TRANSPORT PRELIMINARY EXAM QUESTIONS

1. WHAT IS THE DIFFERENCE BETWEEN:

a) HEAT AND MASS TRANSFER?

- Heat transfer is the transport of heat energy where the driving force is the temperature gradient. Main mechanisms include convection, conduction, and radiation.
- Mass transfer is the transport of material species where the driving force is the concentration gradient. Mass transfer also occurs by bulk flow.

b) HEAT AND MOMENTUM TRANSFER?

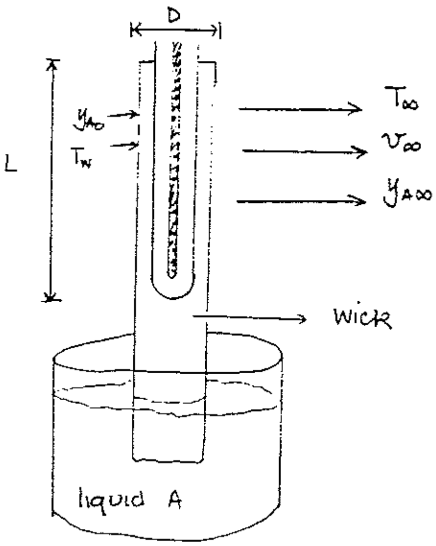
- Momentum transfer is the transfer of momentum where the driving force is the velocity gradient. Main mechanisms include bulk flow, shear stress, and various forces acting on the system such as pressure, work, and body forces.

c) MASS AND MOMENTUM TRANSFER?

- See above

2. SET UP EQUATIONS TO DESCRIBE:

a) A WET BULB THERMOMETER



- energy balance:

$$N_A \pi D L \, \Delta h^{vap} = h (T_\infty - T_W) \pi D L \qquad ①$$

- mass balance

$$N_A = k_G (y_{A_0} - y_{A\infty}) + y_{A_0} N_A$$

$$N_A = \frac{k_G (y_{A_0} - y_{A\infty})}{(1 - y_{A_0})} \qquad ②$$

② → ①

$$\frac{k_G (y_{A_0} - y_{A\infty})}{(1 - y_{A_0})} \Delta h^{vap} = h (T_\infty - T_W)$$

- equilibrium

$$y_{A_0} = \frac{P_A^s (T_W)}{P}$$

- Chilton Colburn analogy to get $\frac{h}{k_G}$

$$j_h = j_m$$

$$St\, Pr^{2/3} = St_m\, Sc^{2/3}$$

$$Pr^{2/3} \frac{Nu}{Re\, Pr} = \frac{Sh}{Re\, Sc} Sc^{2/3}$$

$$Nu = \frac{hL}{k_G} = \frac{hL}{k_{air}}$$

$$Re = \frac{D v_\infty \rho_{air}}{\mu_{air}}$$

$$Pr = \frac{\nu}{\alpha} = \frac{c_p \mu_{air}}{k_{air}}$$

$$Sh = \frac{k_{air} L}{\mathcal{D}_{A\text{-}air}}$$

$$Sc = \frac{\mu_{air}}{\rho_{air} \mathcal{D}_{A\text{-}air}}$$

c) BURNING OF CARBON PARTICLE

$$C + O_2 \rightarrow CO_2$$ assuming complete combustion

- assume transport limitation of O_2 to particle

  - Fick's Law : $N_{O_2} = -D \dfrac{dC_{O_2}}{dr}$

- Shell balance of mass

S.S $$-D \frac{dC_{O_2}}{dr}(4\pi r^2)\Big|_{r+\Delta r} + D \frac{dC_{O_2}}{dr}(4\pi r^2)\Big|_{r} + r_{O_2} 4\pi r^2 \Delta r = 0$$

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  - Fick's Law : $N_{O_2} = -D \frac{dC_{O_2}}{dr}$

equimolar counter current

- shell balance of mass

S.S
$$-D \frac{dC_{O_2}}{dr} (4\pi r^2) \Big|_{r+\Delta r} + D \frac{dC_{O_2}}{dr} (4\pi r^2) \Big|_r + r_{O_2} 4\pi r^2 \Delta r = 0$$

(4) Diffusivity - diffusion only

mass transfer coeff - encompasses convection + diffusion

· primary difference: diffusivity is differential in definition and the mass transfer coefficient has a finite difference in its definition

· the diffusivity is defined as a proportionality factor between mass flux and a concentration gradient $N_A = D_A \frac{c}{d}$

· mass transfer coefficient is defined as a proportionality between mass flux at a boundary or setpoint and conc. in the bulk

$$N_A = k_c (C_0 - C_\infty)$$

(7) no!

periodic table

He
Ne
Ar
Kr
Xe

incr radius

electrons aren't held so tightly

a) $D \propto \frac{1}{\sqrt{MW}}$ → Neon greater diffusivity

b) Neon (looked it up) why?

$\mu = \frac{2}{3\pi^{3/2}} \frac{\sqrt{mk_B}}{\sigma^2}$

c) approx equal b/c both monatomic gas

d) $\frac{C_p \mu}{k}$

means higher k, higher μ ?

$Pr_{Ne} \sim Pr_{Ar}$

$k = \frac{5}{2} C_v \mu$ BSL

what! thats a boltzman "k" isn't it.

$C_v = \frac{5}{2} R.$

c) TEMPERATURE PROFILE

- Find $T_m(z)$, although $T = f(z, r)$

  - energy balance:

$$- \dot{m} c_p T_m \big|_z + \dot{m} c_p T_m \big|_{z+\Delta z} = 2\pi R \Delta z \, q''_{conv}$$

$$q''_{conv} = \frac{\dot{m} C_p}{2\pi R} \frac{dT_m}{dz} = h\,(T_m - T_s)$$

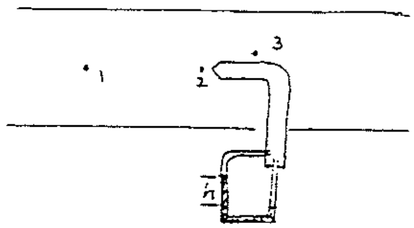
$$\frac{dT_m}{dz} = \frac{2\pi R h}{\dot{m} C_p} (T_m - T_s) = \alpha (T_m - T_s)$$

assuming const T_s:

$T = T_m - T_s$ $\qquad \frac{dT}{dz} = \alpha T$ $\qquad T = T_i - T_s$ at $z = 0$

$\vdots$

c) PITOT TUBE



2 = stagnant pt. $\rightarrow$ $v_2 = 0$

$v_1 = v_3$

• Bernoulli's :

$$\frac{1}{2}v_1^2 + \frac{P_1}{\rho} + gh_1 = \frac{1}{2}v_2^2 + \frac{P_2}{\rho} + gh_2$$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g (h_1 - h_2)$$

$$\frac{1}{2}v_1^2 + \frac{P_1}{\rho} + gh_1 = \frac{1}{2}v_3^2 + \frac{P_3}{\rho} + gh_3$$

$$P_3 = P_1 + \rho g (h_1 - h_3)$$

$$\therefore \quad v_1 = \left\{ \frac{2\left[(P_2 - P_3) + \rho g (z_2 - z_3)\right]}{\rho} \right\}^{1/2}$$

(10) a) $\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{constant}$

restrictions:

1) steady flow
2) incompressible flow
3) inviscid flow
4) flow along a streamline

b) $V_{x_{av}} = \frac{(P_0 - P_L)D^2}{32\mu L}$ - relates pressure drop and average velocity for laminar flow in a horizontal pipe

also written as $Q = \frac{\pi R^4 (P_1 - P_2)}{8\mu L}$

c) viscous force on a small sphere when flow is laminar

valid ~ $Re < 0.1$ $F_{drag} = 6\pi r \mu v$

d) $$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

or $$\frac{D\rho}{Dt} = -\rho(\nabla \cdot v)$$

e) Newtonian fluids, ρ and μ are constant $\nabla \cdot V = 0$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho g + \mu \nabla^2 v \quad \text{rectangular}$$

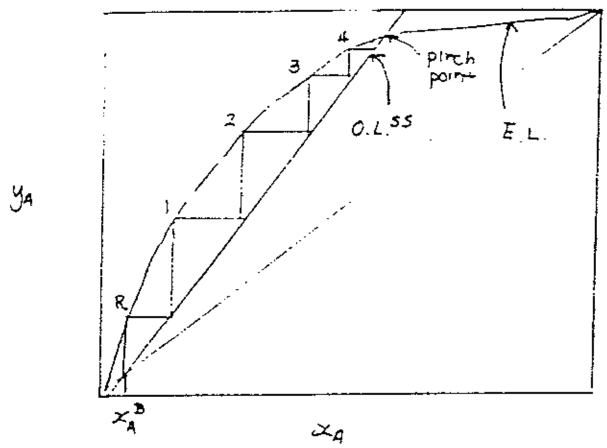
cylindrical

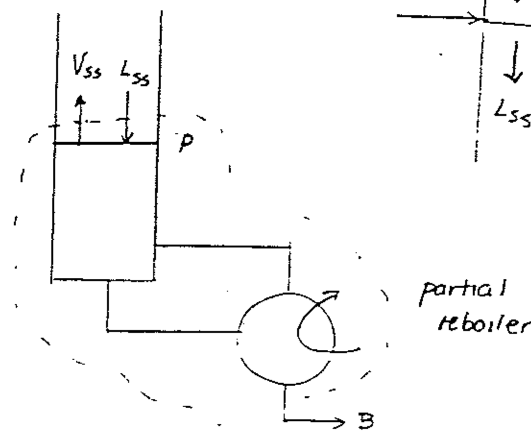
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial \theta}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right) + \rho g_r$$

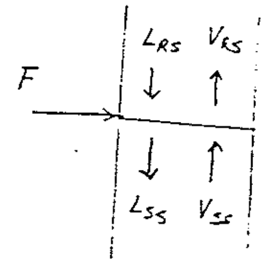
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

- Switch to new mass balance → the stripping section







$$L_{SS} = L_{RS} + L_F$$

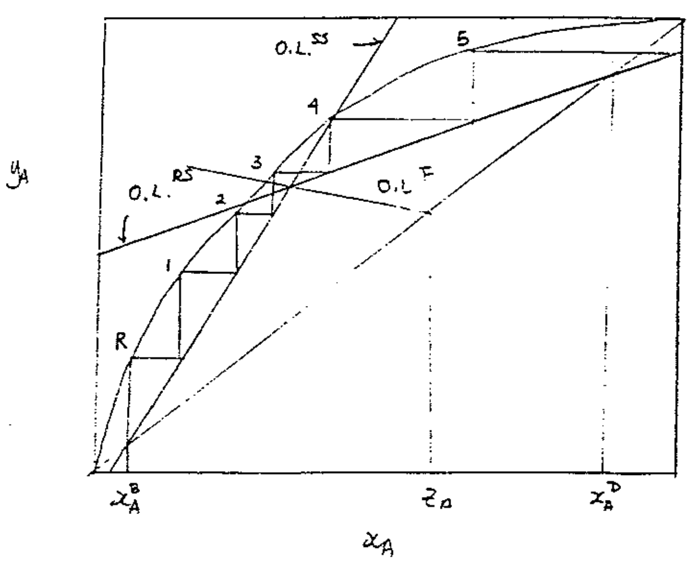
$$V_{SS} = V_{RS} - V_F$$

$$V_{SS}\, y_A^{p} = L_{SS}\, x_A^{p-1} - B x_A^{B}$$

O.L.^{SS} ≡ stripping section operating line

$$y_A^{p} = \underbrace{\frac{L_{SS}}{V_{SS}}}_{\text{slope}} x_A^{p-1} - \underbrace{\frac{B x_A^{B}}{V_{SS}}}_{\text{intercept}}$$

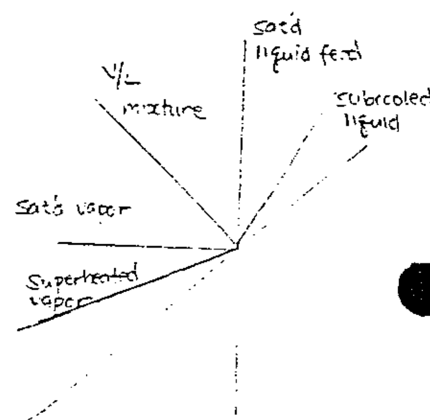
- McCabe-Thiele, put them together



- we need then, reboiler + little less than 5 stages. To get exact R + 5 stages, adjust reflux, r.

- feed stage at 3, where we switch from OL.^{SS} to O.L.^{RS}

- feed O.L.



(13)

a. gas $D_{bulk} \propto \frac{T^{1.75}}{P}$, $D_{Knudsen} \propto \sqrt{T}$

liquid $D \propto T$ ~ indep of P for wide range of P

see Atkins

b. gas $\mu \propto \sqrt{T}$ $\mu \uparrow$ as $P \uparrow$

liquid $\mu \propto \exp(\frac{A}{T})$

c. gas $k \propto \sqrt{T}$ indep of pressure for wide range of P

liquid $k = a + bT$ * $\propto \frac{1}{T}$

d) fit to polynomial

$$C_p = A + BT + CT^2 + \frac{D}{T^2}$$ not a fxn of

either C or D is zero

e) $Nu = f(Pr, Re, L)$ forced convection

$Nu = a(Pr, Gr)^m$ free convection

where $Nu = \frac{hD}{k}$

h is a complex f(T) where k, C_p, μ are all changing

h is indep of pressure until you get to the Knudsen regime

for liq

gases?

f) $\nu = \frac{\mu}{\rho}$ $\rho = \frac{P\, mw}{RT}$

gas $\nu \propto T^{3/2}$

liq $\rho \neq f(T)$

$\rightarrow$ $\nu \propto \exp(B/T)$

15 WHAT IS THE FRICTION FACTOR? THE COEFFICIENT OF FRICTION?

- Consider s.s flow of fluid of constant ρ in two systems: a) flow in straight conduit of constant cross section area A. b) flow around submerged object

$$F_{total} = \underbrace{F_{static}}_{\text{due to stagnant fluid}} + \underbrace{F_{kinetic}}_{\text{due to flowing fluid}}$$

in a) $\vec{F}_k$ in same direction as $\langle \vec{v}_z \rangle$

b) $\vec{F}_k$ in opposite direction as $\vec{v}_\infty$

} → $F_k = AKf$ definition for f

characteristic contact area

characteristic K per unit volume

$$\boxed{f = \frac{F_k}{AK}}$$

- examples

a) flow in conduit, circular tubes of radius R & length L, with elevation diff $h_0 - h_L$

$$F_k = (2\pi RL)\left(\tfrac{1}{2}\rho\langle v\rangle^2\right)f$$

- force balance →

$$F_k = [(p_0 - p_L) + \rho g(h_0 - h_L)]\pi R^2$$

$$\therefore \quad \boxed{f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\mathscr{P}_0 - \mathscr{P}_L}{\frac{1}{2}\rho\langle v\rangle^2}\right)}$$

aka: fanning friction factor

- $f = f(Re, \frac{L}{D})$

$f = f(Re)$ for fully developed flow or $\frac{L}{D} >> 1$

- $f = \frac{16}{Re}$ for $Re < 2.1 \times 10^3$ laminar

$f = \frac{0.0791}{Re^{1/4}}$ for $2.1 \times 10^3 < Re < 10^5$ (Blasius formula) transition

$f = f(Re, \frac{k}{D})$ for turbulent flow, $k \equiv$ height of protuberances due to "roughness" of pipe

(16) Laminar, flat plate boundary layer, exact solution — Blasius

horizontal

2-d, steady, incompressible flow w/ 0 pressure gradient

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

with boundary conditions at $y=0$ $u=0$

at $y=\infty$ $u=U$ $\frac{du}{dy}=0$

sol'n of form $\frac{u}{U} = g(\eta)$ where $\eta \propto \frac{y}{\delta}$

based on Stokes $\delta \propto \sqrt{\frac{\nu x}{U}}$

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

introduce stream fxn ψ where $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

define dimensionless stream fxn

$$f(\eta) = \frac{\psi}{\sqrt{\nu x U}}$$ where $f(\eta)$ = dependent variable, η = independent variable

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = \sqrt{\nu x U}\,\frac{df}{d\eta}\sqrt{\frac{U}{\nu x}} = U\frac{df}{d\eta}$$

$$v = -\frac{\partial \psi}{\partial x} = -\left[\sqrt{\nu x U}\,\frac{\partial f}{\partial x} + \frac{1}{2}\sqrt{\frac{\nu U}{x}}\,f\right]$$

$$= -\left[\sqrt{\nu x U}\,\frac{df}{d\eta}\left(-\frac{1}{2}\eta\frac{1}{x}\right) + \frac{1}{2}\sqrt{\frac{\nu U}{x}}\,f\right]$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left[\eta\frac{df}{d\eta} - f\right]$$

$$\frac{\partial u}{\partial x} = -\frac{U}{2x}\eta\frac{d^2 f}{d\eta^2}$$

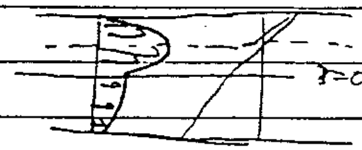
$$\frac{\partial u}{\partial y} = U\sqrt{U/\nu x}\,\frac{d^2 f}{d\eta^2}$$

and

$$\frac{\partial^2 u}{\partial y^2} = \frac{U^2}{\nu x}\frac{d^3 f}{d\eta^3}$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r + \frac{C_1}{r}$$

non linear



$$F = \frac{4}{3}\pi R^3 \rho g + 2\pi\mu R v_\infty + 4\pi\mu R v_\infty$$
$$= \frac{4}{3}\pi R^3 \rho g + 6\pi\mu R v_\infty$$

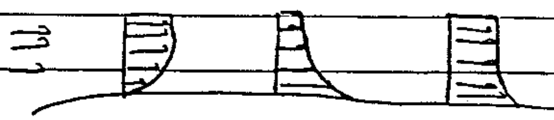
$$N_A = \mathcal{D}\frac{dC_A}{dz} + N_A(x_A - x_B)$$

cylindrical

r $\quad \rho\frac{v_\theta^2}{r} = \frac{\partial P}{\partial r}$

$$0 = \mu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)$$

$$0 = -\frac{\partial P}{\partial z} - \rho g$$



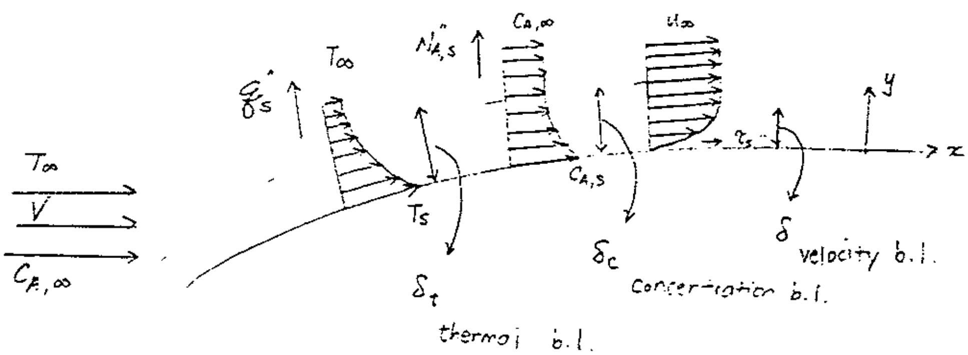
Boundary Layer Eqns

2D

m.B. $\quad \cancel{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0} \qquad \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} = 0$

Mom.B. $\quad \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho v u)}{\partial y} = -\frac{\partial P}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \frac{\partial \tau_{xx}}{\partial x} + \rho g_x$

DERIVE THE BOUNDARY LAYER EQUATIONS



$u = v_x$
$v = v_y$

- mass balance (continuity)

$$\text{mass in}\Big|_x - \text{mass out}\Big|_{x+dx} = 0$$

$$(\rho u)\,dy + (\rho v)\,dx - \left[(\rho u) + \frac{\partial(\rho u)}{\partial x}dx\right]dy - \left[(\rho v) + \frac{\partial(\rho v)}{\partial y}dy\right]dx = 0$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$\text{or}\quad \frac{\partial \rho}{\partial t} = -(\nabla\cdot\rho\vec{v})$$

- momentum balance (x-direction) → velocity boundary layer

$$\frac{\partial(\rho u\cdot u)}{\partial x} + \frac{\partial(\rho v\cdot u)}{\partial y} = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \frac{\partial \tau_{xx}}{\partial x} + \rho g_x$$

- expand out left side and use continuity and these definitions

$$\tau_{xx} = -2\mu\frac{\partial u}{\partial x} + \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$
$$\tau_{yx} = -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial y}\right.\right.$$
$$+ \rho g_x$$

- for constant μ and incompressible fluid, steady-state.

$$\boxed{\begin{aligned} \rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) &= -\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\rho g_x \\ \rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) &= -\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g_y \end{aligned}}$$

x-direction

y-direction

or.

$$\frac{\partial}{\partial t}(\rho\vec{v}) = -(\nabla\cdot\rho\vec{v}\vec{v}) - \vec{\nabla}\cdot p - \vec{\nabla}\cdot\vec{\tau} + \rho\vec{g}$$

- energy balance $\rightarrow$ thermal boundary layer

$$\boxed{\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi+\dot{q}}$$

or

$$\rho c_p\left(\frac{\partial T}{\partial t}+\vec{v}\cdot\vec{\nabla}\cdot T\right) = k\vec{\nabla}^2 T + \mu\Phi - T\left(\frac{\partial p}{\partial T}\right)_\rho(\vec{\nabla}\cdot\vec{v})$$

- species mass balance $\rightarrow$ concentration boundary layer

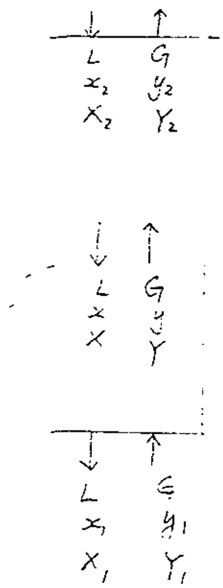
$$\boxed{u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y} = \frac{\partial}{\partial x}\left(D_{AB}\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{AB}\frac{\partial C_A}{\partial y}\right)+\dot{N}_A}$$

or.

$$\frac{\partial C_A}{\partial t}+\vec{v}\cdot\vec{\nabla}C_A = \mathcal{D}_{AB}\nabla^2 C_A + R_A$$

17. SKETCH GOVERNING DIAGRAMS FOR A STRIPPER AND ABSORBER

- Absorber — Countercurrent Multistage Operation (absorption of A into liquid)



$$Y_1 = \frac{y_1}{1-y_1} = \frac{\text{moles A in gas}}{\text{moles insoluble gas}}$$

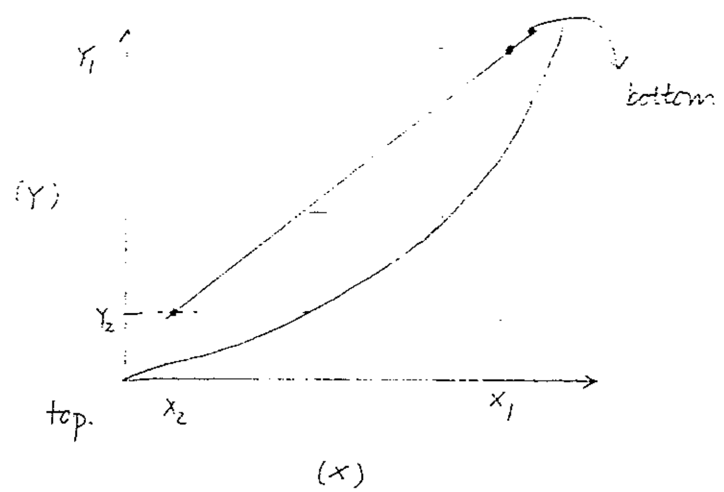
$$X_1 = \frac{x_1}{1-x_1} = \frac{\text{moles of A in liq.}}{\text{moles of solvent}}$$

mole balance.

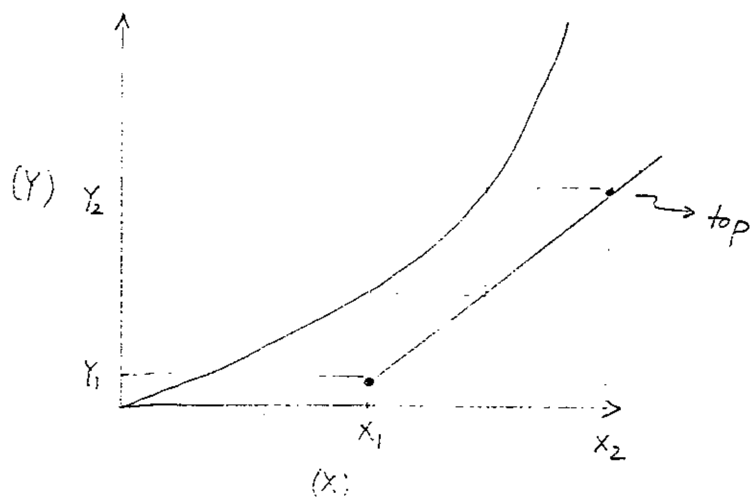
$$LX + GY_1 = LX_1 + GY$$

$$G(Y_1 - Y) = L(X_1 - X)$$

$$Y = \frac{L}{G}X + \frac{GY_1 - LX_1}{G}$$



absorber



Stripper

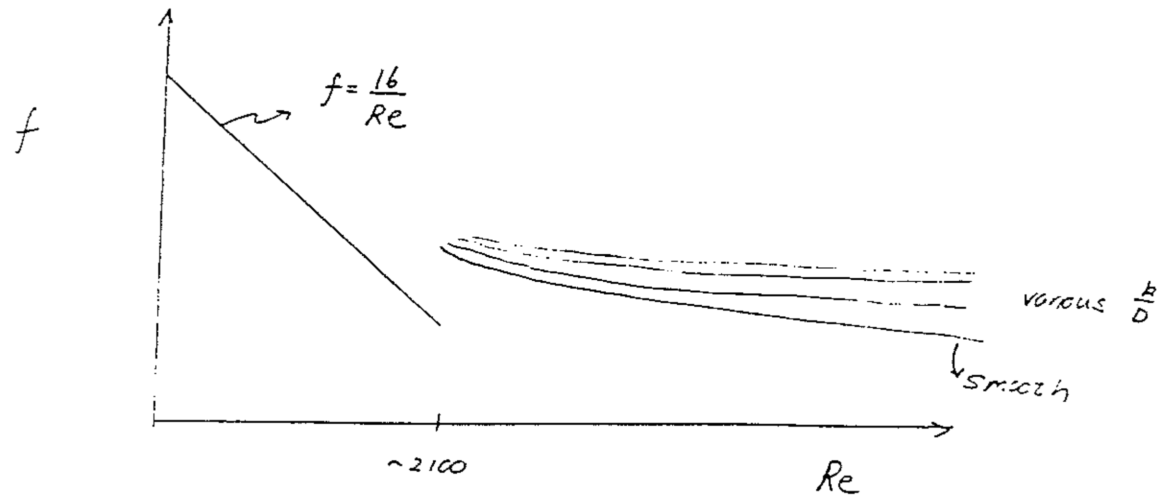
18 DESCRIBE THE FRICTION FACTOR (DRAG COEFF C_D) VS. RE RELATIONSHIP FOR:

a) PIPE

$$f = \frac{1}{4}\frac{D}{L}\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v\rangle^2} \qquad Re = \frac{\rho\langle v\rangle L}{\mu}$$

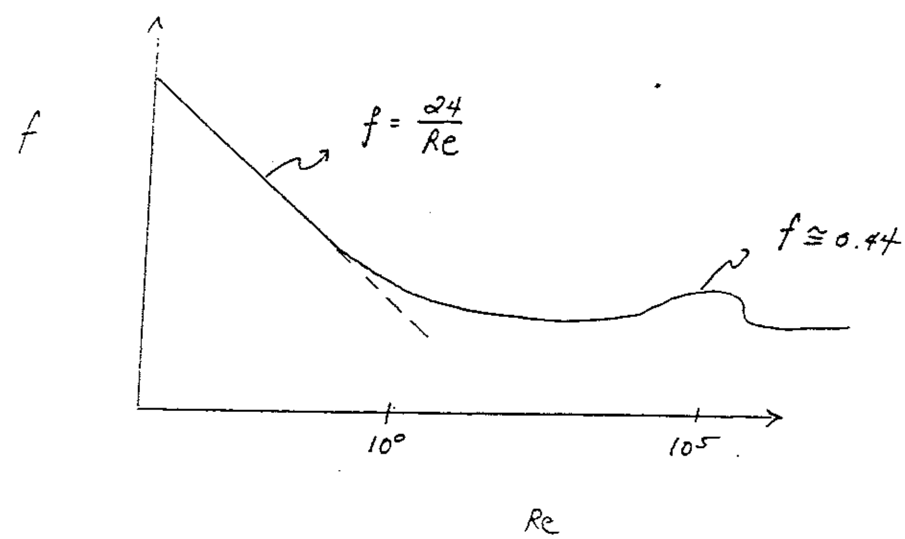
Moody Chart

on log-log charts



c) SPHERE

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sph} - \rho}{\rho}\right). \qquad Re = \frac{\rho v_\infty D}{\mu}$$



c) FLAT PLATE

$$F_k = 2\, L \cdot W \cdot \tfrac{1}{2} \rho v_\infty^2 f$$

- B.S.L p. 203 Eqn 6.G-1 →

$$F_k = 1.328 \sqrt{\rho \mu L W^2 v_\infty^3} \qquad \text{laminar}$$

$$f = \frac{1.328\, W v_\infty \sqrt{\rho \mu L v_\infty}}{L W \rho v_\infty^2}$$

$$f = 1.328 \left(\frac{\mu}{\rho v_\infty L} \right)^{1/2}$$

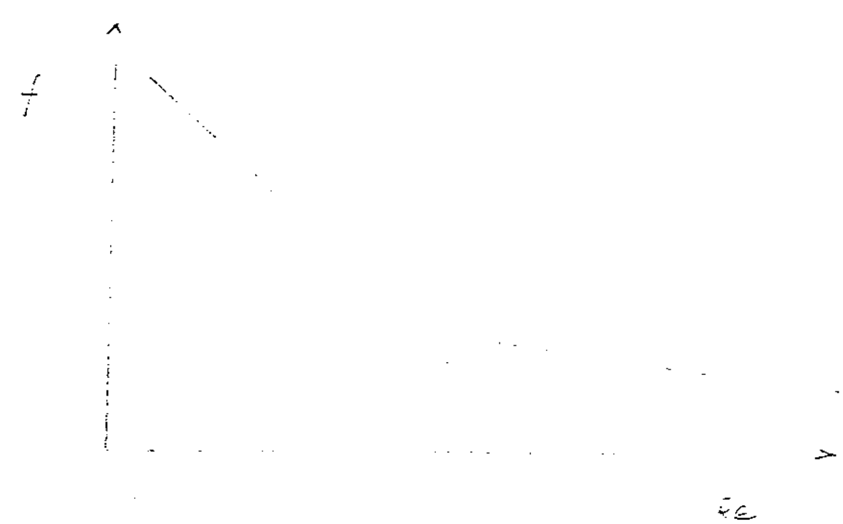
$$f = 1.328\, Re^{-1/2} \qquad Re = \frac{\rho v_\infty L}{\mu}$$

- B.S.L p 203 Eqn 6.G-2 →

$$F_k = 0.072\, \rho v_\infty^2 W L \left(\frac{L v_\infty \rho}{\mu} \right)^{-1/5}$$

$$f = \frac{0.072\, \rho v_\infty^2 W L\, Re^{-1/5}}{L W \rho v_\infty^2}$$

$$f = 0.072\, Re^{-1/5}$$

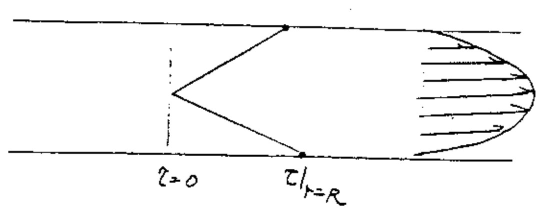


19. SKETCH THE SHEAR STRESS (τ) PROFILE FOR A PIPE.

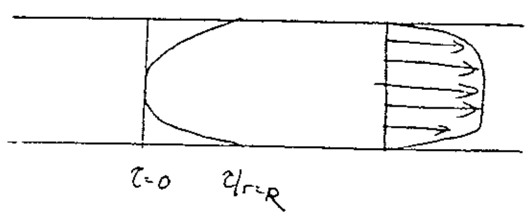
- laminar flow

$$v_z = v_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\tau_{rz} = -\mu \frac{dv}{dr} = 2 v_{max} \cdot r\mu$$ linear τ_{rz} profile.



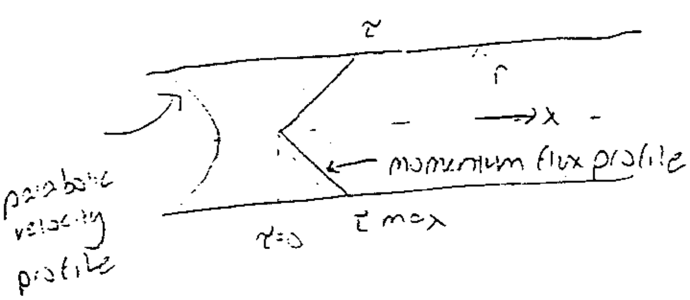
- turbulent flow



(19) using Hagen Poiseuille

$$\tau = \frac{(P_0 - P_L)}{2L} r$$

τ is a max @ the pipe walls



20. DESCRIBE THE MOST COMMONLY USED DIMENSIONLESS PARAMETERS AND THEIR SIGNIFICANCE

- Momentum transfer

$$Re = \frac{\rho V L}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{\text{buoyancy forces}}{\text{viscous forces}}$$

$$Fr = \frac{\rho v^2}{\rho g L} = \frac{\text{inertial forces}}{\text{gravity forces.}}$$

- Heat transfer

$$Nu = \frac{hL}{k} = \frac{\text{convective heat transfer}}{\text{heat transfer by conduction}}$$

$$Pr = \frac{\nu}{\alpha} = \frac{\text{viscous momentum transport}}{\text{diffusional heat transport}}$$

$$Bi = \frac{hL}{k_s} = \frac{\text{resistance to conduction}}{\text{resistance to convection}}$$

$$Fo = \frac{\alpha t}{L^2} = \frac{\text{heat conduction rate}}{\text{rate of thermal energy storage}}$$

- Mass transfer

$$Sh = \frac{h_m L}{\mathcal{D}} = \frac{\text{convective mass transfer}}{\text{diffusive mass transfer}}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{\text{viscous momentum transport}}{\text{diffusive mass transfer.}}$$

$$Bi_m = \frac{h_m L}{D_{AB}} = \frac{\text{resistance to mass transport by diffusion}}{\text{resistance to mass transport by convection}}$$

21 GIVEN A POOL OF ORGANIC LIQUID (eg FROM A SPILL), ESTIMATE ITS RATE OF EVAPORATION

B = air

$z = 0$

A = organic

- ignore heat transfer, isothermal

$$N_{A_z} = -\mathcal{D}_{AB}\frac{\partial C_A}{\partial z} + \underbrace{x_A}_{0}(N_{A_z} + \underbrace{N_{B_z}}_{0})$$

assume $x_A \cong 0$

$$\text{rate} = N_{A_z}\Big|_{z=0} \cdot A_{pool} = -\mathcal{D}_{AB} \cdot A \left.\frac{dC_A}{dz}\right|_{z=0}$$

- governing eqn

$$\frac{\partial C_A}{\partial t} = \mathcal{D}_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

$C_A = 0 \quad z \to \infty$

$C_A = \dfrac{P_A^{sat}(T_{air})}{RT} \quad z = 0$

$C_A = 0 \quad t = 0$

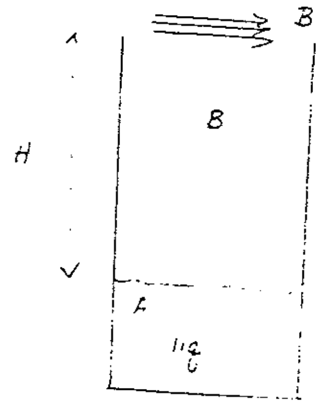
Solve by SOV to get $C_A = f(z,t)$

then get $\left.\dfrac{\partial C_A}{\partial z}\right|_{z=0} \longrightarrow \text{rate} = -\mathcal{D}_{AB} A \left.\dfrac{\partial C_A}{\partial z}\right|_{z=0}$

22. HOW ARE THE DIFFUSIVITY AND VISCOSITY OF A MIXTURE DETERMINED?

• Diffusivity

- can use kinetic theory and corresponding states to obtain D_{AB} for gases at low P.



at S.S $N_B = 0$

$$\therefore \quad N_A = -c\mathcal{D}_{AB}\frac{dC_A}{dz} + x_A N_A$$

$$N_A = \frac{-c\mathcal{D}_{AB}\frac{dC_A}{dz}}{1-x_A}$$

$$\frac{dN_A}{dz} = 0 \quad \rightarrow \text{solve} \quad x_A = f(z)$$

B.C. $\quad z = 0 \quad C_A = \frac{P_A^S}{RT} = C_A$

$\quad\quad z = H \quad C_A = 0$

B.S.L. Eqn 17.2-10

$$\frac{1-x_A}{1-x_{A_0}} = \left(\frac{1}{1-x_{A_0}}\right)^{\frac{z}{H}}$$

$$\therefore \quad N_A\Big|_{z=0} = \frac{c\mathcal{D}_{AB}}{H \cdot x_{B_{ln}}}\, x_{A_0}$$

$$= \frac{P\mathcal{D}_{AB}/RT}{H \cdot (P_B)_{ln}}\, P_{A_1}$$

$$\mathcal{D}_{AB} = \frac{N_A|_{z=0} \cdot H \cdot (P_B)_{ln}}{P/RT \cdot P_{A_1}}$$

• Viscosity

- Capillary viscometer: $\quad Q = \frac{\pi \Delta P R^4}{8\mu L}$ Hagen-Pouiseuille

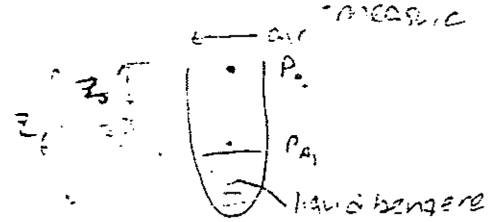
- Couette viscometer $\quad \mu = \frac{Gh}{2\pi R^3 L \Omega}$ $\quad G \equiv$ total torque.

- dropping spherer viscometer Stokes's regime $\quad \mu = \frac{2R^2(\rho_s - \rho)g}{9v_t}$

(22)

experimental determination of diffusivity

- evaporate a pure liquid in a narrow tube with a gas passed over the top.
- measure the fall in liquid level with time

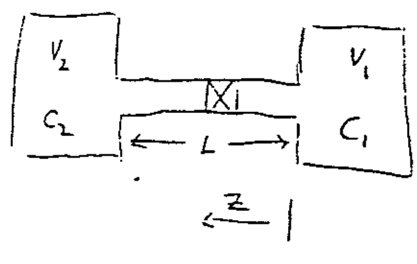


$$t_f = \frac{\rho_A (z_f^2 - z_0^2) RT\, p_{Bm}}{2 M_A D_{AB} P (p_{A_1} - p_{A_2})}$$

- rate of evaporation of sphere

For a gaseous mixture:

2 bulb method



pure $A \rightarrow V_1$
pure $B \rightarrow V_2$

- capillary of area A and length L
- $V_{cap.} \ll V_1$ or V_2
- open valve
- wait
- close valve
- sample chamber contents

$$J_A^* = -D_{AB}\frac{dc}{dz} = -D_{AB}\frac{(c_2 - c_1)}{L}$$

rate of diffusion of A going to V_2 = rate of accumulation in V_2

$$A J_A^* = -D_{AB}\frac{(C_2 - C_1)A}{L} = V_2 \frac{dC_2}{dt} \qquad (1)$$

- calc C_{av}, average value @ eqbm from starting compositions c_1° and c_2° at $t=0$

$$(V_1 + V_2) C_{av} = V_1 C_1^\circ + V_2 C_2^\circ$$

- balance at time t gives

$$(V_1 + V_2) C_{av} = V_1 C_1 + V_2 C_2$$

$$\frac{C_{av} - C_2}{C_{av} - C_2^0} = \exp\left(\frac{-D_{AB}(V_1 + V_2)}{LA(V_2 V_1)} t\right)$$

VISCOSITY

· brookfield viscometer

liquids

· concentric cylinders, measure torque

Newtonian

$$dT = r_i dF$$

$$dT = r_i \tau dA$$

$$\tau = \mu \frac{dv}{dy} \Rightarrow \tau = \frac{\mu \omega r_i}{r_o - r_i} \quad \text{small gap}$$

$$dA = r_i l \, d\theta$$

$$\text{integrate} \quad T = \frac{2\pi r_i^2 l \mu \omega r_i}{r_o - r_i}$$

couette viscometer

gaseous mixture?

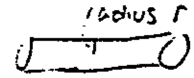
1st estimate - mean viscosity?
using mole fractions

from Atkins:

2 main techniques

1. rate of damping of the torsional oscillations a disk hanging in the gas

2. use Poiseuille



radius r

length L

$$\frac{dV}{dt} = \frac{(P_1^2 - P_2^2)\pi r^4}{16 l \eta P_0}$$

BSL

V = volume flowing

24. WHAT PHENOMENA ARE IMPORTANT DURING AN UNDERGROUND EXPLOSION?

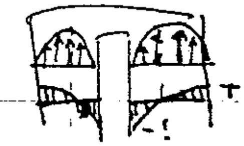
Explosions are fast processes, so diffusion not important. Expansion of gases due to bulk flow. Also important is the void space for gas to expand.

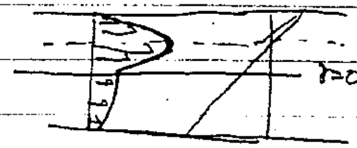
25 CONSIDER A DROP FALLING DOWN A TOWER, T_o, T_{tower} KNOWN. HOW DOES DROP T CHANGE AS IT FALLS.

- forced convection heat transfer as drop falls
- evaporation occurs as drop falls
- rate of evaporation = rate at which heat is transferred to drop

$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r + \frac{C_1}{r}$

non linear





$F = \frac{4}{3}\pi R^3 \rho g + 2\pi\mu R v_\infty + 4\pi\mu R v_\infty$
$\sim \frac{4}{3}\pi R^3 \rho g + 6\pi\mu R v_\infty$

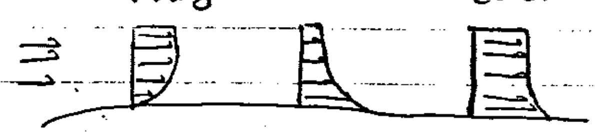
$N_A = \mathcal{D}\frac{dc_A}{dz} + N_A(x_A - x_B)$

cylindrical

$r \qquad \rho\frac{v_\theta^2}{r} = \frac{\partial P}{\partial r}$

$0 = \mu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)$

$0 = -\frac{\partial P}{\partial z} - \rho g$



Boundary Layer Eqns
2D

m.B. $\quad \cancel{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0} \qquad \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} = 0$

Mom. B. $\quad \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho v u)}{\partial y} = -\frac{\partial P}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \frac{\partial \tau_{xx}}{\partial x} + \rho g_x$

② nrg balance

$$\rho C_p V \frac{dT}{dt} = hA\left(T - T_{tower}\right)$$ $\Rightarrow t=0 \quad T=T_i$

? $\frac{hk}{D} = Nu = 2 + 0.6 Re^{1/2} Pr^{1/3}$ ∵ flow past a sphere

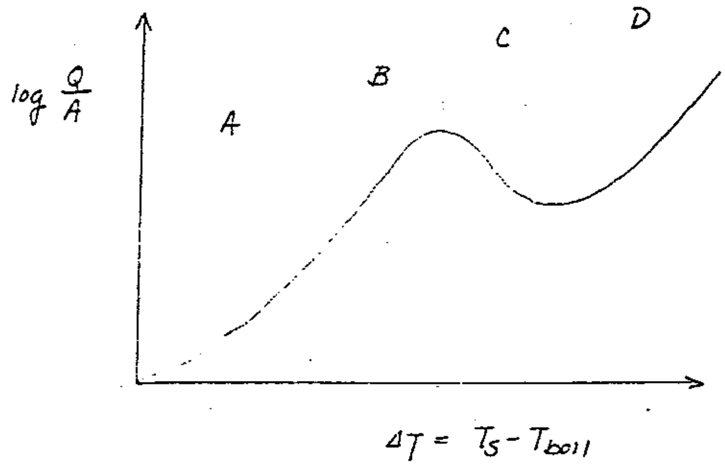
P assume drop @ a uniform T

for velocity assume laminar

$u = \frac{D^2}{18\mu}\left(\rho_{sphere} - \rho_{air}\right)$ from Stokes law

27 DRAW THE BOILING CURVE AND DESCRIBE THE PHYSICAL PHENOMENA RESPONSIBLE FOR THE OBSERVED BEHAVIOR. DRAW AND EXPLAIN THE SIMILAR CURVE FOR CONDENSATION.

- Heat transfer to boiling liquid
  - liquid at T_{boil} and P of equipment
  - heat transfer from surface w/ $T_S > T_{boil}$
  - bubbles of vapor generated at surface



A: Natural convection: few bubbles, mostly natural convection ($\Delta T \lesssim 5°C$)

B. Nucleate boiling: more bubbles ($5 \lesssim \Delta T \leq 30°C$)

C: Transition boiling: many bubbles form so quickly that coalesce and form layer of insulating vapor on surface. As $T \uparrow$, $\frac{Q}{A} = q'' \downarrow$ ($30 \leq \Delta T \lesssim 120°C$)

D. Film boiling: bubbles detach faster than they coalesce radiation thru vapor layer next to surface becom significant

- Heat transfer from condensing vapor
    - $T_s < T_{condensate}$

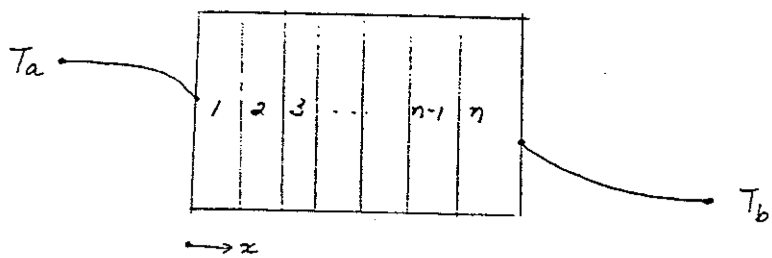
Film condensation: film of condensate forms over surface and causes the main resistance to heat transfer

Dropwise condensation. only drops of liquid are formed on surface more clean surface for heat transfer.

28 ?

30. HOW IS THE OVERALL HEAT TRANSFER COEFFICIENT FOR A HEAT EXCHANGER FOUND ?

- for flat plate geometry:



resistances:

$$R = \frac{\Delta x_i}{k_i A}$$

- all fluxes must equal: $q = Aq_1'' = Aq_2'' = \ldots = Aq_n'' = Aq_0''$

$$q_0'' = -k_1 \frac{dT_1}{dx} \quad \text{integrate} \quad -\frac{q_0''}{k_1}\Delta x_1 = T_2 - T_1$$

$$q_0'' = -k_2 \frac{dT_2}{dx} \quad \rightarrow \quad -\frac{q_0''}{k_2}\Delta x_2 = T_3 - T_2$$

$$\vdots$$

$$q_0'' = -k_n \frac{dT_n}{dx} \quad \rightarrow \quad -\frac{q_0''}{k_n}\Delta x_n = T_{n+1} - T_n$$

- for faces:

$$q_0'' = h_a (T_a - T_1)$$

$$q_0'' = h_b (T_n - T_b)$$

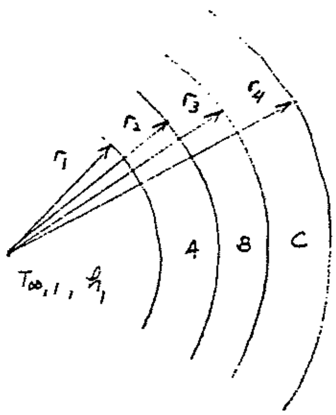
- add all $\rightarrow$

$$q_0'' \left[\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \ldots + \frac{\Delta x_n}{k_n} + \frac{1}{h_a} + \frac{1}{h_b} \right] = T_a - T_b$$

$$q_0'' = U (T_a - T_b)$$

$$U = \frac{1}{\frac{1}{h_a} + \frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \ldots + \frac{\Delta x_n}{k_n} + \frac{1}{h_b}}$$

• for cylindrical heat exchanger



$T_{\infty,4}, h_4$

– energy balance

$$q'' \cdot 2\pi r L \Big|_{r+\Delta r} - q'' \cdot 2\pi r L \Big|_{r} = 0$$

$$\frac{d(rq'')}{dr} = 0 \qquad q'' = -k\frac{dT}{dr}$$

$$\vdots$$

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln\left(\frac{r_1}{r_2}\right)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

$$q'' = \frac{2\pi L k \, (T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

addition of resistances:

$$R_{cond} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k}, \qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$q = 2\pi r_1 L \cdot U_1 \, (T_{\infty,1} - T_{\infty,4}) \rightarrow$$

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k_A}\ln\frac{r_2}{r_1} + \frac{r_1}{k_B}\ln\frac{r_3}{r_2} + \dots + \frac{r_1}{k_n}\ln\frac{r_n}{r_{n-1}} + \frac{r_1}{h_{n+1}}}$$

U_1 based on A_1.

• sphere

$$R_{cond} = \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k}, \qquad \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

(31) NTU method

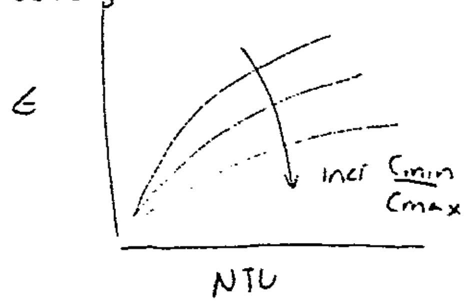
$$q = \epsilon C_{min} (T_{H_i} - T_{c_i})$$

H = hot

c = cold

$$NTU = \frac{UA}{C_{min}} \qquad C = m\bar{c}_p$$

use correlations



combine with $q = mc_p \Delta T_{lm}$ eqn: and solve

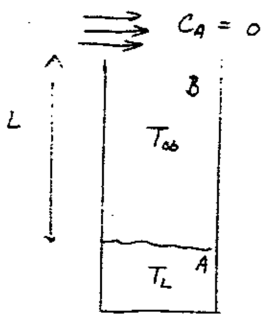
2 eqns 2 unknowns.

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} \quad \text{effectiveness factor}$$

$$NTU = \frac{UA}{C_{min}}$$

33 IS THE ~~HEAT~~ MASS FLUX FROM A LIQUID INTO A GAS USUALLY $\boxed{\text{HIGHER}}$ OR LOWER IF THE GAS IS INSOLUBLE (VS SOLUBLE) IN THE LIQUID ?

- Compare equimolar counter diffusion vs. non-diffusing B.



$$q'' = h(T_L - T_\infty)$$

$$N_A = k(C_{A_s} - C_A^\infty) = kC_{A_s}$$

consider diffusion to find k, then use Chilton-Colburn to find h.

– Non diffusing B

$$N_A = -c\mathcal{D}_{AB}\frac{dx_A}{dz} + x_A(N_A + \cancel{N_B}) \quad \rightarrow \quad N_A = \frac{-c\mathcal{D}_{AB}\frac{dx_A}{dz}}{1-x_A}$$

$$\frac{dN_A}{dz} = 0 \quad \rightarrow \quad N_A = C_1 = -\frac{c\mathcal{D}_{AB}\frac{dx_A}{dz}}{1-x_A}$$

$\vdots$

$$c\mathcal{D}_{AB}\ln(1-x_A) = C_1 z + C_2$$

B.C.
$z = L, \quad x_A = 0$
$z = 0, \quad x_A = \frac{P_A^s}{P} = x_{A_0}$

$$\text{B.C} \Rightarrow \quad C_1 = -\frac{c\mathcal{D}_{AB}\ln(1-x_{A_0})}{L}$$

$$C_2 = c\mathcal{D}_{AB}\ln(1-x_{A_0})$$

$$x_A = 1 - \exp\left[\frac{1}{c\mathcal{D}_{AB}}(C_1 z + C_2)\right]$$

$$\frac{dx_A}{dz} = -\frac{1}{c\mathcal{D}_{AB}}C_1 \exp\left[\frac{1}{c\mathcal{D}_{AB}}(C_1 z + C_2)\right]$$

– Equimolar counter diffusion

$$N_A = -c\mathcal{D}_{AB}\frac{dx_A}{dz} \quad \rightarrow \quad x_A = -\frac{1}{c\mathcal{D}_{AB}}(C_1 z + C_2)$$

$$C_1 = \frac{c\mathcal{D}_{AB}x_{A_0}}{L}$$

$$C_2 = -c\mathcal{D}_{AB}x_{A_0}$$

(34)

$$J_H = \frac{h}{C_p \rho V}(N_{Pr})^{2/3}$$

$$j_H = \frac{f}{2} = \frac{h}{C_p \rho V}\left(\frac{C_p \mu}{k}\right)^{2/3}$$

algebra ↙

$$J_H = \frac{hD}{k}\left(\frac{C_p \mu}{k}\right)^{-1/3}\frac{\mu}{\rho V D}$$

$$J_H = Nu\,(Re)^{-1}(Pr)^{-1/3}$$

$$j_H \equiv St \cdot Pr^{2/3} = \frac{Nu}{Re\,Pr}Pr^{2/3} = \frac{Nu}{Re\,Pr^{1/3}}$$

$$j_D \equiv St_m\,Sc^{2/3} = \frac{Sh}{Re\,Sc}Sc^{2/3} = \frac{Sh}{Re\,Sc^{1/3}}$$

$$Nu = \frac{hL}{k} \qquad Sh = \frac{h_m L}{\mathcal{D}_{AB}}$$

$$Re = \frac{\rho V L}{\mu} \qquad Pr = \frac{\nu}{\alpha} = \frac{\mu}{\rho}\cdot\frac{\rho c_p}{k} = \frac{\mu c_p}{k}$$

36 HOW WOULD YOU SEPARATE O_2 FROM SALT WATER? WHAT VARIABLES AFFECT SOLUBILITY? WHERE IS THE MASS TRANSFER RESISTANCE?

$1 \equiv H_2O$
$2 = O_2$

$$y_2 \phi_2 P = H_{2,1} x_2$$

$$H_{2,1}(P) = H_{2,1}(P_1) \left[\exp \int_{P_1}^{P} \frac{\bar{v}_2^{\infty}}{RT} \, dP \right]$$

$$\therefore \quad x_2 = \frac{y_2 \phi_2 P}{H_{2,1}(P_1) \exp\left[\frac{\bar{v}_2^{\infty}}{RT}(P - P_1) \right]}$$

$\therefore$ decreasing P $\uparrow$ x_2

→ throttle sea water through valve to tank under vaccuum. O_2 bubbles will form.

→ most of the mass transfer resistance is on the liquid phase side because it is in this phase that O_2 is scarce

(40)

$$\rho C_p V \frac{dT}{dt} = WC_p T_{in} - WC_p T$$ nrg balance

- eliminated T_{ref}
- used mass balan

- put in deviation vars
- take Laplace

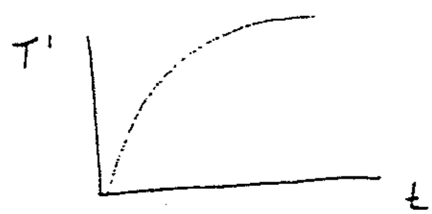
$$\rho C_p V \, s T'(s) = W C_p T_{in}'(s) - W C_p T'(s)$$

$$T'(s) = \frac{T'_{in}(s)}{\tau s + 1}$$

ie 1st order system

$$T_{in}'(s) = \frac{m}{s}$$ m is magnitude of step change

$$T'(t) = m(1 - e^{-t/\tau})$$



would level off @ a new temp

39 WHY ARE ANALOGIES BETWEEN HEAT & MASS TRANSFER MUCH MORE STRAIGHT FORWARD TO USE THAN ANALOGIES BETWEEN MASS & MOMENTUM TRANSFER?

Flux of heat and mass are vectors but momentum is a tensor.

42. WHAT IS THE THEORETICAL BASES FOR ALL THE "FAMOUS" ANALOGIES BETWEEN HEAT, MASS, AND MOMENTUM TRANSPORT?

must ignore — must ignore

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} = -\cancel{\frac{\partial P}{\partial x}}\frac{1}{\rho} + \nu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \cancel{g_x}$$

$$\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \cancel{\Phi} \quad \text{must ignore}$$

$$\frac{\partial C}{\partial t} + v_x\frac{\partial C}{\partial x} + v_y\frac{\partial C}{\partial y} + v_z\frac{\partial C}{\partial y} = \mathcal{D}_{AB}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + \cancel{R_A} \quad \text{must ignore}$$

Write variables in dimensionless forms:

$$v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_0}{T_\infty - T_0}$$

$$C^* = \frac{C - C_0}{C_\infty - C_0}$$

let ξ = dimensionless variable, then all eqns collapse down to

$$\frac{\partial \xi}{\partial t} + \vec{\xi}^* \cdot \nabla \vec{\xi} = \left\{ \begin{array}{ll} \frac{1}{Re} & \text{momentum} \\ \frac{1}{Re\,Pr} & \text{heat} \\ \frac{1}{Re\,Sc} & \text{mass} \end{array} \right\} \nabla^2 \xi$$

(43) Skin friction drag - due to tangential force of fluid on body

form drag = pressure drag - due to normal force of fluid or body

45. WHY DOES FROST NOT FORM UNDER A TREE WHEN IT IS ON THE GROUND ALL AROUND THE TREE?

- insulating stagnant air under foilage keeps convection to a minimal & therefore T is higher.
- also consider radiative insulation. Foilage blocks radiative loss of heat to sky.

48. WRITE THE MOLECULAR TRANSPORT EQUATIONS (CONSTITUTIVE EQUATIONS) FOR: (assume constant properties)

a) MASS TRANSFER

$$\frac{\partial c}{\partial t} + \vec{v} \cdot \vec{\nabla} C = \mathcal{D}_{AB} \vec{\nabla}^2 C + R_A$$

b) MOMENTUM TRANSFER

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = -\vec{\nabla} p + \mu \vec{\nabla}^2 \cdot \vec{v} + \rho \vec{g}$$

c) HEAT TRANSFER

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T \right) = k \vec{\nabla}^2 T - T \left(\frac{\partial p}{\partial T} \right)_\rho (\vec{\nabla} \cdot \vec{v}) + \mu \Phi_v$$

(49) Basic 1-D model

mass $$\frac{d(u_s C_A)}{dz} = r_A \rho_B$$

nrg $$u_s \rho_g C_p \frac{dT}{dz} = (-\Delta H) r_A \rho_B - \frac{4U}{d_t}(T - T_r)$$

momentum $$-\frac{dP_t}{dz} = f \frac{\rho_g u_s^2}{d_p}$$

Ergun eqn:

$$F_k = AKf \Rightarrow \frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho v_0^2} = \frac{L}{D_p} \cdot 4f$$

$v_0 \equiv$ superficial velocity

$D_p \equiv$ partical diameter

recall laminar flow → $$Q = \frac{\pi (\Delta \mathcal{P}) R^4}{8 \mu L} = \langle v \rangle \pi R^2$$

$$\langle v \rangle = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R^2}{8 \mu L}$$

$$\Rightarrow \quad \langle v \rangle = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R_h^2}{8 \mu L}$$

$$R_h \equiv \frac{\text{cross section area for flow}}{\text{wetted perimeter}}$$

$$= \frac{\text{volume for flow}}{\text{wetted surface}} = \frac{\dfrac{\text{volume of voids}}{\text{bed volume}}}{\dfrac{\text{wetted surface}}{\text{volume of bed}}} = \underline{\underline{\frac{\varepsilon}{a}}}$$

$a = a_v(1 - \varepsilon) \qquad D_p = \frac{6}{a_v} \qquad v_0 = \langle v \rangle \varepsilon$

$$\therefore \quad v_0 = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} \frac{D_p^2}{2(36\mu)} \frac{\varepsilon^3}{(1-\varepsilon)^2}$$

(52) need flowing air stream past wick for derivation to be valid
air humidity must be constant

Stagnant air. b.l. may build up w/ higher humidity.

51. WHAT IS INSIDE A LIGHTBULB & WHY ?

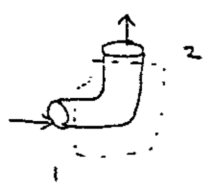
We want radiation of light, so a vacuum exists to prevent Conduction & Convection

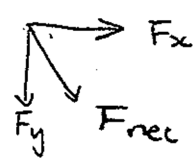
54. HOW DOES A LAWN SPRINKLER WORK ?

- Macroscopic momentum balance

$$\boxed{\frac{dP}{dt} = \rho\langle v_x^2\rangle_1 A_1 - \rho\langle v_x^2\rangle_2 A_2 + P_1 S_1 - P_2 S_2 + m_{tot}\, g_x - F_x}$$

total momentum change — momentum change by flow — momentum change by pressure — force acting on c.v



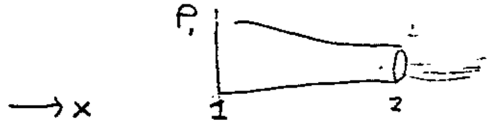


x mom: $0 = \rho\langle v_1^2\rangle A + P_1 A - F_x \rightarrow F_x = \rho\langle v_1^2\rangle A + P_1 A$

y mom: $0 = -\rho\langle v_2^2\rangle A - P_1 A - F_y \rightarrow F_y = -\left(\rho\langle v_2^2\rangle A + P_2 A\right)$

$\therefore$ lawn sprinkler rotates !

22) Steady momentum balance on a horizontal nozzle



control volume selected so that it doesn't include pipe wall

$\dot m_1 = \dot m_2 = \dot m$

$V_2 > V_1$ smaller area

evaluate upstream pressure using mechanical nrg balance

$$\frac{V_1^2}{2} + \frac{P_1}{\rho} = \frac{V_2^2}{2} + \frac{P_2}{\rho}$$

$P_2 = 0$ gauge pressure

$\rho_{H_2O} = 1000 \frac{kg}{m^3}$

$$P_1 = \frac{(V_2^2 - V_1^2)}{2}\rho = \text{+ve large}$$

← -- looking @ numerical examples

for $Q = 0.03154 \frac{m^3}{s}$

$D_1 = 0.0635 m$

$D_2 = 0.0286 m$

$P_1 = 1.156 \times 10^6 \frac{N}{m^2}$

$V_1 = 9.96 m/s$

$V_2 = 49.1 m/s$

$$R_x = \underbrace{\dot m V_2 - \dot m V_1}_{+ve} + \underbrace{P_2 A_2}_{} - \underbrace{P_1 A_1}_{+ve}$$

in numerical examples that I have seen

F_x is $-ve$ = force of nozzle on fluid

to maintain this tension

$\boxed{\text{FIREMEN MUST PULL}}$

57. CONSIDER A DEPARTMENT STORE PING PONG BALL "FLOATING" ABOVE A VACUUM CLEANER DISCHARGE. WHAT DETERMINES HOW HIGH THE BALL WILL BE? WHAT KEEPS THE BALL FROM MOVING LATERALLY OUT OF THE PATH OF THE AIR? WHAT DOES THE VELOCITY PROFILE LOOK LIKE CLOSE TO, AROUND, AND ABOVE THE BALL? WHAT DETERMINES WHETHER THE BALL WILL FALL TO THE GROUND IF THE JET IS POINTING AT AN ANGLE RATHER THAN STRAIGHT UP?