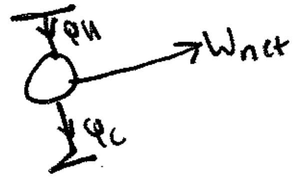
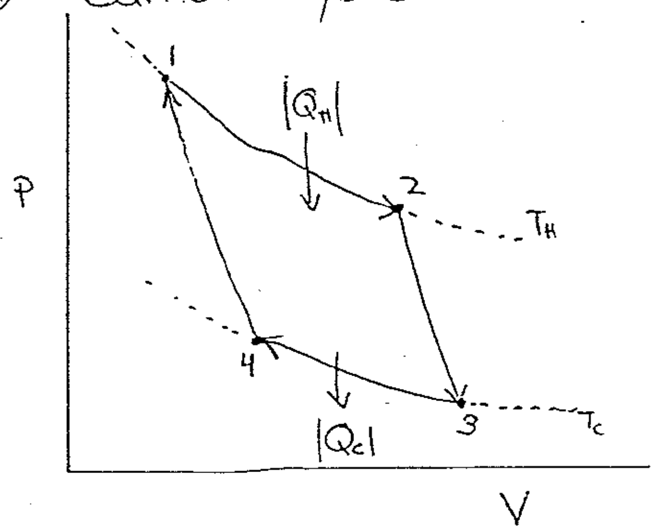
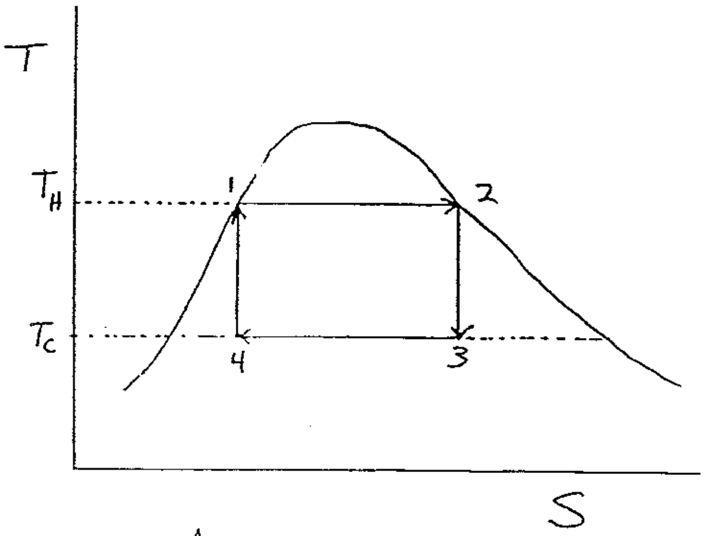
THERMO 1/17



(17) Carnot cycle





Carnot power cycle

[If operated in reverse, Carnot refrigeration or heat pump cycle]

<u>4 Reversible Steps</u>

1) $1 \to 2$ Isothermal expansion (with absorption of heat $|Q_H|$)
2) $2 \to 3$ Adiabatic expansion (temp. drops from T_H to T_C)
3) $3 \to 4$ Isothermal compression (with rejection of heat $|Q_C|$)
4) $4 \to 1$ Adiabatic compression (temp. rises from T_C to T_H)

The net amount of work W produced is

$$W = |Q_H| - |Q_C|$$

The thermal efficiency of the power cycle, η, is defined as

$$\eta \equiv \frac{\text{net work output}}{\text{heat input}} = \frac{W}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|} = \boxed{1 - \frac{T_C}{T_H} = \eta}$$

Carnot cycle

Eqtns governing each step

Isothermal steps ($1\to2$ and $3\to4$)

$dU = dQ - dW$ At constant T $\underline{dU = 0}$

$\therefore\ Q = W = \int P\,dV$ Assuming ideal gas $P = \frac{RT}{V}$

$$\underline{Q = W} = \int RT\frac{dV}{V} = RT\ln\frac{V_2}{V_1}$$

$$\boxed{Q = W = RT\ln\frac{V_2}{V_1} = RT\ln\frac{P_1}{P_2}}$$

since $\frac{P_1}{P_2} = \frac{V_2}{V_1}$ for an isothermal process

Adiabatic steps ($2\to3$ and $4\to1$) $dQ = 0$

$dU = dQ - dW \Rightarrow dU = -dW$

$\underline{dU = C_v\,dT}$

V is not constant but ΔP is much great than ΔV for these two steps

$\underline{dW = P\,dV}$

$dW = -C_v\,dT$

$\boxed{W = -C_v\Delta T}$

$C_v\,dT = -P\,dV$ (subst. $PV = RT$ and rearrange)

$$\frac{dT}{T} = -\frac{R}{C_v}\frac{dV}{V}$$

Recall $\gamma = \frac{C_p}{C_v}$ and $C_p = C_v + R$

$$\frac{R}{C_v} = \gamma - 1$$

$$\frac{dT}{T} = -(\gamma-1)\frac{dV}{V}$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

(Recall $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$) $\Rightarrow \boxed{P_1V_1^{\gamma} = P_2V_2^{\gamma}}$

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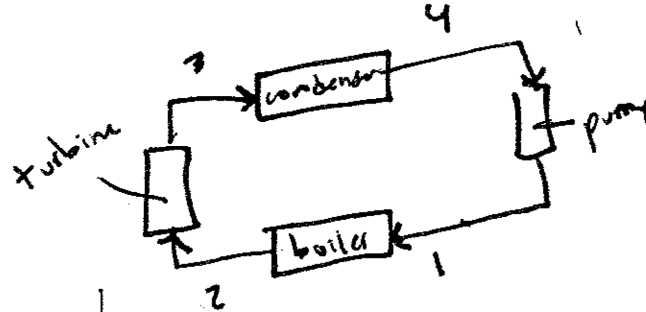
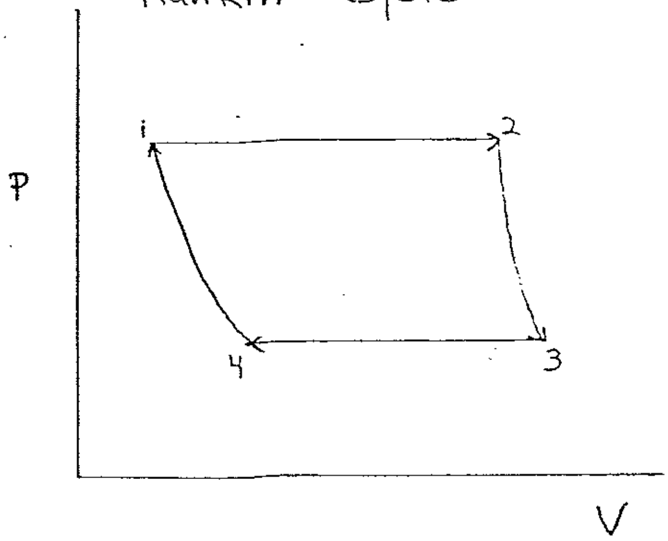
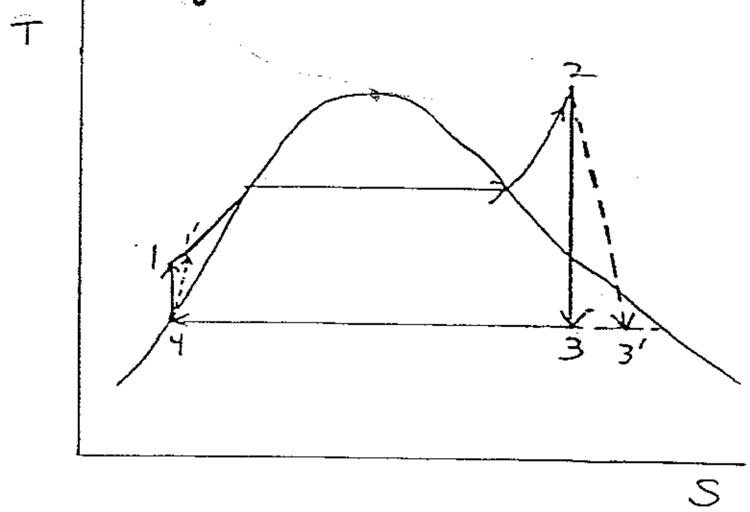
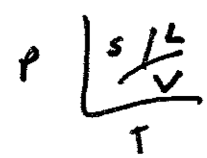


Rankine cycle





<u>4 Steps</u>



1) $1 \to 2$ constant pressure heating in a boiler
(heating of liquid to T_{sat}, vaporization at const. T, P, superheating of the vapor such that T_{final} is much greater than T_{sat})

2) $2 \to 3$ reversible, adiabatic (isentropic) expansion
(crosses saturation curve ∴ wet vapor)

3) $3 \to 4$ constant pressure, const. T condensation (yields saturated liquid)

4) $4 \to 1$ reversible, adiabatic (isentropic) compression to boiler pressure

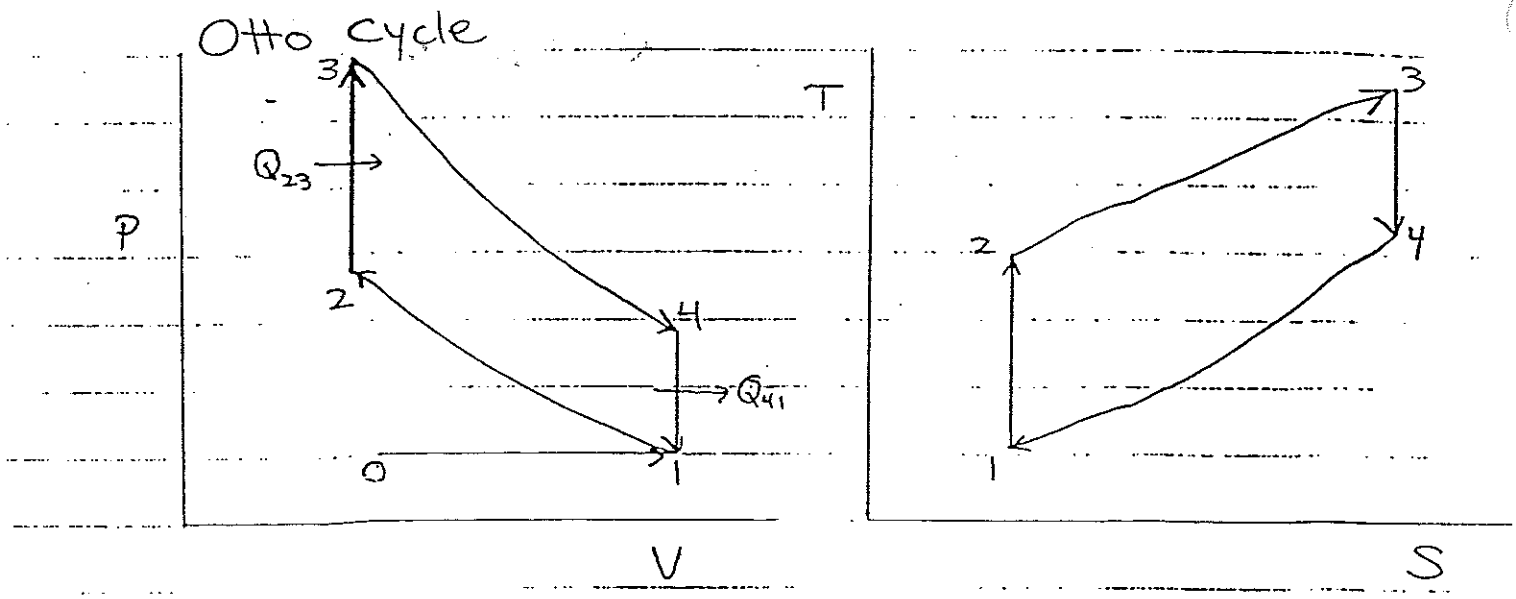
The Rankine cycle is basically the Carnot cycle with two modifications:

① The heating step $(1 \to 2)$ is carried well beyond vaporization producing a super-heated vapor

② The cooling step $(3 \to 4)$ results in complete condensation producing saturated liquid

For real applications, steps 2) and 4) are not reversible
This shifts lines $2 \to 3$ and $4 \to 1$ to $2 \to 3'$ and $4 \to 1'$
(i.e. the paths tend toward increasing entropy)

Otto cycle



The air standard Otto cycle is an ideal cycle for an internal combustion engine

5 Steps

1) $0 \to 1$ Volume expansion, drawing fuel/air mixture into a cylinder
2) $1 \to 2$ adiabatic (isentropic) compression
3) $2 \to 3$ constant volume addition of heat Q_{23} (raising T, P)
4) $3 \to 4$ adiabatic (isentropic) expansion (power stroke)
5) $4 \to 1$ constant volume rejection of heat Q_{41}

Thermal efficiency $\eta \equiv \dfrac{\text{net work}}{\text{heat input}} = \dfrac{W_s(\text{net})}{Q_{23}} = \dfrac{Q_{23}+Q_{41}}{Q_{23}}$

For constant volume steps $Q = C_v \Delta T$

$$\eta = 1 + \frac{Q_{41}}{Q_{23}} = 1 + \frac{C_v(T_1 - T_4)}{C_v(T_3 - T_2)} = \boxed{1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = \eta}$$

Define compression ratio (r) $\quad r = \dfrac{V_1}{V_2}$

Otto cycle

Use $T = \frac{PV}{R}$ $\qquad T_1 = \frac{P_1V_1}{R}$; $T_2 = \frac{P_2V_2}{R}$; $T_3 = \frac{P_3V_3}{R} = \frac{P_3V_2}{R}$; $T_4 = \frac{P_4V_4}{R} = \frac{P_4V_1}{R}$

Then $\quad \eta = 1 - \frac{V_1(P_4-P_1)}{V_2(P_3-P_2)} = 1 - r\frac{(P_4-P_1)}{(P_3-P_2)}$

For adiabatic compression and expansion $\qquad PV^\gamma = \text{const.}$

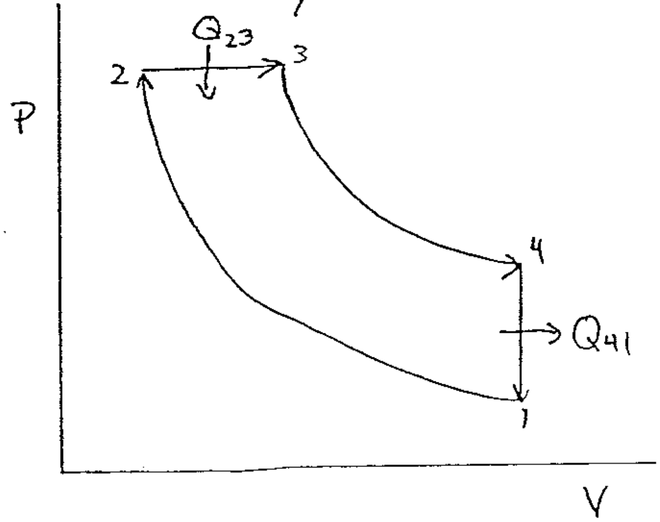
$$\underset{\uparrow}{P_3V_2^\gamma} = \underset{\uparrow}{P_4V_1^\gamma} \quad \text{and} \quad P_1V_1^\gamma = P_2V_2^\gamma \longrightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma = \left(\frac{1}{r}\right)^\gamma$$

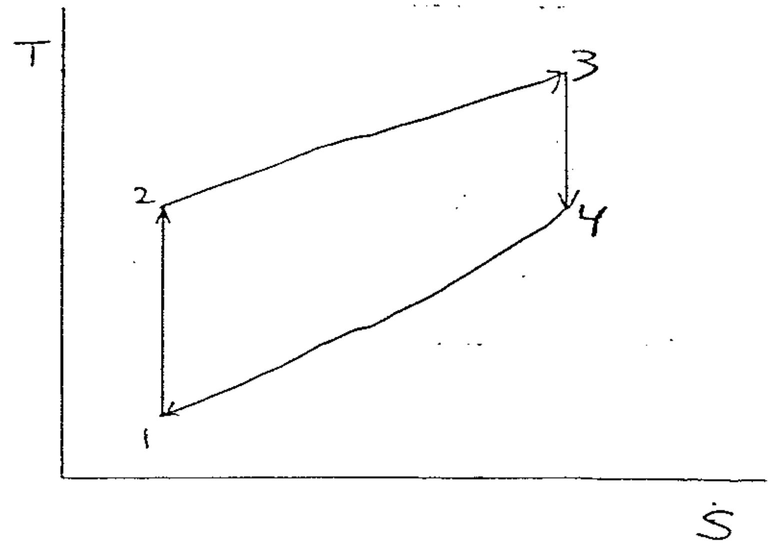
(Since $V_3 = V_2$ and $V_4 = V_1$)

Combining the two expressions eliminates V's $\Rightarrow \frac{P_2}{P_1} = \frac{P_3}{P_4} \Rightarrow \frac{P_4}{P_1} = \frac{P_3}{P_2}$

$$\eta = 1 - r\cdot\frac{P_1}{P_2}\cdot\cancelto{1}{\frac{\left(\frac{P_4}{P_1}-1\right)}{\left(\frac{P_3}{P_2}-1\right)}} = 1 - r\cdot\frac{P_1}{P_2} = 1 - r\left(\frac{1}{r}\right)^\gamma = \boxed{1 - \left(\frac{1}{r}\right)^{\gamma-1} = \eta}$$

Diesel Cycle





4 Steps

1) $1 \to 2$ adiabatic (isentropic) compression
2) $2 \to 3$ constant pressure addition of heat Q_{23} } power stro
3) $3 \to 4$ adiabatic (isentropic) expansion
4) $4 \to 1$ constant volume rejection of heat Q_{41}

Governing eqtns

1) $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ and $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

2) $Q_{23} = C_p(T_3 - T_2)$ and $W_{23} = \int_2^3 P\,dV = P(V_3 - V_2)$

3) $\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$ and $P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$

4) $Q_{41} = C_v(T_1 - T_4)$

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Diesel Cycle

Thermal efficiency $\eta \equiv \frac{W_s}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}} = 1 - \frac{C_V (T_4 - T_1)}{C_p (T_3 - T_2)}$

Define compression ratio $r = \frac{V_1}{V_2}$

expansion ratio $r_e = \frac{V_4}{V_3}$ cutoff ratio $r_c = \frac{V_3}{V_2}$

From governing eqtns

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3 \left(\frac{1}{r_e}\right)^{\gamma-1}$$

$$T_1 = T_2 \left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2 \left(\frac{1}{r}\right)^{\gamma-1}$$

$$\eta = 1 - \frac{1}{\gamma} \left[T_3 \left(\frac{1}{r_e}\right)^{\gamma-1} - T_2 \left(\frac{1}{r}\right)^{\gamma-1} \Big/ T_3 - T_2 \right] = 1 - \frac{1}{\gamma}\left[\left(\frac{1}{r_e}\right)^{\gamma-1} - \frac{T_2}{T_3}\left(\frac{1}{r}\right)^{\gamma-1} \Big/ \left(1 - \frac{T_2}{T_3}\right)\right]$$

$P_2 V_2 = R T_2$ and $P_3 V_3 = R T_3$; $P_2 = P_3$

$$\frac{P_2}{R} = \frac{T_2}{V_2} \qquad \frac{P_2}{R} = \frac{T_3}{V_3} \Rightarrow \frac{T_2}{V_2} = \frac{T_3}{V_3} \Rightarrow \frac{T_2}{T_3} = \frac{V_2}{V_3} = \frac{V_2/V_1}{V_3/V_4} = \frac{r_e}{r} = \frac{T_2}{T_3}$$

$V_1 = V_4$

$$\eta = 1 - \frac{1}{\gamma}\left[\left(\frac{1}{r_e}\right)^{\gamma-1} - \frac{r_e}{r}\left(\frac{1}{r}\right)^{\gamma-1} \Big/ \left(1 - \frac{r_e}{r}\right)\right]$$

$$\boxed{\eta = 1 - \frac{1}{\gamma}\left[\frac{\left(\frac{1}{r_e}\right)^{\gamma} - \left(\frac{1}{r}\right)^{\gamma}}{1/r_e - \frac{1}{r}}\right]}$$

η can also be derived in terms of r and r_c (the cutoff ratio)

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right]$$

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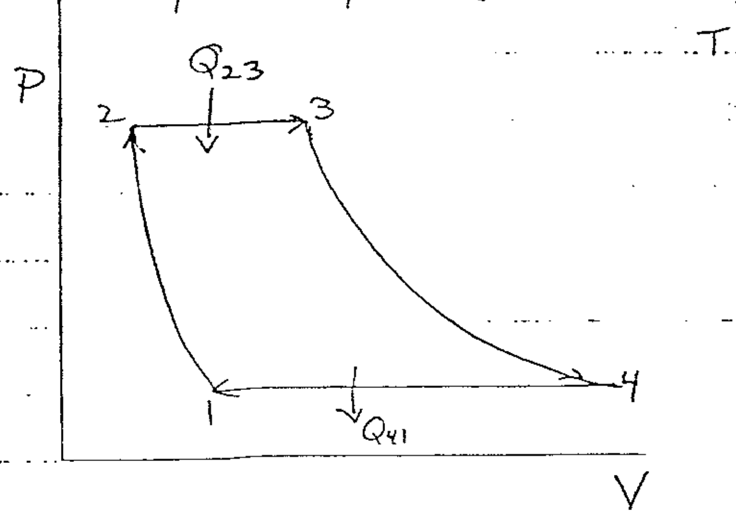
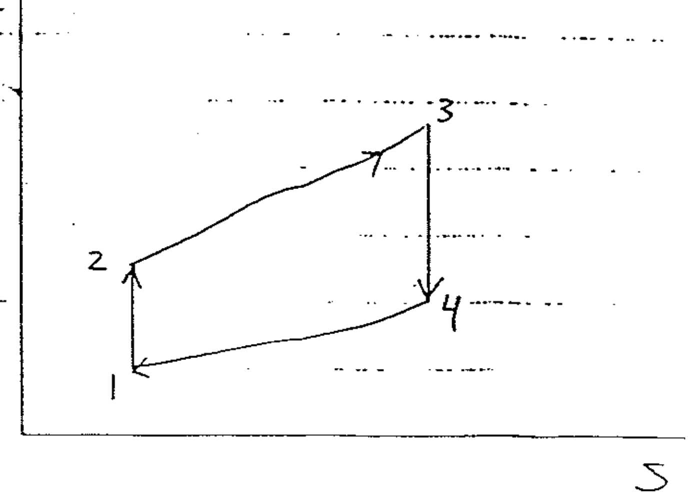
Brayton cycle (air standard)



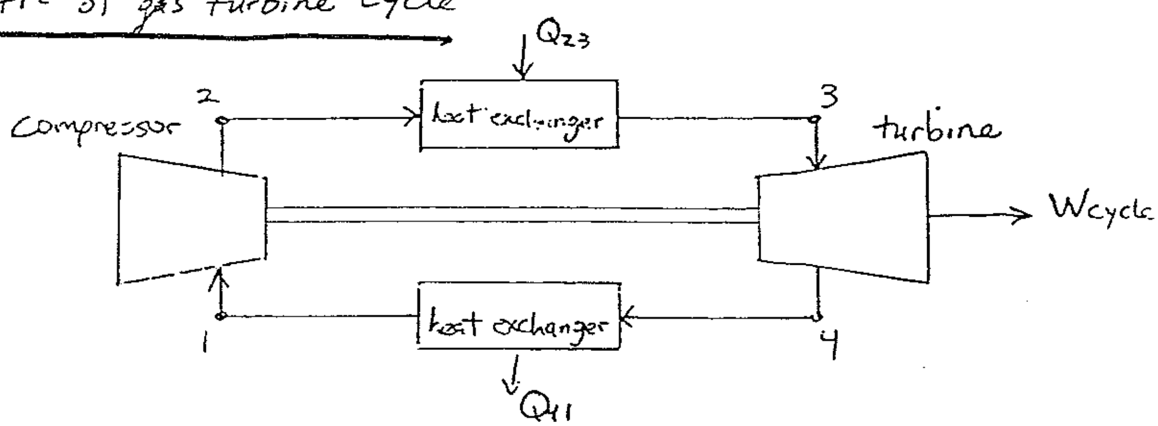


The air standard Brayton cycle is a model cycle for a gas turbine power plant

4 Steps

1) $1 \to 2$ adiabatic (isentropic) compression
2) $2 \to 3$ constant pressure addition of heat Q_{23}
3) $3 \to 4$ adiabatic (isentropic) expansion
4) $4 \to 1$ constant pressure rejection of heat Q_{41}

Schematic of gas turbine cycle



Brayton cycle

Thermal efficiency (on a cold air-standard basis)

C_p, C_v are taken as constant

Governing eqtns

1). $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$ and $-W_{12} = H_2 - H_1 = C_p(T_2 - T_1)$

2). $Q_{23} = C_p(T_3 - T_2)$

3) $\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\gamma-1/\gamma} = \left(\frac{P_1}{P_2}\right)^{\gamma-1/\gamma}$ since $P_4 = P_1$ and $P_3 = P_2$ $-W_{34} = C_p(T_4 - T_3)$

4) $Q_{41} = C_p(T_1 - T_4)$

$$\eta \equiv \frac{W(\text{net})}{Q_{23}} = \frac{W_{12} + W_{34}}{Q_{23}} = 1 + \frac{C_p(T_1 - T_4)}{C_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1}{T_2}\left(\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1}\right)$$

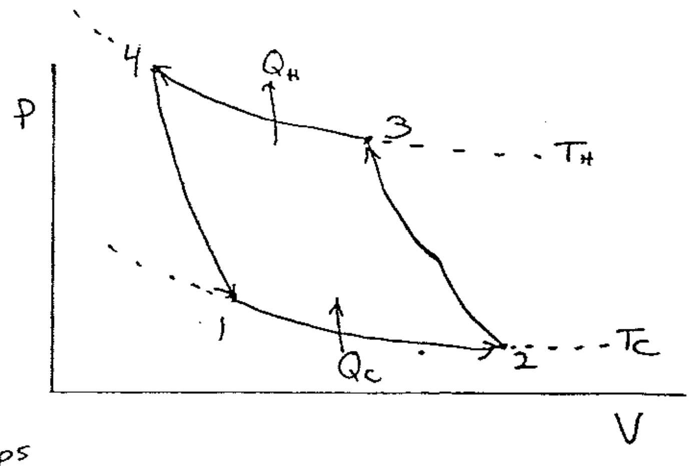
Combining governing eqtns 1) and 3) yields $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

$$\boxed{\therefore \eta = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(P_2/P_1)^{\frac{\gamma-1}{\gamma}}}}$$

Refrigeration

The Carnot Refrigerator

(the reverse of the Carnot power cycle)



2→3 and 4→1 adiabatic steps

Net work $|W|$ required to run system

$\Delta U = 0$ for the cycle $\quad \therefore \quad |W| = |Q_H| - |Q_C|$

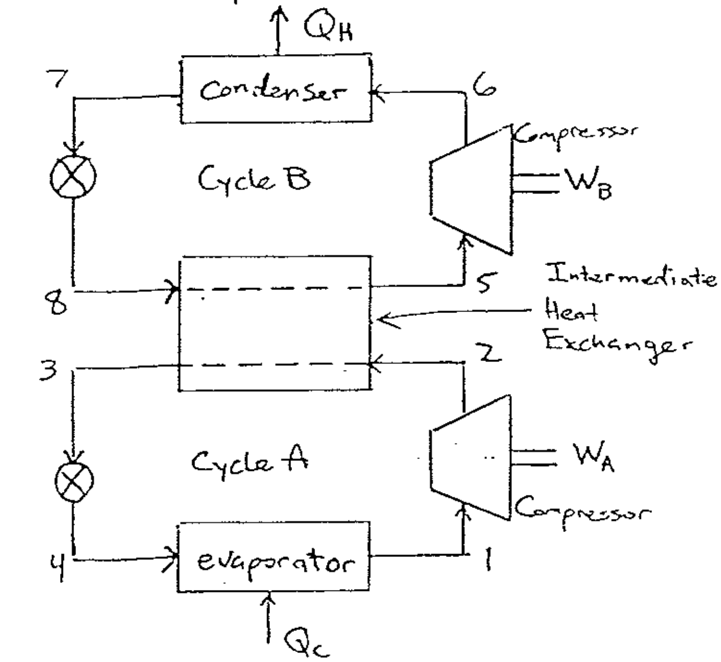
Coefficient of performance $\quad \omega \equiv \dfrac{\text{heat absorbed at lower temp}}{\text{net work}} = \boxed{\dfrac{|Q_C|}{|W|}}$

$$\frac{|W|}{|Q_C|} = \left|\frac{Q_H}{Q_C}\right| - 1 = \frac{T_H}{T_C} - 1 = \frac{T_H - T_C}{T_C} = \frac{1}{\omega}$$

↑ since $\dfrac{|Q_H|}{|Q_C|} = \dfrac{T_H}{T_C}$ for the Carnot cycle

$$\therefore \quad \boxed{\omega = \frac{T_C}{T_H - T_C}}$$

Cascade cycles



$$\omega \equiv \frac{|Q_c|}{|W_A|+|W_B|}$$

Advantage of cascade cycles: Stages can be chosen over temp. ranges that yield reasonable pressures in evaporators and condensers

Multistage cycle

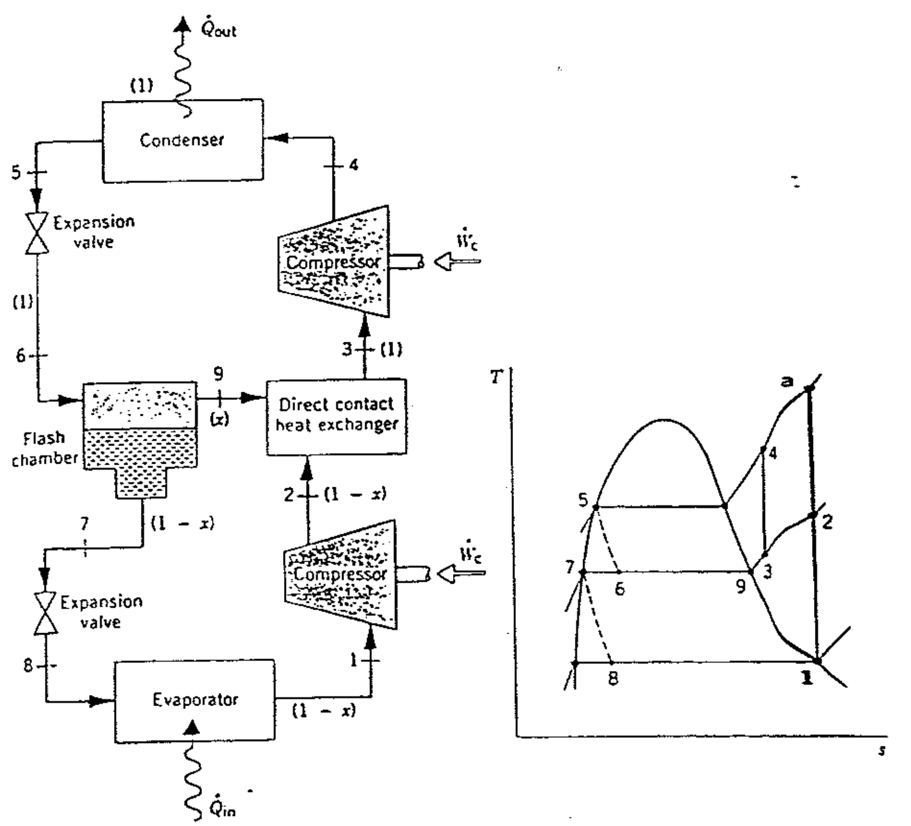
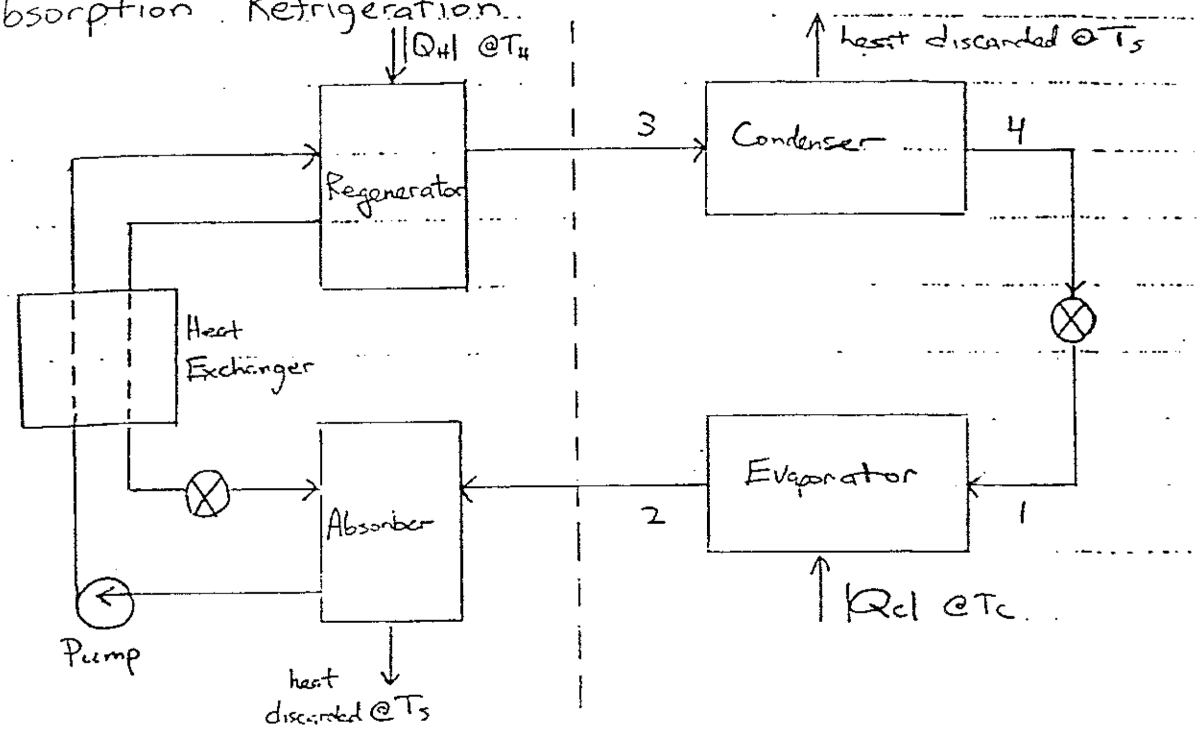


FIGURE 10.7 Refrigeration cycle with two stages of compression and flash intercooling.

Absorption Refrigeration



Absorption refrigeration uses heat directly as the energy source for refrigeration

The right half of the above schematic is identical to a vapor-compression refrigeration unit. The left shows the compression section which is essentially a heat engine

Operation of compression section

Refrigerant vapor is absorbed in a relatively non volatile solvent at evaporator pressure and low T

Heat evolved is discarded to the surroundings (@ T_s)

A pump raises the pressure of the liquid to that of the condenser

Heat from a source @ T_H raises the liquid's temp and causes refrigerant to evaporate (low pressure steam is a common heat source

Refrigerant flows to the condenser and the solvent flow to the absorber

Absorption Refrigeration

Common <u>refrigerant</u> - <u>absorbent</u>

water ... lithium bromide soln. ($T_C >$ freezing pt. of H_2O)

ammonia water (lower T applications)

Work required to drive refrigerator

$$|W| = \frac{T_S - T_C}{T_C}|Q_C| \qquad \text{(see Carnot refrigerator derivation of } \omega\text{)}$$

Heat $|Q_H|$ required to produce work $|W|$

$$\eta = \frac{|W|}{|Q_H|} = 1 - \frac{T_S}{T_H} \qquad \text{(efficiency of a Carnot heat engine)}$$

$$\therefore \quad |Q_H| = |W| \frac{T_H}{T_H - T_S}$$

Subst. expression for $|W|$

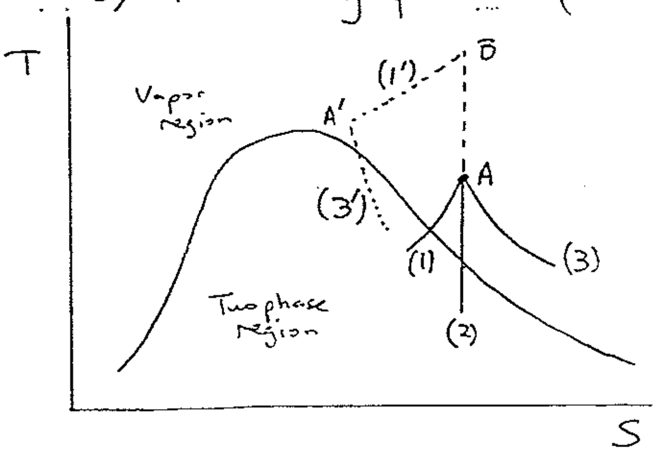
$$\boxed{\frac{|Q_H|}{|Q_C|} = \frac{T_H}{T_H - T_S} \cdot \frac{T_S - T_C}{T_C}} \; *$$

* Minimum value based on assumption of a Carnot refrigerator being driven by a Carnot heat engine. Carnot cycles are unrealizable in real applications

Liquefaction of gases

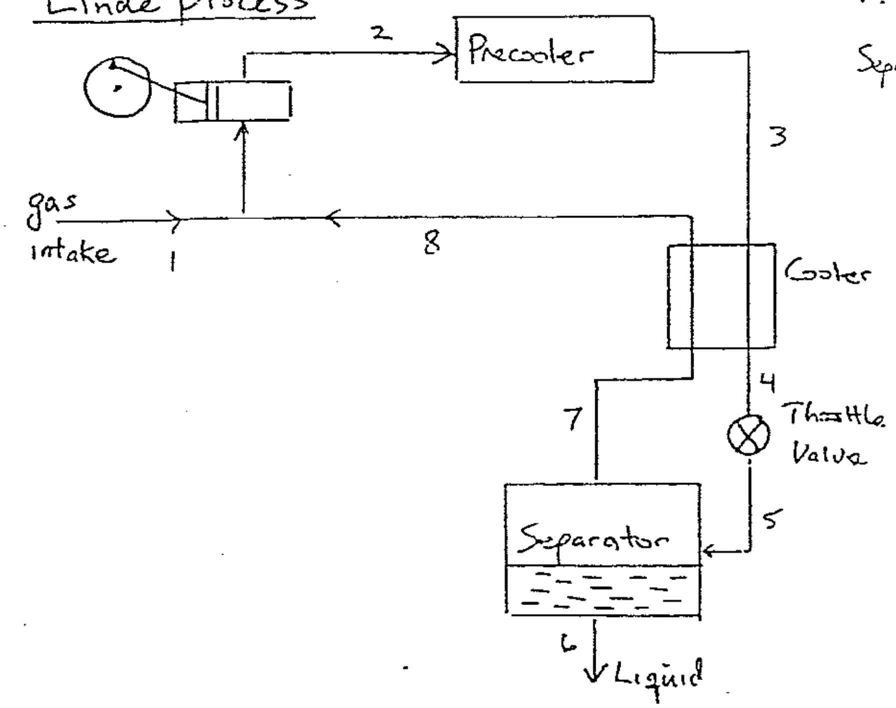
Common liquefaction methods:

1) Heat exchange at const. P
2) Expansion in a turbine from which work is obtained (adiabatic, isentropic)
3) Throttling process (Joule-Thompson liquefaction) (isenthalpic)



From pt. A, gas can be liquefied by either process (1) or (2). Process (3) will not liquefy the gas from point A. To use process (3), gas must be compressed to pt. B, cooled at const. P to pt. A', and then throttle until the two phase region is reached

Linde process



S.S. Energy Balance Around Cooler, Separator, and Throttle Valve

$$H_6 \cdot z + H_8(1-z) = H_3$$

$z \equiv$ fraction of gas liquefied

Energy Balance for Claude Process

$$H_6 z + H_8(1-z) + W_s = H_3$$

$$W_s = -(H_5 - H_4)$$

Claude process - replace throttle valve with an expansion engine operating adiabatically

Polytropic Process

- a process for which mechanical reversibility is the only imposed condition (i.e. no special conditions such as isothermal or adiabatic operation are specified)

Governing eqtns. (ideal gas, nonflow process)

$dU = dQ - dW$ $\qquad$ $\Delta U = Q - W$ $\qquad$ First law

$dW = PdV$ $\qquad$ $W = \int PdV$

$dU = C_v dT$ $\qquad$ $\Delta U = \int C_v dT$

$dH = C_p dT$ $\qquad$ $\Delta H = \int C_p dT$

$dQ = C_v dT + PdV$ $\qquad$ $Q = \int C_v dT + \int PdV$

Irreversible process

1) Find work W for a reversible process that accomplishes the same change of state

2a) If the process produces work, multiply by an efficiency

2b) If the process requires work, divide by an efficiency.