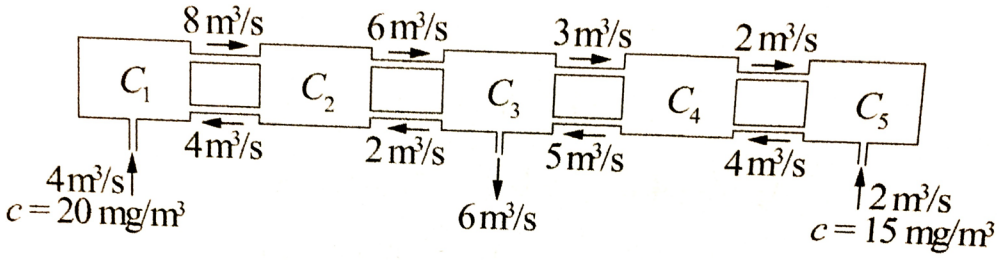
The University of Texas at Austin
Fall 2021 - PGE 310
Homework #6: Solving systems of equations
Due: **December 1, 2021**

---

1. Formulating the systems of equations (By HAND)

The diagram below shows five mixing vessels connected by pipes. Water is pumped through the pipes at the steady rates shown on the diagram. The incoming water contains a chemical, the amount of which is specified by its concentration c in mg/m3



Applying the principle of conservation of mass to each vessel, obtain the equations for the concentrations ci in the vessels. Note that the mass flow rate of the chemical is obtained by multiplying the volume flow rate of the water by the concentration.

Show your work clearly.

---

2. Conceptual questions over Gauss Seidel (By HAND)

Consider the following system of equations:

$$3x_1 - 0.1x_2 + 0.2x_3 = 7.85$$

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

a) Is convergence guaranteed for an iterative method like Gauss Seidel?

b) Perform **two steps** of the Gauss-Seidel iteration starting from the initial guess **(0,0,0).**

Show your work clearly.

---

3. LU decomposition (By HAND)

$$x_1 + 5x_2 - 7x_3 = -3$$

$$-8x_1 + 7x_2 + 6_3 = 2$$

$$x_1 - 6x_2 + 1x_3 = -5$$

a) Solve the above system using LU decomposition. SHOW ALL WORK

b) Compute the inverse of the matrix A using result from upper triangular matrix.

c) Prove the result satisfies $A^{-1}A = I$

Show your work clearly.
