Heat Transfer

Incropera & Dewitt

Heat Transfer - Energy in transit due to a temperature difference

I. Conduction - Transfer through a solid or stationary fluid, (gas or liquid)

No bulk motion, transfer due to molecular activity.

Basic Equation - Fourier's Law - $\frac{dQ}{d\theta} = -kA\left(\frac{dT}{dx}\right)$

heat flux $q''_x = -k\frac{dT}{dx}$ (1-D)

q''_x (W/m^2), k $(W/m \cdot K)$

heat flux is heat transfer in X direction per unit area.

The heat rate $= q_x = q''_x \cdot A$ (1-D)

Heat equation in 3-D

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \underbrace{\dot{q}}_{\text{Energy/Vol, time}} = \rho C_p \frac{\partial T}{\partial t}$$

Cartesian coordinates

(2)

Cylindrical Coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

Thermal Conductivity – k

Varies with temperature

Thermal diffusivity – α

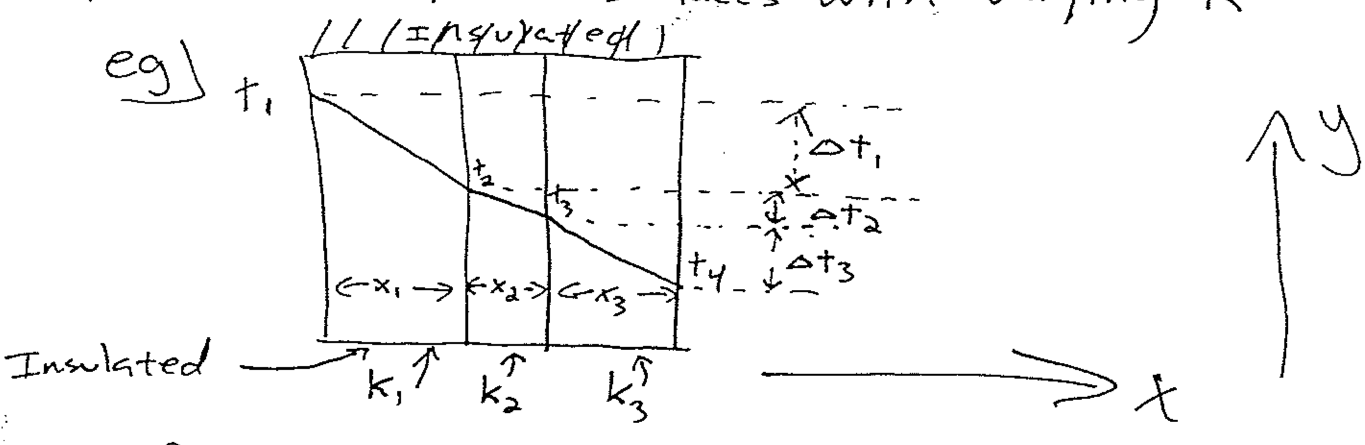
$$\alpha=\frac{k}{\rho C_p}$$

Thermal Resistance

$$R_{t\,conduction}=\frac{T_{s_1}-T_{s_2}}{q_x}=\frac{L}{kA}$$

A useful concept for surfaces with varying k

eg)



Heat flux constant through walls

$$q''=\frac{k_1A_1\Delta t_1}{x_1}=\frac{k_2A_2\Delta t_2}{x_2}=\frac{k_3A_3\Delta t_3}{x_3}$$

but $R = x/kA$

so $\Delta t_1 = qR_1 \quad \Delta t_2 = qR_2 \quad \Delta t_3 = qR_3$

$$q(R_1 + R_2 + R_3) = \Delta t_1 + \Delta t_2 + \Delta t_3 = \Sigma \Delta t$$

$$q = \Sigma[\Delta t / R_t] = (t_1 - t_4) R_t$$

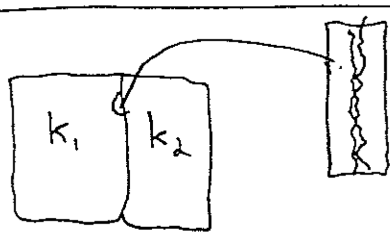
$$R_t = R_1 + R_2 + R_3$$

If the flow of heat was in the y direction

$$q = \Delta t_1/R_1 + \Delta t/R_2 + \Delta t/R_3 = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\Delta t$$

or $q = \Sigma C \Delta t$ where $C = \frac{1}{R} = \frac{kA}{x}$

Contact Resistance



Rough surfaces

×100~~000~~

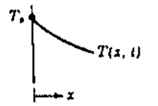
Usually neglected but be aware of it

Temperature Profiles For ~~Various~~ Various Conditions.

2.1 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

onstant surface temperature

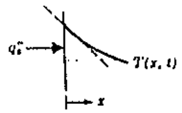
$$T(0,t) = T_s \qquad (2.20)$$



onstant surface heat flux

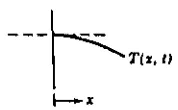
) Finite heat flux

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.21)$$



) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.22)$$



onvection surface condition

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)] \qquad (2.23)$$

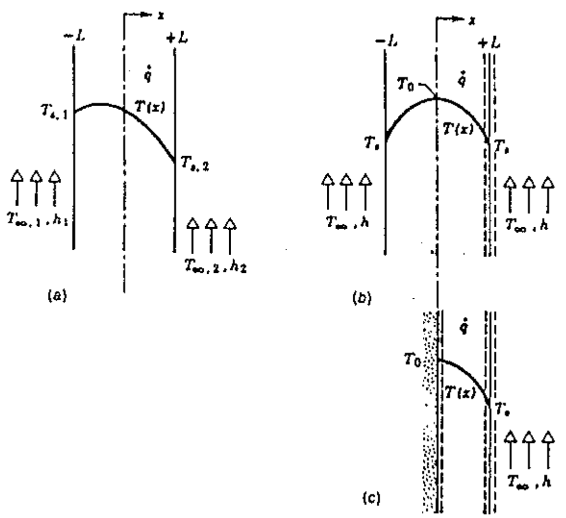


Figure 3.9 Conduction in a plane wall with uniform heat generation. (a) Asymmetrical boundary conditions. (b) Symmetrical boundary conditions. (c) Adiabatic surface at midplane.

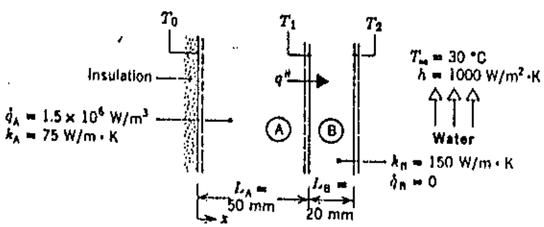
Known:

Plane wall of material A with internal heat generation is insulated on one side and bounded by a second wall of material B, which is without heat generation and is subjected to convection cooling.

Find:

1. Sketch of steady-state temperature distribution in the composite.
2. Inner and outer surface temperatures of the composite.

Schematic:

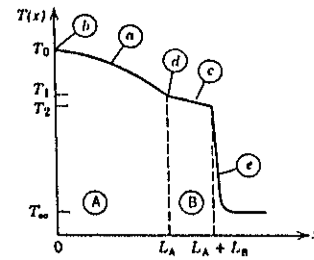


Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction in x direction.
3. Negligible contact resistance between walls.
4. Inner surface of A adiabatic.
5. Constant properties for materials A and B.

Analysis:

1. From the prescribed physical conditions, the temperature distribution in the composite is known to have the following features, as shown.
   (a) Parabolic in material A.
   (b) Zero slope at insulated boundary.
   (c) Linear in material B.
   (d) Slope change $= k_B/k_A = 2$ at interface.
   The temperature distribution in the water is characterized by
   (e) Large gradients near the surface.



2. The outer surface temperature T_2 may be obtained by performing an energy balance on a control volume about material B. Since there is no generation in this material, it follows that, for steady-state conditions and a unit surface area, the heat flux into the material at $x = L_A$ must equal the heat flux from the material due to convection at $x = L_A + L_B$. Hence

$$q'' = h(T_2 - T_\infty) \qquad (1)$$

The heat flux q'' may be determined by performing a second energy balance on a control volume about material A. In particular, since the surface at $x = 0$ is adiabatic, there is no inflow and the rate at which energy is generated must equal the outflow. Accordingly, for a unit surface area,

$$\dot{q}L_A = q'' \qquad (2)$$

Combining Equations 1 and 2, the outer surface temperature is

$$T_2 = T_\infty + \frac{\dot{q}L_A}{h}$$

$$T_2 = 30°\mathrm{C} + \frac{1.5 \times 10^6\ \mathrm{W/m^3} \times 0.05\ \mathrm{m}}{1000\ \mathrm{W/m^2 \cdot K}} = 105°\mathrm{C}$$

2-D Heat Transfer

Use your 230 experience.

If you are lucky, separation of variables will result in some analytical result. Otherwise use Graphical methods or Finite difference analysis.

Graphical Method – just the basics

Lines of constant Temp are perpendicular to lines that indicate direction of heat flow.

Construct a Flux plot of isotherms + heat flow lines.

Determine Heat transfer rate from graph

Shape factors have been determined for common configurations in which case $q = \underset{\text{shape factor}}{S} k \Delta T$

Not a commonly used method.

Finite-Difference Equations + Solutions

They can't ask you to do this on a prelim but you should be familliar with the general approach.

See Incropera + DeWitt p142-158

Unsteady State - Transient Conduction

Lumped Capacitance Method

Assumes temperature of solid is spatially uniform at any instant. Temperature gradients are negligible.

Good approximation if $\frac{R_{cond}}{R_{conv}}$ is small, with $R_{cond} \sim \frac{L}{k}$ and $R_{conv} \sim \frac{1}{h}$

Biot Number $Bi = \frac{hL}{k}$ so, if $Bi \ll 1$ assume uniform T distribution

Eg)

5.1 THE LUMPED CAPACITANCE METHOD

A common transient conduction problem is one in which a solid experiences a sudden change in its thermal environment. Consider a hot metal forging that is initially at a uniform temperature T_i and is quenched by immersing it in a liquid of lower temperature $T_\infty < T_i$ (Figure 5.1). If the quenching is said to begin at time $t = 0$, the temperature of the solid will decrease for time $t > 0$, until it eventually reaches T_∞. This reduction is due to convection heat transfer

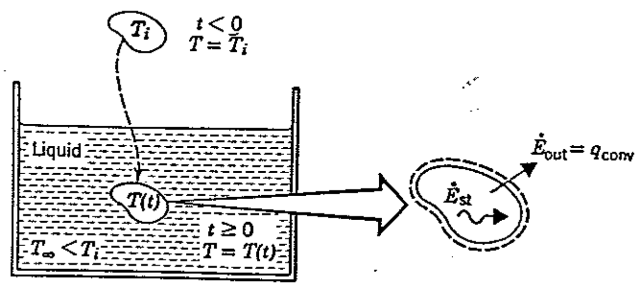


Figure 5.1 Cooling of a hot metal forging.

occurring at the solid–liquid interface. The essence of the lumped capacitance method is the assumption that the temperature of the solid is *spatially uniform* at any instant during the transient process. This assumption implies that temperature gradients within the solid are negligible.

From Fourier's law, heat conduction in the absence of a temperature gradient implies the existence of infinite thermal conductivity. Such a condition is clearly impossible. However, although the condition is never satisfied exactly, it is closely approximated if the resistance to conduction within the solid is small compared with the resistance to heat transfer between the solid and its surroundings. For now we assume that this is, in fact, the case.

In neglecting temperature gradients within the solid, we can no longer consider the problem from within the framework of the heat equation. Instead, the transient temperature response is determined by formulating an overall energy balance on the solid. This balance must relate the rate of heat loss at the surface to the rate of change of the internal energy. Applying Equation 1.10a to the control volume of Figure 5.1, this requirement takes the form

$$-\dot{E}_{out} = \dot{E}_{st} \tag{5.1}$$

or

$$-hA_s(T - T_\infty) = \rho Vc\frac{dT}{dt} \tag{5.2}$$

Introducing the temperature difference

$$\theta \equiv T - T_\infty \tag{5.3}$$

and recognizing that $(d\theta/dt) = (dT/dt)$, it follows that

$$\frac{\rho Vc}{hA_s}\frac{d\theta}{dt} = -\theta$$

Separating variables and integrating from the initial condition, for which $t = 0$ and $T(0) = T_i$, we then obtain

$$\frac{\rho Vc}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

where

$$\theta_i \equiv T_i - T_\infty \tag{5.4}$$

Evaluating the integrals it follows that

$$\frac{\rho Vc}{hA_s}\ln\frac{\theta_i}{\theta} = t \tag{5.5}$$

or

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right] \tag{5.6}$$

Equation 5.5 may be used to determine the time required for the solid to reach some temperature T, or, conversely, Equation 5.6 may be used to compute the temperature reached by the solid at some time t.

The foregoing results indicate that the difference between the solid and fluid temperatures must decay exponentially to zero as t approaches infinity. This behavior is shown in Figure 5.2. From Equation 5.6 it is also evident that the quantity $(\rho Vc/hA_s)$ may be interpreted as a *thermal time constant*. This time constant may be expressed as

$$\tau_t = \left(\frac{1}{hA_s}\right)(\rho Vc) = R_tC_t \tag{5.7}$$

where R_t is the resistance to convection heat transfer and C_t is the *lumped thermal capacitance* of the solid. Any increase in R_t or C_t will cause a solid to respond more slowly to changes in its thermal environment and will increase the time required to reach thermal equilibrium $(\theta = 0)$.

It is useful to note that the foregoing behavior is analogous to the voltage decay that occurs when a capacitor is discharged through a resistor in an electrical RC circuit. Accordingly, the process may be represented by an *equivalent thermal circuit*, which is shown in Figure 5.3. With the switch closed the solid is charged to the temperature θ_i. When the switch is opened, the energy that is stored in the solid is discharged through the thermal resistance and the temperature of the solid decays with time. This analogy suggests that RC electrical circuits may be used to determine the transient behavior of thermal systems. In fact, before the advent of digital computers, RC circuits were widely used to simulate transient thermal behavior.

To determine the total energy transfer Q occurring up to some time t, we

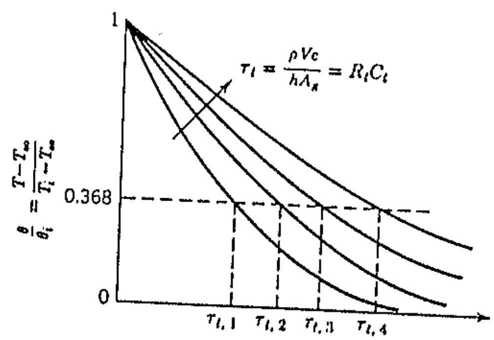


Figure 5.2 Transient temperature response of lumped capacitance solids corresponding to different thermal time constants τ_t.

Boiling - Condensation

Pool Boiling - motion due to free convection and mixing induced by bubble growth and detachment

Forced-Convection Boiling - motion induced by external means as well as pool boiling.

Subcooled Boiling - Temp of liquid < T_{sat}, bubbles formed at surface condense

Saturated Boiling - Bubbles formed @ surface are propelled by buoyancy forces to the free surface.

Condensation - Temp of vapour reduced below saturation Temp.

Surface Condensation - Vapour and cool surface

homogeneous " - Vapour condenses as droplets in gas phase ie. fog

direct Contact " . Vapour contacts cool liquid

For Surface condensation - Film or Dropwise

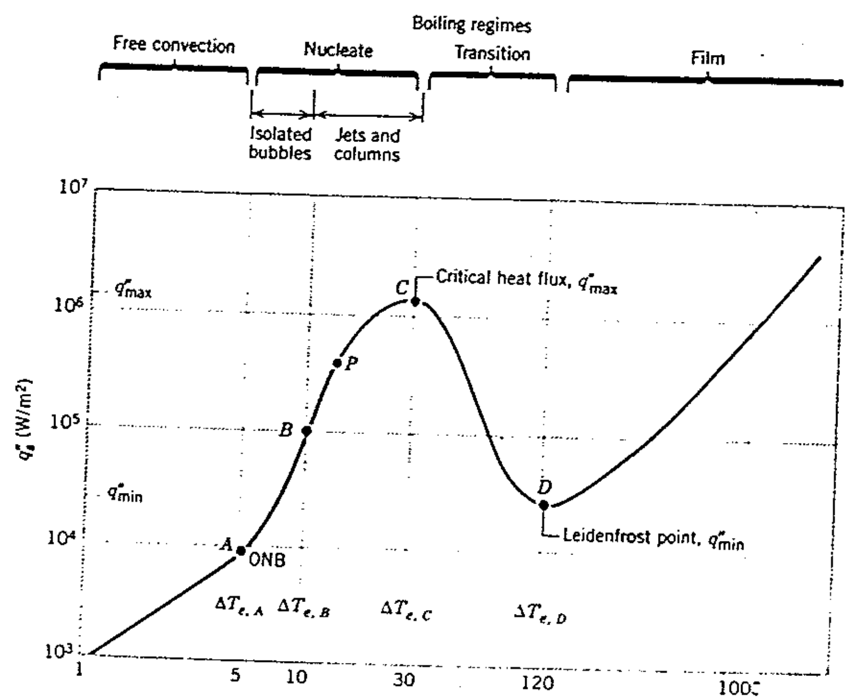


Figure 10.4 Typical boiling curve for water at one atmosphere: surface heat flux

Radiation

Heat exchange between two surfaces
Energy transmitted by electromagnetic waves (photons)

Stefan - Boltzman Law

$$q'' = \sigma T_s^4 \quad (K)$$

(W/m^2) → Ideal Radiator ie. Blackbody

S-B constant $5.67 \times 10^{-8} \, (W/m^2 K^4)$

In reality

$$q'' = \varepsilon \sigma T_s^4$$

emissivity

Net rate of exchange $q'' = \frac{q}{A} = \varepsilon \sigma (T_s^4 - T_{sur}^4)$

often $q_{rad} = h_r A (T_s - T_{sur})$ $\quad h_r = \varepsilon \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2)$

Many terms and ideas which may be confusing.

TERM	DEFINITION
Absorption	The process of converting radiation intercepted by matter to internal thermal energy.
Absorptivity	Fraction of the incident radiation absorbed by matter. Equations 12.41, 12.42, and 12.45. Modifiers: *directional, hemispherical, spectral, total.*
Blackbody	The ideal emitter and absorber. Modifier referring to ideal behavior. Denoted by the subscript b.
Diffuse	Modifier referring to the directional independence of the intensity associated with emitted, reflected, or incident radiation.
Directional	Modifier referring to a particular direction. Denoted by subscript θ.
Directional distribution	Refers to variation with direction.
Emission	The process of radiation production by matter at a finite temperature. Modifiers: *diffuse, blackbody, spectral.*
Emissive power	Rate of radiant energy emitted by a surface in all directions per unit area of the surface, E (W/m^2). Modifiers: *spectral, total, blackbody.*
Emissivity	Ratio of the radiation emitted by a surface to the radiation emitted by a blackbody at the same temperature. Equations 12.32, 12.33, 12.34, and 12.37. Modifiers: *directional, hemispherical, spectral, total.*

TERM	DEFINITION
Gray surface	A surface for which the spectral absorptivity and the emissivity independent of wavelength over the spectral regions of surface irradiation and emission.
Hemispherical	Modifier referring to all directions in the space above a surface.
Intensity	Rate of radiant energy propagation in a particular direction, pe area normal to the direction, per unit solid angle about the dire I (W/m$^2\cdot$sr). Modifier: *spectral.*
Irradiation	Rate at which radiation is incident on a surface from all directio per unit area of the surface, G (W/m^2). Modifiers: *spectral, total, diffuse.*
Kirchhoff's law	Relation between emission and absorption properties for surface irradiated by a blackbody at the same temperature. Equations 1 12.62, 12.63, and 12.64.
Planck's law	Spectral distribution of emission from a blackbody. Equations 1 and 12.26.
Radiosity	Rate at which radiation leaves a surface due to emission and reflection in all directions per unit area of the surface, J (W/m^2). Modifiers: *spectral, total.*
Reflection	The process of redirection of radiation incident on a surface. Modifiers: *diffuse, specular.*
Reflectivity	Fraction of the incident radiation reflected by matter. Equations 12.48, 12.49, and 12.51. Modifiers: *directional, hemispherical, spect total.*
Semitransparent	Refers to a medium in which radiation absorption is a volumetri process.
Solid angle	Region subtended by an element of area on the surface of a sphe with respect to the origin of the sphere, ω (sr). Equations 12.2 and
Spectral	Modifier referring to a single-wavelength (monochromatic) component. Denoted by the subscript λ.
Spectral distribution	Refers to variation with wavelength.
Specular	Refers to a surface for which the angle of reflected radiation is eq to the angle of incident radiation.
Stefan-Boltzmann law	Emissive power of a blackbody. Equation 12.28.
Thermal radiation	Electromagnetic energy emitted by matter at a finite temperature and concentrated in the spectral region from approximately 0.1 to 100 μm.
Total	Modifier referring to all wavelengths.
Transmission	The process of thermal radiation passing through matter.
Transmissivity	Fraction of the incident radiation transmitted by matter. Equations 12.53 and 12.54. Modifiers: *hemispherical, spectral, total.*
Wien's law	Locus of the wavelength corresponding to peak emission by a blackbody. Equation 12.27.

Convection – Heat transfer involving the energy exchange between a surface and an adjacent fluid

2 types:

Forced Convection – a fluid is made to flow past a solid surface by an external agent such as a fan or pump

Natural Convection – a warmer (or cooler) fluid next to the solid boundary layer causes a circulation because of the density difference resulting from the temperature variation throughout a region of the fluid

The rate equation for convective heat transfer is given by Newton's Law of Cooling:

$$q = hA\Delta T$$

Some Considerations:

- Remember, much of the work in this area involves correlations to determine the convective heat transfer coefficient, h
- Flow properties are important in the evaluation of the convective heat transfer coefficient
- Even when a fluid is flowing in a turbulent manner past a surface there is still a layer, sometimes extremely thin, close to the surface where flow is laminar; also, the fluid particles next to the solid boundary are at rest. Since this is always true, the mechanism of heat transfer between a solid surface and a fluid must involve conduction through the fluid layers close to the surface. This film of fluid presents the controlling resistance to convective heat transfer and the coefficient, h, refers to this.
- The heat transfer rate is increased by turbulent flow as opposed to laminar flow. Basically this occurs because in turbulent flow there is bulk mixing of fluid particles between regions at different temperatures.

Significant Parameters in Convective Heat Transfer

momentum diffusivity: $\nu = \dfrac{\mu}{\rho}$ $\quad \dfrac{Kg}{m\cdot S}\ \dfrac{m^3}{Kg}\ \ \dfrac{m^2}{S}$

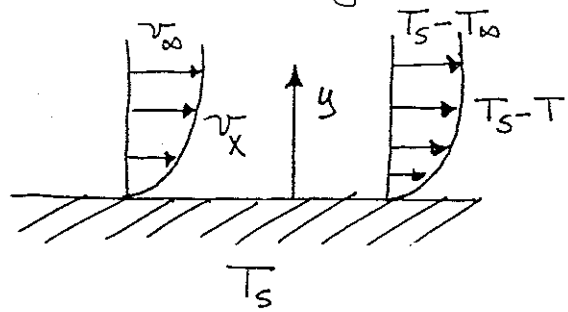
thermal diffusivity: $\alpha = \dfrac{k}{\rho c_p}$ $\quad \dfrac{J}{m^2\cdot{}^\circ C\,S}\ \dfrac{m^3}{Kg}\ \dfrac{m\,{}^\circ C\,Kg}{J}$

the Prandtl number is simply defined as the ratio of the momentum diffusivity to the thermal diffusivity

$$Pr \equiv \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

the Prandtl number is primarily a function of temp.

the temperature profile for a fluid flowing past a surface is (where the surface is at a higher temp. than the fluid):



the heat transfer rate between the surface and the fluid is

$$q = hA(T_s - T_\infty)$$

at the surface the transfer is by conduction, thus,

$$q_y = -kA \frac{\partial}{\partial y}(T - T_s)\Big|_{y=0}$$

these 2 terms must be equal, hence

$$h(T_s - T_\infty) = -k\frac{\partial}{\partial y}(T - T_s)\Big|_{y=0}$$

rearranging,

$$\frac{h}{k} = \frac{\partial(T_s - T)/\partial y\,|_{y=0}}{T_s - T_\infty}$$

nondimensionalizing,

$$\frac{hL}{k} = \frac{\partial(T_s - T)/\partial y\,|_{y=0}}{T_s - T_\infty / L} = \text{Nusselt Number}$$

thus the Nusselt number is simply the ratio of conductive thermal resistance to the ~~[illegible]~~ convective thermal resistance of the fluid- where the thermal conductivity is that of the fluid as opposed to that of the solid

Dimensional Analysis of Convective Energy Transfer

The majority of the empirical correlations for the convective heat transfer coefficient can be expressed in the following forms through the use of dimensionless groups

(a) Forced Convection

$$Nu = f_1(Re, Pr)$$

or

$$St = f_2(Re, Pr)$$

where St is the Stanton number and is defined as

$$St \equiv \frac{h}{\rho v C_p}$$

(b) Natural Convection

$$Nu = f_3(Gr, Pr)$$

where Gr is the Grashof number and is defined as

$$Gr \equiv \frac{\beta g L^3 \Delta T}{\mu^2} \rho^2$$

where β = coefficient of thermal expansion
g = gravitational acceleration

Some further dimensionless parameters

Pe - Peclet Number $\equiv Re\,Pr$

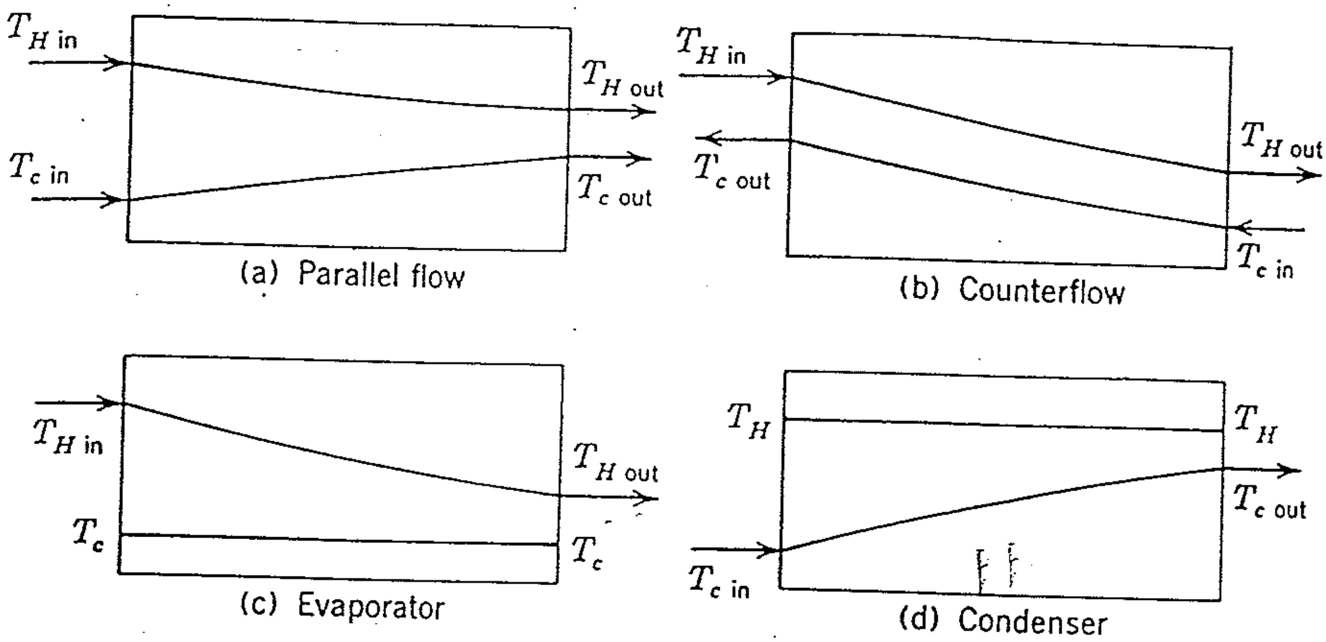
Gz = Graetz Number $\equiv \frac{\pi}{4} \frac{D}{X} Pe$

Ra = Raleigh Number $\equiv Gr\,Pr$

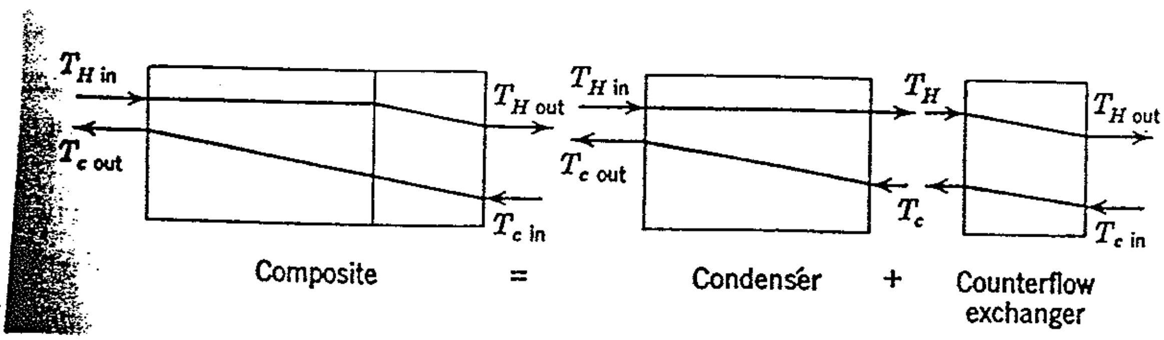
these may come up in specific correlations

Several empirical correlations for the prediction of convective heat transfer coefficients are discussed in all heat transfer books, moreover, a review of exact/analytical techniques may also be helpful. Much of it involves math covered in 230.

Heat Xchangers



Temperature profiles for single-pass, double-pipe heat exchangers.



Temperature profile in a condenser with subcooling.

Note that in the counterflow arrangement it is possible for the hot fluid to leave the exchanger at a temperature below that at which the cold fluid leaves. This situation corresponds to a greater heat transfer per unit area of exchanger surface area than would be obtained if the same fluids entered in a parallel-flow configuration.

Heat transfer rate for exchangers is given by:

$$q = UA\,\Delta T_{lm}$$

where U = overall heat-transfer coefficient
A = Surface area for heat transfer consistent with U
ΔT_{lm} = log mean temperature difference

Log Mean Temperature Difference

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

where ΔT_1 is the temperature difference at one end of the exchanger and ΔT_2 is the temperature difference at the other end of the exchanger

Fouling Factors

Fouling factors take into account the buildup of scale on a heat-transfer surface or the deterioration of the surface by a corrosive fluid due to normal service of the exchanger – these factors represent an additional resistance to heat flow and result in decreased performance

$$\text{Fouling Factor} = R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$$

Types of Xchangers

1) Double-pipe: one fluid flows on the inside of a smaller tube while the other fluid flows in the annular space between the two tubes
2) Shell-and-Tube: an expansion of the double-pipe
3) Cross-Flow: used in air or gas heating and cooling applications
4) Compact Heat Xchangers: primarily used in gas-flow systems where the overall-heat transfer coefficients are low and its desirable to achieve a large surface area in a small volume