In the proof of Lemma 6, $k \geq 1$ and $\Delta = 2\left\lceil \frac{2^\ell n}{\varepsilon^{\ell-1}k} \right\rceil$. Then taking $\varepsilon > 0$ small enough so that $N_{\geq k}(m) \geq \varepsilon n$, we use Lemma 4 to show that whp, after $\Delta/2$ steps we have $\ell - 1$ components together containing at least $\varepsilon n/2$ vertices (this part of the proof actually shows a bit more than this, but it doesn't really matter).

Proposition 1. *For all $\varepsilon > 0$, whp there is a giant component after at most $(1+\varepsilon)n$ steps.*

Proof. We can assume $0 < \varepsilon \leq 2$. Note that if we set $k = 2^\ell/\varepsilon^{\ell-1}$, then $k \geq 1$, so we can set $m = 0$ and apply the same reasoning as in the proof of Lemma 6. Thus after $\Delta/2 = n < (1+\varepsilon)n$ steps, whp we have $\ell - 1$ components together containing at least $\varepsilon n/2$ vertices. This means that we must have at least one component with $\frac{\varepsilon}{2(\ell-1)}n$ vertices. Since ε and ℓ are fixed, this component is a giant component. $\square$