2003 AMC 12B Problems/Problem 1

The following problem is from both the 2003 AMC 12B #1 and 2003 AMC 10B #1, so both problems redirect to this page.

Problem

Which of the following is the same as

$$\frac{2-4+6-8+10-12+14}{3-6+9-12+15-18+21}?$$

$$(\text{A})\ -1 \qquad (\text{B})\ -\frac{2}{3} \qquad (\text{C})\ \frac{2}{3} \qquad (\text{D})\ 1 \qquad (\text{E})\ \frac{14}{3}$$

Solution

The numbers in the numerator and denominator can be grouped like this:

$$\begin{aligned} 2+(-4+6)+(-8+10)+(-12+14) &= 2*4 \\ 3+(-6+9)+(-12+15)+(-18+21) &= 3*4 \\ \frac{2*4}{3*4} = \frac{2}{3} &\Rightarrow (\text{C}) \end{aligned}$$

Alternatively, notice that each term in the numerator is $\frac{2}{3}$ of a term in the denominator, so the quotient has to be $\frac{2}{3}$.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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Preceded by First Question	Followed by Problem 2
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2003 AMC 12B Problems/Problem 2

The following problem is from both the 2003 AMC 12B #2 and 2003 AMC 10B #2, so both problems redirect to this page.

Problem

Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs $\$1$ more than a pink pill, and Al's pills cost a total of $\$546$ for the two weeks. How much does one green pill cost?

$\textbf{(A)}\ \$7 \qquad\textbf{(B)}\ \$14 \qquad\textbf{(C)}\ \$19 \qquad\textbf{(D)}\ \$20 \qquad\textbf{(E)}\ \$39$

Solution

Because there are 14 days in two weeks, Al spends $546/14 = 39$ dollars per day for the cost of a green pill and a pink pill. If the green pill costs x dollars and the pink pill $x-1$ dollars, the sum of the two costs $2x-1$ should equal 39 dollars. Then the cost of the green pill x is $\boxed{\textbf{(D)}\ \$20}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
Preceded by Problem 1	Followed by Problem 3
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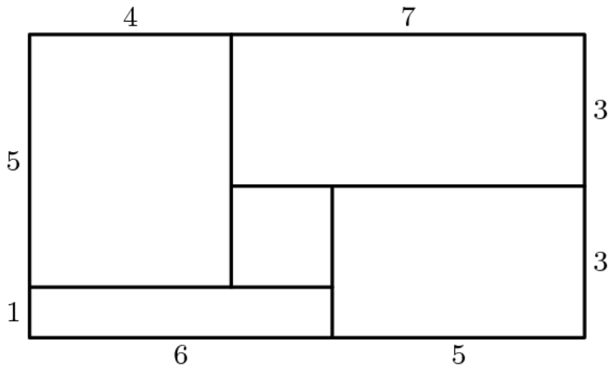
2003 AMC 10B Problems/Problem 4

The following problem is from both the 2003 AMC 12B #3 and 2003 AMC 10B #4, so both problems redirect to this page.

Problem

Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the figure. She plants one flower per square foot in each region. Asters cost $\$1$ each, begonias $\$1.50$ each, cannas $\$2$ each, dahlias $\$2.50$ each, and Easter lilies $\$3$ each. What is the least possible cost, in dollars, for her garden?



$\textbf{(A)}\ 108 \qquad \textbf{(B)}\ 115 \qquad \textbf{(C)}\ 132 \qquad \textbf{(D)}\ 144 \qquad \textbf{(E)}\ 156$

Solution

The areas of the five regions from greatest to least are $21, 20, 15, 6$ and 4.

If we want to minimize the cost, we want to maximize the area of the cheapest flower and minimize the area of the most expensive flower. Doing this, the cost is $1 \cdot 21 + 1.50 \cdot 20 + 2 \cdot 15 + 2.50 \cdot 6 + 3 \cdot 4$, which simplifies to $\$108$. Therefore the answer is $\boxed{\textbf{(A)}\ 108}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 5

The following problem is from both the 2003 AMC 12B #4 and 2003 AMC 10B #5, so both problems redirect to this page.

Problem

Moe uses a mower to cut his rectangular 90-foot by 150-foot lawn. The swath he cuts is 28 inches wide, but he overlaps each cut by 4 inches to make sure that no grass is missed. He walks at the rate of 5000 feet per hour while pushing the mower. Which of the following is closest to the number of hours it will take Moe to mow the lawn.

$\textbf{(A)}\ 0.75 \qquad \textbf{(B)}\ 0.8 \qquad \textbf{(C)}\ 1.35 \qquad \textbf{(D)}\ 1.5 \qquad \textbf{(E)}\ 3$

Solution

Since the swath Moe actually mows is 24 inches, or 2 feet wide, he mows 10000 square feet in one hour. His lawn has an area of 13500, so it will take Moe 1.35 hours to finish mowing the lawn. Thus the answer is $\boxed{\textbf{(C)}\ 1.35}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 6

The following problem is from both the 2003 AMC 12B #5 and 2003 AMC 10B #6, so both problems redirect to this page.

Problem

Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is $4:3$. The horizontal length of a "27-inch" television screen is closest, in inches, to which of the following?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 20.5 \qquad \textbf{(C)}\ 21 \qquad \textbf{(D)}\ 21.5 \qquad \textbf{(E)}\ 22$

Solution

If you divide the television screen into two right triangles, the legs are in the ratio of $4:3$, and we can let one leg be $4x$ and the other be $3x$. Then we can use the Pythagorean Theorem.

$$\begin{aligned}(4x)^2+(3x)^2&=27^2\\16x^2+9x^2&=729\\25x^2&=729\\x^2&=\frac{729}{25}\\x&=\frac{27}{5}\\x&=5.4\end{aligned}$$

The horizontal length is $5.4\times4=21.6$, which is closest to $\boxed{\textbf{(D)}\ 21.5}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 10B Problems/Problem 8

The following problem is from both the 2003 AMC 12B #6 and 2003 AMC 10B #8, so both problems redirect to this page.

Problem

The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

$$\textbf{(A)}\ -\sqrt{3} \qquad \textbf{(B)}\ -\frac{2\sqrt{3}}{3} \qquad \textbf{(C)}\ -\frac{\sqrt{3}}{3} \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 3$$

Solution

Let the first term be a and the common difference be r. Therefore,

$$ar = 2 \ \ (1) \qquad \text{and} \qquad ar^3 = 6 \ \ (2)$$

Dividing (2) by (1) eliminates the a, yielding $r^2 = 3$, so $r = \pm\sqrt{3}$.

Now, since $ar = 2$, $a = \frac{2}{r}$, so $a = \frac{2}{\pm\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$.

We therefore see that $\boxed{\textbf{(B)}\ -\frac{2\sqrt{3}}{3}}$ is a possible first term.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 7

Problem

Penniless Pete's piggy bank has no pennies in it, but it has 100 coins, all nickels,dimes, and quarters, whose total value is $8.35. It does not necessarily contain coins of all three types. What is the difference between the largest and smallest number of dimes that could be in the bank?

$\text{(A)}\ 0 \qquad \text{(B)}\ 13 \qquad \text{(C)}\ 37 \qquad \text{(D)}\ 64 \qquad \text{(E)}\ 83$

Solution

Where a, b, c is the number of nickels, dimes, and quarters, respectively, we can set up two equations:

$$(1)\ 5a + 10b + 25c = 835 \qquad (2)\ a + b + c = 100$$

Eliminate a by subtracting (2) from $(1)/5$ to get $b + 4c = 67$. Of the integer solutions (b, c) to this equation, the number of dimes b is least in $(3, 16)$ and greatest in $(67, 0)$, yielding a difference of $67 - 3 = \boxed{\textbf{(D)}\ 64}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 8

> The following problem is from both the 2003 AMC 12B #8 and 2003 AMC 10B #13, so both problems redirect to this page.

Problem

Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

$\textbf{(A) } 3 \qquad\textbf{(B) } 4 \qquad\textbf{(C) } 6 \qquad\textbf{(D) } 9 \qquad\textbf{(E) } 10$

Solution

Let a and b be the digits of x,

$$\clubsuit(\clubsuit(x)) = a + b = 3$$

Clearly $\clubsuit(x)$ can only be $3, 12, 21,$ or 30 and only 3 and 12 are possible to have two digits sum to.

If $\clubsuit(x)$ sums to 3, there are 3 different solutions : $12, 21, \text{or } 30$

If $\clubsuit(x)$ sums to 12, there are 7 different solutions: $39, 48, 57, 66, 75, 84, \text{or } 93$

The total number of solutions is $3 + 7 = 10 \Rightarrow (E)$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 9

Problem

Let f be a linear function for which $f(6)-f(2)=12.$ What is $f(12)-f(2)?$

$\text{(A) } 12 \qquad \text{(B) } 18 \qquad \text{(C) } 24 \qquad \text{(D) } 30 \qquad \text{(E) } 36$

Solution

Since f is a linear function with slope m,

$$m = \frac{f(6)-f(2)}{\Delta x} = \frac{12}{6-2} = 3$$

$$f(12)-f(2) = m\Delta x = 4(12-2) = 30 \Rightarrow (D)$$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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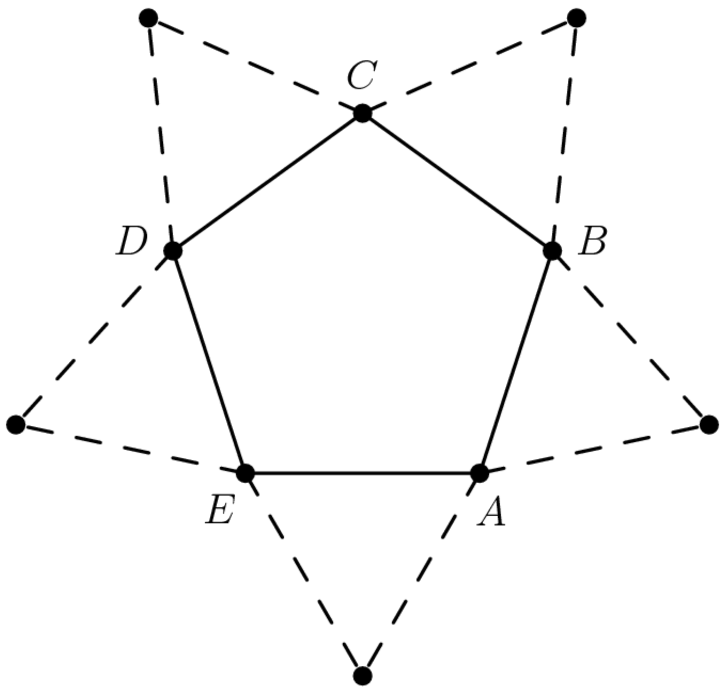
2003 AMC 12B Problems/Problem 10

Problem

Several figures can be made by attaching two equilateral triangles to the regular pentagon ABCDE in two of the five positions shown. How many non-congruent figures can be constructed in this way?



$\textbf{(A)}\ 1 \qquad\textbf{(B)}\ 2 \qquad\textbf{(C)}\ 3 \qquad\textbf{(D)}\ 4 \qquad\textbf{(E)}\ 5$

Solution

Place the first triangle. Now, we can place the second triangle either adjacent to the first, or with one side between them, for a total of $\boxed{\textbf{(B)}\ 2}$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 11

Problem

Cassandra sets her watch to the correct time at noon. At the actual time of 1:00 PM, she notices that her watch reads 12:57 and 36 seconds. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch first reads 10:00 PM?

$\text{(A) 10:22 PM and 24 seconds} \qquad \text{(B) 10:24 PM} \qquad \text{(C) 10:25 PM} \qquad \text{(D) 10:27 PM} \qquad \text{(E) 10:30 PM}$

Solution

For every 60 minutes that pass by in actual time, $57+\frac{36}{60}=57.6$ minutes pass by on Cassandra's watch. When her watch first reads, 10:00 pm, $10(60)=600$ minutes have passed by on her watch. Setting up a proportion,

$$\frac{57.6}{60}=\frac{600}{x}$$

where x is the number of minutes that have passed by in actual time. Solve for x to get 625 minutes, or 10 hours and 25 minutes $\Rightarrow \boxed{\textbf{(C)}\ \text{10:25 PM}}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 12

The following problem is from both the 2003 AMC 12B #12 and 2003 AMC 10B #18, so both problems redirect to this page.

Problem

What is the largest integer that is a divisor of

$$(n+1)(n+3)(n+5)(n+7)(n+9)$$

for all positive even integers n?

$\text{(A) } 3 \qquad \text{(B) } 5 \qquad \text{(C) } 11 \qquad \text{(D) } 15 \qquad \text{(E) } 165$

Solution

Since for all consecutive odd integers, one of every five is a multiple of 5 and one of every three is a multiple of 3, the answer is $3*5=15$, so $\boxed{\text{D}}$.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 13

Problem

An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius?

$\mathrm{(A)}\ 2:1 \qquad \mathrm{(B)}\ 3:1 \qquad \mathrm{(C)}\ 4:1 \qquad \mathrm{(D)}\ 16:3 \qquad \mathrm{(E)}\ 6:1$

Solution

Let r be the common radius of the sphere and the cone, and h be the cone's height. Then

$$75\% \cdot \left(\frac{4}{3}\pi r^3\right) = \frac{1}{3}\pi r^2 h \Longrightarrow h = 3r$$

Thus $h:r = 3:1$ and the answer is $\boxed{B}$.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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Categories: Introductory Algebra Problems | Introductory Geometry Problems

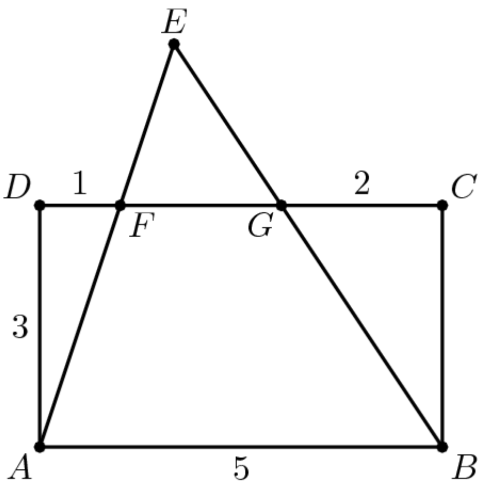
2003 AMC 10B Problems/Problem 20

The following problem is from both the 2003 AMC 12B #14 and 2003 AMC 10B #20, so both problems redirect to this page.

Contents

Problem

In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of $\triangle AEB$.



$\textbf{(A) } 10 \qquad\textbf{(B) } \frac{21}{2} \qquad\textbf{(C) } 12 \qquad\textbf{(D) } \frac{25}{2} \qquad\textbf{(E) } 15$

Solution 1

$\triangle EFG \sim \triangle EAB$ because $FG \parallel AB.$ The ratio of $\triangle EFG$ to $\triangle EAB$ is $2:5$ since $AB = 5$ and $FG = 2$ from subtraction. If we let h be the height of $\triangle EAB$,

$$\frac{2}{5} = \frac{h-3}{h}$$

$$2h = 5h - 15$$

$$3h = 15$$

$$h = 5$$

The height is 5 so the area of $\triangle EAB$ is $\frac{1}{2}(5)(5) = \boxed{\textbf{(D) } \frac{25}{2}}$.

Solution 2

We can look at this diagram as if it were a coordinate plane with point A being $(0,0)$. This means that the equation of the line AE is $y=3x$ and the equation of the line EB is $y=\frac{-3}{2}x+\frac{15}{2}$. From this we can set of the follow equation to find the x coordinate of point E:

$$3x=\frac{-3}{2}x+\frac{15}{2}$$

$$6x=-3x+15$$

$$9x=15$$

$$x=\frac{5}{3}$$

We can plug this into one of our original equations to find that the y coordinate is 5, meaning the area of $\triangle EAB$ is $\frac{1}{2}(5)(5)=\boxed{\textbf{(D)}\ \frac{25}{2}}$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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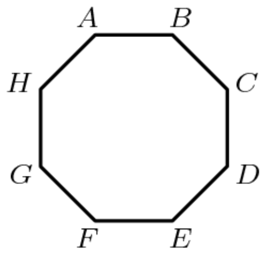
2003 AMC 10B Problems/Problem 23

The following problem is from both the 2003 AMC 12B #15 and 2003 AMC 10B #23, so both problems redirect to this page.

Contents

Problem

A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?



$$\textbf{(A)}\ 1-\frac{\sqrt2}{2}\qquad\textbf{(B)}\ \frac{\sqrt2}{4}\qquad\textbf{(C)}\ \sqrt2-1\qquad\textbf{(D)}\ \frac12\qquad\textbf{(E)}\ \frac{1+\sqrt2}{4}$$

Solution

Solution 1

Here is an easy way to look at this, where p is the perimeter, and a is the apothem:

Area of Octagon: $\frac{ap}{2}=1.$

Area of Rectangle: $\frac{p}{8}\times 2a=\frac{ap}{4}.$

You can see from this that the octagon's area is twice as large as the rectangle's area is $\boxed{\textbf{(D)}\ \frac12}$.

Solution 2

Here is a less complicated way than that of the user above. If you draw a line segment from each vertex to the center of the octagon and draw the rectangle ABEF, you can see that two of the triangles share the same base and height with half the rectangle. Therefore, the rectangle's area is the same as 4 of the 8 triangles, and is $\boxed{\textbf{(D)}\ \frac12}$ the area of the octagon.

Solution 3

Drawing lines AD, BG, CF, and EH, we can see that the octagon is comprised of 1 square, 4 rectangles, and 4 triangles. The triangles each are 45-45-90 triangles, and since their diagonal is x, each of their sides is x*sqrt(2)/2. The area of the entire figure is, likewise, x^2 (the square) + 4*x*x*sqrt(2)/2 (the 4 rectangles) + 2*(x*sqrt(2)/2)^2 (the triangles), which simplifies to 2x^2 + 2*sqrt(2)x^2. The area of ABEF is just x*(x+(2*x*sqrt(2)/2)), = x^2 + sqrt(2)x^2, which we can see is the area of ABCDEFGH/2 = $\boxed{\textbf{(D)}\ \frac{1}{2}}$ the area of the octagon.

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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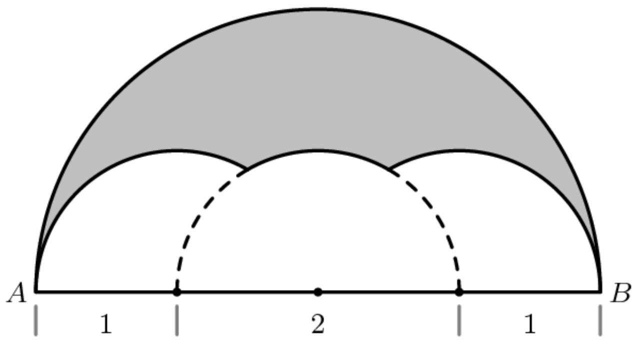
2003 AMC 10B Problems/Problem 19

The following problem is from both the 2003 AMC 12B #16 and 2003 AMC 10B #19, so both problems redirect to this page.

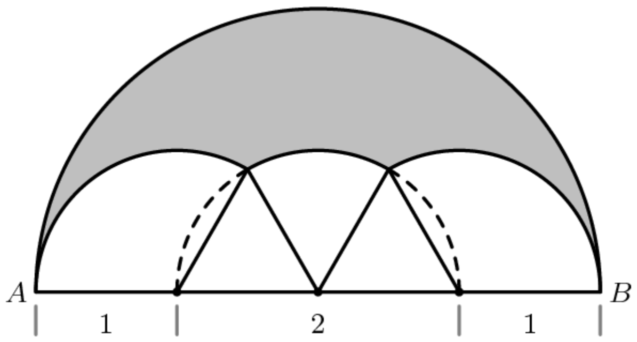
Problem

Three semicircles of radius 1 are constructed on diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?



$\textbf{(A)}\ \pi - \sqrt{3} \qquad \textbf{(B)}\ \pi - \sqrt{2} \qquad \textbf{(C)}\ \frac{\pi + \sqrt{2}}{2} \qquad \textbf{(D)}\ \frac{\pi + \sqrt{3}}{2} \qquad \textbf{(E)}\ \frac{7}{6}\pi - \frac{\sqrt{3}}{2}$

Solution



By drawing four lines from the intersect of the semicircles to their centers, we have split the white region into $\frac{5}{6}$ of a circle with radius 1 and two equilateral triangles with side length 1. This gives the area of the white region as $\frac{5}{6}\pi + \frac{2\cdot\sqrt{3}}{4} = \frac{5}{6}\pi + \frac{\sqrt{3}}{2}$. The area of the shaded region is the area of the white region subtracted from the area of the large semicircle. This is equivalent to $2\pi - (\frac{5}{6}\pi + \frac{\sqrt{3}}{2}) = \frac{7}{6}\pi - \frac{\sqrt{3}}{2}$.

Thus the answer is $\boxed{\textbf{(E)}\ \frac{7}{6}\pi - \frac{\sqrt{3}}{2}}$.

See Also

2003 AMC 12B Problems/Problem 17

Problem

If $\log(xy^3) = 1$ and $\log(x^2y) = 1$, what is $\log(xy)$?

$\mathrm{(A)}\ -\frac{1}{2} \qquad \mathrm{(B)}\ 0 \qquad \mathrm{(C)}\ \frac{1}{2} \qquad \mathrm{(D)}\ \frac{3}{5} \qquad \mathrm{(E)}\ 1$

Solution

Since

$$\begin{array}{ccc} & & \log(xy) + 2\log y = 1 \\ \log(xy) + \log x = 1 & \Longrightarrow & 2\log(xy) + 2\log x = 2 \end{array}$$

Summing gives

$$3\log(xy) + 2\log y + 2\log x = 3 \Longrightarrow 5\log(xy) = 3$$

Hence $\log(xy) = \frac{3}{5} \Rightarrow \mathrm{(D)}$.

It is not difficult to find $x = 10^{\frac{2}{5}}, y = 10^{\frac{1}{5}}$.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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Category: Introductory Algebra Problems

2003 AMC 12B Problems/Problem 18

Problem

Let x and y be positive integers such that $7x^5 = 11y^{13}$. The minimum possible value of x has a prime factorization a^cb^d. What is $a+b+c+d?$

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 33 \qquad \textbf{(E)}\ 34$

Solution

Substitute a^cb^d into x. We then have $7(a^{5c}b^{5d}) = 11y^{13}$. Divide both sides by 7, and it follows that:

$$(a^{5c}b^{5d}) = \frac{11y^{13}}{7}.$$

Note that because 11 and 7 are prime, the minimum value of x must involve factors of 7 and 11 only. Thus, we try to look for the lowest power p of 11 such that $13p+1 \equiv 0 \pmod{5}$, so that we can take 11^{13p+1} to the fifth root. Similarly, we want to look for the lowest power n of 7 such that $13n-1 \equiv 0 \pmod{5}$. Again, this allows us to take the fifth root of 7^{13n-1}. Obviously, we want to add 1 to $13p$ and subtract 1 from $13n$ because 11^{13p} and 7^{13n} are multiplied by 11 and divided by 7, respectively. With these conditions satisfied, we can simply multiply 11^p and 7^n and substitute this quantity into y to attain our answer.

We can simply look for suitable values for p and n. We find that the lowest p, in this case, would be 3 because $13(3)+1 \equiv 0 \pmod{5}$. Moreover, the lowest q should be 2 because $13(2)-1 \equiv 0 \pmod{5}$. Hence, we can substitute the quantity $11^3 \cdot 7^2$ into y. Doing so gets us:

$$(a^{5c}b^{5d}) = \frac{11(11^3 \cdot 7^2)^{13}}{7} = 11^{40} \cdot 7^{25}.$$

Taking the fifth root of both sides, we are left with $a^cb^d = 11^8 \cdot 7^5$.
$a+b+c+d = 11+7+8+5 = \boxed{\textbf{(B)}\ 31}$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 19

Contents

Problem

Let S be the set of permutations of the sequence $1, 2, 3, 4, 5$ for which the first term is not 1. A permutation is chosen randomly from S. The probability that the second term is 2, in lowest terms, is a/b. What is $a+b$?

$\mathrm{(A)}\ 5 \qquad\mathrm{(B)}\ 6 \qquad\mathrm{(C)}\ 11 \qquad\mathrm{(D)}\ 16 \qquad\mathrm{(E)}\ 19$

Solution

There are 4 choices for the first element of S, and for each of these choices there are $4!$ ways to arrange the remaining elements. If the second element must be 2, then there are only 3 choices for the first element and $3!$ ways to arrange the remaining elements. Hence the answer is $\frac{3 \cdot 3!}{4 \cdot 4!} = \frac{18}{96} = \frac{3}{16}$, and $a+b = 19 \Rightarrow \mathrm{(E)}$.

Solution 2

There is a $\frac{1}{4}$ chance that the number 1 is the second term. Let x be the chance that 2 will be the second term. Since $3, 4,$ and 5 are in similar situations as 2, this becomes $\frac{1}{4} + 4x = 1$

Solving for x, we find it equals $\frac{3}{16}$, therefore $3 + 16 = 19 \Rightarrow \mathrm{(E)}$

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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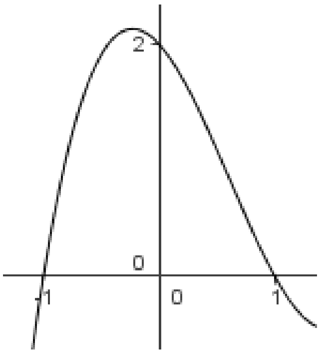
2003 AMC 12B Problems/Problem 20

Problem

Part of the graph of $f(x) = ax^3 + bx^2 + cx + d$ is shown. What is b?



$\text{(A)}\ -4 \qquad \text{(B)}\ -2 \qquad \text{(C)}\ 0 \qquad \text{(D)}\ 2 \qquad \text{(E)}\ 4$

Contents

Solution

Solution 1

Since

$$-f(-1) = a - b + c - d = 0 = f(1) = a + b + c + d$$

It follows that $b + d = 0$. Also, $d = f(0) = 2$, so $b = -2 \Rightarrow \text{(B)}$.

Solution 2

Two of the roots of $f(x) = 0$ are ± 1, and we let the third one be n. Then

$$a(x-1)(x+1)(x-n) = ax^3 - anx^2 - ax + an = ax^3 + bx^2 + cx + d = 0$$

Notice that $f(0) = d = an = 2$, so $b = -an = -2 \Rightarrow \text{(B)}$.

Solution 3

Notice that if $g(x) = 2 - 2x^2$, then $f - g$ vanishes at $x = -1, 0, 1$ and so

$$f(x) - g(x) = ax(x-1)(x+1) = ax^3 - ax$$

implies by x^2 coefficient, $b + 2 = 0, b = -2 \Rightarrow \text{(B)}$.

See also

2003 AMC 12B Problems/Problem 21

Problem

An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians and chosen at random from the interval $(0,\pi)$, and moves 5 cm in a straight line to C. What is the probability that $AC < 7$?

$\mathrm{(A)}\ \frac{1}{6} \qquad \mathrm{(B)}\ \frac{1}{5} \qquad \mathrm{(C)}\ \frac{1}{4} \qquad \mathrm{(D)}\ \frac{1}{3} \qquad \mathrm{(E)}\ \frac{1}{2}$

Solution

By the Law of Cosines,

$$AB^2 + BC^2 - 2AB\cdot BC\cos\alpha = 89 - 80\cos\alpha = AC^2 < 49$$
$$\cos\alpha < \frac{1}{2}$$

It follows that $0 < \alpha < \frac{\pi}{3}$, and the probability is $\frac{\pi/3}{\pi} = \frac{1}{3} \Rightarrow \mathrm{(D)}$.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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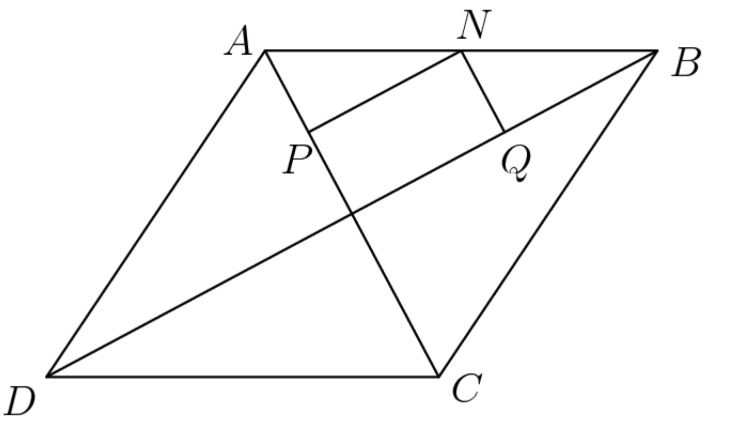
Categories: Introductory Geometry Problems | Introductory Trigonometry Problems

2003 AMC 12B Problems/Problem 22

Problem

Let $ABCD$ be a rhombus with $AC = 16$ and $BD = 30$. Let N be a point on $\overline{AB}$, and let P and Q be the feet of the perpendiculars from N to $\overline{AC}$ and $\overline{BD}$, respectively. Which of the following is closest to the minimum possible value of PQ?



$\mathrm{(A)}\ 6.5 \qquad\mathrm{(B)}\ 6.75 \qquad\mathrm{(C)}\ 7 \qquad\mathrm{(D)}\ 7.25 \qquad\mathrm{(E)}\ 7.5$

Solution

Let $\overline{AC}$ and $\overline{BD}$ intersect at O. Since $ABCD$ is a rhombus, then $\overline{AC}$ and $\overline{BD}$ are perpendicular bisectors. Thus $\angle POQ = 90^\circ$, so $OPNQ$ is a rectangle. Since the diagonals of a rectangle are of equal length, $PQ = ON$, so we want to minimize ON. It follows that we want $ON \perp AB$.

Finding the area in two different ways,

$$\frac{1}{2}AO{\cdot}BO = 60 = \frac{1}{2}ON{\cdot}AB = \frac{\sqrt{8^2+15^2}}{2}{\cdot}ON \Longrightarrow ON = \frac{120}{17} \approx 7.06 \Rightarrow \mathrm{(C)}$$

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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Category: Introductory Geometry Problems

2003 AMC 12B Problems/Problem 23

Problem

The number of x-intercepts on the graph of $y = \sin(1/x)$ in the interval $(0.0001, 0.001)$ is closest to

$\text{(A)}\ 2900 \qquad \text{(B)}\ 3000 \qquad \text{(C)}\ 3100 \qquad \text{(D)}\ 3200 \qquad \text{(E)}\ 3300$

Solution

The function $f(x) = \sin x$ has roots in the form of πn for all integers n. Therefore, we want $\frac{1}{x} = \pi n$ on $\frac{1}{10000} \leq x \leq \frac{1}{1000}$, so $1000 \leq \frac{1}{x} = \pi n \leq 10000$. There are $\frac{10000-1000}{\pi} \approx \boxed{2900} \Rightarrow \text{(A)}$ solutions for n on this interval.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 24

Contents

Problem

Positive integers $a, b,$ and c are chosen so that $a < b < c$, and the system of equations

$$2x + y = 2003 \text{ and } \quad y = |x-a| + |x-b| + |x-c|$$

has exactly one solution. What is the minimum value of c?

$\mathrm{(A)}\ 668 \qquad \mathrm{(B)}\ 669 \qquad \mathrm{(C)}\ 1002 \qquad \mathrm{(D)}\ 2003 \qquad \mathrm{(E)}\ 2004$

Solution 1

Consider the graph of $f(x) = |x-a| + |x-b| + |x-c|$.

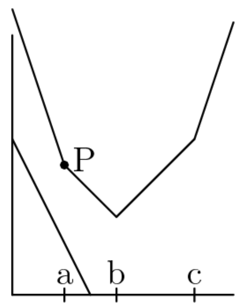
When $x < a$, the slope is -3.

When $a < x < b$, the slope is -1.

When $b < x < c$, the slope is 1.

When $c < x$, the slope is 3.

Setting $x = b$ gives $y = |b-a| + |b-b| + |b-c| = c-a$, so $(b, c-a)$ is a point on $f(x)$. In fact, it is the minimum of $f(x)$ considering the slope of lines to the left and right of $(b, c-a)$. Thus, graphing this will produce a figure that looks like a cup:



From the graph, it is clear that $f(x)$ and $2x + y = 2003$ have one intersection point if and only if they intersect at $x = a$. Since the line where $a < x < b$ has slope -1, the positive difference in y-coordinates from $x = a$ to $x = b$ must be $b - a$. Together with the fact that $(b, c-a)$ is on $f(x)$, we see that $P = (a, c-a+b-a)$. Since this point is on $x = a$, the only intersection point with $2x + y = 2003$, we have $2 \cdot a + (b + c - 2a) = 2003 \implies b + c = 2003$. As $c > b$, the smallest possible value of c occurs when $b = 1001$ and $c = 1002$. This is indeed a solution as $a = 1000$ puts P on $y = 2003 - 2x$, and thus the answer is $\boxed{\mathrm{(C)}\ 1002}$.

This indeed works for the two right segments of slope 1 and 3. We already know that the minimum is achieved between slopes -3 and -1 with $b + c = 2003$:

$$2003 - 2x = -a - b + c + x \longrightarrow 3x \neq a + b - c + 2003 \{b < x < c\} \rightarrow (3b, 3c) \neq a + 2b \rightarrow b > a \ (\text{true})$$

$$2003 - 2x = -a - b - c + 3x \longrightarrow 5x \neq a + b + c + 2003 \{x > c\} \rightarrow (5c, +5\infty) \neq a + 2b + 2c \rightarrow 3c > a + 2b \ (\text{true})$$

Indeed, within the restricted domain of x in each segment, these inequalities prove to be unequal everywhere. So $y = 2003 - 2x$ is strictly below $y = |x-a| + |x-b| + |x-c|$ at these domains.

Solution 2

Step 1: Finding some promising bound

Does the system have a solution where $x \leq a$?

For such a solution we would have $y = (a-x) + (b-x) + (c-x)$, hence $2x + (a+b+c-3x) = 2003$, which solves to $x = a+b+c-2003$. If we want to avoid this solution, we need to have $a+b+c-2003 > a$, hence $b+c > 2003$, hence $c \geq 1003$. In other words, if $c < 1003$, there will always be one solution (x,y) such that $x \leq a$.

Step 2: Showing one solution

We will now find out whether there is a $c < 1003$ for which (and some a, b) the system has only one solution. We already know of one such solution, so we need to make sure that no other solution appears.

Obviously, there are three more theoretically possible solutions: one x in $(a, b]$, one in $(b, c]$, and one in (c, ∞). The first case solves to $x = 2003 + a - b - c$, the second to $3x = 2003 + a + b - c$, and the third to $5x = 2003 + a + b + c$. We need to make sure that the following three conditions hold:

1. $2003 + a - b - c \notin (a, b]$
2. $\dfrac{2003 + a + b - c}{3} \notin (b, c]$
3. $\dfrac{2003 + a + b + c}{5} \notin (c, \infty)$.

Let $c = 1002$ and $b = 1001$. We then have:

1. $2003 + a - b - c = a$
2. $\dfrac{2003 + a + b - c}{3} = \dfrac{2002 + a}{3} \leq \dfrac{2002 + 1000}{3} < 1001 = b$
3. $\dfrac{2003 + a + b + c}{5} = \dfrac{4006 + a}{5} \leq \dfrac{4006 + 1006}{5} < 1002 = a$

Hence for $c = 1002$, $b = 1001$ and any valid a the system has exactly one solution $(x, y) = (a, 2003 - 2a)$.

Step 3: Proving the optimality of our solution

We will now show that for $c < 1002$ the system always has a solution such that $x > a$. This will mean that the system has at least two solutions, and thus the solution with $c = 1002$ is optimal.

1. As we are looking for a $c < 1002$, we have $b + c \leq 2001$, hence $2003 + a - b - c > a$. To make sure that the value falls outside $(a, b]$, we need to make it larger than b, thus $2003 + a - b - c > b$, or equivalently $2003 + a > 2b + c$.
2. The condition we just derived, $2003 + a > 2b + c$, can be rewritten as $2003 + a + b > 3b + c$, then as $2003 + a + b - c > 3b$, which becomes $\dfrac{2003 + a + b - c}{3} > b$. Thus to make sure that the second value falls outside $(b, c]$, we need to make it larger than c. The inequality $\dfrac{2003 + a + b - c}{3} > c$ simplifies to $2003 + a + b > 4c$.
3. To avoid the last solution, we must have $\dfrac{2003 + a + b + c}{5} \leq c$, which simplifies to $2003 + a + b \leq 4c$.

The last two inequalities contradict each other, thus there are no a, b, c that would satisfy both of them.

Conclusion

We just showed that whenever $c < 1002$, the system has at least two different solutions: one with $x \leq a$ and one with $x > a$.

We also showed that for $c = 1002$ there are some a, b for which the system has exactly one solution.

Hence the optimal value of c is $\boxed{\text{(C) } 1002}$.

See also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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2003 AMC 12B Problems/Problem 25

Problem

Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distance between the points are less than the radius of the circle?

$\mathrm{(A)}\ \frac{1}{36} \qquad \mathrm{(B)}\ \frac{1}{24} \qquad \mathrm{(C)}\ \frac{1}{18} \qquad \mathrm{(D)}\ \frac{1}{12} \qquad \mathrm{(E)}\ \frac{1}{9}$

Solution

The first point anywhere on the circle, because it doesn't matter where it is chosen.

The next point must lie within 60 degrees of arc on either side, a total of 120 degrees possible, giving a total $\frac{1}{3}$ chance. The last point must lie within 60 degrees of both.

The minimum area of freedom we have to place the third point is a 60 degrees arc(if the first two are 60 degrees apart), with a $\frac{1}{6}$ probability. The maximum amount of freedom we have to place the third point is a 120 degree arc(if the first two are the same point), with a $\frac{1}{3}$ probability.

As the second point moves farther away from the first point, up to a maximum of 60 degrees, the probability changes linearly (every degree it moves, adds one degree to where the third could be).

Therefore, we can average probabilities at each end to find $\frac{1}{4}$ to find the average probability we can place the third point based on a varying second point.

Therefore the total probability is $1 \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ or $\boxed{\mathrm{(D)}}$

See Also

2003 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2003))	
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