1.3 Properties of Graphs of Functions

If you know the characteristics of a graph you can determine the type of function it represents by eliminating those that do not have these characteristics. Some of the new characteristics are described below.
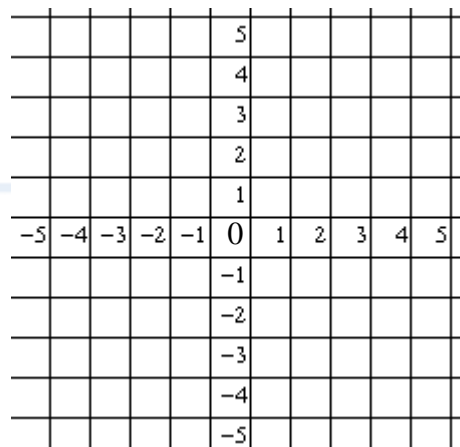
Symmetry

An even function satisfies $f(-x) = f(x)$ for all x in its domain. The graph of an even function is symmetric about the y-axis.

Thus, a function is ***even*** if it is unchanged when x is replaced with $-x$.

For instance, $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

Note: If we graph an even function for $x \geq 0$, we can get the other half by reflecting in the y-axis.
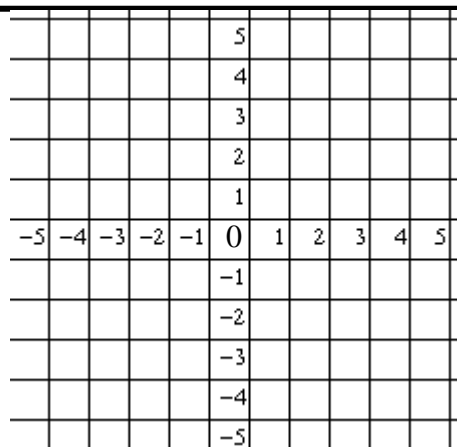
An odd function satisfies $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is symmetric about the origin.

For example, the function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

Note: If we graph an odd function for $x \geq 0$, we can get the other half by rotating through $180°$ about the origin.

Ex. 1 Determine whether each of the following functions is even, odd, or neither.

a) $f(x) = x + \dfrac{1}{x}$

b) $g(x) = x^4$

c) $h(x) = x - x^2$

Continuity and Discontinuity

A continuous function does not have any holes or breaks over its entire domain.

A **discontinuous function** has a value(s) for *x* for which a value for *y* is not defined.

Intervals of Increase and Decrease

An **interval of increase** is an interval where the *y* values get larger, moving from left to right.

An **interval of decrease** is an interval where the *y* values get smaller moving from left to right.

End Behaviours

The end behaviour of a function is the description of the values of *f(x)* as *x* approaches ∞ or $-\infty$ or both.

Interval Notation
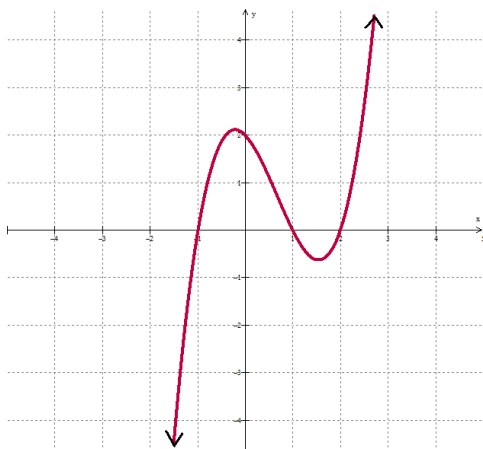
Round brackets: end point is not included
Square brackets: end point is included.

Ex:

Set notation	Interval Notation
$x < 0$	
$2 \leq x < 5$	

Ex. 2. Determine the characteristics of the following graphs:

$r(x) = (x-2)(x-1)(x+1)$

$p(x) = \dfrac{2}{x+1} - 2$

Domain : _____

Range: _____

x-intercepts: _____

y-intercepts: _____

Intervals of Increase: _____

Intervals of Decrease: _____

End Behaviour: _____

_____

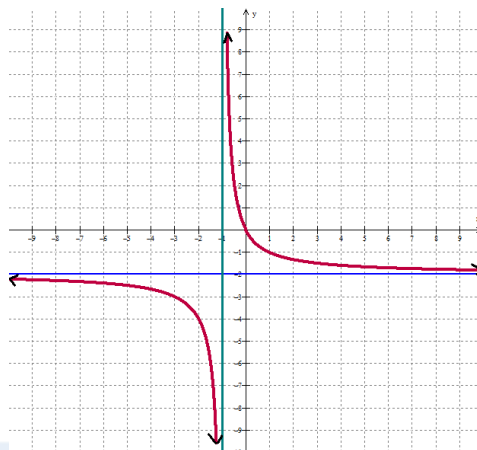Discontinuities/Asymptotes: _____

Symmetry (even/odd): _____

# of turning points _____

Local Max/Min _____

$r(x) > 0$ when x _____

$r(x) < 0$ when x _____

Domain : _____

Range: _____

x-intercepts: _____

y-intercepts: _____

Intervals of Increase: _____

Intervals of Decrease: _____

End Behaviour: _____

_____

Discontinuities/Asymptotes: _____

Symmetry (even/odd): _____

$p(x) > 0$ when x _____

$p(x) < 0$ when x _____