2016 AMC 8 Problems/Problem 1

The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?

$\textbf{(A) }605\qquad\textbf{(B) }655\qquad\textbf{(C) }665\qquad\textbf{(D) }1005\qquad \textbf{(E) }1105$

Solution

It is best to split 11 hours and 5 minutes into 2 parts, one of 11 hours and another of 5 minutes. We know that there is 60 minutes in a hour. Therefore, there are $11 \cdot 60 = 660$ minutes in 11 hours. Adding the second part(the 5 minutes) we get $660 + 5 = \boxed{\textbf{(C)}\ 665}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by First Problem	Followed by Problem 2
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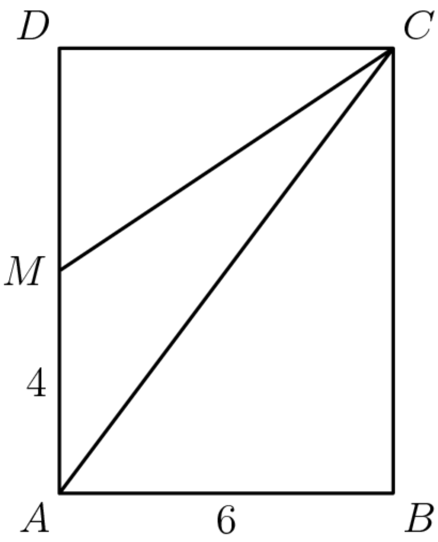
2016 AMC 8 Problems/Problem 2

In rectangle $ABCD$, $AB=6$ and $AD=8$. Point M is the midpoint of $\overline{AD}$. What is the area of $\triangle AMC$?

$\textbf{(A) }12\qquad\textbf{(B) }15\qquad\textbf{(C) }18\qquad\textbf{(D) }20\qquad \textbf{(E) }24$

Solution



Solution 1

Use the area formula for triangles: $A=\frac{bh}{2}$, where A is the area, b is the base, and h is the height.

This equation gives us $A=\frac{4\cdot 6}{2}=\frac{24}{2}=\boxed{\textbf{(A) }12}$.

Solution 2

A triangle with the same height and base as a rectangle is half of the rectangle's area. This means that a triangle with half of the base of the rectangle and also the same height means its area is one quarter of the rectangle's area. Therefore, we get $\frac{48}{4}=\boxed{\textbf{(A) }12}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 1	Followed by Problem 3
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2016 AMC 8 Problems/Problem 3

Four students take an exam. Three of their scores are $70, 80,$ and 90. If the average of their four scores is 70, then what is the remaining score?

$\textbf{(A) }40\qquad\textbf{(B) }50\qquad\textbf{(C) }55\qquad\textbf{(D) }60\qquad \textbf{(E) }70$

Solution

We can call the remaining score r. We also know that the average, 70, is equal to $\frac{70+80+90+r}{4}$. We can use basic algebra to solve for r:

$$\frac{70+80+90+r}{4}=70$$

$$\frac{240+r}{4}=70$$

$$240+r=280$$

$$r=40$$

giving us the answer of $\boxed{\textbf{(A)}\ 40}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 2	Followed by Problem 4
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2016 AMC 8 Problems/Problem 4

When Cheenu was a boy he could run 15 miles in 3 hours and 30 minutes. As an old man he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to walk a mile now compared to when he was a boy?

$\textbf{(A) }6\qquad\textbf{(B) }10\qquad\textbf{(C) }15\qquad\textbf{(D) }18\qquad\textbf{(E) }30$

Solution

When Cheenu was a boy, he could run 15 miles in 3 hours and 30 minutes $= 3 \times 60 + 30$ minutes $= 210$ minutes, thus running $\frac{210}{15} = 14$ minutes per mile. When he is an old man, he can walk 10 miles in 4 hours $= 4 \times 60$ minutes $= 240$ minutes, thus walking $\frac{240}{10} = 24$ minutes per mile. Therefore it takes him $\boxed{\textbf{(B) }10}$ minutes longer to walk a mile now compared to when he was a boy.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 3	Followed by Problem 5
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2016 AMC 8 Problems/Problem 5

The number N is a two-digit number.

- When N is divided by 9, the remainder is 1.
- When N is divided by 10, the remainder is 3.

What is the remainder when N is divided by 11?

$\textbf{(A) }0\qquad\textbf{(B) }2\qquad\textbf{(C) }4\qquad\textbf{(D) }5\qquad\textbf{(E) }7$

Solution

From the second bullet point, we know that the second digit must be 3. Because there is a remainder of 1 when it is divided by 9, the multiple of 9 must end in a 2. We now look for this one:

$$9(1)=9$$
$$9(2)=18$$
$$9(3)=27$$
$$9(4)=36$$
$$9(5)=45$$
$$9(6)=54$$
$$9(7)=63$$
$$9(8)=72$$

The number $72+1=73$ satisfies both conditions. We subtract the biggest multiple of 11 less than 73 to get the remainder. Thus, $73-11(6)=73-66=\boxed{\textbf{(E) }7}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 4	Followed by Problem 6
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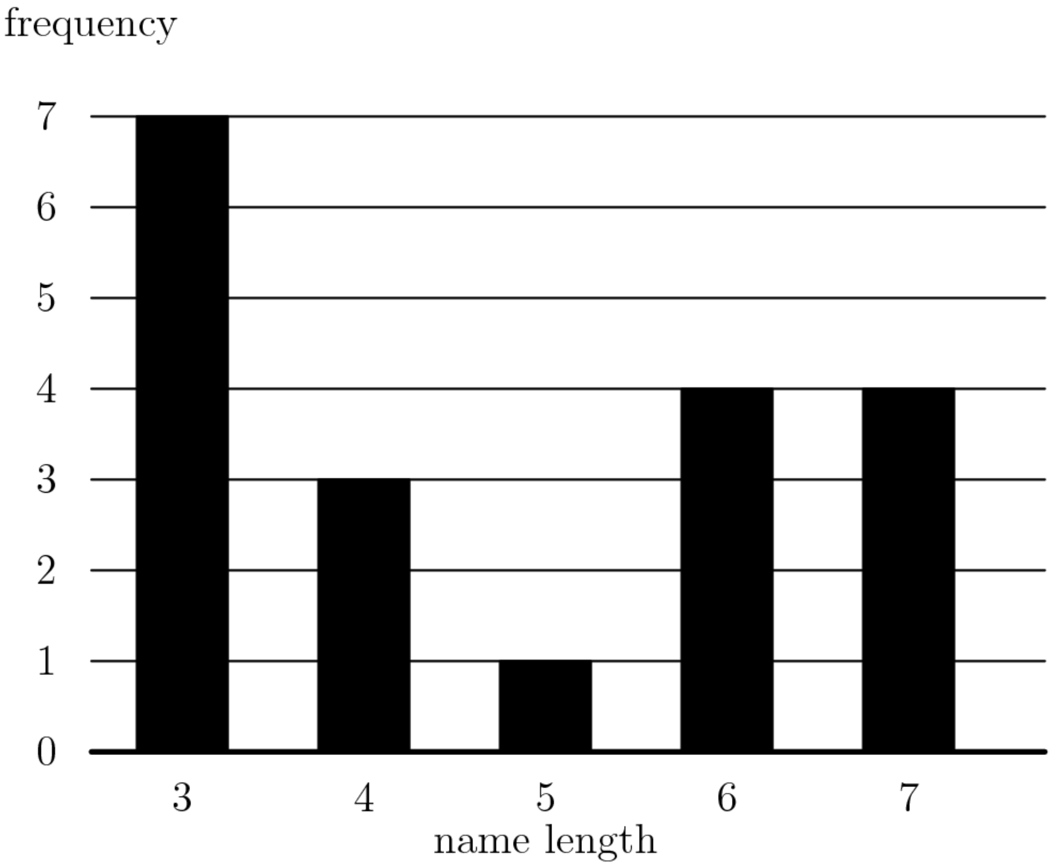
2016 AMC 8 Problems/Problem 6

The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?

$\textbf{(A) }3\qquad\textbf{(B) }4\qquad\textbf{(C) }5\qquad\textbf{(D) }6\qquad\textbf{(E) }7$



Solution

We first notice that the median name will be the 10^{th} name. The 10^{th} name is $\boxed{\textbf{(B)}\ 4}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
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2016 AMC 8 Problems/Problem 7

Which of the following numbers is not a perfect square?

$\textbf{(A) }1^{2016}\qquad\textbf{(B) }2^{2017}\qquad\textbf{(C) }3^{2018}\qquad\textbf{(D) }4^{2019}\qquad \textbf{(E) }5^{2020}$

Solution

We know that our answer must have an odd exponent in order for it to not be a square. Because 4 is a perfect square, 4^{2019} is also a perfect square, so our answer must be $\boxed{\textbf{(B) }2^{2017}}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
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2016 AMC 8 Problems/Problem 8

Find the value of the expression

$$100 - 98 + 96 - 94 + 92 - 90 + \cdots + 8 - 6 + 4 - 2.$$

$\textbf{(A)}\ 20\qquad\textbf{(B)}\ 40\qquad\textbf{(C)}\ 50\qquad\textbf{(D)}\ 80\qquad \textbf{(E)}\ 100$

Solution

We can group each subtracting pair together:

$$(100 - 98) + (96 - 94) + (92 - 90) + \ldots + (8 - 6) + (4 - 2).$$

After subtracting, we have:

$$2 + 2 + 2 + \ldots + 2 + 2 = 2(1 + 1 + 1 + \ldots + 1 + 1).$$

There are 50 even numbers, therefore there are $50/2 = 25$ even pairs. Therefore the sum is $2 \cdot 25 = \boxed{\textbf{(C)}\ 50}$

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 7	Followed by Problem 9
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2016 AMC 8 Problems/Problem 9

What is the sum of the distinct prime integer divisors of 2016?

$\textbf{(A) }9\qquad\textbf{(B) }12\qquad\textbf{(C) }16\qquad\textbf{(D) }49\qquad \textbf{(E) }63$

Solution

The prime factorization is $2016=2^5\times 3^2\times 7$. Since the problem is only asking us for the distinct prime factors, we have $2,3,7$. Their desired sum is then $\boxed{\textbf{(B) }12}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 8	Followed by Problem 10
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2016 AMC 8 Problems/Problem 10

Suppose that $a * b$ means $3a - b$. What is the value of x if

$$2 * (5 * x) = 1$$

$\textbf{(A) }\frac{1}{10}\qquad \textbf{(B) }2\qquad \textbf{(C) }\frac{10}{3}\qquad \textbf{(D) }10\qquad \textbf{(E) }14$

Solution

Let us plug in $(5 * x) = 1$ into $3a - b$. Thus it would be $3(5) - x$. Now we have $2 * (15 - x) = 1$. Plugging $2 * (15 - x)$ into $3a - b$, we have $6 - 15 + x = 1$. Solving for x we have

$$-9 + x = 1$$

$$x = \boxed{\textbf{(D)}\ 10}$$

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
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2016 AMC 8 Problems/Problem 11

Determine how many two-digit numbers satisfy the following property: when the number is added to the number obtained by reversing its digits, the sum is $132.$

$\textbf{(A) }5\qquad\textbf{(B) }7\qquad\textbf{(C) }9\qquad\textbf{(D) }11\qquad \textbf{(E) }12$

Solution

We can write the two digit number in the form of $10a+b$; reverse of $10a+b$ is $10b+a$. The sum of those numbers is:

$$(10a+b)+(10b+a)=132$$

$$11a+11b=132$$

$$a+b=12$$

We can use brute force to find order pairs (a,b) such that $a+b=12$. Since a and b are both digits, both a and b have to be integers less than 10. Thus are ordered pairs are $(3,9); (4,8); (5,7); (6,6); (7,5); (8,4); (9,3)$ or $\boxed{\textbf{(B)} 7}$ ordered pairs.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 10	Followed by Problem 12
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2016 AMC 8 Problems/Problem 12

Jefferson Middle School has the same number of boys and girls. Three-fourths of the girls and two-thirds of the boys went on a field trip. What fraction of the students were girls?

$\textbf{(A)}~\frac{1}{2}\qquad\textbf{(B)}~\frac{9}{17}\qquad\textbf{(C)}~\frac{7}{13}\qquad\textbf{(D)}~\frac{2}{3}\qquad\textbf{(E)}~\frac{14}{15}$

Solution 1

Set the number of children to a number that is divisible by two, four, and three. In this question, the number of children in the school is not a specific number because there are no actual numbers in the question, only ratios.This way, we can calculate the answer without dealing with decimals. 120 is a number that works. There will be 60 girls and 60 boys. So, there will be $60 \cdot \frac{3}{4}$ = 45 girls on the trip and $60 \cdot \frac{2}{3}$ = 40 boys on the trip. The total number of children on the trip is 85, so the fraction of girls on the trip is $\frac{45}{85}$ or $\boxed{\textbf{(B)}\frac{9}{17}}$

Solution 2

Let their be b boys and g girls in the school. We see $g=b$, which mean $\frac{3}{4}b+\frac{2}{3}b=\frac{17}{12}b$ kids went on the trip and $\frac{3}{4}b$ kids are girls. So, the answer is $\frac{\frac{3}{4}b}{\frac{17}{12}b}=\frac{9}{17}$, which is $\boxed{\textbf{(B)}\frac{9}{17}}$

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 11	Followed by Problem 13
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2016 AMC 8 Problems/Problem 13

Two different numbers are randomly selected from the set $-2, -1, 0, 3, 4, 5$ and multiplied together. What is the probability that the product is 0?

$\textbf{(A) }\dfrac{1}{6}\qquad\textbf{(B) }\dfrac{1}{5}\qquad\textbf{(C) }\dfrac{1}{4}\qquad\textbf{(D) }\dfrac{1}{3}\qquad\textbf{(E) }\dfrac{1}{2}$

Solution 1

The product can only be 0 if one of the numbers is 0. Once we chose 0, there are 5 ways we can chose the second number, or $6-1$. There are $\dbinom{6}{2}$ ways we can chose 2 numbers randomly, and that is 15. So, $\dfrac{5}{15}=\dfrac{1}{3}$ so the answer is $\boxed{\textbf{(D) }\dfrac{1}{3}}$.

Solution 2

There are a total of 30 possibilities, because the numbers are different. We want 0 to be the product so one of the numbers is 0. There are 5 possibilities where 0 is chosen for the first number and there are 5 ways for 0 to be chosen as the second number. We seek $\boxed{\textbf{(D) }\dfrac{1}{3}}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 12	Followed by Problem 14
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2016 AMC 8 Problems/Problem 14

Karl's car uses a gallon of gas every 35 miles, and his gas tank holds 14 gallons when it is full. One day, Karl started with a full tank of gas, drove 350 miles, bought 8 gallons of gas, and continued driving to his destination. When he arrived, his gas tank was half full. How many miles did Karl drive that day?

$\textbf{(A)}\ 525 \qquad \textbf{(B)}\ 560 \qquad \textbf{(C)}\ 595 \qquad \textbf{(D)}\ 665 \qquad \textbf{(E)}\ 735$

Solution

Since he uses a gallon of gas every 35 miles, he had used $\frac{350}{35} = 10$ gallons after 350 miles. Therefore, after the first leg of his trip he had $14 - 10 = 4$ gallons of gas left. Then, he bought 8 gallons of gas, which brought him up to 12 gallons of gas in his gas tank. When he arrived, he had $\frac{1}{2} \cdot 14 = 7$ gallons of gas. So he used 5 gallons of gas on the second leg of his trip. Therefore, the second part of his trip covered $5 \cdot 35 = 175$ miles. Adding this to the 350 miles, we see that he drove $350 + 175 = \boxed{\textbf{(A)}\ 525}$ miles.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 13	Followed by Problem 15
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2016 AMC 8 Problems/Problem 15

What is the largest power of 2 that is a divisor of $13^4 - 11^4$?

$\textbf{(A)}\ 8\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 32\qquad\textbf{(D)}\ 64\qquad\textbf{(E)}\ 128$

Solution

First, we use difference of squares on $13^4 - 11^4 = (13^2)^2 - (11^2)^2$ to get $13^4 - 11^4 = (13^2 + 11^2)(13^2 - 11^2)$. Using difference of squares again and simplifying, we get $(169 + 121)(13 + 11)(13 - 11) = 290 \cdot 24 \cdot 2 = (2 \cdot 8 \cdot 2) \cdot (3 \cdot 145)$. Realizing that we don't need the right-hand side because it doesn't contain any factor of 2, we see that the greatest power of 2 that is a divisor $13^4 - 11^4$ is $\boxed{\textbf{(C)}\ 32}$.

2016 AMC 8 Problems/Problem 16

Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead, because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

$\textbf{(A) }1\dfrac{1}{4}\qquad\textbf{(B) }3\dfrac{1}{3}\qquad\textbf{(C) }4\qquad\textbf{(D) }5\qquad \textbf{(E) }25$

Solution

Each lap Bonnie runs, Annie runs another quarter lap, so Bonnie will run four laps before she is overtaken. That means Annie will have run $\boxed{\textbf{(D)}\ 5}$ laps.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
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2016 AMC 8 Problems/Problem 17

An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence $9, 1, 1,$ then how many passwords are possible?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 7290 \qquad \textbf{(C)}\ 9000 \qquad \textbf{(D)}\ 9990 \qquad \textbf{(E)}\ 9999$

Solution 1

For the first three digits, there are $10^3 - 1 = 999$ combinations since 911 is not allowed. For the final digit, any of the 10 numbers are allowed. $999 \cdot 10 = 9990 \rightarrow \boxed{\textbf{(D)}\ 9990}$

Solution 2

Counting the prohibited cases, we find that there are 10 of them. This is because we start with 9,1,1 and we can have any of the 10 digits for the last digit. So our answer is $10^4 - 10 = \boxed{\textbf{(D)}\ 9990}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
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2016 AMC 8 Problems/Problem 18

In an All-Area track meet, 216 sprinters enter a $100-$meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race, the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter?

$\textbf{(A)}\ 36\qquad\textbf{(B)}\ 42\qquad\textbf{(C)}\ 43\qquad\textbf{(D)}\ 60\qquad\textbf{(E)}\ 72$

Solution

From any $n-$th race, only $\frac{1}{6}$ will continue on. Since we wish to find the total number of races, a column representing the races over time is ideal. Starting with the first race:

$$\frac{216}{6}=36$$

$$\frac{36}{6}=6$$

$$\frac{6}{6}=1$$

Adding all of the numbers in the second column yields $\boxed{\textbf{(C)}\ 43}$

Solution 2

Every race eliminates 5 players. The winner is decided when there is only 1 runner left. Thus, 215 players have to be eliminated. Therefore, we need $\frac{215}{5}$ games to decide the winner, or $\boxed{\textbf{(C)}\ 43}$

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
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\end{align}

2016 AMC 8 Problems/Problem 19

The sum of 25 consecutive even integers is $10,000$. What is the largest of these 25 consecutive integers?

$\textbf{(A)}\ 360 \qquad \textbf{(B)}\ 388 \qquad \textbf{(C)}\ 412 \qquad \textbf{(D)}\ 416 \qquad \textbf{(E)}\ 424$

Solution

Let n be the 13th consecutive even integer that's being added up. By doing this, we can see that the sum of all 25 even numbers will simplify to $25n$ since $(n-2k)+\dots+(n-4)+(n-2)+(n)+(n+2)+(n+4)+\dots+(n+2k)=25n$. Now, $25n=10000 \rightarrow n=400$ Remembering that this is the 13th integer, we wish to find the 25th, which is $400+2(25-13)=\boxed{\textbf{(E)}\ 424}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 18	Followed by Problem 20
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2016 AMC 8 Problems/Problem 20

The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?

$\textbf{(A) }20\qquad\textbf{(B) }30\qquad\textbf{(C) }60\qquad\textbf{(D) }120\qquad \textbf{(E) }180$

Solution

We wish to find possible values of a, b, and c. By finding the greatest common factor of 12 and 15, algebraically, it's some multiple of b and from looking at the numbers, we are sure that it is 3, thus b is 3. Moving on to a and c, in order to minimize them, we wish to find the least such that the least common multiple of a and 3 is 12, $\rightarrow 4$. Similarly with 3 and c, we obtain 5. The least common multiple of 4 and 5 is $20 \rightarrow \boxed{\textbf{(A)} 20}$

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 19	Followed by Problem 21
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2016 AMC 8 Problems/Problem 21

A box contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn?

$\textbf{(A) }\dfrac{3}{10}\qquad\textbf{(B) }\dfrac{2}{5}\qquad\textbf{(C) }\dfrac{1}{2}\qquad\textbf{(D) }\dfrac{3}{5}\qquad\textbf{(E) }\dfrac{7}{10}$

Solution 1

We put five chips randomly in order, and then pick the chips from the left to the right. However, we notice that whenever the last chip we draw is red, we pick both green chips before we pick the last (red) chip. Similarly, when the last chip is green, we pick all three red chips before the last (green) chip. Because a green chip will be last 4 out of 10 times and a red chip will be last 6 out of 10 times, our answer is $\boxed{\textbf{(B) } \frac{2}{5}}$.

Solution 2

There are two ways of ending the game, either you picked out all the red chips or you picked out all the green chips. We can pick out 3 red chips, 3 red chips and 1 green chip, 2 green chips, 2 green chips and 1 red chip, and 2 green chips and 2 red chips. Because order is important in this problem, there are $1+4+1+3+6=15$ ways to pick out the chip. But we noticed that if you pick out the three red chips before you pick out the green chip, the game ends. So we need to subtract cases like that to get the total number of ways a game could end, which $15-5=10$. Out of the 10 ways to end the game, 4 of them ends with a red chip. The answer is $\frac{4}{10}=\frac{2}{5}$, or $\boxed{\textbf{(B) } \frac{2}{5}}$.

2016 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2016))	
Preceded by Problem 20	Followed by Problem 22
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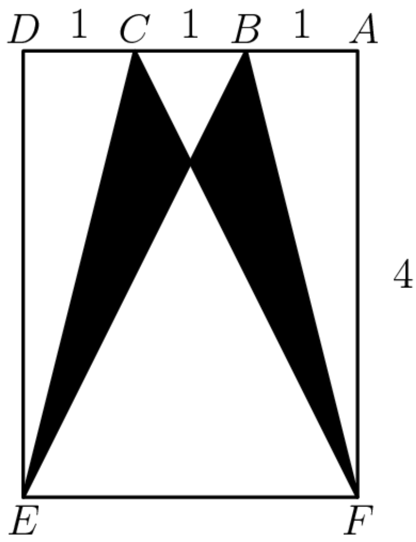
2016 AMC 8 Problems/Problem 22

Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA$. What is the area of the "bat wings" (shaded area)?



$\textbf{(A) }2\qquad\textbf{(B) }2 \frac{1}{2}\qquad\textbf{(C) }3\qquad\textbf{(D) }3 \frac{1}{2}\qquad \textbf{(E) }5$

Solution

The area of trapezoid $CBFE$ is $\frac{1+3}{2}\cdot 4=8$. Next, we find the height of each triangle to calculate their area. The triangles are similar, and are in a $3:1$ ratio, so the height of the larger one is $3,$ while the height of the smaller one is $1.$ Thus, their areas are $\frac{1}{2}$ and $\frac{9}{2}$. Subtracting these areas from the trapezoid, we get $8-\frac{1}{2}-\frac{9}{2} = \boxed{3}$. Therefore, the answer is $\boxed{\textbf{(C) }3}$.

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2016 AMC 8 Problems/Problem 23

Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D. The circles intersect at two points, one of which is E. What is the degree measure of $\angle CED$?

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 105 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 135 \qquad \textbf{(E)}\ 150$

Solution

Drawing the diagram, we see that $\triangle EAB$ is equilateral as each side is the radius of one of the two circles. Therefore, $\widehat{\text{EB}} = m\angle EAB - 60^\circ$. Therefore, since it is an inscribed angle, $m\angle ECB = \frac{60^\circ}{2} = 30^\circ$. So, in $\triangle ECD$, $m\angle ECB = m\angle EDA = 30^\circ$, and $m\angle CED = 180^\circ - 30^\circ - 30^\circ = 120^\circ$. Our answer is $\boxed{\textbf{(C)}\ 120}$.

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2016 AMC 8 Problems/Problem 24

The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P?

$\textbf{(A) }1\qquad\textbf{(B) }2\qquad\textbf{(C) }3\qquad\textbf{(D) }4\qquad \textbf{(E) }5$

Solution

We see that since QRS is divisible by 5, S must equal either 0 or 5, but it cannot equal 0, so $S=5$. We notice that since PQR must be even, R must be either 2 or 4. However, when $R=2$, we see that $T \equiv 2 \pmod{3}$, which cannot happen because 2 and 5 are already used up; so $R=4$. This gives $T \equiv 3 \pmod{4}$, meaning $T=3$. Now, we see that Q could be either 1 or 2, but 14 is not divisible by 4, but 24 is. This means that $S=4$ and $P=\boxed{\textbf{(A)}\ 1}$.

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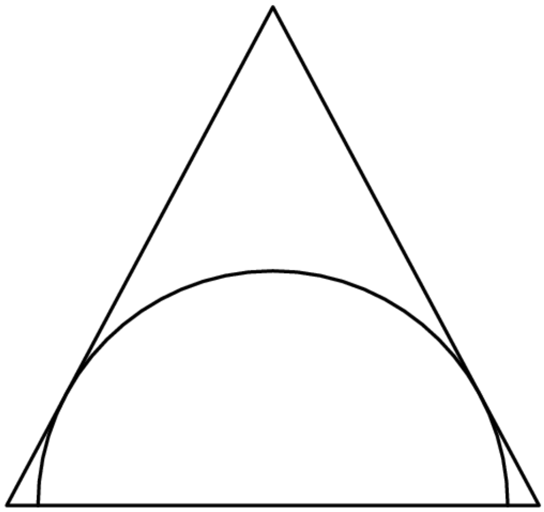
2016 AMC 8 Problems/Problem 25

A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?



$\textbf{(A) }4\sqrt{3}\qquad\textbf{(B) } \dfrac{120}{17}\qquad\textbf{(C) }10\qquad\textbf{(D) }\dfrac{17\sqrt{2}}{2}\qquad\textbf{(E)} \dfrac{17\sqrt{3}}{2}$

Contents

Solution 1

Draw the altitude from the top of the triangle to its base, dividing the isosceles triangle into two right triangles with height 15 and base $\frac{16}{2} = 8$. The Pythagorean triple 8-15-17 tells us that these triangles have hypotenuses of 17.

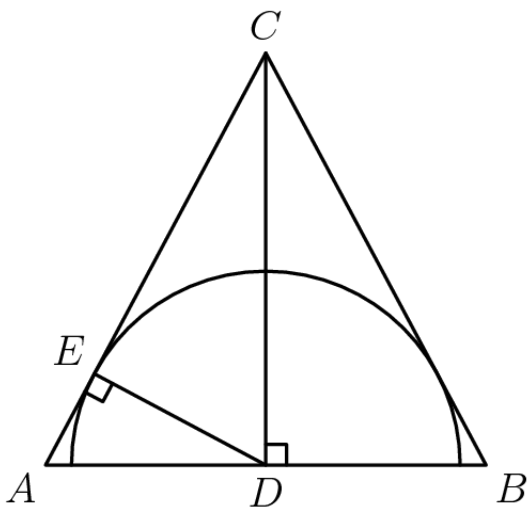
Now draw an altitude of one of the smaller right triangles, starting from the foot of the first altitude we drew (which is also the center of the circle that contains the semicircle) and going to the hypotenuse of the right triangle. This segment is both an altitude of the right triangle as well as the radius of the semicircle (this is because tangent lines to circles, such as the hypotenuse touching the semicircle, are always perpendicular to the radii of the circles drawn to the point of tangency). Let this segment's length be r.

The area of the entire isosceles triangle is $\frac{(16)(15)}{2} = 120$, so the area of each of the two congruent right triangles it gets split into is $\frac{120}{2} = 60$. We can also find the area of one of the two congruent right triangles by using its hypotenuse as its base and the radius of the semicircle, the altitude we drew, as its height. Then the area of the triangle is $\frac{17r}{2}$. Thus we can write the equation $\frac{17r}{2}=60$, so $17r=120$, so $r=\boxed{\textbf{(B) }\frac{120}{17}}$.

Solution 2

First, we draw a line perpendicular to the base of the triangle and cut it in half. The base of the resulting right triangle would be 8, and the height would be 15. Using the Pythagorean theorem, we can find the length of the hypotenuse, which would be 17. Using the two legs of the right angle, we can find the area of the right triangle, 60. $\frac{60}{17}$ times 2 get you the radius, which is the height of the right triangle when using the hypotenuse as the base. The answer is $\boxed{\textbf{(B) }\frac{120}{17}}$.

Solution 3: Similar Triangles

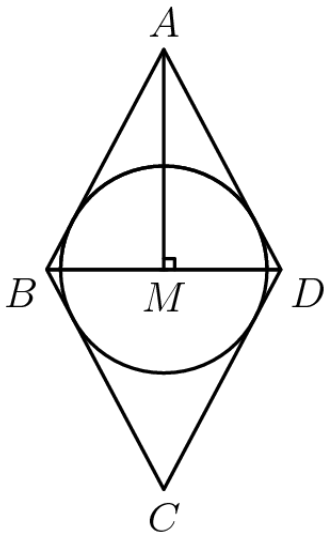


Let's call the triangle $\triangle ABC$, where $AB=16$ and $AC=BC$. Let's say that D is the midpoint of AB and E is the point where AC is tangent to the semicircle. We could also use BC instead of AC because of symmetry.

We notice that $\triangle ACD \cong \triangle BCD$, and are both 8-15-17 right triangles. We also know that we create a right angle with the intersection of the radius and a tangent line of a circle (or part of a circle). So, by AA similarity, $\triangle AED \sim \triangle ADC$, with $\angle EAD \cong \angle DAC$ and $\angle CDA \cong \angle DEA$. This similarity means that we can create a proportion: $\frac{AD}{AB}=\frac{DE}{CD}$. We plug in $AD=\frac{AB}{2}=8, AC=17,$ and $CD=15$. After we multiply both sides by 15, we get $DE=\frac{8}{17}\cdot 15=\boxed{\textbf{(B) }\frac{120}{17}}$.

(By the way, we could also use $\triangle DEC \sim \triangle ADC$.)

Solution 4: Inscribed Circle



We'll call this triangle $\triangle ABD$. Let the midpoint of base BD be M. Divide the triangle in half by drawing a line from A to M. Half the base of $\triangle ABD$ is $\frac{16}{2}=8$. The height is 15, which is given in the question. Using the Pythagorean Triple 8-15-17, the length of each of the legs (AB and DA) is 17.

Reflect the triangle over its base. This will create an inscribed circle in a rhombus $ABCD$. Because $AB \cong DA$, $BC \cong CD$. Therefore $AB=BC=CD=DA$.

The semiperimeter s of the rhombus is $\frac{AB+BC+CD+DA}{2}=\frac{(17)(4)}{2}=34$. Since the area of $\triangle ABD$ is $\frac{bh}{2}$, the area $[ABCD]$ of the rhombus is twice that, which is $bh=(16)(15)=240$.

The Formula for the Incircle of a Quadrilateral (https://en.wikipedia.org/wiki/Incircle_and_excircles_of_a_triangle#Incircle) is sr = $[ABCD]$.

Substituting the semiperimeter and area into the equation, $34r=240$. Solving this, $r=\frac{240}{34}$ = $\boxed{\textbf{(B)}\ \frac{120}{17}}$.

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