Percolation Phase Transitions on Dynamically Grown Graphs

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Background

Dynamically grown graphs and percolation

Dynamically Grown Graphs

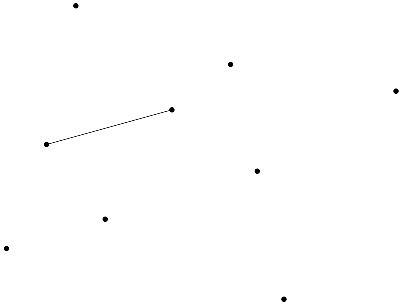
Start with a graph with n vertices and 0 edges

Add edges randomly every $1/n$ units of time

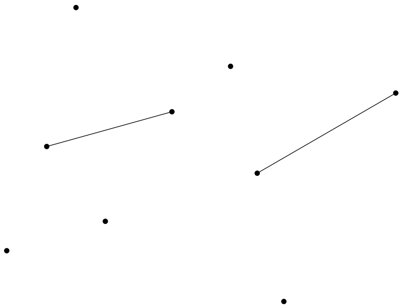
We'll work in the limit as $n \to \infty$

Dynamically Grown Graphs

Dynamically Grown Graphs



Dynamically Grown Graphs



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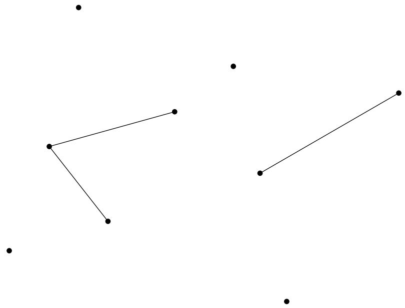

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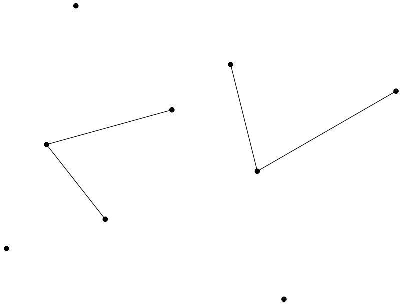

Percolation

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Percolation is when a giant component first emerges (call this time t_c)

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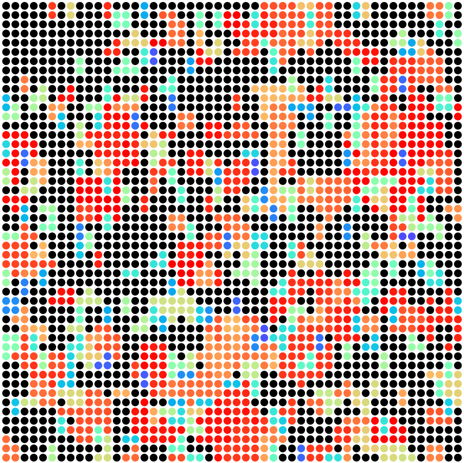
This emergence has lots of different behaviors

Erdős Rényi



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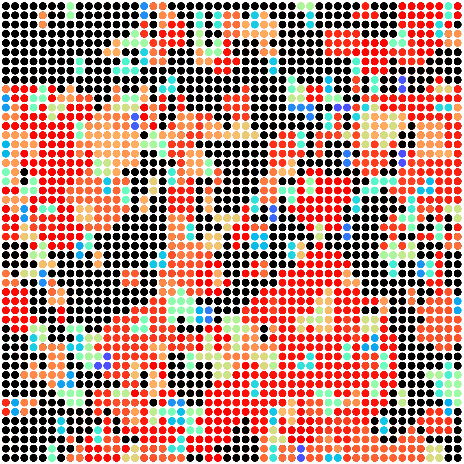

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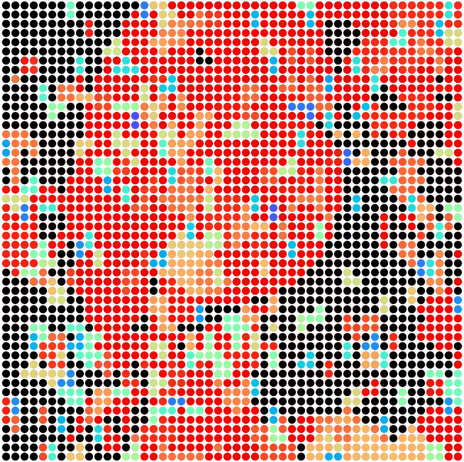
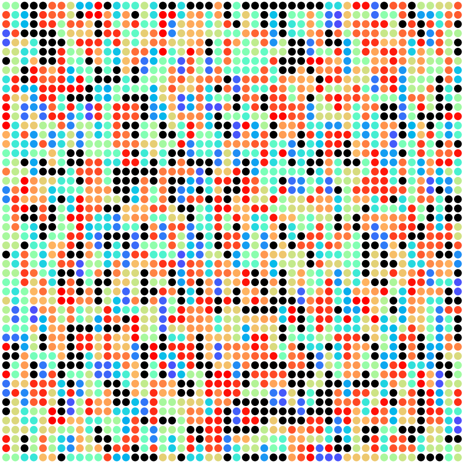


da Costa



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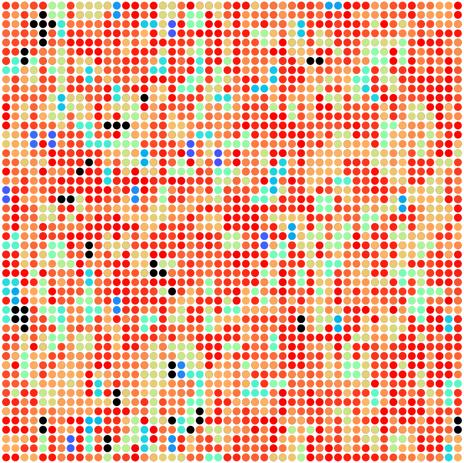

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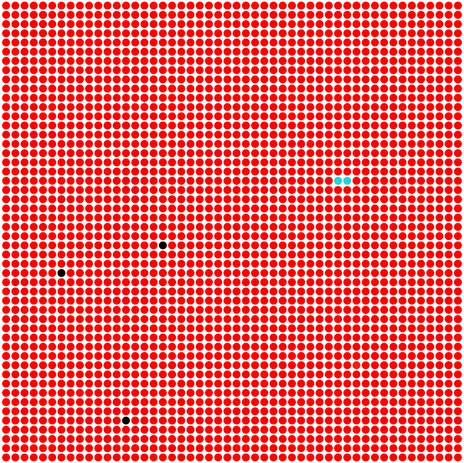

Explosive Percolation

Explosive Percolation is a sudden, seemingly discontinuous emergence of the giant component

Basic Results

Continuous phase transition and scaling behavior

Continuous phase transition

Define Achlioptas rule

Achlioptas claimed to have found a discontinuous emergence of a giant component based on simulations **When?**

Continuous phase transition

Riordan and Warnke (2012)

ℓ-vertex rule: choose ℓ vertices i.i.d., and you're only required to add an edge if all ℓ of them are in distinct clusters (generalizes Achlioptas processes)

All ℓ-vertex rules have a continuous phase transition

Continuous phase transition

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Proof by contradiction...

Scaling behavior

reference da Costa? who tf showed this?

The distribution of vertices belonging to a cluster of size s follows a power law

$$s^{1-\tau} f(s\delta^{1/\sigma})$$

where $\delta = t - t_c$ and f is a scaling function.

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Noticing scaling behavior in rules with explosive percolation was motivation for proving their continuity

Two-Choice Rules

Our Results

Two-Choice Rules

Pick two finite groups of i.i.d. vertices

Follow a deterministic method to choose a representative vertex from each group (can be a different rule for each group)

Add an edge between the two representatives

Two-Choice Rules

Erdős Rényi: Both groups are size 1, so this is the same as sampling edges randomly

Correspondence with Erdős Rényi random graph

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da Costa: both groups are of size m, and pick the vertex with the smallest cluster size from each group