

The original problem is below.

The problem can be solved by considering fractions with denominators that are prime numbers other than 2 or 5. These fractions will give purely repeating decimals. For example  $1/13 = .076923076923...$  where 076923 repeats forever. Now consider

$1/13 = .076923...$	$7/13 = .538461...$
$2/13 = .153846...$	$8/13 = .615384...$
$3/13 = .230769...$	$9/13 = .692307...$
$4/13 = .307692...$	$10/13 = .769230...$
$5/13 = .384615...$	$11/13 = .846153...$
$6/13 = .461538...$	$12/13 = .923076...$

Notice that  $1/13$ ,  $3/13$ ,  $4/13$ ,  $9/13$ ,  $10/13$ ,  $12/13$  are rotations of 076923 and that  $2/13$ ,  $5/13$ ,  $6/13$ ,  $7/13$ ,  $8/13$ ,  $11/13$  are rotations of 153846. Since there are 6 rotations of a 6 digit number (assuming the original number is a rotation of itself), there must be two different sets of rotating numbers in the fractions when dividing by 13 as we can see from the table above. Likewise if you consider  $1/11 = .090909...$  The repeating decimal is two digits long but there are 10 numerators less than 11 so there will be 5 sets of rotations. These are 09, 18, 27, 36, and 45.

If we find a prime so that the number of repeating digits in the decimal is one less than the prime, then all of the fractions will have repeating decimals that are rotations of each other.

$1/7 = .142857...$  so each of  $2/7, 3/7, 4/7, 5/7$ , and  $6/7$  will be rotations of  $1/7$ . Thus the answer to the problem is 142857.

By the way  $1/17 = .0588235294117647...$  which is 16 digits long. So 0588235294117647 multiplied by 2,3,4,5,6,7,8,9,10,11,12,13,14,15, or 16 gives a rotation of the number.

### My Favorite Number

Find a 6 digit nonzero number so that when the number is multiplied by 2,3,4,5, or 6, the product is a rotation of the original number.

A rotation of a number takes digits from the front of the number and puts them at the end in the original order. For example the rotations of 37452 would be 74523, 45237, 52374, and 23745.