

Hashrate Optimization in Proof-of-Work Mining Pools

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ABSTRACT

Despite an increased awareness about networking and systems properties of blockchains, such as Bitcoin, the internal policies and strategies deployed by crypto-currency miners and mining pools remain largely unknown. In this paper, we explore major proof-of-work mining pools, known to have a significant energy consumption, to understand their hashrate allocation policies towards different crypto-currencies. In particular, we focus on the Bitcoin family of crypto-currencies, i.e., BTC, BCH, and BSV. Because directly measuring a mining pool's hashrate is inherently challenging, we indirectly verify the publicly-reported hashrate by actively joining select mining pools. We further evaluate the hashrate allocation strategies, and find that they are sub-optimal. We show that strategically reallocating only a small fraction of existing pool's hashrate, one can come close to an optimal allocation. Finally, we explore the effects of hashrate reallocation strategies in light of important recent events: BSV price surge, BCH and BTC reward halving, and demonstrate their distinct impact on hashrate reallocation benefits.

1 INTRODUCTION

We aim to understand the proof-of-work (PoW) mining pools, i.e., their hashrate allocating decisions, by focusing on the Bitcoin family of crypto-currencies (cryptos). In addition to Bitcoin (BTC), other cryptos (e.g., BCH and BSV) apply exactly the same mining processes, hence the question is how a mining pool should allocate its hashrate across these different cryptos. While some cryptos, i.e., BTC, are approximately an order of magnitude more valuable than others (e.g., BCH and BSV), a significantly larger amount of hash power is required to mine it as well. At the same time, the price of cryptos and the allocated hashrate fluctuate, often widely, over time. Hence, dynamically re-distributing the hashrate appears as a viable policy, yet it is unknown how beneficial this approach might be.

An underlying problem associated with the above goal is that of measuring the hashrate that a mining pool allocates to a particular crypto. While inter-block times provide a ground-truth indicative of the actual mining power, this process is highly influenced by “luck,” i.e., a mining pool with a smaller hash power can mine more blocks over short time scales than a pool with higher hash power. This brings significant variability over short time scales, which blurs the estimates of the actual underlying hash power. An alternative measure is the hashrate published by the pools. These reportings are

unverified and cannot be blindly trusted given that mining pools have incentives to over-report their actual hash power.

Our first contribution is the introduction of a novel way to validating the hashrates reported by the mining pools. By joining the mining pools and actively mining cryptos, we find that mining pools accurately report their hashrate and truthfully compensate their miners.

Our second contribution is finding that the pools' hashrate allocation is largely sub-optimal; yet, varies across the different pools. As a result, a lot of “money is left on the table.” Specifically, most of the mining hashrate is dis-proportionally allocated to BTC, the most-highly priced crypto. We determine the conditions for optimal hashrate allocation and find that a pool need only control a rather small fraction of its hashrate to significantly increase its revenue. Indeed, surveying the leading mining pools, we find that some of the pools in fact control such hash power. As expected, we find that the shorter the time period between reallocations, the higher the benefits. Yet, the marginal loss from less frequent reallocations decreases. Likewise, we show that mining multiple cryptos enables more reallocation opportunities and larger returns.

Finally, we evaluate recent important events: (a) BSV price surge, (b) BCH and BSV halving, and (c) BTC halving. We demonstrate that during such events, when the variance of currency fluctuation is the highest, pools' reallocation strategies may be particularly valuable.

2 BACKGROUND AND RELATED WORK

Crypto Mining. Blockchain-based cryptos, such as Bitcoin, order and confirm transactions based on a consensus mechanism. The PoW typically involves solving some computationally complex problems, which necessarily require a lot of computational resources. Solving the PoW puzzles is referred to as *mining*.

The outcome of the mining is two-fold. First, the result of the PoW puzzle will be included in the next block, such that the PoW of the next block depends on it. In this way, the blocks are chained, and the entire process keeps the transactions trustworthy and secure. Second, the *miners*, who solved the PoW puzzle, will earn a reward in the native crypto along with all the *transaction fees* in the block.

Mining Pools. The high return from block rewards attracts many miners to join the competition and solve the PoW puzzles. As a result, Application Specific Integrated Circuits (ASICs) have been developed specifically for solving the PoW puzzles. Given its high computational capacity and its

power consumption optimization, ASICs quickly replaced CPU-based mining.

Still, nowadays it is virtually impossible for a miner to win a block reward with a single machine. The competition led to the formation of *mining pools*, which consist of many miners. The pools distribute small parts of the PoW to miners, often distributed across the world. This increases the pools' chance to win the reward. Based on the hash power contribution, miners who joined the pool proportionally share the income, as we further discuss below. The mining pools may or may not host mining machines themselves. Usually mining pools keep a small share of the block rewards.

Bitcoin and its forks. Currently, Bitcoin (BTC) is the most valuable crypto with the largest market share. Its PoW is SHA256 based. Several Bitcoin "forks" exist. Most prominently, Bitcoin Cash (BCH) forked from BTC in 2017. BCH later experienced a hard fork itself, which resulted in the creation of Bitcoin SV (BSV). All three cryptos have identical SHA256-based PoW algorithms, and they have the same settings for reward halving, *etc.* Given these similarities, all three cryptos can be mined with the same ASICs. Consequently, many mining pools support two or all of them. Furthermore, switching hash power across them is easy. This paper aims at understanding how well pools utilize this opportunity.

Prior Work. The evolution of Bitcoin mining and in particular the switch from solo to pooled mining was studied in [17]. In [10], the authors collected over 800,000 Bitcoin nodes and presented the scale and geographic distribution of the nodes. Authors of [16] conducted the latest long-term measurement of Bitcoin mining pools— using blocks' information to estimate the hashrate. In comparison, we perform a finer-grained study on multiple cryptos. Mining pools were further analyzed in the context of game theory [5, 7, 12–15, 18] and in similar related contexts [8]. In particular, the authors [7] proved that there exists a singular equilibrium for the resource allocation between any two cryptos, driven by the rewards. And slow and cautious adjustment of resource allocation will lead to equilibrium whereas there could be oscillations otherwise. It is noteworthy that it is proved that in an equilibrium, the miners may not want to devote all the power given the difference of mining cost [15], and the miners or mining pools have no financial incentive to occupy over 50% of the hashrate of a crypto unless one of them has outstandingly low mining cost [18]. Pools' hashrate allocation was examined in [6, 9, 11], taking the perspective of finance and risks; rather than the optimal allocation for profitability, as we do. To the best of our knowledge, our work is the first to analyze hashrate reallocation across multiple cryptos with the same PoW basis at a fine-grained time scale.

Ethics. This paper raises no ethical issues. The actual names of the explored mining pools are anonymized.

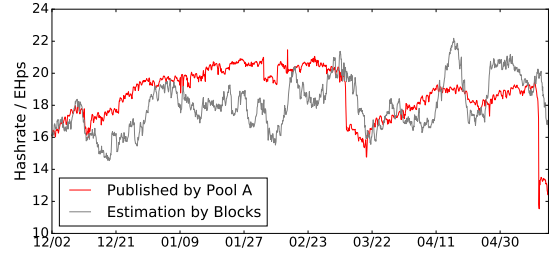


Figure 1: Pool A's estimated and published hashrate (Dec 2019 - May 2020).

3 HASHRATE VALIDATION

The two most popular methods to measure mining pools' hashrate are: 1) based on pools' block rewards [1, 17], and 2) the data published by the pools. While the first method works well when averaged over a long period of time, it is not a good fit for our analysis as we are interested in exploring frequent hashrate reallocations. To this end, below we offer a third unique approach based on miners' compensation.

Methodology. Estimating mining pools' hashrate based on the pools' block rewards has been presented and examined in [1, 17]. The paper uses the pool's inter-block time relative to the default *network difficulty*; i.e., the minimum threshold required for the acceptance of a result of PoW. For BTC, BCH and BSV, the network difficulty is set to a value that results in the creation of a new block, on average, every 10 minutes. Assuming a network difficulty of D and inter-block time T , the expected network hashrate H is then given by:

$$H = \frac{T}{10min} \cdot D \cdot 2^{32}. \quad (1)$$

A miner's or a pool's hashrate can be estimated replacing the inter-block time with that of the actual blocks earned.

While, on average, this method reflects the hashrate of a mining pool, this measure is heavily influenced by the pool's "luck"—i.e., the realization of the probability of mining a block. Consequently, when considering a short period of time, the method results in a large variance between the estimated and actual hashrate. This is especially problematic when considering small mining pools. Figure 1 demonstrates this by presenting the discrepancy between the hashrate reported by pool A and that calculated by method (1). Indeed, on average over a long period of time the two measures are similar. However, given that we want to consider daily hashrate allocation and reallocation, we cannot use this measure for our analysis.

To this end, we have developed a process to validate the truthfulness of the public data published by the mining pools. Specifically, we joined the different pools as miners and took advantage of particular payment methods pools utilize to pay their miners to calculate the pool's actual hashrate. The most

popular compensation methods are: Pay Per Share (PPS), Pay Per Share Plus (PPS+) and Pay Per Last N Share (PPLNS).

Under PPS, mining pools pay miners based on the hashrate the miner devoted to the pool and the network difficulty. For example, given a period of time and network difficulty D , if the miner devoted H hashrate power to the mining pool that was expected to result in N blocks mined, then if the reward from each block is R , the miner's income, I , is calculated as

$$I = \frac{H}{D \cdot 2^{32}} \cdot N \cdot R, \quad (2)$$

Note, that this compensation is paid out regardless of whether the blocks were indeed rewarded to the pool. That is, under PPS, miners earn a guaranteed fixed payment and the mining pool bears the risk of bad luck. Consequently, one cannot infer the hashrate of pools based on the PPS compensation.

Under PPLNS, the mining pool shares the risk with the miners and pays miners based on the actual block rewards the pool gained. For example, consider a time period where overall N blocks were mined on the network and assume the pool received rewards from $M \leq N$ blocks. If a miner devoted an average H hashrate power to the mining pool and pool's total hashrate is HP , the miner will receive

$$I = \frac{H}{HP} \cdot M \cdot R. \quad (3)$$

Note that miners know the hashrate they devote to the pool, H , and their income, I . Moreover, block rewarded to the pool, M , and the rewards, R , are publicly available. Assuming that pools pay miners honestly, as a miner, we can infer each pool's hashrate based on our compensation from each and every pool we joined.

While not all pools offer the PPLNS compensation method, an alternative compensation method that became one of the most common payment methods is Pay Per Share Plus (PPS+), which combines the PPS and PPLNS methods. In general, miners' compensation for mining BTC, BCH and BSV consists of two parts: 1) the base block reward; and 2) the transaction fees associated with the block mined. Under PPS+, the mining pool pays miners the base block reward as in PPS, and their proportional part out of transaction fees associated with the blocks mined. The mining pools usually keep a small portion of the transaction fees as their own profit. Just like with PPLNS, given that each mined block's transaction fees are public information, as a miner compensated by PPS+, we can verify the pool's hashrate.

We joined six different mining pools as miners and received compensation based on PPS or PPS+, depending on the compensation plans the pool offered for the different cryptos.

Data Collection. We collect the hashrates reported by the mining pools from miningpoolStats [4], a leading website for hashrate information that aggregates pools' information by APIs to the pools' websites. We have continuously collected

Table 1: Verification of hashrate of the mining pools with PPS+ and PPS payment methods.

Pool	Expected Income	Actual Income	Diff
Transaction fees in rewarded blocks (PPS+)			
Pool B	0.00007317	0.00007308	-0.12%
Pool C	0.00011046	0.00011185	1.24%
Pool D	0.00008358	0.00008271	-1.05%
Base block reward (PPS)			
Pool A	0.002315953	0.00228921	-1.15%
Pool B	0.002232367	0.00218474	-2.13%
Pool C	0.002271367	0.00225153	-0.87%
Pool D	0.002176690	0.00219443	0.81%
Pool E	0.002165870	0.00219353	1.26%
Pool F	0.002347605	0.002322747	-1.05%

the hashrate published by the mining pools and the block information since Dec 1, 2019.

We obtained data on blocks mined from blockchair.com [2]. The information includes block time, block rewards, transaction fees included with the block, and the guessed miner (mining pool). Though the guessed miner is not presented in every block, we find that for the major pools, the information is accurate comparing against the data published by the pools. We derive the total hashrate each crypto network from the network difficulty stored in the block information. Finally, we collect the cryptos' market price from CoinGeko [3].

In terms of hardware, we have placed an ASIC machine of type AntMiner S9 SE. It has an energy consumption of 1360W and can achieve around 17T hash computations per second for BTC, BCH or BSV mining. We started mining on Mar 18, 2020; and the data in this paper covers the period until May 18, 2020. We joined six major mining pools, whose sum hashrate exceeds half of the total hashrate devoted to BTC, BCH and BSV. We join each pool and mine BTC for one week at a time; iterating through the six mining pools continuously.

Given that we switch pools every day, we do not have continuous data for each pool but rather see windows of operation. Consequently, unfortunately, we cannot use our data for the analysis we want to perform. Instead, we use our data and methodology to test whether the hashrate reported by the pools can be trusted and used in our analysis.

Do pools report truthfully? To answer this question, we take all pools that paid us based on the PPS+ compensation method and compare our actual income to the income we would have expected to gain if we calculate our compensation based on the hashrate published by the pool. The expected income is calculated based on Eq. 3 where before the BTC halving on May 09, 2020, the base block reward was 12.5

coins, and became 6.25 after. As Table 1 shows, the discrepancy between our measure and the values published by the pools is minimal. Moreover, given that we mined for the same period of time and devoted the same level of hashrate to all pools, we can infer that the much higher compensation we received from Pool C implies that during the time that we conducted the experiment, Pool C had the best luck and Pool B got the least luck either in terms of blocks mines or in terms of transaction fees from blocks.

The hashrate verification above assumes that the pools pay their miners honestly. In order to verify that this is indeed the case, we use the compensation we received based on PPS, Table 1 shows PPS income we received from the pools and compares it to the expected income based on Eq.2. As expected, the actual PPS income from all six pools is almost the same. Moreover, the difference between the actual and expected income is minimal.

We, therefore, conclude that the data reported by the six pools we study is reliable.

4 HASHRATE REALLOCATION

Mining pool's profitability depends on the combined hash power of the pool's miners as well as on how the pool allocates this hash power across the different cryptos it mines. For example, while the block reward of BTC is higher than the reward from mining BCH, the probability of winning a block (given a certain amount of hashrate) is lower for BTC as compared to BCH and BSV. This is so because there are more miners competing over mining BTC blocks than miners competing over mining BCH or BSV. While easily transferring hashrate across different cryptos is not always possible, there are "families" of cryptos for which such a switch is easy and almost cost-less. In fact, many pools provide one-click switching for miners, and some even support "smart switching" that enables miners to automatically switch from mining one crypto to another, based on user-defined policies. This suggests that an important decision a mining pool must make is how many cryptos to mine and how to allocate its hash power across these cryptos. Below we first present a model for optimal hashrate allocation and then estimate the potential monetary benefits for the different pools, were they to follow this *optimal allocation* of their hash power. Finally, we test whether the benefits from actively reallocating hash power are higher when the market exhibits exogenous shocks—expected and unexpected—such as a surge in prices or reward halving.

Hashrate Transfer. While many pools offer miners the service of allocating their hashrate, in many cases, it is the miners themselves that choose which crypto to mine. That is, hashrate transfer is not always in the control of the pool. One way for pools to have the flexibility for hashrate transfer is to "own" some of the hashrate used by the pool. Indeed, in a

survey we sent to leading mining pools, we found that some of the pools control a portion of the mining hashrate, *i.e.*, are miners themselves. As we show below, controlling even a small portion of the pool's hashrate in most cases enables the pool to achieve near-optimal allocation and results in substantial reward.

Optimal Allocation Model. Consider N cryptos that miners can freely switch among. Denote by H_i the hashrate devoted to crypto i by a pool P . Denote by H_i^r the total hashrate devoted to crypto i by *other* pools. Finally, denote by R_i the block reward of crypto i . The goal of the mining pool is to maximize its revenues (assuming the cost per unit of hashrate does not depend on the crypto mined), *i.e.*,

$$\textbf{Objective: } \max \sum_i R_i \cdot \frac{H_i}{H_i^r + H_i}. \quad (4)$$

Eq. 4 is at its optimal when:

$$\frac{\sqrt{R_1 H_1^r}}{H_1^r + H_1} = \frac{\sqrt{R_2 H_2^r}}{H_2^r + H_2} = \dots = \frac{\sqrt{R_N H_N^r}}{H_N^r + H_N}. \quad (5)$$

Denote by $H^P = \sum_i^N H_i$ the total hashrate of pool P . The optimal hashrate pool P should devote to crypto i , H_i , is then

$$H_i = \frac{H^P \sqrt{R_1 H_1^r} + \sqrt{R_1 H_1^r} \sum_{j \neq i} H_j^r - H_i^r \sum_j \sqrt{R_j H_j^r}}{\sum_j R_j}. \quad (6)$$

Below, we use this optimal hashrate allocation to perform numerical derivations and simulations to test whether pools allocate optimally. We then estimate the expected monetary benefits for pools from employing the above optimal allocation.

How Sub-Optimally Do Pools Perform? We analyze the hashrate allocation of six pools—all among the largest pools in terms of hashrate devoted to BTC, BCH and BSV. Pool A, Pool B, and Pool C mine all three cryptos, whereas Pool D, Pool E, and Pool F only mine BTC and BCH.

To standardize the data, we take each data point to be the average hashrate published over one hour (pools publish data every 1-3 minutes). Then, we apply the optimal hashrate allocation for that point of time. In the analysis we experiment with the frequency of reallocation where if the pool reallocates every x hours ($x > 1$), we keep optimal hashrate set at time t till time $t + x$. We consider the pool's revenue per block mined to be the crypto's block reward multiplied by the current market price.

To demonstrate the value from following the optimal allocation we derived above, Figure 2 compares the pools' actual revenue to the revenues the pool would have enjoyed under optimal allocation. Given that pools do not control and cannot optimally utilize all their hashrate, we look at the benefits as a function of the percentage of hashrate the pool can transfer across the different cryptos. We present results for optimal

allocations across two cryptos (left panel) and three cryptos (right panel).

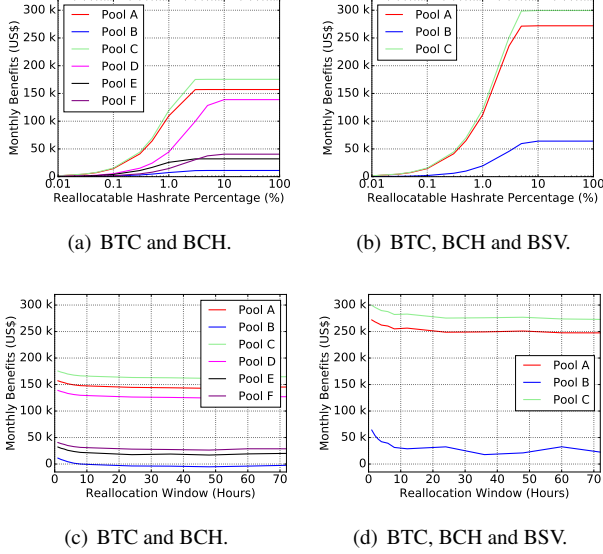


Figure 2: Monthly average benefit from optimal hashrate allocation from Dec 2019 to May 2020.

Two Cryptos. We first look at the scenario where the hashrate is allocated between two cryptos. Since the results for reallocations between BTC and BCH are similar to those achieved when reallocating between BTC and BSV, we only present the results for BTC and BCH case. Figure 2(a) shows that the increased value from following the optimal allocation differs substantially across the different pools and crucially depends on the percentage of hashrate the pool can transfer. This implies that some of pools can be more strategic than others in their hashrate allocation. Specifically, Pools A, C, and D would benefit the most from optimal allocation, whereas Pool B has the least potential benefit—suggesting that Pool B’s allocation policy is relatively close to optimal.

Figure 2(a) also demonstrates that the percent of hashrate pools need to control in order to achieve the optimal allocation does not need to be large. For example, Pool C and B need to control approximately 2% of their hashrate, in order to reach near-optimal performance. In general, controlling between 3% and 10% of a pool’s hashrate enables optimal performance, and provides monthly benefits at the order of hundreds of thousands of USD, as shown on the y-axis.

Our analysis so far assumed that pools reallocate hashrate every hour. This might require too much logistics and thus raises the question of whether the returns from the optimization that is not as frequent would be significantly lower than when reallocating every hour. Figure 2(c) depicts the monthly pool rewards as a function of the reallocation frequency. As

expected, the figure shows that reallocating every hour provide the largest potential benefits such that as the time window between reallocations gets longer, the potential benefits decrease. It is noteworthy that the decrease in the potential benefits due to less frequent hashrate reallocation flattens once re-optimization is performed every 12 hours or more. This suggests that pools that cannot optimize every half a day can re-optimize every other day or two without losing much.

Three Cryptos. Only Pools A, B, and C mine all three cryptos. Figure 2(b) shows that the benefit from reallocating across three cryptos almost doubles the benefits relative to the two-cryptos hashrate reallocation. The pools need to control a smaller share of their hashrate in order to achieve the optimal allocation. Moreover, if the optimal allocation cannot be reached, *i.e.*, the pool controls less than 5% of its hashrate, the benefit from three-way reallocation is still slightly larger than the two-way reallocations. That is, reallocating over more cryptos increases the potential benefits from hashrate reallocation.

Event Magnifier. In this section we explore how exogenous shocks affect the potential benefits from the optimal allocation. Specifically, during the six months for which we have data, three important events occurred: (i) BSV’s market price surpassed that of BCH; (ii) BCH and BSV block reward halving; and (iii) BTC block reward halving. The three events are depicted in Figure 3, which shows the hashrate and market price for the three cryptos over the six months for which we have data. In the analysis below, we assume that the pools can control 10% of their hashrate and reallocate it every hour. We depict the *daily* benefits for the explored mining pools as a function of the time of the event, where Day 0 marks the time of the explored event.

BSV Surpasses BCH. On Jan 14, 2020, the market price of BSV surpassed that of its original BCH fork for the first time (BCH also went through a price surge that day). In order to capture the effect on potential benefits from reallocation, we only study the pools that mine all 3 cryptos. As expected, Figure 4(a) shows that the peak of benefits happens on Day 0, when the given event occurs. The peak is more pronounced for Pool A and Pool C, and is at the order of \$20k. The peak for Pool B is much smaller, which is consistent with the above results—*i.e.*, Pool B likely is more strategic in its hashrate allocation. Interestingly, there is a small peak few days before and after the event. This is consistent with the fact that some in the BTC/BCH/BSV community have anticipated the event and prices of the three cryptos responded accordingly. The peaks in the days following the event suggest that the event has affected the overall balance between BCH and BSV and it has taken the market a while to return to equilibrium in terms of the relative prices and hashrate allocation of these cryptos.

BCH and BSV Halving. On Apr 8 and 10, 2020, the base block rewards of BCH and BSV halved from 12.5 to 6.25

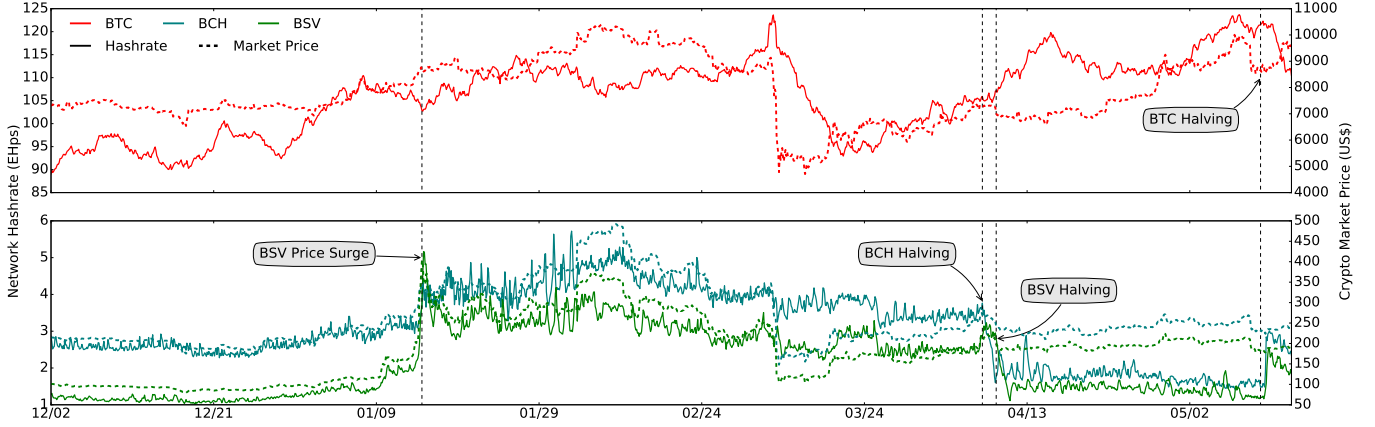


Figure 3: The network hashrates and market prices of BTC, BCH and BSV from Dec 2019 to May 2020.

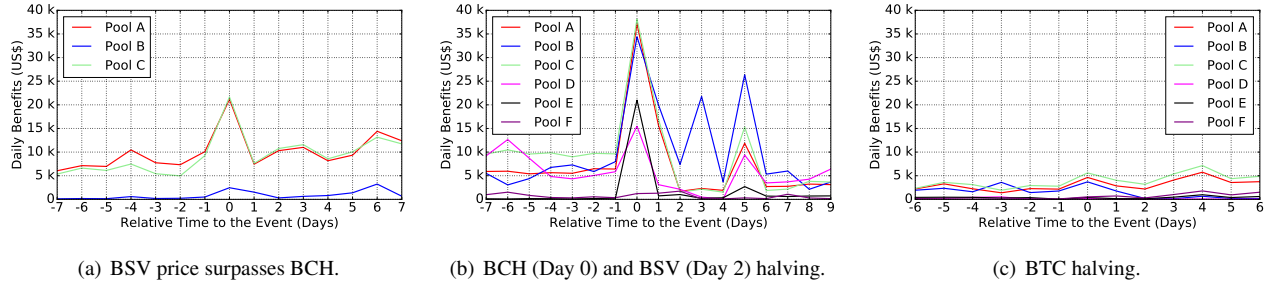


Figure 4: Daily benefit from optimal hashrate allocation for different events.

coins per block. Figure 4(b) shows that the above events lead to a significant departure from optimal hashrate allocation, with a daily benefit peak reaching \$37k on Day 0 (i.e., Apr 8). Contrary to the previous scenarios, Pool B departs the most from the optimal allocation in this case. This suggests that all three pools have mis-estimated the changes in hashrate allocation the halving would cause. In general, the figure shows that potential benefits for different pools follow similar trends. The hashrate devoted to BCH and BSV experience oscillations after the event, resulting in the oscillations in the benefits from optimal allocations. This is likely the result of all pools re-optimizing based on what other pools are doing. According to the figure, it took the market at least a week to get back to equilibrium hashrate allocation.

BTC Halving. On May 10, 2020, the base block rewards of BTC halved from 12.5 to 6.25 coins per block. Our expectation was that this would cause a significant "shock waves" in the mining ecosystem. To our surprise, the results in Figure 4(c) shows the smallest amounts of potential benefits (peak at around \$5k on Day 0), hence the smallest departure from optimal. We believe that there are two main reasons for this result. First, BTC is the most lucrative crypto today—its price is approximately 40 times higher than that of BCH or BSV. Hence, even when the block rewards are halved, the optimal

hashrate allocation does *not* change substantially. Second, since all pools have gone through several BTC halvings, it is likely that the pools can better anticipate the effect of BTC halving on overall hashrate allocation. As before, the figure shows that Pool B adjusts quickly to the new situation, while Pool A and Pool C continue sub-optimally favoring BTC, in the days after its halving.

5 CONCLUSIONS

This paper provided a novel insight into the hashrate allocation policies, and their economic consequences, of the major PoW mining pools. We present an original hashrate measurement methodology and demonstrate that mining pools' hashrates can be indirectly, yet cheaply, accurately, and scalably measured, thus making the mining pools accountable. This provides an opportunity to analyze the pools' hashrate allocations towards different cryptos. By focusing on the Bitcoin family of cryptos (BTC, BCH, and BSV), we found the following: (i) the current hashrate allocation by the mining pools is sub-optimal, mostly due to a large concentration of the hash power towards BTC. (ii) We devise a model for optimal hashrate allocation for an arbitrary number of cryptos, and demonstrate that the mining pools that control even a

relatively small fraction of the pool's hashrate can utilize near-optimal performance. (iii) The achievable benefits are particularly pronounced when the hashrate is allocated towards multiple cryptos, and during important events (i.e., reward halving or price surges), when the fluctuations in rewards and allocated hashrates are the largest.

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