0.1 Moments and Typical Size

Throughout, we'll be focused on solutions to the differential equation

$$\partial_t c(m,t) = \int_{y=0}^{m} K(y, m-y)c(y,t)c(m-y,t) - \int_{z=0}^{\infty} K(z,m)c(z,t)c(m,t). \qquad (\star)$$

Should there be a 1/2 at the beginning? The disrete version of this is

$$\partial_t c(m,t) = \frac{1}{2} \sum_{j+k=m} K(j,k)c(j,t)c(k,t) - \sum_{k} K(k,m)c(k,t)c(m,t).$$

Why the 1/2?

Proposition 1. *if $K(m,m')$ is homogeneous of degree λ and if $c(m,t)$ is a solution to $(\star)$, then*

$$T_{a,b}c(m,t) \doteq a^{\lambda+1} b c(am, bt)$$

is also a solution.

Proof. Since $c(m,t)$ is a solution to $(\star)$, we can expand

$$\partial_t T_{a,b}c(m,t) = a^{\lambda+1} b^2 \partial_{bt} c(am, bt)$$

into its integral expression. Making a change of variables $y = ay'$ and $z = az'$ and then using the homogeneity of K, we recover $(\star)$ but with $T_{a,b}c(m,t)$ in place of $c(m,t)$. $\square$