2017 AMC12A

Problem 1

Pablo buys popsicles for his friends. The store sells single popsicles for $1 each, 3-popsicle boxes for $2, and 5-popsicle boxes for $3. What is the greatest number of popsicles that Pablo can buy with $8?

Pablo 给他的朋友买棒冰，商店单根棒冰卖 1 美元一根，1 盒 3 根棒冰每盒卖 2 美元，1 盒 5 根棒冰每盒卖 3 美元，Pablo 有 8 美元，最多可以买多次根棒冰？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 15$

Problem 2

The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

两个非零实数的和是它们乘积的 4 倍，问这 2 个数的倒数之和是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 12$

Problem 3

Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

Carroll 女士承诺任何人只要在即将到来的考试中做对所有的选择题，此次考试就能得到一个 A，由此，下面哪句话在逻辑上是对的？

(A) If Lewis did not receive an A，then he got all of the multiple choice questions wrong丨如果 Lewis 没有得到 A，那么他的所有选择题做的都是错的。

(B) If Lewis did not receive an A，then he got at least one of the multiple choice questions wrong.丨如果 Lewis 没有得到 A，那么他至少有一道选择题做的是错的。

(C) If Lewis got at least one of the multiple choice questions wrong,then he did not receive an A.丨如果 Lewis 至少有一个选择题做错了，那么他就得不到 A.

(D) If Lewis received an A，then he got all of the multiple choice questions right. 丨 如果 Lewis 得到一个 A，那么他所有的选择题都做对了.

(E) If Lewis reccived an A,then he got at least one of the multiple choice questions right.丨如果 Lewis 得到一个 A，那么他至少做对了一道选择题。

Problem 4

Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

Jerry 和 Silvia 想从一块正方形场地的西南角走到东北角。Jerry 先朝东走，再朝北走，走到达目标，而 Sina 朝着东北角直接走直线到达目标，Silvia 所走的路程和 Jerry 所走路程相比，少走多少，更接近下面哪个数值？

(A) 30% **(B)** 40% **(C)** 50% **(D)** 60% **(E)** 70%

Problem 5

At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

一次聚会有 30 人，其中有 20 人相互之间都认识，还有 10 人任何人都不认识，如果相互之间认识的人拥抱，相互之间不认识的人握手，那么一共握手多少次？

(A) 240 **(B)** 245 **(C)** 290 **(D)** 480 **(E)** 490

Problem 6

Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

Joy 一共有 30 根薄竹竿，每根竹竿的长度是 1cm 到 30cm 之间的不同的整数长度．她把长度为 3cm，7cm 和 15cm 的竹竿放在桌上，然后她想选择第 4 根竹竿，保证能和这 3 根形成一个面积为正的四边形，剩下的竹竿中，有多少根她可以选择作为第 4 根？

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

Problem 7

Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 1$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2017)$?

定义一个在正整数上的函数，满足$f(1) = 2$，当n是偶数时，则$f(n) = f(n-1) + 1$，当n是大于 1 的奇数时，则$f(n) = f(n-2) + 2$，问$f(2017)$是多少？

$\textbf{(A)}\ 2017 \qquad \textbf{(B)}\ 2018 \qquad \textbf{(C)}\ 4034 \qquad \textbf{(D)}\ 4035 \qquad \textbf{(E)}\ 4036$

Problem 8

The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

和线段$\overline{AB}$的距离在 3 个单位长以内的所有点，在三维空间中组成的区域的体积为 216π，AB 的长度是多少？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Problem 9

Let S be the set of points (x,y) in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than the common value. Which of the following is a correct description of S?

S 表示这三个量 3，$x+2$和$y-4$中有 2 个量相等，且第 3 个量不大于这个共同值的坐标平面内所有的点(x,y)所组成的集合，那么下面哪个是对 *S* 的正确描述？

(A) a single point丨1 个点
(B) two intersecting lines丨2 条相交的直线
(C) three lines whose pairwise intersections are three distinct points，丨3 条直线，这 3 条直线两两相交形成 3 个不同的点
(D) a triangle丨1 个三角形
(E) three rays with a common point丨有一个公共点的 3 条射线

Problem 10

Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloe's number?

Chloe 从区间［0，2017］中均匀随机的选择一个实数，Laurent 独立的从区间［0，4034］中均匀随机的选择一个实数，Laurent 的数比 Chloe 的大的概率是多少？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{5}{6} \qquad \textbf{(E)}\ \frac{7}{8}$

Problem 11

Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

Claire 把一个凸多边形的内角的度数相加，得到和为 2017，后来她发现漏加了一个角，那么漏加的这个角的度数是多少度？

$\textbf{(A)}\ 37 \qquad \textbf{(B)}\ 63 \qquad \textbf{(C)}\ 117 \qquad \textbf{(D)}\ 143 \qquad \textbf{(E)}\ 163$

Problem 12

There are 10 horses, named Horse 1, Horse 2, . . ., Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

一共有 10 匹马，名字分别为马 1，马 2，…，马 10，这是根据这些马沿着圆形赛道跑一圈所需要的分钟数命名的：马 k 跑 1 圈需要 k 分钟。在初始时间 0 时，所有的马都在跑道的起点，这些马开始朝着同一方向跑，并且各自速度恒定。$S > 0$（以分钟为单位）表示这 10 匹马再次同时处于起点的最短时间，在这里$S = 2520$。$T > 0$（以分钟为单位）表示至少 5 匹马再次同时处于起点的最短时间，那么 T的各位数字之和是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 13

Driving at a constant speed, Sharon usually takes 180 minutes to drive from her house to her mother's house. One day Sharon begins the drive at her usual speed, but after driving $\frac{1}{3}$ of the way, she hits a bad snowstorm and reduces her speed by 20 miles per hour. This time the trip takes her a total of 276 minutes. How many miles is the drive from Sharon's house to her mother's house?

Sharon 平常以恒定的速度从她家开车到她妈妈家，一般需要 180 分钟。一天 Sharon 以她平常的开车速度开了$\frac{1}{3}$路程后，遭遇暴风雪，接着把速度降低了 20 英里每小时。这一次，全程开完她总共花了 276 分钟。那么从 Sharon 家到她妈妈家总共多少英里？

$\textbf{(A)}\ 132 \qquad \textbf{(B)}\ 135 \qquad \textbf{(C)}\ 138 \qquad \textbf{(D)}\ 141 \qquad \textbf{(E)}\ 144$

Problem 14

Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

Alice 拒绝坐在 Bob 或 Carla 的旁边，Derek 拒绝坐在 Eric 的旁边，在满足以上要求的情况下，他们 5 人要坐在一排 5 张椅子上，一共有多少种坐法？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 40$

Problem 15

Let $f(x) = \sin{x} + 2\cos{x} + 3\tan{x}$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

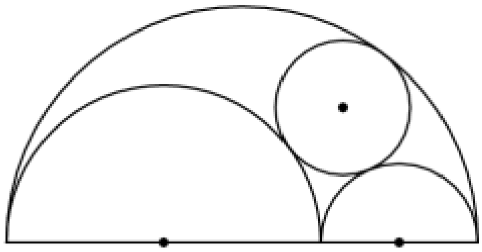
令$f(x) = \sin{x} + 2\cos{x} + 3\tan{x}$，其中未知数 x 是弧度制，使得$f(x) = 0$的最小正数 x 在下面哪个区间内？

$\textbf{(A)}\ (0,1) \qquad \textbf{(B)}\ (1,2) \qquad \textbf{(C)}\ (2,3) \qquad \textbf{(D)}\ (3,4) \qquad \textbf{(E)}\ (4,5)$

Problem 16

In the figure below, semicircles with centers at A and B and with radii 2 and 1, respectively, are drawn in the interior of, and sharing bases with, a semicircle with diameter JK. The two smaller semicircles are externally tangent to each other and internally tangent to the largest semicircle. A circle centered at P is drawn externally tangent to the two smaller semicircles and internally tangent to the largest semicircle. What is the radius of the circle centered at P?

如下图所示，圆心在 *A* 和 *B*，半径分别是 2 和 1 的两个半圆，画在直径为 *JK* 的一个大的半圆内部，并且底部和这个大的半圆的底部重合。这两个小的半圆相互外切，并且都和大的半圆内切。另一个圆心在 *P* 点的圆和这两个小的半圆外切，并和大的半圆内物。那么圆心在 *P* 点的圆的半径是多少？



$\textbf{(A)}\ \frac{3}{4} \qquad \textbf{(B)}\ \frac{6}{7} \qquad \textbf{(C)}\ \frac{\sqrt{3}}{2} \qquad \textbf{(D)}\ \frac{5}{8}\sqrt{2} \qquad \textbf{(E)}\ \frac{11}{12}$

Problem 17

There are 24 different complex numbers z such that $z^{24} = 1$. For how many of these is z^6 a real number?

满足方程$z^{24} = 1$的复数 *z* 共有 24 个。那么其中有多少个能够使得z^6是个实数？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 24$

Problem 18

Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

$S(n)$表示整数 n 的各位上数字之和。例如，$S(1507) = 13$. 对于某个特定的正整数 n，$S(n) = 1274$. 问下面哪个可能是$S(n+1)$的值？

(A) 1 **(B)** 3 **(C)** 12 **(D)** 1239 **(E)** 1265

Problem 19

A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

一个边长为 x 的正方形内接在一个边长分别为 3，4，5 的直角三角形中，这里正方形的一个顶点和三角形的直角顶点重合。一个边长为 y 的正方形内接在另一个边长分别为 3，4，5 的直角三角形中，这里正方形的一条边和直角三角形的斜边重合。问$\frac{x}{y}$是多少?

(A) $\frac{12}{13}$ **(B)** $\frac{35}{37}$ **(C)** 1 **(D)** $\frac{37}{35}$ **(E)** $\frac{13}{12}$

Problem 20

How many ordered pairs (a, b) such that a is a positive real number and b is an integer between 2 and 200, inclusive, satisfy the equation $(\log_b a)^{2017} = \log_b(a^{2017})$?

若 *a* 是一个正实数，*b* 是一个在 2 到 200 之间（包含 2 和 200）的整数，那么有多少个有序对（*a*，*b*）满足方程$(\log_b a)^{2017} = \log_b(a^{2017})$?

(A) 198 **(B)** 199 **(C)** 398 **(D)** 399 **(E)** 597

Problem 21

A set S is constructed as follows. To begin, $S = \{0, 10\}$. Repeatedly, as long as possible, if x is an integer root of some polynomial $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ for some $n \geq 1$, all of whose coefficients a_i are elements of S, then x is put into S. When no more elements can be added to S, how many elements does S have?

集合 S 由如下方法构造。一开始，$S = \{0, 10\}$. 构造某个多项式$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$，其中系数$a_i$都是 *S* 的元素，且$n \geq 1$. 若 *x* 是此多项式的一个整数根，那么就把 *x* 放入 *S* 中。只要有可能，就继续利用 *S* 中的元素作为系数构造多项式，把多项式的整数根放入 *S* 中，如此往复，直到无法再向 *S* 中增添新的元素，那么最终 *S* 中有多少个元素？

(A) 4 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

Problem 22

A square is drawn in the Cartesian coordinate plane with vertices at $(2,2)$, $(-2,2)$, $(-2,-2)$, $(2,-2)$. A particle starts at $(0,0)$. Every second it moves with equal probability to one of the eight lattice points (points with integer coordinates) closest to its current position, independently of its previous moves. In other words, the probability is $1/8$ that the particle will move from (x,y) to each of $(x,y+1)$, $(x+1,y+1)$, $(x+1,y)$, $(x+1,y-1)$, $(x,y-1)$, $(x-1,y-1)$, $(x-1,y)$, or $(x-1,y+1)$. The particle will eventually hit the square for the first time, either at one of the 4 corners of the square or at one of the 12 lattice points in the interior of one of the sides of the square. The probability that it will hit at a corner rather than at an interior point of a side is m/n, where m and n are relatively prime positive integers. What is $m+n$?

在坐标平面内画一个正方形，四个顶点分别为$(2,2)$，$(-2,2)$，$(-2,-2)$，$(2,-2)$。一个粒子从点$(0,0)$出发，每一秒它都以相同概率向和当前位置最近的 8 个格点（坐标为整数的点）移动，和它前一步的移动情况独立。换句话说，粒子从点(x,y)移向$(x,y+1)$，$(x+1,y+1)$，$(x+1,y)$，$(x+1,y-1)$, $(x,y-1)$, $(x-1,y-1)$, $(x-1,y)$，或 $(x-1,y+1)$中的每个点的概率都是$1/8$.这个粒子最终会第一次到达正方形，要么落在正方形的 4 个角落的其中之一，要么落在正方形的边上除了角落的其他 12 个内部格点的其中之一。假设粒子落在正方形的某个角落而不是正方形边上的一个内部格点的概率是m/n，这里 *m* 和 *n* 是互质的正整数。那么 *m* +*n* 是是多少？

(A) 4 **(B)** 5 **(C)** 7 **(D)** 15 **(E)** 39

Problem 23

For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

a，*b*，*c* 为实数，满足以下条件：多项式$g(x) = x^3 + ax^2 + x + 10$的三个不同的根也都是多项式$f(x) = x^4 + x^3 + bx^2 + 100x + c$.的根，问$f(1)$是多少？

$\textbf{(A)}\ -9009 \qquad \textbf{(B)}\ -8008 \qquad \textbf{(C)}\ -7007 \qquad \textbf{(D)}\ -6006 \qquad \textbf{(E)}\ -5005$

Problem 24

Quadrilateral $ABCD$ is inscribed in circle O and has side lengths $AB=3, BC=2, CD=6$, and $DA=8$. Let X and Y be points on $\overline{BD}$ such that $\frac{DX}{BD} = \frac{1}{4}$ and $\frac{BY}{BD} = \frac{11}{36}$. Let E be the intersection of line AX and the line through Y parallel to $\overline{AD}$. Let F be the intersection of line CX and the line through E parallel to $\overline{AC}$. Let G be the point on circle O other than C that lies on line CX. What is $XF\cdot XG$?

四边形 *ABCD* 内接在圆 *O* 中，边长分别为 *AB*=3，*BC*=2，*CD*= 6，*DA*=8，点 *X* 和 *Y* 在线段$\overline{BD}$上，满足通过$\frac{DX}{BD} = \frac{1}{4}$，$\frac{BY}{BD} = \frac{11}{36}$. 通过 *Y* 且和$\overline{AD}$平行的直线交直线 *AX* 于点 E。通过 *E* 且和$\overline{AC}$平行的直线交直线 *CX* 于点 *F*。点 *G* 是圆 *O* 上不同于 *C* 的点，且在直线 *CX* 上。问$XF\cdot XG$是多少？

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ \frac{59-5\sqrt{2}}{3} \qquad \textbf{(C)}\ \frac{91-12\sqrt{3}}{4} \qquad \textbf{(D)}\ \frac{67-10\sqrt{2}}{3} \qquad \textbf{(E)}\ 18$

Problem 25

The vertices V of a centrally symmetric hexagon in the complex plane are given by

$$V=\left\{\sqrt{2}i,-\sqrt{2}i,\frac{1}{\sqrt{8}}(1+i),\frac{1}{\sqrt{8}}(-1+i),\frac{1}{\sqrt{8}}(1-i),\frac{1}{\sqrt{8}}(-1-i)\right\}.$$

For each j, $1\leq j\leq 12$, an element z_j is chosen from V at random, independently of the other choices. Let $P=\prod_{j=1}^{12}z_j$ be the product of the 12 numbers selected. What is the probability that $P=-1$?

一个位于复平面上中心对称的六边形的 6 个顶点由下面的集合 V 给定:

$$V=\left\{\sqrt{2}i,-\sqrt{2}i,\frac{1}{\sqrt{8}}(1+i),\frac{1}{\sqrt{8}}(-1+i),\frac{1}{\sqrt{8}}(1-i),\frac{1}{\sqrt{8}}(-1-i)\right\}.$$

对于$1\leq j\leq 12$的每一个整数 j，从集合 V 中随机独立地选择一个元素z_j，

$$P=\prod_{j=1}^{12}z_j$$

表示选择的这 12 个数的乘积。问$P=-1$的概率是多少？

$\textbf{(A)}\ \frac{5\cdot11}{3^{10}}\qquad\textbf{(B)}\ \frac{5^2\cdot11}{2\cdot3^{10}}\qquad\textbf{(C)}\ \frac{5\cdot11}{3^{9}}\qquad\textbf{(D)}\ \frac{5\cdot7\cdot11}{2\cdot3^{10}}\qquad\textbf{(E)}\ \frac{2^2\cdot5\cdot11}{3^{10}}$

2017 AMC 12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	C	B	A	B	B	B	D	E	C	D	B	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	D	B	D	D	D	E	D	E	C	A	E	