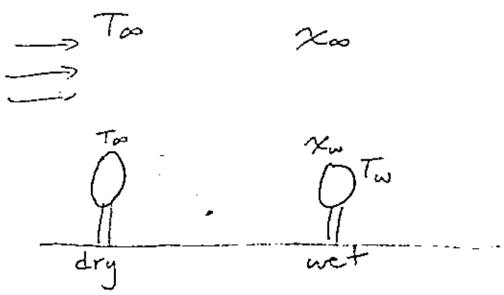
Transport #2(a)

Wet-bulb thermometer



Water evaporates from wet bulb; ΔH^{vap} lowers wet bulb's temperature w.r.t. T_∞.

Fick's Law

$q = h(T_\infty - T_w) =$

$$N_{H_2O} = kC(x_w - x_\infty)$$

$$h(T_\infty - T_w) = (N_{H_2O})(MW_{H_2O})(\Delta H^{vap}_{H_2O})$$

$$\frac{h}{k}(T_\infty - T_w) = C(x_w - x_\infty)(MW_{H_2O})(\Delta H^{vap})$$

— Chilton-Colburn Analogy (betw. Mass, Heat, Momentum)

$$k \Leftrightarrow \frac{h}{\rho \hat{C}_p} \Leftrightarrow \frac{fv}{2}$$

$$\frac{k}{v}\left(\frac{\nu}{D}\right)^{2/3} = \frac{h}{\rho \hat{C}_p v}\left(\frac{\nu}{\alpha}\right)^{2/3} = \frac{f}{2}$$

$\frac{f}{2} \approx \frac{Nu}{Re\,Pr^{1/3}} = \frac{Sh}{Re\,Sc^{1/3}}$

Reynolds $\frac{f}{2} \approx \frac{Nu}{Re} \approx \frac{Sh}{Re}$

$\frac{f}{2} = \frac{h\rho}{k\rho v}$

$$\left(\frac{\nu}{D}\right) \approx \left(\frac{\nu}{\alpha}\right) \approx 1 \quad \text{for gases}$$

$$\text{so} \quad \frac{k}{v} \cong \frac{h}{\rho \hat{C}_p v} \Rightarrow \frac{h}{k} \approx \rho \hat{C}_p$$

$$\rho \hat{C}_p (T_\infty - T_w) = C(MW)(x_w - x_\infty)(\Delta H^{vap})$$

$$x_\infty = x_w - \left(\frac{\hat{C}_p}{\Delta H^{vap}}\right)(T_\infty - T_w) \Rightarrow \text{can get mole fraction of water in air}$$

x_w: x^{sat} at T_w from VLE relation

$\frac{\hat{C}_p}{\Delta H^{vap}}$: known values

T_∞, T_w: measured values

⇓

for relative humidity, divide by x^{sat} at T_∞

Transport #2(b.)

Iceberg being towed in the ocean.

- Don't want detailed velocity profiles
  ↳ friction factor (convective) treatment

Momentum:

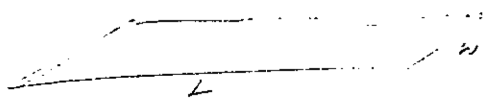
$$F_{towing} = \left(\tfrac{1}{2}\rho v^2\right)(LW)\,f$$

wetted area; assume major surface contacting water is bottom of iceberg, modeled as a flat, rectangular plate

Mass (?)

Iceberg is pure ice water; Sea water is salty ↳ $x_{s\infty}$

↳ $x_s = 0$

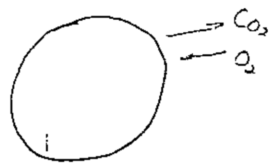


⇓
so?
(how to relate to momentum?)

Heat(?) ΔH^{fusion}

Transport #2 (c.)

Burning Carbon Particle



$C + O_2 \longrightarrow CO_2 \quad \Delta H_{rxn}$

Assumption / B.C.'s

$N_{O_2} = -N_{CO_2}$ at steady state

$x_{O_2} \cong 0$ @ surface of particle (@ $r = R$)
(process diffusion limited)

How to approach?

$x_{O_2} = 0.2$ far from surface (@ $r \Rightarrow \infty$)
↑ composition of air

Diffusion? (Molecular Mechanism)
↳ Concentration gradient

is this an accurate B.C.? Should it be $x_{CO_2} \approx 0.2$?
N_2 still present; no reason for it to be lacking near particle surface.

$x_{CO_2} \cong 1.0$ @ $r = R$

Overall transfer coefficients?
→ assume that outside of particle Boundary Layer, air is <u>well mixed</u>

$x_{CO_2} \approx 0$ @ $r \to \infty$ (0.03% in air)

Assume size of particle doesn't change

How to incorporate energy balance?
$(T_{particle\ surface} - T_\infty)$ driving force for convective heat transfer?
$= (\Delta H_{rxn})(N_{O_2})(MW)$?

Transport

(Heat & mass transfer)

#2 c.) Burning Carbon particle:

rxn: $C + O_2 \rightarrow CO_2 + \text{heat}$

model as a <u>Sphere</u>

<u>Issues:</u>

Heat generation (ΔH_{rxn})

diffusion of reactant (O_2) in, $N_{CO_2} = -N_{O_2}$

product (CO_2) out

Changing particle size?)

ΔH fusion

(#2 b.) (Heat) (Momentum & Mass transfer)

Iceberg being towed in ocean.

model as fluid flowing over dissolving (melting) flat plate?

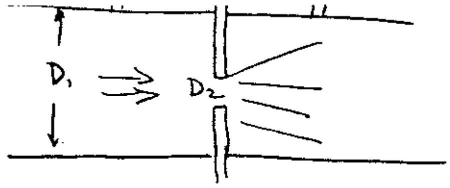
(#2a) → see Cussler, <u>Diffusion</u>

Transport #6

$$\text{Sherwood \#} = Sh = \frac{k\,l}{D} \equiv \frac{\text{mass transfer velocity}}{\text{diffusion velocity}}$$

mass transfer coeff.
chas. length
diffusion coeff.

a) [redacted]

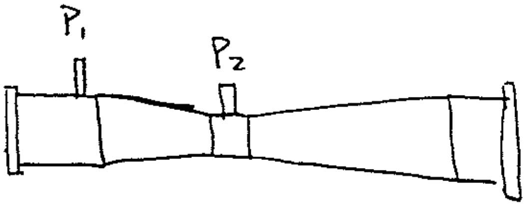


large permanent pressure drop
cheap, easy to maintain

$$q_f\left(\frac{ft^3}{sec}\right) = C_d S_c \sqrt{\frac{2\rho(p_1-p_2)}{1-\left(\frac{S_c}{S_1}\right)^2}}$$

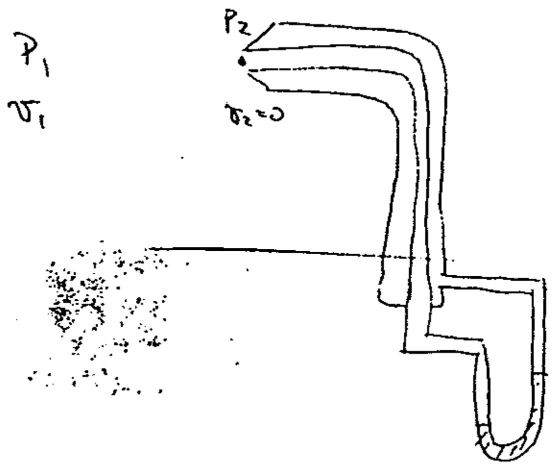
C_d = coefficient of discharge
S_c = cross-sectional area at point of minimum
S_1 = cross-sectional area in upstream duct.

b) Venturi Meter



$$q_1 = \frac{KYA_2}{\rho_1}\sqrt{\frac{2(p_1-p_2)\rho_1}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

) Pitot Tube



$$(V_1)^2 = 2\left(-\frac{\Delta P}{\rho} + \Sigma F\right)$$

$$V_1 = \sqrt{\frac{2(-\Delta P)}{\rho}}$$

Transport #10

(a) Bernoulli's Equation. (Denn p 90, BSL p 211)
(for viscous isothermal fluids)

Assumes: - Steady state
- single phase, uniform properties (ρ constant)
- uniform equivalent pressure (the same over entire cross-section)

$$\frac{\alpha_2}{2}\langle V\rangle_2^2 + gh_2 = \frac{\alpha_1}{2}\langle V\rangle_1^2 + gh_1 - \int_{P_1}^{P_2}\frac{dP}{\rho} + \delta W_s - l_v$$

Differential form:

$$\tfrac{1}{2}d(\alpha\langle V\rangle^2) + g\,dh + \frac{1}{\rho}dP = dW_s - \underbrace{(T\,ds - dQ_H)}_{dl_v\ (\geq 0)}$$

= 0 only for reversible processes.

Notation:

$V \equiv$ component of velocity vector normal to surface.

$\alpha \equiv \dfrac{\langle V^3\rangle}{\langle V\rangle^3}$

W_s, $Q_H \equiv$ work, heat added to system

$l_v \equiv$ "viscous losses" per unit mass (rate at which mech. energy converted to heat).

Bernoulli equation comes from (general equation): (cons. of energy)

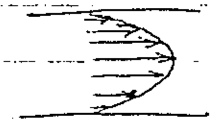
$$\frac{d}{dt}\int_{z_1}^{z_2}\langle\rho(e + \tfrac{1}{2}v^2 + gh)\rangle A\,dz$$

$$= \langle\rho(\overset{\text{internal}}{e} + \overset{\text{kinetic}}{\tfrac{1}{2}v^2} + \overset{\text{potential}}{gh})V\rangle_1 A_1 - \langle\rho(e + \tfrac{1}{2}v^2 + gh)V\rangle_2 A_2$$ } Total energy variation

$$+ \underbrace{\langle pV\rangle_1 A_1 - \langle pV\rangle_2 A_2}_{\text{rate of doing work to move fluid into and out of control volume.}} + \underbrace{\dot{Q}_H + \dot{W}_s}_{\text{heat + shaft work added to system.}}$$

Transport #10

(b.) Hagen - Poiseuille Law



flow through cylinder
Laminar, $Re < 2100$

Derivation:

Assumptions: $v_z = v_z(r)$ only
$v_\theta, v_r = 0$

$$N\text{-}S: \quad 0 = -\frac{\partial P}{\partial z} = \mu \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right), \quad P = P(z) \text{ only}$$

$$\frac{1}{\mu}\frac{dP}{dz} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \text{const} = \frac{1}{\mu}\frac{\Delta P}{L}$$

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{1}{\mu}\frac{\Delta P}{L} r$$

$$r\frac{dv_z}{dr} = \frac{1}{2\mu}\frac{\Delta P}{L}r^2 + C_1$$

$$\frac{dv_z}{dr} = \frac{1}{2\mu}\frac{\Delta P}{L}r + \frac{C_1}{r} \Rightarrow C_1 \to 0 \text{ since } \frac{dv_z}{dr} \text{ finite @ } r = 0$$

$$v_z = \frac{1}{4\mu}\frac{\Delta P}{L}r^2 + C_2$$

no-slip B.C.: @ $r = R$, $v_z = 0$ $\Rightarrow$

$$v_z(r) = \frac{R^2}{4\mu}\left(-\frac{\Delta P}{L}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$(-\Delta P > 0)$

$$\langle v_z \rangle = \frac{1}{\pi R^2}\int_{area} v_z \, dA = \frac{R^2}{8\mu}\left(\frac{-\Delta P}{L}\right)$$

$$\int_{area} v_z \, dA = \boxed{Q = \frac{\pi R^4}{8\mu}\left(\frac{-\Delta P}{L}\right) = \frac{\pi D^4}{128\mu}\left(\frac{-\Delta P}{L}\right)}$$

Note: $f = \left(\frac{-\Delta P}{L}\right)\frac{D}{2\rho v^2} \Rightarrow \frac{D}{2}\left(\frac{-\Delta P}{L}\right) = \frac{8\mu\langle v_z \rangle}{R} = \frac{8\mu\langle v_z \rangle}{D/2}$

$$f = \frac{16\mu\langle v_z \rangle}{\rho\langle v_z \rangle^2 D} = \frac{16\mu}{\rho\langle v_z \rangle D} = \frac{16}{Re}$$

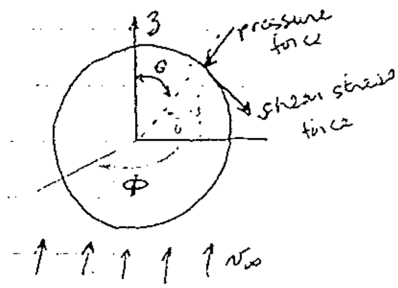
Fanning friction factor from 1st principles

Transport #10

(c.) Stokes's Law — flow around a sphere at $Re < 1$ (inertialess regime)

drag coefficient, $C_D = \frac{24}{Re}$

drag force, $F_D = 3\pi\mu v_\infty D = 6\pi\mu v_\infty R$



In Denn p248–254 ⇒ derivation (cont, N.S., sol'n of DE's to get v_r, v_θ, $\tau_{r\theta}$, and pressure)

In BSL p56 → above expressions given w/o derivation.

force/area acting tangentially in θ-dir. → $\tau_{r\theta} = \frac{3}{2}\frac{\mu v_\infty}{R}\left(\frac{R}{r}\right)^4 \sin\theta$

$p = p_0 - \rho g z - \frac{3}{2}\frac{\mu v_\infty}{R}\left(\frac{R}{r}\right)^2 \cos\theta$

$v_r = v_\infty\left[1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\cos\theta$

$v_\theta = v_\infty\left[1 - \frac{3}{4}\left(\frac{R}{r}\right) - \frac{1}{4}\left(\frac{R}{r}\right)^3\right]\sin\theta$

B.C.'s/assumptions

no slip: $v_r, v_\theta = 0$ @ $r = R$

no φ dependence: $\frac{d}{d\phi} = 0$, $v_\phi = 0$

Creeping flow (low Re, <1)

z-component of pressure force per unit area: $-p\cos\theta$

unit area $= (R\sin\theta\, d\phi)(R\, d\theta) = R^2 \sin\theta\, d\phi\, d\theta$

normal force $F_n = \int_0^{2\pi}\int_0^{\pi} \left(-p|_{r=R} \cos\theta\right) R^2 \sin\theta\, d\theta\, d\phi$

↖ plug in for this

$F_n = \frac{4}{3}\pi R^3 \rho g + 2\pi\mu R v_\infty$

z-component of shear stress $= (-\tau_{r\theta})(-\sin\theta)$

$(-\cos(\frac{\pi}{2} - \theta))$

friction force $F_t = \int_0^{2\pi}\int_0^{\pi} \left(+\tau_{r\theta}|_{r=R} \sin\theta\right) R^2 \sin\theta\, d\theta\, d\phi$

↖ plug in for this

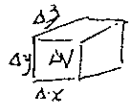
$F_t = 4\pi\mu R v_\infty$

$$F = \underbrace{\frac{4}{3}\pi R^3 \rho g}_{\text{buoyant force}} + \underbrace{\underbrace{2\pi\mu R v_\infty}_{\text{form drag}} + \underbrace{4\pi\mu R v_\infty}_{\text{friction drag}}}$$

Stokes's Law: $F_k = F_D = 6\pi\mu R v_\infty$ (force assoc. w/ fluid mvmt.)

Transport #10

(d) Continuity equation (conservation of mass in control volume)



rate of change of mass in ΔV = $\frac{d}{dt}(\underset{\text{avg. density}}{\rho}\,\Delta x\,\Delta y\,\Delta z)$

= (mass in − mass out)@ each face

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial \rho v_y}{\partial y} - \frac{\partial \rho v_z}{\partial z}$$

$$\Downarrow$$

$$\underbrace{\frac{\partial \rho}{\partial t} + v_x\frac{\partial \rho}{\partial x} + v_y\frac{\partial \rho}{\partial y} + v_z\frac{\partial \rho}{\partial z}}_{\frac{D\rho}{Dt}} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

(rectangular)

(Cylindrical):

$$-\frac{\partial \rho}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z)$$

(Spherical):

$$-\frac{\partial \rho}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi)$$

Transport #10

(e) Momentum Equation (Cauchy), Navier-Stokes Equns

Basis: conservation of momentum in control volume

rate of change of momentum = mom. entering − mom. leaving + Σ forces acting on control volume.

stresses (shear, ~~body, gravity~~), pressure → normal forces, + body forces (gravity)

$$(x\text{-eqn})\quad \frac{\partial}{\partial t}\underbrace{(\rho v_x)}_{x\text{-mom/volume}}\underbrace{\Delta x\Delta y\Delta z}_{\text{volume}} = (\rho v_x)(\Sigma v_i \cdot \text{area})\Big|_x - (\ )(\ )\Big|_{x+\Delta x}$$

$$+ \underbrace{\text{stresses}(\text{area})}_{(\text{in}-\text{out on faces})} + \text{body force}$$

in x-dir:

$$\begin{cases} \sigma_{xx}(\Delta y\Delta z) + \tau_{xy} + \tau_{xz} = \sigma_x \\ \tau_{yx}(\Delta x\Delta z) \\ \tau_{zx}(\Delta y\Delta z) \end{cases}$$

σ_x: stress acting on x-face

(usu. only gravity, sometimes electrical)

$$\rho g_x \Delta x\Delta y\Delta z$$

→ Stresses are symmetric: $\tau_{xy} = \tau_{yx}$ (from conservation of angular momentum) → no internal couples arising from fluid structure.

→ $\sigma_{xx} = \tau_{xx} - P \Rightarrow \tau_{xx} = \sigma_{xx} + P$

<u>Cauchy momentum equation:</u> (in rectangular coords.)

$$(x\text{-dir})\quad \rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho g_x$$

<u>Stress constitutive equation</u>

$$\tau_{xx} = \mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla\cdot\underline{V})\right] \qquad \left(\nabla\cdot\underline{V} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$

$$\mathcal{P} = p + \rho g h - \tfrac{1}{3}\mu\,\nabla\cdot\underline{V}$$

<u>Navier-Stokes:</u>

$$\rho\frac{D\underline{V}}{Dt} = -\nabla\mathcal{P} + \mu\nabla^2\underline{V} \qquad (\text{general})$$

$$\left(\begin{matrix}x\text{-dir,}\\ \text{rectangular}\end{matrix}\right) \Rightarrow \rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial \mathcal{P}}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right),$$

Other coords, other components ⇒ pp 159-162 in Denn

Transport #14 (and # 42)

$b = \frac{k}{v} = \frac{h\Delta T}{v} = \frac{1}{2}\rho v$

$= \frac{h}{\rho c_p}$

N_1 (mass flux) $= k \Delta c_i$
$= (a + b v)\Delta c_1$

q (heat flux) $= h \Delta T = \frac{h}{\rho \hat{C}_p} \Delta(\rho \hat{C}_p T)$
$= (a' + b' v) \Delta (T \rho \hat{C}_p)$

τ (momentum flux) $= f(\frac{1}{2}\rho v^2) = \left(\frac{f v}{2}\right)(\rho v)$
$= (a'' + b'' v)(\rho v)$

$\frac{k}{v} = \frac{h}{\rho c_p v} = \frac{f}{2R}$

$\frac{k}{v} = \frac{h}{\rho c_p v} = \frac{f}{2}$

$a, a', a'' \rightarrow$ due to diffusion (molecular process, indep of velocity
$b, b', b'' \rightarrow$ due to convection (effect of eddies)

When eddies dominate (very rapid turbulent flow) —

$\Downarrow$

<u>Reynolds Analogy:</u> $b = b' = b''$

So:

$$\frac{k}{v} = \frac{h}{\rho \hat{C}_p v} = \frac{f}{2}$$

(only good for gases where $D \approx \alpha \approx \nu \approx 0.1\ \text{cm}^2/\text{sec.}$)

<u>Chilton-Colburn Analogy</u>

(Changes in liquids best represented as $Pr \left(\frac{\nu}{\alpha}\right)$ & $Sc \left(\frac{\nu}{D}\right)$ numbers.)

So:

$$b = \frac{k}{v}\left(\frac{\nu}{D}\right)^{2/3}$$

$$b' = \frac{h}{\rho \hat{C}_p v}\left(\frac{\nu}{\alpha}\right)^{2/3}$$

$\left(\alpha = \frac{k}{\rho \hat{C}_p}\right.$ ← thermal conductivity $\left.\right)$

$$b'' = \frac{f}{2}\left(\frac{\nu}{\nu}\right)^{2/3} = \frac{f}{2}$$

$\Downarrow$

$$\frac{k}{v}\underbrace{\left(\frac{\nu}{D}\right)^{2/3}}_{Sc} = \frac{h}{\rho \hat{C}_p v}\underbrace{\left(\frac{\nu}{\alpha}\right)^{2/3}}_{Pr} = \frac{f}{2}$$

(18) Friction factor → flow in conduits
(Transport) Drag coefficient → flow around submerged objects

$$F_{Kinetic} = (Area_{(char.)})(\text{Kinetic energy per volume})\, f$$

$$F_k = A K f \Rightarrow f = \text{friction factor, drag coefficient.}$$

<u>flow in conduits</u> (ρ = constant density of fluid)

A = wetted surface area

$K = \frac{1}{2}\rho\langle v\rangle^2$

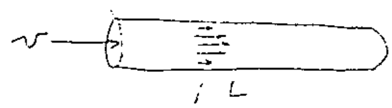
$F_k = [(P_0 - P_L) + \rho g(h_0 - h_L)](\text{cross-sec. area})$ ← A_{cs}

(a) <u>for pipe</u>: (length L, radius R, diam. D)

$A = 2\pi R L$

$K = \frac{1}{2}\rho\langle v\rangle^2$

$A_{cs} = \pi R^2$



$$F_K = (P_0 - P_L) + \rho g(h_0 - h_L)(\pi R^2) = (\mathcal{P}_0 - \mathcal{P}_L)\pi R^2 = (2\pi R L)(\tfrac{1}{2}\rho\langle v\rangle^2) f$$

$$f = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{4}\left(\frac{D}{L}\right)\frac{1}{(\frac{1}{2}\rho\langle v\rangle^2)}$$ getting f from experimental data

$$F_K = \int_0^L \int_0^{2\pi} \left(-\mu \frac{\partial v_z}{\partial r}\right)_{r=R} R\, d\theta\, dz$$

for fully developed flow, $\frac{\partial v_z}{\partial r} \neq f(z)$; $\frac{\partial v_z}{\partial r} \neq f(\theta)$

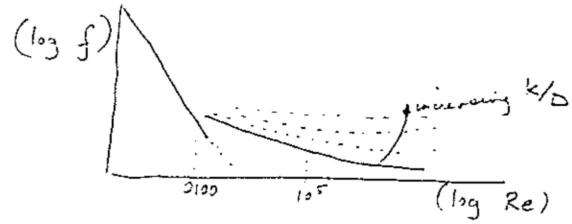
$$F_K = (2\pi R L)(-\mu)\left(\frac{\partial v_z}{\partial r}\right)_{r=R} = (2\pi R L)(\tfrac{1}{2}\rho\langle v\rangle^2) f$$

$$f = \frac{-2\mu}{\rho\langle v\rangle^2}\left(\frac{\partial v_z}{\partial r}\right)_{r=R} = \left(\frac{\mu}{\rho\langle v\rangle D}\right)\left(-\frac{2D}{\langle v\rangle}\left(\frac{\partial v_z}{\partial r}\right)_{r=R}\right)$$

contributions to f:
friction drag only
- no form drag -

$$\boxed{f = \left(\frac{1}{Re}\right)\left(\frac{-2D}{\langle v\rangle}\left(\frac{\partial v_z}{\partial r}\right)_{r=R}\right)} \Rightarrow f = f(Re)$$

for pipe flow.



T, 18, cont.

pipe

$$f = \frac{16}{Re} \quad \text{for } Re < 2100 \qquad \text{(Hagen-Poiseuille)}$$

$$f = \frac{0.0791}{(Re)^{1/4}} \quad \text{for } 2100 < Re < 10^5 \qquad \text{(Blausius)}$$

also to consider: - tube wall roughness $\Rightarrow \frac{k}{D}$ → height of roughnesses, → diameter

- non-circular cross section

↳ mean hydraulic radius $R_h = \frac{S}{Z}$ → cross section area, → wetted perimeter

$D = 4 R_h$

(useful for turbulent flow only.)

flow around submerged objects (ρ = constant)

A = area of solid projected onto plane $\perp$ to v_∞

$K = \frac{1}{2}\rho v_\infty^2$

F_K = gravitational - buoyant forces = $(\text{Volume})\rho_{sphere}\, g - (\text{Volume})\rho g$

↳(at terminal velocity.)

(b) for flow around a sphere (radius R, diam D)

$A = \pi R^2$

$K = \frac{1}{2}\rho v_\infty^2$

$Vol = \frac{4}{3}\pi R^3$

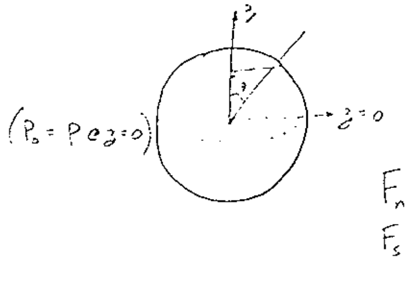
$$F_K = \left(\tfrac{4}{3}\pi R^3\right)(g)(\rho_{sph} - \rho) = (\pi R^2)\left(\tfrac{1}{2}\rho v_\infty^2\right) f$$

$$\boxed{f = \frac{4}{3}\,\frac{D g\,(\rho_{sph} - \rho)}{v_\infty^2\ (\rho)}}$$

v_∞ is terminal velocity (quantity measured for experimental determination of f)

contributions to $f \Rightarrow$ friction drag and form drag.

friction drag: shear
form drag: pressure variations at different surfaces.



$$F_K = \underbrace{F_n - F_s}_{\text{form}} + \underbrace{F_t}_{\text{friction}}$$

$$F_n = \int_0^{2\pi}\int_0^{\pi} \left(-P|_{r=R} \overbrace{\cos\theta}^{\text{in } z\text{-direction}}\right) R^2 \sin\theta \, d\theta \, d\phi$$

$$F_s = \int_0^{2\pi}\int_0^{\pi} \left\{-(P_0 - \rho g z)|_{r=R} \cos\theta\right\} R^2 \sin\theta \, d\theta \, d\phi$$

$$F_t = \int_0^{2\pi}\int_0^{\pi} \left\{-\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]\Big|_{r=R} \sin\theta\right\} R^2 \sin\theta \, d\theta \, d\phi$$

$\Rightarrow f = f(Re)$

$$\text{Viscosity} = \frac{\text{Shear stress}}{\text{Shear rate}} = \frac{\tau_{rz}}{\left(\frac{\partial v_z}{\partial r}\right)}$$

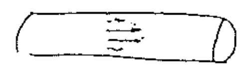
<u>Viscometry</u> – capillary, cone and plate, coaxial cylinders

- <u>Capillary Viscometer</u> → need to measure: flow rate Q, Pressure drop $\Delta \mathcal{P}$; Know: radius R, length L ⇓ get μ
  - laminar flow ($Re < 2100$)
  - steady state, fully-developed flow

  Solve N-S equation, get velocity profile:

$$v_z(r) = \frac{R^2}{4\mu}\left(-\frac{\Delta \mathcal{P}}{L}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$$

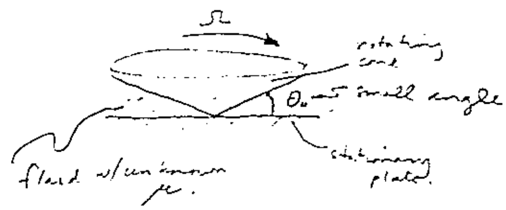
$$\langle v_z \rangle = \frac{1}{\pi R^2}\int_{area} v_z \, dA = \frac{R^2}{8\mu}\left(-\frac{\Delta \mathcal{P}}{L}\right)$$



$$\mu = \frac{\pi R^4}{8}\left(-\frac{\Delta \mathcal{P}}{L}\right)\frac{1}{Q}$$ ← volumetric flow rate

- <u>Cone and Plate Viscometer</u>

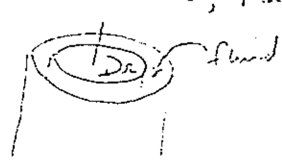
  cone rotated with <u>known</u> angular velocity Ω; <u>Torque</u> required <u>measured</u>



only $\tau_{\theta\phi}$ important

Rigorous derivation → BSL pp 98 – 101

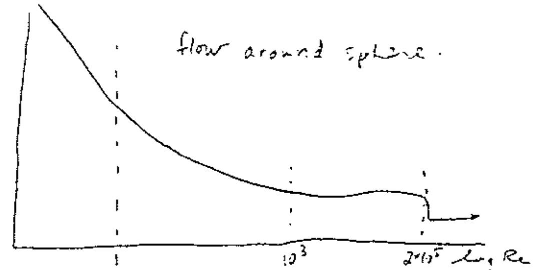
- <u>Coaxial cylinder Viscometer</u> (Couette-Hatschek, MacMichael viscometer)



either inner or outer cylinder rotated while other remains stationary → essentially Couette flow if $R_o - R_i$ small

Known: Ω (angular velocity) Measured: torque required to turn @ Ω.

T, 18, cont



Sphere

$C_D = f \cong \dfrac{24}{Re}$ for $Re < 0.1\ (1.0)$ Stokes's Law / Creeping flow
10% error

$C_D = f \cong \dfrac{18.5}{Re^{3/5}}$ for $2 < Re < 5\times10^2$ (10^3) intermediate region

$C_D = f \cong 0.44$ for $5\times10^2 < Re < 2\times10^5$ (10^3) Newton's "Law" region

(c.) flow around submerged flat plate (p 203 BSL Problem 6.G)
width W, length L, wetted on both sides

(i) $F_k = 1.328\sqrt{\rho\mu L W^2 v_\infty^3}$ friction drag only (plate is flat)

$= (WL\rho v_\infty^2) f$ $F_k = (2WL)(\tfrac{1}{2}\rho v_\infty^2) f$ → definition of f
(wetted area)

$= 1.328\sqrt{\mu W v_\infty}\sqrt{\rho L W v_\infty^3}$

$f = 1.328\dfrac{\sqrt{\mu \cancel{W} v_\infty}}{\sqrt{\rho L \cancel{W} v_\infty^2}} = 1.328\sqrt{\dfrac{\mu}{\rho L v_\infty}}$ $\boxed{f = 1.328\, Re^{-1/2}}$ Laminar flow.

(ii) $F_k = 0.074\,\rho v_\infty^2 W L\left(\dfrac{L v_\infty \rho}{\mu}\right)^{-1/5}$ → accurate for $5\times10^5 < \dfrac{L\rho v_\infty}{\mu} < 2\times10^7$
(turbulent flow)

$= \rho v_\infty^2 W L f$

$f = 0.074\left(\dfrac{L v_\infty \rho}{\mu}\right)^{-1/5}$ $\boxed{f = 0.074\, Re^{-1/5}}$ Turbulent flow
$5\times10^5 < Re < 2\times10^7$

Flow around other objects:

$f = \dfrac{F_k}{(\tfrac{1}{2}\rho v_\infty^2)\, A_{projected}}$

e.g. flow over very long cylinder $(L \gg D)$

$f = \dfrac{F_k}{(\tfrac{1}{2}\rho v_\infty^2)(LD)} = \dfrac{F_k/L}{\tfrac{1}{2}\rho v_\infty^2 D}$

drag force per unit cylinder length.

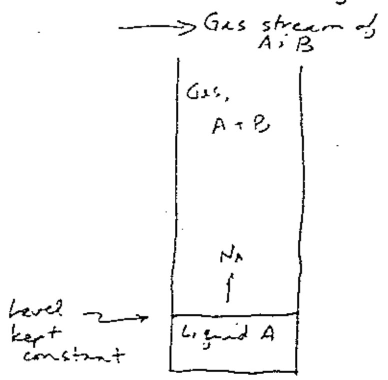
Transport #22

How are diffusivity and viscosity of a mixture determined?
math 230 notes

Diffusivity:

from BSL p523-25

one possibility: @ steady state, $N_B = 0$



$$N_{A_z} = -c\mathcal{D}_{AB}\frac{\partial x_A}{\partial z} + x_A\left(N_{A_z} + \cancel{N_{B_z}}^{\,0}\right)$$

$$N_{A_z} = \frac{-c\mathcal{D}_{AB}}{1-x_A}\frac{\partial x_A}{\partial z}$$

$$-\frac{\partial N_{A_z}}{\partial z} = 0$$

if know concentration profile of A, (mole fraction) from solution of D.E.
C (overall gas concentration), can get $\mathcal{D}_{AB}$.

another possibility: discussed in ChE 230

1. Know rate of Evaporation → rate evap.

$$N_{CCl_4} = (\Delta Vol)\rho(MW)A\left(\frac{1}{time}\right)$$

$$N_A = \frac{P}{RT}\frac{\bar{D}_{AB}}{z_2 - z_1}\ln\left(\frac{P_{B2}}{P_{B1}}\right)$$

→ Air

μ:

correlations (BSL 18)

based on P_c', T_c',

$$\mu_e = \sum x_i \mu_c$$

→

Viscometry → (...)

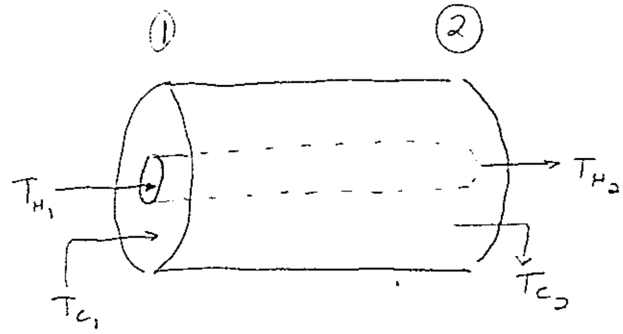
Transport #26

What is the angular dependence of Nu for a falling drop?

$$Nu = \frac{hD}{k}$$

dimensionless temperature gradient at the surface.

Transport # 20



$$dQ = U\,dA\,(T_H - T_c)$$

$$= -(wC_p)_H\, dT_H$$

$$= (wC_p)_c\, dT_c$$

$$dT_H = \frac{-dQ}{(wC_p)_H}$$

$$dT_c = \frac{dQ}{(wC_p)_c}$$

$$d(T_H - T_c) = dT_H - dT_c = -\left(\frac{1}{(wC_p)_H} + \frac{1}{(wC_p)_c}\right) dQ$$

$$= -\left(\frac{1}{(wC_p)_H} + \frac{1}{(wC_p)_c}\right)(U\,dA)(T_H - T_c)$$

$$\frac{d(T_H - T_c)}{T_H - T_c} = \int_1^2 d\ln(T_H - T_c) = -\int_A U\,dA\left(\frac{1}{(wC_p)_H} + \frac{1}{(wC_p)_c}\right)$$

$$\ln\left(\frac{(T_H - T_c)_2}{(T_H - T_c)_1}\right) = \ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{(wC_p)_H} + \frac{1}{(wC_p)_c}\right)$$

$$Q = -(wC_p)_H (T_{H_2} - T_{H_1}) \Rightarrow \frac{1}{(wC_p)_H} = \frac{T_{H_1} - T_{H_2}}{Q}$$

$$= (wC_p)_c (T_{c_2} - T_{c_1}) \Rightarrow \frac{1}{(wC_p)_c} = \frac{T_{c_2} - T_{c_1}}{Q}$$

$$-UA\left(\frac{1}{(wC_p)_H} + \frac{1}{(wC_p)_c}\right) = -UA\left(\frac{T_{H_1} - T_{H_2}}{Q} + \frac{T_{c_2} - T_{c_1}}{Q}\right)$$

$$= -\frac{UA}{Q}\left(-(T_H - T_c)_2 + (T_H - T_c)_1\right)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \frac{UA}{Q}(\Delta T_2 - \Delta T_1)$$

$$\boxed{Q = UA\left(\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}\right)}$$

overall heat transfer coefficient for cocurrent or countercurrent tube & shell heat exchangers.

Transport #34

$$j_H = \left(\frac{h}{\rho \hat{C}_p v}\right)\left(\frac{\nu}{\alpha}\right)^{2/3}$$ from experimental data, approximating $j_H \propto \frac{1}{Re}$

from Reynolds Analogy

$Re \equiv \frac{\text{inertial forces}}{\text{viscous forces}}$

$$= \frac{h}{\rho \hat{C}_p v}\left(\frac{\hat{C}_p \mu}{k}\right)^{2/3}$$

$$= \left(\frac{h}{k}\right)\left(\frac{\mu}{\rho v}\right)\left(\frac{k}{\hat{C}_p \mu}\right)^{1/3}$$

$$= \left(\frac{hD}{k}\right)\left(\frac{\mu}{\rho v D}\right)\left(\frac{\alpha}{\nu}\right)^{1/3}$$

$$= Nu\ Re^{-1}\ Pr^{-1/3}$$

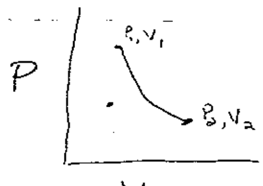
#38

Lewis # $\quad \frac{\alpha}{\mathscr{D}} = \frac{k}{\rho \hat{C}_p \mathscr{D}}$ ← independent of gas-phase velocity (same effect on heat and mass transfer, effects cancel)

$\frac{k}{\rho c_p D_{AB}}$

Transport #50

Derive equations for gas undergoing isentropic expansion.



1st Law for non-flow: $dU = dQ - dW$

isentropic, so $dQ = 0$

Assume ideal gas. Equation reduces to:

$$dU = -dW$$

$$C_v dT = -P dV = -\frac{RT}{V} dV$$

$$C_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{V_1}^{V_2} \frac{dV}{V}$$

$$C_v \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{V_2}{V_1}\right)$$

$$\ln \frac{V_2}{V_1} = \ln\left(\frac{T_2}{T_1}\right)^{-\frac{C_v}{R}} \Rightarrow \left(\frac{V_2}{V_1}\right) = \left(\frac{T_2}{T_1}\right)^{-\frac{C_v}{R}}$$

For ideal gas,

$$H = U + PV = U + RT \Rightarrow dH = dU + RdT \Rightarrow C_p dT = C_v dT + RdT$$

So: $C_p = C_v + R$ also (notation) $\frac{C_p}{C_v} = \gamma$

$$\gamma = \frac{C_v + R}{C_v} = 1 + \frac{R}{C_v}$$

$$1 - \gamma = -\frac{R}{C_v}$$

$$-\frac{C_v}{R} = \frac{1}{1-\gamma} \Rightarrow \boxed{\left(\frac{V_2}{V_1}\right) = \left(\frac{T_2}{T_1}\right)^{\left(\frac{1}{1-\gamma}\right)}} \text{ or } \left(\frac{T_2}{T_1}\right) = \left(\frac{V_2}{V_1}\right)^{(1-\gamma)}$$

$$\frac{V_2}{V_1} = \frac{\frac{RT_2}{P_2}}{\frac{RT_1}{P_1}} = \frac{P_1 T_2}{P_2 T_1} \Rightarrow \left(\frac{P_1}{P_2}\right) = \left(\frac{T_1}{T_2}\right)\left(\frac{T_2}{T_1}\right)^{\left(\frac{1}{1-\gamma}\right)} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{1-(1-\gamma)}{1-\gamma}\right)}$$

$$\boxed{\left(\frac{P_1}{P_2}\right) = \left(\frac{T_2}{T_1}\right)^{\left(\frac{\gamma}{1-\gamma}\right)}}$$

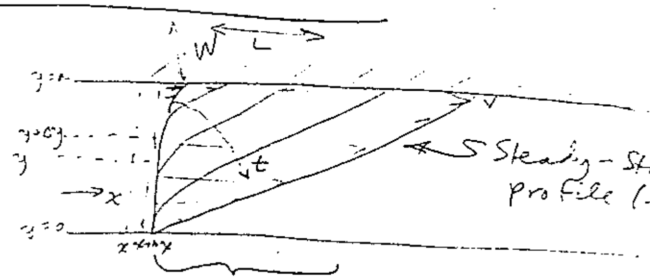
$$\frac{T_2}{T_1} = \frac{P_2 V_2 / R}{P_1 V_1 / R} = \frac{P_2 V_2}{P_1 V_1} \Rightarrow \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_1}\right)^{(1-\gamma)} = \left(\frac{V_2}{V_1}\right)^{(1-\gamma-1)}$$

$$\boxed{\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma}} \Rightarrow P_2 V_2^{\gamma} = P_1 V_1^{\gamma} = PV^{\gamma} = \text{const.}$$

Transport #58

push

+1



Why is s.s. profile linear?

Mom. Balance

$$-\mu\left(\frac{\partial v_x}{\partial y}\right)\Big|_y (LW) + \mu\left(\frac{\partial v_x}{\partial y}\right)\Big|_{y+\Delta y} (LW) = 0$$

$$\xrightarrow{\text{const}} \mu\left(\frac{\partial^2 v_x}{\partial y^2}\right) = 0$$

B.C.'s

@ $y=0$ $v_x = 0$

@ $y=a$ $v_x = V$

Integrate $\int$

$$\frac{\partial v_x}{\partial y} = C_1$$

Integrate $\int$

$$v_x = C_1 y + C_2$$

Apply B.C.'s:

$$0 = C_1(0) + C_2 \Rightarrow C_2 = 0$$

$$V = C_1(a) \Rightarrow C_1 = \frac{V}{a}$$

So we see that $v_x = \frac{V}{a} y$ is linear, because shear stress $-\mu \frac{\partial v_x}{\partial y} = \frac{-\mu V}{a}$ is constant.

Part 2 driving force for fluid flow in a pipe: pressure drop + gravity (if not horizontal)

driving force here: Force applied to keep top plate moving at V.

force $= \left((\tau_{yx})\big|_{y=a}\right)(\text{Area} = LW)$

$\Rightarrow \frac{\text{force}}{\text{unit area of plate}} = -\mu \frac{V}{a}$

Momentum balance of above.
More generalized:

$$\underbrace{\frac{\partial}{\partial t}(\rho \underline{v})}_{0} = -\underbrace{[\nabla\cdot\rho\underline{v}\underline{v}]}_{0} - \underbrace{\nabla p}_{0} - [\nabla\cdot\underline{\tau}] + \underbrace{\rho g}_{0} \Rightarrow$$

momentum transport in y-direction only ?

$$-\frac{\partial \tau_{yx}}{\partial y} = \mu \frac{\partial^2 v_x}{\partial y^2} = 0$$