2018 AMC 12B

Problem 1

Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

Kate 烤了一块 20 英寸乘以 18 英寸的玉米面包，这块玉米面包被切分成 2 英寸乘以 2 英寸的多个小块面包。问这大块玉米面包包含多少块小块面包？

$\textbf{(A) } 90 \qquad \textbf{(B) } 100 \qquad \textbf{(C) } 180 \qquad \textbf{(D) } 200 \qquad \textbf{(E) } 360$

Problem 2

Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

Sam 在 90 分钟内开了 96 英里。他在前 30 分钟内的平均速度为 60 英里每小时，在第二个 30 分钟内的平均速度为 65 英里每小时。他在最后 30 分钟内的平均速度是多少英里每小时？

$\textbf{(A) } 64 \qquad \textbf{(B) } 65 \qquad \textbf{(C) } 66 \qquad \textbf{(D) } 67 \qquad \textbf{(E) } 68$

Problem 3

A line with slope 2 intersects a line with slope 6 at the point $(40, 30)$. What is the distance between the x-intercepts of these two lines?

一条斜率为 2 的直线和一条斜率为 6 的直线交于点（40，30）。那么这 2 条直线的 *x* 截距的距离是多少？

$\textbf{(A) } 5 \qquad \textbf{(B) } 10 \qquad \textbf{(C) } 20 \qquad \textbf{(D) } 25 \qquad \textbf{(E) } 50$

Problem 4

A circle has a chord of length 10, and the distance from the center of the circle to the chord is 5. What is the area of the circle?

一个圆的一条弦长为 10，圆心到弦的距离是 5，那么这个圆的面积是多少？

$\textbf{(A) } 25\pi \qquad \textbf{(B) } 50\pi \qquad \textbf{(C) } 75\pi \qquad \textbf{(D) } 100\pi \qquad \textbf{(E) } 125\pi$

Problem 5

How many subsets of $\{2,3,4,5,6,7,8,9\}$ contain at least one prime number?

集合$\{2,3,4,5,6,7,8,9\}$有多少个子集包含至少 1 个质数？

$\textbf{(A) } 128 \qquad \textbf{(B) } 192 \qquad \textbf{(C) } 224 \qquad \textbf{(D) } 240 \qquad \textbf{(E) } 256$

Problem 6

Suppose S cans of soda can be purchased from a vending machine for Q quarters. Which of the following expressions describes the number of cans of soda that can be purchased for D dollars, where 1 dollar is worth 4 quarters?

假设从一个自动售卖机上买 *S* 瓶苏打水需要 *Q* 个 25 美分硬币，下面哪个表达式是用 *D* 美元可以买到的苏打水的瓶数？这里 1 美元等于 4 个 25 美分硬币。

$\textbf{(A) } \frac{4DQ}{S} \qquad \textbf{(B) } \frac{4DS}{Q} \qquad \textbf{(C) } \frac{4Q}{DS} \qquad \textbf{(D) } \frac{DQ}{4S} \qquad \textbf{(E) } \frac{DS}{4Q}$

Problem 7

What is the value of

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

下面表达式的值是多少

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

$\textbf{(A) } 3 \qquad \textbf{(B) } 3\log_7 23 \qquad \textbf{(C) } 6 \qquad \textbf{(D) } 9 \qquad \textbf{(E) } 10$

Problem 8

Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

线段$\overline{AB}$是一个圆的直径且 *AB*=24.不同于 *A* 或 *B* 的点 *C* 在周上，当 *C* 在圆上移动时，$\triangle ABC$的重心（质心）的轨迹是一条缺少两个点的封闭曲线，和这条曲线所包围的区域的面积最接近的正整数是多少？

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 38 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 63 \qquad \textbf{(E)}\ 75$

Problem 9

What is $\sum\limits_{i=1}^{100}\sum\limits_{j=1}^{100}(i+j)$?

下面表达式的值是多少

$$\sum_{i=1}^{100}\sum_{j=1}^{100}(i+j)?$$

$\textbf{(A)}\ 100{,}100 \qquad \textbf{(B)}\ 500{,}500 \qquad \textbf{(C)}\ 505{,}000 \qquad \textbf{(D)}\ 1{,}001{,}000 \qquad \textbf{(E)}\ 1{,}010{,}000$

Problem 10

A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

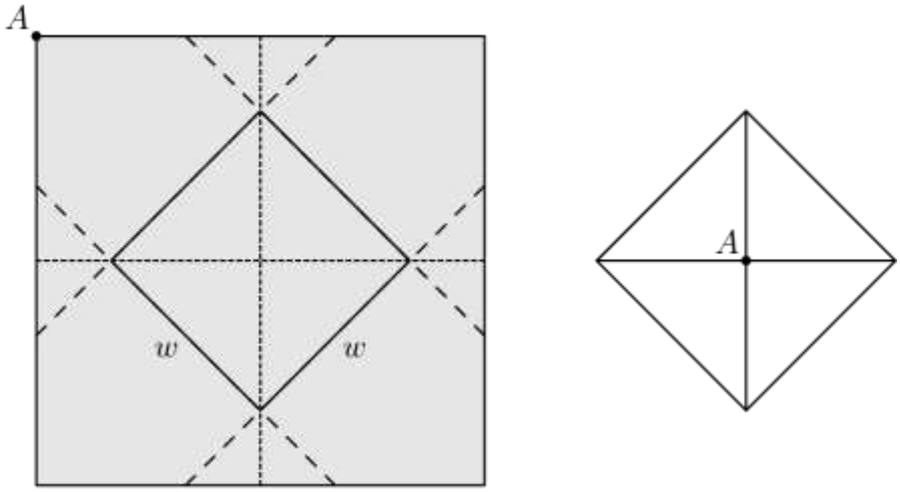
一列由 2018 个正整数组成的数列中有唯一的众数，并且出现了恰好 10 次，这列数字中，最少可能有多少个不同的数？

$\textbf{(A)}\ 202 \qquad \textbf{(B)}\ 223 \qquad \textbf{(C)}\ 224 \qquad \textbf{(D)}\ 225 \qquad \textbf{(E)}\ 234$

Problem 11

A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?

一个底面为正方形的封闭的盒子要用一张正方形的包装纸包裹起来，如下图左图所示，这个盒子放在包装纸的中心，底面的四个顶点位于正方形包装纸的中间线上。如下图右图所示，包装纸的四个角落沿着盒子的侧面竖着折起来，并在盒子顶面的中心 A 点处重合。盒子的底面长度为 w，盒子高为 h，这张包装纸的面积是多少？



(A) $2(w+h)^2$ **(B)** $\frac{(w+h)^2}{2}$ **(C)** $2w^2+4wh$ **(D)** $2w^2$ **(E)** w^2h

Problem 12

Side $\overline{AB}$ of $\triangle ABC$ has length 10. The bisector of angle A meets $\overline{BC}$ at D, and $CD=3$. The set of all possible values of AC is an open interval (m,n). What is $m+n$?

$\triangle ABC$的边$\overline{AB}$的长度为 10，角 *A* 的角平分线和$\overline{BC}$交于 *D*，且 *CD*=3，*AC* 所有可能的取值形成一个开区间(m,n)，问$m+n$是多少？

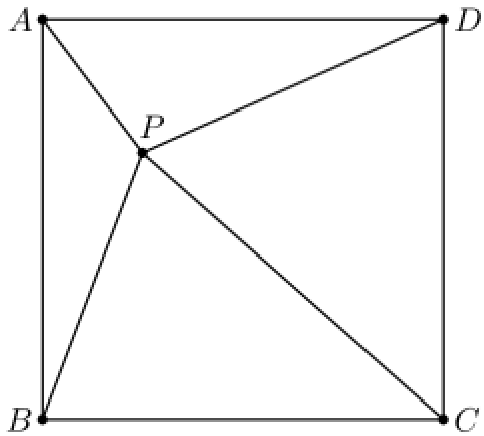
(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

Problem 13

Square $ABCD$ has side length 30. Point P lies inside the square so that $AP=12$ and $BP=26$. The centroids of $\triangle ABP$, $\triangle BCP$, $\triangle CDP$, and $\triangle DAP$ are the vertices of a convex quadrilateral. What is the area of that quadrilateral?

正方形 *ABCD* 的边长为 30，点 *P* 在正方形内，满足 *AP*=12，*BP*=26.已知△ABP，△BCP，

△CDP 和△DAP 的重心是一个凸四边形的 4 个顶点，这个四边形的面积是多少?



$\textbf{(A) }100\sqrt{2}\qquad\textbf{(B) }100\sqrt{3}\qquad\textbf{(C) }200\qquad\textbf{(D) }200\sqrt{2}\qquad\textbf{(E) }200\sqrt{3}$

Problem 14

Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

Joey 和 Chloe 还有他们的女儿 Zoe 的生日是同一天，Joey 比 Chloe 大 1 岁，Zoe 今天恰好 1 岁。一共有 9 次这样的生日，生日当天 Chloe 的年龄是 Zoe 的年整数倍，而今天是第 1 次。等到下次 Joey 的年龄是 Zoe 的年整数倍的生日那天，Joey 年龄的 2 位数字之和是多少?

$\textbf{(A) }7\qquad\textbf{(B) }8\qquad\textbf{(C) }9\qquad\textbf{(D) }10\qquad\textbf{(E) }11$

Problem 15

How many odd positive 3-digit integers are divisible by 3 but do not contain the digit 3?

有多少个 3 位正奇数可以被 3 整除但不包含数字 3?

$\textbf{(A) } 96 \qquad \textbf{(B) } 97 \qquad \textbf{(C) } 98 \qquad \textbf{(D) } 102 \qquad \textbf{(E) } 120$

Problem 16

The solutions to the equation $(z+6)^8=81$ are connected in the complex plane to form a convex regular polygon, three of whose vertices are labeled $A, B,$ and C. What is the least possible area of $\triangle ABC$?

把方程$(z+6)^8=81$的所有解在复平面内连接起来，形成了一个凸的正多边形，其中三个顶点标记为 *A*，*B* 和 *C*，那么△ABC 的最小可能面积是多少？

$\textbf{(A) } \frac{1}{6}\sqrt{6} \qquad \textbf{(B) } \frac{3}{2}\sqrt{2}-\frac{3}{2} \qquad \textbf{(C) } 2\sqrt3-3\sqrt2 \qquad \textbf{(D) } \frac{1}{2}\sqrt{2} \qquad \textbf{(E) } \sqrt 3-1$

Problem 17

Let p and q be positive integers such that $\frac{5}{9} < \frac{p}{q} < \frac{4}{7}$ and q is as small as possible. What is $q-p$?

正整数p和q满足$\frac{5}{9} < \frac{p}{q} < \frac{4}{7}$，且$q$取尽可能小的值。则$q-p$的值是多少？

$\textbf{(A) } 7 \qquad \textbf{(B) } 11 \qquad \textbf{(C) } 13 \qquad \textbf{(D) } 17 \qquad \textbf{(E) } 19$

Problem 18

A function f is defined recursively by $f(1)=f(2)=1$ and$f(n)=f(n-1)-f(n-2)+n$ for all integers $n\geq 3$. What is $f(2018)$?

一个函数f有如下递推式定义：$f(1)=f(2)=1$，且对所有整数$n\geq 3$，有 $f(n)=f(n-1)-f(n-2)+n$，问$f(2018)$是多少？

$\textbf{(A)}\ 2016 \qquad \textbf{(B)}\ 2017 \qquad \textbf{(C)}\ 2018 \qquad \textbf{(D)}\ 2019 \qquad \textbf{(E)}\ 2020$

Problem 19

Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1,2,\ldots,\dfrac{n}{2},n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

Mary 选择了一个 4 位偶数 *n*，她按照从小到大的次序依次从左向右写下 *n* 的所有因子：$1,2,\ldots,\dfrac{n}{2},n$.在此过程中的某一时刻，Mary 写下了 *n* 的一个因子 323，那么写在 323 右边的下一个因子的最小可能值是多少？

$\textbf{(A)}\ 324 \qquad \textbf{(B)}\ 330 \qquad \textbf{(C)}\ 340 \qquad \textbf{(D)}\ 361 \qquad \textbf{(E)}\ 646$

Problem 20

Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

ABCDEF 是一个边长为 1 的正六边形。*X*，*Y* 和 *Z* 分别是边 $\overline{AB}$, $\overline{CD}$和 $\overline{EF}$的中点，问△ACE 和△XYZ 的内部相交所形成的凸六边形的面积是多少？

$\textbf{(A)}\ \frac{3}{8}\sqrt{3} \qquad \textbf{(B)}\ \frac{7}{16}\sqrt{3} \qquad \textbf{(C)}\ \frac{15}{32}\sqrt{3} \qquad \textbf{(D)}\ \frac{1}{2}\sqrt{3} \qquad \textbf{(E)}\ \frac{9}{16}\sqrt{3}$

Problem 21

In $\triangle ABC$ with side lengths $AB = 13$, $AC = 12$, and $BC = 5$, let O and I denote the circumcenter and incenter, respectively. A circle with center M is tangent to the legs AC and BC and to the circumcircle of $\triangle ABC$. What is the area of $\triangle MOI$?

△*ABC* 的三条边长分别为 *AB*=13，*AC*=12，*BC*=5，点O和点 *I* 分别表示外心和内心。一个圆心为 *M* 的圆和直角边 *AC*，*BC* 都相切，并且和△ABC 的外接圆相切。问△*MOI* 的面积是多少？

$\textbf{(A)}\ \frac{5}{2} \qquad \textbf{(B)}\ \frac{11}{4} \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ \frac{13}{4} \qquad \textbf{(E)}\ \frac{7}{2}$

Problem 22

Consider polynomials $P(x)$ of degree at most 3, each of whose coefficients is an element of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many such polynomials satisfy $P(-1) = -9$?

考虑次数最多为 3 的多项式$P(x)$，它的每一项的系数都是集合｛0，1，2，3，4，5，6，7，8，9｝的一个元素。满足$P(-1) = -9$的这样的多项式有多少个？

$\textbf{(A)}\ 110 \qquad \textbf{(B)}\ 143 \qquad \textbf{(C)}\ 165 \qquad \textbf{(D)}\ 220 \qquad \textbf{(E)}\ 286$

Problem 23

Ajay is standing at point A near Pontianak, Indonesia, 0° latitude and 110° E longitude. Billy is standing at point B near Big Baldy Mountain, Idaho, USA, 45° N latitude and 115° W longitude. Assume that Earth is a perfect sphere with center C. What is the degree measure of $\angle ACB$?

Ajay 站在靠近印度尼西亚庞蒂亚纳克市的点 *A* 处，经纬度是纬度0°，东经 110°E。而 Billy 站在靠近美国爱达荷州大秃顶山，经纬度是北纬 45°N，西经 115°W。假设地球是个完美的球体，球心为 C，那么∠ACB 是多少度？

$\textbf{(A)}\ 105 \qquad \textbf{(B)}\ 112\frac{1}{2} \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 135 \qquad \textbf{(E)}\ 150$

Problem 24

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

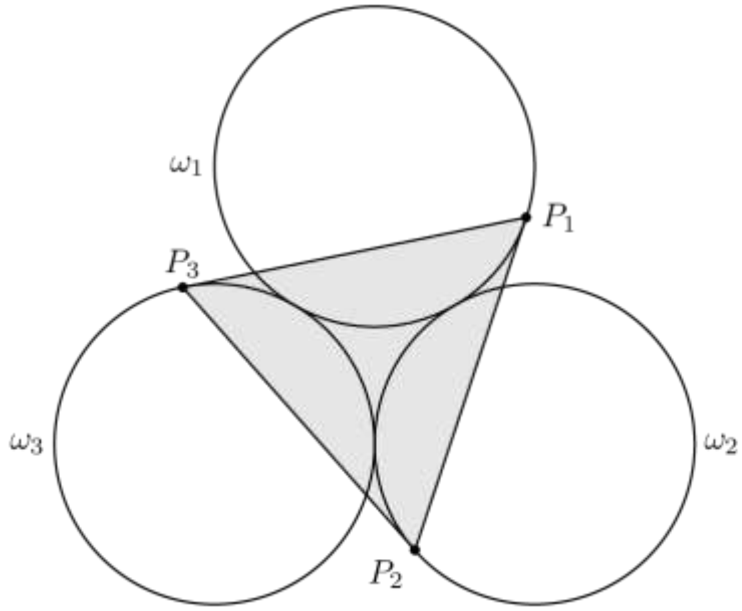
令$\lfloor x \rfloor$表示小于等于 x 的最大整数，有多少个实数 x 满足方程$x^2 + 10,000\lfloor x \rfloor = 10,000x$？

$\textbf{(A) } 197 \qquad \textbf{(B) } 198 \qquad \textbf{(C) } 199 \qquad \textbf{(D) } 200 \qquad \textbf{(E) } 201$

Problem 25

Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3 respectively such that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. See the figure below. The area of $\triangle P_1P_2P_3$ can be written in the form $\sqrt{a} + \sqrt{b}$ for positive integers a and b. What is $a + b$?

圆ω_1, ω_2和 ω_3的半径均为 4，两两外切的放在平面内。点 P_1, P_2, P_3分别在ω_1, ω_2和 ω_3上，满足$P_1P_2 = P_2P_3 = P_3P_1$，并且P_iP_{i+1}和ω_i相切，这里$i = 1, 2, 3$，$P_4 = P_1$。$\triangle P_1P_2P_3$的面积可以写成$\sqrt{a} + \sqrt{b}$，a 和 b 都是正整数，那么 a+b 是多少？



$\textbf{(A) } 546 \qquad \textbf{(B) } 548 \qquad \textbf{(C) } 550 \qquad \textbf{(D) } 552 \qquad \textbf{(E) } 554$

2018 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
A	D	B	B	D	B	C	C	E	D	A	C	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	A	B	A	B	C	C	E	D	C	C	D	