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Ministry of Education
General Directorate of Curricula

9

Part One

Series of Maths Books for Intermediate Stage

Mathematics

Third Intermediate

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This series (Maths for Intermediate stage) has been edited by a special team of specialists in Ministry of Education/ General Directorate of curricula with participation of specialists from universities professors in Ministry of higher Education according to international standards to achieve the goals of designing modern syllabus which helps the students to be:

Successful learners long life

Self-esteem individuals

Iraqi citizens feeling proud

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INTRODUCTION

The Maths subject is considered one of the basic courses that helps students to acquire educational abilities to develop his thinking and solving problems and it helping to deal with difficult situations in his life.

As a starting point of attention by the Ministry of Education represented by the General Directorate of curricula to develop the curricula in general and specially of Maths in order to go a long with the technological and scientific development in different fields of life. A plan has set up to edit the series of Maths books for the three stages. Primary stage has been achieved and the work started to continue the series by editing the books of intermediate stage.

The series of new Iraqi Maths Books as a part of General frame work of curricula that reinforces the basic values as Iraqi identity, forgiveness, respecting different opinions, social justice and offering equal chance for creativity and it also reinforces abilities of thinking and learning, self-efficiency, action and citizen ship efficiency.

The series of Iraqi Maths books has been built on student- centered learning according to international standards.

The series of Iraqi maths books for intermediate stage has been built on six items: learn, make sure of your understanding, solve the exercises, solve the problems, think and write. The Maths book for third intermediate stage contains four basic fields: The numbers and the operations, Algebra, Geometry and Measurement, Statistics and Probabilities for each field. The books consists of two parts. Part one which consists of three chapters ,each chapter has its own exercises.and part two also consists of three chapters and each chapter has its own exercise.

The maths books have distinguished by presenting material in modern styles that attract and help the student to be active through presenting drills, exercises and environmental problems in addition there are extra exercises at the end of the book that are different from the exercises and drills in the lessons because they are objective so the student can answer through multiple choices and that prepare the student to participate the international competitions.

This book is an expansion for the series of developed Maths books for primary stage and it is also considered as support for the developed syllabus in maths and it also has a teachers book so we hope in applying them, the student will gain scientific and practical skills and develop their interest to study Maths.

We hope God help us to serve our country and our sons

Authors

Chapter 1

Relations and Inequalities in Real Numbers

Tsunami wave is moved in a great rapidity in the Seas, but its rapidity becomes greater when it reaches to beach due to the effect of its huge energy. It terribly strikes beach to cause mass destruction. We can calculate speed of Tsunami by using ($v = \sqrt{9.6\, d}$ m/s, where (d) represents the deep of water in metre).

Pretest

Classify the number if it is rational or irrtional number:

1 $\sqrt{25}$

2 $\sqrt{7}$

3 $\frac{0}{\sqrt{3}}$

4 $\sqrt{\frac{16}{25}}$

5 $\sqrt{\frac{49}{5}}$

6 $\frac{30}{4}$

7 $-6\frac{3}{2}$

8 $-\sqrt{8}$

Estimate the following square roots by near them to the nearest tenth, then represent them on the straight line of numbers:

9 $\sqrt{2}$ ≈.......

10 $-\sqrt{3}$ ≈......

11 $\sqrt{\frac{6}{25}}$ ≈......

12 $\sqrt{\frac{81}{49}}$ ≈......

Compare between the real numbers by using the symbols (< , > , =):

13 $\sqrt{5}$ $\boxed{}$ $2\frac{1}{3}$

14 1.25 $\boxed{}$ $\sqrt{2.25}$

15 $\sqrt{\frac{0}{3}}$ $\boxed{}$ $\frac{0}{6}$

16 $\frac{\sqrt{12}}{\sqrt{3}}$ $\boxed{}$ $\frac{\sqrt{5}}{\sqrt{20}}$

17 Ordering the following real numbers from the least to the greatest.

$\sqrt{7}$, 2.25 , $\sqrt{5}$

18 Ordering the following real numbers from the greatest to the least.

$-3\frac{1}{5}$, $-\frac{7}{3}$, -3.33

Solve the following inequalities in R by using properties of inequalities in the real numbers:

19 $3x + \frac{2}{5} \geq 4x - \frac{3}{5}$

20 $\frac{3}{7} > z - \frac{9}{14}$

21 $\frac{3y}{8} \geq \frac{2}{7}$

22 $\frac{-4m}{11} < \frac{9}{22}$

23 $6(z - 3) > 5(z + 1)$

24 $4\left(\frac{1}{2}v + \frac{3}{8}\right) > 0$

Simplify the following numerical sentences by using ordering operations on the real numbers:

25 $\sqrt{2}\,(1 - \sqrt{18}) =$ ....

26 $3\sqrt{12} + 2\sqrt{3} - 4\sqrt{3} =$ ....

27 $\frac{\sqrt{7} - 8\sqrt{7}}{2\sqrt{7}} =$ ....

28 $\frac{6\sqrt{44}}{\sqrt{5}} \div \frac{18\sqrt{11}}{\sqrt{5}}$

Lesson [1-1] Ordering Operations in Real Numbers

Idea of the lesson:

* Simplify the numerical sentences which contain real numbers by using ordering opertions.

Vocabulary:

* Real number.
* Rationalizing the Denominator
* Conjugate

Learn

Tsunami quake, which occurred in Japan in 2011 is considered one of the greatest quake which happened over the ages. Its speed can be calculated by using the law $v = \sqrt{9.6\,d}$ m/s, where d represents the deep of water. What's the approximate speed of Tsunami if the deep of water is 1000m?

[1-1-1] Using ordering operations to simplify numerical sentences .

You have previously learned the natural numbers, whole numbers, integers, rational numbers and real numbers. We can ordering them in the following:

$N \subset W \subset Z \subset Q \subset R$

You have also learned how to simplify the numerical sentences by using the ordering of operations in these numbers. We will develop your skills in simplifying the numerical sentences which contain different real numbers include real roots and perfect squares roots, and also simplifying fractions contain roots by applying the properties on them and by using the ordering of operations in the real numbers. We use also (Rationalizing) denominator to simplify sentences by multiplying the conjugate numbers (the result of multiplying two conjugate numbers is a rational number, (the number $2 - \sqrt{3}$ is conjugate to $2+\sqrt{3}$ because the product them is rational number).

Example (1) Find the approximate speed of Tsunami if the water deep is 1000m.

$v = \sqrt{9.6\,d}$ *Law of calculating Tsunami speed, where d represents the deep of water*

$= \sqrt{9.6 \times 1000} = \sqrt{9600} = 98$ m/sec *The approximate speed of Tsunami*

Example (2) **Simplifying the following numerical sentences by using the ordering of operations on the real numbers:**

i) $(\sqrt{12} - \sqrt{18})(\sqrt{12} + \sqrt{18}) = (2\sqrt{3} - 3\sqrt{2})(2\sqrt{3} + 3\sqrt{2})$ *by using the distribution*

$= 2\sqrt{3}\,(2\sqrt{3} + 3\sqrt{2}) - 3\sqrt{2}\,(2\sqrt{3} + 3\sqrt{2}) = 12 + 6\sqrt{6} - 6\sqrt{6} - 18 = -6$

ii) $\left(\sqrt[3]{\frac{8}{27}} - \sqrt{\frac{2}{3}}\right) \div \left(\frac{3\sqrt{2} - 2\sqrt{3}}{\sqrt{27}}\right) = \left(\frac{2}{3} - \sqrt{\frac{2}{3}}\right) \div \left(\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{3}}\right)$

$= \frac{2\sqrt{3} - 3\sqrt{2}}{3\sqrt{3}} \times \frac{-3\sqrt{3}}{2\sqrt{3} - 3\sqrt{2}} = -1$

Example (3) **Simplifying the following numerical sentences by using the ordering of operations on the real numbers, then write the result to nearest tenth:**

iii) $\sqrt{12}\,(\sqrt{3} - \sqrt{8}) - 6 = 2\sqrt{3}\,(\sqrt{3} - 2\sqrt{2}) - 6 = 2\sqrt{3} \times \sqrt{3} - 2\sqrt{3} \times 2\sqrt{2} - 6$

$= 6 - 4\sqrt{3 \times 2} - 6 = -4\sqrt{6} \approx -4 \times 2.4 = -9.6$

iv) $(-27)^{\frac{1}{3}}\left(\frac{1}{9}\sqrt{7} - \frac{1}{9}\sqrt{28}\right) = \sqrt[3]{-27}\left(\frac{1}{9}\sqrt{7} - \frac{2}{9}\sqrt{7}\right) = -3\left(\frac{1}{9}\sqrt{7} - \frac{2}{9}\sqrt{7}\right)$

$= -\frac{1}{3}\sqrt{7} + \frac{2}{3}\sqrt{7} = \frac{1}{3}\sqrt{7} \approx 0.9$

Note : $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Example (4) **Simplify the following numerical sentences by using rooting the denominator and ordering operations on the real numbers.**

i) $\frac{7-\sqrt{5}}{\sqrt{5}} = \frac{7-\sqrt{5}}{\sqrt{5}} \times 1 = \frac{7-\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}(7-\sqrt{5})}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}-\sqrt{5}\sqrt{5}}{5} = \frac{7\sqrt{5}-5}{5}$

ii) $\frac{\sqrt{21}}{2\sqrt{3}-\sqrt{7}} = \frac{\sqrt{21}}{2\sqrt{3}-\sqrt{7}} \times \frac{2\sqrt{3}+\sqrt{7}}{2\sqrt{3}+\sqrt{7}} = \frac{\sqrt{3}\sqrt{7}(2\sqrt{3}+\sqrt{7})}{(2\sqrt{3}-\sqrt{7})(2\sqrt{3}+\sqrt{7})}$ *Multiplying by conjugates*

$= \frac{6\sqrt{7}+7\sqrt{3}}{12-7} = \frac{6\sqrt{7}+7\sqrt{3}}{5}$ *The denominator is a difference between two squares*

[1-1-2] Using calculator and approximation to Simplify Numerical Sentence

You have previously learned how to simplify the numerical sentences which contain integer negative powers and scientific form for number by using calculator. Now, you will develop your skills by simplify the
numerical sentences which contain numbers raising to rational powers, in addition to the integers by using calculator to write the result in approximated way.

Example (5) **Calculate the powers for each of the following, then write the result which should be approximated to two decimal places, if it is not an integer:**

i) $9^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27} \approx 0.04$

ii) $(\sqrt{7})^2 = (7^{\frac{1}{2}})^2 = 7$

iii) $2^{\frac{5}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{-3}{2}} = 2^{\frac{10+2-9}{6}} = 2^{\frac{1}{2}} = \sqrt{2} \approx 1.41$

iv) $5^2 \div 5^{\frac{3}{2}} = 5^{\frac{4}{2}-\frac{3}{2}} = 5^{\frac{1}{2}} = \sqrt{5} \approx 2.24$

Use the ordering of operations and write the result which should be approximated to two decimal places by using a calculator for each of the following:

v) $(\frac{1}{2})^2 + 3^{-2} - 2^{\frac{3}{2}} = \frac{1}{2^2} + \frac{1}{3^2} - \sqrt{2^3} = \frac{1}{4} + \frac{1}{9} - \sqrt{8} \approx 0.25 + 0.11 - 2.83 = -2.47$

vi) $8^{\frac{1}{3}} - (-8)^0 + 3^2 \times 3^{\frac{1}{2}} = \sqrt[3]{8} - 1 + 3^{\frac{5}{2}} = \sqrt[3]{8} - 1 + \sqrt{3^5} \approx 2 - 1 + 9 \times 1.73 = 16.57$

Example (6)

Use calculator to write the result in the scientific form for the number which should be aproximated to the nearest two decimal places:

i) $7.6 \times 10^{-4} - 0.41 \times 10^{-3} = 7.6 \times 10^{-4} - 4.135 \times 10^{-4} = 3.465 \times 10^{-4} \approx -3.47 \times 10^{-4}$

ii) $0.052 \times 10^4 + 7.13 \times 10^2 = 5.2 \times 10^2 + 7.13 \times 10^2 = 12.33 \times 10^2 \approx 1.23 \times 10^3$

iii) $(7.83 \times 10^{-5})^2 = (7.83 \times 10^{-5})(7.83 \times 10^{-5}) = 61.3089 \times 10^{-10} \approx 6.13 \times 10^{-9}$

iv) $4.86 \times 10^2 \div 0.55 \times 10^5 = (4.86 \div 0.55) \times 10^2 \times 10^{-5} \approx 8.84 \times 10^{-3}$

Make sure of your understanding

Simplify the following numerical sentences :

1. $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = ...$
2. $(\sqrt{7} - \sqrt{2})^2 = ...$
3. $(\sqrt{125} - \sqrt{20})(\sqrt[3]{\frac{8}{27}}) = ...$
4. $\frac{4\sqrt{12}}{5\sqrt[3]{-27}} \div \frac{2\sqrt{24}}{\sqrt{8}} = ...$

Questions 1-4 are similar to example 2

Simplify the following numerical sentences , and write the result to the nearest tenth:

5. $\sqrt{7}(\sqrt{28} - \sqrt{2}) - 5 \approx ......$
6. $(-125)^{\frac{1}{3}}(\frac{1}{10}\sqrt{3} - \frac{1}{4}\sqrt{12}) \approx .....$

Questions 5-6 are similar to example 3

Simplify the following numerical sentences by rooting the denominator and ordering operations on the real numbers:

7. $\frac{1-\sqrt{3}}{4\sqrt{3}} = ...$
8. $\frac{1-\sqrt{20}}{\sqrt{5}} = ...$
9. $\frac{\sqrt{50} - \sqrt{3}}{2\sqrt{3}} - \frac{10 - \sqrt{6}}{2\sqrt{6}} = ...$

Questions 7-9 are similar to example 4

Use the ordering of operations and write the result which should be approximated to two decimal places by using the calculator for each of the following:

10. $(\frac{1}{3})^2 + 3^{-3} - 3^{\frac{3}{2}} \approx ......$
11. $27^{\frac{1}{3}} - (-9)^0 + 3^2 \times 5^{\frac{1}{2}} \approx ......$

Questions 10-11 are similar to example 5

Use the calculator to write the result in the scientific form of the number which should be approximated to the nearest two decimal places:

12. $6.43 \times 10^{-5} - 0.25 \times 10^{-3} = ...$
13. $(9.23 \times 10^{-3})^2 = ...$

Questions 12-13 are similar to example 6

Solve the Exercises

Simplify the following numerical sentences:

14. $(\sqrt{18} - \sqrt{50})(\frac{-27}{64})^{\frac{1}{3}} = ...$.
15. $\frac{\sqrt{12}}{3\sqrt[3]{125}} \div \frac{5\sqrt[3]{8}}{\sqrt{25}} = ...$.

Simplify the following numerical sentences , and write the result to the nearest tenth.

16. $7\sqrt{\frac{2}{49}} - 3\sqrt{\frac{8}{81}} + \sqrt{\frac{18}{36}} \approx ......$

Simplify the following the numerical sentences by using rooting the denominator and the ordering of operations in the real numbers :

17. $\frac{\sqrt{7} - 3\sqrt{5}}{\sqrt{7} + 3\sqrt{5}} = ...$.
18. $\frac{\sqrt{33} - \sqrt{11}}{\sqrt{99}} - \frac{\sqrt{60} - \sqrt{5}}{5\sqrt{15}} = ...$

Solve the problems

19 Satellites: We essentialy use satellite in communications, such us TV signals, telephone calls in all over the world, weather forecasts and tracking of hurricanes. The satellites rotate around earth in limited speed and special orbits. The orbital speed of moon is calculated by the following relation $v = \sqrt{\frac{4\times10^{14}}{r}}$ m/sec, where r represents the radius of orbit (the distance of moon from the earth centre), what is the speed of moon if the orbit radius is 300 km?

20 Fighting fires: We can calculate the speed of flowing water which releases by fire trucks by using the following law $v = \sqrt{2hg}$ foot/sec, where h represents the maximum height of water, and (g) represents the acceleration speed of earth (32 foot/sec^2). To fire fighting in the forests, the firefighters in the Civil Defence need to huge bumper to pump water in height of 80 foot. Is it enough to use a pumper releases water in a speed of 72 foot/sec,?

1 foot = 30cm
Measuring unit in French system

21 Geometry: Find the area of triangle which topped a front of house if its height $(\sqrt{18} - \sqrt{3})$ meter and its base length is $(3\sqrt{2} + \sqrt{3})$ meter.

Think

22 Challenge: **Prove the correct of the following:**

$$(7^{\frac{1}{3}} - 5^{\frac{1}{3}})\ (7^{\frac{2}{3}} + 7^{\frac{1}{3}}\ 5^{\frac{1}{3}} + 5^{\frac{2}{5}}) = 2$$

23 Correct the mistake: Shaker wrote the result of adding two numbers as follow:

$$8.4 \times 10^{-3} + 0.52 \times 10^{-2} = 1.36 \times 10^{-3}$$

Determin Shaker's mistake and correct it.

24 Numerical sense: Does the number $\sqrt{125}$ locate between the two numbers 10.28 and 11.28?

Write

The result of adding by approximation to the nearest tenth:

$6^{\frac{3}{2}} + 5^{\frac{3}{2}} = ...$

Lesson [1-2] Mappings

Idea of the lesson:
*Identify the mapping and its types and how it can be represented graphically in the coordinate plane and Identify the composition of mappings

Vocabulary:
*The relation.
*Ordered pair .
*Cartesian product .
*The mapping.
*Domain and Co-domain and the Range.
*The composition of mapping.

Learn

the group X represents the archaeological locations in Iraq x = {Ishtar gate, Awr , Al- Hadar}.Assume the group B represents some Iraqi cities y = {Baghdad, Al- Hila, Al- Nasiriya, Al-Mosul , Arbil}. The relation R:X→ y which represents the connection of each archaeological location with the city which it sits in:

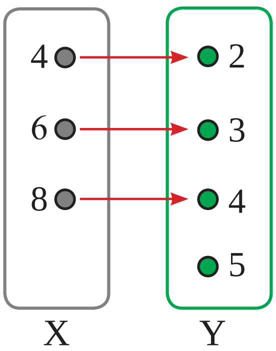
R={(Al-Nasiriya, Awr),(Al-Mosul,Al-Hadar),(Babylon, Istar gate)}, X represents its domain and Y represents its Co-domain.

[1-2-1] Mapping and its representation in the coordinate plane

You have previously learned the relation from set X to the set Y and it is subset (set of ordered pairs (x,y), where the first projection (the first coordinate) belongs to the set X and the second projection (the second coordinate) belongs to the set Y according to the Cartesian product X x Y. you will learn the mapping R:X→Y and how to represent it in an arrowy diagram and represent it (graphically) and identify its types.

The mapping: Let R relation from the set X to the set Y and each element in set X has one from in Y then the relation R can be called the mapping from X to Y ,R:X→Y . We called the set X the (domain) , and the set Y (Co-domain) , each elemeat in Y connected with element from X and represeents a from for it the set of all form in the Co-domain is called the (Range) , and rule which teansfers the element into its form is called the connection rule (mapping rule) and we refer to it by R (X) , (X,Y).

Example (1) If R: X→Y represents a mapping of connection rule ($y=\frac{1}{2}x$) from the set X= {4.6, 8} to the set, Y= {2,3,4,5}, and write the mapping in ordered pairs form ,then represent the mapping in an arrowy diagram and determine the domain and the range of the mapping.



The arrowy diagram explains the relation of connecting the elements of the two sets within the connection rule

Y=R(X)= $y = R(x) = \frac{1}{2}x$

4→2 , 6→3 , 8→4

So the set of mapping R = {(4,2),(6,3),(8.4)} Domain: is the set of first coordinates of the ordered pairs in R, and it is the set {4,6,8}

Range: is the set of the second coordinates of the ordered pairs in R, and it is the set {2,3,4}

Note: the range is a subset from the Co-domain of the mapping we see that the range $\neq$ Co-domain $(R_R) \neq (Y)$

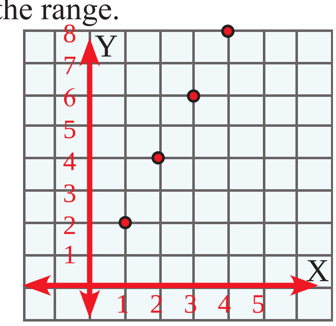
Example (2) The following table represents the relation between the weight (kg) and the price of fish (y = f(x)).

Does the relation represent a mapping?

If it is a mapping, then write a connection rule and Determine the domain and the range.

and represent in the plane.

x = weight (kg)	y = price in thousands dinars
1	2
2	4
3	6
4	8



The connection rule $y = 2x$

The domain = $\{1,2,3,4\}$ the range = $\{2,4,6,8\}$

[1-2-2] The types of mappings

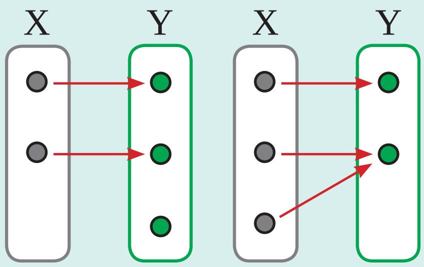
The mapping will be $f: X \to Y$

i) Surjective mapping

If the range=the co-domain

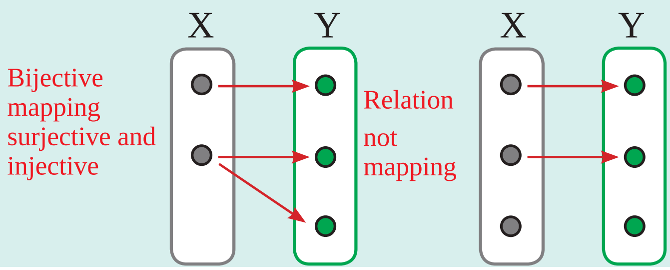
ii) Injective mapping : If each element in Y connects with only one element only in X

iii) Bijective mapping If the mapping is surjective and Injective at the same time



Surjective mapping and not injective

Injective mapping and not surjective

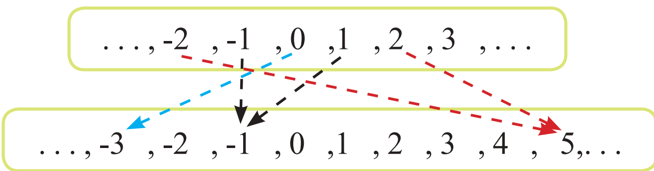


Example (3) If $f: Z \to Z$, where $f(x) = 2x^2 - 3$, show the type of the mapping, where Z represents the set of the integers.

$f(x) = 2x^2 - 3$ $f(-2) = 5$, $f(-1) = -1$, $f(0) = -3$, $f(1) = -1$, $f(2) = 5$

First: The mapping is not surjective because the range does not equal the co-domain.

Second: The mapping is not injective because $f(1) = f(-1) = -1$ while $1 \neq -1$

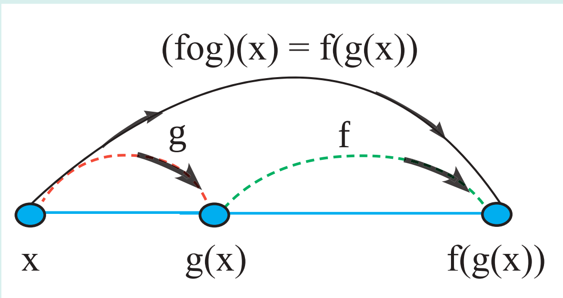


[1-2-3] Composition of mappings

We study a method to find a new mapping from two known mappings which are f(x),g(x) and they are:-

i) The mapping $(fog)(x)=f(g(x))$ and it can be read as f composite g (f after g), and it is a result of finding g(x) at first and then finding its image in the mapping f.

ii) The mapping $(gof)(x) = g(f(x))$ and it can be read as g composite f, and it is the result of finding f(x) at first, and then finding its image in the mapping g.



Example (4) **If f :N→N ,f (x) = 2x + 1 ,and g:N→N , g(x) = x^2 , Find the two mappings composition ,i) (fog)(3) ii) (gof)(3) , what you see? iii) if (fog)(x) = 33, find the value of x**

i) Find (fog)(3)

$(fog)(3) = f(g(3)) = f(3^2)$
$= f(9) = 2 \times 9 + 1$
$= 19$

ii) $(gof)(3) = g(f(3))$
$= g(2 \times 3 + 1)$
$= g(7) = 7^2 = 49$

Note: $(fog)(3) \neq (gof)(3)$

iii) $(fog)(x) = f(g(x)) = f(x^2) = 2x^2 + 1$

$2x^2 + 1 = 33 \Rightarrow 2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = 4$ or $x = -4$ negteet

Make sure of your understanding

Write a connection rule for the mapping and represent it in an arrowy diagram and write the domain and the range of it:

Questions 1-2 are similar to example 1

1 f ={(1,2), (2,3), (3,4), (4,5)}

2 g ={(1,3), (2,5), (3,7), (4,9)}

Write a connection rule for the following mappings and represent them in the coordinate plane and write their domain and range:

Questions 3-4 are similar to example 2

3 f ={(1,0), (2,0), (3,0), (4,0)}

4 g ={(0,0), (1,-1), (2,-2), (3,-3)}

Questions 5 are similar to example 3

5 If the mapping f: N→N, where f(x)=3x+2. Show if the mapping is surjective or not.

Questions 6-7 are similar to example 4

6 Assume the two mappings f:Z→Z,where f(x)= 3x+1 and g:Z→Z, where g(x)=2x+5. Find the value of x if (fog)(x)=28.

7 If f: N→N, where f(x)=5x+2 and g:N→N, where g(x)=x+3.

Write the mapping (fog)by writing its ordered pairs.

Solve the Exercises

8 If A={1,2,3} and B={4,5,6}, and f:A→B is defined as follow:

f ={(1,4), (2,5), (3,6)} , Draw the arrowy diagram of the mapping and represent it in the coordinate plane.

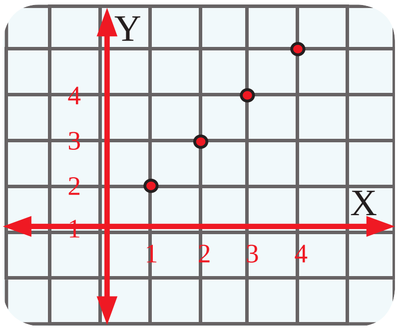
9 If f: A→ Z , where f (x) = $x^2$and the set A={-2,-1,0,1,2}. Represent the mapping in the coordinate plane, and show if the mapping is injective or not?

10 **Assume f:N→N, and g: N→N, f(x)=x^2, and g(x)=x+1. It is required to find:**

i) (gof)(x) , (fog)(x) , ii) (fog)(2) , (gof)(2)

Solve the problems

11 Temperatures: In a day of winter, the temperatures recorded as shown in the following relation $R = \{(6,-2), (9,-3), (12,-4), (15,-5)\}$, where the first coordinate represents the time in hours , and the second coordinate represents the temperature in celsius degrees. Represent the relation in a table and represent it in the coordinate plane . Does the relation represent a mapping or not? Explen your answer.



12 Coordinate plane: The nearby graphic figure represents the mapping f:N→N.
Write the coordinates of the ordered pairs which can be represented by the mapping points in the graphic. Write a connection rule of the mapping, is the mapping an injective or not?

13 Health: The relation $W_r = 2(\frac{W_b}{3})$ represents the mass of water in human body, where Wr represents the mass of water and W_b represents the mass of human body. Hassan's mass is 150 kg, he follows a diet to reduce the mass for three months, he lost 6kg in the first month and 12kg in the second month and 12kg in the third month. Write all the ordered pairs for the relation between Hassan's mass and the mass of water in his body. Does it represent a mapping or not ?

Think

14 Challenge: If A={1,2,3,} and g:A→A is a defined mapping, as follow:

g = {(3,1),(1,2), (2,3)} , f = {(1,3),(3,3), (2,3)}
show does fog = gof ?

15 Correct the mistake: Yaseen saide that the relation f:Z→Z, where $f(x) = x^3$ does not represent an injective mapping. Determine Yaseen's mistake and correct it.

16 Numerical sense: **Determine if each of the following relations f:X→Y represents a mapping or not? Explain that.**

X	1	2	3	4	5
Y	3	6	11	18	27

Write

The value of x, if the mapping f: N → N , and where f(x) = 4x - 3 , (fof) (x) = 33 .

Lesson [1-3] The Sequences

Idea of the lesson:
*Identify the sequence and the arithmetic sequence and their properties.

Vocabulary:
*Sequence
*Arithmetic sequence
*General term
*Constant sequence
*Common difference of sequence

Learn

Bashar works in a gallery for five days a week. He produces one painting each three days. Arrange a table to connect btween the number of days and the number of paintings which were painted by Bashar, if you know that Bashar worked for 4 weeks. Write set of the odered pairs from the table. Does the table represent a pattern? Does it represent a sequence?

[1-3-1] Sequence and Function

You have previously learned the function and how to determine its domain and range. Now, you will learn the sequence as a function and how to express it and how to write its terms, as follow: The sequence

$f: N \longrightarrow R$ is a function represented by a set of the ordered pairs $\{(1,f(1)), (2,f(2)), (3,f(3)), \ldots, (n,f(n)), \ldots\}$,where the first progection represent the natural numbers set (infinite sequence) and we can refer to it by ${}_{n=1}^{\infty}\{(f(n)\}$,$({}_{n=1}^{\infty}\{u_n\})$ or as a subset (finite sequence) and we can refer to it by ${}_{n=1}^{m}\{(f(n)\}$,$({}_{n=1}^{m}\{u_n\})$, so it is enough to write the second progection (images) $\{ (f(1)), (f(2)), (f(3)), \ldots, (f(n)), \ldots\}$ and it is called u_n the general term of the sequence $f(n) = u_n$.

The sequence can be written $\{u_1, u_2, u_3, u_4, \ldots, u_i, \ldots\}$.

Example (1) Arrange a table to connect between the number of days and the number of paintings. Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence?

Number of paintings	1	2	3	4	5	6
Number of days	3	6	9	12	15	18

Ordered pairs :

$\{(1,3),(2,6),(3,9),(4,12),(5,15),(6,18)\}$

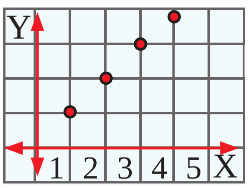
Yes it represents a pattern and the relation represents "three times" , and the relation represents a sequence, its general term is $u_n = 3n$, $n \in \{1,2,3,4,5,6\}$

and it can be written as follow $\{u_n\} = \{3n\} = \{3,6,9,12,15,18\}$

Example (2) **Write the first five ordered pairs of the sequence $\{u_n\}$ and represent it in the coordinate plane:**

i) $\{n\} = \{1, 2, 3, 4, 5, \ldots\}$

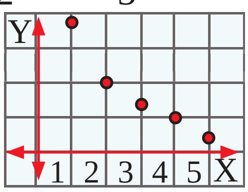
$\{(1,1), (2,2), (3,3), (4,4), (5,5), \ldots\}$



$u_n = n$

ii) $\{\frac{1}{n}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}$

$\{(1,1), (2, \frac{1}{2}), (3, \frac{1}{3}), (4, \frac{1}{4}), (5, \frac{1}{5}), \ldots\}$



$u_n = \frac{1}{n}$

[1-3-2] Arithmetic Sequence

Arithmetic sequence is the sequence in which the difference between each two consecutive terms represents a constant number, and it is called the base of the sequence (the common difference of sequence), its symbol is $d = u_{n+1} - u_n$. We can write a sequence by knowing its first term $a = u_1$ and the base d The general term law of the arithmetic sequence is $u_n = a + (n-1)\,d$, where $n \in N$. and in general we can defined the kind of the sequence as following :

i) Increasing sequence in which $d > 0$, example $\{1, 3, 5, 7, 9, \ldots\}$,

ii) Decreasing sequence in which $d < 0$, example $\{4, 2, 0, -2, -4, \ldots\}$

iii) Constant sequence in which $d = 0$, example $\{5, 5, 5, 5, 5, ..\}$.

Example (3) **Write the first five terms for each of the following arithmetic sequences:**

i) An arithmetic sequence in which the first term is 3 and its common difference is 6.

$\{3, 9, 15, 21, 27, \ldots\}$

ii) An arithmetic sequence in which the first term is 1 and its common difference is -3.

$\{1, -2, -5, -8, -11, \ldots\}$

iii) An arithmetic sequence in which the seventh term is 36 and its common difference is 4

$u_n = a + (n-1)\,d \Rightarrow u_7 = a + 6d \Rightarrow 36 = a + 6\times 4 \Rightarrow a = 12$

$\{ 12, 16, 20, 24, 28, \ldots \}$

$u_1 \overset{+d}{\Rightarrow} u_2 \overset{+d}{\Rightarrow} u_3 \overset{+d}{\Rightarrow} \ldots\ldots \overset{+d}{\Rightarrow} u_n$

Example (4) **Write terms to the following sequences:**

i) An arithmetic sequence in which the third term is 8 and $d = -3$. Find the terms between u_7 and u_{11} .

$u_n = a + (n-1)\,d \Rightarrow u_3 = a + 2d \Rightarrow 8 = a - 6 \Rightarrow a = 8 + 6 = 14$

$u_n = a + (n-1)\,d \Rightarrow u_7 = a + 6d \Rightarrow u_7 = 14 + 6(-3) \Rightarrow u_7 = -4$

By finding the values of (a) , we can get the value of the term 7 and the terms which follow

$u_8 = u_7 + d = -4 - 3 = -7$

$u_9 = u_8 + d = -7 - 3 = -10$

$u_{10} = u_9 + d = -10 - 3 = -13$

$\{ -7 , -10 , -13 \}$

ii) Write the twentieth term of the arithmetic sequence $\{6, 1, -4, -9, \ldots\}$, and determine if is a decreasing or increasing .

$d = u_{n+1} - u_n \Rightarrow d = 1 - 6 = -5 , a = 6$

$u_n = a + (n-1)\,d \Rightarrow u_{20} = a + 19d \Rightarrow u_{20} = 6 + 19(-5) \Rightarrow u_{20} = -89$

$\because d < 0$, so the sequence is decreasing

Example (5) **Write the first five terms for each of the following sequences:**

i) $\{2n - 1\} = \{1, 3, 5, 7, 9\}$, ii) $\{(-1)^n\} = \{-1, 1, -1, 1, -1\}$

iii) $\{7\} = \{7, 7, 7, 7, 7\}$, iv) $\{\frac{n}{3}\} = \{\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}\}$

v) $\{n^2\} = \{1, 4, 9, 16, 25\}$, vi) $\{n^3\} = \{1, 8, 27, 64, 125\}$

Make sure of your understanding

Write the first four ordered pairs for the sequence which its general term is given:

1 $u_n = 3n$

2 $u_n = n - 4$

3 $u_n = n^2$

4 $u_n = \frac{1}{2n}$

5 $u_n = 3n - 1$

Questions 1-5 are similar to example 2

Write the first five terms of each of the following sequences:

6 An arithmetic sequence in which the first term is 1 and its common difference is 5.

7 An arithmetic sequence in which the first term is -5 and its common difference is 2.

Questions 6-8 are similar to example 3

8 An arithmetic sequence in which the first term is -3 and its common difference is -4.

Write the terms for the following sequences:

9 Find the terms between u_8 and u_{12} for an arithmetic sequence in which the third term is (9) and d= -2.

10 Find the terms between u_{10} and u_6 for an arithmetic sequence in which the second term is -11 and d = -3.

11 Write 23rd term for the arithmetic sequence {3, -1, -5, -9, …}

Questions 9-11 are similar to example 4

Write the first five terms for each of the following sequences:

12 $\{4n\}$ =………

13 $\{2n - 5\}$ =………

14 $\{\frac{1}{n+1}\}$=……

15 $\{9\}$ =………

Questions 12-15 are similar to example 5

Solve the Exercises

Write the first four ordered pairs for the sequence which its general term is given:

16 $u_n = 10 - 4n$

17 $u_n = n^2 - 1$

18 $u_n = \frac{1}{3n+1}$

Write the first five terms for each of the following sequences:

19 An arithmetic sequence in which the seventh term is $\frac{1}{24}$ and the common difference is $\frac{1}{3}$

Write terms for the following sequences:

20 Find the terms between u_{10} and u_{13} for an arithmetic sequence in which the seventh term is $\frac{13}{2}$ and d = 1 .

21 Find the terms between u_{20} and u_{23} for an arithmetic sequence in which the second term is (0) and d = -1.

Determine the type of sequences (increasing, decreasing, constant) for each of the following:

22 $\{u_n\} = \{3 - 2n\}$

23 $\{u_n\} = \{n^3 - 1\}$

24 $\{u_n\} = \{\frac{1}{n+2}\}$

Write the first five terms for each of the following sequences:

25 $\{\frac{3n}{2}\}$ =…

26 $\{\sqrt{3}\}$ =…

27 $\{\frac{n}{n+1}\}$=…

Solve the problems

28 Running sport: In one of running competitions of the times of the first winner are recorded in the following table

Distance in km	1	2	3	4	5
Time in minutes and seconds	3.12	6.32	9.52	12.72	15.92

Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence? Explain your answer.

29 Sport of pole vault: The following table shows the attempts of one of the world champions in the sport of pole vaul.

Attempts	1	2	3	4	5
Height in metre	5.90	5.95	6.00	6.05	6.10

Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence? Explain your answer.

30 Agriculture: Hassan had bought a farm for breeding cows,and after one year which had 20 cows. This number was increased year by year as a result to the new births which were in a constant rate. After six years, the number of cows had become the double. Represent the proplem in a table and Write the ordered pairs set from the table. Does the table represent a pattern? Does it represent a sequence? Explain your answer..

Think

31 Challenge:**Find the value of x which makes the first three terms of the arithmetic sequences** as follow:

$\{2x, x + 1, 3x + 11, ...\}$

32 Correct the mistake: Rabiha said that the sequence which general term is $u_n = 8 - 2n$, considered an increasing sequence because $d > 0$, discover the mistake of Rabiha and then correct it.

33 Numerical sense: What is the eleventh term for a sequence in which the third term is (4) and the common difference is $-\frac{1}{2}$.

Write

The term which occupies the place 101 in the arithmetic sequence which its fifth term is -4 and the common difference is 12.

Lesson [1-4] Compound Inequalities

Idea of the lesson:
*Solving the inequalities which contain connecting tools (and) , (or), then representing the solution on the numbers line

Vocabulary:
*Compound inequalities
*Intersection
*Union
*Solution set

Learn

We use the minimum and maximum celsius degrees to measure the weather temperature during a day because it is variable from time to time. If the minimum celsius temperature in Baghdad is 8c̊ and the maximum one is 15c̊ during December month, writ an ineqalities reperesents the tempereathure in Baghdad , then find its solution

[1-4-1] Compound inequalities which contain "and"

You have previously learned the algebraic inequalities and their properties. You have also learned how to find the solution set and how to represent it in the line of numbers. Now, you will learn the compound inequalities which contain the connecting tool "and" and how to find their solution set and how to represent them in the line of the real numbers. As the compound inequality contains the connecting tool "and" and it consists of two inequalities, so it will be true only when the two inequalities are true. According to that, its solution set will be a set of intersection for solution of the two inequalities. We can do that by two methods, the first is graphically by representing the solution of the two inequalities in the line of numbers and then determine the intersection area. The second method is algebraically by finding the solution set for each inequality, then take their intersection set ($S = S_1 + S_2$).

Example (1) Write the compound inequality which represents the small and big celsius temperatures in Baghdad, then find its solution.

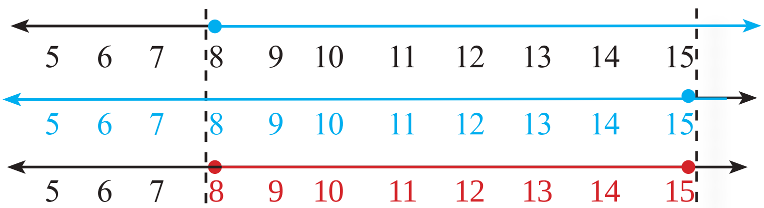
The temperature (minimum) is not less than 8^0 ($8 \le x$), while the temperature (maximum) is not greater than 15°($x \le 15$). The temperature is not less than 8 and not greater than 15° ($8 \le x$ and $x \le 15$). It can be solved in any of the two methods:

The first method : graphically

$x \ge 8$

$x \le 15$

$8 \le x \le 15$



It can be read x greater than or equals to 8 and less than or equals to 15

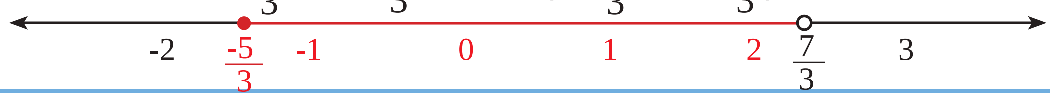
The second method: Algebraically : $8 \le x \le 15 \Rightarrow 8 < x$ and $x \le 15$

$\Rightarrow S = S_1 + S_2 = \{x: x \ge 8\} + \{x: x \le 15\} = \{x: 8 \le x \le 15\}$

Example (2) Solve the compound inequality which contains (and) $-3 \le 3x+2 < 9$ algebraically, then represent the solution on the straight line of numbers.

$-3 \le 3x+2 < 9 \Rightarrow -3 - 2 \le 3x+2 - 2 < 9 - 2 \Rightarrow -5 \le 3x < 7 \Rightarrow \frac{-5}{3} \le \frac{3x}{3} < \frac{7}{3}$

$\Rightarrow \frac{-5}{3} \le x < \frac{7}{3} \Rightarrow S = \{x: \frac{-5}{3} \le x < \frac{7}{3}\}$



[1-4-2] Compound inequalities which contain "or"

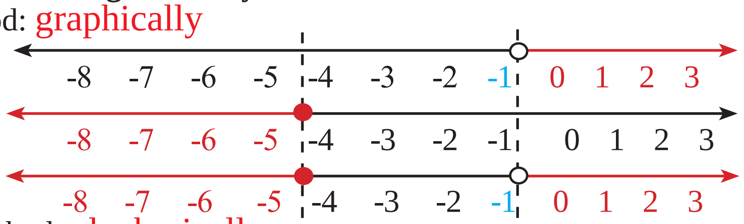
After you have already learned the compound inequality which contains the connecting tool (and), you will learn the compound inequality which contains the connecting tool (or) and it will be true only when, at least one of its two inequalities is true. Accordingly its solution set is a set of the two qualities solution union. It can be found in two methods, first graphically by representing the two inequalities solution in the line of numbers, then determining union area. Second method, algebraically by finding the solution set for each inequality, then taking their union set ($S = S_1 \cup S_2$).

Example (3) **Solve the compound inequality $2 < x+3$ or $x+3 \le -2$ graphically and algebraically.**

First method: graphically

$x+3 > 2$ — $x > -1$

$x+3 \le -2$ — $x \le -5$



$x \le -5$ or $x > -1$

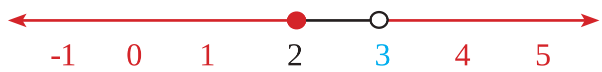
Second method: algebraically

$$x+3 \le -2 \text{ or } x+3 > 2 \Rightarrow \left\{\begin{array}{l} x+3 > 2 \text{ or } x+3 \le -2 \\ x > -1 \text{ or } x \le -5 \end{array}\right\} \Rightarrow S = S_1 \cup S_2 = \{x: x > -1\} \cup \{x: x \le -5\}$$

Example (4) **Solve the compound inequality which contains (or) algraphically and represent the solution on the line of numbers.**

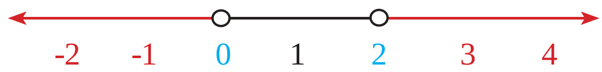
i) $y-3 \le -1$ or $y+3 > 6 \Rightarrow y \le 2$ or $y > 3$

$\Rightarrow S = S_1 \cup S_2 = \{y: y \le 2\} \cup \{y: y > 3\}$



ii) $\dfrac{2v+1}{3} > \dfrac{5}{3}$ or $\dfrac{2v+1}{3} < \dfrac{1}{3} \Rightarrow v > 2$ or $v < 0$

$\Rightarrow S = S_1 \cup S_2 = \{v : v > 2\} \cup \{v : v < 0\}$



[1-4-3] Triangular Inequality

One of the subjects which connects algebra to geometry is the triangular inequality "in each triangle, the sum of two sides length is greater than the length of the third side" , it is used in the geometrical constructions and designs. If the lengths side of a triangle is (A,B,C), then the following three inequalities should be true : $A+B > C$, $A+C > B$, $B+C > A$

Example (5) i) Can the three sides of a triangle with length 13cm ,10cm and 2cm compose a triangle ?

No, they can't because: $2 + 10 \boxtimes 13$ is false , $10 + 13 > 2$ is true , $2 + 13 > 10$ is true .

ii) Write a compound inequality which shows the length of the third side in a triangle which has two sides with length 8cm and 10cm.

Suppose that the length of the third side is x, then:

$$\left.\begin{array}{l} 8+10 > x \Rightarrow 18 > x \Rightarrow \text{The third side is less than 18} \\ 8+x > 10 \Rightarrow x > 2 \Rightarrow \text{The third side is greater than 2} \\ 10+x > 8 \Rightarrow x > -2 \Rightarrow \text{Doesn't give any useful data} \end{array}\right\} \Rightarrow$$

So the length of this side must be less than18 and greater than 2 and by the compound inequality, we see that the range of the third side length is $2 < x < 18$

Make sure of your understanding

Solve the compound inequalities which include (and) graphically:

Questions 1-2 are similar to example 1

1 $-4 \leq y - 1 < 3$

2 $8 \geq z + 2 \geq -4$

Solve the compound inequalities which include (and) algebraically, then represent the solution set on the line of numbers:

Questions 3-4 are similar to example 2

3 $12 \leq x + 6$ and $x + 6 < 15$

4 $-9 < 2x - 1 \leq 3$

Solve the compound inequalities which includes (or) graphically:

Questions 5-6 are similar to example 3

5 $8y \leq 32$ or $8y \geq 64$

6 $\frac{2z}{3} < \frac{2}{3}$ or $\frac{2z}{3} \geq \frac{8}{9}$

Solve the compound inequalities which include (or) algebraically, then represent the solution on the line of numbers:

Questions 7-8 are similar to example 4

7 $3n - 7 > -5$ or $3n - 7 \leq -9$

8 $x + 15 < 22$ or $x + 15 \geq 30$

Can the three sides, which shown below, compose a triangle?

Questions 9-12 are similar to example 5

9 1cm, 2cm, $\sqrt{3}$ cm

10 5cm ,4cm, 9cm

11 1cm, $\sqrt{2}$ cm, $\sqrt{2}$ cm

12 3cm, 4cm, $2\sqrt{3}$ cm

Solve the Exercises

Solve the compound inequalities which include (and) graphically:

13 $-12 < x$ and $x \leq -7$

14 $2 \leq y + 4 < 6$

Solve the compoun inequalities which include (and) algebraically, then represent the solution set on the line of numbers:

15 $14 \leq 3x + 7$ and $3x + 7 < 26$

16 $\frac{1}{15} \geq \frac{z+3}{5} \geq \frac{1}{25}$

Solve the compound inequalities which includes (or) graphically:

17 $z - 2 < -7$ or $z - 2 > 4$

18 $x - 6 \leq -1$ or $x - 6 > 4$

Solve the compound inequalities which include (or) algebraically, then represent the solution set on the line of numbers:

19 $x + 8 < 22$ or $x + 10 \geq 30$

20 $y < -1$ or $y + 3 > 2$

21 $\frac{y}{2} < 3\frac{1}{2}$ or $\frac{y}{2} > 7\frac{1}{2}$

22 $5x \leq -1$ or $5x \geq 4$

Write the compound inequality which shows the length of the third side in the triangle which has two known-length sides:

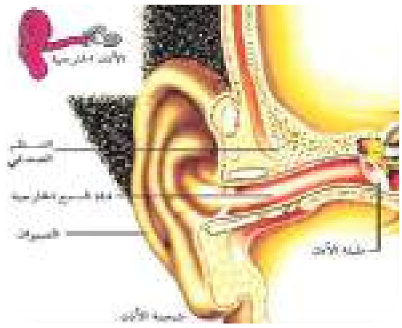
23 3cm , 10cm

24 6cm, 4cm

25 1cm, 3cm

Solve the problems

26 **Sound:** Human's ear can hear sound which its frequency is not less than 20 Hz and not more than 20000 Hz. Write a compound inequality represents the frequencies which human's ear can not hear them, then represent on line of numbers.



27 **Cars tyre:** The ideal air pressure which is recommended for tyres of saloon cars is not less than 28 pascal (kg/ ing^2) and not more than 36 pascal. Write a compound inequality which represents the pressure, then represent on line of numbers.

Cars tyre: The ideal air pressure which is recommend

Note: Pascal is unit for measuring the pressure of air which is Kg $ling^2$.

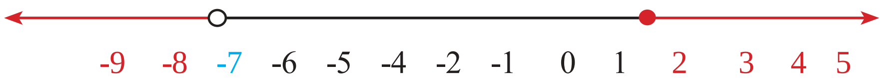
28 **Magnetic train:** Hanging magnetic train which operates in the magnetic lifting force, briefly it is called (Maglev). Different types of the magnetic trains were designed all over the world, the speed of those train is not less than 300 k/h and not more than 550 k/h. Write an equality represents the speed of train, then represent on line of numbers.

Think

29 **Challenge:** Write a compound inequality shows the range of the third side length in each triangle:

7 cm , 12 cm , x cm

30 **Correct the mistake:** Sawsen said that the compound inequality $-4 < x+3$ and $x+3 \leq 5$ represents the set of solution in the following line of numbers.



Show Sawsen's mistake, then correct it.

31 **Numerical sense:** **Mention if the three lengths are for a triangle or not? Explain that.**

i) 3.2cm, 5.2cm, 6.2cm ii) 1cm, 1cm, $\sqrt{2}$ cm

Write

A compound inequality represents the minimum degree temperature of which is 18° and the maximum degree temperature of which is 27°.

Lesson [1-5] Absolute Value Inequalities

Idea of the lesson:
*Solving inequalities which contain an absolute value.

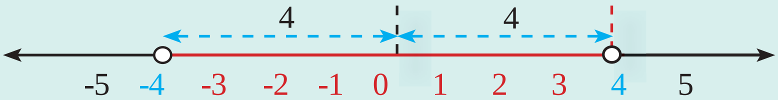
Vocabulary:
* Absolute value

Learn

Babylon hotel is one of the tourist hotels in Baghdad. It locates in Al-jadriya area. The ideal temperature of water in the swimming pool is 25 celsius, with increas or decrease of one degree. Write an absolute value inequality represents the range of water temperature in the swimming pool.

[1-5-1] Absolute value Inequalities $|g(x)| \le a$, $|g(x)| < a$, where $X \in R$

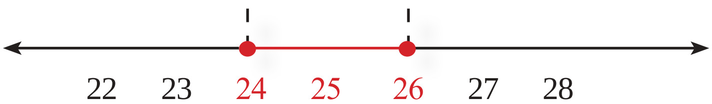
You have previously learned about the compound inequalities which contain (and) and (or), and how to solve them graphically and algebraically, and how to represent the solution set in the line of numbers. Now, you will learn the absolute value inequality with from $|g(x)| \le a$, $|g(x)| < a$, $a \in R$
for example : $|x| < 4$ which means: What are the values of x which is far of zero in less than 4 units? they include all numbers between -4 and 4, then represent them in the line of numbers which is:



We note that the solution of this quality is:$\{-4 < x$ and $x < 4\}$
That means that the absolute value inequality was connect to relation of less than (less or equals to) represents a compound inequality which includes (and).
In general form : $|x| \le a \Rightarrow -a \le x \le a$, $a > 0$

Example (1) Write the absolute value inequality which represents the temperature of water in the swimming pool, then represent it graphically.

Assume that the temperature of water is (x) celsius, so the inequality which represents the temperature of pool when it is not more than 26 celsius is:
$x \le 25 + 1 \Rightarrow x - 25 \le 1$
and the inequality which represents the temperature of pool when it is not less than 24 celsius is:
$x \ge 25 - 1 \Rightarrow x - 25 \ge -1$
So the absolute value inequality is the compound inequality which represents the range of water temperature in the swimming pool. $x - 25 \ge -1$ and $x - 25 \le 1 \Rightarrow -1 \le x - 25 \le 1 \Rightarrow |x-25| \le 1$
The representation of the solution set and the line of numbers is:



Example (2) **Solve the absolute value inequalities, then represent the solution in the line of numbers.**

i) $|x + 6| < 3 \Rightarrow -3 < x + 6 < 3 \Rightarrow -3-6 < x < 3-6$
$\Rightarrow -9 < x < -3$

ii) $|y| - 5 \le 1 \Rightarrow |y| \le 1 + 5 \Rightarrow |y| \le 6$
$\Rightarrow -6 \le y \le 6$

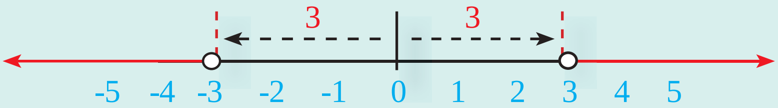
[1-5-2] Absolute value Inequalities which is in from of $|g(x)| \geq a$, $|g(x)| > a$ where $X \in R$

After you have learned the absolute value inequality which contains the form of $|g(x)| \leq a$, $|g(x)| < a$ where $x \in R$. Now, you will learn the absolute value inequality which contains is in the form of $|g(x)| \geq a$ $|g(x)| > a$ where $x \in R$ for example: $|x| > 3$, which means: distance between x and zero greater than 3 that is

$x > 3$ or $x < -3$ and the inequality solution set is $\{x: x < -3\} \cup \{x: x > 3\}$

So the absolute value inequality with the relation greater than (greater or equal) is a compound relation which includes (or).

In general form : $|x| \geq a \Rightarrow |x \geq a$ or $x \leq -a$, $a > 0$



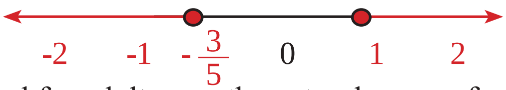
Example (3) **Solve the absolute value inequality, then represent the solution on the line of numbers.**

i) $|x+4| > 2 \Rightarrow -2 > x+4$ or $x+4 > 2 \Rightarrow -6 > x$ or $x > -2$

$$\Rightarrow S = S_1 \cup S_2 = \{x: x < -6\} \{x: x > -2\}$$

ii) $|5y-1| \geq 4 \Rightarrow -4 \geq 5y-1$ or $5y-1 \geq 4 \Rightarrow -\frac{3}{5} \geq y$ or $y \geq 1$

$$\Rightarrow S = S_1 \cup S_2 = \{y: y \leq -\frac{3}{5}\} \cup \{y: y \geq 1\}$$



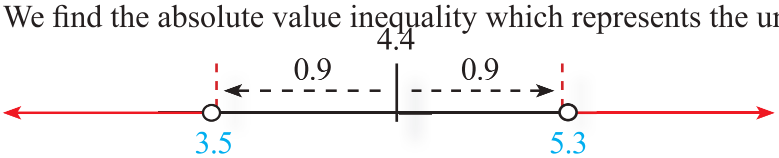
iii) In analysis of a blood for adult men, the natural range of potassium is (3.5 – 5.3) mmol/L . Write the absolute value inequality which represents tha unnatural range of potassium in human blood.

The inequality which represents the unnatural quantity of potassium and less than the lowest value of average is: $x < 3.5$

The inequality which represents the unnatural quantity of potassium and more than the greatest average is: $x > 5.3$

Tha unnatural range of potassium is the compound inequality solution: $x > 5.3$ or $x < 3.5$

We find the absolute value inequality which represents the unnatural range of potassium:



we take the middle between two points , then we substract and add the radius of the distance.

$3.5 > x$ or $x > 5.3 \Leftrightarrow 4.4 - 0.9 > x$ or $x > 4.4 + 0.9$

$$\Leftrightarrow -0.9 > x - 4.4 \text{ or } x - 4.4 > 0.9 \Leftrightarrow |x - 4.4| > 0.9$$

Example (4) **Find the solution set for the following absolute value inequalities:**

i) $|2x-5| + 3 < 11 \Rightarrow |2x-5| < 8 \Rightarrow -8 < 2x-5 < 8 \Rightarrow -3 < 2x < 13$

$$\Rightarrow -\frac{3}{2} < x < \frac{13}{2} \Rightarrow \{x: x > -\frac{3}{2}\} \cap \{x: x < \frac{13}{2}\} \Rightarrow \{x : \frac{-3}{2} < x < \frac{13}{2}\}$$

ii) $|7-y| < 8 \Rightarrow -8 < 7-y < 8 \Rightarrow -15 < -y < 1 \Rightarrow -1 < y < 15 \Rightarrow \{y: y > -1\} \cap \{y: y < 15\}$

iii) $\left|\frac{2t-8}{4}\right| \geq 9 \Rightarrow \left|\frac{2(t-4)}{4}\right| \geq 9 \Rightarrow \left|\frac{t-4}{2}\right| \geq 9 \Rightarrow |t-4| \geq 18$

$t-4 \leq -18$ or $t-4 \geq 18 \Rightarrow t \leq -14$ or $t \geq 22 \Rightarrow \{t : t \leq -14\} \cup \{t : t \geq 22\}$

iv) $\left|\frac{5-3v}{2}\right| \geq 6 \Rightarrow |5-3v| \geq 12 \Rightarrow -12 \geq 5-3v$ or $5-3v \geq 12 \Rightarrow -3v \geq -17$ o r $-3v \geq 7$

$$\Rightarrow v \geq \frac{17}{3} \text{ or } v \leq \frac{-7}{3} \Rightarrow \{v : v, \frac{17}{3}\} \cup \{v: v \leq \frac{-7}{3}\}$$

Make sure of your understanding

Write the absolute value inequality which represents the following problems:

Questions 1-2 are similar to examples 1,3

1 The ideal temperature inside flats is 22° celsius with increase or decrease of 2° celsius.

2 The right angle changes to an acute angle or obtuse angle if the spinner of angle moves to the right or left, at least one degree

Solve the absolute value inequalities, then represent the solution on the line of numbers.

Questions 3-6 are similar to example 2

3 $|x+1|<5$

4 $|3z-7| \leq 2$

5 $|x|+8<9$

6 $|5y|-2 \leq 8$

Questions 7-10 are similar to example 3

7 $|x+4|>6$

8 $|5z-9|>1$

9 $|2x|+7 \geq 8$

10 $|4y|-2>3$

Questions 11-14 are similar to example 4

11 $|5-x|<10$

12 $|4z-14|>2$

13 $\left|\frac{x-12}{4}\right| \leq 9$

14 $\left|\frac{6-2y}{4}\right| \geq 9$

Solve the Exercises

Write the absolute value inequality which represents the following problems:

15 The temperature inside fridge should be 8° celsius with increasing or decreasing of 0.5° celsius. Write the range of ideal temperature inside fridge.

16 The boiling degree of water is 100° celsius at the sea surface level. It increases or decreases in the mountainous areas and valleys in no more than 20° celsius. Write the range of vipration in the boiling degree of water.

Solve the following absolute value inequalities:

17 $|x+3|<6$

18 $|x|-6<5$

19 $|2z|-5<2$

20 $|y-3| \geq \frac{1}{3}$

21 $2|x|-7 \geq 1$

22 $|9y|-6>3$

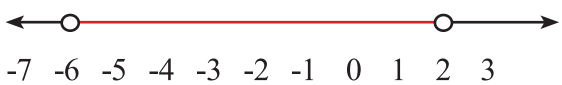
23 $|11z|-2 \geq 9$

24 $|1-x|<1$

25 $\left|\frac{4}{5}z-1\right|>\frac{4}{5}$

26 $\left|\frac{z-1}{7}\right| \leq 2$

Write an inequality includes an absolute value for all of the following graphic inequalities:

27 

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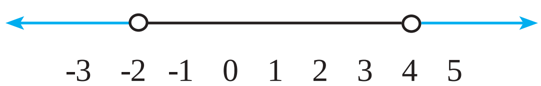
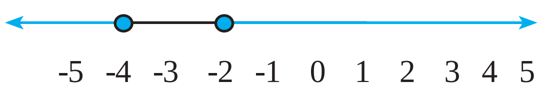
29 

30 

Solve the problems

Write the absolute value inequality which represents each of the following problems:

31 Badger: The animal, Badger is one of mammals which belongs to the division of preanants. It has short legs. It lives in holes which the Badger itself made. The length of its body, from head to tail, is from 68cm to 76cm. Write the range of Badger length.

32 Health: The natural pulse rate (number of heart (beats) for adult men is from 60 to 90 beats in minute. Write the range of unnatural heart beats of human.

33 Transportation: The civilian plane flies in height from 8 km to 10 km where it is considered a moderate area. Write the range of the civilian aviation area.

Think

34 Challenge: **Solve the absolute value inequalities and represent the solution on the line of numbers .**

i) $\left| \dfrac{\sqrt{3}\,(x+1)}{\sqrt{3}} \right| \le \sqrt{6}$ ii) $\left| \dfrac{\sqrt{12} - \sqrt{3}\,y}{\sqrt{5}} \right| \ge \sqrt{15}$

35 Correct the mistake: Khulood said that the absolute value inequality $|6 - 3y| \ge 7$ represents a compound inequality with a relation (and). and with its solution : $\{ y : -\frac{1}{3} \le y \le \frac{13}{2} \}$
Show the mistake of Khulood, then correct it.

36 Numerical sense: **Write the solution set for the following absolute value inequalities in the real numbers set:**

i) $|z| - 1 < 0$ ii) $|x - 1| > 0$

Write

An absolute value inequality represents a situation from life, then represent the solution set on the straight line of numbers.

Lesson [1-6] Problem solving plan (understand the problem)

Idea of the lesson:
* Using a strategy to understand and solve the problem.

Learn

A survey shows that 62% of young people practise the sport of football. If the margin of mistake is within 4 points percentage. Find the percentage range for young people who practise the sport of football.

UNDERSTAND

What are the given data in the problem: 62% of young people practise football game, the margin of mistake is 4 points.

what is required ? Find the percentage range which represents young people who practise football.

PLAN

How can you solve the problem?
Since the actual percentage of young people who practise football is 62%, and the percentage of the survey, is less than or equals to 4% , then, $|x - 62| \leq 4$, where (x) represents the actual percentage of young people who practise football.

SOLVE

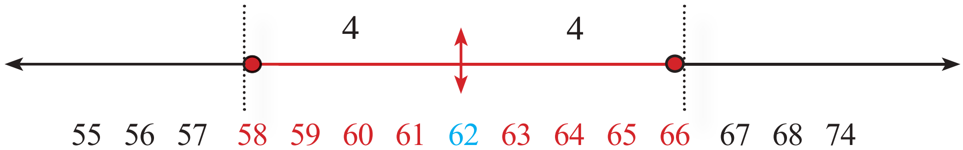
Find the solution set for the absolute value inequality:

$$|x - 62| \leq 4 \Rightarrow -4 \leq x - 62 \quad \text{and} \quad x - 62 \leq 4$$
$$\Rightarrow -4 + 62 \leq x \quad \text{and} \quad x \leq 4 + 62$$
$$\Rightarrow 58 \leq x \quad \text{and} \quad x \leq 66$$
$$\Rightarrow \{x: 58 \leq x\} \cap \{x: x \leq 66\}$$
$$\Rightarrow \{x: 58 \leq x \leq 66\}$$

The percentage range of young people who practise football.

CHECK

Use the line of numbers to check the correcting of solution.



Problems

Solve the following problems using the strategy (Understanding the problem)

1 Salmon fish: The average of Salmon age is from two to eight years. It will be in dangerous situation when the temperature of water is high. Salmon lives in a temperature between 20 to 23 celsius. Write an inequality represents the temperature of water in which Salmon does not live.

2 Panda bear: Afemal panda gives birth one or two babies which need to milk of the mother panda for more than 6-14 times in a day. The babies of panda, the giant, weight from 40 kg to 60 kg in one year. They live with their mothers up to two years. Write an inequality represents the weight of panda's baby when it is one year

3 Beehive: By a survey study on a beehive, Anwar noticed that 88% of the male of bees were expelled in the end of summer. If the margin of mistake is 3 percentage, find the percentage range of the male of bees which are expelled from the hive.

4 Telpherage: Telpherage or the cable car is the cheapest and simplest type of transporting means. It operates by electricity. It is an important means of transportation in the countrieswhich have lots of mountains and rough surfaces. Some countries use telpherage as a means for entertainment and to see the beautifull sightseeings as in the north of Iraq. The lowest speed for telpherage is 20 km/h and the greatest speed is 40 km/h. Write the absolute value inequality which shows the speed of telpherage's cars.

Chapter Review

vocabulary

English	عربي	English	عربي
General Term	الحد العام	Real Number	العدد الحقيقي
Constant Sequence	المتتابعة الثابتة	Rationalizing	تجذير
Common difference	أساس المتتابعة	Conjugate	المرافق
Increasing Sequence	المتتابعة المتزايدة	Relation	العلاقة
Decreasing Sequence	المتتابعة المتناقصة	Ordered Pair	الزوج المرتب
Compound Inequality	المتباينة المركبة	Function	الدالة
Absolute Value	القيمة المطلقة	Surjective function	دالة شاملة
Absolute Value Inq.	متباينة القيمة المطلقة	Injective Function	دالة متباينة
Intersection	التقاطع	Bijective Function	دالة متقابلة
Union	الاتحاد	Domain	المجال
Solution set	مجموعة الحل	Co-Domain	المجال المقابل
Less Than	أقل من	Range	المدى
Less Than or Equal	أقل من أو يساوي	Composite Function	تركيب الدوال
Greater Than	أكبر من	Sequence	المتتابعة
Greater Than or Equal	أكبر من أو يساوي	Arithmetic Sequence	المتتابعة الحسابية

[1 - 1] Ordering the operations in the Real numbers

Example (1): Simplify the following numerical sentences by using the ordering of operations on the real numbers. Then, write the result to the nearest tenth:

$$(-8)^{\frac{1}{3}}\left(\frac{1}{4}\sqrt{2} - \frac{1}{3}\sqrt{18}\right)$$

$$= -2\left(\frac{1}{4}\sqrt{2} - \sqrt{2}\right) = -\frac{1}{2}\sqrt{2} + 2\sqrt{2}$$

$$= \frac{3}{2}\sqrt{2} \approx \frac{3}{2} \times 1.4 = 2.1$$

Example (2): Using a calculator to write the result in the scientific form for the number which should be written to the nearest two decimal places.

$$0.016 \times 10^4 + 1.95 \times 10^3$$

$$= 0.16 \times 10^3 + 1.95 \times 10^3$$

$$\approx 2.1 \times 10^3$$

Exercise (1): Simplify the following numerical sentences by using the ordering of operations on the real numbers. Then, write the result to the nearest tenth.

$$\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \dots\dots\dots\dots\dots\dots$$

....

Exercise (2):Using a calculator to write the result in the scientific form for the number which should be written to the nearest two decimal places.

$6.25 \times 10^3 \div 0.05 \times 10^6 =$ ....

....

[1 - 2] Mappings

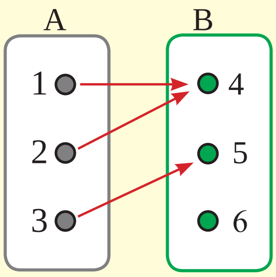
Example: If the mapping $R : A \to B$ was given as follow:

$R = \{(1,4), (2,4), (3,5)\}$

Where, $B = \{4,5,6\}$, $A = \{1,2,3\}$

represent the relation by an arrowy diagram, then determine the domain and range of the mapping

the arrowy diagram



domain: $\{1, 2, 3\}$

range: $\{4,5\}$

Exercise: If $A=\{1, 2, 3\}$ and the two mappings

$f: A \to A$ and

$g: A \to A$ were known as follow:

$f = \{(1,2), (2,3), (3,1)\}$

$g = \{(1,1), (2,2), (3,3)\}$

Find the composition of the two mappings:

i) fog ii) gof

....

....

[1 - 3] The Sequences

Example (1): **Write the first five terms for the sequence $\{ u_n \}$:**

i) $u_n = \frac{1}{n}$

$\{\frac{1}{n}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...\}$

ii) $u_n = \frac{2n-1}{n}$

$\{\frac{2n-1}{n}\} = \{1, \frac{3}{2}, \frac{5}{3}, \frac{7}{4}, \frac{9}{5}, ...\}$

Example (2): Write the first five terms for an arithmetic sequence which its seventh term is 6 and its common difference is 3.

$u_n = a + (n-1)d \to u_4 = a + 6d$

$\to 6 = a + 6 \times 3 \to a = -12$

$\{-12, -9, -6, -3, 0, ...\}$

Exercise (1): **Write the first five terms for each of the following sequences:**

i) $\{3n - 2\} =$ ....

ii) $\{(-2)^n\} =$ ....

Exercise (2): Write the twentieth term of the arithmetic sequences:

$\{12, 6, 0, -6, -12, ...\}$

....

....

....

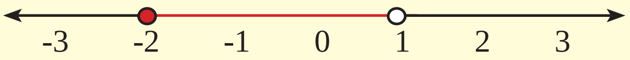
[1 - 4] compound Inequality

Example (1): Solve the compound inequality which includes (and) algebraically, then represent the solution on the line of numbers:

$-6 \le 2x-2$ and $2x-2 < 0 \Rightarrow -6 \le 2x-2 < 0$

$\Rightarrow -4 \le 2x < 2 \Rightarrow -2 \le x < 1$

$\Rightarrow S = \{x : -2 \le x < 1\}$

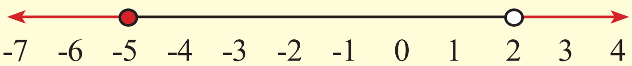


Example (2): Solve the compound inequality algebraically, then represent it on the line of numbers:

$x+1 > 3$ or $x+1 \le -4$

$\Rightarrow x > 2$ or $x \le -5$

$\Rightarrow \{x: x > 2\} \cup \{x: x \le -5\}$



Exercise (1): Solve the compound inequality which includes (and) algebraically, then represent the solution set on the line of numbers.

$-9 < 2x - 1 \le 3$

....

....

....

Exercise (2): Solve the compound inequality which includes (or) algebraically, then represent the solution on the line of numbers.

$2y-6 > -3$ or $2y-6 \le -7$

....

....

....

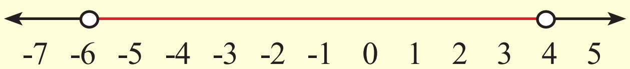
[1 - 5] The Absolute Value Inequalies

Example (1): Solve the following absolute value inequality, then represent the solution on the line of numbers.

$|x+1| < 5 \Rightarrow -5 < x+1 < 5$

$\Rightarrow -5-1 < x < 5-1 \Rightarrow -6 < x < 4$

$\Rightarrow S = \{x: -6 < x < 4\}$

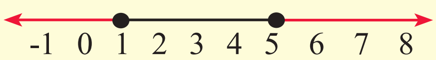


Example (2):Solve the following absolute value inequality, then represent the solution on the line of numbers.

$\left|\frac{3z-9}{6}\right| \ge 1 \Rightarrow \left|\frac{3(z-3)}{6}\right| \ge 1 \Rightarrow \left|\frac{z-3}{2}\right| \ge 1$

$\Rightarrow |z-3| \ge 2 \Rightarrow -2 \ge z-3$ or $z-3 \ge 2$

$\Rightarrow 1 \ge z$ or $z \ge 5 \Rightarrow \{z: 1 \ge z\} \cup \{z: z \ge 5\}$



Exercise (1): Solve the following absolute value inequality, then represent the solution on the line of numbers.

$|3y| - 1 \le 8$

....

....

....

Exercise (2):Solve the following absolute value inequality, then represent the solution on the line of numbers.

$\left|\frac{6-2x}{8}\right| \ge 3$

....

....

....

Chapter Test

Simplify the following numerical sentences by using the ordering of operations in the real numbers:

1 $(\sqrt{3} + \sqrt{5})\ (\sqrt{3} + \sqrt{5}) = ...$

2 $\dfrac{\sqrt{3} - \sqrt{6}}{\sqrt{3}} - \dfrac{\sqrt{8} - 5}{3\sqrt{2}} = ...$

Use the ordering of operations and the calculator to write each of the following which should be written to the nearest tenth.

3 $(\frac{1}{125})^{\frac{1}{3}} - (-\frac{1}{2})^{0} + (121)^{\frac{1}{2}} \times (\frac{1}{9})^{\frac{1}{2}} = ...$

4 If $f : z \rightarrow R$, where $f(x) = x^2$. Draw an arrowy diagram for the mapping, then show if the mapping is injuctive, surjective or bijective ?

5 If the mapping $f: N \rightarrow N$, where $f(x) = 3x + 1$, $g : N \rightarrow N$, where, $g(x) = x^2$

Find: (gof)(5) , (fog)(5) , (gof)(2) , (fog)(2) .

6 If the mapping $f: R \rightarrow R$ where $f(x)= 3x+1$ and the mapping $g: R \rightarrow R$, since $g(x)= 2x+5$.

Does (fog) (x) = (gof) (x) ? find the value of x if (fog) (x) = 28

Write the terms of the following sequences:

7 Find the terms between u_3 and u_8 for an arithmetic sequence which its second term is $\frac{-3}{2}$ and $d = 2$.

8 Find the terms between u_4 and u_9 for an arithmetic sequence which its third term is 6 and $d = \frac{-5}{2}$.

Determine the type of the following sequences (increasing, decreasing, constant):

9 $u_n = 9 - 3n$

10 $u_n = n^2 - 2$

11 $u_n = \frac{1}{3n+1}$

Write the first five terms for each of the following sequences

12 $\{\frac{n}{n+2}\} = ...$

13 $\{4\sqrt{2}\} = ...$

14 $\{\frac{-n}{n+5}\} = ...$

Solve the compound inequalities, then represent the solution set on the line of numbers.

15 $12 \leq x + 6$ و $x+6 < 20$

16 $\frac{1}{8} \geq \frac{z+2}{2} > \frac{1}{16}$

17 $x-3 \leq -5$ or $x-3 > 5$

18 $7t-5 > -1$ or $7t-5 \leq -14$

19 $y \leq 0$ or $y + 7 \geq 16$

20 $\frac{y}{3} < 1\frac{1}{3}$ or $\frac{y}{3} > 9\frac{1}{3}$

Write the compound inequality shows the range of the third side in each triangle:

21 4cm , 9cm

22 5cm ,12cm

23 7cm ,15cm

Solve the following abslute value inegualities:

24 $|x - 6| \leq 3$

25 $|3z| - 5 < 4$

26 $|x + 1| > \frac{1}{2}$

27 $6|x| - 8 \geq 3$

28 $|3y| - 2 > 9$

29 $|8z| - 1 > 7$

30 $|4 - 3y| \geq 14$

31 $|\frac{6-3y}{9}| \geq 5$

Chapter 2

Algebraic Expressions

Al-Mustansiriya school is an ancient one .It was established in the reign of Abbassiyn in Baghdad in 1233 .It was an important cultural and scientific center. It locates in AL- Risafa side of Baghdad . There is a rectangular area in the middle of school which has a great fountain and o'clock of school .If we assume that the length of the internal area of school is (x+14) meters and its width is(x+2)meters,then we can calculate the area by multiplying the tow algebraic expressions (x+14) (x+2).

Pretest

Find the result of adding or subtracting the following algebraic expressions:

1. $(3x^2 + 4x - 12) + (2x^2 - 6x + 10)$
2. $(\frac{1}{2} zy + 5z - 7y) - (\frac{1}{4} zy - 3z + 2y)$

Find the result of multiplying the following algebraic terms:

3. $7x^2 \times \frac{1}{14x}$
4. $\sqrt{2}\, yz \times \sqrt{2}\, yz^2$
5. $\frac{3}{4} v^2 t \times \sqrt{12}\, t^{-1}$
6. $3h\,(\frac{1}{6} v - \frac{1}{3} h^{-2})$

Find the result of multiplying two algebraic expressions:

7. $(x+2)\,(x-2)$
8. $(5-2z)\,(3+3z)$
9. $(\frac{1}{2} x^2 + 6)\,(\frac{4}{3} x^2 + 12)$
10. $(2\sqrt{3}\, t - 4)^2$
11. $(x+3)\,(x^2 - 3x + 9)$
12. $(xy + 1)\,(x^{-1}y - xy^{-1} - 1)$

Find the result of multiplying by using the vertical method:

13. $(y-1)\,(y+1)$
14. $(2x+3)\,(4x^2 - x - 5)$
15. $(3-z)\,(3 + 5z - z^2)$

Find the result of dividing the following algebraic expressions:

16. $\frac{3xy^2}{15x^2y}$
17. $\frac{-47z^{-2}}{7z^2}$
18. $\frac{8x^3 + 4x^2 - 2x}{2x}$
19. $\frac{21 - 14a + 7a^2}{7a}$

Factoring the algebraic expressions by using the greater common factor :

20. $3y^3 + 6y^2 - 9y$
21. $\frac{1}{2} zx^2 - 2z^2 x + 4zx$

Lesson [2-1] Multiplying Algebraic Expressions

Idea of the lesson:
* Multiplying an algebraic expression by other algebraic expression which represents special cases.

Vocabulary:
* Square of sum
* Square of difference
* Cubic of sum
* Cubic of difference

Learn

A square-shaped home garden was surrounded by a fence. The length of its side is h meter with an aisle of one meter width. What is the area of aisle according to h ?

[2-1-1] Multiplying two Algebraic Expressions each one Contains two terms.

You have previously learned how to multiply two algebraic terms with each other and how to multiply an algebraic expression by another one. Now, you will learn how to multiply two algebraic expressions which each one of them has two terms with each other , and they represent a square of sum or a square of difference or sum multiply by difference, by using the properties that you previously studied which are distributing ,substituting and ordering.

Example (1) Find the area of the aisle which surrounds the square-shaped garden.

The area of the aisle is the difference between the two area of the big square (garden with aisle) and the small square (the garden)

$(h+2)^2 = (h+2)(h+2) = h^2 + 2h + 2h + 4 = h^2 + 4h + 4$ *The area of garden with aisle*

$h \times h = h^2$ *The area of garden*

$(h^2 + 4h + 4) - h^2 = h^2 - h^2 + 4h + 4 = 4h + 4$ *The area of aisle*

Example (2) **Find the result of multiplying an algebraic expressions by another algebrail expression where each one has two terms :**

i) $(x + y)^2 = (x+ y)(x+ y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$ *Square of sum for two terms*

ii) $(x - y)^2 = (x- y)(x- y) = x^2 - xy - yx + y^2 = x^2 - 2xy + y^2$ *Square of difference between two terms*

iii) $(x + y)(x-y) = x^2 - xy + yx - y^2 = x^2 - y^2$ *Sum of two terms × the difference between them.*

iv) $(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$ *Sum of two terms × sum of two terms.*

v) $(x + 2)(x - 6) = x^2 - 6x + 2x - 12 = x^2 - 4x - 12$ *Sum of two terms ×the difference between two terms.*

vi) $(x -1)(x - 4) = x^2 - 4x - x + 4 = x^2 - 5x + 4$ *difference between two terms×difference between two terms.*

Example (3) **Find the result of multiplying the following algebraic expressions:**

i) $(z + 3)^2 = z^2 + 6z + 9$

ii) $(h - 5)^2 = h^2 - 10h + 25$

iii) $(2x - 7)(2x + 7) = 4x^2 - 49$

iv) $(3y + 1)(y + 2) = 3y^2 + 7y + 2$

v) $(v + \sqrt{2})(v - \sqrt{2}) = v^2 - 2$

vi) $(n - \sqrt{3})(5n - \sqrt{3}) = 5n^2 - 6\sqrt{3}n + 3$

[2-1-2] Multiplying algebraic expressions from two terms by another three terms

You have previously learned the multiplying of algebraic expression which have many terms. Now, you will learn special cases of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of there terms by using the properties that you studied in distributing , substituting and ordering.

Example (4) **Find the result of multiplying an algebraic expression which consists of two terms by an algebraic expression which consists of three terms:**

i) $(x+2)(x^2 - 2x + 4) = x^3 - 2x^2 + 4x + 2x^2 - 4x + 8 = x^3 + 8 = x^3 + 2^3$

The result of multiplying is the sum of two cubes

ii) $(y-3)(y^2 + 3y + 9) = y^3 + 3y^2 + 9y - 3y^2 - 9y - 27 = y^3 - 27 = y^3 - 3^3$

The result of multiplying is the difference between two cubes

iii) $(y+2)^3 = (y+2)(y+2)^2 = (y+2)(y^2 + 4y + 4)$

Cube of two terms sum

$= y^3 + 4y^2 + 4y + 2y^2 + 8y + 8 = y^3 + 6y^2 + 12y + 8$

iv) $(z-3)^3 = (z-3)(z-3)^2 = (z-3)(z^2 - 6z + 9)$

Cube of the difference between two terms

$= z^3 - 6z^2 + 9z - 3z^2 + 18z - 27 = z^3 - 9z^2 + 27z - 27$

Example (5) **Find the result of multiplying the following algebraic expressions :**

i) $(2v+5)(4v^2 - 10v + 25) = 8v^3 - 20v^2 + 50v + 20v^2 - 50v + 125 = 8v^3 + 125 = (2v)^3 + 5^3$

ii) $(\frac{1}{3} - z)(\frac{1}{9} + \frac{1}{3}z + z^2) = \frac{1}{27} + \frac{1}{9}z + \frac{1}{3}z^2 - \frac{1}{9}z - \frac{1}{3}z^2 - z^3 = \frac{1}{27} - z^3 = (\frac{1}{3})^3 - z^3$

iii) $(x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}\,x + \sqrt[3]{4}) = x^3 + \sqrt[3]{2}\,x^2 + \sqrt[3]{4}\,x - \sqrt[3]{2}\,x^2 - \sqrt[3]{4}\,x - \sqrt[3]{8}$

$= x^3 + \sqrt[3]{2}\,x^2 - \sqrt[3]{2}\,x^2 + \sqrt[3]{4}\,x - \sqrt[3]{4}\,x - 2 = x^3 - 2$

iv) $(x + \frac{1}{2})^3 = (x + \frac{1}{2})(x + \frac{1}{2})^2 = (x + \frac{1}{2})(x^2 + x + \frac{1}{4}) = x^3 + x^2 + \frac{1}{4}x + \frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{8}$

$= x^3 + x^2 + \frac{1}{2}x^2 + \frac{1}{4}x + \frac{1}{2}x + \frac{1}{8} = x^3 + \frac{3}{2}x^2 + \frac{3}{4}x + \frac{1}{8}$

v) $(y - \sqrt{5})^3 = (y - \sqrt{3})(y - \sqrt{5})^2 = (y - \sqrt{5})(y^2 - 10\sqrt{5}\ y^2 + 25)$

$= y^3 - 10y^2 + 25y - 5y^2 + 50y - 125 = y^3 - 15y^2 + 75y - 125$

Make sure of your understanding

Find the result of multiplying an algebraic expression by another algebraic expression where both of them have two terms:

Questions 1- 7 are similar to examples 2-3

1. $(x + 3)(x - 3)$
2. $(\sqrt{7} - h)^2$
3. $(z + \sqrt{5})(z - \sqrt{5})$
4. $(v + 5)(v + 1)$
5. $(x - 3)(x - 2)$
6. $(3x - 4)(x + 5)$
7. $(\frac{1}{3}y + 3)(\frac{1}{3}y + 2)$

Find the result of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms:

Questions 8-13 are similar to examples 4-5

8. $(y+2)(y^2 - 2y+4)$
9. $(2z + 4)(4z^2 - 8z + 16)$
10. $(v - \sqrt[3]{3})(v^2 + \sqrt[3]{3}\ v + \sqrt[3]{9})$
11. $(\sqrt[3]{\frac{2}{7}} + m)(\sqrt[3]{\frac{4}{49}} - \sqrt[3]{\frac{2}{7}}\ m + m^2)$
12. $(x + 5)^3$
13. $(y - 4)^3$

Solve the Exercises

Find the result of multiplying an algebraic expression by another algebraic expression where both of them have two terms:

14. $(n - 6)^2$
15. $(y + 5)(y - 5)$
16. $(x + \sqrt{8})^2$
17. $(y + \sqrt{6})(y - \sqrt{6})$
18. $(8 + h)(3 + h)$
19. $(4 - y)(5 - y)$
20. $(2x - 3)(x + 9)$
21. $(z - 2\sqrt{7})(2z - \sqrt{7})$

Find the result of multiplying an algebraic expression which consists of two terms by another algebraic expression which consists of three terms:

22. $(x+6)(x^2 - 6x+36)$
23. $(y - 1)(y^2 + y + 1)$
24. $(z - 3)^3$
25. $(\frac{2}{3} - r)(\frac{4}{9} + \frac{2}{3}r + r^2)$
26. $(x - \sqrt[3]{4})(x^2 + \sqrt[3]{4}\ x + \sqrt[3]{16})$
27. $(z - \sqrt{5})^3$
28. $(\sqrt[3]{\frac{1}{5}} + n)(\sqrt[3]{\frac{1}{25}} - \sqrt[3]{\frac{1}{5}}\ n + n^2)$
29. $(\sqrt[3]{\frac{1}{9}} + \frac{1}{h})(\sqrt[3]{\frac{1}{81}} - \sqrt[3]{\frac{1}{9}}\ \frac{1}{h} + \frac{1}{h^2})$

Solve the problems

30 Swimming pool: Baghdad hotel is one of the important tourist hotels in Baghdad, the capital of Iraq. The length of the swimming pool is $(x+9)$ meter and the width is $(x+1)$ meter.
It is surrounded by an aisle which its width is 1 meter.
Write the area of the swimming pool with the aisle in simplest form.

31 History: Babylon city locates to the north of Al-Hila city in Iraq. Babylonians lived there since about 3000 years BC. In 575 , they built the gate of Ishtar which considers the eighth gate of Babylon wall. Wa'el drew a painting represents the gate of Ishtar. The dimensions of the painting was $(y+7)$, $(y-4)$ cm.
Write the painting area which was drawn by Wa'el in simplest form by y .

32 Ornament at fish : A cubic-shoped aquarium, the length of its side is $(v+3)$ cm.
Write the volume of the aquarium in simplest form.

Think

33 Challenge: Find the result of the following in simplest form:

$(x + 1)^2 - (x - 2)^2$

34 Correct the mistake: :Nisreen wrote the result of multiplying the two algebraic expressions, as follow:

$(\sqrt{5}\, h - 4)(h - 6) = 5h^2 + 10h - 24$

Determine Nisreen's mistake , then correct it.

35 Numerical sense: Which of the following two numbers is greater.

$(\sqrt{3} - \sqrt{2})^2$ or the number $(\sqrt{3} + \sqrt{2})^2$? Clarify your answer.

Write

The result of multiplying the two algebraic expressions :

$(2z + \frac{1}{2})(2z - \frac{1}{2})$

Lesson [2-2] Factoring the Algebraic Expression by using a Greater Common Factor

Idea of the lesson:
*Factoring the algebraic expression by using the greater common factor.

Vocabulary:
*Factoring the algebraic expression
*The greater common factor
*Binomial terms
*Inverse
*Checking the correction of solution.

Learn

The monument of Kahrmana square in the middle of Baghdad is one of the distinctive civilizational landmarks in Iraq . It locates in the center of the square in Al-Karada. The radius of the circular statue is (r) meter. It is surrounded by a basin which is like a circular aisle. If the radius and the basin of the statue is r +2 meter, find the basin area.

[2-2-1] Factoring the algebraic expression by using a greater common factor

You have previously learned how to find the greater common factor for numbers . You have also learned how to factor the algebraic expression by using the greater common factor (GCF) . Now, you will increase your skills by learning the factoring of algebraic expression which consist of two or three terms by using the greater common factor, then checking the correction of solution.

Example (1) The radius of the base of kahrmana statue is r meter, and the radius of the statue with the basin is r+2 meter. Find the basin area.

$A_1 = r^2\pi$ *the area of statue .*
$A_2 = (r + 2)^2\ \pi = (r^2 + 4r + 4)\pi = r^2\pi + 4r\pi + 4\pi$ *the area of statue with basin*
$A = A_2 - A_1 = r^2\ \pi + 4r\pi + 4\pi - r^2\ \pi$ *the area of basin*
$= 4r\pi + 4\pi = 4\pi\ (r + 1)$ *($4\ \pi$)the greater common factor*

The area of basin which surrounds the statue is $4\pi\ (r + 1)$ squre meters

Example (2) **Factoring each expression by using the greater common factor (GCF), then checking the correction of solution :**

i) $6x^3 + 9x^2 - 18x = 3x\ (2x^2 + 3x - 6)$ *The greater common factor is 3x*

Checking :

$3x\ (2x^2 + 3x - 6) = 3x\ (2x^2) + 3x\ (3x) - 6(3x)$

$= 6x^3 + 9x^2 - 18x$

To check by using the multiplication of algebraic expressions.

ii) $\sqrt{12}\ y^2z + \sqrt{2}\ (\sqrt{6}\ yz^2 - \sqrt{24}\ yz)$

$= 2\sqrt{3}\ y^2z + 2\sqrt{3}\ yz^2 - 4\sqrt{3}\ yz$ *Open the bracket with simplify the numerical roots.*

$= 2\sqrt{3}\ yz\ (y + z - 2)$ *The greater common factor is $2\sqrt{3}\ yz$*

$2\sqrt{3}\ yz\ (y+z-2) = 2\sqrt{3}\ y^2z + 2\sqrt{3}\ yz^2 - 4\sqrt{3}\ yz$

Checking :

To check , we use the multiplication of the algebraic expression .

We see that the variables are equaled in terms with the original. expression and it is also with numerical factors because:

$2\sqrt{3} = \sqrt{12}\ ,\ 2\sqrt{3} = \sqrt{2}\ \sqrt{6}\ ,\ 4\sqrt{3} = \sqrt{2}\ \sqrt{24}$

Example (3) **Factoring each expression by using the binomial as a greater common factor :**

i) $5x (x +3) - 7(x+3) = (x+3) (5x -7)$ — *The greater common factor is* $(x+3)$

ii) $\frac{1}{2} (y -1) + \frac{1}{3} y^2 (y-1) = (y-1) (\frac{1}{2} + \frac{1}{3} y^2)$ — *The greater common factor is* $(y-1)$

iii) $\sqrt{3}\ v^2 (z +2) - \sqrt{5}\ v (z +2) = v(z +2) (\sqrt{3}\ v- \sqrt{5}\ )$ — *The greater common factor is* $v(z+2)$

[2-2-2] Factoring an algebraic expression by using the property of grouping

You have previously learned in the previous how to factor the algebraic expression which consists of two or three terms by using the greater common factor. Now, you will learn how to factor an algebraic expression which consists of four terms or more by using the grouping of the terms ,where the grouping terms have common factors.

Example (4) **Factoring each algebraic expression by using the grouping, then check the correction of the solution:**

i) $4x^3 - 8x^2 + 5x - 10 = (4x^3 - 8x^2) + (5x - 10)$ — *Grouping terms which have common factors.*

$= 4x^2 (x - 2) + 5(x - 2)$ — *Factoring the grouping terms*

$= (x-2) (4x^2 + 5)$ — *The greater common factor is(x-2)*

checking:

$(x-2) (4x^2 +5) = x(4x^2 +5) - 2(4x^2 + 5)$ — *Using the property of distributing*

$= 4x^3 + 5x - 8x^2 - 10 = 4x^3 - 8x^2 + 5x - 10$ — *Using multiplication and ordering*

ii) $\sqrt{2}\ h^2 t+ \sqrt{3}\ t^2v -\sqrt{8}\ h^2v -\sqrt{12}\ v^2t$

$= (\sqrt{2}\ h^2t -\sqrt{8}\ h^2v) + (\sqrt{3}\ t^2v -\sqrt{12}\ v^2t)$ — *Grouping terms*

$= \sqrt{2}\ h^2(t-2v) + \sqrt{3}\ tv (t-2v)$ — *Factoring the grouping terms*

$= (t-2v) (\sqrt{2}\ h^2 + \sqrt{3}\ tv)$ — *The greater common factor is(t- 2v)*

checking:

$(t-2v) (\sqrt{2}\ h^2 + \sqrt{3}\ tv) = t(\sqrt{2}\ h^2 + \sqrt{3}\ tv) - 2v(\sqrt{2}\ h^2 + \sqrt{3}\ tv)$ — *Using the property of distributing*

$= \sqrt{2}\ h^2 t + \sqrt{3}\ t^2 v - \sqrt{8}\ h^2 v - \sqrt{12}\ v^2 t$ — *Using multiplication and ordering*

Example (5) Factoring the algebraic expression by using the grouping with the inverse:

$14x^3 - 7x^2 + 3 - 6x = (14x^3 - 7x^2) + (3 - 6x)$ — *Grouping the terms*

$= 7x^2 (2x -1) + 3(1- 2x)$ — *Factoring the grouping terms*

$= 7x^2 (2x -1) + 3(-1) (2x -1)$ — *Using the inverse*

$= 7x^2 (2x -1) - 3(2x -1)$ — *Writing +3(-1) as -3*

$= (2x -1) (7x^2 -3)$ — *The greater common factor is (2x-1)*

Make sure of your understanding

Factor each expression by using the greater common factor (GCF) , then check the correction of solution:

1. $9x^2 - 21x$
2. $10 - 15y + 5y^2$
3. $14z^4 - 21z^2 - 7z^3$
4. $\sqrt{8}\ t^2r + \sqrt{2}\ (tr^2 - \sqrt{3}\ tr)$

Questions 1- 4 are similar to example 2

Factor each expression by using the binomial as a greater common factor:

5. $3y\ (y - 4) – 5(\ y - 4)$
6. $\frac{1}{4}(t + 5) + \frac{1}{3}t^2\ (t + 5)$
7. $\sqrt{2}\ n\ (x + 1) - \sqrt{3}\ m\ (x + 1)$
8. $2x\ (x^2 - 3) + 7(x^2 - 3)$

Questions 5-8 are similar to example 3

Factor each expression by using the property of grouping , then check the correction of solution:

9. $3y^3 - 6y^2 + 7y - 14$
10. $21 - 3x + 35x^2 - 5x^3$
11. $2r^2\ k + 3k^2\ v - 4r^2\ v - 6v^2\ k$
12. $3z^3 - \sqrt{18}\ z^2 + z - \sqrt{2}$

Questions 9 -12 are similar to example 4

Factor each expression by using the property of grouping with the inverse:

13. $21y^3 - 7y^2 + 3 - 9y$
14. $\frac{1}{2}x^4 - \frac{1}{4}x^3 + 5 - 10x$
15. $6z^3 - 9z^2 + 12 - 8z$
16. $5t^3 - 15t^2 - 2t + 6$

Questions 13-16 are similar to example 5

Solve the Exercises

Factor each expression by using the greater common factor(GCF), then check the correction of solution

17. $12y^3 - 21y^2$
18. $5t^3 + 10t^2 - 15t$
19. $6v^2\ (3v - 6) + 18v$
20. $\sqrt{12}\ n^3r + \sqrt{3}\ (nr^3 - \sqrt{2}\ nr)$

Factor each expression by using the binomial as a greater common factor:

21. $\frac{1}{7}(y + 1) + \frac{1}{3}y^2\ (y + 1)$
22. $\sqrt{3}\ k\ (x^2 + 1) - \sqrt{5}\ v\ (x^2 + 1)$

Factor each expression by using the property of grouping, then check the correction solution:

23. $5x^3 - 10x^2 + 10x - 20$
24. $49 - 7z + 35z^2 - 5z^3$
25. $3t^3\ k + 9k^2\ s - 6t^3\ s - 18s^2\ k$
26. $2y^4 - \sqrt{12}\ y^3 + \sqrt{2}\ y - \sqrt{6}$

Factor the expression by using the property of grouping with inverse:

27. $12x^3 - 4x^2 + 3 - 9x$
28. $4r^3 - 16r^2 - 3r + 12$

Solve the problems

29 Solar energy : The solar panels are the main component in the solar energy systems which generate electricity .The cells are manufactured from semiconducting materials such as silicon. They absorb the light of sun . What are the dimensions of the solar panel , if its area was $3x(x-4)-22(x-4)$ square meter.

30 Flamenco bird: Flamenco bird is one of the migratory birds which has beautiful shape . Its color is pink. These birds travel for long distances during the season of the annual migration passing by the marshes in the south of Iraq to get food from the water pools. If the area of the water pool which was covered by the flamenco birds in one of the Iraqi marshes is $4y^2 + 14y + 7(2y+7)$ square meter.
What is the shape of that pool, and what are its dimensions ?

31 Baghdad o·clock : It is a high building which has a Four_ Faces o·clock at the top of it.This building locates in the celebration park in Baghdad . It was established in 1994 .What is the radius of the internal circle of the o·clock if you know that its area is $z^2\pi - 3z\pi + (3z - 9)\pi$.

Think

32 Challenge: Factor of the following expressions in a simplest form:

$5x^5 y + 7y^3 z - 10x^5 z - 14z^2 y^2$

33 Correct the mistake: Ibtisam had written the result of factoring the following expression, as follow:

$\sqrt{2}\, t^4 - \sqrt{24}\, t^3 + t^2 - \sqrt{12}\, t = (t + 2\sqrt{3})(\sqrt{2}\, t^2 - t)$

Discover the mistake of Ibtisam and correct it.

34 Numerical sense: What is the unknown number in this expression.

$x^2 + 3x + 5x + 15 = (x + 3)(x + \boxed{})$

Write

The result of subtraction the expression (x + y) (x - y) from the expression (x + y) (x + y) in simplest form .

Lesson [2-3] Factoring the Algebraic Expression by using Special Identities

Idea of the lesson:
*Factoring the algebraic expression as a difference between two squares and a complete squre.

Vocabulary:
*The difference between two squares
*Perfect square
*Complet square
*The missing term

Learn

Al-Shaab international stadium in Baghdad is one of the important stadiums in Iraq. It was established in1966 . If the area of the playground, which was allocated for playing football , is x^2 -400 square meter , what are the dimensions of the playground ?

[2-3-1] Factoring the algebraic expression by the difference between two squares.

You have previously learned how to find the result of multiplying an algebraic expression which represents the sum of two terms by another algebraic expression which represents the difference between them , and the result represents the difference between their two squares . Now, you will learn the inverse operation of multiplication which is factoring the algebraic expression which is as a difference between two squares $(x^2 - y^2) = (x + y)(x - y)$.
The expression $x^2 + y^2$ can not be factored in this stage .

Example (1) Find the dimensions of football playground which its area is x^2 - 400 square meter ?

$x^2 - 400 = (x)^2 - (20)^2$ *Write each terms as a perfect square*
$= (x + 20)(x - 20)$ *Write the factoring*

The first bracket: the square root of the first term + the square root of the second term.
The second bracket: the square root of the first term – the square root of the second term.

So the length of the football playground is x+20 meter and its width is x-20 meter.

Example (2) **Factoring each of the following algebraic expressions as a difference between two squares**

i) $x^2 - 9 = (x + 3)(x - 3)$

ii) $36y^2 - z^2 = (6y + z)(6y - z)$

iii) $49 - v^2 = (7 + v)(7 - v)$

iv) $2x^2 - z^2 = (\sqrt{2}\,x + z)(\sqrt{2}\,x - z)$

v) $5h^2 - 7v^2 = (\sqrt{5}\,h + \sqrt{7}\,v)(\sqrt{5}\,h - \sqrt{7}\,v)$

vi) $12 - t^2 = (2\sqrt{3} + t)(2\sqrt{3} - t)$

vii) $8x^3y - 2xy^3 = 2xy(4x^2 - y^2)$ *Factoring by using the common factor*
$= 2xy(2x + y)(2x - y)$ *Factoring by using the difference between two squares.*

viii) $\frac{1}{16}z^4 - \frac{1}{81} = (\frac{1}{4}z^2 + \frac{1}{9})(\frac{1}{4}z^2 - \frac{1}{9}) = (\frac{1}{4}z^2 + \frac{1}{9})(\frac{1}{2}z + \frac{1}{3})(\frac{1}{2}z - \frac{1}{3})$

[2 -3-2] Factoring the Algebraic Expression by the Perfect square

you have previously learned how to find the result of multiplying a square of sum two terms and a square of difference between two terms , the result was consisted of three terms. Now , you will learn the inverse operation of multiplication which is factoring an expression consists of three terms in form of perfect square .

$x^2 + 2xy + y^2 = (x + y)^2$, $x^2 - 2xy + y^2 = (x - y)^2$ The alagebric expression $a x^2 \pm bx + c$ will be a perfect square if $bx = \pm 2\sqrt{(ax^2)(c)}$ where $a \neq 0$

Example (3) **Factor each of the following algebraic expressions which are in a form of a perfect square.**

i) $x^2 + 6x + 9 = (x)^2 + 2(x \times 3) + (3)^2$

$= (x + 3)(x + 3)$

$= (x + 3)^2$

ii) $y^2 - 4y + 4 = (y)^2 - 2(y \times 2) + (2)^2$

$= (y - 2)^2$

iii) $16z^2 - 8z + 1 = (4z)^2 - 2(4z \times 1) + (1)^2 = (4z - 1)^2$

Write the first and last terms as a perfect square
Write the middle term as a double of the first term root multiplying by the root of the last term.
Write factoring of expression.
The final factoring as ²(root of last term +first term root)
Note the sign between the two numbers is the sign of the middle term.

Example (4) **Determine which of the following algebraic expressions represents a perfect square and factor it**

i) $x^2 + 10x + 25$

$(x)^2$ $(5)^2$

$2(x)(5) = 10x$ perfect square

$x^2 + 10x + 25 = (x+5)^2$

ii) $y^2 + 14x + 36$

$(y)^2$ $(6)^2$

$2(y)(6) = 12y \neq 14y$ Not perfect square

iii) $4 - 37v + 9v^2$

$(2)^2$ $(3v)^2$

$-2(2)(3v) = -12v \neq -37v$

Not perfect square

iv) $9h^2 - 6h + 3$

$(3h)^2$ $(\sqrt{3})^2$

$-2(3h)(\sqrt{3}) = -6\sqrt{3}h \neq -6h$

Not perfect square

Example (5) **Write the missing term in the algebraic expression $a x^2 + bx + c$ to make it a perfect squre , then factor it**

To become a perfect square, , we apply the law of the middle term.

i) $25x^2 - \dots\dots + 49$ $bx \pm \sqrt{(ax^2)(c)}\ 2$

$bx = 2\sqrt{(ax^2)(c)}$ $\Rightarrow bx = 2\sqrt{(25x^2)(49)}$ $\Rightarrow bx = 70x$

$\Rightarrow 25x^2 - 70x + 49 = (5x - 7)^2$

ii) $\dots\dots + 8x + 16$

$bx = 2\sqrt{(ax^2)(c)}$ $\Rightarrow 8x = 2\sqrt{(ax^2)(16)}$ $\Rightarrow 64x^2 = 4 \times 16 \times ax^2 \Rightarrow ax^2 = x^2$

$\Rightarrow x^2 + 8x + 16 = (x + 4)^2$

iii) $y^2 + 14y + \dots\dots$

$by = 2\sqrt{(ay^2)(c)}$ $\Rightarrow 14y = 2\sqrt{(y^2)(c)}$ $\Rightarrow 196y^2 = 4 \times y^2 \times c \Rightarrow c = 49$

$\Rightarrow y^2 + 14y + 49 = (y + 7)^2$

Make sure of your understanding

Factor each of the following algebraic expressions as a difference between two squares:

1. $x^2 - 16$
2. $36 - 4x^2$
3. $h^2 - v^2$
4. $9m^2 - 4n^2$
5. $27x^3z - 3xz^3$
6. $\frac{1}{4}y^4 - \frac{1}{16}$

Questions 1-6 are similar to example 2

Factor each of the following algebraic expressions as a perfect square:

7. $y^2 - 8y + 16$
8. $9z^2 - 6z + 1$
9. $v^2 + 2\sqrt{3}\,v + 3$
10. $4h^2 - 20h + 25$

Questions 7-10 are similar to example 3

Determine which one of the following algebraic expressions represents a perfect square, then factor it.

11. $x^2 + 18x + 81$
12. $16 - 14v + v^2$
13. $64h^2 - 48h - 9$
14. $3 - 4\sqrt{3}\,t + 4t^2$

Questions 11 -14 are similar to example 4

Write the missing term in the algebraic expression ax^2+bx+c to become a perfect square, then factor it.

15. …. $+ 14y + 49$
16. $z^2 + 4z +$ ……
17. $3 -$ ….. $+ 9x^2$
18. $4x^2 + 2\sqrt{5}\,x +$ …

Questions 15-18 are similar to example 5

Solve the Exercises

Factor each of the following algebraic expressions in simplest form :

19. $25 - 4x^2$
20. $y^2 - 121$
21. $x^2 - 16z^2$
22. $12 - 3t^2$
23. $8y^3x - 2x^3y$
24. $\frac{1}{4}y^2 - \frac{1}{8}$
25. $\frac{1}{3}z^5 - \frac{1}{12}z$
26. $4x^2 + 20x + 25$
27. $3z^2 - 6z + 3$
28. $16n^2 + 8\sqrt{3}\,n + 3$
29. $4t^3 - 12t^2 + 9t$
30. $1 - 4m + 4m^2$

Determine which of the following algebraic expressions represents a perfect square, then factor it:

31. $4x^2 + 18x + 16$
32. $y^2 + 10y + 25$
33. $49 - 7v + v^2$
34. $2h^2 - 12h - 18$
35. $4v^2 + 4v + 4$
36. $3 - 2\sqrt{3}\,z + z^2$

Write the missing term in the algebraic expression ax^2+bx+c to become a perfect square, then factor it:

37. $y^2 +$ …. $+ 36$
38. $25 - 20x +$ ....
39. $4v^2 + 8v +$ …….
40. $5 -$ ….. $+ 16x^2$
41. $81 + 18z +$ ….
42. $9h^2 + 6\sqrt{2}\,h +$ …

Solve the problems

43 Al-Malwiya minaret : It locates in Samara city, Iraq . It is one of the Iraqi distinctive landmark because of its unique shape. It is also one of the Iraqi famous ancient landmarks which belongs to the reign of Abbassiyyn . It based on a square base which its area is $x^2 - 8x + 16$ square meter What is the length of the base side which the minaret based on according to x?

44 Farm for breeding cows : Saad has a squre-shaped farm for breeding cows. The length of its side , is X meter. He extended his farm to became in a rectangular shped farm According to that, the area of the farm became $x^2 - 81$ square meter, What are the length and width of the farm after extension according to x?

45 Painting : Bashar drew a painting represents the marshes in the south of Iraq. The expression which represents the area of painting was $4x^2 - 8x + 9$ cm^2. Does the expression of the painting area represent a perfect square or not ?

Think

46 Challenge: Determine which the following algebraic expressions represent a perfect square and factor it:

$$\frac{1}{9}x^2 - \frac{1}{6}x + \frac{1}{16}$$

47 Correct the mistake: Muntaha said that the expression $(2x+1)(2x-1)$ is a factoring to the perfect squre $4x^2 - 4x + 1$. Determine the mistake of Muntaha and correct it.

48 Numerical sense: Dose the expression $9x^2 + 12x - 4$ represent a perfect square or not ? Clarify your answer.

Write

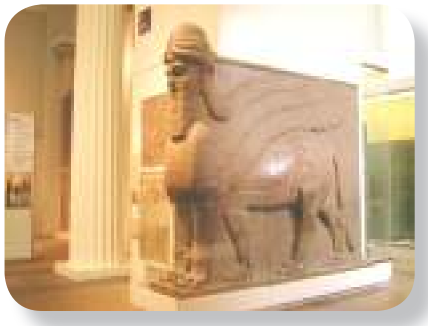
A factoring for the algebraic expression $4x^2 - 8x + 4$.

Lesson [2-4] Factoring the Algebraic Expression of three terms by Probe and Error (Experiment).

Idea of the lesson:
*Factoring the algebraic expression which consists of three terms by using
*Probe and Error.
Vocabulary:
*The two middles.
*The two parties.
*The middle term.

Learn

Assyrian winged bull (shido lamas). It is the way in which this name is written in the Assyrian writings. The origin of the word lamas is derived from the Summerian word Lammu.
There is a statue of it in AL-Moosal,s monument. What are the dimensions of the painting of winged bull which its area is $x^2 + 10x + 21$ centimeter square ?



[2-4-1] Factoring the algebraic expression x^2+bx+c

You have previously learned how to find the result of multiplying an algebraic expression by another algebraic expression which both of them consists of two terms:
i) $(x+2)(x+3) = x^2+5x+6$, ii) $(x+3)(x-5) = x^2-2x-15$, iii) $(x-1)(x-4) = x^2-5x+4$
Now you will learn the inverse operation of multiplication which is factoring the algebraic expression which consists of three terms x^2+bx+c by using the probe and error (experiment) for factoring the algebraic expression, We find two real.
m, n, where $b = m + n, c = nm$ and write $x^2 + bx + c = (x + n)(x + m)$.

Example (1) What are the dimensions of the painting of winged bull which its area is $x^2 + 10x + 21$ centimeter squar?

For factoring the algebraic expression, we follow the following steps:
Factoring the algebraic expression:

Factors of number 21	Sum of the two factors
(1) (21)	1 + 21 = 22
(3) (7)	3 + 7 = 10
(-3) (-7)	(-3) + (-7) = -10

Result of multiplying the two parties $+7x$
Result of multiplying the two middles $+3x$
The middle term $+10x$

Two parties
$x^2 + 10x + 21 = (x + 3)(x + 7)$
Two middles

The width of the painting is $(x + 3)$cm
The length of the painting is $(x + 7)$cm

Example (2) Factoring the following algebraic expression $y^2 + y - 12$

Factors of number 12	Sum of two Factors
(1) (-12)	1 - 12 = -11
(12) (-1)	12 - 1 = 11
(2) (-6)	2 - 6 = -4
(6) (-2)	6 - 2 = -4
(3) (-4)	3 - 4 = -1
(4) (-3)	4 - 3 = 1

The result of multiplying the two parties $+4y$
The result of multiplying $-3x$
The middle term $+y$

$y^2 + y - 12 = (y - 3)(y + 4)$

Example (3) **Factoring the following algebraic expressions :**

i) $z^2 - z - 6 = (z - 3)(z + 2)$ The middle term $2z - 3z = -z$

ii) $x^2 - 9x + 18 = (x - 3)(x - 6)$ The middle term $-6x - 3x = -9x$

iii) $y^2 + 6y - 27 = (y + 9)(y - 3)$ The middle term $-3y + 9y = +6y$

iv) $x^2 - xy + 20 = (x - 5y)(x + 4y)$ The middle term $+4xy - 5xy = -xy$

v) $15 - 8z + z^2 = (5 - z)(3 - z)$ The middle term $-5z - 3z = -8z$

[2-4-2] Factoring the algebraic expression ax^2+bx+c where $a \neq 0$

Now, you will learn how to factor an algebraic expression which consists of three terms as in the form of $a x^2 + bx + c$ and that $a \neq 0$.

Example (4) **Factoring each of the following algebraic expressions :**

i) $\mathbf{6x^2 + 17x + 7}$

$6 = \begin{cases} (1)(6) \\ (2)(3) \end{cases}$, $7 = (1)(7)$

We find the factors of 6, 7, as follow

$(1)(6) \quad (1)(7) \Rightarrow (1)(1) + (6)(7) = 43$

$(1)(6) + (7)(1) = 13$

$(2)(3) \quad (1)(7) \Rightarrow (2)(1) + (3)(7) = 23$

$(2)(7) + (3)(1) = 17$

The result of multiplying the two parties	$+14x$
The result of multiplying the two middles	$+3x$
The middle term	$+17x$

$6x^2 + 17x + 7 = (2x + 1)(3x + 7)$

ii) $\mathbf{7y^2 - 26y - 8}$

We find the factors of 7, 8, as follow

$8 = \begin{cases} (1)(8) \\ (2)(4) \end{cases}$, $7 = (1)(7)$

The result of multiplying the two parties	$-28y$
The result of multiplying the two middles	$+2y$
The middle term	$-26y$

$(1)(1) - (8)(7) = -55$

$(1)(7) - (8)(1) = -1$

$(2)(1) - (4)(7) = -26$

$(2)(7) - (4)(1) = -10$

$7y^2 - 26y - 8 = (7y + 2)(y - 4)$

Example (5) **Factor each of the following algebraic expressions in a simplest form:**

i) $3z^2 - 17z + 10 = (3z - 2)(z - 5)$ The middle term $-15z - 2z = -17z$

ii) $4v^2 - v - 3 = (4v + 3)(v - 1)$ The middle term $-4v + 3v = -v$

iii) $15 + 11h + 2h^2 = (5 + 2h)(3 + h)$ The middle term $+5h + 6h = 11h$

iv) $6x^2 - 51x + 63 = 3(2x^2 - 17x + 21) = 3(x - 7)(2x - 3)$ The middle term $-3x - 14x = -17x$

v) $3x^2 - 10xy - 3y^2 = (3x - y)(x + 3y)$ The middle term $-9xy - xy = -10xy$

Make sure of your understanding

Factor each of the following algebraic expressions in simplest form:

Questions 1-6 are similar to examples 1,3

1. $x^2 + 6x + 8$
2. $1 - 2z + z^2$
3. $x^2 - 13x + 12$
4. $3 + 2z - z^2$
5. $x^2 - 2x - 3$
6. $15 - 8z + z^2$

Factor each of the following algebraic expressions in simplest form:

Questions 7-14 are similar to examples 4-5

7. $2x^2 + 5x + 3$
8. $3y^2 - 14y + 8$
9. $3x^2 - 10x + 8$
10. $8 - 25z + 3z^2$
11. $5y^2 - y - 6$
12. $6 + 29z - 5z^2$
13. $x^2 - 9xy + 20y^2$
14. $3y^2 - 19yx - 14x^2$

Put the signs between the terms in brackets to make the factoring of the algebraic expression correct:

Questions 15-18 are similar to example 4

15. $x^2 + 9x + 20 = (x…4)(x…5)$
16. $y^2 - 12y + 20 = (y…2)(y…10)$
17. $6x^2 - 7x + 2 = (2x…1)(3x…2)$
18. $20 - 7y - 3y^2 = (5 ... 3y)(4 ... y)$

Solve the Exercises

Factor each of the following algebraic expressions in simplest form:

19. $x^2 + 9x + 14$
20. $y^2 - 5y + 6$
21. $24 - 2z - z^2$
22. $3 + 2z - z^2$
23. $x^2 - 2x - 3$
24. $36 - 15z + z^2$

Factor each of the following algebraic expressions in simplest form:

25. $2x^2 + 12x - 14$
26. $4y^2 - 6y + 2$
27. $10 + 9z - 9z^2$
28. $2x^2 + 3x + 1$
29. $13y^2 - 11y - 2$
30. $50 - 20z + 2z^2$
31. $30x^2 - xy - y^2$
32. $16y^2 - 2yx - 3x^2$
33. $6z^2 - 2zx - 4x^2$

Put the signs between the terms in brackets to make the factoring of the algebraic expression correct:

34. $x^2 + x - 20 = (x…4)(x…5)$
35. $x^2 - x - 56 = (x…7)(x…8)$
36. $35 + 3y - 2y^2 = (5…y)(7…2y)$
37. $3x^2 - 5x + 2 = (x ... 1)(3x ... 2)$

Solve the problems

38 **Al-Ikhidher Castle:** It is an ancient castle locates in Karbala governorate in the middle of Iraq. The ruins of the castle are still existed nowadays. It represents a unique defensive fortress which surrounded by a great rectangular -shaped wall. What are the dimensions of external wall by x , if the castle area with wall represents by $6x^2 - 39x + 60$ square meter .

39 **Entertaining games:** Discovery swing considers one of the dangerous games in the fun city. The expression $5t^2 + 5t - 30$ represents the path of Discovery in the fun city , where (t) represents the time of movemment , and the factoring of expression helps to know the time which the swinging takes in the first time. Factor the expression.

40 **The subway:** The subway considers a system of under ground railway. It is one of the fast means for transportation in the big cities and in those cities which have a high density of population. Each train consists of several vehicles. If the expression $14y^2 - 23y + 3$ represents the ground area of the vehicle in square meter. What are the dimensions of the vehicle by y ?

Think

41 **Challenge:** Factor of the following algebraic expressions in simplest form:

$4x^3 + 4x^2 - 9x - 9$

42 **Correct the mistake:** Sa,ad factored the expression $6z^2 - 16z - 6$, as follow:

$6z^2 - 16z - 6 = (3z - 1)(2z + 6)$

Discover Sa,ad's mistake, then correct it.

43 **Numerical sense:** Can we determine if the signs of the two bracket in factoring the expression $x^2 - 12x + 35$ are different or similar and without factoring the expression ?

Clarify you answer.

Write

The signs between the terms inside brackets to make the factoring of the algebraic expression correct.

$6z^2 + 5z - 56 = (3z \ldots 8)(2z \ldots 7)$

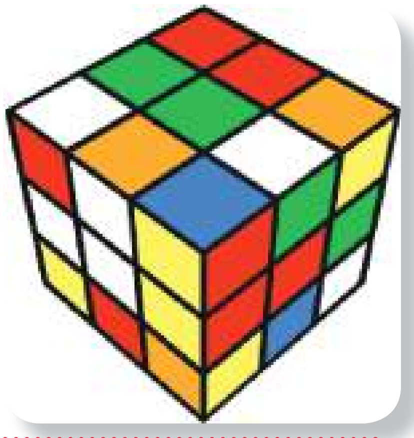
Lesson [2-5] Factoring the Algebraic Expression, sum of two cubes or difference between two cubes.

Idea of the lesson:
* Factoring the algebraic expression of three terms which represents sum (difference between) two cubes.

Vocabulary:
* Sum of two cubes
*Difference between two cubes

Learn

Rubik's cube, which was invented by the Hungarian sculptor and architecture Arnw Rubik in 1974, is a three – dimensions mechanical puzzle. What is the sum volume of two cubus of Rubik's , the length of the first cube side is 3 dcm, and the length of the side of the second cube is 4 dcm?



[2-5-1] Factoring the algebraic expression, sum of two cubes.

You have previously learned in the first lesson of this chapter the multiplication of two –terms algebraic expression by three-terms algebraic expression . The result of their multiplication represents an expression as a sum of two cubes, like $((x + 2)(x^2 - 2x + 4) = x^3 + 8 = x^3 + 2^3)$ Now, you will learn the inverse operation which is factoring the two- terms algebraic expression which represents the sum of two cubes: $x^3 + y^3 = (x + y)(x^2 - x y + y^2)$ where $x = \sqrt[3]{x^3}$, $y = \sqrt[3]{y^3}$

Example (1) What is the sum of two volumes of two cubes of Rubik . The side length of the first cube is 3dcm, while the side length of the second one is 4dcm.

$v_1 + v_2 = 3^3 + 4^3$ The cube volume = length x width x hight = (length of side)^3

$= (3 + 4)(3^2 - 3 \times 4 + 4^2)$ Law of factoring sum of two cubes.

$= 7(9 - 12 + 16) = 7 \times 13 = 91 dcm^3$

Example (2)

Factor each of the following expressions in simplest form:

i) $x^3 + 5^3 = (x + 5)(x^2 - 5x + 5^2) = (x + 5)(x^2 - 5x + 25)$

ii) $y^3 + 8 = y^3 + 2^3 = (y + 2)(y^2 - 2y + 4)$

iii) $8z^3 + 27 = 2^3z^3 + 3^3 = (2z)^3 + 3^3 = (2z + 3)(4z^2 - 6z + 9)$

iv) $\frac{1}{a^3} + \frac{1}{64} = \frac{1}{a^3} + \frac{1}{4^3} = (\frac{1}{a} + \frac{1}{4})(\frac{1}{a^2} - \frac{1}{4a} + \frac{1}{16})$

v) $\frac{27}{x^3} + \frac{8}{125} = \frac{3^3}{x^3} + \frac{2^3}{5^3} = (\frac{3}{x})^3 + (\frac{2}{5})^3 = (\frac{3}{x} + \frac{2}{5})(\frac{9}{x^2} - \frac{6}{5x} + \frac{4}{25})$

vi) $\frac{1}{2} t^3 + 4 = \frac{1}{2}(t^3 + 8) = \frac{1}{2}(t^3 + 2^3) = \frac{1}{2}(t + 2)(t^2 - 2t + 4)$

vii) $0.008 + v^3 = (0.2)^3 + v^3 = (0.2 + v)(0.04 - 0.2v + v^2)$

[2-5-2] Factoring the Algebraic Expression difference between two cubes

You have learned in the first lesson of this chapter the multiplying of an algebraic expression which consists of two terms by another one which consists of three terms
$(x - 3)(x^2 + 3x + 9) = x^3 - 27 = x^3 - 3^3$, and the result of their multiplying represents an expression as a difference between two cubes,like
Now , you will learn the inverse operation which is factoring the algebraic expression which consists of two terms and in the form of difference between two cubes $x^3 - y^3 = (x - y)\ (x^2 + x\ y + y^2)$
where $x = \sqrt[3]{x^3}$, $y = \sqrt[3]{y^3}$.

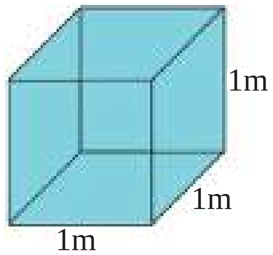
Example (3) Cube –shaped basin filled with water, the length of its side is 1m .
The water was transferred to another cube – shaped basin which is bigger than the first one, the length of its side is 1.1 m.
What is the additional quantity of water thah we need to fill the big basin ?

the additional quantity of water = the volume of big cube - the volume of small cube.

$v_2 - v_1 = (1.1)^3 - 1^3$

$= (1.1 - 1)\ (\ (1.1)^2 + 1.1 \times 1 + 1^2\ )$ Law of factoring the difference between two cubes

$= 0.1\ (1.21 + 1.1 + 1) = 0.1 \times 3.31 = 0.331\ m^3$



Example (4)

Factor each of the following algebraic expressions in simplest form:

i) $x^3 - 3^3 = (x - 3)\ (x^2 + 3x + 3^2) = (x - 3)\ (x^2 + 3x + 9)$

ii) $y^3 - 64 = y^3 - 4^3 = (y - 4)\ (y^2 + 4y + 16)$

iii) $27z^3 - 8 = 3^3z^3 - 2^3 = (3z)^3 - 2^3 = (3z - 2)\ (9z^2 + 6z + 4)$

iv) $\frac{1}{b^3} - \frac{1}{125} = \frac{1}{b^3} - \frac{1}{5^3} = (\frac{1}{b} - \frac{1}{5})(\frac{1}{b^2} + \frac{1}{5b} + \frac{1}{25})$

v) $\frac{1}{3} t^3 - 9 = \frac{1}{3} (t^3 - 27) = \frac{1}{3} (t^3 - 3^3) = \frac{1}{3} (t - 3)\ (t^2 + 3t + 9)$

vi) $0.216 - n^3 = (0.6)^3 - n^3 = (0.6 - n)\ (0.36 + 0.6n + n^2)$

vii) $1 - 0.125\ z^3 = 1 - (0.5)^3 z^3 = (1 - 0.5z)\ (1 + 0.05z + 0.25z^2)$

viii) $32 - \frac{1}{2} m^3 = \frac{1}{2} (64 - m^3) = \frac{1}{2} (4^3 - m^3) = \frac{1}{2} (4 - m)\ (16 + 4m + m^2)$

Make sure of your understanding

Factor each of the following algebraic expressions in simplest form :

Questions 1- 8 are similar to examples 1-2

1. $y^3 + 216$
2. $x^3 + z^3$
3. $125 + 8z^3$
4. $\frac{1}{27}x^3 + \frac{1}{8}$
5. $\frac{1}{a^3} + \frac{1}{64}$
6. $\frac{1}{3}t^2 + 9$
7. $0.125 + v^3$
8. $1 + 0.008z^3$

Factor each of the following algebraic expressions in simplest form :

Questions 9-16 are similar to examples 3-4

9. $a^3 - 8^3$
10. $8y^3 - 64$
11. $\frac{1}{c^3} - \frac{1}{8}$
12. $\frac{1}{2}v^3 - 4$
13. $0.125 - m^3$
14. $25 - \frac{1}{5}n^3$
15. $3b^3 - 81$
16. $0.216v^3 - 0.008t^3$

Solve the Exercises

Factor each of the following algebraic expressions in simplest form :

17. $6^3 + x^3$
18. $27 + 64x^3$
19. $125y^3 + 1$
20. $\frac{1}{64} + \frac{8}{125}y^3$
21. $\frac{1}{5}v^3 + 25$
23. $0.027 + 27n^3$
24. $0.125x^3 + 0.008y^3$

Factor each of the following algebraic expressions in simplest form :

25. $y^3 - 64$
26. $27y^3 - 8$
27. $\frac{1}{x^3} - \frac{27}{8}$
28. $9 - \frac{1}{3}n^3$
29. $0.001 - v^3$
30. $4 - \frac{1}{2}t^3$
31. $25c^3 - \frac{1}{5}$
32. $0.01x^3 - 0.0084y^3$

Solve the problems

33 Library: Shtotgart's library, in Germany, is one of the most beautiful libraries in the world. It is also one of the largest libraries in line with the requirements of the modern education in Germany. The side length of the library building is $\frac{1}{2}y^3 - 13\frac{1}{2}$ meter. Factor the expression which represents the side length.

34 Aquarium: The volume of aquarium of ornamental fish is $25x^3$ cubic meter.
A cube-shaped stone was put inside the aquarium. The size of stone was $\frac{1}{5}$ cubic meter . The aquarium was filled with water . write the expression which represents the volume of water then factor it .

35 Residence : The designs of new house – buildings start to take different shapes which are more complicated in architecture. These houses were designed in shape of cubes If the volume of the first house is $\frac{8}{a^3}$ cubic meter and the second house is $\frac{27}{b^3}$ cubic meter. Write is the volume of the two houses?

Think

36 Challenge: Factor of the following algebraic expressions in simplest form:
$0.002z^3 - 0.016y^3$

37 Correct the mistake: Bushra factored the expression $8v^3 - 0.001$,as follow
$8v^3 - 0.001 = (2v + 0.1)(4v^2 - 0.4v + 0.01)$
Discover Bushra's mistake, then correct it..

38 Numerical sense: Is it possible to add 27 , 8 by using the method of factoring the sum of two cubes ? clarify your answer.

Write

The signs between the terms inside brackets to make the factoring of algebraic expression correct:
$125 - x^3 = (5 \dots x)(25 \dots 5x \dots x^2)$

Lesson [2-6] Simplifying Rational Algebraic Expressions

Idea of the lesson:
*Multiplying and dividing the rational algebraic expressions then write them in simplest form.
*Adding and subtracting the rational algebraic expressions then write them in simplest form.

Vocabulary:
* Ratio
*Fraction

Learn

Hassan had bought a group of flowers bouquets in $x^2 - x - 6$ dinars.
The cost of one bouquet was $2x - 6$ dinars. Write the ratio of one bouquet cost to the total cost of all bouquets, then write it in simplest form.

[2-6-1] Simplifying the multiplying and dividing of rational algebraic expressions

You have previously learned the properties of the rational and real numbers, you have also learned how to simplify the numerical sentences by using the least common multipler (L.C.M) and ordering operations. Now, you will learn how to simplify the rational algebraic expressions (Fractional) by dividing each of numerator and dominator by a common factor, and repeat it so that no way stay for that, and then, we can say that the expression is in a simplest form.

Example (1) Write the cost ratio of one flower bouqued to the total cost of the bouquets in a simplest form.

$$\frac{\text{cost of one bouquet}}{\text{cost of total bouquet}} = \frac{2x - 6}{x^2 - x - 6} = \frac{2(x - 3)}{(x - 3)\,(x + 2)}$$

Factor the numerator and dominator

$$= \frac{2\cancel{(x - 3)}}{\cancel{(x - 3)}\,(x + 2)} = \frac{2}{x + 2}$$

By dividing the numerator and dominator on the common factor

Example (2) **Write each of the following expressions in simplest form:**

i) $\dfrac{x^2 - 4}{(x^2 - 4x + 4)} = \dfrac{(x + 2)\,(x - 2)}{(x - 2)^2} = \dfrac{(x + 2)\,\cancel{(x - 2)}}{(x - 2)\,\cancel{(x - 2)}} = \dfrac{x + 2}{x - 2}$

ii) $\dfrac{5z + 10}{z - 3} \times \dfrac{z^3 - 27}{(z^2 + 6z + 8)} = \dfrac{5\cancel{(z + 2)}}{\cancel{z - 3}} \times \dfrac{\cancel{(z - 3)}\,(z^2 + 3z + 9)}{\cancel{(z + 2)}\,(z + 4)} = \dfrac{5(z^2 + 3z + 9)}{z + 4}$

iii) $\dfrac{16 - x^2}{3x + 5} \times \dfrac{(3x^2 + 2x - 5)}{(x^2 + 3x - 4)} = \dfrac{\cancel{(4 + x)}\,(4 - x)}{\cancel{(3x - 5)}} \times \dfrac{\cancel{(3x + 5)}\,\cancel{(x - 1)}}{\cancel{(x + 4)}\,\cancel{(x - 1)}} = 4 - x$

iv) $\dfrac{8 + t^3}{4 - 2t + t^2} \div \dfrac{(2 + t)^3}{t^2 - 9t + 14} = \dfrac{8 + t^3}{4 - 2t + t^2} \times \dfrac{t^2 - 9t + 14}{(2 + t)^3}$

Multiply the first by the second which is inverted

$= \dfrac{\cancel{(2 + t)}\,\cancel{(4 - 2t + t^2)}}{\cancel{4 - 2t + t^2}} \times \dfrac{\cancel{(t + 2)}\,(t + 7)}{(2 + t)^{\cancel{3}}} = \dfrac{t + 7}{2 + t} = \dfrac{t + 7}{t + 2}$

Factoring the numerator and dominator and dividing by the common factor

[2-6-2] Simplyfing adding and subtracting of the rational algebraic expressions.

You have previously learned how to factor the algebraic expressions and how to find the least common multipler (L.C.M): represents the result of multiplying the common factors by the biggest power and unjoined factors), when simplifying fractional numerical sentences.
Now, you will learn how to simplify the adding and subtracting of the (fractional) rational algebraic expressions by factoring each of the numerator and dominator of the faction to simplest form, then adding and subtracting the fractional expressions by using the common multiplier and simplify the expression to simplest form.

Example (3) Write the rational algebraic expression in simplest form.

$$\frac{y^2}{(y+2)} - \frac{4}{(y+2)}$$

$$= \frac{y^2 - 4}{(y+2)} = \frac{\cancel{(y+2)}\,(y-2)}{\cancel{(y-2)}} = y - 2$$

The least common multipler (x + 2)

Factoring the numerator as a form of a difference between two squares by dividing each of numerator and dominator by y + 2

Example (4) **Write each of the following expressions in simplest form:**

i) $$\frac{7x-14}{x^2-4} + \frac{5}{(x+2)} = \frac{7(x-2)}{(x+2)\,(x-2)} + \frac{5}{x+2}$$

$$= \frac{7}{x+2} + \frac{5}{x+2}$$

$$= \frac{7+5}{x+2} = \frac{12}{x+2}$$

By factoring the numenator and dominator

The least common multipler (x + 2)

ii) $$\frac{4z}{2z-5} - \frac{z}{z+3} = \frac{4z}{2z-5} \times \left(\frac{z+3}{z+3}\right) - \frac{z}{z+3} \times \left(\frac{2z-5}{2z-5}\right)$$

The least common multipler

$$= \frac{4z(z+3) - z(2z-5)}{(2z-5)\,(z+3)} = \frac{2z^2 + 17z}{(2z-5)\,(z+3)} = \frac{z(2z+17)}{(2z-5)\,(z+3)}$$

iii) $$\frac{t^2+2t+4}{t^3-8} + \frac{12}{3t-6} = \frac{t^2+2t+4}{(t-2)\,(t^2+2t+4)} + \frac{12}{3(t-2)} = \frac{1}{(t-2)} + \frac{4}{(t-2)} = \frac{5}{(t-2)}$$

iv) $$\frac{8}{v+4} + \frac{2}{v-4} - \frac{2}{v^2-4} = \frac{8}{v+4} + \frac{2}{v-4} - \frac{1}{(v+4)\,(v-4)} = \frac{8(v-4) + 2(v+4) - 1}{(v+4)\,(v-4)}$$

$$= \frac{8v - 32 + 2v + 8 - 1}{(v+4)\,(v-4)} = \frac{10v - 25}{(v+4)\,(v-4)} = \frac{5\,(2v-5)}{(v+4)\,(v-4)}$$

Make sure of your understanding

Write each of the following expressions in simplest form:

Questions 1-6 are similar to examples 1-2

1. $\dfrac{2z^2 - 4z + 2}{z^2 - 7z + 6}$

2. $\dfrac{y^3 + 27}{y^3 - 3y^2 + 9y}$

3. $\dfrac{5x + 3}{2x + 43} \times \dfrac{x^2 + 5x + 6}{25x^2 - 9}$

4. $\dfrac{z^2 + 7z - 8}{z-1} \times \dfrac{z^2 - 4}{z^2 + 6x - 16}$

5. $\dfrac{x^2 - 9}{x^3 + 4x + 4} \times \dfrac{x^2 - 4}{x^2 - x - 6}$

6. $\dfrac{2y^2 - 2y}{y^2 - 9} \div \dfrac{y^2 + y - 2}{y^2 + 2y - 3}$

Write each of the following expressions in simplest form:

Questions 7-12 are similar to examples 3-4

7. $\dfrac{2}{x^2 - 9} + \dfrac{3}{x^2 - 4x + 3}$

8. $\dfrac{2y^3 - 128}{y^3 + 4y^2 + 16y} - \dfrac{y - 1}{y}$

9. $\dfrac{z^2 + z + 1}{z^4 - z} - \dfrac{z + 3}{z^2 + 2z - 3}$

10. $\dfrac{x^2 - 1}{x^2 - 2x + 1} - 1$

11. $\dfrac{3}{z - 1} + \dfrac{2}{z + 3} + \dfrac{8}{z^2 + 2z - 3}$

12. $\dfrac{y - 3}{y - 1} + \dfrac{5y - 15}{(y - 3)^2} - \dfrac{3y + 1}{y^2 - 4v + 3}$

Solve the Exercises

Write each of the following expressions in simplest form:

13. $\dfrac{x + 5}{12x} \times \dfrac{6x - 30}{x^2 - 25}$

14. $\dfrac{y + 3}{2y^2 + 6y + 18} \times \dfrac{y^3 - 27}{y^2 - 9}$

15. $\dfrac{3 - x}{4 - 2x} \times \dfrac{x^2 + x - 6}{9 - x^2}$

16. $\dfrac{y + 2}{2y - 4} \times \dfrac{y^3 + 8}{y - 2}$

17. $\dfrac{y^2 - 7y}{y^3 - 27} \div \dfrac{y^2 - 49}{y^2 + 3y + 9}$

18. $\dfrac{64 - z^3}{32 + 8z + 2z^2} \div \dfrac{(4 - z)^2}{16 - z^2}$

Write each of the following expressions in simplest form:

19. $\dfrac{5}{x^2 - 36} - \dfrac{2}{x^2 - 12x + 36}$

20. $\dfrac{y^2 - y}{y^3 - 1} - \dfrac{1}{y^2 + y + 1}$

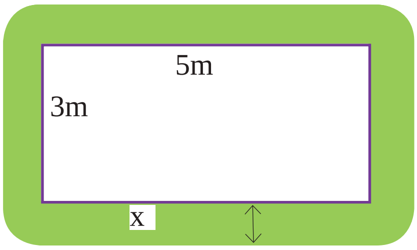
21. $\dfrac{3}{x - 2} - \dfrac{2}{x - 2} - \dfrac{4 + 2x + x^2}{x^3 - 8}$

22. $\dfrac{y - 5}{y + 1} + \dfrac{y -1}{y + 5} - \dfrac{25}{y^2 + 6y + 5}$

Solve the problems

23 Library: If the algebraic expression $x^2 - 4$ represents the number of scientific books in the library, and the algebraic expression $x^2 + x - 6$ represents the number of literary books in the library. Write the ratio of the scientific books to the literary books in a simplest form.

24 Geometry: Dimensions of a rectangle are 3,5 meters. It was extended to bigger one by surrounding it by an aisle with width of x meter.Write the algebraic expression which represents the sum of the two ratios of the rectangle length before and after the extension, in a simplest form.



25 Fireworks: The algebraic expression $20 + 15t - 5t^2$ represents the height, in meters, of a fireworks shell which was shot from a 20 meter-high building roof, where t represents the time of reaching the shell to the target, in seconds. And the algebraic expression $4 + 19t - 5t^2$ represents the height of another shell which was shot from a roof of a 4 meter-high building.
Write the ratio of the first shell height to the height of the second shell, in simplest form.

Think

26 Challenge: Factoring of the following algebraic expression:

$$\frac{y^2 - 5}{2y^3 - 16} \div \frac{y - \sqrt{5}}{(2y^2 + 4y + 8)}$$

27 Correct the mistake: Samah simplified the algebraic expression and wrote it in a simplest form, as follow

$$\frac{z^2 - z - 30}{5 + z} \times \frac{2z + 12}{z^2 - 36} = 1$$

Discover samah's mistake and correct it.

28 Numerical sense:What is the result of adding the two algebraic expressions without using paper and pen ? clarify your answer.

$$\frac{5}{x^2 - 49} \times \frac{-4}{(x - 7)(x + 7)}$$

Write

The value of the algebraic expression in a simplest form:

$$\frac{z^2 - 4}{2z + 8} \div \frac{z^2 - z - 12}{2z^2 + 2z - 12}$$

Lesson [2-7] Problem solving plan (The four steps)

Learn

Idea of the lesson:
*Using stratey of the four steps to solve problem.

The modern buildings take different geometric shapes.In the nearby figure , a hotel in a shape of right cylinder. Its sides were covered with glass. If the radius of the hotel base is x - 8 meter and Its height is x + 12 meter.
What is the lateral area of the hotel?

UNDERSTAND

What are the facts in the problem: A cylinder-shaped hotel, the radius of its base is x - 8 meter and its height is x +12 meter.

What is required? finding the lateral area of the hotel.

PLAN

How can you solve the problem? Since the shape of the hotel is similar to a right cylinder, then we use the law of the lateral area of the vertical cylinder which is :

lateral area = 2 × π × base radius × height (LA = 2 π rh)

SOLVE

$LA = 2\pi \times r \times h$ — **R base radius = x - 8 meter**, **Height = x + 12 meter**

$= 2\pi(x - 8)(x + 12)$ — **Substitute in the law with facts**

$= 2\pi(x^2 + 4x - 96)$ — **Use the multiplying of algebraic expressions**

$= 2\pi x^2 + 8\pi x - 192\pi$ — **Use the property of distribution**

The lateral area of the hotel by the square meters.

CHECK

Use factoring of algebraic expressions to check the correct of solution.

Getting a common factor $LA = 2\pi x^2 + 8\pi x - 192\pi$

$= 2\pi(x^2 + 4x - 96)$

Factoring the algebraic expressions by experiment. $= 2\pi(x - 8)(x + 12)$

That is r = 8 - x , h = x +12, so the solution is correct.

Problems

Solve the following problems by the strategy of (the four steps)

1 Fun city: Some games in the fun city occupy an area larger than it when it is stopping. The swing, for example,occupies a circular area which its diameter is x meter when it is rotating. But when it is stopping the diameter of its area becomes less in about 8 m. what is the difference between the area of stopping and rotating of the swing? then factor it .

2 Panda Bear: The natural home of panda bear is the mountain range of the middle of China. Panda needs a wide region to live in within the zoo. The area which was allocated to panda was extanded about 6m, length and width. So the length of area became 8 + x meter and the width became 4+x meter. What is the measuring of the area which allocates to panda to live in before the extension ?

3 Snowball: It is a transparent sphere which is made of glass to form a landscape. The snowball is also contained water which use as a medium for falling snow. If the radius of the snow sphere is (y - 3) cm. What is the sphere volume?

4 Geometry: A cube-shaped box, its side is x cm. A smaller cube which its side is 3cm put onside the box. Factor the algebraic expressions which represents the difference between the two cubes volume.

Chapter Review

vocabulary

English	عربي	English	عربي
Perfect square	مربع كامل	Square of sum	مربع مجموع
The lost term	الحد المفقود	Square of difference	مربع فرق
The unknown term	الحد المجهول	Cubic of sum	مكعب مجموع
The middle	الوسطين	Cubic of difference	مكعب فرق
The parties	الطرفين	Factoring	تحليل
The middle term	الحد الأوسط	Algebraic expression	مقدار جبري
Sum of two cubes	مجموع مكعبين	Greater common factor	عامل مشترك أكبر
Difference between two cubes	فرق بين مكعبين	Least common multipler	مضاعف مشترك أصغر
Numerator	بسط الكسر	Grouping	تجميع
Dominator	مقام الكسر	Inverse	معكوس
Simplify form	أبسط صورة	Check	تحقق
Divide	يقسم	Correct solution	الحل الصحيح
Multiply	مضاعف	Difference between two squares	فرق بين مربعين
Completing the square	إكمال المربع	Inverse operation	عملية عكسية

[2 - 1] Multiplying Algebraic Expressions

Example (1): **Find the result of multiplying the Following algebraic expressions :**

i) $(x - 3)^2 = x^2 - 6x + 9$

ii) $(\sqrt{2} + z)(\sqrt{2} - z) = 2 - z^2$

iii) $(x - 7)(x^2 + 7x + 49) = x^3 - 343 = x^3 - 7^3$

Exercise (1): **Find the result of multiplying the Following algebraic expressions:**

i) $(z + 6)^2 =$ ....…

ii) $(4x - 3)(4x + 3) =$ ……………..

iii) $(5 + z)(25 - 5z + z^2) =$ ……………..

[2-2]Factoring the Algebraic Expression by using greater common.Factor

Example (1):Factor expression by using the greater common factor , then check the correction of solution:

$4x^2 + 14x - 30 = 2(2x - 3)(x + 5)$

$2(2x - 3)(x + 5) = 2(2x^2 + 7x - 15)$

ckeck: $= 4x^2 + 14x - 30$

Exercise (1):Factor expression by. using the greater common factor , the check the cor

$\sqrt{8}\ x^2z + \sqrt{3}\ (\sqrt{6}\ xz^2 - \sqrt{12}\ xz) =$…

………………………………………

ckeck : ....

[2 - 3] Factoring the Algebraic Expression by using Special Identities.

Example (1): **Factor each of the following algebraic expressions as a differnce between two squares:**

i) $x^2 - 16 = (x + 4)(x - 4)$

ii) $25y^2 - 49 = (5y + 7)(5y - 7)$

Example (2): Factor of the following algebraic expression as a perfect squares:

$x^2 - 12x + 36 = (x)^2 - 2(x \times 6) + (6)^2$

$= (x - 6)(x - 6) = (x - 6)^2$

Exercise (1): **Factor each of the following algebraic expression as a differnce between two squares:**

i) $4x^2 - 49 =$ ……………………

ii) $3x^2 - y^2 =$ ……………………..

Exercise (2): Factoralgebraic expression as a perfect square.

$81z^2 - 18z + 1 =$ ……………………

[2 - 4] Factoring the Algebraic Expression of three terms by Trial and Experiment.

Example (1): **Factor each of the following algebraic expressions in simplest form:**

i) $x^2 - x - 12 = (x - 4)(x + 3)$

$3x - 4x = -x$ The middle term

ii) $y^2 - 8y + 15 = (y - 3)(y - 5)$

$- 3x - 5x = - 8x$ The middle term

Example (2): Factor of the following algebraic expressions in simplest form:

$5x^2 + 13x - 6 = (5x - 2)(x + 3)$

$15x - 2x = 13x$ The middle term

Exercise (1): **Factor each of the following algebraic expressions in simplest form:**

i) $y^2 - y - 20 =$ ....

.... The middle term

ii) $x^2 - 17x + 30 =$.......................

.... The middle term

Exercise (2): Factor algebraic expressions in simplest form:

$7 - 23z + 6z^2 =$ …………………

………………. The middle term

[2 - 5] Factoring the Algebraic Expression as a Sum of two Cubes or a Difference between two Cubes:

Example (1): **Factor each of the following algebraic expressions in simplest form:**

i) $x^3 + 5^3 = (x + 5)(x^2 - 5x + 5^2)$

$= (x + 5)(x^2 - 5x + 25)$

ii) $27z^3 + 8 = (3z)^3 + 2^3$

$= (3z + 2)(9z^2 - 6z + 4)$

iii) $y^3 - 125 = y^3 - 5^3 = (y - 5)(y^2 + 5y + 25)$

Exercise (1): **Factor each of the following algebraic expressions in simplest form:**

i) $x^3 + 27 =$ ……………………………………………

……………………………………………

ii) $8z^3 + 125 =$ ……………………………

……………………………………………

iii) $x^3 - 64 =$ ……………………………………………

……………………………………………

iv) $\frac{1}{z^3} - \frac{1}{27} =$……………………………………

……………………………………………

[2 - 6] Simplify the Rational Algebraic Expressions .

Example (1): **Write each expression in a simplest form:**

i) $\frac{x + 3}{2x - 6} \times \frac{x^3 - 27}{x^2 + 3x + 9}$

$= \frac{x + 3}{2(x - 3)} \times \frac{(x - 3)(x^2 + 3x + 9)}{x^2 + 3x + 9} = \frac{x + 3}{2}$

ii) $\frac{125 + y^3}{25 - 5y + y^2} \div \frac{(5 + y)^3}{y^2 + 10y + 25}$

$= \frac{(5 + y)(25 - 5y + y^2)}{25 - 5y + y^2} \times \frac{(x + 2)^2}{(5 + y)^3} = 1$

iii) $\frac{3x - 15}{x^2 - 25} + \frac{2}{x + 5} = \frac{3(x - 5)}{(x + 5)(x - 5)} + \frac{2}{x + 5}$

$= \frac{3}{x + 5} + \frac{2}{x + 2}$

$= \frac{3 + 2}{x + 5} = \frac{5}{x + 5}$

Exercise (1): : **Write each expression in a simplest form:**

i) $\frac{z^2 - 4}{z + 2} \times \frac{z^2 + 9z + 20}{z^2 + 2z - 8} =$ ……………

……………………………………………

ii) $\frac{27 - x^3}{2x^2 - 6x + 18} \div \frac{(3 - x)^2}{x^2 - x - 6} =$ …………

……………………………………………

iii) $\frac{4z}{2z - 5} - \frac{z}{z + 3} =$ ………………………

……………………………………………

……………………………………………

Chapter Test

Find the result of multiplying an algebraic expression by another algebraic expression, each one of them consists of two terms:

1. $(x + 5)^2$
2. $(v - \sqrt{2})(v + \sqrt{2})$
3. $(2 - x)(5 - x)$
4. $(2y - 3)(y + 9)$

Find the result of multiplying a two terms algebraic expression by another algebraic expression, which consists of three terms:

5. $(x + 11)(x^2 - 11x + 121)$
6. $(\frac{1}{3} - y)(\frac{1}{9} + \frac{1}{3}y + y^2)$
7. $(y - 1)^3$
8. $(z + \frac{1}{4})^3$

Factor the expression by using the grater common factor (GCF) , then check the correction of solution:

9. $8x^2 - 12x$
10. $7y^3 + 14y^2 - 21y$
11. $\sqrt{18}\, z^3r + \sqrt{2}\,(zr^2 - zr)$

Factor the expression by using the binomial as a greatr common factor:

12. $\frac{2}{3}(y + 5) + \frac{1}{3}y(y + 5)$
13. $\sqrt{5}\, z(z^2 - 1) - \sqrt{2}\, z^2(z^2 - 1)$

Factor the expression by using the property of grouping:

14. $6x^4 - 18x^3 + 10x - 30$
15. $56 - 8y + 14y^2 - 2y^3$

Factor the expression by grouping with inverse:

16. $9x^3 - 6x^2 + 8 - 12x$
17. $\sqrt{11}\, z^3 - \sqrt{44}\, z^2 + 5(2 - z)$

Factor each of the following algebraic expressions:

18. $16 - x^2$
19. $\frac{1}{3}z^2 - \frac{1}{27}$
20. $\frac{1}{16}v - \frac{1}{2}v^4$
21. $8x^3 - \frac{1}{125}$
22. $81 - 18y + y^2$
23. $7z^3 - 36z + 5$

Determine which of the following algebraic expressions represent a perfect square, then factor it:

24. $25x^2 + 30x + 9$
25. $49 - 4y + y^2$
26. $4v^2 + 4\sqrt{5}\, v + 5$

Write the missing term in the algebraic expression ax²+bx+c to become a perfect square,then factor it:

27. $x^2 + ... + 81$
28. $36 - 12y + ...$
29. $7 - .... + 4z^2$

Factor each of the following algebraic expressions:

30. $x^2 + 7x + 10$
31. $x^2 - 5\sqrt{3}\, x + 18$
32. $2v^2 + 9v + 7$
33. $32 - 16x + 2x^2$
34. $\frac{1}{4}y^2 - 2y + 3$
35. $12 - 7\sqrt{2}\, v + 2v^2$
36. $8 + 27x^3$
37. $125y^3 - 1$
38. $\frac{1}{v^3} - \frac{8}{27}$
39. $1 + 0.125y^3$
40. $z^3 - 0.027$
41. $3 - \frac{1}{9}v^3$

Write each of the following algebraic expressions in a simplest form:

42. $\frac{27 - 8z^3}{4z^2 - 9} \div \frac{9 + 6z + 4z^2}{9 + 6z}$
43. $\frac{7}{x^2 - 25} - \frac{6}{x^2 + 10x + 25}$
44. $\frac{y^2 - 1}{1 - y^3} + \frac{1 + y}{1 + 2y + y^2}$
45. $\frac{z + 3}{z + 5} - \frac{z - 5}{z - 3} + \frac{1}{z^2 + 2z - 15}$

Chapter 3

Equations

Basil and Sa'ad had travelled in tourist tours by Baghdad International airport. Basil's group was less than sa'ad group in 22 persons. If the number of all travelers is 122 persons. We can calculate the number of each group by solving the two linear equations of first degree $x + y = 122$, $x - y = 22$, where the variable x represents the number of persons in Sa'ad·s group and the variable y represents the number of persons in Basil·s group.

Pretest

Find the result of multiplying an algebraic expression by another algebraic expression, each one consists of two terms:

1 $(y - 5)^2$

2 $(z + 2)(z - 2)$

3 $(x - \sqrt{5})(x + \sqrt{5})$

4 $(4 - y)(6 - y)$

5 $(3z - 2)(z + 8)$

Find the result of multiplying an algebraic expression of two terms by another algebraic expression of three terms:

6 $(x + 3)(x^2 - 3x + 9)$

7 $(\frac{1}{2} - y)(\frac{1}{4} + \frac{1}{2}y + y^2)$

Factor the expression by using the greater common factor (GCF), then check the correction of solution:

8 $5x^2 - 10x$

9 $9y^3 + 6y^2 - 3y$

10 $\sqrt{12}\, z^2 + \sqrt{3}\, z$

Factor the expression by using the binomial as a greater common factor:

11 $x(5 - x) - 3(5 - x)$

12 $\frac{1}{2}(y + 1) + \frac{1}{2}y(y + 1)$

13 $\sqrt{3}\, z(z - 1) - \sqrt{2}(z - 1)$

Factor the expression by grouping:

14 $6x^3 - 12x^2 + 5x - 10$

15 $9 - 18y + 7y^2 - 14y^3$

16 $\sqrt{2}\, z^4 - \sqrt{6}\, z^3 + z - \sqrt{3}$

Factor the expression by grouping with the inverse:

17 $4x^3 - 2x^2 + 9 - 6x$

18 $\frac{3}{4}y^3 - \frac{1}{4}y^2 + 4 - 12y$

19 $\sqrt{4}\, z^3 - \sqrt{25}\, z^2 + 3(5 - 4z)$

Factor each of the following algebraic expressions:

20 $y^2 - 25$

21 $\frac{1}{2}z^2 - \frac{1}{8}$

22 $36 - 12x + x^2$

23 $y^2 - 2y - 15$

Determine which of the following algebraic expressions represents a perfect square, then factor it:

24 $16x^2 + 40x + 25$

25 $64 - 16y + y^2$

26 $z^2 - 6z - 9$

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square, then factor it.

27 $x^2 + \dots + 64$

28 $9 - 24y + \dots$

29 $5 - \dots + 4z^2$

Factor each of the following algebraic expressions:

30 $18 - 3y - y^2$

31 $z^2 - 2\sqrt{3}\, z + 3$

32 $4 - 21x + 5x^2$

33 $1 + 27z^3$

34 $y^3 - 125$

35 $y^3 - \frac{1}{8}$

36 $\frac{1}{x^3} - \frac{1}{64}$

37 $1 - 0.125z^3$

Lesson [3-1] Solving the System of two Linear Equations with two variables.

Idea of the lesson:
*Solve the system of two linear equations graphically and by substitution and elimination.

Vocabulary:
*Linear equation.
*Linear Equations system.
*Solving the system.

Learn

Ahmed has a factory for canning dates. The cost of the empty cans is 100000 dinars. The process of filling one can with dates costs 500 dinars, then it sells in 1000 dinars. Ahmed would like to know the number of cans that he should sell to get profit.

[3-1-1] Solving a System of two Linear Equations graphically.

Assume $\overleftrightarrow{L_1} = a_1x + b_1y = c_1$, $\overleftrightarrow{L_2} = a_2x + b_2y = c_2$ ، represent two equations of first degree (linear) with two variables x,y. To solve this system graphically we follow:

1) Representing each of the two lines in the coordinate plane.

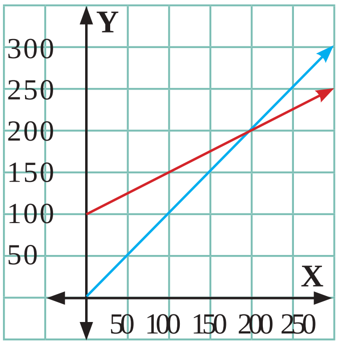
2) Finding a point of intersection the two lines by drawing two colums from the point on the two axis X - axis and Y - axis then the intersection point will represent the solution set.

Example (1)

Finding the number of cans that Ahmed should sell to get profit.

$y = 500x + 100000$ ….. (1)

$y = 1000x$ ……. (2)



Assuming that the costs of the production is the variable (y),and the number of cans which were sold is the variable (x), accordingly :

An equation represents the total production costs.

An equation represents the total value of sales.

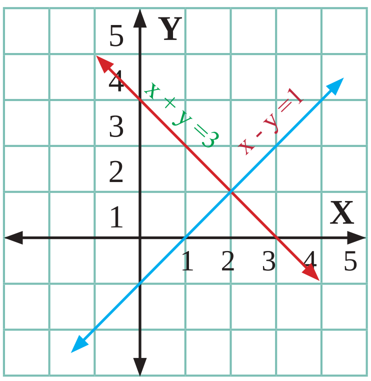
The scale of axis (y) is by thousands of dinars.

We represent the two equations graphically and determine the intersection point of the two lines (200,200) which represents selling 200 cans. The profit starts when more than 200 cans have being sold.

The ordered pair (200,200) which represents a solution for two equations is called a solution of the system.

Example (2)

Find a solution set for the system graphically in R .



$x - y = 1$ ……. (1)

$x + y = 3$ ……. (2)

We represent the two equations graphically and determine the intersection point of the two lines (2, 1).

The solution set of the system is S = { (2,1) }

To check the correction of solution, we substitute the value of the two variables x,y in both equations to get two correct statements.

$x - y = 1 \rightarrow 2 - 1 = 1 \rightarrow 1 = 1$ *Substituting by the equations…(1).*

$x + y = 3 \rightarrow 2 + 1 = 3 \rightarrow 3 = 3$ *Substituting by the equations....(2).*

[3-1-2] Solving two Linear Equations by Substitution method.

We can summarize this method to solve a system of two linear with tow variables x , y by transforming one of the two equations to an equation with only one variable by finding a relation between x,y from one of the two equations, then substituting it in the other equation.

Example (3) **Find the solution set of the system by using the substitution:**

i) $\left.\begin{array}{ll} y = 4x & \ldots\ldots (1) \\ y = x + 6 & \ldots\ldots (2) \end{array}\right\} \Rightarrow 4x = x + 6$

$\Rightarrow 4x - x = 6 \Rightarrow x = 2$

$y = x + 6 \Rightarrow y = 2 + 6 \Rightarrow y = 8$

we Substitute the value y from the equation…(1) in the equation ….(2) solving the equation, then find the variable value x.

Substituting the value of (x) by the equation…(2) to find the variable value (y).

So the solution set for the system is $\{(2 , 8)\}$

ii) $\left.\begin{array}{ll} x + 8y = 10 & \ldots (1) \\ x - 4y = 2 & \ldots\ldots (2) \end{array}\right\} \Rightarrow x = 2 + 4y$

$2 + 4y + 8y = 10 \Rightarrow 12y = 8 \Rightarrow y = \frac{2}{3}$

$x + 8y = 10 \Rightarrow x + 8 \times \frac{2}{3} = 10 \Rightarrow x = 10 - \frac{16}{3} \Rightarrow x = \frac{14}{3}$

We find the value of x from the equation…(2), then Substituting in the equation…(1).

Substituting the value of y by the equation…(1).

So the solution set of the system is $\{(\frac{2}{3}, \frac{14}{3})\}$

[3-1-3] Solving the system of two linear Equations by Elimiation method.

This method can be summarized to solve a system of two equations with two varibles x , y by eliminating one of the two variables through making the coefficient of one of them equaled in value and different in sign in both equations.

Example (4) **Find the solution set for the system by using the elimination method:**

i) $\left.\begin{array}{ll} x + 2y = 5 & \ldots\ldots (1) \\ 3x - y = 1 & \ldots\ldots (2) \end{array}\right\}$

$\Rightarrow \left\{\begin{array}{ll} x + \cancel{2y} = 5 & \ldots\ldots (1) \\ 6x - \cancel{2y} = 2 & \ldots\ldots (2) \end{array}\right.$ by adding

$7x = 7 \Rightarrow x = 1$

$x + 2y = 5 \Rightarrow 1 + 2y = 5 \Rightarrow 2y = 4 \Rightarrow y = 2$

Multiplying the equation…(2) by the number 2 then adding them to the equation….(1).

Substituting the value x in one of two equations (simplest equation)

Substituting in the equation …(1)

So the solution set of the system is $\{(1 , 2)\}$

ii) $\left.\begin{array}{ll} 3x + 4y = 10 & \ldots\ldots (1) \\ 2x + 3y = 7 & \ldots\ldots (2) \end{array}\right\}$

$\Rightarrow \left\{\begin{array}{ll} 6x + 8y = 20 & \ldots\ldots (2) \\ 6x \pm 9y = \pm 21 & \ldots\ldots (1) \end{array}\right.$ by subracting

$y = 1$

$2x + (3 \times 1) = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$

Multiplying the equation…(1) by the number 2 and the equation…(2) by the number3, then we subtract the two equations.

We substitute the value x in the one of the two equations (before changing the sign)

substitute in the equation…(2)

So the solution set of the system is $\{(2 , 1)\}$

Make sure of your understanding

Find a solution set of the system in R graphically:

1. $\left.\begin{array}{r} 3x - y = 6 \\ x - y = 3 \end{array}\right\}$

2. $\left.\begin{array}{r} y - x = 3 \\ y + x = 0 \end{array}\right\}$

3. $\left.\begin{array}{r} y = x - 2 \\ y = 3 - x \end{array}\right\}$

Questions 1-3 are similar to examples 1-2

Find the solution set of the system by using the method of substitution, for each of the following :

4. $\left.\begin{array}{r} 2x + 3y = 1 \\ 3x - 2y = 0 \end{array}\right\}$

5. $\left.\begin{array}{r} x - 2y = 11 \\ 2x - 3y = 18 \end{array}\right\}$

6. $\left.\begin{array}{r} y - 5x = 10 \\ y - 3x = 8 \end{array}\right\}$

Questions 4-6 are similar to example 3

Find the solution set of the system by using the elimination method, for each of the following :

7. $\left.\begin{array}{r} 3x - 4y = 12 \\ 5x + 2y = -6 \end{array}\right\}$

8. $\left.\begin{array}{r} x - 3y = 6 \\ 2x - 4y = 24 \end{array}\right\}$

9. $\left.\begin{array}{r} 3y - 2x - 7 = 0 \\ y + 3x + 5 = 0 \end{array}\right\}$

Questions 7-9 are similar to example 4

Find the solution set of the system, then check the e correction of the solution:

10. $\left.\begin{array}{r} \frac{2x}{3} - \frac{y}{2} = 1 \\ \frac{3y}{3} - \frac{x}{3} = 4 \end{array}\right\}$

11. $\left.\begin{array}{r} 0.2x - 6y = 4 \\ 0.1x - 7y = -2 \end{array}\right\}$

12. $\left.\begin{array}{r} \frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4} \\ \frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4} \end{array}\right\}$

Solve the Exercises

Find the solution set of the system graphically:

13. $\left.\begin{array}{r} x - y = -4 \\ y + x = 6 \end{array}\right\}$

14. $\left.\begin{array}{r} y = x - 4 \\ x = 2 - y \end{array}\right\}$

Find the solution set of the system by using the method of substitution, for each of the following:

15. $\left.\begin{array}{r} 3x + 2y = 2 \\ x - y = 8 \end{array}\right\}$

16. $\left.\begin{array}{r} 2x - y = -4 \\ 3x - y = 3 \end{array}\right\}$

Find the solution set of the system by using the method of elimination, for each of the following:

17. $\left.\begin{array}{r} 3x = 22 - 4y \\ 4y = 3x - 14 \end{array}\right\}$

18. $\left.\begin{array}{r} 5x - 3y = 6 \\ 2x + 5y = -10 \end{array}\right\}$

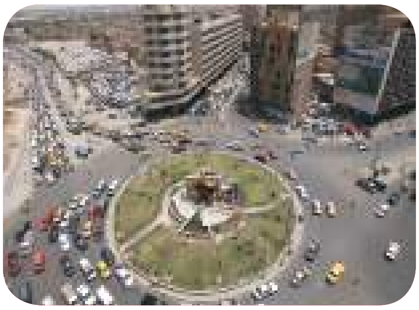
Find the solution set of the system, then check the correction of the solution:

19. $\left.\begin{array}{r} \frac{x}{3} - \frac{y}{3} = 2 \\ 2x + 3y = 6 \end{array}\right\}$

20. $\left.\begin{array}{r} 0.2x - 3y = 3 \\ 0.1x - 6y = -3 \end{array}\right\}$

Solve the problems

21 **Weather :** During January, the number of days (x) in which the temperature goes down in Baghdad decreases about 10 Celsius , is nearly less in 9 days than the number of days (y) in which the temperature goes up about 10 Celsius. Write two equations represent this situation, then find their solution by using the method of elimination to find the number of days in each situation.



22 **Trade :** A commercial shop had sold 25 fridges and washing machines. The price of one fridge was million dinars while price of one washer was 500000 dinars. If the total cost of fridges and washers was 20millions dinars, then how many appliances did the seller sell from each type?
Write two equations represent the problem, then solve them by using the substitution method.

23 **Graduation party :** Sajad and Anwer held a party on the occasion of their graduation from college. The number of friends who were invited by Sajad is more in three from the friends who were invited by Anwer. The total number of friends who came to the party is 23 persons. How many persons did each of Sajad and Anwer invite? write two equations represent the problem , then solve them to find the required.

Think

24 **Challenge:** Find the solution set for the system:

$$\left.\begin{array}{l} \frac{2}{6}x - \frac{1}{3}y = 1 \\ \frac{1}{2}x + \frac{1}{2}y = 3 \end{array}\right\}$$

25 **Correct the mistake:** Ahmed said that the solution set of the system:

Is the set $\{(\frac{5}{16}, \frac{5}{9})\}$

$$\left.\begin{array}{l} 2x + 3y = 6 \\ 3x + 2y = 1 \end{array}\right\}$$

Discover Ahmed's mistake, then correct it.

Write

Asolution set for the system : $5x - 6y = 0$
$x + 2y = 4$

Lesson [3-2] Solving Quadratic Equations with one variable

Idea of the lesson:
*Solving the equation which. consists of two terms by factoring the difference between two squares.

Vocabulary:
*Equation
*Second degree
*One variable
*Difference between two squares

Learn

Zaqura is one of the Iraqi civilized landmarks. It sites in south of Iraq. Basil drew a square-shaped picture its area is 9 m^2.
Find the side length of the picture.

[3-2-1] Using difference between two squres to solve equations

The general equation of second degree with one variable $ax^2 + bx + c = 0$ where, $a \neq 0$ and $a,b,c \in R$ and solving it means finding a values set of the variable (x) which satisfies the equation , by making it correct statement.We will study in this item solving the equations consist of two terms by using the greater common factor, difference between two squares and the property of zero-product.

Example (1) Write an equation represents the area of picture, then solve it to find the side length of the picture.

Assume that the side length of picture is the variable (x) and the equation which represents the area of picture is: $x^2 = 9$

$x^2 - 9 = 0 \Rightarrow \boxtimes \Rightarrow (x + 3)(x - 3) = 0$ *Factoring by using the difference between two squares*

$\Rightarrow \; x + 3 = 0$ or $x - 3 = 0$ *property of zero-product*

$\Rightarrow \; x = -3$, neglect *The wall picture length is 3 m*

Example (2) solve the following equation by using the difference between two squares, then check the correct of solution.

$16 - y^2 = 0 \Rightarrow (4 + y)(4 - y) = 0$ *Factoring by using the difference between two squares*

$4 + y = 0 \Rightarrow$ or $4 - y = 0 \Rightarrow y = -4$ or $y = 4 \Rightarrow S = \{-4 , 4\}$ *solution set.*

Check: each value in the solution set for the variable (y) must satisfies the equation

$L.S = 16 - y^2 = 16 - (-4)^2 = 16 - 16 = 0 = R.S$ *By substitution y = -4*

$L.S = 16 - y^2 = 16 - 4^2 = 16 - 16 = 0 = R.S$ *By substitution y = 4*

Example (3) **Solve the following equations by using the difference between two squares:**

i) $4x^2 - 25 = 0 \Rightarrow (2x + 5)(2x - 5) = 0 \Rightarrow 2x + 5 = 0$ or $2x - 5 = 0$

$\Rightarrow x = \frac{-5}{2}$ or $x = \frac{5}{2} \Rightarrow s = \{\frac{5}{2} , \frac{-5}{2}\}$

ii) $3z^2 - 12 = 0 \Rightarrow 3(z^2 - 4) = 0 \Rightarrow (z + 2)(z - 2) = 0$ *By dividing the two parties on 3, then by factoring.*

$\Rightarrow z + 2 = 0$ or $z - 2 = 0 \Rightarrow s = \{-2 , 2\}$

iii) $2y^2 - 6 = 0 \Rightarrow y^2 - 3 = 0 \Rightarrow (y + \sqrt{3})(y - \sqrt{3}) \Rightarrow y = -\sqrt{3}$ or $y = \sqrt{3} \Rightarrow s = \{-\sqrt{3} , -\sqrt{3}\}$

iv) $x^2 - 5 = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0 \Rightarrow x = -\sqrt{5}$ or $x = \sqrt{5} \Rightarrow s = \{-\sqrt{5} , -\sqrt{5}\}$

v) $(z + 1)^2 - 36 = 0 \Rightarrow (z + 1 + 6)(z + 1 - 6) = 0 \Rightarrow (z + 7)(z - 5) = 0 \Rightarrow s = \{-7 , 5\}$

[3-2-2] **Using Square** root Property to Solve the equations root

You have learned in the previous item how to solve the equation of second degree with one variable by the factoring method using the difference between two squares. Now, we will find the solution set for the second –degree equation with one variable by using the method of square root property .

$\sqrt{x^2} = |x| \geq 0$

$25 = 5^2 \Rightarrow \sqrt{25} = \sqrt{5^2} = |5| = 5$

$25 = (-5)^2 \Rightarrow -\sqrt{(-5)^2} = |-5| = 5$

In general form : If a is positive real number then $x^2 = a \Rightarrow x = \pm\sqrt{a}$

Example (4) Solve the following equation by using the property of square root, then check the correction of solution:

$x^2 = 9 \Rightarrow x = \pm\sqrt{9} \Rightarrow x = \pm 3$ *By using the property of the square root*

$\Rightarrow S = \{3, -3\}$ *Solution set of the equation*

Check: Each value in the solution set of the variable (x) must satisfy the equation

$L.S = x^2 = 3^2 = 9 = R.S$ *By subtitution $x = 3$*

$L.S = x^2 = (-3)^2 = -3 \times -3 = 9 = R.S$ *By subtitution $x = -3$*

Example (5) **Solve the following equation by using the square root rule:**

i) $y^2 = 36 \Rightarrow y = \pm\sqrt{36} \Rightarrow x = \pm 6 \Rightarrow S = \{6, -6\}$

ii) $z^2 = \frac{9}{25} \Rightarrow z = \pm\sqrt{\frac{9}{25}} \Rightarrow z = \pm\frac{3}{5} \Rightarrow S = \{\frac{3}{5}, -\frac{3}{5}\}$

iii) $x^2 + 81 = 0 \Rightarrow x^2 = -81$ There is no solution for this equation in the real numbers
(there is no a real number has a negative square)

iv) $3y^2 = 7 \Rightarrow y^2 = \frac{7}{3} \Rightarrow y = \pm\sqrt{\frac{7}{3}} \Rightarrow y = \pm\frac{\sqrt{7}}{\sqrt{3}} \Rightarrow S = \{\frac{\sqrt{7}}{\sqrt{3}}, -\frac{\sqrt{7}}{\sqrt{3}}\}$

v) $4x^2 - 5 = 0 \Rightarrow 4x^2 = 5 \Rightarrow x^2 = \frac{5}{4} \Rightarrow x = \pm\sqrt{\frac{5}{4}} \Rightarrow x = \pm\frac{\sqrt{5}}{2} \Rightarrow S = \{\frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}\}$

Notice: If the two sides of a correct equation were quadrted, then the resulted equation stills correct $(y = x \Rightarrow y^2 = x^2)$ example $\sqrt{x} = 5 \Rightarrow (\sqrt{x})^2 = 5^2 \Rightarrow x = 25$

The inverted is not correct $x^2 = y^2 \not\Rightarrow y = x$

Example (6) **Solve the following equations :**

i) $3\sqrt{x} = 18 \Rightarrow \sqrt{x} = 6 \Rightarrow (\sqrt{x})^2 = 6^2 \Rightarrow x = 36 \Rightarrow S = \{36\}$ *By squared the both sides of the cquation*

ii) $\sqrt{y+8} = 3 \Rightarrow (\sqrt{y+8})^2 = 3^2 \Rightarrow y + 8 = 9 \Rightarrow y = 9 - 8 \Rightarrow y = 1 \Rightarrow S = \{1\}$

iii) $\sqrt{5z} = 7 \Rightarrow (\sqrt{5z})^2 = 7^2 \Rightarrow 5z = 49 \Rightarrow z = \frac{49}{5} \Rightarrow S = \{\frac{49}{5}\}$

iv) $\sqrt{\frac{x}{13}} - 1 = 0 \Rightarrow \sqrt{\frac{x}{13}} = 1 \Rightarrow (\sqrt{\frac{x}{13}})^2 = 1^2 \Rightarrow \frac{x}{13} = 1 \Rightarrow x = 13 \quad S = \{13\}$

Make sure of your understanding

Solve the following equations by using the difference between two squares, then check the correction of solution:

Questions 1-3 are similar to example 2

1 $x^2 - 16 = 0$ 2 $81 - y^2 = 0$ 3 $2z^2 - 8 = 0$

Solve the following equations by using the difference between two squares:

Questions 4-9 are similar to example 3

4 $4x^2 - 9 = 0$ 5 $5y^2 - 20 = 0$

6 $(y + 2)^2 - 49 = 0$ 7 $(3 - z)^2 - 1 = 0$

8 $x^2 - 3 = 0$ 9 $y^2 - \frac{1}{9} = 0$

Solve the following equations by using the rule of the square root:

Questions 10-15 are similar to example 4

10 $x^2 = 64$ 11 $z^2 = 7$

12 $2y^2 = \frac{49}{8}$ 13 $6z^2 - 5 = 0$

14 $4(x^2 - 12) = 33$ 15 $z^2 + \frac{2}{3} = \frac{5}{9}$

Questions 16-18 are similar to example 5

Solve the following equations:

16 $3\sqrt{x} = 15$ 17 $\sqrt{y - 5} = 2$ 18 $\sqrt{2z} = 6$

Solve the Exercises

Solve the following equations, then check the correction of solution:

19 $x^2 = 49$ 20 $5y^2 - 10 = 0$ 21 $3z^2 - 27 = 0$

Solve the following equations in R by using the difference between two squares:

22 $9x^2 - 36 = 0$ 23 $7y^2 - 28 = 0$

24 $9(x^2 - 1) - 7 = 0$ 25 $(y + 5)^2 - 64 = 0$

26 $x^2 - 2 = 0$ 27 $y^2 - \frac{1}{36} = 0$

Solve the following equations by using the rule of the square root:

28 $x^2 = 121$ 29 $50 - 2y^2 = 0$

30 $x^2 = \frac{1}{64}$ 31 $3y^2 = \frac{25}{3}$

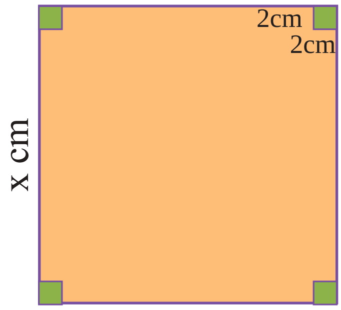
32 $7(x^2 - 2) = 50$ 33 $\frac{1}{5}y^2 = \frac{1}{3}$

Solve the following equations:

34 $6\sqrt{x} = 30$ 35 $\sqrt{y - 9} = 4$ 36 $\sqrt{4z} = 8$

Solve the problems

37 Carpets: A rectangular- shaped carpet, its length is 12 m and width 3m. It was divided into parts to cover the floor of a square- shaped room. write the equation which represent the problem and find the side length of the room?



38 Geometry: A piece of cardboord which was square-shaped, its side length is x cm. four equaled equaled pieces were cut fromits four corners.The side length of each square is 2cm. It was folded to form a box without cover. which was a rectangular parallele surfaces-shaped box, its volme 32 cm^3 write the equation which represent the problem and. Find the side length of the origin cardboord.

39 Fountain: A square- shaped swimming pool was designed in the center of a square-shaped garden. It's side length is 3m.The remained area of the garden which was surrounded the pool was $40m^2$. write the equation which represent the problem and the side length of the garden?

Think

40 Challenge: **Solve the following equations:**

i) $9(x^2 + 1) = 34$ ii) $4x^2 - 3 = 0$

41 **Does the given set represent the solution set for the equation or not?**

i) $(2y + 1)^2 = 16$, $\{\frac{3}{\sqrt{2}} , -\frac{3}{\sqrt{2}}\}$ ii) $3x^2 - 7 = 0$, $\{\frac{7}{\sqrt{3}} , -\frac{7}{\sqrt{3}}\}$

42 Correct the mistake:Salah said that the set ($\{\frac{4}{\sqrt{5}} , -\frac{4}{\sqrt{5}}\}$) represents the solution set for the equation $5x^2 = 4$ Discover Salah's mistake and correct it.

43 Numerical sense: A positive integer consists of one digit, If one was subtracted from it's square, the result would be a number from the multiplying of ten.What is the number?

Write

The solution set for the equation:
$(8 - 3y)^2 - 1 = 0$

Lesson [3-3] Using Probe and Error to Solve the Quadratic Equations(Experiment)

Idea of the lesson:
*Solving the equations of the second degree which consist of three terms by factoring in experiment.

Vocabulary:
*Quadratic equation
*Experiment

Learn

If the length of the basketball court increases in about 2m more than the double of its width, and its area is 480m².
Find the two dimensions of the court.

[3-3-1] Solving the equation $x^2 + bx + c = 0$.

You have previously learned how to find the factoring of an algebraic expression which consists of three terms by experiment. Now, you will use the factoring to solve the equations of second degree which consist of three terms $x^2 + bx + c = 0$ where b , c are real numbers (factoring the expression to two brackets with two different signs or two similar signs according to the sign of the absolute term and the middle term).

Example (1) Finding the two dimensions of a basketball court.

Assume that the width of the court is the variable x, so the length of the court will be 2x+2.

Court area = length × width

$x(2x + 2) = 480 \Rightarrow x^2 + 2x - 480 = 0 \Rightarrow x^2 + x - 240 = 0$

$$\Rightarrow \begin{cases} (x + 16)(x - 15) = 0 \\ x + 16 = 0 \Rightarrow x = -16 \end{cases}$$

or $x - 15 = 0 \Rightarrow x = 15$

The middle term $-15x + 16x = x$.

We neglect it because there is no negative length.

So the width of the court is 15m and the length is $2 \times 15 + 2 = 32$m

Example (2) **Solve the following equations by factoring in experiment:**

i) $x^2 - 7x + 12 = 0 \Rightarrow (x - 3)(x - 4) = 0 \Rightarrow x = 3$ or $x = 4 \Rightarrow S = \{3 , 4\}$

ii) $y^2 + 8y + 15 = 0 \Rightarrow (y + 3)(y + 5) = 0 \Rightarrow y = -3$ or $y = -5 \Rightarrow S = \{-3 , -5\}$

iii) $z^2 + z - 30 = 0 \Rightarrow (z + 6)(z - 5) = 0 \Rightarrow z = -6$ or $z = 5 \Rightarrow S = \{-6 , 5\}$

iv) $x^2 - 2x - 63 = 0 \Rightarrow (x - 9)(x + 7) = 0 \Rightarrow x = 9$ or $x = -7 \Rightarrow S = \{9 , -7\}$

v) What is the number which its square increases in 12?

Assume that the number is x, then the square of the number will be x^2, and the numerical sentence which represents the problem is

$x^2 - x = 12 \Rightarrow x^2 - x - 12 = 0 \Rightarrow (x - 4)(x + 3) = 0 \Rightarrow x = 4$ or $x = -3$

So, the number is either 4 or -3

[3 -3-2] Solving the equation $ax^2 + bx + c = 0$, $a \neq 0$

You have previously learned how to solve an equation by experiment method and the variable x^2 without cofficients. Now, you will learn how to solve the same equation but with existence of cofficients for the variable x^2.

Example (3) **A swimming pool which its length is less in three times of its width in 1 m. If the area of the swimming pool is 140m2, find its dimensions.**

Assume that the width of the swimming pool is the variable x so the length of the pool is $3x - 1$

The equation which represents the problem is

We solve the equation $x(3x-1) = 140$:

$x(3x-1) = 140 \Rightarrow 3x^2 - x - 140 = 0$

$\Rightarrow (3x + 20)(x - 7) = 0$ *The middle term* $-21x + 20x = -x$

$\Rightarrow 3x + 20 = 0 \Rightarrow x = -\frac{20}{3}$ We neglect it because there is no negative length

or $x - 7 = 0 \Rightarrow x = 7$

So the width of the swimming pool is 7m and its length is 20m.

Example (4) **Solve the following equations by factoring in experiment:**

i) $4y^2 - 14y + 6 = 0 \Rightarrow (4y - 2)(y - 3) = 0$ $-12y - 2y = -14y$ *The middle term*

$\Rightarrow \begin{cases} 4y - 2 = 0 \Rightarrow y = \frac{1}{2} \\ \text{or } y - 3 = 0 \Rightarrow y = 3 \Rightarrow S = \{\frac{1}{2}, 3\} \end{cases}$

ii) $3x^2 - x - 21 = 0 \Rightarrow (3x - 3)(x + 7) = 0$ $21x - 3x = 18x$ *The middle term*

$\Rightarrow \begin{cases} 3x - 3 = 0 \Rightarrow x = 1 \\ \text{or } x + 7 = 0 \Rightarrow x = -7 \Rightarrow S = \{1, -7\} \end{cases}$

iii) $20 + 13z + 2z^2 = 0 \Rightarrow (4 + z)(5 + 2z) = 0$ $8z + 5z = 13z$ *The middle term*

$\Rightarrow \begin{cases} 4 + z = 0 \Rightarrow z = -4 \\ \text{or } 5 + 2z = 0 \Rightarrow z = -\frac{5}{2} \Rightarrow S = \{-4, -\frac{5}{2}\} \end{cases}$

iv) $9x^2 - 69x - 24 = 0 \Rightarrow 3(3x^2 - 23x - 21) = 0 \Rightarrow 3x^2 - 23x - 21 = 0$

$\Rightarrow (3x + 1)(x - 8) = 0$ $-24x - x = -23x$ *The middle term*

$\Rightarrow \begin{cases} 3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \\ \text{or } x - 8 = 0 \Rightarrow x = 8 \Rightarrow S = \{-\frac{1}{3}, 8\} \end{cases}$

Make sure of your understanding

Solve the following equations by factoring in experiment:

Questions 1-6 are similar to example 2

1. $x^2 - 9x + 18 = 0$
2. $x^2 - 4x - 32 = 0$
3. $y^2 + 48y - 49 = 0$
4. $y^2 + 9y - 36 = 0$
5. $x^2 - 3x + 2 = 0$
6. $y^2 - 8y - 33 = 0$

Questions 7-9 are similar to example 1

7. What is the number which its square is more greater than its double in 35?
8. What is the number that if we add its fourfold to it's square, the result will be 45?
9. A carpet ,its length is more than its width in about 2m and its area is $48m^2$, what are the dimensions of the carpet?

Questions 10-14 are similar to example 4

10. $15x^2 - 11x - 14 = 0$
11. $6 + 7x - 5x^2 = 0$
12. $42 + 64y + 24y^2 = 0$
13. $36 - 75x + 6x^2 = 0$
14. $70 - 33y + 4y^2 = 0$

Questions 15 is similar to example 3

15. A rectangular- shaped land, its length is more than its width in 4m, what are the two dimensions of the land if its area is $60m^2$?

Solve the Exercises

Solve the following equations by factoring in experiment:

16. $x^2 - 15x + 56 = 0$
17. $y^2 + 16y + 63 = 0$
18. $x^2 + 15x - 16 = 0$
19. $y^2 - y - 42 = 0$
20. $x^2 - 4x + 3 = 0$
21. $y^2 - 6y - 55 = 0$
22. A rectangular- shaped metal, its width decreases in 2m from its length. What are the two dimensions of the piece of metal? If its area is $24m^2$?
23. $12x^2 - 20x + 7 = 0$
24. $28 + 2z - 8z^2 = 0$
25. $81 - 9x - 12x^2 = 0$
26. $50z^2 + 10z - 4 = 0$
27. A dining hall, its length less from the twice of its width in 3m and its area is$54m^2$, what are the dimensions of the hall?

Find the solution set for the following equations, then check the correction of solution:

28. $x^2 - 4x + 3 = 0$
29. $y^2 - 9y - 36 = 0$
30. $4 - 26x + 12x^2 = 0$
31. $80 - 38y + 3y^2 = 0$

Solve the problems

32 Sport: If the length of a picture of football stadium increases in 4m more than the twice of its width, its area was $160m^2$.
What are the two dimensions of the picture?

33 Field of ostriches: If the length of a field for breeding ostriches decreases in 4m than the twice of its width. If its area was $96m^2$, will a 44m length fence be enough to surround the field?

24 Picture frame: Samir bought a picture frame, its length is twice of its width.
Samir needs to shorten the frame in 2cm from both length and width to make it suitable for picture. What are the dimensions of the frame which Samir bought if the picture area is $40\ cm^2$?

Think

35 Challenge: **Solve the following equations by factoring in experiment.**

i) $(x - 3)(x + 2) = 14$ ii) $3y^2 - 11y + 10 = 80$

36 **Clarify,Does the given set represents a solution set for the equation or not?**

i) $4x^2 + 2x = 30$, $\{\frac{-2}{5}, 3\}$ ii) $42 - 33y + 6y^2 = 0$, $\{2, \frac{7}{2}\}$

37 Correct the mistake: Rana said that the solution set for the equation $2x^2 - 34x + 60 = 0$ is $\{3, 15\}$.

Determine Rana's mistake, then correct it.

Write

An equation represents the following problem, then find its solution:

What is the integer number which its square is less than its twice in 35?

Lesson [3-4] Solving the Quadratic Equations by Perfect square.

Learn

Idea of the lesson:
*Solving the quadratic equations by method of perfect square.

Vocabulary:
*First term
*Last term
*Perfect square
*Completing the square

Jaguar (Panthera onca) is a kind of tigers.
The equation $x^2 - 20x + 100 = 0$ represents a square region area in square meters which is allocated to the tiger inside a zoo.
What is the expression which represented the side length of the squared area ?

[3-4-1] Solving the equations by the Perfect square.

You have previaosly learned how to factor the algebraic expression which is as a perfect square. Now, we will use the factoring in solving equations by factoring the complete square to find the solution set of the equation.

Example (1) What is the expression which represented by the side length of the square area?

$x^2 - 20x + 100 = 0$ To factor the left side of the equation, we should be sure that the expression represents a perfect square.

$2(x + 10)$ A perfect square because: the middle term $=2\times$ (the first term root $\times$ the last term root).

$x^2 - 20x + 100 = 0 \Rightarrow (x - 10)^2 = 0 \Rightarrow (x - 10)(x - 10) = 0$ Factoring of the expression

$\Rightarrow x - 10 = 0 \Rightarrow x = 10$

or $x - 10 = 0 \Rightarrow x = 10$

So the side length of the square region area which is allocated to tiger is 10m .

Example (2) **Solving the following equations by the complete square:**

i) $4x^2 + 20x + 25 = 0$ The middle term $2 \times (2x \times 5) = 20x$

$\Rightarrow (2x + 5)^2 = 0 \Rightarrow 2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$ We take one of the repeated factors

ii) $y^2 - y + \frac{1}{4} = 0$ The middle term $2 \times (yx \times \frac{1}{2}) = y$

$\Rightarrow (y - \frac{1}{2})^2 = 0 \Rightarrow y - \frac{1}{2} = 0 \Rightarrow y = \frac{1}{2}$ We take one of the repeated factors

iii) $3 - 3\sqrt{3}\,z + 9z^2 = 0$ The middle term $2 \times (\sqrt{3} \times 3z) = 6\sqrt{3}\,z$

$\Rightarrow (\sqrt{3} - 3z)^2 = 0 \Rightarrow \sqrt{3} - 3z = 0 \Rightarrow 3z = \sqrt{3}$ We take one of the repeated factors

$z = \frac{1}{\sqrt{3}}$

[3-4-2] Solving Equations by Completing the square

Now, you will learn how to solve an equation of second degree by completing the square:

1- We put the quadratic equation as follow: $ax^2 + bx = -c$ ، where $a \neq 0$

2- If $1 \neq a$, the equation will be divided by a.

3- We add the expression (quadrate of the half factor x) to the two sides of the equation.

4- We factor the left side which becomes a perfect square after step 3 , then we simplify the right part.

5- We take the square root for the two sides, then we find the values of x.

Example (3) **Solve the following equations by completing the square:**

i) $x^2 - 4x - 12 = 0 \Rightarrow x^2 - 4x = 12$ we write the equation as in the first step

$\Rightarrow x^2 - 4x + 4 = 12 + 4$ Adding the expression $(\frac{1}{2} \times -4) = 4$ to the two sides

$\Rightarrow x^2 - 4x + 4 = 16 \Rightarrow (x-2)^2 = 16$ of the equation.

$$\Rightarrow x - 2 = \pm 4 \Rightarrow \begin{cases} x - 2 = 4 \Rightarrow x = 6 \\ \text{or } x - 2 = -4 \Rightarrow x = -2 \end{cases} \Rightarrow S = \{6, -2\}$$

We take the square root of the two sides of the equation

ii) $2y^2 - 3 = 3y \Rightarrow 2y^2 - 3y = 3$ We write the equation as in the first step

$\Rightarrow y^2 - \frac{3}{2}y = \frac{3}{2}$ By dividing the two sides of the equation by 2

$\Rightarrow y^2 - \frac{3}{2}y + \frac{9}{16} = 3 + \frac{9}{16}$ Adding the expression$(\frac{1}{2} \times -\frac{3}{2})^2 = \frac{9}{16}$ to the two sides of the equation by factoring the left side and simplify the right side of the equation.

$\Rightarrow (y - \frac{3}{4})^2 = \frac{33}{16}$

$\Rightarrow y - \frac{3}{4} = \pm \frac{\sqrt{33}}{4}$ We take the square root of the two sides of the equation.

$$\Rightarrow \begin{cases} y - \frac{3}{4} = \frac{\sqrt{33}}{4} \Rightarrow y = \frac{3 + \sqrt{33}}{4} \\ \text{or} \\ y - \frac{3}{4} = -\frac{\sqrt{33}}{4} \Rightarrow y = \frac{3 - \sqrt{33}}{4} \end{cases} \Rightarrow S = \left\{ \frac{3 - \sqrt{33}}{4}, \frac{3 + \sqrt{33}}{4} \right\}$$

Example (4) The length of a rectangle is greater than its width in 2cm estimate, the length and width of the rectangle by nearing to the nearest integer n if its area was $36cm^2$.

$x(x + 2) = 36$ Assume that the rectangular width is the variable x, then the rectangle length is x+2.

The equation which represent the problem:

$\Rightarrow x^2 + 2x = 36$

We solve the equation by completing the square

$\Rightarrow x^2 + 2x + 1 = 36 + 1$ We add $(\frac{1}{2} \times 2)^2 = 1$ to the two sides of the equation . neglect $x \approx -1$

$\Rightarrow (x + 1)^2 = 37 \Rightarrow x + 1 = \pm\sqrt{37} \Rightarrow x + 1 \approx \pm 6 \Rightarrow x + 1 \approx 6 \Rightarrow x \approx 5$

Note : $\sqrt{37} \approx \sqrt{36} = 6$ $\Rightarrow x + 1 \approx -6 \Rightarrow$ neglect $x \approx -7$

So the approximate width of the rectangle is 5 cm and its length is 7 cm.

Make sure of your understanding

Solve the following equations by the perfect square:

1. $x^2 + 12x + 36 = 0$
2. $y^2 - 10y + 25 = 0$
3. $4x^2 - 4x + 1 = 0$
4. $y^2 + 2\sqrt{7}\,y + 7 = 0$
5. $x^2 + 16x = -64$
6. $\frac{1}{16} - \frac{1}{2}x + x^2 = 0$

Questions 1- 6 are similar to example 2

Solve the following equations by complete square :

7. $x^2 - 10x - 24 = 0$
8. $y^2 - 3 = 2y$
9. $4x^2 - 3x - 16 = 0$
10. $3y^2 + 2y = 1$
11. $x^2 - \frac{6}{5}x = \frac{1}{5}$
12. $5y^2 + 15y - 30 = 0$

Questions 7-12 are similar to example 3

Solve the Exercises

Solve the following equations by the perfect square:

13. $x^2 + 12x + 144 = 0$
14. $y^2 - 20y + 100 = 0$
15. $y^2 + 4\sqrt{2}\,y + 8 = 0$
16. $7 - 2\sqrt{7}\,z + z^2 = 0$
17. $3y^2 + 36 - 12\sqrt{3}\,y = 0$
18. $9z^2 - 10z + \frac{25}{9} = 0$

Solve the following equations by complete square

19. $y^2 + 2\sqrt{3}\,y = 3$
20. $4z^2 - 12z - 27 = 0$
21. $x^2 - 2x = 0$
22. $y^2 - 8y = 24$
23. $x^2 - \frac{2}{3}x = 4$
24. $8y^2 + 16y - 64 = 0$

Solve the following equations by complete square, then find the result by nearing to the nearest integer:

25. $x^2 - 6x = 15$
26. $y(2y + 28) = 28$
27. $z^2 - 10z + 10 = 0$

Solve the problems

28 **Babylon city:** It is Babylon in Latin. It is an Iraqi city which sites nearby the river of Euphrates. It was the capital of Babylonians during the reign of Hamoraby (1750-1792) BC. Find the from the equation $x^2 - 28x + 196 = 0$ which represents the lenght of the side of one of the square – shaped hall.

39 **Panda bear:** The area which was allocated to the Panda bear in a zoo is a rectangular- shaped area. which is 126 m^2 . its width is less than the length in 8 m.find the dimensions of the allocated area for panda by nearing to the nearest integer .

30 **Whales:** Some whales are swimming in groups towards the beach and no one know why because there is no scientific illustration to this phenomena. Those who interested in protecting the environment try to return them to the sea. Solve the equation $x^2 + 20x = 525$ by the method of completing the square to find the value of x which represents the number of whales which swam toward the beach in Australia.

Think

31 **Challenge: Solve the following equations by completing the square, then find the result by nearing to the nearest integer :**

i) $4x(x - 6) = 27$ ii) $6y^2 - 48y = 6$

32 **Correct the mistake:** Sawsen solved the equation $4x^2 - 4\sqrt{3}x + 3 = 0$ by the method of completing the square, then wrote the solution set for the equation as follows $S = \{\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}\}$.

Discover Sawsen's mistake and correct it.

33 **Numerical sense:** Does the solution set of the equation $y^2 - 4y + 4 = 0$ contains two equaled values in the expression which one of them is positive and the other is negative? Clarify your answer.

Write

The solution set for the equation:

$\frac{1}{81} - \frac{2}{9}z + z^2 = 0$

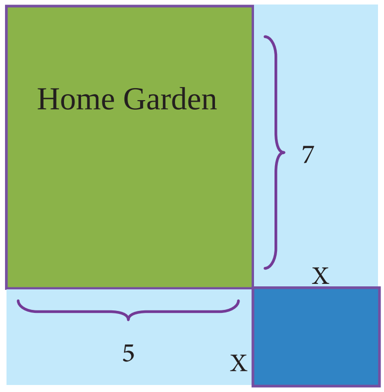
Lesson [3-5] Using General Law to Solve Equations.

Idea of the lesson:
* Solving the linear equtions from the second degree by using the general law.

Vocabulary:
* Coefficient
* Absolute term
* General law

Learn

An aisle was required to be paved on the two sides of a home garden. It was paved with ceramics. The length of the garden is 7m and its width is 5m. And the area of paving is 45 m^2 find the width of the aisle which was required to be paved with ceramics.



[3-5-1] Solve the Equation by Using the Law $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and $a \neq 0$.

You have previously learned how to solve an equation of second degree by many methods, but there are equations which cannot be solved by the previous methods, so we will solve them by the general law (constitution) by finding the real roots for the quadratic equation, as follow:
1) We put the quadratic equation in the general form (standard) $ax^2 + bx + c = 0$.
2) We write the values of coefficients : a coefficient x^2, b coefficient x with its sign, c represents the absolute term with its sign.

Supstitutiong by the general law to find the two values of the variable.

Example (1) From learn paragraph , what is the width of the aisle which needs to be paved on the two sides of the garden?

Assume that the width of the aisle is x, then the area of the right part of the aisle equals 7x, the area of the front part =5x , the area of the aisle angle = x^2 and the sum of the two area of the paving is $45m^2$.

$x^2 + 7x + 5x = 45 \Rightarrow x^2 + 12x = 45$ The equation which represents the problem

$x^2 + 12x - 45 = 0$ Put the equation in the general form

$a = 1 \ , \ b = 12 \ , \ c = -45$ Determine the coefficients and substituting, in the general law

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-12 \pm \sqrt{144 - 4\times1\times(-45)}}{2\times1} \Rightarrow x = \frac{-12 \pm \sqrt{144 + 180}}{2}$$

$$\Rightarrow x = \frac{-12 \pm \sqrt{324}}{2} \Rightarrow x = \frac{-12 \pm 18}{2} \Rightarrow \begin{cases} x = \frac{-12 + 18}{2} \Rightarrow x = 3 \\ \text{or } x = \frac{-12 - 18}{2} \Rightarrow x = -15 \end{cases}$$

The width of the aisle is3m

$x = -15$ Neglect because it is impossible

Example (2) Find the solution set for the following equations by using the general law:

$x^2 - 3x - 5 = 0 \ , \ a = 1 \ , \ b = -3 \ , \ c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{3 \pm \sqrt{9 + 20}}{2} \Rightarrow x = \frac{3 \pm \sqrt{29}}{2} \Rightarrow \begin{cases} x = \frac{3 + \sqrt{29}}{2} \\ \text{or } x = \frac{3 - \sqrt{29}}{2} \end{cases} \Rightarrow S = \left\{\frac{3 + \sqrt{29}}{2}, \frac{3 - \sqrt{29}}{2}\right\}$$

[3-5-2] The Discriminative ($\Delta = b^2 - 4ac$)

In the first part of this lesson, you have learned how to solve the equation by the general law to find the real roots of the equation. Now, we will talk about the discriminate of the quadratic equation $ax^2 + bx + c = 0$ which is $\Delta = b^2 - 4ac$

and the type of the two roots of the equation determines as follow:

Roots type :	$\Delta = b^2 - 4ac$.
1- Two rational real roots.	1)positive and perfect square (prameter rational number
2- Two irr rational real roots.	2)Positive and not a complete square
3- Two equaled real roots ($\frac{-b}{2a}$).	3)Zero
4-Two irrational real roots. (the solution set in R = Ø)	4)Negative

Example (3) **Determine the equation roots, firstly, then find the solution set if it is possible:**

i) $2x^2 + 3x - 2 = 0$, $a = 2$, $b = 3$, $c = -2$

The discriminative expression is a perfect square that means the equation has two rational roots.

$\Delta = b^2 - 4ac \Rightarrow \Delta = 9 - 4 \times 2 \times (-2) = 25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-3 \pm \sqrt{9 + 16}}{4} \Rightarrow x = \frac{-3 \pm \sqrt{25}}{4} \Rightarrow x = \frac{-3 + 5}{4} = \frac{1}{2} \text{ or } x = \frac{-3 - 5}{4} = -2$$

ii) $y^2 - 4y - 9 = 0$, $a = 1$, $b = -4$, $c = -9$

The discriminative expression is not a perfect square, so the equation has two irrational roots.

$\Delta = b^2 - 4ac \Rightarrow \Delta = 16 - 4 \times 1 \times (-9) = 52$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{4 \pm \sqrt{16 + 36}}{2} \Rightarrow x = \frac{4 \pm \sqrt{52}}{2} \Rightarrow \{x = 2 + \sqrt{13} \text{ or } x = 2 - \sqrt{13}\}$$

iii) $z^2 + 8z = -16 \Rightarrow z^2 + 8z + 16 = 0$, $a = 1$, $b = 8$, $c = 16$

The discriminative expression is zero , that means the equation has two equaled real roots.

$\Delta = b^2 - 4ac \Rightarrow \Delta = 64 - 4 \times 1 \times 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-8 \pm \sqrt{64 - 64}}{2} = -4$$

iv) $x^2 - 2x + 10 = 0$, $a = 1$, $b = -2$, $c = 10$

The discriminative expression is negative, therefore the equation doesn't have a solution in R.

$\Delta = b^2 - 4ac \Rightarrow \Delta = 4 - 4 \times 1 \times 10 = -36$

Example (4) What is the value of the constant (K) which makes the two roots of the equation $x^2 - (k+1)x + 4 = 0$ equaled? Check your answer.

The two roots of the equation will be equaled when the value of the discriminative expression

Δ equals zero.

$a = 1$, $b = -(k+1)$, $c = 4$ Determine the values of the coefficients

$\Delta = b^2 - 4ac \Rightarrow \Delta = (k+1)^2 - 4 \times 1 \times 4 \Rightarrow \Delta = (k+1)^2 - 16$

$\Delta = 0 \Rightarrow (k+1)^2 - 16 = 0$

We substitute the value of the discriminant by zero because the two roots of the equation are equal to the root of the two sides of the equation.

$\Rightarrow (k+1)^2 - 16 = 0 \Rightarrow (k+1)^2 = 16$

$$\Rightarrow k + 1 = \pm 4 \Rightarrow \begin{cases} k + 1 = +4 \Rightarrow k = 3 \\ \text{or } k + 1 = -4 \Rightarrow k = -5 \end{cases}$$

Check:

We substitute by the value K= 3 in the original equation, then we find the roots of the equation:

$x^2 - (k+1)x + 4 = 0 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x - 2)^2 = 0 \Rightarrow x = 2$

We substitute by the value K=-5 in the original equation, then we find the equation roots..

$x^2 - (k+1)x + 4 = 0 \Rightarrow x^2 + 4x + 4 = 0 \Rightarrow (x + 2)^2 = 0 \Rightarrow x = -2$

Make sure of your understanding

Find the solution set for the following equations by using the general law:

Questions 1-6 are similar to examples 1-2

1. $x^2 - 4x - 5 = 0$
2. $y^2 + 5y - 1 = 0$
3. $3x^2 - 9x = -2$
4. $4y^2 + 8y = 6$
5. $4x^2 - 12x + 9 = 0$
6. $2y^2 - 3 = -5y$

Determine the roots of equation at firstly, then find the solution set if it is possible:

Questions 7-10 are similar to example 3

7. $2x^2 + 3x = 5$
9. $y^2 - 8y - 12 = 0$
8. $3x^2 - 7x + 6 = 0$
10. $y^2 + 12 = -9y$

Questions 11-14 are similar to examples 3-4

11. What is the value of the constant (k) which makes the two roots of the equation $x^2 - (k + 2)x + 36 = 0$ equaled? Check your answer.
12. What is the value of the constant (K)) which makes the two roots of the equation $4y^2 + 25 = (k - 5)y$ equaled? Check your answer.
13. What is the value of the constant (K)) which makes the two roots of the equation $z^2 + 16 = (k + 4)z$ equaled? Check your answer.
14. Show that the equation $z^2 - 6z + 28 = 0$ doesn't have a solution set in real numer.

Solve the Exercises

Find the solution set for the following equations by using the general law:

15. $x^2 - 7x - 14 = 0$
16. $y^2 + 3y - 9 = 0$
17. $2x^2 - 8(3x + 2) = 0$
18. $2y^2 - 2 = -10y$

Determine the equation roots at firstly, then find the solution set in R if it is possible:

19. $x^2 + 4x = 5$
20. $y^2 - 2y - 10 = 0$
21. $2x^2 - 5x + 7 = 0$
22. $y^2 - 14y + 49 = 0$
23. What is the value of the constant (K) which makes the two roots of the equation $x^2 - (k + 6)x + 49 = 0$ equaled? Check your answer.
24. What is the value of the constant (K) which makes the two roots of the equation $4y^2 + 36 = (k - 6)y$ equaled? Check your answer.
25. What is the value of the constant (K) which makes the two roots of the equation $z^2 + 81 = (k + 9)z$ equaled? Check your answer.
26. Show that the equation $2z^2 - 3z + 10 = 0$ doesn't have a solution set in real numer.

Solve the problems

27 Fireworks: In one of the occasions, a group of fireworks was shot vertically, they reached acertain height of 140m. Calculate the time (t) second in which the fireworks reached up to that height, if the following equation $5t^2 + 60t = 140$.

28 Trade: Samir calculates the cost of one men's suit, then he adds amount of profit and sell it in 120,000 dinars. If (P) in the equation $p^2 - 30p + 225 = 0$ represents the amount of Samir's profit in one suit which is in thousands of dinars. What is the cost of one suit?

Think

29 Challenge: **Determine the roots of the equation at firstly, then find the solution set if it is possible:**

i) $x^2 + 8x = 10$

ii) $3y^2 - 6y - 42 = 0$

30 Correct the mistake:Sa'ad said that the equation $2x^2 - 3x - 9 = 0$ doesn't have a solution in the set of the real numbers.
Discover sa·ad·s mistake and correct it.

31 Numerical sense:Marrwa used the discriminative expression for writing the two roots of the equation $z^2 - 8z + 16 = 0$ without factoring. illustrate how Marrwa was able to write the two roots of the equation.

Write

The type of the two roots of the equation $x^2 + 100 = 20x$ by using the discriminative expression without solving it.

Lesson [3-6] Solving the Fractional Equations

Idea of the lesson:
*Solving the fractional equations of second degree.

Vocabulary:
* Numerator
*Denominator
*Fractional equation

Learn

If the price of a masterwork is $2x + 3$ thousands dinars, and the price of buying 6 pieces of masterworks is $x^2 + 3x - 1$ thousands dinars.So if the ratio of one masterwork price to the price of three masterworks is $\frac{1}{3}$, what is the price of buying one masterwork?.

You have previously learned how to simplify the fractional algebraic expressions by dividing both the numerator and denominator by a common factor. Now you will use the factoring of algebraic expressions to solve fractional equations which have a variable in its denominator by get rid of fractions, then solve them by using one of the methods that you have previously learned.

Example (1) Write the price of buying one masterwork.

$$\frac{\text{one master work price}}{\text{three master work price}} = \frac{1}{3} \Rightarrow \frac{2x+3}{(x^2+3x-1)} = \frac{1}{3}$$

Simplify the fraction by multiplying the two sides by the two middles.

$$\Rightarrow x^2 + 3x - 1 = 6x + 9 \Rightarrow x^2 - 3x - 10 = 0$$

Simplify the equation for factoring.

$$\Rightarrow (x-5)(x+2) = 0 \Rightarrow \begin{cases} x - 5 = 0 \Rightarrow x = 5 \\ \text{or } x + 2 = 0 \Rightarrow x = -2 \end{cases}$$

Neglect because there is no price in negative

Then the price of buying one masterwork is($2x + 3 = 13$) 13000 dinars.

Example (2) Find the solution set for the following equation, then check the correction of the solution.

$$5x + \frac{x-2}{3x} = \frac{2}{3}$$

We multiply the two sides of the equation by the least common multiple (LCM) to get rid of fractions

$$3x(5x) + 3x\left(\frac{x-2}{3x}\right) = 3x\left(\frac{2}{3}\right) \Rightarrow 15x^2 + x - 2 = 2x \Rightarrow 15x^2 - x - 2 = 0$$

Simplify the equation

$$\Rightarrow (3x+1)(5x-2) = 0$$

Factoring by experiment

$$\Rightarrow \left\{\begin{array}{l} 3x + 1 = 0 \Rightarrow x = \frac{-1}{3} \\ \text{or } 5x - 2 = 0 \Rightarrow x = \frac{2}{5} \end{array}\right\} \Rightarrow S = \left\{\frac{-1}{3}, \frac{2}{5}\right\}$$

Solution set

Check:Substituting by the original equation when $x = \frac{-1}{3}$

$$LHS = 5\left(\frac{-1}{3}\right) + \frac{\frac{-1}{3} - 2}{3 \times \frac{-1}{3}} = \frac{-5}{3} + \frac{-1}{-3} + 2 = \frac{-5}{3} + \frac{1}{3} + 2 = \frac{-5+1+6}{3} = \frac{2}{3} = RHS$$

it is also easy to check when $x = \frac{2}{5}$ (leave it to students)

You have previously learned how to simplify the adding of the (fractional) relative algebraic expressions and subtract them by factoring each of the numerator and denominator of the fraction to simplest form, then doing the operation of adding and subtracting the fractional expressions by using the least common multiple and simplify the expressions to the simplest form. Now, you will use that to solve the fractional equations to find the solutions set of the fractional equation.

Example (3) Find the solution set for the equation:

$$\frac{x}{x-3} + \frac{4x}{x+3} = \frac{18}{x^2-9}$$

Factoring the denominators to the possible

$$\Rightarrow \frac{x}{x-3} + \frac{4x}{x+3} = \frac{18}{(x-3)(x+3)}$$

simplest form by multiplying the two sides of the equation by LCM (x-3) (x+3)

$$\Rightarrow x(x+3) + 4x(x-3) = 18$$

Simplifying the equation and solve it to find the values of the variable.

$$\Rightarrow x^2 + 3x + 4x^2 - 12x - 18 = 0 \Rightarrow 5x^2 - 9x - 18 = 0$$

$$\Rightarrow (5x+6)(x-3) = 0 \Rightarrow x = -\frac{6}{5} \text{ or } x = 3$$

Note:

We have to exclude the values which make the denominator of any fractional term from the original equation terms, zero because it leads to divide by zero and that considers impossible.

So we exclude x=3 from the solution because ($\frac{x}{x-3} = \frac{3}{0}$), and the solution will be only $x = -\frac{6}{5}$

Check:Substituting by the original equation $x = -\frac{6}{5}$ to see if the two sides of the equation are equaled or not ?

$$LHS = \frac{x}{x-3} + \frac{4x}{x+3} = \frac{-\frac{6}{5}}{-\frac{6}{5}-3} + \frac{4 \times \frac{-6}{5}}{-\frac{6}{5}+3} = \frac{6}{21} - \frac{8}{3} = -\frac{50}{21}$$

$$RHS = \frac{18}{x^2-9} = \frac{18}{(-\frac{6}{5})^2 - 9} = \frac{18}{(\frac{36}{25}) - 9} = -\frac{450}{189} = -\frac{50}{21}$$

LHS = RHSF

So the value $x= -\frac{6}{5}$ satisfy the equation

Example (4) Find the solution set for the equation:

$$\frac{2}{x+2} - \frac{x}{2-x} = \frac{x^2+4}{x^2-4}$$

Before multiplying the two sides of the equation by LCM for the denominators we try to

$$\Rightarrow \frac{2}{x+2} + \frac{x}{x-2} = \frac{x^2+4}{(x+2)(x-2)}$$

factor the denominator of the fraction for the right side and change 2-x = - (x-2)

by using the information a-b= - (b-a)

$\Rightarrow 2(x-2) + x(x+2) = x^2 + 4$ By multiplying the two sides of the equation by LCM(x + 2) (x - 2)

$$\Rightarrow 2x - 4 + x^2 + 2x - x^2 - 4 = 0 \Rightarrow 4x - 8 = 0 \Rightarrow x = 2$$

When we substitute x=2 in the original equation , we get a process of dividing by zero and it is impossible ($\frac{x}{2-x} = \frac{6}{5}$), so the equation doesn't have a solution in the real number set (R), that means the solution set in R is an empty set (Ø).

Make sure of your understanding

Find the solution set for each of the following equations and check the correct of solution:

Questions 1-6 are similar to examples 1-2

1. $\frac{1}{x} + \frac{1}{2} = \frac{6}{4x^2}$

2. $\frac{y}{2} - \frac{7}{5} = \frac{3}{10y}$

3. $\frac{x+4}{2} = \frac{-3}{2x}$

4. $\frac{y+1}{y^2} = \frac{3}{4}$

5. $\frac{9x-14}{x-5} = \frac{x^2}{x-5}$

6. $\frac{1}{y^2-6} = \frac{2}{y+3}$

Find the solution set for each of the following equations:

Questions 7-10 are similar to examples 3-4

7. $\frac{y-4}{y+2} - \frac{2}{y-2} = \frac{17}{y^2-4}$

8. $\frac{9}{x^2-x-6} - \frac{5}{x-3} = 1$

9. $\frac{12}{y^2-16} + \frac{6}{y+4} = 2$

10. $\frac{3y}{y-1} + \frac{2y}{y-6} = \frac{2x^2-15y+20}{y^2-7y+6}$

Solve the Exercises

Find the solution set for each of the following equations and check the correct of solution:

11. $\frac{4}{6x^2} + \frac{1}{3} = \frac{1}{x}$

12. $\frac{3y}{4} - \frac{6}{12y} + \frac{1}{4} = 0$

13. $\frac{9x+22}{x^2} = 1$

13. $\frac{9}{(y+2)^2} = \frac{3y}{y+2}$

Find the solution set for each of the following equations:

15. $\frac{3}{x-4} - \frac{2}{x-3} = 1$

16. $\frac{y-5}{y+5} - \frac{y+5}{y-5} = \frac{4y^2-24}{y^2-25}$

17. $\frac{6-x}{x^2+x-12} - \frac{2}{x+4} = 1$

18. $\frac{4+8y}{y^2-9} + \frac{6}{y-3} = 3$

Solve the problems

19 Sports: If a cyclist wanted to cut a distance of 60 km between the two cities A and B in certain speed. If his speed increases in about 10 km/h , then he will be able to cut this distance in an hour less than the first time. Find his speed at first.

20 Transporting passengers: One of the Iraqi airlines planes cuts a distance of 350 km from Baghdad to Erbil in a certain speed. If the speed of the plane increases in 100 km/h, then the plane will be able to cut the distance in time which will be less in 12 minutes from the first time. Find approximate speed of the plane at first.

21 Racing: Nawfel participated in triple race which includes swimming, riding bicycles and running, and he took two hours to finish the race, as shown in the nearby table, considering x represents Nawfel's speed average in swimming. Find the average of his approximate speed in swimming racing .

	Distance km	Speed h/km	Time
Swimming	$d_s = 1$	x	t_s
Riding bicycles	$d_b = 20$	$5x$	t_b
running	$d_r = 4$	$x + 4$	t_r

Note: use the equation of total time which Nawfel spent in the race, in term of his speed in swimming is $T(x) = t_s + t_b + t_r$

Think

22 Challenge: Find the solution set for each of the following equations:

$$\frac{3}{x+5} + \frac{4}{5-x} = \frac{x^2 - 15x + 14}{x^2 - 25}$$

23 Correct the mistake: Nammeer used the discriminative expression to show the roots of equation,

$$\frac{2}{x-7} \times \frac{1}{x-1} = 1$$

he said that the equation has two relative real roots. Discover Nammeer's mistake and correct it.

Write

The solution set in real numer .

$$\frac{1}{x+6} - \frac{5}{x-6} = 2$$

Lesson [3-7] problem solving plan (Writing Equation)

Learn

Idea of the lesson:
*Using the strategy of writing an equation to solve problem.

A ship cuts a distance of 240 km between the port A and the port B in certain speed. If its speed increases in 10 km/h, It will be able to cut the distance in time which is less in 2 hours from the first time.
Find the speed of ship at first.

UNDERSTAND

What is the data in the problem: A ship cuts a distance of 240 km between the two cities A and B in certain speed. It cuts the same distance in time which is less in two hours from the first time if the speed of the ship increases in 10 km/h.

What is wanted in the problem: Finding the speed of ship at first.

PLAN

How can you solve the problem? Write an equation represents the problem, then solve it to find the speed of liner at first.

SOLVE

Assume that the first speed of ship = v , The first time = $\frac{240}{v}$

So its second speed = v + 10 , the second time = $\frac{240}{v + 10}$

$\frac{240}{v} - \frac{240}{v+10} = 2$ **The first time – the second time equals 2**

By multiplying the two sides of the equation by LCM v(v + 10)

$240v + 2400 - 240v = 2v(v + 10)$

$2400 = 2v^2 + 20v$

$v^2 + 10v - 1200 = 0 \Rightarrow (v + 40)(v - 30) = 0$

$\Rightarrow \begin{cases} v + 40 = 0 \Rightarrow v = -40 & \text{Neglect} \\ \text{or } v - 30 = 0 \Rightarrow v = 30 \text{ km/h} & \text{The speed of ship at first} \end{cases}$

CHECK

The first time of the ship = $\frac{240}{v} = \frac{240}{30} = 8$ h

The second time of the ship = $\frac{240}{v + 10} = \frac{240}{40} = 6$ h

The second time of ship is less than its first time in about two hours (8-6=2h), so the solution is correct.

Problems

Solve the following problems by the strategy of (writing an equation)

1 Fountain: Asquare–shaped area was planted with flowers in the middle of a square- shaped garden of a hotel, its side length is 4m. The remaining area which surrounds it is 84 m^2. What is the side length of the garden?

2 Babylon lion: It is a statue which was found in Babylonian archeological city in Iraq in 1776.It was made of the solid black basalt stone. It locates on a base in the middle of a rectangular- shaped area which its length is greater than its width in 2m and its area is 15 m^2. What are its dimensions?

3 Lion: Lion is one of the strongest animals on earth. It is the king of forest according to its strength. If the equation $x^2 - 30x$ represents the area which is under the control of the lion.What is the side length of the area? which represnted by x , if the area 175 km^2 ?

4 Fireworks: In one occasion, a group of fireworks was shot vertically. They reached up to 200m height. Calculate the time in which the fireworks reached up to that height if the following equation $2t^2 + 30t = h$ represents the relation between height, in meters (h), in which the fireworks reach after t second.

Chapter Review

vocabulary

English	عربي	English	عربي
Coefficient	معامل	Linear equation	معادلة خطية
Absolute term	الحد المطلق	System of equations	نظام معادلات
Absolute value	القيمة المطلقة	Solution set	مجموعة الحل
General law	القانون العام	Quadratic equation	معادلة تربيعية
Discriminative Expression	المقدار المميز	Factoring	تحليل
Nominator	بسط الكسر	One variable	متغير واحد
Dominator	مقام الكسر	Analysis by experiment	التحليل بالتجربة
Right side	الطرف الأيمن	First term	الحد الأول
Left side	الطرف الأيسر	The middle term	الحد الأوسط
Fractional equation	معادلة كسرية	The last term	الحد الأخير
Plan	خطة	Perfect square	مربع كامل
Problem	مسألة	Completing a square	إكمال مربع
Linear equations system	أنظمة معادلات خطية	Difference between two squares	فرق بين مربعين
		Greater common factor	مضاعف مشترك أصغر

[3 - 1] Solving a system of two Linear Equations in two variables

Example: Find the solution set for the system by using the elimination for each of the following:

$x + 3y = 7$ ... (1) By adding the two equations

$x - 3y = 1$ ... (2)

$2x = 8 \Rightarrow x = 4$ Substituting x in one of the two equations

$x + 3y = 7 \Rightarrow 4 + 3y = 7 \Rightarrow y = 1$

So, the solution set for system is $\{(4 , 1)\}$

Exercise: Find the solution set of the system by using the elimination for each of the following.

$x + y = 2$ .... (1)

$x + 5y = 4$ ... (2)

....

....

[3 - 2] Solving Quadratic Equations with one variable.

Example (1): Solve the following equation by the difference between two squares.

$25 - x^2 = 0$

$\Rightarrow (5+x)(5-x) = 0 \Rightarrow 5 + x = 0$ or $5 - x = 0$

$\Rightarrow x = -5$ or $x = 5$

$S = \{-5 , 5\}$ Solution set

Example (2): Solve the following equation by the property of the square root.

$y^2 = \frac{16}{25} \Rightarrow y = \pm\sqrt{\frac{16}{25}} \Rightarrow y = \pm\frac{4}{5}$

$S = \{\frac{4}{5} , -\frac{4}{5}\}$ Solution set

Exercise (1): Solve the following equation by the difference between two squares.

$x^2 - 64 = 0$

....

....

Exercise (2): Solve the following equation by the property of the square root.

$y^2 = 49$

....

....

[3 - 3] Using Prob. and Error to Solve the quadratic Equation.

Example (1): Solve the following equation by factoring in experiment

$x^2 - 2x - 15 = 0$

Since the sign of the absolute term is negative, then the sign of the two brackets is different, and the negative sign is for the greatest.

$\Rightarrow (x - 5)(x + 3) = 0 \quad x = 5$ or $x = -3$

$\Rightarrow S = \{5, -3\}$ Solution set

Example (2): Solve the following equation by factoring in experiment.

$3y^2 - 11y + 5 = 0 \Rightarrow (3y - 5)(y - 2) = 0$

the middle term.

$-6y - 5y = -11y$

$$\Rightarrow \left\{\begin{array}{l} 3y - 5 = 0 \Rightarrow y = \frac{5}{3} \\ \text{or } y - 2 = 0 \Rightarrow y = 2 \end{array}\right\}$$

$\Rightarrow S = \{\frac{5}{3}, 2\}$ Solution set

Exercise (1): Solve the following equation by factoring in experiment

$x^2 - 10x + 21 = 0$

....

....

....

Exercise (2): Solve the following equation by factoring in experiment

$4y^2 + 16y - 9 = 0$

....

....

....

[3 - 4] Solving the Quadratic Equations by the Perfect Square.

Example (1): Solve the following equation by the perfect square

$9x^2 - 36x + 36 = 0$

the middle term.

$\Rightarrow (3x - 6)^2 = 0 \quad 3x - 6 = 0 \quad 2 \times (3x \times 6) = 36x$

$\Rightarrow 3x = 6 \Rightarrow x = 2$ We take one of the repeated roots

Example (2): Solve the equation by the method of completing the square

$x^2 - 6x = 27$

Add the expression $(\frac{1}{2} \times -6)^2 = 9$ to the two sides of the equation.

$\Rightarrow x^2 - 6x + 9 = 27 + 9$

$\Rightarrow x^2 - 6x + 9 = 36 \Rightarrow (x - 3)^2 = 36$

We take the square root for the two sides of the equation.

$\Rightarrow (x - 3) = \pm\sqrt{36} \Rightarrow x - 3 = \pm 6$

$$\Rightarrow \left\{\begin{array}{l} x - 3 = 6 \Rightarrow x = 9 \\ \text{or } x - 3 = -6 \Rightarrow x = -3 \Rightarrow S = \{9, -3\} \end{array}\right.$$

Exercise (1): Solve the following equations in the perfect square.

$4x^2 - 28x + 49 = 0$

....

....

Exercise (2): Solve the equation by the method of completing the square.

$x^2 - 12x = 28$

....

....

....

....

[3 - 5] Using General Law to Solve Equations.

Example (1):Find the solution set for the equation by using the general law.

$x^2 - 5x - 7 = 0$, $a = 1$, $b = -5$, $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{5 \pm \sqrt{25 + 28}}{2}$$

$$x = \frac{5 \pm \sqrt{53}}{2} \Rightarrow x = \frac{5 + \sqrt{53}}{2} \text{ or } x = \frac{5 - \sqrt{53}}{2}$$

$$S = \left\{ \frac{5 + \sqrt{53}}{2} , \frac{5 - \sqrt{53}}{2} \right\}$$

Example (2): Determine the roots of the equation

$3x^2 + 5x - 2 = 0$, $a = 3$, $b = 5$, $c = -4$

$\Delta = b^2 - 4ac \Rightarrow \Delta = 25 - 4 \times 3 \times (-2) = 49$

The distinctive expression is a perfect square that means the equation has two rational roots.

Exercise (1): Find the solution set for the equation by using the general law:

$x^2 - 3x - 8 = 0$

....

....

....

Exercise (2): Determine the equation roots.

$2x^2 - 7x - 3 = 0$

....

....

....

[3 - 6] Solving the Fractional Equations.

Example:Find the solution set of the equation and check the solution.

$$\frac{x}{x - 1} + \frac{3x}{x + 1} = \frac{12}{x^2 - 1}$$ Factoring the denominators

$$\Rightarrow \frac{x}{x - 1} + \frac{3x}{x + 1} = \frac{12}{(x - 1)(x + 1)}$$

LCM $(x - 1)(x + 1)$ By multiplying the two sides of equation by

$\Rightarrow x(x + 1) + 3x(x - 1) = 12$

Simplify and solve the equation to find the values of the variable

$\Rightarrow x^2 + x + 3x^2 - 3x - 12 = 0$

$\Rightarrow 4x^2 - 2x - 12 = 0$

$\Rightarrow (4x + 6)(x - 2) = 0 \Rightarrow x = -\frac{3}{2}$ or $x = 2$

Checking:Substituting by the original equation by one of the values and leaving the second to students. Substitute x=2 and see if the two sides of the equation are equaled or not?

$$\left.\begin{array}{l} LHS = \frac{2}{1} + \frac{6}{3} = 4 \\ RHS = \frac{12}{3} = 4 \end{array}\right\} \quad LHS = RHS$$

Exercise: Find the solution set for the equation and check the solution:

$$\frac{2x}{x - 4} + \frac{x}{x + 4} = \frac{32}{x^2 - 16}$$

....

....

....

....

....

Checking:

....

....

....

Chapter Test

Find the solution set for the two equations graphically:

1 $\begin{cases} y = 1 + x \\ y = 2 - x \end{cases}$

2 $\begin{cases} y + x = 0 \\ y - x = 0 \end{cases}$

3 $\begin{cases} y - x - 5 = 0 \\ y + x - 1 = 0 \end{cases}$

Find the solution set for the two equations by using the substitution or elimination for each of the following:

4 $\begin{cases} 2x + y = 1 \\ x - y = 8 \end{cases}$

5 $\begin{cases} 4x - 2y = -4 \\ x + y = 6 \end{cases}$

6 $\begin{cases} \frac{x}{3} + \frac{y}{2} = 1 \\ x + y = 2 \end{cases}$

Solve the following equations by using the greater common factor and the difference between two squares:

7 $9x^2 - 25 = 0$

8 $3y^2 - 12 = 0$

9 $(7 - z)^2 - 1 = 0$

Solve the following equations by using the rule of square root:

10 $x^2 = 49$

11 $81 - y^2 = 0$

12 $z^2 = \frac{36}{9}$

Solve the following equation by factoring in experiment:

13 $x^2 + 9x + 18 = 0$

14 $z^2 - 2z - 48 = 0$

15 $3x^2 - x - 10 = 0$

16 $7z^2 - 18z - 9 = 0$

17 What is the number which its square decreases from its four times in 3 ?

18 Aswimming pool , its length is gerter than the twice of its width in 4 m and its area is $48m^2$.
What are the dimensions of the pool?

Solve the following equations by the perfect square:

19 $x^2 - 16x + 64 = 0$

20 $\frac{1}{9} - \frac{1}{3}z + \frac{1}{4}z^2 = 0$

Solve the following equations by completing the square:

21 $x^2 - 14x = 32$

22 $4y^2 + 20y - 11 = 0$

23 $z^2 - \frac{2}{3}z = 1$

Find the solution set for the following equations by using the general law:

24 $x^2 - 3x - 7 = 0$

25 $3y^2 - 12y = -3$

26 $5z^2 + 6z = 9$

Determine the equation roots at first, then find the solution set if it is possible:

27 $2x^2 + 8x + 8 = 0$

28 $y^2 - 6y - 9 = 0$

29 $4z^2 - 3z + 7 = 0$

30 What is the value of constant (K) which makes the two roots of the equation $x^2 - (k + 6)x + 9 = 0$
Equaled? Check your answer.

Find the solution set for each of the following equations and check the correction of solution:

31 $\frac{6x}{5} = \frac{5}{6x}$

32 $\frac{1}{6y^2} + \frac{1}{2} = \frac{1}{y}$

33 $\frac{z + 4}{z^2} = \frac{1}{2}$

Find the solution set for each of the following equations :

34 $\frac{4}{x - 5} - \frac{3}{x - 2} = 1$

35 $\frac{2y}{y + 2} + \frac{y}{2 - y} = \frac{7}{y^2 - 4}$

Exercises of chapters

Multiple Choice

[1-1] Ordering Operations in Real Numbers

Choose the correct answer for each of the following:

Simplify the following numerical sentences by using the ordering of operations in the real numbers:

1. $(\sqrt{2}+\sqrt{7})(\sqrt{2}+\sqrt{7}) = \ldots.$ a) $2+9\sqrt{7}$ b) $2+9\sqrt{2}$ c) $9+2\sqrt{14}$ d) $2+9\sqrt{14}$

2. $(\sqrt{18}-\sqrt{8})(\sqrt[3]{\frac{-27}{125}}) = \ldots.$ a) $\frac{3\sqrt{2}}{\sqrt{5}}$ b) $\frac{-3\sqrt{2}}{5}$ c) $\frac{2\sqrt{3}}{\sqrt{5}}$ d) $\frac{-2\sqrt{3}}{5}$

3. $\frac{6\sqrt{50}}{3\sqrt[3]{-8}} \div \frac{2\sqrt{14}}{\sqrt{7}} = \ldots.$ a) $\frac{-5}{2}$ b) $\frac{-2}{2}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{-\sqrt{2}}{5}$

4. $\sqrt{8}(\sqrt{2}-\sqrt{3}) - 3\sqrt{6} \approx \ldots.$ a) $5-4\sqrt{6}$ b) $5+4\sqrt{6}$ c) $4-5\sqrt{6}$ d) $4+5\sqrt{6}$

5. $(-27)^{\frac{1}{3}}(\frac{1}{6}\sqrt{2}-\frac{1}{4}\sqrt{32}) = \ldots.$ a) $\frac{-5}{\sqrt{2}}$ b) $\frac{5}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{-\sqrt{2}}{5}$

Simplify the following numerical sentences by using ratioralizing the denominator and ordering theoperations in the real numbers:

6. $\frac{1-\sqrt{5}}{\sqrt{5}-1} = \ldots.$ a) $\frac{1}{\sqrt{5}}$ b) $\frac{-1}{\sqrt{5}}$ c) 1 d) -1

7. $\frac{(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})} = \ldots$ a) $5+6\sqrt{2}$ b) $5-6\sqrt{2}$ c) $2\sqrt{6}-5$ d) $2\sqrt{6}+5$

Use the ordering of operations and write the result neared to two decimal places using the calculator for each of the following:

8. $(\frac{1}{3})^2 - 3^{-2} - (5)^{\frac{3}{2}} \approx \ldots$ a) -18.11 b) 18.11 c) 11.18 d) -11.18

9. $8^{\frac{-1}{3}} - (-7)^0 + \frac{1}{6} \times 4^{\frac{1}{2}} \approx \ldots.$ a) - 0.16 b) - 0.17 c) 0.16 d) - 0.17

Use the calculator to write the result in a scientific form for each number, it should be neared to two decimal places.

10. $(7.46 \times 10^{-2})^2 \approx \ldots.$ a) 5.56×10^{-5} b) 5.57×10^{-4} c) 5.56×10^{-4} d) 5.57×10^{-5}

Multiple Choice

[1-2] Mappings

Choose The Correct Answer for each of the following:

1 If the mapping $f : A \rightarrow B$, defined as following : $x \rightarrow x + 1$ where

$A = \{1 , 3 , 5\}$ ، $B = \{2 , 4 , 6 , 8\}$ then the range is :

a) $\{2 , 4 , 8\}$ b) $\{4 , 6 , 8\}$

c) $\{2 , 4 , 6\}$ d) $\{2 , 6 , 8\}$

2 If , $A = \{1 , 2 , -2 , -3\}$ and $g: A \rightarrow Z$. then the range of mapping $g(x) = 5x - 3$ is:

a) $\{2 , 9 , 13 , 18\}$ b) $\{2 , 7 , -13 , -18\}$

c) $\{9 , 13 , 18 , 21\}$ d) $\{7 , 13 , 15 , 18\}$

3 If , $f: Z \rightarrow B$ where $f(x)= 3x -2$ then the number 10 is the image of the number:

a) 5 b) 4 c) 3 d) 2

4 If , $f: A \rightarrow R$, where $A = \{2 , 3 , 4 , 5\}$, $B = \{4 , 6 , 8\}$ and $f = \{(2 , 4), (3 , 6), (4 , 8), (5 , 8)\}$

then f is surjection mapping because:

a) Range $\neq$ co-domain b) f does not injection

c) The range is the domain of A d) Range = co-domain

5 If , $f: Z \rightarrow Z$ where $f(x) = 2x - 3$, $g:Z \rightarrow Z$ where $g(x) = x + 1$ then the mapping $(gof) (x)$ is:

a) $2x - 2$ b) $2x - 4$ c) $2x + 2$ d) $2x + 4$

6 Let $f: \{2 , 3 , 5\} \rightarrow N$ where $f(x) = 3x - 1$

and $g: N \rightarrow N$ where $g(x) = x + 1$ then the range of (gof) is:

a) $= \{5 , 8 , 14\}$ b) $= \{5 , 6 , 9\}$

c) $= \{6 , 12 , 15\}$ d) $= \{6 , 9 , 12\}$

7 If the mapping $f: Q \rightarrow Q$ where $f(x) = 4x + 1$ and the mapping $g: Q \rightarrow Q$ where

$g(x) = \frac{1}{3} x^2 - 1$ if $(fog) (x) = 45$ then the value of x is :

a) ± 5 b) ± 6 c) ± 7 d) ± 8

Multiple Choice

[1-3] The sequences

Choose The Correct Answer for each of the following:

Write the first five terms for each of the following sentences:

1 $\{5n - 2\} = \ldots$

a) $\{2, 6, 12, 16, 20\ldots\}$ b) $\{3, 8, 13, 18, 23\ldots\}$

c) $\{4, 8, 12, 18, 22\ldots\}$ d) $\{5, 10, 16, 20, 24\ldots\}$

2 $\{\frac{n}{2} + 1\} = \ldots..$

a) $\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}\}$ b) $\{\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \ldots\}$

c) $\{\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2} \ldots\}$ d) $\{2, \frac{5}{2}, 3\, \frac{7}{2}, 4 \ldots\}$

3 $\{(\frac{-1}{2+n})\} = \ldots\ldots$

a) $\{\frac{-1}{2}, \frac{-1}{3}, \frac{-1}{4}, \frac{-1}{5}, \frac{-1}{6}, \ldots\}$ b) $\{\frac{-1}{3}, \frac{-1}{4}, \frac{-1}{5}, \frac{-1}{6}, \frac{-1}{7}, \ldots\}$

c) $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\}$ d) $\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \ldots\}$

Write the first five terms for each of the following sentences:

4 An arithmetic sequence, its second term is 3 and its comman difference is 3 .

a) $\{0,3,6,9,12,\ldots\}$ b) $\{2,5,8, 11,14,\ldots\}$ c) $\{3,6,9, 12,15,\ldots\}$ d) $\{1,4,7,10,13,\ldots\}$

5 An arithmetic sequence, its third term is -8 and its comman difference is 2 .

a) $\{-14,-12,-10,-8,-6,..\}$ b) $\{-12,-10,-8,-6,-4,..\}$ c) $\{-10,-8,-6,-4,-2,..\}$ d) $\{-8,-6,-4,-2,0,..\}$

6 An arithmetic sequence the ninth term and fifteenth term which its second term is 2 and common difference 2 is :

a) $u_9 = 12$, $u_{15} = 20$ b) $u_9 = 14$, $u_{15} = 24$ c) $u_9 = 16$, $u_{15} = 28$ d) $u_9 = 18$, $u_{15} = 32$

7 The terms between U_2 and U_6 for an arithmetic sequence which its second term is $\frac{9}{5}$ and its comuen difference 2 .

a) $\{\frac{9}{2}, \frac{19}{2}, \frac{29}{2},\ldots\}$ b) $\{\frac{19}{2}, \frac{29}{2}, \frac{39}{2},\ldots\}$ c) $\{\frac{9}{5}, \frac{19}{5}, \frac{29}{5},\ldots\}$ d) $\{\frac{19}{5}, \frac{29}{5}, \frac{39}{5},\ldots\}$

Multiple Choice

[1-4] Compound Inequalities

Choose The Correct Answer for each of the following:

Solve the compound inequalities which include (and) algebraically:

1 $-10 < x$ and $x \le -2$

a) $\{x: -10 \le x\} \cap \{x: x \le -2\}$ b) $\{x: -10 < x\} \cap \{x: x \le -2\}$

c) $\{x: -10 \le x\} \cup \{x: x \le -2\}$ d) $\{x: -10 < x\} \cup \{x: x \le -2\}$

2 $0 \le y - 3$ and $y - 3 < 12$

a) $\{y: 3 < y < 15\}$ b) $\{y: -3 \le y \le 15\}$

c) $\{y: 3 \le y < 15\}$ d) $\{y: 3 \le y \le 15\}$

3 $16 < 3z + 9$ and $3z + 9 < 30$

a) $\{z: \frac{3}{7} \le z < 7\}$ b) $\{z: \frac{7}{3} < z \le 7\}$

c) $\{z: \frac{3}{7} < z < 7\}$ d) $\{z: \frac{7}{3} < z < 7\}$

Solve the compound inequalities which include (or) algebraically:

4 $2t - 4 > -8$ or $2t - 4 \le -12$

a) $\{t: t > -2\} \cap \{t: t \le -4\}$ b) $\{t: t > -2\} \cup \{t: t \le -4\}$

c) $\{t: t < -2\} \cap \{t: t \ge -4\}$ d) $\{t: t < -2\} \cup \{t: t \ge -4\}$

5 $\frac{y+5}{3} < \frac{1}{3}$ or $\frac{y+5}{3} > \frac{7}{3}$

a) $\{y: y < 4\} \cap \{y: y > 2\}$ b) $\{y: y > -4\} \cup \{y: y < 2\}$

c) $\{y: y < -4\} \cap \{y: y > -2\}$ d) $\{y: y < -4\} \cup \{y: y > 2\}$

Write the compound inequality which shows the range of the third side length in a triangle which the lengths of its other two sides are known:

6 5cm , 12cm a) $7 < z < 17$ b) $7 \le z < 17$ c) $7 \le z \le 17$ d) $7 < z \le 17$

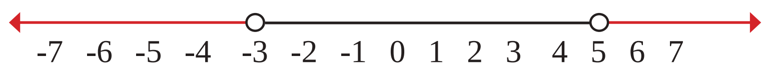
7 8cm , 2cm a) $6 \le x < 10$ b) $6 \le x \le 10$ c) $6 < x < 10$ d) $6 < x \le 10$

Write the inequalities represent the solution set in the line of numbers:

8

a) $-4 < x < 3$ b) $-4 \le x < 3$ c) $-4 \le x \le 3$ d) $-4 < x \le 3$

9



a) $y \le -3$ or $y > 5$ b) $y \le -3$ or $y \ge 5$ c) $y < -4$ or $y \ge 5$ d) $y < -3$ or $y > 5$

Multiple Choice

[1-5] Absolute Value Inequalities

Choose The Correct Answer for each of the following:

Solve the following absolute value inequalities:

1 $|y - 8| < 13$

a) $5 < y < -21$ b) $-5 \leq y \leq 21$ c) $-5 < y < 21$ d) $-5 < y \leq 21$

2 $|3x| - 7 < 1$

a) $-\frac{8}{3} \leq x < \frac{8}{3}$ b) $-\frac{8}{3} < x \leq \frac{8}{3}$ c) $-\frac{8}{3} \leq x \leq \frac{8}{3}$ d) $-\frac{8}{3} < x < \frac{8}{3}$

3 $|3 - x| < 3$

a) $-6 < x < 0$ b) $0 < x < 6$ c) $-6 < x < 6$ d) $0 \leq x \leq 6$

4 $|5t - 5| > 0$

a) $t \leq 1$ or $t > 1$ b) $t \geq 1$ or $t < -1$

c) $t > 1$ or $t < 1$ d) $t > -1$ or $t < -1$

5 $|v - 3| \geq \frac{1}{2}$

a) $v \leq \frac{7}{2}$ or $v \leq \frac{-5}{2}$ b) $v \geq \frac{7}{2}$ or $v \geq \frac{-5}{2}$

c) $v \geq \frac{7}{2}$ or $v \leq \frac{5}{2}$ d) $v \leq \frac{7}{2}$ or $v \geq \frac{-5}{2}$

6 $|6 - 3y| \geq 9$

a) $y \leq 1$ or $y \geq -5$ b) $y < -1$ or $y > 5$

c) $y > -1$ or $y < 5$ d) $y \leq -1$ or $y \geq 5$

7 $\left|\frac{7 - 2y}{3}\right| \geq 3$

a) $y \leq -1$ or $y \geq 8$ b) $y < -1$ or $y \geq 8$

c) $y < -1$ or $y > 8$ d) $y < -1$ or $y > 8$

8 $\left|\frac{z - 1}{7}\right| \leq 2$

a) $-13 < z \leq 15$ b) $-13 \leq z < 15$

c) $-13 \leq z \leq 15$ d) $-13 < z < 15$

Multiple Choice

[2-1] Multiplying Algebraic Expressions

Choose The Correct Answer for each of the following:

Find the result of multiplying and algebraic expression by another algebraic expression:

1. $(x + 5)^2$
   a) $x^2 - 10x + 25$ b) $x^2 + 10x + 25$ c) $x^2 + 5x + 25$ d) $x^2 - 5x + 25$

2. $(z - \sqrt{7})^2$
   a) $z^2 - 7z + 49$ b) $z^2 + 7y + 49$ c) $z^2 - \sqrt{7}\, z + 7$ d) $z^2 - 2\sqrt{7}\, z + 7$

3. $(x + 8)(x - 8)$
   a) $x^2 - 64$ b) $x^2 + 64$ c) $x^2 + 16$ d) $x^2 - 16$

4. $(3 - 2z)(3 + 2z)$
   a) $6 - 4z^2$ b) $9 - 4z^2$ c) $6 + 4z^2$ d) $9 + 4z^2$

5. $(y + \sqrt{6})(y - \sqrt{6})$
   a) $y^2 - \sqrt{12}$ b) $y^2 - 6$ c) $y^2 + \sqrt{12}$ d) $y^2 + 6$

6. $(2x - 3)(x + 9)$
   a) $2x^2 + 15x - 27$ b) $2x^2 - 5x - 27$ c) $2x^2 - 15x + 27$ d) $2x^2 + 15x + 27$

7. $(y - 2)(y^2 + 2y + 4)$
   a) $y^3 + 8$ b) $y^3 - 8$ c) $y^3 - 4$ d) $y^3 - 16$

8. $(\frac{1}{3} - x)(\frac{1}{9} + \frac{1}{3}x + x^2)$
   a) $\frac{1}{27} - x^3$ b) $\frac{1}{27} + x^3$ c) $\frac{1}{9} + x^3$ d) $\frac{1}{9} - x^3$

9. $(z - 2)^3$
   a) $z^3 + 6z^2 + 12z + 8$ b) $z^3 - 6z^2 + 12z - 8$
   c) $z^3 + 6z^2 - 12z - 8$ d) $z^3 - 6z^2 - 12z + 8$

10. $(y + \frac{1}{5})^3$
   a) $y^3 - \frac{3}{3}y^2 + \frac{3}{25}y - \frac{1}{125}$ b) $y^3 + \frac{3}{5}y^2 - \frac{3}{25}y + \frac{1}{125}$
   c) $y^3 + \frac{3}{5}y^2 + \frac{3}{25}y + \frac{1}{125}$ d) $y^3 - \frac{3}{5}y^2 - \frac{3}{25}y - \frac{1}{125}$

Multiple Choice

[2-2]Factoring the Algebraic Expression by using agreater Common Factor .

Choose The Correct Answer for each of the following:

Factoring each expression by using the greatest common factor:

1 $12x^3 + 9x^2 - 3x$

a) $3x(4x^2 + 3x +1)$ b) $3x(4x^2 + 3x -1)$

c) $9x(3x^2 + x +1)$ d) $9x(3x^2 + x -1)$

2 $6y^2(3y - 4) + 36y$

a) $6y(3y^2 + 4y +6)$ b) $6y(3y^2 + 4y -6)$

c) $6y(3y^2 - 4y -6)$ d) $6y(3y^2 - 4y +6)$

Factoring each each expression by using the binomial term as a greatest common factor:

3 $3z(z - 3) - 7(z - 3)$

a) $(z + 3)(3z - 7)$ b) $(z - 3)(3z + 7)$

c) $(z - 3)(3z - 7)$ d) $(z + 3)(3z + 7)$

4 $\frac{1}{4}(x + 9) - \frac{1}{2}x^2(x + 9)$

a) $(x + 9)(\frac{1}{4} - \frac{1}{2}x^2)$ b) $(x - 9)(\frac{1}{4} - \frac{1}{2}x^2)$

c) $(x + 9)(\frac{1}{4} + \frac{1}{2}x^2)$ d) $(x + 9)(\frac{1}{2} - \frac{1}{4}x^2)$

5 $\sqrt{2}\,v(x -1) - \sqrt{3}\,t(x -1)$

a) $(x + 1)(\sqrt{2}\,v - \sqrt{3}\,t)$ b) $(x - 1)(\sqrt{2}\,v - \sqrt{3}\,t)$

c) $(x - 1)(\sqrt{2}\,v + \sqrt{3}\,t)$ d) $(x + 1)(\sqrt{2}\,v + \sqrt{3}\,t)$

Factoring each expression by using the property of grouping, then check the correct of solution:

6 $3y^3- 9y^2+ 5y - 15$

a) $(y + 3)(3y^2 + 5)$ b) $(y - 3)(3y^2 - 5)$

c) $(y - 3)(3y^2 + 5)$ d) $(y - 3)(3y^2 - 5)$

Factoring the expression by using the property of grouping with the inverse:

7 $20y^3 - 4y^2 + 3 - 15y$

a) $(5y + 1)(4y^2 - 3)$ b) $(5y - 1)(4y^2 + 3)$

c) $(5y - 1)(4y^2 - 3)$ d) $(5y+ 1)(4y^2 + 3)$

8 $\frac{1}{6}x^4 - \frac{1}{3}x^3 + 4 - 2x$

a) $(x - 2)(\frac{1}{6}x^3 - 2)$ b) $(x + 2)(\frac{1}{6}x^3 - 2)$

c) $(x + 2)(\frac{1}{6}x^3 - 2)$ d) $(x - 2)(\frac{1}{6}x^3 + 2)$

Multiple Choice

[2-3] Factoring the Algebraic Expression by using Special Identities.

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions:

1. $9 - 4x^2$
 a) $(3 + 2x)(3 + 2x)$ b) $(3 + 2x)(3 - 2x)$
 c) $(9 - x)(9 + 4x)$ d) $(3 + x)(3 - 4x)$

2. $12y^3z - 3yz^3$
 a) $3y(2y - z)(y + 2z)$ b) $3z(2y - z)(2y + z)$
 c) $3yz(2y - z)(2y + z)$ d) $3yz(y - 2z)(y + 2z)$

3. $\frac{1}{6}x^3 - x\frac{1}{24}$
 a) $\frac{x}{6}(x + \frac{1}{2})(x - \frac{1}{2})$ b) $\frac{x}{6}(x + \frac{1}{4})(x - \frac{1}{4})$
 c) $\frac{x}{3}(\frac{1}{2}x + \frac{1}{2})(\frac{1}{2}x - \frac{1}{2})$ d) $\frac{x}{6}(\frac{1}{4}x + \frac{1}{4})(\frac{1}{4}x - \frac{1}{4})$

4. $4x^2 + 24x + 36$
 a) $(x + 6)^2$ b) $(x - 6)^2$ c) $4(x - 3)^2$ d) $4(x + 3)^2$

5. $16 - 8y + y^2$
 a) $(4 + 2y)^2$ b) $(4 - 2y)^2$ c) $(4 - y)^2$ d) $(4 + y)^2$

Determine which of the following algebraic expressions represents a perfect square:

6. $4x^2 - 20x + 25$
 a perfect square because a) $2(x)(5) = 10x$
 a perfect square because b) $-2(2x)(5) = -20x$
 a perfect square because c) $- 4(x)(5) \neq 10x$
 it is not a perfect square because d) $- 4(x)(5) \neq 20x$

7. $64 - 48y + 9y^2$
 it is not a perfect square because a) $2(4)(3y) \neq - 48y$
 a perfect square because b) $2(8)(4y) = 48y$
 it is not a perfect because c) $-2(8)(3y) \neq - 48y$
 a perfect square because d) $-4(4)(3y) = 48y$

Write the missing term in the algebraic expression $ax^2 + bx + c$ to become a perfect square:

8. $z^2 + \dots + 49$
 a) $14z$ b) $- 10z$ c) $7z$ d) $-7z$

9. $36 - 24x + \dots$
 a) $2x^2$ b) $-2x^2$ c) $4x^2$ d) $- 4x^2$

10. $16y^2 + 40y + \dots$
 a) 9 b) 25 c) -9 d) -25

Multiple Choice

[2-4] Factoring the Algebraic Expression of three terms by Trial and Error .

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions:

1. $x^2 + 7x + 12$
   a) $(x - 3)(x + 4)$ b) $(x + 3)(x + 4)$
   c) $(x - 1)(x + 7)$ d) $(x - 3)(x - 4)$

2. $x^2 - 5x - 36$
   a) $(x - 6)(x + 6)$ b) $(x + 12)(x - 3)$
   c) $(x - 9)(x + 4)$ d) $(x + 9)(x - 4)$

3. $y^2 + 4y - 21$
   a) $(y - 7)(y + 3)$ b) $(y + 7)(y - 3)$
   c) $(y – 7)(y - 3)$ d) $(y + 7)(y + 3)$

4. $4x^2 + 10x + 6$
   a) $(x – 6)(4x + 1)$ b) $(4x + 2)(x - 3)$
   c) $(4x – 6)(x - 1)$ d) $(2x + 3)(2x + 2)$
   c) $(5 + 8z)(3 - z)$ d) $(3 + 8z)(5 - z)$

5. $24y^2 - 2y - 1$
   a) $(4y - 1)(6y + 1)$ b) $(2y - 1)(12y - 1)$
   c) $(4y + 1)(6y - 1)$ d) $(3y – 1)(8y + 1)$

6. $10x^2 - 11x + 1$
   a) $(5x - 1)(2x + 1)$ b) $(10x + 1)(x - 1)$
   c) $(5x + 1)(2x - 1)$ d) $(10x - 1)(x - 1)$

7. $22 +3z - 4z^2$
   a) $(11 + 4z)(2 - z)$ b) $(22 - 4 z)(1 + z)$
   c) $(11 - 4z)(2 + z)$ d) $(22 + 8z)(1 - z)$

Put signs between the terms inside brackets to make the factoring of the algebraic expression correct:

8. $x^2 + 15x + 26 = (x…2)(x…13)$
   a) $(x - 2)(x - 13)$ b) $(x - 2)(x + 13)$
   c) $(x + 2)(x + 13)$ d) $(x + 2)(x - 13)$

9. $4y^2 -2y -12 = (2y…3)(2y…4)$
   a) $(2y - 3)(2y + 4)$ b) $(2y + 3)(2y + 4)$
   c) $(2y - 3)(2y - 4)$ d) $(2y + 3)(2y - 4)$

10. $48 - 30z + 3z^2 = (6…3z)(8… z)$
    a) $(6 - 3z)(8 - z)$ b) $(6 + 3z)(8 + z)$
    a) $(6 - 3z)(8 + z)$ b) $(6 + 3z)(8 - z)$

Multiple Choice

[2-5] Factoring Algebraic Expressions Contains Sum of Two Cubes or difference Between Two Cubes.

Choose The Correct Answer for each of the following:

Factor each of the following algebraic expressions in simplest form:

1. $8 + x^3$
   a) $(2 - x)(4 + 2x + x^2)$ b) $(2 + x)(4 - 2x + x^2)$
   c) $(2 - x)(4 - 2x + x^2)$ d) $(2 + x)(4 + 2x + x^2)$

2. $8y^3 + 27$
   a) $(2y + 3)(4y^2 + 6y + 9)$ b) $(2y - 3)(4y^2 + 6y + 9)$
   c) $(2y + 3)(4y^2 - 6y + 9)$ d) $(2y - 3)(4y^2 - 6y + 9)$

3. $\frac{1}{z^3} + \frac{1}{64}$
   a) $(\frac{1}{z} + \frac{1}{4})(\frac{1}{z^2} + \frac{1}{4z} + \frac{1}{16})$ b) $(\frac{1}{z} - \frac{1}{4})(\frac{1}{z^2} - \frac{1}{4z} + \frac{1}{16})$
   c) $(\frac{1}{z} - \frac{1}{4})(\frac{1}{z^2} + \frac{1}{4z} + \frac{1}{16})$ d) $(\frac{1}{z} + \frac{1}{4})(\frac{1}{z^2} - \frac{1}{4z} + \frac{1}{16})$

4. $\frac{27}{125} + \frac{8}{x^3}$
   a) $(\frac{3}{5} - \frac{2}{x})(\frac{9}{25} + \frac{6}{5x} + \frac{4}{x^2})$ b) $(\frac{3}{5} - \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} + \frac{4}{x^2})$
   c) $(\frac{3}{5} + \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} + \frac{4}{x^2})$ d) $(\frac{3}{5} + \frac{2}{x})(\frac{9}{25} - \frac{6}{5x} - \frac{4}{x^2})$

5. $0.027 + z^3$
   a) $(0.03 + z)(0.09 - 0.3z + z^2$ b) $(0.03 + z)(0.009 - 0.03z + z^2)$
   c) $(0.3 + z)(0.9 - 0.3z + z^2)$ d) $(0.3 + z)(0.09 - 0.3z + z^2)$

6. $\frac{8}{y^3} - \frac{1}{27}$
   a) $(\frac{2}{y} - \frac{1}{3})(\frac{4}{y^2} - \frac{2}{3y} + \frac{1}{9})$ b) $(\frac{2}{y} + \frac{1}{3})(\frac{4}{y^2} - \frac{2}{3y} + \frac{1}{9})$
   c) $(\frac{2}{y} - \frac{1}{3})(\frac{4}{y^2} + \frac{2}{3y} + \frac{1}{9})$ d) $(\frac{2}{y} - \frac{1}{3})(\frac{4}{y^2} + \frac{2}{3y} - \frac{1}{9})$

7. $9 - \frac{1}{3} z^3$
   a) $\frac{1}{3}(3 - z)(9 + 3z - z^2)$ b) $\frac{1}{3}(3 - z)(9 + 3z + z^2)$
   c) $\frac{1}{3}(3 + z)(9 + 3z + z^2)$ d) $\frac{1}{3}(3 - z)(9 - 3z + z^2)$

8. $0.008x^3 - 1$
   a) $(0.02x - 1)(0.04x^2 + 0.002x + 1)$ b) $(0.02x - 1)(0.04x^2 + 0.02x + 1)$
   c) $(0.2x + 1)(0.4x^2 - 0.2x + 1)$ d) $(0.2x - 1)(0.04x^2 + 0.2x + 1)$

Multiple Choice

[2-6] Simplifying Rational Algebraic Expressions

Choose The Correct Answer for each of the following:

Write each of the following expressions in simplest form:

1. $\frac{x+3}{4x} \times \frac{4x-12}{x^2-9}$ a) $\frac{3}{x}$ b) $\frac{x}{4}$ c) $\frac{1}{4}$ d) $\frac{1}{x}$

2. $\frac{y+2}{y^2+2y+4} \times \frac{y^3-8}{y^2-4}$ a) $\frac{1}{y-2}$ b) 1 c) $\frac{1}{y+2}$ d) -1

3. $\frac{z^2-2z-15}{9+3z} \times \frac{5}{z^2-25}$ a) $\frac{5}{z+5}$ b) $\frac{3}{5(z+5)}$ c) $\frac{5}{3(z+5)}$ d) $\frac{3}{z+5}$

4. $\frac{x^2-49}{2x^2+9x-35} \div \frac{x-7}{4x^2-25}$ a) $x-7$ b) $2x-5$ c) $x+7$ d) $2x+5$

5. $\frac{1-z^3}{1+z+z^2} \div \frac{(1-z)^2}{1-z^2}$ a) $1-z$ b) $1+z$ c) $1+z+z^2$ d) $1-z+z^2$

Write each of the following expressions in simplest form:

6. $\frac{2y^2+1}{y^3-1} - \frac{y}{y^2+y+1}$ a) $\frac{y}{y+1}$ b) $\frac{1}{y+1}$ c) $\frac{1}{y-1}$ d) $\frac{y}{y-1}$

7. $\frac{5-4z^2}{8z^3+1} + \frac{2z-1}{4z^2-2z+1}$ a) $\frac{2z-1}{(2z+1)(4z^2-2z+1)}$ b) $\frac{2z+1}{(2z+1)(4z^2-2z+1)}$ c) $\frac{2}{(2z+1)(4z^2-2z+1)}$ d) $\frac{4}{(2z+1)(4z^2-2z+1)}$

8. $\frac{3}{x-5} - \frac{2}{5-x} - \frac{130+24x+5x^2}{x^3-125}$ a) $\frac{2x}{(x^2+5x+25)}$ b) $\frac{-2x}{(x^2+5x+25)}$ c) $\frac{1}{(x^2+5x+25)}$ d) $\frac{8}{x^2+5x+25}$

9. $\frac{3y+1}{y+4} - \frac{y-4}{3y-1} - \frac{10+8y^2}{3y^2+11y-4}$ a) $\frac{5}{(y+4)(3y-1)}$ b) $\frac{3}{(y+4)(3y-1)}$ c) $\frac{-3}{(y+4)(3y-1)}$ d) $\frac{-5}{(y+4)(3y-1)}$

Multiple Choice

[3-1] Solving a system of two linear equations with two variables

Choose The Correct Answer for each of the following:

Find the set of solution of the system graphically in R:

1. $\left.\begin{array}{l} y = 4x - 6 \\ y = x \end{array}\right\}$ a) {(-2 , -2)} b) {(-2 , 2)} c) {(2 , -2)} d) {(2 , 2)}

2. $\left.\begin{array}{l} y = x - 3 \\ y = 3 - x \end{array}\right\}$ a) {(-3 , 0)} b) {(3, 0)} c) {(0 , -3)} d) {(0 , 3)}

Find the set of solution of the system in R by using the substitution for each of the following:

3. $\left.\begin{array}{l} 3x + 4y = 26 \\ 5x - 2y = 0 \end{array}\right\}$ a) {(2 , 5)} b) {(-2 , -5)} c) {(2 , -5)} d) {(-2 , 5)}

4. $\left.\begin{array}{l} y = 6x + 12 \\ 3y = 2x - 8 \end{array}\right\}$ a) $\{(\frac{-11}{4} , \frac{9}{2})\}$ b) $\{(\frac{11}{4} , \frac{-9}{2})\}$ c) $\{(\frac{-11}{4} , \frac{-9}{2})\}$ d) $\{(\frac{11}{4} , \frac{9}{2})\}$

5. $\left.\begin{array}{l} \frac{3x}{4} - \frac{y}{2} = 4 \\ \frac{y}{2} - \frac{x}{4} = 2 \end{array}\right\}$ a) {(12 , -10)} b) {(-12, -10)} c) {(12 , 10)} d) {(-12 , 10)}

Find the set of solution of the system in R by using the elimination for each of the following:

6. $\left.\begin{array}{l} 7x - 4y = 12 \\ 3x - y = 5 \end{array}\right\}$ a) $\{(-\frac{8}{5} , \frac{1}{5})\}$ b) $\{(-\frac{8}{5} , -\frac{1}{5})\}$ c) $\{(\frac{8}{5} , \frac{1}{5})\}$ d) $\{(\frac{8}{5} , -\frac{1}{5})\}$

7. $\left.\begin{array}{l} 6y - 2x - 8 = 0 \\ y + x - 12 = 0 \end{array}\right\}$ a) {(8 , -4)} b) {(8, 4)} c) {(-8 , 4)} d) {(-8 ,-4)}

8. $\left.\begin{array}{l} \frac{2}{3}x - \frac{1}{6}y = 2\frac{1}{3} \\ \frac{1}{4}x - \frac{1}{2}y = 3\frac{1}{2} \end{array}\right\}$ a) {(-2 , -6)} b) {(-2, 6)} c) {(2 , -6)} d) {(2 , 6)}

Multiple Choice

[3-2] Solving Quadratic Equations with one variable

Choose The Correct Answer for each of the following:

Solve the following equations in R by using the greatest common factor and the difference between two squares:

1. $3x^2 - 12x = 0$ a) $s = \{4 , -4\}$ b) $s = \{3 , -3\}$ c) $s = \{0 , 4\}$ d) $s = \{0 , 3\}$
2. $7z^2 - 21 = 0$ a) $s = \{7 , -7\}$ b) $s = \{3 , -3\}$ c) $s = \{\frac{1}{3} , -\frac{1}{3}\}$ d) $s = \{\sqrt{3} , -\sqrt{3}\}$
3. $4(x^2 - 1) - 5 = 0$ a) $s = \{\frac{3}{2} , -\frac{3}{2}\}$ b) $s = \{\frac{1}{2} , -\frac{1}{2}\}$ c) $s = \{\frac{3}{2} , \frac{3}{2}\}$ d) $s = \{\frac{1}{2} , \frac{1}{2}\}$
4. $(y + 7)^2 - 81 = 0$ a) $s = \{2 , -2\}$ b) $s = \{16 , -16\}$ c) $s = \{2 , -16\}$ d) $s = \{-2 , 16\}$
5. $3x^2 - 6 = 0$ a) $s=\{\sqrt{3} ,- \sqrt{3}\}$ b) $s=\{\sqrt{2} ,- \sqrt{2}\}$ c) $s=\{6 ,- 6\}$ d) $s=\{ 2,- 2\}$

Solve the following equations in R by using the rule of square root:

6. $x^2 = 144$ a) $s = \{7 , -7\}$ b) $s = \{14 , -14\}$ c) $s = \{12 , -12\}$ d) $s = \{12 , 12\}$
7. $32 - 2y^2 = 0$ a) $s = \{6 , 6\}$ b) $s = \{4 , -4\}$ c) $s = \{6 , -6\}$ d) $s = \{4 , 4\}$
8. $5z^2 = 9$ a) $s = \{\frac{3}{5} ,- \frac{3}{5}\}$ b) $s = \{\frac{5}{3} ,- \frac{5}{3}\}$ c) $s = \{\frac{3}{\sqrt{5}} ,- \frac{3}{\sqrt{5}}\}$ d) $s = \{\frac{3}{\sqrt{5}} , \frac{3}{\sqrt{5}}\}$
9. $4(y^2 - 1) = 45$ a) $s = \{\frac{7}{2} ,- \frac{7}{2}\}$ b) $s = \{\frac{7}{2} , \frac{7}{2}\}$ c) $s = \{\frac{2}{7} ,- \frac{2}{7}\}$ d) $s = \{\frac{7}{4} ,- \frac{7}{4}\}$
10. $\frac{1}{2} z^2 = \frac{1}{9}$ a) $s = \{\frac{2}{3} ,- \frac{2}{3}\}$ b) $s = \{\frac{\sqrt{2}}{3} ,- \frac{\sqrt{2}}{3}\}$ c) $s = \{\frac{3}{\sqrt{2}} ,- \frac{3}{\sqrt{2}}\}$ d) $s = \{\frac{3}{2} ,- \frac{3}{2}\}$
11. $x^2 - \frac{13}{16} = \frac{3}{16}$ a) $s = \{\frac{3}{4} ,- \frac{3}{4}\}$ b) $s = \{\frac{\sqrt{3}}{4} ,- \frac{\sqrt{3}}{4}\}$ c) $s = \{2 ,- 2\}$ d) $s = \{1 ,- 1\}$

Multiple Choice

[3-3] Solving the quadratic equations by the experiment

Choose The Correct Answer for each of the following:

Solve the following equations in R by factoring in the experiment:

1 $y^2 + 10y + 21 = 0$ a) s ={3,-7} b) s ={-3,7} c) s ={-3,-7} d) s ={3,7}

2 $x^2 - 5x - 36 = 0$ a) s ={7, -8} b) s ={-4,9} c) s ={4,-9} d) s ={-4,-9}

3 $x^2 - 8x - 48 = 0$ a) s ={4,12} b) s ={4,-12} c) s ={-4,12} d) s ={-4,-12}

4 $4y^2 + 18y + 18 = 0$ a) s $=\{-3, \frac{3}{4}\}$ b) s $=\{3, \frac{3}{4}\}$ c) s $=\{3, \frac{3}{2}\}$ d) s $=\{-3, \frac{-3}{2}\}$

5 $6z^2 + 36z - 42 = 0$ a) s ={1, 7} b) s ={-1,7} c) s ={-1,-7} d) s ={1,-7}

6 $22 - 20y - 2y^2 = 0$ a) s ={11,1} b) s ={1,-11} c) s ={11,-1} d) s ={-1,-11}

7 $32 + 12x - 9x^2 = 0$ a) s $=\{\frac{4}{3}, \frac{8}{3}\}$ b) s $=\{\frac{-4}{3}, \frac{-8}{4}\}$ c) s $=\{\frac{4}{3}, \frac{-8}{3}\}$ d) s $=\{\frac{-4}{3}, \frac{8}{3}\}$

8 What is the number which its square increases in 42?

a) s ={7, 6} b) s ={7, - 6} c) s ={-7, 6} d) s ={-7, - 6}

9 Two numbers its product is 54 , one of them increases in 3 to the other number , what are the two numbers?

a) s ={6 , 9} b) s ={6 , -9} c) s ={-6 , 9} d) s ={-6 , - 9}

10 Two numbers its product is 48 , one of them decreases in 8 to the other number , what are the two numbers?

a) s ={8 , 6} b) s ={12 , - 4} c) s ={10 , 4} d) s ={-12 , - 4}

Multiple Choice

[3-4] Solving the quadratic equations by completing the square

Choose The Correct Answer for each of the following:

Solve the following equations in R by the perfect Square:

1	$x^2 + 6x + 9 = 0$	a) $x = 6$	b) $x = -3$	c) $x = 4$	d) $x = 3$
2	$4z^2 - 20z + 25 = 0$	a) $z = \frac{-5}{2}$	b) $z = \frac{-2}{5}$	c) $z = \frac{5}{2}$	d) $z = \frac{2}{5}$
3	$\frac{1}{16} - \frac{1}{2}x + x^2 = 0$	a) $x = \frac{1}{4}$	b) $x = \frac{-1}{4}$	c) $x = \frac{1}{2}$	d) $x = \frac{-1}{2}$
4	$y^2 - 2\sqrt{3}\,y + 3 = 0$	a) $y = -3$	b) $y = 3$	c) $y = -\sqrt{3}$	d) $y = \sqrt{3}$

Solve the following equations in R by completing the square:

5	$x^2 - 12x = 13$	a) $s = \{13, 1\}$	b) $s = \{13, -1\}$	c) $s = \{-13, 1\}$	d) $s = \{-13, -1\}$
6	$4y^2 - 32y = 17$	a) $s = \{\frac{1}{2}, \frac{17}{2}\}$	b) $s = \{\frac{-1}{2}, \frac{2}{17}\}$	c) $s = \{\frac{1}{2}, \frac{2}{17}\}$	d) $s = \{\frac{-1}{2}, \frac{17}{2}\}$
7	$16z^2 - 40z - 11 = 0$	a) $s = \{\frac{11}{4}, \frac{1}{4}\}$	b) $s = \{\frac{-11}{4}, \frac{-1}{4}\}$	c) $s = \{\frac{11}{4}, \frac{-1}{4}\}$	d) $s = \{\frac{-11}{4}, \frac{1}{4}\}$

8 $y^2 - \frac{1}{3}y = \frac{2}{9}$

a) $\{\frac{3}{2}, \frac{1}{3}\}$ b) $\{\frac{-3}{2}, \frac{1}{3}\}$

c) $\{\frac{2}{3}, \frac{-1}{3}\}$ d) $\{\frac{-2}{3}, \frac{1}{3}\}$

9 $z^2 + 2\sqrt{5}\,z = 4$

a) $s = \{3 + \sqrt{5}, 3 - \sqrt{5}\}$ b) $s = \{\sqrt{5} - 3, 3 - \sqrt{5}\}$

c) $s = \{3 - \sqrt{5}, -3 - \sqrt{5}\}$ d) $s = \{\sqrt{5} + 3, \sqrt{5} - 3\}$

Solve the following equations in R by completing the square, and find the result by near it to a nearest integer:

10	$x^2 - 8x = 9$	a) $s \approx \{9, 1\}$	b) $s \approx \{9, -1\}$	c) $s \approx \{-9, 1\}$	d) $s \approx \{-9, -1\}$

Multiple Choice

[3-5] Using General Law to Solve the Equations.

Choose The Correct Answer for each of the following:

The solution set for the following equations by using the general law in R:

1 $x^2 - 3x - 4 = 0$ a) $s = \{4, 1\}$ b) $s = \{4, -1\}$ c) $s = \{-4, 1\}$ d) $s = \{-4, -1\}$

2 $y^2 - 5y - 5 = 0$ a) $s = \{\frac{3+5\sqrt{5}}{2}, \frac{3-5\sqrt{5}}{2}\}$ b) $s = \{\frac{5+3\sqrt{5}}{4}, \frac{3-5\sqrt{5}}{4}\}$

c) $s = \{\frac{5+3\sqrt{5}}{2}, \frac{5-3\sqrt{5}}{2}\}$ d) $s = \{\frac{5+3\sqrt{3}}{2}, \frac{3-3\sqrt{3}}{2}\}$

3 $2x^2 - 8x = -3$ a) $s = \{\frac{4+\sqrt{10}}{2}, \frac{4-\sqrt{10}}{2}\}$ b) $s = \{\frac{2+\sqrt{10}}{2}, \frac{4+\sqrt{10}}{2}\}$

c) $s = \{\frac{4+\sqrt{5}}{4}, \frac{4-\sqrt{5}}{4}\}$ d) $s = \{\frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2}\}$

4 $3x^2 - 6(2x+1) = 0$ a) $s = \{2+\sqrt{3}, 2-\sqrt{3}\}$ b) $s = \{2+\sqrt{2}, 2-\sqrt{2}\}$

c) $s = \{2+\sqrt{6}, 2-\sqrt{6}\}$ d) $s = \{6+\sqrt{6}, 6-\sqrt{6}\}$

Determine the roots of equation by using the distinctive:

5 $x^2 - 6x - 7 = 0$

a) Two rational real roots b) Two irrational real roots

c) One real root $(\frac{-b}{2a})$ d) Two unreal roots (the solution set in R $= \varnothing$)

6 $2y^2 - 3y - 8 = 0$

a) Two rational roots b) Two irrational roots

c) One real root $(\frac{-b}{2a})$ d) Two unreal roots (the solution set in R $= \varnothing$)

7 $8x^2 - 8x + 2 = 0$

a) Two rational real roots b) Two irrational real roots

c) One real root $(\frac{-b}{2a})$ d) Two unreal roots (the solution set in R $= \varnothing$)

8 What is the value of the constant K which makes the two roots of the equation

$y^2 - (k + 10)y + 16 = 0$ are equaled?

a) $k = 2, -18$ b) $k = -2, -18$ c) $k = 6, 18$ d) $k = -6, -18$

Multiple Choice

[3-6] Solving the Rational Equations

Choose The Correct Answer for each of the following:

Find the solution for each of following equations in R:

1. $\frac{2}{12x^2} - \frac{1}{6} = \frac{1}{4x}$ a) $s = \{2, \frac{1}{2}\}$ b) $s = \{-2, \frac{1}{2}\}$ c) $s = \{2, \frac{-1}{2}\}$ d) $s = \{-2, \frac{-1}{2}\}$

2. $\frac{5}{6} - \frac{7}{63} + \frac{4}{3} = 0$ a) $s = \{1, \frac{-7}{2}\}$ b) $s = \{-1, \frac{-7}{2}\}$ c) $s = \{1, \frac{7}{2}\}$ d) $s = \{-1, \frac{7}{2}\}$

3. $\frac{8x}{5} = \frac{5}{8x}$ a) $s = \{\frac{5}{8}, \frac{-8}{5}\}$ b) $s = \{\frac{5}{8}, \frac{8}{5}\}$ c) $s = \{\frac{5}{8}, \frac{-5}{8}\}$ d) $s = \{\frac{8}{5}, \frac{-8}{5}\}$

4. $\frac{1+2y}{3y+9} = \frac{y}{2}$ a) $s = \{1, \frac{1}{3}\}$ b) $s = \{-1, \frac{1}{3}\}$ c) $s = \{2, \frac{1}{3}\}$ d) $s = \{-2, \frac{1}{3}\}$

5. $\frac{16x - 64}{x^2} = 1$ a) $x = -8$ b) $x = 8$ c) $x = -6$ d) $x = 6$

Find the solution for each of following equations in R:

6. $\frac{2}{x-2} - \frac{3}{x-1} = 1$ a) $s = \{2 + \sqrt{7}, 2 - \sqrt{7}\}$ b) $s = \{1 + \sqrt{3}, 1 - \sqrt{3}\}$ c) $s = \{1 + \sqrt{7}, 1 - \sqrt{7}\}$ d) $s = \{2 + \sqrt{3}, 2 - \sqrt{3}\}$

7. $\frac{y-6}{y+6} - \frac{y+6}{y-6} = \frac{24y+6}{y^2-36}$ a) $y = -\frac{1}{3}$ b) $y = -\frac{1}{2}$ c) $y = \frac{1}{3}$ d) $y = -\frac{1}{3}$

8. $\frac{x}{x+3} - \frac{x}{x-3} = \frac{x^2+2x+81}{x^2-9}$ a) $x = -9$ b) $x = 9$ c) $x = -8$ d) $x = 8$

9. $\frac{3y}{y-4} + \frac{y}{y-2} = \frac{-4y+8}{y^2-6y+8}$ a) $s = \{4, -2\}$ b) $s = \{-4, -2\}$ c) $s = \{-4, 2\}$ d) $s = \{4, 2\}$

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