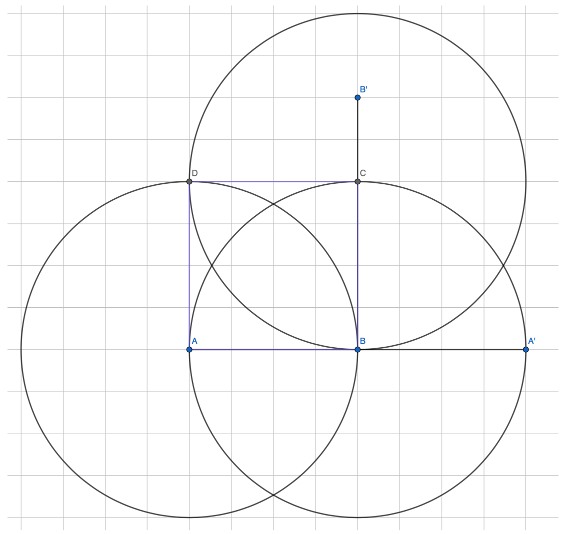
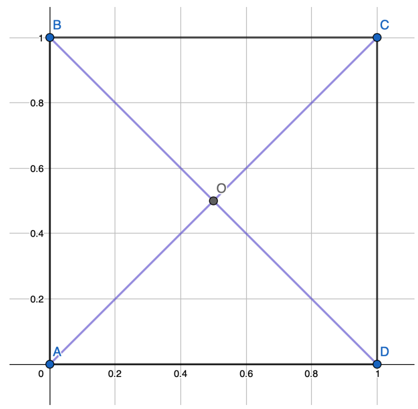
**Exercise 1** (3.5)**.** Construct a square with side length 1.



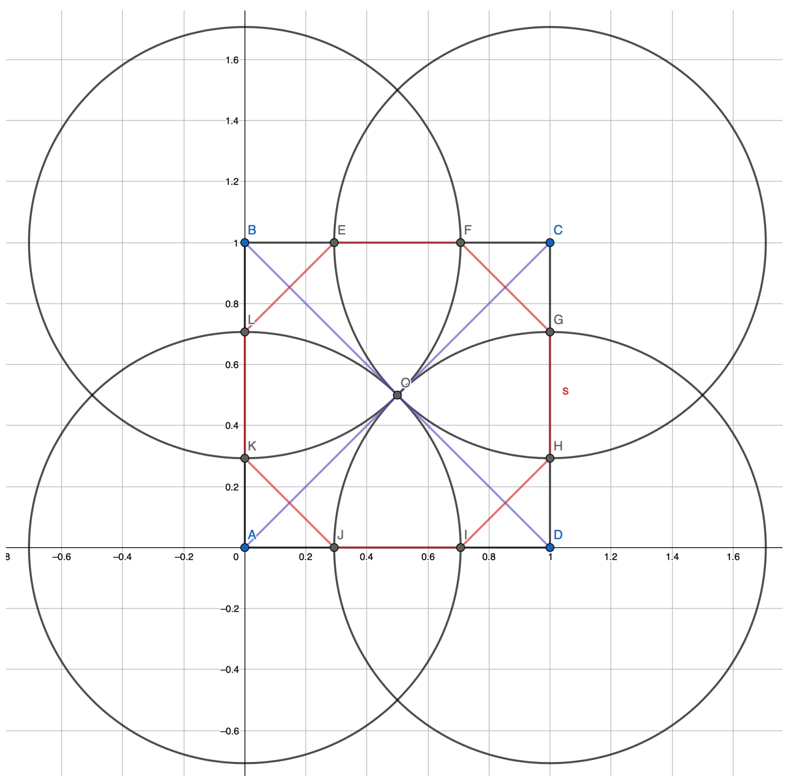

Suppose $|AB| = 1$, then extend this line, then construct the point $A'$ that is on the intersection of $\mathcal{C}_B(|AB|)$. By Lemma 3.7, we can construct the perpendicular bisector to this line, which must necessarily pass through $B$. Let $C$ be one of the two points on the perpendicular bisector of $AA'$ that intersects $\mathcal{C}_B(|AB|)$. Now let $D$ be the point of intersection (other than $B$) of $\mathcal{C}_C(|BC|)$ and $\mathcal{C}_A(|AB|)$. The quadrilateral $ABCD$ is then a square with side length $|AB| = 1$.

**Exercise 2** (3.16)**.** Construct a regular octagon.

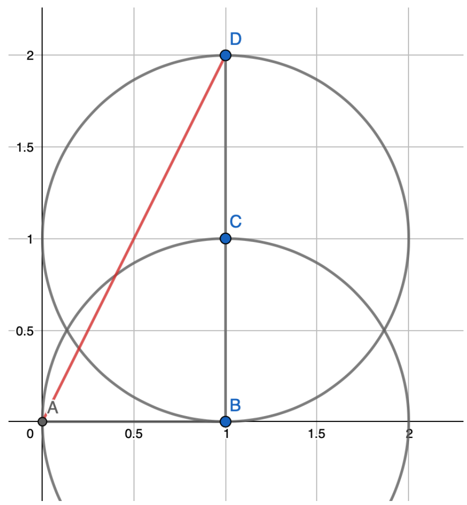
By the previous exercise, we know we can construct a square, so do that and label its edges $A, B, C, D$. Construct the diagonals and label their intersection $O$.




Now construct the four circles $\mathcal{C}_A(|AO|), \mathcal{C}_B(|BO|), \mathcal{C}_C(|CO|)$, and $\mathcal{C}_D(|DO|)$. The 8 intersection points of these circles with the square form a regular hexagon, pictured in red below.



**Exercise 3** (3.24)**.** Construct $\sqrt{5}$.

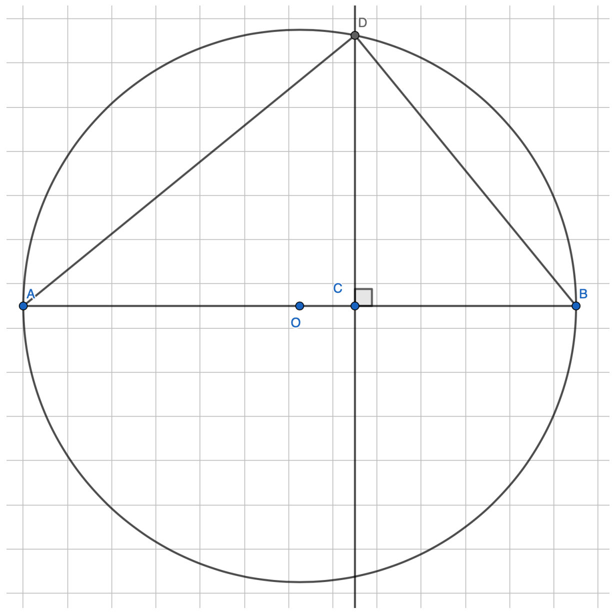



Take points $A$ and $B$ that are a distance 1 apart from each other. Construct the perpendicular to $AB$ through $B$, then choose a point at distance 2 from $B$ on it:

- First choose an intersection point of the perpendicular bisector and $\mathcal{C}_B(|AB|)$, and label it $C$.
- The circle $\mathcal{C}_C(|BC|)$ intersects the perpendicular bisector at two points: $B$ and some other point. Label this other point $D$. Since $|AB| = |BC| = 1$, the point $D$ is distance 2 from $B$.

By the Pythagorean Theorem, $|AC| = \sqrt{5}$.

**Exercise 4** (3.26)**.** Construct the geometric mean $\sqrt{ab}$.

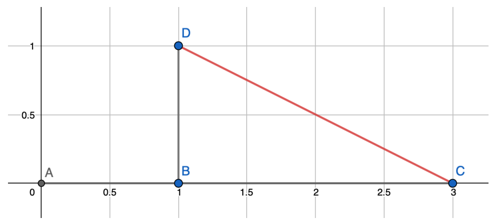



Take points $A$ and $C$ that are a distance $a$ away, then find a point $B$ on the extension of $AC$ that is a distance $b$ away from $C$. Now find the midpoint $O$ of $AB$, and use it to construct the circle $\mathcal{C}_O(|AO|)$.

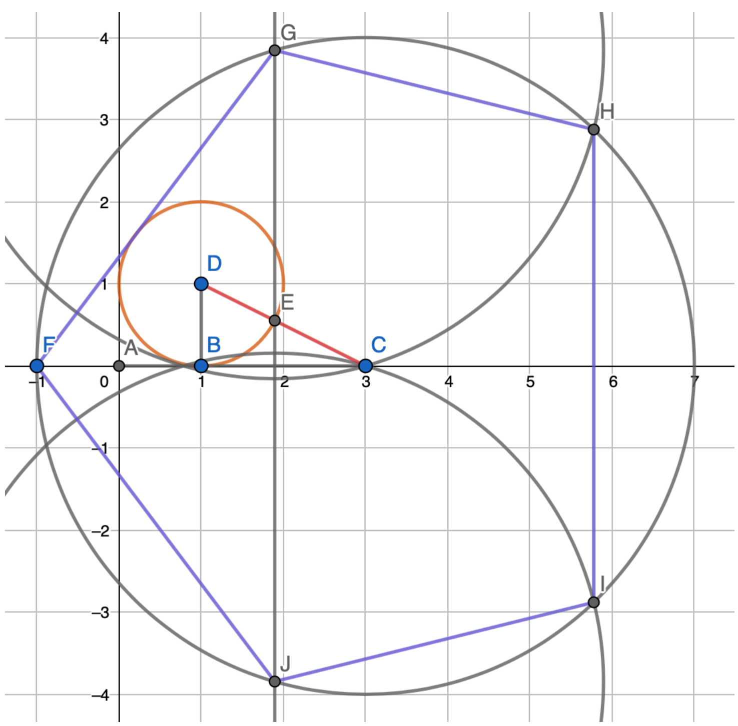
Construct the perpendicular to $AB$ through $C$ and call one of its intersections with the circle $D$. By symmetry, the power of the point $C$ is $|CD|^2 = |AC||BC| = ab$, so $|CD| = \sqrt{ab}$.

**Exercise 5** (3.33)**.** Construct a regular pentagon inscribed in a radius 4 circle.

Suppose $|AB| = 1$, then as in Exercise 3.24, we can construct a triangle with legs 1 and 2 and hypotnuse $\sqrt{5}$, as pictured below.




Now construct the point $E$ by taking the intersection of $\mathcal{C}_D(|BD|)$ with $CD$. Note that $|CE| = \sqrt{5} - 1$. Now take a perpendicular line of $FC$ through $E$, and mark its intersections with $\mathcal{C}_C(|CF|)$ as $G$ and $J$. Now mark the farther intersection points of $\mathcal{C}_J(|CJ|)$ and $\mathcal{C}_G(|CG|)$ with $\mathcal{C}_C(|CF|)$ as $H$ and $I$. Then $FGHIJ$ is our desired pentagon.



**Exercise 6** (3.40)**.** Prove the angle sum formulas with Euler's formula.

On one hand, we have

$$e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i\sin(\alpha+\beta).$$

But this also evalutes to

$$\begin{aligned} e^{i(\alpha+\beta)} &= e^{i\alpha}e^{i\beta} \\ &= \big(\cos\alpha + i\sin\alpha\big)\big(\cos\beta + i\sin\beta\big) \\ &= \cos\alpha\cos\beta - \sin\alpha\sin\beta + i\big(\sin\alpha\cos\beta + \cos\alpha\sin\beta\big). \end{aligned}$$

Equating these two then gives

$$\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta, \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta, \end{aligned}$$

as desired.

**Exercise 7** (3.55)**.** Is it possible to construct a triangle $\Delta ABC$ with $\angle BAC = 20°$, $|AB| = 4$, and $|AC| = 2\sqrt{3}$?

No, it is not possible. Suppose $20°$ were constructible, then we could also construct $40°$. But $40°$ is the innermost angle of each triangle composing a 9-gon, so the 9-gon would also be constructible. But by Theorem 3.49, the 9-gon cannot be constructed. Thus we cannot construct our desired $\Delta ABC$.