2014 AMC 12A Problems

2014 AMC 12A (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have 75 minutes working time to complete the test.

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Contents

Problem 1

What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ \frac{25}{2} \qquad \textbf{(D)}\ \frac{170}{3} \qquad \textbf{(E)}\ 170$

Solution

Problem 2

At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is 24.50. How much would 8 adult tickets and 6 child tickets cost?

$\textbf{(A)}\ 35 \qquad \textbf{(B)}\ 38.50 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 42.50$

Solution

Problem 3

Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

Problem 4

Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

$\textbf{(A)}\ \frac{bde}{ac} \qquad \textbf{(B)}\ \frac{ac}{bde} \qquad \textbf{(C)}\ \frac{abde}{c} \qquad \textbf{(D)}\ \frac{bcde}{a} \qquad \textbf{(E)}\ \frac{abc}{de}$

Solution

Problem 5

On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Problem 6

The difference between a two-digit number and the number obtained by reversing its digits is 5 times the sum of the digits of either number. What is the sum of the two digit number and its reverse?

$\textbf{(A)}\ 44 \qquad \textbf{(B)}\ 55 \qquad \textbf{(C)}\ 77 \qquad \textbf{(D)}\ 99 \qquad \textbf{(E)}\ 110$

Solution

Problem 7

The first three terms of a geometric progression are $\sqrt{3}$, $\sqrt[3]{3}$, and $\sqrt[6]{3}$. What is the fourth term?

$\textbf{(A) }1\qquad\textbf{(B) }\sqrt[7]{3}\qquad\textbf{(C) }\sqrt[8]{3}\qquad\textbf{(D) }\sqrt[9]{3}\qquad\textbf{(E) }\sqrt[10]{3}$

Solution

Problem 8

A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least 50

Coupon 2: 20 dollars off the listed price if the listed price is at least 100

Coupon 3: 18% off the amount by which the listed price exceeds 100

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

$\textbf{(A) }179.95\qquad\textbf{(B) }199.95\qquad\textbf{(C) }219.95\qquad\textbf{(D) }239.95\qquad\textbf{(E) }259.95$

Solution

Problem 9

Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

$\textbf{(A)}\ a+3\qquad\textbf{(B)}\ a+4\qquad\textbf{(C)}\ a+5\qquad\textbf{(D)}\ a+6\qquad\textbf{(E)}\ a+7$

Solution

Problem 10

Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle. What is the length of one of the two congruent sides of one of the isosceles triangles?

$\textbf{(A) }\dfrac{\sqrt3}4\qquad\textbf{(B) }\dfrac{\sqrt3}3\qquad\textbf{(C) }\dfrac23\qquad\textbf{(D) }\dfrac{\sqrt2}2\qquad\textbf{(E) }\dfrac{\sqrt3}2$

Solution

Problem 11

David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

$\textbf{(A) }140\qquad\textbf{(B) }175\qquad\textbf{(C) }210\qquad\textbf{(D) }245\qquad\textbf{(E) }280$

Solution

Problem 12

Two circles intersect at points A and B. The minor arcs AB measure 30° on one circle and 60° on the other circle. What is the ratio of the area of the larger circle to the area of the smaller circle?

$\textbf{(A) }2\qquad\textbf{(B) }1+\sqrt3\qquad\textbf{(C) }3\qquad\textbf{(D) }2+\sqrt3\qquad\textbf{(E) }4$

Solution

Problem 13

A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

$\textbf{(A) } 2100 \qquad \textbf{(B) } 2220 \qquad \textbf{(C) } 3000 \qquad \textbf{(D) } 3120 \qquad \textbf{(E) } 3125$

Solution

Problem 14

Let $a < b < c$ be three integers such that a, b, c is an arithmetic progression and a, c, b is a geometric progression. What is the smallest possible value of c?

$\textbf{(A) } -2 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 6$

Solution

Problem 15

A five-digit palindrome is a positive integer with respective digits $abcba$, where a is non-zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S.

$\textbf{(A) } 9 \qquad \textbf{(B) } 18 \qquad \textbf{(C) } 27 \qquad \textbf{(D) } 36 \qquad \textbf{(E) } 45$

Solution

Problem 16

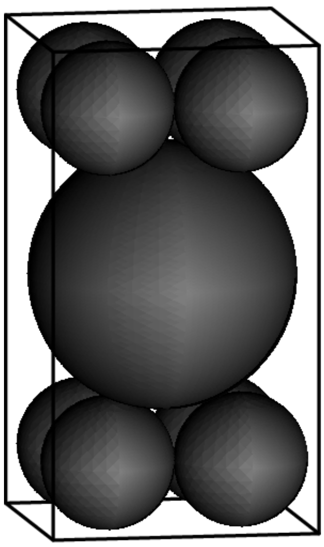
The product $(8)(888\dots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

$\textbf{(A) } 901 \qquad \textbf{(B) } 911 \qquad \textbf{(C) } 919 \qquad \textbf{(D) } 991 \qquad \textbf{(E) } 999$

Solution

Problem 17

A $4 \times 4 \times h$ rectangular box contains a sphere of radius 2 and eight smaller spheres of radius 1. The smaller spheres are each tangent to three sides of the box, and the larger sphere is tangent to each of the smaller spheres. What is h?



$\textbf{(A) }2+2\sqrt7\qquad\textbf{(B) }3+2\sqrt5\qquad\textbf{(C) }4+2\sqrt7\qquad\textbf{(D) }4\sqrt5\qquad\textbf{(E) }4\sqrt7$

Solution

Problem 18

The domain of the function $f(x)=\log_{\frac12}(\log_4(\log_{\frac14}(\log_{16}(\log_{\frac1{16}}x))))$ is an interval of length $\tfrac mn$, where m and n are relatively prime positive integers. What is $m+n$?

$\textbf{(A) }19\qquad\textbf{(B) }31\qquad\textbf{(C) }271\qquad\textbf{(D) }319\qquad\textbf{(E) }511$

Solution

Problem 19

There are exactly N distinct rational numbers k such that $|k|<200$ and

$$5x^2+kx+12=0$$

has at least one integer solution for x. What is N?

$\textbf{(A) }6\qquad\textbf{(B) }12\qquad\textbf{(C) }24\qquad\textbf{(D) }48\qquad\textbf{(E) }78$

Solution

Problem 20

In $\triangle BAC$, $\angle BAC=40^\circ$, $AB=10$, and $AC=6$. Points D and E lie on $\overline{AB}$ and $\overline{AC}$ respectively. What is the minimum possible value of $BE+DE+CD$?

$\textbf{(A) }6\sqrt3+3\qquad\textbf{(B) }\dfrac{27}2\qquad\textbf{(C) }8\sqrt3\qquad\textbf{(D) }14\qquad\textbf{(E) }3\sqrt3+9$

Solution

Problem 21

For every real number x, let $\lfloor x\rfloor$ denote the greatest integer not exceeding x, and let

$$f(x)=\lfloor x\rfloor(2014^{x-\lfloor x\rfloor}-1).$$

The set of all numbers x such that $1\leq x<2014$ and $f(x)\leq 1$ is a union of disjoint intervals. What is the sum of the lengths of those intervals?

$\textbf{(A) }1\qquad\textbf{(B) }\dfrac{\log 2015}{\log 2014}\qquad\textbf{(C) }\dfrac{\log 2014}{\log 2013}\qquad\textbf{(D) }\dfrac{2014}{2013}\qquad\textbf{(E) }2014^{\frac1{2014}}$

Solution

Problem 22

The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m,n) are there such that $1\leq m\leq 2012$ and

$$5^n<2^m<2^{m+2}<5^{n+1}?$$

$\textbf{(A) }278\qquad\textbf{(B) }279\qquad\textbf{(C) }280\qquad\textbf{(D) }281\qquad\textbf{(E) }282$

Solution

Problem 23

The fraction

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\ldots b_2b_1b_0},$$

where n is the length of the period of the repeating decimal expansion. What is the sum $b_0 + b_1 + \cdots + b_{n-1}$?

$\textbf{(A)}\ 874 \qquad \textbf{(B)}\ 883 \qquad \textbf{(C)}\ 887 \qquad \textbf{(D)}\ 891 \qquad \textbf{(E)}\ 892$

Solution

Problem 24

Let $f_0(x) = x + |x-100| - |x+100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?

$\textbf{(A)}\ 299 \qquad \textbf{(B)}\ 300 \qquad \textbf{(C)}\ 301 \qquad \textbf{(D)}\ 302 \qquad \textbf{(E)}\ 303$

Solution

Problem 25

The parabola P has focus $(0,0)$ and goes through the points $(4,3)$ and $(-4,-3)$. For how many points $(x,y) \in P$ with integer coordinates is it true that $|4x + 3y| \leq 1000$?

$\textbf{(A)}\ 38 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 44 \qquad \textbf{(E)}\ 46$

Solution