TRANSPORT EQUATIONS

Cons. of mass

Cons. of momentum

Cons. of energy

Cons. of mass of α

Navier-Stokes

ϕ, n_α, N_α, e

Fick's Law

Fourier's Law

Newton's Law

Kinetic theory of gases : μ, K, D

Hagen-Poiseuille Law / v-profile

Stokes Law

Stokes Flow Equation

Euler's Equation

Bernoulli's Equation - rigorous deft

Potential Flow Eqns

Prandtl BL Equations

how acount for turbulent flow?

von-Kármán-Prandtl equations?

Eddy viscosity / Prandtl mixing length

fanning friction factor

drag coefficient

laminar f

turbulent f : Blasius formula

f for creeping flow around sphere

f for packed colums

Ergun equation

Macroscopic mass balance

Macroscopic momentum balance

Macroscopic energy balance

Boussinesq approximation

Reynold's analogy (turbulent flow)

Colburn analogy

radiation - Stefan-Boltzmann

Lambert's cosine Law

Fick's 2nd law

Eddy Diffusivity / Prandtl mix length

Leibnitz rule

McCabe-Thiele operating line equations

NTU - ht and mt

Chapman-Enskog

overall energy balance

$$\boxed{\rho C_p \frac{DT}{Dt} = -(\nabla\cdot q) - (\tau : \nabla \vec{v}) - \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_p \frac{Dp}{Dt}}$$

viscous dissipation term usu. negligible

$$\boxed{\rho C_p \frac{DT}{Dt} = \rho C_p \left(\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T\right)}$$

$$\frac{d(\rho \cdot v)}{dt} = -\nabla p \cancel{+} - [\nabla\cdot(\rho v v)] - \nabla\cdot\tau + \rho g$$

Stokes Eqn: Assumes no acceleration

$$0 = -\nabla p + \mu \nabla^2 v + \rho g$$

Euler's Eqn: assumes negligible viscosity and const ρ.

$$\rho \frac{Dv}{dt} = -\nabla p + \rho g$$

~~cylindrical~~ cartesian coordinates

$$\nabla\cdot\vec{v} = \frac{dv_x}{dx} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \quad ; \quad \nabla^2\vec{v} = \nabla^2 v_x + \nabla^2 v_y + \nabla^2 v_z$$

$$\nabla^2 \vec{v}_x = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}$$

cylindrical ~~spherical~~ coordinates

$$\nabla\cdot\vec{v} = \frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla^2 \vec{v}]_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}$$

Spherical

$$\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$[\nabla^2\vec{v}]_r = \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}$$

$$- \frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) - \frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

KEY EQUATIONS

Conservation of Mass:

$$\frac{\partial}{\partial t}\rho = -(\vec{\nabla}\cdot\rho\vec{v})$$

Overall momentum balance

$$\frac{\partial}{\partial t}(\rho\vec{v}) = -[\nabla\cdot\rho\vec{v}\vec{v}] - \nabla\cdot\tau - \nabla p + \rho g$$

⇓ reduces to

Navier-Stokes, Stokes flow and Euler's eqn.

Conservation of Momentum:

$$\frac{\partial}{\partial t}\rho\vec{v} = -[\vec{\nabla}\cdot\vec{\vec{\phi}}] + \rho\vec{g}$$

$$\rho\frac{D\vec{v}}{Dt} = -\vec{\nabla}p - [\vec{\nabla}\cdot\vec{\vec{\tau}}] + \rho\vec{g}$$

$$\rho\frac{D\vec{v}}{Dt} = -\vec{\nabla}p + \mu\vec{\nabla}^2\vec{v} + \rho\vec{g}$$

Navier-Stokes (const. ρ + μ)

Conservation of Energy

$$\frac{\partial}{\partial t}\rho(\hat{U} + \tfrac{1}{2}v^2) = -(\vec{\nabla}\cdot\vec{e}) + (\rho\vec{v}\cdot\vec{g})$$

$$\rho\frac{D}{Dt}(\hat{U} + \tfrac{1}{2}v^2) = -(\vec{\nabla}\cdot\vec{q}) - (\vec{\nabla}\cdot p\vec{v}) - (\vec{\nabla}\cdot[\vec{\tau}\cdot\vec{v}]) + (\rho\vec{v}\cdot\vec{g})$$

$$\rho\hat{C}_p\frac{DT}{Dt} = \underbrace{k\vec{\nabla}^2T}_{-(\vec{\nabla}\cdot\vec{q})} + \mu\phi_v$$

(Newtonian, const. ρ + k)

unit of each term is energy/volume

V = fluid velocity

Conservation of Mass of α:

$$\frac{\partial}{\partial t}\rho w_\alpha = -(\vec{\nabla}\cdot\vec{n}_\alpha) + r_\alpha$$

$$\frac{\partial}{\partial t}c_\alpha = -(\vec{\nabla}\cdot\vec{N}_\alpha) + R_\alpha$$

– Viscous dissipation term is only useful for flow with huge velocity gradients

– $\tau = q = 0$: adiabatic flow & high spd flows around streamlined objects.

$$\rho\frac{Dw_\alpha}{Dt} = -(\vec{\nabla}\cdot\vec{j}_\alpha) + r_\alpha$$

$$c\frac{Dx_\alpha}{Dt} = -(\vec{\nabla}\cdot\vec{J}^*_\alpha) + R_\alpha - x_\alpha\sum_{\beta=1}^{N}R_\beta$$

$$\rho\frac{Dw_\alpha}{Dt} = \rho D_{AB}\nabla^2 w_\alpha + r_\alpha$$

$$c\frac{Dx_\alpha}{Dt} = cD_{AB}\nabla^2 x_\alpha + R_\alpha - x_\alpha\sum^{N}R_\beta$$

} const. ρD_{AB}

Newton's Law of Viscosity

$$\vec{\vec{\tau}} = -\mu(\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^{+}) + (\tfrac{2}{3}\mu - \kappa)(\vec{\nabla}\cdot\vec{v})\vec{\delta}$$

cylindrical : $\vec{\nabla}\vec{v} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} & \frac{\partial v_r}{\partial z} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} & \frac{\partial v_\theta}{\partial z} \\ \frac{\partial v_z}{\partial r} & \frac{1}{r}\frac{\partial v_z}{\partial \theta} & \frac{\partial v_z}{\partial z} \end{bmatrix}$

Spherical : $\vec{\nabla}\vec{v} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} & \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} & \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} - \frac{v_\phi}{r}\cot\theta \\ \frac{\partial v_\phi}{\partial r} & \frac{1}{r}\frac{\partial v_\phi}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta}{r}\cot\theta \end{bmatrix}$

- Mechanical energy balance; reduces to Bernoulli's eqn

Fourier's Law

$$\vec{q} = -k\nabla T$$

* know what ∇ is for cylindrical and spherical coord.

Fick's Law

$\omega_A = $ mass/time

$$\vec{J}_A = -\rho D_{AB}\nabla\omega_A$$

$$\vec{J}_A^{*} = -c D_{AB}\nabla x_A$$

Eqn of continuity for multi-component mixture

$\frac{\partial \rho_\alpha}{\partial t} = -\nabla(\rho_\alpha v) - (\nabla\cdot j_\alpha) + r_\alpha$; much better form is

$$\rho\left[\frac{\partial \omega_\alpha}{\partial t} + (\vec{v}\cdot\nabla\omega_\alpha)\right] = -(\nabla\cdot\vec{j}_\alpha) + r_\alpha$$

$\frac{\partial C_A}{\partial t} = D_{AB}\nabla^2 C_A$: Fick's 2nd law of diffusion

Overall energy balance

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U}\right) = -\left(\nabla\cdot\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U}\right)\vec{v}\right) - (\nabla\cdot\vec{q}) - (\nabla\cdot p\vec{v}) - (\nabla\cdot[\tau\cdot\vec{v}]) + \rho(\vec{v}\cdot\vec{g})$$

KEY EQUATIONS

Equations of Change (Combined Fluxes)

Mass : $\frac{\partial}{\partial t} \rho = -(\nabla \cdot \rho \vec{v})$

Mass of α : $\frac{\partial}{\partial t} \rho w_\alpha = -(\nabla \cdot \vec{n}_\alpha) + r_\alpha$ or $\frac{\partial c_\alpha}{\partial t} = -(\nabla \cdot \vec{N}_\alpha) + R_\alpha$

Momentum : $\frac{\partial}{\partial t} \rho \vec{v} = -[\nabla \cdot \vec{\phi}] + \rho \vec{g}$

Energy : $\frac{\partial}{\partial t} \rho (\hat{U} + \frac{1}{2} v^2) = -(\nabla \cdot \vec{e}) + (\rho \vec{v} \cdot \vec{g})$

Combined Fluxes (Molecular + Convective)

Mass : $\vec{n}_\alpha = \vec{j}_\alpha + \rho \vec{v} w_\alpha$

Momentum : $\vec{\phi} = \vec{\pi} + \rho \vec{v}\vec{v}$ (tensors) $\vec{\pi} = p\vec{\delta} + \vec{\tau}$

Energy : $\vec{e} = (\vec{q} + [\vec{\pi} \cdot \vec{v}]) + \rho \vec{v} (\hat{U} + \frac{1}{2} v^2)$

Equations of Change (Molecular Fluxes)

Mass : $\frac{D\rho}{Dt} = -\rho (\nabla \cdot \vec{v})$

Mass of α : $\rho \frac{D w_\alpha}{Dt} = -(\nabla \cdot \vec{j}_\alpha) + r_\alpha$

Momentum : $\rho \frac{D \vec{v}}{Dt} = -\nabla p - [\nabla \cdot \vec{\tau}] + \rho \vec{g}$

Energy : $\rho \frac{D}{Dt} (\hat{U} + \frac{1}{2} v^2) = -(\nabla \cdot \vec{q}) - (\nabla \cdot p \vec{v}) - (\nabla \cdot [\vec{\tau} \cdot \vec{v}]) + (\rho \vec{v} \cdot \vec{g})$

Newton's Law

$$\vec{\tau} = -\mu (\nabla \vec{v} + (\nabla \vec{v})^{+}) + (\frac{2}{3}\mu - \kappa)(\nabla \cdot \vec{v}) \vec{\delta}$$

Fourier's Law

$$\vec{q} = -k \nabla T$$

Fick's First Law

$$\vec{j}_A = -\rho D_{AB} \nabla w_A \quad , \quad \vec{J}_A^{*} = -c D_{AB} \nabla x_A$$

Other Equations of Change

Angular Momentum : $$\frac{\partial}{\partial t} \rho[\vec{r}\times\vec{v}] = -[\nabla\cdot \rho\vec{v}[\vec{r}\times\vec{v}]] - [\nabla\cdot[r\times p\vec{\delta}]^{+}] - [\nabla\cdot[\vec{r}\times\vec{\tau}^{+}]^{+}] + [\vec{r}\times\rho\vec{g}] - [\vec{\varepsilon}:\vec{\tau}]$$

Internal Energy : $$\frac{\partial}{\partial t}\rho\hat{U} = -(\nabla\cdot\rho\hat{U}\vec{v}) - (\nabla\cdot\vec{q}) - p(\nabla\cdot\vec{v}) - (\vec{\tau}:\nabla\vec{v}$$

Temperature : $$\rho\hat{C}_p\frac{DT}{Dt} = -(\nabla\cdot\vec{q}) - (\vec{\tau}:\nabla\vec{v}) - \left(\frac{\partial \ln\rho}{\partial \ln T}\right)_p \frac{Dp}{Dt}$$

Mechanical Energy : $$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho\hat{\Phi}\right) = -\left(\nabla\cdot\left(\tfrac{1}{2}\rho v^2 + \rho\hat{\Phi}\right)\vec{v}\right) - (\nabla\cdot p\vec{v}) - p(-\nabla\cdot\vec{v}) - (\nabla\cdot[\vec{\tau}\cdot\vec{v}]) - (-\vec{\tau}:\nabla\vec{v})$$

Macroscopic Balances

Mass : $$\frac{d}{dt}m_{tot} = -\Delta w + w_o \qquad \Delta w = \overset{\text{exit}}{w_2} - \overset{\text{entrance}}{w_1}$$

$$w_o = \Sigma_\alpha w_{\alpha,o}$$

Mass of α : $$\frac{dm_{\alpha,tot}}{dt} = -\Delta w_\alpha + w_{\alpha,o} + r_{\alpha,tot} \qquad \Sigma_\alpha r_{\alpha,tot} = 0$$

Moles of α: $$\frac{dM_{\alpha,tot}}{dt} = -W_\alpha + W_{\alpha,o} + R_{\alpha,tot}$$

Moles : $$\frac{dM_{tot}}{dt} = -\Delta W + W_o + \sum_{\alpha=1}^{N} R_{\alpha,tot}$$

Momentum : $$\frac{d\vec{P}_{tot}}{dt} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle}w + pS\right)\vec{u} + \vec{F}_{s\to f} + \vec{F}_o + m_{tot}\vec{g}$$

Angular Momentum : $$\frac{d\vec{L}}{dt} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle}w + pS\right)[\vec{r}\times\vec{u}] + \vec{T}_{s\to f} + \vec{T}_o + \vec{T}_{ext}$$

Energy : $$\frac{d}{dt}(U_{tot} + K_{tot} + \Phi_{tot}) = -\Delta(\hat{U} + p\hat{V} + \tfrac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi})w] + Q_o + Q + W_m$$

Mechanical Energy : $$\frac{d}{dt}(K_{tot} + \Phi_{tot}) = -\Delta\left[\left(\tfrac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \frac{p}{\rho}\right)w\right] + B_o + W_m - E_c - E_v$$

Especially with gases

$$\boxed{\begin{aligned} &\underset{\text{total}}{N_A} = -cD_{AB}\frac{dx_A}{dz} + x_A(N_{Az} + N_{Bz}) \\ &\frac{\partial C_A}{\partial t} = -(\nabla\cdot N_A) + R_A \end{aligned}}$$

DIFFERENTIAL OPERATIONS

s = scaler
$\vec{v}$ = vector
$\vec{\tau}$ = tensor

Cartesion Coordinates

$$[\nabla s]_i = \frac{\partial s}{\partial i} \qquad i = x, y, z$$

$$[\nabla \times \vec{v}]_x = \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \qquad [\nabla \times \vec{v}]_y = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}$$

$$[\nabla \times \vec{v}]_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \qquad [\nabla \cdot \vec{v}] = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla^2 s = \nabla \cdot \nabla s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$[\nabla^2 \vec{v}]_i = [\nabla \cdot \nabla \vec{v}]_i = \frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2} + \frac{\partial^2 v_i}{\partial z^2} \qquad i = x, y, z$$

$$[\nabla \cdot \vec{\tau}]_i = \frac{\partial \tau_{xi}}{\partial x} + \frac{\partial \tau_{yi}}{\partial y} + \frac{\partial \tau_{zi}}{\partial z} \qquad i = x, y, z$$

$$(\vec{\tau} : \nabla \vec{v}) = \tau_{xx} \frac{\partial v_x}{\partial x} + \tau_{xy} \frac{\partial v_x}{\partial y} + \tau_{xz} \frac{\partial v_x}{\partial z}$$
$$+ \tau_{yx} \frac{\partial v_y}{\partial x} + \tau_{yy} \frac{\partial v_y}{\partial y} + \tau_{yz} \frac{\partial v_y}{\partial z}$$
$$+ \tau_{zx} \frac{\partial v_z}{\partial x} + \tau_{zy} \frac{\partial v_z}{\partial y} + \tau_{zz} \frac{\partial v_z}{\partial z}$$

Cylindrical Coordinates

$$[\nabla s]_r = \frac{\partial s}{\partial r} \qquad [\nabla s]_\theta = \frac{1}{r} \frac{\partial s}{\partial \theta} \qquad [\nabla s]_z = \frac{\partial s}{\partial z}$$

$$[\nabla \times \vec{v}]_r = \frac{1}{r} \frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \qquad [\nabla \times \vec{v}]_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}$$

$$[\nabla \times \vec{v}]_z = \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) - \frac{1}{r} \frac{\partial v_r}{\partial \theta} \qquad [\nabla \cdot \vec{v}] = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla^2 s = \nabla \cdot \nabla s = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial s}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$[\nabla^2 \vec{v}]_r = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta}$$

$$[\nabla^2 \vec{v}]_\theta = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta}$$

$$[\nabla^2 \vec{v}]_z = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}$$

$$[\nabla\cdot\vec{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial\theta}(\tau_{\theta r}) + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}$$

$$[\nabla\cdot\vec{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}) + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}$$

$$[\nabla\cdot\vec{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial\theta}(\tau_{\theta z}) + \frac{\partial}{\partial z}\tau_{zz}$$

$$(\vec{\tau}:\nabla\vec{v}) = \tau_{rr}\left(\frac{\partial v_r}{\partial r}\right) + \tau_{r\theta}\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r}\right) + \tau_{rz}\left(\frac{\partial v_r}{\partial z}\right)$$
$$+ \tau_{\theta r}\left(\frac{\partial v_\theta}{\partial r}\right) + \tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) + \tau_{\theta z}\left(\frac{\partial v_\theta}{\partial z}\right)$$
$$+ \tau_{zr}\left(\frac{\partial v_z}{\partial r}\right) + \tau_{z\theta}\left(\frac{1}{r}\frac{\partial v_z}{\partial\theta}\right) + \tau_{zz}\left(\frac{\partial v_z}{\partial z}\right)$$

Spherical Coordinates

$$[\nabla s]_r = \frac{\partial s}{\partial r} \qquad [\nabla s]_\theta = \frac{1}{r}\frac{\partial s}{\partial\theta} \qquad [\nabla s]_\phi = \frac{1}{r\sin\theta}\frac{\partial s}{\partial\phi}$$

$$[\nabla\times\vec{v}]_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta) - \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} \qquad [\nabla\times\vec{v}]_\theta = \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(r v_\phi)$$

$$[\nabla\times\vec{v}]_\phi = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial\theta} \qquad [\nabla\cdot\vec{v}] = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial s}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial\phi^2}$$

$$[\nabla^2\vec{v}]_r = \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}$$
$$- \frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta) - \frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$[\nabla^2\vec{v}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$[\nabla^2\vec{v}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial\phi}$$

$$[\nabla\cdot\vec{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla\cdot\vec{\tau}]_\theta = \frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}$$

$$[\nabla\cdot\vec{\tau}]_\phi = \frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}$$

$$(\vec{\tau}:\nabla\vec{v}) = \tau_{rr}\left(\frac{\partial v_r}{\partial r}\right) + \tau_{r\theta}\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r}\right) + \tau_{r\phi}\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{v_\phi}{r}\right)$$
$$+ \tau_{\theta r}\left(\frac{\partial v_\theta}{\partial r}\right) + \tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) + \tau_{\theta\phi}\left(\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} - \frac{v_\phi}{r}\cot\theta\right)$$
$$+ \tau_{\phi r}\left(\frac{\partial v_\phi}{\partial r}\right) + \tau_{\phi\theta}\left(\frac{1}{r}\frac{\partial v_\phi}{\partial\theta}\right) + \tau_{\phi\phi}\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta}{r}\cot\theta\right)$$

Momentum Transport

Ch. 1 Viscosity

Newton's Law : $\tau_{yx} = -\mu \dfrac{dv_x}{dy}$ → definition of newtonian fluid

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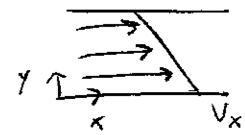
steady state, laminar flow



Newtonian fluids

flux of x momentum in positive y direction

Generalization of Newton's Law → for flow in more than 1 direction

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$$\tau_{ij} = -\mu\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + \left(\frac{2}{3}\mu - \kappa\right)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\delta_{ij}$$

$$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

$\kappa = 0$ ideal, monatomic gas

$$\underline{\underline{\tau}} = -\mu\left(\nabla\underline{v} + (\nabla\underline{v})^{+}\right) + \left(\frac{2}{3}\mu - \kappa\right)(\nabla\cdot\underline{v})\,\underline{\underline{\delta}}$$

$\nabla\cdot\underline{v} = 0$ incompressible liquid

Molecular Theory of Gases (low ρ)

Rigid sphere model :

p. 23

$$\mu = \frac{1}{3} n m \bar{u} \lambda = \frac{1}{3}\rho \bar{u}\lambda$$

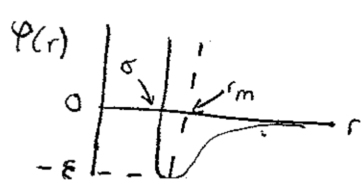
$$\mu = \frac{2}{3}\frac{\sqrt{m\kappa_B T/\pi}}{\pi d^2} = \frac{2}{3\pi}\frac{\sqrt{\pi m \kappa_B T}}{\pi d^2} \propto \frac{\sqrt{m}}{d^2} T^{1/2}$$

Lennard-Jones Potential (nonpolar molecules)

$$F(r) = \frac{-d\varphi}{dr}$$

$$\varphi(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

$$\mu = \frac{5}{16}\frac{\sqrt{\pi m \kappa_B T}}{\pi\sigma^2\Omega_\mu} = 2.6693\times10^{-5}\frac{\sqrt{MT}}{\sigma^2\Omega_\mu}$$



M = mol. wt.

For gas mixtures:
$$\mu_{mix} = \sum_{\alpha=1}^{N} \frac{x_\alpha \mu_\alpha}{\Sigma_\beta x_\beta \phi_{\alpha\beta}}$$

$$\phi_{\alpha\beta} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_\alpha}{M_\beta}\right)^{-1/2}\left[1 + \left(\frac{\mu_\alpha}{\mu_\beta}\right)^{1/2}\left(\frac{M_\beta}{M_\alpha}\right)^{1/4}\right]^2$$

Molecular Theory of Liquids

p. 29

$$\mu = \frac{\tilde{N} h}{\tilde{V}} \exp\left(3.8\, T_b / T\right)$$

Suspensions and Emulsions

Einstein: $\frac{\mu_{eff}}{\mu_0} = 1 + \frac{5}{2}\phi$ (spheres)

if $\phi > 0.05$, Mooney eqn
$$\frac{\mu_{eff}}{\mu_0} = \exp\left(\frac{5/2\,\phi}{1 - (\phi/\phi_0)}\right)$$

Graham Eqn for conc. solns
$$\frac{\mu_{eff}}{\mu_0} = 1 + \frac{5}{2}\phi + \frac{9}{4}\left(\frac{1}{\psi\,(1 + \frac{1}{2}\psi)(1+\psi)^2}\right)$$

Krieger-Dougherty, Taylor, Smoluchowski Eqns

Convective Momentum Transport

p. 34

Momentum flux (9 components)
$$\rho\vec{v}\vec{v} = \left(\sum_i \delta_i \rho v_i\right)\vec{v} = \left(\sum_i \delta_i \rho v_i\right)\left(\sum_j \delta_j v_j\right)$$

see Table 1.7-1

flux through a plane of arbitrary orientation $\vec{n}$

volume rate of flow through surface $(\vec{n}\cdot\vec{v})\,dS$

rate of flow of momentum across surface
$$(\vec{n}\cdot\vec{v})\,\rho\vec{v}\,dS$$

Combined momentum flux
$$\vec{\phi} = \vec{\pi} + \rho\vec{v}\vec{v} = p\,\vec{\delta} + \vec{\tau} + \rho\vec{v}\vec{v}$$

Ch. 2 Shell Balances (Laminar Flow) (steady state)

Flow of a Falling Film (Inclined Plate)

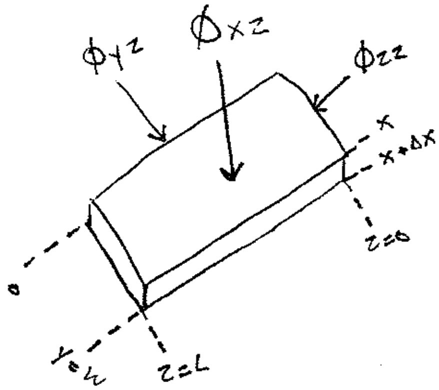
See Figure 2.2-2 p. 42

$$LW(\phi_{xz}|_x - \phi_{xz}|_{x+\Delta x}) + W\Delta x(\phi_{zz}|_{z=0} - \phi_{zz}|_{z=L})$$

fluxes — convective fluxes

$$+ (LW\Delta x)(\rho g \cos\beta) = 0$$

force of gravity



$\phi_{yz} = \rho v_y v_z + \tau_{yz}$

$\phi_{xz} = \rho v_x v_z + \tau_{xz}$

$\phi_{zz} = \rho v_z v_z + p + \tau_{zz}$

$p = p(x)$

p and $\rho v_z v_z$ are the same at $z=0$ and $z=L$

Since v_x, v_y are 0 and $v_z = v_z(x)$

then $\tau_{yz} = \tau_{zz} = 0$

Divide by $LW\Delta x$ and take $\lim_{\Delta x \to 0}$.

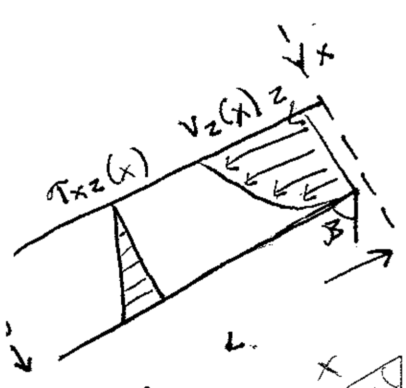
Substitute in definitions of ϕ.

$$\frac{d\tau_{xz}}{dx} = \rho g \cos\beta$$

integrate and use BC: $x=0$, $\tau_{xz} = 0$

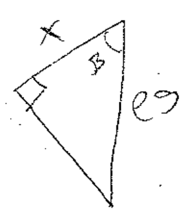
$$\tau_{xz} = (\rho g \cos\beta)x = -\mu \frac{dv_z}{dx}$$

integrate and use BC: $x = \delta$, $v_z = 0$ (no-slip)



$$v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

parabolic velocity distribution



$\cos\beta = x/\rho g$

$x = \rho g \cos\beta$

$$\langle v_z \rangle = \frac{\int_0^W \int_0^\delta v_z \, dx\, dy}{\int_0^W \int_0^\delta dx\, dy} = \frac{1}{\delta}\int_0^\delta v_z \, dx$$

mass flow rate $w = \int_0^W \int_0^\delta \rho v_z \, dx\, dy$

$\delta = \sqrt{\frac{3\mu\langle v_z\rangle}{\ldots}}$ $\quad Re = 4\delta\langle v_z\rangle\rho/\mu$

$$\pi(r+\Delta r)^2 - \pi r^2 = \pi(r^2 + 2r\Delta r + \Delta r^2) - \pi r^2$$
$$= 2r\Delta r\pi$$

Flow through a Circular Tube

p. 48

see Figure 2.3-1

$$(2\pi r L \phi_{rz})|_r - (2\pi r L)(\phi_{rz})|_{r+\Delta r} + (2\pi r \Delta r)\phi_{zz}|_{z=0}$$

mol. flux in — mol. flux out — conv. flux in

$$-(2\pi r \Delta r)\phi_{zz}|_{z=L} + (2\pi r \Delta r L)\rho g = 0$$

conv. flux out — gravity force

$\phi_{rz} = \tau_{rz} + \cancel{\rho v_r v_z}^{\,0}$

$\phi_{zz} = \cancel{\tau_{zz}}^{\,0} + \rho v_z v_z + p$

divide by $2\pi L \Delta r$ and take limit $\Delta r \to 0$

$$-\frac{\partial r\phi_{rz}}{\partial r} = -r\,\phi_{zz}|_{z=0} + r\,\phi_{zz}|_{z=L} - r\rho g$$

$v_z = v_z(r)$, $v_r = v_\theta = 0$, $p = p(z)$

$$\frac{d}{dr}(r\tau_{rz}) = \left(\frac{(p_0 - \rho g 0) - (p_L - \rho g L)}{L}\right) r = \frac{P_0 - P_L}{L} r$$

integrate and use BC : $r = 0$, τ_{rz} = finite

$$\tau_{rz} = \frac{P_0 - P_L}{2L} r = -\mu \frac{dv_z}{dr}$$

integrate and use BC : $r = R$, $v_z = 0$ (no-slip)

$$v_z = \frac{(P_0 - P_L)R^2}{4\mu L}\left[1 - (r/R)^2\right]$$

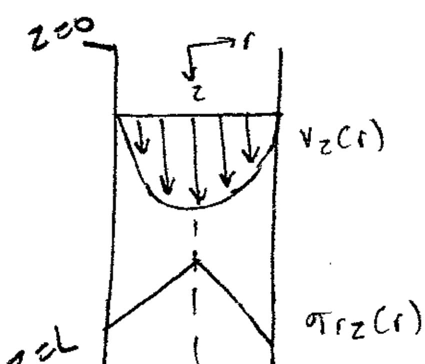
parabolic

$P_0 = p_0 - \rho g(z=0)$

$P_L = p_L - \rho g L$

$Re = D\langle v_z\rangle \rho / \mu$

laminar flow in a tube



Hogen - Pousielle

$$\boxed{Q = \frac{(P_0 - P_L) R^4}{8\mu L}}$$

Flow Through An Annulus

steady-state flow between 2 cylinders of radii κR and R, flowing upward

$v_z = v_z(r)$, $v_\theta = 0 = v_r$, $p = p(z)$

$$\frac{d}{dr}(r\tau_{rz}) = \left(\frac{(p_0 + \rho g 0) - (p_L + \rho g L)}{L}\right) r = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} r\right)$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r + \frac{C_1}{r}$$

v_z is max at some $r = \lambda R$ $(\tau_{rz} = 0)$

$\longrightarrow$ solve for C_1

$$\tau_{rz} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R}{2L}\left[\left(\frac{r}{R}\right) - \lambda^2\left(\frac{R}{r}\right)\right] = -\mu\left(\frac{dv_z}{dr}\right)$$

integrate and evaluate constants C_2 and λ

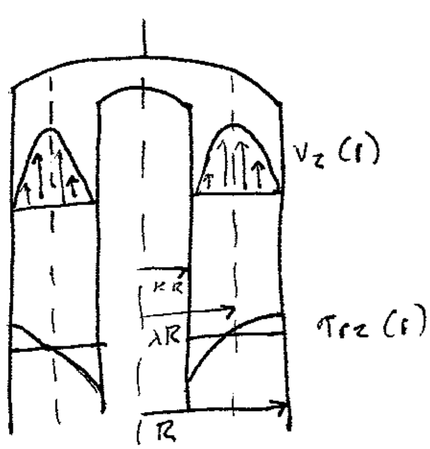
BC: $r = \kappa R$, $v_z = 0$

$r = R$, $v_z = 0$ (no-slip)

$$\tau_{rz} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R}{2L}\left[\left(\frac{r}{R}\right) - \frac{1-\kappa^2}{2\ln(1/\kappa)}\left(\frac{R}{r}\right)\right]$$

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2 - \frac{1-\kappa^2}{\ln(1/\kappa)}\ln\left(\frac{R}{r}\right)\right]$$

$$Re = 2R(1-\kappa)\langle v_z\rangle \rho/\mu$$



Flow of Two Adjacent Immiscible Fluids

driven by pressure gradient $(p_L - p_0)/L$

$$W\Delta x(\phi_{zz}|_{z=L} - \phi_{zz}|_{z=0}) + wL(\phi_{xz}|_{x+\Delta x} - \phi_{xz}|_{x})$$

$$+ \, wL\Delta x\left(\frac{p_L - p_0}{L}\right) = 0$$

$p = p(x)$, $v_z = v_z(x)$, $v_x = 0$

$$\frac{d\tau_{xz}}{dx} = \frac{p_0 - p_L}{L} \quad \text{valid for both regions } (I + II)$$

$$\tau_{xz}^{I} = \left(\frac{p_0 - p_L}{L}\right)x + C_1^{I} = -\mu^{I}\, dv_z^{I}/dx$$

$$\tau_{xz}^{II} = \left(\frac{p_0 - p_L}{L}\right)x + C_1^{II} = -\mu^{II}\, dv_z^{II}/dx$$

BC: $x = 0$ (fluid-fluid interface), $\tau_{xz}^{I} = \tau_{xz}^{II}$

$\longrightarrow \; C_1^{I} = C_1^{II}$

integrate and use BCs

$x = 0$, $v_z^{I} = v_z^{II}$

$x = -b$, $v_z^{I} = 0$

$x = +b$, $v_z^{II} = 0$

$$\tau_{xz} = \frac{(p_0 - p_L)b}{L}\left[\left(\frac{x}{b}\right) - \frac{1}{2}\left(\frac{\mu^{I} - \mu^{II}}{\mu^{I} + \mu^{II}}\right)\right]$$

$$v_z^{I} = \frac{(p_0 - p_L)b^2}{2\mu^{I}L}\left[\left(\frac{2\mu^{I}}{\mu^{I} + \mu^{II}}\right) + \left(\frac{\mu^{I} - \mu^{II}}{\mu^{I} + \mu^{II}}\right)\left(\frac{x}{b}\right) - \left(\frac{x}{b}\right)^2\right]$$

$$v_z^{II} = \frac{(p_0 - p_L)b^2}{2\mu^{II}L}\left[\left(\frac{2\mu^{II}}{\mu^{I} + \mu^{II}}\right) + \left(\frac{\mu^{I} - \mu^{II}}{\mu^{I} + \mu^{II}}\right)\left(\frac{x}{b}\right) - \left(\frac{x}{b}\right)^2\right]$$

Creeping Flow Around a Sphere

V_r and V_θ → c/n use shell balance method

creeping flow = very slow flow = Stokes flow

$Re = D v_\infty \rho / \mu < 0.1$

absence of eddy formation downstream of the sphere

From ch. 4 : V_r, V_θ, V_ϕ, p solved

τ_{rr}, $\tau_{\theta\theta}$, $\tau_{\phi\phi}$, $\tau_{r\theta}$ solved (all other = 0)

Normal force :

at each point fluid exerts force $-(p+\tau_{rr})|_{r=R}$

$$F^{(n)} = \int_0^{2\pi}\int_0^{\pi} \underbrace{\left(-(p+\tau_{rr})|_{r=R}\cos\theta\right)}_{\text{z-comp of force}} \underbrace{R^2 \sin\theta\, d\theta\, d\phi}_{\text{surface element}}$$

$$= \frac{4}{3}\pi R^3 \rho g + 2\pi\mu R v_\infty$$

Tangential Force :

at each point, $\tau_{r\theta}|_{r=R}$

$$F^{(t)} = \int_0^{2\pi}\int_0^{\pi} \underbrace{(\tau_{r\theta}|_{r=R})\sin\theta}_{\text{z-comp of force}} \underbrace{R^2 \sin\theta\, d\theta\, d\phi}_{\text{surface element}}$$

$$= 4\pi\mu R v_\infty$$

$$F = \underbrace{\frac{4}{3}\pi R^3 \rho g}_{\text{buoyant force}} + \underbrace{2\pi\mu R v_\infty}_{\text{form drag}} + \underbrace{4\pi\mu R v_\infty}_{\text{friction drag}}$$

form drag ↓ normal to surface

friction drag ↓ tangential

$$F = \underbrace{\frac{4}{3}\pi R^3 \rho g}_{\text{buoyant force}} + \underbrace{6\pi\mu R v_\infty}_{\text{kinetic force}}$$

Stokes' Law

kinetic force → Stoke's law

useful for $Re < 0.1$

See Appendix B for all coordinates

Ch. 3 Equations of Change (Isothermal Systems)

Continuity Equation p. 77

$$\frac{\partial \rho}{\partial t} = -(\vec{\nabla} \cdot \rho \vec{v}) = -\left(\frac{\partial}{\partial x} \rho v_x + \frac{\partial}{\partial y} \rho v_y + \frac{\partial}{\partial z} \rho v_z\right)$$

incompressible fluid : $(\vec{\nabla} \cdot \vec{v}) = 0$

Equation of Motion p. 78

$$\begin{cases} \frac{\partial}{\partial t} \rho v_x = -\left(\frac{\partial}{\partial x} \phi_{xx} + \frac{\partial}{\partial y} \phi_{yx} + \frac{\partial}{\partial z} \phi_{zx}\right) + \rho g_x \\ \frac{\partial}{\partial t} \rho v_y = -\left(\frac{\partial}{\partial x} \phi_{xy} + \frac{\partial}{\partial y} \phi_{yy} + \frac{\partial}{\partial z} \phi_{zy}\right) + \rho g_y \\ \frac{\partial}{\partial t} \rho v_z = -\left(\frac{\partial}{\partial x} \phi_{xz} + \frac{\partial}{\partial y} \phi_{yz} + \frac{\partial}{\partial z} \phi_{zz}\right) + \rho g_z \end{cases}$$

$$\frac{\partial}{\partial t} \rho v_i = -[\vec{\nabla} \cdot \vec{\phi}]_i + \rho g_i \qquad i = x, y, z$$

$$\frac{\partial}{\partial t} \rho \vec{v} = -[\vec{\nabla} \cdot \vec{\phi}] + \rho \vec{g}$$

$$\underbrace{\frac{\partial}{\partial t} \rho \vec{v}}_{\text{rate of } \uparrow \text{ of momentum per unit vol.}} = \underbrace{-[\vec{\nabla} \cdot \rho \vec{v}\vec{v}]}_{\text{convective momentum transport}} \underbrace{- \vec{\nabla} p - [\vec{\nabla} \cdot \vec{\tau}]}_{\text{molecular momentum transport}} + \underbrace{\rho \vec{g}}_{\text{external force}}$$

Equation of Mechanical Energy p. 81

Kinetic Energy :

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2\right) = -(\vec{\nabla} \cdot \tfrac{1}{2}\rho v^2 \vec{v}) - (\vec{\nabla} \cdot p\vec{v}) - p(-\vec{\nabla} \cdot \vec{v}) - (\vec{\nabla} \cdot (\vec{\tau} \cdot \vec{v})) - (-\vec{\tau} : \vec{\nabla}\vec{v}) + \rho(\vec{v} \cdot \vec{g})$$

Kinetic + Potential Energy :

$$\boxed{\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho \hat{\Phi}\right) = -(\vec{\nabla} \cdot (\tfrac{1}{2}\rho v^2 + \rho \hat{\Phi})\vec{v}) - (\vec{\nabla} \cdot p\vec{v}) - p(-\vec{\nabla} \cdot \vec{v}) - (\vec{\nabla} \cdot [\vec{\tau} \cdot \vec{v}]) - (-\vec{\tau} : \vec{\nabla}\vec{v})}$$

Equation of Angular Momentum p. 82

$$\frac{\partial}{\partial t} \rho[\vec{r} \times \vec{v}] = -[\vec{\nabla} \cdot \rho\vec{v}\,[\vec{r} \times \vec{v}]] - [\vec{\nabla} \cdot [\vec{r} \times p\vec{\delta}]^{+}]$$
$$- [\vec{\nabla} \cdot [\vec{r} \times \vec{\tau}^{+}]^{+}] + [\vec{r} \times \rho\vec{g}] - [\vec{\varepsilon} : \vec{\tau}]$$

if $\vec{\tau}$ is symmetric, last term $= 0$

Substantial derivative p. 83

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_x \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} + v_z \frac{\partial c}{\partial z}$$

velocity of observer same as velocity of system

$$\rho\frac{Df}{Dt} = \frac{\partial}{\partial t}(\rho f) + \left(\frac{\partial}{\partial x}\rho v_x f\right) + \left(\frac{\partial}{\partial y}\rho v_y f\right) + \left(\frac{\partial}{\partial z}\rho v_z f\right)$$

Equations of change:

$$\frac{D\rho}{Dt} = -\rho(\vec{\nabla} \cdot \vec{v})$$

$$\rho\frac{D\vec{v}}{Dt} = -\vec{\nabla}p - [\vec{\nabla} \cdot \tau] + \rho\vec{g}$$

$$\rho\frac{D}{Dt}\left(\tfrac{1}{2}v^2\right) = -(\vec{v} \cdot \vec{\nabla}p) - (\vec{v} \cdot [\vec{\nabla} \cdot \vec{\tau}]) + \rho(\vec{v} \cdot \vec{g})$$

$$\rho\frac{D}{Dt}[\vec{r} \times \vec{v}] = -[\vec{\nabla} \cdot [\vec{r} \times p\vec{\delta}]^{+}] - [\vec{\nabla} \cdot [\vec{r} \times \vec{\tau}]^{+}]$$
$$+ [\vec{r} \times \rho\vec{g}]$$

Constant ρ and μ: Navier-Stokes Eqn p. 85

$$\rho\frac{D}{Dt}\vec{v} = -\vec{\nabla}p + \mu\vec{\nabla}^2\vec{v} + \rho\vec{g}$$

Creeping flow: neglect acceleration : Stokes Flow Eqn

$$0 = -\vec{\nabla}p + \mu\vec{\nabla}^2\vec{v} + \rho\vec{g}$$

Also Hagen - Poiseuille tube flow at any speed
(term drops out)

p. 85

Neglect viscous forces $(\vec{\nabla}\cdot\vec{\tau} = 0)$: Euler equation for inviscid fluids

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p + \rho \vec{g}$$

$Re >> 1$

Bernoulli Equation for steady flow of inviscid fluids p. 86

$$\boxed{\frac{1}{2}(v_2^2 - v_1^2) + \int_{p_1}^{p_2} \frac{1}{\rho}\, dp + g(h_2 - h_1) = 0}$$

also irrotational - potential flow

derived from Euler equation

Using the equations of change :

ex - steady flow in a long circular tube p. 88

let $\vec{v} = v_z(r, \theta)$

continuity eqn : $\frac{\partial v_z}{\partial z} = 0$ (see Appendix B)

eqns of motion : $0 = -\frac{d\mathcal{P}}{dr}$

$$0 = -\frac{d\mathcal{P}}{d\theta}$$

$$0 = -\frac{d\mathcal{P}}{dz} + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right)$$

$\mathcal{P} = p + \rho g h$

if $d\mathcal{P}/dr = d\mathcal{P}/d\theta = 0$, then $\mathcal{P} = \mathcal{P}(z)$ only

let $\mu \frac{1}{r}\frac{d}{dr}\left(r \frac{dv_z}{dr}\right) = \frac{d\mathcal{P}}{dz} = C_0$ (constant)

integrate both sides and use BC :

$z = 0$, $\mathcal{P} = \mathcal{P}_0$

$z = L$, $\mathcal{P} = \mathcal{P}_L$

$r = R$, $v_z = 0$ (no-slip)

$r = 0$, $v_z =$ finite

$$\mathcal{P} = \mathcal{P}_0 - (\mathcal{P}_0 - \mathcal{P}_L)(z/L)$$

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu L}\left[1 - (r/R)^2\right]$$

same as w/ shell balance

ex - Falling film with varying μ, steady state p. 89

look at Table B.5:

$$0 = -\frac{\partial p}{\partial x} + \rho g \sin \beta$$

$$0 = -\frac{\partial p}{\partial y}$$

$$0 = -\frac{\partial p}{\partial z} - \frac{d}{dx}\tau_{xz} + \rho g \cos \beta$$

First eqn: $p = p(x) \longrightarrow \frac{\partial p}{\partial z} = 0$

now 3rd eqn same as before

ex - Couette viscometer steady state p. 89

$V_\theta = V_\theta(r)$, $V_r = 0$, $V_z = 0$, $p = p(r, z)$

$$-\rho \frac{V_\theta^2}{r} = -\frac{\partial p}{\partial r}$$

$$0 = \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r V_\theta)\right)$$

$$0 = -\frac{\partial p}{\partial z} - \rho g$$

use BC (no-slip): $r = KR$, $V_\theta = 0$

$r = R$, $V_\theta = \Omega_o R$

$$V_\theta = \Omega_o R \frac{\left(\frac{r}{KR} - \frac{KR}{r}\right)}{\left(\frac{1}{K} - K\right)}$$

ex - Shape of the Surface of a Rotating Liquid p. 93

$v_r = v_z = 0 \qquad v_\theta = v_\theta(r) \quad , \quad p = p(z, r) \quad$, steady state

$$-\rho \frac{v_\theta^2}{r} = -\frac{\partial p}{\partial r}$$

$$0 = \mu \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right)$$

$$0 = -\frac{\partial p}{\partial z} - \rho g$$

BC: $r = 0$, v_θ is finite
$r = R$, $v_\theta = R\Omega$

ex - Flow near a slowly rotating sphere p. 95
use creeping flow equation of motion

$v = v_\phi(r, \theta) \quad , \quad P = P(r, \theta) \quad$ steady state

symmetric about z-axis → no dependence on ϕ

$$0 = -\frac{\partial p}{\partial r}$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi \sin\theta)\right)$$

how do you get this?

Ch. 4 Velocity Distributions with More Than 1 Ind. Variable

Time-Dependent Flow of Newtonian Fluids p. 114

Similarity solution - ex. 4.1-1
Separation of Variables - ex. 4.1-2
Sinusoidal oscillations - long time behavior - ex 4.1-3

Multiple Non-Vanishing Components of Fluid Velocity p. 122

For viscous flow problems use continuity equation and equation of change for vorticity $(\vec{\nabla} \times \vec{v})$:

$$\frac{\partial}{\partial t}[\vec{\nabla} \times \vec{v}] - [\vec{\nabla} \times [\vec{v} \times [\vec{\nabla} \times \vec{v}]]] = \nu \nabla^2 [\vec{\nabla} \times \vec{v}]$$

Then use Navier-Stokes to find pressure distribution

Stream Function Method p. 122

$$v_x = -\frac{\partial \psi}{\partial y} \qquad \frac{\partial}{\partial t}(\nabla^2 \psi) + \frac{\partial(\psi, \nabla^2 \psi)}{\partial (x, y)} = \nu \nabla^2 \psi$$

$$v_y = \frac{\partial \psi}{\partial x}$$

See Table 4.2-1 for other coordinate systems

ex - Creeping Flow Around a sphere p. 122

Re << 1 : Stoke's Equation

set LHS = 0

$$0 = E^4 \psi = \left[\frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]^2 \psi = 0$$

BC: $r = R$, $v_r = -\dfrac{1}{r^2 \sin\theta}\dfrac{\partial \psi}{\partial \theta} = 0$

$r = R$, $v_\theta = \dfrac{1}{r\sin\theta}\dfrac{\partial\psi}{\partial\theta} = 0$ (no-slip)

$r \to \infty$, $\psi \longrightarrow \frac{1}{2} V_\infty r^2 \sin^2\theta$

postulate $\psi(r,\theta) = f(r)\sin^2\theta$
equidimensional - use trial solution $f(r) = Cr^n$
solve for ψ, v_r and v_θ

Flow of Inviscid Fluids Using the Velocity Potential

Euler eqn valid if $Re \gg 1$ (low viscosity)
- omit term containing viscosity

If steady and 2D flow:
- $\frac{\partial}{\partial t}$ and $[\vec{\omega} \cdot \nabla \vec{v}]$ vanish

Thus vorticity $\vec{\omega} = \vec{\nabla} \times \vec{v}$ is constant along a streamline

Irrotational - $\vec{\omega} = 0$ through entire flow field

If ρ = constant and $\vec{\omega} = 0$, potential flow

This flow description not valid near solid surfaces
- need boundary layer theory

Potential Flow:

p.126

$$\boxed{\vec{\nabla} \cdot \vec{v} = 0} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \nabla \tfrac{1}{2} v^2 - [\vec{v} \times [\vec{\nabla} \times \vec{v}]] \right) = -\nabla p$$

$$\boxed{\vec{\nabla} \times \vec{v} = 0} = \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}$$

Integrate eqn of motion to get:

$$\frac{1}{2} \rho (v_x^2 + v_y^2) + p = \text{constant}$$

Bernoulli eqn for incompressible, potential flow
the constant is the same along all streamlines

use velocity potential ϕ:

$$v_x = -\frac{\partial \phi}{\partial x} \qquad v_y = -\frac{\partial \phi}{\partial y}$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad \text{Cauchy-Riemann}$$

Solutions with the velocity potential

$$w(z) = \phi(x,y) + i\psi(x,y), \quad z = x + iy$$

$$\nabla^2 \phi = 0, \quad \nabla^2 \psi = 0$$

$\phi(x,y)$ = constant – equipotential lines
$\psi(x,y)$ = constant – streamlines
} $\perp$ to each other

$$\frac{dw}{dz} = -v_x + iv_y$$

or use inverse function

$$z(w) = x(\phi,\psi) + iy(\phi,\psi)$$

$$F(x, y, \phi) = 0 \quad \text{or} \quad G(x, y, \psi) = 0$$

$$-\frac{dz}{dw} = \frac{v_x + iv_y}{v_x^2 + v_y^2}$$

* only valid away from solid surfaces. separation, departure of streamlines from a boundary surface

ex – Potential Flow Around a Cylinder 4.3-1
ex – Flow into a rectangular channel 4.3-2
ex – Flow near a corner 4.3-3

Flow near solid surfaces – boundary layer theory p. 133

potential flow solutions don't satisfy no-slip BC at wall

method – obtain an approximate solution in a thin BL near wall taking viscosity into account. Then match this solution to the potential flow solution

→ works at high Re (thin BL)

p. 139 order of magnitude arguments

Prandtl boundary layer equations:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu \frac{\partial^2 v_x}{\partial y^2}$$

$y = 0 \quad v_x = 0$ (no-slip)
$y = 0 \quad v_y = 0$ (no mass transfer from wall)
$v_x(x,y) \rightarrow v_e(x)$ potential flow at outer edge

$$-\frac{1}{\rho}\frac{dP}{dx} = v_e \frac{dv_e}{dx}$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = v_e \frac{\partial v_e}{\partial x} + \nu \frac{\partial^2 v_x}{\partial y^2}$$

use continuity equation to substitute in v_y

$$v_x \frac{\partial v_x}{\partial x} - \left(\int_0^y \frac{\partial v_x}{\partial x} dy \right) \frac{\partial v_x}{\partial y} = v_e \frac{\partial v_e}{\partial x} + \nu \frac{\partial^2 v_x}{\partial y^2}$$

Von Kármán momentum balance:

$$\mu \left. \frac{\partial v_x}{\partial y} \right|_{y=0} = \frac{d}{dx} \int_0^\infty \rho v_x (v_e - v_x) dy + \frac{d v_e}{dx} \int_0^\infty \rho (v_e - v_x) dy$$

ex - Laminar Flow along a flat plate 4.4-1

guess a velocity distribution

4.4-2 exact solution - use stream function and combination of variables

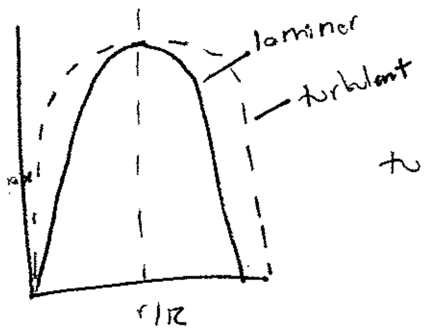
ex - Flow near a corner 4.4-3

Ch. 5 Velocity Distributions in Turbulent Flow

compare laminar and turbulent flow p. 154

- circular tube

laminar: $\dfrac{v_z}{v_{z,max}} = 1 - (r/R)^2 \qquad \dfrac{\langle v_z \rangle}{v_{z,max}} = \dfrac{1}{2} \qquad Re < 2100$



$P_0 - P_L = \left(\dfrac{8 \mu L}{\pi \rho R^4} \right) w$

turbulent: $\dfrac{\bar{v}_z}{v_{z,max}} = (1 - r/R)^{1/7} \qquad \dfrac{\langle \bar{v}_z \rangle}{v_{z,max}} = \dfrac{4}{5}$

$10^4 < Re < 10^5$

$P_0 - P_L = 0.198 \, (2/\pi)^{7/4} \left(\dfrac{\mu^{1/4} L}{\rho R^{19/4}} \right) w^{7/4}$

- plate (flat)

laminar: $F = 1.328 \sqrt{\rho \mu L W^2 v_\infty^3}$

turbulent: $F = 0.74 \sqrt[5]{\rho^4 \mu L^4 W^5 v_\infty^9}$

Time Smoothed Eqns of Change (Incompressible Fluids)

$v_z = \bar{v}_z + v_z' \rightarrow$ fluctuations from $\bar{v}_z$ p. 156

$\bar{v}_z = \dfrac{1}{t_0} \int_{t - \frac{1}{2} t_0}^{t + \frac{1}{2} t_0} v_z(s) \, ds$

if $\bar{v}_z \neq \bar{v}_z(t)$, steadily driven turbulent flow

$\overline{v_z'} = 0, \quad \overline{\bar{v}_z} = \bar{v}_z, \quad \overline{\bar{v}_z v_z'} = 0, \quad \overline{\frac{\partial}{\partial x} v_z} = \frac{\partial}{\partial x} \bar{v}_z, \quad \overline{\frac{\partial}{\partial t} v_z} = \frac{\partial}{\partial t} \bar{v}_z$

local motion in x + y direction correlated

substitute $v_z = \bar{v}_z + v_z'$ and $p = \bar{p} + p'$, and use above relations to simplify eqns of change

$\dfrac{\partial}{\partial x} \bar{v}_x + \dfrac{\partial}{\partial y} \bar{v}_y + \dfrac{\partial}{\partial z} \bar{v}_z = 0 \qquad$ (incompressible)

x-component: (also have y + z)

$$\frac{\partial}{\partial t} \rho \bar{v}_x = -\frac{\partial}{\partial x} \bar{p} - \left(\frac{\partial}{\partial x} \rho \bar{v}_x \bar{v}_x + \frac{\partial}{\partial y} \rho \bar{v}_y \bar{v}_x + \frac{\partial}{\partial z} \rho \bar{v}_z \bar{v}_x \right) \underbrace{- \left(\frac{\partial}{\partial x} \rho \overline{v_x' v_x'} + \frac{\partial}{\partial y} \rho \overline{v_y' v_x'} + \frac{\partial}{\partial z} \rho \overline{v_z' v_x'} \right)}_{\text{momentum transport associated w/ turbulent fluctuations}} + \mu \nabla^2 \bar{v}_x + \rho g_x$$

$\overline{\tau}_{xx}^{(t)} = \rho \overline{v_x' v_x'}$ $\overline{\tau}_{xy}^{(t)} = \rho \overline{v_x' v_y'}$ $\overline{\tau}_{xz}^{(t)} = \rho \overline{v_x' v_z'}$

Turbulent momentum flux tensor $\overline{\tau}^{(t)}$ Reynolds stresses

Time-smoothed viscous momentum flux $\overline{\tau}^{(v)}$

- use Newton's law with $\overline{v}_x$ instead of v_x, etc.

* Time-smoothed eqns of change:

1. replace all v_i with $\overline{v}_i$ and p with $\overline{p}$
2. replace τ_{ij} with $\overline{\tau}_{ij} = \overline{\tau}_{ij}^{(v)} + \overline{\tau}_{ij}^{(t)}$

but, can't evaluate $\overline{\tau}_{ij}^{(t)}$ easily

Near a wall: p. 159

viscous sublayer, buffer layer, inertial layer, turbulent stream

Inertial sublayer: von-Kármán-Prandtl universal logarithmic velocity distribution

$$\boxed{\frac{\overline{v}_x}{v_*} = 2.5 \ln\left(\frac{y v_*}{\nu}\right) + 5.5} \qquad \frac{y v_*}{\nu} > 30$$

$v_* =$ friction velocity $= \sqrt{\tau_0/\rho}$ $\tau_0 = -\overline{\tau}_{yx}|_{y=0}$

Barenblatt-Chorin universal velocity distribution:

$$\frac{\overline{v}_x}{v_*} = \left(\frac{1}{\sqrt{3}} \ln Re + \frac{5}{2}\right)\left(\frac{y v_*}{\nu}\right)^{3/(2\ln Re)}$$

Viscous sublayer:

$$\boxed{\frac{\overline{v}_x}{v_*} = \frac{y v_*}{\nu}\left[1 - \frac{1}{2}\left(\frac{\nu}{v_* B}\right)\left(\frac{y v_*}{\nu}\right) - \frac{1}{4}\left(\frac{y v_*}{14.5\nu}\right)^3 + \dots\right]}$$

$$0 < \frac{y v_*}{\nu} < 5$$

$5 < \frac{y v_*}{\nu} < 30$ no analytical derivations available

Empirical Expressions for Turbulent Momentum Flux

Eddy Viscosity

p.162

$$\bar{\tau}_{yx}^{(t)} = -\mu^{(t)} \frac{d\bar{v}_x}{dy}$$

$\mu^{(t)}$ = eddy viscosity = ε
property of flow
(μ is a property of a fluid)

wall turbulence : $\mu^{(t)} = \mu \left(\frac{y v_*}{14.5 \nu} \right)^3 \quad 0 < \frac{y v_*}{\nu} < 5$

free turbulence : $\mu^{(t)} = \rho \kappa_0 b (\bar{v}_{z,max} - \bar{v}_{z,min})$

κ_0 exp. parameter, b width of mixing zone

Prandtl Mixing Length

$$\bar{\tau}_{yx}^{(t)} = -\rho l^2 \left| \frac{d\bar{v}_x}{dy} \right| \frac{d\bar{v}_x}{dy}$$

wall : $l = \kappa_1 y$
free : $l = \kappa_2 b$
κ_1, κ_2 constants

von Driest Equation

describe $\bar{\tau}$ from wall to turbulent stream

$$l = (0.4y) \frac{1 - \exp(-y v_* / 26\nu)}{\sqrt{1 - \exp(-0.26 y v_* / \nu)}}$$

Turbulent Flow in Ducts

p.165

<u>ex</u> - Circular tube 5.5-1, 5.5-2

Turbulent Flow in Jets (free turbulence)

p.168

<u>ex</u> - Circular wall jet 5.6-1

Ch. 6 Interphase Transport in Isothermal systems

In systems where the velocity and pressure profiles c/n be easily calculated, construct correlations of dimensionless variables from exp. data to estimate the flow behavior in geometrically similar systems. p.177

Definition of Friction Factors

$$\boxed{f = \frac{\tau_{wall}}{\frac{1}{2}\rho\langle v\rangle^2}}$$

F = force exerted on solid by fluid

$= F_s + F_K$

F_s: force if stationary

F_K: force associated with fluid motion

$F_K = AKf$

A = char. area

K = char kinetic energy

f = proportionality = friction factor

Flow through circular tube:

$$F_K = (2\pi RL)\left(\tfrac{1}{2}\rho\langle v\rangle^2\right)f = (P_0 - P_L)\pi R^2$$

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{P_0 - P_L}{\frac{1}{2}\rho\langle v\rangle^2}\right) \quad \text{Fanning friction factor}$$

Flow around a sphere:

$$F_K = (\pi R^2)\left(\tfrac{1}{2}\rho v_\infty^2\right)f = \tfrac{4}{3}\pi R^3\rho_{sph}g - \tfrac{4}{3}\pi R^3\rho g$$

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sph} - \rho}{\rho}\right) \quad \text{drag coefficient } (C_D)$$

Flow in Tubes p.179

$f = f(Re, L/D)$

laminar: $f = \frac{16}{Re}$ ($Re < 2100$ stable) Hagen-Poiseuille eqn

turbulent: <u>Blasius formula</u>

$$\underline{f = 0.0791 / Re^{1/4}} \quad 2.1\times10^3 < Re < 10^5$$

Prandtl formula:

$$\frac{1}{\sqrt{f}} = 4.0\log_{10} Re\sqrt{f} - 0.4 \quad 2.3\times10^3 < Re < 4\times10^6$$

Barenblatt formula: $f = \frac{2}{\psi^{2/(\alpha+1)}}$ $\psi = \frac{e^{3/2}(\sqrt{3} + 5\alpha)}{2^{\alpha}\alpha(\alpha+1)(\alpha+2)}$

$\alpha = 3/2\ln Re$

$3.07 \times 10^{3} < Re < 3.23 \times 10^{6}$

Rough pipes: Haaland eqn

$$\frac{1}{\sqrt{f}} = -3.6 \log_{10}\left[\frac{6.9}{Re} + \left(\frac{K/D}{3.7}\right)^{10/9}\right]$$ $4\times10^{4} < Re < 10^{8}$, $0 < K/D < 0.05$

non-circular tubes: (turbulent flow)

$R_h = S/Z$

$$f = \left(\frac{R_h}{L}\right)\left(\frac{P_0 - P_L}{\frac{1}{2}\rho\langle v_z\rangle^2}\right)$$

$Re_h = 4R_h\langle v_z\rangle\rho/\mu$

Flow Around Spheres

p. 185

$f = f(Re)$

$f = \frac{24}{Re}$ $Re < 0.1$ (from Stokes law)
creeping flow

$f = \left(\sqrt{\frac{24}{Re}} + 0.5407\right)^2$ $Re < 6000$

$f \approx 0.44$ $5 \times 10^{2} < Re < 1 \times 10^{5}$

Compare flow in tubes and around spheres:

Flow In Tubes	Flow Around Spheres
· laminar - turbulent transition at $Re = 2100$	· no well defined laminar - turbulent transition
- only contribution to f is friction drag	· contributions to f from friction and form drag
- no BL separation	· kink in f vs. Re curve associated with shift in separation zone

Packed Columns

$$\boxed{f = \frac{1}{4}\left(\frac{D_p}{L}\right)\left(\frac{P_0 - P_L}{\frac{1}{2}\rho v_0^2}\right)}$$

D_p = eff part diameter

v_0 = superficial velocity

$$\boxed{P_0 - P_L = \frac{1}{2}\rho \langle v \rangle^2 \left(\frac{L}{R_h}\right) f_{tube}}$$

$$Re_h = 4 R_h \langle v \rangle \rho / \mu$$

$$f = \frac{1}{4} \frac{D_p}{R_h} \frac{\langle v \rangle^2}{v_0^2} = \frac{1}{4\varepsilon^2} \frac{D_p}{R_h} f_{tube}$$

ε = void fraction

$$R_h = \varepsilon / a, \quad a_v = \frac{a}{1-\varepsilon}, \quad D_p = \frac{6}{a_v}$$

$$\boxed{f = \frac{3}{2} \frac{(1-\varepsilon)}{\varepsilon^3} f_{tube}}$$

laminar flow: $f_{tube} = 16/Re_h$

$$\boxed{f = \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{75}{D_p G_0/\mu}}$$

$$\frac{P_0 - P_L}{L} = 150 \left(\frac{\mu v_0}{D_p^2}\right) \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Blake-Kozeny Eqn

turbulent flow: $f_{tube} = 7/12$

$$\boxed{f = 7/8 \left(\frac{1-\varepsilon}{\varepsilon^3}\right)}$$

$$\frac{P_0 - P_L}{L} = \frac{7}{4} \left(\frac{\rho v_0^2}{D_p}\right) \frac{1-\varepsilon}{\varepsilon^3}$$

Burke-Plummer eqn

transition region

$$\frac{P_0 - P_L}{L} = 150 \left(\frac{\mu v_0}{D_p^2}\right) \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right) \frac{1-\varepsilon}{\varepsilon^3}$$

$$\boxed{\frac{(P_0 - P_L)\rho}{G_0^2}\left(\frac{D_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 150\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right) + \frac{7}{4}}$$

Ergun eqn

$$\frac{(P_0 - P_L)\rho}{G_0^2}\left(\frac{D_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 150\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right) + 4.2\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right)^{1/6}$$

Tallmadge equation

Ch. 7 Macroscopic Balances for Isothermal Flow Systems

Macroscopic Mass Balance

p. 198

$$\frac{d}{dt} m_{tot} = \rho_1 \langle v_1 \rangle S_1 - \rho_2 \langle v_2 \rangle S_2 = -\Delta w$$

Macroscopic Momentum Balance

p. 200

$$\frac{d}{dt} \vec{P}_{tot} = \rho_1 \langle v_1^2 \rangle S_1 \vec{u}_1 - \rho_2 \langle v_2^2 \rangle S_2 \vec{u}_2 + p_1 S_1 \vec{u}_1 - p_2 S_2 \vec{u}_2 + F_{s \to f} + m_{tot} \vec{g}$$

$\vec{u}_1, \vec{u}_2$ = direction of flow at planes 1, 2

$\vec{P}_{tot} = \int \rho \vec{v} \, dV$

$$\frac{d}{dt} \vec{P}_{tot} = -\Delta\left(\frac{\langle v^2 \rangle}{\langle v \rangle} w + pS\right)\vec{u} + \vec{F}_{s \to f} + m_{tot} \vec{g}$$

steady state:

$$F_{f \to s} = -\Delta\left(\frac{\langle v^2 \rangle}{\langle v \rangle} w + pS\right)\vec{u} + m_{tot} \vec{g}$$

Macroscopic Angular Momentum Balance

p. 202

$$\frac{d}{dt} \vec{L}_{tot} = \rho_1 \langle v_1^2 \rangle S_1 [\vec{r}_1 \times \vec{u}_1] - \rho_2 \langle v_2^2 \rangle S_2 [\vec{r}_2 \times \vec{u}_2] + p_1 S_1 [\vec{r}_1 \times \vec{u}_1] - p_2 S_2 [\vec{r}_2 \times \vec{u}_2] + \vec{T}_{s \to f} + \vec{T}_{ext}$$

$\vec{L}_{tot} = \int \rho [\vec{r} \times \vec{v}] dV$ total angular mom.

$\vec{T}_{ext} = \int [\vec{r} \times \rho \vec{g}] dV$ torque from grav force

Macroscopic Mechanical Energy Balance

p. 203

(Engineering Bernoulli Equation)

$$\underbrace{\frac{d}{dt}(K_{tot} + \Phi_{tot})}_{\text{rate of increase of KE + PE}} = \underbrace{\left(\tfrac{1}{2}\rho_1 \langle v_1^3 \rangle + \rho_1 \hat{\Phi}_1 \langle v_1 \rangle\right) S_1}_{\text{rate KE + PE enter at plane 1}} - \underbrace{\left(\tfrac{1}{2}\rho_2 \langle v_2^3 \rangle + \rho_2 \hat{\Phi}_2 \langle v_2 \rangle\right) S_2}_{\text{rate KE + PE leave at plane 2}}$$

$$+ \underbrace{(p_1 \langle v_1 \rangle S_1 - p_2 \langle v_2 \rangle S_2)}_{\text{net rate at which surroundings do work on fluid at planes 1 + 2}} + \underbrace{W_m}_{\text{rate of work on fluid by moving surfaces}} + \underbrace{\int_{V(t)} p(\vec{\nabla} \cdot \vec{v}) dV}_{\text{rate mech energy} \uparrow \text{ or } \downarrow \text{ from expansion or compression}} + \underbrace{\int_{V(t)} (\vec{\tau} : \vec{\nabla}\vec{v}) dV}_{\downarrow \text{ of mech energy from viscous dissipation}}$$

$$K_{tot} = \int \frac{1}{2} \rho v^2 \, dV \quad , \quad \Phi_{tot} = \int \rho \hat{\Phi} \, dV$$

$$\frac{d}{dt}(K_{tot} + \Phi_{tot}) = -\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \frac{p}{\rho}\right)w + W_m - E_c - E_v$$

$$E_c = -\int_{V(t)} p(\nabla \cdot \vec{v})\, dV$$

+ compression

− expansion

= 0 incompressible flow

$$E_v = -\int_{V(t)} (\vec{\tau} : \nabla\vec{v})\, dV$$

+ Newtonian fluids

if steady state and approximate $\Delta\left(\frac{p}{\rho}\right)w + E_c \approx w\int_1^2 \frac{1}{\rho}\, dp$

$$\boxed{\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle}\right) + g\Delta h + \int_1^2 \frac{1}{\rho}\, dp = \hat{W}_m - \hat{E}_v}$$

$$\hat{W}_m = W_m / w \quad , \quad \hat{E}_v = E_v / w$$

Estimation of Viscous Loss (E_v)

incompressible Newtonian fluid : $E_v = \int \mu \Phi_v \, dV$

steady state flow:

$$\hat{E}_v = \frac{1}{2}\langle v\rangle^2 e_v$$

$$e_v = e_v(Re, \text{ dimensionless geometric ratios})$$

steady flow in straight conduit:

$$\hat{E}_v = \frac{F_{f\to s}}{\rho S}$$

turbulent : $\hat{E}_v = \frac{1}{2}\langle v\rangle^2 \frac{L}{R_h} f \qquad e_v = \frac{L}{R_h} f$

For turbulent flow calculations in a straight conduit:

$$\frac{1}{2}(v_2^2 - v_1^2) + g(z_2 - z_1) + \int_{p_1}^{p_2} \frac{1}{\rho}\, dp = \hat{W}_m - \sum_i \left(\frac{1}{2} v^2 \frac{L}{R_h} f\right)_i - \sum_i \left(\frac{1}{2} v^2 e_v\right)_i$$

ex - 7.6-1 Pressure Rise + Friction Loss in a sudden Enlargement

ex - 7.6-2 Liquid-Liquid Ejector

ex - 7.6-3 Thrust on a Pipe Bend

ex - 7.6-4 Impinging Jet

ex - 7.6-5 Isothermal Flow of Liquid Through an Orifice

ex - 7.7-1 Acceleration Effects in Unsteady Flow from a Cylindrical Tank

ex - 7.7-2 Manometer Oscillations

Ch. 8 Polymeric Liquids (non-Newtonian Fluids)

steady laminar flow in circular tube p. 232

$$\frac{v_z}{v_{z,max}} = 1 - \left(\frac{r}{R}\right)^{(1/n)+1} \qquad \frac{v_z}{v_{z,max}} = \frac{(1/n)+1}{(1/n)+3}$$

n = positive parameter for the fluid

$n < 1$

$$P_0 - P_L = w^n$$

polymer Newtonian fluid

Weissenberg rod-climbing effect

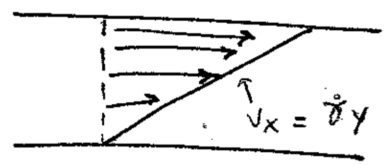
Polymers do not obey Newton's law of viscosity

Steady simple shear Flow p. 237

$$\tau_{yx} = -\eta \frac{dv_x}{dy}$$

$$\tau_{xx} - \tau_{yy} = -\Psi_1 \left(\frac{dv_x}{dy}\right)^2$$

$$\tau_{yy} - \tau_{zz} = -\Psi_2 \left(\frac{dv_x}{dy}\right)^2$$

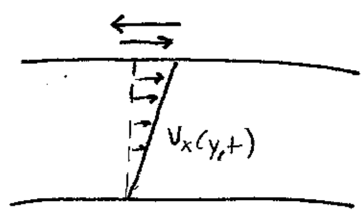


Small Amplitude Oscillatory Motion p. 238

$$\tau_{yx} = -\eta' \dot{\gamma}^0 \cos \omega t - \eta'' \dot{\gamma}^0 \sin \omega t$$

$$v_x(y,t) = \dot{\gamma}^0 y \cos \omega t$$

$$\eta^* = \underbrace{\eta'}_{\text{viscous response}} - i\underbrace{\eta''}_{\text{elastic response}}$$



steady state Elongational Flow p. 238

$$v_x = -\tfrac{1}{2}\dot{\varepsilon}x, \quad v_y = -\tfrac{1}{2}\dot{\varepsilon}y, \quad v_z = \dot{\varepsilon}z$$

$\dot{\varepsilon}$ = elongational rate

$$\tau_{zz} - \tau_{xx} = -\bar{\eta}\frac{dv_z}{dz}$$

Newtonian : $\bar{\eta} = 3\mu$ (Trouton viscosity)

Generalized Newtonian Models

- only describes Non-Newtonian viscosity
- does not describe time-dep or elastic effects

$$\vec{\tau} = -\eta\left(\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^{+}\right) = -\eta\,\dot{\gamma}$$

$$\eta = \eta(\dot{\gamma})$$

power law : $\eta = m\,\dot{\gamma}^{\,n-1}$

Carreau eqn : $\dfrac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + (\lambda\dot{\gamma})^2\right]^{(n-1)/2}$

Viscoelastic Models

Maxwell : $\vec{\tau} + \lambda_1 \frac{\partial}{\partial t}\vec{\tau} = -\eta_0\,\dot{\gamma}$ p. 245

relaxation time (λ_1) η_0: 0 shear rate viscosity

Jeffreys : $\vec{\tau} + \lambda_1 \frac{\partial}{\partial t}\vec{\tau} = -\eta_0\left(\dot{\gamma} + \lambda_2 \frac{\partial}{\partial t}\dot{\gamma}\right)$

λ_2: retardation time

Generalized Maxwell :

$$\vec{\tau}(t) = \sum_{k=1}^{\infty} \vec{\tau}_k(t) \qquad \vec{\tau}_k + \lambda_k \frac{\partial}{\partial t}\vec{\tau}_k = -\eta_k\,\dot{\gamma}$$

$$\eta_k = \eta_0 \frac{\lambda_k}{\sum_j \lambda_j} \qquad \lambda_k = \frac{\lambda}{k^\alpha}$$

$$\vec{\tau}(t) = -\int_{-\infty}^{t} \left\{\sum_{k=1}^{\infty} \frac{\eta_k}{\lambda_k} \exp\left[-(t-t')/\lambda_k\right]\right\} \dot{\gamma}(t')\,dt'$$

$$= -\int_{-\infty}^{t} G(t-t')\,\dot{\gamma}(t')\,dt'$$

Energy Transport

Ch. 9 Thermal Conductivity and other Mechanisms

Fourier's Law of Conduction p. 266

$$q_y = -k\frac{dT}{dy}, \quad q_x = -k\frac{dT}{dx}, \quad q_z = -k\frac{dT}{dz}$$

$$\vec{q} = -k\nabla T \quad \text{(isotropic)}$$

$$\vec{q} = -[\kappa \cdot \nabla T] \quad \text{(anisotropic)}$$

↳ thermal conductivity tensor

$$\alpha = \frac{k}{\rho\hat{C}_p} \qquad Pr = \frac{\nu}{\alpha} = \frac{\hat{C}_p\,\mu}{k} \qquad Pe = RePr$$

Thermal Conductivity of Gases at Low Density p. 274

$$k = \frac{1}{2}nK\bar{u}\lambda = \frac{1}{3}\rho\hat{C}_v\bar{u}\lambda \quad \text{(monatomic)}$$

$$k = \frac{2}{3\pi}\frac{\sqrt{\pi m K T}}{\pi d^2}\hat{C}_v \quad \text{(monatomic)}$$

$$k \propto \frac{\sqrt{mT}}{d^2}$$

Chapman - Enskog:

$$k = 1.9891 \times 10^{-4}\frac{\sqrt{T/M}}{\sigma^2\Omega_k} \quad \text{(monatomic)}$$

$$k = \frac{5}{2}\hat{C}_v\,\mu \quad \text{(monatomic)}$$

Eucken:

$$k = \left(\hat{C}_p + \frac{5}{4}\frac{R}{M}\right)\mu \quad \text{(polyatomic)}$$

$$k_{mix} = \sum_{\alpha=1}^{N}\frac{x_\alpha k_\alpha}{\sum_\beta x_\beta \Phi_{\alpha\beta}} \quad \text{(mixture)}$$

Thermal Conductivity of Liquids p. 279

$$k = 3(\tilde{N}/\tilde{V})^{2/3}K v_s$$

Thermal Conductivity of Solids p. 280

pure metals: $\dfrac{k}{k_e T} = L = \text{constant}$

Thermal Conductivity of Composites p. 281

small ϕ:

$$\frac{k_{eff}}{k_0} = 1 + \frac{3\phi}{\left(\frac{k_1+2k_0}{k_1-k_0}\right)-\phi}$$

large ϕ:

$$\frac{k_{eff}}{k_0} = 1 + \frac{3\phi}{\left(\frac{k_1+2k_0}{k_1-k_0}\right)-\phi+1.569\left(\frac{k_1-k_0}{3k_1-4k_0}\right)\phi^{10/3}+\ldots}$$

Convective Transport p. 283

$$(\tfrac{1}{2}\rho v^2+\rho\hat{U})\vec{\delta}_x v_x + (\tfrac{1}{2}\rho v^2+\rho\hat{U})\vec{\delta}_y v_y + (\tfrac{1}{2}\rho v^2+\rho\hat{U})\vec{\delta}_z v_z = (\tfrac{1}{2}\rho v^2+\rho\hat{U})\vec{v}$$

across a surface with $\vec{n}$: $\vec{n}\cdot(\tfrac{1}{2}\rho v^2+\rho\hat{U})\vec{v}$

Work Associated with Molecular Motions p. 284

Combined energy flux:

$$\vec{e} = (\tfrac{1}{2}\rho v^2+\rho\hat{U})\vec{v} + [\vec{\pi}\cdot\vec{v}] + \vec{q}$$

$$\vec{\pi} = p\delta + \vec{\tau}$$

$$\vec{\pi}\cdot\vec{v} = p\vec{v} + [\vec{\tau}\cdot\vec{v}]$$

$$\rho U\vec{v} + p\vec{v} = \rho(\hat{U} + (p/\rho)\vec{v}) = \rho(\hat{U}+p\hat{V})\vec{v}$$

$$= \rho\hat{H}\vec{v} \quad \text{enthalpy}$$

$$\vec{e} = (\tfrac{1}{2}\rho v^2+\rho\hat{H})\vec{v} + [\vec{\tau}\cdot\vec{v}] + \vec{q}$$

for $\vec{n}$ orientation: $\vec{n}\cdot\vec{e}$

$$\hat{H}-\hat{H}^\circ = \int_{T^\circ}^{T}\hat{C}_p\,dT + \int_{p^\circ}^{p}\left[\hat{V}-T\left(\frac{\partial\hat{V}}{\partial T}\right)_p\right]dp$$

Ch. 10 Shell Balances (Solids and Laminar Flow)

$$\begin{pmatrix}\text{rate in by}\\ \text{convection}\end{pmatrix} - \begin{pmatrix}\text{rate out by}\\ \text{convection}\end{pmatrix} + \begin{pmatrix}\text{rate in by}\\ \text{molecular}\\ \text{transport}\end{pmatrix} - \begin{pmatrix}\text{rate out by}\\ \text{molecular}\\ \text{transport}\end{pmatrix} +$$ p. 291

$$\begin{pmatrix}\text{rate of work}\\ \text{done by}\\ \text{molec. transport}\\ \text{on system}\end{pmatrix} - \begin{pmatrix}\text{rate of work}\\ \text{done by}\\ \text{molec. transport}\\ \text{by system}\end{pmatrix} + \begin{pmatrix}\text{rate of work}\\ \text{done on system}\\ \text{by ext. forces}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{energy}\\ \text{production}\end{pmatrix} = 0$$

energy production : degradation of electrical energy into heat, fission, viscous dissipation, chemical reaction

Boundary Conditions :

1. Temp specified at a surface
2. Heat flux normal to a surface is given
3. At interfaces, continuity of temp and heat flux normal to surface are required
4. solid - fluid interfaces : Newton's law of cooling
   $q = h(T_0 - T_b)$

Heat Conduction with an Electrical Heat Source p. 292

rate of heat production per unit vol = $S_e = I^2/k_e$

$$(2\pi r L) q_r|_r - (2\pi (r+\Delta r) L)(q_r|_{r+\Delta r}) = (2\pi r \Delta r L) S_e$$

$$\frac{d}{dr}(r q_r) = S_e r \qquad \text{at } r=0,\ q_r \text{ is finite}$$

$$q_r = \frac{S_e r}{2} = -k\frac{dT}{dr} \qquad \text{(Fourier)}$$

$$\text{at } r=R,\quad T=T_0$$

$$T - T_0 = \frac{S_e R^2}{4k}\left[1 - (r/R)^2\right]$$

T_{max} at $r=0$

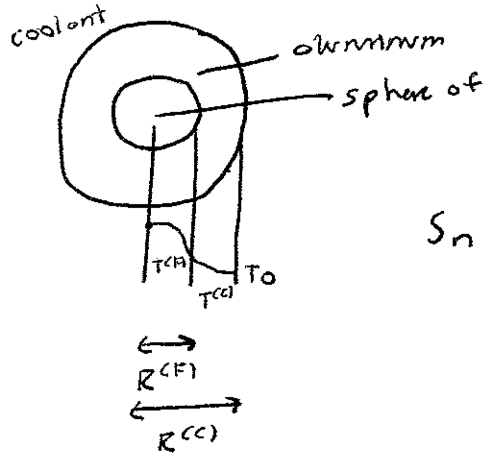
$$\langle T\rangle - T_0 = \frac{\int_0^{2\pi}\int_0^R (T(r) - T_0)\, r\, dr\, d\theta}{\int_0^{2\pi}\int_0^R r\, dr\, d\theta} = \frac{S_e R^2}{8k}$$

$$Q|_{r=R} = 2\pi R L \cdot q_r|_{r=R} = \pi R^2 L \cdot S_e$$

Heat Conduction with Nuclear Heat Source

$$q_r^{(F)}\Big|_r \cdot 4\pi r^2 - q_r^{(F)}\Big|_{r+\Delta r} \cdot 4\pi (r+\Delta r)^2 = S_n \cdot 4\pi r^2 \Delta r$$

S_n = energy from fission inside sphere



$$S_n = S_{no}\left[1 + b\left(\frac{r}{R^{(F)}}\right)^2\right]$$

$$\frac{d}{dr}\left(r^2 q_r^{(F)}\right) = S_{no}\left[1 + b\left(\frac{r}{R^{(F)}}\right)^2\right] r^2$$

$$\frac{d}{dr}\left(r^2 q_r^{(C)}\right) = 0$$

BC: $r = 0$, $q_r^{(F)}$ finite

$r = R^{(F)}$, $q_r^{(F)} = q_r^{(C)}$

$$q_r^{(F)} = S_{no}\left(\frac{r}{3} + \frac{b}{R^{(F)2}}\frac{r^3}{5}\right) = -k^{(F)}\frac{dT^{(F)}}{dr}$$

$$q_r^{(C)} = S_{no}\left(\frac{1}{3} + \frac{b}{5}\right)\frac{R^{(F)3}}{r^2} = -k^{(C)}\frac{dT^{(C)}}{dr}$$

$r = R^{(F)}$, $T^{(F)} = T^{(C)}$

$r = R^{(C)}$, $T^{(C)} = T_0$

$$T^{(F)} = \frac{S_{no}R^{(F)2}}{6k^{(F)}}\left\{\left[1 - \left(\frac{r}{R^{(F)}}\right)^2\right] + \frac{3}{10}b\left[1 - \left(\frac{r}{R^{(F)}}\right)^4\right]\right\}$$
$$+ \frac{S_{no}R^{(F)2}}{3k^{(C)}}\left(1 + \frac{3}{5}b\right)\left(1 - \frac{R^{(F)}}{R^{(C)}}\right)$$

$$T^{(C)} = \frac{S_{no}R^{(F)2}}{3k^{(C)}}\left(1 + \frac{3}{5}b\right)\left(\frac{R^{(F)}}{r} - \frac{R^{(F)}}{R^{(C)}}\right)$$

Heat Conduction with a Viscous Heat Source p. 298

$$WL\, e_x|_x - WL\, e_x|_{x+\Delta x} = 0$$

$$\frac{de_x}{dx} = 0 \quad , \quad e_x = C_1$$

$$\vec{e} = \underbrace{\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U}\right)\vec{v}}_{=0 \text{ since } v_x = 0} + \underbrace{[\vec{\pi}\cdot\vec{v}]}_{\tau_{xx}v_x + \tau_{xy}v_y + \tau_{xz}v_z} + \vec{q}$$

$v_x = v_y = 0$

$\tau_{xz} = -\mu \dfrac{dv_z}{dx}$

$\vec{q} \rightarrow -k\dfrac{dT}{dx}$

$$-k\frac{dT}{dx} - \mu v_z \frac{dv_z}{dx} = C_1 \quad , \quad v_z = v_b\,(x/b)$$

$$-k\frac{dT}{dx} - \mu x\left(\frac{v_b}{b}\right)^2 = C_1$$

$x = 0$, $T = T_0$

$S_v = \mu\left(\dfrac{v_b}{b}\right)^2$ $\qquad$ $x = b$, $T = T_b$

$$\frac{T - T_0}{T_b - T_0} = \frac{1}{2}Br\,\frac{x}{b}\left(1 - \frac{x}{b}\right) + \frac{x}{b} \qquad Br = \frac{\mu v_b^2}{k(T_b - T_0)}$$

Viscous heating important when large velocity gradients

Heat Conduction with Chemical Heat Source p. 300

Zone I + III : inert , Zone II : catalyst

S_c = energy prod from rxn $= S_{c1}F(\Theta)$ $\qquad \Theta = {T - T_0}/{T_1 - T_0}$

$$\pi R^2 e_z|_z - \pi R^2 e_z|_{z+\Delta z} + (\pi R^2 \Delta z) S_c = 0$$

$$\frac{de_z}{dz} = S_c = \frac{d}{dz}\left(\left(\tfrac{1}{2}\rho v^2 + \rho\hat{H}\right)v_z + \tau_{zz}v_z + q_z\right)$$

$$\rho\hat{C}_p v_0 \frac{dT}{dz} = k_{eff,zz}\frac{d^2T}{dz^2} + S_c \qquad (\text{zone II})$$

Zone I + III : same eqn but $S_c = 0$

BC : $z = -\infty$, $T^{I} = T_1$ $\qquad$ $z = L$, $T^{II} = T^{III}$

$z = 0$, $T^{I} = T^{II}$ $\qquad$ $z \to \infty$, T^{III} = finite

$z = 0$, $k_{eff,zz}\dfrac{dT^{I}}{dz} = k_{eff,zz}\dfrac{dT^{II}}{dz}$

$z = L$, $k_{eff,zz}\dfrac{dT^{II}}{dz} = k_{eff,zz}\dfrac{dT^{III}}{dz}$

Heat Conduction Through Composite walls

$$q_x = q_0 \text{ (constant)}$$

$$-k_{01}\frac{dT}{dx} = q_0 = -k_{12}\frac{dT}{dx} = -k_{23}\frac{dT}{dx}$$

$$T_0 - T_1 = q_0\left(\frac{x_1 - x_0}{k_{01}}\right)$$

$$T_1 - T_2 = q_0\left(\frac{x_2 - x_1}{k_{12}}\right)$$

$$T_2 - T_3 = q_0\left(\frac{x_3 - x_2}{k_{23}}\right)$$

Constant flux through the walls

$$T_a - T_0 = \frac{q_0}{h_0}$$

$$T_3 - T_b = \frac{q_0}{h_3}$$

$$q_0 = \frac{T_a - T_b}{\left(\frac{1}{h_0} + \sum_{j=1}^{3}\frac{x_j - x_{j-1}}{k_{j-1,j}} + \frac{1}{h_3}\right)}$$

$$q_0 = U(T_a - T_b)$$

In a cylinder slab:

$$(2\pi r L q_r)|_r - (2\pi r L q_r)_{r+\Delta r} = 0$$

$$\frac{d}{dr}(r q_r) = 0 \longrightarrow r q_r = r_0 q_0$$

$$-k_{01} r\frac{dT}{dr} = r_0 q_0 = -k_{12} r\frac{dT}{dr} = -k_{23} r\frac{dT}{dr}$$

$$T_0 - T_1 = r_0 q_0 \frac{\ln(r_1/r_0)}{k_{01}}$$

$$T_1 - T_2 = r_0 q_0 \frac{\ln(r_2/r_1)}{k_{12}}$$

$$T_2 - T_3 = r_0 q_0 \frac{\ln(r_3/r_2)}{k_{23}}$$

$$T_a - T_0 = q_0/h_0$$

$$T_3 - T_b = q_3/h_3$$

Heat Conduction in a Fin

p. 307

Assumptions : $T = T(z)$, no heat loss from edges,

$q_z = h(T - T_a)$ with constant h

$$2BWq_z|_z - 2BWq_z|_{z+\Delta z} - h(2W\Delta z)(T - T_a) = 0$$

$$\frac{-dq_z}{dz} = \frac{h}{B}(T - T_a) \quad , \quad q_z = -k\frac{dT}{dz}$$

$$\frac{d^2T}{dz^2} = \frac{h}{kB}(T - T_a) \qquad z = 0, \; T = T_w$$
$$z = L, \; dT/dz = 0$$

$$\eta = \frac{\text{actual rate of heat loss from fin}}{\text{rate of heat loss from isothermal fin at } T_w}$$

$$= \frac{\int_0^W \int_0^L h(T - T_a)\,dz\,dy}{\int_0^W \int_0^L h(T_w - T_a)\,dz\,dy} = \frac{\tanh N}{N} \quad , \quad N^2 = \frac{hL^2}{kB}$$

Forced Convection

p. 310

- Flow patterns determined by an external force
- First find velocity profiles, then find temp profiles

Free Convection

- when fluid is heated, $\rho \downarrow$ and fluid rises

do energy balance to find T-distribution

do momentum balance and expand ρ is Taylor's series about $\bar{T} = \frac{1}{2}(T_1 + T_2)$

substitute in T-distribution and solve

Grashof number $= Gr = \left[\!\left[\dfrac{\bar{\rho} g B^3 \Delta \rho}{\mu^2} \right]\!\right]$

Ch 11 Equations of Change - Nonisothermal Systems

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U}\right) = -\left(\nabla\cdot\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U}\right)\vec{v}\right) - \left(\nabla\cdot\vec{q}\right)$$ p. 335

rate of increase of energy per unit volume — convective transport — heat conduction

$$-\left(\nabla\cdot p\vec{v}\right) - \left(\nabla\cdot[\vec{\tau}\cdot\vec{v}]\right) + \rho\left(\vec{v}\cdot\vec{g}\right)$$

work done by pressure forces — work done by viscous forces — work done by external forces

if $\vec{g}$ is ind. of time:

$$\rho(\vec{v}\cdot\vec{g}) = -(\nabla\cdot\rho\vec{v}\,\hat{\Phi}) - \frac{\partial}{\partial t}(\rho\,\hat{\Phi})$$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U} + \rho\hat{\Phi}\right) = -\left(\nabla\cdot\left(\tfrac{1}{2}\rho v^2 + \rho\hat{U} + \rho\hat{\Phi}\right)\vec{v}\right)$$

$$-(\nabla\cdot\vec{q}) - (\nabla\cdot p\vec{v}) - (\nabla\cdot(\tau\cdot\vec{v}))$$

special forms: p. 336

subtract mechanical energy equation:

equation of change for internal energy

$$\frac{\partial}{\partial t}\rho\hat{U} = -(\nabla\cdot(\rho\hat{U}\vec{v})) - (\nabla\cdot\vec{q}) - p(\nabla\cdot\vec{v}) - (\vec{\tau}:\nabla\vec{v})$$

convection — conduction — reversible compression — irreversible viscous dissipation

-or-

$$\rho\frac{D\hat{U}}{Dt} = -(\nabla\cdot\vec{q}) - p(\nabla\cdot\vec{v}) - (\tau:\nabla\vec{v})$$

let $\hat{U} = \hat{H} - p\hat{V} = \hat{H} - (p/\rho)$

equation of change for enthalpy

$$\rho\frac{D\hat{H}}{Dt} = -(\nabla\cdot\vec{q}) - (\tau:\nabla\vec{v}) + \frac{Dp}{Dt}$$

For Newtonian fluids, $\hat{H} = \hat{H}(p, T)$:

equation of change for temperature

$$\rho\hat{C}_p\frac{DT}{Dt} = -(\nabla\cdot\vec{q}) - (\vec{\tau}:\nabla\vec{v}) - \left(\frac{\partial\ln\rho}{\partial\ln T}\right)_p\frac{Dp}{Dt}$$

Fourier: $-(\nabla\cdot\vec{q}) = \nabla\cdot k\nabla T = k\nabla^2 T$

Newton: $-(\vec{\tau}:\nabla\vec{v}) = \mu\Phi_v + \kappa\Psi_v$

Ideal gas : $\frac{\partial \ln \rho}{\partial \ln T} = -1$: $\rho \hat{C}_p \frac{DT}{Dt} = k\nabla^2 T + \frac{Dp}{Dt}$

Fluid at const p : $\frac{Dp}{Dt} = 0$: $\rho \hat{C}_p \frac{DT}{Dt} = k\nabla^2 T$

Fluid w/ const ρ : $\frac{\partial \ln \rho}{\partial \ln T} = 0$: $\rho \hat{C}_p \frac{DT}{Dt} = k\nabla^2 T$

Stationary solid : $\vec{v} = 0$: $\rho \hat{C}_p \frac{\partial T}{\partial t} = k\nabla^2 T$

Boussinesq Equation p. 338

approximate : $\rho(T) = \bar{\rho} - \bar{\rho}\bar{\beta}(T - \bar{T})$

$$\rho \frac{D\vec{v}}{Dt} = (-\nabla p + \bar{\rho}\vec{g}) - [\vec{\nabla} \cdot \vec{\tau}] - \bar{\rho}\vec{g}\bar{\beta}(T - \bar{T})$$

neglected in free convection

buoyancy term neglected in forced convection

Summary of Equations of Change Table 11.4-1 p. 340

<u>ex</u> - 11.4-1 Steady-state Forced Convection Heat Transfer in Laminar Flow in a Circular Tube p. 342

<u>ex</u> - 11.4-2 Tangential Flow in an Annulus with Viscous Heat Gen p. 342
- Find vel. distn and subst into energy equation

<u>ex</u> - 11.4-3 Steady Flow in a Nonisothermal Film p. 343

<u>ex</u> - 11.4-4 Transpiration Cooling p. 344

<u>ex</u> - 11.4-5 Free-Convection Heat Transfer from a Vertical Plate p. 346
- Momentum and energy equations coupled

<u>ex</u> - 11.4-6 Adiabatic Frictionless Processes in an Ideal Gas p. 349

<u>ex</u> - 11.4-7 One-dim Compressible Flow - Stationary Shock Wave p. 350

Table 11.5-2 Dimensionless Groups p. 355

Ch. 12 Temp Distributions with More than One Ind Variable

Unsteady Heat Conduction in Solids — p. 374

$$\rho \hat{C}_p \frac{\partial T}{\partial t} = (\nabla \cdot k \nabla T) = k \nabla^2 T \quad \text{if } k = \text{constant}$$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

ex - 12.1-1 Heating a semi-infinite slab — p. 375
use similarity variable

ex - 12.1-2 Heating of a Finite slab — p. 376
separation of variables

ex - 12.1-3 Unsteady Heat Conduction near a wall with sinusoidal heat flux — p. 379
asymptotic solution

ex - 12.1-4 Cooling of a Sphere in Contact with a well-stirred Fluid — p. 379
Laplace Transform method

Steady Heat Conduction in Laminar, Incompressible Flow
(Newtonian fluids with const. fluid properties)

$$(\nabla \cdot \vec{v}) = 0 \qquad \text{p. 381}$$

$$\rho(\vec{v} \cdot \nabla \vec{v}) = \mu \nabla^2 \vec{v} - \nabla p$$

$$\rho \hat{C}_p (\vec{v} \cdot \nabla T) = k \nabla^2 T + \mu \Phi_v$$

ex - Laminar Tube Flow with constant wall heat flux — p. 383
see 10.8 for asymptotic solution for large distances down tube

12.2-1 full solution w/ sep. of variables

12.2-2 asymptotic solution for short distances down tube
combination of variables

Steady Potential Flow of Heat in Solids — p. 385

$$\vec{q} = -k \nabla T$$

$\nabla^2 T = 0$ heat cond. eqn, in 2D: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$

$w(z) = f(x,y) + i g(x,y)$ where f + g are solutions

heat flux: $ik \frac{dw}{dz} = q_x - i q_y$

Boundary Layer Theory for Nonisothermal Flow p. 387

2-D flow around a submersed object

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\boxed{\begin{aligned} &\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y}\right) = \rho v_e \frac{dv_e}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} + \rho g_x \beta (T - T_\infty) \\ &\rho \hat{C}_p \left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}\right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial v_x}{\partial y}\right)^2 \end{aligned}}$$

$v_x = v_y = 0$ solid surface

at edge of BL: $v_x \to v_e(x)$ pot. flow

integrate eqns to get Von Kármán balances p. 388

$$\mu \frac{\partial v_x}{\partial y}\Big|_{y=0} = \frac{d}{dx}\int_0^\infty \rho v_x (v_e - v_x)\, dy + \frac{dv_e}{dx}\int_0^\infty \rho (v_e - v_x)\, dy + \int_0^\infty \rho g_x \beta (T - T_\infty)\, dy$$

$$k \frac{\partial T}{\partial y}\Big|_{y=0} = \frac{d}{dx}\int_0^\infty \rho \hat{C}_p v_x (T_\infty - T)\, dy$$

__ex__ - 12.4-1 Heat Transfer in Laminar Forced Convection Along a Heated Flat Plate p. 388

use von Kármán Integral

__ex__ - 12.4-2 above problem, exact asymptotic solution p. 391

Ch. 13 Temperature Distributions in Turbulent Flow

p. 407

Time-Smoothed Equations of Change for Incomp., Non-iso. Flow

$$T = \bar{T} + T'$$

With constant ρ, μ, $\hat{C}_p$ + k: using Fourier's and Newton's Laws

$$\frac{\partial}{\partial t}\rho \hat{C}_p \bar{T} = -\left(\frac{\partial}{\partial x}\rho \hat{C}_p \bar{v}_x \bar{T} + \frac{\partial}{\partial y}\rho \hat{C}_p \bar{v}_y \bar{T} + \frac{\partial}{\partial z}\rho \hat{C}_p \bar{v}_z \bar{T}\right)$$

$$-\left(\frac{\partial}{\partial x}\rho \hat{C}_p \overline{v_x'T'} + \frac{\partial}{\partial y}\rho \hat{C}_p \overline{v_y'T'} + \frac{\partial}{\partial z}\rho \hat{C}_p \overline{v_z'T'}\right)$$

$$+ k\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} + \frac{\partial^2 \bar{T}}{\partial z^2}\right)$$

$$+ \mu\left[2\left(\frac{\partial \bar{v}_x}{\partial x}\right)^2 + \left(\frac{\partial \bar{v}_x}{\partial y}\right)^2 + 2\left(\frac{\partial \bar{v}_x}{\partial x}\right)\left(\frac{\partial \bar{v}_y}{\partial y}\right) + \dots\right]$$

$$+ \mu\left[2\,\overline{\left(\frac{\partial v_x'}{\partial x}\right)\left(\frac{\partial v_x'}{\partial x}\right)} + \overline{\left(\frac{\partial v_x'}{\partial y}\right)\left(\frac{\partial v_x'}{\partial y}\right)} + 2\,\overline{\left(\frac{\partial v_x'}{\partial y}\right)\left(\frac{\partial v_y'}{\partial x}\right)} + \dots\right]$$

let $\bar{q}_x^{(t)} = \rho \hat{C}_p \overline{v_x'T'}$, $q_y^{(t)} = \rho \hat{C}_p \overline{v_y'T'}$, $q_z^{(t)} = \rho \hat{C}_p \overline{v_z'T'}$

$$\Phi_v^{(t)} = \sum_{i=1}^{3}\sum_{j=1}^{3}\overline{\left(\left(\frac{\partial v_i'}{\partial x_j}\right)\left(\frac{\partial v_i'}{\partial x_j}\right) + \left(\frac{\partial v_i'}{\partial x_j}\right)\left(\frac{\partial v_j'}{\partial x_i}\right)\right)}$$

Summary of Time-Smoothed Eqns of Change: const ρ, μ, $\hat{C}_p$, k

$$\boxed{\begin{aligned} &(\nabla \cdot \vec{\bar{v}}) = 0 \\ &\rho \frac{D\vec{\bar{v}}}{Dt} = -\nabla \bar{p} - [\nabla \cdot (\bar{\tau}^{(v)} + \bar{\tau}^{(t)})] + \rho \vec{g} \\ &\rho \hat{C}_p \frac{D\bar{T}}{Dt} = -(\nabla \cdot (\bar{q}^{(v)} + \bar{q}^{(t)})) + \mu(\bar{\Phi}_v^{(v)} + \bar{\Phi}_v^{(t)}) \end{aligned}}$$

Time-Smoothed Temp Profile Near a Wall p. 410

$$-\frac{d\bar{T}}{dy} = \frac{\beta q_0}{\kappa \rho \hat{C}_p v_* y} \qquad v_* = \sqrt{\frac{\tau_0}{\rho}}$$

in inertial sublayer

$$T_0 - \bar{T} = \frac{\beta q_0}{\kappa \rho \hat{C}_p v_*}\left[\ln\left(\frac{y v_*}{\nu}\right) + f(Pr)\right]$$

Empirical Expressions for Turbulent Heat Flux — p. 410

$$\bar{q}_y^{(t)} = -k^{(t)} \frac{d\bar{T}}{dy}$$

$$Pr^{(t)} = \frac{\nu^{(t)}}{\alpha^{(t)}} \qquad \text{Reynolds' analogy : } Pr^{(t)} = 1$$

Mixing length of Prandtl + Taylor :

$$\bar{q}_y^{(t)} = -\rho \hat{C}_p \, l^2 \left| \frac{d\bar{v}_x}{dy} \right| \frac{d\bar{T}}{dy}$$

predicts $Pr^{(t)} = 1$

Taylor : $Pr^{(t)} = 1/2$

<u>ex</u> – Heat flux at the wall in turbulent flow in a tube — p. 411
13.3-1

Turbulent Flow in Tubes — p. 411

$$\rho \hat{C}_p \bar{v}_z \frac{d\bar{T}}{dz} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\bar{q}_r^{(v)} + \bar{q}_r^{(t)}\right)\right)$$

$$\bar{q}_r = -(k + k^{(t)})\frac{d\bar{T}}{dr} = -\left(1 + \frac{\alpha^{(t)}}{\alpha}\right) k \frac{d\bar{T}}{dr}$$

$$\bar{v}_z \frac{d\bar{T}}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(r(\alpha + \alpha^{(t)})\frac{d\bar{T}}{dr}\right)$$

BC : $r=0$, $\bar{T}$ = finite

$r=R$, $k\frac{\partial \bar{T}}{\partial r} = q_0$

$z=0$, $\bar{T} = T_1$

asymptotic solution for large z

Temperature Distribution for Turbulent Flow in Jets — p. 415

$$\rho \hat{C}_p \left(\bar{v}_r \frac{\partial \bar{T}}{\partial r} + \bar{v}_z \frac{\partial \bar{T}}{\partial z}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r \bar{q}_r^{(t)}\right)$$

neglect $q_r^{(v)}$
viscous dissipation negligible

$$\bar{q}_r^{(t)} = -k^{(t)} \frac{\partial \bar{T}}{\partial r}$$

non-dimensionalize $\Theta = \dfrac{\bar{T} - T_1}{T_0 - T_1}$

BC : $z=0$, $\Theta = 1$, $r=0$ Θ is finite, $r \to \infty$ $\Theta = 0$

Ch. 14 Heat Transfer Coefficients

$$Q = hA\Delta T$$ p. 423

3 conventions: (surface: T_{01} to T_{02}, bulk fluid: T_{b1} to T_{b2})

$$Q = h_1 (\pi DL)(T_{01} - T_{b1}) = h_1 (\pi DL)\Delta T_1$$

$$Q = h_a (\pi DL)\left(\frac{(T_{01}-T_{b1}) + (T_{02}-T_{b2})}{2}\right) = h_a (\pi DL)\Delta T_a$$

$$Q = h_{ln} (\pi DL)\left(\frac{(T_{01}-T_{b1}) - (T_{02}-T_{b2})}{\ln (T_{01}-T_{b1}) - \ln (T_{02}-T_{b2})}\right) = h_{ln} (\pi DL)\Delta T_{ln}$$

local heat transfer coefficient:

$$dQ = h_{loc} (\pi D dz)(T_0 - T_b) = h_{loc} (\pi D dz)\Delta T_{loc}$$

flow around a submersed sphere: p. 424

$$Q = h_m (4\pi R^2)(T_0 - T_\infty) \quad \text{(mean } h)$$

$$dQ = h_{loc} (dA)(T_0 - T_\infty)$$

Two coaxial streams T_h and T_c seperated by a cylindrical tube:

$$dQ = U_0 (\pi D_0 dz)(T_h - T_c)$$ p. 425

$$\frac{1}{D_0 U_0} = \left(\frac{1}{D_0 h_0} + \frac{\ln(D_1/D_0)}{2k^{01}} + \frac{1}{D_1 h_1}\right)_{loc}$$

$$Nu = hD/k$$

Forced Convection Through Tubes and Slits p. 428

see Table 14.2-1 p. 430

laminar flow in a tube: with constant wall flux

$$(T_0 - T_b) = \frac{11}{48}\frac{q_0 D}{k}$$

$$q_0 = \frac{48}{11}\left(\frac{k}{D}\right)(T_0 - T_b)$$

$$h_{loc} = \frac{48}{11}\left(\frac{k}{D}\right)$$

$$Nu_{loc} = \frac{hD}{k} = \frac{48}{11}$$

To calculate physical properties, use film temp p. 432

Tubes, slits, other ducts: $T_f = \frac{1}{2}(T_{0a} + T_{ba})$

$T_{0a} = \frac{1}{2}(T_{01} + T_{02})$, $T_{L0} = \frac{1}{2}(T_{L1} + T_{L2})$

$$Re = \frac{D(\rho v)}{\mu} = \frac{Dw}{S\mu}$$

submerged objects: $T_f = \frac{1}{2}(T_0 + T_\infty)$

Forced Convection in Tubes

p. 433

$Nu = Nu(Re, Pr, L/D, \mu_b/\mu_0)$

highly turbulent flow, $L/D > 10$:

$$Nu_{ln} = 0.026\, Re^{0.8}\, Pr^{1/3} \left(\frac{\mu_b}{\mu_0}\right)^{0.14}$$

laminar flow:

$$Nu_{ln} = 1.86 \left(Re\, Pr \frac{D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_0}\right)^{0.14}$$

highly turbulent, long smooth tube:

$$j_{H,ln} = \frac{1}{2} f \qquad (Re > 10{,}000)$$

$$j_{H,ln} = \frac{Nu_{ln}}{Re\, Pr^{1/3}} = \frac{h_{ln}}{\langle \rho v \rangle \hat{C}_p} \left(\frac{\hat{C}_p \mu}{k}\right)^{2/3} = \frac{h_{ln} S}{w \hat{C}_p} \left(\frac{\hat{C}_p \mu}{k}\right)^{2/3}$$

Forced Convection around Submerged Objects

p. 438

Flat Plate:

$$j_{H,loc} = \frac{1}{2} f_{loc} = 0.332\, Re_x^{-1/2} \quad \text{Colburn analogy}$$

Spheres:

$$Nu_m = 2 + 0.60\, Re^{1/2}\, Pr^{1/3}$$

$$Nu_m = 2 + (0.4\, Re^{1/2} + 0.06\, Re^{2/3})\, Pr^{0.4} \left(\frac{\mu_\infty}{\mu_0}\right)^{1/4}$$

Cylinder:

$$Nu_m = (0.4\, Re^{1/2} + 0.06\, Re^{2/3})\, Pr^{0.4} \left(\frac{\mu_\infty}{\mu_0}\right)^{1/4}$$

$$Nu_m = (0.376\, Re^{1/2} + 0.057\, Re^{2/3})\, Pr^{1/3} + 0.92\left[\ln\left(\frac{29050}{Re}\right) + 4.18\, Re\right]^{-1/3} Re^{1/3}\, Pr^{1/3}$$

other objects : $Nu_m - Nu_{m,0} = 0.6\, Re^{1/2} Pr^{1/3}$

$\hookrightarrow$ at $Re = 0$

Forced Convection Through Packed Beds p. 441

$d\varphi = h_{loc}(a S dz)(T_0 - T_L)$

$j_H = 2.19\, Re^{-2/3} + 0.78\, Re^{-0.381}$

$$j_H = \frac{h_{loc}}{\hat{C}_p G_0}\left(\frac{\hat{C}_p \mu}{k}\right)^{2/3}, \quad Re = \frac{D_p G_0}{(1-\varepsilon)\mu\Psi} = \frac{6 G_0}{a \mu \Psi}$$

$T_f = \frac{1}{2}(T_0 - T_L)$, $G_0 = w/S$, Ψ = shape factor

small Re : $j_H = 2.19\, Re^{-2/3}$

$$Nu_{loc} = \frac{h_{loc} D_p}{k(1-\varepsilon)\Psi} = 2.19(Re\,Pr)^{1/3}$$

Free and Mixed Convection p. 442

$Nu_m = C(Gr\,Pr)^{1/4}$, $Gr = Ra / Pr$

no bouyant forces : $Nu_m^{cond} = K(\text{shape})$

spheres : $Nu_m^{cond} = 2$

Thin Laminar BL's :

$Nu_m^{lam} = C(Pr, \text{shape})(Gr\,Pr)^{1/4}$

$C = C_1(\text{shape})\, C_2(Pr)$

$$C_2 = \frac{0.671}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$$

heated horiz flat surface facing down or cooled facing up

$$Nu_m^{lam} = \frac{0.527}{\left[1 + (1.9/Pr)^{9/10}\right]^{2/9}}(Gr\,Pr)^{1/5}$$

combined conduction + convection

$$Nu_m^{comb} = \left[(Nu_m^{lam})^n + (Nu_m^{cond})^n\right]^{1/n}$$

$n \rightarrow$ shape (1.07)

Turbulent BL's:

$$Nu_m^{free} = [(Nu_m^{lam})^m + (Nu_m^{turb})^m]^{1/m}$$

Mixed Free and Forced Convection

$$Nu_m^{total} = [(Nu_m^{free})^3 + (Nu_m^{forced})^3]^{1/3}$$

Condensation of Pure Vapors on Solid Surfaces

Film Condensation

$$Q = h_m A (T_d - T_o) = w \Delta\hat{H}_{vap}$$

laminar, nonrippling flow at const surface temp T_o:

$$h_m = 0.954 \left(\frac{k^3 \rho^2 g L}{\mu w}\right)^{1/3}$$

laminar, moderate temp differences

$$h_m = 0.725 \left(\frac{k^3 \rho^2 g \Delta\hat{H}_{vap}}{\mu D (T_d - T_o)}\right)^{1/4}$$

vertical tubes, laminar, T_o:

$$h_m = \frac{4}{3}\left(\frac{k^3 \rho^2 g}{3 \mu \Gamma}\right)^{1/3}$$

vertical tubes, laminar temp diff:

$$h_m = \frac{2\sqrt{2}}{3}\left(\frac{k^3 \rho^2 g \Delta\hat{H}_{vap}}{\mu L (T_d - T_o)}\right)^{1/4}$$

turbulent flow:

$$h_m = 0.003 \left(\frac{k^3 \rho^2 g (T_d - T_o) L}{\mu^3 \Delta\hat{H}_{vap}}\right)^{1/2}$$

Small $T_d - T_o$:

$$h_m = 0.021 \left(\frac{k^3 \rho^2 g \Gamma}{\mu^3}\right)^{1/3}$$

vertical tube: $\Gamma = \frac{w}{\pi D}$

Ch. 15 Macroscopic Balances for Nonisothermal Systems

same assumptions as in ch. 7 p. 435

also, neglect $\vec{q}$ by conduction, neglect $[\vec{\tau}\cdot\vec{v}]$ relative to $\rho\vec{v}$

$$\frac{d}{dt}\left(U_{tot} + K_{tot} + \Phi_{tot}\right) = \left(\rho_1 \hat{U}_1 \langle v_1\rangle + \tfrac{1}{2}\rho_1\langle v_1^3\rangle + \rho_1\hat{\Phi}_1\langle v_1\rangle\right)S_1$$

rate of increase of KE, PE + internal energy — rate KE, PE + IE enter system

$$-\left(\rho_2\hat{U}_2\langle v_2\rangle + \tfrac{1}{2}(\rho_2)\langle v_2^3\rangle + \rho_2\hat{\Phi}_2\langle v_2\rangle\right)S_2$$

rate KE, PE + IE leave system

$$+\,Q \quad + \quad W_m \quad + \quad \left(p_1\langle v_1\rangle S_1 - p_2\langle v_2\rangle S_2\right)$$

heat added to system — work done on system by moving surfaces — rate work is done on system at planes 1 + 2

$$U_{tot} = \int \rho\hat{U}\,dV, \quad K_{tot} = \int \tfrac{1}{2}\rho v^2\,dV, \quad \Phi_{tot} = \int \rho\hat{\Phi}\,dV$$

$$w_1 = \rho_1\langle v_1\rangle S_1, \quad w_2 = \rho_2\langle v_2\rangle S_2$$

$$\frac{d}{dt}E_{tot} = -\Delta\left[\left(\hat{U} + p\hat{V} + \frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi}\right)w\right] + Q + W_m$$

$$\hat{\Phi}_1 = gh_1, \quad \hat{\Phi}_2 = gh_2$$

$$\hat{H}_1 = \hat{U}_1 + p_1\hat{V}_1, \quad \hat{H}_2 = \hat{U}_2 + p_2\hat{V}_2$$

steady state: $\Delta\left(\hat{H} + \frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + gh\right) = \hat{Q} + \hat{W}_m$

Macroscopic Mechanical Energy Balance

from section 7.4

$$\frac{d}{dt}\left(K_{tot} + \Phi_{tot}\right) = -\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \frac{p}{\rho}\right)w + W_m - E_c - E_v$$

steady state: $\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle}\right) + g\Delta h = -\int_1^2 \frac{1}{\rho}dp + \hat{W}_m - \hat{E}_v$

isothermal: ideal gas: $\int_1^2 \frac{1}{\rho}dp = \frac{RT}{M}\int_{p_1}^{p_2}\frac{1}{p}dp = \frac{RT}{M}\ln\frac{p_2}{p_1}$

incompressible liquids: $\int_1^2 \frac{1}{\rho}dp = \frac{1}{\rho}(p_2 - p_1)$

adiabatic gases with const C_p, $p + \rho$: $\gamma = \hat{C}_p/\hat{C}_v$

$$\int_1^2 \frac{1}{\rho}dp = \frac{p_1}{\rho_1}\frac{\gamma}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\gamma-1} - 1\right]$$

subtract mechanical energy balance from total energy balance

internal energy balance: p. 458

$$\frac{dU_{tot}}{dt} = -\Delta \hat{U} w + Q + E_c - E_v$$

no conservation law

Steady-state Problems with Flat Velocity Profiles:

Sec Table 15.3-1 turbulent flow

<u>ex</u> - 15.3-1 Cooling of an Ideal Gas

<u>ex</u> - 15.3-2 Mixing of Two Ideal Gas Streams

Differential Form of the Mechanical Energy Balance p. 461

$$d(\tfrac{1}{2}v^2) + g\,dh + \frac{1}{\rho}dp = d\hat{W} - d\hat{E}_v$$

$$v\,dv + g\,dh + \frac{1}{\rho}dp = d\hat{W} - \frac{1}{2}v^2 \frac{f}{R_h}\,dl$$

Differential Form of the Total Energy Balance p. 462

$$d(\tfrac{1}{2}v^2) + g\,dh + d\hat{H} = d\hat{Q} + d\hat{W}$$

$$v\,dv + g\,dh + \hat{C}_p\,dT + [\hat{V} - T(\frac{\partial \hat{V}}{\partial T})_p]\,dp = \frac{U_{loc} Z \Delta T}{w}\,dl + d\hat{W}$$

<u>ex</u> - 15.4-1 Parallel or Counter-Flow Heat Exchangers

<u>ex</u> - 15.4-2 Power Requirement for Pumping a Compressible Fluid through a long pipe

Table 15.5-1 Summary of Unsteady-state Nonisothermal Macroscopic Balances

<u>ex</u> - 15.5-1 Heating of a Liquid in an Agitated Tank

<u>ex</u> - 15.5-2 Operation of a simple Temperature Controller

<u>ex</u> - 15.5-3 Flow of Compressible Fluids Through Head Meters

<u>ex</u> - Free Batch Expansion of a Compressible Fluid 15.5-4

Ch 16 Energy Transport by Radiation

atoms or molecules return to lower energy states from excited states, energy is emitted as electromagnetic radiation

$\lambda = c/\nu$, photon : $\varepsilon = h\nu$

increasing wavelength of EM radiation = decreasing energy of photons

Absorption and Emission at Solid Surfaces p. 490

a = fraction of incident radiation absorbed

a_ν = fraction absorbed with frequency ν

$$a = \frac{q^{(a)}}{q^{(i)}} \qquad a_\nu = \frac{q_\nu^{(a)}}{q_\nu^{(i)}}$$ → amount absorbed

real body : $a_\nu < 1$, varies with ν

gray body : a_ν is constant and < 1 over range of ν

black body : $a_\nu = 1$ all ν's

emissivity $e_\nu = \dfrac{q_\nu^{(e)}}{q_{b\nu}^{(e)}}$ $\qquad e = \dfrac{q^{(e)}}{q_b^{(e)}}$ → amount emitted

cavity radiation : ind. of the nature of the walls , $q^{(cav)}$

put a black body in a cavity : $q^{(cav)} = q_b^{(e)}$

put a nonblack body in a cavity : $a\, q^{(cav)} = q^{(e)}$

$$a = \frac{q^{(e)}}{q_b^{(e)}} = e \qquad \text{Kirchhoff's law}$$

$$e_\nu = a_\nu$$

Total emitted flux from a black surface:

$$q_b^{(e)} = \sigma T^4 \qquad \text{Stefan - Boltzmann law}$$

non-black surface:

$$q^{(e)} = e\sigma T^4$$

Planck Distribution Law

$$q_{b\lambda}^{(e)} = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{ch/\lambda kT} - 1}$$

to prove this, Planck had to introduce quantization of energy - beginning of QM!

integrate to get Stepfon - Boltzmann constant:

$$\sigma = \frac{2}{15} \frac{\pi^5 k^4}{c^2 h^3}$$

Wien's displacement law: $\lambda_{max} T = 0.2884$ cm K

esti temp of remote objects

Direct Radiation Between Black Bodies in Vacuo at Different Temps

Lambert's cosine law: $q_{b\theta}^{(e)} = \frac{q_b^{(e)}}{\pi} = \frac{\sigma T^4}{\pi} \cos\theta$ p. 497

$$\int_0^{2\pi}\int_0^{\pi/2} q_{b\theta}^{(e)} \sin\theta \, d\theta \, d\phi = \frac{\sigma T^4}{\pi} \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= \sigma T^4 = q_b^{(e)}$$

$$Q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4) = A_2 F_{21} \sigma (T_1^4 - T_2^4)$$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{1}{\pi A_1} \iint \frac{\cos\theta_1 \cos\theta_2}{r_{12}^2} dA_1 dA_2$$

$$F_{21} = \frac{1}{\pi A_2} \iint \frac{\cos\theta_1 \cos\theta_2}{r_{12}^2} dA_1 dA_2$$

For $i = 1, 2 \ldots n$ surfaces:

$$Q_{ie} = \sigma A_i \sum_{j=1}^{n} F_{ij} (T_i^4 - T_j^4)$$

if $i = 3, 4 \ldots n$ adiabatic, $(Q = 0)$

$$Q_{12} = A_1 \overline{F}_{12} \sigma (T_1^4 - T_2^4) = A_2 \overline{F}_{21} \sigma (T_1^4 - T_2^4)$$

Radiation Between Nonblack Bodies at Different Temps p. 502

Small convex surface in a large, isothermal enclosure (cavity)

$$Q_{12} = \sigma A_1 (e_1 T_1^4 - a_1 T_2^4)$$

enclosure formed by n gray, opaque diffuse-reflecting surfaces

$$q_{ik} = A_i F_{ik} (J_i - J_k)$$

J_i = radiosity = sum of fluxes of reflected and emitted radiant energy

$$J_i = (1 - e_i) I_i + e_i \sigma T_i^4$$

$$\frac{Q_{ie}}{A_i} = J_i - I_i = J_i - \frac{J_i - e_i \sigma T_i^4}{1 - e_i}$$

$$Q_i = Q_{ie} = \sum_{k=1}^{n} Q_{ik}$$

ex - 16.5-2 Radiation and Free-Convection Heat Losses from a Horizontal Pipe

ex - 16.5-3 Combined Radiation and Convection

Radiant Energy Transport in Absorbing Media p. 506

$$\frac{\partial}{\partial t} \rho \hat{U} = -(\nabla \cdot \rho \hat{U} \vec{v}) - (\nabla \cdot \vec{q}) - (\nabla \cdot p \vec{v}) - (\tau : \nabla \vec{v}) - (\mathcal{E} - A)$$

$\mathcal{E}$ = photon emission rate

A = photon absorption rate

$u^{(r)}$ = radiant energy density

$$\frac{\partial}{\partial t} u^{(r)} = -(\nabla \cdot \vec{q}^{(r)}) + (\mathcal{E} - A) \quad \text{(photons)}$$

no convection - photons move ind. of material velocity

$$\frac{\partial}{\partial t} u_\nu^{(r)} = -(\nabla \cdot \vec{q}_\nu^{(r)}) + (\mathcal{E}_\nu - A_\nu)$$

steady-state nonflow, radiation in z direction

$$0 = -\frac{d}{dz} q_z + A$$

$$0 = -\frac{d}{dz} q_z^{(r)} - A \qquad A = m_a q^{(r)}$$

m_a = extinction coeff.

Lamberts law: integrate $0 = -\frac{d}{dz} q_\nu^{(r)} - m \alpha_\nu q_\nu^{(r)}$

$$q_\nu^{(r)}(z) = q_\nu^{(r)}(0) \exp(-m \alpha_\nu z)$$

widely used in spectroscopy

Mass Transport

Ch. 17 Diffusivity and Mechanisms of Mass Transport

Fick's Law p. 514

$$j_{Ay} = -\rho D_{AB} \frac{dw_A}{dy}$$ binary solid, liquid or gas solution

$$v_y = w_A v_{Ay} + w_B v_{By}$$ mass ave velocity

$$j_{Ay} = \rho w_A (v_{Ay} - v_y)$$

$$j_{Ay} + j_{By} = 0$$

$$\vec{j}_A = -\rho D_{AB} \nabla w_A \quad , \quad \vec{j}_B = -\rho D_{BA} \nabla w_B$$

$$D_{AB} = D_{BA}$$

have same dimensions $\left(\frac{\text{length}^2}{\text{time}}\right)$: $\alpha = \frac{k}{\rho \hat{C}_p}$, $\nu = \frac{\mu}{\rho}$, D_{AB}

dimensionless groups :

$$Pr = \frac{\nu}{\alpha} = \frac{\hat{C}_p \mu}{k}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

$$Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho \hat{C}_p D_{AB}}$$

non-isotropic : $\vec{j}_A = -(\rho \Delta_{AB} \cdot \nabla w_A)$

low pressure gases : p. 521

$$\frac{p D_{AB}}{(p_{cA} p_{cB})^{1/3} (T_{cA} T_{cB})^{5/12} \left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2}} = a \left(\frac{T}{\sqrt{T_{cA} T_{cB}}}\right)^b$$

self-diffusion : $$(c D_{AA^*})_c = 2.96 \times 10^{-6} \left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2} \frac{p_{cA}^{2/3}}{T_{cA}^{1/6}}$$

low pressures : $$(c D_{AB})_c = 2.96 \times 10^{-6} \left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2} \frac{(p_{cA} p_{cB})^{1/3}}{(T_{cA} T_{cB})^{1/12}}$$

Theory of Diffusion in Gases at Low Density

p. 525

Rigid sphere model

$$\boxed{D_{AA^*} = \frac{1}{3}\bar{u}\lambda = \frac{2}{3\pi}\frac{\sqrt{\pi m_A k_B T}}{\pi d_A^2}\frac{1}{\rho}} \quad \propto \frac{\sqrt{T}}{d_A^2\, m}$$

unequal masses and diameters:

$$\boxed{D_{AB} = \frac{2}{3}\sqrt{\frac{k_B T}{\pi}}\sqrt{\frac{1}{2}\left(\frac{1}{m_A}+\frac{1}{m_B}\right)}\frac{1}{\pi\left(\frac{1}{2}(d_A+d_B)\right)^2}\frac{1}{n}}$$

Chapman - Enskog kinetic theory

$$cD_{AB} = \frac{3}{16}\sqrt{\frac{2KT}{\pi}\left(\frac{1}{m_A}+\frac{1}{m_B}\right)}\frac{1}{\tilde{N}\sigma_{AB}^2\,\Omega_{D,AB}}$$

non-polar gas pairs: $\sigma_{AB} = \frac{1}{2}(\sigma_A+\sigma_B)$, $\varepsilon_{AB} = \sqrt{\varepsilon_A\varepsilon_B}$

isotopic pairs: $\sigma_{AA^*} = \sigma_A = \sigma_{A^*}$, $\varepsilon_{AA^*} = \varepsilon_A = \varepsilon_{A^*}$

$$\frac{\mu}{\rho D_{AA^*}} = \frac{\nu}{D_{AA^*}} = \frac{5}{6}\frac{\Omega_{D,AA^*}}{\Omega_\mu}$$

$$D_{AA^*} \approx 1.32\nu$$

Theory of Diffusion in Binary Liquids

p. 528

Nerst - Einstein equation: $D_{AB} = k_B T\,(u_A / F_A)$

A is spherical w/ slip at interface:

$$\frac{u_A}{F_A} = \left(\frac{3\mu_B + R_A\beta_{AB}}{2\mu_B + R_A\beta_{AB}}\right)\frac{1}{6\pi\mu_B R_A}$$

↳ coeff of sliding friction

$\beta_{AB} = \infty$ (no-slip) ⟶ Stokes Law

$$\frac{D_{AB}\,\mu_B}{k_B T} = \frac{1}{6\pi R_A}$$

$\beta_{AB} = 0$ (complete slip)

only applies to dilute solutions

$$\frac{D_{AB}\,\mu_B}{k_B T} = \frac{1}{4\pi R_A}$$

Eyring Activated State theory

$$\frac{D_{AB}\,\mu_B}{k_B T} = \frac{1}{\xi}\left(\frac{\hat{N}_A}{\hat{V}_B}\right)^{1/3}$$

Empirical : Wilke - Chang eqn

$$D_{AB} = 7.4\times10^{-8}\,\frac{\sqrt{\psi_B M_B}\,T}{\mu\,\hat{V}_A^{0.6}}$$

Colloidal Suspensions p. 531

Brownian motion : Langevin Eqn : $m\,\frac{d\vec{v}_A}{dt} = -\zeta\,\vec{v}_A + F(t)$

$\zeta = 6\pi\mu_B R_A$

$F(t)$ = force

Einstein : $D_{AB} = \frac{k_B T}{\zeta} = \frac{k_B T}{6\pi\mu_B R_A}$

Polymers : $D_{AB} \sim \frac{1}{\sqrt{M}}$, $D_{AA^*} \sim \frac{1}{M^2}$ p. 532

Molar Units p. 533

$c_\alpha = \rho_\alpha / M_\alpha$

$x_\alpha = c_\alpha / c$ $\qquad w_\alpha = \rho_\alpha/\rho$

Binary systems :

$\vec{J}_A^* = c_A(\vec{v}_A - \vec{v}^*) = -c\,D_{AB}\nabla x_A$

$\vec{j}_A = \rho_A(\vec{v}_A - \vec{v}) = -\rho\,D_{AB}\nabla w_A$

Convective Flux

$\rho_\alpha\,\delta_x v_x + \rho_\alpha\,\delta_y v_y + \rho_\alpha\,\delta_z v_z = \rho_\alpha\vec{v}$

$c_\alpha\,\delta_x v_x^* + c_\alpha\,\delta_y v_y^* + c_\alpha\,\delta_z v_z^* = c_\alpha\vec{v}^*$

combined flux

$\vec{n}_\alpha = \vec{j}_\alpha + \rho_\alpha\vec{v}$, $\vec{N}_\alpha = \vec{J}_A^* + c_\alpha\vec{v}^*$ p. 536

See Table 17.8-1 Molar + Mass Fluxes p. 537

Multicomponent Diffusion p. 538

Maxwell - Stefan eqn

$$\nabla x_\alpha = - \sum_{\beta=1}^{N} \frac{x_\alpha x_\beta}{D_{\alpha\beta}} (\vec{v}_\alpha - \vec{v}_\beta)$$

$$= - \sum_{\beta=1}^{N} \frac{1}{c D_{AB}} (x_\beta \vec{N}_\alpha - x_\alpha \vec{N}_\beta)$$

Ch. 18 Conc. Distributions in Solids and Laminar Flow

$$\boxed{N_{Az} = \underbrace{-cD_{AB}\frac{\partial x_A}{\partial z}}_{\text{molecular flux}} + \underbrace{x_A(N_{Az}+N_{Bz})}_{\text{convective flux}}}$$ p. 543

$R_A = k_n'' C_A^n$

homogeneous rxn : occurs in entire volume, add into diffeq

heterogeneous rxn : takes place in restricted region, BC

$N_{Az}|_{surface} = k_n'' C_A^n|_{surface}$

Shell Mass Balances

rate in - rate out + prod = 0 p. 545

BC :
$$\boxed{\begin{aligned} x_A &= x_{A0} \\ N_{Az} &= N_{A0} \\ N_{A0} &= k_c(C_{A0} - C_{Ab}) \\ N_{A0} &= k_1'' C_{A0} \end{aligned}}$$

Diffusion Through a Stagnant Gas Film p. 545

$$SN_{Az}|_z - SN_{Az}|_{z+\Delta z} = 0$$

$$-\frac{dN_{Az}}{dz} = 0 \qquad N_{Az} = \frac{-cD_{AB}}{1-x_A}\frac{dx_A}{dz}$$

$$\frac{d}{dz}\left(\frac{1}{1-x_A}\frac{dx_A}{dz}\right) = 0 \qquad p = cRT \text{ , } D_{AB} \text{ const.}$$

BC : $z = z_1$, $x_A = x_{A1}$, $z = z_2$, $x_A = x_{A2}$

ex - 18.2-1 Diffusion with a moving interface

rate of evap. of A = rate moles A enter gas phase

$$-S\frac{\rho^{(A)}}{M_A}\frac{dz_1}{dt} = \frac{cD_{AB}}{(z_2-z_1)(x_B)_{ln}}(x_{A1}-x_{A2})S$$

ex - 18.2-3 Diffusion through a non-iso. spherical film

Diffusion with a Heterogeneous Chemical Reaction p. 551

(0) catalyst surrounded by stagnant gas film

$2A \rightarrow B$ instan. rxn , isothermal

$N_{Bz} = -\frac{1}{2} N_{Az}$ steady state "diffusion controlled".

$$N_{Az} = -\frac{c D_{AB}}{1-\frac{1}{2}x_A} \frac{dx_A}{dz}$$

$$\frac{dN_{Az}}{dz} = 0 \qquad BC: \quad z=0 \quad x_A = x_{A0}; \qquad z=\delta \quad x_A = 0$$

$$\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}x_A} \frac{dx_A}{dz}\right) = 0$$

ex - 18.3-1 Slow Heterogeneous Rxn

new BC : $z=\delta$, $x_A = \frac{N_{Az}}{k_1'' c}$ $\quad (N_{Az} = k_1'' c x_A)$

Second Damköhler # : Da^{II}

$$Da^{II} = \frac{k_1'' \delta}{D_{AB}}$$

as $Da^{II} \rightarrow 0$, get instan. rxn result

Diffusion with Homogeneous Chemical Reaction p. 554

pseudo binary assumption

$$N_{Az}|_z S - N_{Az}|_{z+\Delta z} S - k_1''' c_A S \Delta z = 0$$

$$\frac{dN_{Az}}{dz} + k_1''' c_A = 0$$

If c_A is small, $N_{Az} = -D_{AB}\frac{dc_A}{dz}$

$$D_{AB}\frac{d^2 c_A}{dz^2} - k_1''' c_A = 0$$

$z=0$, $c_A = c_{A0}$

$z=L$, $N_{Az} = 0$ $\left(\frac{dc_A}{dz}=0\right)$

ex - 18.4-1

Gas Absorption with Chemical Rxn in an Agitated Tank

Diffussion into a Falling Liquid Film (Gas Absorption) p. 558

slow diffusion - A will penetrate very far into film

gas A into laminar falling film of liquid B

momentum problem (2.2):

$$V_z(x) = V_{max}\left[1-\left(\frac{x}{\delta}\right)^2\right]$$

mass balance:

$$0 = N_A|_z W\Delta x - N_{Az}|_{z+\Delta z} W\Delta x + N_{Ax}|_x W\Delta z - N_{Ax}|_{x+\Delta x} W\Delta z$$

$$\frac{\partial N_{Az}}{\partial z} + \frac{\partial N_{Ax}}{\partial x} = 0$$

$$N_{Az} = \underbrace{-D_{AB}\frac{\partial C_A}{\partial z}}_{\text{neglect compared to convection}} + X_A(N_{Az}+N_{Bz}) \approx C_A V_z(x)$$

$$\sum_{\alpha=1}^{N} N_\alpha = cv^*$$

stationary axes

$v \approx v^*$ dilute soln

$$N_{Ax} = -D_{AB}\frac{\partial C_A}{\partial x} + \underbrace{X_A(N_{Ax}+N_{Bx})}_{\text{neglect compared to diffusion}}$$

$$= -D_{AB}\frac{\partial C_A}{\partial x}$$

$$V_z\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2} = V_{max}\left[1-\left(\frac{x}{\delta}\right)^2\right]\frac{\partial C_A}{\partial z}$$

$z=0,\ C_A=0$

$x=0,\ C_A=C_{A0}$

$x=\delta,\ \frac{\partial C_A}{\partial x}=0$

Since A can only penetrate small distance + didn't see wall:

$$V_{max}\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$$

$z=0,\ C_A=0$

$x=0,\ C_A=C_{A0}$

$x\to\infty,\ C_A=0$

use combination of variables

Diffusion into a Falling Liquid Film (Solid Dissolution) p. 562

$v_z = v_z(y)$

$0 < z < L$ wall is only A (from 2.2-18)

$$v_z = \frac{\rho g \delta^2}{2\mu} \left[1 - \left(1 - \frac{y}{\delta}\right)^2\right] = \frac{\rho g \delta^2}{2\mu} \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]$$

at and adjacent to wall, $(y/\delta)^2 << (y/\delta)$

$$v_z = (\rho g \delta / \mu) y = a y$$

$$a y \frac{\partial c_A}{\partial z} = D_{AB} \frac{\partial^2 c_A}{\partial y^2}$$

$z = 0$, $c_A = 0$

$y = 0$, $c_A = c_{A0}$

$y \to \infty$, $c_A = 0$ (approximation)

combination of variables

Diffusion and Rxn inside a porous catalyst p. 563

$$N_{Ar}|_r \cdot 4\pi r^2 - N_{Ar}|_{r+\Delta r} 4\pi (r+\Delta r)^2 + R_A 4\pi r^2 \Delta r = 0$$

$$\frac{d}{dr}(r^2 N_{Ar}) = r^2 R_A \quad , \quad N_{Ar} = -D_A \frac{dc_A}{dr}$$

1st order rxn

$$D_A \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{dc_A}{dr}\right) = k_1'' a\, c_A \qquad c_A = c_{AR} \text{ at } r = R$$

c_A is finite at $r = 0$

change of variables $c_A/c_{AR} = (1/r) f(r)$

Diffusion in a Three-Component Gas System p. 567

$$\frac{d N_{\alpha z}}{dz} = 0 \quad \alpha = 1, 2, 3 \qquad N_{2z} = N_{3z} = 0 \text{ (not moving)}$$

$$x_1 + x_2 + x_3 = 1$$

$$\frac{dx_2}{dz} = \frac{N_{1z}}{c D_{12}} x_2 \qquad \frac{dx_3}{dz} = \frac{N_{1z}}{c D_{13}} x_3$$

using 17.9-1

Ch. 19 Equations of Change for Multicomponent Systems

in mass quantities

p. 582

$$\frac{\partial \rho_\alpha}{\partial t} = -(\nabla \cdot \vec{n}_\alpha) + r_\alpha$$

$$\frac{\partial \rho_\alpha}{\partial t} = -(\nabla \cdot \rho_\alpha \vec{v}) - (\nabla \cdot \vec{j}_\alpha) + r_\alpha$$

conv. diffusion rxn

$$\frac{\partial \rho}{\partial t} = -(\nabla \cdot \rho \vec{v}) \qquad \text{(for mixture)}$$

at const. ρ : $(\nabla \cdot \vec{v}) = 0$

in molar quantities

$$\frac{\partial c_\alpha}{\partial t} = -(\nabla \cdot N_\alpha) + R_\alpha$$

$$\frac{\partial c_\alpha}{\partial t} = -(\nabla \cdot c_\alpha v^*) - (\nabla \cdot J_\alpha^*) + R_\alpha$$

$$\frac{\partial c}{\partial t} = -(\nabla \cdot c \vec{v}^*) + \sum_{\alpha=1}^{N} R_\alpha$$

$\longrightarrow$ moles are not conserved in chem rxn

at const c : $(\nabla \cdot \vec{v}^*) = \frac{1}{c} \sum_{\alpha=1}^{N} R_\alpha$

2 equivalent forms

$$\rho \left(\frac{\partial w_\alpha}{\partial t} + (\vec{v} \cdot \nabla w_\alpha) \right) = -(\nabla \cdot \vec{j}_\alpha) + r_\alpha$$

$$c \left(\frac{\partial x_\alpha}{\partial t} + (\vec{v}^* \cdot \nabla x_\alpha) \right) = -(\nabla \cdot J_\alpha^*) + R_\alpha - x_\alpha \sum_{\beta=1}^{N} R_\beta$$

Binary systems with constant ρD_{AB} :
dilute solns, const T + p

$$\rho \left(\frac{\partial w_A}{\partial t} + (\vec{v} \cdot \nabla w_A) \right) = \rho D_{AB} \nabla^2 w_A + r_A$$

Binary systems with constant $c D_{AB}$:
low density gases

$$c \left(\frac{\partial x_A}{\partial t} + (\vec{v}^* \cdot \nabla x_A) \right) = c D_{AB} \nabla^2 x_A + (x_B R_A - x_A R_B)$$

Binary systems with 0 velocity and no chem rxn :

$$\frac{\partial c_A}{\partial t} = D_{AB} \cdot \nabla^2 c_A \qquad \text{Fick's 2nd law}$$

Table 19.2-1 Equations of Change for Multicomponent Mixtures

Table 19.2-2 Combined Fluxes

Table 19.2-3 Equations of Change for Multicomponent Mixtures

Table 19.2-4 Equation of Energy w/ gravity

Summary of Multicomponent Fluxes p. 590

mass : $\vec{J}_A = -\rho D_{AB} \nabla w_A$ (binary only)

momentum : $\vec{\tau} = -\mu [\nabla \vec{v} + (\nabla \vec{v})^{+}] + (\frac{2}{3}\mu - \kappa)(\nabla \cdot \vec{v})\vec{\delta}$

energy : $\vec{q} = -k \nabla T + \sum_{\alpha=1}^{N} \frac{\bar{H}_\alpha}{M_\alpha} \vec{J}_\alpha$

combined energy flux :

$$\vec{e} = \rho(\hat{U} + \tfrac{1}{2}v^2)\vec{v} + \vec{q} + \rho\vec{v} + [\vec{\tau} \cdot \vec{v}]$$

$$= \rho(\hat{U} + \tfrac{1}{2}v^2)\vec{v} - k\nabla T + \sum_{\alpha=1}^{N} \frac{\bar{H}_\alpha}{M_\alpha} \vec{J}_\alpha + \rho\vec{v} + [\vec{\tau} \cdot \vec{v}]$$

$$= -k\nabla T + \sum_{\alpha=1}^{N} \bar{H}_\alpha \vec{J}_\alpha + \rho(\hat{U} + \rho\hat{V})\vec{v} + \underbrace{\tfrac{1}{2}\rho v^2 \vec{v} + [\vec{\tau} \cdot \vec{v}]}$$

discard last terms in films + low-vel. BC's

$$\vec{e} = -k\nabla T + \sum_{\alpha=1}^{N} \bar{H}_\alpha \vec{J}_\alpha + \rho \hat{H}_\alpha \vec{v}$$

$$= -k\nabla T + \sum_{\alpha=1}^{N} \bar{H}_\alpha \vec{N}_\alpha$$

Using the Equations of Change for Mixtures p. 592

ex - 19.4-1 Simultaneous Heat and Mass Transport

ex - 19.4-2 Concentration Profile in a Tubular Reactor

ex - 19.4-3 Multicomponent Diffusion - catalytic oxidation of CO

ex - 19.4-4 Thermal Conductivity of a Polyatomic Gas

Ch. 20 Conc Distributions with More Than One Ind Variable

Table 20.0-1 Analogies Between Conduction and Diffusion Eqns

Time - Dependent Diffusion p. 613

<u>ex</u> - 20.1-1 Unsteady State Evaporation of a Liquid

$$\frac{\partial v_z^*}{\partial z} = 0 \quad , \quad v_z^* = v_{z0}^*(t)$$

$$v_z^* = \frac{N_{Az0} + N_{Bz0}}{c} \quad , \quad N_{Bz0} = 0$$

$$v_z^* = -\frac{D_{AB}}{1 - x_{A0}} \left.\frac{\partial x_A}{\partial z}\right|_{z=0}$$

$$\frac{\partial x_A}{\partial t} - \left(\frac{D_{AB}}{1 - x_{A0}} \left.\frac{\partial x_A}{\partial z}\right|_{z=0} \right) \frac{\partial x_A}{\partial z} = D_{AB} \frac{\partial^2 x_A}{\partial z^2}$$

$t = 0 \;,\; x_A = 0$

$z = 0 \;,\; x_A = x_{A0}$ Use combination

$z \to \infty \;,\; x_A = 0$ of variables

<u>ex</u> - 20.1-2 Gas Absorption with Rapid Reaction

Fick's 2nd law applies

$$\frac{\partial c_A}{\partial t} = D_{AS} \frac{\partial^2 c_A}{\partial z^2} \qquad 0 < z \le z_R(t)$$

$$\frac{\partial c_B}{\partial t} = D_{BS} \frac{\partial^2 c_B}{\partial z^2} \qquad z_R(t) \le z < \infty$$

$t = 0 \qquad c_B = c_{B\infty} \quad z > 0$

$z = 0 \qquad c_A = c_{A0}$

$z = z_R(t) \qquad c_A = c_B = 0$

$$z = z_R(t) \qquad -\frac{1}{a} D_{AS} \frac{\partial c_A}{\partial z} = \frac{1}{b} D_{BS} \frac{\partial c_B}{\partial z}$$

$z \to \infty \qquad c_B = c_{B\infty}$

combination of variables

ex - 20.1-3 Unsteady Diffusion with 1st-order Homogeneous Rxn

$$\frac{\partial w_A}{\partial t} + (\vec{v} \cdot \nabla w_A) = \mathcal{D}_{AB} \nabla^2 w_A - k_1''' w_A$$

$t=0$, $w_A = w_{A1}(x, y, z)$

$w_A = w_{A0}(x\ y, z)$

solve by superposition and Laplace

ex - 20.1-4 Influence of Changing Interfacial Area on Mass Transfer at An Interface

$$\frac{\partial c_A}{\partial t} = \mathcal{D}_{AB} \frac{\partial^2 c_A}{\partial z^2}$$ if area not changing

Varying interfacial area :

$v_x = \frac{1}{2} a x \qquad v_y = \frac{1}{2} a y \qquad v_z = -a z$

$$a = \frac{d \ln s}{dt}$$

$$\frac{\partial c_A}{\partial t} - \left(\frac{d}{dt} \ln s\right) z \frac{\partial c_A}{\partial z} = \mathcal{D}_{AB} \frac{\partial^2 c_A}{\partial z^2}$$

combination of variables

Steady State Transport in Binary Boundary Layers p. 623

<u>BL equations with $\rho, \mu, k, \hat{C}_p, \mathcal{D}_{AB}$ constant and neglecting viscous dissipation :</u>

continuity : $$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

motion : $$\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = \rho v_e \frac{dv_e}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} + \bar{\rho} g_x \bar{\beta}(T - T_\infty) + \underbrace{\bar{\rho} g_x \bar{\xi}(w_A - w_{A\infty})}_{\text{binary buoyant force}}$$

energy : $$\rho \hat{C}_p \left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}\right) = k \frac{\partial^2 T}{\partial y^2} - \underbrace{\left(\frac{\bar{H}_A}{M_A} - \frac{\bar{H}_B}{M_B}\right) r_A}_{\text{chemical heat source}}$$

continuity of A : $$\rho\left(v_x \frac{\partial w_A}{\partial x} + v_y \frac{\partial w_A}{\partial y}\right) = \rho \mathcal{D}_{AB} \frac{\partial^2 w_A}{\partial y^2} + r_A$$

BC's:

$v_x = 0$ at solid surface, $v_x = v_e(x)$ at edge of velocity BL

$T = T_0(x)$ at solid surface, $T = T_\infty$ at edge of thermal BL

$w_A = w_{A0}(x)$ at solid surface, $w_A = w_{A\infty}$ at edge of diffusional BL

$v_y = v_0(x)$ at $y = 0$

von Kármán balances:

integrate above equations with eqn of continuity:

Continuity + motion:

$$\mu \frac{\partial v_x}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\int_0^\infty \rho v_x (v_e - v_x)\,dy + \frac{dv_e}{dx}\int_0^\infty \rho (v_e - v_x)\,dy - \int_0^\infty \rho g_x \beta (T - T_\infty)\,dy - \int_0^\infty \rho g_x \xi (w_A - w_{A\infty})\,dy + \rho v_0 v_e$$

continuity + energy:

$$k \frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\int_0^\infty \rho v_x \hat{C}_p (T_\infty - T)\,dy - \int_0^\infty \left(\frac{\bar{H}_A}{M_A} - \frac{\bar{H}_B}{M_B}\right) r_A\,dy - \rho v_0 \hat{C}_p (T_\infty - T_0)$$

Continuity + continuity of A:

$$\rho D_{AB} \frac{\partial w_A}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\int_0^\infty \rho v_x (w_{A\infty} - w_A)\,dy + \int_0^\infty r_A\,dy - \rho v_0 (w_{A\infty} - w_{A0})$$

ex - 20.2-1 Diffusion and Chemical Reaction in Isothermal Laminar Flow Along a Soluble Flat Plate

let $\Delta = \delta_c / \delta$

use von Kármán motion + continuity of A balances

integrate first and substitute δ into 2nd

$$Sc = \mu / \rho D_{AB}$$

$$Da^{I} = \frac{k_n''' C_{A0}^{n-1} x}{v_\infty}$$

ex - 20.2-2 Forced Convection from a Flat Plate at High Mass Transfer Rates

$\rho, \mu, k, \hat{C}_p, \mathcal{D}_{AB}$ constant, no viscous dissipation, no chemical rxn

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$v_x \frac{\partial w_A}{\partial x} + v_y \frac{\partial w_A}{\partial y} = \mathcal{D}_{AB} \frac{\partial^2 w_A}{\partial y^2}$$

$x \le 0$ or $y = \infty$, $v_x = v_\infty$, $T = T_\infty$, $w_A = w_{A\infty}$

$y = 0$, $v_x = 0$, $T = T_0$, $w_A = w_{A0}$, $v_y = v_0(x)$

integrate 1st eqn and insert into others (v_y)

dimensionless ratios, combination of variables

Steady State Boundary Layer Theory for Flow Around Objects p.633

$h_y = 1$, $h_x = h_x(x, z)$, $h_z = h_z(x, z)$

$h_\alpha^2 = \sum_i \left(\frac{\partial x_i}{\partial q_\alpha} \right)^2$ x_i = cartesian coordinates

q_α = curvilinear coordinates

$$\frac{\partial}{\partial x}(h_z v_x) + h_x h_z \frac{\partial}{\partial y} v_y = 0$$

$$v_x \frac{1}{h_x} \frac{\partial c_A}{\partial x} + v_y \frac{\partial c_A}{\partial y} = \mathcal{D}_{AB} \frac{\partial^2 c_A}{\partial y^2}$$

Boundary Layer Mass Transport with Complex Interfacial Motion p.637

instantaneous interfacial area $= dS = s(u, w, t)\, du\, dw$

$$\frac{\partial w_A}{\partial t} + \left(v_{y0} - y \frac{\partial \ln s}{\partial t}\right) \frac{\partial w_A}{\partial y} = \mathcal{D}_{AB} \frac{\partial^2 w_A}{\partial y^2} + \frac{1}{\rho} r_A$$

$$\frac{\partial c_A}{\partial t} + \left(v_{z0} - z \frac{\partial \ln s}{\partial t}\right) \frac{\partial c_A}{\partial z} = \mathcal{D}_{AB} \frac{\partial^2 c_A}{\partial z^2} + R_A$$

$$\frac{\partial x_A}{\partial t} + \left(v_{z0}^* - z \frac{\partial \ln s}{\partial t}\right) \frac{\partial x_A}{\partial z} = \mathcal{D}_{AB} \frac{\partial^2 x_A}{\partial z^2} + \frac{1}{c}\left[R_A - x_A (R_A + R_B\right.$$

"Taylor Dispersion" in Laminar Tube Flow

solute pulse of A introduced into fluid B in steady laminar flow through a long, straight tube of radius R

Poiseuille Flow

$$\frac{\partial w_A}{\partial t} + v_{z,max}\left[1-(r/R)^2\right]\frac{\partial w_A}{\partial z} = D_{AB}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_A}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right)$$

$$r=0,\quad r=R,\quad \frac{\partial w_A}{\partial r}=0$$

approximate analysis:

$$\langle w_A\rangle = \frac{\int_0^{2\pi}\int_0^R w_A\, r\,dr\,d\theta}{\int_0^{2\pi}\int_0^R r\,dr\,d\theta} = \frac{2}{R^2}\int_0^R w_A r\,dr$$

neglect axial molecular diffusion

shifted axial coordinate: $\bar{z} = z - \langle v_z\rangle t$

$$\frac{\partial w_A}{\partial t} + v_{z,max}\left(\tfrac{1}{2}-\xi^2\right)\frac{\partial w_A}{\partial \bar{z}} = \frac{D_{AB}}{R^2}\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial w_A}{\partial \xi}\right)$$

$$\xi = r/R$$

neglect $\frac{\partial w_A}{\partial t}$ compared to radial diffusion term

- quasi-steady state

$$w_A(\xi,\bar{z},t) = \langle w_A\rangle + \underbrace{w_A'(\xi,\bar{z},t)}_{\text{neglect this}}$$

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial w_A}{\partial \xi}\right) = \frac{R^2 v_{z,max}}{D_{AB}}\left(\tfrac{1}{2}-\xi^2\right)\frac{\partial\langle w_A\rangle}{\partial \bar{z}}$$

integrate

$$w_A(\xi,\bar{z}) = \frac{R^2 v_{z,max}}{8 D_{AB}}\frac{\partial\langle w_A\rangle}{\partial \bar{z}}\left(\xi^2-\tfrac{1}{2}\xi^4\right) + w_A(0,\bar{z})$$

$$\langle w_A\rangle = \frac{\int_0^1 w_A\,\xi\,d\xi}{\int_0^1 \xi\,d\xi} = \frac{R^2 v_{z,max}}{24 D_{AB}}\frac{\partial\langle w_A\rangle}{\partial \bar{z}} + w_A(0,\bar{z})$$

$$w_A - \langle w_A\rangle = \frac{R^2\langle v_z\rangle}{2D_{AB}}\frac{\partial\langle w_A\rangle}{\partial \bar{z}}\left(-\tfrac{1}{3}+\xi^2-\tfrac{1}{2}\xi^4\right)$$

Taylor's approximate solution

K = axial dispersion coefficient

$$= \frac{R^2 \langle v_z \rangle^2}{48\, D_{AB}} = \frac{1}{48} D_{AB}\, Pe_{AB}^{2}$$

$$\langle \rho_A \rangle = \frac{m_A}{2\pi R^2 \sqrt{\pi K t}} \exp\left(- \frac{(z - \langle v_z \rangle t)^2}{4Kt} \right)$$

extract D_{AB} from conc data of a traveling pulse

Ch. 21

Concentration Distributions in Turbulent Flow

$$c_A = \bar{c}_A + c_A' \qquad v_i = \bar{v}_i + v_i' \qquad \text{p. 657}$$

$$\frac{\partial \bar{c}_A}{\partial t} = -\left(\frac{\partial}{\partial x}\bar{v}_x \bar{c}_A + \frac{\partial}{\partial y}\bar{v}_y \bar{c}_A + \frac{\partial}{\partial z}\bar{v}_z \bar{c}_A\right)$$
$$- \left(\frac{\partial}{\partial x}\overline{v_x' c_A'} - \frac{\partial}{\partial y}\overline{v_y' c_A'} - \frac{\partial}{\partial z}\overline{v_z' c_A'}\right)$$
$$+ \mathscr{D}_{AB}\left(\frac{\partial^2 \bar{c}_A}{\partial x^2} + \frac{\partial^2 \bar{c}_A}{\partial y^2} + \frac{\partial^2 \bar{c}_A}{\partial z^2}\right) - \begin{cases} k_1''' \bar{c}_A \text{ or} \\ k_2''' \bar{c}\bar{c}_A^2 + \overline{c_A'^2} \end{cases}$$

Turbulent molar flux : $\bar{J}_{Ai}^{(t)} = \overline{v_i' c_A'}$

Turbulent momentum flux : $\bar{\tau}_{ij}^{(t)} = \rho\,\overline{v_i' v_j'}$

Turbulent heat flux : $\bar{q}_i^{(t)} = \rho c_p \overline{v_i' T'}$

2^{nd} order rxn has an additional term : $-k_2''' \overline{c_A'^2}$

- interaction between chemical kinetics and turbulent fluctuations

Summary of time-smoothed eqns for turbulent flow:

Continuity $\quad (\nabla \cdot \bar{\vec{v}}) = 0$

motion $\quad \rho \dfrac{D\bar{\vec{v}}}{Dt} = -\nabla \bar{p} - [\nabla \cdot (\bar{\vec{\tau}}^{(v)} + \bar{\tau}^{(t)})] + \rho g$

Continuity of A $\quad \dfrac{D\bar{c}_A}{Dt} = -(\nabla \cdot (\bar{J}_A^{(v)} + \bar{J}_A^{(t)})) - \begin{cases} k_1''' \bar{c}_A \text{ or} \\ k_2''' c\bar{c}_A^2 + \overline{c_A'^2} \end{cases}$

$$\bar{J}_A^{(v)} = -\mathscr{D}_{AB}\,\bar{c}_A$$

Semi-Empirical Expressions for $\bar{J}_A^{(t)}$

p 659

Eddy Diffusivity

$$\bar{J}_{Ay}^{(t)} = -\mathscr{D}_{AB}^{(t)} \frac{d\bar{c}_A}{dy}$$

$$Sc^{(t)} = \frac{\nu^{(t)}}{\mathscr{D}_{AB}^{(t)}} \sim 1 \qquad \text{Turbulent Schmidt Number}$$

Prandtl Mixing Length

$$\bar{J}_{Ay}^{(t)} = -\ell^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{c}_A}{dy}$$

satisfies Reynolds Analogy:
$\nu^{(t)} = \alpha^{(t)} = \mathscr{D}_{AB}^{(t)}$
$Pr^{(t)} = Sc^{(t)} = 1$

Enhancement of Mass Transfer by 1st Order Rxn in Turbulent Flow

axial symmetry and $\bar{C}_A$ is ind of time:

$$\bar{v}_z \frac{\partial \bar{C}_A}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(D_{AB} + D_{AB}^{(t)}\right)\frac{\partial \bar{C}_A}{\partial r}\right) - k_1''' \bar{C}_A$$

need to find mass transfer rate at wall:

$$D_{AB} \frac{\partial \bar{c}_A}{\partial r}\Big|_{r=R} = k_c\left(C_{A0} - \underset{\downarrow\ 0}{\bar{C}_{A,\,axis}}\right)$$

$C = \bar{C}_A / C_{A0}$ for large z, $C_A \neq C_A(z)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(D_{AB} + D_{AB}^{(t)}\right)\frac{\partial c}{\partial r}\right) = k_1''' c$$

$C = 0$ at $r=0$, $C=1$ at $r = R$

$y = R - r$, $\sigma = y/R$

$$\frac{1}{2}\left(\frac{k_c D}{D_{AB}}\right)\left(\frac{D_{AB}}{\nu}\right)\int_0^1 \frac{1}{(D_{AB}/\nu) + K\sigma^3}\, d\sigma - 1 = \left(\frac{k_1''' R^2}{\nu}\right) x$$

better way: use $r=0$, $\frac{\partial c}{\partial r} = 0$ $\quad \int_0^1 \frac{1}{(D_{AB}/\nu) + K\sigma^3}\left[\int_0^\sigma c(\bar{\sigma})\right. d\sigma$

$v^+ = \bar{v}_z / v_*$, $z^+ = z v_* / \nu$, $r^+ = r v_* / \nu$, $\ell^+ = \ell v_* / \nu$

$$\frac{dv^+}{dy^+} = \begin{cases} \dfrac{-1 + \sqrt{1 + 4(\ell^+)^2\left(1 - y^+/R^+\right)}}{2(\ell^+)^2} & y^+ > 0 \\ 1 & y^+ = 0 \end{cases}$$

Turbulent Mixing and Turbulent Flow with 2nd-order rxn

$$\frac{DC_A}{Dt} = D_{AS}\nabla^2 C_A + R_A \qquad \frac{DC_B}{Dt} = D_{BS}\nabla^2 C_B + R_B$$

no rxn: $\Gamma = \dfrac{C_{A0} - C_A}{C_{A0}} = \dfrac{C_B}{C_{B0}}$, $\dfrac{D\Gamma}{Dt} = D_{is}\nabla^2\Gamma$

rxn: $\Gamma_{rxn} = \dfrac{C_{A0} - (C_A - C_B)}{C_{A0} + C_{B0}} = \left(\dfrac{C_{A0} - C_A}{C_{A0}}\right)_{no\ rxn}\left(\dfrac{C_B}{C_{B0}}\right)_{no\ rxn}$

$$\left(\frac{C_A' - C_B'}{C_{A0} + C_{B0}}\right)_{no\ rxn} = \left(C_A'/(C_{A0})\right)_{no\ rxn}$$

fast rxn: $(C_A'/C_{A0})_{rxn} = (C_A'/C_{A0})_{no\ rxn}$

slow rxn: $\bar{R}_A = -k'''\left(\bar{C}_A \bar{C}_B + \overline{C_A' C_B'}\right)$

Ch. 22 Interphase Transport in Nonisothermal Mixtures

Transfer Coefficients in One Phase

p. 672

$$N_{A0} - x_{A0}(N_{A0} + N_{B0}) = -\left(c\mathcal{D}_{AB}\frac{\partial x_A}{\partial y}\right)\Big|_{y=0}$$

$$e_0 - (N_{A0}\bar{H}_{A0} + N_{B0}\bar{H}_{B0}) = -\left(k\frac{\partial T}{\partial y}\right)\Big|_{y=0}$$

$$W_{A0} - x_{A0}(W_{A0} + W_{B0}) = k_{xA} A \Delta x_A$$

$$E_0 - (W_{A0}\bar{H}_{A0} + W_{B0}\bar{H}_{B0}) = hA\Delta T$$

} heat and mass transfer

local transfer coeffs for dA:

$$N_{A0} - x_{A0}(N_{A0} + N_{B0}) = k_{x,loc}\,\Delta x_A$$

$$e_0 - (N_{A0}\tilde{C}_{pA,0} + N_{B0}\tilde{C}_{pB,0})(T_0 - T^\circ) = h_{loc}\,\Delta T$$

Apparent mass transfer coefficient:

$$N_{A0} = k^\circ_{x,loc}\,\Delta x_A \qquad k^\circ_{x,loc} = \frac{k_{x,loc}}{[1 - x_{A0}(1+r)]}$$

$$N_{A0} = k^\circ_{c,loc}\,\Delta c_A$$

$$n_{A0} = k^\circ_{\rho,loc}\,\Delta \rho_A \qquad r = N_{B0}/N_{A0}$$

$$N_{A0} = k^\circ_{p,loc}\,\Delta p_A$$

Sherwood number $Sh = \dfrac{k_x l_0}{c\mathcal{D}_{AB}}$

Analytical Expressions for Mass Transfer Coefficients

p. 676

Analogies between heat and mass transfer Table 22.2-1

Also has dimensionless groups

Gas Absorption by falling film: $Sh_m = 1.128\,(ReSc)^{1/2}$

Solid Dissolution into falling film: $Sh_m = 1.017\sqrt[3]{\left(\frac{L}{\delta}\right)}\,(ReSc)^{1/3}$

Flow around spheres (creeping flow): $Sh_m = 0.6415\,(ReSc)^{1/2}$

Steady, non-separated BL on arbitrarily-shaped objects - see book

Rotating Disk - $Sh_m = 0.620\,Re^{1/2}\,Sc^{1/3}$

Binary Transfer Coefficients in One Phase

(analogous to heat transfer correlations)

heat

$$Q(t) = \int_0^L \int_0^{2\pi} \left(k \frac{\partial T}{\partial r}\Big|_{r=R}\right) R\, d\theta\, dz = h_1 (\pi D L)(T_0 - T_1)$$

$$h_1(t) = \frac{1}{\pi D L (T_0 - T_1)} \int_0^L \int_0^{2\pi} \left(k \frac{\partial T}{\partial r}\Big|_{r=R}\right) R\, d\theta\, dz$$

$\breve{r} = r/D$, $\breve{z} = z/D$, $\breve{T} = \dfrac{T - T_0}{T_1 - T_0}$

mass

$$W_{A0}(t) - x_{A0}\left(W_{A0}(t) + W_{B0}(t)\right) = \int_0^L \int_0^{2\pi} \left(c D_{AB} \frac{\partial x_A}{\partial r}\Big|_{r=R}\right) R\, d\theta\, dz = k_{x1} (\pi D L)(x_{A0} - x_{A1})$$

$$k_1(t) = \frac{1}{\pi D L (x_{A0} - x_{A1})} \int_0^L \int_0^{2\pi} \left(c D_{AB} \frac{\partial x_A}{\partial r}\Big|_{r=R}\right) R\, d\theta\, dz$$

$$\breve{x}_A = \frac{(x_A - x_{A0})}{x_{A1} - x_{A0}}$$

$$Nu_1(t) = \frac{h_1 D}{k} = \frac{1}{2\pi L/D} \int_0^{L/D} \int_0^{2\pi} \left(-\frac{\partial \breve{T}}{\partial \breve{r}}\Big|_{\breve{r}=\frac{1}{2}}\right) d\theta\, d\breve{z}$$

$$Sh_1(t) = \frac{k_{x1} D}{c D_{AB}} = \frac{1}{2\pi L/D} \int_0^{L/D} \int_0^{2\pi} \left(-\frac{\partial \breve{x}_A}{\partial \breve{r}}\Big|_{\breve{r}=\frac{1}{2}}\right) d\theta\, d\breve{z}$$

For forced convection:

$$Nu_1 = G(Re, Pr, L/D) \qquad Sh_1 = G(Re, Sc, L/D)$$

G is the same function in both cases

For free convection:

$$Nu_m = H(Gr, Pr) \qquad Sh_m = H(Gr_x, Sc)$$

H is the same function in both cases

Forced convection around spheres:

$$Nu_m = 2 + 0.60\, Re^{1/2} Pr^{1/3} \qquad Sh_m = 2 + 0.60\, Re^{1/2} Sc^{1/3}$$

Forced convection along a flat plate:

$$j_{H,loc} = j_{D,loc} = \tfrac{1}{2} f_{loc} = 0.332\, Re_x^{-1/2}$$

$$j_{H,loc} = \frac{Nu_{loc}}{Re\, Pr^{1/3}} = \frac{h_{loc}}{\rho \hat{C}_p v_\infty} \left(\frac{\hat{C}_p \mu}{k}\right)^{2/3}$$

$$j_{D,loc} = \frac{Sh_{loc}}{Re\, Sc^{1/3}} = \frac{k_{x,loc}}{c\, v_\infty} \left(\frac{\mu}{\rho D_{AB}}\right)^{2/3}$$

Chilton-Colburn Analogy

$j_H = j_D = f(Re$, geometry, BC's)

to evaluate, use $x_{Af} = \frac{1}{2}(x_{A0} + x_{A\infty})$

$$T_f = \frac{1}{2}(T_0 + T_\infty)$$

ex - 22.3-1 Evaporation from a free falling drop

ex - 22.3-2 Wet and Dry Bulb Psychrometer

ex - 22.3-3 Mass Transfer in Creeping Flow Through Packed Beds

ex - 22.3-4 Mass Transfer to Drops and Bubbles

Definition of Transfer Coefficients in Two Phases p. 687

$$N_{A0}|_{gas} = N_{A0}|_{liquid} = N_{A0}$$

$$N_{A0} = k^{\circ}_{y,loc}(y_{Ab} - y_{A0}) = k^{\circ}_{x,loc}(x_{A0} - x_{Ab})$$

assume equilibrium across interface

- solubility data gives $y_{A0} = f(x_{A0})$

$$N_{A0} = K^{\circ}_{y,loc}(y_{Ab} - y_{Ae}) \qquad y_{Ae} \text{ in equil. with } x_{Ab}$$

$$N_{A0} = K^{\circ}_{x,loc}(x_{Ae} - x_{Ab}) \qquad x_{Ae} \text{ in equil. with } y_{Ab}$$

$K^{\circ}_{y,loc}$ = overall mass transfer coeffs based on gas phase

$$m_x = \frac{y_{Ab} - y_{A0}}{x_{Ae} - x_{A0}}, \quad m_y = \frac{y_{A0} - y_{Ae}}{x_{A0} - x_{Ab}}$$

$$\frac{k^{\circ}_{x,loc}}{K^{\circ}_{x,loc}} = 1 + \frac{k^{\circ}_{x,loc}}{m_x k^{\circ}_{y,loc}}, \quad \frac{k^{\circ}_{y,loc}}{K^{\circ}_{y,loc}} = 1 + \frac{m_y k^{\circ}_{y,loc}}{k^{\circ}_{x,loc}}$$

if equil. curve is linear: $m_y = m_x = m$

if $\dfrac{k^{\circ}_{x,loc}}{m k^{\circ}_{y,loc}} \ll 1$, mass transfer is liquid phase controlled

if $\dfrac{k^{\circ}_{x,loc}}{m k^{\circ}_{y,loc}} \gg 1$, mass transfer is gas phase controlled

if bulk conc. d/n change over mass transfer surface S:

$$(N_{A0})_m = K^{\circ}_{xm}(x_{Ae} - x_{Ab})$$

$$K^{\circ}_{xm} = \frac{1}{S}\int_S \frac{1}{(1/k^{\circ}_{x,loc}) + (1/m_x k^{\circ}_{y,loc})}\, dS$$

Mass Transfer and Chemical Reactions p. 694

ex - 22.5-1 Estimation of Interfacial Area in a Packed Column

absorption of CO into caustic solution

long times : $w_{A0} = A c_{A0} \sqrt{D_{AB} k_1'''}$

$$A = \frac{1}{c_{A0}\sqrt{D_{AB} k_1'''}} \quad \frac{d M_{A,tot}}{dt}$$

ex - 22.5-2 Estimation of Volumetric Mass Transfer Coefficients

$$k_c = \frac{D_{AB}}{\delta} \qquad \text{Stagnant film model}$$

$$\phi = \sqrt{\frac{k_1''' \delta^2}{D_{AB}}} \Rightarrow \sqrt{\frac{k_1''' D_{AB}}{k_c^2}}$$

$$\frac{V}{A\delta} \rightarrow \frac{V k_c}{A D_{AB}}$$ compare to penetration model

ex - 22.5-3 Model-Insensitive Correlations for Absorption with Rapid Reaction

Hatta number $= Ha = \dfrac{N_{A0}}{N_{A0}^{phys}}$ $\longrightarrow$ w/ rxn, $\longrightarrow$ w/o rxn

$$N_{A0}^{phys} = c_{A0}\sqrt{\frac{D_{A}s}{\pi t}}$$

Combined Heat and Mass Transfer by Free Convection p. 698

ex - 22.6-1 Additivity of Grashof Numbers

$$Nu_m = 0.518\ [0.73\ (Gr + Gr_\omega)]^{1/4}$$
$$Sh_m = 0.518\ [0.61\ (Gr + Gr_\omega)]^{1/4}$$

ex - 22.6-2 Free Convection Heat Transfer as a source of Forced-Convection Mass Transfer

$$\underset{\text{thermal}}{Gr} > \underset{\text{mass}}{Gr_\omega}, \quad Sc > Pr$$

thermal buoyant forced provide a momentum source

Effects of Interfacial Forces on Heat and Mass Transfer (Marangoni Effects) p. 699

Stresses in gas phase ignored

$$[(\vec{\delta} - \vec{n}\vec{n}) \cdot [\vec{n} \cdot \vec{\tau}]] = -\nabla^s \sigma$$

$\vec{n}$ is unit vector in z direction:

$$\tau_{zx} = -\frac{\partial \sigma}{\partial x}, \quad \tau_{zy} = -\frac{\partial \sigma}{\partial y}$$

- droplets and bubbles surrounded by liquid continuum
  - surfactants and microscopic particles that can eliminate Hadamard - Rybczinski circulation
  - gas absorbers and liquid extractors
- Sprays of droplets in a gaseous continuum
  - no effect
- supported liquid films in a gaseous or liquid continuum
- Foams of gas bubbles in a liquid continuum

ex - 22.7-1 Elimination of Circulation in a Rising Gas Bubble

surfactants reduce surface tension

$$\tau_{r\theta,s}|_{r=R} = \frac{1}{R}\frac{\partial \sigma}{\partial \theta} \text{ is induced}$$

$$= \underbrace{\frac{3}{2}\frac{\mu v_\infty}{R}\sin\theta}_{\text{rising solid sphere}} \text{ stops circulation}$$

Transfer Coefficients at High Net Mass Transfer Rates p. 703

- distort BL profiles and alter BL thicknesses
- ↑ friction factors
- ↑ h and k_c if mass transfer is toward the boundary
- trends are reversed in free convection and rotating surfaces

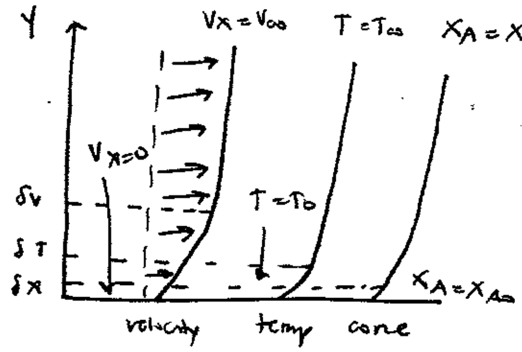
$$N_{A0} - x_{A0}(N_{A0} + N_{B0}) = k^{\bullet}_{x,loc}\,\Delta x_A$$

$$e_0 - (N_{A0}\tilde{C}_{pA,0} + N_{B0}\tilde{C}_{pB,0})(T_0 - T^{\circ}) = h^{\bullet}_{loc}\,\Delta T$$

$$k_{x,loc} = \lim_{N_{A0}+N_{B0}\to 0} k^{\bullet}_{x,loc}$$

$$h_{loc} = \lim \quad \text{"} \quad \text{"} \quad h^{\bullet}$$

Stagnant Film Model



$$\Theta = \frac{\phi}{R} = \frac{\phi}{e^{\phi}-1} = \frac{\ln(\;)}{R}$$

$$\Pi = \frac{e^{\phi m}-1}{e^{\phi}-1} = \frac{(1+R)^n}{R}$$

see Table 22.8-1 for definitions

Penetration Model

Gas absorption in a falling film (18.5)

unsteady-state vaporation (20.1)

$$\Pi_x = \frac{X_A - X_{A0}}{X_{A\infty} - X_A} = \frac{\text{erf}\,(m_x - \varphi) + \text{erf}\,\rho}{1 + \text{erf}\,\rho}$$

$$\Pi_T = \frac{T - T_0}{T_\infty - T_0} = \frac{\text{erf}\,(m_T - \rho) + \text{erf}\,\rho}{1 + \text{erf}\,\rho}$$

$$\rho = \frac{N_{A0} + N_{B0}}{c}\sqrt{\frac{t}{D_{AB}}}$$

Flat Plate Boundary Layer Model (see 20.2)

$$R = \frac{K\Lambda}{\Pi'(0,\Lambda,K)}, \quad \phi = \frac{K\Lambda}{\Pi'(0,\Lambda,0)}, \quad \Theta = \frac{\Pi'(0,\Lambda,K)}{\Pi'(0,\Lambda,0)}$$

Ch. 23

Macroscopic Mass Balances

p. 727

$$\frac{dm_{\alpha,tot}}{dt} = -\Delta w_\alpha + w_{\alpha,0} + r_{\alpha,tot} \qquad \alpha = 1, 2, 3 \ldots N$$

$$\frac{dm_{tot}}{dt} = -\Delta w + w_0 \qquad \left(\textstyle\sum_\alpha r_{\alpha,tot} = 0\right)$$

$$\frac{dM_{\alpha,tot}}{dt} = -\Delta W_\alpha + W_{\alpha,0} + R_{\alpha,tot} \qquad \alpha = 1, 2, 3 \ldots N$$

$$\frac{dM_{tot}}{dt} = -\Delta W + W_0 + \sum_{\alpha=1}^{N} R_{\alpha,tot}$$

Macroscopic Momentum and Angular Momentum Balances

p. 738

$$\frac{d\vec{P}_{tot}}{dt} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle} w + pS\right)\vec{u} + \vec{F}_{s\to f} + \vec{F}_0 + m_{tot}\vec{g}$$

$\vec{F}_0$ is the net influx of momentum by mass transfer

$$\frac{d\vec{L}}{dt} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle} w + pS\right)[\vec{r}\times\vec{u}] + \vec{T}_{s\to f} + \vec{T}_0 + \vec{T}_{ext}$$

$\vec{T}_0$ is the net influx of angular momentum by mass transfer

Macroscopic Energy Balance

p. 738

$$\frac{d}{dt}\left(U_{tot} + K_{tot} + \Phi_{tot}\right) = -\Delta\left[\left(\hat{U} + p\hat{V} + \frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi}\right)w\right] + Q_0 + Q + W_m$$

Q_0 is the addition of energy from mass transfer

Macroscopic Mechanical Energy Balance

p. 739

$$\frac{d}{dt}\left(K_{tot} + \Phi_{tot}\right) = -\Delta\left[\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \frac{p}{\rho}\right)w\right] + B_0 + W_m - E_c - E_v$$

B_0 is the addition from mass transport

See Table 23.5-1 for Macroscopic Balances

Steady-State Problems p. 739

ex - 23.5-1 Energy Balances for a Sulfur Dioxide Converter

use mass balance to find exit temp, use energy balance to find heat removal

ex - 23.5-2 Height of a Packed Tower Absorber

use overall macro. mass balance to find exit liquid comp. and relation of bulk comp. of the two phases

use diff. form of macro. mass balance to find interfacial conditions and tower height

Unsteady-State Problems p. 752

ex - 23.6-1 Start-Up of a Chemical Reactor

ex - 23.6-2 Unsteady Operation of a Packed Column

Ch. 24 Other Mechanisms of Mass Transport

Equation of Change for Entropy p. 765

$$\rho \frac{D\hat{S}}{Dt} = -(\nabla \cdot \vec{s}) + g_s$$

↳ rate of entropy prod per volume

$$\vec{s} = \frac{1}{T}\vec{q}^{(h)} + \sum_{\alpha=1}^{N} \frac{\bar{S}_\alpha}{M_\alpha} \vec{j}_\alpha$$

$$T g_s = -(\vec{q}^{(h)} \cdot \nabla \ln T) - \sum_{\alpha=1}^{N} (\vec{j}_\alpha \cdot \frac{cRT}{\rho_\alpha} \vec{d}_\alpha) - (\tau : \nabla \vec{v})$$

$$\vec{q}^{(h)} = \vec{q} - \sum_{\alpha=1}^{N} \frac{\bar{H}_\alpha}{M_\alpha} \vec{j}_\alpha \qquad - \sum_{\alpha=1}^{N} \frac{\bar{G}_\alpha}{M_\alpha} r_\alpha$$

Generalized Fick Equations

$$\vec{j}_\alpha = -D_\alpha^T \nabla \ln T + \rho_\alpha \sum_{\beta=1}^{N} \mathbb{D}_{\alpha\beta} \vec{d}_\beta \qquad \alpha = 1, 2 \ldots N$$

$$\mathbb{D}_{\alpha\beta} = -cRT\, a_{\alpha\beta} / \rho_\alpha \rho_\beta$$

Generalized Maxwell-Stefan equations

$$\vec{d}_\alpha = -\sum_{\beta \neq \alpha} \frac{x_\alpha x_\beta}{D_{\alpha\beta}} \left(\frac{D_\alpha^T}{\rho_\alpha} - \frac{D_\beta^T}{\rho_\beta}\right)(\nabla \ln T) - \sum_{\beta \neq \alpha} \frac{x_\alpha x_\beta}{D_{\alpha\beta}} \left(\frac{\vec{j}_\alpha}{\rho_\alpha} - \frac{\vec{j}_\beta}{\rho_\beta}\right.$$