Digit Counting

Find a 10 digit number of the form

$c_0\, c_1\, c_2\, c_3\, c_4\, c_5\, c_6\, c_7\, c_8\, c_9$

such that each c_i is the number of i's in the number. i.e. if there are four zeros in the number then $c_0=4$.

Small example:

Suppose we were looking for a 4 digit number: $c_0\, c_1\, c_2\, c_3$ This means that c_0 is the number of 0's in the number, c_1 is the number of 1's in the number, c_2 is the number of 2's in the number, and c_3 is the number of 3's in the number. There are 2 solutions: 2020 which has 2 zeros, 0 ones, 2 twos, and 0 threes; and 1210 which has 1 zero, 2 ones, 1 two and 0 threes.