.2579	0.0679	0.0579	7.2886
17	0.8444	15.5623	120.7834	1.1843	18.4304	0.0643	0.0543	7.7613
18	0.8360	16.3983	134.9957	1.1961	19.6147	0.0610	0.0510	8.2323
19	0.8277	17.2260	149.8950	1.2081	20.8109	0.0581	0.0481	8.7017
20	0.8195	18.0456	165.4664	1.2202	22.0190	0.0554	0.0454	9.1694
21	0.8114	18.8570	181.6950	1.2324	23.2392	0.0530	0.0430	9.6354
22	0.8034	19.6604	198.5663	1.2447	24.4716	0.0509	0.0409	10.0998
23	0.7954	20.4558	216.0660	1.2572	25.7163	0.0489	0.0389	10.5626
24	0.7876	21.2434	234.1800	1.2697	26.9735	0.0471	0.0371	11.0237
25	0.7798	22.0232	252.8945	1.2824	28.2432	0.0454	0.0354	11.4831
30	0.7419	25.8077	355.0021	1.3478	34.7849	0.0387	0.0277	13.7557
40	0.6717	32.8347	596.8561	1.4889	48.8864	0.0305	0.0205	18.1776
50	0.6080	39.1961	879.4176	1.6446	64.4632	0.0255	0.0155	22.4363
60	0.5504	44.9550	1,192.8061	1.8167	81.6697	0.0222	0.0122	26.5333
100	0.3697	63.0289	2,605.7758	2.7048	170.4814	0.0159	0.0059	41.3426

Factor Table - $i = 1.50\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9852	0.9852	0.0000	1.0150	1.0000	1.0150	1.0000	0.0000
2	0.9707	1.9559	0.9707	1.0302	2.0150	0.5113	0.4963	0.4963
3	0.9563	2.9122	2.8833	1.0457	3.0452	0.3434	0.3284	0.9901
4	0.9422	3.8544	5.7098	1.0614	4.0909	0.2594	0.2444	1.4814
5	0.9283	4.7826	9.4229	1.0773	5.1523	0.2091	0.1941	1.9702
6	0.9145	5.6972	13.9956	1.0934	6.2296	0.1755	0.1605	2.4566
7	0.9010	6.5982	19.4018	1.1098	7.3230	0.1516	0.1366	2.9405
8	0.8877	7.4859	26.6157	1.1265	8.4328	0.1336	0.1186	3.4219
9	0.8746	8.3605	32.6125	1.1434	9.5593	0.1196	0.1046	3.9008
10	0.8617	9.2222	40.3675	1.1605	10.7027	0.1084	0.0934	4.3772
11	0.8489	10.0711	48.8568	1.1779	11.8633	0.0993	0.0843	4.8512
12	0.8364	10.9075	58.0571	1.1956	13.0412	0.0917	0.0767	5.3227
13	0.8240	11.7315	67.9454	1.2136	14.2368	0.0852	0.0702	5.7917
14	0.8118	12.5434	78.4994	1.2318	15.4504	0.0797	0.0647	6.2582
15	0.7999	13.3432	89.6974	1.2502	16.6821	0.0749	0.0599	6.7223
16	0.7880	14.1313	101.5178	1.2690	17.9324	0.0708	0.0558	7.1839
17	0.7764	14.9076	113.9400	1.2880	19.2014	0.0671	0.0521	7.6431
18	0.7649	15.6726	126.9435	1.3073	20.4894	0.0638	0.0488	8.0997
19	0.7536	16.4262	140.5084	1.3270	21.7967	0.0609	0.0459	8.5539
20	0.7425	17.1686	154.6154	1.3469	23.1237	0.0582	0.0432	9.0057
21	0.7315	17.9001	169.2453	1.3671	24.4705	0.0559	0.0409	9.4550
22	0.7207	18.6208	184.3798	1.3876	25.8376	0.0537	0.0387	9.9018
23	0.7100	19.3309	200.0006	1.4084	27.2251	0.0517	0.0367	10.3462
24	0.6995	20.0304	216.0901	1.4295	28.6335	0.0499	0.0349	10.7881
25	0.6892	20.7196	232.6310	1.4509	30.0630	0.0483	0.0333	11.2276
30	0.6398	24.0158	321.5310	1.5631	37.5387	0.0416	0.0266	13.3883
40	0.5513	29.9158	524.3568	1.8140	54.2679	0.0334	0.0184	17.5277
50	0.4750	34.9997	749.9636	2.1052	73.6828	0.0286	0.0136	21.4277
60	0.4093	39.3803	988.1674	2.4432	96.2147	0.0254	0.0104	25.0930
100	0.2256	51.6247	1,937.4506	4.4320	228.8030	0.0194	0.0044	37.5295

Factor Table - $i = 2.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9804	0.9804	0.0000	1.0200	1.0000	1.0200	1.0000	0.0000
2	0.9612	1.9416	0.9612	1.0404	2.0200	0.5150	0.4950	0.4950
3	0.9423	2.8839	2.8458	1.0612	3.0604	0.3468	0.3268	0.9868
4	0.9238	3.8077	5.6173	1.0824	4.1216	0.2626	0.2426	1.4752
5	0.9057	4.7135	9.2403	1.1041	5.2040	0.2122	0.1922	1.9604
6	0.8880	5.6014	13.6801	1.1262	6.3081	0.1785	0.1585	2.4423
7	0.8706	6.4720	18.9035	1.1487	7.4343	0.1545	0.1345	2.9208
8	0.8535	7.3255	24.8779	1.1717	8.5830	0.1365	0.1165	3.3961
9	0.8368	8.1622	31.5720	1.1951	9.7546	0.1225	0.1025	3.8681
10	0.8203	8.9826	38.9551	1.2190	10.9497	0.1113	0.0913	4.3367
11	0.8043	9.7868	46.9977	1.2434	12.1687	0.1022	0.0822	4.8021
12	0.7885	10.5753	55.6712	1.2682	13.4121	0.0946	0.0746	5.2642
13	0.7730	11.3484	64.9475	1.2936	14.6803	0.0881	0.0681	5.7231
14	0.7579	12.1062	74.7999	1.3195	15.9739	0.0826	0.0626	6.1786
15	0.7430	12.8493	85.2021	1.3459	17.2934	0.0778	0.0578	6.6309
16	0.7284	13.5777	96.1288	1.3728	18.6393	0.0737	0.0537	7.0799
17	0.7142	14.2919	107.5554	1.4002	20.0121	0.0700	0.0500	7.5256
18	0.7002	14.9920	119.4581	1.4282	21.4123	0.0667	0.0467	7.9681
19	0.6864	15.6785	131.8139	1.4568	22.8406	0.0638	0.0438	8.4073
20	0.6730	16.3514	144.6003	1.4859	24.2974	0.0612	0.0412	8.8433
21	0.6598	17.0112	157.7959	1.5157	25.7833	0.0588	0.0388	9.2760
22	0.6468	17.6580	171.3795	1.5460	27.2990	0.0566	0.0366	9.7055
23	0.6342	18.2922	185.3309	1.5769	28.8450	0.0547	0.0347	10.1317
24	0.6217	18.9139	199.6305	1.6084	30.4219	0.0529	0.0329	10.5547
25	0.6095	19.5235	214.2592	1.6406	32.0303	0.0512	0.0312	10.9745
30	0.5521	22.3965	291.7164	1.8114	40.5681	0.0446	0.0246	13.0251
40	0.4529	27.3555	461.9931	2.2080	60.4020	0.0366	0.0166	16.8885
50	0.3715	31.4236	642.3606	2.6916	84.5794	0.0318	0.0118	20.4420
60	0.3048	34.7609	823.6975	3.2810	114.0515	0.0288	0.0088	23.6961
100	0.1380	43.0984	1,464.7527	7.2446	312.2323	0.0232	0.0032	33.9863

Factor Table - $i = 4.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9615	0.9615	0.0000	1.0400	1.0000	1.0400	1.0000	0.0000
2	0.9246	1.8861	0.9246	1.0816	2.0400	0.5302	0.4902	0.4902
3	0.8890	2.7751	2.7025	1.1249	3.1216	0.3603	0.3203	0.9739
4	0.8548	3.6299	5.2670	1.1699	4.2465	0.2755	0.2355	1.4510
5	0.8219	4.4518	8.5547	1.2167	5.4163	0.2246	0.1846	1.9216
6	0.7903	5.2421	12.5062	1.2653	6.6330	0.1908	0.1508	2.3857
7	0.7599	6.0021	17.0657	1.3159	7.8983	0.1666	0.1266	2.8433
8	0.7307	6.7327	22.1806	1.3686	9.2142	0.1485	0.1085	3.2944
9	0.7026	7.4353	27.8013	1.4233	10.5828	0.1345	0.0945	3.7391
10	0.6756	8.1109	33.8814	1.4802	12.0061	0.1233	0.0833	4.1773
11	0.6496	8.7605	40.3772	1.5395	13.4864	0.1141	0.0741	4.6090
12	0.6246	9.3851	47.2477	1.6010	15.0258	0.1066	0.0666	5.0343
13	0.6006	9.9856	54.4546	1.6651	16.6268	0.1001	0.0601	5.4533
14	0.5775	10.5631	61.9618	1.7317	18.2919	0.0947	0.0547	5.8659
15	0.5553	11.1184	69.7355	1.8009	20.0236	0.0899	0.0499	6.2721
16	0.5339	11.6523	77.7441	1.8730	21.8245	0.0858	0.0458	6.6720
17	0.5134	12.1657	85.9581	1.9479	23.6975	0.0822	0.0422	7.0656
18	0.4936	12.6593	94.3498	2.0258	25.6454	0.0790	0.0390	7.4530
19	0.4746	13.1339	102.8933	2.1068	27.6712	0.0761	0.0361	7.8342
20	0.4564	13.5903	111.5647	2.1911	29.7781	0.0736	0.0336	8.2091
21	0.4388	14.0292	120.3414	2.2788	31.9692	0.0713	0.0313	8.5779
22	0.4220	14.4511	129.2024	2.3699	34.2480	0.0692	0.0292	8.9407
23	0.4057	14.8568	138.1284	2.4647	36.6179	0.0673	0.0273	9.2973
24	0.3901	15.2470	147.1012	2.5633	39.0826	0.0656	0.0256	9.6479
25	0.3751	15.6221	156.1040	2.6658	41.6459	0.0640	0.0240	9.9925
30	0.3083	17.2920	201.0618	3.2434	56.0849	0.0578	0.0178	11.6274
40	0.2083	19.7928	286.5303	4.8010	95.0255	0.0505	0.0105	14.4765
50	0.1407	21.4822	361.1638	7.1067	152.6671	0.0466	0.0066	16.8122
60	0.0951	22.6235	422.9966	10.5196	237.9907	0.0442	0.0042	18.6972
100	0.0198	24.5050	563.1249	50.5049	1,237.6237	0.0408	0.0008	22.9800

Factor Table - $i = 6.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9434	0.9434	0.0000	1.0600	1.0000	1.0600	1.0000	0.0000
2	0.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854
3	0.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612
4	0.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272
5	0.7473	4.2124	7.9345	1.3382	5.6371	0.2374	0.1774	1.8836
6	0.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304
7	0.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676
8	0.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1952
9	0.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133
10	0.5584	7.3601	29.6023	1.7908	13.1808	0.1359	0.0759	4.0220
11	0.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213
12	0.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113
13	0.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920
14	0.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5463
15	0.4173	9.7122	57.5546	2.3966	23.2760	0.1030	0.0430	5.9260
16	0.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794
17	0.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240
18	0.3505	10.8276	75.3569	2.8543	30.9057	0.0924	0.0324	6.9597
19	0.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867
20	0.3118	11.4699	87.2304	3.2071	36.7856	0.0872	0.0272	7.6051
21	0.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151
22	0.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166
23	0.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099
24	0.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951
25	0.2330	12.7834	115.9732	4.2919	54.8645	0.0782	0.0182	9.0722
30	0.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422
40	0.0972	15.0463	185.9568	10.2857	154.7620	0.0665	0.0065	12.3590
50	0.0543	15.7619	217.4574	18.4202	290.3359	0.0634	0.0034	13.7964
60	0.0303	16.1614	239.0428	32.9877	533.1282	0.0619	0.0019	14.7909
100	0.0029	16.6175	272.0471	339.3021	5,638.3681	0.0602	0.0002	16.3711

Factor Table - $i = 8.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9259	0.9259	0.0000	1.0800	1.0000	1.0800	1.0000	0.0000
2	0.8573	1.7833	0.8573	1.1664	2.0800	0.5608	0.4808	0.4808
3	0.7938	2.5771	2.4450	1.2597	3.2464	0.3880	0.3080	0.9487
4	0.7350	3.3121	4.6501	1.3605	4.5061	0.3019	0.2219	1.4040
5	0.6806	3.9927	7.3724	1.4693	5.8666	0.2505	0.1705	1.8465
6	0.6302	4.6229	10.5233	1.5869	7.3359	0.2163	0.1363	2.2763
7	0.5835	5.2064	14.0242	1.7138	8.9228	0.1921	0.1121	2.6937
8	0.5403	5.7466	17.8061	1.8509	10.6366	0.1740	0.0940	3.0985
9	0.5002	6.2469	21.8081	1.9990	12.4876	0.1601	0.0801	3.4910
10	0.4632	6.7101	25.9768	2.1589	14.4866	0.1490	0.0690	3.8713
11	0.4289	7.1390	30.2657	2.3316	16.6455	0.1401	0.0601	4.2395
12	0.3971	7.5361	34.6339	2.5182	18.9771	0.1327	0.0527	4.5957
13	0.3677	7.9038	39.0463	2.7196	21.4953	0.1265	0.0465	4.9402
14	0.3405	8.2442	43.4723	2.9372	24.2149	0.1213	0.0413	5.2731
15	0.3152	8.5595	47.8857	3.1722	27.1521	0.1168	0.0368	5.5945
16	0.2919	8.8514	52.2640	3.4259	30.3243	0.1130	0.0330	5.9046
17	0.2703	9.1216	56.5883	3.7000	33.7502	0.1096	0.0296	6.2037
18	0.2502	9.3719	60.8426	3.9960	37.4502	0.1067	0.0267	6.4920
19	0.2317	9.6036	65.0134	4.3157	41.4463	0.1041	0.0241	6.7697
20	0.2145	9.8181	69.0898	4.6610	45.7620	0.1019	0.0219	7.0369
21	0.1987	10.0168	73.0629	5.0338	50.4229	0.0998	0.0198	7.2940
22	0.1839	10.2007	76.9257	5.4365	55.4568	0.0980	0.0180	7.5412
23	0.1703	10.3711	80.6726	5.8715	60.8933	0.0964	0.0164	7.7786
24	0.1577	10.5288	84.2997	6.3412	66.7648	0.0950	0.0150	8.0066
25	0.1460	10.6748	87.8041	6.8485	73.1059	0.0937	0.0137	8.2254
30	0.0994	11.2578	103.4558	10.0627	113.2832	0.0888	0.0088	9.1897
40	0.0460	11.9246	126.0422	21.7245	259.0565	0.0839	0.0039	10.5699
50	0.0213	12.2335	139.5928	46.9016	573.7702	0.0817	0.0017	11.4107
60	0.0099	12.3766	147.3000	101.2571	1,253.2133	0.0808	0.0008	11.9015
100	0.0005	12.4943	155.6107	2,199.7613	27,484.5157	0.0800		12.4545

Factor Table - $i = 10.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9091	0.9091	0.0000	1.1000	1.0000	1.1000	1.0000	0.0000
2	0.8264	1.7355	0.8264	1.2100	2.1000	0.5762	0.4762	0.4762
3	0.7513	2.4869	2.3291	1.3310	3.3100	0.4021	0.3021	0.9366
4	0.6830	3.1699	4.3781	1.4641	4.6410	0.3155	0.2155	1.3812
5	0.6209	3.7908	6.8618	1.6105	6.1051	0.2638	0.1638	1.8101
6	0.5645	4.3553	9.6842	1.7716	7.7156	0.2296	0.1296	2.2236
7	0.5132	4.8684	12.7631	1.9487	9.4872	0.2054	0.1054	2.6216
8	0.4665	5.3349	16.0287	2.1436	11.4359	0.1874	0.0874	3.0045
9	0.4241	5.7590	19.4215	2.3579	13.5735	0.1736	0.0736	3.3724
10	0.3855	6.1446	22.8913	2.5937	15.9374	0.1627	0.0627	3.7255
11	0.3505	6.4951	26.3962	2.8531	18.5312	0.1540	0.0540	4.0641
12	0.3186	6.8137	29.9012	3.1384	21.3843	0.1468	0.0468	4.3884
13	0.2897	7.1034	33.3772	3.4523	24.5227	0.1408	0.0408	4.6988
14	0.2633	7.3667	36.8005	3.7975	27.9750	0.1357	0.0357	4.9955
15	0.2394	7.6061	40.1520	4.1772	31.7725	0.1315	0.0315	5.2789
16	0.2176	7.8237	43.4164	4.5950	35.9497	0.1278	0.0278	5.5493
17	0.1978	8.0216	46.5819	5.5045	40.5447	0.1247	0.0247	5.8071
18	0.1799	8.2014	49.6395	5.5599	45.5992	0.1219	0.0219	6.0526
19	0.1635	8.3649	52.5827	6.1159	51.1591	0.1195	0.0195	6.2861
20	0.1486	8.5136	55.4069	6.7275	57.2750	0.1175	0.0175	6.5081
21	0.1351	8.6487	58.1095	7.4002	64.0025	0.1156	0.0156	6.7189
22	0.1228	8.7715	60.6893	8.1403	71.4027	0.1140	0.0140	6.9189
23	0.1117	8.8832	63.1462	8.9543	79.5430	0.1126	0.0126	7.1085
24	0.1015	8.9847	65.4813	9.8497	88.4973	0.1113	0.0113	7.2881
25	0.0923	9.0770	67.6964	10.8347	98.3471	0.1102	0.0102	7.4580
30	0.0573	9.4269	77.0766	17.4494	164.4940	0.1061	0.0061	8.1762
40	0.0221	9.7791	88.9525	45.2593	442.5926	0.1023	0.0023	9.0962
50	0.0085	9.9148	94.8889	117.3909	1,163.9085	0.1009	0.0009	9.5704
60	0.0033	9.9672	97.7010	304.4816	3,034.8164	0.1003	0.0003	9.8023
100	0.0001	9.9993	99.9202	13,780.6123	137,796.1234	0.1000		9.9927

Factor Table - $i = 12.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.8929	0.8929	0.0000	1.1200	1.0000	1.1200	1.0000	0.0000
2	0.7972	1.6901	0.7972	1.2544	2.1200	0.5917	0.4717	0.4717
3	0.7118	2.4018	2.2208	1.4049	3.3744	0.4163	0.2963	0.9246
4	0.6355	3.0373	4.1273	1.5735	4.7793	0.3292	0.2092	1.3589
5	0.5674	3.6048	6.3970	1.7623	6.3528	0.2774	0.1574	1.7746
6	0.5066	4.1114	8.9302	1.9738	8.1152	0.2432	0.1232	2.1720
7	0.4523	4.5638	11.6443	2.2107	10.0890	0.2191	0.0991	2.5515
8	0.4039	4.9676	14.4714	2.4760	12.2997	0.2013	0.0813	2.9131
9	0.3606	5.3282	17.3563	2.7731	14.7757	0.1877	0.0677	3.2574
10	0.3220	5.6502	20.2541	3.1058	17.5487	0.1770	0.0570	3.5847
11	0.2875	5.9377	23.1288	3.4785	20.6546	0.1684	0.0484	3.8953
12	0.2567	6.1944	25.9523	3.8960	24.1331	0.1614	0.0414	4.1897
13	0.2292	6.4235	28.7024	4.3635	28.0291	0.1557	0.0357	4.4683
14	0.2046	6.6282	31.3624	4.8871	32.3926	0.1509	0.0309	4.7317
15	0.1827	6.8109	33.9202	5.4736	37.2797	0.1468	0.0268	4.9803
16	0.1631	6.9740	36.3670	6.1304	42.7533	0.1434	0.0234	5.2147
17	0.1456	7.1196	38.6973	6.8660	48.8837	0.1405	0.0205	5.4353
18	0.1300	7.2497	40.9080	7.6900	55.7497	0.1379	0.0179	5.6427
19	0.1161	7.3658	42.9979	8.6128	63.4397	0.1358	0.0158	5.8375
20	0.1037	7.4694	44.9676	9.6463	72.0524	0.1339	0.0139	6.0202
21	0.0926	7.5620	46.8188	10.8038	81.6987	0.1322	0.0122	6.1913
22	0.0826	7.6446	48.5543	12.1003	92.5026	0.1308	0.0108	6.3514
23	0.0738	7.7184	50.1776	13.5523	104.6029	0.1296	0.0096	6.5010
24	0.0659	7.7843	51.6929	15.1786	118.1552	0.1285	0.0085	6.6406
25	0.0588	7.8431	53.1046	17.0001	133.3339	0.1275	0.0075	6.7708
30	0.0334	8.0552	58.7821	29.9599	241.3327	0.1241	0.0041	7.2974
40	0.0107	8.2438	65.1159	93.0510	767.0914	0.1213	0.0013	7.8988
50	0.0035	8.3045	67.7624	289.0022	2,400.0182	0.1204	0.0004	8.1597
60	0.0011	8.3240	68.8100	897.5969	7,471.6411	0.1201	0.0001	8.2664
100		8.3332	69.4336	83,522.2657	696,010.5477	0.1200		8.3321

Factor Table - $i = 18.00\%$

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.8475	0.8475	0.0000	1.1800	1.0000	1.1800	1.0000	0.0000
2	0.7182	1.5656	0.7182	1.3924	2.1800	0.6387	0.4587	0.4587
3	0.6086	2.1743	1.9354	1.6430	3.5724	0.4599	0.2799	0.8902
4	0.5158	2.6901	3.4828	1.9388	5.2154	0.3717	0.1917	1.2947
5	0.4371	3.1272	5.2312	2.2878	7.1542	0.3198	0.1398	1.6728
6	0.3704	3.4976	7.0834	2.6996	9.4423	0.2859	0.1059	2.0252
7	0.3139	3.8115	8.9670	3.1855	12.1415	0.2624	0.0824	2.3526
8	0.2660	4.0776	10.8292	3.7589	15.3270	0.2452	0.0652	2.6558
9	0.2255	4.3030	12.6329	4.4355	19.0859	0.2324	0.0524	2.9358
10	0.1911	4.4941	14.3525	5.2338	23.5213	0.2225	0.0425	3.1936
11	0.1619	4.6560	15.9716	6.1759	28.7551	0.2148	0.0348	3.4303
12	0.1372	4.7932	17.4811	7.2876	34.9311	0.2086	0.0286	3.6470
13	0.1163	4.9095	18.8765	8.5994	42.2187	0.2037	0.0237	3.8449
14	0.0985	5.0081	20.1576	10.1472	50.8180	0.1997	0.0197	4.0250
15	0.0835	5.0916	21.3269	11.9737	60.9653	0.1964	0.0164	4.1887
16	0.0708	5.1624	22.3885	14.1290	72.9390	0.1937	0.0137	4.3369
17	0.0600	5.2223	23.3482	16.6722	87.0680	0.1915	0.0115	4.4708
18	0.0508	5.2732	24.2123	19.6731	103.7403	0.1896	0.0096	4.5916
19	0.0431	5.3162	24.9877	23.2144	123.4135	0.1881	0.0081	4.7003
20	0.0365	5.3527	25.6813	27.3930	146.6280	0.1868	0.0068	4.7978
21	0.0309	5.3837	26.3000	32.3238	174.0210	0.1857	0.0057	4.8851
22	0.0262	5.4099	26.8506	38.1421	206.3448	0.1848	0.0048	4.9632
23	0.0222	5.4321	27.3394	45.0076	244.4868	0.1841	0.0041	5.0329
24	0.0188	5.4509	27.7725	53.1090	289.4944	0.1835	0.0035	5.0950
25	0.0159	5.4669	28.1555	62.6686	342.6035	0.1829	0.0029	5.1502
30	0.0070	5.5168	29.4864	143.3706	790.9480	0.1813	0.0013	5.3448
40	0.0013	5.5482	30.5269	750.3783	4,163.2130	0.1802	0.0002	5.5022
50	0.0003	5.5541	30.7856	3,927.3569	21,813.0937	0.1800		5.5428
60	0.0001	5.5553	30.8465	20,555.1400	114,189.6665	0.1800		5.5526
100		5.5556	30.8642	15,424,131.91	85,689,616.17	0.1800		5.5555

CHEMICAL ENGINEERING

CHEMICAL THERMODYNAMICS

Vapor-Liquid Equilibrium

For a multi-component mixture at equilibrium

$$\hat{f}_i^V = \hat{f}_i^L, \text{ where}$$

$\hat{f}_i^V =$ fugacity of component i in the vapor phase, and

$\hat{f}_i^L =$ fugacity of component i in the liquid phase.

Fugacities of component i in a mixture are commonly calculated in the following ways:

For a liquid $\quad \hat{f}_i^L = x_i \gamma_i f_i^L$, where

$x_i =$ mole fraction of component i,

$\gamma_i =$ activity coefficient of component i, and

$f_i^L =$ fugacity of pure liquid component i.

For a vapor $\quad \hat{f}_i^V = y_i \hat{\Phi}_i P$, where

$y_i =$ mole fraction of component i in the vapor,

$\hat{\Phi}_i =$ fugacity coefficient of component i in the vapor, and

$P =$ system pressure.

The activity coefficient γ_i is a correction for liquid phase non-ideality. Many models have been proposed for γ_i such as the Van Laar model:

$$\ln \gamma_1 = A_{12} \left(1 + \frac{A_{12} x_1}{A_{21} x_2} \right)^{-2}$$
$$\ln \gamma_2 = A_{21} \left(1 + \frac{A_{21} x_2}{A_{12} x_1} \right)^{-2}, \text{ where}$$

$\gamma_1 =$ activity coefficient of component 1 in a two-component system,

$\gamma_2 =$ activity coefficient of component 2 in a two-component system, and

$A_{12}, A_{21} =$ constants, typically fitted from experimental data.

The pure component fugacity is calculated as:

$$f_i^L = \Phi_i^{sat} P_i^{sat} \exp\{v_i^L (P - P_i^{sat})/(RT)\}, \text{ where}$$

$\Phi_i^{sat} =$ fugacity coefficient of pure saturated i,

$P_i^{sat} =$ saturation pressure of pure i,

$v_i^L =$ specific volume of pure liquid i, and

$R =$ Ideal Gas Law Constant.

Often at system pressures close to atmospheric:

$$f_i^L \cong P_i^{sat}$$

The fugacity coefficient $\hat{\Phi}_i$ for component i in the vapor is calculated from an equation of state (e.g., Virial). Sometimes it is approximated by a pure component value from a correlation. Often at pressures close to atmospheric, $\hat{\Phi}_i = 1$. The fugacity coefficient is a correction for vapor phase non-ideality.
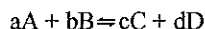
For sparingly soluble gases the liquid phase is sometimes represented as

$$\hat{f}_i^L = x_i k_i$$

where k_i is a constant set by experiment (Henry's constant). Sometimes other concentration units are used besides mole fraction with a corresponding change in k_i.

Chemical Reaction Equilibrium

For reaction

$$aA + bB \rightleftharpoons cC + dD$$

$$\Delta G^o = -RT \ln K_a$$

$$K_a = \frac{\left(\hat{a}_C^c\right)\left(\hat{a}_D^d\right)}{\left(\hat{a}_A^a\right)\left(\hat{a}_B^b\right)} = \prod_i \left(\hat{a}_i\right)^{v_i}, \text{ where}$$

$\hat{a}_i =$ activity of component $i = \dfrac{\hat{f}_i}{f_i^o}$

$f_i^o =$ fugacity of pure i in its standard state

$v_i =$ stoichiometric coefficient of component i

$\Delta G^o =$ standard Gibbs energy change of reaction

$K_a =$ chemical equilibrium constant

For mixtures of ideal gases:

$\quad\quad f_i^o =$ unit pressure, often 1 bar

$$\hat{f}_i = y_i P = p_i$$

where $p_i =$ partial pressure of component i.

Then $K_a = K_p = \dfrac{\left(p_C^c\right)\left(p_D^d\right)}{\left(p_A^a\right)\left(p_B^b\right)} = P^{c+d-a-b} \dfrac{\left(y_C^c\right)\left(y_D^d\right)}{\left(y_A^a\right)\left(y_B^b\right)}$

For solids $\quad \hat{a}_i = 1$

For liquids $\quad \hat{a}_i = x_i \gamma_i$

The effect of temperature on the equilibrium constant is

$$\frac{d \ln K}{dT} = \frac{\Delta H^o}{RT^2}$$

where $\Delta H^o =$ standard enthalpy change of reaction.

For additional information concerning heat transfer and fluid mechanics, refer to the **HEAT TRANSFER, THERMODYNAMICS,** or **FLUID MECHANICS** sections.

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HEATS OF REACTION

For a chemical reaction the associated energy can be defined in terms of heats of formation of the individual species $\left(\Delta \hat{H}_f^o\right)$ at the standard state .

$$\left(\Delta \hat{H}_r^o\right) = \sum_{products} v_i \left(\Delta \hat{H}_f^o\right)_i - \sum_{reactants} v_i \left(\Delta \hat{H}_f^o\right)_i$$

The standard state is 25°C and 1 bar.

The heat of formation is defined as the enthalpy change associated with the formation of a compound from its atomic species as they normally occur in nature (i.e., $O_{2(g)}$, $H_{2(g)}$, $C_{(solid)}$, etc.)

The heat of reaction for a combustion process using oxygen is also known as the heat of combustion. The principal products are $CO_{2(g)}$ and $H_2O_{(e)}$.

CHEMICAL REACTION ENGINEERING

A chemical reaction may be expressed by the general equation

$$aA + bB \leftrightarrow cC + dD.$$
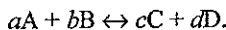
The rate of reaction of any component is defined as the moles of that component formed per unit time per unit volume.

$$-r_A = -\frac{1}{V}\frac{dN_A}{dt} \qquad \text{[negative because A disappears]}$$

$$-r_A = \frac{-dC_A}{dt} \qquad \text{if V is constant}$$

The rate of reaction is frequently expressed by

$$-r_A = kf_r(C_A, C_B, ...), \text{ where}$$

k = reaction rate constant and

C_I = concentration of component I.

The Arrhenius equation gives the dependence of k on temperature

$$k = Ae^{-E_a/\overline{R}T}, \text{ where}$$

A = pre-exponential or frequency factor,

E_a = activation energy (J/mol, cal/mol),

T = temperature (K), and

$\overline{R}$ = gas law constant = 8.314 J/(mol·K).

In the conversion of A, the fractional conversion X_A, is defined as the moles of A reacted per mole of A fed.

$$X_A = (C_{Ao} - C_A)/C_{Ao} \qquad \text{if V is constant}$$

Reaction Order

If $-r_A = kC_A^x C_B^y$

the reaction is x order with respect to reactant A and y order with respect to reactant B. The overall order is

$$n = x + y$$

BATCH REACTOR, CONSTANT T AND V

Zero-Order Reaction

$$-r_A = kC_A^o = k(1)$$
$$-dC_A/dt = k \qquad \text{or}$$
$$C_A = C_{Ao} - kt$$
$$dX_A/dt = k/C_{Ao} \qquad \text{or}$$
$$C_{Ao}X_A = kt$$

First-Order Reaction

$$-r_A = kC_A$$
$$-dC_A/dt = kC_A \qquad \text{or}$$
$$\ln(C_A/C_{Ao}) = -kt$$
$$dX_A/dt = k(1 - X_A) \qquad \text{or}$$
$$\ln(1 - X_A) = -kt$$

Second-Order Reaction

$$-r_A = kC_A^2$$
$$-dC_A/dt = kC_A^2 \qquad \text{or}$$
$$1/C_A - 1/C_{Ao} = kt$$
$$dX_A/dt = kC_{Ao}(1 - X_A)^2 \qquad \text{or}$$
$$X_A/[C_{Ao}(1 - X_A)] = kt$$

Batch Reactor, General

For a well-mixed, constant-volume, batch reactor

$$-r_A = dC_A/dt$$
$$t = -C_{Ao}\int_o^{X_A} dX_A/(-r_A)$$

If the volume of the reacting mass varies with the conversion according to

$$V = V_{X_{A=0}}(1 + \varepsilon_A X_A)$$

$$\varepsilon_A = \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}}$$

then

$$t = -C_{Ao}\int_o^{X_A} dX_A/[(1 + \varepsilon_A X_A)(-r_A)]$$

FLOW REACTORS, STEADY STATE

Space-time τ is defined as the reactor volume divided by the inlet volumetric feed rate. Space-velocity SV is the reciprocal of space-time, $SV = 1/\tau$.

Plug-Flow Reactor (PFR)

$$\tau = \frac{C_{Ao}V_{PFR}}{F_{Ao}} = C_{Ao}\int_o^{X_A}\frac{dX_A}{(-r_A)}, \text{ where}$$

F_{Ao} = moles of A fed per unit time.

Continuous Stirred Tank Reactor (CSTR)

For a constant volume, well-mixed, CSTR

$$\frac{\tau}{C_{Ao}} = \frac{V_{CSTR}}{F_{Ao}} = \frac{X_A}{-r_A} \text{ , where}$$

$-r_A$ is evaluated at exit stream conditions.

<u>Continuous Stirred Tank Reactors in Series</u>

With a first-order reaction $A \rightarrow R$, no change in volume.

$$\tau_{N\text{-reactors}} = N\tau_{individual}$$

$$= \frac{N}{k}\left[\left(\frac{C_{Ao}}{C_{AN}}\right)^{1/N} - 1\right], \text{ where}$$

N = number of CSTRs (equal volume) in series, and

C_{AN} = concentration of A leaving the Nth CSTR.

DISTILLATION

Flash (or equilibrium) Distillation

Component material balance:

$$Fz_F = yV + xL$$

Overall material balance:

$$F = V + L$$

Differential (Simple or Rayleigh) Distillation

$$\ln\left(\frac{W}{W_o}\right) = \int_{x_o}^{x}\frac{dx}{y-x}$$

When the relative volatility α is constant,

$$y = \alpha x/[1 + (\alpha - 1)x]$$

can be substituted to give

$$\ln\left(\frac{W}{W_o}\right) = \frac{1}{(\alpha-1)}\ln\left[\frac{x(1-x_o)}{x_o(1-x)}\right] + \ln\left[\frac{1-x_o}{1-x}\right]$$

For binary system following Raoult's Law

$$\alpha = (y/x)_a/(y/x)_b = p_a/p_b, \text{ where}$$

p_i = partial pressure of component i.

Continuous Distillation (binary system)

Constant molal overflow is assumed (trays counted downward)

<u>Overall Material Balances</u>

Total Material:

$$F = D + B$$

Component A:

$$Fz_F = Dx_D + Bx_B$$

<u>Operating Lines</u>

Rectifying Section

Total Material:

$$V_{n+1} = L_n + D$$

Component A:

$$V_{n+1}y_{n+1} = L_n x_n + Dx_D$$

$$y_{n+1} = [L_n/(L_n + D)] x_n + Dx_D/(L_n + D)$$

Stripping Section

Total Material:

$$L_m = V_{m+1} + B$$

Component A:

$$L_m x_m = V_{m+1}y_{m+1} + Bx_B$$

$$y_{m+1} = [L_m/(L_m - B)] x_m - Bx_B/(L_m - B)$$

Reflux Ratio

Ratio of reflux to overhead product

$$R_D = L/D = (V - D)/D$$

Minimum reflux ratio is defined as that value which results in an infinite number of contact stages. For a binary system the equation of the operating line is

$$y = \frac{R_{min}}{R_{min} + 1}x + \frac{x_D}{R_{min} + 1}$$

Feed Condition Line

slope = $q/(q - 1)$, where

$$q = \frac{\text{heat to convert one mol of feed to saturated vapor}}{\text{molar heat of vaporization}}$$

Murphree Plate Efficiency

$$E_{ME} = (y_n - y_{n+1})/(y^*_n - y_{n+1}), \text{ where}$$

y = concentration of vapor above plate n,

y_{n+1} = concentration of vapor entering from plate below n, and

y^*_n = concentration of vapor in equilibrium with liquid leaving plate n.

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A similar expression can be written for the stripping section by replacing n with m.

Definitions:

α = relative volatility,

B = molar bottoms-product rate,

D = molar overhead-product rate,

F = molar feed rate,

L = molar liquid downflow rate,

R_D = ratio of reflux to overhead product,

V = molar vapor upflow rate,

W = weight in still pot,

x = mole fraction of the more volatile component in the liquid phase, and

y = mole fraction of the more volatile component in the vapor phase.

Subscripts

B = bottoms product,

D = overhead product,

F = feed,

m = any plate in stripping section of column,

$m+1$ = plate below plate m,

n = any plate in rectifying section of column,

$n+1$ = plate below plate n, and

o = original charge in still pot.
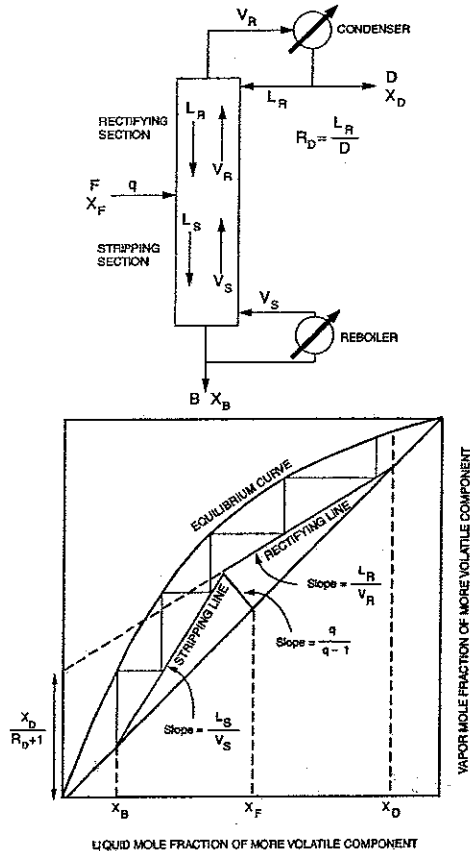
MASS TRANSFER

Diffusion

Molecular Diffusion

Gas: $\dfrac{N_A}{A} = \dfrac{p_A}{P}\left(\dfrac{N_A}{A} + \dfrac{N_B}{A}\right) - \dfrac{D_m}{RT}\dfrac{\partial p_A}{\partial z}$

Liquid: $\dfrac{N_A}{A} = x_A\left(\dfrac{N_A}{A} + \dfrac{N_B}{A}\right) - CD_m\dfrac{\partial x_A}{\partial z}$

in which $(p_B)_{lm}$ is the log mean of p_{B2} and p_{B1},

Unidirectional Diffusion of a Gas A Through a Second Stagnant Gas B ($N_b = 0$)

$$\dfrac{N_A}{A} = \dfrac{D_m P}{\overline{R}T(p_B)_{lm}} \times \dfrac{(p_{A2} - p_{A1})}{z_2 - z_1}$$

in which $(p_B)_{lm}$ is the log mean of p_{B2} and p_{B1},

N_I = diffusive flow of component I through area A, in z direction, and

D_m = mass diffusivity.

EQUIMOLAR COUNTER-DIFFUSION (GASES)

($N_B = -N_A$)

$$N_A/A = D_m/(\overline{R}T) \times [(p_{A1} - p_{A2})/(z_2 - z_1)]$$

Unsteady State Diffusion in a Gas

$$\partial p_A/\partial t = D_m (\partial^2 p_A/\partial z^2)$$

CONVECTION

Two-Film Theory (for Equimolar Counter-Diffusion)

$$
\begin{aligned}
N_A/A \quad &= \quad k'_G (p_{AG} - p_{Ai}) \\
&= \quad k'_L (C_{Ai} - C_{AL}) \\
&= \quad K'_G (p_{AG} - p_A^*) \\
&= \quad K'_L (C_A^* - C_{AL})
\end{aligned}
$$

where p_A^* is partial pressure in equilibrium with C_{AL}, and

$C_A^* =$ concentration in equilibrium with p_{AG}.

Overall Coefficients

$$1/K'_G = 1/k'_G + H/k'_L$$

$$1/K'_L = 1/Hk'_G + 1/k'_L$$

Dimensionless Group Equation (Sherwood)

For the turbulent flow inside a tube the Sherwood number

$\left(\dfrac{k_m D}{D_m}\right)$ is given by: $\left(\dfrac{k_m D}{D_m}\right) = 0.023\left(\dfrac{D\nu\rho}{\mu}\right)^{0.8}\left(\dfrac{\mu}{\rho D_m}\right)^{1/3}$

where,

D = inside diameter,

D_m = diffusion coefficient,

V = average velocity in the tube,

ρ = fluid density, and

μ = fluid viscosity.