

<u>Unit 2 Nonlinear Classification</u>, <u>Linear regression, Collaborative</u>

3. Introduction to Non-linear

> <u>Lecture 6. Nonlinear Classification</u> > Classification

Course > Filtering (2 weeks)

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# 3. Introduction to Non-linear Classification Introduction to Non-linear Classification



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## **Counting Dimensions of Feature Vectors**

1/1 point (graded)

Let  $x\in\mathbf{R}^{150}$ , i.e.  $x=\left[x_1,x_2,\ldots,x_{150}\right]^T$  where  $x_i$  is the i-th component of x. Let  $\phi\left(x\right)$  be an **order** 3 polynomial feature vector. This means, for example,  $\phi\left(x\right)$  can be

$$\phi\left(x\right) = \underbrace{\left[\underbrace{x_{1}, \ldots, x_{i}, \ldots, x_{150}}_{\deg 1}, \underbrace{x_{1}^{2}, x_{1}x_{2}, \ldots, x_{i}x_{j}, \ldots x_{150}^{2}}_{\deg 2}, \underbrace{x_{1}^{3}, x_{1}^{2}x_{2}, \ldots, x_{i}x_{j}x_{k}, \ldots, x_{150}^{3}}_{\deg 3}\right]}_{\text{where } 1 \leq i \leq j \leq k \leq 150.$$

Note that the components of  $\phi(x)$  forms a basis of the space of all polynomials with zero constant term and of degree at most 3.

What is the dimension of the space that  $\phi\left(x\right)$  lives in? That is,  $\phi\left(x\right)\in\mathbb{R}^{d}$  for what d?

*Hint*: The number of ways to select a multiset of k non-unique items from n total is  $\binom{n+k-1}{k}$ . For example, if a ball can be any of 3 colors,

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then the number of color configurations of 2 balls is  $\binom{3+2-1}{2}=\binom{4}{2}=6.$ 

d= 585275 ightharpoonup Answer: 585275

#### Solution:

For each of the feature transformations (power 1, power 2, power 3), there are n-multichoose-power combinations. Thus  $\binom{150}{1}+\binom{151}{2}+\binom{152}{3}=585275$ . **Remark:** We see that the dimension of the space that the feature vectors live grows quickly as a function of d, the dimension we started with if  $x\in\mathbb{R}^d$ .

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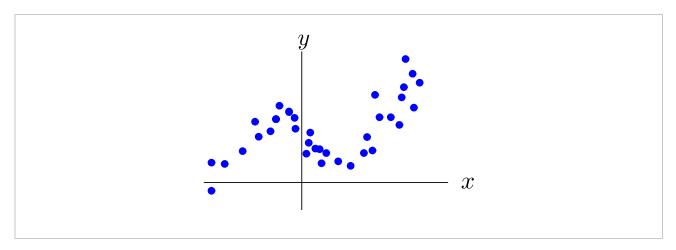
You have used 1 of 3 attempts

• Answers are displayed within the problem

## Regression using Higher Order Polynomial feature

1/1 point (graded)

Assume we have n data points in the training set  $\left\{\left(x^{(t)},y^{(t)}\right)\right\}_{t=1,\dots,n}$  where  $\left(x^{(t)},y^{(t)}\right)$  is the t-th training example:



We want to find a non-linear regression function f that predicts y from x, given by

$$f(x; \theta, \theta_0) = \theta \cdot \phi(x) + \theta_0$$

where  $\phi(x)$  is a polynomial feature vector of some order. What (loosely) is the minimum order of  $\phi(x)$ ?

3 **✓** Answer: 3

### Solution:

The relationship between y and x can be roughly described by a cubic function, so a feature vector  $\phi(x)$  of minimum order 3 can minimize structural errors.

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You have used 1 of 2 attempts

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## Effect of Regularization on Higher Order Regression

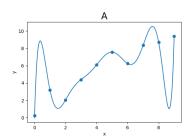
2/2 points (graded)

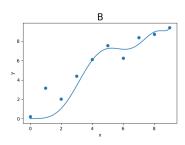
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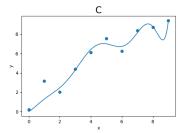
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Let us go back to explore the effect of regularizaion on Higher Order regression.

The three figures below show the fitting result of a 9th order polynomial regression with different regularization parameter  $\lambda$  on the same training data.







Which figure above corresponds to the smallest regularization parameter  $\lambda$ ?

	ullet	) A





Which figure corresponds to the largest regularization parameter  $\lambda$ ?









## Solution:

The effect of regularization is to restrict the parameters of a model to freely take on large values. This will make the model function smoother, leveling the 'hills' and filling the 'vallyes'. It will also make the model more stable, as a small perturbation on x will not change y significantly with smaller  $\|\theta\|$ .

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**1** Answers are displayed within the problem

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Enquiry per Question 1
Why the calculation has to be done by the (n,k) formula? If I consider it as a counting problem divided by sequence (as sequence does not matter for polynomial), why isn't th...

Non-linear regression function - minimum order Polynomial feature
I am confuse with something that looks obvious. Should an order 2 Polynomial feature be not linear? I would say so since we introduce non-linear terms (such as x1x2 or x1^2...

Q1 Counting Dimensions of Feature Vectors, another way to look at it

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