ifferences between momentum, heat, + mass transfer #1

Ms —

More complicated than viscous flow or heat conduction because for the first time have to deal w/ mixtures

momentum - Newton's Law $\tau_{yx} = -\mu \frac{dv_x}{dy}$

heat - Fourier's Law $q = -k \frac{dT}{dy}$

mass - Fick's Law $N_A = -D_{AB} \frac{dC}{dy}$

heat vs. mass

mass deals w/ mixtures while heat transfer deals w/ single component - energy

mass has Δ B.C.

heat vs. momentum

τ is a tensor + has Δ B.C. than h.t.

mass vs. momentum —

same problems / Δ's as above

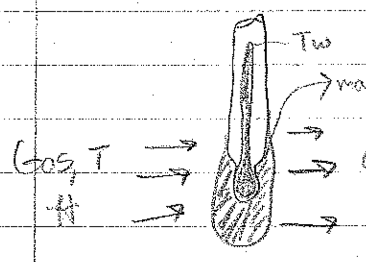
Transport Questions: 2a

2a. Wet bulb thermometer: (MSH - pg 748)

Defn (MSH): S.S., non-equilibrium temperature reached by a small mass of liquid immersed under adiabatic conditions in a continuous stream of gas. function (T_{gas}, + $Humidity_{gas}$)

Also:

T_w is the S.S. liquid temperature at which the heat needed to evaporate the liquid + heat the vapor to gas temperature = the sensible heat flowing from the gas to the liquid



Theory of wet bulb temp:

1. neglecting radiation (large enough gas velocity so that sensible heat transfer >>> radiation Q)

q, rate of sensible heat transfer to liquid = rate of vaporization (λ_w + $C_p \Delta T$)

latent heat of liq. at T_w — sensible heat of vapor

(1) $q = M_A N_A [\lambda_w + c_{p_A}(T - T_w)]$

↳ molal rate of vaporization

To find q:

(2) $q \rightsquigarrow$ $\quad q = hA(T - T_i)$

$T \quad T_i = T_w$

↳ h.t.c. between gas + liquid surface.

To find N_A: Assuming one-way diffusion

$N_A \rightsquigarrow$ $\quad N_A = \dfrac{k_y (y_i - y) A}{\overline{(1-y)}_L}$ (3)

$y \quad y_i$

↳ one-way diffusion factor (log mean of $(1-y_A)$ + $(1-y_{Ai})$)

where $\overline{(1-y)}_L = \dfrac{(1-y_A) - (1-y_{Ai})}{\ln \dfrac{(1-y_A)}{(1-y_{Ai})}}$

Since A in (2) + (3) the same

$\therefore \dfrac{q}{h(T - T_w)} = \dfrac{N_A \overline{(1-y)}_L}{k_y (y_i - y)}$

in the usual range of temp + humidities ~ 1 so can be omitted

& plugging in (1) for q_w

$$h(T-T_w) = \frac{\overbrace{K_y}^{\text{mass transfer coefficient (mol/area/unit mole fraction)}}(y_i-y)}{\cancel{N_A}(\overline{1-y})_L}\left[M_A \cancel{N_A}\left[\lambda_w + C_p(T-T_w)\right]\right]$$

Assumptions: $(\overline{1-y})_L \sim 1$

$C_p \Delta T \ll \lambda_w$ ∴ sensible heat neglected

$$\therefore h_y(T-T_w) = K_y(y_i-y)M_A[\lambda_w]$$

y_i = mole fraction of vapor in sat'd gas at T_w (no conc. gradient in liquid)

Now: to put in terms of humidity

& From Eqn 23.1 (MSH)

$$\mathcal{H} = \frac{M_A P_A}{M_B(1-P_A)}$$

M_A: MW of A; P_A: partial pressure of vapor

= mass of vapor, A / mass of dry gas, B

At 1 atm!! (since $p_A = y_A P$)

$$\mathcal{H} = \frac{M_A y_A}{M_B(1-y_A)}$$

$$\mathcal{H} M_B - \mathcal{H} y_A = M_A y_A$$

$$y_A = \frac{\mathcal{H} M_B}{M_A + \mathcal{H}}$$

mole fraction of vapor in the gas phase

But MSH gives $y = \dfrac{\mathcal{H}/M_A}{1/M_B + \mathcal{H}/M_A}$ $\left(= \dfrac{\mathcal{H} M_B}{M_A + \mathcal{H}} \right)$

$y \simeq \dfrac{\mathcal{H} M_B}{M_A}$ since $\mathcal{H}/M_A \ll 1/M_B$

$$\text{Now: } h_y(T-T_w) = K_y[\lambda_w]\left(\mathcal{H}_w M_B - \mathcal{H} M_B\right)$$

$$\text{or } \boxed{\frac{\mathcal{H}-\mathcal{H}_w}{T-T_w} = \frac{-h_y}{M_B K_y \lambda_w}} = \frac{-C_p}{\lambda_w}\left(\frac{N_{Sc}}{N_{Pr}}\right)^m = \frac{C_s}{\lambda}$$

$N_{Sc} \to \mu/\rho D_v$; $N_{Pr} \to \dfrac{C_p \mu}{k}$

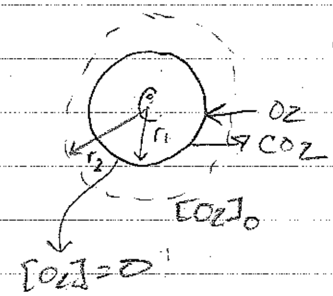
$$\therefore \frac{h_y}{M_B K_y} = C_p\left(\frac{N_{Sc}}{N_{Pr}}\right)^n$$

if $m = 2/3$ close to experimental value for air in water (but slightly less because it does not account for radiation)

Burning Carbon Particle

$A + B \rightarrow C$

$C + O_2 \rightarrow CO_2$



$N_r = -N_B$ e.m.c.D

1. Mole balance on O_2

In - out + Gen = Acc.

$N_{B,r} 4\pi r^2 \big|_r - N_{B,r} 4\pi r^2 \big|_{r+\Delta r} + O = 0$

$\div$ by $-4\pi \Delta r$

$$\lim_{\Delta r \to 0} \frac{\left(N_{B,r} r^2 |_{r+\Delta r} - N_{B,r} r^2 |_r \right)}{\Delta r} = 0$$

$$\frac{d(r^2 N_{B,r})}{dr} = 0$$

2. Find expression for $N_{B,r}$

Fick's Law

A diffuses in the direction of decreasing mole fraction of A

$$N_{A,r} = x_A(\cancel{N_A + N_B}) - cD_{AB}\frac{dx_A}{dr}$$

$$N_{B,r} = -cD_{BA}\frac{dx_B}{dr}$$

Assume $(cD_{AB} \neq f(r))$ i.e. constant temp. (900°C - combustion)

$$\therefore cD_{AB}\frac{d}{dr}\left(r^2 \frac{dx_B}{dr}\right) = 0$$

integrating once

$$r^2 \frac{dx_B}{dr} = C_1$$

" again:

$$X_B = \int \frac{C_1}{r^2} dr = -\frac{C_1}{r} + C_2$$

2C cont.

$$x_B = -\frac{C_1}{r} + C_2$$

→ decrease w/ time since Carbon is consumed

B.C. At $r = r_1(t)$ $x_B = 0$

$r = r_2$ $x_B = x_{B,0}$

$$0 = -\frac{C_1}{r_1} + C_2$$

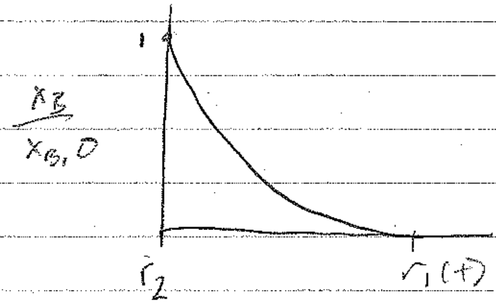
$$C_1 = C_2(r_1)$$

$$x_{B,0} = -\frac{C_2(r_1)}{r_2} + C_2 = C_2\left(1 - \frac{r_1}{r_2}\right)$$

$$C_2 = \frac{x_{B,0}}{(1 - r_1/r_2)} \quad \& \quad C_1 = \frac{x_{B,0}(r_1)}{(1 - r_1/r_2)}$$

$$\therefore x_B = -\frac{C_2(r_1)}{r} + C_2$$

$$\therefore \frac{x_B}{x_{B,0}} = \frac{1}{(1 - r_1/r_2)}\left(1 - \frac{r_1}{r}\right) = C\left(1 - \frac{r_1(t)}{r}\right)$$



$$\therefore N_{B,r} = -C D_{BA}\frac{dx_B}{dr} = -C D_{BA}\left(\frac{C_1}{r^2}\right) = -\frac{C D_{BA}}{r^2}\left(\frac{x_{B,0}(r_1)}{1 - r_1/r_2}\right)$$

$$N_{B,r} = -C_{B_0} D_{BA}\left(\frac{r_1(t)}{1 - r_1(t)/r_2}\right)\frac{1}{r^2}$$

Part 3 - Mole Balance on Carbon: 2c - contin.

In - Out + Gen. = Acc.

$$0 - 0 + r_c 4\pi r_1(t)^2 = \frac{d}{dt}\left(\frac{4}{3}\pi r_1^3 \rho_c\right)$$

$$\frac{1}{\rho_c} r_c = \frac{dr_1}{dt}$$

(4) Relate r_c to $N_{B,r}$

$$-r_c\ (\text{mol}/\text{m}^2) = -N_{B,r}\Big|_{r=r_1} = \boxed{C_{B,0} D_{BA}\left(\frac{1}{r_1 - \frac{r_1^2}{r_2}}\right)}$$

$$\therefore -\frac{dr_1}{dt} = \frac{C_{B,0} D_{BA}}{\rho_c}\left(\frac{1}{r_1 - \frac{r_1^2}{r_2}}\right)$$

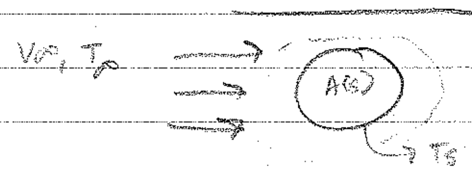
B.C. at $t = 0$, $r_1 = r_{1,0}$
$t = t_f$ $r_1 = 0$

$$-\int\left(r_1 - \frac{r_1^2}{r_2}\right) dr_1 = \frac{C_{B,0} D_{BA}}{\rho_c} t$$

$$-\frac{r_1^2}{2} + \frac{r_1^3}{3r_2} + C_1 = \frac{C_{B,0} D_{BA}}{\rho_c} t$$

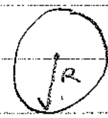
$$C_1 = -\frac{r_{1,0}^3}{3r_2} + \frac{r_{1,0}^2}{2}$$

$$t_f = \frac{\rho_c}{C_{B,0} D_{BA}}\left(\frac{r_{1,0}^2}{2} - \frac{r_{1,0}^3}{3r_2}\right)$$



Forced convection

Assume: sphere, entirely under water.



Mole balance:

In - Out + Gen = Acc.

$$0 - N_A 4\pi r^2 = \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho_c\right) \qquad N_A = A k_m (C - C_\infty)$$

$$N_A = \frac{\pi \rho_c}{3} \frac{dr}{dt}$$

Energy balance

$$q = hA(T_\infty - T_s) = \lambda_w N_A A$$

energy transfer to iceberg = energy used in melting iceberg.

For forced convection heat transfer around a single sphere:

$$\underset{(Nu)}{\frac{h_m D}{k_f}} = 2.0 + 0.6 \underset{(Re)^{1/2}}{\left(\frac{D v_\infty \rho_f}{\mu_f}\right)^{1/2}} \underset{(Pr)^{1/3}}{\left(\frac{C_p \mu}{k}\right)^{1/3}}$$

solve for $h = f(r)$

$$N_A = \frac{h(r)}{\lambda_w}(T_\infty - T_s) = \frac{\pi \rho_c}{3} \frac{dr}{dt}$$

$$t = \frac{\pi \rho_c \lambda_w}{3(T_\infty - T_s)} \int_R^0 \frac{1}{h(r)}\, dr$$

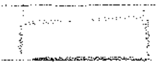
Use Momentum Transport

to get Drag

$$F_k = A f \tfrac{1}{2} v^2 \quad \text{skin + body drag}$$

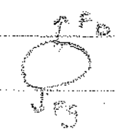
Friction factor: 3

friction factor - assuming constant ρ

$F_k = AKf \Rightarrow$ for conduit 

$F_k = (p_0 - p_L + \rho g(h_0 - h_L))A$

$f = \text{func.}(Re, \text{system shape})$

for sphere: 

$F_k =$

$= 4/3 \pi R^3 (\rho_{sp} g - \rho_{fluid} g)$

Experimentally, for flow in conduits:

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{P_0 - P_L}{\frac{1}{2}\rho\langle v\rangle^2}\right)$$

For flow in spheres:

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sp} - \rho}{\rho}\right)$$

Heat transfer will affect friction factor since it will affect fluid properties, ρ, μ etc.

$\uparrow Q$, $\uparrow T$, for liquid - $\downarrow \rho$, $\downarrow \mu$, $\uparrow f$?

Differences between diffusivity + mass transfer coef. 4.

Diffusivity: D_{AB} (cm^2/sec) $\uparrow T$, $\downarrow P$

For liq. $\uparrow x_A$

Analogous to $\nu = \mu/\rho$ for gases, $\neq f(x_A)$

+ $\alpha = \frac{k}{\rho c_p}$

Diffusivity is a molecular property + describes flux for local (infinitesimal) concentration gradients. $D_{AB} = f(T, P, x)$
mole fract.

→ between phase:

Mass transfer coefficients describe flux over measurable concentration gradients + the bounds associated with a boundary. Accounts for mass transfer by diffusion or convection. $k = f(Re, Sc)$

k_c - convective mass transfer coefficient why is $Pr_{liq} > Pr_{gas}$ 5

$Pr = \frac{C_p \mu}{K}$

$H_2O > Air$

$\mu_{liq} > \mu_{gas}$

$1.794 cp > .018$

$Cp_{liq} > Cp_{gas}$

1 Btu/lb°F > 0.2

$k_{liq} > k_{gas}$ but not much

0.320 > .014 Btu/ft·hr·°F for Air, for H_2O

6. Sherwood Number = Nusselt # for mass transfer at slow mass transfer rates 6

$\frac{K_x D}{c D_{AB}}$ in BSL $\frac{(mol/L^2 t) L}{(mol/L^3)(L^2/t)}$

$N_{Sh} = \frac{K_c D}{D_v}$ (in MSH) $\frac{(cm/s)(cm)}{cm^2/sec}$

↑ vol. diffusivity

$N_{Sh} = \frac{K_c D}{D_{AB}} = \frac{\text{convective transport}}{\text{diffusive transport}}$

$\mu [=] \frac{lb}{ft \cdot sec}$

used to find mass transfer coefficients from making analogies between N_{Nu} + N_{Sh} in order to use heat transfer correlations such as $N_{Nu} = 2.0 + 6 Re^{1/2} Pr^{1/3}$ $Pr = \frac{C_p \mu}{k}$

$N_{Sh} = 2.0 + 6 Re^{1/2} Sc$ $Sc = \frac{\mu}{\rho D_{AB}}$

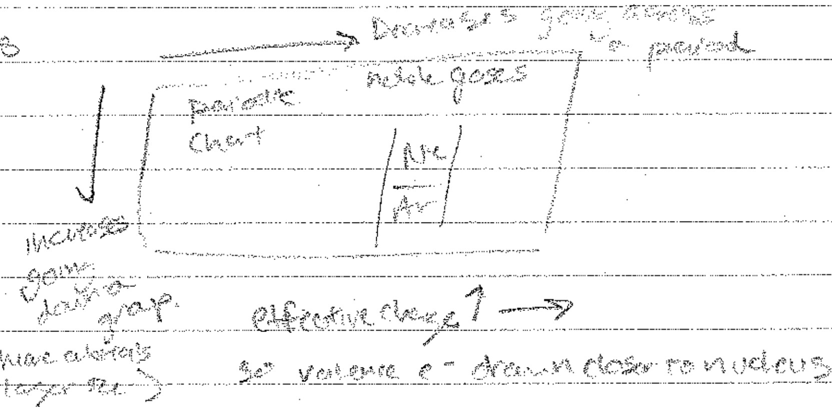
$j_H = j_D$

a. the Chilton - Colburn analogy

$Nu/Re Pr^{1/3} = Sh/Re Sc^{1/3}$

For Ne + Ar, which has the larger: 7.

(a) Atomic radius



∴ Ar > Ne's atomic radius

(b) Diffusivity = f(Temp, pressure, composition)

D_{AB} for smaller Ne > D_{AB} for Ar

since $D_{AB} \propto 1/M.W.$

The Chapman-Enskog formula $D_{AB} \propto (1/M_A)$

(b) viscosity $\propto$ MW

$\mu_{Ar} > \mu_{Ne}$

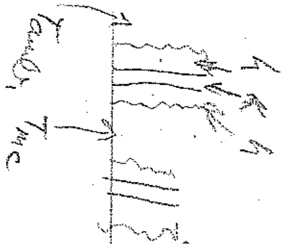
(c) heat capacity are the <u>same</u> for monatomic ideal gases

based on structure - $C_p = 20.786$ J/mol K

for He, Ne, Ar, Kr,

(d) $N_{Pr} = \frac{C_p \mu}{k}$ since $\mu_{Ar} > \mu_{Ne}$

assume $N_{Pr}(Ar) > N_{Pr}(Ne)$



cavitation = $P_{fluid} = P_{vap}$

$P\downarrow$ until $= P_{vap}$. causing bubble formation (boiling)

Consider the problem of pumping oil down AK pipeline. 8
Given D, L & oil properties. How would you calculate

(a) pump size

Pump = (MOH)

Applying Bernoulli's Eqn:

$$\eta W_p = \left(\frac{P_b}{\rho} + \frac{g z_b}{g_c} + \frac{V^2}{2g_c}\right) - \left(\frac{P_a}{\rho} + \frac{g z}{g_c} + \frac{V_a^2}{2g_c}\right)$$

From notes:
Need to know flow rate Q.
$Q \to Re \to f \to \Delta p \to$ Chart of ΔP vs $Q \to$ size of pump.

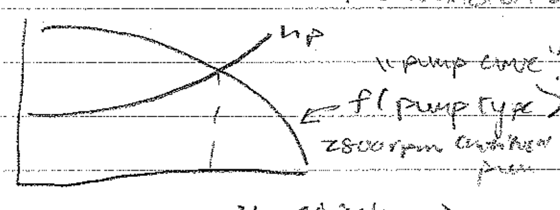
$W_p = g\,h_p$

head loss

$$h_p = \frac{\Delta P}{\rho g} + \frac{\Delta V^2}{2g} + \Delta z + \sum K_L \frac{V^2}{2g} + \sum \frac{fL}{D}\frac{V^2}{2g}$$

system curve

loss coefficients



For given Q + ~~Re~~ h_p find pump.

f = friction factor

(Bernoulli's)

$$F_K = AKf = Af\frac{1}{2}\rho v^2 = (\Delta P + \rho g \Delta h)A \quad \text{for tube}$$
$$= \tfrac{4}{3}\pi R^3(\rho_{sp} g - \rho g)$$

weight - buoyancy

Laminar flow = $f = 16/Re$ in tube

Turbulent => calculate ε/D + use chart of f vs Re

For spheres:

$10^{-3} < Re < 1$ => Stokes Law $f = 24/Re$

$1 < Re < 1000$ = "intermediate law" $f = 18.5/Re$

(8b) Heat loss from a pipe

$$\frac{q}{A_o} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{X_w}{k_m}\left(\frac{D_o}{\bar{D}_L}\right) + \frac{1}{h_o}} = U_o\,(T_h - T_c)$$

when $A_o \approx A_i$ (thin walled) $U_o = U_i$

$$\therefore \; U_o = U_i = \frac{1}{1/h_o + x_w/k + 1/h_i}$$

Calculate Re to determine laminar / turbulent

$$Re = \frac{\rho D v}{\mu}$$

For laminar flow:

$$N_{Nu} = \frac{h_i D}{k} = \frac{N_{Gz}}{\pi} \ln\left(\frac{T_w - T_i}{T_w - T_o}\right)$$

assume const. wall temperature + log mean inlet outlet temp.

For turbulent flow, $Re > 2100$

Dittus - Boelter Eqn.

$$\frac{h_i D}{k} = 0.023 \left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{c_p \mu}{k}\right)^{1/3}$$

For long tubes w/ sharp-edged entrances,

$$N_{Nu} = 0.023\,(Re)^{0.8}\,(Pr)^{1/3}$$

if $\mu \neq \mu_w$ throughout use Sieder-Tate

$$\frac{h_i D}{k} = 0.023\,Re^{0.8}\,Pr^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

μ_w evaluated at T_w

To find T_w: Iterative calculation based on

$$\frac{\Delta T_{overall}}{1/U} = \frac{\Delta T_i}{D_o/D_i h_i} = \frac{\Delta T_w}{(X_w/k_m)\,D_o/\bar{D}_L} = \frac{\Delta T_o}{1/h_o}$$

Method: $\Delta T_i = \dfrac{D_o/D_i h_i}{1/U_o}\,\Delta T = \dfrac{1/h_i}{1/h_i + D_i/D_o h_o}\,\Delta T$

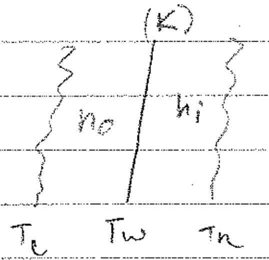
Free convection:

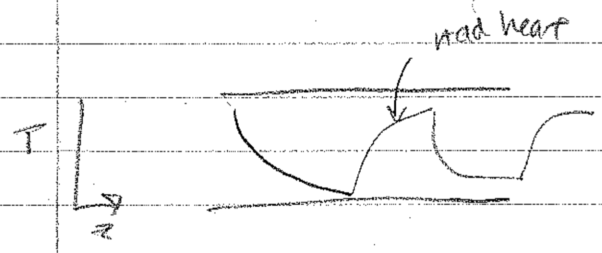
$$Nu = f(Pr, Gr) \approx 0.53 \left(N_{Gr} N_{Pr}\right)_f^{0.25}$$

$$Gr = \frac{D_o^3 \rho^2 \beta g \Delta T}{\mu^2}$$

$$\beta = \frac{(\partial V / \partial T)_P}{V} \quad I.G = \frac{1}{T} \quad \text{fluid} = \frac{\Delta v / \Delta T}{\bar{v}} = \frac{\rho_1 - \rho_2}{\bar{\rho}_{avg}(T_2 - T_1)}$$

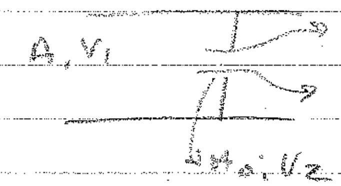
Temperature Profile





Give the governing equations for

(a) orifice meter: Find Flow, Q



1. Continuity:

$$A V_1 = A_o V_2$$

2. Bernoulli's Eqn:

$\frac{N \cdot m}{kg}$

$kg \cdot \frac{m^2}{s^2}$

Denn's Bernoulli Equation:

isothermal ideal gas: $= \frac{RT}{M_w} \ln \frac{P_2}{P_1}$

incompressible $= \frac{P_2 - P_1}{\rho}$

1 = turbulent
2 = laminar in pipes

(A) $$\frac{\alpha_2}{2} \langle v \rangle_2^2 + g h_2 = \frac{\alpha_1}{2} \langle v_1 \rangle_1^2 + g h - \int_{P_1}^{P_2} \frac{dP}{\rho} + \delta W_s - l_v$$

$\frac{m^2}{s^2}$ $\boxed{\frac{m^2}{s^2} = \frac{J}{kg}}$ $\frac{N \cdot m^2}{kg \, s^2}$ viscous losses

In terms of heads: ($\div$ it by g.)

$$\frac{\alpha_2 \langle V \rangle_2^2}{2g} + h_2 + \frac{P_2}{\rho_2 g} = \frac{\alpha_1 \langle v_1 \rangle_1^2}{2g} + h_1 + \frac{P_1}{\rho_1 g} + \frac{\delta W_s}{g} - \frac{l_v}{g}$$

$$l_v = \sum_{\text{pipe lengths}} \frac{\overset{?}{2} \langle v \rangle^2 L f}{D} + \sum_{\text{fittings}} \tfrac{1}{2} V^2 K$$

K = # of velocity heads

Thus the B.E. is:
For incompressible fluid

$$l_v = \frac{1}{2} V_2^2 \left(1 - \left(\frac{A_o}{A}\right)^2 \right) + \frac{\Delta P}{\rho} \approx \tfrac{1}{2} K V^2$$

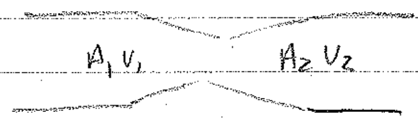
$$K = 1.6 \left[1 - \left(\frac{A_o}{A}\right)^2 \right]$$

$$V_2^2 = \frac{2(P_1 - P_2)}{\rho \left(1 - (A_o/A)^2 + 1.6 - 1.6 (A_o/A)^2\right)}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho (1 + K - (A_o/A)^2)}} \quad + \quad Q = A_o V_2$$

Venturi Meter

like orifice but less head loss



0.62 for orifice

From Fluids txtbk: $Q = Q_{\text{orifice meter}} = K A_2 \sqrt{2g\Delta h}$ $\qquad \Delta h = \Delta P/\rho g$

$$\frac{1}{2}V_1^2 + g\cancel{h_1} = \frac{1}{2}V_2^2 + g\cancel{h_2} + \int \frac{dP}{\rho} + \cancel{\delta W_s}^{\,0} + \underset{\frac{1}{2}KV^2}{l_v}$$

$$\frac{1}{2}V_2^2\left(1 - (A_2/A_1)^2 + K\right) = \frac{P_1 - P_2}{\rho}$$

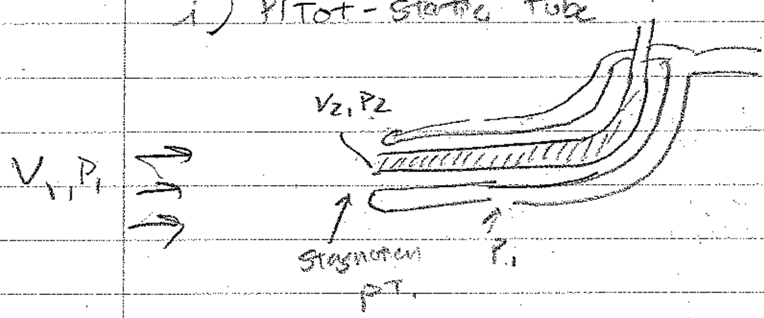
$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left(1 - (A_2/A_1)^2 + K\right)}}$$

K: look up for venturi meters.

$$Q = A_2 V_2 =$$

9c - Pitot tube - measures velocity by Δp

i) Pitot - static tube



$$\frac{1}{2}\cancel{V_1}^2 + gz_1 = \frac{1}{2}\cancel{V_2^2}^{\,0} + gz_2 + \int\frac{dP}{\rho}$$

$$\frac{1}{2}V_1^2 = \frac{P_2 - P_1}{\rho}$$

$$V_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

$(P_2 - P_1)$: from manometer.

can neglect l_v except at low Re

From Denn's book

(10) Bernoulli's equation

$$\frac{\alpha}{2} V_1^2 + g z_1 = \frac{\alpha}{2} V_2^2 + g z_2 + \int_{P_1}^{P_2} \frac{dp}{\rho} + \delta W_s + l_v$$

$$\int \frac{dp}{\rho} = \frac{P_2 - P_1}{\rho}$$ incomp. fluids

Assuming turbulent flow
$\alpha = 1$ uniform velocity over cross-section.

$l_v = \frac{1}{2} K V^2$ viscous losses

$$\frac{1}{2} V_1^2 + g z_1 + \frac{P_1}{\rho} = \frac{1}{2} V_2^2 + g z_2 + \frac{P_2}{\rho} + l_v$$

(b) Hagen-Poiseuille Eqn

For laminar flow in a pipe, $Re < 2100$

$$Q = \frac{\pi}{128} \frac{|\Delta P| D^4}{L \eta} \quad \left(\text{from } f = \frac{16}{Re}\right)$$

(c) Stokes Law

For $Re < 1$, Drag Force on a sphere is $F_D = 3\pi \eta V D$ (Denn's book)

$D = 2r$

⇕ same

$F_K = 6\pi \mu R V_\infty$ (BSL)

(d) continuity equation (Denn) Pg 88

$$\frac{d}{dt} \int_{z_1}^{z_2} \langle \rho \rangle A\, dz = \langle \rho V \rangle_1 A_1 - \langle \rho V \rangle_2 A_2$$

single fluid

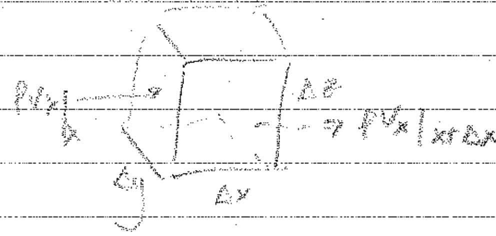
$$\frac{d}{dt} \int_{z_1}^{z_2} \rho A\, dz = \rho V_1 A_1 - \rho V_2 A_2$$

if incompressible $\frac{d}{dt} \int_{z_1}^{z_2} A\, dz = A_1 V_1 - A_2 V_2$

Give the following: (10)

(d) Equation of Continuity

Acc = In - Out



$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \Delta y \Delta z \left(\rho V_x|_x - \rho V_x|_{x+\Delta x} \right) + \Delta x \Delta z \left(\rho V_y|_y - \rho V_y|_{y+\Delta y} \right)$$

$$+ \Delta x \Delta y \left(\rho V_z|_z - \rho V_z|_{z+\Delta z} \right)$$

$\div$ by $\Delta x \Delta y \Delta z$, & take limit as $\Delta x, \Delta y, \Delta z \to 0$

$$\boxed{\frac{\partial \rho}{\partial t} = -\left(\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} \right) \quad \text{rate of } \Delta \text{ of density at a fixed point.} \qquad \text{or } \frac{\partial \rho}{\partial t} = -\nabla \cdot \rho V \quad \text{"divergence of } \rho V\text{"}}$$

or, for the Δ in ρ as seen from a path following the fluid motion:

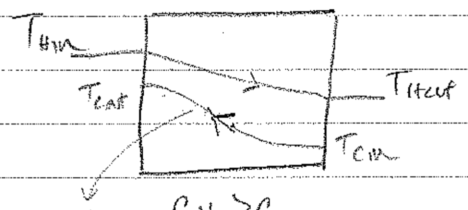
$$\frac{D\rho}{Dt} = -\rho \left(\nabla \cdot V \right)$$

Simplifications:

ρ = const. (incompressible) then $\boxed{\nabla \cdot V = 0}$ - when ρ = const.

11. NTU? How to calculate.

(WWW - pg 414)



$C_H > C_C$

$C_{min} = C_C$

For counterflow

ΔT_{max} occurs in fluid with

smallest C_p

$q = m C_p (\Delta T)$

$\downarrow \quad \uparrow$

effectiveness factor $\epsilon = \dfrac{\text{Actual h.t.}}{\text{max. h.t. possible}} = \dfrac{C_{max} \left(\begin{matrix} \Delta T_H \Rightarrow C_{min} = C_C \\ \text{OR} \\ \Delta T_C \Rightarrow C_{min} = C_H \end{matrix} \right)}{C_{min} (T_{Hin} - T_{cin})}$

OR $\boxed{\epsilon = \dfrac{q}{C_{min} (T_{Hin} - T_{cin})}}$

↳ occurs in fluid w/ max. ΔT.

To find ϵ

$q = C_c (T_{c_2} - T_{c_1}) = UA \, \Delta T_{lm}$

$NTU = \dfrac{AU}{C_{min}}$

1. To find q
   1. Calculate C_{min} + C_{max}
   2. Calculate $NTU = UA / C_{min}$
   3. Use figure to calculate ϵ (or derive eqn)
   4. then calc. $q = \epsilon \, C_{min} (T_{Hin} - T_{cin})$
      then use q to calculate T_{Hout} + T_{cout}.

McCabe Thiele Diagram - compute # of ideal plates needed to achieve certain concent. Δ 12

2 operating lines:

1. Rectifying section (Above feed plate)

$$y_{n+1} = \frac{L_n}{L_n + D} x_n + \frac{D x_D}{L_n + D}$$

For $R_D = \frac{L}{D}$

Assumes constant molal overflow (similar ΔH_{vap}'s)

$R_k = \frac{1}{2}\rho v^2$

Gases

	T	P	13
D_{AB}	↑	↓	
μ	↑ (liq ↓)	↑ (liq ⟷)	
(BSL) K	↑ (liq ↓)	↑	
C_p	↑	⟺	$C_p = \left(\frac{\partial H}{\partial T}\right)_P$ $C_p \neq f$(pressure) whenever the substance is independent of pressure. (valid for i.g., liquids solids, + low pressure gases
h	↑ eq ↑	↑ eq ⟺	
v	↑	↑	

Reynold's Analogy ⇒ transport energy dimensionless / " " across the solid-fluid interface is similar ratio for momentum transport. 14.

Assumes 1. $\frac{\nu}{\alpha} \equiv N_{Pr} = \frac{\varepsilon_m}{\varepsilon_H}$

2. q/A varies linearly w/ r

3. the point where $T_{loc} = \bar{T}_{bulk}$ is the same as where $u = \bar{v}$

Then

$$\frac{Nu}{Re\,Pr} = \frac{f}{2} \quad \text{for } Pr = 1$$

$$\frac{hD}{K\rho Dv} \times \frac{K}{C_p\mu}$$

$$\boxed{\frac{h}{C_p \rho v} = N_{St} = \frac{f}{2}}$$

flow at high Re in straight round tubes

Colburn Analogy: Better than Re Analogy ($Pr = 1$) since applies to $0.6 < Pr < 100$

For $Re > 10{,}000$ long smooth tubes

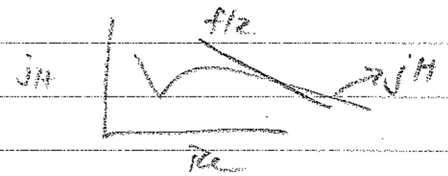
Colburn j factor:

$$j_H = \frac{h}{C_p \rho V} N_{Pr}^{2/3} \cancel{\left(\frac{\mu_w}{\mu}\right)^{0.14}} = \frac{f}{2} \quad \text{to find } f$$

$$j_H = 0.023\, N_{Re}^{-0.2} \quad (\text{for finding } h)$$

j_H vs. Re at $Re > 10{,}000$

the same as $f/2$ vs Re



$$\frac{f}{2} = \frac{h}{C_p \rho V} Pr^{2/3}$$

Power = $\dot{P} = Q|\Delta p|$

friction factor? 15

$$\boxed{F_k = f K A} = f K \frac{1}{2} v^2 \quad (BSL)$$

definition of f K = kinetic energy/volume $= \frac{1}{2}\rho v^2$
A = wetted surface

Also from a force balance
on a tube:

$$F_k = \Delta p A + \rho g \Delta h (A) = (\Delta p + \rho g \Delta h)\pi r^2$$

$$f = \frac{(\Delta P + \rho g \Delta z)}{\frac{1}{2}\rho V^2} \frac{\pi R^2}{2\pi R L} = \frac{(\Delta P + \rho g \Delta z)}{\frac{1}{2}\rho V^2}$$

Boundary Layer Equations 16.

Dem (202, 220, 286, 325)

Prandtl B.L. equations (BSL - 146, 331, 367, 609

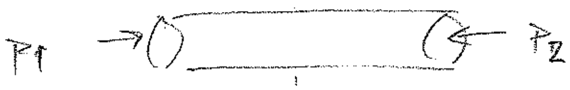
Describe the friction factor vs Re relation for a

Solve for f
Solve for v (Hagen-Poiseuille)
Relate f to v → Re

a) pipe

$$F_K = fKA = f\frac{1}{2}\rho v^2(2\pi RL)$$

$$\boxed{K = \tfrac{1}{2}\rho v^2}$$



$$F_K = (P_1 - P_2)A + \rho g(z_1 - z_2)A$$

$$f\frac{1}{2}v^2(2\pi RL) = \pi R^2((P_1 - P_2) + \rho g\Delta z))$$

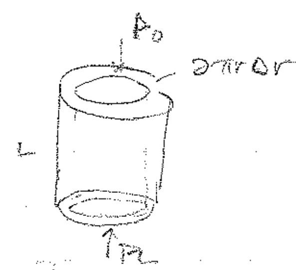
$$f = \frac{R(\Delta P + \rho g\Delta z)}{(2L)\ \rho\,\tfrac{1}{2}v^2} = \frac{D}{4L}\frac{(\Delta P + \rho g\Delta z)}{(\tfrac{1}{2}\rho v^2)}$$

$$Re = \frac{\rho D v}{\mu}$$

Need to find v_z

Assume s.s.

0 = in - out + Force



$$\frac{\partial(r\tau_{rz})}{dr} = \left[\left(\frac{P_0 - P_L}{L}\right) + \rho g\right]r$$

For Newtonian Fluids: $\tau_{rz} = -\mu\frac{dv_z}{dr}$

Integrate

$$r\tau_{rz} = \left(\frac{P_0 - P_L}{L} + \rho g\right)\frac{r^2}{2} + C_1$$

$$\tau_{rz} = \left(\frac{P_0 - P_L}{L} + \rho g\right)\frac{r}{2} + \cancel{C_1/r}^{\,0}$$

$$\frac{dv_z}{dr} = \frac{-1}{\mu}\left(\frac{P_0 - P_L}{L} + \rho g\right)\frac{r}{2}$$

$$v_z = \frac{-1}{\mu}\left(\left(\frac{P_0 - P_L}{L}\right) + \rho g\right)\frac{r^2}{4} + C_1 \qquad r = R,\ v = 0$$

$$v_z = \frac{1}{4\mu}\left(\frac{\Delta P}{L} + \rho g\right)(R^2 - r^2)$$

$$\langle v_z \rangle = \frac{\int_0^{2\pi}\int_0^R v_z\, r\,dr\,d\theta}{\int_0^{2\pi}\int_0^R r\,dr\,d\theta} = \left(\frac{\Delta P + \rho g}{8\mu}\right) R^2$$

$$f = \frac{D}{4} \frac{(\Delta P/L + \rho g)\; 8\mu}{\frac{1}{2}\rho v \,(\Delta P/L + \rho g) R^2} = \frac{4\mu}{\rho v R^2} = \frac{16\mu}{\rho v D}$$

$$f = \boxed{\frac{16}{Re}}$$

(b) Sphere

$$F_K = \tfrac{1}{2}\rho v^2 f A = \pi r^2 (\tfrac{1}{2}\rho v_\infty^2) f$$

using Stokes Law $6\pi \mu R v_\infty = F_K$

$$f = \frac{6\pi \mu R v_\infty}{\pi r^2 (\frac{1}{2}\rho v_\infty^2)} = \frac{24}{Re}$$ for $Re < 0.1$ Creeping Flow

(c) Using (BSL 4.4-27)

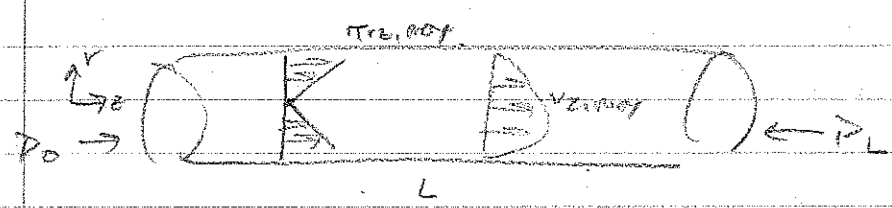
$$F_K = \text{drag force} = 1.328 \sqrt{\rho \mu L W^2 v_\infty^2}$$

width

$$F_K = \tfrac{1}{2}\rho v_\infty^2 f A$$

$$f = \frac{F_K}{\frac{1}{2}\rho v_\infty^2 A}$$

Shear stress profile for a pipe



momentum balance - shell balance on $\Delta r(2\pi r)L$

Acc = in - out + forces

$$0 = (2\pi r L)(\tau_{rz}|_r - \tau_{rz}|_{r+\Delta r}) + (P_0 - P_L)(2\pi r \Delta r)$$

÷ by $2\pi r L \Delta r$ lim $\Delta r \to 0$

$$-\frac{\partial (r\tau_{rz})}{\partial r} = \frac{(P_L - P_0)r}{L}$$

$$\frac{\partial (r\tau_{rz})}{\partial r} = \frac{(P_0 - P_L)r}{L}$$

$$r\tau_{rz} = (P_0 - P_L)L\, r^2/2 + C_1$$

$$\tau_{rz} = \frac{(P_0 - P_L)r^2}{2L} = \frac{\Delta P}{2L} r^2$$

Dimensionless Parameters

$Re = \frac{\rho D v}{\mu}$ = $\frac{\text{inertial forces}}{\text{viscous forces}}$

h.t.

$Pr = \frac{C_p \mu}{k}$ = $\frac{\text{diffusivity of momentum}}{\text{thermal diffusivity}}$

$Nu = \frac{hL}{k}$ =

$Sh = \frac{k' L}{D_{AB}}$ = $\frac{\text{mass transfer vel}}{\text{diffusion velocity}}$

mass.

$Sc = \frac{\mu}{\rho D}$ = $\frac{\text{diff. of momentum}}{\text{Diff. of mass}}$

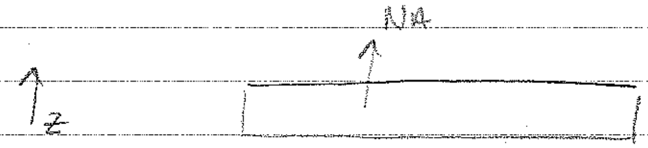
$Gr = \frac{D^3 \Delta\rho\, g}{\rho v^2}$ free convection

Evap - 587

come back to.

Pool of organic liquid - find rate of evaporation. 21

B



Fick's Law

$$N_A = -C D_{AB} \frac{dx_A}{dz} + x_A (N_A + \cancel{N_B}^{0})$$

Assume B insoluble in A

$$N_A = \frac{-C D_{AB}}{(1-x_A)} \frac{dx_A}{dz}$$

$\frac{mol}{m^3} \cdot \frac{L^2}{t} \cdot \frac{}{L}$

Continuity eqn.

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial z} (N_{Az} + \cancel{N_{Bz}}^{0}) = 0$$ (Assume S.S.

$$0 = +\frac{\partial}{\partial z} \left(\frac{C D_{AB}}{1-x_A} \frac{dx_A}{dz} \right)$$

$$C_1 = \frac{C D_{AB}}{(1-x_a)} \frac{dx_A}{dz}$$

$$C_1 z = -C D_{AB} \ln(1-x_A) + C_2$$

$z = 0$, $x =$

Diffusivity - 22

525, 526, 552,

Stokes-Ei - 514,

502, 567

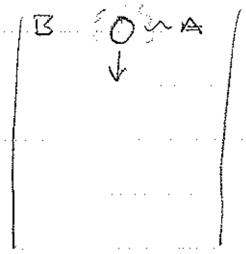
Temp profile in heat exchanger

Important phenomena during an underground explosion? 24

radiation =

$$q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

25.



Continuity

$$\frac{\partial c_A}{\partial t} = -\frac{\partial N_A}{\partial z} \qquad \frac{\partial c_B}{\partial t} = -\frac{\partial N_B}{\partial z}$$

T_T = tower temp

T_i = initial drop temp.

How does drop temp Δ as it falls

$$\frac{\partial c}{\partial T} = \frac{\partial N_A}{\partial z} \ldots$$

For no fluid motion, All heat transfer by conduction

$$\nabla \cdot k \nabla T = \rho C_p \frac{\partial T}{\partial t}$$

$$k \nabla T = k \frac{\partial T}{\partial r} \vec{e}_r + \frac{k}{r} \frac{\partial T}{\partial \theta} \vec{e}_\theta + \frac{k}{r \sin\theta} \frac{\partial T}{\partial \phi} \vec{e}_\phi$$

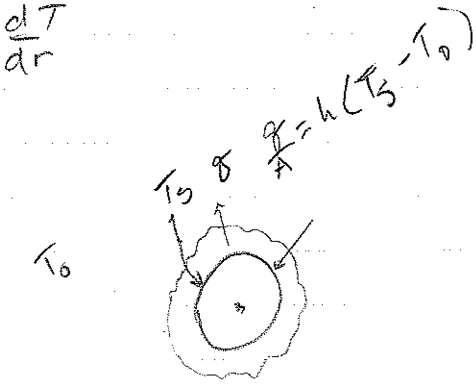
$$\nabla \cdot k \nabla T = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}\left(\frac{k}{r} \frac{\partial T}{\partial \theta} \sin\theta\right)}}_{0} + \underbrace{\cancel{\frac{1}{r \sin\theta} \frac{\partial}{\partial \phi}\left(\frac{k}{r \sin\theta} \frac{\partial T}{\partial \phi}\right)}}_{0}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right) = \rho C_p \frac{\partial T}{\partial t}$$

I.C. $t = 0$, $T = T_i$ for drop.

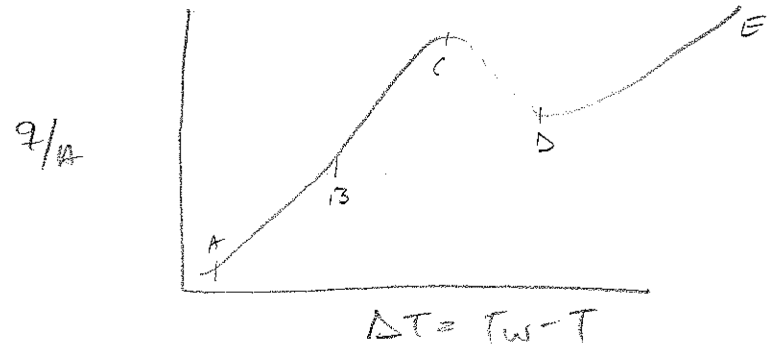
B.C. $r = 0$ finite

$r = R$ $\quad -h_A(T_s - T_T) = -k \frac{dT}{dr}$



What is the angular dependence of the Nusselt number for a falling drop? 26.

Boiling Curve



AB - natural convection $q = h A \Delta T$

BC - nucleate Boiling

CD - transition boiling - so many bubbles form they create insulating layer of vapor on heating surface

DE - film boiling

28.

Given the free stream velocity + particle diameter calculate the B.L. thickness at a 45° Angle. What is the pressure at the forward + backward stagnation points? Why the difference?

(Model as a flat plate?)

Derive the S.S. momentum balance for fully developed laminar flow. in a pipe. 29.

To find U for shell + tube heat exchangers. pg 30

$q = U_o A \Delta T_{lm}$ (MSH 434)

$$U_o = \frac{1}{\frac{1}{h_i}\frac{D_i}{D_o} + \frac{x}{k}\left(\frac{D_o}{D_L}\right) + \frac{1}{h_o}}$$

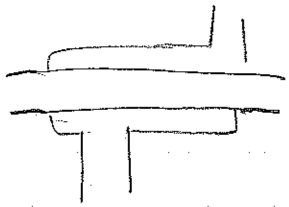
for thin walled

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{x}{k} + \frac{1}{h_o}$$

for h_i ⇒ Dittus Boelter

for h_o ⇒ Donohue Eqn.

M3H (313) 31

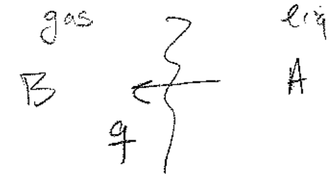


$$q = \dot{m}_c c_{p_c}(T_{c_o} - T_{c_{in}}) = \dot{m}_h c_{p_H}(T_{h_{in}} - T_{h_{out}}) = \frac{UA(\Delta T_1 - \Delta T_2)}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

Wet Bulb/Dry Bulb Psychrometer

~~24~~ 32

more heat flux if gas is soluble in liquid



~~$N_A = -N_B$~~

Ficks Law:

$$N_A = -c D_{AB} \frac{dx_A}{dz} + x_A (N_A + N_B)$$

For EMCD $N_A = -N_B$

$$N_A = -c D_{AB} \frac{dx_A}{dz}$$

For stationary comp.

$$N_A = \frac{-c D_{AB}}{(1-x)} \frac{dx_A}{dz} > \text{EMCD}$$

$$\frac{\vec{q}}{A} = -k \nabla T + \sum N_i H_i$$

losing enthalpy of B

$N_A (H_A - H_B)$ EMCD

$N_A H_A + \cancel{N_B H_B}$ Stationary

$\quad\quad\quad\quad 0$

$$q_s > q_{EMCD}$$

Chilton - Colburn

$$j_H \equiv \frac{Nu}{Re\, Pr^{1/3}} = j_D \equiv \frac{Sh}{Re\, Sc} = f/2$$

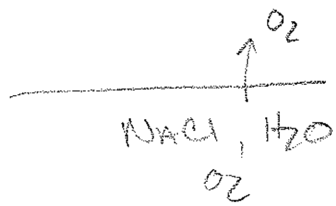
$$\frac{hL/k}{\frac{\rho D v}{\mu}\,\frac{C_p \mu}{k}} \qquad = \frac{k_L/D_{AB}}{\frac{\rho D v}{\mu}\,\frac{\mu}{\rho D_{AB}}}$$

Joule Thompson coefficient =

$$= \left(\frac{\partial T}{\partial P}\right)$$

H_2 is negative

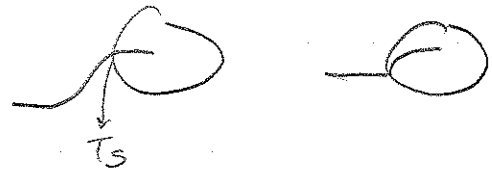
O_2 + N_2 is positive


O_2
$NaCl$, H_2O
O_2

Assume

$k > hD$

low Biot high Biot 60



$Bi = \frac{hD}{k}$

low Biot #:

$$q = h 4\pi r^2 (T_s - T_B) = \rho C_p \frac{4}{3} \pi R^3 \frac{dT_s}{dt}$$

Lewis # $= \frac{k}{\rho c_p D}$

From Naver-Stokes Eqn, 63

$$\rho \frac{Dv}{Dt} = -\nabla P + \mu^2 \nabla^2 v + \rho g$$

$$\rho \left[\frac{dv_z}{dt} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{dv_z}{dz} \right]$$

$$= -\frac{dP}{dz} + \mu \left(\frac{d^2 v_z}{dz^2} + \frac{1}{r} \frac{d}{dr} r \frac{dv_z}{dr} \right.$$

$$\frac{dP}{dz} = \frac{\mu}{r} \frac{d}{dr}\left(r \frac{dv_z}{dr} \right)$$

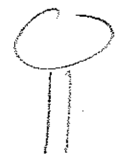
$$v^* = v/V \qquad r^* = r/L$$

$$\frac{1}{\mu} \frac{\Delta P}{L} = \frac{1}{r} \frac{d}{dr}\left(r \frac{dv_z}{dr} \right)$$

$$=$$

$$\frac{1}{\mu} \frac{\Delta P L}{V} = \frac{1}{r^*} \frac{d}{dr^*}\left(r^* \frac{dV^*}{dr^*} \right)$$

Bubble leaving Straw



Summing Forces

$$4/3 \pi R^3 (\qquad) g = \sigma \cdot 2\pi r_H$$