1. What is the difference between:
   a. heat and mass transfer
   b. heat and momentum transfer
   c. mass and momentum transfer

2. Set up equations to describe:
   a. wet bulb thermometer
   b. iceberg being towed in the ocean
   c. burning carbon particle

3. Will heat transfer affect the friction factor? In what way?

4. What is the difference between diffusivity and a mass transfer coefficient?

5. Why is the Prandtl number greater for liquids than for gases?

6. What is the Sherwood number?

7. Do Neon and Argon have the same atomic radius? If not, which is larger? Which has the larger (a) diffusivity; (b) viscosity; (c) heat capacity; and (d) Prandtl number?

8. Consider the problem of pumping oil down the Alaskan pipeline. Given the pipe diameter and length, and the properties of the oil, how would you calculate the (a) pump sizes; (b) heat loss; (c) temperature profile?

9. Describe and give the governing equations for (a) an orifice meter; (b) a venturi meter; (c) the pitot tube.

10. Give the following:
    a. Bernoulli's equation
    b. Hagen-Poiseuille law
    c. Stokes law
    d. Continuity equation
    e. Navier-Stokes equations

11. What is an NTU and how do you calculate it?

12. Describe the use of a McCabe-Thiele diagram.

13. How do the following vary with temperature and pressure?
    a. diffusivity
    b. dynamic viscosity
    c. thermal conductivity
    d. heat capacity
    e. heat transfer coefficient
    f. kinematic viscosity

14. What is the Reynolds analogy? the Chilton-Colburn analogy?

15. What is the friction factor? the coefficient of friction?

16. Derive the boundary-layer equations.

17. Sketch the governing diagrams for a stripper and an absorber p 617 Geankoplis

18. Describe the friction factor (drag coefficient) vs. Re relation for a (a) pipe, (b) sphere, (c) flat plate, etc.

19. Sketch the shear stress profile for a pipe.

20. Define the most commonly used dimensionless parameters and describe their significance.

21. Given a pool of organic liquid (such as from a spill), how would you estimate its rate of evaporation?

22. How are the diffusivity and viscosity of a mixture determined?

23. Sketch the temperature profile in a heat exchanger.

24. What phenomena are important during an underground explosion? (e.g. bulk flow, diffusion, etc.)

25. Consider a drop falling down a tower, initial temperature and tower temperature are given. How does the drop temperature change as it falls?

26. What is the angular dependence of the Nusselt number for a falling drop?

27. Draw the boiling curve and describe the physical phenomena responsible for the observed behavior. Draw and explain the similar curve for condensation.

28. Given the free stream velocity and particle diameter, calculate the boundary layer thickness at a 45 degree angle. What is the pressure at the forward and backward stagnation points? What causes the difference?

29. Derive the steady state momentum balance for fully developed laminar flow in a pipe.

30. How is the overall heat transfer coefficient for a heat exchanger found?

31. Given two temperatures and a knowledge of all the fluids' properties in a double pipe countercurrent heat exchanger, how do you calculate the other two temperatures?

32. Derive equations describing the wet-bulb/dry-bulb psychrometer. Obtain a relation between the wet-bulb temperature and air humidity in terms of dimensionless groups.

33. Is the heat flux from a liquid into a gas usually higher or lower if the gas is insoluble (versus soluble) in the liquid? This compares "diffusion through a stationary component" with the extreme case of "equimolar counterdiffusion".

34. The Chilton-Colburn j-factor for heat transfer is proportional to h, the convective heat transfer coefficient. Why is j proportional to $(Pr)^{-1/3}$? Why is j only a fraction of Re? Why does j decrease as Re increases?

35. O_2 and N_2 leaks from pressurized tanks are often considered less dangerous than

H_2 leaks. Why? (Answer is best described using Joule-Thompson coefficient).

36. How would you separate oxygen from salt water? Suppose you were processing fairly large volumes so that energy efficiency is a strong consideration. What thermodynamic variables affect solubility? Where is the mass transfer resistance? What type of unit operation would you use? How would you design it?

37. What area is used when defining friction factor for a wetted wall column?

? 38. What is the Lewis relation? Is it dependent on the gas phase velocity? Why or why not?

39. Why are analogies between mass and heat transfer much more straightforward to use than analogies between mass and momentum transfer?

40. Given a CSTR at temperature T with no reaction what would happen if the inlet temperature were suddenly increased?

? 41. Analogies between heat, mass and momentum transport are important. Give examples of when they don't hold.

42. What is the theoretical basis for all the "famous" analogies between heat, mass and momentum transport? What are the mass and heat transfer equivalents of the momentum transport equation?

43. What is the difference between skin friction drag and form drag?

44. How would you determine a mass transfer coefficient experimentally?

45. Why does frost not form under a tree when it is on the ground all around the tree? [illegible]

46. Draw a McCabe-Thiele diagram for a distillation column that uses a reacting absorbent.

47. What are the most commonly used (3) correlations describing heat and mass transfer?

48. Give: $\tau_{zx} = -\eta \frac{\partial v_x}{\partial z} = \frac{\eta}{\rho} \frac{\partial (\rho v_x)}{\partial z}$

a. Newton's law

b. Fick's law $\rightarrow J_A = -D \frac{dc_A}{dz}$

c. Fourier's law $\rightarrow Q = -k \frac{dT}{dz} = \frac{k}{\rho C_p} \frac{d(\rho C_p T)}{dz}$

49. Give the equations describing flow in a packed bed.

? 50. Derive the equations for gas undergoing an isentropic expansion.

51. What is inside a light bulb, and why?

52. Why do you have to whirl a wet-bulb / dry-bulb psychrometer in the air prior to reading it?

53. In which direction is the momentum flux from a fluid flowing over a flat plate?

54. How does a lawn sprinkler work?

Why? ... holding a high pressure hose, must they pull or push the hose?

56. For a double plate window with insulating gas between the panes draw the temperature profile from inside the warm room, through the windows and to the outdoors. Allow for natural convection both in the room and in the gas between the two plates. What gas would you recommend using and why?

57. Consider the department store ping-pong ball "floating" above a vacuum cleaner discharge. What determines how high the ball will be? What keeps the ball from moving laterally out of the path of the air? What does the velocity profile look like close to, around and above the ball? What determines whether the ball will fall to the ground if the jet is pointing at an angle rather than straight up?

path of the air? What does the velocity profile look like close to, around and above the ball? What determines whether the ball will fall to the ground if the jet is pointing at an angle rather than straight up?

58. You have two infinite parallel plates initially at rest with a fluid between them. One plate remains fixed, the other is set in motion at velocity V. What do the transient velocity profiles look like? What does the steady state look like? Why? What is the driving force for fluid flow in a pipe? What is the driving force here? Describe a momentum balance. What equiation would you use to describe this. Simplify the equation to obtain a differential equation. How would one determine the force necessary to keep the top plate moving at V? (Graves)

J 59. For the system in number 58, determine a characteristic time. Which would take longer for the steady state profile to be reached, molasses or water and why?

60. You have a small sphere of moltem metal. How far will it drop (in air) before it solidifies? What does the Biot number tell you here? How do you find the convective heat transfer coefficient? (Blanch)

61. For a particle dropping in a fluid field derive the equation for the terminal velocity and discuss the friction factor coefficient.

62. Why do they put dimples on a golf ball?

J 63. Consider laminar flow in a pipe. Write out the momentum equation appropriate for this geometry. Drop all terms which are identically zero. You should end up with one equation and only two terms in it, if you neglect gravity (or include it in the pressure term). Reduce the equation to a non-dimensional form. Use L for a characteristic length and V for a characteristic velocity. One should notice that Re does not appear in the non-dimensional form. Why then is the Re number so important in determining if a flow in a pipe is laminar or turbulent?

TRANSPORT PRELIM QUESTIONS

1. Find: the difference between →

ⓐ heat and mass transfer?

viscous dissipation

① Heat transfer is the transfer of energy due to a temperature difference (or gradient).

② Mass transfer is the transfer of chemical species due to a concentration difference (or gradient) or due to a difference in chemical potential.

ⓑ heat and momentum transfer?

① +

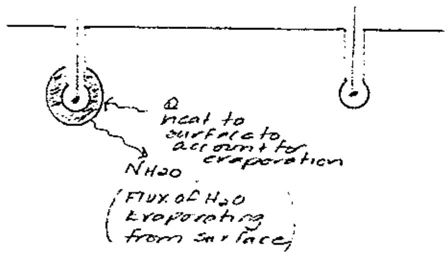
③ Momentum transfer is the transfer of momentum due to a velocity difference (or gradient).

ⓒ mass and momentum transfer?

② + ③

2. Find: set up equations to describe →

ⓐ wet bulb thermometer (psychrometer)



(Measures the relative humidity in air)

Wet wick → measures the colder T caused by evaporation of water.

Dry bulb → measures ambient temperature.

relative humidity → the amount of water actually in the air divided by the amount at saturation at the dry-bulb T.

EQUATIONS FOR
MASS + ENERGY FLUX →

surface

sat at i

$N_1 = k(C_{1i} - C_1) = kc(y_{1i} - y_1)$

bulk
water vapor
concentration

saturation
T @ wet bulb

$q = h(T_i - T)$

dry
bulb

ALSO → $N_1 \Delta\tilde{H}_{vap} = -q$

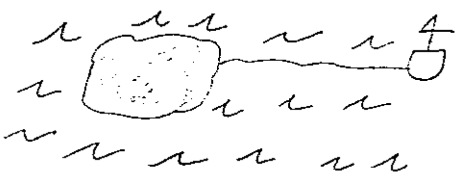
HENCE → $k c \Delta\tilde{H}_{vap}(y_{1i} - y_1) = h(T - T_i)$

USE CHILTON-COLBURN
TO RELATE →
h and k

$$(Pr)^{2/3} \cdot \frac{h}{\rho \hat{C}_p v} = \frac{f}{2} = \frac{k}{v}(Sc)^{2/3}$$

solve for relative humidity → $\frac{p_1}{p_1(\text{sat at } T)} = \frac{y_1}{y_1(\text{sat at } T)}$

ⓑ iceberg being towed in the ocean



$2r = d$

d_H = hydraulic diameter

$= \frac{4 * CSA}{WP}$ → Cross Sectional Area → wetted perimeter

$\begin{pmatrix}\text{Amount} \\ \text{melted}\end{pmatrix}\begin{pmatrix}\text{latent} \\ \text{heat}\end{pmatrix} = \begin{pmatrix}\text{Rate of heat} \\ \text{transferred to} \\ \text{iceberg}\end{pmatrix}$

$m \Delta H^{sl} = h A \Delta T$

$Q = m \Delta H$

$\rho \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) \Delta H^{sl} = h \, \pi d_H^2 (T_w - T)$

h and k are for the liquid

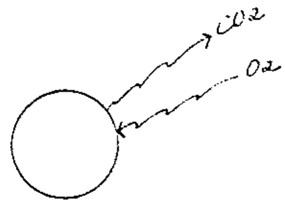
Also →

Force to tow = drag on particle → STOKES LAW

$F = 6 \pi \mu r v$

use $Nu = \frac{h d_H}{k} = 2.0 + 0.6 Re^{1/2} Pr^{1/3}$

6) A burning carbon particle → $C + O_2 \rightarrow CO_2$



Here, assume that the flux of O_2 to the surface is equal to $(-)$ the flux of CO_2 away from the surface

$$N_{O_2} = -N_{CO_2}$$

Fick's Law → $$N_{O_2} = -D_{AB}\frac{d[O_2]}{dr} + x_A(N_{O_2} + \underbrace{N_{CO_2}}_{})^{0}$$

Rate of O_2 consumption → $$Q = -4\pi r^2 N_{O_2} = 4\pi r^2 D_{AB}\frac{d[O_2]}{dr}$$

Integrating →

$$\frac{Q}{4\pi}\int_R^\infty \frac{dr}{r^2} = D_{AB}\int_{C_S}^{C_b} d[O_2] = D_{AB}(C_b - C_s)$$

$$-\frac{Q}{4\pi}\left(\frac{1}{\infty} - \frac{1}{R}\right) = \frac{Q}{4\pi R} = D_{AB}(C_b - C_s)$$

$$Q = 4\pi R D_{AB}(C_b - C_s)$$

BUT Q also is →

$$Q = 4\pi R^2 r_{O_2} = 4\pi r^2 R C_{O_2}^{S}$$ ← on a per area

$$C_{O_2}^{S} = \frac{Q}{4\pi r^2 R}$$

$$Q = 4\pi R D_{AB} C_b - \frac{Q\, 4\pi R C_s}{4\pi r R}$$

... affect the friction factor? In what way?

$$F_K = AKf = (2\pi RL)\left(\tfrac{1}{2}\rho\langle v\rangle^2\right)f$$

$$F_K = [\Delta P + \rho g \cancel{\Delta h}^{0}]\pi R^2$$

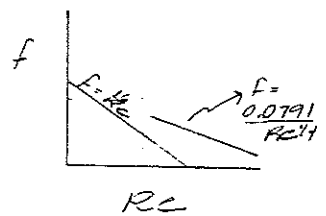
$$\Delta P \pi R^2 = (2\pi RL)\left(\tfrac{1}{2}\rho\langle v\rangle^2\right)f$$

$$\boxed{f = \frac{F_K}{AK}}$$

KNOW THIS!

$$f = \frac{\Delta P \cancel{\pi} R^{\cancel{2}}}{\cancel{2}\cancel{\pi} RL\left(\frac{1}{\cancel{2}}\rho\langle v\rangle^2\right)} = \frac{\Delta P R}{L(\rho\langle v\rangle^2)}$$

Fanning Friction factor for flow in a pipe



yes, heat transfer will lead to changes in the temperature which leads to changes in fluid properties like ρ, μ because $f = f(Re, L/D)$. Can also cause changes in the fluid configuration.

FOR LAMINAR FLOW tube $\rightarrow \boxed{f = \frac{16}{Re}}$

FOR TURBULENT FLOW tube $\rightarrow \boxed{f = \frac{0.0791}{Re^{1/4}}}$

4. Find: the difference between diffusivity and a mass transfer coefficient.

D_{AB} = diffusivity = the flux of A in B, that is how well A moves through B per time

$\tilde{K}_c$ = mass transfer coefficient = $\frac{D_{AB}}{\delta}$ = Ratio of the diffusivity to the film thickness at a surface.

$$W_A = -D_{AB} \nabla C_A = -D_{AB} \frac{dC_A}{dx}$$
$$W_A = k_c (C_{A0} - C_{A\delta})$$

5. Find: why is the Prandtl number greater for liquids than for gases?

$Pr\# = \frac{\hat{C}_p \mu}{k}$

the viscosity of liquids is greater than that of gases

PROBABLY GIVES HIGHER PRANDTL # → $C_p^L > C_p^G$

$\mu^L > \mu^G$

NOT SURE ABOUT THIS { the thermal conductivity of liquids is larger than gases }?

the specific heat capacity of the liquid is higher

$N_A = -D \frac{dC_A}{dx} + x_A (N_A + N_B)$

6. Find: what is the Sherwood number?

$$\boxed{Sh = \frac{k_c d_e}{D_{AB}}}$$

The Sherwood number is used in estimating the mass transfer coefficient.

CAN CALCULATE k_c if a correlation for the Sherwood # is known

Like the Nusselt number which is used in figuring out h, the heat transfer coefficient. This is the mass transfer coefficient equivalent.

7. Find: do Ne and Ar have the same atomic radius? If not, which is larger? Which has a larger →

ⓐ diffusivity?
ⓑ viscosity?
ⓒ heat capacity?
ⓓ Prandtl number? $Pr = \frac{\hat{C}_p \mu}{K}$ (About the same)

$K \rightarrow Ne > Ar$

$\frac{\hat{C}_p \mu}{K} = \frac{\hat{C}_p \mu}{K}$

$\frac{Ar}{Ne} = \frac{K_{Ar} \hat{C}_{pNe}}{K_{Ne} \hat{C}_{pAr}}$

$\frac{Ar}{Ne} = 0.625$

$\mu_{Ne} > \mu_{Ar}$

No, Argon, being higher on the periodic table has a larger atomic radius.

ⓐ $D_{AB}^{Ne} > D_{AB}^{Ar}$ Argon is larger, more difficult to move.

$\mu_{Ar} > \mu_{Ne}$ $\mu \propto \frac{\sqrt{A}}{d^2}$

ⓑ $\cancel{\mu_{Ne} > \mu_{Ar}}$ Does this make sense?

ⓒ $C_p^{Ar} > C_p^{Ne}$ It's bigger, requires more energy

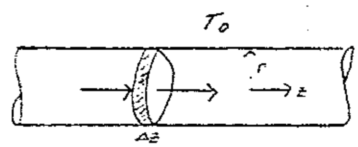
ⓓ $Pr^{Ne} \approx Pr^{Ar}$ Don't know why! $Pr\# = \frac{\hat{C}_p \mu}{K}$

Designing of a pump for this system requires determining the following information →

① the desired flow rate
② the pressure drop due to friction in the tube
③ the total head (hydrostatic ($\rho g h$) and other)

probably need pumps at several points in the pipeline.

ⓑ heat loss in the pipe →



Energy Balance →

$$\text{Rate of Accum. of Energy} = \text{Rate of Energy in} - \text{Rate of energy out} + \overset{\to 0}{\text{Rate of energy generation}}$$

Assume $L \gg R$, then temperature variations in the r direction are small vs. those in the z-direction.

CONDUCTION

$\left(\frac{\text{mass}}{\text{vol}}\right)\left(\frac{\text{energy}}{\text{mass}\,T}\right)$ $\left(\frac{T}{\text{time}}\right)(\text{vol})$

$$\rho C_p \frac{dT}{dt} \pi r^2 \Delta z = -K \frac{dT}{dz} \pi r^2 \Big|_z + K \frac{dT}{dz} \pi r^2 \Big|_{z+\Delta z}$$

$$+ \; 2\pi r \Delta z \, h (T - T_0) + \rho C_p v_z$$

Must know →

① Flow rate, Q

② Then calculate $f(Re)$ $\qquad Re(Q) = \frac{D v \rho}{\mu}$

e.g. $f = \frac{0.0791}{Re^{1/4}}$ for turbulent flow $\qquad v = \frac{Q}{\pi D^2/4}$

$f = \frac{16}{Re}$ for laminar flow $\qquad$ find Re, f, Δp!

Know $F_K = AKf = 2\pi r L(\frac{1}{2}\rho \langle v \rangle^2) f = (\frac{\Delta P}{\rho} + \rho g h)\pi r^2$

solve for $\Delta p = f(f)$

Look up ΔP vs. Q and choose most appropriate pump!

③ Heat loss in the tube →

$Q = hA(T - T_0)$

8. Find: Consider the problem of pumping oil down the Alaskan pipeline. Given the pipe diameter and length, and the properties of the oil, how would you calculate →

ⓐ pump sizes

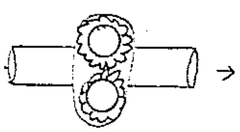
pump - transfers fluid from one location to another by increasing the pressure of the fluid and, thereby, supplying the driving force necessary for flow.

TYPES → ① Reciprocating or positive-displacement pumps with valve action: piston pumps, diaphragm pumps, plunger pumps.

R[illegible]ciprocating
P[illegible]on pump →
P[illegible]ton through a cylinder,
s[illegible]m is often the energy
so[illegible]rce, rotating crankshaft
or electric motor also.
simplex, duplex, triplex (# of cylinders)
single or double acting

② Rotary positive-displacement pumps with no valve action: gear pumps, lobe pumps, screw pumps, eccentric-cam pumps, metering pumps.

Rotary positive displacement Pumps →



· gears rotate in opposite directions and push fluid out. No priming required.

③ Rotary centrifugal pumps with no valve action: open impeller, closed impeller, volute pumps, turbine pumps.

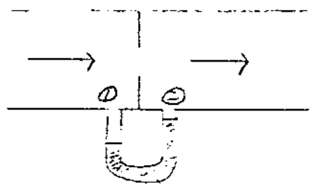
Rotary centrifugal pumps → the fluid is fed into the pump at the center of a rotating impeller and is thrown outward by centrifugal force. The fluid at the outside attains a high velocity which in turn yields a high kinetic energy. The kinetic energy converts to a pressure energy.

④ Air-displacement systems: air lifts, acid eggs or blow cases, jet pumps, barometric pumps.

A[illegible] displacement systems → [illegible]se use air to cause a pressure difference.

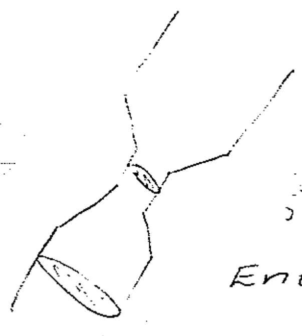
9. Find: describe and give the governing equations for →

(a) an orifice meter



A type of flowmeter, the orifice meter works in the following way → an obstruction to flow is placed in a tube as shown. The fluid pressure drops from the upstream side to the downstream side. The pressure drops with the flow rate, the greater the flow rate the greater the pressure drop.

Mass Balance → Steady state



$$0 = \rho_1 v_1 \cancel{A_c} - \rho_2 v_2 \cancel{A_c} + 0$$

$$\rho_1 v_1 = \rho_2 v_2 \qquad \text{incompressible, } \rho_1 = \rho_2$$

$$v_1 = v_2 = v$$

Energy Balance → $-\cancel{W_s}^{\,0} = \frac{1}{2}\Delta u^2 + \cancel{g\Delta z}^{\,0} + \int_{P_1}^{P_2} V\,dP + F$

Energy loss due to friction on a mass basis

$$\frac{1}{2}\Delta u^2 + \underset{\text{specific volume}}{V}\Delta P + F = 0$$

$$\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + \frac{P_2 - P_1}{\rho} + F = 0$$

Putting together → $\cancel{\frac{1}{2} v^2} - \cancel{\frac{1}{2} v^2} + \frac{P_2 - P_1}{\rho} + F = 0$

The fanning equation relates the friction term to the friction coefficient. →

$$dF = \frac{2fu^2\,dL}{D} \qquad F = \frac{2fu^2 L}{D}$$

$$\frac{2fu^2L}{D} = -\frac{(P_2 - P_1)}{\rho} \rightarrow u = \left\{\frac{(P_1 - P_2)/\rho}{2f}\,\frac{D}{L}\right\}^{\frac{1}{2}}$$

$$u = \underbrace{\left(\frac{D}{4Lf}\right)^{1/2}}_{C}\left\{2(P_1 - P_2)/\rho\right\}^{1/2}$$

10. Give the following →

(a) Bernoulli's equation

Starting with an energy balance → Rate of accum. of energy = Rate of energy in − Rate of energy out + Rate of energy accum.

IE KE PE

'L on a per mass basis'

$$\frac{d(mu)}{dt} + \Delta[(U + \tfrac{1}{2}u^2 + zg)\dot{m}] = \dot{Q} - \dot{W}$$

$$\dot{W} = \dot{W}_s + \Delta[(PV)\dot{m}]$$

$$\frac{d(mu)}{dt} + \Delta[(U + \tfrac{1}{2}u^2 + zg)\dot{m}] = \dot{Q} - \dot{W}_s - \Delta(PV)\dot{m}$$

$U + PV = H$

$$\frac{d(mu)}{dt} + \Delta[(U + PV) + \tfrac{1}{2}u^2 + zg)\dot{m}] = \dot{Q} - \dot{W}_s$$

$$\frac{d(mu)}{dt} + \Delta(H + \tfrac{1}{2}u^2 + zg)\dot{m} = \dot{Q} - \dot{W}_s$$

FOR S.S. FLOW $\frac{d(mu)}{dt} = 0$

$$\Delta(H + \tfrac{1}{2}u^2 + zg)\dot{m} = \dot{Q} - \dot{W}_s$$

$$\Delta H + \tfrac{1}{2}\Delta u^2 + \Delta z g = Q - W_s$$

TO GET THE MECHANICAL ENERGY BALANCE →

use $dH = TdS + VdP$

for a reversible change of state → $dS = \frac{dQ}{T}$

$TdS = dQ$

Integrate → $dH = dQ + VdP$

$$\Delta H = Q + \int_{P_1}^{P_2} V\,dP \rightarrow Q = \Delta H - \int_{P_1}^{P_2} V\,dP$$

Plugging in → $\Delta H + \frac{1}{2}\Delta u^2 + \Delta z g = \Delta H - \int_{P_1}^{P_2} V dP - W_s$

Solving for W_s → $-W_s = \frac{1}{2}\Delta u^2 + \Delta z g + \int_{P_1}^{P_2} V dP$
For reversible

To correct for irreversibility → $-W_s = \frac{1}{2}\Delta u^2 + g\Delta z + \int_{P_1}^{P_2} V dP + F$

MECHANICAL ENERGY BALANCE

Bernoulli's Equation Assumes → ① nonviscous $(F=0)$
② incompressible fluid ($V=$ Ct)
③ $W_s = 0$

Hence → ② gives

$$\int_{P_1}^{P_2} V dP = V\Delta P = \frac{\Delta P}{\rho}$$

Therefore →

$$\boxed{0 = \frac{1}{2}\Delta u^2 + g\Delta z + \frac{\Delta P}{\rho}}$$

-or-

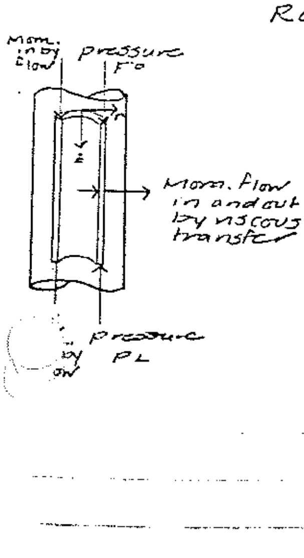
$$\boxed{\text{constant} = \frac{1}{2}u^2 + gz + \frac{P}{\rho}}$$

BERNOULLI'S EQUATION

ⓑ Hagen-Poiseuille Law

Constructing a momentum balance on flow in a cylindrical tube at steady state →

Rate of momentum into system − Rate of momentum out of system + sum of forces acting on system = 0



Mom. in by viscous transfer $= (2\pi r L \tau_{rz})|_r$

Mom. out by viscous transfer $= (2\pi r L \tau_{rz})|_{r+\Delta r}$

Mom. in by flow $= (2\pi r v_z)(\rho v_z)|_{z=0}$

Mom. out by flow $= (2\pi r v_z)(\rho v_z)|_{z=L}$

these 2 are equal becau: incompression fluid is assumed
$\nabla \cdot v = 0$

Pressure force at $z = 0$ $= (2\pi r \Delta r) p_0$

Pressure force at $z = L$ $= -(2\pi r \Delta r) p_L$

gravity force acting on cylindrical shell $= (2\pi r \Delta r L) \rho g$

Hence $\rightarrow$ $(2\pi r L \tau_{rz})|_r - (2\pi r L \tau_{rz})|_{r+\Delta r}$
$+ \underbrace{(2\pi r v_z)(\rho v_z)|_{z=0}}_{\text{equal}} - \underbrace{(2\pi r v_z)(\rho v_z)|_{z=L}}_{\text{equal}}$
$+ (2\pi r \Delta r) p_0 - (2\pi r \Delta r) p_L$
$+ (2\pi r \Delta r L) \rho g = 0$

Divide by Δr (also by $2\pi L$), take limit as $r \rightarrow 0$

$$-\frac{[(r\tau_{rz})_{r+\Delta r} - (r\tau_{rz})_r]}{\Delta r} + \frac{r(p_0 - p_L)}{L} + r\rho g = 0$$

$$-\frac{d(r\tau_{rz})}{dr} + r\left[\frac{(p_0 - p_L)}{L} + \rho g\right] = 0$$

$$\frac{d(r\tau_{rz})}{dr} = \left[\frac{(p_0 - p_L)}{L} + \rho g\right] r$$

INTEGRATING $\rightarrow$ $r\tau_{rz} = \frac{1}{2}\left[\frac{(p_0 - p_L)}{L} + \rho g\right] r^2 + C_1$

$$-\mu\frac{dv_z}{dr} = \tau_{rz} = \frac{1}{2}\left[\frac{(p_0 - p_L)}{L} + \rho g\right] r + \frac{C_1}{r}$$

CONSTANT $\mu \rightarrow$

$$v_z = -\frac{1}{4\mu}\left[\frac{(p_0 - p_L)}{L} + \rho g\right] r^2 + C_1 \ln r + C_2$$

at $r = 0$ v_z is finite $\quad C_1 \rightarrow 0$
at $r = R$ $v_z = 0$ $\qquad C_2 = \frac{1}{4\mu}\left[\frac{(p_0 - p_L)}{L} + \rho g\right] R^2$

$$v_z = \frac{1}{4\mu}\left[\frac{(p_0 - p_L)}{L} + \rho g\right] R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

The Hagen-Poiseulle Law is derived from the flow rate (volumetric) in a cylinder.

$$Q = \int_0^{2\pi}\int_0^{R} v_z \, r\,dr\,d\theta$$

$$Q = 2\pi \int_0^{R} \frac{1}{4\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g\right] R^2 \left[r - \frac{r^3}{R^2}\right] dr$$

$$Q = \frac{2\pi}{4\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g\right] R^2 \left[\frac{1}{2} r^2 - \frac{1}{4}\frac{r^4}{R^2}\right]_0^R$$

$$Q = \frac{2\pi}{24\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g\right] R^2 \left[\frac{1}{2} R^2 - \frac{1}{4} R^2\right]$$

$$\boxed{Q = \frac{\pi}{8\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g\right] R^4}$$

Relation between volume rate of flow and the forces causing the flow.

HAGEN-POISEULLE LAW

(c) Stokes Law

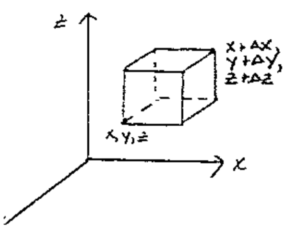
the drag force on a sphere!

$$\boxed{F_d = 6\pi\mu r v_\infty}$$

v_∞ = velocity of fluid around particle

r = radius of particle

(d) continuity equation - a mass balance



NO Rxn.

0

Rate of Accum. of mass = Rate of Mass in - Rate of Mass out + Rate of Mass accum.

$$\frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z = \rho v_x|_x \Delta z \Delta y - \rho v_x|_{x+\Delta x}\Delta z \Delta y$$

$$+ \rho v_y|_y \Delta x \Delta z - \rho v_y|_{y+\Delta y}\Delta x \Delta z$$

$$+ \rho v_z|_z \Delta x \Delta y - \rho v_z|_{z+\Delta z}\Delta x \Delta y$$

Divide by $\Delta x \Delta y \Delta z$, take limit as $\Delta x, \Delta y$ and Δz approach zero

$$\frac{\partial \rho}{\partial t} = -\frac{\partial (v_x \rho)}{\partial x} - \frac{\partial (v_y \rho)}{\partial y} - \frac{\partial (v_z \rho)}{\partial z}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(v_x \rho) + \frac{\partial}{\partial y}(v_y \rho) + \frac{\partial}{\partial z}(v_z \rho) = 0}$$

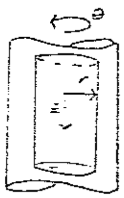
EQUATION OF CONTINUITY IN RECTANGULAR COORD.

- or -

the divergence

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (v \rho) = 0$$

What about cylindrical coordinates?

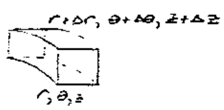


$x = r\cos\theta$

$y = r\sin\theta$

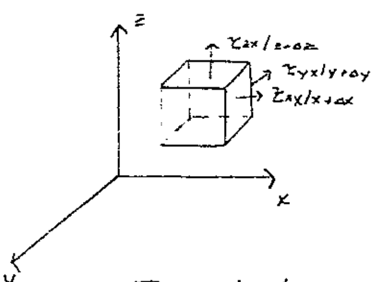
$y/x = \tan\theta$

BLUH!



(e) Navier-Stokes equations

Rate of Mom. accum. = Rate of Mom. in − Rate of Mom. out + Sum of forces acting on system



Mom. in and out due to flow

$$\frac{\partial(\rho v_x)}{\partial t}\Delta x \Delta y \Delta z = (\rho v_x) v_x\big|_x \Delta y \Delta z$$

$$- (\rho v_x) v_x\big|_{x+\Delta x} \Delta y \Delta z + (\rho v_y) v_x\big|_y \Delta x \Delta z$$

$$- (\rho v_y) v_x\big|_{y+\Delta y} \Delta x \Delta z + (\rho v_z) v_x\big|_z \Delta x \Delta y$$

Divide by $\Delta x \Delta y \Delta z$, take limit as Δx, Δy and $\Delta z \to 0$.

$$-(\rho v_z) v_x\big|_{z+\Delta z} \Delta x \Delta y \quad + \tau_{xx}\big|_x \Delta y \Delta z$$

Flux of x Mom.

$$- \tau_{xx}\big|_{x+\Delta x} \Delta y \Delta z + \tau_{yx}\big|_y \Delta x \Delta z - \tau_{yx}\big|_{y+\Delta y} \Delta x \Delta z$$

$$+ \tau_{zx}\big|_z \Delta x \Delta y - \tau_{zx}\big|_{z+\Delta z} \Delta x \Delta y$$

Sum of forces

$$+ P\big|_x \Delta y \Delta z - P\big|_{x+\Delta x} \Delta y \Delta z + \rho g_x \Delta x \Delta y \Delta z$$

$$\frac{\partial(\rho v_x)}{\partial t} = -\frac{\partial(\rho v_x v_x)}{\partial x} - \frac{\partial(\rho v_y v_x)}{\partial y} - \frac{\partial(\rho v_z v_x)}{\partial x}$$

$$- \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \frac{\partial \tau_{zx}}{\partial z} - \frac{\partial p}{\partial x}$$

$$+ \rho g_x$$

FOR $\rho = CT$

$$\frac{\partial(\rho v_x)}{\partial t} = -\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right)$$

$$- \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) - \frac{\partial p}{\partial x}$$

$$+ \rho g_x$$

FOR NEWTONIAN FLUID, $\mu = CT$

USING SIMPLIFICATIONS!

$$\frac{\partial(\rho v_x)}{\partial t} = -\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right)$$

$$+ \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) - \frac{\partial p}{\partial x}$$

$$+ \rho g_x$$

NAVIER-STOKE COMPONENT IN X.

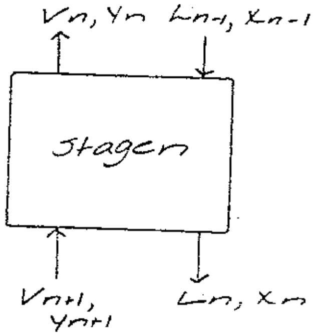
OR $\rightarrow$ $$\boxed{\rho \frac{D\bar{v}}{Dt} = -\nabla \bar{p} + \mu \nabla^2 \bar{v} + \rho \bar{g}}$$

NAVIER-STOKES EQUATION

McCabe-Thiele Diagram

McCabe-Thiele Analysis → Mass Balances

Single Equilibrium Stage →



Mass Balance →

$$L_n + V_n = L_{n-1} + V_{n+1}$$

$$L_n x_n + V_n y_n = L_{n-1} x_{n-1} + V_{n+1} y_{n+1}$$

For constant Molal Overflow →

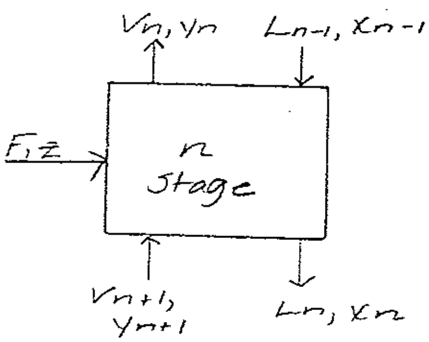
$$V_n = V_{n+1}$$

OR → $L_n = L_{n-1}$

EQUILIBRIUM → $y_n = f(x_n)$

Given → L_{n-1}, V_{n+1}, x_{n-1}, y_{n+1}

Feed Stage →

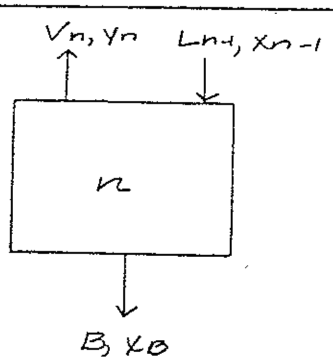


Same as above → BUT

$$L_n + V_n = L_{n-1} + V_{n+1} + F$$

$$L_n x_n + V_n y_n = L_{n-1} x_{n-1} + V_{n+1} y_{n+1} + F z$$

Partial Reboiler →



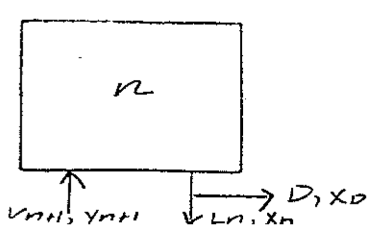
Mass Balance →

$$L_{n-1} = V_n + B$$

$$L_{n-1} x_{n-1} = V_n y_n + B x_B$$

Equilibrium → $y_n = f(x_B)$

Total Condensor →



Mass Balance →

$$D + L_n = V_{n+1}$$

$$D x_D + L_n x_n + V_{n+1} y_{n+1}$$

No equilibrium → $y_{n+1} = x_n = x_D$

DISTILLATION →

Assumption →

CMO = constant molal overflow → Molar flowrates in vapor and liquid streams are constant in column section (between inlets and outlets). This is a valid assumption if →

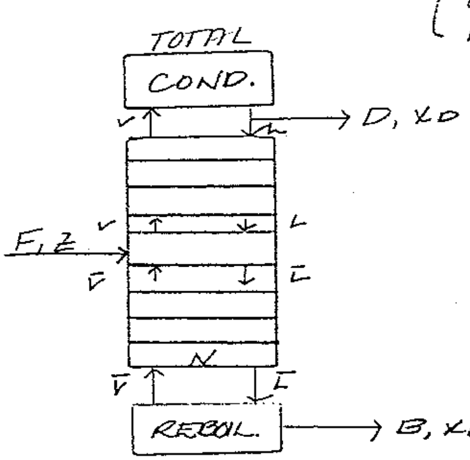
ⓐ column is adiabatic, specific heat changes are negligible compared to the latent heat changes, and molar heat of vaporization is independent of concentration.

-or-

ⓑ column is adiabatic and saturated liquid and vapor lines on an enthalpy/concentration diagram are parallel. For some systems, such as hydrocarbons, mass heat of vaporization is constant so CMO is valid.

THE BIG PICTURE

{COND. NOT AN EQUILIBRIUM CONTACT IN THIS PROBLEM, BUT THE REBOILER IS}



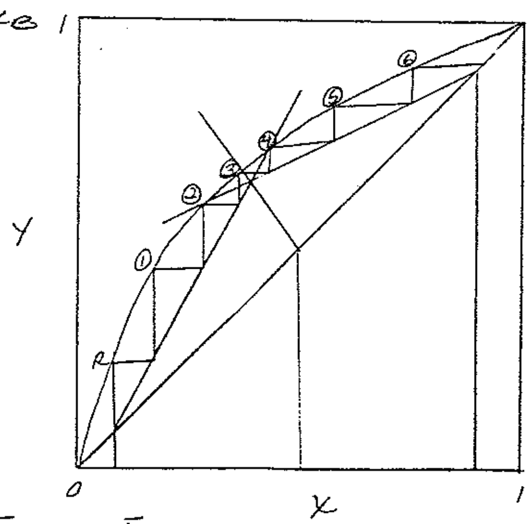
MASS BALANCES →

$F = D + B$

$V = L + D$

$\bar{L} = \bar{V} + B$

$Fz = Dx_D + Bx_B$



UPPER OPERATING LINE →

$$\left.\begin{aligned} Vy &= Dx_D + Lx \\ V &= D + L \end{aligned}\right\} \text{BAL'S}$$

$D = V - L$

$Vy = (V - L)x_D + Lx$

$$y = \frac{(V-L)}{V}x_D + \frac{L}{V}x$$

$$y = \left(1 - \frac{L}{V}\right)x_D + \frac{L}{V}x$$

BY THE SAME ANAL FOR BOT. → $y = \frac{\bar{L}}{\bar{V}}x + \left(\frac{\bar{L}}{\bar{V}} - 1\right)x_B$

13. Find: how do the following vary with Temperature and Pressure →

(a) diffusivity → $\mathcal{D}_{AB}$, the mass diffusivity for a binary system is a function of Temperature, Pressure and composition.

$\mathcal{D}_{AB} \propto \frac{1}{P}$ as pressure increases, the diffusivity decreases.

$\mathcal{D}_{AB} \rightarrow T$ as temperature increases, the diffusivity increases.

(b) dynamic viscosity → μ, the dynamic viscosity is a function of Temperature and pressure only.

$\mu^{G} \rightarrow T$ as the temperature of a gas at low density increases, so does the viscosity.

$\mu^{L} \rightarrow \frac{1}{T}$ as the temperature of a liquid increases, the viscosity decreases.

$\mu \rightarrow P$ as the pressure increases, the viscosity increases.

(c) thermal conductivity → k is a function of temperature and pressure only.

$k^{G}_{L} \rightarrow T$ as the temperature of a gas at low density increases, the thermal conductivity increases.

$k_l \rightarrow \frac{1}{T}$ as the temperature of a liquid increases, the thermal conductivity decreases.

(d) heat capacity → C_P is a function of temperature only.

$C_P, C_V \rightarrow T$ the heat capacity of a substance increases with increases with increasing temperature.

(e) heat transfer coefficient →

$h \rightarrow T$

$h \rightarrow P$

(f) Kinematic viscosity → $\nu = \frac{\mu}{\rho}$, this kind of viscosity is also a function of temperature and pressure.

$\rho = (MW)\frac{P}{RT}$

$\nu = \mu(P,T)\overset{\text{CONST.}}{\left(\frac{R}{MW}\right)}\left(\frac{T}{P}\right)$

$\nu^G \rightarrow T$ as the temperature of a gas at low density is increased, the dynamic viscosity increases, hence the kinematic viscosity will increase.

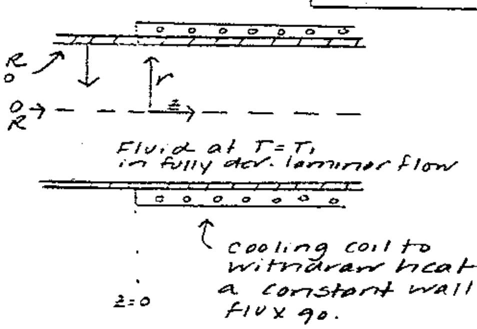
$\nu^L \rightarrow T$ as the temperature of a liquid is increased, the dynamic viscosity decreases, so probably the μ decreases slower than the T increases, hence ν^L will slowly increase with T.

$\nu \rightarrow P$ as the pressure of a fluid increases the dynamic viscosity increases, this probably will keep the kinematic viscosity approximately constant.

14. Find: what is the Reynolds analogy? What is the Chilton-Colburn analogy?

$$\boxed{\frac{\rho \hat{C}_P \langle \bar{v}_z \rangle \left[\frac{\langle \bar{v}_z \bar{T} \rangle}{\langle \bar{v}_z \rangle} - T_0 \right]}{q_0} = \frac{\rho \langle \bar{v}_z{}^2 \rangle}{\tau_0}}$$

A TUBE:



Energy Balance in cylindrical coordinates is conducted and other stuff, to get the above.

This is the Reynolds analogy between heat and momentum transfer. This equation states that the ratio of the transport of energy downstream to the transport of energy across the solid fluid interface is equal to a similar ratio for momentum transport.

NO, too hard, see next page →

REYNOLDS ANALOGY → [illegible] analogy deals with situations in which mass transfer, heat transfer, and fluid flow occur at the same rate.

(Basically, the rates of mass, heat and momentum transfer can be essentially the same for fluids in turbulent flow)

MASS FLUX → $N_1 = k\,\Delta C_1$
$= [a + b\,v]\,\Delta C_1$ (a = due to [illegible], b = due to eddies)

HEAT FLUX → $q = h\,\Delta T$
$= [a' + b'\,v]\,\Delta(\rho \hat{C}_p T)$ (a' = conduction, b' = eddies)

MOMENTUM FLUX → $\tau = [a'' + b''\,v]\,\rho v$ (a'' = viscous, b'' = eddies)
$\tau = f(\tfrac{1}{2}\rho v^2) = [\tfrac{f v}{2}]\,\rho v$

Now, in turbulent flow, a, a' and a'' are very small compared to the other terms, hence they are neglected, also →

$$b = b' = b''$$

THUS → $k = b\,v$
$h = b' v \rho \hat{C}_p \rightarrow b' v = \dfrac{h}{\rho \hat{C}_p}$
$\dfrac{f v}{2} = b'' v$

SO → $b\,v = k$
$b' v = \dfrac{h}{\rho \hat{C}_p}$
$b'' v = \dfrac{f v}{2}$

$$j_H = \frac{C_f}{2} = St\,Pr^{2/3} = \frac{Nu}{Re\,Pr}$$
$$j_m = \frac{C_f}{2} = St_m\,Sc^{2/3} = \frac{Sh}{Re\,Sc}$$

St Stm

HENCE → $k = \dfrac{h}{\rho \hat{C}_p} = \dfrac{f v}{2}$

OR → $$\frac{h}{\rho \hat{C}_p v} = \frac{f}{2} = \frac{k}{v}$$

gases is more accurate

THE REYNOLDS ANALOGY! KNOW THIS EQUATION...
(valid when transport occurs by means of turbulent eddies)

FOR THIS TO BE TRUE →

$$Pr = \frac{\hat{C}_p \mu}{k} = \frac{\mu}{\rho D_{AB}} = Sc = 1$$ $Pr = Sc = 1$

$$Sc = \frac{\nu}{D_{AB}} \qquad Pr = \frac{\nu}{\alpha}$$

CHILTON – COLBURN ANALOGY → compensates for the fact that Sc >> Pr in liquids.

They said that → $$b'v = \frac{h}{\rho \hat{C}_p}\left(\frac{\nu}{\alpha}\right)^{2/3}$$

$$= \frac{h}{\rho \hat{C}_p}(Pr)^{2/3}$$

$$bv = K\left(\frac{\nu}{D_{AB}}\right)^{2/3}$$

$$= K\,(Sc)^{2/3}$$

HENCE → $$\frac{h}{\rho \hat{C}_p v}(Pr)^{2/3} = \frac{f}{2} = \frac{K}{v}(Sc)^{2/3}$$ CHILTON-COLBURN ANALOGY, used for Liquids

... the friction factor; the coefficient of friction?

the friction factor is some ratio of shear stress to kinetic energy $\rightarrow$ $f = \dfrac{F}{AK} = \dfrac{\tau}{\rho v^2}$

wetted area or shadow area

coefficient of friction is that due to fanning for pipe flow $\rightarrow$

$$f = \frac{F}{AK} \qquad F = \Delta P \frac{\pi D^2}{4}$$

$$K = \frac{1}{2}\rho v^2$$

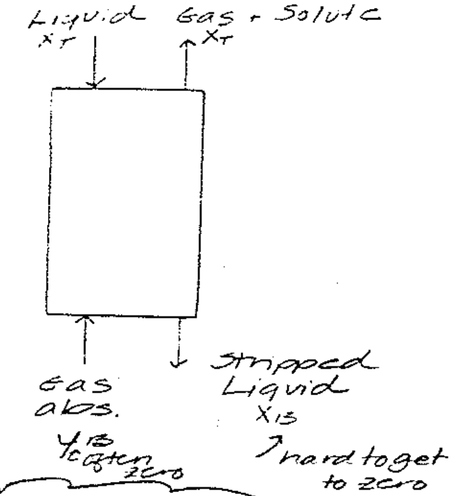
$$A = 2\pi r L$$

$$C_f = f = \frac{\Delta P \cancel{\pi} \cancel{D^2} D / \cancel{4}\, 2}{\frac{1}{\cancel{2}}\rho \langle v\rangle^2 \cancel{2}\cancel{\pi} \frac{\cancel{D}}{\cancel{2}} L} = \frac{\Delta P}{2\rho\langle v\rangle^2}\frac{D}{L}$$

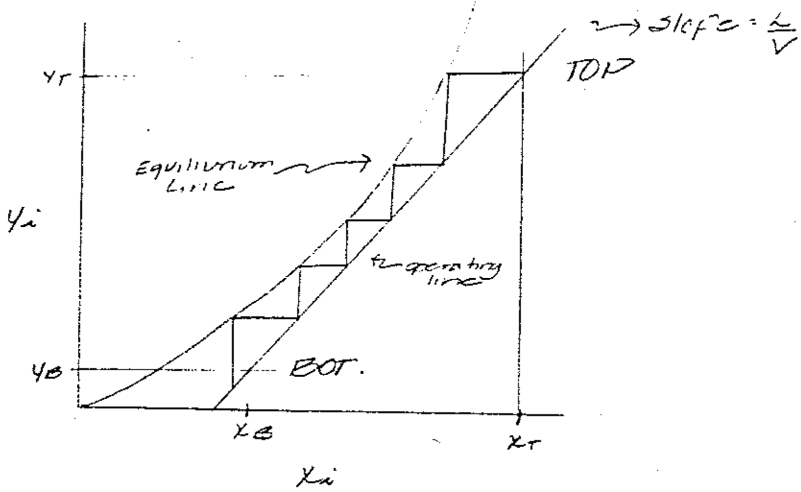
⑯ <u>Derive the boundary Layer Equations</u>

an absorber →

(a) STRIPPER →

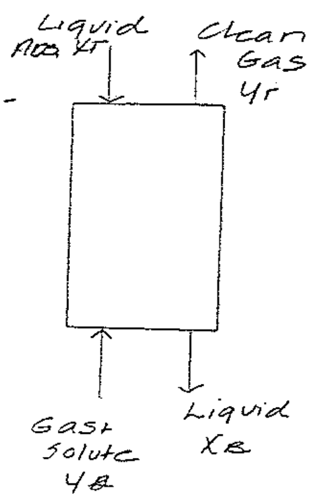


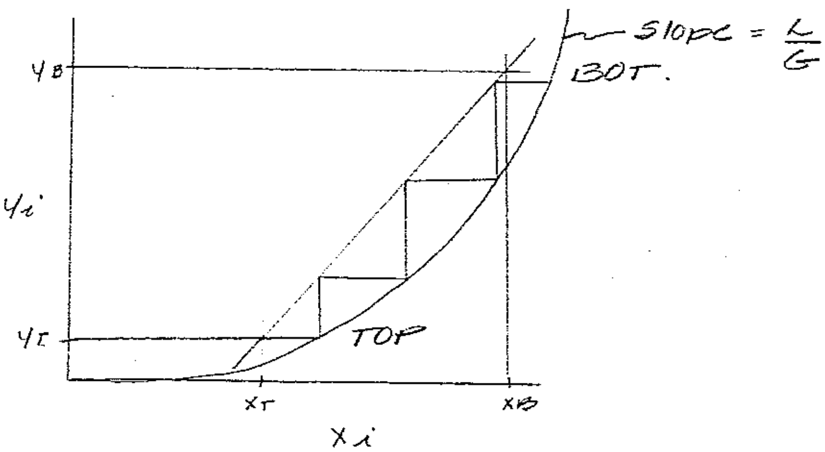
STRIPPING FROM LIQUID TO GAS



(EQUILIBRIUM LINE ABOVE THE OPERATING LINE)

(b) ADSORBER





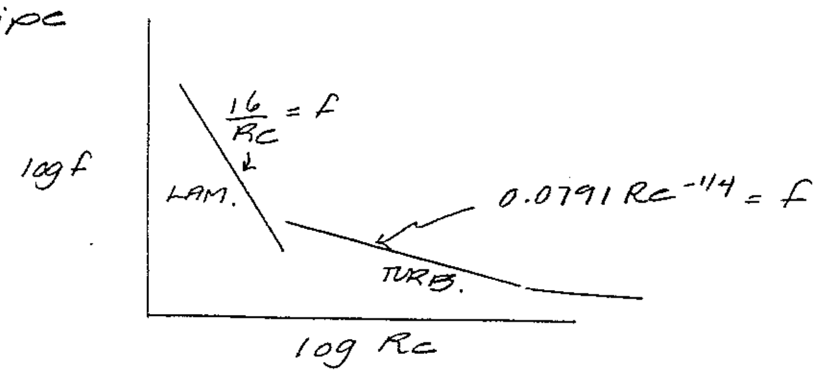
(x_T often zero)

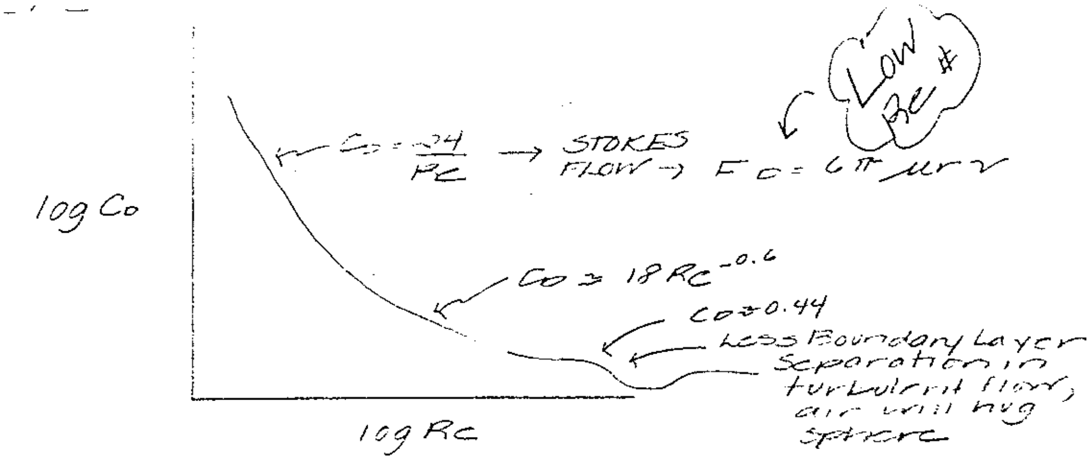
Easier to get $y_T = 0$ for Adsorber than $x_B = 0$ for stripper

ADSORBING FROM GAS TO LIQUID

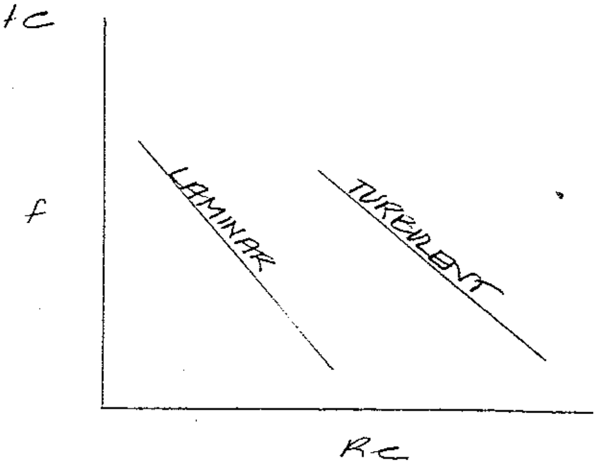
(18) Describe the friction factor (C_D) vs. Re relation for a →

(a) pipe

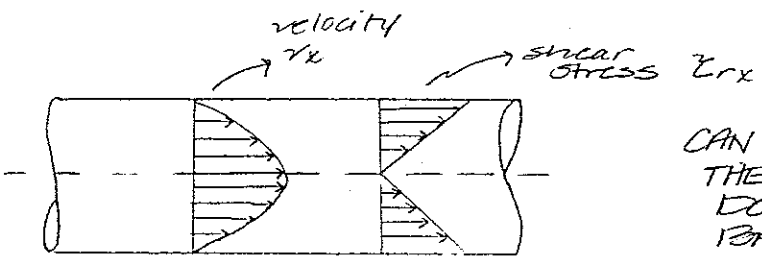




© flat plate



⑲ Sketch the shear stress profile in a pipe → (assume laminar flow, steady state regime)



CAN PROVE THESE 2, DO MOM. BAL.

$v_x = v_m\left(1-\left(\frac{r}{R}\right)^2\right)$

$\tau_{rx} = \frac{2 v_m \mu}{R^2} r$

⑳ Define the most commonly used dimensionless parameters and describe their significance.

$Re = \frac{\rho V D}{\mu} = \frac{\text{inertial}}{\text{viscous}}$ forces

$Pr = \frac{C_p \mu}{k} = \frac{\text{mol. diff. of mom.}}{\text{mol. diff. of heat.}}$

$Sc = \frac{\mu}{\rho D_{AB}} = \frac{\text{mol. diff. of mom.}}{\text{mol. diff. of mass}}$

$$Nu = \frac{hD}{K} = \frac{\text{temp. grad. at wall}}{\text{temp. grad. in bulk fluid}}$$

$$Sh = \frac{k_m L}{D_{AB}} = \frac{\text{conc. grad. at wall}}{\text{conc. grad. in bulk fluid}}$$

$$Pe = RePr = \left(\frac{\rho v D}{\mu}\right)\left(\frac{C_p \mu}{K}\right) = \frac{\rho v D C_p}{K} = \frac{\text{heat x-fer by conv.}}{\text{heat x-fer by cond.}}$$

$$= ReSc = \left(\frac{\rho v D}{\mu}\right)\left(\frac{\mu}{\rho D_{AB}}\right) = \frac{vD}{D_{AB}} = \frac{\text{mass x-fer by conv.}}{\text{mass x-fer by diff.}}$$

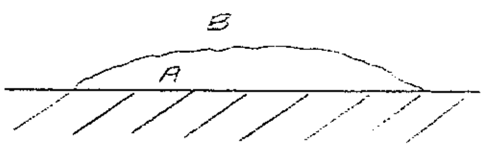
$$St = \frac{Nu}{RePr} = \frac{hD/K}{\rho v D/\mu \; C_p \mu/K} = \frac{h}{\rho v C_p} = \frac{\text{wall heat x-fer rate}}{\text{heat x-fer by convection}}$$

$$Fr = \frac{v^2}{Lg} = \frac{\text{inertial force}}{\text{gravitational force}}$$

$$Bi = Biot = \frac{hL}{k} = \frac{\text{heat convection across bound.}}{\text{heat conduction in solid}}$$

$$Br = \frac{\mu v^2}{K(T_b - T_0)} =$$ measure of the extent to which viscous heating is important relative to the heat flow resulting from the impressed temperature difference

(21) Given a pool of organic liquid (such as from a spill), how would you estimate its rate of evaporation?



- Assume pool rapidly equilibrates to ambient temperature

- Assume gas adjacent to liquid surface is in equilibrium with liquid $C^* \rightarrow$

$$f_i^v = f_i^L$$

$$y_i \cancel{\phi_i} P = \cancel{x_i \gamma_i} P_i^{sat} \cancel{\phi_i^{sat} (PE)}$$

$$y_i P = P_i^{sat}$$

$$y_i = \frac{P_i^{sat}}{P} = C^*$$

Flux of organic $\rightarrow$ $N_A = k_g (C^* - C_b)$

$$\underset{\text{moles/time}}{Q} = k_g A (C^* - C_b)$$ (if breeze is blowing $C_b = 0$)

Use $Sh = \frac{kL}{D}$ to find k $\quad Sh = f(Sc, Re)$

$$\left(\frac{\mu}{\rho D}, \frac{\rho v D}{\mu}\right)$$

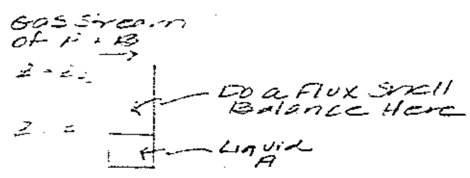
1130, 1 mol. → $N_A = -D\frac{dC_A}{dx}\Big|_{x=0}$

$$\frac{\partial C_A}{\partial t} = -D\frac{d^2 C_A}{dx^2} \qquad \begin{array}{l} @\ t=0 \quad C=C_b \\ @\ x=0 \quad C=C^* \\ @\ x\to\infty \quad C=C_b \end{array}$$

Get $c(x,t)$ → Find N_A

(22) How are the diffusivity and viscosity of a mixture determined?

DIFFUSIVITY → (1)

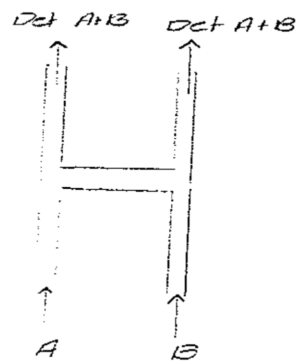


Can solve for the flux of A up, to determine D_{AB}

$$D_{AB} = \frac{N_A\,(z_2 - z_1)\,RT}{P \ln(P_{B2}/P_{B1})}$$

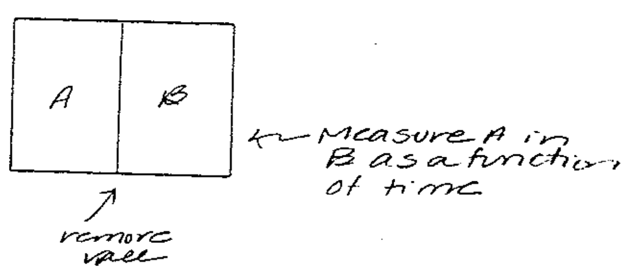
(2) Could also use the Stephan method and measure weight loss of the liquid.

(3) For 2 noncondensable gases →



(4) For solid in liquid measure the time required the dissolution.

(5) For liquid in liquid use a diffusion cell



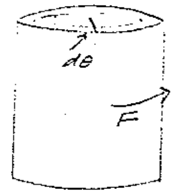
$$\eta = \frac{\tau}{\Gamma} = \frac{F/A}{V/H}$$

= viscosity

VISCOMETERS
1. CAPPILARY
2. CONE & PLATE
3. COUETTE
4. FALLING CYLINDER
5. PARALLEL DISC
6. ROLLING BALL

$$dF = \tau dA = \eta \Gamma dA$$

$$\Gamma = \frac{R\Omega}{H} \qquad dA = RL\,d\theta$$



HENCE → $dF = \dfrac{\eta R^2 \Omega L\, d\theta}{H}$

$$R\,dF = \frac{\eta R^3 \Omega L\, d\theta}{H}$$

differential torque

INTEGRATE FROM $0 \to 2\pi$, SOLVE!

$$G = \text{Total torque} = \int_0^{2\pi} R\,dF = \frac{2\pi \eta R^3 \Omega L}{H}$$

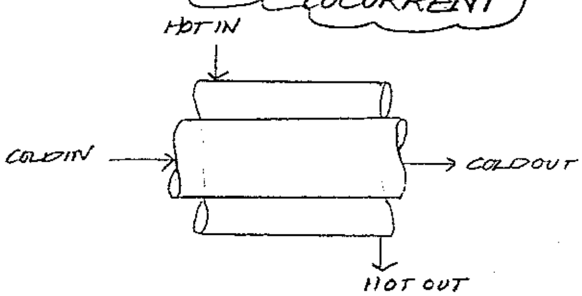
HENCE → $\eta = \dfrac{GH}{2\pi R^3 \Omega L}$

② Cone + Plate viscometer

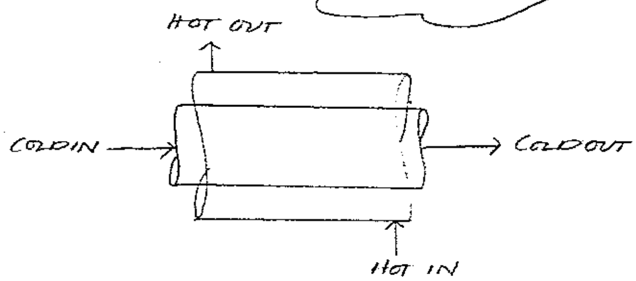
③ Falling Sphere viscometer (HMM... what's this?)
Uses Stokes Law

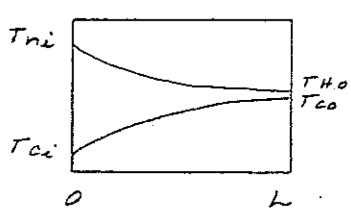
Sketch the temperature profile in a heat exchanger →

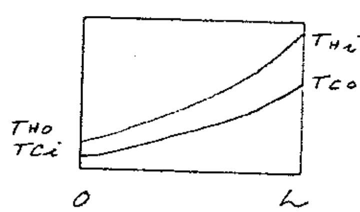
DOUBLE PIPE COCURRENT



DOUBLE PIPE COUNTERCURRENT







$$\Delta T_{lm} = \frac{(T_{Ho} - T_{co}) - (T_{Hi} - T_{ci})}{\ln\left[\frac{(T_{Ho} - T_{co})}{(T_{Hi} - T_{ci})}\right]}$$

$$\Delta T_{lm} = \frac{(T_{Hi} - T_{co}) - (T_{Ho} - T_{ci})}{\ln\left[\frac{(T_{Hi} - T_{co})}{(T_{Ho} - T_{ci})}\right]}$$

explosion? (e.g. bulk flow, diffusion, etc...)

① Bulk flow (convection) of expanding gases as well as thermal expansion, things getting hot...

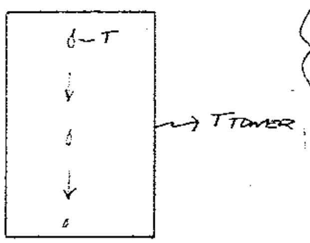
IMPT. ② Tortuosity and porosity of the matrix

③ Diffusion is not important

④ Melting points and subsequent heat transfer rates.

Flame speed
Dimensions of void space
Speed of Flame

㉕ Consider a drop falling down a tower, initial T and tower T are given. How does the drop T change as it falls.



Energy Balance →

Convective heat transfer = evaporation heat + temperature change

$$\rho C_p \frac{dT}{dt} = -N_A \Delta H^v + hA(T_d - T_\infty)$$

Friction will heat drop up

Evaportation will cool the drop

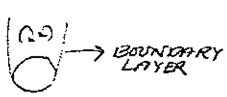
(Get h from Nu correlation for flow around a sphere)

If drop $T > T_w$, and $H_{air} < 100\%$ drop will evaporate and cool to T_∞

㉖ What is the angular dependence of the Nu number for a falling drop?

For a sphere → $\boxed{Nu = 2 + 0.7 Re^{1/2} Pr^{2/3}} = \dfrac{hD}{K}$

$$C_D = \frac{24}{Re}\left(1 + 0.15 Re^{0.587}\right)$$

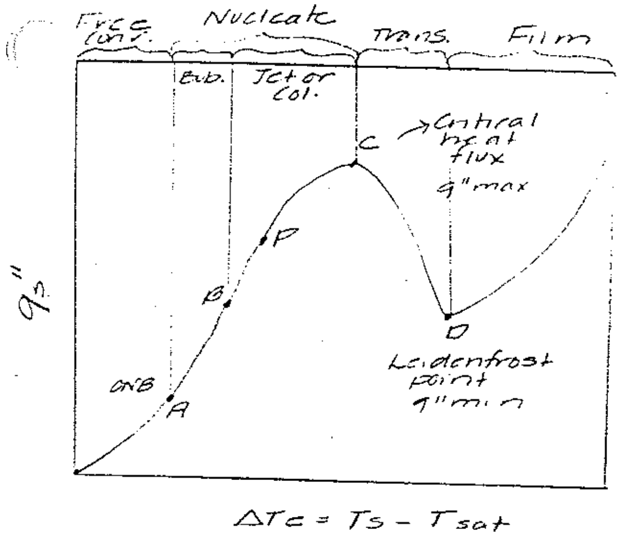


h is high here

Hmm...
don't know about this one

phenomena responsible for the observed behavior.
Draw and explain a similar curve for condensation.

BOILING CURVE



q_s'' = surface heat flux

0 – A → free convection
≈ 5°C range, insufficient vapor in contact with the liquid phase to cause boiling at T^{sat}

ONB → onset of nucleate boiling

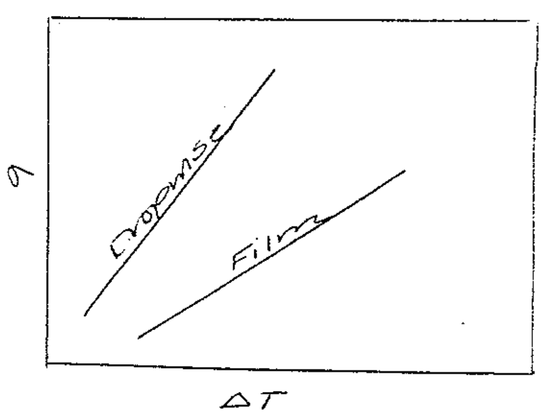
A – B → Nucleate boiling
isolated bubbles

B – C → Nucleate boiling
jets or columns
→ slugs of vapor

C – D → Transition boiling, unstable film boiling, or partial film boiling

D → Film Boiling, a vapor blanket, heat transfer from surface to liquid occurs by cond. through vapor.

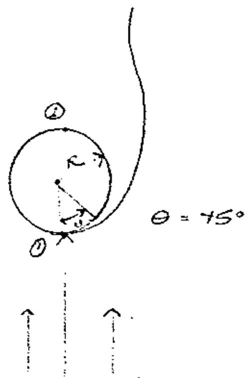
CONDENSATION CURVE



Film → a liquid film forms on which further condensation occurs

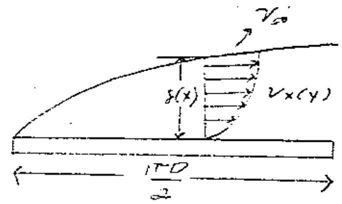
Dropwise → drops form directly on surface, not as much resistance to heat transfer as through a liquid film.

Calculate the boundary layer thickness at a 45 degree angle. What is the pressure at the forward and backward stagnation points? What causes the difference?



To calculate the thickness of the boundary layer at 45°, look at the surface as a flat plate →

Length $= \frac{\pi D}{2}$ (circumference of ½ of a circle)



B.L.E. → $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$ CONTINUITY

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right)$$ MOMENTUM

solve ① for v_y
assume $\frac{\partial^2 v_x}{\partial x^2}$ can be neglected (it is much smaller than $v_x \frac{\partial v_x}{\partial x}$)

$\frac{v_x}{v_\infty} = \phi(\eta)$

$\eta = \frac{y}{\delta(x)}$

$$v_x \frac{\partial v_x}{\partial x} - \left(\int_0^y \frac{\partial v_x}{\partial x} dy \right) \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

at $y=0$, $v_y=0$
$y=0$, $v_x=0$
$y \to \infty$, $v_x = v_\infty$

THEN, solving for a flat plate →

$$\delta(x) = 4.64 \sqrt{\frac{\nu x}{v_\infty}} \approx 5 \sqrt{\frac{\nu x}{v_\infty}}$$

$$\delta(x) = 5\sqrt{\frac{\nu x}{v_\infty}}$$

HENCE, find $\delta(x)$ at $x = \frac{\pi D}{8} = \frac{\pi 2R}{8} = \frac{\pi R}{4}$

$$\delta(45°) = 5\sqrt{\frac{\nu \pi R}{4 v_\infty}}$$

At the forward stagnation point →

① $p - p_\infty = \frac{1}{2}\rho v^2$

At the backward stagnation point →

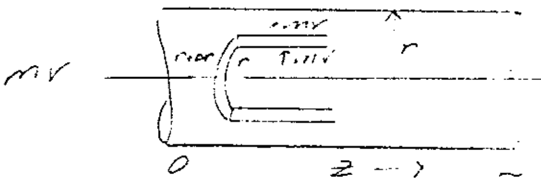
② $p - p_\infty = -\frac{1}{2}\rho v^2$

$p^{①} - p_\infty - p^{②} + p_\infty = \Delta p$

$\Delta p = \frac{1}{2}\rho v^2 - (-\frac{1}{2}\rho v^2)$

$\Delta p = \rho v^2$

This difference is cause by acceleration and decelleration around the sphere developed laminar flow in a pipe →



Rate of Mom. Acc. = Rate in − Rate out + Sum of Forces

Rate of Mom. in with flow at $z=0$ $= (2\pi r \Delta r\, v_z)\,\rho v_z|_{z=0}$

Rate of Mom. out with flow at $z=L$ $= (2\pi r \Delta r\, v_z)\,\rho v_z|_{z=L}$

Rate of Mom. at r due to shear stress $= (2\pi r L)\,\tau_{rz}|_r$

Rate of Mom. at $r+\Delta r$ due to shear stress $= (2\pi r L)\,\tau_{rz}|_{r+\Delta r}$

Force of Pressure at $z=0$ $= (2\pi r \Delta r)\,p_0$

Force of pressure at $z=L$ $= (2\pi r \Delta r)\,p_L$

Gravity force acting on annular volume $= (2\pi r \Delta r L)\,\rho g$

HENCE →

These are equal; v_z same at 0 and L

$$0 = (2\pi r \Delta r v_z)\rho v_z|_{z=0} - (2\pi r \Delta r v_z)\rho v_z|_{z=L} + (2\pi r L)\tau_{rz}|_r - (2\pi r L)\tau_{rz}|_{r+\Delta r} + (2\pi r \Delta r)p_0 - (2\pi r \Delta r)p_L + 2\pi r \Delta r L \rho g$$

$$0 = -\frac{\partial (r\tau_{rz})}{\partial r} - \frac{r\,\Delta P}{L} + \rho g r$$

$$\frac{\partial (r\tau_{rz})}{\partial r} = \left(\frac{P_0 - P_L}{L} + \rho g\right) r$$

INTEGRATE →

$$\tau_{rz} = \left(\frac{P_0 - P_L}{L} + \rho g\right)\frac{r}{2} + \frac{C_1}{r}$$

heat exchanger?

U = the total thermal resistance to heat transfer between two fluids.

R_f = fouling factor → accounts for fluid impurities

$$\boxed{\frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{fi}''}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{fo}''}{A_o} + \frac{1}{h_oA_o}}$$

Correlation for h_i → $Nu = 0.023Re^{0.8}Pr^{0.4}$

Correlation for h_o → Donohue Equation

FIND

$h_i \rightarrow Nu = 0.023Re^{0.8}Pr^{0.3}$

$h_o \rightarrow Nu = 0.023Re^{0.8}Pr^{0.\ldots}$

(Dittus-Boelter Correlation)

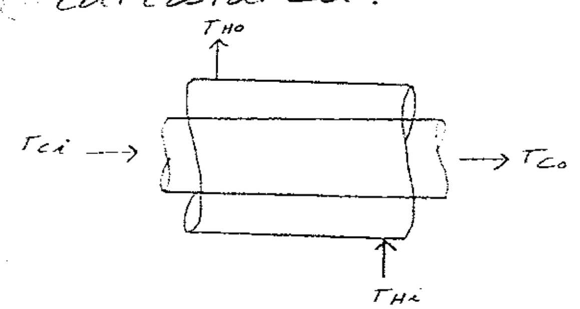
$h_o \rightarrow St = 0.023Re^{-0.2}Pr^{-2/3}$ use D_o

h_i use D_i $\rightarrow St = 0.023Re^{-0.2}Pr^{-2/3}$

(Colburn Correlation)

$Q = UA\Delta T_{lm}$ Find Q from → $Q = mC_p\Delta T$

(31) Given two T's and a knowledge of all fluids' properties for a double pipe heat exchanger (countercurrent), how are the other two T's calculated?



$Q_c = \dot{m}_c C_{pc}(T_{co} - T_{ci})$

$Q_H = \dot{m}_H C_{pH}(T_{Ho} - T_{Hi})$

3 EQN'S
3 UNKS
(2T'S, Q)

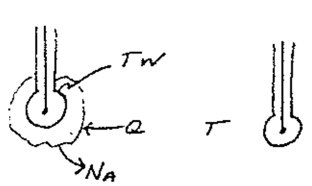
$Q = Q_c = -Q_H = UA\Delta T_{lm}$

$= UA \dfrac{(T_{co} - T_{Hi}) - (T_{ci} - T_{Ho})}{\ln\left[\dfrac{(T_{co} - T_{ci})}{(T_{ci} - T_{Ho})}\right]}$

Know: C_{pc}, C_{pH}, $\dot{m}_c$, $\dot{m}_H$, 2 T'S, U, A

Find: 2 T'S, Q

(32) Derive equations describing the wet-bulb/dry bulb psychrometer. Obtain a relation between the wet bulb temperature and air humidity in terms of Dimensionless groups.



$Q = h(T - T_W) = \Delta H_{vap} N_A$

$N_A = k_m(C_w - C_b)$

$h(T - T_W) = \Delta H_{vap} k_m (C_w - C_b)$

$= \Delta H_{vap} \rho k_m (H_w - H)$

psychrometric ratio $= \dfrac{h}{k_m \Delta H_{vap} \rho}$

$\dfrac{(H_w - H)}{(T - T_W)} = \dfrac{h}{k_m \Delta H_{vap} \rho} \rightarrow \dfrac{(H_w - H)}{(T_w - T)} = -\dfrac{h}{k_m \Delta H_{vap} \rho}$

33) Is the heat flux from a liquid into a gas usually higher or lower if the gas is insoluble (vs. soluble) in the liquid? This compares "diffusion through a stationary component" with the extreme case of "EMCD."

$$\text{liquid} \xrightarrow{q} \text{gas}$$

does q → increase or when the gas becomes insoluble in the liquid

$$N_A = -D_{AB}\nabla C_A + x_A(N_A + \cancelto{0}{\vec{N}_B}) \qquad N_A = -N_B$$

$$\boxed{N_A = -D_{AB}\frac{dC_A}{dx} \quad \text{EMCD}}$$

But diffusion through a stagnant medium →

$$N_A = -D_{AB}\frac{dC_A}{dx} + x_A(N_A + \cancelto{0}{N_B})$$

$$N_A = -D_{AB}\frac{dC_A}{dx} + x_A N_A$$

$$\boxed{N_A(1-x_A) = -D_{AB}\frac{dC_A}{dx} \quad \text{STAGNANT}}$$

$$N_A = -\frac{D_{AB}}{x_B}\frac{dC_A}{dx} \qquad \frac{1}{x_B} > 1 \Rightarrow$$ Flux of species A is faster if B is stagnant

34) The Chilton-Colburn j-factor for heat transfer is proportional to h, the convective heat transfer coefficient. Why is j proportional to $Pr^{-1/3}$? Why is j only a fraction of Re? Why does j decrease as Re increases?

$$j_H = \frac{h}{\rho C_p u} = \frac{C_f}{2} = St\,Pr^{2/3} \qquad \boxed{St = \frac{Nu}{Re\,Pr}}$$

$$j_D = \frac{k_m}{u} = \frac{C_f}{2} = St_m = Sh\,Sc^{2/3} \qquad \boxed{St_m = \frac{Sh}{Re\,Pr}}$$

$$j_H = St\,Pr^{2/3} = \frac{Nu}{Re\,Pr^{1/3}} \Rightarrow j_H = \frac{Nu}{Re}Pr^{-1/3}$$

BY DEFINITION...

$j_H = 0.0395\,Re^{-1/4}$ for turbulent flow in a pipe considered less dangerous than H_2 leaks. Why?

$$\mu = \left(\frac{\partial T}{\partial P}\right)_H = \frac{\left[T \left.\frac{\partial V}{\partial T}\right)_P - V\right]}{C_P}$$

Because $\mu < 0$ for H_2 but not for N_2 and O_2. That is as H_2 expands it's T increases at constant H

H_2 more explosive

(36) How would you separate oxygen from salt water? Suppose you were processing fairly large volumes so that energy efficiency is a strong consideration. What thermodynamic variables affect solubility? Where is the mass transfer resistance? What type of unit operation would you use? How would you design it?

H_2O
O_2
$NaCl$

- Henry's law is strongly dependent on pressure
- hence, decreasing the pressure would allow bubbles to form (O_2 to vaporize)

$f_i^L = x_i H$

→ pass stream through an orifice to drop its pressure

→ mass transfer resistance is the formation of bubbles

→ could score the pipe to form bubbles

UNIT OPERATION →
gas liquid setting tank downstream from a choking orifice

(37) What area is used when defining friction factor for a wetted wall surface?

$$C_f = \frac{F}{A \frac{1}{2} \rho \langle v \rangle^2}$$

FOR A GAS → use internal area based on → (Diameter − 2 film)

FOR A LIQUID → use vessel diameter for wall friction phase velocity? why or why not?

$$C_s = \frac{h_Y}{M K_Y}$$ C_s = humid heat capacity

McCABE & SMITH

- holds for air water
- psychometric line and adiabatic line are the same when this relation holds
- for air-organic systems, psychometric lines are much steeper than ad-saturation lines
- not dependent on gas-phase velocity because

$\frac{C_f}{2} = St\, Pr^{2/3}$?

velocity cancels out...

(39) Why are analogies between mass and heat transfer much more straightforward to use than analogies between mass and momentum transfer?

- Mass and heat transfer are related by film resistance theory

Also → momentum is a vector modified by tensor laws whereas mass and heat are scalars.

Also → mass + heat follow simpler and more analogous driving force laws

(40) Given a CSTR at temperature T with no reaction what would happen if the inlet T were suddenly increased?

new T, out

Energy Balance → $\rho C_p V \frac{dT}{dt} = \rho \dot{V} C_p (T_1 - T)$

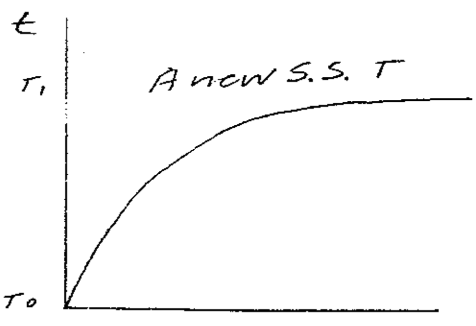
$$\int_{T_0}^{T_1} \frac{dT}{(T_1 - T)} = \frac{\dot{V}}{V} dt \rightarrow -\ln(T_1 - T)\Big|_{T_0}^{T} = \frac{1}{\tau} t$$

$$-\ln(T_1 - T_0) + \ln(T_1 - T) = -\frac{1}{\tau} t$$

$$\ln \frac{(T_1 - T)}{(T_1 - T_0)} = -\frac{1}{\tau} t$$

$$T_1 - T = (T_1 - T_0) e^{-\frac{1}{\tau} t}$$

$$T = T_1 - (T_1 - T_0) e^{-\frac{1}{\tau} t}$$



transport are important. Give examples of when they don't hold.

Reynolds Analogy (heat and mass) does not hold for liquids

need to have turbulent flow, where the effect of eddies will dominate and diffusion, conduction or viscosity. "Mixing up"

Chilton-Colburn → holds for liquids

(42) What is the theoretical basis for all the "famous" analogies between heat, mass and momentum transport? What are mass and heat transfer equivalents of the momentum transport equation?

The General Equations for radial "diffusion" and 'convection' are →

Mass $N = -(D+\epsilon)\dfrac{d\bar{c}}{dy}$

Heat $q = -(k+\rho C_p \epsilon)\dfrac{d\bar{\theta}}{dy}$

Mom. $\tau = (\mu + \rho\epsilon)\dfrac{d\bar{u}}{dy}$

ϵ = eddy diffusivity
q_0 = wall heat transfer
N_0 = wall mass transfer
τ_0 = wall shear stress

NON-DIMENSIONALIZE BY PUTTING →

$u^+ = \dfrac{u}{\sqrt{\tau_0/\rho}} = \dfrac{u}{u^*}$

$y^+ = \dfrac{y\sqrt{\tau_0/\rho}}{\mu} = y\dfrac{u^*}{\mu}$

$\theta^+ = \dfrac{\theta}{\theta^*} = \dfrac{q_0}{\rho u^* C_p}$

$c^+ = \dfrac{c}{c^*} = \dfrac{c}{N_0 u^*}$

Mass
$1 = -\left(\dfrac{1}{Sc} + \dfrac{\epsilon}{\nu}\right)\dfrac{dc^+}{dy^+}$

Heat
$1 = -\left(\dfrac{1}{Pr} + \dfrac{\epsilon}{\nu}\right)\dfrac{d\theta^+}{dy^+}$

Mom
$1 = \left(1 + \dfrac{\epsilon}{\nu}\right)\dfrac{du^+}{dy^+}$

By substituting the form for the velocity profile into the momentum equation, we can find $\frac{\epsilon}{\nu}$ and hence derive conc. and T profiles

FROM THESE EQUATIONS IT CAN BE SEEN THAT THE Mass, heat and Mom. profiles all have a similar form (hence the analogies) and the form drag: ... friction drag

Skin friction Drag → arises from no-slip boundary condition at a fluid solid boundary (tangential)

Form Drag → arises from acceleration or decelleration of a fluid as it passes an obstruction

(14) How would you determine as mass transfer coefficient experimentally?

1. wetted wall column experiments
2. Packed bed experiments (HTU → k_g)
3. time for evaporation
4. dissolution time

(15) Why does frost not form under a tree when it is on the ground all around a tree?

Most important → a tree insulated the ground from radiative heat loss.

At night, the heat gained by the ground during the day radiates away.

Draw a McCabe-Thiele diagram for a distillation column that uses a reactive adsorbant.

Advantage → Distillation and reaction can take place simultaneously in the same vessel and the products can be removed to drive the reversible reaction to completion

... describing heat and mass transfer?

$$Nu = 0.023 Re^{0.8} Pr^{0.3} \text{ or } Pr^{0.4}$$
$$Nu = 0.023 Re^{0.8} Pr^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$
$$Nu = 0.023 Re^{0.8} Pr^{1/3}$$

Dittus-Boelter
Turb flow in pipe
Colburn

$Sh = 0.026 Re^{0.8} Sc^{1/3}$

(48) Write the molecular transport equations (constitutive) for →

(a) Mass → $\left(\frac{\partial C_A}{\partial t} + \underline{v} \cdot \nabla C\right) = D \nabla^2 C_A + R_A$

(b) Momentum → $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = \mu \nabla^2 \underline{v} - \nabla P + \rho g$

(c) Heat transfer → $\rho C_p\left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T\right) = K \nabla^2 T + R_A(-\Delta H_{rxn}) + \underline{\tau} : \nabla \underline{v}$

(49) Give the equations describing flow in a packed bed.

The Equation used to calculate pressure drop → $v_s = \frac{\Delta P k}{\Delta L \mu}$

DARCY'S LAW → $Q = \text{flow rate} = \frac{K \Delta P}{L}$

this is for flow through porous media

ERGUN EQN →

$$\frac{dP}{dL} = -\frac{G}{\rho g_c D_p}\left(\frac{1-\phi}{\phi^3}\right)\left[\frac{150(1-\phi)\mu}{D_p} + 1.75G\right]$$

(50) Derive an equations for a gas undergoing isentropic expansion.

$du = \cancel{dQ}^{\,0} - dW \qquad du = -W$

$Pv = CT$
$W = \int P dV$

$du = T\cancel{dS}^{\,0} - PdV$
$dH = T\cancel{dS}^{\,0} + VdP$

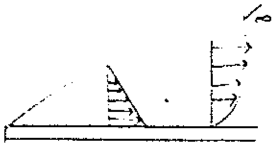
$v = CT \qquad \Delta u = mC_v\Delta T$

$$C_v dT = -PdV$$

(51) What is inside a light bulb and why? Tungsten
No medium for conduction and no currents for conduction, want all heat flux to be purely by radiation.

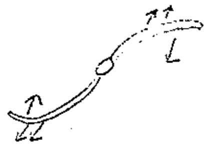
(52) Why do you have to whirl a wet-bulb/dry-bulb psychrometer in the air previous to reading it?
If you don't whirl the psychrometer first, you get a small layer of air saturated with water and you wouldn't ...

fluid flowing over a flat plate?



Increasing Momentum Flux

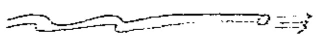
(54) How does a lawn sprinkler work?



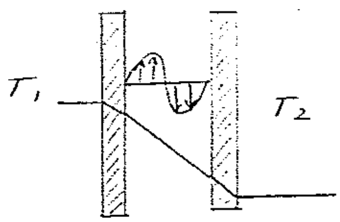
Water going through a curved pipe puts a force on the convex side, causing the sprinkler to rotate

(55) Consider firefighters holding a high pressure hose, must they pull or push the hose?

push



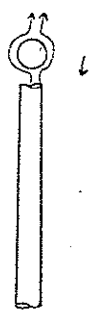
(56)



Air → Low h, cheap

Grasoff # → $\frac{L^3 \rho^2 \beta \Delta T g}{\mu^2}$ = $\frac{\text{Bouyant forces}}{\text{viscous forces}}$

(57) The Golf Ball →



$\Sigma F = ma$

$m_B g = F_g$

$m_{air} g = F_B$

Stokes $= F_D$

$F_D = 6\pi\mu v r$

$F_g = V \rho_B g$

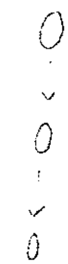
$F_B = V \rho_{air} g$

$\cancel{m a}^{0} = Vg(\rho_b - \rho_{air}) - 6\pi\mu v r$

WHAT ABOUT ΔP ACROSS BALL?

$\Delta P = \rho v^2$

it drop (in air) before it solidifies. What does the Biot number tell you here? How do you find the convective heat transfer coefficient?



Sphere heat transfer from the air to the sphere at a rate

To get to fus T: ① $q = hA(T - T_\infty) = \rho C_p \frac{dT}{dt} \frac{4}{3}\pi R^3$

② $qt = \Delta H_{fus} \rho \frac{4}{3}\pi R^3$

$$hA(T - T_\infty) = \rho C_p \frac{d(T - T_\infty)}{dt} \frac{4}{3}\pi R^3$$

$$\Delta t_1 = \frac{\rho C_p \frac{4}{3}\pi R^3}{h\,4\pi R^2} \ln (T - T_\infty)\Big|_{T_1}^{T_f}$$

$$\Delta t_1 = \frac{\rho C_p R}{h} \ln\left[\frac{(T_f - T_\infty)}{(T_1 - T_\infty)}\right]$$

$$q\,\Delta t_2 = \Delta H_{fus} \rho \frac{4}{3}\pi R^3$$

$$\Delta t_2 = \frac{\Delta H_{fus} \rho \frac{4}{3}\pi R^3}{q}$$

$$\Delta t = \Delta t_1 + \Delta t_2 \qquad v_t = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_t \Delta t$$

BIOT NUMBER →

$Bi = \frac{hD}{K}$ $\frac{\text{convective heat out}}{\text{conductive heat in}}$

Bi small → $K \gg h$
high conductivity
T inside has small or no gradient

Bi large → can't neglect T grad. in sphere

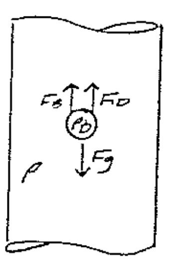
$Bi < 0.1$

Find the convective h →

$$\frac{h}{\rho C_p u} = \frac{C_D}{2}$$

$$\frac{hD}{K} = Nu = 0.023\, Re^{0.8} Pr^{1/3}$$

(61) For a particle dropping in a fluid field derive the equation for the terminal velocity and discuss the friction factor coefficient.



Σ Forces $= ma$

$$m\vec{a}^{\,0} = \frac{4}{3}\pi R^3 g(\rho - \rho_f) + 6\pi\mu v r$$

a v_t

$F_G = mg = \rho \frac{4}{3}\pi R^3 g$

$F_B = m_f g = \rho_f \frac{4}{3}\pi R^3 g$

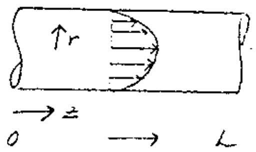
$F_D = 6\pi\mu v r$

$$v_t = \frac{\frac{4}{3}\pi R^{3} g(\rho_f - \rho)}{6\pi\mu v r} = \frac{2}{3}\frac{R^2 g}{\mu v}(\rho_f - \rho)$$

$$F = \frac{F_D}{A\frac{1}{2}\rho v^2}$$

momentum equation appropriate for this geometry.

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = \mu\nabla^2\underline{v} - \nabla p + \rho g$$



(Steady State)

CONS. OF MASS $\quad \rho = CT$

$$0 = (\rho v_z) v_z 2\pi r\Delta r\big|_0 - (\rho v_z) v_z 2\pi r \Delta r\big|_L$$
$$+ \tau_{rz}\cancel{2\pi} rL\big|_r - \tau_{rz}\cancel{2\pi} rL\big|_{r+\Delta r} + \rho g_z \cancel{2\pi} r\Delta r L$$
$$+ p_z \cancel{2\pi} r\Delta r\big|_0 - p_z \cancel{2\pi} r\Delta r\big|_L$$

$$0 = -\cancel{L}\frac{\partial(\tau_{rz} r)}{\partial r} + \rho g_z \cancel{L} + \frac{(P_0 - P_L)}{L} r$$

$$\frac{\partial(\tau_{rz} r)}{\partial r} = r\rho g_z + \frac{(P_0 - P_L)}{L} r = r\left[\frac{(P_0 - P_L)}{L} + \rho g_z\right]$$

$$r\frac{\partial \tau_{rz}}{\partial r} + \tau_{rz}\cancel{\frac{\partial r}{\partial r}} = r\left[\frac{(P_0 - P_L)}{L} + \rho g_z\right]$$

$$\tau_{rz} = -\mu\frac{\partial v_z}{\partial r}$$

$$-r\mu\frac{\partial^2 v_z}{\partial r^2} - \mu\frac{\partial v_z}{\partial r} = r\left[\frac{(P_0 - P_L)}{L} + \rho g_z\right]$$

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{1}{\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g_z\right] = 0$$

$$v^* = \frac{v_z}{v} \qquad \therefore v_z = v^* v$$

$$r^* = \frac{r}{R} \qquad r = r^* R$$

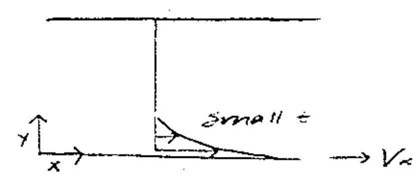
$$\frac{\partial^2(v^* v)}{\partial(r^{*2}R^2)} + \frac{1}{r^*R}\frac{\partial v^* v}{\partial r^* R} + \frac{1}{\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g_z\right] = 0$$

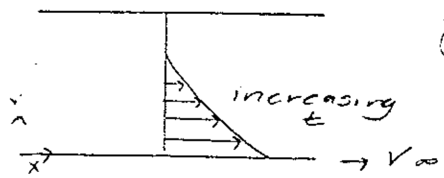
$$\frac{v}{R^2}\frac{\partial^2 v^*}{\partial r^{*2}} + \frac{1\, v}{r^* R^2}\frac{\partial v^*}{\partial r^*} + \frac{R^2}{v\mu}\left[\frac{(P_0 - P_L)}{L} + \rho g_z\right] = 0$$

The Reynolds # does not show up because $\frac{\partial v_z}{\partial z} = 0$ the inertial term drops out.
Does not mean Re is not important.

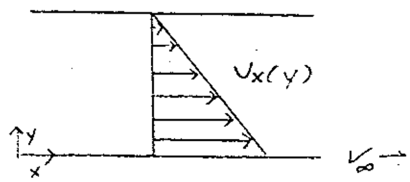
with a fluid between them. One plate remains fixed, the other is set in motion at a velocity V. What do the transient velocity profiles look like?

$t=0$





UNSTEADY STATE



STEADY STATE ← what does the steady state look like? why?

What is the driving force for fluid flow in a pipe?

A pressure difference is the driving force for fluid flow in a pipe.

What is the driving force here?

A velocity gradient is the driving force for momentum transport.

Momentum is fluxing opposite to the direction of increasing velocity!

Describe a Momentum Balance →

Rate of acc. of mom = Rate of Mom. in − Rate of Mom. out + Sum of Forces acting on system

$\nabla \cdot \underline{v} = 0$

<u>IN GENERAL</u> →

N.S. assumes $\rho, \mu = CT$ inc. fluid

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = \mu\nabla^2\underline{v} + \rho\underline{g} - \nabla P$$

FOR x-comp.

$$\rho\left(\cancelto{S.S.}{\frac{\partial v_x}{\partial t}} + v_x\cancelto{0}{\frac{\partial v_x}{\partial x}} + v_y\cancelto{0}{\frac{\partial v_x}{\partial y}} + v_z\cancelto{0}{\frac{\partial v_x}{\partial z}}\right) = \mu\left(\cancelto{0}{\frac{\partial^2 v_x}{\partial x^2}} + \frac{\partial^2 v_x}{\partial y^2} + \cancelto{0}{\frac{\partial^2 v_x}{\partial z^2}}\right) + \cancelto{0}{\rho g_x} - \cancelto{0}{\frac{dP}{dx}}$$

$$\mu\frac{\partial^2 v_x}{\partial y^2} = 0 \qquad \frac{d}{dy}\left(\frac{dv_x}{dy}\right) \rightarrow \frac{dv_x}{dy} = C_1$$

$$dv_x = C_1\,dy \rightarrow v_x = C_1 y + C_2 \qquad \begin{array}{l}\text{at } y=0\ V_x = V_\infty \\ \text{at } y=L\ V_x = 0\end{array}$$

$$C_2 = V_\infty \qquad 0 = C_1 L + V_\infty$$

$$C_1 = \frac{-V_\infty}{L}$$

$$v_x = -\frac{V_\infty}{L}y + V_\infty \qquad v_x = V_\infty\left(1 - \frac{y}{L}\right)$$

the top plate moving at V?

$$\tau_{yx} = -\mu \frac{dv_x}{dy} = \frac{F}{A} \qquad F = \tau_{yx} A$$

eval. at $y=0$

$$F = -\mu \frac{dv_x}{dy} A$$

$$\frac{dv_x}{dy} = V_0\left(-\frac{1}{L}\right) \rightarrow F = \frac{\mu V_0}{L} A$$

$$\tau_{yx} = \frac{\mu V_0}{L}$$

(59) For the system in number (58) determine the characteristic time. Which would take longer for the S.S. profile to be reached, molasses or water and why?

time to reach S.S. →

$$\rho\left(\frac{\partial v_x}{\partial t}\right) = \mu \frac{\partial^2 v_x}{\partial y^2} \rightarrow \frac{\partial v_x}{\partial t} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

$v^* = v_x/v \qquad y^* = y/L$

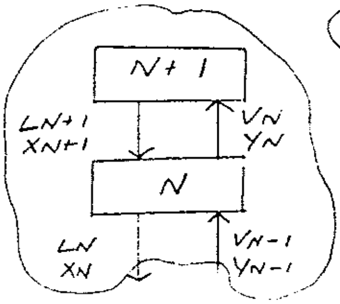
$t^* = t/t_c$

$$\frac{\partial(v v^*)}{\partial(t_c t^*)} = \frac{\mu}{\rho} \frac{\partial^2(v v^*)}{\partial(y^{*2} L^2)} = \frac{\mu v}{\rho L^2} \frac{\partial^2 v^*}{\partial y^{*2}}$$

$$\frac{1}{t_c} \frac{\partial v^*}{\partial t^*} = \frac{\mu}{\rho L^2} \frac{\partial^2 v^*}{\partial y^{*2}}$$

$$\frac{1}{t_c} = \frac{\mu}{\rho L^2} \qquad t_c = \frac{\rho L^2}{\mu}$$

Min. # of stages at total reflux



Assumes
CMO
$y_N = x_{N+1}$
$x_n = y_{N-1}$

$L_N x_N = V_{N-1} y_{N-1}$

$K_N = \frac{y_N}{x_N}$

$x_N = \frac{y_N}{K_N}$

$y_N = x_N K_N$

$y_N = x_{N+1}$

$x_{N+1} = x_N K_N$

$K_{N-1} = \frac{y_{N-1}}{x_{N-1}}$

$= \frac{x_N}{x_{N-1}}$

$x_N = K_{N-1} x_{N-1}$

$y_N = x_N K_N$

$y_N = K_{N-1} K_N x_{N-1}$

$y_N = K_N K_{N-1} \ldots K_2 x$

$= x, \pi$