TRANSPORT PRELIM QUESTIONS

1. What is the difference between:
   a. ~~heat and mass transfer?~~
   b. heat and momentum transfer?
   c. mass and momentum transfer?

2. Set up equations to describe:
   a. wet bulb thermometer
   b. iceberg being towed in the ocean
   c. burning carbon particle

3. Will heat transfer affect the friction factor? In what way?

4. What is the difference between diffusivity and a mass transfer coefficient? $\frac{\nu}{\alpha}$ $\frac{\mu C_p}{k}$

5. Why is the Prandtl number greater for liquids than for gases?

6. What is the Sherwood number?

7. Do neon and argon have the same atomic radius? If not, which is larger? Which has the larger (a) diffusivity; (b) viscosity; (c) heat capacity; and (d) Prandtl number? Ne $\frac{\mu C_p}{k}$

8. Consider the problem of pumping oil down the Alaskan pipeline. Given the pipe diameter and length, and the properties of the oil, how would you calculate the (a) pump sizes; (b) heat loss; (c) temperature profile?

9. Describe and give the governing equations for (a) and orifice meter; (b) a venturi meter; (c) a pitot tube.

10. Give the following:
    a. Bernoulli's equation
    b. Hagen-Poiseuille law
    c. Stokes law
    d. Continuity equation
    e. Navier-Stokes equations

11. What is an NTU and how do you calculate it?

12. Describe the use of a McCabe-Thiele diagram.

13. How do the following vary with temperature and pressure?
    a. diffusivity
    b. dynamic viscosity
    c. thermal conductivity
    d. heat capacity

e. ~~heat transfer coefficient~~
f. kinematic viscosity

14. What is the Reynolds analogy? the Chilton-Colburn analogy?

J 15. ~~What is the friction factor? the coefficient of friction?~~

16. Derive the boundary-layer equations.

17. Sketch the governing diagrams for a stripper and an absorber.

18. Describe the friction factor (drag coefficient) vs. Re relation for a (a) pipe, (b) sphere, (c) flat plate, etc.

J 19. Sketch the shear stress profile for a pipe.

20. Define the most commonly used dimensionless parameters and describe their significance.

21. Given a pool of organic liquid (such as from a spill), how would you estimate its rate of evaporation?

22. How are the diffusivity and viscosity of a mixture determined?

J 23. Sketch the temperature profile in a heat exchanger.

24. What phenomena are important during an underground explosion? (e.g. bulk flow, diffusion, etc.)

25. Consider a drop falling down a tower, initial temperature and tower temperature are given. How does the drop temperature change as it falls?

26. What is the angular dependence of the Nusselt number for a falling drop?

J 27. Draw the boiling curve and describe the physical phenomena responsible for the observed behavior. Draw and explain the similar curve for condensation.

28. Given the free stream velocity and particle diameter, calculate the boundary layer thickness at a 45 degree angle. What is the pressure at the forward and backward stagnation points? What causes the difference?

29. Derive the steady state momentum balance for fully developed laminar flow in a pipe. [illegible] equation

J

30. How is the overall heat transfer coefficient for a heat exchanger found?

J 31. ~~Given two temperatures and a knowledge of all the fluids'~~ properties in a double pipe countercurrent heat exchanger, how do you calculate the other two temperatures?

32. Derive equations describing the wet-bulb/dry-bulb psychrometer. Obtain a relation between the wet-bulb temperature and air humidity in terms of dimensionless groups.

33. Is the heat flux from a liquid into a gas usually higher or lower if the gas is insoluble (versus soluble) in the liquid? This compares "diffusion through a stationary component" with the extreme case of "equimolar counterdiffusion." with Ficks law

34. The Chilton-Colburn j-factor for heat transfer is proportional to h, the convective heat transfer coefficient. Why is j proportional to $(Pr)^{-1/3}$? Why is j only a fraction of Re? Why does j decrease a Re increases?

J 35. O_2 and N_2 leaks from pressurized tanks are often considered less dangerous than H_2 leaks. Why? (Answer is best described using Joule-Thompson coefficient.)

36. How would you separate oxygen from salt water? Suppose you were processing fairly large volumes so that energy efficiency is a strong consideration. What thermodynamic variables affect solubility? Where is the mass transfer resistance? What type of unit operation would you use? How would you design it?

37. What area is used when defining friction factor for a wetted wall column?

38. What is the Lewis relation? Is it dependent on the gas phase velocity? Why or why not?

J 39. Why are analogies between mass and heat transfer much more straightforward to use than analogies between mass and momentum transfer?

40. Given a CSTR at temperature T with no reaction what would happen if the inlet temperature were suddenly increased?

WT 41. Analogies between heat, mass and momentum transport are important. Give examples of when they don't hold.

42. What is the theoretical basis for all the "famous" analogies between heat, mass and momentum transport? What are the

mass and heat transfer equivalents of the momentum transport equation?

J 43. What is the difference between skin friction drag and form drag?

tangential forces

due to flow ΔP induced. normal to object

44. How would you determine a mass transfer coefficient experimentally?

45. Why does frost not form under a tree when it is on the ground all around the tree?

46. Draw a McCabe-Thiele diagram for a distillation column that uses a reacting absorbent.

? J 47. What are the most commonly used (3) correlations describing heat and mass transfer?

48. Write the molecular transport equations (constitutive equations) for:
   a. mass transfer
   b. momentum transfer
   c. heat transfer.

49. Give the equations describing flow in a packed bed. Ergun

50. Derive the equations for gas undergoing an isentropic expansion.

J 51. What is inside a light bulb, and why?

52. Why do you have to whirl a wet-bulb/dry-bulb psychrometer in the air prior to reading it?

53. In which direction is the momentum flux from a fluid flowing over a flat plate?

54. How does a lawn sprinkler work?

J 55. Consider firefighters holding a high pressure hose, must they pull or push the hose? Why?

56. For a double plate window with insulating gas between the panes draw the temperature profile from inside the warm room, through the windows and to the outdoors. Allow for natural convection both in the room and in the gas between the two plates. What gas would you recommend using and why?

57. Consider the department store ping-pong ball "floating" above a vacuum cleaner discharge. What determines how high the ball will be? What keeps the ball from moving laterally out of the

path of the air? What does the velocity profile look like close to, around and above the ball? What determines whether the ball will fall to the ground if the jet is pointing at an angle rather than straight up?

58. You have two infinite parallel plates initially at rest with a fluid between them. One plate remains fixed, the other is set in motion at velocity V. What do the transient velocity profiles look like? What does the steady state look like? Why? What is the driving force for fluid flow in a pipe? What is the driving force here? Describe a momentum balance. What equiation would you use to describe this. Simplify the equation to obtain a differential equation. How would one determine the force necessary to keep the top plate moving at V? (Graves)

59. For the system in number 58, determine a characteristic time. Which would take longer for the steady state profile to be reached, molasses or water and why?

60. You have a small sphere of moltem metal. How far will it drop (in air) before it solidifies? What does the Biot number tell you here? How do you find the convective heat transfer coefficient? (Blanch)

61. For a particle dropping in a fluid field derive the equation for the terminal velocity and discuss the friction factor coefficient.

62. Why do they put dimples on a golf ball?

63. Consider laminar flow in a pipe. Write out the momentum equation appropriate for this geometry. Drop all terms which are identically zero. You should end up with one equation and only two terms in it, if you neglect gravity (or include it in the pressure term). Reduce the equation to a non-dimensional form. Use L for a characteristic length and V for a characteristic velocity. One should notice that Re does not appear in the non-dimensional form. Why then is the Re number so important in determining if a flow in a pipe is laminar or turbulent?

TRANSPORT

① What is the difference between :

(a) Heat and mass transfer ?

Heat transfer is transferring of energy in the form of heat. The driving force is a temp difference. Transfer by conduction, convection, and radiation.

Mass transfer is transfer of a species. Driving force for mass transfer is concentration gradient. However, mass is also transferred by bulk flow.

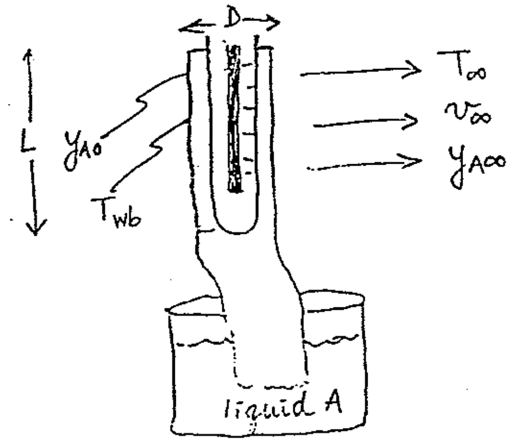
(b) Momentum transfer is the transfer of momentum. It is transferred by bulk flow, by shear stress (where driving force is velocity gradient), and by any forces acting on the system.

(c) See above

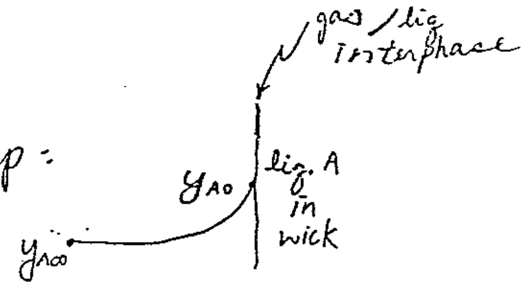
Van't Hoff
Moody Chart
Fenske

② Set up equations to describe :

(a) Wet bulb thermometer



Close-up :



Energy balance :

$$\boxed{N_A \cancel{\pi} \cancel{D} \cancel{L}\, \Delta h^{vap} = h\,(T_\infty - T_{wb})\cancel{\pi}\cancel{D}\cancel{L}}$$

From mass balance :

$$N_A = k_G(y_{A0} - y_{A\infty}) + y_{A0} N_A$$

$$N_A(1 - y_{A0}) = k_G\,(y_{A0} - y_{A\infty})$$

$$N_A = \frac{k_G(y_{A0} - y_{A\infty})}{(1 - y_{A0})}$$

$$\frac{k_G(y_{A0} - y_{A\infty})}{(1 - y_{A0})}\Delta h^{vap} = h\,(T_\infty - T_{wb})$$

Assume y_{A0} is in equil. with liq. A $\Rightarrow$ $y_{A0} = \dfrac{P_A^S(T_{wb})}{P}$

Also, can use one of the analogies (like Chilton-Colburn) to get the ratio of $\frac{h}{k_G}$:

$$j_H = j_D$$

$$\frac{Nu}{Pe_H} Pr^{2/3} = \frac{Sh}{Pe_D} Sc^{2/3}$$

$$Pr^{2/3} \frac{Nu}{Re\,Pr} = \frac{Sh}{Re\,Sc} Sc^{2/3}$$

where $Nu = \dfrac{hL}{k_{air}}$; $Sh = \dfrac{k_G L}{\mathcal{D}_{A-air}}$

$Re = \dfrac{v_\infty \rho_{air} D}{\mu_{air}}$; $Pr = \dfrac{\mu C_p}{k_{air}}$; $Sc = \dfrac{\mu}{\rho \mathcal{D}_{A-air}}$

(b) Iceberg towed in the ocean

Since iceberg not spherical, assume hydraulic radius:

$$\boxed{R_h = \frac{\text{volume of iceberg}}{\text{total wetted area}}}$$

Consider the amount of ice melting:

Δh_m = heat needed to melt unit mass of ice

heat balance: $\dot{m}\,\Delta h_m = h\,(\text{area})(T_\infty - T_s)$ ← heat transfer by convection

h: heat xfer coeff. area $\approx \pi D_h^2$ where $D_h = 4R_h$ $\quad T_s \approx 32°F$

Force needed to drag iceberg:

If we assume iceberg ≈ spherical, use Stoke's law ($Re < 1$)

$$\boxed{Re = \frac{\rho v D_h}{\mu}}$$ (see if Stoke's Law holds)

moving slowly: assume creeping flow

If Stoke's law holds: $\boxed{F_d = 6\pi\mu R_h v}$

We can estimate h (heat transfer coeff) for flow past a sphere:

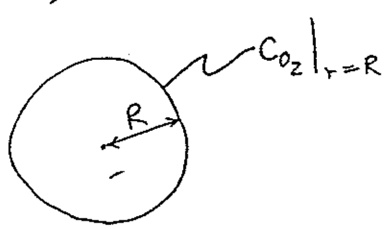
$$Nu = 2.0 + 0.60\,Re^{1/2}\,Pr^{1/3}$$

where $Nu = \dfrac{h D_h}{k_f}$, $Re = \dfrac{\rho_f D_h v}{\mu_f}$, and $Pr = \left(\dfrac{C_p \mu}{k}\right)_f$

(c) Burning of a carbon particle

$$C + O_2 \longrightarrow CO_2 \quad \text{(assume complete combustion)}$$

Assume that the process is diffusion limited (of O_2 into the particle).



Fick's law : $N_{O_2} = -\mathcal{D} \frac{dC_{O_2}}{dr}$

<u>Mole balance over spherical shell :</u>

reaction of O_2

$$-\mathcal{D} \frac{dC_{O_2}}{dr}(4\pi r^2)\Big|_{r+\Delta r} + \mathcal{D} \frac{dC_{O_2}}{dr}(4\pi r^2)\Big|_{r} + r_{O_2} 4\pi r^2 \Delta r$$

$$= 0 \quad \text{at S.S.}$$

We have a second order DE and assuming that r_{O_2} depends only on C_{O_2}, we can solve the DE for C_{O_2} and thus get r_{O_2}. Then we know how fast the particle is reacting.

③ Will heat transfer affect the friction factor? In what way?

The friction factor $f = f(Re, L/D)$ for a flow in pipe.

$$Re = \frac{\rho v D}{\mu} = f(T)$$

Since heat transfer affects the T of the fluid inside the pipe, it will affect density and viscosity as well as velocity (indirectly).

④ What is the difference between diffusivity and a mass transfer coefficient?

Diffusivity is the proportionality constant for molecular diffusion as described by Fick's law. The driving force is the concentration gradient whithin one phase.

The mass transfer coeff. is a proportionality const. for both diffusive & convective mass transfer between a solid and a moving phase or between two relatively immiscible fluids. The driving force is the difference in concentration between the 2 phases. The mass Xfer is $\perp$ to the phase boundary

⑤ Why is the Prandtl number greater for liquids than for gases?

$$Pr = \frac{\mu/\rho}{\frac{k}{C_p\rho}} = \frac{\text{molecular momentum transport}}{\text{molecular heat transport}}$$

$$= \frac{\mu C_p}{k} = \frac{\text{viscous discipation of energy}}{\text{heat conduction}}$$

Viscosities and heat capacities are always higher for liquids than for gases.

⑥. What is the Sherwood number?

It is the "Nusselt number" for mass transfer.

$$Sh = \frac{kL}{\mathcal{D}} = \frac{\text{total mass transfer}}{\text{molecular mass transfer}}$$ $\boxed{?}$

⑦. Do neon and argon have the same atomic radius?

No, <u>Argon has a larger radius</u>.

Which has a larger <u>viscosity</u>?

assuming ideal gas, according to kinetic theory of gases:

$$\mu \propto \frac{\sqrt{m}}{d^2}$$

m ← molec. weight

d ← collision diameter

So <u>μ is more strongly dependent on d</u> $\Rightarrow$ $\boxed{\mu_{Ne} > \mu_{Ar}}$

Which has a larger <u>diffusivity</u>?

Also, according to kinetic theory of gases,

$$\mathcal{D}_{AB} \propto \frac{\left[(m_A+m_B)/(m_A \cdot m_B)\right]^{1/2}}{\sigma_{AB}^2}$$

σ_{AB} ← something like avg. collision diameter

$$\mathcal{D}_{AA} \propto \frac{(2/m_A)^{1/2}}{d_A^2} = \frac{4}{\sqrt{m_A}\, d_A^2}$$

$$\therefore \boxed{\mathcal{D}_{Ne-Ne} > \mathcal{D}_{Ar-Ar}}$$

Which has larger heat capacity?

For ideal gas, $\boxed{C_v = \frac{3}{2}R}$ for monoatomic gases

Therefore, the heat capacities should be very close.

$$\boxed{C_{p,Ar} \approx C_{p,Ne}}$$

Which has a larger Pr? $\boxed{Pr = \frac{\mu C_p}{k}}$

where $k = \frac{5}{2} C_v \mu$ for monoatomic gases

$$Pr_{Ar} = \frac{\cancel{\mu_{Ar}} C_{p,Ar}}{\frac{5}{2} C_{v,Ar} \cancel{\mu_{Ar}}} \qquad Pr_{Ne} = \frac{C_{p,Ne}}{\frac{5}{2} C_{v,Ne}} \Rightarrow \boxed{Pr_{Ar} \approx Pr_{Ne}}$$

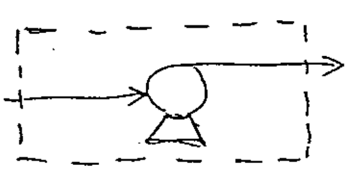
(8) Consider problem of pumping oil down Alaskan pipeline. Given pipe diameter and length, and properties of oil, calculate:

(a) pump sizes

Pumps: ΔP, $Q \rightarrow$ Power

friction is important

Find the ΔP that a pump must provide in order to pump the oil through pipe of length L and diameter D.



2 turbulent
1 laminar

acting on

$$\frac{\Delta P}{\rho} + g\Delta h + \alpha \frac{\Delta v^2}{2} + \frac{E_k|^{A_{cs}}}{\rho} = 0$$

We must also know $Q \Rightarrow$ which gives $\langle v \rangle$.

Get $Re = \frac{\rho \langle v \rangle D}{\mu} \Rightarrow$ look up chart to get f

Then, f is related to ΔP thru: $\boxed{F_k = A f K} = (2R\pi L)(\frac{1}{2}\rho\langle v \rangle^2) f$

and $F_k = \pi R^2 p_0 - \pi R^2 p_L + (h_0 - h_L)\pi R^2 \rho g$

After finding ΔP, plot ΔP vs. Q on a pump chart and select most appropriate pump.

(b) heat loss

Need to get Re and Pr.

$$Re = \frac{\rho \langle v \rangle D}{\mu}$$

$$Pr = \frac{\text{momentum transfer due to shear force}}{\text{heat transfer due to conduction}}$$

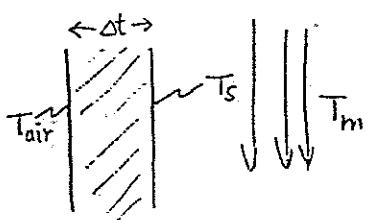
$$= \frac{\mu/\rho}{k/C_p\rho} = \frac{\mu C_p}{k}$$

Then use correlation to get $Nu = \frac{hD}{k_f} = \frac{\text{total heat xfer}}{\text{diffusive heat xfer}}$

$$q'' = h\,(T_m - T_s)$$

to find this, we need temp. profile as function of r

where $T_m = \frac{2}{\langle v \rangle R^2} \int_0^R v\,T\,r\,dr$



Total heat Xfer by convection from flowing fluid to inner wall of pipe :

$$h\,(T_m - T_s)(2\pi R L) = q$$

Total heat Xfer by conduction through pipe thickness Δt :

$$\frac{2\pi \Delta t\, k_p\,(T_s - T_{air})}{\ln\left(\frac{R}{R+\Delta t}\right)} = q$$

Solve for T_s and then find q !!

(c) temperature profile (temp is a function of x and r)

Pretty difficult to find $T(x,r)$. So only find $T_m(x)$.
↑ mean temp.

Energy bal:

$$-P\Delta x\, q''_{conv} = \dot{m}c_p T_m\big|_x - \dot{m}c_p T_m\big|_{x+\Delta x}$$

$$P q''_{conv} = \dot{m}c_p \, + \frac{dT_m}{dx}$$

where $q''_{conv} = h(T_s - T_m)$

$$\therefore \; +Ph(T_s - T_m) = \dot{m}c_p \frac{dT_m}{dx}$$

$$\frac{dT_m}{dx} = \underbrace{\left(\frac{Ph}{\dot{m}c_p}\right)}_{=\alpha}(T_s - T_m)$$

$$\frac{dT_m}{dx} + \alpha T_m = \alpha T_s$$

let $T_m = uv \Rightarrow \frac{dT_m}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = \alpha T_s - \alpha u v$

let $-\alpha v = \frac{dv}{dx} \Rightarrow \ln v = -\alpha x \Rightarrow v = e^{-\alpha x}$

$$e^{-\alpha x}\frac{du}{dx} = \alpha T_s \Rightarrow du = \alpha T_s e^{\alpha x} dx \Rightarrow u = T_s e^{\alpha x} + C_1$$

$$\therefore \; T_m = e^{-\alpha x}(T_s e^{\alpha x} + C_1)$$

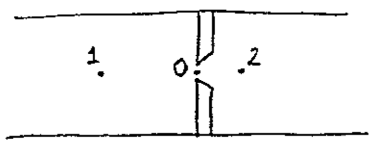
BC: $T_m = T_{m,i}$ at $x = 0$

etc...

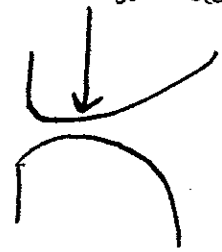
1. Bernoulli's
2. continuity

(9) Describe and give governing equations for the following meters (all for measuring flow rates)

(a) $\boxed{\text{orifice meter}}$



Suppose pt. 2 is at vena contracta



Bernoulli's eqn (assume incompressible):

$$\boxed{\frac{v_1^2}{2} + \frac{P_1}{\rho} + \cancel{gh_1} = \frac{v_2^2}{2} + \frac{P_2}{\rho} + \cancel{gh_2}}$$

do not know A_c

know A_0

Continuity eqn:

$$\boxed{v_1 A_1 = v_2 A_2 = v_2 A_0 C_c}$$

A_2 is unknown

elim. w/ a coefficient

where C_c = contraction coeff. $= \frac{A_2}{A_0}$

$$\therefore v_1 = C_c \frac{A_0}{A_1} v_2 = C_c \frac{D_0^2}{D_1^2} v_2$$

substitute v_1

$$v_2 = \left[\frac{2(P_1 - P_2)}{\rho\left(1 - C_c^2 (D_0/D_1)^4\right)}\right]^{1/2}$$

At vena contracta, $\underline{v_2 C_v = v_{2a}}$ (accounting for frictional losses).

flow rate $= Q = v_{2a} A_2 = v_{2a} C_c A_0$

$$\therefore Q = C_c C_v \left\{\frac{2\frac{\Delta P}{\rho}}{1 - C_c^2 (D_0/D_1)^4}\right\}^{1/2}$$

let $C_d = C_c C_v$

But: usually C_c and C_d not determined separately:

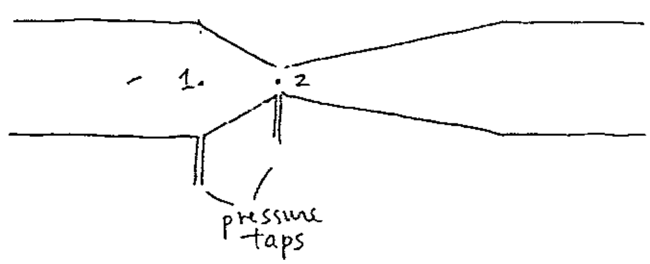
$$Q = CA_0\sqrt{\frac{2\Delta P}{\rho}}$$

$\uparrow$ experimentally determined

(b) $\boxed{\text{venturi meter}}$

– d/n have vena contracta

– now I know A_2



Assume for the moment friction losses ≈ 0

Bernoulli's eqn:

$$\boxed{\frac{1}{2}v_1^2 + \frac{P_1}{\rho} + \cancel{gh} = \frac{1}{2}v_2^2 + \frac{P_2}{\rho} + \cancel{gh} + \cancel{e_v}^{\,0}}$$

Continuity:

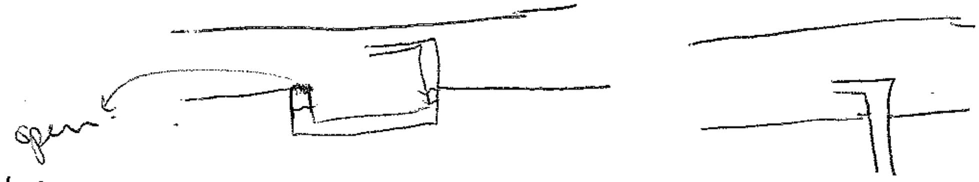
$$\boxed{A_1 v_1 = A_2 v_2}$$

$$v_1 = \frac{A_2}{A_1} v_2$$

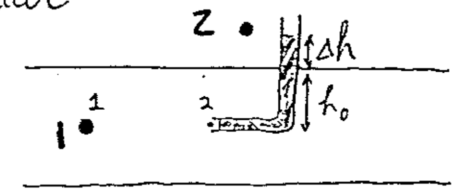
$$\frac{1}{2}\left(\frac{A_2}{A_1}\right)^2 v_2^2 + \frac{P_1}{\rho} = \frac{1}{2}v_2^2 + \frac{P_2}{\rho}$$

solve for v_2 and then $\underline{Q = v_2 A_2 C_v}$

coeff that accounts for friction losses



(c) pitot tube



pt. 2 is stagnation point

Bernoulli's eqn:

$$\frac{1}{2}v_1^2 + \frac{P_1}{\rho} = \frac{1}{2}\cancel{v_2^2}^{\,0} + \frac{P_2}{\rho} + \cancel{e_v}\ \text{negligible}$$

density of manometer fluid

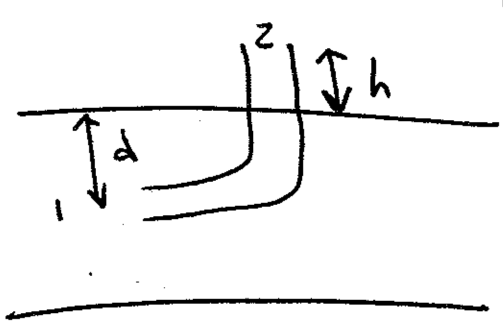
P_1 is simply the static pressure $= \rho g h_0$

$P_2 = \rho g(h_0 + \Delta h)$

solve for $v_1 \Rightarrow$ get Q

See other sheet

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} + 0 = \frac{p_2}{\rho} + \frac{0^2}{2} + g(h+d)$$



$p_1 = p_2 + \rho g d$

$u_1 = \sqrt{2gd}$

(10): Give the following:

(a) Bernoulli's equation: Steady-state Macroscopic mechanical energy balance

$$\tfrac{1}{2}v_1^2 + \frac{P_1}{\rho} + gh_1 = \tfrac{1}{2}v_2^2 + \frac{P_2}{\rho} + gh_2 \qquad \boxed{(\rho = \text{const.})}$$

$$\boxed{\frac{1}{2}\frac{\langle v_1^3\rangle}{\langle v_1\rangle} + gh_1 = \frac{1}{2}\frac{\langle v_2^3\rangle}{\langle v_2\rangle} + gh_2 + \int_{P_1}^{P_2}\frac{dP}{\rho}}$$

more rigorous

(b) Hagen–Poiseuille law: relates Q to ΔP
Obtained from momentum balance.

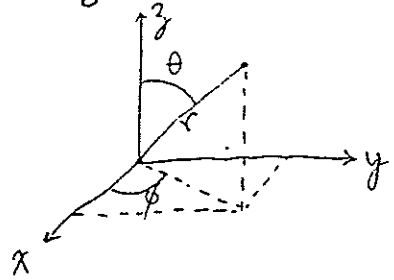
$$Q = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4}{8\mu L}$$

Assumptions:
(1) laminar flow
(2) ρ = const.
(3) v only function of r
(4) Newtonian fluid
(5) $v = 0$ at wall (no slip)

(c) Stokes law: drag force on sphere when creeping flow

$$F_k = 6\pi\mu R v_\infty$$

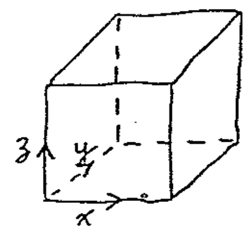
Derivation: Integration of the normal (pressure forces) in the z-direction.



$$\int_0^{2\pi}\int_0^{\pi}\left[p\cos\theta\Big|_{r=R}\right]\left[R^2\sin\theta\, d\theta\, d\phi\right]$$

Plus integration of tangential viscous forces.

(d). Continuity eqn: (in rectangular coord.)



$$v_x\rho\big|_x \Delta y\Delta z - v_x\rho\big|_{x+\Delta x}\Delta y\Delta z + v_y\rho\big|_y \Delta x\Delta z - v_y\rho\big|_{y+\Delta y}\Delta x\Delta z$$

$$+ v_z\rho\big|_z \Delta x\Delta y - v_z\rho\big|_{z+\Delta z}\Delta x\Delta y = \frac{\partial\rho}{\partial t}\Delta x\Delta y\Delta z$$

$$-\left[\frac{\partial(v_x\rho)}{\partial x} + \frac{\partial(v_y\rho)}{\partial y} + \frac{\partial(v_z\rho)}{\partial z}\right] = \frac{\partial\rho}{\partial t}$$

In vector notation,

$$\boxed{\frac{\partial\rho}{\partial t} = -(\vec{\nabla}\cdot\rho\vec{v})}$$

(e) Navier-Stokes : momentum balance

Consider x-mom. by bulk flow:

$$\rho v_x v_x\big|_x \Delta y\Delta z - \rho v_x v_x\big|_{x+\Delta x}\Delta y\Delta z$$

$$\rho v_x v_y\big|_y \Delta x\Delta z - \rho v_x v_y\big|_{y+\Delta y}\Delta x\Delta z$$

$$\rho v_x v_z\big|_z \Delta x\Delta y - \rho v_x v_z\big|_{z+\Delta z}\Delta x\Delta y$$

Consider x-mom. by velocity gradients :

$$\tau_{xx} \, \Delta y \Delta z \big|_x - \tau_{xx} \Delta y \Delta z \big|_{x+\Delta x}$$

$$\tau_{yx} \, \Delta x \Delta z \big|_y - \tau_{yx} \Delta x \Delta z \big|_{y+\Delta y}$$

$$\tau_{zx} \, \Delta x \Delta y \big|_z - \tau_{zx} \Delta x \Delta y \big|_{z+\Delta z}$$

Consider pressure forces and gravity:

$$p\big|_x \Delta y \Delta z - p\big|_{x+\Delta x} \Delta y \Delta z + \rho g_x \Delta x \Delta y \Delta z$$

Rate of accum. of mom. :

$$\frac{\partial (\rho v_x)}{\partial t} \Delta x \Delta y \Delta z$$

Combine all :

$$-\left[\frac{\partial}{\partial x}(\rho v_x v_x) + \frac{\partial}{\partial y}(\rho v_x v_y) + \frac{\partial}{\partial z}(\rho v_x v_z) \right]$$

$$-\left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx} \right] - \frac{\partial p}{\partial x} + \rho g_x = \frac{\partial (\rho v_x)}{\partial t}$$

In vector notation :

$$\boxed{-\vec{\nabla} \odot \rho \vec{v}\vec{v} - \vec{\nabla} \odot \underline{\underline{\tau}} - \vec{\nabla} p + \rho \vec{g} = \frac{\partial}{\partial t} \rho \vec{v}}$$

(11) What is an NTU and how do you calculate it?

NTU ≡ number of transfer units
(for the overall gas or liquid phases; used in continuous countercurrent contactors. Also used in heat exchanger analysis)

$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$

$$\varepsilon = 1 - e^{-NTU}$$

In heat exchanger analysis,

pacity Ratio

$R_C = \frac{(\dot{m}C_p)_{min}}{(\dot{m}C_p)_{max}}$

$$\boxed{NTU \equiv \frac{UA}{(\dot{m}C_p)_{min}}}$$

$\frac{kg}{s}\ \frac{J}{kg\,K}$

⇐ The larger the heat xfer coeff. and area, the more transfer units → thus more heat exchanged!!

NTU is used along with the effectiveness factor for heat exchangers, ε, to determine the outlet temperatures if only the inlet temps are known.

In mass transfer,

N_t is a measure of the difficulty of absorption. The larger the required change in conc. $y_{in} - y_{out}$, the more transfer units is required.

$$\boxed{N_{tOG} \approx \int_{y_{out}}^{y_{in}} \frac{dy}{y - y^*}}$$

$$\boxed{Z = N_{tOG}\, H_{tOG}}$$

height of column packing

$H_{tOG} =$

$C_f \equiv \dfrac{\tau_s}{\rho v_\infty^2/2}$ ⇐ gives us wall shear stress

(14) What is the Reynold's analogy? Chilton-Colburn analogy?

Reynold's analogy: Assumption: $Pr = 1$

Relates heat to momentum transfer.

$$\frac{C_f}{2} Re = Nu$$

replace Nu with $St_H = \dfrac{Nu}{Re\,Pr}$

Prove $C_f = f$

$$\boxed{\frac{C_f}{2} = St_H}$$

where $\boxed{C_f = \dfrac{2}{Re} \dfrac{\partial v^*}{\partial y^*}\Big|_{y=0}}$ $\quad * =$ dimensionless

this gives us shear stress at wall and thus force to keep plate from moving

$v^* = v/v_\infty$

$y^* = y/L$

Relationship between C_f and f (Fanning friction factor):

$$\boxed{F_k = \tau_{wall}(\text{Area})} = \tau_{wall}\cancel{LW} = AKf = (LW)(\tfrac{1}{2}\rho v_\infty^2)f$$

$$C_f = \frac{2}{Re}\frac{\partial\left(\frac{v}{v_\infty}\right)}{\partial\left(\frac{y}{L}\right)}\Bigg|_{wall} = \frac{2\mu}{\rho v_\infty \cancel{L}}\frac{\cancel{L}}{v_\infty}\frac{\partial v}{\partial y}\Bigg|_{wall} = \frac{2}{\rho v_\infty^2}\tau_{wall}$$

$$\cancel{\tfrac{1}{2}\rho v_\infty^2}\, C_f = f\,\cancel{\tfrac{1}{2}\rho v_\infty^2} \qquad \boxed{C_f = f} \;!!$$

Chilton-Colburn analogy:

Added a empirical parameter $Pr^{2/3}$ to account for $Pr \neq 1$

$\boxed{\dfrac{C_f}{2} = St_H\, Pr^{2/3}}$ // Also in mass xfer: $\boxed{St_D\, Sc^{2/3} = St_H\, Pr^{2/3}}$

(15) What is the friction factor? the coefficient of friction?

$$\boxed{f = \frac{F_k}{AK}}$$

where F_k = force exerted by fluid on pipe or submerged object

A = area over which the force acts or for submerged objects, the area projected onto plane $\perp$ to direction of fluid flow

K = some kind of kinetic energy per unit volume

$\boxed{Ex}$: flow in circular pipe

$$\boxed{p_0 \pi R^2 - p_L \pi R^2 + (h_0 - h_L)\rho g \pi R^2 - F_k = 0}$$

$$F_k = \pi R^2 \left[p_0 - p_L + (h_0 - h_L)\rho g \right]$$

$A = 2\pi R L$ $\qquad$ $K = \frac{1}{2}\rho \langle v \rangle^2$

Coeff. of friction

$$\boxed{C_f = f}$$

coeff. of friction

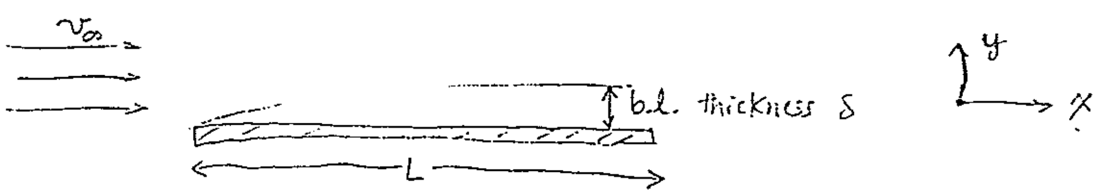
(16) Derive the boundary layer eqns

For example, start with 2-D eqns of motion: for Newtonian, incompressible fluid.

x-dir $$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$ N-S

-dir $$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

Consider fluid flowing over a flat plate:



$L \gg \delta$

Perform magnitude analysis on the various terms in the eqns of motion:

x-dir $$\rho\left(\frac{\partial v_x}{\partial t} + \frac{v_\infty^2}{L} + \cancel{\delta}\frac{v_\infty}{\cancel{\delta}}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{v_\infty^2}{L^2} + \frac{v_\infty^2}{\delta^2}\right) + \cancel{\rho g_x}^{\,0}$$

neglect ?

y-dir $$\rho\left(\frac{\partial v_y}{\partial t} + v_\infty\frac{\delta}{L} + \delta\frac{\cancel{\delta}}{\cancel{\delta}}\right) = -\cancel{\frac{\partial p}{\partial y}} + \mu\left(\frac{\delta^2}{L^2} + \frac{\delta^2}{\delta^2}\right) + \cancel{\rho g_y}$$ neglig-

We have,

x-dir $$\boxed{\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = -\frac{dp}{dx} + \mu\left(\frac{\partial^2 v_x}{\partial y^2}\right)}$$

All the terms in the eqn. of motion for y-dir. are smaller.

Along with eqn of continuity:

$$\vec{\nabla} \odot \rho\vec{v} = 0$$

$\rho = \text{const.}$

$$\vec{\nabla} \odot \vec{v} = 0$$

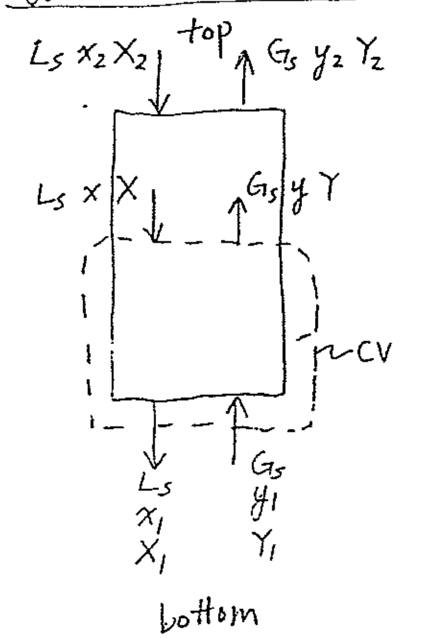
for x, y dir. only

$$\boxed{\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0}$$

⑰ Sketch governing diagrams for a stripper and absorber

Absorber → high y_i, low x_i

Countercurrent Multistage Operation (absorption of A into liq)



$$\boxed{\begin{aligned} Y_1 &= \frac{y_1}{1-y_1} \\ X_1 &= \frac{x_1}{1-x_1} \end{aligned}}$$

$= \frac{\text{moles of A in gas}}{\text{moles insoluble gas}}$

$= \frac{\text{moles of A in liq}}{\text{moles of solvent}}$

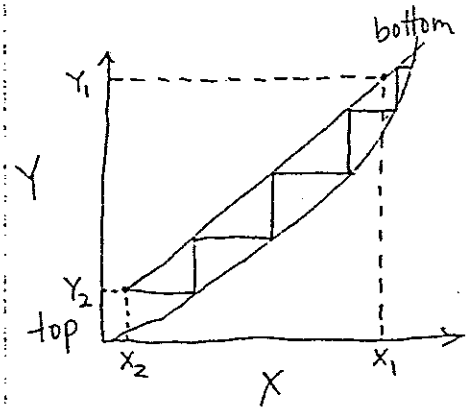
mole balance about CV:

$$L_s X + G_s Y_1 = L_s X_1 + G_s Y$$

$$G_s(Y_1 - Y) = L_s(X_1 - X)$$

$$G_s Y = L_s X + (G_s Y_1 - L_s X_1)$$

$$Y = \frac{L_s}{G_s} X + \left(\frac{G_s Y_1 - L_s X_1}{G_s}\right)$$ ⇐ straight operating line on Y-X diagram



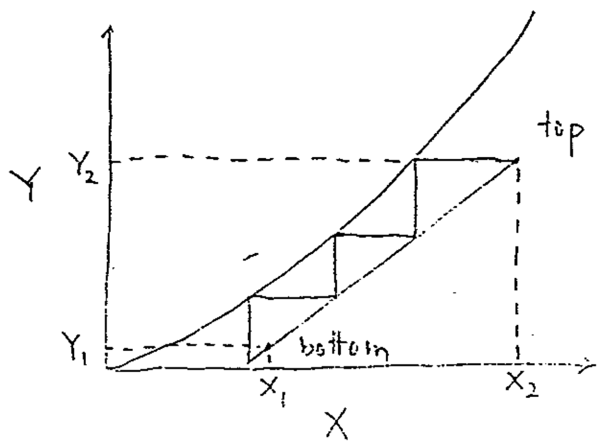
Assume that the gas & liq. leaving each tray are in equil.

Operating line above eq. line

- solute is transferred from gas to the liquid low Y_i, high X_i

Stripper : countercurrent multistage operation

Same as the absorber except operating line is below the equil. line.



- solute transferred from liquid to gas

- like the stripping section in a distillation column

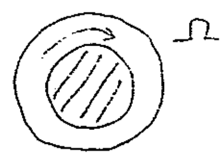
Find N_A by finding $N_A\big|_{z=0}$. Then if we know the rate at which A leaves the interface, we can find $\mathcal{D}_{AB}$.

Viscosity

Capillary viscometer: Hagen-Poiseuille law because of laminar flow.

$$Q = \frac{\pi \Delta P R^4}{8 \mu L}$$ We know $Q, R, \Delta P, L \Rightarrow$ get μ!!

Couette viscometer



Momentum balance:

$$\tau_{r\theta} \cancel{2\pi} r \cancel{L}\Big|_r - \tau_{r\theta} \cancel{2\pi} r \cancel{L}\Big|_{r+\Delta r} = 0$$

$$\frac{d(\tau_{r\theta} r)}{dr} = 0$$

$$r \frac{d\tau_{r\theta}}{dr} + \tau_{r\theta} = 0 \qquad \text{where } \tau_{r\theta} = -\mu \frac{dv_\theta}{dr}$$

$$-r\cancel{\mu} \frac{d^2 v_\theta}{dr^2} - \cancel{\mu} \frac{dv_\theta}{dr} = 0$$

solve for $v_\theta(r)$

Then get $\tau_{r\theta}\Big|_{r=r_i} = -\mu \frac{dv_\theta}{dr}\Big|_{r=r_i}$ if inner cylinder is turned

If we know $\Omega \Rightarrow$ get torque $\Rightarrow$ get μ!!

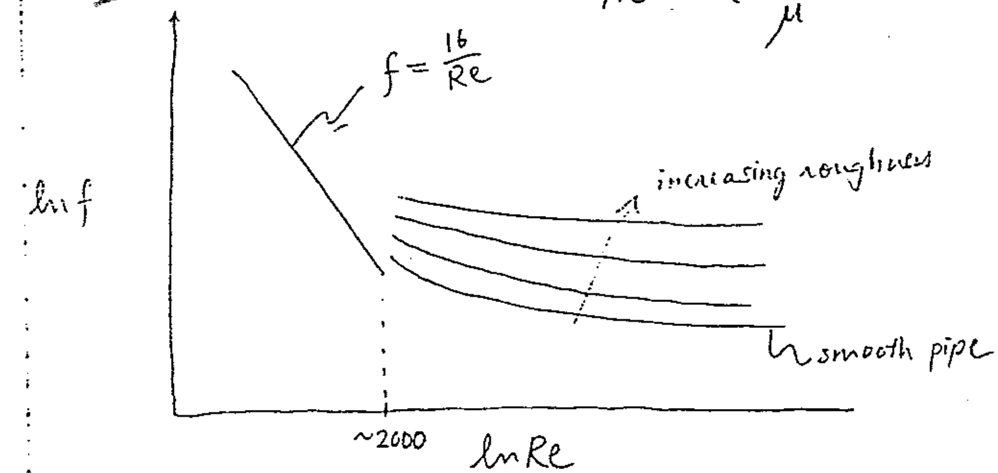
Dropping sphere viscometer (Stokes' regime)

(18) Describe the friction factor (drag coeff C_D) vs. Re relation for a:

(a) pipe

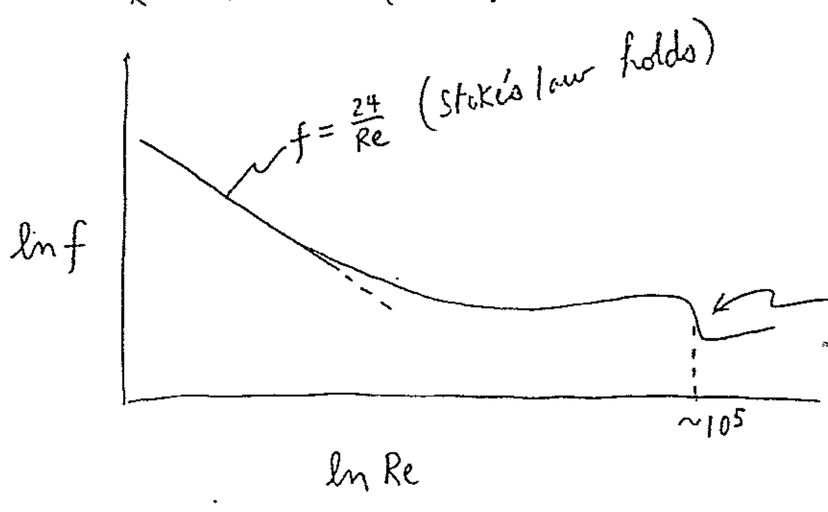
Moody chart: $f = \left(\frac{1}{4}\right)\left(\frac{D}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v\rangle^2}\right)$

$$Re = \frac{\rho\langle v\rangle D}{\mu}$$



(b) sphere

$$F_k = (\pi R^2)\left(\tfrac{1}{2}\rho v_\infty^2\right) f$$



No clear transition region from laminar to turbulent

caused by b.l. separation

Force bal on sphere:

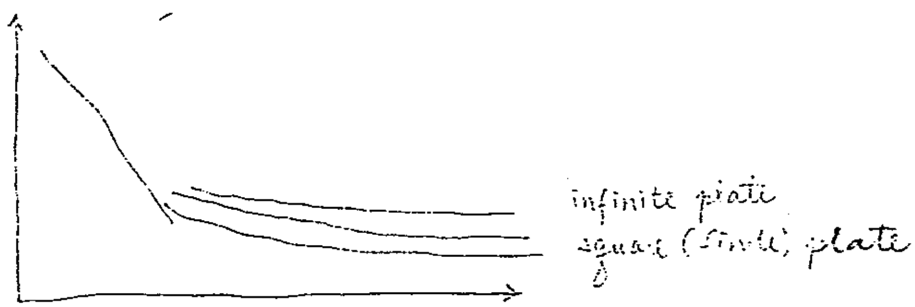
$$-\tfrac{4}{3}\pi R^3 \rho g + \tfrac{4}{3}\pi R^3 \rho_s g - F_k = 0$$

$$F_k = \tfrac{4}{3}\pi R^3 g\,(\rho_s - \rho)$$

$$\frac{2\,\frac{4}{3}\cancel{\pi} R^{\cancel{3}} g\,(\rho_s-\rho)}{(\cancel{\pi}\cancel{R^2})(\rho\, v_\infty^2)} = f$$

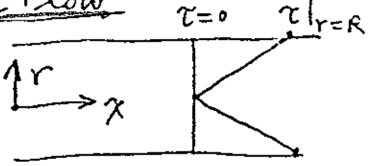
$$\boxed{f = \frac{8}{3}\,\frac{Rg}{v_\infty^2}\left(\frac{\rho_s-\rho}{\rho}\right)}$$

(c) <u>*flat plate*</u>



(19) Sketch the shear stress (τ) profile for a pipe.

__Laminar Flow__



τ_{rx} 0 at center

max at wall

__H-P:__ $Q = \dfrac{\pi(P_0-P_L)R^4}{8\mu L}$

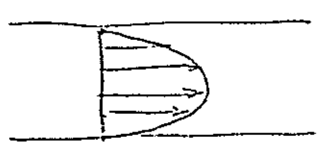
$$\boxed{\tau_{rx} = \left(\frac{P_0-P_L}{2L}\right) r}$$ linear τ_{rx} profile!!

$$-\mu \frac{dv_x}{dr} = \frac{P_0-P_L}{2L} r$$

$$\int_{v_x}^{0} dv_x = \int_{r}^{R} \frac{-(P_0-P_L)}{2\mu L} r\,dr$$

$$-v_x = \frac{-(P_0-P_L)}{2\mu L} \left.\frac{r^2}{2}\right|_r^R = \frac{-(P_0-P_L)}{2\mu L}\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$

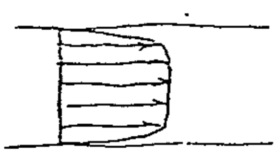
$$v_x = \frac{(P_0-P_L)}{4\mu L}(R^2-r^2)$$ parabolic profile!!



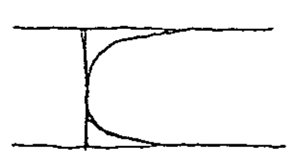
max at center

0 at wall

__Turbulent Flow__ __much flatter velocity profile__



velocity



shear stress

(20) Describe most commonly used dimensionless parameters and their significance.

<u>Momentum transfer</u>

$$Re = \frac{\rho \langle v \rangle D}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

D ← some charact. length

<u>Heat transfer</u>

$$Nu = \frac{hL}{k} = \frac{\text{overall heat transfer}}{\text{heat xfer by conduction}}$$

$$Pr = \frac{\mu/\rho}{k/\rho C_p} = \frac{C_p \mu}{k} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$$ simultaneous momentum and heat transfer

$$Bi = \frac{R_{cond}}{R_{conv}} = \frac{L/k\cancel{A}}{1/h\cancel{A}} = \frac{hL}{k} = \frac{\text{resistance to conduction (internal)}}{\text{resistance to b.l. convection}}$$

$$Fo = \frac{kt}{\rho C_p L^2} = \text{dimensionless time (how fast object's T changes)} = \frac{\text{heat conduction rate}}{\text{~~rate of~~ thermal energy rate}}$$

<u>Mass transfer</u>

$$Sh = \frac{kL}{\mathcal{D}} = \frac{\text{total mass transfer}}{\text{diffusive mass transfer}}$$

<u>Momentum / Mass transfer</u>

$$Sc = \frac{\mu/\rho}{\mathcal{D}} = \frac{\mu}{\rho \mathcal{D}} = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}}$$

(21) Given a pool of organic liquid (e.g. from spill), estimate its rate of evaporation.

Ignore heat transfer by assuming the liq. has the same T as air.

Consider diffusion through air:

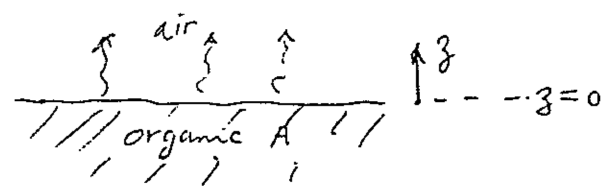
neglect this term because net flux ≈ 0 (N)

$$N_A = \cancel{N x_A} + J_A$$

$$N_A = -\mathcal{D}_{AB} \frac{dC_A}{dz}$$

A = organic
B = air

$$-\mathcal{D}_{AB} \left.\frac{dC_A}{dz}\right|_{z=z} \longrightarrow \qquad \longrightarrow -\mathcal{D} \left.\frac{dC_A}{dz}\right|_{z=z+\Delta z}$$



~~at SS~~, $\mathcal{D}_{AB} \frac{\partial^2 C_A}{\partial z^2} = \frac{\partial C_A}{\partial t}$

BC 1 at $z=0$, C_A given by $P_A^s(T_{air})$

BC 2 at $z=\infty$, $C_A = 0$

BC 3 at $t=0$, $C_A = 0$

or fix this distance

Solve by similarity (∞)

Assuming ideal gas, $PV = nRT \Rightarrow \frac{n}{V} = \frac{P}{RT}$

$$C_A = \frac{P_A^s}{RT} \quad \text{at } z = 0$$

...ind at infinite BC

let $C_A = Z(z)T(t)$ (separation of variables)

$$\frac{\partial C_A}{\partial z} = T\frac{dZ}{dz} \qquad \frac{\partial^2 C_A}{\partial z^2} = T\frac{d^2 Z}{dz^2}$$

$$\frac{\partial C_A}{\partial t} = Z\frac{dT}{dt}$$

$$\mathcal{D}_{AB} T \frac{d^2 Z}{dz^2} = Z\frac{dT}{dt}$$

$$\frac{1}{Z}\frac{d^2 Z}{dz^2} = \frac{1}{\mathcal{D}}\frac{1}{T}\frac{dT}{dt} = \text{const.} = -1$$

solve for $c_A(z,t)$

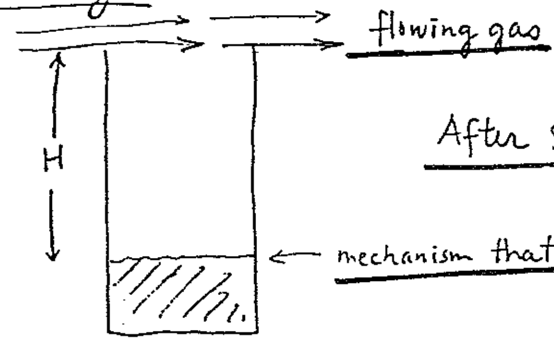
$$\boxed{\text{rate of evaporation} = -\mathcal{D}_{AB}\left.\frac{\partial c_A}{dz}\right|_{z=0}} \cdot (\text{area of organic})$$

$$\boxed{\text{flux at surface}}$$

$N_A|_{z=0}$

(22) <u>How are the diffusivity and viscosity of a mixture determined?</u>

<u>Diffusivity</u>



After SS has been achieved, $N_B = 0$

$$\boxed{N_A = -\mathscr{D}_{AB}\frac{dC_A}{dz} + x_A(N_A + \cancelto{0}{N_B})}$$

$$N_A = -c\mathscr{D}_{AB}\frac{dx_A}{dz} + x_A N_A$$

$$N_A(1 - x_A) = -c\mathscr{D}_{AB}\frac{dx_A}{dz}$$

$$\boxed{N_A = \frac{-c\mathscr{D}_{AB}\frac{dx_A}{dz}}{(1-x_A)}}$$

Now, mole bal on Δz :

$$A N_A\Big|_z = A N_A\Big|_{z+\Delta z}$$

$$\boxed{\frac{dN_A}{dz} = 0}$$

BC at $z = 0$, $C_A = \frac{P_A^S}{RT}$, $x_A = \frac{P_A^S}{CRT}$

at $z = H$, $C_A = 0$

$$\frac{d}{dz}\left[\frac{+c\mathscr{D}_{AB}\frac{dx_A}{dz}}{(1-x_A)}\right] = 0$$

} solve for x_A

$$x_A = f(z)$$

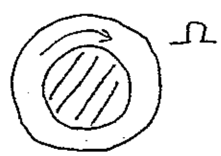
Find N_A by finding $N_A|_{z=0}$. Then if we know the rate at which A leaves the interface, we can find $\mathcal{D}_{AB}$.

Viscosity

Capillary viscometer: Hagen-Poiseuille law because of laminar flow.

$$Q = \frac{\pi \Delta P R^4}{8 \mu L} \qquad \text{We know } Q, R, \Delta P, L \Rightarrow \text{get } \mu\,!!$$

Couette viscometer



Momentum balance:

$$\tau_{r\theta} \cancel{2\pi} r \cancel{L}\Big|_r - \tau_{r\theta} \cancel{2\pi} r \cancel{L}\Big|_{r+\Delta r} = 0$$

$$\frac{d(\tau_{r\theta} r)}{dr} = 0$$

$$r\frac{d\tau_{r\theta}}{dr} + \tau_{r\theta} = 0 \qquad \text{where} \quad \tau_{r\theta} = -\mu \frac{dv_\theta}{dr}$$

$$-r\cancel{\mu}\frac{d^2 v_\theta}{dr^2} - \cancel{\mu}\frac{dv_\theta}{dr} = 0$$

solve for $v_\theta(r)$

Then get $\tau_{r\theta}\Big|_{r=r_i} = -\mu \frac{dv_\theta}{dr}\Big|_{r=r_i}$ if inner cylinder is turned

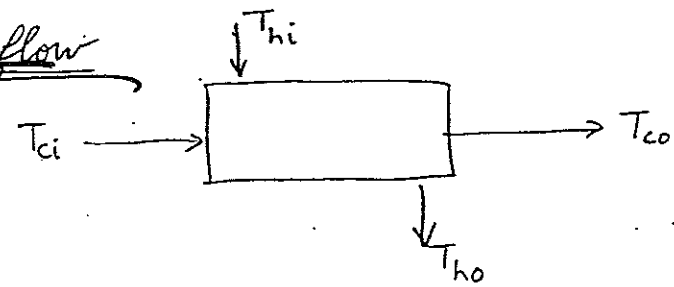
If we know $\Omega \Rightarrow$ get torque $\Rightarrow$ get μ!!

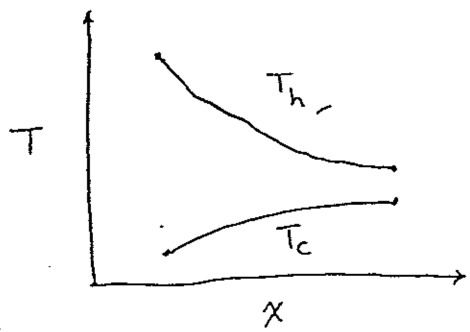
Dropping sphere viscometer (Stokes' regime)

Can get μ if we know ρ of sphere ... terminal velocity

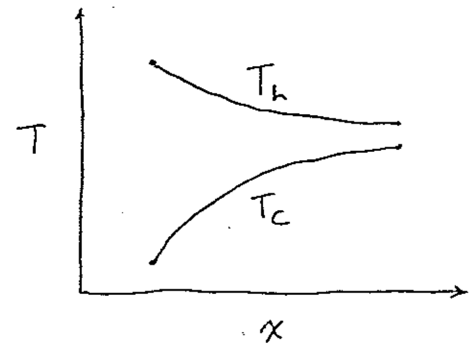
(23) Sketch the temp. profile in a heat exchanger.

Parallel flow



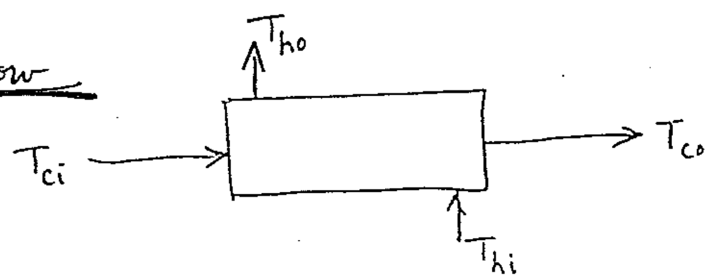


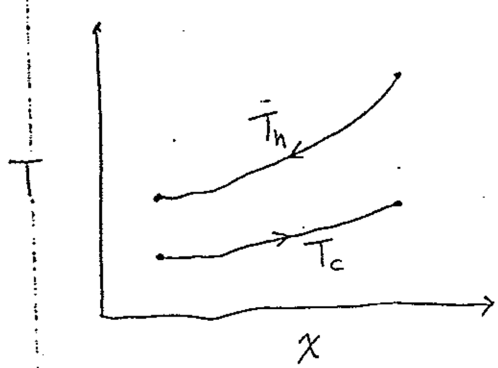
(a) $(\dot{m}C_p)_c > (\dot{m}C_p)_h$



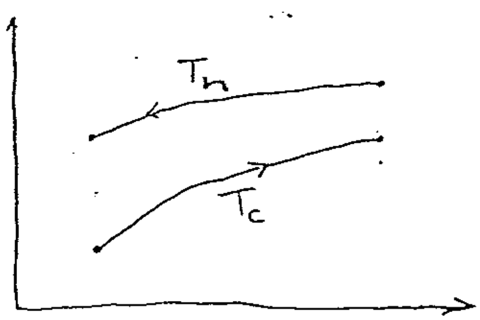
(b) $(\dot{m}C_p)_c < (\dot{m}C_p)_h$

Counter flow





(a) $(\dot{m}C_{pc}) > (\dot{m}C_{ph})$

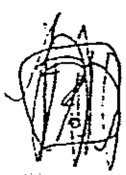


(b) $(\dot{m}C_p)_c < (\dot{m}C_p)_h$

(24) What phenomena are important during an underground explosion?

Explosions are fast processes — so diffusion is not important. Expansion of gases due to bulk flow. Also important is the void space available for gases to expand.

(25) Consider a drop falling down a tower, initial temp. and tower temp. are given. How does the drop temp. change as it falls?



- forced convection heat xfer as drop falls.
- evaporation occurs as it falls.
- rate of evaporation = rate at which heat is transferred to drop.

Set up (non-steady state) heat conduction through spherical shells. The boundary conditions will be:

at $r = R(t)$, $T = T^s$ ← temp. at which liq. evaporates will be constant

at $r = 0$, $\frac{\partial T}{\partial r} = 0$

at $t = 0$, $T = T_0$ at all r

Another eqn. will be the heat flux to surface:

$$q'' = h(T_\infty - T^s)$$

This will determine the rate of evaporation of the liquid and will establish $R(t)$. h can be determined from correlations. The unsteady state DE will be solved by similarity solutions.

Another way of solving the problem is by lumped capacitance ⇒ that is, assume that the temp. is uniform throughout the droplet. Therefore T is only a function of time. To determine validity of this method, check $Bi = \frac{hD}{k_{liq}}$ and see if it is $\ll 1$. If it is, then:

$$\rho V(t) C_p \frac{dT}{dt} = h\,(T_\infty - T)\,[4\pi R^2(t)]$$

Much easier to solve !!

(26) What is the angular dependence of Nu for a falling drop?

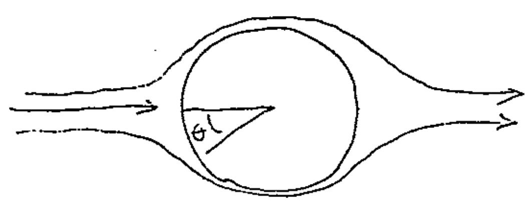
Consider Stokes' regime ($Re < 0.1$)

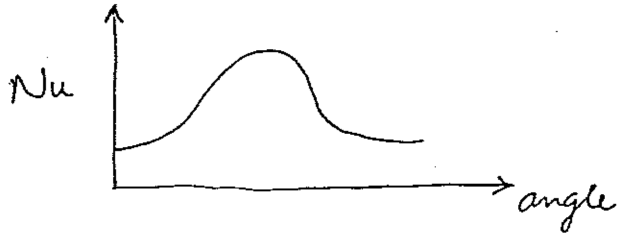
$$Nu = \frac{\text{overall heat xfer}}{\text{diffusive heat xfer}} = \frac{hL}{k}$$

$$Nu = f(Re, Pr)$$

$$Re = \frac{\rho v_{\theta} D_p}{\mu}$$

Heat xfer more effective where there is less thermal b.l. ⇒ that would be also where there is less velocity b.l. (stagnant).

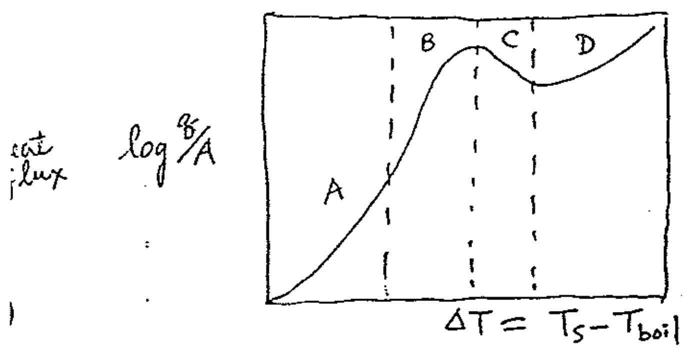




(27) Draw the boiling curve and describe the physical phenomena responsible for the observed behavior. Draw and explain the similar curve for condensation.

Heat xfer to boiling liquid

- liq. at boiling T at P of equipment
- heat xfer from surface with $T_s > T_{boil}$
- bubbles of vapor generated at surface



A : natural convection - few bubbles, but mostly natural convection

B : nucleate boiling - more bubbles

C : transition boiling - many bubbles form so quickly that they coalesce and form layer of insulating vapor on surface. That's why as $T\uparrow$, $q/A \downarrow$.

D : film boiling - bubbles detach faster than they coalesce. Radiation thru vapor layer next to surface becomes significant.

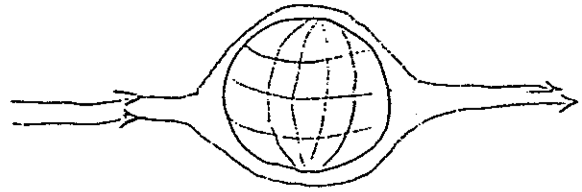
Heat transfer from condensing vapor

- $T_s < T_{condense}$

Film condensation - film of condensate forms over surface and causes the main resistance to heat xfer.

Dropwise condensation - only drops of liquid are formed on surface; therefore more clean surface for heat xfer.

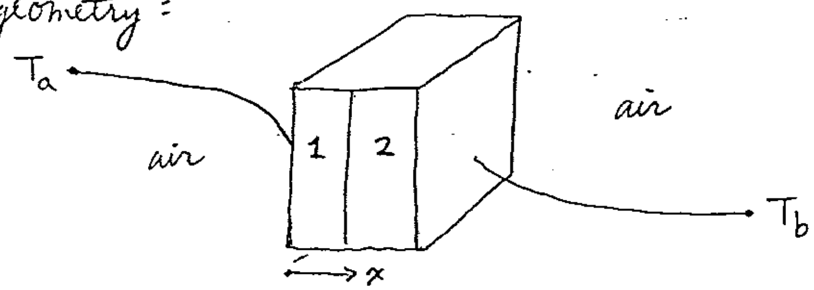
(28) Given the free stream velocity (v_∞) and particle diameter (D_p), calculate b.l. thickness at 45° angle. What is P at forward and backward stagnation pts? What causes the difference?



?

(30). How is the overall heat transfer coeff. for a heat exchanger found?

First find overall heat xfer coeff. for a flat plate geometry:



$$q = Aq_1'' = Aq_2'' = Aq_o''$$

$\therefore \; q_o'' = \text{const.} \quad \Leftarrow$ const. heat flux

$$q_o'' = -k_1 \frac{dT_1}{dx}$$

$$\int_{x_o}^{x_1} -\frac{q_o''}{k_1}\,dx = \int_{T_1}^{T_{1-2}} dT_1 \quad \Rightarrow \quad -\frac{q_o''}{k_1}\Delta x_1 = T_{1-2} - T_1$$

Similarly,

$$-\frac{q_o''}{k_2}\Delta x_2 = T_2 - T_{1-2}$$

For the two faces exposed to air (convection):

$$q_o'' = h_a (T_a - T_1)$$

$$q_o'' = h_b (T_2 - T_b)$$

We have :

$$q_o''\left(\frac{\Delta x_1}{k_1}\right) = \cancel{T_1} - \cancel{T_{1-2}}$$

$$q_o''\left(\frac{\Delta x_2}{k_2}\right) = \cancel{T_{1-2}} - \cancel{T_2}$$

$$+ \quad q_o''\left(\frac{1}{h_a}\right) = T_a - \cancel{T_1}$$

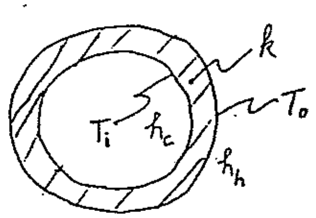
$$q_o''\left(\frac{1}{h_b}\right) = \cancel{T_2} - T_b$$

$$q_o''\left[\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{1}{h_a} + \frac{1}{h_b}\right] = T_a - T_b$$

$$q_o'' = U\,(T_a - T_b)$$

where $\boxed{U = \dfrac{1}{\left[\dfrac{\Delta x_1}{k_1} + \dfrac{\Delta x_2}{k_2} + \dfrac{1}{h_a} + \dfrac{1}{h_b}\right]}}$

For cylindrical heat exchanger,



energy bal. on cylindrical shell :

$$q''\,\cancel{2\pi}\, r\, \cancel{L}\Big|_{r+\Delta r} - q''\,\cancel{2\pi}\, r\, \cancel{L}\Big|_{r} = 0$$

$$\frac{d(rq'')}{dr} = 0$$

$$r \frac{dq''}{dr} + q'' = 0$$

$$\frac{dq''}{dr} = -\frac{1}{r} q''$$

$$\int \frac{dq''}{q''} = \int -\frac{dr}{r}$$

$$\ln q'' = -\ln r + \text{const.}$$

$$q'' = C_0' \left(\frac{1}{r}\right)$$

$$-k \frac{dT}{dr} = C_0 \left(\frac{1}{r}\right)$$

$$\int dT = \int \frac{-C_0}{k} \frac{dr}{r}$$

where $C_1 = \frac{C_0}{k}$

$$T = -C_1 \ln r + C_2$$

BC: $T = T_i$ at $r = r_i$

$T = T_o$ at $r = r_o$

$$T_i = -C_1 \ln r_i + C_2$$

$$- \quad T_o = -C_1 \ln r_o + C_2$$

$$T_i - T_o = C_1 \ln r_o - C_1 \ln r_i = C_1 \ln \frac{r_o}{r_i}$$

$$C_1 = \frac{T_i - T_o}{\ln \frac{r_o}{r_i}}$$

$$T_i = \frac{T_o - T_i}{\ln \frac{r_o}{r_i}} \ln r_i + C_2$$

$$C_2 = T_i - \frac{T_o - T_i}{\ln \frac{r_o}{r_i}} \ln r_i$$

$$\therefore \quad T = \frac{T_o - T_i}{\ln \frac{r_o}{r_i}} \ln r + T_i - \frac{T_o - T_i}{\ln \frac{r_o}{r_i}} \ln r_i$$

$$T = \frac{T_o - T_i}{\ln \frac{r_o}{r_i}} \ln\left(\frac{r}{r_i}\right) + T_i$$

Now, $q = q'' A$

$$q = q_i'' A_i = \frac{k(T_i - T_o)}{\ln \frac{r_o}{r_i}} \frac{1}{\cancel{r_i}} (2\pi \cancel{r_i} L)$$

Also, by convection from inner and outer surfaces,

$$q = 2\pi r_i L h_i (T_c - T_i)$$

$$q = 2\pi r_o L h_o (T_o - T_h)$$

$$q \left(\frac{1}{2\pi r_i L h_i}\right) = T_c - T_i$$

$$+ \quad q \left(\frac{1}{2\pi r_o L h_o}\right) = T_o - T_h$$

$$q \left(\frac{\ln \frac{r_o}{r_i}}{k 2\pi L}\right) = T_o - T_i$$

$$q \left[\frac{1}{2\pi r_i L h_i} + \frac{1}{2\pi r_o L h_o} + \frac{\ln (r_o/r_i)}{2\pi L k}\right] = T_c - T_h$$

$$\therefore \boxed{R_{Tot} = \frac{1}{\left[\frac{1}{2\pi r_i L h_i} + \frac{1}{2\pi r_o L h_o} + \frac{\ln (r_o/r_i)}{2\pi L k}\right]}}$$

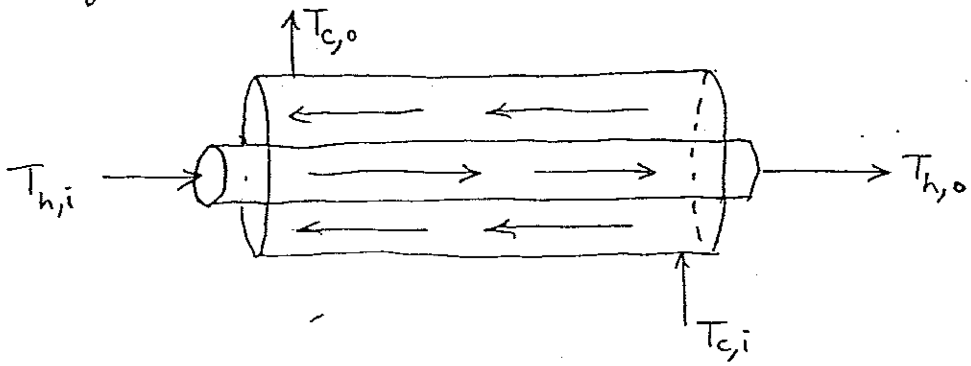
for overall heat xfer. coeff. based on outside radius r_o =

$$q = \underbrace{U_o A_o}_{R_{tot}} (T_c - T_h)$$

$$U_o = \frac{R_{tot}}{A_o} = R_{tot} \frac{1}{2\pi r_o L}$$

$$\boxed{U_o = \frac{1}{\left[\frac{r_o}{r_i}\frac{1}{h_i} + \frac{1}{h_o} + \frac{r_o}{k} \ln \frac{r_o}{r_i} \right]}}$$

(31) Given two temp. and a knowledge of the fluids' properties in a double pipe countercurrent heat exchanger, how do you calculate the other two temps?



For example, if given $T_{h,i}$ and $T_{c,i}$:

$$\boxed{q = \dot{m}_c \overline{C_{p,c}} \cdot (T_{c,o} - T_{c,i})}$$

3 unknowns : $q, T_{c,o}, T_{h,o}$

average C_p's

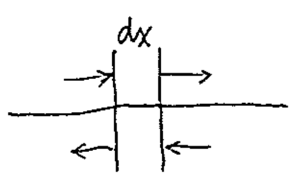
$$\boxed{q = \dot{m}_h \overline{C_{p,h}} \; (T_{h,i} - T_{h,o})}$$

$$\boxed{q = UA \Delta T_{lm} = UA \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}}$$

where $\Delta T_1 = T_{h,i} - T_{c,o}$

$\Delta T_2 = T_{h,o} - T_{c,i}$

How do we know it's a log mean temp. diff?



$$dq = \dot{m}_c C_{p,c}\,(dT_c) \quad \Rightarrow \quad q = \dot{m}\, C_{p,c}\, \Delta T_c$$

$$dq = -\dot{m}_h\, C_{p,h}\,(dT_h) \quad \Rightarrow \quad q = -\dot{m}\, C_{p,h}\, \Delta T_h$$

$$dq = U(T_h - T_c)\,dA \;=\; U\,\Delta T\, dA$$

$$\Delta T = T_h - T_c$$

$$d(\Delta T) = dT_h - dT_c = -\frac{dq}{\dot{m}_h C_{p,h}} - \frac{dq}{\dot{m}_c C_{p,c}}$$

$$\boxed{d(\Delta T) = -U\,\Delta T\, dA\left(\frac{1}{\dot{m}_h C_{p,h}} + \frac{1}{\dot{m}_c C_{p,c}}\right)}$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \int -U\left(\;\cdots\;\right) dA$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -U\left(\;\cdots\;\right) A$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left[-\frac{\Delta T_h}{q} + \frac{\Delta T_c}{q}\right] = -\frac{UA}{q}\left(\Delta T_c - \Delta T_h\right)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = +\frac{UA}{q}\left(T_{h,o} - T_{h,i} - T_{c,o} + T_{c,i}\right)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \frac{UA}{q}\left(\Delta T_2 - \Delta T_1\right)$$

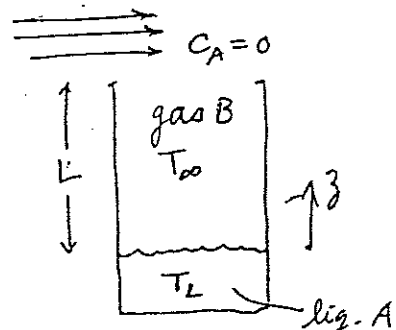
$$q = UA\left(\frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)}\right) \quad \checkmark$$

$$\rho C_p \frac{dT}{dt} = k\nabla^2 T$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla\cdot\vec{q}$$

(33) Is the heat flux from a liq. into a gas usually higher or lower if the gas is insoluble (vs. soluble) in the liquid?

Compare equimolar counterdiffusion vs. non-diffusing B.



$$q'' = -h(T_\infty - T_L)$$

look at direction of mass transfer + heat transfer. If same way: good. If opposite way, bad.

We can consider diffusion and find k (mass xfer coeff). Then by Chilton-Colburn analogy, compare to h.

Non-diffusing B:

$$N_A = -c\mathcal{D}_{AB}\frac{dx_A}{dz} + x_A(N_A)$$

$$N_A(1-x_A) = -c\mathcal{D}_{AB}\frac{dx_A}{dz}$$

$$\frac{dN_A}{dz} = 0$$ For steady conditions, no rxn, absolute molar flux of A must be constant throughout column

$$N_A = C_1$$

BC $z = L$, $x_A = 0$

$z = 0$, $x_A = \frac{P_A^S}{P} = x_{A0}$

$$\frac{-c\mathcal{D}_{AB}\frac{dx_A}{dz}}{(1-x_A)} = C_1$$

$$-c\mathcal{D}_{AB}\frac{dx_A}{1-x_A} = C_1\,dz$$

$$c\mathcal{D}_{AB}\ln(1-x_A) = C_1 z + C_2$$

Apply BC:

$$0 = C_1 L + C_2$$

$$c\mathcal{D}_{AB}\ln(1-x_{A0}) = C_2$$

$$C_1 = -\frac{C_2}{L} = \frac{-c\mathcal{D}_{AB}\ln(1-x_{A0})}{L}$$

We know that,

$$-c\mathcal{D}_{AB}\left.\frac{dx_A}{dz}\right|_{z=0} = k\,(C_{A0} - 0)$$

$$\ln(1-x_A) = \frac{1}{c\mathcal{D}_{AB}}\left(c_1 z + c_2\right)$$

$$1-x_A = \exp\left[\frac{1}{c\mathcal{D}_{AB}}(c_1 z + c_2)\right]$$

$$x_A = 1 - \exp\left[\frac{1}{c\mathcal{D}_{AB}}(c_1 z + c_2)\right]$$

$$\frac{dx_A}{dz} = \frac{-1}{c\mathcal{D}_{AB}}\,c_1\,\exp\left[\frac{1}{c\mathcal{D}_{AB}}(c_1 z + c_2)\right]$$

<u>Equimolar counter diffusion:</u>

$$N_A = -c\mathcal{D}_{AB}\frac{dx_A}{dz}$$

$$-c\mathcal{D}_{AB}\frac{dx_A}{dz} = c_1$$

$$-c\mathcal{D}_{AB}\,dx_A = c_1\,dz$$

$$-c\mathcal{D}_{AB}\,x_A = c_1 z + c_2$$

$$x_A = \frac{-1}{c\mathcal{D}_{AB}}\left(c_1 z + c_2\right)$$

$$\frac{dx_A}{dz} = -\frac{1}{c\mathcal{D}_{AB}}\,c_1$$

<u>Apply BC:</u>

$$0 = Lc_1 + c_2$$

$$-c\mathcal{D}_{AB}\,x_{A0} = c_2$$

$$c_1 = -\frac{c_2}{L} = \frac{c\mathcal{D}_{AB}\,x_{A0}}{2}$$

Actually, we can say if $T_\infty > T_L$, then counterdiffusion is better for larger heat flux. If $T_L > T_\infty$, then non-diffusing B is better.

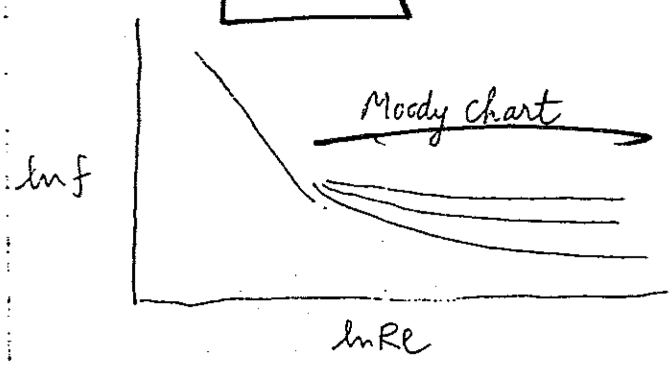
(34) The Chilton-Colburn j-factor is proportional to h, the convective heat transfer coeff. Why is j proportional to $Pr^{-1/3}$? Why is j only a fraction of Re? Why does j decrease as Re increases?

$$\boxed{j_H = St_H \, Pr^{2/3}} = \frac{Nu}{Re\,Pr} Pr^{2/3} = \frac{Nu}{Re\,Pr^{1/3}}$$

The $Pr^{2/3}$ term is an empirical addition that represents data better at $Pr \neq 1$.

We can see that j decreases as Re increases by:

$$\boxed{j_H = \frac{f}{2}} = f(Re) \quad \text{for fully developed flow}$$



(35) O_2 and N_2 leaks are often considered less dangerous than H_2 leaks. Why?

$$\text{J-T coeff} \equiv \mu = \left(\frac{\partial T}{\partial P}\right)_H$$

As H_2 leaks from a hole:

$$\Delta h = Q - \cancel{W_s}^{\,0}$$

$Q \approx 0$ since the time that it takes to leak is very fast and not much heat has chance to transfer.

$\Delta h \approx 0$ isenthalpic!!

For ideal gases, $\mu = 0$. But for real gases,

$\mu > 0$ if $T < T_I$ (inversion temperature)

$\mu < 0$ if $T > T_I$

For O_2, $T_I >$ normal ambient temp. $\Rightarrow$ So $\mu > 0$ and as O_2 leaks, its temp. drops. as $P\downarrow$, $T\downarrow$
N_2 is virtually inert.

For H_2, $T_I <$ normal ambient temp. $\Rightarrow$ So $\mu < 0$ and as H_2 leaks, $P\downarrow \Rightarrow T\uparrow$ and danger of combustion.

as $P\downarrow$, $T\uparrow \longrightarrow$ combustion

(36). How would you separate O_2 from salt water? What variables affect solubility? Where is the mass transfer resistance?

Look at thermo equil. between air and sea water:

$1 = H_2O$
$2 = O_2$

$$y_2 \varphi_2 P = H_{2,1} x_2$$

but $H_{2,1}$ depends on P as follows:

$$H_{2,1}(P) = H_{2,1}(P_1)\left[\exp \int_{P_1}^{P} \frac{\bar{v}_2^{\infty}}{RT} dP\right]$$

$$\therefore \quad x_2 = \frac{y_2 \varphi_2 P}{H_{2,1}(P_1) \exp\left[\frac{\bar{v}_2^{\infty}}{RT}(P-P_1)\right]}$$

Since the exponential P dependence is stronger, as P decreases, $x_2 \uparrow$.

So throttle sea water through a valve to a tank under vaccuum. O_2 bubbles will form.

Most of mass transfer resistance in this interphase mass transfer is in the liquid side because it is in this phase that O_2 is scarce.

(37) What area is used in defining friction factor for a wetted wall column?

I would think the wetted area of the column.

(38) What is the Lewis relation? Is it dependent on the gas phase velocity? Why or why not?

The wet-bulb temperature equation resembles the equation that follows an adiabatic saturation curve. This curve is followed by humidification processes where the gas that leaves the humidifier is saturated.
The only difference between the wet-bulb temp. eqn. and the adiabatic satur. eqn. is that h_G/k_y in wet bulb eqn. is replaced by C_s in adiabatic satur. eqn.

heat required to raise the temp. of unit mass of gas and vapor one degree at const. P.

Lewis relation says: $\frac{h_G}{k_y} = C_s$

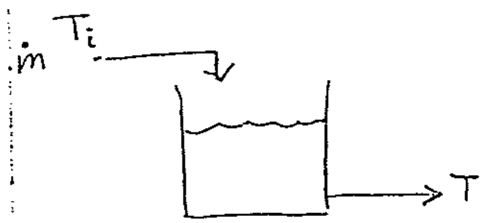
It holds when $Le = \frac{Sc}{Pr} = 1$ which is the case for air-water vapor system.

There is no dependence on gas phase velocity as it does not appear in Le.

(39) Why are analogies between mass and heat transfer much more straighforward to use than analogies between mass and momentum transfer?

Because the flux of heat and mass are vectors while that of momentum are tensors.

(40) Given a CSTR at temp. T with no reaction, what would happen if the inlet T were suddenly increased?



$$\dot{m} C_p T_i - \dot{m} C_p T = m_{tot} C_p \frac{dT}{dt}$$

$$\dot{m} \cancel{C_p} (T_i - T) = m_{tot} \cancel{C_p} \frac{dT}{dt}$$

Assume $C_p = \text{const.}$

BC at $t=0$, $T=T_0$

$$\int \frac{dT}{T_i - T} = \int \frac{\dot{m}}{m_{tot}} dt$$

$$-\ln(T_i - T) = \frac{\dot{m}}{m_{tot}} t + C_1$$

$$C_1 = -\ln(T_i - T_0)$$

$$\ln\left(\frac{T_i - T}{T_i - T_0}\right) = -\frac{\dot{m}}{m_{tot}} t$$

$$\frac{T_i - T}{T_i - T_0} = \exp\left(\frac{-\dot{m}}{m_{tot}} t\right)$$

$$T_i - T = (T_i - T_o)\exp\left(-\frac{\dot{m}}{m_{tot}}t\right)$$

$$\boxed{T = T_i - (T_i - T_o)\exp\left(-\frac{\dot{m}}{m_{tot}}t\right)}$$

(41) Analogies between heat, mass, and momentum transport are important. Give examples of when they don't hold.

- For momentum transfer, there is no dimensionless number analogous to Nu or Sh or St.

- Reynold's analogy does not hold when $Pr \neq 1$

- When the fluid properties change because T, P change drastically.

- When there are extra terms like $\frac{dP}{dx}$, $\underbrace{\dot{Q}}_{\text{heat gener.}}$, and $\underbrace{R_A}_{\text{reaction of A}}$.

- The geometry is not the same. The boundary conditions are not analogous.

- The turbulent diffusivities are not the same: $E_v \neq E_H \neq E_D$

- The ratios $\frac{E_H}{\alpha} \neq \frac{E_D}{\mathcal{D}_{AB}} \neq \frac{E_v}{\nu}$

- When there is form as well as skin drag.

non-dimensionlize eqns + omit key terms

(42) What is the theoretical basis for for all the "famous" analogies between heat, mass and momentum transport?

Analogies begin because the DEs look similar, in fact, identical in some cases if written in dimensionless variables.

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = \underbrace{\cancel{-\frac{\partial p}{\partial x}}}_{\text{must ignore}} + \nu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \underbrace{\cancel{g_x}}_{\text{must ignore}}$$

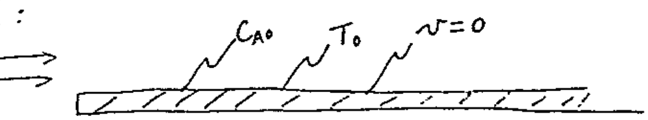
$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \underbrace{\cancel{\Phi}}_{\text{must ignore}}$$

$$\frac{\partial C_A}{\partial t} + v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y} + v_z \frac{\partial C_A}{\partial z} = \mathcal{D}_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right) + \underbrace{\cancel{R_A}}_{\text{must ignore}}$$

Write variables in dimensionless forms:

Example of:

$C_\infty, T_\infty, v_\infty$



$$v^* = \frac{v}{v_\infty} \qquad T^* = \frac{T - T_0}{T_\infty - T_0} \qquad C^* = \frac{C - C_0}{C_\infty - C_0}$$

Note that if $T_0 > T_\infty$, then for anology to work, $C_{A0} > C_\infty$.

(43) What is the difference between skin friction drag and form drag?

Skin friction drag is drag caused by friction when a fluid flows over a solid.

Form drag is caused by having to move fluid particles out of the way as the solid object moves through the fluid.

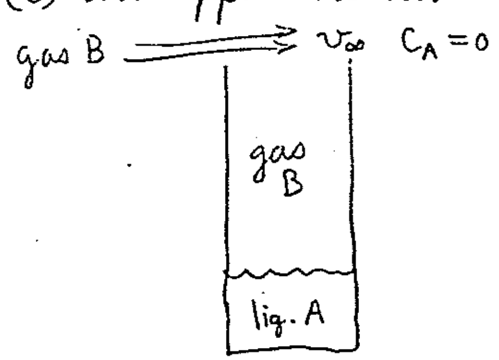
(44) How would you determine a mass transfer coeff. experimentally?

Perform experiments like:

(a) wet wall column where fluid flows down the inside surface of a column and a gas flows up through the middle.

(b) volatile or soluble solid spheres or pipes with fluid flowing around them.

(c) An apparatus like:

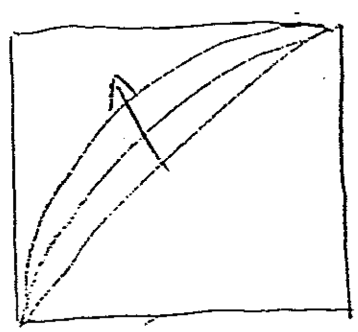


The key is to be able to find the flux or rate of mass leaving the liquid, solid or gas. Easily done in (a) and (b) by measuring conc. of A in gas at beginning and end of column and by weighing the solid sphere or pipe for part (b). For (c), could have device that keeps the liq. level the same and keep track of how much liq. is injected to keep level the same.

(45) Why does frost not form under a tree when it is on the ground all around the tree?

Radiation between tree branches and ground?

(46) Draw McCabe-Thiele diagram for distillation column that uses reacting absorbent.



Equivalent to moving the equil. line as shown. Therefore less stages for given separation.

(47) What are the most commonly used (3) correlations for heat and mass transfer?

~~Dittus-Boelter: turbulent flow in tubes~~

$$\cancel{Nu = 0.023\, Re_D^{0.8}\, Pr^{n}}$$

$n = 0.4$ for heating
$n = 0.3$ for cooling

$$\cancel{Sh = 0.023\, Re_D^{0.8}\, Sc^{n}}$$

Others ---

$$Nu = 2 + 0.6\, Re^{1/2}\, Pr^{1/3} \quad \text{(spheres)}$$

$j_D \approx j_H$

(49) Give the equations describing flow in packed bed.

Key is to treat bed as a bundle of tubes that are gnarled and twisted $\Rightarrow$ modify H-P eqn:

$$\langle v \rangle = \frac{\Delta P \, R_h^2}{8 \mu L}$$

$\hookrightarrow$ steady state laminar const ρ + μ

where R_h = hydraulic radius

$$= \frac{\text{cross-sect. area of flow}}{\text{wetted perimeter}}$$

$$= \frac{\varepsilon}{a}$$

$\Rightarrow$ Get eqn for laminar flow in packed beds.

Then analyze situation for turbulent flow
$\Rightarrow$ Get eqn for turbulent flow.

Add together to get Ergun equation.

(50) Derive eqns. for gas undergoing <u>isentropic</u> expansion.

$$\frac{P_1}{T_1} \longrightarrow \frac{P_2}{T_2}$$

$$ds = \left(\frac{\partial s}{\partial P}\right)_T dP + \left(\frac{\partial s}{\partial T}\right)_P dT$$

$$ds = -\left(\frac{\partial v}{\partial T}\right)_P dP + \frac{C_P}{T}\, dT$$

For ideal gas,

$$Pv = RT$$

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P}$$

$$ds = -\frac{R}{P}\, dP + \frac{C_P}{T}\, dT$$

$$0 = \int_{P_1}^{P_2} -R\frac{dP}{P} + \bar{C}_P \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\boxed{0 = -R \ln \frac{P_2}{P_1} + \bar{C}_P \ln \frac{T_2}{T_1}}$$

This is how P's and T's are related.

$$du = Tds - Pdv$$

$$dh = Tds + vdP$$

$$g = h - Ts$$

$$dg = dh - Tds - sdT$$

$$= vdP - sdT$$

$$da = u - Ts$$

$$= du - Tds - sdT$$

$$= -Pdv - sdT$$

<u>Maxwell</u>:

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial v}{\partial s}\right)_P$$

$$\left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$$

$$\left(\frac{\partial P}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

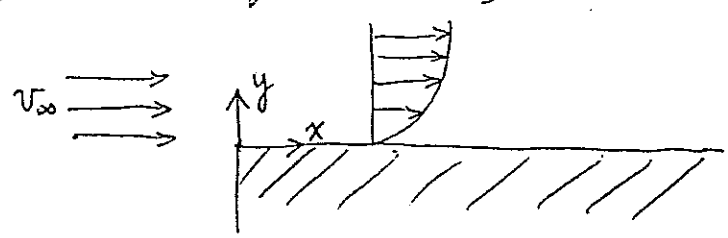
(51) What is inside a lightbulb and why?

There is a vacuum inside the bulb because we don't want any conduction or convection because we don't want bulb surface to get too hot. We only want radiation of visible light.

(52) Why do you have to whirl a wet-bulb / dry-bulb psychrometer in the air prior to using it?

Because we don't want to measure the humidity of the stagnant air in the vicinity of the thermometer. We want the humidity of the bulk air.

(53) In which direction is the momentum flux from a fluid flowing over a flat plate?



Down the velocity gradient $\Rightarrow$ in the $-y$-direction.

Conservation of Momentum (x-dir.)

54

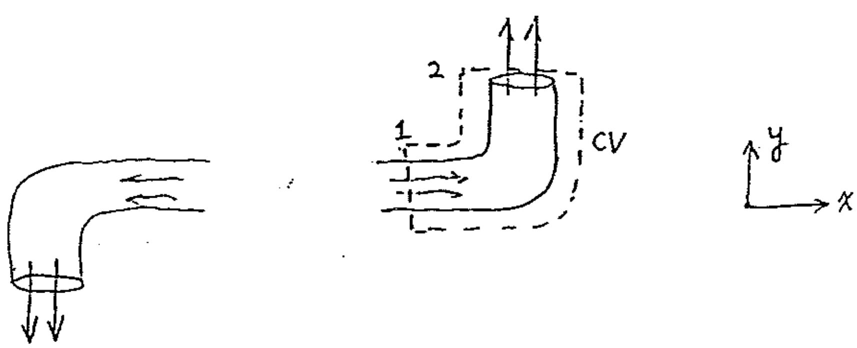
$$\frac{d}{dt}\int_{x_1}^{x_2} \rho \langle v_x \rangle A\,dx = \rho \langle v_x^2 \rangle_1 A_1 - \rho \langle v_x^2 \rangle_2 A_2 + P_1 A_{x,1} - P_2 A_{x,2} + \rho V g_x - \vec{F}$$

force exerted by fluid on surrounding surfaces

At S.S.

$$\boxed{0 = \rho \langle v_x^2 \rangle_1 A_1 - \rho \langle v_x^2 \rangle_2 A_2 + P_1 \underline{S}_1 - P_2 \underline{S}_2 + m_{tot}\, g_x - F}$$

How does a lawn sprinkler work?



x-mom. bal. on CV:

force exerted by fluid on inside pipe surface in x-dir

$$0 = \rho \langle v_1^2 \rangle A + P_1 A - F_x$$

y-mom. bal. on CV:

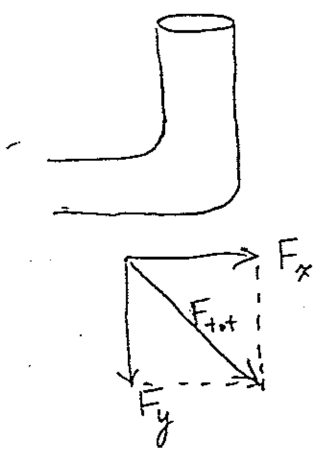
$$0 = -\rho \langle v_2^2 \rangle A - P_2 A - F_y$$

$F_x = \rho\langle v_1^2\rangle A + P_1 A$

$F_y = -\left(\rho\langle v_2^2\rangle A + P_2 A\right)$

these two forces approximately same magnitude because $\langle v_2\rangle \approx \langle v_1\rangle$ and $P_1 \approx P_2 \approx P_{atm}$

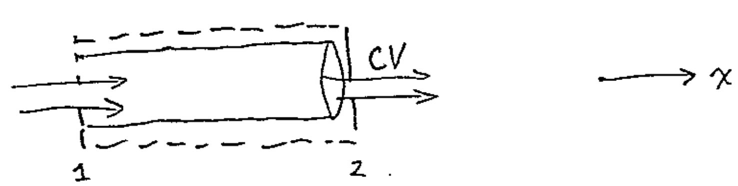
So forces on sprinkler:



That's why sprikler rotates!!

(55) Consider firefighters holding a high pressure hose, must they pull or push the hose? Why?

$\boxed{?}$



$\tau_{rz} = -\mu \frac{\partial v_z}{\partial r} = +$

so need force − pull!

x-mom bal:

$$0 = \rho\langle v_1^2\rangle A - \rho\langle v_2^2\rangle A + P_1 A - P_2 A - F_x$$

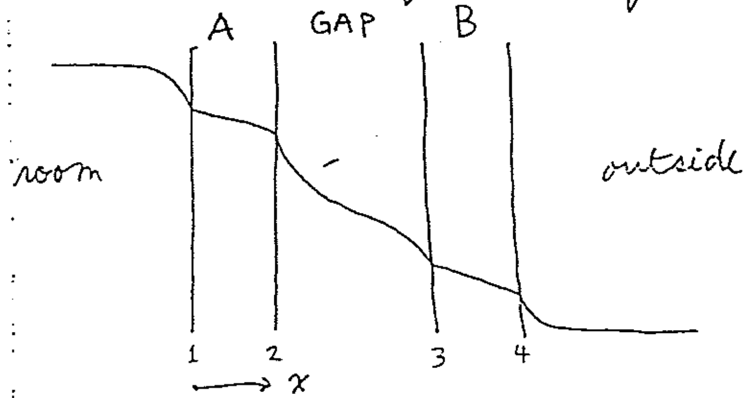
By continuity, $\rho \approx$ const. $\Rightarrow \langle v_1\rangle = \langle v_2\rangle$

$$0 = (P_1 - P_2)A - F_x \quad \Rightarrow \quad F_x = (\overbrace{P_1 - P_2}^{+})A$$

This is the force exerted on the hose.

F_x

(56). For a double plate window with insulating gas between the panes, draw the temp. profile from inside the warm room, through the windows and to the outdoors. Allow for natural convection both in the room and in the gas between the two plates. What gas would you recommend using and why?



$q'' = $ const. through all layers

$q'' = h_1 (T_{room} - T_1)$

$q'' = -k_A \frac{dT}{dx}$

$q'' = h_2 (T_2 - \overline{T}_{gap})$

$q'' = h_3 (\overline{T}_{gap} - T_3)$

$q'' = -k_B \frac{dT}{dx}$

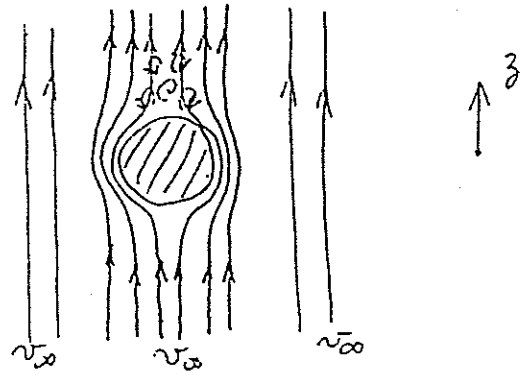
$q'' = h_4 (T_4 - T_{out})$

I wouldn't use a gas $\Rightarrow$ use vacuum so less conduction and convection.

(57) Consider the department ping-pong ball "floating" above a vacuum cleaner discharge. What determines how high the ball will be? What keeps the ball from moving laterally out of the path of the air? What does the velocity profile look like close to, around and above the ball? What determines whether the ball will fall to the ground if the jet is pointing at an angle rather than straight up?

The mass of the ball, the velocity of the air stream, the viscosity of air (thus the temp. of air), the volume of ball. The volume and mass combine into the density of ball.

Since the velocity near the sides of the ball is greater than farther away from the ball, the pressure is less there. Therefore there is a net pressure force keeping the ball in the air stream.



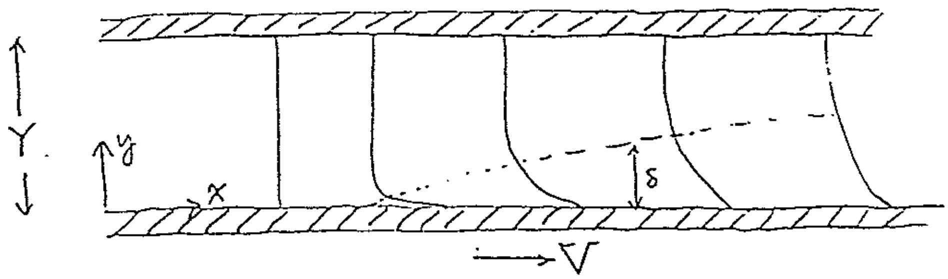
Forces acting on ball (Stokes regime model):

bouyancy: $\frac{4}{3}\pi R^3 \rho_{air} g$ friction: $6\pi\mu R v_\infty$

gravity: $\frac{4}{3}\pi R^3 \rho_s g$

If stream is at an angle, the velocity of the stream and the density of the ball will affect whether the ball stays up or not!

(58) You have two infinite plates initially at rest with a fluid between them. One plate remains fixed, the other is set in motion at velocity V. What do the transient velocity profiles look like?



Eqn of motion:

$$\frac{\partial v_x}{\partial t} + \cancel{v_x \frac{\partial v_x}{\partial x}}^{0} + \cancel{v_y \frac{\partial v_x}{\partial y}}^{0} + \cancel{v_z \frac{\partial v_x}{\partial z}}^{0} = \cancel{-\frac{\partial p}{\partial x}}^{0} + \nu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}}^{0} + \frac{\partial^2 v_x}{\partial y^2} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}^{0} \right) + \cancel{g_x}^{0}$$

$$\boxed{\frac{\partial v_x}{\partial t} = \nu \frac{\partial^2 v_x}{\partial y^2}}$$

Solve this by similarity: let $\varphi(\eta) = \frac{v_x}{V}$ and $\eta = \frac{y}{\delta(t)}$

Continuity:

$$\nabla \cdot \rho \underline{v} = 0 \overset{\rho = \text{const}}{\Longrightarrow} \nabla \cdot \underline{v} = 0$$

$$\frac{\partial v_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}}^{0} = 0$$

v_x does not vary with x!!

BC: $\boxed{\begin{array}{l} v_x = 0 \quad \text{at} \quad y = Y \\ v_x = V \quad \text{at} \quad y = 0 \\ v_x = 0 \quad \text{at} \quad t = 0 \end{array}}$

IC:

Plug $v_x = V\varphi(\eta)$ into DE and solve for $\delta(t)$
Then, we can get $\varphi(\eta) \Rightarrow$ and thus v_x.

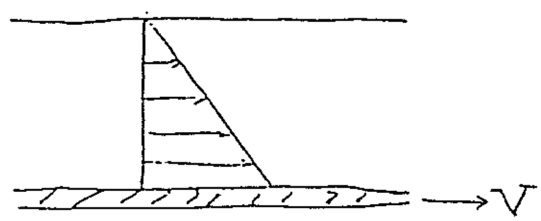
At SS,

$$\boxed{\frac{\partial^2 v_x}{\partial y^2} = 0} \Rightarrow \frac{d^2 v_x}{\partial y^2} = 0$$

$$\frac{dv_x}{dy} = C_1$$

$$dv_x = C_1 dy$$

$$v_x = C_1 y + C_2$$



What is the driving force for fluid flow in a pipe?

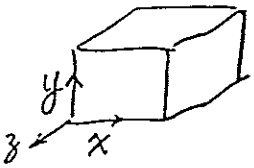
It's the pressure gradient ΔP. And also gravity if not horizontal.

What is the driving force here?

It is the movement of a plate and the viscosity of the fluid that transfers x-momentum.

Describe a momentum balance.

For example for x-momentum:



$$\left\{\begin{matrix}\text{momentum in} \\ \text{by bulk flow}\end{matrix}\right\} = \rho v_x v_x \Delta y \Delta z \Big|_x + \rho v_x v_y \Delta x \Delta z \Big|_y + \rho v_x v_z \Delta x \Delta y \Big|_z$$

$$\left\{\begin{matrix}\text{momentum out} \\ \text{by bulk flow}\end{matrix}\right\} = \rho v_x v_x \Delta y \Delta z \Big|_{x+\Delta x} + \cdots\cdots$$

$$\left\{\begin{matrix}\text{momentum in} \\ \text{by velocity gradient}\end{matrix}\right\} = \tau_{xx} \Delta y \Delta z \Big|_x + \tau_{yx} \Delta x \Delta z \Big|_y + \tau_{zx} \Delta x \Delta y \Big|_z$$

$$\left\{\begin{matrix}\text{momentum out} \\ \text{by velocity gradient}\end{matrix}\right\} = \tau_{xx} \Delta y \Delta z \Big|_{x+\Delta x} + \cdots\cdots$$

$$\left\{\begin{matrix}\text{forces acting on} \\ \text{fluid element}\end{matrix}\right\} = P \Delta y \Delta z \Big|_x - P \Delta y \Delta z \Big|_{x+\Delta x} + \rho \Delta x \Delta y \Delta z \, g_x$$

How would one determine the force necessary to keep the top plate moving at V?

Find shear stress at surface: $\tau_{yx} = -\mu \dfrac{dv_x}{dy}\Big|_{y=0}$

$$\boxed{\text{Force} = \tau_{xy} \text{ (area)}}$$

Determine a characteristic time for this system.

$\dfrac{\mu}{\rho} [=] \dfrac{m^2}{s}$

$$\boxed{t^* = \frac{\rho Y^2}{\mu}}$$

$\mu [=] \dfrac{\cancel{m^2}}{s} \dfrac{kg}{\cancel{m^3}} = \dfrac{kg}{m \cdot s}$

To find the heat transfer coefficient h, we can look up correlations that have been developed for turbulent flow around spheres.

Probably like: $Nu = f(Re, Pr)$

If we want to know the distance dropped, we have to find terminal velocity v_t. Assuming Stoke's regime: ($Re < 0.1$)

friction force : $6\pi\mu R v_t$

bouyancy force : $\frac{4}{3}\pi R^3 \rho_{air} g$

gravity force : $\frac{4}{3}\pi R^3 \rho_s g$

Σ forces $= 0$

$$6\pi\mu R v_t + \frac{4}{3}\pi R^3 \rho_{air} g + \frac{4}{3}\pi R^3 \rho_s g = 0$$

Solve for v_t !!

If Stoke's regime is not assumed, we can use the f vs. Re charts to get f. Then get $F_k = fAK$, where:

usually $K = \frac{1}{2}\rho v_t^2$ $\quad A = \pi R^2$

Which would take longer to reach steady state, molasses or water? Why?

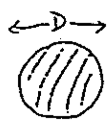
$$\frac{\partial v_x}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2} \qquad \text{dim. time} = \frac{\nu t}{L^2}$$

I think molasses will since $\left(\frac{\mu}{\rho}\right)_{molasses} > \left(\frac{\mu}{\rho}\right)_{H_2O}$.

This means $\frac{\partial v_x}{\partial t}$ is larger $\Rightarrow$ velocity profile changes faster with time.

(60) You have a small sphere of molten metal. How far will it drop (in air) before it solidifies? What does the Biot number tell you here? How do you find the convective heat transfer coefficient?

$$Bi = \frac{hD}{k_m} = \frac{\text{internal thermal diffusion resistance}}{\text{external convection resistance}}$$



So if $Bi \ll 1 \Rightarrow$ we can assume that T is uniform inside sphere.

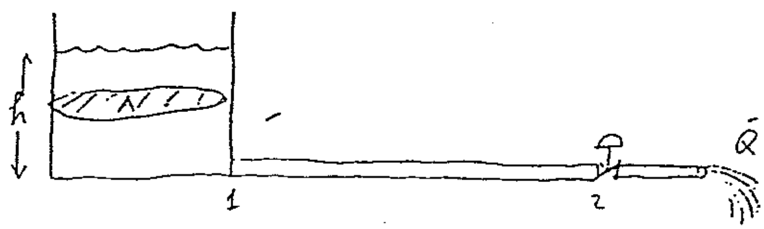
$$\frac{\cancel{4}}{3}\cancel{\pi}\left(\frac{D}{2}\right)^{\cancel{3}} \rho_s C_p \frac{dT}{dt} = -h\,(T - T_{air})\,(\cancel{4}\cancel{\pi})\left(\frac{D}{\cancel{2}}\right)^{\cancel{2}}$$

$$\frac{D}{6}\rho_s C_p \frac{dT}{dt} = -h\,(T - T_{air})$$

$$\int_{T_0}^{T_s} \frac{dT}{T - T_{air}} = \int_0^{t_s} -\frac{6h}{D\rho_s C_p}\,dt$$

$$\ln\left(\frac{T_s - T_{air}}{T_0 - T_{air}}\right) = -\frac{6h}{D\rho_s C_p}\,t_s$$

Derive the DE for height of liquid in a tank w.r.t. time when the tank is connected to a long straight pipe with a valve on it.



Use viscous losses

Mass bal:

$$A\frac{dh}{dt} = -\dot{Q} = -kP$$

pressure exerted on valve

proportionality const.

Bernoulli's eqn:

$$[P_0 + \rho g h] = P_2$$

$$A\frac{dh}{dt} = -kP_0 - \rho g h k$$

$$\frac{dh}{dt} = -\frac{\rho g k}{A}h - \frac{kP_0}{A}$$

let $\alpha = -\frac{\rho g k}{A}$

$\beta = -\frac{kP_0}{A}$

$$\frac{dh}{dt} = \alpha h - \beta$$

let $h = u(t)\,v(t)$

$$u\frac{dv}{dt} + v\frac{du}{dt} = \alpha u v - \beta$$