Math 502 Final Project

For your final project, you will pick a topic related to this course to explore in-depth. Potential project ideas are below, but feel free to come up with your own. The graded work of this project consists of two parts:

1. A paper written in LaTeX satisfying the following:
   - Aim for the paper to be around 10 pages in length (measured with 1.5 spacing). This is a suggestion: the formatting is less important than the writing/content.
   - Include a references section with at least three references. Let me know if you want tips on how to do this in LaTeX. Make sure your main references are library resources (edited articles, books) rather than random online sources which might not be accurate.
   - Include a couple examples or exercises that you worked out yourself.
   - Feel free to explore the history of your topic, but make sure the majority of the paper is mathematical.
2. A 30 minute lecture on your topic the last week of class. The goal is to teach your classmates about your topic. You are welcome to meet with me sometime the week before to discuss your plan and/or paper draft.

Timeline:

Tell me your topic before	March 26
(Optional) Draft of paper due before	April 9
(Optional) Meet with me	April 13-16
Presentations	April 20 and 22
Final Paper Due	April 23

Potential Project Ideas

- Constructible numbers, Straightedge and Compass constructions
  Understand the algebraic approach to classical geometric questions. See Section 13.3 and the end of section 14.5 of your book for more information. Exercise 17 in 14.5 about constructing a regular 17-gon would be great to include.
- Polynomials in Several Variable over a field and Gröbner Bases
  We mostly focused on one variable polynomials in this class but there's much to be said about polynomials in many variables. Gröbner bases are used in computational algebraic geometry to understand ideals of polynomial rings in several variables over a field. Possible resources to start with include section 9.6 in your book and the book *Ideals, Varieties, and Algorithms* by Cox, Little and O'Shea (I have a copy if you'd like to borrow it).

- Hilbert's Theorem 90 and Kummer Theory
  There are a series of exercises in section 14.2 of your book outlining Hilbert's Theorem 90 (starting with problem 17). A first goal would be to understand the norm and trace associated to elements in the larger field of a Galois extension. These are basically products and sums of Galois conjugates. Hilbert's Theorem 90 is a specific result about cyclic extensions under a specific norm property. We'll see this week that, under a few assumptions about the base field, cyclic extensions are simple radical extensions. Such extensions generalize to Kummer theory. Your book says a little about it in 14.7 and 17.3.

- The Kronecker-Weber Theorem and Class Field Theory
  Recall that the Kronecker-Weber theorem states that abelian extensions of the rational numbers are contained in cyclotomic extensions. There are multiple proofs to this in the literature, some more difficult than others. Class Field Theory, a way to study abelian field extensions, is involved in one approach the theorem. This topic gets very difficult quickly so a goal would be to understand the basic concepts involved.

- The Inverse Galois Problem
  It's still an open question in math whether every finite group can be realized as the Galois group of an extension of the rationals. We talked about some partial results in class, but more is known. There's a lot to explore here, but a possible resource is *Groups as Galois Groups: An Introduction* by Völklein (available through the library).