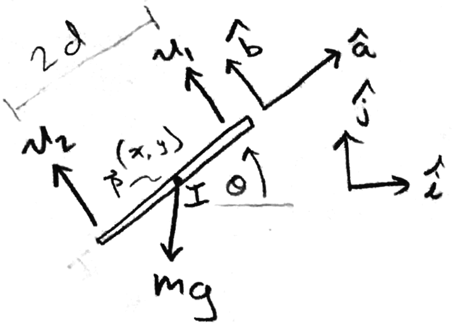
# Simple Quad-Rotor

Dec 9, 2017




$$\Sigma \vec{F} = m\ddot{\vec{p}}$$

$$\Sigma \vec{\tau} = I\ddot{\vec{\alpha}} = I\ddot{\theta}\hat{k}$$

$$\hat{a} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{b} = \hat{k}\times\hat{a} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\vec{\omega} = \dot{\theta}\hat{k}$$

$$\dot{\hat{a}} = \vec{\omega}\times\hat{a} = \dot{\theta}\hat{b} \longrightarrow \ddot{\hat{a}} = \ddot{\theta}\hat{b} - \dot{\theta}^2\hat{a}$$

$$\dot{\hat{b}} = \vec{\omega}\times\hat{b} = -\dot{\theta}\hat{a} \longrightarrow \ddot{\hat{b}} = -\ddot{\theta}\hat{a} - \dot{\theta}^2\hat{b}$$

$$\vec{P} = x\hat{i} + y\hat{j}$$

$$\Sigma F = (u_1 + u_2)\hat{b}$$

$$\Sigma \tau = (d\hat{a})\times(u_1\hat{b}) + (-d\hat{a})\times(u_2\hat{b}) = d(u_1 - u_2)\hat{k}$$

$$\Sigma \vec{F} = m\ddot{\vec{p}}$$

$$(u_1 + u_2)\hat{b} = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$\begin{cases} \cdot\hat{i} & (u_1 + u_2)(-\sin\theta) = m\ddot{x} \\ \cdot\hat{j} & (u_1 + u_2)(\cos\theta) = m\ddot{y} \end{cases}$$

← add gravity terms!

$$\Sigma \tau = I\ddot{\theta}\hat{k}$$

$$d(u_1 - u_2) = I\ddot{\theta}$$

hmm... 3 DoF, 2 actuators... "under-actuated"

↳ Idea: looks like we can decouple system: $\theta$ can be controlled without paying attention to $x, y$.

↳ Let $\boxed{\tau = d(u_1 - u_2)}$

$\tau = I\ddot{\theta}$ ← look familiar? use 2nd-ord. Sys. Ctrl.

↳ let's use a "fast" control on $\theta$ → treat $\theta$ as control

↳ Let $\boxed{F = u_1 + u_2}$

$$\left.\begin{cases} m\ddot{x} = -f\sin\theta \\ m\ddot{y} = f\cos\theta + mg \end{cases}\right\}$$

hmm... feedback linearization

Let $F_x = -f\sin\theta$

Let $F_y = f\cos\theta + mg$

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↳ Continued.

$$\left.\begin{array}{l} m\ddot{x} = F_x \\ m\ddot{y} = F_y \end{array}\right\} \text{Solve as } 2^{nd} \text{ order System}$$

Now solve for: $f, \theta$

$$\left.\begin{array}{l} -F_x = f \sin\theta \\ F_y + mg = f\cos\theta \end{array}\right] \text{Polar Coordinates!}$$

$$\frac{-F_x}{F_y + mg} = \tan(\theta) \qquad F_x^2 + (F_y + mg)^2 = f^2$$

**What Goes Wrong?**

- Bad Model
- Too far away -> Saturate Motors.

Extensions of these ideas:

- "convert to $2^{nd}$-order System trick" → General MIMO Linear Control
- "feedback linearization" → Lie Derivatives and other hard math (see Wikipedia...)

Equation Summary

$$\begin{cases} \ddot{x} = -F\sin\theta / m \\ \ddot{y} = (F\cos\theta / m) - g \\ \ddot{\theta} = \tau / I \end{cases} \qquad \begin{cases} F = u_R + u_L \\ \tau = d(u_R - u_L) \end{cases} \qquad \begin{cases} u_R = \frac{1}{2}(F + \tau/d) \\ u_L = \frac{1}{2}(F - \tau/d) \end{cases}$$

$$\begin{cases} m\ddot{x} = F_x = -F\sin\theta \\ m\ddot{y} = F_y = F\cos\theta - mg \\ I\ddot{\theta} = \tau \end{cases} \qquad \begin{cases} -F_x = F\sin\theta \\ F_y + mg = F\cos\theta \end{cases}$$

$$\begin{cases} \tan\theta = \dfrac{-F_x}{F_y + mg} \\ F^2 = F_x^2 + (F_y + mg)^2 \end{cases}$$

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