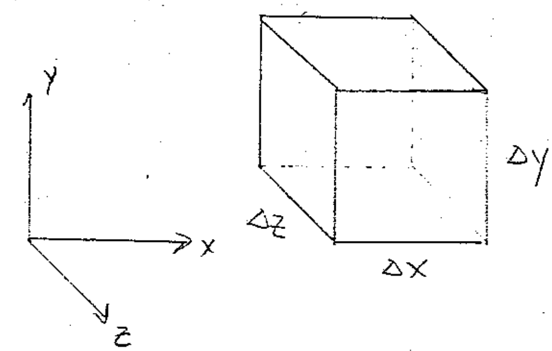
Coordinate Systems

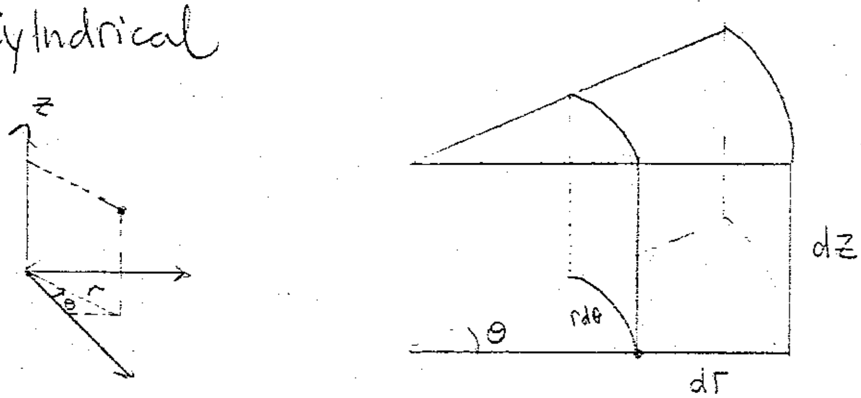
Cartesian Coordinates:



$V = xyz$

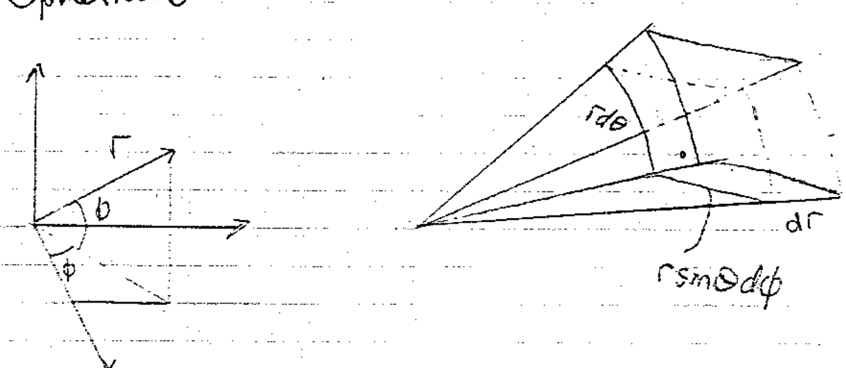
$\Delta V = \Delta x \Delta y \Delta z$

Cylindrical



$dV = r\,dr\,d\theta\,dz$

Spherical



$dV = r^2 \sin\theta \, dr\, d\theta\, d\phi$

Heat transfer

Energy Balance:

$\left\{ \begin{array}{l} \text{rate of accum. of} \\ \text{internal and kinetic} \\ \text{energy} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of internal} \\ \text{and kinetic energy} \\ \text{in by convection} \end{array} \right\} - \left\{ \begin{array}{l} \text{rate of internal} \\ \text{and kinetic energy} \\ \text{out by convection} \end{array} \right\}$

$+ \left\{ \begin{array}{l} \text{net rate of} \\ \text{heat addition} \\ \text{done by} \\ \text{conduction} \end{array} \right\} - \left\{ \begin{array}{l} \text{net rate of} \\ \text{work done by} \\ \text{system on surroundings} \end{array} \right\} + \left\{ \begin{array}{l} \text{net rate of} \\ \text{heat generation} \end{array} \right\}$

General form of the eqn. of energy:

$$\underbrace{\frac{\partial}{\partial t}\rho\left(\hat{U}+\tfrac{1}{2}v^2\right)}_{\text{rate of gain of energy per unit volume}} = \underbrace{-\left(\nabla\cdot\rho\vec{v}\left(\hat{U}+\tfrac{1}{2}v^2\right)\right)}_{\text{rate of energy input per unit volume by convection}} \underbrace{-\left(\nabla\cdot\vec{q}\right)}_{\text{rate of energy input per unit volume by conduction}} + \underbrace{\rho\left(\vec{v}\cdot\vec{g}\right)}_{\text{rate of work done on fluid per unit volume by gravitational forces}}$$

$$\underbrace{-\left(\nabla\cdot\vec{p}\,\vec{v}\right)}_{\text{rate of work done on fluid per unit volume by pressure forces}} \quad \underbrace{-\left(\nabla\cdot\left[\vec{\vec{\tau}}\cdot\vec{v}\right]\right)}_{\text{rate of work done on fluid per unit volume by viscous forces}} \quad + \underbrace{\dot{q}'''}_{\text{generation per unit volume}}$$

Rewriting

$$\underbrace{\rho\frac{D\hat{U}}{dt}}_{\text{includes convection}} = \underbrace{-(\nabla\cdot\vec{q})}_{\text{conduction}} - \underbrace{p(\nabla\cdot\vec{v})}_{\text{PV work}} - \underbrace{(\vec{\tau}:\nabla\vec{v})}_{\text{viscous dissipation}} + \underbrace{\dot{q}'''}_{\text{gen.}} \qquad \ell$$

Then, simplifying this form using the following relationship

$$d\hat{U} = \left(\frac{\partial\hat{U}}{\partial\hat{V}}\right)_T d\hat{V} + \left(\frac{\partial\hat{U}}{\partial T}\right)_{\hat{V}} dT = \left[-p + T\left(\frac{\partial p}{\partial T}\right)_{\hat{V}}\right] d\hat{V} + \hat{C}_V\, dT \quad + \dot{q}'''$$

we get

$$\rho \hat{C}_v \frac{DT}{Dt} = -(\nabla\cdot\vec{q}) - T\left(\frac{\partial P}{\partial T}\right)_{\hat{V}}(\nabla\cdot\vec{v}) - (\vec{\tau}:\nabla\vec{v}) + \dot{q}'''$$

or

$$\rho \hat{C}_v \frac{DT}{Dt} = k\nabla^2 T - T\left(\frac{\partial P}{\partial T}\right)_{\rho}(\nabla\cdot\vec{v}) + \mu\Phi_v + \dot{q}'''$$

where $$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right)^2$$

$$+ \left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 - \frac{2}{3}\underbrace{(\nabla\cdot\vec{v})^2}_{0 \text{ for an incompressible fluid}}$$

for an incompressible fluid

$C_v = C_p \quad \nabla\cdot\vec{v} = 0$

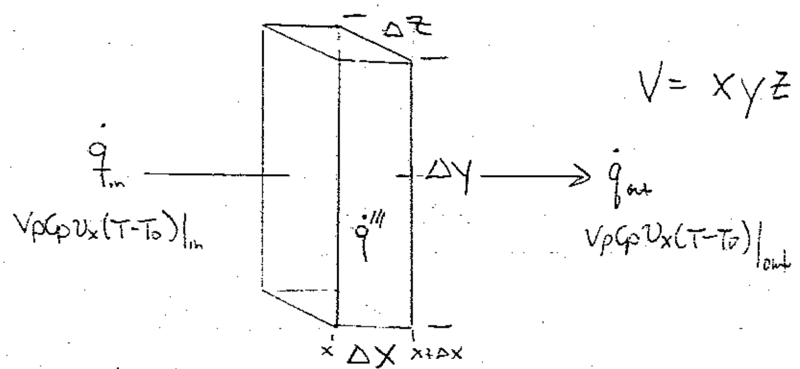
$$\rho C_p \frac{DT}{Dt} = k\nabla^2 T + \mu\Phi_v + \dot{q}'''$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}$$

$$\rho C_p\left[\underbrace{\frac{\partial T}{\partial t}}_{\text{accumulation}} + \underbrace{v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}}_{\text{convection}}\right] = \underbrace{k\nabla^2 T}_{\text{conduction}} + \underbrace{\mu\Phi_v}_{\text{viscous heating}} + \overbrace{\dot{q}'''}^{\text{generation}}$$

General Shell balance for Heat Transfer

Cartesian:



$V = xyz \qquad \Delta V = \Delta x \Delta y \Delta z$

consider x-direction only

In − out + gen = acc.

$$\underbrace{A q_x \Big|_x}_{\text{cond. in}} - \underbrace{A q_x \Big|_{x+\Delta x}}_{\text{cond. out}} + \underbrace{A \rho C_p v_x (T-T_0)\Big|_x}_{\text{conv. in}} - \underbrace{A \rho C_p v_x (T-T_0)\Big|_{x+\Delta x}}_{\text{conv. out}}$$

$$+ \dot{q}''' V = \frac{\partial}{\partial t}\left(\rho C_p V (T-T_0)\right)$$

$V \neq V(t) \quad T_0 \neq T_0(x,y,z,t) \quad v_x \neq v_x(x)$ ← from continuity

$$\left(q_x|_x - q_x|_{x+\Delta x}\right) yz + yz\,\rho C_p v_x \left((T-T_0)|_x - (T-T_0)|_{x+\Delta x}\right) + \dot{q}''' \Delta x y z$$

$$= \rho C_p \Delta x y z \frac{\partial (T-T_0)}{\partial t}$$

Divide by $\Delta x y z$
and take the limit

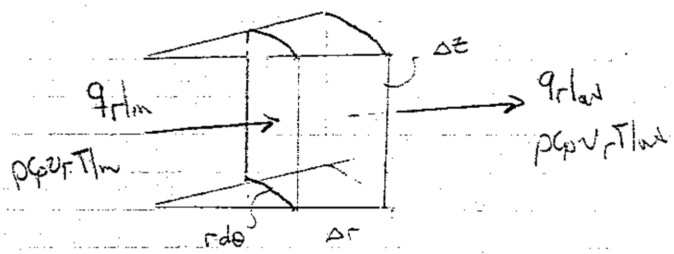
$$\lim_{\Delta x \to 0} \frac{\left(q_x|_x - q_x|_{x+\Delta x}\right)}{\Delta x} + \rho C_p v_x \frac{\left((T-T_0)|_x - (T-T_0)|_{x+\Delta x}\right)}{\Delta x} + \dot{q}''' = \rho C_p \frac{\partial T}{\partial t}$$

$$\rho c_p \frac{\partial T}{\partial t} = -\rho c_p v_x \frac{\partial T}{\partial x} - \frac{\partial q_x}{\partial x} + \dot{q}'''$$

$$q_x = -k \frac{\partial T}{\partial x}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} \right) = k \frac{\partial^2 T}{\partial x^2} + \dot{q}'''$$

Cylindrical :



$dA = r\,d\theta\,dz$

$\Delta A = r\,\Delta\theta\,\Delta z$

in − out + gen = acc.

$$A q_r|_r - A q_r|_{r+\Delta r} + A\rho c_p v_r T|_r - A\rho c_p v_r T|_{r+\Delta r} + \dot{q}''' V = \frac{\partial}{\partial t}(\rho c_p V T)$$

$$r\Delta\theta\Delta z\, q_r|_r - (r+\Delta r)\Delta\theta\,\Delta z\, q_r|_{r+\Delta r} + r\Delta\theta\Delta z\,\rho c_p v_r T|_r - (r+\Delta r)\Delta\theta\Delta z\,\rho c_p v_r T|_{r+\Delta r}$$

$$+ \dot{q}''' r\Delta\theta\Delta z\Delta r = \rho c_p \frac{\partial}{\partial t}(r\Delta\theta\Delta z\Delta r\, T)$$

Divide by ΔV and take limit

$$\lim_{\Delta r \to 0} \frac{(r q_r|_r - r q_r|_{r+\Delta r})}{r\,\Delta r} + \frac{\rho c_p}{r}\frac{(r v_r T|_r - r v_r T|_{r+\Delta r})}{\Delta r} + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

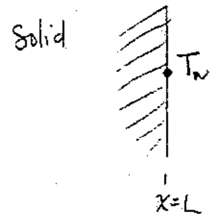
$$-\frac{1}{r}\frac{\partial\, r q_r}{\partial r} - \frac{\rho c_p}{r}\frac{\partial (r v_r T)}{\partial r} + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{\rho c_p T}{r}\underbrace{\frac{\partial r v_r}{\partial r}}_{0} - \rho c_p v_r \frac{\partial T}{\partial r} + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

cont.

Boundary Conditions for Heat Transfer

1^{st} Kind:



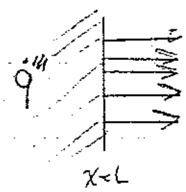
at $x=L \quad T = T_w = T_\infty$

$$-k\frac{\partial T}{\partial x}\Big|_{x=L} = h\left(T\big|_{x=L} - T_\infty\right) \qquad \text{for } h \to \infty \qquad T_w = T_\infty = T\big|_{x=L}$$

how can you maintain const. $T_{surface}$ (condensation of steam)

2^{nd} Kind:

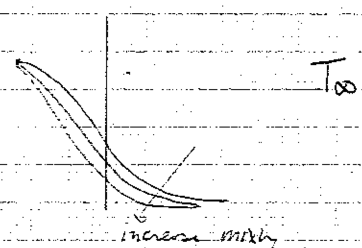
$q|_{x=L} = q_0$ ← const. known quantity



$$-k\frac{\partial T}{\partial x} = q_0$$

Electricity & Radiation can be used to maintain const. flux

3^{rd} Kind:



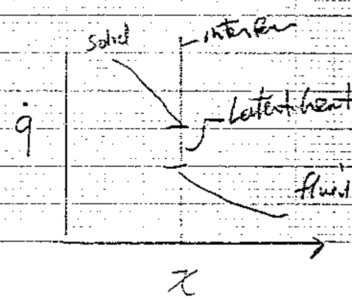
$$q|_{x=L} = -k\frac{\partial T}{\partial x} = h(T - T_\infty)$$

Heat Balance at interface:

in − out + gen = accum → 0 surface = 0 thickness so it has no volume (cannot accumulate energy)

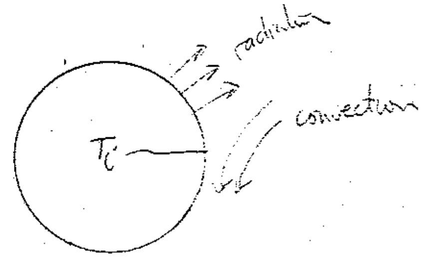
gen ⇒ Jump conditions (Crystalization, heat of mixing

in + gen = out



$$-k\frac{\partial T}{\partial x} + \underbrace{\rho \Delta H \frac{\partial X}{\partial t}}_{\text{latent heat}} = h(T - T_\infty)$$

Other Boundary Conditions:



first Law:

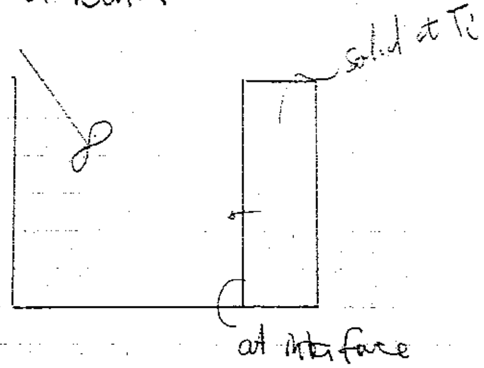
$$\frac{dU}{dt} = \delta\dot{Q} + \delta\dot{W} \qquad dU = \rho V c_p \frac{dT}{dt} = \dot{Q}$$

Heat can be lost by convection & radiation

$$Q_{conv} = Ah(T - T_\infty) \qquad Q_{rad} = A\varepsilon(T_s^4 - T_w^4)\sigma$$

$$\rho V c_p \frac{dT}{dt} = Ah(T - T_\infty) + A\varepsilon(T_s^4 - T_w^4)\sigma$$

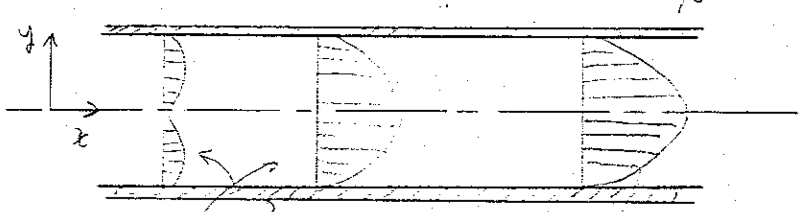
Fluid Bath



$$\rho_f c_{p_f} V_f \frac{dT_f}{dt} = -k\frac{\partial T}{\partial x}A$$ to include convection need additional B.C.

$$-k\frac{\partial T}{\partial x} = h(T - T_f(t))$$

$\boxed{\text{Viscous Heating}}$



$$v_x = \frac{(P_0 - P_L)}{2\mu L} B^2 \left[1 - \left(\frac{y}{B}\right)^2\right]$$

continuity $\frac{\partial v_x}{\partial x} = 0$

equation of Energy becomes:

$$\rho C_p v_x \frac{\partial T}{\partial x} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left(\frac{\partial v_x}{\partial y}\right)^2$$

look at Peclet #.

$P_e = \frac{vL}{\alpha} = \frac{\text{convective heat transfer}}{\text{conductive heat transfer}}$ if $P_e \gg 1$ then $\frac{\partial^2 T}{\partial x^2} \ll v_x \frac{\partial T}{\partial x}$

so, $\frac{\partial^2 T}{\partial x^2} \approx 0$

B.C.

at $y=0$ $\frac{\partial T}{\partial y} = 0$ at $y = B$ $T = T_0$ at $x = 0$ $T = T_0$

Non-linear ~ solve numerically

in the fully developed region the heat generated is equal to the heat transfered out of the walls and $\frac{\partial T}{\partial x} = 0$ T become invariant with position

The eqn. of Energy then becomes

$$0 = k\frac{\partial^2 T_\infty}{\partial y^2} + \frac{4\mu V_{x,max}^2}{B^4} y^2$$

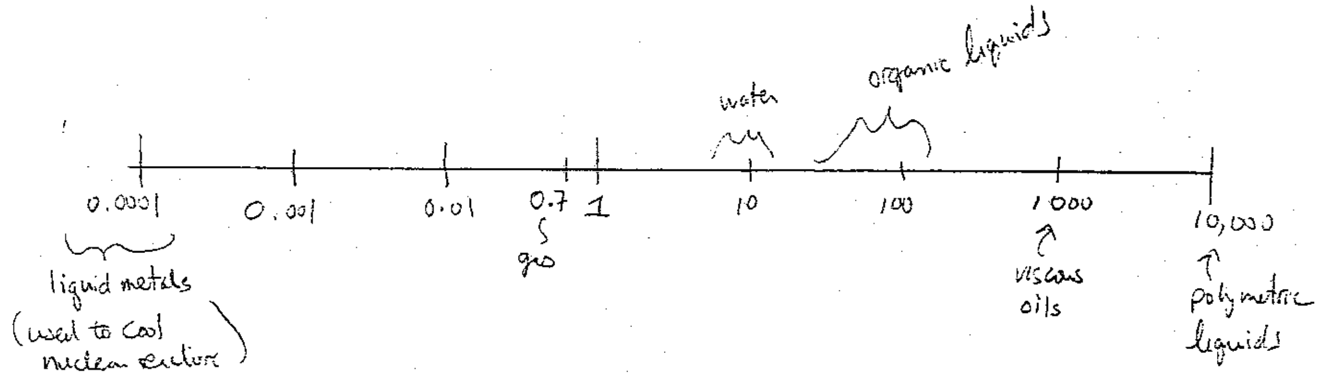
$$\text{Brinkman \#} = \frac{\text{heat generated due to viscous dissipation}}{\text{heat transport by Conduction}} = \frac{\mu \frac{\langle v_z \rangle^2}{L^2}}{k \frac{\Delta T}{L^2}}$$

$$= \frac{\mu \langle v_z \rangle^2}{k \Delta T}$$

Ignore viscous dissipation when $Br \ll 1$

Prandtl Number

$$P_r = \frac{\nu}{\alpha} = \frac{C_p \mu}{k} = \frac{\text{ability of fluid to transfer momentum}}{\text{ability of fluid to transfer heat}}$$



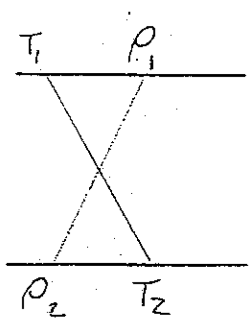
Na & Bi

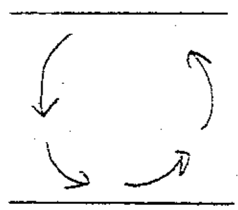
(can use magnetic pumps)

$\frac{L}{V}$

Free Convection

free convection occurs because of density variations, which are due to temperature gradients

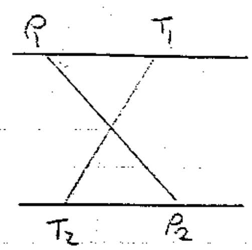


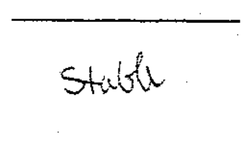


unstable circulation

$T_2 > T_1 \quad \rho_2 < \rho_1$

$\frac{\partial T}{\partial x} > 0 \quad \frac{\partial \rho}{\partial x} < 0$





$T_1 > T_2 \quad \rho_1 < \rho_2$

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$$\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

$$= \frac{1}{T} \text{ for an ideal gas}$$

Use correlations to determine heat transfer coefficient forced vs. free convection

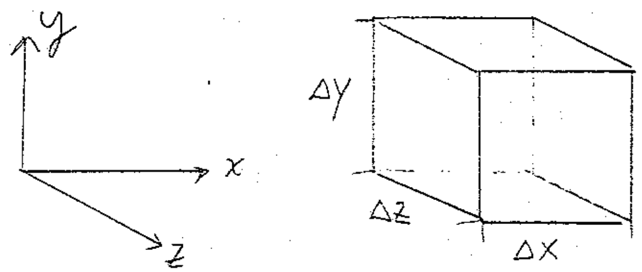
$\frac{Gr}{Re^2} \ll 1$ free convection may be neglected

$\frac{Gr}{Re^2} \approx 1$ need to consider both free & forced convection

$\frac{Gr}{Re^2} \gg 1$ forced convection may be neglected

Mass Transfer

Eqn. of Continuity for a binary mixture of A and B



n_A = total flux of a due to convective and diffusive flow $= \left[\frac{\text{mass}}{cm^2\ sec}\right]$

Mass Balance eqn. for A is:

rate of mass of A in − rate of mass of A out + rate of generation of mass of A by homogeneous chemical rxn = rate of accumulation of mass of A

$$n_{Ax}|_x \Delta Y \Delta Z - n_{Ax}|_{x+\Delta x} \Delta Y \Delta Z + n_{Ay}|_y \Delta Z \Delta X - n_{Ay}|_{y+\Delta y} \Delta Z \Delta X$$

$$+ n_{Az}|_z \Delta X \Delta Y - n_{Az}|_{z+\Delta z} \Delta X \Delta Y + \underset{\sim}{r_A}^{\frac{gm}{sec\ cm^3}} \Delta X \Delta Y \Delta Z = \frac{\partial \rho_A}{\partial t} \Delta X \Delta Y \Delta Z$$

Divide by $\Delta X \Delta Y \Delta Z$ and Take the limit

$$\lim_{\Delta X, \Delta Y, \& \Delta Z \to 0} \frac{n_{Ax}|_x - n_{Ax}|_{x+\Delta x}}{\Delta X} + \frac{n_{Ay}|_y - n_{Ay}|_{y+\Delta y}}{\Delta Y} + \frac{n_{Az}|_z - n_{Az}|_{z+\Delta z}}{\Delta Z} + r_A = \frac{\partial \rho_A}{\partial t}$$

$$\frac{\partial \rho_A}{\partial t} + \frac{\partial n_{Ax}}{\partial x} + \frac{\partial n_{Ay}}{\partial y} + \frac{\partial n_{Az}}{\partial z} = r_A \quad (1)$$

analogously for component B

$$\frac{\partial \rho_B}{\partial t} + \frac{\partial n_{Bx}}{\partial x} + \frac{\partial n_{By}}{\partial y} + \frac{\partial n_{Bz}}{\partial z} = r_B \quad (2)$$

combining (1) & (2)

$$\frac{\partial(\rho_A+\rho_B)}{\partial t} + \frac{\partial(n_{Ax}+n_{Bx})}{\partial x} + \frac{\partial}{\partial y}(n_{Ay}+n_{By}) + \frac{\partial}{\partial z}(n_{Az}+n_{Bz}) = r_A + r_B$$

$\rho_A+\rho_B=\rho \qquad n_{Ax}+n_{Bx} = v_x\rho$ (mass average velocity) $\qquad v_x = \dfrac{\rho_A v_{Ax}+\rho_B v_{Bx}}{\rho}$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\vec{n}_A+\vec{n}_B) = r_A + r_B$$

$$\rho\vec{v} = \vec{n}_A + \vec{n}_B = \rho_A\vec{v}_A + \rho_B\vec{v}_B$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = r_A + r_B$$

$r_A + r_B = 0$ conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$ ← continuity eqn.

for an incompressible fluid $\rho = $ const.

$\nabla\cdot\vec{v} = 0$

for species A & B

$$\frac{\partial \rho_A}{\partial t} + \nabla\cdot\vec{n}_A = r_A \qquad \vec{n}_A = \vec{j}_A + w_A(\vec{n}_A+\vec{n}_B)$$
$$= -\rho \mathcal{D}_{AB}\nabla w_A + \rho_A\vec{v}$$

for const. ρ $\nabla\cdot\vec{v}=0$ and const. $\mathcal{D}_{AB}$

$$\frac{\partial \rho_A}{\partial t} + \vec{v}\cdot\nabla\rho_A = \mathcal{D}_{AB}\nabla^2\rho_A + r_A$$

$$\frac{\partial \rho_B}{\partial t} + \vec{v}\cdot\nabla\rho_B = \mathcal{D}_{AB}\nabla^2\rho_B + r_B$$

Molar form :

$$\frac{\partial C_A}{\partial t} + \nabla \cdot \vec{N}_A = R_A \qquad \frac{\partial C_B}{\partial t} + \nabla \cdot \vec{N}_B = R_B$$

$$\vec{N}_A + \vec{N}_B = C\vec{V}^* \qquad \vec{N}_A = C_A\vec{V_A} \qquad \vec{N}_B = C_B\vec{V_B}$$

$$\vec{N}_A = \vec{J}_A + x_A(\vec{N}_A + \vec{N}_B)$$

$$= -\mathcal{D}_{AB}\nabla C_A + x_A(\vec{N}_A + \vec{N}_B)$$

$$\vec{N}_B = \vec{J}_B + x_B(\vec{N}_A + \vec{N}_B)$$

$$= -\mathcal{D}_{AB}\nabla C_B + x_B(\vec{N}_A + \vec{N}_B)$$

$$\frac{\partial C}{\partial t} + \nabla \cdot (C\vec{V}^*) = R_A + R_B$$ ← can use to determine $\vec{V}^*$ if const. C and $R_A + R_B = 0$ then $\nabla \vec{V}^* = 0$

for const. C and $\mathcal{D}_{AB}$

$$\frac{\partial C_A}{\partial t} + \vec{V}^* \nabla C_A = \mathcal{D}_{AB}\nabla^2 C_A + R_A - x_A(R_A + R_B)$$

$$\vec{V}^* = x_A\vec{V_A} + x_B\vec{V_B}$$

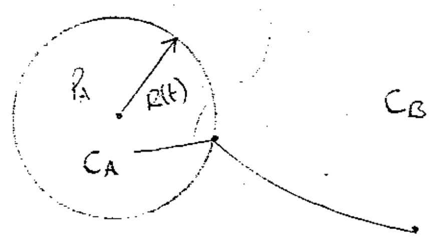
Fick's Law

$$\frac{\partial C_A}{\partial t} = \mathcal{D}_{AB}\nabla^2 C_A$$ ←
1) dilute systems $x_A \approx 0$
2) Stationary liquids $\vec{v} = 0$
3) Equimolar counter diffusion in gases
4) Solids

Application of Fick's 2nd Law

Dissolution of a solid sphere or Evaporation of a bubble

assume concentration of A in sphere = const. (C_B = low vapor pressure, well mixed)



The diffusion eqn. will apply outside the bubble

Fick's Second Law Molar forms:

Overall $\qquad \dfrac{\partial C}{\partial t} + \nabla\cdot(C\vec{v}^*) = \cancel{R_A+R_B}^{\,0} \qquad C = \text{const.}$

$$\nabla\cdot\vec{v}^* = 0 \qquad \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial\theta}(v_\theta \sin\theta)}^{\,0} + \frac{1}{r\sin\theta}\cancel{\frac{\partial v_\phi}{\partial\phi}}^{\,0} = 0$$

Component A

$$\frac{\partial C_A}{\partial t} + \nabla N_A = \cancel{R_A}^{\,0}$$

$$N_A = -\mathcal{D}_{AB}\frac{\partial C_A}{\partial r} + \cancel{x_A}^{\,\text{dilute}}(N_A + N_B)$$

$$\frac{\partial C_A}{\partial t} = \mathcal{D}_{AB}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right)$$

Boundary Conditions: $\quad$ from Henry's Law $P_{Ai} = \dfrac{C_{Ai}}{H}$

at $r = R(t) \quad C_A = C_{Ai} \quad t > 0$

at $r \to \infty \quad C_A \to 0 \quad t > 0$

at $t = 0 \quad C_A = 0 \quad \forall\, r > R(t)$

Solve using similarity transforms with the following Boltzmann factor

$$\eta = \frac{r - R(t)}{\sqrt{4 \mathcal{D}_{AB} t}}$$

to find the flux at the surface

$$N_A\Big|_{r=R(t)} = -\mathcal{D}_{AB} \frac{\partial C_A}{\partial r}\Big|_{r=R(t)}$$

find that from $\left[C_A(r,t) \text{ (solve assuming } R(t) \approx \text{const)} \right]$

then consider a mass balance at the interface

Flux at interface x Area = Change in Moles with time

$$N_A\Big|_{r=R(t)} A = \frac{d \overset{\text{const.}}{\widetilde{C_A}} V}{dt}$$

$$A = 4\pi R(t)^2$$

$$V = \frac{4}{3}\pi R(t)^3$$

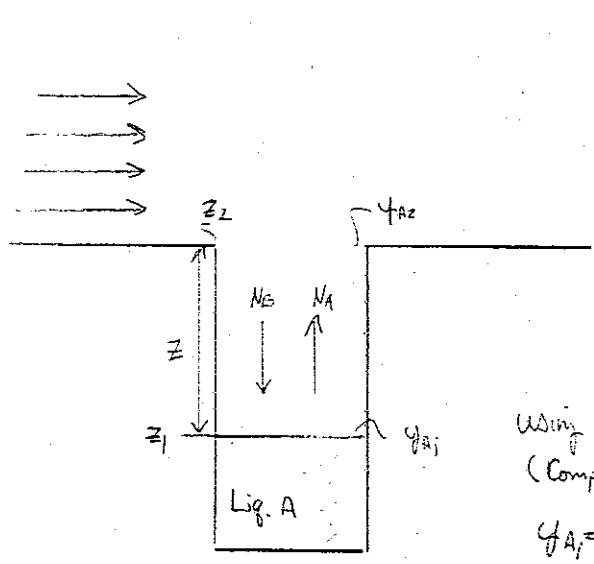
$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d(R(t)^3)}{dt}$$

$$N_A\Big|_{r=R(t)} 4\pi R(t)^2 = \frac{4}{3}\pi C_A \frac{d R(t)^3}{dt}$$

$$N_A\Big|_{r=R(t)} \cancel{4\pi} \cancel{R(t)^2} dt = \cancel{\frac{4}{3}}\pi C_A \cancel{3} \cancel{R(t)^2} dR(t)$$

$$\frac{N_A\big|_{r=R(t)}}{C_A} = \frac{dR(t)}{dt}$$

Stephan Tube:



using Raoult's Law & Assuming $x_A \approx 1$
(Component B is not soluble in A)

$$y_{A_i} = \frac{P_A^{sat}}{P_{tot}}$$

Pseudo-Steady State approximation $Z(t) \approx$ const.

(This assumption is good for chemicals that will not evaporate quickly)
need to consider P^{sat} & T

Mass Balance:

$$\cancel{\frac{\partial C_A}{\partial t}} + \nabla \vec{N_A} = \cancel{R_A}^{\,0} \qquad \cancel{\frac{\partial C_B}{\partial t}} + \nabla \vec{N_B} = \cancel{R_B}^{\,0}$$

Assume: Diffusion in z-direction only

$$\frac{dN_A}{dz} = 0 \quad \text{and} \quad \frac{dN_B}{dz} = 0$$

molar flux of Both species is not a function of position (const.)

if B is insoluble in A then the flux of B at the gas-Liquid inter-face will be zero, and since $N_B \neq N_B(z)$ then the flux must be zero everywhere

$$N_A = -\mathcal{D}_{AB}\frac{dC_A}{dz} + y_A(N_A + \cancel{N_B}^{\,0})$$

$$= -C\mathcal{D}_{AB}\frac{dy_A}{dz} + y_A N_A$$

$$N_A(1-y_A) = -C\mathcal{D}_{AB}\frac{dy_A}{dz}$$

$$N_A = -\frac{C\mathcal{D}_{AB}}{(1-y_A)}\frac{dy_A}{dz}$$

from mass balance:

$$\frac{dN_A}{dz} = 0 \qquad N_A = \text{const.} = A_1$$

Boundary Conditions:

at $z = z_1$ $y_A = y_{A_1} = \frac{P^{sat}}{P_T}$

at $z = z_2$ $y_A = y_{A_2}$

$$\frac{dN_A}{dz} = -\frac{d}{dz}\left(\frac{C\mathcal{D}_{AB}}{1-y_A}\right)\frac{dy_A}{dz} = 0$$

If C and $\mathcal{D}_{AB}$ = const

$$\frac{d}{dz}\left(\frac{1}{1-y_A}\frac{dy_A}{dz}\right) = 0$$

$$\frac{1}{1-y_A}\frac{dy_A}{dz} = C_1$$

$$-\ln(1-y_A) = C_1 z + C_2$$

B.C. #1 at $z = z_1$ $y_A = y_{A1}$

$$-\ln(1-y_{A1}) = C_1 z_1 + C_2$$

$$C_2 = -C_1 z_1 - \ln(1-y_{A1})$$

B.C. #2 at $z = z_2$ $y_A = y_{A2}$

$$-\ln(1-y_{A2}) = C_1 z_2 - C_1 z_1 - \ln(1-y_{A1})$$

$$\ln(1-y_{A1}) - \ln(1-y_{A2}) = C_1(z_2 - z_1)$$

$$C_1 = \frac{1}{(z_2-z_1)} \ln\left(\frac{1-y_{A1}}{1-y_{A2}}\right)$$

$$-\ln(1-y_A) = \ln\left(\frac{1-y_{A1}}{1-y_{A2}}\right)\frac{z}{z_2-z_1} - \frac{z_1}{z_2-z_1}\ln\left(\frac{1-y_{A1}}{1-y_{A2}}\right) - \ln(1-y_{A1})$$

$$\ln\left(\frac{1-y_{A1}}{1-y_A}\right) = \left(\frac{z-z_1}{z_2-z_1}\right)\ln\left(\frac{1-y_{A1}}{1-y_{A2}}\right)$$

$$\frac{1-y_A}{1-y_{A1}} = \left(\frac{1-y_{A2}}{1-y_{A1}}\right)^{\left(\frac{z-z_1}{z_2-z_1}\right)}$$

$$y_A = 1 - (1-y_{A1})\left(\frac{1-y_{A2}}{1-y_{A1}}\right)^{\left(\frac{z-z_1}{z_2-z_1}\right)}$$

flux is most important

$$N_A = \frac{-C\mathcal{D}_{AB}}{(1-y_A)}\frac{dy_A}{dz}$$

$$= C\mathcal{D}_{AB}\frac{d}{dz}\ln(1-y_A)$$

$$\ln(1-y_A) = \frac{z-z_1}{(z_2-z_1)}\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right) + \ln(1-y_{A1})$$

$$\frac{d\ln(1-y_A)}{dz} = \frac{1}{(z_2-z_1)}\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right)$$

$$N_A = \frac{C\mathcal{D}_{AB}}{z_2-z_1}\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right)$$

radius of the tube

$N_A = \frac{\text{moles}}{\text{cm}^2\text{ sec}}$ $\qquad \frac{\text{moles}}{\text{sec}} = N_A A = N_A \pi R^2$

if the density is known the flux and Δz can be found from measuring the mass of the tube with time. Need to also measure y_{A2} (if pure g_B is passing by you could assume $y_{A2} = 0$)

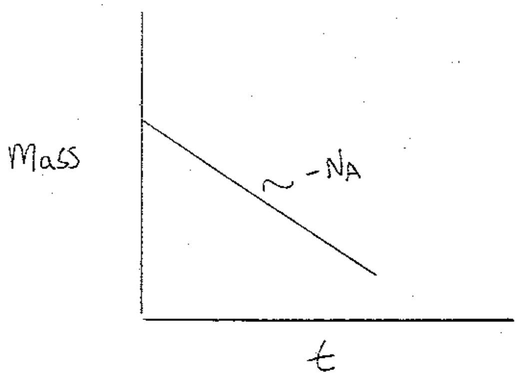
$\frac{\text{Mass}}{\text{sec.}} = N_A \pi R^2 M_W$ $\qquad \rho V = \text{mass} \qquad V = \pi R^2 \Delta z$

$\Delta\text{mass} = \rho\pi R^2 \Delta z \qquad \Delta z = \frac{\Delta\text{mass}}{\rho\pi R^2}$

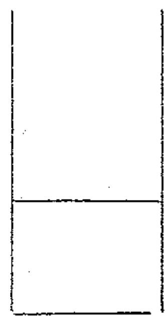
mass of fluid

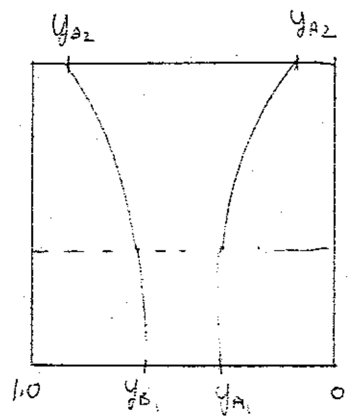
$$N_A = -\frac{dM}{dt}\frac{1}{\pi R^2 M_W}$$

Plot:



measuring ΔZ gives $\mathcal{D}_{AB}$





$\frac{dy_B}{dz} \neq 0$ so why is the flux zero ??

$$N_A = \underbrace{-c\mathcal{D}_{AB}\frac{dy_A}{dz}}_{\text{Diffusion}} + y_A(N_A + N_B)$$

$$+ \; C_A \overbrace{V_z^*}^{\text{convection due to bulk diffusion}}$$

$V_z^* \uparrow$ ~ is in positive z-direction

$N_A =$ positive + positive

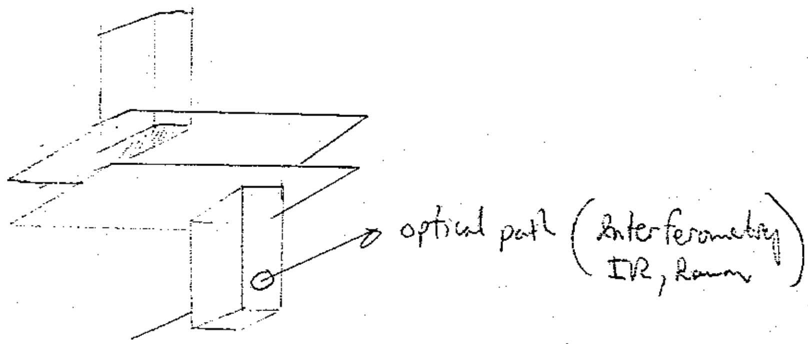
$$N_B = \underbrace{-c\mathcal{D}_{AB}\frac{dy_B}{dz}}_{\text{diff.}} + y_B(N_A + N_B)$$

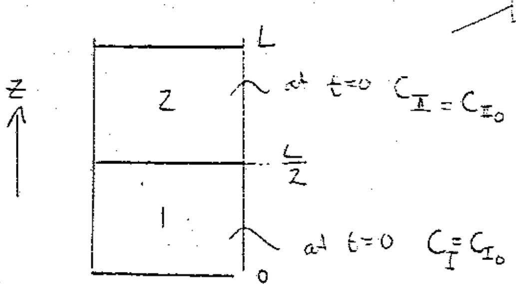
$$+ \; C_B \overbrace{V^*}^{\text{conv.}}$$

$N_B =$ Negative + positive $\Leftarrow$ the diffusion term is exactly equal to the convection term for species B, which confirms the original assumption that

$N_B = 0$

How else can you measure diffusion?



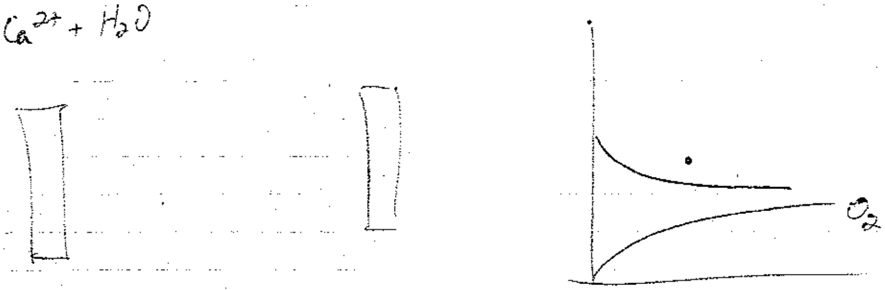


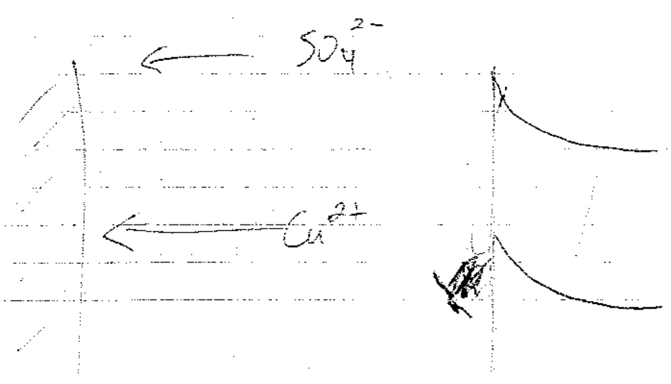
$$\frac{\partial C_A}{\partial t} = \mathcal{D} \frac{\partial^2 C_A}{\partial z^2}$$

at $z = L$ $\quad \frac{\partial C_A}{\partial z} = 0$

at $z = 0$ $\quad \frac{\partial C_A}{\partial z} = 0$

$O_2 + Ca^{2+} + H_2O$





Dimensionless Numbers

$$Re = \frac{\rho V^2/D}{\mu V/D^2} = \frac{\rho V D}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$ Reynolds #

$$Fr = \frac{\rho V^2/D}{\rho g} = \frac{V^2}{gD} = \frac{\text{inertial forces}}{\text{gravity forces}}$$ Froude #

$$Bo = \frac{g(\rho_\ell - \rho_v)L^2}{\sigma} = \frac{\text{gravitational forces}}{\text{surface tension forces}}$$ Bond #

$$f = \frac{\Delta P}{(L/D)(\rho v^2/2)}$$ = Dimensionless pressure drop for internal flow — Friction factor

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{\text{Buoyancy forces}}{\text{viscous forces}}$$ Grashof #

$$We = \frac{\rho V^2 L}{\sigma} = \frac{\text{inertial forces}}{\text{surface tension forces}}$$ Weber #

$$Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{k} = \frac{\text{ability of fluid to transfer momentum}}{\text{ability of fluid to transfer heat}}$$ Prandtl #

$$Sc = \frac{\nu}{\mathcal{D}_{AB}} = \frac{\text{ability of fluid to transfer momentum}}{\text{ability of fluid to transfer mass}}$$ Schmidt #

$$Le = \frac{\alpha}{\mathcal{D}_{AB}} = \frac{\text{ability of fluid to transfer heat}}{\text{ability of fluid to transfer mass}}$$ Lewis #

D.

$$Bi_h = \frac{hL}{k_s} = \frac{\text{internal resistance to heat x-fer}}{\text{Boundary Layer resistance to heat x-fer}} = \text{Biot \# (heat)}$$

$$Bi_m = \frac{kL}{\mathcal{D}_{AB}} = \frac{\text{internal mass x-fer resistance}}{\text{Boundary Layer mass x-fer resistance}} = \text{Biot \# (mass)}$$

$$Nu = \frac{hL}{k_f} = \text{dimensionless temp. gradient at the surface} \qquad \text{Nusselt \#}$$

$$Sh = \frac{kL}{\mathcal{D}_{AB}} = \text{dimensionless conc. gradient at the surface} \qquad \text{Sherwood \#}$$

$$Pe_h = \frac{VL}{\alpha} = Re\,Pr = \frac{\text{heat x-fer by convection}}{\text{heat x-fer by conduction}} \qquad \text{Peclet \# (heat)}$$

$$Pe_m = \frac{VL}{\mathcal{D}_{AB}} = Re\,Sc = \frac{\text{mass x-fer by convection}}{\text{mass x-fer by diffusion}} \qquad \text{Peclet \# (mass)}$$

$$St = \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr} = \text{Modified Nusselt \#} \qquad \text{Stanton \#}$$

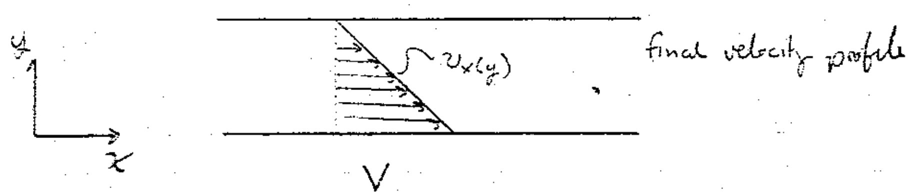
$$Br = \frac{\mu \langle v \rangle^2}{k \Delta T} = \frac{\text{heat generated due to viscous dissipation}}{\text{heat transport by conduction}} \qquad \text{Br \#}$$

$$Ra = Gr\,Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \text{Rayleigh \#}$$

Fluid Properties

viscosity:

consider 2 plates

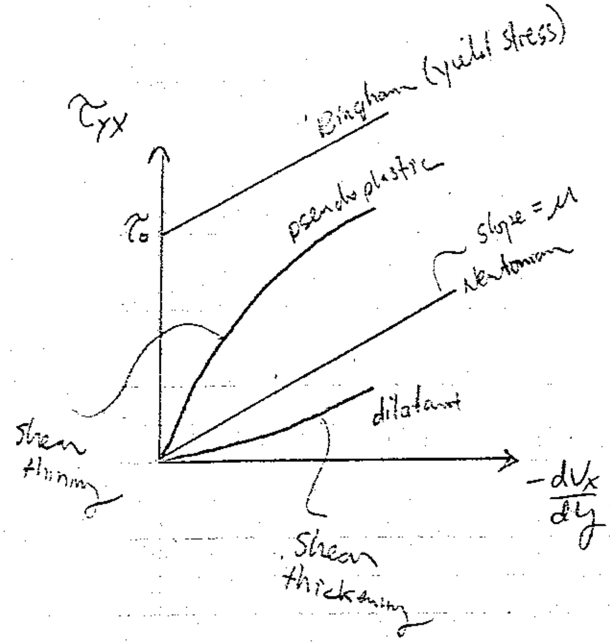


The viscosity is defined as the proportionality constant between the Force to maintain constant velocity of the lower plate per unit area and the velocity decrease in the y-direction.

$$\frac{F}{A} = \mu \frac{V}{Y} = \mu \frac{\Delta V}{\Delta Y} = \mu \frac{dv_x}{dy}$$

$$\tau_{yx} = -\mu \frac{dv_x}{dy} \sim \text{Newton's Law of viscosity}$$

momentum is transfered from high to low velocity and occurs as a layer of fluid with a certain velocity transmits a portion of its momentum to the adjacent layer. The momentum is transfered in the direction of decreasing velocity. The ability of a fluid to transfer momentum is essentially its viscosity.

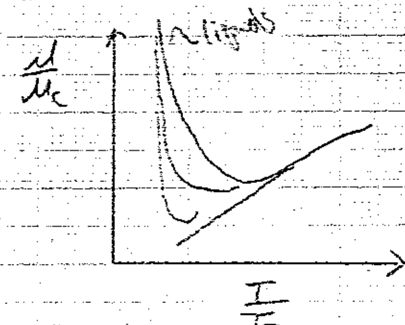


Liquids	Gases (at low p)
$P\uparrow$ $\mu \rightarrow$ (incompressible)	$\mu\uparrow$ $T\uparrow$
$T\uparrow$ $\mu\downarrow$	$\mu\uparrow$ $P\uparrow$

	μ
air	0.02 cp
H_2O	1.0 cp
Glycerol	1000 cp

cp = Pa·s

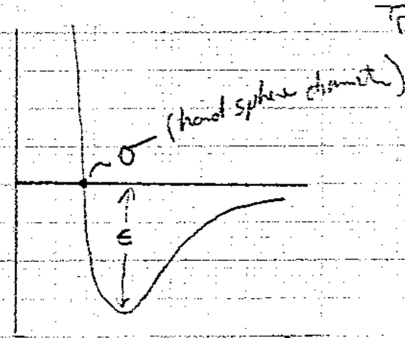
For liquids μ can be determined from reduced plots (corresponding states)



For a monatomic gas:

$$\mu \propto \frac{\sqrt{MT}}{\sigma^2 \Omega_\mu}$$

$$\Omega = \Omega\left(\frac{kT}{\epsilon}\right)$$



Thermal conductivity:

Fourier's Law of Heat Conduction

$$q_y = -k \frac{dT}{dy}$$

for a monatomic Gas:

$$k \propto \frac{\sqrt{T/M}}{\sigma^2 \Omega_k}$$

In general:

Gases	Liquids	Solids
$k\uparrow$ $T\uparrow$	$k\downarrow$ $T\uparrow$	$k\downarrow$ $T\uparrow$
$k\uparrow$ $P\uparrow$		

Diffusivity:

Fick's Law of Diffusion

$$J_{Ay} = -\mathcal{D}_{AB} \frac{dC_A}{dy}$$

Gases

at low ρ the diffusivity is almost composition independent

$\mathcal{D}_{AB}\uparrow$ $T\uparrow$

$\mathcal{D}_{AB}\downarrow$ $P\uparrow$

Liquids

Strong function of concentration

$\mathcal{D}_{AB}\uparrow$ $T\uparrow$

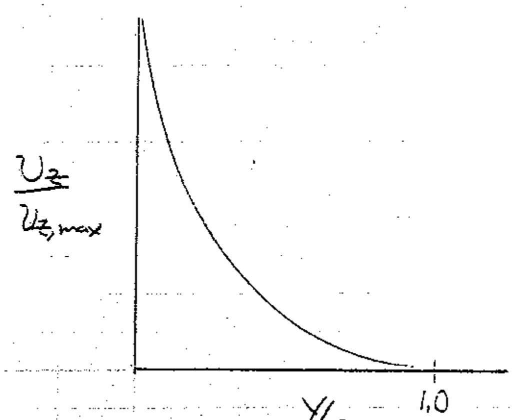
Gases $$\mathcal{D}_{AB} \propto \frac{\sqrt{T^3\left(\frac{1}{M_A}+\frac{1}{M_B}\right)}}{P\,\sigma_{AB}^2\,\Omega_{\mathcal{D},AB}}$$

Liquids $$\mathcal{D}_{AB} = \frac{kT}{\mu_B\, 6\pi R_A}$$ ← stoke-Einstein eqn.

α α

General Transport

Universal Velocity Profile:



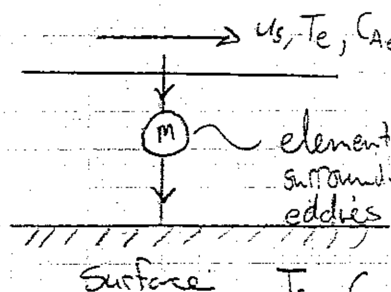
Cross section can be divided into three regions:

(1) turbulent core ~ contribution of eddy transport is much greater than that of molecular transport

(2) buffer layer two mechanisms are comparable in magnitude

(3) laminar sub-layer turbulent edcies have died out so only molecular transport need be considered.

Reynolds Analogy



element of mass, which doesn't mix with surrounding fluid is transfered by turbulent eddies, and an equivalent mass is transfered in the opposite direction

assumptions

1) element instantly reaches equil. with interfacial layers

2) existence of buffer & laminar sublayer is neglected

as the element enters the surface region the heat transfered will be

$$-q_0 = \frac{M C_p (T_e - T_s)}{A t}$$

The rate of change of momentum will be

$$\frac{M u_s}{A t} = -R_0$$

for mass transfer:

$$-N_A|_{y=0} = \frac{M}{\rho}(C_{Ae} - C_{Aw})\frac{1}{At}$$

$$\frac{-q_0}{-R_0} = \frac{C_p(T_e - T_w)}{u_s} \qquad \frac{-R_0}{u_s} = \frac{-q_0}{C_p(T_w - T_s)}$$

and

$$\frac{-N_A|_{y=0}}{-R_0} = \frac{C_{Ae} - C_{Aw}}{\rho\, u_s} \qquad \frac{-R_0}{\rho u_s} = \frac{-N_A|_{y=0}}{C_{Ae} - C_{Aw}}$$

$$\frac{-q_0}{(T_w - T_s)} = h \qquad \frac{-N_A|_{y=0}}{(C_{Ae} - C_{Aw})} = k$$

$$\frac{R}{\rho u_s^2} = \frac{h}{C_p \rho u_s} = St$$ ~ heat transfer

$$\frac{R}{\rho u_s^2} = \frac{k}{u_s} = St$$ ~ mass transfer

thus $St_h = St_m$

Lewis Relation

$$\frac{h}{C_p \rho u_s} = \frac{k}{u_s} \quad \text{or} \quad \frac{h}{C_p \rho} = k$$ ← suggests a direct relation between heat & mass x-fer

good if Pr & Sc ≈ 1

Chilton - Colburn Analogy:

by defining a j-factor one can relate mass and heat transfer

$$j_H = j_D = \left\{ \begin{array}{c} \text{function of Re,} \\ \text{geometry, and} \\ \text{boundary conditions} \end{array} \right\}$$ ← empirical analogy

$$j_H = \frac{Nu}{Re\, Pr^{1/3}} \qquad j_D = \frac{Sh}{Re\, Sc^{1/3}}$$

from this h and k are not independent, so by knowing one you can find the other.