1. Write down the molecular transport equations (constitutive equations) for mass, momentum, and heat transfer.

First, let's examine the general transport equation:

$$\text{rate of transfer process} = \frac{\text{driving force}}{\text{resistance}}$$

This equation has three major applications in chemical engineering:

1) Newton's Law: $\tau_{zx} = -\nu \dfrac{d(v_x \rho)}{dz}$ for constant ρ

$\nu = \dfrac{\mu}{\rho} \equiv$ momentum diffusivity $[=] \dfrac{m^2}{s}$

$\tau_{zx} \equiv$ flux of x-directed momentum in z-direction $[=] \dfrac{kg \cdot m/s}{s \cdot m^2}$

2) Fourier's Law; $\dfrac{q_z}{A} = -\alpha \dfrac{d(\rho C_p T)}{dz}$ for constant ρ, C_p

$\alpha = \dfrac{k}{\rho C_p} \equiv$ thermal diffusivity $[=] \dfrac{m^2}{s}$

$\dfrac{q_z}{A} \equiv$ heat flux $[=] \dfrac{J}{s \cdot m^2}$

3) Fick's Law: $J^*_{Az} = -D_{AB} \dfrac{dC_A}{dz}$ for constant c_A

$D_{AB} \equiv$ molecular diffusivity $[=] \dfrac{m^2}{s}$

$J^*_{Az} \equiv$ mass flux $[=] \dfrac{kg\,mol\,A}{s \cdot m^2}$

2. Derive the equation of continuity and equation of motion in rectangular coordinates for a fluid element of dimension $\Delta x \Delta y \Delta z$. Also write for curvilinear coordinates.

We can write a mass balance on the element

$$\left\{\begin{matrix}\text{rate of mass}\\ \text{accumulation}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of mass}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of mass}\\ \text{out}\end{matrix}\right\}$$

$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \left[(\rho v_x)|_x - (\rho v_x)|_{x+\Delta x}\right] \Delta y \Delta z$$

$$+ \Delta x \Delta z \left[(\rho v_y)|_y - (\rho v_y)|_{y+\Delta y}\right] + \Delta x \Delta y \left[(\rho v_z)|_z - (\rho v_z)|_{z+\Delta z}\right]$$

If we divide by $\Delta x \Delta y \Delta z$ and take the limit as $\Delta x, \Delta y, \Delta z \rightarrow 0$, we get

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}\rho v_x + \frac{\partial}{\partial y}\rho v_y + \frac{\partial}{\partial z}\rho v_z\right)$$

We simplify to vector notation

$$\frac{\partial \rho}{\partial t} = -(\underline{\nabla} \cdot \rho \underline{v})$$

This is the equation of continuity in rectangular coordinates. For curvilinear coordinates (r, θ, z) & (r, θ, ϕ)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

For an incompressible fluid, the equation of continuity reduces to $(\underline{\nabla} \cdot \underline{v}) = 0$.

We can write a momentum balance on the element in x

$$\left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{accumulation}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{momentum}\\ \text{out}\end{matrix}\right\} + \left\{\begin{matrix}\text{sum of forces}\\ \text{acting on}\\ \text{system}\end{matrix}\right\}$$

We have convective flow:

$$\Delta y \Delta z(\rho v_x v_x|_x - \rho v_x v_x|_{x+\Delta x}) + \Delta x \Delta z(\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y}) + \Delta x \Delta y(\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z})$$

We have molecular flow (velocity gradients):

$$\Delta y \Delta z(\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x}) + \Delta x \Delta z(\tau_{yx}|_y - \tau_{yx}|_{y+\Delta x}) + \Delta x \Delta y(\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z})$$

We have fluid pressure p and gravitational force per unit mass g:

$$\Delta y \Delta z(p|_x - p|_{x+\Delta x}) + \rho g_x \Delta x \Delta y \Delta z$$

The rate of accumulation in the element $\Delta x \Delta y \Delta z$ is

$$\Delta x \Delta y \Delta z\left(\frac{\partial \rho v_x}{\partial t}\right)$$

We plug these expressions into the momentum balance, divide by $\Delta x \Delta y \Delta z$ and take the limit of $\Delta x, \Delta y, \Delta z$ as $\to 0$

$$\frac{\partial}{\partial t}\rho v_x = -\left(\frac{\partial}{\partial x}\rho v_x v_x + \frac{\partial}{\partial y}\rho v_y v_x + \frac{\partial}{\partial z}\rho v_z v_x\right)$$
$$-\left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

The x and y components may be obtained similarly. Put them all together in vector notation

$$\frac{\partial}{\partial t}\rho \underset{\sim}{v} = -[\nabla \cdot \rho \underset{\sim}{v}\underset{\sim}{v}] - [\underset{\sim}{\nabla} \cdot \underset{\approx}{\tau}] - \underset{\sim}{\nabla} p + \rho \underset{\sim}{g}$$

rate of increase of momentum per unit volume	rate of momentum gain by convection per unit volume	rate of momentum gain by viscous transfer per unit volume	pressure force on element per unit volume	gravitational force on element per unit volume

For an incompressible fluid, we can use the equation of continuity and some rearrangement to get

$$\rho \frac{D\underset{\sim}{v}}{Dt} = -\underset{\sim}{\nabla} p - [\underset{\sim}{\nabla} \cdot \underset{\approx}{\tau}] + \rho \underset{\sim}{g}$$

mass per unit volume times acceleration	pressure force on element per unit volume	viscous force on element per unit volume	gravitational force on element per unit volume

In terms of cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$
$$-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z$$

In terms of spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$$
$$= -\frac{\partial p}{\partial r} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi}\right.$$
$$\tau_{\theta\theta} + \tau_{\phi\phi}\ )$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial\phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}\right) + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r}\cot\theta\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial\phi} + \frac{\tau_{r\phi}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}\right) + \rho g_\phi$$

3. Derive the equation of energy for a fluid element of dimension $\Delta x \Delta y \Delta z$. Also write for curvilinear coordinates.

We start with an energy balance.

$$\begin{Bmatrix}\text{rate of}\\ \text{accumulation}\\ \text{of internal}\\ \text{and kinetic}\\ \text{energy}\end{Bmatrix} = \begin{Bmatrix}\text{rate of}\\ \text{internal and}\\ \text{kinetic energy}\\ \text{in by}\\ \text{convection}\end{Bmatrix} - \begin{Bmatrix}\text{rate of}\\ \text{internal and}\\ \text{kinetic energy}\\ \text{out by}\\ \text{convection}\end{Bmatrix} + \begin{Bmatrix}\text{net rate of}\\ \text{heat}\\ \text{addition by}\\ \text{conduction}\end{Bmatrix} - \begin{Bmatrix}\text{net rate}\\ \text{of work}\\ \text{done by system}\\ \text{on surroundings}\end{Bmatrix}$$

We know the rate of accumulation of KE & PE:

$$\Delta x \Delta y \Delta z \frac{\partial}{\partial t}\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)$$

We know the net rate of convection:

$$\Delta y \Delta z \left\{ v_x\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)\Big|_x - v_x\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)\Big|_{x+\Delta x}\right\}$$
$$+ \Delta x \Delta z \left\{ v_y\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)\Big|_y - v_y\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)\Big|_{y+\Delta y}\right\}$$
$$+ \Delta x \Delta y \left\{ v_z\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)\Big|_z - v_z\left(\rho\hat{U} + \tfrac{1}{2}\rho v^2\right)\Big|_{z+\Delta z}\right\}$$

We know the rate of conduction:

$$\Delta y \Delta z \left\{ q_x|_x - q_x|_{x+\Delta x}\right\} + \Delta x \Delta z \left\{ q_y|_y - q_y|_{y+\Delta y}\right\} + \Delta x \Delta y \left\{ q_z|_z - q_z|_{z+\Delta z}\right\}$$

The rate of doing work against gravitational force

$$-\rho\,\Delta x \Delta y \Delta z\left(v_x g_x + v_y g_y + v_z g_z\right)$$

The rate of doing work against static pressure:

$$\Delta y \Delta z \{(pv_x)|_{x+\Delta x} - (pv_x)|_x\} + \Delta x \Delta z \{(pv_y)|_{y+\Delta y} - (pv_y)|_y\}$$
$$+ \Delta x \Delta y \{(pv_z)|_{z+\Delta z} - (pv_z)|_z\}$$

The rate of doing work against viscous forces:

$$\Delta y \Delta z \{(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z)|_{x+\Delta x} - (\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z)|_x$$
$$+ \Delta x \Delta z \{(\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z)|_{y+\Delta y} - (\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z)|_y$$
$$+ \Delta x \Delta y \{(\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z)|_{z+\Delta z} - (\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z)|_z$$

Substituting the equations into the energy balance, dividing by $\Delta x \Delta y \Delta z$ and taking the limit as $\Delta x, \Delta y, \Delta z \to 0$

$$\frac{\partial}{\partial t}(\rho \hat{U} + \tfrac{1}{2}\rho v^2) = -\left(\frac{\partial}{\partial x} v_x(\rho \hat{U} + \tfrac{1}{2}\rho v^2) + \frac{\partial}{\partial y} v_y(\rho \hat{U} + \tfrac{1}{2}\rho v^2)\right.$$
$$\left. + \frac{\partial}{\partial z} v_z(\rho \hat{U} + \tfrac{1}{2}\rho v^2)\right) - \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)$$
$$+ \rho(v_x g_x + v_y g_y + v_z g_z) - \left(\frac{\partial}{\partial x} p v_x + \frac{\partial}{\partial y} p v_y + \frac{\partial}{\partial z} p v_z\right)$$
$$-\left(\frac{\partial}{\partial x}(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z) + \frac{\partial}{\partial y}(\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z)\right.$$
$$\left. + \frac{\partial}{\partial z}(\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z)\right)$$

We write this in vector notation

$$\underbrace{\frac{\partial}{\partial t}(\rho \hat{U} + \tfrac{1}{2}\rho v^2)}_{\text{rate of gain of energy per unit volume}} = \underbrace{-(\nabla \cdot \underline{v}(\rho \hat{U} + \tfrac{1}{2}\rho v^2))}_{\text{rate of energy input per unit volume by convection}} \underbrace{- (\nabla \cdot \underline{q})}_{\text{rate of energy input per unit vol by conduction}} + \underbrace{\rho(\underline{v} \cdot \underline{g})}_{\text{rate of work done on fluid per unit vol by gravity}}$$
$$\underbrace{- (\nabla \cdot p\underline{v})}_{\text{rate of work done on fluid per unit volume by pressure forces}} \underbrace{- (\nabla \cdot [\underline{\underline{\tau}} \cdot \underline{v}])}_{\text{rate of work done on fluid per unit volume by viscous forces}}$$

We can rearrange this and with the aid of the continuity

$$\rho \frac{D}{Dt}(\hat{U} + \tfrac{1}{2}v^2) = -(\nabla \cdot \underline{q}) + \rho(\underline{v} \cdot \underline{g}) - (\nabla \cdot p\underline{v}) - (\nabla \cdot [\underline{\underline{\tau}} \cdot \underline{v}$$

We can substitute in an equation derived for $\frac{1}{2}v^2$

$$\rho \frac{D}{Dt}\left(\tfrac{1}{2}v^2\right) = p(\underline{\nabla}\cdot\underline{v}) - (\underline{\nabla}\cdot p\underline{v}) + \rho(\underline{v}\cdot\underline{g}) - (\underline{\nabla}\cdot[\underline{\underline{\tau}}\cdot\underline{v}]) + (\underline{\underline{\tau}}:\underline{\nabla}\underline{v})$$

We get the equation of thermal energy

$$\underbrace{\rho\frac{D\hat{U}}{Dt}}_{\substack{\text{rate of gain}\\ \text{of internal}\\ \text{energy per unit}\\ \text{volume}}} = \underbrace{-(\underline{\nabla}\cdot\underline{q})}_{\substack{\text{rate of input}\\ \text{of internal}\\ \text{energy by}\\ \text{conduction}}} \underbrace{-p(\underline{\nabla}\cdot\underline{v})}_{\substack{\text{reversible rate}\\ \text{of internal energy}\\ \text{increase per unit}\\ \text{volume by compression}}} \underbrace{-(\underline{\underline{\tau}}:\underline{\nabla}\underline{v})}_{\substack{\text{irreversible rate}\\ \text{of internal energy}\\ \text{increase per unit}\\ \text{volume by viscous dissipation}}}$$

We can write this out in rectangular coordinates (x, y, z):

$$\rho\hat{C}_v\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)$$

$$-T\left(\frac{\partial p}{\partial T}\right)_\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) - \left(\tau_{xx}\frac{\partial v_x}{\partial x} + \tau_{yy}\frac{\partial v_y}{\partial y} + \tau_{zz}\frac{\partial v_z}{\partial z}\right)$$

$$-\left\{\tau_{xy}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) + \tau_{xz}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) + \tau_{yz}\left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right)\right.$$

We can also write this for cylindrical coordinates (r, θ, z):

$$\rho\hat{C}_v\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}(rq_r) + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \right.$$

$$-T\left(\frac{\partial p}{\partial T}\right)_\rho\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}\right) - \left\{\tau_{xx}\frac{\partial v_r}{\partial r} + \tau_{\theta\theta}\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + \right.\right.$$

$$+\tau_{zz}\frac{\partial v_z}{\partial z}\right\} - \left\{\tau_{r\theta}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] + \tau_{rz}\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]\right.$$

$$\left. + \tau_{\theta z}\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]\right\}$$

We can write for spherical coordinates (r, θ, ϕ):

$$\rho\hat{C}_v\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_r)\right.$$

$$\left. + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(q_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial q_\phi}{\partial \phi}\right] - T\left(\frac{\partial p}{\partial T}\right)_\rho\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right.$$

$$\left. + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right) - \left\{\tau_{rr}\frac{\partial v_r}{\partial r} + \tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right.$$

$$\left. + \tau_{\phi\phi}\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)\right\} - \left\{\tau_{r\theta}\left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r}\right)\right.$$

$$+\tau_{r\phi}\left(\frac{\partial v_\phi}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{v_\phi}{r}\right)+\tau_{\theta\phi}\left(\frac{1}{r}\frac{\partial v_\phi}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}-\frac{\cot\theta}{r}v_\phi\right.$$

4. Derive the equations of continuity for a binary mixture of A & B for a fluid element $\Delta x\Delta y\Delta z$. Also give curvilinear coordinates.

We write a mass balance on the fluid element

$$\begin{Bmatrix}\text{rate of change}\\ \text{of mass } A\end{Bmatrix}=\begin{Bmatrix}\text{rate of mass}\\ A \text{ in}\end{Bmatrix}-\begin{Bmatrix}\text{rate of mass}\\ A \text{ out}\end{Bmatrix}+\begin{Bmatrix}\text{rate of}\\ \text{production}\\ \text{of } A \text{ by}\\ \text{chemical reaction}\end{Bmatrix}$$

$$\Delta x\Delta y\Delta z\frac{\partial\rho_A}{\partial t}=\Delta y\Delta z\left(n_{Ax}|_x-n_{Ax}|_{x+\Delta x}\right)+\Delta x\Delta z\left(n_{Az}|_z-n_{Az}|_z\right.$$

$$+\Delta x\Delta y\left(n_{Az}|_z-n_{Az}|_{z+\Delta z}\right)+r_A\,\Delta x\Delta y\Delta z$$

Dividing the expression by $\Delta x\Delta y\Delta z$ and taking the limit as $\Delta x,\Delta y,\Delta z$ go to zero, we get

$$\frac{\partial\rho_A}{\partial t}+\left(\frac{\partial n_{Ax}}{\partial x}+\frac{\partial n_{Ay}}{\partial y}+\frac{\partial n_{Az}}{\partial z}\right)=r_A$$

In vector notation we may write this as

$$\frac{\partial\rho_A}{\partial t}+(\nabla\cdot\mathbf{n}_A)=r_A$$

Similarly the equation of continuity for B is

$$\frac{\partial\rho_B}{\partial t}+(\nabla\cdot\mathbf{n}_B)=r_B$$

The sum of these two gives the continuity equation for the mixture

$$\frac{\partial\rho}{\partial t}+(\nabla\cdot\rho\mathbf{v})=0$$

In cylindrical coordinates we have

$$\frac{\partial c_A}{\partial t}+\left(\frac{1}{r}\frac{\partial}{\partial r}(rN_{Ar})+\frac{1}{r}\frac{\partial N_{A\theta}}{\partial\theta}+\frac{\partial N_{Az}}{\partial z}\right)=R_A$$

In spherical coordinates we have

$$\frac{\partial c_A}{\partial t}+\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2N_{Ar})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(N_{A\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial N_{A\phi}}{\partial\phi}\right)=R_A$$

1. What is the difference between:

a. heat and mass transfer?

$$J_A^* = \mathcal{D}_{AB}\frac{d(c_A)}{dz} \rightarrow$$ concentration gradient leads to mass transfer

$$\frac{q_z}{A} = \alpha\frac{d(\rho\hat{C}_pT)}{dz} \longrightarrow$$ temperature gradient leads to heat transfer

b. heat and momentum transfer?

$$\frac{q_z}{A} = \alpha\frac{d(\rho\hat{C}_pT)}{dz} \longrightarrow$$ temperature gradient leads to heat transfer

$$\tau_{zx} = \nu\frac{d(\rho v_x)}{dz} \longrightarrow$$ velocity gradient leads to momentum transfer

c. mass and momentum transfer?

$$\tau_{zx} = \nu\frac{d(\rho v_x)}{dz} \longrightarrow$$ velocity gradient leads to momentum transfer

$$J_A^* = \mathcal{D}_{AB}\frac{d(c_A)}{dz} \longrightarrow$$ concentration gradient leads to mass transfer

Also, note the dimensionless numbers that follow

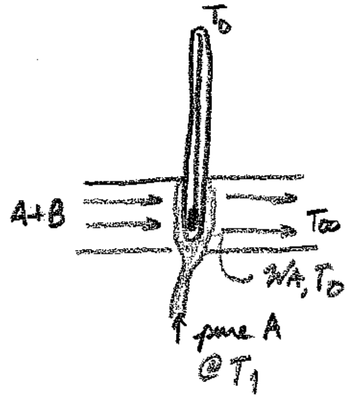
$$Pr = \frac{\nu}{\alpha} = \frac{(\mu/\rho)}{(k/\rho\hat{C}_p)} = \frac{\mu\hat{C}_p}{k} = \frac{\text{momentum}}{\text{heat}}$$

$$Sc = \frac{\nu}{\mathcal{D}_{AB}} = \frac{\mu}{\rho\mathcal{D}_{AB}} = \frac{\text{momentum}}{\text{mass}}$$

$$Le = \frac{Pr}{Sc} = \frac{(\mu\hat{C}_p/k)}{(\mu/\rho\mathcal{D}_{AB})} = \frac{\mathcal{D}_{AB}\rho\hat{C}_p}{k} = \frac{\text{mass}}{\text{heat}}$$

2. Set up equations to describe:

a. wet bulb thermometer



A stream of condensable A and noncondensable B flow across the thermometer covered with a wick saturated in liquid A. Pure A continually evaporates from the wick, lowering the temperature of the bulb until a steady-state temperature is reached. This is the wet-bulb temperature.

assumptions:

1. Velocity of fluid is high enough that temp readings are unaffected by radiation or thermal conduction up thermometer.
2. Velocity is not so high as to effect viscous dissipation.

We can perform an energy balance:

$$\mathcal{W}_A^{(m)} \left(\bar{H}_{Ag0} - \tilde{H}_{A1} \right) = Q$$

molar flow rate of A into gas; enthalpy of A as soon as released; enthalpy of liquid A; heat transfer

$$Q = h_m \pi D L \left(T_\infty - T_0 \right)$$

$$\Delta \tilde{H}_{A,vap} = \left(\bar{H}_{Ag0} - \tilde{H}_{A1} \right)$$

$$\mathcal{W}_A^{(m)} = k_{xm} \pi D L \left(x_{A0} - x_{A\infty} \right) + x_{A0} \left(\mathcal{W}_A^{(m)} + \cancelto{0}{\mathcal{W}_B^{(m)}} \right)$$

$$\mathcal{W}_A^{(m)} \left(1 - x_{A0} \right) = k_{xm} \pi D L \left(x_{A0} - x_{A\infty} \right)$$

$$k_{xm} \cancel{\pi D L} \frac{(x_{A0} - x_{A\infty})}{(1 - x_{A0})} \Delta \tilde{H}_{A,vap} = h_m \cancel{\pi D L} \left(T_\infty - T_0 \right)$$

$$\frac{(x_{A0} - x_{A\infty})}{(T_\infty - T_0)(1 - x_{A0})} = \frac{h_m}{k_{xm} \Delta \tilde{H}_{A,vap}}$$

$$j_H = \frac{h_m}{\cancel{\rho_f} \hat{C}_{pf} \cancel{V_\infty}} Pr_f^{2/3} = \frac{k_{mx}}{\cancel{c_f} \cancel{V_\infty}} Sc_f^{2/3} = j_D$$

$$\frac{h_m}{k_{xm}} = \tilde{C}_{pf} \left(\frac{Sc}{Pr} \right)^{2/3} = \tilde{C}_{pf} Le^{-2/3}$$

$$x_{A0} = x_{A0}(T_0, p) \approx \frac{p_{A,vap}}{p}$$

Thus we have a solution of the form

$$\frac{(x_{A0} - x_{A\infty})}{(T_\infty - T_0)(1 - x_{A0})} = \frac{\tilde{C}_{pf}}{\Delta \tilde{H}_{A,vap}} \left(\frac{Sc}{Pr} \right)_f^{2/3}$$

b. iceberg being towed in the ocean

We can approximate a solution by modelling the berg as a sphere in a stream at constant temperature.

Assumptions:

1. Assume the berg is spherical.
2. Neglect portion of iceberg that will be above water.
3. Neglect the ocean's thermal gradient.
4. Assume that the velocity of the boat is high enough that the bulk temperature of the ice remains constant, neglecting radiation or thermal conduction. Assume the velocity is low enough to neglect viscous dissipation.

We can write an overall energy balance:

$$\dot{w}\lambda = Q$$

$$Q = hA(T_{ice} - T_w)$$

$$\dot{w} = \frac{d}{dt}(\rho V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho\right)$$

We can use a hydraulic radius to describe the berg

$$r_H = \frac{\text{SA of channel}}{\text{WP of channel}}$$

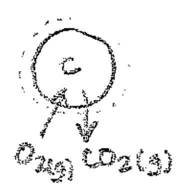
To estimate h_m, use correlation for submerged object

$$\frac{h_m D}{k_f} = 2.0 + 0.60\, Re^{1/2} Pr^{1/3}$$

Combining these equations, we get

$$\lambda \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho\right) = h\left(4\pi r^2\right)(T_{ice} - T_{water})$$

c. a burning carbon particle



The reaction at hand is the following:

$$C_{(s)} + O_{2(g)} \longrightarrow CO_{2(g)}$$

So we have equimolar counterdiffusion to and from the surface of the sphere. We use Fick's law and simplify

$$N_A = -N_B$$

$$N_A = -\mathcal{D}_{AB}\frac{dC_A}{dr} + x_A \underbrace{(\cancel{N_A + N_B})}_{0}$$

We can perform a mass balance on the particle:

$$\left\{\begin{matrix}\text{rate}\\ O_2\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate}\\ CO_2\\ \text{out}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of}\\ \text{generation}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{accumulation}\end{matrix}\right\}$$

$$W_{Ar}\,4\pi r^2\big|_r - W_{Ar}\,4\pi r^2\big|_{r+\Delta r} + 0 = 0$$

Dividing by $4\pi\Delta r$ and taking the limit as $\Delta r \to 0$

$$\frac{d}{dr}\left(W_{Ar}\, r^2\right) = 0$$

We have already defined the flux of O_2

$$W_{Ar} = -\mathcal{D}_e \frac{dC_{O_2}}{dr}$$

$$\frac{d}{dr}\left(-\mathcal{D}_e r^2 \frac{dC_{O_2}}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r^2 \frac{dC_A}{dr}\right) = 0$$

We apply the boundary conditions that

$C_A = C_0$ @ $r = r_0$

$C_A = 0$ @ $r = R(t)$ (rapid reaction)

$$r^2 \frac{dC_A}{dr} = C_1$$

$$C_A = -\frac{C_1}{r} + C_2$$

$$0 = -\frac{C_1}{R} + C_2$$

$$C_0 = -\frac{C_1}{r_0} + C_2$$

$$C_1 = \frac{C_0}{\left(\frac{1}{R} - \frac{1}{r_0}\right)}$$

$$C_2 = \frac{C_0}{\left(1 - \frac{R}{r_0}\right)} = \frac{RC_0}{\left(\frac{1}{R} - \frac{1}{r_0}\right)}$$

Eventually, you get

$$\frac{C_A}{C_{A0}} = \frac{\frac{1}{R} - \frac{1}{r}}{\frac{1}{R} - \frac{1}{R_0}}$$

3. Will heat transfer affect the friction factor? In what way?

For laminar flow, $f = 16/Re$. Since $f \sim 1/Re$, and

$Re = \frac{\rho \langle v \rangle D}{\mu}$, we say that the friction factor is dependent on the temperature through the viscosity and density of the fluid. Since

$$Re \sim \frac{\rho(T)}{\mu(T)}, \quad f \sim \frac{\mu(T)}{\rho(T)}$$

4. What is the difference between diffusivity and a mass transfer coefficient?

Diffusivity is a molecular property and describes flux in terms of local (differential) concentration gradients. It is a function of T, P, and the components only.

Mass transfer coefficients deal with measurable concentration (macroscopic) gradients. These are functions of the system's geometry and flow conditions. Usually these are concerned with flux at a boundary.

5. Why is the Prandtl number greater for liquids than for gases?

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{K} = \frac{\text{molecular diffusivity of mass}}{\text{molecular diffusivity of heat}}$$

We know that the viscosity of liquids is much higher for liquids than for gases. This is the dominating factor in the expression.

6. What is the Sherwood number?

The Sherwood number is a dimensionless group frequently found in mass transfer situations.

$$Sh = \frac{K_m L}{D_{AB}} = \frac{\text{concentration gradient at boundary}}{\text{concentration gradient from bulk to boundary}}$$

It describes the relative importance of the concentration gradient at the boundary with respect to that between the bulk and the boundary. The analog of the Sherwood number in heat transfer is the Nusselt number.

$$Nu = \frac{hL}{K} = \frac{\text{temperature gradient at boundary}}{\text{temperature gradient from bulk to boundary}}$$

7. Do Neon and Argon have the same atomic radius? If not, which is larger? Which is larger (a) diffusivity (b) viscosity (c) heat capacity; and (d) Prandtl number?

Argon is larger than neon (and heavier too).

(a) According to kinetic theory

$$\mathcal{D}_{AB} = \frac{1}{3}\bar{u}\lambda \quad \text{where} \quad \bar{u} = \sqrt{\frac{8KT}{\pi m}} \equiv \text{mean molecular speed}$$

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n} \equiv \text{mean free path}$$

$$\mathcal{D}_{AB} \propto \frac{1}{m^{1/2}}, \quad \mathcal{D}_{AB} \propto \frac{1}{d^2}$$

For an ideal gas

$$\mathcal{D}_{AA^*} = \frac{2}{3}\left(\frac{K^3}{\pi^3 m_A}\right)^{1/2} \frac{T^{3/2}}{p\, d_A^{\,2}}$$

Since both the mass and the diameter of argon are higher than neon, its diffusion coefficient is lower.

(b) According to kinetic theory

$$\mu = \frac{1}{3} n m \bar{u} \lambda \quad \text{where} \quad \bar{u} = \sqrt{\frac{8KT}{\pi m}}, \quad \lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$

For an ideal gas, Maxwell said

$$\mu = \frac{2}{3\pi^{3/2}} \frac{\sqrt{mKT}}{d^2}$$

$$\mu \propto \sqrt{m}, \quad \mu \propto \frac{1}{d^2}$$

Since the mass of Argon is larger than neon its viscosity is also higher.

(c) Since both Ar and Ne are ideal gases, their heat capacities are roughly the same.

(d) The Prandtl number is defined as

$$Pr = \frac{\nu}{\alpha} = \frac{\mu \hat{C}_p}{k} = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}}$$ NO. p. 156 BSL

Using kinetic theory, we know that

$$k = \frac{1}{d^2}\sqrt{\frac{K^3 T}{\pi^3 m}}, \quad k \propto \frac{1}{d^2}, \quad k \propto m^{-1/2}$$

$$\mu \propto \frac{1}{d^2}, \quad \mu \propto m^{1/2}$$

$Pr \propto m$

The prandtl number for Argon is higher.

8. Consider the problem of pumping oil down the Alaska pipeline. Given the pipe diameter and length, and the properties of the oil, how would you calculate the (a) pump sizes (b) heat loss (c) temperature profile?

(a) In order to size these pumps, must be able to solve the Bernoulli equation for W_p

$$\frac{P_a}{\rho} + \frac{g Z_a}{g_c} + \frac{\alpha_a \bar{V}_a^2}{2 g_c} + \eta W_p = \frac{P_b}{\rho} + \frac{g Z_b}{g_c} + \frac{\alpha_b \bar{V}_b^2}{2 g_c} + h_f$$

(for turbulent flow, $\alpha_a = \alpha_b = 1.0$)

If we assume negligible kinetic and potential energy losses, we can rewrite as

$$\eta W_p = \frac{\Delta P}{\rho} + h_f$$

If we assume turbulent flow, we know that

$$h_f = (2\pi R L)(\tfrac{1}{2}\rho \langle v \rangle^2) f = (\mathcal{P}_0 - \mathcal{P}_L)\pi R^2$$

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v \rangle^2}\right) = f(Re)$$

In order to get the Reynolds number, we must know the flowrate.

$$Q = \pi \frac{D^2}{4} \langle v \rangle$$

$$\langle v \rangle = \frac{Q}{\pi \frac{D^2}{4}}$$

$$Re = \frac{\rho \langle v \rangle D}{\mu}$$

Once we determine the Reynolds number, we have the friction factor and pressure drop. We can then size our pumps.

(b) In order to find the heat loss, we need to know some value for the heat transfer coefficient. We can use the Nusselt number for highly turbulent flow

$$Nu = \frac{hD}{k} = \frac{\text{convective energy transport}}{\text{diffusive energy transport}}$$

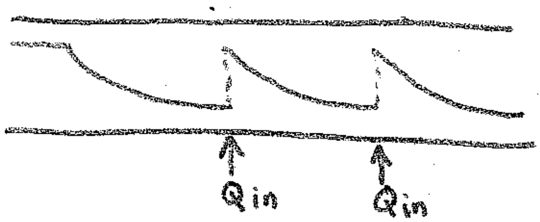
$$= 0.026\, Re^{0.8} Pr^{1/3} \left(\frac{\mu_b}{\mu_0}\right)^{0.14}$$

$$\frac{h_{lm} D}{k_b} = 0.026\left(\frac{DG}{\mu_b}\right)^{0.8}\left(\frac{\hat{C}_p \mu}{k}\right)^{1/3}\left(\frac{\mu_b}{\mu_o}\right)^{0.14}$$

From this we have the h_i inside wall heat transfer coefficient. If we assume a constant wall temperature we calculate the heat loss

$$Q = h(\pi DL)\,\Delta T_{lm}$$

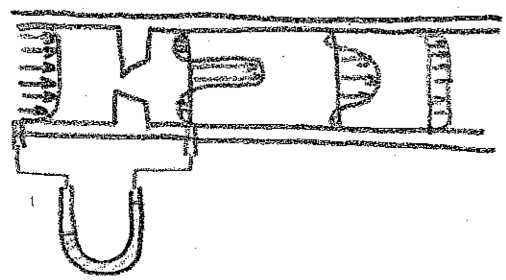
(c) The temperature profile will be shaped like a standard tube heat exchanger with constant outside temperature.



Since the viscosity goes up with decreasing temperature, so you want lots of heaters in the stream.

9. Describe and give the governing equations for (a) an orifice meter (b) a venturi meter (c) a pitot tube

(a)



We can perform a mass balance on the pipe:

$$\rho_1 \langle v_1 \rangle S_1 = \rho_2 \langle v_2 \rangle S_2$$

However, we know $\rho_1 = \rho_2$, $S_1 = S_2$

$$\langle v_1 \rangle = \langle v_2 \rangle = \langle v \rangle$$

We can use Bernoulli's equation

$$\Delta \frac{1}{2}\frac{\langle \bar{v}^3 \rangle}{\langle \bar{v} \rangle} + \Delta\hat{\Phi} + \int_{p_1}^{p_2} \frac{1}{\rho}\,dp + \hat{W} + \hat{E}_v = 0$$

This reduces to

$$\frac{\langle v_2 \rangle^2}{2} - \frac{\langle v_1 \rangle^2}{2} + \frac{p_2 - p_1}{\rho} + \frac{1}{2}\langle v_2 \rangle^2 e_v = 0$$

$$\frac{\langle v\rangle^2}{2\alpha_2} - \frac{\langle v\rangle^2}{2\alpha_1} + \frac{p_2-p_1}{\rho} + \frac{1}{2}\langle v\rangle^2 e_v = 0$$

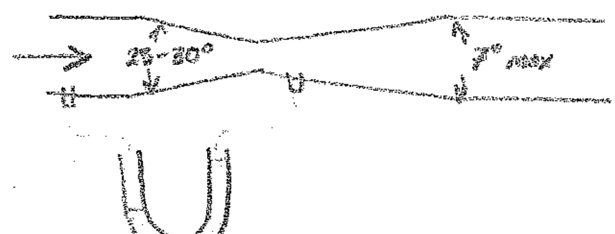
$$\langle v\rangle = \sqrt{\frac{2(p_2-p_1)/\rho}{\frac{1}{\alpha_2} - \frac{1}{\alpha_1} + e_v}}$$

$$\dot{W} = \rho\langle v\rangle S = \rho S\sqrt{\frac{2(p_2-p_1)/\rho}{\frac{1}{\alpha_2} - \frac{1}{\alpha_1} + e_v}}$$

If we assume that e_v is zero, $\alpha_1 = 1$ (velocity profile at 1 is flat), and $\frac{1}{\alpha_2} = \left(\frac{S}{S_0}\right)^2$

$$\dot{W} = \rho\langle v\rangle S = S_0\rho\sqrt{\frac{2(P_2-P_1)/\rho}{\left(\frac{S_0}{S}\right)^2 - 1}} = C_d S_0\sqrt{\frac{2\rho(p_1-p_2)}{1-\left(\frac{S_0}{S}\right)^2}}$$

(b)



We can perform a mass balance on the fluid:

$$W = \rho_1\langle v_1\rangle S_1 = \rho_2\langle v_2\rangle S_2$$

We can use the Bernoulli equation:

$$\Delta\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \Delta\hat{\Phi} + \int_{p_1}^{p_2}\frac{1}{\rho}dp + \hat{W} + \hat{E}_v = 0$$

This reduces to

$$\frac{\langle v_2\rangle^2}{2\alpha_2} - \frac{\langle v_1\rangle^2}{2\alpha_1} + \int_1^2\frac{1}{\rho}dp + \frac{1}{2}\langle v_2\rangle^2 e_v = 0$$

$$\frac{\langle v_1\rangle^2}{2\alpha_2}\left(\frac{\rho_2 S_2}{\rho_1 S_1}\right)^2 - \frac{\langle v_1\rangle^2}{2\alpha_1} + \int_1^2\frac{1}{\rho}dp + \frac{1}{2}\langle v_1\rangle^2\left(\frac{\rho_2 S_2}{\rho_1 S_1}\right)^2 e_v = 0$$

$$\langle v_1\rangle = \sqrt{\frac{-2\int_1^2\frac{1}{\rho}dp}{\frac{1}{\alpha_2}\left(\frac{\rho_2 S_2}{\rho_1 S_1}\right)^2 - \frac{1}{\alpha_1} + \left(\frac{\rho_2 S_2}{\rho_1 S_1}\right)^2 e_v}}$$

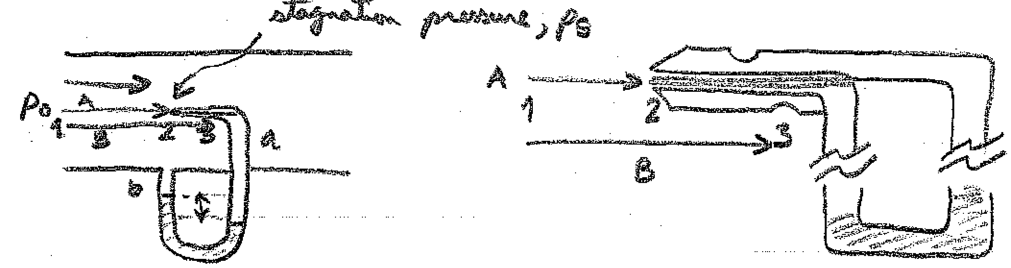
$$\frac{\langle v_2\rangle^2}{2\alpha_2} - \frac{\langle v_2\rangle^2}{2\alpha_1}\left(\frac{\rho_1 S_1}{\rho_2 S_2}\right)^2 + \int_1^2 \frac{1}{\rho}\,dp + \frac{1}{2}\langle v_2\rangle^2 e_v = 0$$

$$\langle v_2\rangle = \sqrt{\frac{-2\int_1^2 1/\rho\,dp}{\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\left(\frac{\rho_1 S_1}{\rho_2 S_2}\right)^2 + e_v}} = \sqrt{\frac{-2\alpha_2\int_1^2 1/\rho\,dp}{1 - \frac{\alpha_2}{\alpha_1}\left(\frac{\rho_1 S_1}{\rho_2 S_2}\right)^2 + \alpha_2 e_v}}$$

If we assume that e_v is zero, $\alpha_1 = 1$ (flat velocity profile), and $1/\alpha_2 = (S_2/S_0)^2$

$$w = \rho_2\langle v_2\rangle S_2 = C_d\,\rho_2 S_0\sqrt{\frac{-2\int_1^2 (1/\rho)\,dp}{1 - \left(\frac{\rho_2 S_0}{\rho_1 S_1}\right)^2}}$$

(c)



We can use Bernoulli's equation along a streamline A

$$\frac{1}{2}V_2^2 + gh_2 = \frac{1}{2}V_1^2 + gh_1 - \int_{p_1}^{p_2}\frac{dp}{\rho} - l_v$$

We assume incompressible fluid, no friction (small loss over short distance),

$$\frac{1}{2}V_1^2 + \frac{p_1 - p_2}{\rho} = 0$$

If we look at streamline 3 and apply Bernoulli equation

$$\frac{1}{2}V_3^2 = \frac{1}{2}V_1^2 + \frac{p_1 - p_3}{\rho} = \frac{p_2 - p_3}{\rho}$$

since $V_1 = V_3$

$$V_1 = \sqrt{\frac{2(p_2 - p_3)}{\rho}}$$

This is the equation for the orifice meter.

10. Give the following: (a) Bernoulli's equation (b) Hagen-Poiseuille law (c) Stokes law (d) Continuity equation (e) Navier-Stokes equation

(a) The Bernoulli's equation may be written in a couple of ways (derived from steady-state equation of energy)

McCabe, Smith, & Harriott:

$$\frac{p_a}{\rho} + \frac{g Z_a}{g_c} + \frac{\alpha_a \bar{V}_a^2}{2g_c} = \frac{p_b}{\rho} + \frac{g Z_b}{g_c} + \frac{\alpha_b \bar{V}_b^2}{2g_c} + h_f$$

Bird, Stewart, & Lightfoot:

$$\underbrace{\Delta \frac{1}{2} \frac{\langle \bar{v}^3 \rangle}{\langle \bar{v} \rangle}}_{\text{change in kinetic energy}} + \underbrace{\Delta \hat{\Phi}}_{\text{change in potential energy}} + \underbrace{\int_{p_1}^{p_2} \frac{1}{\rho}\, dp}_{\text{isothermal system pressure work}} + \underbrace{\hat{W}}_{\text{rate at which system performs mechanical work on surroundings}} + \underbrace{\hat{E}_v}_{\text{friction loss rate at which energy is irrev lost to therm}} = 0$$

(b) The Hagen-Poiseuille law is derived from the velocity distribution for laminar, incompressible flow in a tube

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

The max velocity occurs at $r = 0$

$$v_{z,max} = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R^2}{4\mu L}$$

The average velocity is found by summing all the velocities over a cross section and dividing by the cross-sectional area

$$\langle v_z \rangle = \frac{\int_0^{2\pi}\int_0^R v_z\, r\, dr\, d\theta}{\int_0^{2\pi}\int_0^R r\, dr\, d\theta} = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R^2}{8\mu L} = \frac{1}{2} v_{z,max}$$

The volume rate of flow Q is the product of area and the average velocity

$$Q = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L) R^4}{8\mu L}$$

This is the Hagen-Poiseuille Law giving the relationship between the volume rate of flow and the forces causing the flow – the forces associated with the pressure drop and gravitational acceleration.

(c) Stokes law is used to describe creeping flow around a sphere. For these conditions, we have

$$\tau_{r\theta} = \frac{3}{2}\frac{\mu V_\infty}{R}\left(\frac{R}{r}\right)^4 \sin\theta$$

$$p = p_0 - \rho g z - \frac{3}{2}\frac{\mu V_\infty}{R}\left(\frac{R}{r}\right)^2 \cos\theta$$

$$v_r = v_\infty\left[1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\cos\theta$$

$$v_\theta = -v_\infty\left[1 - \frac{3}{4}\left(\frac{R}{r}\right) - \frac{1}{4}\left(\frac{R}{r}\right)^3\right]\sin\theta$$

If we do a force balance in the normal direction

$$F_n = \int_0^{2\pi}\int_0^{\pi}\left(-p|_{r=R}\cos\theta\right)R^2\sin\theta \, d\theta \, d\phi$$

$$= \frac{4}{3}\pi R^3 \rho g + 2\pi\mu R v_\infty$$

If we do a force balance in the tangential direction

$$F_t = \int_0^{2\pi}\int_0^{\pi}\left(+\tau_{r\theta}|_{r=R}\sin\theta\right)R^2\sin\theta \, d\theta \, d\phi$$

$$= \underbrace{\frac{4}{3}\pi R^3 \rho g}_{\text{buoyant force}} + \underbrace{2\pi\mu R v_\infty}_{\text{form drag}} + \underbrace{4\pi\mu R v_\infty}_{\text{friction drag}}$$

$F_s = \frac{4}{3}\pi R^3 \rho g \equiv$ buoyant force, exerted even if fluid is stationary

$F_k = 6\pi\mu R v_\infty \equiv$ force associated with fluid movement, kinetic contribution

This last equation is Stokes' law.

(d) We can derive the equation of continuity by performing a mass balance on element $\Delta x \Delta y \Delta z$

$$\left\{\begin{matrix}\text{rate of}\\ \text{mass}\\ \text{accumulation}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{mass}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass}\\ \text{out}\end{matrix}\right\}$$

$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \Delta y \Delta z\left[(\rho v_x)|_x - (\rho v_x)|_{x+\Delta x}\right]$$

$$+ \Delta x \Delta z\left[(\rho v_y)|_y - (\rho v_y)_{y+\Delta y}\right] + \Delta x \Delta y\left[(\rho v_z)|_z - (\rho v_z)|_{z+\Delta z}\right]$$

Dividing through by $\Delta x \Delta y \Delta z$ and taking the limit as $\Delta x, \Delta y, \Delta z \rightarrow 0$

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}\rho v_x + \frac{\partial}{\partial y}\rho v_y + \frac{\partial}{\partial z}\rho v_z\right)$$

$$\frac{\partial \rho}{\partial t} = -(\nabla \cdot \rho \mathbf{v})$$

We can expand this to get a total derivative

$$\frac{\partial \rho}{\partial t} + v_x\frac{\partial \rho}{\partial x} + v_y\frac{\partial \rho}{\partial y} + v_z\frac{\partial \rho}{\partial z} = \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \mathbf{v})$$

We can also write the equation of continuity in curvilinear coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

(e) The Navier-Stokes equation is the equation of motion for constant ρ and constant μ, also using the equation of continuity

$$\frac{\partial}{\partial t}\rho \mathbf{v} = -[\nabla \cdot \rho \mathbf{v}\mathbf{v}] - \nabla p - [\nabla \cdot \boldsymbol{\tau}] + \rho \mathbf{g}$$

rate of increase of momentum per unit volume; rate of momentum gain by convection per unit volume; pressure force on element per unit volume; rate of momentum gain by viscous transfer per unit volume; gravitational force on element per unit volume

Rearranging, using equation of continuity (constant ρ)

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p - [\nabla \cdot \boldsymbol{\tau}] + \rho \mathbf{g}$$

mass per unit volume times acceleration; pressure force on element per unit volume; viscous force on element per unit volume; gravitational force on element per unit volume

If we assume constant μ ($\tau_{zx} = \mu\frac{dv_x}{dz}$, etc.), we get the Navier-Stokes Equation

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p - \mu\nabla^2\mathbf{v} + \rho \mathbf{g}$$

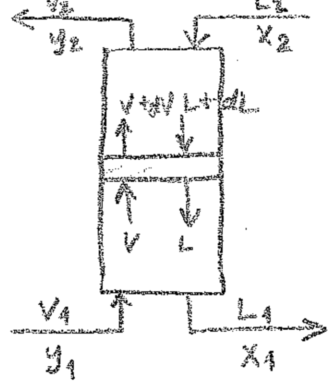
11. What is an NTU and how do you calculate it?

The height of a packed tower, gas absorber may be obtained more easily by multiplying the number of transfer units by the height of a transfer unit. In reality, each of these terms is a complex quantity and difficult to evaluate.

NTU = number of transfer units is a measure of the difficulty of separation (for high purity, many transfer units)

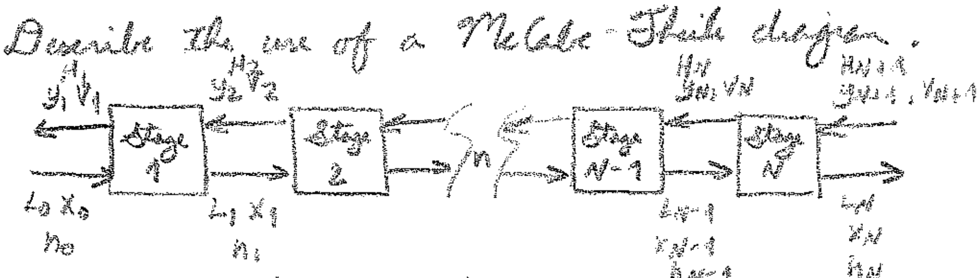
HTU = measure of the separation effectiveness of packing for particular chemical species (rapid mass transfer, small height)

As an example, we consider a countercurrent packed tower



$$dV = dL$$
$$d(Vy) = d(Lx)$$
$$Vy - V_1 y_1 = Lx - L_1 x_1$$
$$Vy - L_1 x_1 = Lx + V_1 y_1$$
$$d(Vy) = k_y'(y_i - y)\,dA = k_y'(y_i - y)\,aS\,dz$$
$$\int_0^z dz = \frac{V}{k_y' aS}\int_{y_1}^{y_2}\frac{dy}{(y_i - y)}$$
$$= (HTU)(NTU) = H_G N_G$$
$$d(Lx) = k_x'(x - x_i)\,dA = k_x'(x - x_i)\,aS\,dz$$
$$\int_0^z dz = \frac{L}{k_x' aS}\int_{x_1}^{x_2}\frac{dx}{(x - x_i)}$$
$$= (HTU)(NTU) = H_L N_L$$

12. Describe the use of a McCabe-Thiele diagram.



A mass balance around any stage n gives

$L_0 + V_{n+1} = L_n + V_1$ (total mass balance)

$L_0 x_0 + V_{n+1} y_{n+1} = L_n x_n + V_1 y_1$ (component mass balance)

$$y_{n+1} = \frac{L_n}{\;} x_n + \frac{V_1 y_1 - L_0 x_0}{\;}$$

This equation relates the compositions of a V-phase (y_{n+1}) and a L-phase (x_n) flowing past each other between stages. If plotted on an x-y diagram, we call this an operating line. The equation yields a straight line if L and V are constant throughout the cascade. This is usually true for a distillation

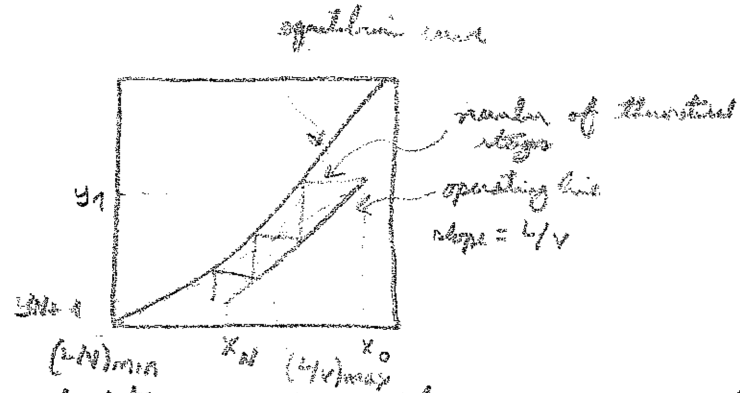
$$y_{n+1} = \frac{L}{V}x_n + \frac{Vy_1 - Lx_0}{V}$$

column, where molar heat of vaporization for mixture is constant. For liquid-liquid extraction, the solvent components are completely insoluble (b) & (c), component (a) is transferred from one solvent to another. The total mass flow of raffinate (L) decreases as the solute is transferred from it to the extract phase (V) and the extract decreases in mass. Must redefine L and V

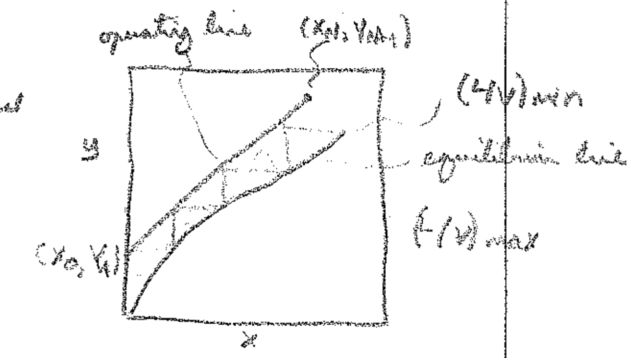
$$Y_{n+1} = \frac{L'}{V'}X_n + \frac{V'Y_1 - L'X_0}{V'}$$

where L' = mass of unextracted raffinate component (b) not including solute (a), V' = mass of extract solvent (c), not including solvent (a), Y_{n+1} = mass of (a) per unit mass (c), X_n = mass of solute (a) per unit mass of unextracted raffinate (b). We also use this equation for gas absorption, adsorption. If both phases are dilute in component (a) the former equation will work.

In a McCabe-Thiele diagram, the location of the operating line below the equilibrium curve indicates that the net mass transfer is from the L-phase to the V-phase. On the other hand, the operating line above the equilibrium curve shows the net mass transfer is from the V-phase to the L-phase in simple countercurrent flow.



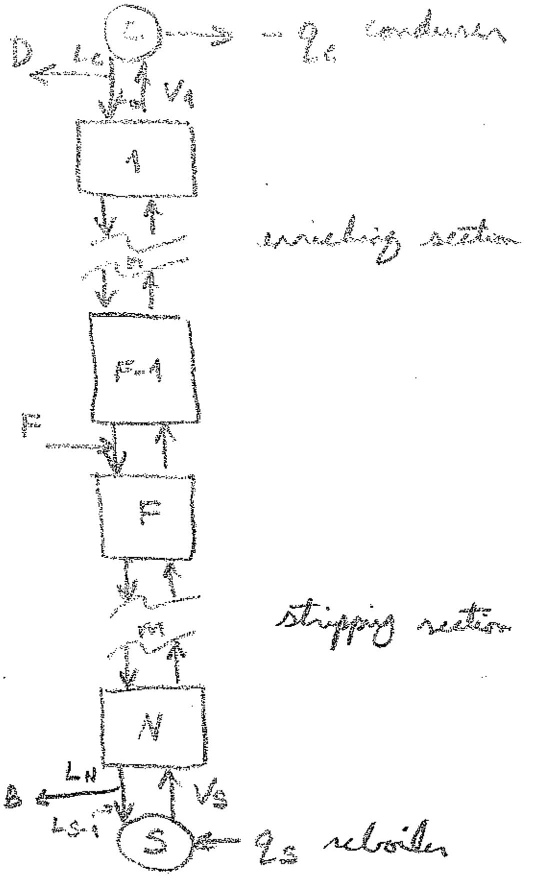
Calculations on the equil. diagram for simple countercurrent flow with transfer from the L-phase to the V-phase.



Countercurrent absorption from V phase to L-phase.

The McCabe Thiele diagram may be used to calculate the theoretical number of stages (plates, transfer units) required to effect a separation.

The introduction of a reflux at one or both ends results in two net flows and therefore two operating lines.



$$\frac{L}{L+D} = \frac{L_0/D}{L_0/D+1}$$

$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D \quad \text{enriching}$$

$$y_m = \frac{\bar{L}}{\bar{V}} x_{m-1} - \frac{B}{\bar{V}} x_B \quad \text{stripping}$$

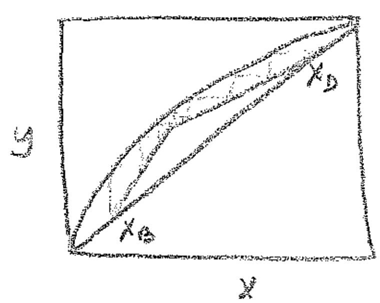
$$V y_i = L x_i + D x_D$$

$$\bar{V} y_i = \bar{L} x_i - B x_B$$

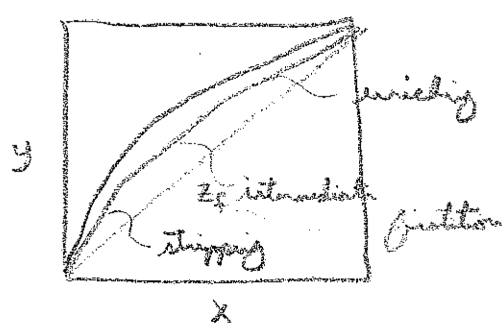
$$(\bar{V} - V) y_i = (\bar{L} - L) x_i - B x_B - D x_D$$

$$F z_F = B x_B + D x_D$$

$$F = B + D$$



The stepping-off shifts from one operating line to the other at the stage which the feed is added, regardless of whether or not the stage is in the vicinity of the intersection.



13. How do the following vary with temperature and pressure? (a) diffusivity (b) dynamic viscosity (c) thermal conductivity (d) heat capacity (e) heat transfer coefficient (f) ν

(a) $\mathcal{D}_{AB_g} \propto \dfrac{\sqrt{T^3\left(\frac{1}{M_A}+\frac{1}{M_B}\right)}}{p\,\sigma_{AB}^2\,\Omega_{\mathcal{D},AB}}$ $\qquad \mathcal{D}_{AB_L} \propto \dfrac{\sqrt{\Psi_B M_B}\;T}{\mu\,\tilde{V}^{0.6}}$

Diffusivity increases with temperature for gases and liquids. An increase in pressure decreases the diffusivity.

(b) $\mu_g \propto \dfrac{\sqrt{MT}}{\sigma^2\,\Omega_\mu}$ $\qquad \mu_L \propto \dfrac{\tilde{N}h}{\tilde{V}}\,e^{3.8T_b/T}$

The viscosity of gases increases with increasing T. The opposite occurs for liquids.

(c) $k_g \propto \dfrac{\sqrt{T/M}}{\sigma^2\,\Omega_k}$ $\qquad k_L \propto \left(\dfrac{\tilde{N}}{\tilde{V}}\right)^{2/3} K v_s$

Thermal conductivity increases with increasing T & P.

(d) The heat capacity of gases increase with increasing temperature and pressure.

(e) The heat transfer coefficient is a function of Pr.

(f) $\nu = \dfrac{\mu}{\rho}$

Kinematic viscosity increases with increasing temperature for gases and decreases for liquids.

14. What is the Reynolds analogy? the Chilton - Colburn analogy?

Reynolds argued that mass or heat transport into a flowing fluid must involve two simultaneous processes:

1. The natural diffusion of the fluid when at rest
2. the eddies caused by visible motion which mixes the fluid up and brings fresh particles in contact with the surface.

$N_1 = k\Delta c_1 = [a + b v]\,\Delta c_1$

$q = h\Delta T = [a' + b' v]\,\Delta(\rho \hat{C}_p T)$

$\tau = f\left(\tfrac{1}{2}\rho v^2\right) = \left[\dfrac{f v}{2}\right]\rho v = [a'' + b'' v]\,\rho v$

In Reynold's words, these lead to an equivalency

$$b = b' = b''$$

$$k = bv \qquad h = \rho \hat{C}_p b' v \qquad \frac{fv}{2} = b'' v$$

$$\frac{h}{\rho \hat{C}_p} = b' v$$

$$\frac{k}{v} = \frac{h}{\rho \hat{C}_p v} = \frac{f}{2}$$

This is the Reynold's analogy. It is accurate for gases, but not for liquids. For gases

$$D = \alpha = \nu$$

$$\frac{\nu}{D} = \frac{\nu}{\alpha} = 1$$

$$Sc = Pr = 1$$

Chilton - Colburn tried to extend the theory to liquids.

$$b' = \frac{h}{\rho \hat{C}_p v}\left(\frac{\nu}{\alpha}\right)^{2/3} = j_H$$

$$b = \frac{k}{v}\left(\frac{\nu}{D}\right)^{2/3} = j_D$$

$$\frac{k}{v}\left(\frac{\nu}{D}\right)^{2/3} = \frac{h}{\rho \hat{C}_p v}\left(\frac{\nu}{\alpha}\right)^{2/3} = \frac{f}{2}$$

$$St_{AB}\, Sc^{2/3} = St\, Pr^{2/3} = \frac{f}{2}$$

$$Sh\, Re^{-1} Sc^{-1/3} = Nu\, Re^{-1} Pr^{-1/3} = \frac{f}{2}$$

$$j_D = j_H = \frac{f}{2}$$

An example of the application of this analogy is found in forced convection around a sphere:

$$Nu = \frac{h_m D}{k_f} = 2.0 + 0.60\, Re^{1/2} Pr^{1/3}$$

$$Sh = \frac{k_m D}{D_{AB}} = 2.0 + 0.60\, Re^{1/2} Sc^{1/3}$$

15. What is the friction factor? the coefficient of friction?

The friction factor is the ratio of the shear stress to kinetic energy

$$\frac{F}{A} = fK$$

For flow in conduits, we have

$$F = (2\pi RL)\left(\tfrac{1}{2}\rho\langle v\rangle^2\right)f$$

$$F = [(p_0 - p_L) + \rho g(h_0 - h_L)]\pi R^2 = (\mathcal{P}_0 - \mathcal{P}_L)\pi R^2$$

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v\rangle^2}\right) \equiv \text{Fanning friction factor}$$

Around submerged objects (sphere)

$$F_K = (\pi R^2)\left(\tfrac{1}{2}\rho v_\infty^2\right)f$$

$$F_K = \frac{4}{3}\pi R^3\rho_{sphere}\, g - \frac{4}{3}\pi R^3\rho g$$

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sph} - \rho}{\rho}\right)$$

For laminar flow in long tubes ($Re < 2100$)

$$f = \frac{16}{Re}$$

For turbulent flow in long tubes

$$f = 0.0791\, Re^{-1/4} = \text{Blasius formula}$$

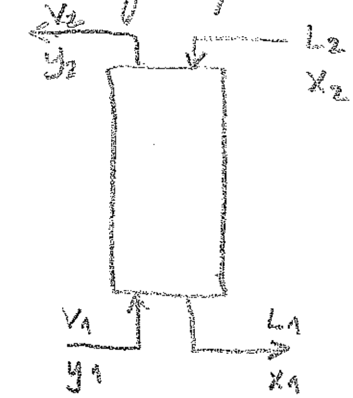
For creeping flow around a sphere

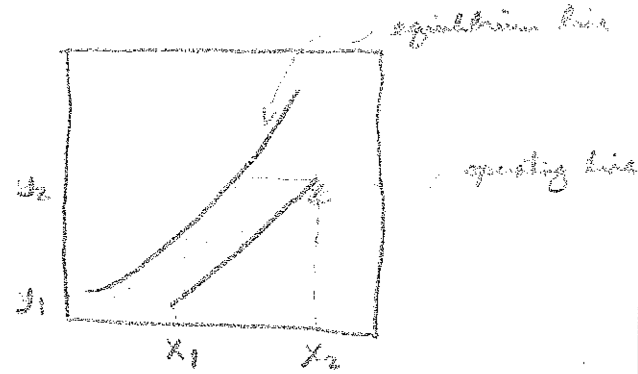
$$f = \frac{24}{Re}$$

The friction factor is sometimes called the coefficient of drag.

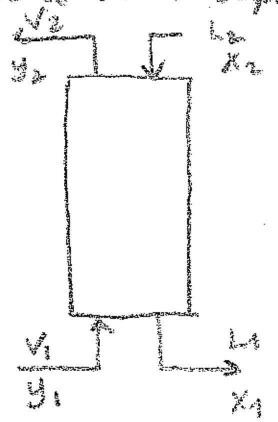
17. Sketch the governing diagrams for a stripper and an absorber.

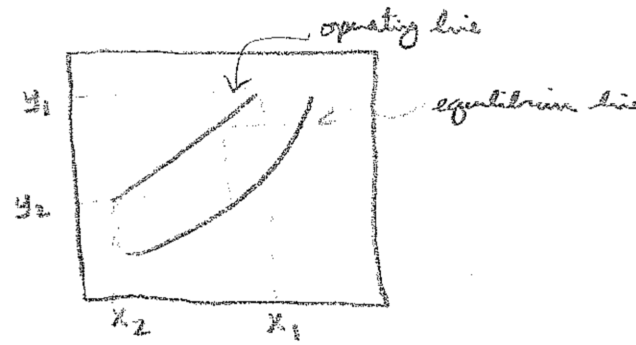
In a stripper, you are removing solute from the solute rich liquid phase into a solute-poor vapor phase.





In an absorber, you are removing solute from the solute rich-vapor phase to a solute-poor liquid phase

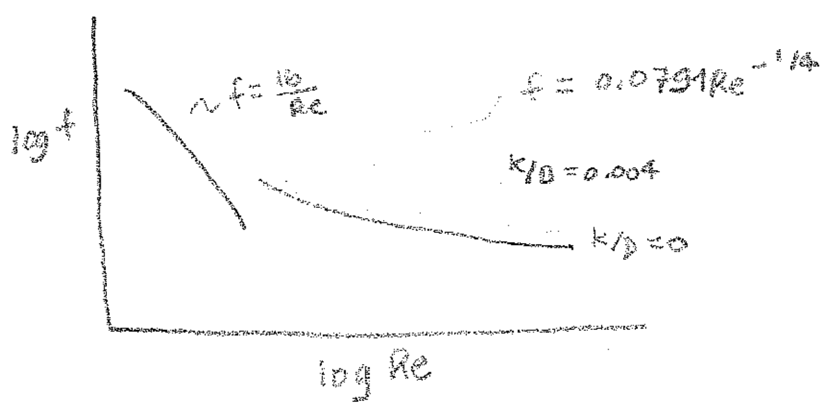




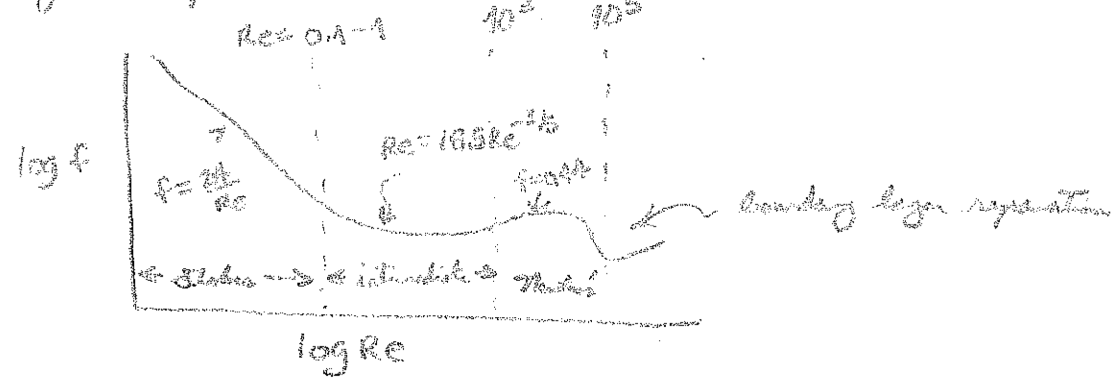
18. Describe the friction factor vs. Re relation for a (a) pipe (b) sphere (c) flat plate

(a) For a pipe, $f = \frac{16}{Re}$ for $Re < 2100$ and

$f = 0.0791 Re^{-1/4}$ for $Re > 2100$



(b) for a sphere



(c) For the friction factor on a flat plate, we have to solve the boundary layer equations

for laminar flow : $F_k = 1.328\sqrt{\rho \mu L W^2 v_\infty^3}$

for turbulent flow : $F_k = 0.072\rho v_\infty^2 W L \left(\frac{L v_\infty \rho}{\mu}\right)^{-1/5}$

19. Sketch the shear stress profile for a pipe.
The shear stress profile follows Newton's law of viscosity.

$\tau_{yx} = -\mu \dfrac{d v_x}{dy}$

We can do a shell balance on a vertical pipe to determine its velocity profile. We select a cylindrical shell of thickness Δr and length L and list the contributions to the momentum balance in the z-direction

$$\underbrace{2\pi r L\left(\tau_{rz}|_r - \tau_{rz}|_{r+\Delta r}\right)}_{\substack{\text{rate of momentum in}\\ \text{at } r \text{ and out at } r+\Delta r}} + \underbrace{2\pi r \Delta r\, v_z\left[(\rho v_z)|_{z=0} - (\rho v_z)_{z=L}\right]}_{\substack{\text{rate of momentum in across}\\ \text{annular surface at } z=0 \text{ minus}\\ \text{out at } z=L}}$$

$$+\underbrace{2\pi r \Delta r L(\rho g)}_{\substack{\text{gravity force acting}\\ \text{on cylindrical shell}}} + \underbrace{2\pi r \Delta r(p_0 - p_L)}_{\substack{\text{pressure force}\\ \text{acting on annular surface}}} = 0$$

If we divide by $2\pi \Delta r L$ and take the limit as $\Delta r \to 0$

$$\lim_{\Delta r \to 0}\left(\frac{(r\tau_{rz})|_{r+\Delta r} - (r\tau_{rz})|_r}{\Delta r}\right) = \left(\frac{p_0 - p_L}{L} + \rho g\right) r$$

$$\frac{d}{dr}(r\tau_{rz}) = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right) r$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r + \frac{C_1}{r}$$

At $r = 0$, $\frac{dv_r}{dz} = \tau_{rz}$ is finite

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r$$

Using Newton's law of viscosity

$$\tau_{rz} = -\mu \frac{dv_z}{dr}$$

$$-\mu \frac{dv_z}{dr} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r$$

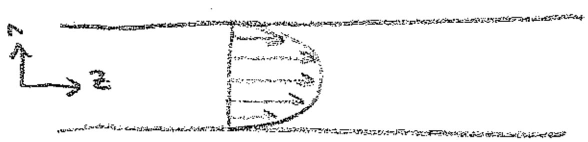
$$\frac{dv_z}{dr} = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu L}\right) r$$

$$v_z = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu L}\right) r^2 + C_2$$

At $r = R$, $v_z = 0$ (no slip condition)

$$v_z = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu L}\right)(R^2 - r^2) = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Thus, our velocity profile is parabolic.



20. Define the most commonly used dimensionless parameters and describe their significance.

$$Re = \frac{D\langle v\rangle\rho}{\mu} = \frac{\rho\langle v\rangle^2}{\mu\frac{\langle v\rangle}{L}} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\text{convective momentum transp}}{\text{diffusive momentum transp}}$$

$$Pr = \frac{\nu}{\alpha} = \frac{(\mu/\rho)}{(k/\hat{C}_p\rho)} = \frac{\mu\hat{C}_p}{k} = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}}$$

$$Sc = \frac{\nu}{\mathcal{D}_{AB}} = \frac{(\mu/\rho)}{\mathcal{D}_{AB}} = \frac{\mu}{\rho\mathcal{D}_{AB}} = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of mass}}$$

$$Le^{-1} = \frac{\mathcal{D}_{AB}}{\alpha} = \frac{\mathcal{D}_{AB}}{(k/\rho\hat{C}_p)} = \frac{\rho\hat{C}_p\mathcal{D}_{AB}}{k} = \frac{\text{molecular diffusivity of mass}}{\text{molecular diffusivity of heat}}$$

$$Nu = \frac{hD}{k} = \frac{\text{convective heat transport}}{\text{diffusive heat transport}} = \frac{\text{temp. grad. at boundary layer}}{\text{temp. grad. between bulk and b}}$$

$$Sh = \frac{kD}{\mathcal{D}_{AB}} = \frac{\text{convective mass transport}}{\text{diffusive mass transport}} = \frac{\text{conc. grad. at boundary layer}}{\text{conc. grad. between bulk and}}$$

$$Pe_{heat} = RePr = \left(\frac{D\langle v\rangle\rho}{\mu}\right)\left(\frac{\mu\hat{C}_p}{k}\right) = \frac{D\langle v\rangle\rho\hat{C}_p}{k} = \frac{\text{convective momentum tr}}{\text{diffusive heat transp}}$$

$$Pe_{mass} = ReSc = \left(\frac{D\langle v\rangle\rho}{\mu}\right)\left(\frac{\mu}{\rho\mathcal{D}_{AB}}\right) = \frac{D\langle v\rangle}{\mathcal{D}_{AB}} = \frac{\text{convective momentum transp}}{\text{diffusive mass transp}}$$

$$St_{heat} = \frac{Nu}{RePr} = \left(\frac{hD}{k}\right)\left(\frac{\mu}{\rho\langle v\rangle D}\right)\left(\frac{k}{\hat{C}_p\mu}\right) = \frac{h}{\rho\langle v\rangle\hat{C}_p} = \frac{\text{convective heat transp}}{\text{convective momentum tra}}$$

$$St_{mass} = \frac{Sh}{ReSc} = \left(\frac{kD}{\mathcal{D}_{AB}}\right)\left(\frac{\mu}{\rho\langle v\rangle D}\right)\left(\frac{\rho\mathcal{D}_{AB}}{\mu}\right) = \frac{k}{\langle v\rangle} = \frac{\text{convective mass transp}}{\text{convective momentum tra}}$$

$$Bi = \frac{hL}{k} = \frac{\text{surface convective resistance}}{\text{internal conductive resistance}}$$

$$Fr = \frac{\langle v\rangle^2}{Lg} = \frac{\rho\langle v\rangle^2}{\rho GL} = \frac{\text{inertial forces}}{\text{gravity forces}}$$

$$Gr = \frac{\beta g(T_0 - T_\infty)L^3}{\nu^2}, \text{ natural convection} = \frac{\text{buoyancy forces}}{\text{viscous forces}}$$

$$Gz = \frac{wc}{Lk}, \text{ Graetz problem}$$

$$Br = \frac{\mu(\langle v\rangle/D)^2}{k(T_0-T_1)/D^2} = \frac{\text{heat production by viscous dissipation}}{\text{heat transport by conduction}}$$

$$\frac{Gr}{Re^2} = \frac{\rho\beta g(T_0 - T_\infty)}{\rho V^2/D} = \frac{\text{buoyancy forces}}{\text{inertial forces}}$$

$$\text{Damkohler Number} = \frac{\text{max reaction rate}}{\text{max m.t. rate}}$$

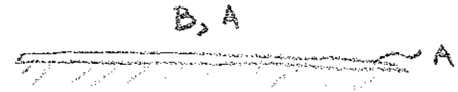
$$\text{Thiele Parameter} = \frac{\text{reaction rate}}{\text{diffusion rate}}$$

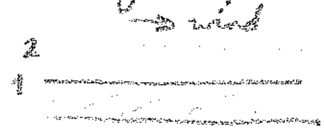
$$j_D = Sh\,Re^{-1}Sc^{-1/3}$$

$$j_H = Nu\,Re^{-1}Pr^{-1/3}$$

$$\frac{f}{2} = j_D = j_H$$

21. Given a pool of organic liquid (such as from a spill), how would you estimate its rate of evaporation?





We have diffusion through a stagnant film.

Assumptions:

1. Pool rapidly equilibrates to ambient temperature.
2. Gas adjacent to liquid surface is in equilibrium with C^*.

$$\bar{f}_i^V = \bar{f}_i^L$$

$$y_i \phi_i P = x_i \gamma_i \Phi_i^{sat} P_i^{sat}$$

Using Raoult's law

$$y_i = \frac{P_i^{sat}}{P} = C^*$$

We know the flux at the surface of the pool:

$$N_{Az} = c\mathcal{D}_{AB}\frac{dx_A}{dz} + x_A(N_{Az} + \cancel{N_{Bz}}^{\,0})$$

$$N_{Az} = \frac{c\mathcal{D}_{AB}}{1-x_A}\frac{dx_A}{dz}$$

If we assume that the wind is continuously taking A

$$-\frac{dN_A}{dz} = 0$$

$$\frac{d}{dz}\left(\frac{c\mathcal{D}_{AB}}{1-x_A}\frac{dx_A}{dz}\right) = 0$$

$$\frac{d}{dz}\left(\frac{1}{1-x_A}\frac{dx_A}{dz}\right) = 0$$

$$\frac{1}{1-x_A}\frac{dx_A}{dz} = C_1$$

$$\frac{1}{1-x_A} x_A = C_1 z + C_2$$

We impose the boundary conditions

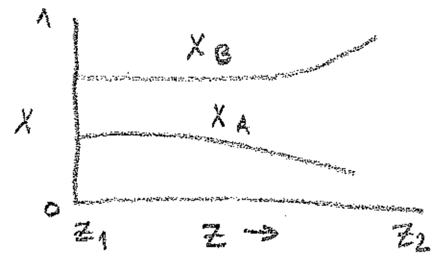
$x_A = x_{A1}$ @ $z = z_1$

$x_A = x_{A2}$ @ $z = z_2$

We obtain the following profile

$$\left(\frac{1-x_A}{1-x_{A1}}\right) = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\frac{z-z_1}{z_2-z_1}}$$

$$\left(\frac{x_B}{x_{B1}}\right) = \left(\frac{x_{B2}}{x_{B1}}\right)^{\frac{z-z_1}{z_2-z_1}}$$



22. How are the diffusivity and viscosity of a mixture determined?

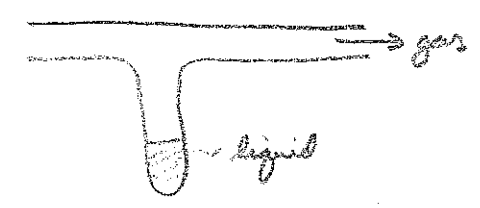
For a mixture we define the effective n-comp. diffusivity

$$N_i = -c\mathcal{D}_{AB}\nabla x_i + x_i \Sigma N_j$$

We can use the Stefan-Maxwell equation

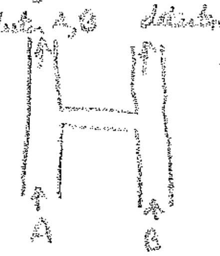
$$\nabla x_i = \Sigma \frac{c_i c_j}{c^2 \mathcal{D}_{ij}}(v_j - v_i) = \Sigma \frac{1}{c\mathcal{D}_{ij}}(x_i N_j - x_j N_i)$$

$$\frac{1}{c\mathcal{D}_{im}} = \frac{\Sigma (1/c\mathcal{D}_{ij})(x_j N_i - x_i N_j)}{N_i - x_i \sum_{j=1}^{n} N_j}$$



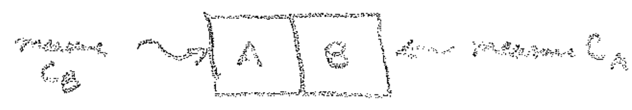
measure weight loss of liquid / solid

For two noncondensable gases



For liquid / solid measure dissolution time.

For liquid in a liquid use a diffusion cell



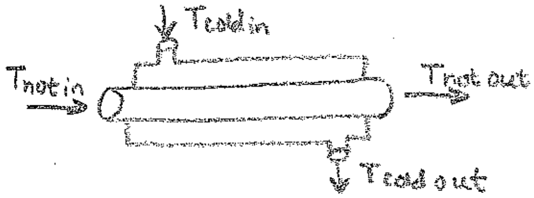
For viscosity measurements, we can use the Wilke formula

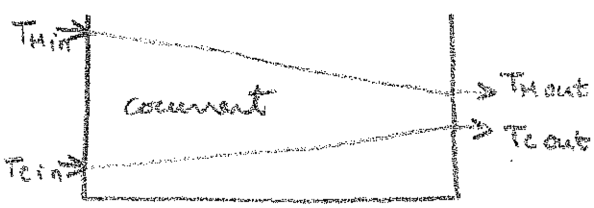
$$\mu_{mix} = \sum_i \frac{x_i \mu_i}{\sum_j x_j \Phi_{ij}} \quad \text{where} \quad \Phi_{ij} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_i}{M_j}\right)^{-1/2}\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{1/2}\left(\frac{M_j}{M_i}\right)^{1/4}\right]$$

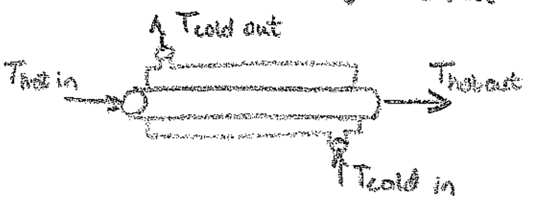
We could use a Couette viscometer. By measuring the torque required to rotate one cylinder at a known speed, the viscosity may be readily calculated from

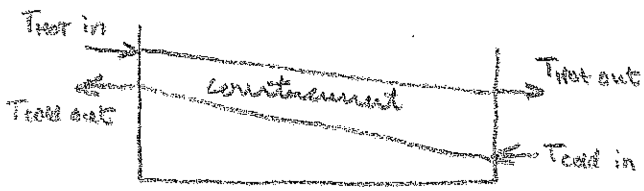
$$F_s = \frac{\mu A u_0}{g_c B}$$

23. Sketch the temperature profile in a heat exchanger.









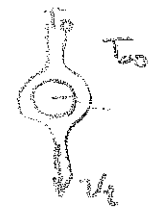
24. What phenomena are important during an underground explosion?

When we have an underground explosion, the following are important phenomena:

1) Bulk flow (convective momentum transport), thermal expansion of gas
2) the tortuosity and porosity of the soil is important in allowing the exploding gas to escape
3)

25. Consider a drop falling from a tower; initial temperature and tower temperature are given. How does the drop temperature change as it falls?

We can use the lumped heat capacity method to solve this problem by assuming that the internal resistance of the falling drop (a small sphere) is negligible compared to its external resistance. We can then model the energy loss as



$$q = hA(T - T_\infty) = -\hat{C}_p \rho V \frac{dT}{dt}$$

We need one boundary value

$$T = T_0 \quad @ \quad t = 0$$

$$-\frac{hA}{\hat{C}_p \rho V}\, dt = \frac{dT}{T - T_\infty}$$

$$\ln(T - T_\infty) = -\frac{hA}{\hat{C}_p \rho V} t + c_1$$

$$= -\frac{hA}{\hat{C}_p \rho V} t + \ln(T_0 - T_\infty)$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hA}{\hat{C}_p \rho V}\right) t$$

This equation only holds if $\frac{hD}{k} = Bi < 0.1$

We could estimate h using the correlation

$$Nu = \frac{hD}{k} = 2.0 + 0.6 Re^{1/2} Pr^{1/3}$$

If this were evaporation of a free falling drop:

$$W_A^{(m)} = K_m A(X_{A0} - X_{A\infty}) + X_{A0}\left(W_A^{(m)} + \cancel{W_B^{(m)}}^{\,0}\right) \quad \text{(for low diffusion)}$$

$$W_A^{(m)} = k_m A \frac{(X_{A0} - X_{A\infty})}{(1 - X_A)}$$

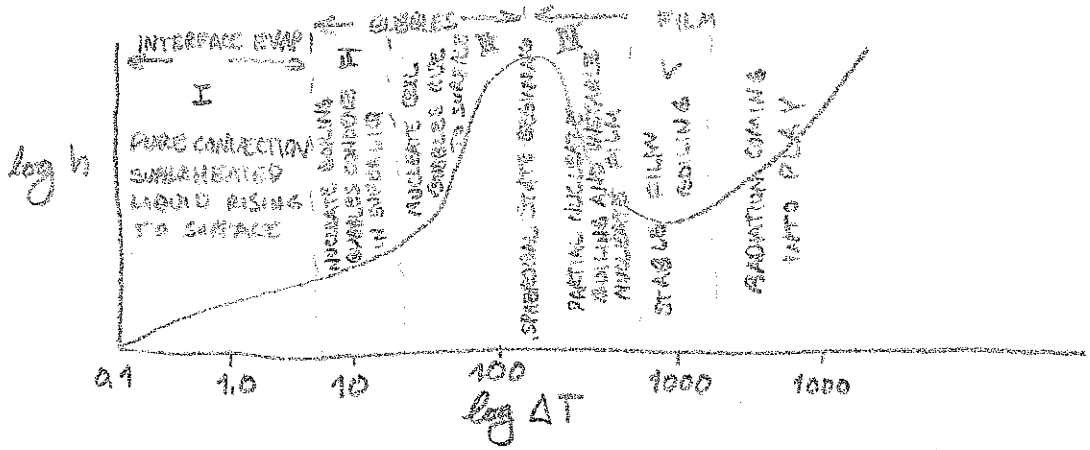
again we could use the correlation

$$Sh = \frac{k_m D}{\mathcal{D}_{AB}} = 2.0 + 6.0 Re^{1/2} Sc^{1/3}$$

26. What is the angular dependence of the Nusselt number for a falling drop?

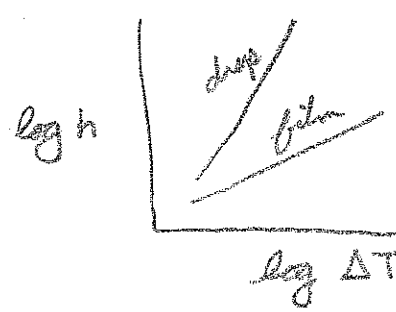
$$Nu = \frac{hD}{k} = 2.0 + 0.6\, Re^{1/2} Pr^{1/3}$$

27. Draw the boiling curve and describe the physical phenomena responsible for the observed behavior. Draw and explain the similar curve for condensation.



where $\Delta T = T_w - T_{sat}$

As boiling begins, warm water rises in convection eddies (pool boil). Bubbles begin to form in the superheated liquid, and eventually rise to bulk boiling. There comes a point in which the bubbles form so fast that they use up all of the available area for heat transfer. This is an unstable region that coalesces in film boiling.



where $\Delta T = T_w - T_{sat}$

In condensation, drops begin to form on the cooled surface. This provides the most area for heat transfer and thus the best heat transfer. As drops coalesce, they form a film which provides less heat transfer.

29. Derive the steady-state momentum balance for fully developed laminar flow in a pipe.

We can derive this from Navier-Stokes

$$\rho \frac{D\underline{v}}{dt} = -\nabla \underline{p} - [\nabla \cdot \underline{\underline{\tau}}] + \rho \underline{g}$$

$$= -\nabla \underline{p} - \mu^2 \nabla^2 \underline{v} + \rho \underline{g}$$

We use cylindrical coordinates for flow in z-direction

$$\rho\left(\cancelto{0}{\frac{\partial v_z}{\partial t}} + \cancelto{0}{v_r}\frac{\partial v_z}{\partial r} + \cancelto{0}{\frac{v_\theta}{r}}\frac{\partial v_z}{\partial \theta} + v_z \cancelto{0}{\frac{\partial v_z}{\partial z}}\right) = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancelto{0}{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$

Since we are in fully developed flow, $\frac{d v_z}{dz} = \frac{d^2 v_z}{dz^2} = 0$. Also $v_r = v_\theta = 0$. We are left with

$$\frac{\partial p}{\partial z} - \rho g_z = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$$

$$-\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{\mu} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$$

$$-\frac{(\mathcal{P}_0 - \mathcal{P}_L)r^2}{2\mu} + c_1 = r\frac{\partial v_z}{\partial r}$$

$$-\frac{(\mathcal{P}_0 - \mathcal{P}_L)r}{2\mu} + \frac{c_1}{r} = \frac{\partial v_z}{\partial r} \quad \left(v_z \text{ finite @ } r = 0\right)$$

$$-\frac{(\mathcal{P}_0 - \mathcal{P}_L)r^2}{4\mu} + C_2 = v_z \quad \left(v_z = 0 \text{ @ } r = R\right)$$

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

30. How is the overall heat transfer coefficient for a heat exchanger found?

In order to find the overall heat transfer coefficient for a heat exchanger, we must combine both convective and conductive resistances to heat transfer.

$$q = h_i A (T_1 - T_2) = \frac{k_A A}{\Delta x}(T_2 - T_3) = h_o A (T_3 - T_4)$$

$$q = \frac{T_1 - T_4}{1/h_i A + \Delta x_A / k_A A + 1/h_o A} = UA\Delta T_{overall} = \frac{T_1 - T_4}{\Sigma R}$$

$$U = \frac{1}{1/h_i + \Delta x_A / k_A + 1/h_o} \quad [=] \quad \frac{W}{m^2 \cdot K}, \ \frac{btu}{h \cdot ft^2 \cdot {}^\circ F}$$

The effect of fouling and deposits on the wall of the pipe are taken care of by adding an additional resistance

$$U = \frac{1}{1/h_i + 1/h_{di} + (r_o - r_i)/k_A A_{A\,lm} + A_i/A_o h_o + A_i/A_o h_{do}}$$

where h_{di} & h_{do} are inside and outside fouling factors, respectively. We can also use the Nusselt equation to approximate h

$$Nu = \frac{h_i D_i}{k} = 0.023\, Re^{0.8} Pr^{0.3} \text{ for } h_i \text{ (Dittus - Boelter)}$$

$$Nu = \frac{h_o D_o}{k} = 0.023\, Re^{0.8} Pr^{0.4} \text{ for } h_o$$

or can use Colburn relationship

$$St = 0.023 Re^{0.2} Pr^{-2/3} \quad (h_o, \text{ use } D_o)$$

$$St = 0.023 Re^{-0.2} Pr^{-2/3} \quad (h_i, \text{ use } D_i)$$

31. Given two temperatures and a knowledge of all the fluids' properties in a double pipe countercurrent heat exchanger, how do you calculate the other two temperatures?

$$q = U_i A_i \Delta T_m = U_o A_o \Delta T_m \quad \text{where } \Delta T_m = F_T \Delta T_{ln}$$

$$\Delta T_{ln} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln[(T_{hi} - T_{co}) / (T_{ho} - T_{ci})]}$$

32. Derive equations describing the wet-bulb / dry bulb psychrometer. Obtain a relation between the wet bulb temperature and the air humidity in terms of dimensionless numbers.

We assume that the velocity of air past the wet bulb thermometer is high enough such that we can neglect radiation, conduction but low enough so that we can neglect viscous dissipation.

$$W_A\left(\bar{H}_{gA0} - \bar{H}_{A1}\right) = Q$$

$$Q = h_m \pi DL\left(T_\infty - T_0\right)$$

$$\Delta H_{vap} = \bar{H}_{gA0} - \bar{H}_{A1}$$

$$W_A^{(m)} = k_{xm}\pi DL\left(x_{A0} - x_{A\infty}\right) + x_{A0}\left(W_A^{(m)} + \cancel{W_B^{(m)}}^{\,0}\right)$$

$$W_A^{(m)} = k_{xm}\pi DL\,\frac{(x_{A0} - x_{A\infty})}{(1 - x_{A0})}$$

$$k_{xm}\cancel{\pi DL}\,\frac{(x_{A0} - x_{A\infty})}{(1 - x_{A0})}\,\Delta H_{vap} = h_m\cancel{\pi DL}\left(T_\infty - T_0\right)$$

$$\frac{\mathcal{H} - \mathcal{H}_w}{T_\infty - T_w} = -\frac{h_m}{k_{xm}\Delta H_{vap}}$$

33. Is the heat flux from a liquid into a gas usually higher or lower if the gas is insoluble (versus soluble) in a liquid? This compares "diffusion through a stationary component" with the extreme case of "equimolar counterdiffusion". We can solve this by looking at Fick's law

$$N_{Az} = -c\mathcal{D}_{AB}\frac{dx_A}{dz} + x_A\left(N_{Az} + N_{Bz}\right)$$

For stagnant, $N_{Bz} = 0$ For EMCD, $N_{Az} = -N_{Bz}$

$$N_{Az} = -\frac{c\mathcal{D}_{AB}}{(1-x_A)}\frac{dx_A}{dz} \qquad N_{Az} = -c\mathcal{D}_{AB}\frac{dx_A}{dz}$$

Since $\dfrac{1}{(1-x_A)} > 1$, diffusion and therefore driving force is maximized.

34. The Chilton - Colburn j-factor for heat transfer is proportional to h, the convective heat transfer coefficient. Why is j proportional to $Pr^{-1/3}$? Why is j only a fraction of Re? Why does j decrease as Re increases?

Chilton - Colburn were trying to extend the theory of Reynold's analogy to liquids. The Reynolds' analogy argued that mass or heat transport in a fluid must involve two processes: natural diffusion and turbulent eddies that mix the fluid.

$$N_1 = k\Delta c_1 = [a + bv]\Delta c_1$$

$$q = h\Delta T = [a' + b'v]\Delta(\rho\hat{C}_p T)$$

$$\tau = f\left(\tfrac{1}{2}\rho v^2\right) = \left[\frac{fv}{2}\right]\rho v = [a'' + b''v]\rho v$$

Reynold states the equivalency

$$b = b' = b''$$

$$k = bv$$
$$h = \rho\hat{C}_p b' v$$
$$\frac{fv}{2} = b'' v$$

$$\frac{k}{v} = \frac{h}{\rho\hat{C}_p v} = \frac{f}{2}$$

This is Reynolds' analogy. It is accurate for gases, but not for liquids. For gases

$$\mathcal{D}_{AB} = \alpha = \nu$$

$$\frac{\nu}{\mathcal{D}_{AB}} = \frac{\nu}{\alpha} = 1$$

$$Sc = Pr = 1$$

Chilton - Colburn extended this theory to liquids

$$b' = \frac{h}{\rho\hat{C}_p v}\left(\frac{\nu}{\alpha}\right)^{2/3} = j_H \qquad j_H = j_D = \frac{f}{2}$$

$$b = \frac{k}{v}\left(\frac{\nu}{\mathcal{D}_{AB}}\right)^{2/3} = j_D$$

$$\frac{k}{v}\left(\frac{\nu}{\mathcal{D}_{AB}}\right)^{2/3} = \frac{h}{\rho\hat{C}_p v}\left(\frac{\nu}{\alpha}\right)^{2/3} = \frac{f}{2}$$

$$St_{AB}\,Sc^{2/3} = St\,Pr^{2/3} = \frac{f}{2}$$

35. O_2 and N_2 leaks from pressurized tanks are often considered less dangerous than H_2 leaks. Why?

First of all, H_2 is very explosive in air

$2H_2 + O_2 \rightarrow 2H_2O$

whereas N_2 & O_2 are not as explosive. Second of all, the Joule-Thompson Coefficient for H_2, defined as

$\mu = \left(\frac{\partial T}{\partial P}\right)_H$

is positive for both N_2 & O_2, and negative for H_2. This means that N_2 & O_2 cool as they expand, while H_2 heats up. Look out.

36. How would you separate oxygen from salt water? Suppose you were processing fairly large volumes so that energy efficiency is a strong consideration. What thermodynamic variables affect solubility? Where is the mass transfer resistance? What type of unit operation would you use? How would you design it?

If energy was cheap, we could heat the water to drive off the oxygen. If energy efficiency is a consideration, we can look at the solubility of O_2. Henry's law is highly dependent on pressure. We can reduce the pressure through a vacuum process (expand through a valve or pull a vacuum). The main mass transfer resistance is in bubble formation. We could add surfactant to lower the surface tension of the fluid. We could design a vacuum settling vessel with an upstream expansion valve.

$x_i H = P$

37. What area is used when defining friction factor for a wetted wall column?

For the liquid flowing down the walls, we will simply use the diameter of the wall to calculate the friction factor. For the gas moving upward, we simply use the diameter minus twice the thickness of the film.

$D' = D - 2\delta$ for gas $\quad A = \frac{\pi}{4}(D - 2\delta)^2$

38. What is the Lewis relation? Is it dependent on the gas phase velocity? Why or why not?

The Lewis relation relates the molecular diffusivity of mass to the molecular diffusivity of heat

$$\frac{h}{\rho k_m} = c_p \left(\frac{N_{Sc}}{N_{Pr}}\right)^m$$

for gases $N_{Sc} = N_{Pr} = 1$

$$\frac{h}{\rho C_p k_m} = \frac{\text{molecular diffusivity of heat}}{\text{molecular diffusivity of mass}}$$

Although momentum is present in both Sc and Pr used to derive Le, the terms cancel each other out.

39. Why are the analogies between mass and heat transfer much more straightforward to use than analogies between mass and momentum transfer?

Momentum is written in terms of a vector, whereas heat and mass transfer are scalars. The momentum flux is in tensor form, whereas heat and mass fluxes are vectors. Mass and heat transfer are easily reflected by film theory.

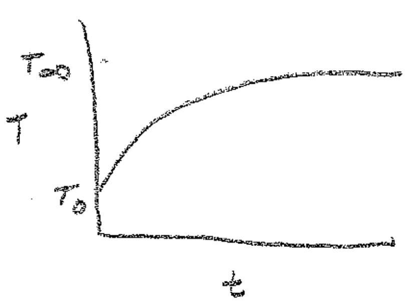
40. Given a CSTR at temperature T with no reaction what would happen if the inlet temperature were suddenly increased?

We can perform an energy balance on the CSTR, assuming perfect insulation, and mass in = mass out, well-mixed

$$\rho \dot{v} C_p (T_i - T) = \rho V C_p \frac{dT}{dt}$$

$$\frac{\dot{v}}{V} dt = \frac{dT}{(T_i - T)}$$

$$\frac{\dot{v}}{V} t = -\ln(T_i - T) + C_1$$



assume $T = T_0$ @ $t = 0$

$$-\frac{\dot{v}}{V} t = \ln\left(\frac{T_i - T}{T_i - T_0}\right)$$

$$\frac{T_i - T}{T_i - T} = \exp\left(-\frac{\dot{v}}{V} t\right)$$

41. Analogies between heat, mass, and momentum transport are important. Give examples of when they don't hold.

These analogies break down when

1. not constant physical properties
2. large rates of mass transfer
3. chemical reactions
4. 3-D equations in momentum
5. different boundary conditions
6. viscous dissipation
7. radiation
8. P, T forced diffusion

42. What is the theoretical basis for all "famous" analogies between heat, mass, and momentum transport? What are the mass and heat transfer equivalents of the momentum transport equation?

We have the Reynolds' analogy, which says that heat, mass, and momentum transport occurs by two processes: natural diffusion and forced convection through turbulent eddies.

$N_1 = k\Delta C_1 = [a + bv]\Delta C_1$

$q = h\Delta T = [a' + b'v]\Delta(\rho\hat{C}_p T)$

$\tau = f(\frac{1}{2}\rho v^2) = [a'' + b''v]\rho v$

Reynolds states the equivalency

$b = b' = b''$

$$\frac{k}{v} = \frac{h}{\rho\hat{C}_p v} = \frac{f}{2}$$

$$St_{AB} = St = \frac{f}{2}$$

Chilton-Colburn make a correction for liquids

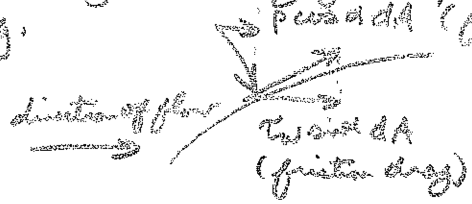
$$St_{AB}\, Sc^{2/3} = St\, Pr^{2/3} = \frac{f}{2}$$

$$Sh\, Re^{-1} Sc^{-1/3} = Nu\, Re^{-1} Pr^{-1/3} = f/2$$

From these equations it is easy to see that heat, mass, and momentum transfer have the same form.

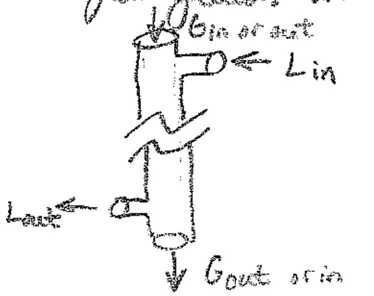
43. What is the difference between skin friction drag and form drag?

Drag is defined as the force in the direction of flow exerted by the fluid on the solid. When the wall of the body is parallel to the direction of flow, we have friction drag. The total integrated drag from wall shear is called friction drag. The fluid pressure, which acts in the direction normal to the flow, creates form drag. The total integrated drag from pressure is called the form drag.



44. How would you determine a mass transfer coefficient experimentally?

1. The coefficient k has been studied in experimental devices in which the area of contact between phases is known and where boundary-layer separation does not take place. The wetted-wall tower is one of the devices used. Gives info on mass transfer to and from fluids in turbulent flow.



In this tower the interfacial area is known and form drag is absent.

$$k = \psi(D_v, D, \mu, u, \rho)$$
$$k = \psi(Re, Sh)$$

2. Mass transfer in turbulent pipe flow studied using tubes made from slightly soluble solid and measuring rate of dissolution of solid for various flowrates. Can also make portion of wall electrode and carry out an electrochemical reduction under conditions where current is limited by rate of mass transfer of reacting ion to the wall.

3. External mass transfer (diffusion to particles or to outside of pipes or cylinders) requires different correlations due to BL. Can study mass-transfer coefficients by studying evaporation from wet solids. Not easy to ensure no internal resistance, helps if solid is made from a slightly soluble substance

4. Experiments can be made with actual mass transfer devices (packed towers, sieve trays, bubble columns). Mass transfer rates are converted to volumetric mass transfer coefficient Ka, where a is the transfer area per unit volume of equipment.

45. Why does frost not form under a tree when it is on the ground all around the tree?

Blackbody radiation is the process at work here. The ground around the tree is radiating into space (which is at a much lower temperature) while the ground under the tree is radiating upward to the tree. The tree is radiating back, and the ground is too warm to form frost.

46. Draw a McCabe Thiele diagram for a distillation column which uses a reacting absorbent.

The basic equations for the distillation column are

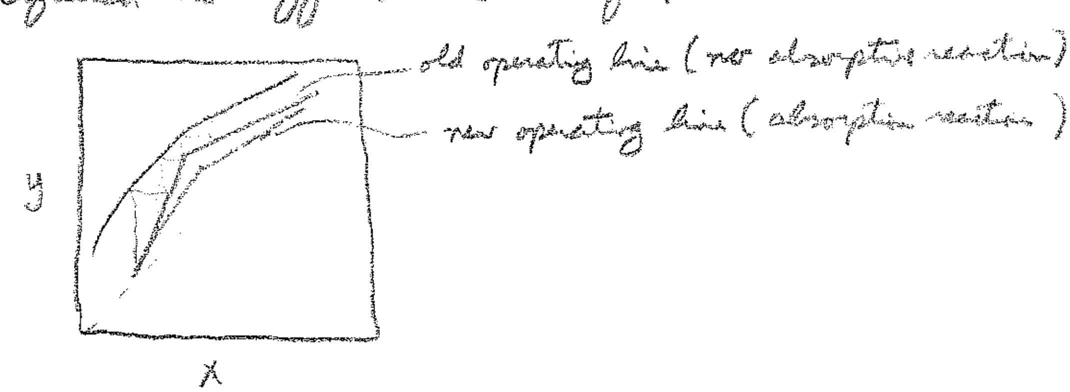
$$F = B + D$$

$$F z_F = B x_B + D x_D$$

$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D \quad \text{enriching}$$

$$y_m = \frac{\bar{L}}{\bar{V}} x_{m-1} - \frac{B}{\bar{V}} x_B \quad \text{stripping}$$

If we use a reacting absorbent, the operating lines for each equation will shift due to an increase in mass transfer coefficient (due to greater effective interfacial area, since absorption can take place in stagnant regions). Therefore fewer stages will be required to effect the transfer.



47. What are the most commonly used (3) correlations describing heat and mass transfer?

(1) $j_H = j_D = \frac{f}{2}$

$$Nu\,Re^{-1}Pr^{-1/3} = Sh\,Re^{-1}Sc^{-1/3}$$

$$\frac{h}{\rho \hat{C}_p V}\left(\frac{\hat{C}_p \mu}{k}\right)^{2/3} = \frac{k_x}{cV}\left(\frac{\mu}{\rho D_{AB}}\right)^{2/3}$$

(2) Forced convection around a sphere of diameter D:

$$\frac{h_m D}{k_f} = Nu = 2.0 + 0.6\,Re^{1/2}Pr^{1/3}$$

$$\frac{h_{xm} D}{c\mathcal{D}_{AB}} = Sh = 2.0 + 0.6\,Re^{1/2}Sc^{1/3}$$

(3) Forced convection in pipes of diameter D

laminar: $Nu = 1.86\left(Re\,Pr\,\frac{D}{L}\right)^{1/3}\left(\frac{\mu}{\mu_0}\right)^{0.14}$

$Sh = 1.86\left(Re\,Sc\,\frac{D}{L}\right)^{1/3}\left(\frac{\mu}{\mu_0}\right)^{0.14}$

turbulent: $Nu = 0.026\,Re^{0.8}Pr^{1/3}\left(\frac{\mu}{\mu_0}\right)^{0.14}$

$Sh = 0.026\,Re^{0.8}Pr^{1/3}\left(\frac{\mu}{\mu_0}\right)^{0.14}$

48. Write out the constitutive equations for:
(a) mass transfer (b) momentum transfer (c) heat transfer

(a) $N_{Az} = \mathcal{D}_{AB}\frac{dc_A}{dz}$

(b) $\tau_{xz} = \frac{\mu}{\rho}\frac{d(\rho v_x)}{dz}$

(c) $q_z = \frac{k}{\hat{C}_p \rho}\frac{d(\hat{C}_p \rho T)}{dz}$

49. Derive the equations for gas undergoing an isentropic expansion.

In order to derive the Ergun equation, we begin with the Hagen Poiseuille equation

$$\langle v \rangle = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R^2}{8 \mu L}$$

For flow in a packed bed we modify this equation with a hydraulic radius

$$R = 2R_H = 2\left(\frac{\text{cross sectional area available for flow}}{\text{wetted perimeter}}\right)$$

$$= 2\left(\frac{\text{volume available for flow}}{\text{total wetted surface}}\right) = 2\,\frac{\left(\frac{\text{volume of voids}}{\text{volume of bed}}\right)}{\left(\frac{\text{wetted surface}}{\text{volume of bed}}\right)}$$

$$= 2\,\frac{\epsilon}{a}$$

We can redefine a by relating it to the specific surface a_v (total particle surface / volume of the particles)

$a = a_v(1-\epsilon)$

We define the mean particle diameter D_p

$D_p = 6/a_v$

If we combine terms and plug back into the Hagen-Poiseuille equation

$$\langle v \rangle = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R_H^2}{2 \mu L}$$

The average velocity in the interstices $\langle v \rangle$ is not of interest, but the superficial velocity $v_0 = \langle v \rangle \epsilon$

$$v_0 = \frac{(\mathcal{P}_0 - \mathcal{P}_L)\epsilon^3}{2\mu L a^2} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)\epsilon^3}{2\mu L a_v^2 (1-\epsilon)^2} = \frac{(\mathcal{P}_0 - \mathcal{P}_L) D_p^2}{2\mu L (36)} \frac{\epsilon^3}{(1-\epsilon)^2}$$

$$v_0 = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} \frac{D_p^2}{2(36\mu)} \frac{\epsilon^3}{(1-\epsilon)^2}$$

We correct this by replacing the 2 with a value 25/6

$$v_0 = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} \frac{D_p^2}{150\mu} \frac{\epsilon^3}{(1-\epsilon)^2}$$

This Blake-Kozeny equation corresponds to a bed friction factor of

$$f = \left(\frac{(1-\epsilon)^2}{\epsilon^3}\right)\frac{75}{D_p G_0/\mu}$$

For turbulent flow, we need a correction

$$\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} = \frac{1}{D}\cdot\frac{1}{2}\rho\langle v\rangle^2\cdot 4f_0 = 6f_0\frac{1}{D_p}\cdot\frac{1}{2}\rho v_0^2\frac{1-\epsilon}{\epsilon^3}$$

Experimental data indicate that $6f_0 = 3.50$. Hence

$$\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} = 3.50\frac{1}{D_p}\frac{1}{2}\rho v_0^2\frac{1-\epsilon}{\epsilon^3}$$

This corresponds to a friction factor of

$$f = 0.875\,\frac{1-\epsilon}{\epsilon^3}$$

We combine the Blake-Kozeny equation for laminar flow and the Burke-Plummer for turbulent flow

$$\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} = \frac{150\mu v_0}{D_p}\frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75\rho v_0^2}{D_p}\frac{(1-\epsilon)}{\epsilon^3}$$

This is the Ergun equation. We can rewrite it in terms of dimensionless groups:

$$\left(\frac{(\mathcal{P}_0 - \mathcal{P}_L)\rho}{G_0^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right) = 150\frac{(1-\epsilon)}{(D_p G_0/\mu)} + 1.75$$

50. Derive the equation for a gas undergoing an isentropic expansion.

We begin with the macroscopic energy balance (Bernoulli Equation)

$$\frac{d}{dt}\left(K_{tot} + \cancel{\Phi_{tot}}^{0} + U_{tot}\right) = -\Delta\left[\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \cancel{\hat{\Phi}}^{0} + \hat{G}\right)w\right] - \cancel{W}^{0} - \cancel{E_v}^{0}$$

$$0 = -\Delta\left[\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{H}\right)w\right] - W$$

$$dU = -dW$$

$$C_v\,dT = -P\,dV = -\frac{RT}{V}dV$$

$$\frac{dT}{T} = -\frac{R}{C_v}\frac{dV}{V}$$

$$\gamma = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

$$\frac{R}{C_V} = \gamma - 1$$

$$\ln\left(\frac{T_2}{T_1}\right) = (1-\gamma)\ln\left(\frac{V_2}{V_1}\right)$$

$$\left(\frac{V_2}{V_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{1-\gamma}}$$

51. What is inside a light bulb, and why?

Inside the light bulb is a vacuum. This is to reduce the heat that could be dissipated through, say, air if it were inside the bulb. This includes conduction with the air and natural convection. Also, O_2 could react with the filament or even cause an arc.

52. Why do you have to whirl a wet-bulb/dry-bulb psychrometer in the air prior to reading it?

One must insure that the air velocity across the wet bulb thermometer is high enough to neglect radiation and thermal conduction in the length. Also, sufficient air must flow past the wick and long enough to achieve the steady-state mass flux. Otherwise one cannot be sure that the wet bulb temperature has been reached in the thermometer.

53. In which direction is the momentum flux from a fluid flowing over a flat plate?

We can use Navier Stokes equation to solve this problem:

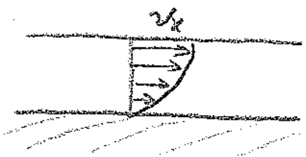
$$\rho \frac{D\underline{v}}{Dt} = -\nabla p + \mu \nabla^2 \underline{v} + \rho g$$

If we assume fully developed laminar flow

$$0 = -\nabla p + \mu \nabla^2 \underline{v}$$

$$0 = -\frac{\Delta p}{L} + \mu \frac{\partial^2 v_x}{dy^2}$$

$$\frac{\partial^2 v_x}{dy^2} = \frac{\Delta p}{\mu L}$$



$$\frac{\partial v_x}{\partial y} = \frac{\Delta p}{\mu L} y + c_1$$

$$v_x = \frac{\Delta p}{2\mu L} y^2 + c_1 y + c_2$$

Apply the boundary conditions:

$$v_x = 0 \text{ @ } y = 0 \quad \therefore c_2 = 0$$

$$\frac{\partial v_x}{\partial y} = \tau_{xy} = 0 \text{ @ } y = \delta$$

$$c_1 = \frac{\Delta p}{\mu L}\delta$$

The velocity profile looks like

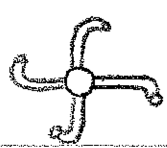
$$v_x = \frac{\Delta p}{2\mu L}\left(y^2 + 2\delta y\right)$$

The momentum flux looks like

$$\tau_{xy} = -\mu\left(\frac{\partial v_x}{dy}\right) = \frac{\Delta p}{L}(\delta - y)$$

The momentum is normal to the x direction.

54. How does a lawn sprinkler work?



We can perform a force balance on the nozzle of one of the sprinkler heads. From this we can see that the pressure