2001 AMC 12 Problems

Contents

Problem 1

The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

$\text{(A)}\ 2S + 3 \qquad \text{(B)}\ 3S + 2 \qquad \text{(C)}\ 3S + 6 \qquad \text{(D)} 2S + 6 \qquad \text{(E)}\ 2S + 12$

Solution

Problem 2

Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N?

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 9$

Solution

Problem 3

The state income tax where Kristin lives is levied at the rate of $p\%$ of the first $\$28000$ of annual income plus $(p + 2)\%$ of any amount above $\$28000$. Kristin noticed that the state income tax she paid amounted to $(p + 0.25)\%$ of her annual income. What was her annual income?

(A) \$28000 (B) \$32000 (C) \$35000 (D) \$42000 (E) \$56000

Solution

Problem 4

The mean of three numbers is 10 more than the least of the numbers and 15 less than the greatest. The median of the three numbers is 5. What is their sum?

(A) 5 (B) 20 (C) 25 (D) 30 (E) 36

Solution

Problem 5

What is the product of all positive odd integers less than 10000?

(A) $\frac{10000!}{(5000!)^2}$ (B) $\frac{10000!}{2^{5000}}$ (C) $\frac{9999!}{2^{5000}}$ (D) $\frac{10000!}{2^{5000}\cdot 5000!}$ (E) $\frac{5000!}{2^{5000}}$

Solution

Problem 6

A telephone number has the form ABC-DEF-GHIJ, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Solution

Problem 7

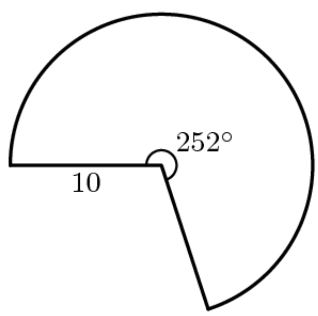
A charity sells 140 benefit tickets for a total of $\$2001$. Some tickets sell for full price (a whole dollar amount), and the rest sells for half price. How much money is raised by the full-price tickets?

(A) \$782 (B) \$986 (C) \$1158 (D) \$1219 (E) \$1449

Solution

Problem 8

Which of the cones listed below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?



(A) A cone with slant height of 10 and radius 6

(B) A cone with height of 10 and radius 6

(C) A cone with slant height of 10 and radius 7

(D) A cone with height of 10 and radius 7

(E) A cone with slant height of 10 and radius 8

Solution

Problem 9

Let f be a function satisfying $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y. If $f(500) = 3$, what is the value of $f(600)$?

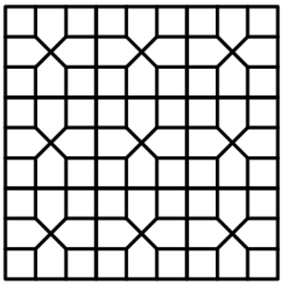
$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ \frac{5}{2} \qquad \text{(D)}\ 3 \qquad \text{(E)}\ \frac{18}{5}$

Solution

Problem 10

The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to

$\text{(A)}\ 50 \qquad \text{(B)}\ 52 \qquad \text{(C)}\ 54 \qquad \text{(D)}\ 56 \qquad \text{(E)}\ 58$



Solution

Problem 11

A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

$\text{(A)}\ \frac{3}{10} \qquad \text{(B)}\ \frac{2}{5} \qquad \text{(C)}\ \frac{1}{2} \qquad \text{(D)}\ \frac{3}{5} \qquad \text{(E)}\ \frac{7}{10}$

Solution

Problem 12

How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

$\text{(A)}\ 768 \qquad \text{(B)}\ 801 \qquad \text{(C)}\ 934 \qquad \text{(D)}\ 1067 \qquad \text{(E)}\ 1167$

Solution

Problem 13

The parabola with equation $y = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

$\text{(A)}\ 2b \qquad \text{(B)}\ 2c \qquad \text{(C)}\ 2a + 2b \qquad \text{(D)}\ 2h \qquad \text{(E)}\ 2k$

Solution

Problem 14

Given the nine-sided regular polygon $A_1A_2A_3A_4A_5A_6A_7A_8A_9$, how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $\{A_1, A_2, \ldots, A_9\}$?

$\text{(A) } 30 \qquad \text{(B) } 36 \qquad \text{(C) } 63 \qquad \text{(D) } 66 \qquad \text{(E) } 72$

Solution

Problem 15

An insect lives on the surface of a regular tetrahedron with edges of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge. What is the length of the shortest such trip? (Note: Two edges of a tetrahedron are opposite if they have no common endpoint.)

$\text{(A) } \frac{1}{2}\sqrt{3} \qquad \text{(B) } 1 \qquad \text{(C) } \sqrt{2} \qquad \text{(D) } \frac{3}{2} \qquad \text{(E) } 2$

Solution

Problem 16

A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

$\text{(A) } 8! \qquad \text{(B) } 2^8 \cdot 8! \qquad \text{(C) } (8!)^2 \qquad \text{(D) } \frac{16!}{2^8} \qquad \text{(E) } 16!$

Solution

Problem 17

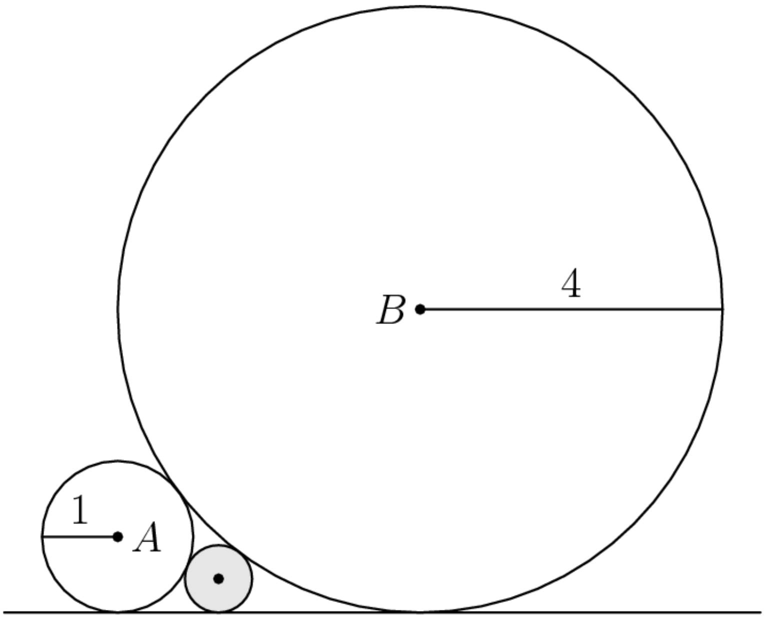
A point P is selected at random from the interior of the pentagon with vertices $A=(0,2)$, $B=(4,0)$, $C=(2\pi+1,0)$, $D=(2\pi+1,4)$, and $E=(0,4)$. What is the probability that $\angle APB$ is obtuse?

$\text{(A) } \frac{1}{5} \qquad \text{(B) } \frac{1}{4} \qquad \text{(C) } \frac{5}{16} \qquad \text{(D) } \frac{3}{8} \qquad \text{(E) } \frac{1}{2}$

Solution

Problem 18

A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. The radius of the third circle is



(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{12}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

Solution

Problem 19

The polynomial $P(x)=x^3+ax^2+bx+c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y=P(x)$ is 2, what is b?

(A) -11 (B) -10 (C) -9 (D) 1 (E) 5

Solution

Problem 20

Points $A=(3,9)$, $B=(1,1)$, $C=(5,3)$, and $D=(a,b)$ lie in the first quadrant and are the vertices of quadrilateral $ABCD$. The quadrilateral formed by joining the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ is a square. What is the sum of the coordinates of point D?

(A) 7 (B) 9 (C) 10 (D) 12 (E) 16

Solution

Problem 21

Four positive integers a, b, c, and d have a product of $8!$ and satisfy:

$$ab+a+b=524$$
$$bc+b+c=146$$
$$cd+c+d=104$$

What is $a-d$?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Solution

Problem 22

In rectangle $ABCD$, points F and G lie on $\overline{AB}$ so that $AF = FG = GB$ and E is the midpoint of $\overline{DC}$. Also, $\overline{AC}$ intersects $\overline{EF}$ at H and $\overline{EG}$ at J. The area of the rectangle $ABCD$ is 70. Find the area of triangle EHJ.

$\text{(A) }\frac {5}{2} \qquad \text{(B) }\frac {35}{12} \qquad \text{(C) }3 \qquad \text{(D) }\frac {7}{2} \qquad \text{(E) }\frac {35}{8}$

Solution

Problem 23

A polynomial of degree four with leading coefficient 1 and integer coefficients has two zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

$\text{(A) }\frac {1 + i \sqrt {11}}{2} \qquad \text{(B) }\frac {1 + i}{2} \qquad \text{(C) }\frac {1}{2} + i \qquad \text{(D) }1 + \frac {i}{2} \qquad \text{(E) }\frac {1 + i \sqrt {13}}{2}$

Solution

Problem 24

In $\triangle ABC$, $\angle ABC = 45^\circ$. Point D is on $\overline{BC}$ so that $2 \cdot BD = CD$ and $\angle DAB = 15^\circ$. Find $\angle ACB$.

$\text{(A) }54^\circ \qquad \text{(B) }60^\circ \qquad \text{(C) }72^\circ \qquad \text{(D) }75^\circ \qquad \text{(E) }90^\circ$

Solution

Problem 25

Consider sequences of positive real numbers of the form $x, 2000, y, \dots$ in which every term after the first is 1 less than the product of its two immediate neighbors. For how many different values of x does the term 2001 appear somewhere in the sequence?

$\text{(A) }1 \qquad \text{(B) }2 \qquad \text{(C) }3 \qquad \text{(D) }4 \qquad \text{(E) more than }4$

Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2001 AMC 12
- Mathematics competition resources