DIMENSIONLESS GROUPS

Nu	hL/k_f
Bi	hL/k_s
Fo	$\alpha t/L^2$
Bo	vL/D
Sh	kL/D
Br	$\mu v^2/k\Delta T$
Da	τk, $\frac{(-r_A)V}{F_{A0}}$
Eu	$\frac{p}{\rho v^2}$
Sc	$\frac{\nu}{D}$
Ma	$\frac{v}{c}$
jD	$Sh\, Re^{-1}\, Sc^{-1/3}$
jH	$Nu\, Re^{-1}\, Pr^{-1/3}$
Le	$\frac{D}{\alpha}$
Gz	$\frac{\pi}{4}\, Re\, Pr\, \frac{D}{L}$

De	λ / T
– St (okes)	$\rho s D^2 / \mu v$
Re	$\rho v D / \mu$
Pr	ν / α
Ra	Gr Pr
Gr	$\dfrac{L^3 \rho^2 g \beta \Delta T}{\mu^2}$
C_D	$\dfrac{F_D}{\frac{1}{2} \rho v^2 A_c}$
f	$\dfrac{F_K}{A \cdot \frac{1}{2} \rho v^2}$
– Fr	$v^2 / g D$ — Fronde
– St (anton)	$h / \rho v C_p$
– We	$\rho v^2 D / \sigma$
Pe	$D v / \alpha$
ϕ	$\dfrac{V}{(SA)_{ext}} \sqrt{\dfrac{K}{D_{AB}}}$
– Kn	$\dfrac{\lambda}{L}$

Dimensionless Groups

Nusselt
Biot
Fourier
Bodenstein

Nusselt

$$= \frac{1/K_f}{1/h} = \frac{\text{resistence to conduction}}{\text{resistence to convection}} \text{ (of the fluid)}$$

$$Nu = \frac{hL}{K_f} = \frac{\text{convective heat transport}}{\text{diffusive heat transport - conduction}}$$

K_f → fluid

- used in correlations for determing h

$Nu = Nu(Re, Pr, Br, L/D)$

is viscous dissipation is small:

$Nu = Nu(Re, Pr, L/D)$

sphere: $Nu = 2 + 0.6\, Re^{1/2} Pr^{1/3}$

$= 2$ if stationary fluid

$Nu < 1$ → cond. dominates — resistence to convection limiting

$Nu > 1$ resistence to conduction limiting → Conv. dominates

Biot

$$Bi = \frac{hL}{K_s} = \frac{L/K_s}{1/h} = \frac{\text{convective resistance}}{\text{conductive resistance}}$$

K_s → solid

- use to determine what is more important in limiting heat transfer

Fourier

$$Fo = \frac{\alpha t}{L^2} = \frac{\text{heat conduction rate}}{\text{rate of internal energy storage}}$$

- non-dimensionless time parameter

Bodenstein

$$Bo = \frac{Vd}{D_{AB}} = (Re)(Sc) = \frac{\text{total momentum transfer}}{\text{molecular mass transfer}}$$

(like Pe)

measures flow contribution made by molecular diffusion

Sherwood

$$Sh = \frac{KL}{D_{AB}} = \frac{\text{conv. m.t}}{\text{diffusive m.t.}}$$

Mass transfer to/from bubbles : $Sh = \frac{K_c D}{D_{AB}} = 0.991\, Pe^{1/3}$

turbulent tube flow :

$$Sh_D = 0.04\, Re_D^{0.75}\, Sc^{0.33}$$

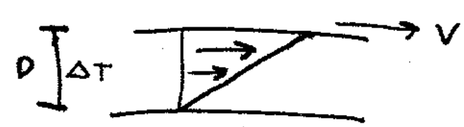
Convective transport off of a dissolving plate :

$$\overline{Sh} = 0.66\, Re_L^{1/2}\, Sc^{1/3}$$

- use to find D_{AB} from empirical correlations

Brinkman

$$Br = \frac{\mu (V/D)^2}{K \Delta T / D^2} = \frac{\text{heat produced by viscous dissipation}}{\text{heat transport by conduction}}$$



if $\mu \downarrow$, $Br \downarrow$, toss out viscous heating terms

$$= \frac{\mu V^2}{K \Delta T}$$

Damköhler

$Da = \tau K$ 1^{st} order rxn, CSTR

$$= \frac{(-r_A)V}{F_{A0}} = \frac{\text{rate of rxn}}{\text{rate of convective transport}}$$

if $Da > 1$ convective transfer limited

$Da < 1$ rxn rate limited

CSTR: $Da = 0.1$ ~ 10% conversion

$Da = 10^{+}$ ~ 90% conversion

Euler

$$Eu = \frac{\text{pressure force}}{\text{inertial force}} = \frac{P}{\rho v^2}$$

- comes from non-dimensionlizing Navier-Stokes

Schmidt

$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

momentum

$$\frac{\delta}{\delta_c} = Sc^{1/3}$$

mass

Mach

$$Ma = \frac{v}{c}$$

speed of sound

j_D - Chilton-Colburn j-factor

$$j_D = \frac{Sh}{Re\, Sc^{1/3}}$$

$j_H = j_D = f(Re, \text{geometry}, BC\text{'s})$

- relate heat and mass transfer

flat plate: $j_H = j_D = 0.332\, Re_x^{-1/2}$
(forced convection)

length of plate $= x$

- use to determine mass transfer coefficient

Lewis

$$Le = \frac{D_{AB}}{\alpha} = \frac{\text{mass diffusivity}}{\text{thermal diffusivity}}$$

- use in BL theory

δ_c vs δ_t

- if need to heat to get a rxn going
(overcome ΔE)

Graetz

$$Gz = \frac{\pi}{4} Re\, Pr \frac{D}{L} = f(Re, Pr, D/L)$$

Laminar pipe flow + free conv. in horizontal pipe

Deborah

$$De = \frac{\lambda}{T} = \frac{\text{char. time of material}}{\text{char. time of process}}$$

"silly putty" example

- visco-elastic fluids

De small - flow

De large - break

Stokes

$$St = \frac{\rho g D^2}{\mu V} = \frac{\text{gravitational force}}{\text{viscous force}}$$

Reynolds

$$Re = \frac{\rho V D}{\mu} = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\text{conv mom trans}}{\text{diff mom trans}}$$

- like Nu, Sc
- Correlations for friction factor

non-dimen. of N-S

$$\frac{D\vec{V}^+}{Dt^+} = -\nabla^+ p^+ + \underbrace{\left(\frac{\mu}{\rho V D}\right)}_{1/Re} \nabla^{+2} \vec{V}^+ + \underbrace{\left(\frac{g D}{V^2}\right)}_{1/Fr} \frac{\vec{g}}{g}$$

divide each term by Eu to get this

Prandtl

$$Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{k} = \frac{\text{momentum diffusion}}{\text{thermal diffusion}}$$

comes from non-dimensionalization of energy eqn.

$$\frac{DT^*}{Dt^*} = \frac{1}{Re\,Pr} \nabla^{+2} T + \frac{Br}{Re\,Pr} \underbrace{\Phi^+}_{}$$

→ dissipation

$Pr_{H_2O} = 7$

$Pr_{gases} \sim 0.6 - 1$

j_H Colburn Factor

$$j_H = St\,Pr^{2/3} = \frac{C_f}{2} = Nu\,Re^{-1}\,Pr^{-1/3}$$

$C_f \to f$

$$= \frac{h}{\rho v C_p}\left(\frac{C_p \mu}{k}\right)^{2/3}$$

$\frac{C_f}{2} = St$ (only valid if $Pr = 1$)

Reynold's said : $\cancel{j_H = Pr^{2/3}}$ (exp't)

Colburn said : $j_H = St\,Pr^{2/3}$ (exp't)

added this

- we can compute h from frictional and drag information of the fluid

Rayleigh

$Ra = Gr\,Pr$

- useless

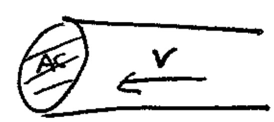
Groschoff

$$Gr = \frac{L^3 \rho^2 g \overset{\text{thermal expansion } -\frac{1}{\rho}\frac{d\rho}{dt}}{\beta} \Delta T}{\mu^2} = \frac{\text{buoyancy force}}{\text{viscous force}}$$

- free convection
- cigarette smoke rises until cools and viscous forces are not important anymore - then viscous forces take over and it spreads out

Drag Coefficient C_D

$$C_D = \frac{\overset{\text{drag force}}{F_D}}{\frac{1}{2}\rho v^2 \underset{\text{face area}}{A_c}}$$



$$F_D = C_D \tfrac{1}{2}\rho v^2 A_c = C_D (\pi R^2)(\tfrac{1}{2}\rho v^2)$$

Fanning Friction Factor f

$$F_K = (2\pi R L)(\tfrac{1}{2}\rho v^2) f$$

$$f = \frac{F_K}{2\pi R L \frac{1}{2}\rho v^2} = \frac{F_K}{A \cdot \frac{1}{2}\rho v^2}$$

Froude

$$Fr = \frac{V^2}{gD} = \frac{\text{inertial force}}{\text{gravitational force}}$$

– non-dimensionalizing N-S

Stanton

$$St = \frac{h}{\rho V C_p} = \frac{\text{convective heat transport}}{\text{momentum transport (convective)}}$$

– only can be used in correlating forced convection data

$$St = f(Re, Pr)$$

Reynold's analogy : $St = \frac{C_f}{2}$

only if $Pr = 1$ and no form drag

Colburn analogy : valid for wide range of Pr

$$\frac{C_f}{2} = St \cdot Pr^{2/3}$$

Weber

$$We = \frac{\text{Inertial forces}}{\text{Surfaces forces}} = \frac{\rho V^2 D}{\sigma}$$

char length

surface tension

– bubble and drop formation

Peclet

$$Pe = \frac{Dv}{\alpha} = \frac{\text{conv. momentum transport}}{\text{molecular heat transport}}$$

$$Pe = Re\,Pr$$

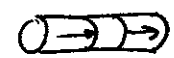
Thiele

Thiele : $\frac{V}{SA_{ext}}\sqrt{\frac{K}{D_{AB}}} = \phi = \frac{\text{reaction rate}}{\text{rate of diffusion into catalyst}}$

– diffusion into porous catalysts

From : 1st order rxn

cylindrical pore



no convective

flux in - flux out + rxn = 0

$$\frac{d^2 C_A}{dx^2} - \frac{K}{D_{AB}} C_A = 0$$

$$\frac{C_A}{C_{A\,surface}} = \frac{\cosh\left(\sqrt{\frac{K}{D_{AB}}}(L-x)\right)}{\cosh\sqrt{\frac{K}{D_{AB}}}\,L}$$

high ϕ : rate of diffusion slower than rxn rate
mass transfer limiting

Knudson

$$Kn = \frac{\text{mean free path}}{\cancel{\text{distance of pore}}\ \text{characteristic length}}$$