CFA® Program
Level I

FORMULA SHEET (2024) Version 1.0

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FINANCE | RISK | SUSTAINABILITY

FOR REFERENCE ONLY

(Note: Formula Sheet is not provided in the CFA exam)

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CFA Level 1 – Formula Sheet (2024)

Setting Up the Texas BA II Plus Financial Calculator

Video: https://youtu.be/0MS8d8QOFmc

Using Texas BA II Plus Financial Calculator

Video: https://youtu.be/LWmTTiZz8BU

Video (Requires Login to Facebook): https://fb.watch/nci5V7Dwtj/

QUANTITATIVE METHODS

Learning Module 1: Rates and Returns

Determinants of Interest Rates

Interest rate, r = Real risk-free rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

(1 + Nominal risk-free rate) = (1 + Real risk-free rate) × (1 + Inflation premium)

Nominal risk-free rate = Real risk-free rate + Inflation premium

Maturity premium = Interest rate on longer-maturity, liquid Treasury debt − Interest rate on short-term Treasury debt

Holding Period Return

$$R = \frac{P_1 - P_0 + I_1}{P_0}$$

where:
P_0 = Price at the beginning of the period
P_1 = Price at the end of the period
I_1 = Income

If given holding period returns $R_1, R_2, \ldots, R_T$ over the holding period:

$$R = (1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_T) - 1$$

Arithmetic Return

$$\bar{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it} = \frac{1}{T}(R_{i1} + R_{i2} + \cdots + R_{iT})$$

Geometric Mean Return

$$\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^{T}(1+R_t)} - 1 = \sqrt[T]{(1+R_{i1}) \times (1+R_{i2}) \times \ldots \times (1+R_{iT})} - 1$$

Harmonic Mean

$$\bar{X}_{Hi} = \frac{n}{\sum_{i=1}^{n}(1/X_i)} \qquad for\ X_i > 0$$

Relationship between Arithmetic Mean, Geometric Mean, and Harmonic Mean

$$(Geometric\ mean)^2 = Arithmetic\ mean \times Harmonic\ mean$$

Money-Weighted Return (MWR)

$$\sum_{t=0}^{T} \frac{CF_t}{(1+MWR)^t} = 0$$

Time-Weighted Return (TWR)

Given the holding period returns for each sub-period, R_1, R_2, …, R_T

If T > 1 year, then

$$Annualized\ TWR = [(1+R_1) \times (1+R_2) \times \ldots \times (1+R_T)]^{1/T} - 1$$

If $T = 1$ year, then

$$Annualized\ TWR = (1+R_1) \times (1+R_2) \times \ldots \times (1+R_T) - 1$$

If $T < 1$ year, then

$$TWR\ for\ holding\ period = (1+R_1) \times (1+R_2) \times \ldots \times (1+R_T) - 1$$

Non-Annual Compounding

$$PV = FV_N\left(1 + \frac{R_S}{m}\right)^{-mN}$$

where:

m = Number of compounding periods per year

R_s = Quoted annual interest rate

N = Number of years

Annualizing Returns

$R_{annual} = (1 + R_{weekly})^{52} - 1$

$R_{annual} = (1 + R_{monthly})^{12} - 1$

$R_{annual} = (1 + R_{daily})^{252} - 1$ assuming 252 trading days per year

$R_{weekly} = (1 + R_{daily})^{5} - 1$ assuming 5 trading days per week

Continuously Compounded Returns

$$P_t = P_0 e^{r_{0,T}}$$

$$r_{0,T} = \ln\left(\frac{P_t}{P_0}\right)$$

$$r_{0,T} = r_{0,1} + r_{1,2} + \cdots + r_{T-2,T-1} + r_{T-1,T}$$

Real Returns

(1 + real return) = (1 + real risk-free rate) × (1 + risk premium)

Pre-Tax and After-Tax Nominal Return

After-tax nominal return = Pre-tax nominal return × (1 – Tax rate)

$$After\text{-}tax\ real\ return = \frac{[1 + Pre\text{-}Tax\ nominal\ return \times (1 - Tax\ rate)]}{1 + Inflation\ premium} - 1$$

Leveraged Return

Return on a leveraged portfolio

$$R_L = R_P + \frac{V_B}{V_E}(R_P - r_D)$$

where:

R_P = Return on the investment portfolio (unleveraged)

r_D = Cost of debt

V_B = Debt/borrowed funds

V_E = Equity of the portfolio

Learning Module 2: The Time Value of Money in Finance

$$FV_t = PV(1+r)^t \qquad\qquad PV = \frac{FV_t}{(1+r)^t}$$

where:

FV_t = Future value at time t

PV = Present value

r = Discount rate per period

t = Number of compounding periods

As compounding frequency becomes very large (i.e., continuous compounding)

$$FV_t = PVe^{rt} \qquad\qquad PV = FV_t e^{-rt}$$

Present Value of Zero-Coupon Bond

$$PV(Discount\ Bond) = \frac{FV}{(1+r)^t}$$

where:

FV = Principal (or Face Value)

r = Market discount rate per period

t = Maturity of bond

$$r = \left(\frac{FV_t}{PV}\right)^{1/T} - 1$$

Present Value of Coupon Bond

$$PV(Coupon\ Bond) = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N}$$

where:

PV = Bond's price

PMT = Periodic coupon payment

FV = Face value

N = Number of periods

r = Market discount rate per period

Present Value of a Perpetual Bond (Perpetuity)

$$PV(Perpetual\ Bond) = \frac{PMT}{r}$$

Annuity Instruments (e.g., Mortgage)

$$A = \frac{rPV}{1-(1+r)^{-t}}$$

where:

A = Periodic cash flow

r = Market interest rate per period

PV = Present value or principal amount of loan/bond

t = Number of payment periods

Price of a Preferred Share

$$PV_t = \frac{D_t}{r}$$

where:

D_t = Fixed periodic dividend

r = Expected rate of return

Price of a Common Share

Constant Dividend Growth Rate into Perpetuity

$$PV_t = \frac{D_t(1+g)}{r-g} = \frac{D_{t+1}}{r-g} \qquad r > g$$

where:

D_t = Common dividend at time t

g = Constant growth rate

r = Expected rate of return

$$r = \frac{D_{t+1}}{PV_t} + g$$

$$\frac{PV_t}{E_t} = \frac{\frac{D_t}{E_t} \times (1+g)}{r-g}$$

$$\frac{PV_t}{E_{t+1}} = \frac{\frac{D_{t+1}}{E_{t+1}}}{r-g}$$

where:

E_t = Earnings per share for period t

$\frac{PV_t}{E_t}$ = Trailing price-to-earnings ratio

$\frac{PV_t}{E_{t+1}}$ = Forward price-to-earnings ratio

Two-stage Dividend Discount Model

$$PV_t = \sum_{i=1}^{n} \frac{D_t(1+g_s)^i}{(1+r)^i} + \frac{E(S_{t+n})}{(1+r)^n}$$

where:

g_s = Higher short-term dividend growth rate

g_L = Lower long-term dividend growth rate

n = Initial growth phase

$E(S_{t+n})$ = Stock value in n periods (Terminal value)

$$= \frac{D_{t+n+1}}{r - g_L}$$

Forward Rate

$$F_{1,1} = \frac{(1+r_2)^2}{(1+r_1)} - 1$$

where:

$F_{1,1}$ = One-year forward rate one year from now

r_1 = Discount rate on one-year risk-free discount bond

r_2 = Discount rate on two-year risk-free discount bond

Learning Module 3: Statistical Measures of Asset Returns

Measures of Central Tendency

$$Sample\ Mean, \qquad \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

where:

X_i = Observation i ($i = 1, 2, 3, \dots, n$)

Median

$$\text{Position of median} = \frac{Number\ of\ observations + 1}{2}$$

Quantiles

$$\text{Interquartile range} = Q_3 - Q_1$$

where: Q_1 = First quartile

Q_3 = Third quartile

Box and Whisker Plot

$$Upper\ fence = Q_3 + 1.5 \times IQR$$

$$Lower\ fence = Q_1 - 1.5 \times IQR$$

Measures of Dispersion

Range = Maximum value – Minimum value

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n} |X_i - \bar{X}|}{n}$$

Sample Variance

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}$$

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}}$$

Sample Target Semideviation

$$s_{Target} = \sqrt{\frac{\sum_{X_i \le B}^{n} (X_i - B)^2}{n - 1}}$$

where:
B = target
n = total number of sample observations

Coefficient of Variation

$$CV = \frac{s}{\bar{X}}$$

Sample Skewness

$$Skewness \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^{n} (X_i - \bar{X})^3}{s^3}$$

Sample Excess Kurtosis

$$K_E \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^{n} (X_i - \bar{X})^4}{s^4} - 3$$

Sample Covariance

$$s_{XY} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})$$

Sample Correlation Coefficient

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

Learning Module 4: Probability Trees and Conditional Expectations

Expected Value of a Discrete Random Variable

$$E(X) = \sum_{i=1}^{n} P(X_i)\, X_i$$

Variance of a Random Variable

$$\sigma^2(X) = E[X - E(X)]^2$$
$$= \sum_{i=1}^{n} P(X_i)\, [X - E(X)]^2$$

Conditional Expected Value of a Random Variable

$$E(X|S) = P(X_1|S)\, X_1 + P(X_2|S)\, X_2 + \cdots + P(X_n|S)\, X_n$$

Conditional Variance of a Random Variable

$$\sigma^2(X|S) = P(X_1|S)[X_1 - E(X_1|S)]^2 + P(X_2|S)[X_2 - E(X_2|S)]^2 + \cdots$$
$$+ P(X_n|S)[X_n - E(X_n|S)]^2$$

Total Probability Rule for Expected Value

$$E(X) = E(X|S_1)P(S_1) + E(X|S_2)P(S_2) + \cdots + E(X|S_n)P(S_n)$$

where: S_1, S_2, …, S_n are mutually exclusive and exhaustive events.

Bayes' Formula

$$P(A|B) = \frac{P(B|A)}{P(B)} \times P(A)$$

$$P(Event|Information) = \frac{P(Information|Event)}{P(Information)} \times P(Event)$$

Video (Bayes' Formula and Total Probability Rule): https://youtu.be/9_h0EzssPZ4

Learning Module 5: Portfolio Mathematics

For n assets in a portfolio

Expected return on portfolio

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_n E(R_n)$$

Variance on portfolio

$$\sigma^2(R_P) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(R_i, R_j)$$

Requires n variances and $\frac{n(n-1)}{2}$ distinct covariances to estimate portfolio variance.

Covariance

$$Cov(R_i, R_j) = E\left[\left(R_i - E(R_i)\right)\left(R_j - E(R_j)\right)\right]$$
$$= \frac{1}{n-1} \sum_{t=1}^{n} (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)$$

For a two-asset ($n = 2$) portfolio:

$$\sigma^2(R_P) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 Cov(R_1, R_2)$$

where: $Cov(R_1, R_2) = \rho(R_1, R_2) \times \sigma(R_1) \times \sigma(R_2)$

Video: https://youtu.be/lUwulZ9ONC0

For a three-asset ($n = 3$) portfolio:

$$\sigma^2(R_P) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2 w_1 w_2 Cov(R_1, R_2)$$
$$+ 2 w_1 w_3 Cov(R_1, R_3) + 2 w_2 w_3 Cov(R_2, R_3)$$

Covariance Given a Joint Probability Function

$$Cov(R_A, R_B) = \sum_{i=1} \sum_{j=1} P(R_{A,i}, R_{B,j}) \times [R_{A,i} - E(R_A)] \times [R_{B,j} - E(R_B)]$$

If X and Y are uncorrelated, then $E(XY) = E(X)E(Y)$

If X and Y are independent, then $P(X, Y) = P(X)P(Y)$

Safety-First Optimal Portfolio

Safety-First Ratio

$$SFRatio = \frac{E(R_P) - R_L}{\sigma_P}$$

$$Shortfall\ risk = \Pr[E(R_P) < R_L] = Normal(-SFRatio)$$

where:
R_L = Investor's threshold level
$E(R_P)$ = Expected portfolio return
σ_P = Portfolio standard deviation

Video: https://youtu.be/S3x5JrGIOUA

Learning Module 6: Simulation Methods

Lognormal Distribution

Mean of a lognormal random variable

$$\mu_L = \exp(\mu + 0.50\sigma^2)$$

Variance of a lognormal random variable

$$\sigma_L^2 = \exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]$$

where:
μ = Mean of the normal random variable
σ^2 = Variance of the normal random variable

Continuously Compounded Rates of Return

$$P_T = P_0 \exp(r_{0,T})$$

where:
P_0 = Current asset price
P_T = Asset price at time T
$r_{0,T}$ = Continuously compounded return from 0 to T

If returns are independently and identically distributed (i.i.d.), then

$$r_{0,T} = r_{0,1} + r_{1,2} + \cdots + r_{T-2,T-1} + r_{T-1,T}$$

If the one-period continuously compounded returns are i.i.d. random variables with mean μ and σ^2, then

$$E(r_{0,T}) = \mu T$$
$$\sigma^2(r_{0,T}) = \sigma^2 T$$

$$\sigma(r_{0,T}) = \sigma\sqrt{T}$$

Learning Module 7: Estimation and Inference

$$Sharpe\ ratio = \frac{R_P - R_F}{\sigma_P}$$

where:
R_P = Portfolio return
R_F = Risk-free rate
σ_P = Portfolio standard deviation of return

$$\begin{matrix} Variance\ of\ the\ sampling\ distribution \\ of\ the\ sample\ means \end{matrix} = \frac{\sigma^2}{n}$$

$$\begin{matrix} Standard\ error\ of \\ the\ sample\ mean \end{matrix} = \frac{\sigma}{\sqrt{n}}$$

where:
σ = Population standard deviation
n = Sample size

Note: If σ is not known, use s, the sample standard deviation.

Bootstrap Resampling

$$s_{\bar{X}} = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}_b - \bar{\theta}\right)^2}$$

where:
$s_{\bar{X}}$ = Estimate of the standard error of the sample mean
B = Number of resamples drawn from the original sample
$\hat{\theta}_b$ = Mean of a resample
$\bar{\theta}$ = Mean across all the resample means

Learning Module 8: Hypothesis Testing

$$\text{Confidence level} = 1 - \alpha$$

$$\text{Power of the test} = 1 - \beta$$

where:
α = Significance level (Probability of Type I error)
β = Probability of Type II error

Test of a Single Mean

Test statistic

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Degrees of freedom $= n - 1$

$$(1-\alpha)\%\ Confidence\ Interval = \bar{X} + Critical\ value \times \left(\frac{s}{\sqrt{n}}\right)$$

Test of the Difference in Means

Test statistic

$$t = \frac{(\bar{X}_{d1} - \bar{X}_{d2}) - (\mu_{d1} - \mu_{d2})}{\sqrt{\frac{s_p^2}{n_{d1}} + \frac{s_p^2}{n_{d2}}}}$$

Degrees of freedom $= n_{d1} + n_{d2} - 2$

$$s_p^2 = \frac{(n_{d1} - 1)s_{d1}^2 + (n_{d2} - 1)s_{d2}^2}{n_{d1} + n_{d2} - 2}$$

Test of the Mean of Differences

Test statistic

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

Degrees of freedom $= n - 1$

Test of a Single Variance

Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Degrees of freedom $= n - 1$

Test of the Difference in Variances

Test statistic

$$F = \frac{s_{Before}^2}{s_{After}^2}$$

Degrees of freedom $= n_1 - 1,\ n_2 - 1$

Test of a Correlation

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom $= n - 2$

Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^{m} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

Degrees of freedom $= (r-1)(c-1)$

where:
$m =$ Number of cells in the table
$O_{ij} =$ Number of observations in each cell of row i and column j
$E_{ij} =$ Expected number of observations in each cell of row i and column j

Learning Module 9: Parametric and Non-Parametric Tests of Independence

Test of a Correlation

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom $= n - 2$

Pearson Correlation (or Bivariate Correlation)

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

Spearman Rank Correlation Coefficient

$$r_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$$

where:

d = Difference in ranks

Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^{m} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

Degrees of freedom $= (r-1)(c-1)$

where:

m = Number of cells in the table

O_{ij} = Number of observations in each cell of row i and column j

E_{ij} = Expected number of observations in each cell of row i and column j

$$= \frac{(Total\ row\ i) \times (Total\ column\ j)}{Overall\ total}$$

Standardized Residual (or Pearson Residual)

$$Standardized\ Residual = \frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}}}$$

Learning Module 10: Simple Linear Regression

$$Y_i = b_0 + b_1X_1 + \cdots + b_nX_n + \varepsilon_i, \quad i = 1, 2, \ldots, n$$

where:
Y = Dependent variable
X = Independent variable
b_0 = Intercept
b_i = Slope coefficient, $i = 1, 2, \ldots, n$
ε_i = Error term
$b_0, b_1, \ldots, b_n$ = Regression coefficients

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1X_i + e_i$$

where:
$\hat{Y}_i$ = Estimated value on the regression line for the *i*th observation
$\hat{b}_0$ = Intercept
$\hat{b}_1$ = Slope
e_i = Residual for the *i*th observation

$$\hat{b}_1 = \frac{Covariance\ of\ X\ and\ Y}{Variance\ of\ X} = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1\bar{X}$$

Sum of Squares Total, $SST = \sum_{i=1}^{n}(Y_i - \bar{Y})^2 = SSR + SSE$

Sum of Squares Regression, $SSR = \sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2$

Sum of Squares Error, $SSE = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n} e_i^2$

Coefficient of Determination

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

Correlation coefficient

$$r = \frac{Covariance\ of\ X\ and\ Y}{(Standard\ deviation\ of\ X)(Standard\ deviation\ of\ Y)}$$

Note: (Correlation coefficient)^2 = Coefficient of determination

Sample standard deviation of X

$$S_X = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

Sample standard deviation of Y

$$S_Y = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}}$$

Homoskedasticity

$$E(\varepsilon_i^2) = \sigma_\varepsilon^2, \quad i = 1, 2, \ldots, n$$

ANOVA F-Test

Mean square regression (MSR)

$$MSR = \frac{SSR}{k}$$

Mean square error (MSE)

$$MSE = \frac{SSE}{n-k-1}$$

F-distributed test statistic

$$F = \frac{MSR}{MSE}$$

where:
n = Number of observations
k = Number of independent variables

Standard error of estimate

$$s_e = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n-k-1}}$$

Hypothesis Test of the Slope Coefficient

$$t = \frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$$

Degrees of freedom, $df = n - k - 1$

where:

B_1 = Hypothesized population slope

$s_{\hat{b}_1}$ = Standard error of the slope coefficient

$$= \frac{s_e}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

Hypothesis Test of the Intercept

$$t_{intercept} = \frac{\hat{b}_0 - B_0}{s_{\hat{b}_0}}$$

Standard error of the intercept, $s_{\hat{b}_0}$

$$s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

Prediction Intervals

$$\hat{Y}_f \pm t_{\alpha/2} \times s_f$$

where: $\hat{Y}_f = \hat{b}_0 + \hat{b}_1 X_f$

Variance of the prediction error of Y, given X

$$s_f^2 = s_e^2 \left[1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{(n-1)s_X^2}\right]$$

Standard error of the forecast

$$s_f = s_e \sqrt{1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{(n-1)s_X^2}}$$

The Log-Lin Model

$$\ln Y_i = b_0 + b_1 X_i$$

The Lin-Log Model

$$Y_i = b_0 + b_1 \ln X_i$$

The Log-Log Model

$$\ln Y_i = b_0 + b_1 \ln X_i$$

Learning Module 11: Introduction to Big Data Techniques

No formula.

ECONOMICS

Learning Module 1: Firms and Market Structures

Total profit = Total revenue – Total cost

Economic profit = Total revenue – Total **economic** costs

Accounting profit = Total revenue – Total **accounting** costs

$$Total\ revenue = Price \times Quantity = P \times Q$$

$$Average\ revenue = \frac{Total\ revenue}{Quantity}$$

$$Marginal\ cost = \frac{\Delta TC}{\Delta Q}$$

$$Average\ variable\ cost = \frac{Total\ variable\ cost}{Quantity}$$

$$Average\ fixed\ cost = \frac{Total\ fixed\ cost}{Quantity}$$

Total cost = Total fixed cost + Total variable cost

Average total cost = Average fixed cost + Average variable cost

Concentration Ratio

$$Concentration\ ratio = \sum_{i=1}^{n} (Market\ share)_i$$

Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{n} (Market\ share)_i^2$$

Learning Module 2: Understanding Business Cycles

No formula

Learning Module 3: Fiscal Policy

$$Budget\ \ surplus/(deficit) = G - T + B$$

where:
G = Government spending
T = Taxes
B = Payments of transfer benefits

Disposable Income

$$YD = Y - NT = (1 - t)Y$$

where:
t = Net tax rate
NT = Net taxes = Taxes – Transfers
tY = Total tax revenue

The Fiscal Multiplier

$$Fiscal\ multiplier = \frac{1}{1 - c(1 - t)}$$

where:
c = Marginal propensity to consume
t = Tax rate

Learning Module 4: Monetary Policy

$$Neutral\ rate = Trend\ growth + Inflation\ target$$

Learning Module 5: Introduction to Geopolitics

No formula

Learning Module 6: International Trade

No formula

Learning Module 7: Capital Flows and the FX Market

$$Real\ exchange\ rate_{d/f} = S_{d/f} \times \frac{P_f}{P_d}$$

$$\begin{aligned}\%\ Change\ in\ real\ exchange\ rate &= \left(1 + \%\Delta S_{d/f}\right) \times \frac{\left(1 + \%\Delta P_f\right)}{\left(1 + \%\Delta P_d\right)} - 1 \\ &\approx \%\Delta S_{d/f} + \%\Delta P_f - \%\Delta P_d\end{aligned}$$

Percentage change in base currency f (vs currency d)

$$\frac{E\left(S_{d/f}\right) - S_{d/f}}{S_{d/f}}$$

where:
$S_{d/f}$ = Spot exchange rate
P_f = General price level of goods indexed in currency f
P_d = General price level of goods indexed in currency d

Learning Module 8: Exchange Rate Calculations

Cross-Rate

$$\frac{A}{B} = \frac{A}{C} \times \frac{C}{D}$$

Forward Rate

$$F_{A/B} = S_{A/B} \times \left[\frac{1 + r_A \times T}{1 + r_B \times T}\right]$$

$$\begin{aligned}Forward\ points &= F_{A/B} - S_{A/B} \\ &= S_{A/B}\left(\frac{r_A - r_B}{1 + r_B}\right)T\end{aligned}$$

where:
$S_{A/B}$ = Spot exchange rate
$F_{A/B}$ = Forward exchange rate
T = Time to maturity

CORPORATE ISSUERS

Learning Module 1: Organizational Forms, Corporate Issuer Features, and Ownership

No formula

Learning Module 2: Investors and Other Stakeholders

No formula

Learning Module 3: Working Capital and Liquidity

No formula

Learning Module 4: Corporate Governance: Conflicts, Mechanisms, Risks, and Benefits

$$\begin{matrix}\textit{Cash}\\ \textit{conversion}\\ \textit{cycle}\end{matrix} = \begin{matrix}\textit{Days of inventory}\\ \textit{on hand}\end{matrix} + \begin{matrix}\textit{Days sales}\\ \textit{outstanding}\end{matrix} - \begin{matrix}\textit{Days payables}\\ \textit{outstanding}\end{matrix}$$

$$\begin{matrix}\textit{EAR of Supplier}\\ \textit{Financing}\end{matrix} = \left(1 + \frac{Discount\%}{100\% - Discount\%}\right)^{\frac{\textit{Days in Year}}{\textit{Payment Period} - \textit{Discount Period}}} - 1$$

$$\textit{Total working capital} = \textit{Current assets} - \textit{Current Liabilities}$$

$$\begin{matrix}\textit{Net working}\\ \textit{capital}\end{matrix} = \textit{Current assets (excluding cash and marketable securities)}$$
$$-\textit{Current Liabilities (excluding short-term and current debt)}$$

Cash flow from operations
= Cash received from customers
+ Interest and dividends received on financial investments
− Cash paid to employees and suppliers
− Taxes paid to governments
− Interest paid to lenders

Free cash flow = Cash flow from operations – Investments in long-term assets

$$\textit{Current ratio} = \frac{\textit{Current assets}}{\textit{Current liabilities}}$$

$$Quick\ ratio = \frac{Cash + Short\text{-}term\ marketable\ instruments + Receivables}{Current\ liabilities}$$

$$Cash\ ratio = \frac{Cash + Short\text{-}term\ marketable\ instruments}{Current\ liabilities}$$

Learning Module 5: Capital Investments and Capital Allocation

Net Present Value

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_T}{(1+r)^T} = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$$

where:

CF_t = After-tax cash flow at time t

r = Required rate of return

CF_0 = Initial outlay

Internal Rate of Return

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

Video: https://youtu.be/bzck7QLhICw

Return on Invested Capital

$$ROIC = \frac{After\text{-}tax\ operating\ profit}{Average\ invested\ capital}$$
$$= \frac{Operating\ profit_t \times (1 - Tax\ rate)}{Average\ total\ long\text{-}term\ liabilities\ and\ equity_{t-1,t}}$$

$$ROIC = \frac{After\text{-}tax\ operating\ profit}{Sales} \times \frac{Sales}{Average\ invested\ capital}$$

Real Options in Capital Budgeting

$$\begin{matrix} Project\ NPV \\ (with\ option) \end{matrix} = \begin{matrix} Project\ NPV \\ (without\ option) \end{matrix} - Option\ cost + Option\ value$$

Learning Module 6: Capital Structure

Weighted Average Cost of Capital

$$WACC = w_d r_d (1 - t) + w_e r_e$$

where:

w_d = Target weight of debt in capital structure = $\frac{D}{D+E}$

w_e = Target weight of common stock in capital structure = $\frac{E}{D+E}$

r_d = Before-tax marginal cost of debt

t = Marginal tax rate

$r_d(1 - t)$ = After-tax marginal cost of debt

r_e = Marginal cost of common stock

Operating Leverage

$$Operating\ leverage = \frac{Fixed\ costs}{Total\ costs}$$

Interest Coverage

$$Interest\ coverage = \frac{Profit\ before\ interest\ and\ taxes}{Interest\ expense}$$

Modigliani-Miller Capital Structure Propositions

$$V_L = V_U + tD$$

$$r_e = r_0 + (r_0 - r_d)(1 - t)\frac{D}{E}$$

$$E = \frac{(CF_e - r_d D)(1 - t)}{r_e}$$

$$V_L = \frac{CF_e(1 - t)}{r_{WACC}}$$

where:

V_L = Value of levered firm

V_U = Value of unlevered firm

t = Marginal tax rate

r_e = Cost of equity

r_d = Cost of debt

r_0 = Cost of capital (for a 100% equity-financed company)

D = Market value of debt

E = Market value of equity

CF_e = After-tax cash flows to shareholders

$r_d D$ = Interest expense on debt

Static Trade-off Theory of Capital Structure

$$V_L = V_U + tD - PV(Costs\ of\ Financial\ Distress)$$

Learning Module 7: Business Models

No formula

FINANCIAL STATEMENT ANALYSIS

VOLUME 2

Learning Module 1: Introduction to Financial Statement Analysis

No formula

Learning Module 2: Analyzing Income Statements

Gross profit = Revenue – Cost of Goods Sold

Operating income = Gross margin – Selling, General, and Administrative Expense

Taxable income = Operating income – Interest expense

Net income = Taxable income − Taxes

Ending shareholders' equity = Beginning shareholders' equity + Net income
+ Other comprehensive income
− Dividends
+ Net capital contributions from shareholders

Ending retained earnings = Beginning retained earnings + Net income – Dividends

Return on Equity

$$ROE = \frac{Net\ income}{Average\ shareholders'\ equity}$$

Net Profit Margin

$$Net\ profit\ margin = \frac{Net\ income}{Revenue}$$

Basic EPS

$$Basic\ EPS = \frac{Net\ income - Preferred\ dividends}{Weighted\ average\ number\ of\ shares\ outstanding}$$

Diluted EPS (for convertible preferred stock)

$$Diluted\ EPS = \frac{Net\ income}{\begin{matrix} Weighted\ average\ number \\ of\ shares\ outstanding \end{matrix} + \begin{matrix} New\ common\ shares\ that\ would \\ have\ been\ issued\ at\ conversion \end{matrix}}$$

Diluted EPS (for convertible debt)

$$Diluted\ EPS = \frac{Net\ income - Preferred\ dividends + \begin{matrix} After\ tax\ interest\ expense \\ on\ convertible\ debt \end{matrix}}{\begin{matrix} Weighted\ average\ number \\ of\ shares\ outstanding \end{matrix} + \begin{matrix} New\ common\ shares\ that\ would \\ have\ been\ issued\ at\ conversion \end{matrix}}$$

Diluted EPS (for options)

$$Diluted\ EPS = \frac{Net\ income - Preferred\ dividends}{\begin{matrix} Weighted\ average\ number \\ of\ shares\ outstanding \end{matrix} + \begin{matrix} Additional\ common \\ shares\ issued\ upon \\ conversion \end{matrix}}$$

Treasury stock method

$$\begin{matrix} Additional\ common \\ shares\ issued\ upon \\ conversion \end{matrix} = \left(\begin{matrix} New\ shares \\ issued\ at \\ option\ exercise \end{matrix} - \begin{matrix} Shares\ repurchased \\ with\ cash\ received \\ from\ option\ exercised \end{matrix} \right) \times \begin{matrix} Proportion\ of\ year \\ during\ which\ options \\ were\ outstanding \end{matrix}$$

Video (Basic & Diluted EPS): https://youtu.be/2C-mwVqO2SQ

Learning Module 3: Analyzing Balance Sheets

$$Working\ capital = Current\ assets - Current\ liabilities$$

Liquidity Ratios

$$Current\ ratio = \frac{Current\ assets}{Current\ liabilities}$$

$$Quick\ (acid\ test)\ ratio = \frac{Cash + Marketable\ securities + Receivables}{Current\ liabilities}$$

$$Cash\ ratio = \frac{Cash + Marketable\ securities}{Current\ liabilities}$$

Solvency Ratios

$$Long\text{-}term\ debt\text{-}to\text{-}equity = \frac{Long\text{-}term\ debt}{Total\ equity}$$

$$Debt\text{-}to\text{-}equity = \frac{Total\ debt}{Total\ equity}$$

$$Total\ debt = \frac{Total\ debt}{Total\ assets}$$

$$Financial\ leverage = \frac{Total\ assets}{Total\ equity}$$

Learning Module 4: Analyzing Statements of Cash Flows I

$$\begin{matrix}Ending\\cash\end{matrix} = \begin{matrix}Beginning\\cash\end{matrix} + \begin{matrix}Cash\ flow\\from\ operating\\activities\end{matrix} + \begin{matrix}Cash\ flow\\from\ investing\\activities\end{matrix} + \begin{matrix}Cash\ flow\\from\ financing\\activities\end{matrix}$$

$$\begin{matrix}Ending\ accounts\\receivable\end{matrix} = \begin{matrix}Beginning\ accounts\\receivable\end{matrix} + Revenue - \begin{matrix}Cash\ collected\\from\ customers\end{matrix}$$

$$\begin{matrix}Ending\\inventory\end{matrix} = \begin{matrix}Beginning\\inventory\end{matrix} + Purchases - \begin{matrix}Cost\ of\\goods\ sold\end{matrix}$$

$$\begin{matrix}Ending\ accounts\\payable\end{matrix} = \begin{matrix}Beginning\ accounts\\payable\end{matrix} + Purchases - \begin{matrix}Cash\ paid\\to\ suppliers\end{matrix}$$

$$\begin{matrix}Ending\ wages\\payable\end{matrix} = \begin{matrix}Beginning\ wages\\payable\end{matrix} + \begin{matrix}Wages\\expense\end{matrix} - \begin{matrix}Cash\ paid\\to\ employees\end{matrix}$$

$$\begin{matrix}Ending\ interest\\payable\end{matrix} = \begin{matrix}Beginning\ interest\\payable\end{matrix} + \begin{matrix}Interest\\expense\end{matrix} - \begin{matrix}Cash\ paid\\for\ interest\end{matrix}$$

$$\begin{matrix}Ending\ income\\tax\ payable\end{matrix} = \begin{matrix}Beginning\ income\\tax\ payable\end{matrix} + \begin{matrix}Income\ tax\\expense\end{matrix} - \begin{matrix}Cash\ paid\\for\ income\ taxes\end{matrix}$$

$$Ending\ PP\&E = Beginning\ PP\&E + \begin{matrix}Equipment\\purchased\end{matrix} - \begin{matrix}Equipment\\sold\end{matrix}$$

$$\begin{matrix}Ending\ accumulated\\depreciation\end{matrix} = \begin{matrix}Beginning\ accumulated\\depreciation\end{matrix} + \begin{matrix}Depreciation\\expense\end{matrix} - \begin{matrix}Accumulated\\depreciation\ on\\equipment\ sold\end{matrix}$$

<u>Note</u>:

$$\begin{matrix}Gain\ on\ sale\\of\ equipment\end{matrix} = \begin{matrix}Cash\ received\ from\\sale\ of\ equipment\end{matrix} - \begin{matrix}Book\ value\ of\\equipment\ sold\end{matrix}$$

$$\begin{matrix}Ending\ retained\\earnings\end{matrix} = \begin{matrix}Beginning\ retained\\earnings\end{matrix} + \begin{matrix}Net\\income\end{matrix} - Dividends$$

Learning Module 5: Analyzing Statements of Cash Flows II

Free Cash Flow To Firm (FCFF)

$$FCFF = NI + NCC + Int(1 - Tax\ rate) - FCInv - WCInv$$
$$= CFO + Int(1 - Tax\ rate) - FCInv$$

where:

NI = Net income

NCC = Non-cash charges (e.g., depreciation and amortization)

Int = Interest expense

$FCInv$ = Capital expenditures

$WCInv$ = Working capital expenditures

CFO = Cash flow from operating activities = $NI + NCC - WCInv$

Free Cash Flow to Equity (FCFE)

$$FCFE = CFO - FCInv + Net\ Borrowing$$

where:

$Net\ Borrowing = Debt\ issued - Debt\ repaid$

Performance Ratios

$$Cash\ flow\ to\ revenue = \frac{CFO}{Revenue}$$

$$Cash\ return\ on\ assets = \frac{CFO}{Average\ total\ assets}$$

$$Cash\ return\ on\ equity = \frac{CFO}{Average\ shareholders\ equity}$$

$$Cash\ to\ income = \frac{CFO}{Operating\ income}$$

$$Cash\ flow\ per\ share = \frac{CFO - Preferred\ dividends}{Number\ of\ common\ shares\ outstanding}$$

Coverage Ratios

$$Debt\ coverage\ ratio = \frac{CFO}{Total\ debt}$$

$$Interest\ coverage\ ratio = \frac{CFO + Interest\ paid + Taxes\ paid}{Interest\ paid}$$

$$Reinvestment\ ratio = \frac{CFO}{Cash\ paid\ for\ long\ term\ assets}$$

$$Debt\ payment\ ratio = \frac{CFO}{Cash\ paid\ for\ long\ term\ debt\ repayment}$$

$$Dividend\ payment\ ratio = \frac{CFO}{Dividends\ paid}$$

$$Investing\ and\ financing\ ratio = \frac{CFO}{Cash\ flow\ for\ investing\ and\ financing\ activities}$$

Learning Module 6: Analysis of Inventories

IFRS

$Inventories = Lower\ of\ Cost\ and\ Net\ Realizable\ Value\ (NRV)$

NRV = Estimated selling price less estimated costs of completion and costs necessary to complete the sale

US GAAP

$Inventories = Lower\ of\ Cost\ and\ NRV$

For last-in, first-out (LIFO) method or retail inventory methods

$$Inventories = Lower\ of\ Cost\ and\ Market\ Value$$

$Market\ value = Current\ replacement\ cost\ (subject\ to\ lower\ and\ upper\ limits)$

Lower limit $= NRV - Normal\ profit\ margin$
Upper limit $= NRV$

Video: https://youtu.be/V8C31msIBzs

$$Inventory\ turnover\ ratio = \frac{Cost\ of\ sales}{Average\ inventory}$$

$$Days\ of\ inventory\ on\ hand = \frac{Number\ of\ days\ in\ period}{Inventory\ turnover\ ratio}$$

$$Ending\ inventory\ (FIFO) = Ending\ inventory\ (LIFO) + LIFO\ reserve$$

$$COGS\ (FIFO) = COGS\ (LIFO) - Change\ in\ LIFO\ reserve$$

Learning Module 7: Analysis of Long-Term Assets

$$Net\ book\ value = Historical\ cost - Accumulated\ depreciation$$

$$Gain\ on\ sale\ of\ asset = Sale\ proceeds - Net\ book\ value$$

$$\begin{matrix} Estimated\ total \\ useful\ life \end{matrix} = \begin{matrix} Estimated\ age \\ of\ equipment \end{matrix} + \begin{matrix} Estimated \\ remaining\ life \end{matrix}$$

$$\begin{matrix} Estimated\ total \\ useful\ life \end{matrix} = \frac{Gross\ PP\&E}{Annual\ depreciation\ expense}$$

$$\begin{matrix} Estimated\ age \\ of\ equipment \end{matrix} = \frac{Accumulated\ depreciation}{Annual\ depreciation\ expense}$$

$$\begin{matrix} Estimated \\ remaining\ life \end{matrix} = \frac{Net\ PP\&E}{Annual\ depreciation\ expense}$$

Straight-line Depreciation

$$Annual\ depreciation\ expense = \frac{Historical\ cost - Salvage\ value}{Estimated\ useful\ life}$$

Fixed Asset Turnover

$$Fixed\ asset\ turnover = \frac{Revenue}{Average\ net\ PP\&E}$$

Impairment of Long-Lived Assets

IFRS

Impairment = Carrying amount − Recoverable amount

where:
Recoverable amount = max(Fair value less costs to sell, Value in use)

US GAAP

If asset's carrying amount > undiscounted expected future cash flows:

Impairment = Carrying amount − Fair value

Learning Module 8: Topics in Long-Term Liabilities and Equity

Lessee Accounting – Finance Lease (IFRS)

$$\begin{matrix} \text{Interest expense} \\ \text{on lease} \end{matrix} = \text{Implied interest rate} \times \text{Beginning lease liability}$$

$$\text{Principal repayment} = \text{Lease payment} - \text{Interest expense}$$

$$\begin{matrix} \text{Ending lease} \\ \text{liability} \end{matrix} = \begin{matrix} \text{Beginning lease} \\ \text{liability} \end{matrix} + \text{Interest expense} - \text{Lease payment}$$

If ROU asset is amortized on a straight-line basis:

$$\begin{matrix} \text{Amortization} \\ \text{expense} \end{matrix} = \frac{\text{Initial ROU asset value} - \text{Salvage value}}{\text{Lease term}}$$

$$\begin{matrix} \text{Ending ROU} \\ \text{asset} \end{matrix} = \begin{matrix} \text{Beginning ROU} \\ \text{asset} \end{matrix} - \begin{matrix} \text{Amortization} \\ \text{expense} \end{matrix}$$

Lessee Accounting – Operating Lease (US GAAP)

$$\begin{matrix} \text{Amortization} \\ \text{expense} \end{matrix} = \text{Lease payment} - \text{Interest expense}$$

$$\begin{matrix} \text{Ending ROU} \\ \text{asset} \end{matrix} = \begin{matrix} \text{Beginning ROU} \\ \text{asset} \end{matrix} - \begin{matrix} \text{Amortization} \\ \text{expense} \end{matrix}$$

$$\begin{matrix} \text{Ending lease} \\ \text{liability} \end{matrix} = \begin{matrix} \text{Beginning lease} \\ \text{liability} \end{matrix} - \begin{matrix} \text{Amortization} \\ \text{expense} \end{matrix}$$

Stock Options

$$\text{Compensation expense} = \frac{\text{Fair value of options granted}}{\text{Vesting period}}$$

VOLUME 3

Learning Module 1: Analysis of Income Taxes

Deferred Tax Asset/Liability

For **Assets**:

$$\begin{matrix}\text{Deferred tax} \\ \text{liability/(asset)}\end{matrix} = \text{Tax rate} \times \left(\begin{matrix}\text{Carrying amount} \\ \text{of asset}\end{matrix} - \begin{matrix}\text{Tax base} \\ \text{of asset}\end{matrix}\right)$$

For **Liabilities**:

$$\begin{matrix}\text{Deferred tax} \\ \text{liability/(asset)}\end{matrix} = \text{Tax rate} \times \left(\begin{matrix}\text{Tax base} \\ \text{of liability}\end{matrix} - \begin{matrix}\text{Carrying amount} \\ \text{of liability}\end{matrix}\right)$$

$$\text{Income tax expense} = \text{Income tax payable} + \begin{matrix}\text{Changes in deferred tax} \\ \text{assets and liabilities}\end{matrix}$$

$$\text{Effective tax rate} = \frac{\text{Income tax expense}}{\text{Pre-tax income}}$$

$$\text{Cash tax rate} = \frac{\text{Cash tax}}{\text{Pre-tax income}}$$

Learning Module 2: Financial Reporting Quality

Adjusted EBITDA

$$\begin{matrix}\text{Adjusted} \\ \text{EBITDA}\end{matrix} = \begin{matrix}\text{Adjusted} \\ \text{EBIT}\end{matrix} + \begin{matrix}\text{Software} \\ \text{and R\&D} \\ \text{amortization}\end{matrix} + \text{Depreciation} + \begin{matrix}\text{Post-IPO} \\ \text{share-based} \\ \text{amortization}\end{matrix}$$

Straight-line method of depreciation

$$\text{Depreciation expense} = \frac{\text{Cost} - \text{Salvage value}}{\text{Useful life}}$$

Double-Declining Balance method

$$\text{Depreciaton expense} = \frac{2}{\text{Useful life}} \times (\text{Cost} - \text{Accumulated depreciation})$$

Video: https://youtu.be/6RskYAxdAFk

Units-of-Production method

$$\text{Depreciation expense} = \frac{\text{Units produced}}{\text{Total units over useful life}} \times (\text{Cost} - \text{Salvage value})$$

Learning Module 3: Financial Analysis Techniques

Activity Ratios

$$Inventory\ turnover = \frac{Cost\ of\ sales}{Average\ inventory}$$

$$Days\ of\ inventory\ on\ hand = \frac{Number\ of\ days\ in\ the\ period}{Inventory\ turnover}$$

$$Receivables\ turnover = \frac{Revenue}{Average\ receivables}$$

$$Days\ of\ sales\ outstanding = \frac{Number\ of\ days\ in\ the\ period}{Receivables\ turnover}$$

$$Payables\ turnover = \frac{Purchases}{Average\ payables}$$

$$Number\ of\ days\ of\ payables = \frac{Number\ of\ days\ in\ the\ period}{Payables\ turnover}$$

$$Working\ capital\ turnover = \frac{Revenue}{Average\ working\ capital}$$

$$Fixed\ asset\ turnover = \frac{Revenue}{Average\ net\ fixed\ assets}$$

$$Total\ asset\ turnover = \frac{Revenue}{Average\ total\ assets}$$

Liquidity Ratios

$$Current\ ratio = \frac{Current\ assets}{Current\ liabilities}$$

$$Quick\ ratio = \frac{Cash + Short\ term\ marketable\ investments + Receivables}{Current\ liabilities}$$

$$Cash\ ratio = \frac{Cash + Short\ term\ marketable\ investments}{Current\ liabilities}$$

$$\begin{matrix} Defensive\ interval \\ ratio \end{matrix} = \frac{Cash + Short\ term\ marketable\ investments + Receivables}{Daily\ cash\ expenditures}$$

$$\begin{array}{c}\text{Cash conversion}\\ \text{cycle}\end{array} = \begin{array}{c}\text{Days of inventory}\\ \text{on hand}\end{array} + \begin{array}{c}\text{Days of sales}\\ \text{outstanding}\end{array} - \begin{array}{c}\text{Number of days}\\ \text{of payables}\end{array}$$

Video (Cash Conversion Cycle): https://youtu.be/IFsI9c4wUD4

Solvency Ratios

$$\begin{array}{c}\text{Debt-to-assets ratio}\\ \text{("Total debt ratio")}\end{array} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Debt-to-capital ratio} = \frac{\text{Total debt}}{\text{Total debt} + \text{Total equity}}$$

$$\text{Debt-to-equity ratio} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Financial leverage ratio} = \frac{\text{Average total assets}}{\text{Average total equity}}$$

$$\text{Debt-to-EBITDA ratio} = \frac{\text{Total or net debt}}{\text{EBITDA}}$$

Coverage Ratios

$$\text{Interest coverage ratio} = \frac{\text{EBIT}}{\text{Interest payments}}$$

$$\text{Fixed charge coverage ratio} = \frac{\text{EBIT} + \text{Lease payments}}{\text{Interest payments} + \text{Lease payments}}$$

Profitability Ratios

$$\text{Gross profit margin} = \frac{\text{Gross profit}}{\text{Revenue}}$$

$$\text{Operating profit margin} = \frac{\text{Operating income}}{\text{Revenue}}$$

$$\text{Pretax margin} = \frac{\text{EBT}}{\text{Revenue}}$$

$$\text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

$$Operating\ ROA = \frac{Operating\ income}{Average\ total\ assets}$$

$$ROA = \frac{Net\ income}{Average\ total\ assets}$$

$$Return\ on\ invested\ capital = \frac{EBIT \times (1 - Effective\ tax\ rate)}{Average\ total\ debt\ and\ equity}$$

$$ROE = \frac{Net\ income}{Average\ total\ equity}$$

$$Return\ on\ common\ equity = \frac{Net\ income - Preferred\ dividends}{Average\ common\ equity}$$

DuPont Analysis

$$ROE = ROA \times Financial\ Leverage$$

$$ROE = Net\ profit\ margin \times Total\ asset\ turnover \times Financial\ leverage$$

$$ROE = \begin{matrix} Tax \\ burden \end{matrix} \times \begin{matrix} Interest \\ burden \end{matrix} \times \begin{matrix} EBIT \\ margin \end{matrix} \times \begin{matrix} Total\ asset \\ turnover \end{matrix} \times \begin{matrix} Financial \\ leverage \end{matrix}$$

where:

$$Tax\ burden = \frac{Net\ income}{EBT}$$

$$Interest\ burden = \frac{EBT}{EBIT}$$

Business Risk

$$\begin{matrix} Coefficient\ of\ variation \\ of\ operating\ income \end{matrix} = \frac{Standard\ deviation\ of\ operating\ income}{Average\ operating\ income}$$

$$\begin{matrix} Coefficient\ of\ variation \\ of\ net\ income \end{matrix} = \frac{Standard\ deviation\ of\ net\ income}{Average\ net\ income}$$

$$\begin{matrix} Coefficient\ of\ variation \\ of\ revenue \end{matrix} = \frac{Standard\ deviation\ of\ revenue}{Average\ revenue}$$

Financial Sector Ratios

$$\begin{array}{c}\textit{Monetary reserve requirement} \\ \textit{(Cash reserve ratio)}\end{array} = \frac{\textit{Reserves held at central bank}}{\textit{Specified deposit liabilities}}$$

$$\textit{Net interest margin} = \frac{\textit{Net interest income}}{\textit{Total interest earning assets}}$$

$$\textit{Liquid asset requirement} = \frac{\textit{Approved readily marketable securities}}{\textit{Specified deposit liabilities}}$$

$$\textit{Net interest margin} = \frac{\textit{Net interest income}}{\textit{Total interest earning assets}}$$

Learning Module 4: Introduction to Financial Statement Modeling

Nothing new.

EQUITY INVESTMENTS

Learning Module 1: Market Organization and Structure

$$Maximum\ leverage\ ratio = \frac{1}{\text{Minimum margin requirement}}$$

Total return on leveraged stock investment:

$$Total\ Return = \frac{\begin{matrix}Sales\\proceeds\end{matrix} + Dividends - Loan - \begin{matrix}Margin\\interest\end{matrix} - \begin{matrix}Sales\\commission\end{matrix}}{\begin{matrix}Initial\\equity\end{matrix} + \begin{matrix}Purchase\\commission\end{matrix}} - 1$$

$$Initial\ equity = \begin{matrix}Minimum\ margin\\requirement\end{matrix} \times Total\ purchase\ price$$

Video (Return on Leveraged Stock Position): https://youtu.be/tZd4XtvjjlI

$$Margin\ Call\ Price = \frac{P_0(1 - Initial\ Margin)}{(1 - Maintenance\ Margin)}$$

Learning Module 2: Security Market Indexes

$$\text{Price Return Index,} \qquad V_{PRI} = \frac{\sum_{i=1}^{N} n_i P_i}{D}$$

where: n_i = the number of units of constituent security *i* held in the index portfolio
N = the number of constituent securities in the index
P_i = the unit price of constituent security *i*
D = value of the divisor

$$\text{Price return of an index,} \qquad PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

$$\text{Total Return Index,} \qquad TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:
V_{PRI1} = value of the price return index at the **end** of the period
V_{PRI0} = value of the price return index at the **beginning** of the period
Inc_I = total income (dividends and/or interest) from all securities in the index held over the period

Weighting Methods

$$\text{Price weighting,} \qquad w_i^P = \frac{P_i}{\sum_{j=1}^{N} P_j}$$

Video (Recalculating the divisor of a price weighted index): https://youtu.be/eYiZNK-ETrg

$$\text{Equal weighting,} \qquad w_i^E = \frac{1}{N}$$

$$\text{Market-capitalization weighting,} \qquad w_i^M = \frac{Q_i P_i}{\sum_{j=1}^{N} Q_j P_j}$$

$$\text{Float-adjusted market capitalization weighting,} \qquad w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^{N} f_j Q_j P_j}$$

where:
f_i = fraction of shares outstanding in the market float
Q_i = number of shares outstanding of security *i*
P_i = share price of security *i*
N = number of securities in the index

$$\text{Fundamental weighting,} \qquad w_i^F = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

where F_i denotes a fundamental size measure of company *i*

Learning Module 3: Market Efficiency

No formula

Learning Module 4: Overview of Equity Securities

Return on Equity (using **average** total book value of equity)

$$ROE_t = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Return on Equity (using **beginning** book value of equity)

$$ROE_t = \frac{NI_t}{BVE_{t-1}}$$

where BVE = book value (Assets – Liabilities)

Learning Module 5: Company Analysis: Past and Present

$$Market\ share = \frac{Revenue}{Market\ size}$$

$$Sales\ potential = 100\% - Market\ share\%$$

$$Net\ sales = Average\ selling\ price \times Quantity\ sold$$

$$Take\ rate = \frac{Revenue\ earned\ from\ transactions}{Total\ transaction\ volume} \times 100\%$$

$$Operating\ income = Q \times (P - VC) - FC$$

where:
Q = Units sold in a period
P = Price per unit
VC = Variable operating cost per unit
FC = Fixed operating costs
$P - VC$ = Contribution margin per unit

$$Degree\ of\ operating\ leverage\ (DOL) = \frac{\%\Delta \text{Operating income}}{\%\Delta \text{Sales}}$$

$$Degree\ of\ financial\ leverage\ (DFL) = \frac{\%\Delta \text{Net income}}{\%\Delta \text{Operating income}}$$

$$WACC = \begin{matrix} Weight \\ of\ debt \end{matrix} \times \begin{matrix} Gross\ cost \\ of\ debt \end{matrix} \times (1 - tax\ rate) + \begin{matrix} Weight \\ of\ equity \end{matrix} \times \begin{matrix} Cost\ of \\ equity \end{matrix}$$

Learning Module 6: Industry and Company Analysis

Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{\infty} s_i^2$$

where:
s_i = Market share of participant i (stated as a whole number)

Learning Module 7: Company Analysis: Forecasting

$$\%Variable\ cost \approx \frac{\%\Delta(Cost\ of\ revenue + Operating\ expense)}{\%\Delta Revenue}$$

$$\%Fixed\ cost \approx 1 - \%Variable\ cost$$

$$\begin{matrix} Number\ of\ units\ sold \\ post\text{-}cannibalization \end{matrix} = \begin{matrix} Number\ of\ units\ sold \\ pre\text{-}cannibalization \end{matrix} - \begin{matrix} Expected \\ cannibalization \end{matrix}$$

$$\begin{matrix} Expected \\ cannibalization \end{matrix} = \begin{matrix} Number\ of\ units\ sold \\ pre\text{-}cannibalization \end{matrix} \times \begin{matrix} Cannibalization \\ factor \end{matrix}$$

Learning Module 8: Equity Valuation: Concepts and Basic Tools

Dividend Discount Model (DDM)

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

where:
V_0 = Intrinsic value of a share at $t = 0$
D_t = expected dividend in year t
r = required rate of return on stock
P_n = expected price per share at $t = n$ (terminal value)

Free-cash-flow-to-equity (FCFE) Valuation Model

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}$$

where:
$FCFE = CFO - FCInv + Net\ Borrowing$
$FCInv$ = Fixed capital investment
$Net\ Borrowing$ = Borrowings minus repayments

Value of preferred stock (non-callable, non-convertible, perpetual)

$$V_0 = \frac{D_0}{r}$$

Value of preferred stock (non-callable, non-convertible, maturity at time *n*)

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{Par\ value}{(1+r)^n}$$

Gordon Growth Model

$$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

where:
D_0 = Most recent annual dividend
D_1 = Expected dividend in the next period
g = Constant growth rate
r = Required return on equity

Sustainable growth rate

$$g = b \times ROE$$

where:
b = earnings retention rate (= 1 – Dividend payout ratio)
ROE = Return on equity

Video: https://youtu.be/MnfRRRhuGpA

Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^{n} \frac{D_0(1+g_s)^t}{(1+r)^t} + \frac{V_n}{(1+r)^t}$$

where:
g_L = Long-term stable growth rate
g_s = Short-term growth rate
$V_n = \frac{D_{n+1}}{r-g_L} = \frac{D_0(1+g_s)^t(1+g_L)}{r-g_L}$

Justified forward P/E

$$\frac{P_0}{E_1} = \frac{Dividend\ payout\ ratio}{r-g}$$

Enterprise Value

$$EV = \begin{matrix} Market\ value \\ of\ equity \end{matrix} + \begin{matrix} Market\ value \\ of\ preferred\ stock \end{matrix} + \begin{matrix} Market\ value \\ of\ debt \end{matrix} - \begin{matrix} Cash\ and \\ short\ term \\ investments \end{matrix}$$

Asset-based Valuation

$$\begin{matrix} Adjusted \\ book\ value \end{matrix} = \begin{matrix} Market\ value \\ of\ assets \end{matrix} - \begin{matrix} Market\ value \\ of\ liabilities \end{matrix}$$

FIXED INCOME

Learning Module 1: Fixed-Income Instrument Features

$$Current\ yield = \frac{Annual\ coupon}{Bond\ price}$$

$$Bond\ price = \frac{Coupon}{(1+r)^1} + \frac{Coupon}{(1+r)^2} + \cdots + \frac{Coupon + Face\ value}{(1+r)^n}$$

where:
$Coupon\ per\ period =$ Coupon rate per period $\times$ Face value
$r =$ Yield to maturity per period
$n =$ Number of payments

Floating-rate Note (FRN) coupon rate $=$ MRR $+$ Spread

Learning Module 2: Fixed-Income Cash Flows and Types

Fully Amortizing Loan with Level Payment

$$A = \frac{r \times Principal}{1 - (1+r)^{-N}}$$

where:
$A =$ Periodic payment
$r =$ Market interest rate per period
$N =$ Number of payment periods

If the periodic payment is monthly:

Monthly interest payment = Interest rate per month × Beginning principal of loan

Monthly principal payment = Total monthly payment − Monthly interest payment

Ending principal of loan = Beginning principal of loan − Monthly principal payment

Capital-Index Bond (e.g., TIPS)

Inflation-adjusted principal = Principal amount × (1 + Inflation adjustment)

Coupon per period = Coupon rate per period × Inflation-adjusted principal

Deferred Coupon Bond

Video: https://youtu.be/erRbAUOGIyM

Convertible Bonds

$$\begin{matrix}\text{Conversion} \\ \text{ratio}\end{matrix} = \frac{\text{Convertible bond par value}}{\text{Conversion price}}$$

$$\begin{matrix}\text{Conversion} \\ \text{value}\end{matrix} = \begin{matrix}\text{Conversion} \\ \text{ratio}\end{matrix} \times \begin{matrix}\text{Current share} \\ \text{price}\end{matrix}$$

Zero-Coupon Bond

Original issue discount = Bond par value – Issuance price

Learning Module 3: Fixed-Income Issuance and Trading

No formula

Learning Module 4: Fixed-Income Markets for Corporate Issuers

Repurchase Agreements

$$\text{Repurchase price} = \text{Price of bond} \times \left[1 + \text{Repo rate} \times \frac{\text{Repo term (in days)}}{\text{Number of days in a year}}\right]$$

$$\text{Initial margin} = \frac{\text{Security price}_0}{\text{Purchase price}_0}$$

$$\text{Haircut} = \frac{\text{Security price}_0 - \text{Purchase price}_0}{\text{Security price}_0}$$

$$\text{Variation margin} = (\text{Initial margin} \times \text{Purchase price}_t) - \text{Security price}_t$$

Learning Module 5: Fixed-Income Markets for Government Issuers

No formula.

Learning Module 6: Fixed-Income Bond Valuation: Prices and Yields

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \cdots + \frac{PMT_N + FV_N}{(1+r)^N}$$

where:

PMT_t = Coupon that occurs in t periods

r = Market discount rate per period

N = Number of periods to maturity

FV = Face value of bond

Full Price, Flat Price, and Accrued Interest (Video: https://youtu.be/l7G075JAu5w)

$PV^{Full} = PV^{Flat}$ + Accrued Interest

$\quad = PV_{BOP} \times (1+r)^{t/T}$

where:

Accrued Interest $= \frac{t}{T} \times PMT$

t = number of days from the last coupon payment to the settlement date

T = number of days in the coupon period

t/T = fraction of the coupon period that has gone by since the last payment

PV_{BOP} = price on the previous coupon date (before the settlement date)

Matrix Pricing

$$Interpolated\ yield = Yield_S + \left(\frac{Tenor_{Target} - Tenor_S}{Tenor_L - Tenor_S}\right) \times (Yield_L - Yield_S)$$

where:

$Yield_S$ = Yield of shorter-term bond

$Yield_L$ = Yield of longer-term bond

$Tenor_S$ = Tenor of shorter-term bond

$Tenor_L$ = Tenor of longer-term bond

$Tenor_{Target}$ = Tenor of the subject bond

$Tenor_S < Tenor_{Target} < Tenor_L$

Required yield spread = Bond YTM – Government Bond YTM (Similar maturity)

Learning Module 7: Yield and Yield Spread Measures for Fixed Rate Bonds

Periodicity Conversion

$$\left(1+\frac{APR_m}{m}\right)^m=\left(1+\frac{APR_n}{n}\right)^n$$

where:
APR_m = Annual percentage rate for m periods per year
APR_n = Annual percentage rate for n periods per year

$$Current\ yield_t=\frac{Annual\ coupon_t}{Bond\ price_t}$$

$$Government\ equivalent\ yield,\qquad Yield_{ACT/ACT}=\frac{365}{360}\times Yield_{30/360}$$

$$Simple\ yield=\frac{Coupon\ +\left(\frac{FV-PV}{N}\right)}{Flat\ price}$$

Callable Bonds

$$PV=\frac{PMT}{(1+YTC)^1}+\frac{PMT}{(1+YTC)^2}+\cdots+\frac{PMT+Call\ price}{(1+YTC)^N}$$

where:
PV = Price of the callable bond
PMT = Coupon payment per period
YTC = Yield to call per period
N = Number of periods to when the bond can be called at the call price

Option-adjusted price = Flat price of bond + Value of embedded call option

Value of call option = Price of option-free bond – Price of callable bond

G-spread = Bond YTM – Interpolated sovereign bond YTM

I-spread = Bond YTM – Swap rate

Z-Spread

$$PV=\frac{PMT}{(1+z_1+Z)^1}+\frac{PMT}{(1+z_2+Z)^2}+\cdots+\frac{PMT+FV}{(1+z_N+Z)^N}$$

where:
Z = Z-spread
z_N = Spot rate for N periods

OAS = Z-spread – Option value (in basis points per year)

Learning Module 8: Yield and Yield Spread Measures for Floating-Rate Instruments

Value of Floating Rate Note (FRN)

$$PV = \frac{\left(\frac{MRR + QM}{m}\right) \times FV}{\left(1 + \frac{MRR + DM}{m}\right)^1} + \frac{\left(\frac{MRR + QM}{m}\right) \times FV}{\left(1 + \frac{MRR + DM}{m}\right)^2} + \cdots + \frac{\left(\frac{MRR + QM}{m}\right) \times FV + FV}{\left(1 + \frac{MRR + DM}{m}\right)^n}$$

where:
QM = Quoted Margin
DM = Discount Margin
MRR = Market reference rate
m = Periodicity (i.e., number of payment periods per year)
FV = Face Value of FRN
N = Number of evenly spaced periods to maturity

Video: https://youtu.be/zqYOtVLkYR8

Yield Measures for Money Market Instruments

Discount Rate Basis

$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right)$$

$$DR = \frac{Year}{Days} \times \left(\frac{FV - PV}{FV}\right)$$

where:
PV = present value of money market instrument
FV = future value paid at maturity
$Days$ = number of days between settlement and maturity
$Year$ = number of days in the year
DR = discount rate (stated as annual percentage rate)

Add-on Rate Basis

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} \times AOR\right)}$$

$$AOR = \frac{Year}{Days} \times \left(\frac{FV - PV}{PV}\right)$$

$$Bond\ equivalent\ yield = \frac{365}{Days} \times \left(\frac{FV - PV}{PV}\right)$$

Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves

Calculation of Bond Price Using Spot Rates

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+FV}{(1+Z_N)^N}$$

where:
PV = Price of bond
PMT = Bond coupon payment
Z_N = Spot rate (or zero-coupon yield or zero rate) for period N
FV = Face value of bond

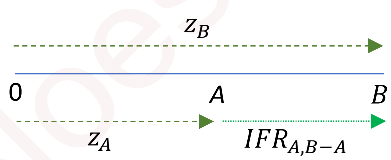
Given a Par Rate, $FV = PV$ and $PMT = Par\ Rate\ (\%) \times FV$

$$100 = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+100}{(1+Z_N)^N}$$

Forward Rates, *IFR*

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

where:
$IFR_{A,B-A}$ = Forward rate for $(B-A)$ periods that starts in period A



Learning Module 10: Interest Rate Risk and Return

Duration Gap

$$Duration\ gap = Macaulay\ duration - Investment\ horizon$$

Macaulay Duration

$$Macaulay\ duration = \left(1-\frac{t}{T}\right)\left[\frac{\frac{PMT}{(1+r)^{1-t/T}}}{PV^{Full}}\right] + \left(2-\frac{t}{T}\right)\left[\frac{\frac{PMT}{(1+r)^{2-t/T}}}{PV^{Full}}\right] + \cdots + \left(N-\frac{t}{T}\right)\left[\frac{\frac{PMT+FV}{(1+r)^{N-t/T}}}{PV^{Full}}\right]$$

$$Macaulay\ duration = \left\{\frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r}\right\} - \frac{t}{T}$$

where:
r = Yield per period
c = Coupon rate per period
N = Number of evenly spaced periods to maturity as of the beginning of the current period
t = Number of days from the last coupon payment to the settlement date
T = Number of days in the coupon period

Video: https://youtu.be/USgjcdCk7Fs

Learning Module 11: Yield-Based Bond Duration Measures and Properties

Modified Duration

$$Modified\ Duration = \frac{Macaulay\ Duration}{1+r}$$

Approximate Modified Duration

$$AnnModDur \approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta Yield) \times (PV_0)}$$

$$\%\Delta PV^{Full} \approx -AnnModDur \times \Delta Yield$$

Money Duration

Money duration $= AnnModDur \times PV^{full}$

$\Delta PV^{Full} \approx -MoneyDur \times \Delta Yield$

Duration of Zero-Coupon Bond

$$MacDur = Time\ to\ maturity$$

$$ModDur = \frac{Time\ to\ maturity}{1+r}$$

Duration of Perpetual Bond

$$MacDur = \frac{1+r}{r}$$

$$ModDur = \frac{1}{r}$$

Duration of Floating-Rate Notes

$$MacDur = \frac{T-t}{T} = \begin{array}{c} Fraction\ of\ period\ remaining\ until \\ the\ next\ reset\ date \end{array}$$

Learning Module 12: Yield-Based Bond Convexity and Portfolio Properties

Convexity

$$Convexity = \sum_{t=1}^{N} \frac{t(t+1) \times \frac{PV_t}{PV^{Full}}}{(1+YTM)^2}$$

Approximate Annualized Convexity

$$ApproxConv \approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta Yield)^2 \times (PV_0)}$$

$$\%\Delta PV^{Full} \approx -AnnModDur \times \Delta Yield + \frac{1}{2} \times AnnConvexity \times (\Delta Yield)^2$$

Money Convexity

$$MoneyCon = AnnConvexity \times PV^{Full}$$

$$\Delta PV^{Full} \approx -(MoneyDur \times \Delta Yield) + \left[\frac{1}{2} \times MoneyCon \times (\Delta Yield)^2\right]$$

Portfolio Duration and Convexity

$$Portfolio\ Modified\ Duration = \sum_{i=1}^{N} w_i \times ModDur_i$$

$$Portfolio\ Convexity = \sum_{i=1}^{N} w_i \times Convexity_i$$

where:

w_i = Weight of bond i, measured in market value

Learning Module 13: Curve-Based and Empirical Fixed-Income Risk Measures

Effective Duration

$$EffDur = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta Curve) \times PV_0}$$

Effective Convexity

$$EffCon = \frac{(PV_{-}) + (PV_{+}) - 2 \times PV_0}{(\Delta Curve)^2 \times PV_0}$$

$$\%\Delta PV^{Full} \approx -EffDur \times \Delta Curve + \frac{1}{2} \times EffCon \times (\Delta Curve)^2$$

Key Rate Duration

$$KeyRateDur_k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k}$$

$$\%\Delta PV = -KeyRateDur_k \times \Delta r_k$$

$$\sum_{k=1}^{n} KeyRateDur_k = EffDur$$

where:
$r_k = k$th key rate

Learning Module 14: Credit Risk

Expected Loss

$$EL = LGD \times POD$$

$$LGD = EE \times (1 - RR)$$

where:
POD = Probability of default
LGD = Loss given default
EE = Expected exposure
RR = Recovery rate
$1 - RR$ = Loss severity

$$Credit\ spread \approx POD \times LGD$$

Decomposing Bond Yields

$$Yield\ spread = Bond\ YTM - Government\ bond\ YTM \quad (Similar\ maturity)$$

$$Liquidity\ spread = Bond\ YTM\ (Bid) - Bond\ YTM\ (Offer)$$

$$Credit\ spread = Yield\ spread - Liquidity\ spread$$

Price Impact Given a Change in Yield Spread

$$\%\Delta PV^{Full} \approx -AnnModDur \times \Delta Spread + \frac{1}{2} \times AnnConvexity \times (\Delta Spread)^2$$

where:

$$AnnModDur \approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta Yield) \times (PV_0)}$$

$$AnnConvexity \approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta Yield)^2 \times (PV_0)}$$

Learning Module 15: Credit Analysis for Government Issuers

No formula.

Learning Module 16: Credit Analysis for Corporate Issuers

$$EBIT\ margin = \frac{Operating\ income}{Revenue}$$

$$EBIT\ to\ interest\ expense = \frac{Operating\ income}{Interest\ expense}$$

$$Debt\ to\ EBITDA = \frac{Debt}{EBITDA}$$

$$RCF\ to\ net\ debt = \frac{Retained\ cash\ flow}{Debt - Cash\ and\ marketable\ securities}$$

$$FFO\ to\ debt = \frac{FFO}{Debt}$$

where:

FFO = Net income from continuing operations + Depreciation & amortization
+ Deferred income taxes + Other non-cash items

Learning Module 17: Fixed-Income Securitization

No formula.

Learning Module 18: Asset-Backed Security (ABS) Instrument and Market Features

No formula.

Learning Module 19: Mortgage-Backed Security (MBS) Instrument and Market Features

Loan-to-value (LTV) ratio

$$LTV = \frac{Loan\ amount}{House\ price}$$

Debt-to-income (DTI) ratio

$$DTI = \frac{Monthly\ debt\ payment}{Monthly\ pre\text{-}tax\ gross\ income}$$

Mortgage Pass-Through Securities

$$WAC = \sum_{i=1}^{N} c_i \left(\frac{CB_i}{CB}\right)$$

$$WAM = \sum_{i=1}^{N} MM_i \left(\frac{CB_i}{CB}\right)$$

where:
WAC = Weighted average coupon
WAM = Weighted average maturity
c_i = Coupon rate on mortgage i
MM_i = Number of months to maturity for mortgage i
N = Number of mortgages in MBS
CB_i = Current balance on mortgage i
CB = Total current balance of mortgages in MBS

Commercial Mortgage-Backed Securities (CMBS)

Debt Service Coverage Ratio (DSCR)

$$DSCR = \frac{Net\ operating\ income}{Debt\ service}$$

Net Operating Income (NOI)

$$NOI = (Rental\ income - Cash\ operating\ expenses) - Replacement\ reserves$$

DERIVATIVES

Learning Module 1: Derivative Instrument and Derivatives Market Features

No formula.

Learning Module 2: Forward Commitments and Contingent Claim Features and Instruments

Forward Contract

$$\text{Buyer (Long) payoff} = S_T - F_0(T)$$

$$\text{Seller (Short) payoff} = -[S_T - F_0(T)]$$

where:
S_T = Spot price on contract's maturity
$F_0(T)$ = Forward price with maturity of T

Futures Contract

For one futures contract:

$$\text{Long Futures daily mark-to-market} = f_t(T) - f_{t-1}(T)$$

$$\text{Short Futures daily mark-to-market} = -[f_t(T) - f_{t-1}(T)]$$

where:
$f_t(T)$ = Closing price of futures contract on day t
$f_{t-1}(T)$ = Closing price of futures contract on day $t-1$
T = Maturity of futures contract

If margin balance < maintenance margin:

$$Variation\ Margin = Initial\ margin - Margin\ balance$$

Options Contract

LONG Call option

$$Payoff\ or\ Value\ at\ expiration, c_T = \max(0, S_T - X)$$

$$Profit\ at\ expiration, \Pi = \max(0, S_T - X) - c_0$$

where:
c_0 = Call premium
X = Exercise/Strike price
S_T = Spot price at expiration

SHORT Call option

$$Payoff\ or\ Value\ at\ expiration, c_T = -\max(0, S_T - X)$$

$$Profit\ at\ expiration, \Pi = -[\max(0, S_T - X) - c_0]$$

LONG Put option

$$Payoff\ or\ Value\ at\ expiration, p_T = \max(0, X - S_T)$$

$$Profit\ at\ expiration, \Pi = \max(0, X - S_T) - p_0$$

SHORT Put option

$$Payoff\ or\ Value\ at\ expiration, p_T = -\max(0, X - S_T)$$

$$Profit\ at\ expiration, \Pi = -[\max(0, X - S_T) - p_0]$$

Credit Default Swap (CDS)

$$CDS\ MTM\ Change = \Delta CDS\ Spread \times CDS\ Notional \times EffDur_{CDS}$$

In a credit event, payment from CDS seller to CDS buyer $\approx LGD\ (\%) \times Notional$

Learning Module 3: Derivative Benefits, Risks, and Issuer and Investor Uses

No formula.

Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

If there are no underlying costs or benefits:

$$\text{Forward price, } F_0(T) = S_0(1+r)^T$$

If there are underlying costs or benefits in present value terms:

$$\text{Forward price, } F_0(T) = [S_0 - PV_0(Income) + PV_0(Cost)](1+r)^T$$

where:
S_0 = Current spot price
r = Risk-free rate
T = Tenor of forward contract

Under continuous compounding, $F_0(T) = S_0 e^{rT}$

Under continuous compounding, with income (i) and cost (c) expressed in %:

$$F_0(T) = S_0 e^{(r+c-i)T}$$

Foreign Exchange Forward Contract

$$F_{0,f/d}(T) = S_{0,f/d}(T) e^{(r_f - r_d)T}$$

where:
$F_{0,f/d}$ = Forward exchange rate
$S_{0,f/d}$ = Spot exchange rate
r_f = Continuously compounded risk-free rate (for price/quote currency)
r_d = Continuously compounded risk-free rate (for base currency)
T = Maturity of forward contract

Learning Module 5: Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Value of LONG Forward Prior to Expiration

$$V_0(T) = 0$$

$$V_t(T) = S_t - \frac{F_0(T)}{(1+r)^{T-t}} = S_t - F_0(T) \times (1+r)^{-(T-t)}$$

$$V_T(T) = S_0 - F_0(T)$$

If the asset incurs cost or generates income from time t through maturity,

$$V_t(T) = [S_t - PV_t(Income) + PV_t(Cost)] - F_0(T) \times (1+r)^{-(T-t)}$$

For foreign exchange forward contract,

$$V_t(T) = S_{t,f/d} - F_{0,f/d}(T) \times e^{-(r_f - r_d)(T-t)}$$

Value of SHORT Forward Prior to Expiration

$$V_0(T) = 0$$

$$V_t(T) = -\left[S_t - \frac{F_0(T)}{(1+r)^{T-t}}\right]$$

$$V_T(T) = -[S_0 - F_0(T)]$$

Interest Rate Forward Contracts (Forward Rate Agreements (FRA))

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

where:

z_A = Spot rate for A periods

z_B = Spot rate for B periods

$IFR_{A,B-A}$ = Implied forward rate for $(B-A)$ periods, starting in A periods

Payoff for a Long FRA = $\left(MRR_{B-A} - IFR_{A,B-A}\right) \times Notional\ principal \times Period$

Payoff for a Short FRA = $-\left(MRR_{B-A} - IFR_{A,B-A}\right) \times Notional\ principal \times Period$

Learning Module 6: Pricing and Valuation of Futures Contracts

If there are no underlying costs or benefits:

Futures price, $f_0(T) = S_0(1+r)^T$

If there are underlying costs or benefits in present value terms:

$$f_0(T) = [S_0 - PV_0(Income) + PV_0(Cost)](1+r)^T$$

Under continuous compounding, $f_0(T) = S_0 e^{rT}$

Under continuous compounding, with income (i) and cost (c) expressed in %:

$$f_0(T) = S_0 e^{(r+c-i)T}$$

Foreign Exchange Forward Contract

$$f_{0,f/d}(T) = S_{0,f/d}(T)e^{(r_f - r_d)T}$$

Interest Rate Futures Contract

$$f_{A,B-A} = 100 - \left(100 \times MRR_{A,B-A}\right)$$

where:

$f_{A,B-A}$ = Futures price for a market reference rate for $(B - A)$ periods that begins in A periods

Futures contract basis point value, $BPV = Notional\ principal \times 0.01\% \times Period$

Learning Module 7: Pricing and Valuation of Interest Rates and Other Swaps

For a **fixed-rate payer** in an interest rate swap:

$$Periodic\ settlement\ value = (MRR - s_N) \times Swap\ Notional \times Period$$

For a **fixed-rate receiver** in an interest rate swap:

$$Periodic\ settlement\ value = (s_N - MRR) \times Swap\ Notional \times Period$$

where:

s_N = Fixed swap rate

MRR = Market reference rate

Calculating Par Swap Rate

$$\sum_{i=1}^{N} \frac{IFR}{(1 + z_i)^i} = \sum_{i=1}^{N} \frac{s_N}{(1 + z_i)^i}$$

where:

IFR = Implied forward rates

s_N = Fixed swap rate

N = Tenor of swap contract

Valuation of Interest Rate Swap

Value of a **pay-fixed** interest rate swap on a settlement date after inception

$$= \begin{matrix} Current\ settlement \\ value \end{matrix} + \Sigma(Floating\ payments) - \Sigma(Fixed\ payments)$$

Value of a **receive-fixed** interest rate swap on a settlement date after inception

$$= \begin{matrix} Current\ settlement \\ value \end{matrix} + \Sigma(Fixed\ payments) - \Sigma(Floating\ payments)$$

Learning Module 8: Pricing and Valuation of Options

Option value = Exercise value + Time value

At time t (prior to option expiration):

$$Call\ option\ exercise\ value = Max\left[0, S_t - X(1+r)^{-(T-t)}\right]$$

$$Call\ option\ time\ value = c_t - Max\left[0, S_t - X(1+r)^{-(T-t)}\right]$$

$$Put\ option\ exercise\ value = Max\left[0, X(1+r)^{-(T-t)} - S_t\right]$$

$$Put\ option\ time\ value = p_t - Max\left[0, X(1+r)^{-(T-t)} - S_t\right]$$

$$\text{Lower bound of call option value} = Max\left[0, S_t - X(1+r)^{-(T-t)}\right]$$

$$\text{Upper bound of call option value} = S_t$$

$$\text{Lower bound of put option value} = Max\left[0, X(1+r)^{-(T-t)} - S_t\right]$$

$$\text{Upper bound of put option value} = X$$

where:
S_t = Spot price at time t
X = Exercise price (or strike price)
T = Maturity of option
r = Risk-free rate

Learning Module 9: Option Replication Using Put-Call Parity

Put-Call Parity

$$S_0 + p_0 = c_0 + X(1+r)^{-T}$$

Put-Call Forward Parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

Value of the Firm

$$V_0 = c_0 + PV(Debt) - p_0$$

Value of debt $= PV(Debt) - p_0$
Value of equity $= c_0$

Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model

Risk-neutral probability of a price increase in underlying

$$\pi = \frac{1 + r - R^d}{R^u - R^d}$$

where:

$R^u = Up\ factor = \frac{S_1^u}{S_0} > 1$

$R^d = Down\ factor = \frac{S_1^d}{S_0} < 1$

S_0 = Current asset price

S_1^u = One-period asset price when price moves up

S_1^d = One-period asset price when price moves down

Video: https://youtu.be/ymUlKgz-rAw

Hedge ratio

$$h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

where:

$c_1^u = \max(0, S_1^u - X)$

$c_1^d = \max\left(0, S_1^d - X\right)$

Riskless portfolio with a Call: h of the underlying, S, and short call position, c

$V_0 = hS_0 - c_0$

$V_1^u = hS_1^u - c_1^u$

$V_1^d = hS_1^d - c_1^d$

Riskless portfolio with a Put: h of the underlying, S, and long put position, p

$V_0 = hS_0 + p_0$

$V_1^u = hS_1^u + p_1^u$

$V_1^d = hS_1^d + p_1^d$

Value of a one-period call option

$$c_0 = \frac{\pi c_1^u + (1-\pi)c_1^d}{1+r}$$

Value of a one-period put option

$$p_0 = \frac{\pi p_1^u + (1-\pi)p_1^d}{1+r}$$

where:

$p_1^u = \max(0, X - S_1^u)$

$p_1^d = \max(0, X - S_1^d)$

Video: https://youtu.be/bXEC-78y_AU

ALTERNATIVE INVESTMENTS

Learning Module 1: Alternative Investment Features, Methods, and Structures

GP Compensation Structure

Ignoring management fee; no catch-up clause

$$r_{GP} = \max[0, p(r - r_h)]$$

Ignoring management fee; with catch-up clause

$$r_{GP} = \max[0, r_{cu} + p(r - r_h - r_{cu})]$$

where:

r_{GP} = GP's rate of return

p = Performance fee as a percentage of total return

r = Single-period rate of return

r_h = Hard hurdle rate

r_{cu} = Catch-up clause

Learning Module 2: Alternative Investment Performance and Returns

Multiple on Invested Capital

$$MOIC = \frac{Realized\ value\ of\ investment + Unrealized\ value\ of\ investment}{Total\ amount\ of\ invested\ capital}$$

Leveraged Portfolio Return

$$r_L = r + \frac{V_b}{V_c}(r - r_b)$$

where:

r = Periodic rate of return on invested funds

r_b = Periodic cost of borrowing

V_b = Amount of borrowed funds

V_c = Amount of cash (investor's own capital)

Investor's Return Net of Fees

$$r_i = \frac{P_1 - P_0 - R_{GP}}{P_0}$$

$$R_{GP} = (P_1 \times r_m) + \max[0, (P_1 - P_0) \times p]$$

where:
P_0 = Beginning-of-period asset value
P_1 = End-of-period asset value
p = GP performance fee
R_{GP} = GP's return in current terms
r_m = GP's management fees as a percentage of assets under management

Calculating Hedge Fund Fees and Returns

*Management Fee Based on **Beginning** Market Value*

$$\begin{matrix} Management \\ Fee \end{matrix} = \begin{matrix} \%Management \\ Fee \end{matrix} \times \begin{matrix} Beginning\ Market \\ Value \end{matrix}$$

*Management Fee Based on **Ending** Market Value*

$$\begin{matrix} Management \\ Fee \end{matrix} = \begin{matrix} \%Management \\ Fee \end{matrix} \times \begin{matrix} Ending\ Market \\ Value \end{matrix}$$

*Incentive Fee Calculated **Independent** of Management Fee*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times Gain$$

*Incentive Fee Calculated **Net** of Management Fee*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times (Gain - Management\ Fee)$$

*Incentive Fee with **Hard Hurdle** (**Independent** of Management Fee)*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times (Gain - Hurdle)$$

*Incentive Fee with **Hard Hurdle** (**Net** of Management Fee)*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times (Gain - Management\ Fee - Hurdle)$$

$$Hurdle = Hurdle\ Rate \times Beginning\ market\ value$$

Note: 1) No incentive is paid if hedge fund incurs loss for the year.
2) Gain may be subject to high watermark.

Video: https://youtu.be/0DKmCgsAAdc

Learning Module 3: Investments in Private Capital: Equity and Debt

No formula.

Learning Module 4: Real Estate and Infrastructure

Loan-to-Value (LTV) Ratio

$$LTV = \frac{Mortgage\ liability}{Portfolio\ value}$$

Required reduction in mortgage liability = Mortgage liability – Required mortgage liability

Learning Module 5: Natural Resources

No formula.

Learning Module 6: Hedge Funds

No formula.

Learning Module 7: Introduction to Digital Assets

No formula.

PORTFOLIO MANAGEMENT

VOLUME 2

Learning Module 1: Portfolio Risk and Return: Part I

Expected Return on Asset

$$1 + E(R) = (1 + r_{rF}) \times [1 + E(\pi)] \times [1 + E(RP)]$$

where:

r_{rF} = Real risk-free rate

$E(\pi)$ = Expected inflation

$E(RP)$ = Expected risk premium for the asset

Utility on Investment

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:

U = Utility of investment

$E(R)$ = Expected return of investment

A = Risk aversion coefficient

σ^2 = Variance of investment (Note: Substitute σ in decimals)

Capital Allocation Line (CAL)

For a portfolio of risky assets (Weight: w_i) and risk-free asset:

$$E(R_p) = R_f + \left[\frac{E(R_i) - R_f}{\sigma_i}\right]\sigma_p$$

where:

R_f = Rate of return on risk-free asset

$E(R_i)$ = Expected return of risky asset

$E(R_p)$ = Expected return of portfolio

σ_i = Standard deviation of risky asset's returns

σ_p = Standard deviation of portfolio's returns = $w_i \times \sigma_i$

$\frac{E(R_i) - R_f}{\sigma_i} = Market\ price\ of\ risk$

Two-asset portfolio

Portfolio expected return, $E(R_p) = w_1R_1 + w_2R_2$

Portfolio variance, $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)$

Portfolio standard deviation, $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)}$

Note: 1) $Cov(R_1, R_2) = \rho_{12}\sigma_1\sigma_2$

2) *n* securities requires *n* variances and $\frac{n(n-1)}{2}$ covariances

Video: https://youtu.be/lUwulZ9ONC0

Foreign Asset

Return of a foreign asset in domestic currency

$$R_D = (1 + R_{lc}) \times (1 + R_{FX}) - 1$$

Standard deviation of return of a foreign asset in domestic currency

$$\sigma_D = \sqrt{\sigma_{lc}^2 + \sigma_{FX}^2 + 2 \times \rho \times \sigma_{lc} \times \sigma_{FX}}$$

where:

R_{lc} = Return of foreign asset (in local currency)

R_{FX} = Change in exchange rate (FX rate quoted as domestic currency/foreign currency)

σ_{lc} = Standard deviation of foreign asset's returns

σ_{FX} = Standard deviation of the exchange rate (DC/FC)

ρ = Correlation coefficient between returns on foreign asset and exchange rate

Portfolio of Many Risky Assets

$$\sigma_p^2 = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\overline{Cov} = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\rho\bar{\sigma}^2$$

where:

N = Number of assets in portfolio

$\bar{\sigma}^2$ = Average variance

$\overline{Cov}$ = Average covariance

Learning Module 2: Portfolio Risk and Return: Part II

Capital Market Line (CML)

$$E(R_p) = w_f R_f + (1 - w_f)E(R_m) = R_f + \left[\frac{E(R_m) - R_f}{\sigma_m}\right]\sigma_p$$

$$\sigma_p = (1 - w_f)\sigma_m$$

Return-Generating Models

$$E(R_i) - R_f = \beta_{i1}[E(R_m) - R_f] + \sum_{j=2}^{k} \beta_{ij} E(F_j)$$

where:
$E(R_i) - R_f$ = Expected excess return on asset i
k = Number of factors
β_{ij} = Factor weights (also called factor loadings)
$E(R_m)$ = Expected return on market

The Single-Index Model

$$E(R_i) - R_f = \left(\frac{\sigma_i}{\sigma_m}\right)[E(R_m) - R_f]$$

where:
$\frac{\sigma_i}{\sigma_m} = Factor\ loading\ (or\ factor\ weight)$

Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Beta of security i

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

Portfolio beta, $\beta_p = \sum_{i=1}^{n} w_i \beta_i$

Total variance = Systematic variance + Nonsystematic variance

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_e^2$$

Total risk, $\sigma_i = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_e^2}$

Arbitrage Pricing Theory (APT) Model

$$E(R_P) = R_F + \lambda_1 \beta_{P,1} + \cdots + \lambda_K \beta_{P,K}$$

where:
$E(R_P)$ = Expected return on portfolio
R_F = Risk-free rate
λ_j = Risk premium for factor j
$\beta_{P,1}$ = Sensitivity of the portfolio to factor j
K = Number of risk factors

Fama-French Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t$$

Carhart Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,UMD} UMD_t$$

where:
$E(R_i)$ = Return on an asset in excess of the one-month T-bill return
MKT = Excess return on the market portfolio
SMB = Difference in returns between small-capitalization stocks and large-capitalization stocks (Size)
HML = Difference in returns between high-book-to-market stocks and low-book-to-market stocks (Value versus growth)
UMD = Difference in returns of the prior year's winners versus losers (Momentum)

Portfolio Performance Appraisal Measures

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p}$$

$$M^2 = \left(R_p - R_f\right)\frac{\sigma_m}{\sigma_p} + R_f = Sharpe\ ratio \times \sigma_m + R_f$$

$$M^2\ alpha = M^2 - R_m$$

Jensen's Alpha, $\alpha_p = R_p - \left[R_f + \beta_p\left(R_m - R_f\right)\right]$

Security Characteristic Line (SCL)

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f)$$

$$\text{Information ratio} = \frac{\alpha_i}{\sigma_{ei}}$$

VOLUME 6

Learning Module 1: Portfolio Management: An Overview

No formula.

Learning Module 2: Basics of Portfolio Planning and Construction

No formula.

Learning Module 3: The Behavioral Biases of Individuals

No formula.

Learning Module 4: Introduction to Risk Management

No formula.