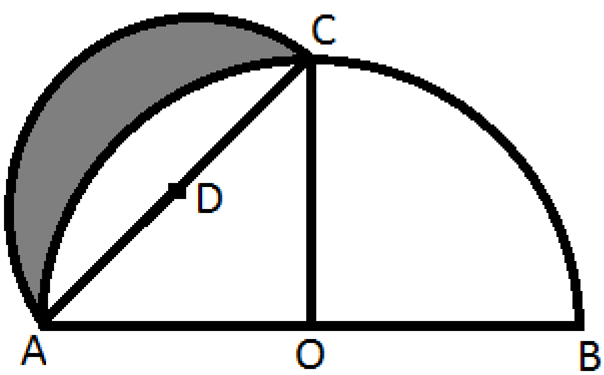
The original problem is below. This problem was first solved in the 5^{th} century b.c. by Hippocrates of Chios who proved that the area shaded is equal to the area of the triangle AOC. In the case of our problem AO and CO are both 6 so using the formula area of triangle = ½ base x height gives exactly 18 for the shaded area.

A lune is a plane figure bounded by two circular arcs, so the work of Hippocrates became known as Quadrature of the Lune. An excellent exposition of this work and his proof can be found in <u>Journey Through Genius: The Great Theorems of Mathematics</u> by William Dunham.

Another way to solve the problem is to find the area of the semi-circle centered at D with radius AD and subtract off the unshaded area. AO and CO are both 6 and from a right triangle so using the Pythagorean Theorem we know that AC = $\sqrt{72}$= $6\sqrt{2}$. Thus AD = $3\sqrt{2}$. So the area of the semi-circle centered at D with radius AD is $\frac{1}{2}\pi(3\sqrt{2})^2 = 9\pi$. The area of the ¼ circle centered at O with radius OA is $\frac{1}{4}\pi(6)^2=9\pi$ while the area of the triangle AOC is 18 so the area of the unshaded portion of the semi-circle ADC is 9π-18. Subtracting this from the area of the semi-circle gives that the area of the shaded part is 9π- (9π-18) = 18.

Lunar Area

Let AB be the diameter of a semicircle with center O. Construct OC perpendicular to AB. Let D be the midpoint of AC. Construct a semicircle with center D and diameter AC. Find the area of the shaded region if the length of AB is 12.



Please give your answer as an exact value. For example: π instead of 3.14.