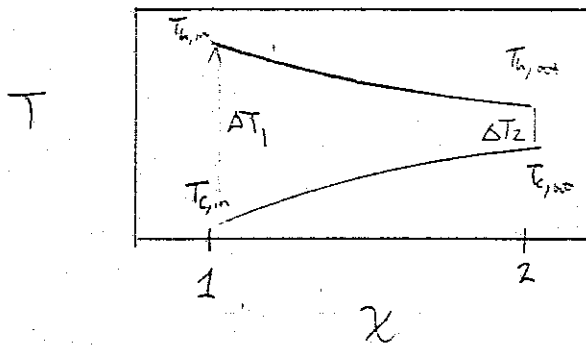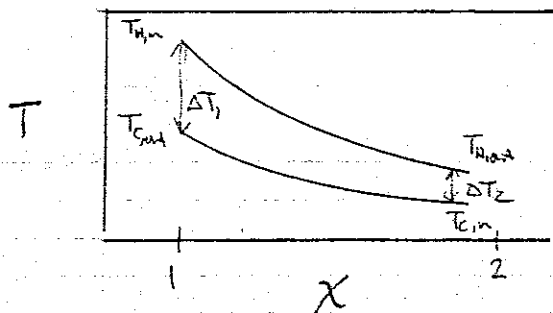Parallel flow

Counter Current

$$\Delta T_1 = T_{h,in} - T_{c,out}$$

$$\Delta T_2 = T_{h,out} - T_{c,in}$$

Rule of Thumb:

ΔT approach should be no less than $10\,°C$
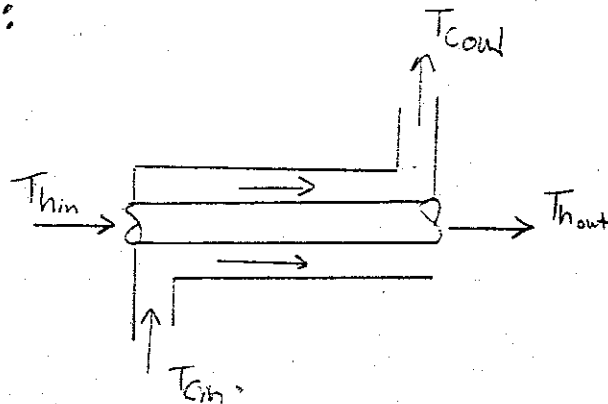
Heat Exchangers

Parallel flow:

Overall Energy Balance:

$$\dot{m}\left(H_{h_{in}} - H_{h_{out}}\right) = q$$

$$\dot{m} C_{ph}\left(T_{h_{in}} - T_{h_{out}}\right)$$

$$= \dot{m} C_{pc}\left(T_{out} - T_{c_{in}}\right) = q$$

for a condensing Process:

$$\dot{m}_h \lambda = \dot{m}_c C_{pc}\left(T_{c_{out}} - T_{c_{in}}\right) = q$$

(latent heat of vaporization)

Overall heat transfer coefficient:

$$d\dot{q} = U \Delta T \, dA$$

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} + \frac{\ln(D_o/D_i)}{2\pi k L}$$

wall ↗

↑
Can also include
fouling

$$d(\Delta T) = dT_h - dT_c$$

$$d(\Delta T) = -dq\left(\frac{1}{C_{ph}} + \frac{1}{C_{pc}}\right)$$

$$\int_{\substack{\Delta T_1 \\ T_{h_{in}} - T_{c_{in}}}}^{\substack{\Delta T_2 \\ T_{h_{out}} - T_{c,out}}} d(\Delta T) = -U \Delta T \left(\frac{1}{C_{ph}} + \frac{1}{C_{pc}}\right) dA$$

$$q = UA \ \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

for parallel $\Delta T_2 = T_{h,i} - T_{c,i}$

$$\Delta T_1 = T_{h,o} - T_{c,o}$$

combining the energy balance:

$$N_{air} \frac{d}{dz}\left(\widetilde{C}_{p,air} T_{air} + \Delta\widetilde{H}_{vap} y_{H_2O}\right) = ha(T_{H_2O} - T_{air}) +$$

$$k_a c \, \Delta\widetilde{H}_{vap}\left(y_{H_2O,i} - y_{H_2O}\right)$$

Chilton-Colburn analogy

$$\frac{k}{v}\left(\frac{v}{\mathcal{D}}\right)^{2/3} = \frac{h}{\rho\widetilde{C}_{p,air}v}\left(\frac{v}{\alpha}\right)^{2/3} \qquad \text{for Gases } Le \approx 1 = \frac{\alpha}{\mathcal{D}}$$

$$h = \left(c\,\widetilde{C}_{p,air}\right)k$$

Then:

$$N_{air}\frac{d}{dz}\left(\widehat{H}\right) = k_a c\left(\widehat{H}_i - \widehat{H}\right)$$

where

$$\widehat{H} = \widetilde{C}_{p,air} T_{air} + \Delta\widehat{H}_{vap} y_{H_2O}$$

$$\widehat{H}_i = \widetilde{C}_{p,air} T_{H_2O} + \Delta\widetilde{H}_{vap} y_{H_2O,i}$$

$$\boxed{\text{Cooling Towers}}$$

$G_{out} \uparrow \qquad \longrightarrow L_{in} (\text{Hot})$

$z + \Delta z$

z

$\longrightarrow L_{out} (\text{Cold})$

G_{out}
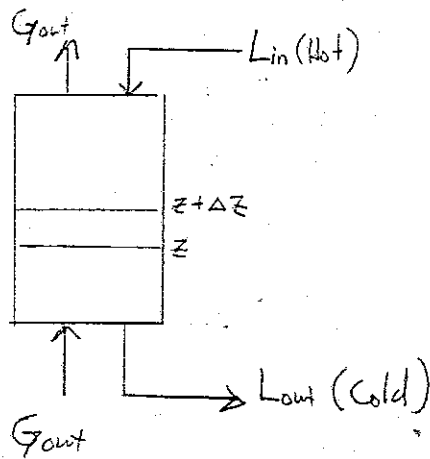
Need to consider simultaneous Heat and Mass Transfer

Mass balance on tower:

$$accum = In - Out + gen$$
$$0$$

$$0 = G y_{H_2O}\Big|_z - G y_{H_2O}\Big|_{z+\Delta z} + (\Delta z a) k \left(C_{H_2O,i} - C_{H_2O} \right)$$

$$0 = -\frac{d(G y_{H_2O})}{dz} + kaC \left(y_{H_2O,i} - y_{H_2O} \right)$$

$$0 = -\underbrace{n_{air}}_{\substack{\text{moles} \\ \overline{\text{Area time}}(\text{flux})}} \frac{d y_{H_2O}}{dz} + kaC \left(\underbrace{y_{H_2O,i}}_{\substack{\text{interface} \\ \text{conc.}}} - \underbrace{y_{H_2O}}_{\substack{\text{bulk conc.}}} \right)$$

Energy Balance:

Air: $\qquad 0 = -n_{air} \tilde{C}_p \frac{d T_{air}}{dz} + ha \left(T_i - T_{air} \right)$

H_2O + wet air:

$$0 = -n_{H_2O} \tilde{C}_{p, H_2O} \frac{d T_{H_2O}}{dz} - n_{air} \frac{d \tilde{H}}{dz}$$

$$k\left(C_{A_i} - C_A\right)\left(\Delta H_{vap}\right) = h\left(T_\infty - T_i\right)$$

$$\frac{k}{c\,\hat{C}_p}\left(\frac{\alpha}{D}\right)^{2/3} c\left(y_{A_i} - y_A\right)\left(\Delta H_{vap}\right) = h\left(T_\infty - T_i\right)$$

$$y_A = y_{A_i} - \frac{\hat{C}_p}{\left(\Delta H_{vap}\right)}\left(T_\infty - T_i\right)$$

$$\binom{relative}{humidity} = \frac{P_A}{P_A(sat\ at\ T_\infty)} = \frac{y_A}{y_A(sat\ at\ T_\infty)}$$

$$= \frac{1}{y_A(sat\ at\ T_\infty)}\left[y_{A_i}(sat\ at\ T_i) - \frac{\hat{C}_p}{\Delta H_{vap}}\left(T_\infty - T_i\right)\right]$$

find $y_{A_i}(sat\ at\ T_i)$ and $y_A(sat\ at\ T_\infty)$ from Raoult's Law

$$y_A P = x_A^{\,1} P^{sat}$$

$$y_A = \frac{P^{sat}(T)}{P} \qquad P^{sat}(T)\ \text{is found from chart or Antoine}$$
$$\text{eqn.}$$

Wet-bulb Thermometer

T_∞

T_∞
P_{H_2O}

$T_i, P_{H_2O}(sat)$
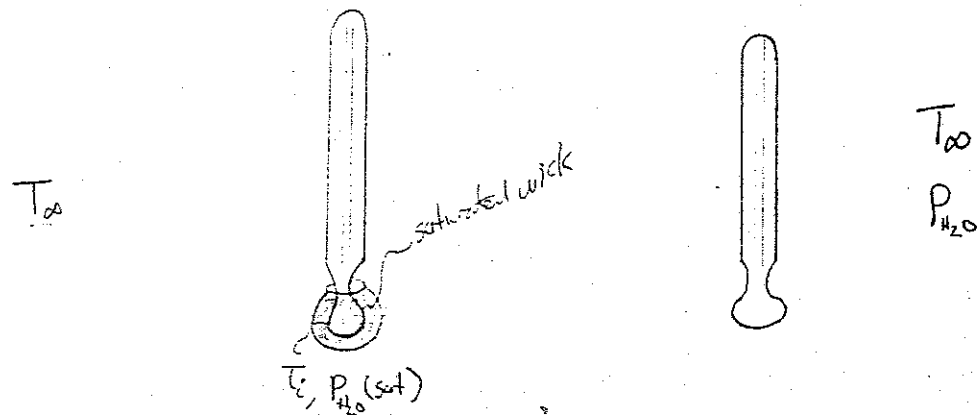
The wet bulb measures the colder temperature, which is caused by evaporation of the H_2O

T_∞ C_{Ai} T = const.

C_A

T_i H_2O
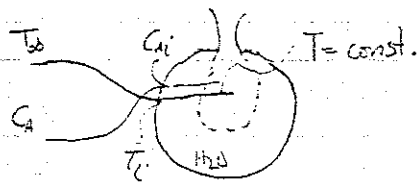
Mass flux:

$$N_A = k(C_{Ai} - C_A)$$

Energy:

$$q = h(T_i - T_\infty)$$

C_A = bulk concentration C_{Ai} = Saturation concentration at T_i

T_i = wet bulb temperature T_∞ = dry bulb temperature

The mass and energy balances are coupled:

$$N_A \Delta H_{vap} = h(T_\infty - T_i)$$

(Negative)(Negative)   (positive)

from Chilton-Colburn analogy

$$k = \frac{h}{\rho \hat{C}_p}\left(\frac{D}{\alpha}\right)^{2/3} = \frac{h}{c \hat{C}_p}\left(\frac{D}{\alpha}\right)^{2/3}$$

$x, y =$ wt fraction C

1.0 C

x_P

F
y_E^*
x_W
x_R

0 A

L
R
M
P
E
K
S

Wt fraction B

(a)

B A L

R
x_R
x_P
M
x_W
P
E
y_E^*
K B

C

(b)

Figure 10.9 Rectangular coordinates.

plait point
where two phases
become one

CH_3COCH_3

Conjugate line

conjugate line ex find on the
tie line end pts

a

b

c

TCE

does not ... d upon the prior specification of a separation between the two components. The degrees of freedom available for a single-feed column were listed in Sec. 7-5, and inspection of the tabulation there shows that only one product concentration can be specified at any given solvent treat. Therefore, it is not possible to locate both V_N and L_1 arbitrarily by specification and then find the solvent treat which requires an infinite number of stages to accomplish the specified separation. The solvent treat is fixed by the location of V_N and L_1, and there is only one set of V_N and L_1 locations (plus the corresponding solvent treat) which will require an infinite number of stages.

FIG. 7-26. Determination of minimum solvent treat in system where the tie lines have a negative slope. Tie line A gives the intersection closest to I_{N+1}. The operating Δ must lie between I_{N+1} and Δ_{min}.

The location of the minimum solvent treat is illustrated in Fig. 7-26 for a single-feed column. If the tie lines have a negative slope as shown in this figure, the minimum treat will occur while L_{N+1} is still larger than V_N and the difference point still lies to the right of the diagram. At infinite solvent treat, Δ is coincident with I_{N+1}. As the treat is decreased, Δ moves to the right on a straight line through V_N and L_{N+1}, since

$$\Delta_R = L_{N+1} - V_N$$

The minimum treat as defined above will occur when Δ reaches the first intersection which any extrapolated tie line within the diagram makes with the line through V_N and L_{N+1}. All tie lines, or rather tie lines from

all parts of the diagram, must be extrapolated to find that intersection lies closest to L_{N+1}. In Fig. 7-26, tie line A is the one with which the operating lines first become coincident and its intersection determines the minimum solvent treat. The operating Δ must lie between I_{N+1} and Δ_{min}.

If the tie lines have a positive slope, their intersections with the line through V_N and I_{N+1} must occur to the left of the diagram as shown in Fig. 7-27. The location of Δ at infinite solvent treat is still at I_{N+1}, and Δ still moves to the right as the treat is decreased. No intersections with

FIG. 7-27. Determination of minimum solvent treat in system where the tie lines have a positive slope. Tie line A gives intersection farthest to the left. The operating Δ must lie to the left of Δ_{min}.

the extrapolated tie lines occur to the right, and therefore the decrease in the treat can be continued until V_N equals L_{N+1} in rate. When $V_N = I_{N+1}$, Δ falls at an infinite distance to the right of the diagram. Further decrease in the treat causes Δ to reappear to the left of the diagram (Δ switches from Δ_R to Δ_L). Now Δ moves toward V_N as the treat is decreased and must eventually reach the first intersection with a tie line. Usually the tie line through F gives the intersection farthest from V_N, but the tie lines in Fig. 7-27 have been located in such a manner as to show that this need not always be true.

The feed point F in Fig. 7-27 may lie high enough so that even the tie line at the plait point will fall below it. If this is the case, and if the tie
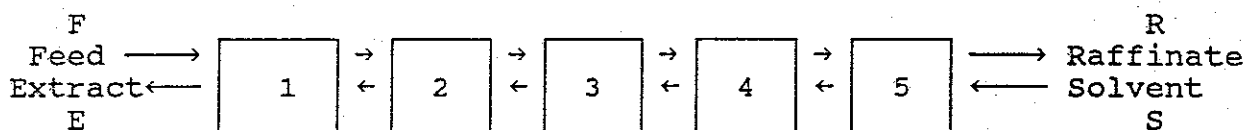
Reference: Chapter 18 of Wankat Equilibrium Staged Separations

Definition: Extraction is the separation process wherein two immiscible (or partially miscible) liquid phases are contacted to cause the transfer of at least one component between them. An example of such a process is the transfer of acetic acid from an aqueous phase to a hydrocarbon phase.

    This process is done without a vapor phase, at constant T and P. It is used where the high T's of distillation are undesirable, such as in temperature sensitive biochemical separations. Because it involves at least three components extraction is usually followed by another separation to purify the desired component. An extraction battery may be thought of as analogous to a distillation column "turned on its side".

```
 F                                                              R
 Feed ──→  ┌─────┐ → ┌─────┐ → ┌─────┐ → ┌─────┐ → ┌─────┐ ──→ Raffinate
 Extract←─ │  1  │ ← │  2  │ ← │  3  │ ← │  4  │ ← │  5  │ ←── Solvent
 E         └─────┘   └─────┘   └─────┘   └─────┘   └─────┘      S
```

No Temp Change no pressure change

Two Major Concepts of Extraction

I. Phase Equilibria Equilateral or Right Triangles at T & P
    A. Compositions May be Determined Directly from Graph
    B. The Two Phase Liquid Region
        1. Tie Lines relate Equilibrium Compositions
            a. Plait Point is where Miscibility Occurs
        2. Conjugate Line Enables Drawing any Tie Line
            a. One Stage Lever Rule Problem (Handout #3) =CMB
        3. PE Example 413 (Handout) illustrates:
            a. 2 Stage Crossflow Extraction
            b. Mixing Point & Separation Along Tie Line
II. Counter Current Extraction (Most Common Sort)
    A. Tie Lines Represent Equilibrium Phases Leaving Each Stage
    B. Mixing Point (M) for entire Extraction Battery
        1. Represents Mixing Feed and Solvent
        2. M Does not Separate along Tie Line (due to >1 stage)
        3. Allows Determination of R given E (or E given R)
    C. Delta (Δ) Point Connects Phases Passing Between Stages
        1. Δ Determined by Flow Difference (F-E) or (S-R)
            a. lies at intersection of $\overline{FE}$ and $\overline{SR}$
            b. analogous with operating lines in distillation
    D. Step Off Stages - Use Tie Lines & Δ Point Between R and E
        1. If Number of Stages Given - Trial & Error
    E. Minimum Solvent Treat $(S/F)_{min}$ (Feed close to Hypotenuse)
        1. Given F, S(comp), and R extend line $\overline{SR}$ graphically
        2. Extend all Tie Lines below F to intersect with $\overline{SR}$
        3. For (+) sloping tie lines Δ_{min} is closest inter-
section with $\overline{SR}$ to triangle. Δ must lie to the right of Δ_{min}
        4. For (-) sloping tie lines Δ_{min} is furthest inter-

18-D2.   Since S/F is not given, we cannot find a mixing point. However, with 184 stages must
          have a pinch point in the column. This means we are essentially at minimum reflux.
          Therefore, find Δ_{MIN} and $(S/F)_{MIN}$. This was already done in Figure 18-18. E_N is 0.058
          wt. frac. water and 0.186 wt. frac. acetic acid. $S = 1296$ kg/hr. This problem requires
          the student to think about the meaning of very large numbers of stages.

18-D3.   The solution is shown in the figure. Operating lines which extend from Δ to R_j to E_{j+1}
          are not shown. Mixing point is found from lever arm rule

$$\frac{S}{F} = \frac{\overline{MF}}{\overline{MS}} = \frac{85.5}{250} = .342$$

          To find exiting extract flow rate

$$E + R = S + F = 85.5 + 250 = 335.5$$

          From lever arm rule $\dfrac{E}{R} = \dfrac{\overline{MR}}{\overline{ME}} = 1.05$

          Solving, $E_{out} = 171.8$ kg/hr
          Outlet extract is 48.0% acetone, 47.9% CH_2Cl_2 and 4.1% water.
          Need about 7 1/2 equilibrium stages
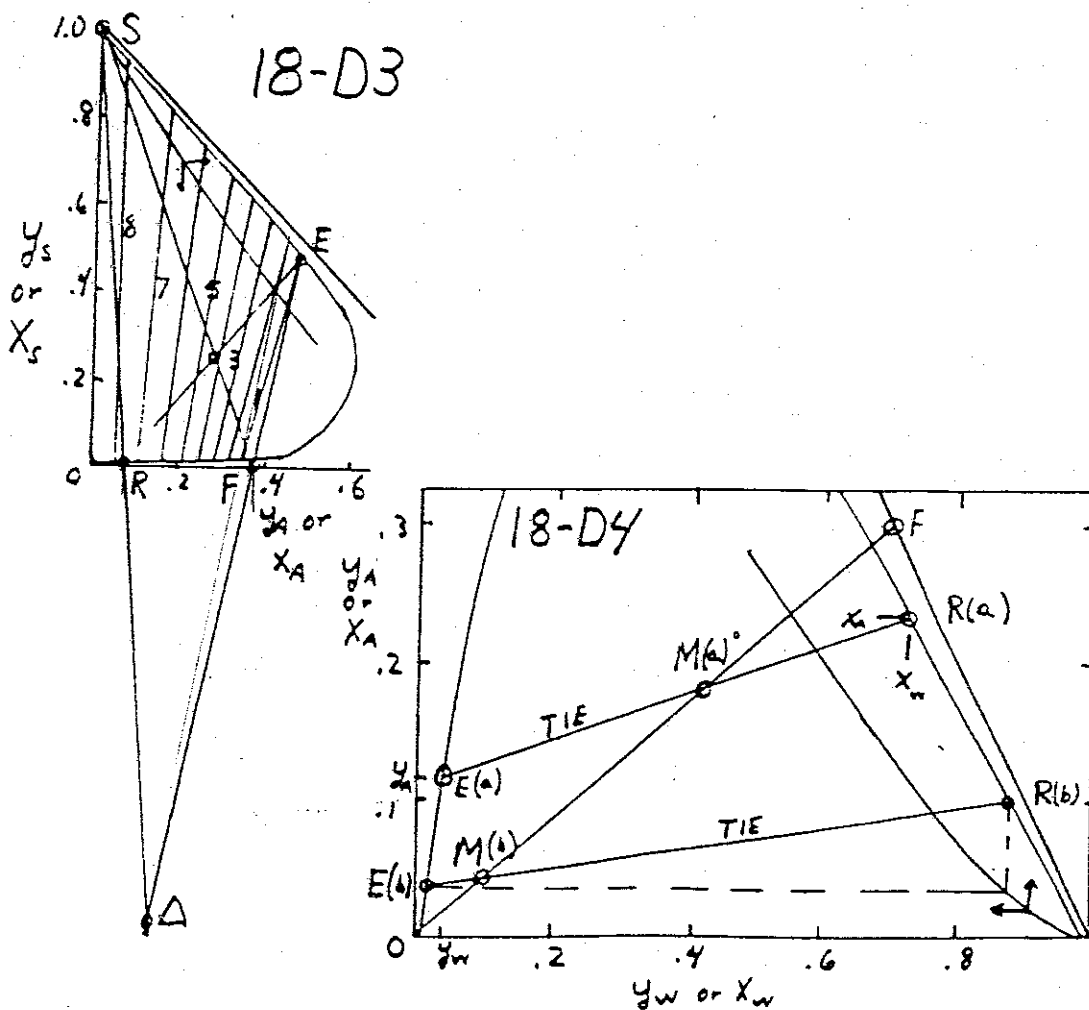
18-D3

18-D4

18-D5.   Plot points F, S, R₁.

Mixing Point: $F/(S+F) = 1000/(1500 + 1000) = .4 = \overline{SM}/\overline{SF}$

Find M. Can also calculate coordinates of M. Draw line R₁M to find point E_N. Lines R_{N+1}E_N and R₁S intersect at Δ. From Eq. (18-15),

$$R_{J+1}/E_J = \overline{E_J\Delta}/\overline{\Delta R_{J+1}}$$

$\overline{E\Delta}/\overline{\Delta R_1} = 74/190 = R_1/E_o = R_1/1500$. Thus, $R_1 = 584$ kg/hr

External balance: $R_1 + E_N = S + F$

which gives, $E_N = 1920$ kg/hr

Results from Figure are need 8 1/4 stages. Extract is 20 wt% acetic acid and 5.7% water. Raffinate is 7 wt% acetic acid and 91% water.

18-D5

18-D5. Plot points F, S, R_1.

Mixing Point: $F/(S+F) = 1000/(1500 + 1000) = .4 = \overline{SM}/\overline{SF}$

Find M. Can also calculate coordinates of M. Draw line R_1M to find point E_N. Lines $R_{N+1}E_N$ and R_1S intersect at Δ. From Eq. (18-15),

$$R_{J+1}/E_J = \overline{E_J\Delta}/\overline{\Delta R_{J+1}}$$

$\overline{E\Delta}/\overline{\Delta R_1} = 74/190 = R_1/E_o = R_1/1500$. Thus, $R_1 = 584$ kg/hr

External balance: $R_1 + E_N = S + F$

which gives, $E_N = 1920$ kg/hr

Results from Figure are need 8 1/4 stages. Extract is 20 wt% acetic acid and 5.7% water. Raffinate is 7 wt% acetic acid and 91% water.

18-D6. Guess a value for M and step off stages. Repeat until need 3 stages. After three trials found M shown in figure. This required 3 1/10 stages which is close enough.

Extract Composition: Acetic Acid = 10.5%, Water = 3.5%.
Raffinate Composition: Acetic Acid = 5%, Water = 93%

Solvent Flow Rate: $F/(S+F) = \overline{SM}/\overline{SF} = 15/57 = 2000/(S + 2000)$

$$S = 5600 \text{ kg/hr}$$

Raffinate Flow Rate: $R_1/E_o = E_o\Delta/\Delta R_1 = R_1/5600$, $R_1 = 770$ kg/hr.

Extract Flow Rate: $E_N = F + S - R_1 = 2000 + 5600 - 772 = 6830$

18-D6
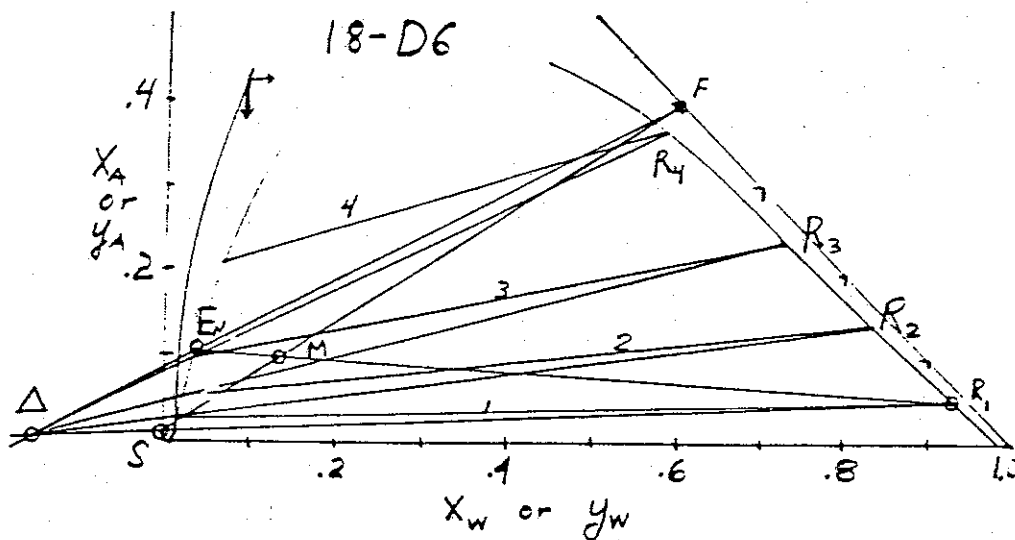
18-D7. Need to determine mixing points and raffinate flow rates stage-by-stage. Mass balances for stage j are,

$$R_{J-1} + E_{J,in} = M_J = R_J + E_J$$

$$R_{J-1} x_{A_{J-1}} + E_{J,in} y_{A_{J,in}} = R_J x_{A_J} + E_J y_{A_J} = M_J x_{A_{M_J}}$$

then coordinate of mixing point is

$$\text{Stage 3:} \quad x_{A_3} = \frac{R_2 x_{A_2} + E_{3,\text{in}} \, y_{\text{in}}}{R_2 + E_{3,\text{in}}} = \frac{(1376.2)(.205) + 25}{3876.2} = 0.079$$

From tie line: $x_{A_3} = .15$, $y_{A_3} = .04$

$$R_3 = M_3 \left[\frac{x_{A_3} - y_{A_3}}{x_{A_3} - y_{A_3}} \right] = (3876.2) \left[\frac{.079 - .04}{.15 - .04} \right] = 1382.5$$
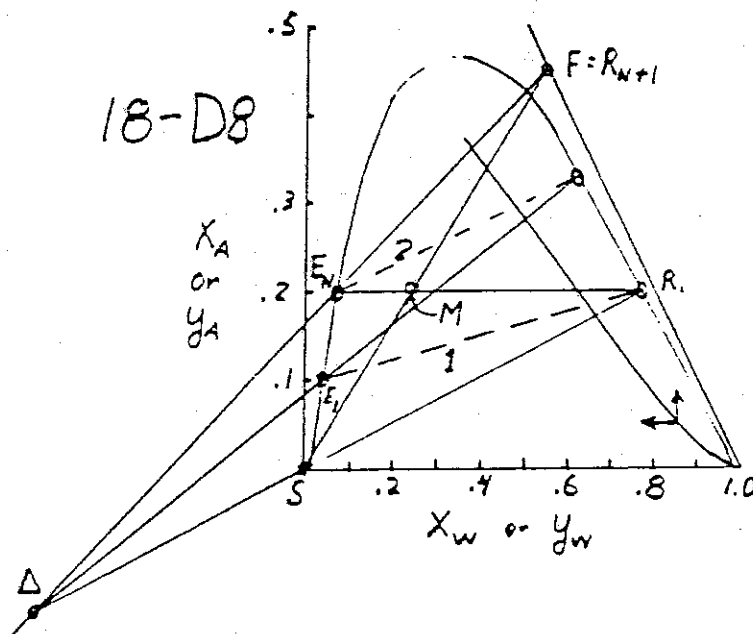
18-D8.   a. Draw lines from S to F and from R_1 to E_N. Intersection gives point M (see Figure). Then from lever-arm the,

$$\frac{S}{F} = \frac{\overline{FM}}{\overline{SM}} = 1.222$$

$$S = (1.222)(2000) = 2444$$

b. Δ is at intersection of lines $E_N R_{N+1}$ and $E_o R_1$. Then step off stages as shown. Need 2 stages.

18-D9.   To find Δ: 1) Plot E_N and $R_{N+1} = F$.

     2) $\Delta = E_J - R_{J+1} = E_N - R_{N+1} = 1500$

$$x_{A\Delta} = \frac{E_N x_{A_N} - R_{N+1} x_{A_{N+1}}}{\Delta} = 0.06666$$

     3) Δ is on line through points E_N and R_{N+1}.
Plot Δ. Or, use lever-rule

$$\frac{\Delta}{R_{N+1}} = \frac{\overline{R_{N+1} E_N}}{\overline{\Delta E_N}} = 1.5$$

Extraction
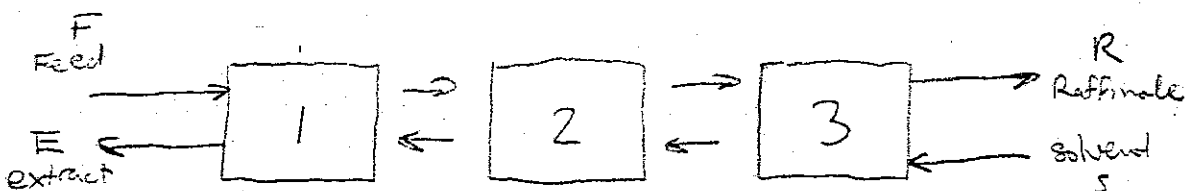
(1) Lever Rule:

Midpoint:

$$X_{A,m} = \frac{F_i X_{A,F_i} + S Y_{A,S}}{F_i + S}$$

$$X_{D,m} = \frac{F_i X_{D,F_i} + S Y_{D,S}}{F_i + S}$$

$$M = F + S$$
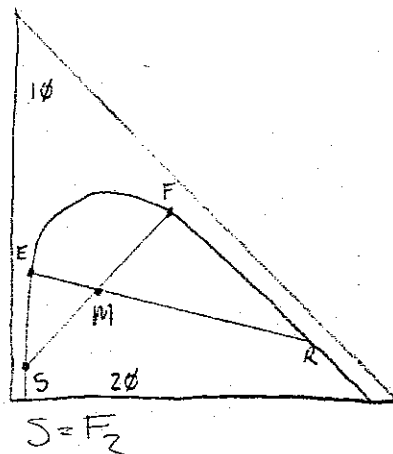
(2) Find $Y_{A,E}$; $Y_{D,E}$; $X_{A,R}$; $X_{D,R}$ by trial and error of a construction line through M

(3) Find E & R

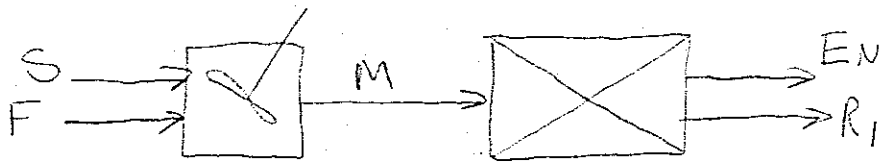$$M = E + R \qquad \text{\&} \qquad M X_{A,m} = E Y_{A,E} + R X_{A,R}$$

(4) Check:

$$S Y_{A,S} + F \cdot X_{A,S} = E Y_{A,E} + R X_{A,R}$$
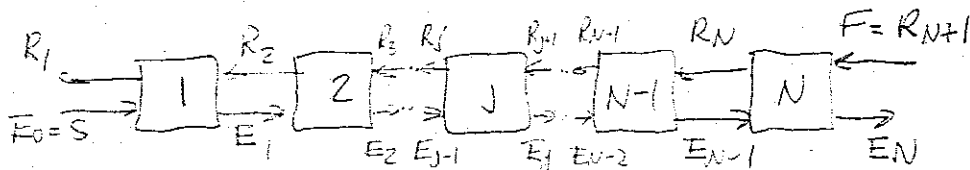
Cocurrent (1 stage is equivalent to ∞ # of stages)
Cross Current (more efficient than Cocurrent)
Counter Current (most efficient)

Counter Current Extraction :

$$E_0 + R_{N+1} = M = E_N + R_1$$

$$E_0 \, y_{A,0} + R_{N+1} \, X_{A,N+1} = M \, X_{A,m} = E_N \, y_{A,N} + R_1 \, X_{A,1}$$

For Midpoint :

$$X_{A,m} = \frac{E_0 \, y_{A,0} + R_{N+1} \, X_{A,N+1}}{E_0 + R_{N+1}}$$

$$X_{D,m} = \frac{E_0 \, y_{D,0} + R_{N+1} \, X_{D,N+1}}{E_0 + R_{N+1}}$$

Difference Point : (Passing streams)

$$\boxed{\Delta = E_0 - R_1 = \dots = E_j - R_{j+1} = \dots = E_N - R_{N+1}}$$

for solvent A

$$\Delta X_{A,\Delta} = E_0 \, y_{A,0} - R_1 \, X_{A,1} = \dots = E_j \, y_{A,j} - R_{j+1} \, X_{A,j+1}$$

$$= \dots = E_N \, y_{A,N} - R_{N+1} \, X_{A,N+1}$$

for Diluent D

$$\Delta X_{D,\Delta} = E_0 \, y_{D,0} - R_1 \, X_{D,1} = \dots = E_j \, y_{D,j} - R_{j+1} \, X_{A,j+1}$$

$$= \dots = E_N \, y_{D,N} - R_{N+1} \, X_{D,N+1}$$

Cordinates of Δ Point

$$\boxed{X_{A,\Delta} = \frac{E_0 \, y_{A,0} - R_1 \, X_{A,1}}{\Delta} = \frac{E_N \, y_{A,N} - R_{N+1} \, X_{A,N+1}}{\Delta}}$$

$$X_{D,\Delta} = \frac{E_0 \, y_{D,0} - R_1 \, X_{D,1}}{\Delta} = \frac{E_N \, y_{D,N} - R_{N+1} \, X_{D,N+1}}{\Delta}$$

3 Methods for Finding location of △

1) Graphical Construction

lines $\triangle, E_0, R$ & $\triangle, E_N + R_N + 1$

intersect @ $\triangle$

E_N can be found from external balances

2) Coordinates, eg. (18-14)

3) Lever arm Rule.

$$\frac{R_{j+1}}{E_j} = \frac{\overline{E_j \triangle}}{\overline{R_{j+1} \triangle}} \qquad \text{see page 592}$$

Minimum Solvent Treat

Page 602 & 603

follow step in book

closest to diagram (left side)
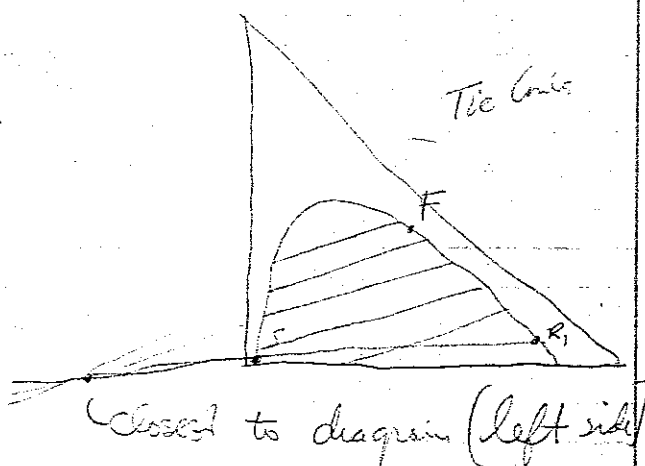
Tie lines to conjugate curve

Opacity Line

Tie Line

E

F

R₃

R₂

R₁

D
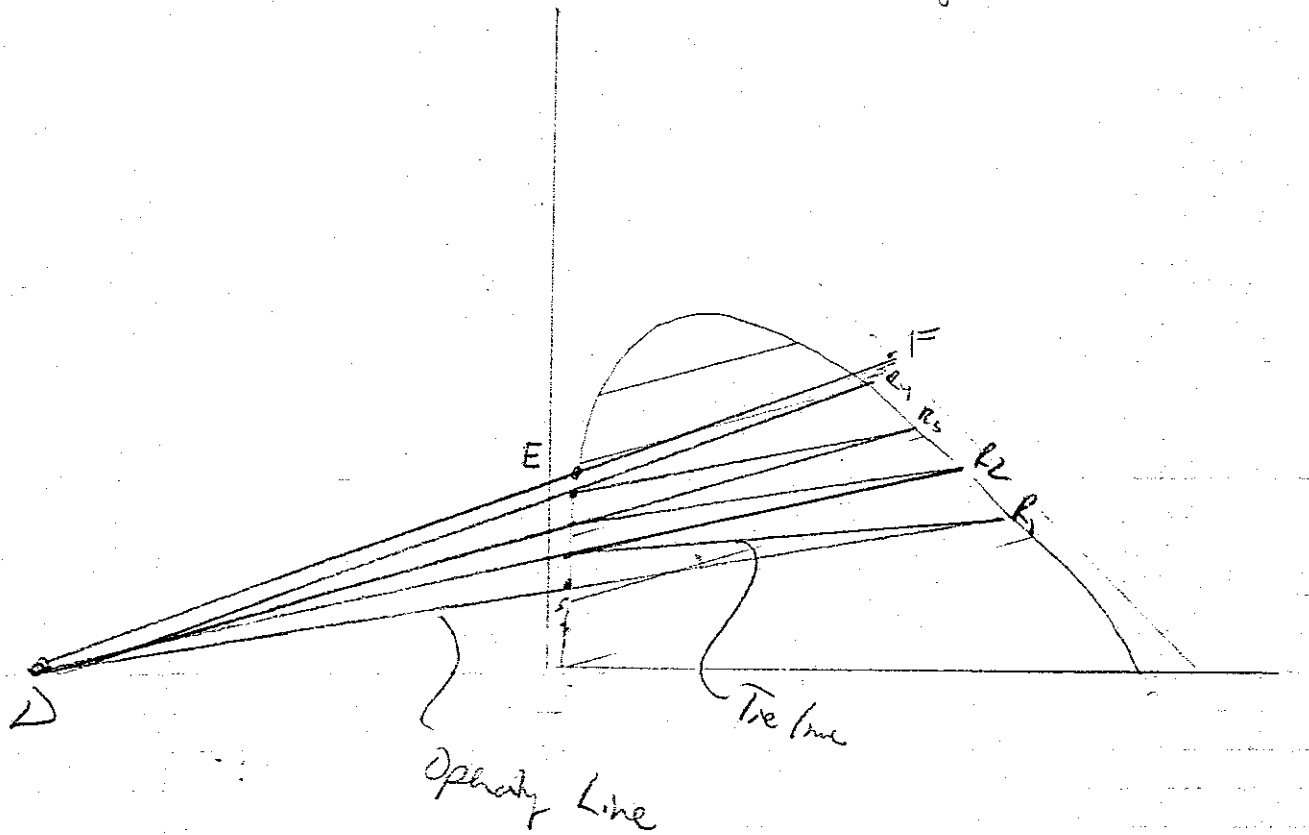
Steps in Extraction design Problem

1) Draw 2 phase region, tie lines & conjugate curve
2) With F, x_F, S, x_S find M_0 (overall mixing point)
     a) Given R_N (or E_1) use $R_N M_0 E_1$ line to get $F, (or R_N)$

3) Find $\Delta = F - E_1 = R_N - S$

4) Using conjugate curve & R_N find E_N along tie line

5) use $\Delta E_N R_N - 1$ line to get $R_N - 1$

6) repeat steps 4 & 5 until tie line crosses E_1

tie lines determine # of stages

Rating Problem :

Given 3 Stage Extraction Battery S, x_S, F, x_F
find E, x_E, R, x_R

1) Guess R_N & use $R_N M_0 E$ to get E_1
2) Find $\Delta = F - E_j = R_N - S$
4) 2 use steps 4 & 5 from above to determine
5) 3 # of steps

if # of steps is different
iterate (Guess new R_N)