Macroeconomic Model Building: Impulse-response functions

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Agents & decisions

Agents

- Household
  - Utility maximization
  - Labor supply
  - Decision on capital accumulation, consumption, saving
- Firm
  - Profit maximization, capital demand, labor demand, production

Markets

- Goods market
- Assets market
- Capital market
- Labor market

Model equations

- Euler equation: $u_{C_t} = u_{C_{t+1}} \beta (1 + r_{t+1})$
- Capital supply: $1 + r^K_{t+1} - \delta = 1 + r_{t+1}$
- Labor supply: $g_{L_t} = u_{C_t} w_t$
- Budget constraint:
  $r^K_t K_t + w_t L_t + \pi_t + (1 + r_t) B_t = C_t + B_{t+1} + I_t$
- Investment function: $I_t = K_{t+1} - (1 - \delta) K_t$
- Production function: $Y_t = f(K_t, L_t)$
- Capital demand: $MPK_t = r^K_t$
- Labor demand: $MPL_t = w_t$
- Goods market: $Y_t = C_t + I_t$
- Assets market: $B_t = 0$
- Capital market: $K^S_t = K^D_t$
- Labor market: $L^S_t = L^D_t$

Steady state

- Euler equation: $u_C = u_C \beta(1 + r)$
- Capital supply: $1 + r^K - \delta = 1 + r$
- Labor supply: $g_L = u_C w$
- Budget constraint: $r^K K + wL + \pi + (1 + r)B = C + B + I$
- Investment function: $I = K - (1 - \delta)K$
- Production function: $Y = f(K, L)$
- Capital demand: $MPK = r^K$
- Labor demand: $MPL = w$
- Goods market: $Y = C + I$
- Assets market: $B = 0$
- Capital market: $K^S = K^D$
- Labor market: $L^S = L^D$

Description of the economy

Our economy of interest is characterized by the following equations:

$$U = \sum_{t=1}^{\infty} 0.98^{t-1}(\ln C_t - 3.22L_t^2)$$

$$I_t = K_{t+1} - (1 - 0.05)K_t$$

$$Y_t = A_t K_t^{0.3} L_t^{0.7}$$

We are in 2017, the economy is in steady state and the total factor productivity is one. Assume that in 2018 the TFP increases to 1.1, but in 2019 it decreases back to one and stays on that level for the rest of the periods. Write a code that presents how this shock affects consumption, production and investment.

First step

The economy is in its steady state in 2017, thus we have to calculate the steady state first. It will also return to the steady state after the shock in a reasonable time period (let's say 100 years), thus it will be useful later as well. Remember the most important steady state equations!

- Euler equation: $u_C = u_C \beta(1 + r)$
- Capital supply: $1 + r^K - \delta = 1 + r$
- Labor supply: $g_L = u_C w$
- Budget constraint: $r^K K + wL + \pi + (1 + r)B = C + B + I$
- Investment function: $I = K - (1 - \delta)K$
- Production function: $Y = f(K, L)$
- Capital demand: $MPK = r^K$
- Labor demand: $MPL = w$
- Goods market: $Y = C + I$
- Assets market: $B = 0$

Steady state equations

- Euler equation: $1 = \beta(1 + r)$
- Capital supply: $1 + r^K - \delta = 1 + r$
- Labor supply: $3.22 \cdot 2L = \frac{w}{C}$
- Budget constraint: $r^K K + wL + \pi + (1 + r)B = C + B + I$
- Investment function: $I = K - (1 - \delta)K$
- Production function: $Y = K^{0.3}L^{0.7}$
- Capital demand: $0.3\frac{Y}{K} = r^K$
- Labor demand: $0.7\frac{Y}{L} = w$
- Goods market: $Y = C + I$
- Assets market: $B = 0$

Steady state step-by-step

$$1 + r = \frac{1}{\beta}$$

$$r^K = 1 + r - (1 - \delta)$$

$$\frac{Y}{K} = \frac{r^K}{0.3}$$

$$\frac{L}{K} = \left(\frac{Y}{K}\right)^{\frac{1}{0.7}}$$

$$\frac{Y}{L} = \frac{Y/K}{L/K}$$

$$w = 0.7\frac{Y}{L}$$

We already know r, r^K, w and a good number of ratios, like $\frac{Y}{K}$, $\frac{L}{K}$ and $\frac{Y}{L}$. Let's use them, to calculate L!

Steady state step-by-step

$$3.22 \cdot 2L = \frac{w}{C} \qquad \text{(Labor supply)}$$

$$C = \frac{w}{3.22 \cdot 2L}$$

$$I = \delta K \qquad \text{(Investment function)}$$

$$Y = \frac{w}{3.22 \cdot 2L} + \delta K \qquad \text{(Goods market)}$$

$$\frac{Y}{L} = \frac{w}{3.22 \cdot 2L^2} + \delta \frac{K}{L} \qquad \text{(Divide by L)}$$

$$L = \left[\frac{w}{3.22 \cdot 2 \cdot \left(\frac{Y}{L} - \delta \frac{K}{L} \right)} \right]^{\frac{1}{2}} \qquad \text{(Rearrange)}$$

We know w, $\frac{Y}{L}$ and $\frac{K}{L}$ as well, thus we can calculate L.

Steady state step-by-step

From here on, it's easy.

$$Y = \frac{Y}{L} \cdot L$$
$$K = \frac{L}{L/K}$$
$$I = \delta \cdot K$$
$$C = Y - I$$

In MATLAB

First make sure that nothing interferes

```
close all;
clear all;
clc;
```

And define some globals that we will use later

```
global KS LS alpha;
```

In MATLAB

Set the parameters

```
beta = 0.98;
delta = 0.05;
Psi = 3.22;
eta = 2;
a = 1;
alpha = 0.3;
```

In MATLAB

Use the previous equations to calculate steady state

```
R = 1/beta;
rk = R-(1-delta);
yk = rk/alpha;
lk = (yk/a)^(1/(1-alpha));
yl = yk/lk;
w = (1-alpha)*yl;
LS = ((w/(eta*Psi))/(yl-delta/lk))^(1/eta);
YS = yl*LS;
KS = LS/lk;
IS = delta*KS;
CS = YS-IS;
```

The transition

In 2018, there is a TFP shock. Let's assume that by 2118, the economy returns to the steady state. Thus any endogenous variable takes it's steady state value in all periods before 2018 and also in 2118 (and after). For 2018, 2019, 2020 … 2117 we need to solve a system of equations.

The transition - model equations

- Euler equation: $\frac{1}{C_t} = \frac{1}{C_{t+1}}\beta(1 + r_{t+1})$
- Capital supply: $1 + r^K_{t+1} - 0.05 = 1 + r_{t+1}$
- Labor supply: $6.44L_t = \frac{w_t}{C_t}$
- Investment function: $I_t = K_{t+1} - (1 - 0.05)K_t$
- Production function: $Y_t = A_t K_t^{0.3} L_t^{0.7}$
- Capital demand: $K_t = 0.3\frac{Y_t}{r^K_t}$
- Labor demand: $L_t = 0.7\frac{Y_t}{w_t}$
- Goods market: $Y_t = C_t + I_t$

We don't want to solve so many equations, let's reduce them!

Model reduction

Express the real wage and the real rental rate of capital from the labor and capital demand functions, and substitute them in all other functions!

- Capital demand: $r_t^K = 0.3\frac{Y_t}{K_t}$
- Labor demand: $w_t = 0.7\frac{Y_t}{L_t}$

Then

- Euler equation: $\frac{1}{C_t} = \frac{1}{C_{t+1}}\beta(1 + r_{t+1})$
- Capital supply: $1 + 0.3\frac{Y_{t+1}}{K_{t+1}} - 0.05 = 1 + r_{t+1}$
- Labor supply: $6.44L_t = \frac{0.7\frac{Y_t}{L_t}}{C_t}$
- Investment function: $I_t = K_{t+1} - (1 - 0.05)K_t$
- Production function: $Y_t = A_t K_t^{0.3} L_t^{0.7}$
- Goods market: $Y_t = C_t + I_t$

Model reduction

Substitute Y_t with the production function!

- Production function: $Y_t = A_t K_t^{0.3} L_t^{0.7}$

Thus

- Euler equation: $\frac{1}{C_t} = \frac{1}{C_{t+1}} \beta(1 + r_{t+1})$
- Capital supply: $1 + 0.3 \frac{A_{t+1} K_{t+1}^{0.3} L_{t+1}^{0.7}}{K_{t+1}} - 0.05 = 1 + r_{t+1}$
- Labor supply: $6.44 L_t = \frac{0.7 \frac{A_t K_t^{0.3} L_t^{0.7}}{L_t}}{C_t}$
- Investment function: $I_t = K_{t+1} - (1 - 0.05) K_t$
- Goods market: $A_t K_t^{0.3} L_t^{0.7} = C_t + I_t$

Model reduction

Substitute the investment in the Goods market clearing

- Investment function: $I_t = K_{t+1} - (1 - 0.05)K_t$

Thus

- Euler equation: $\frac{1}{C_t} = \frac{1}{C_{t+1}}\beta(1 + r_{t+1})$
- Capital supply: $1 + 0.3\frac{A_{t+1}K_{t+1}^{0.3}L_{t+1}^{0.7}}{K_{t+1}} - 0.05 = 1 + r_{t+1}$
- Labor supply: $6.44L_t = \frac{0.7\frac{A_tK_t^{0.3}L_t^{0.7}}{L_t}}{C_t}$
- Goods market: $A_tK_t^{0.3}L_t^{0.7} = C_t + K_{t+1} - (1 - 0.05)K_t$

Model reduction

Merge the capital supply into the Euler equation

- Capital supply: $= 1 + r_{t+1}$

Thus

- Euler equation: $\frac{1}{C_t} = \frac{1}{C_{t+1}}\beta\left(1 + 0.3\frac{A_{t+1}K_{t+1}^{0.3}L_{t+1}^{0.7}}{K_{t+1}} - 0.05\right)$
- Labor supply: $6.44L_t = \frac{0.7\frac{A_t K_t^{0.3}L_t^{0.7}}{L_t}}{C_t}$
- Goods market: $A_t K_t^{0.3} L_t^{0.7} = C_t + K_{t+1} - (1 - 0.05)K_t$

Model reduction

Express C_t from the labor supply, and substitute it everywhere.

- Labor supply: $C_t = 0.7\frac{A_t K_t^{0.3} L_t^{0.7}}{6.44 L_t^2}$

Thus

- Euler equation:

$$\frac{1}{0.7\frac{A_t K_t^{0.3} L_t^{0.7}}{6.44 L_t^2}} = \frac{1}{0.7\frac{A_{t+1} K_{t+1}^{0.3} L_{t+1}^{0.7}}{6.44 L_{t+1}^2}} \beta \left(1 + 0.3\frac{A_{t+1} K_{t+1}^{0.3} L_{t+1}^{0.7}}{K_{t+1}} - 0.05\right)$$

- Goods market:
$$A_t K_t^{0.3} L_t^{0.7} = 0.7\frac{A_t K_t^{0.3} L_t^{0.7}}{6.44 L_t^2} + K_{t+1} - (1 - 0.05) K_t$$

Model reduction

Let's simplify them!

- Euler equation:

$$\frac{A_{t+1}K_{t+1}^{0.3}L_{t+1}^{-1.3}}{A_tK_t^{0.3}L_t^{-1.3}} = 0.98\left(1 - 0.05 + 0.3A_{t+1}K_{t+1}^{-0.7}L_{t+1}^{0.7}\right)$$

- Goods market:
$A_tK_t^{0.3}L_t^{0.7} = \frac{0.7}{6.44}A_tK_t^{0.3}L_t^{-1.3} + K_{t+1} - (1 - 0.05)K_t$

Assigning new numbers

To avoid using 2018, 2019, etc. let's denote:

$$2018 = 1$$

$$2019 = 2$$

$$2020 = 3$$

$$\vdots$$

$$2117 = 100$$

We want to solve

$$\frac{A_2 K_2^{0.3} L_2^{-1.3}}{A_1 K_1^{0.3} L_1^{-1.3}} = 0.98 \left(1 - 0.05 + 0.3 A_2 K_2^{-0.7} L_2^{0.7}\right)$$

$$A_1 K_1^{0.3} L_1^{0.7} = \frac{0.7}{6.44} A_1 K_1^{0.3} L_1^{-1.3} + K_2 - (1 - 0.05)1$$

$$\vdots$$

$$\frac{A_{101} K_{101}^{0.3} L_{101}^{-1.3}}{A_{100} K_{100}^{0.3} L_{100}^{-1.3}} = 0.98 \left(1 - 0.05 + 0.3 A_{101} K_{101}^{-0.7} L_{101}^{0.7}\right)$$

$$A_{100} K_{100}^{0.3} L_{100}^{0.7} = \frac{0.7}{6.44} A_{100} K_{100}^{0.3} L_{100}^{-1.3} + K_{101} - (1 - 0.05)1$$

We know that

$$A_1 = 1.1$$
$$A_2 = 1$$
$$\vdots$$
$$A_{101} = 1$$

And also $K_1 = KS$ and $L_{101} = LS$.

Let's denote

$$L_1 = x(1)$$
$$L_2 = x(2)$$
$$\vdots$$
$$L_{100} = x(100)$$

and

$$K_2 = x(101)$$
$$K_3 = x(102)$$
$$\vdots$$
$$K_{101} = x(200)$$

Equations

The first 100 equations will be of type

$$\frac{A_{t+1}K_{t+1}^{0.3}L_{t+1}^{-1.3}}{A_tK_t^{0.3}L_t^{-1.3}} = 0.98\left(1 - 0.05 + 0.3A_{t+1}K_{t+1}^{-0.7}L_{t+1}^{0.7}\right)$$

the second 100 equations will be of type

$$A_tK_t^{0.3}L_t^{0.7} = \frac{0.7}{6.44}A_tK_t^{0.3}L_t^{-1.3} + K_{t+1} - (1 - 0.05)K_t$$

Equations in MATLAB

The first equation in MATLAB will be

$$\frac{x_{101}^{0.3}x_2^{-1.3}}{1.1KS^{0.3}x_1^{-1.3}} - 0.98\left(1 - 0.05 + 0.3x_{101}^{-0.7}x_2^{0.7}\right)$$

The next 98 equations will be (if i is the number of equation)

$$\frac{x_{i+100}^{0.3}x_{i+1}^{-1.3}}{x_{i+99}^{0.3}x_i^{-1.3}} - 0.98\left(1 - 0.05 + 0.3x_{i+100}^{-0.7}x_{i+1}^{0.7}\right)$$

The 100th equation will be

$$\frac{x_{200}^{0.3}LS^{-1.3}}{x_{199}^{0.3}x_{100}^{-1.3}} - 0.98\left(1 - 0.05 + 0.3x_{200}^{-0.7}LS^{0.7}\right)$$

Equations in MATLAB

The 101th equation in MATLAB will be

$$1.1KS^{0.3}x_1^{0.7} - \frac{0.7}{6.44}1.1KS^{0.3}x(1)^{-1.3} + x_{101} - (1-0.05)KS$$

The next 99 equations will be (if i is the number of equation)

$$x_{i-1}^{0.3}x_{i-100}^{0.7} = \frac{0.7}{6.44}x_{i-1}^{0.3}x_{i-100}^{-1.3} + x_i - (1-0.05)x_{i-1}$$

To solve the system, we use

```
op = optimset('MaxFunEvals',10000,'MaxIter',10000);
xstart = [LS*ones(100,1) KS*ones(100,1)];

[x fval] = fsolve(@myfunction,xstart);

K = [KS x(101:200) KS];
L = [x(1:100) LS LS];
```

Production path

We use the production function

$$Y_t = A_t K_t^{0.3} L_t^{0.7}$$

```
A = [1.1 ones(101,1)];

for i=1:102
    Y(i)=A(i)*K(i)^(0.3)*L(i)^(0.7);
end
```

Investment path

We use the investment function

$$I_t = K_{t+1} - (1 - 0.05)K_t$$

```
for i=1:101
    I(i)=K(i+1)-0.95*K(i);
end
```

Consumption path

We use the goods market clearing

$$C_t = Y_t - I_t$$

```
for i=1:101
    C(i)=Y(i)-I(i);
end
```

Real wage path

We use the labor demand function

$$w_t = 0.7\frac{Y_t}{L_t}$$

```
for i=1:101
    w(i)=0.7*Y(i)/L(i);
end
```

Real rental rate of capital path

We use the capital demand function

$$r_t^K = 0.3\frac{Y_t}{K_t}$$

```
for i=1:101
    rk(i)=0.3*Y(i)/K(i);
end
```

Interest rate path

We use the capital supply function

$$r_t^K - \delta = r_t$$

```
for i=1:101
    r(i)=rk(i)-0.05;
end
```

Plotting IRFs

For 30 periods

```
periods=[1:30];
figure();
subplot(3,3,1);
plot(periods,A(1:30)-1);
legend('A');
subplot(3,3,2);
plot(periods,K(1:30)-KS);
legend('K');
subplot(3,3,3);
plot(periods,L(1:30)-LS);
legend('L');
```

Plotting IRFs - Continued

```
subplot(3,3,4);
plot(periods,Y(1:30)-YS);
legend('Y');
subplot(3,3,5);
plot(periods,I(1:30)-IS);
legend('I');
subplot(3,3,6);
plot(periods,C(1:30)-CS);
legend('C');
subplot(3,3,7);
plot(periods,w(1:30)-wS);
legend('w');
```

Plotting IRFs - Continued

```
subplot(3,3,8);
plot(periods,rk(1:30)-rkS);
legend('rk');
subplot(3,3,9);
plot(periods,r(1:30)-rS);
legend('r');
```