The original problem is below. Since we are looking for a 10 digit number where each c_i is the number of i's in the number we know that the sum of the digits must be 10. Thus we only need to look at the following possibilities for the digits:

9 1 0 0 0 0 0 0 0 0	5 3 1 1 0 0 0 0 0 0	3 3 2 2 0 0 0 0 0 0
8 2 0 0 0 0 0 0 0 0	5 2 2 1 0 0 0 0 0 0	3 3 2 1 1 0 0 0 0 0
8 1 1 0 0 0 0 0 0 0	5 2 1 1 1 0 0 0 0 0	3 3 1 1 1 1 0 0 0 0
7 3 0 0 0 0 0 0 0 0	5 1 1 1 1 1 0 0 0 0	3 2 2 2 1 0 0 0 0 0
7 2 1 0 0 0 0 0 0 0	4 4 2 0 0 0 0 0 0 0	3 2 2 1 1 1 0 0 0 0
7 1 1 1 0 0 0 0 0 0	4 4 1 1 0 0 0 0 0 0	3 2 1 1 1 1 1 0 0 0
6 4 0 0 0 0 0 0 0 0	4 3 3 0 0 0 0 0 0 0	3 1 1 1 1 1 1 1 0 0
6 3 1 0 0 0 0 0 0 0	4 3 2 1 0 0 0 0 0 0	2 2 2 2 2 0 0 0 0 0
6 2 2 0 0 0 0 0 0 0	4 3 1 1 1 0 0 0 0 0	2 2 2 2 1 1 0 0 0 0
6 2 1 1 0 0 0 0 0 0	4 2 2 2 0 0 0 0 0 0	2 2 2 1 1 1 1 0 0 0
6 1 1 1 1 1 0 0 0 0	4 2 2 1 1 0 0 0 0 0	2 2 1 1 1 1 1 1 0 0
5 5 0 0 0 0 0 0 0 0	4 2 1 1 1 1 0 0 0 0	2 1 1 1 1 1 1 1 1 0
5 4 1 0 0 0 0 0 0 0	4 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 1 1 1
5 3 2 0 0 0 0 0 0 0	3 3 3 1 0 0 0 0 0 0	

Now for each of these possibilities count the number of each digit found in the number. For example 9 1 0 0 0 0 0 0 0 0 gives

8 1 0 0 0 0 0 0 0 1. (8 zeros, 1 one, 1 nine, 0 for everything else.) If the digits are the same in the original number and the count then the count is the solution to the problem. The only one that works is 6 2 1 1 0 0 0 0 0 0 which gives

6 2 1 0 0 0 1 0 0 0 so this is the only solution.

Digit Counting

Find a 10 digit number of the form

$c_0\ c_1\ c_2\ c_3\ c_4\ c_5\ c_6\ c_7\ c_8\ c_9$

such that each c_i is the number of i's in the number. i.e. if there are four zeros in the number then $c_0=4$.

Small example:

Suppose we were looking for a 4 digit number: $c_0\ c_1\ c_2\ c_3$ This means that c_0 is the number of 0's in the number, c_1 is the number of 1's in the number, c_2 is the number of 2's in the number, and c_3 is the number of 3's in the number. There are 2 solutions: 2020 which has 2 zeros, 0 ones, 2 twos, and 0 threes; and 1210 which has 1 zero, 2 ones, 1 two and 0 threes.