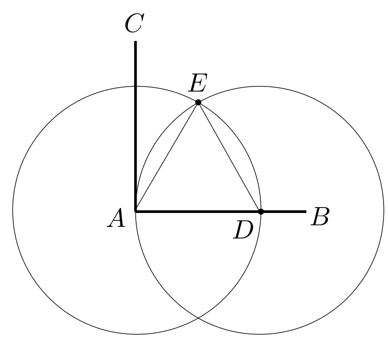
Exercise 1 (3.60)**.** Describe how to trisect an obtuse angle with a compass and twice notched straightedge.

Any obtuse angle can be decomposed into a right angle and an acute angle. We can easily trisect a right angle: Suppose $\angle CAB = 90°$ (as shown below), then form a circle $\mathcal{C}_A$ with radius less than $|AB|$. It intersects AB at a point D. Now construct $\mathcal{C}_D(|AD|)$, and call the point of intersection of the two circles that lies inside of $\angle CAB$ the point E. Since E is on both circles, and since both circles have the same radius, $|AE| = |ED| = |DE|$, i.e. ΔAED is an equilateral triangle. Thus $\angle EAD = 60°$, which means $\angle CAD = 30° = 90°/3$, so we have trisected $\angle CAB$.



So given an obtuse angle, we first split it into a right angle and an acute angle. We trisect the right angle as above, then we trisect the acute angle as in the proof of Theorem 3.59. This gives us a trisection of the whole obtuse angle.

Exercise 2 (5.3)**.** Show that each interior angle of a regular n-gon is $\frac{n-2}{n} \cdot 180°$.

We will first show by induction that the interior angles of *any* polygon (not necessarily regular) sum to $(n-2) \cdot 180°$. In the case of equilateral triangles ($n = 3$), we know this to be true: the angles in a triangle must sum to $180° = (n-2) \cdot 180°$.

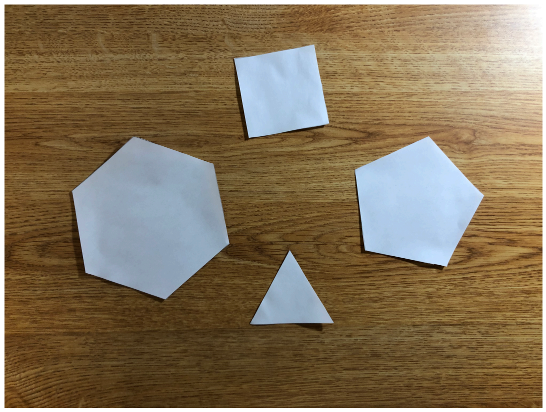
Now suppose the result holds for n-gons, then we want to show it also must hold for $(n+1)$-gons. Let $X_1 \cdots X_{n+1}$ be a regular $(n+1)$-gon with points labeled clockwise, then we can decompose it into the n-gon $X_1 \cdots X_n$ and the triangle $X_n X_{n+1} X_1$. By our inductive hypothesis and the fact that the interior angles of a triangles sum to $180°$, the sum of our $(n+1)$-gon's interior angles is

$$(n-2) \cdot 180° + 180° = ((n+1) - 2) \cdot 180°.$$

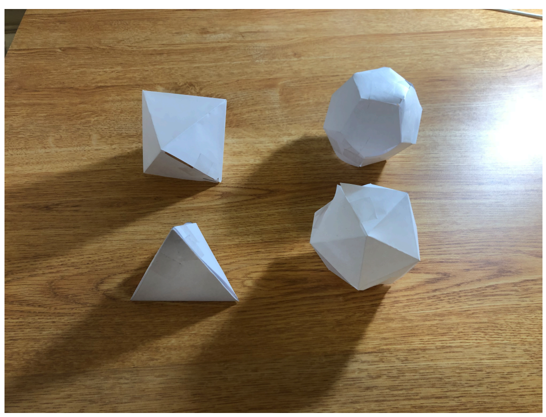
Now since we know we're working with *regular* polygons, each of the interior angles must be the same. Thus each angle is

$$\frac{n-2}{n} \cdot 180°.$$

Exercise 3 (5.4). Construct a regular triangle, square, pentagon, and hexagon, each with side length 2 inches.



Exercise 4 (5.5). Construct models of the Platonic solids using the templates from the previous exercise.



We can answer the next three questions using the same method. Suppose we have a semiregular polyhedron represented by

$$(\underbrace{3,\ldots,3}_{m \text{ times}},\underbrace{N,\ldots,N}_{n \text{ times}}).$$

Let F_3 denote the number of triangular faces, and let f_N denote the number of n-gonal faces. The number of vertices is

$$V = \frac{NF_N}{n} = \frac{3F_3}{m},$$

which implies $F_3 = \frac{Nm}{3n}F_N$. Using this identity, the number of edges is

$$E = \frac{3F_3 + NF_N}{2} = \frac{Nm + Nn}{2n}F_N$$

and the number of faces is

$$F = F_3 + F_N = \frac{3n + Nm}{3n}F_N.$$

Finally, we know that the Euler characteristic $\chi = V - E + F$ of this polyhedron is 2, so

$$\begin{aligned} V - E + F &= \frac{N}{n}F_N - \frac{Nm + Nn}{2n}F_N + \frac{3n + Nm}{3n}F_N \\ 2 &= \left(\frac{N(6 - m - 3n) + 6n}{6n}\right)F_N \\ F_N &= \frac{12n}{N(6 - m - 3n) + 6n}. \end{aligned}$$

Exercise 5 (5.17)**.** How many pentagonal faces are on a snub dodecadehron?

The snub dodecahedron has representation $(3,3,3,3,5)$, so $n = 1, N = 5$, and $m = 4$. Thus $F_5 = 12$.

Exercise 6 (5.18)**.** How many square faces are on a snub cube?

The snub cube has representation $(3,3,3,3,4)$, so $n = 1, N = 4$, and $m = 4$. Thus $F_4 = 6$.

Exercise 7 (5.19)**.** How many square faces are on a semiregular polyhedron represented by $(3,4,4,4)$?

$n = 3, N = 4$, and $m = 1$, so $F_4 = 18$.