2011 AMC 8 Problems/Problem 1

Problem

Margie bought 3 apples at a cost of 50 cents per apple. She paid with a 5-dollar bill. How much change did Margie recieve?

$\textbf{(A)}\ \$1.50 \qquad \textbf{(B)}\ \$2.00 \qquad \textbf{(C)}\ \$2.50 \qquad \textbf{(D)}\ \$3.00 \qquad \textbf{(E)}\ \$3.50$

Solution

50 cents is equivalent to $\$0.50.$ Then the three apples cost $3\times \$0.50 = \$1.50.$ The change Margie receives is $\$5.00 - \$1.50 = \boxed{\textbf{(E)}\ \$3.50}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 2

Problem

Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

$\textbf{(A)}$ Karl's garden is larger by 100 square feet.

$\textbf{(B)}$ Karl's garden is larger by 25 square feet.

$\textbf{(C)}$ The gardens are the same size.

$\textbf{(D)}$ Makenna's garden is larger by 25 square feet.

$\textbf{(E)}$ Makenna's garden is larger by 100 square feet.

Solution

The area of a rectangle is given by the formula length times width. Karl's garden is $20 \times 45 = 900$ square feet and Makenna's garden is $25 \times 40 = 1000$ square feet. Since $1000 > 900$, Makenna's garden is larger by $1000 - 900 = 100$ square feet.

$\Rightarrow \boxed{\textbf{(E)}\text{ Makenna's garden is larger by 100 square feet.}}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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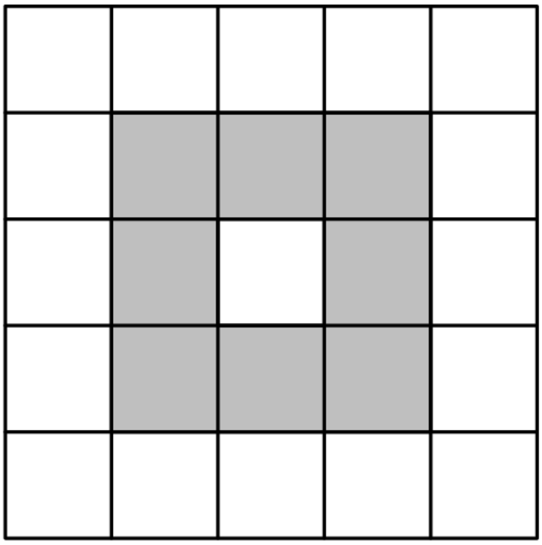
2011 AMC 8 Problems/Problem 3

Contents

Problem

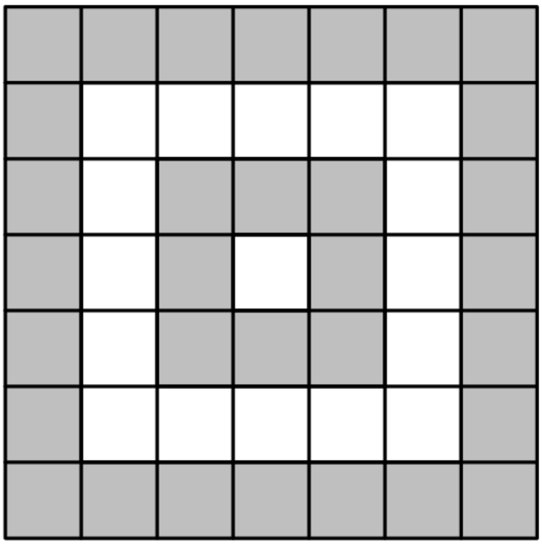
Extend the square pattern of 8 black and 17 white square tiles by attaching a border of black tiles around the square. What is the ratio of black tiles to white tiles in the extended pattern?



$\textbf{(A)}\ 8:17 \qquad\textbf{(B)}\ 25:49 \qquad\textbf{(C)}\ 36:25 \qquad\textbf{(D)}\ 32:17 \qquad\textbf{(E)}\ 36:17$

Solution 1

One way of approaching this is drawing the next circle of boxes around the current square.



We can now count the number of black and white tiles; 32 black tiles and 17 white tiles. This means the answer is $\boxed{\textbf{(D)}\ 32:17}$.

Solution 2

If we did not want to draw a diagram (though that is probably simpler in this case, we can imagine the last border of black tiles. We see that each side length (if the side length of a square is 1) increases by 2 each time we go out one layer. Therefore, we know that the next border will have a side length of 7. That border has $7^2 - 5^2 = 24$ black squares in it. The other black border has $3^2 - 1^2 = 8$ squares in it, so there are 32 black squares in all. The other ones must all be white, so there are $49 - 32 = 17$ white squares. Thus, we get our answer of $\boxed{\textbf{(D)}\ 32:17}$.

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 4

Problem

Here is a list of the numbers of fish that Tyler caught in nine outings last summer:

$$2, 0, 1, 3, 0, 3, 3, 1, 2.$$

Which statement about the mean, median, and mode is true?

$\textbf{(A)}\ \text{median} < \text{mean} < \text{mode} \qquad \textbf{(B)}\ \text{mean} < \text{mode} < \text{median}$

$\textbf{(C)}\ \text{mean} < \text{median} < \text{mode} \qquad \textbf{(D)}\ \text{median} < \text{mode} < \text{mean}$

$\textbf{(E)}\ \text{mode} < \text{median} < \text{mean}$

Solution

First, put the numbers in increasing order.

$$0, 0, 1, 1, 2, 2, 3, 3, 3$$

The mean is $\frac{0+0+1+1+2+2+3+3+3}{9} = \frac{15}{9}$, the median is 2, and the mode is $3.$

Because, $\frac{15}{9} < 2 < 3,$ the answer is $\boxed{\textbf{(C)}\ \text{mean} < \text{median} < \text{mode}}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 5

Problem

What time was it 2011 minutes after midnight on January 1, 2011?

$\textbf{(A)}$ January 1 at 9:31PM

$\textbf{(B)}$ January 1 at 11:51PM

$\textbf{(C)}$ January 2 at 3:11AM

$\textbf{(D)}$ January 2 at 9:31AM

$\textbf{(E)}$ January 2 at 6:01PM

Solution

There are 60 minutes in an hour. $2011/60 = 33\text{r}31$, or 33 hours and 31 minutes. There are 24 hours in a day, so the time is 9 hours and 31 minutes after midnight on January 2, 2011.
$\Rightarrow \boxed{\textbf{(D)}\ \text{January 2 at 9:31AM}}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 6

Contents

Problem

In a town of 351 adults, every adult owns a car, motorcycle, or both. If 331 adults own cars and 45 adults own motorcycles, how many of the car owners do not own a motorcycle?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 306 \qquad \textbf{(E)}\ 351$

Solution 1

By PIE, the number of adults who own both cars and motorcycles is $331+45-351=25.$ Out of the 331 car owners, 25 of them own motorcycles and $331-25=\boxed{\textbf{(D)}\ 306}$ of them don't.

Solution 2

There are 351 total adults, and 45 own a motorcycle. The number of adults that don't own a motorcycle is $351-45=306.$ Since everyone owns a car or motorcycle and one who doesn't own a motorcycle owns a car, the answer is $\boxed{\textbf{(D)}\ 306}$.

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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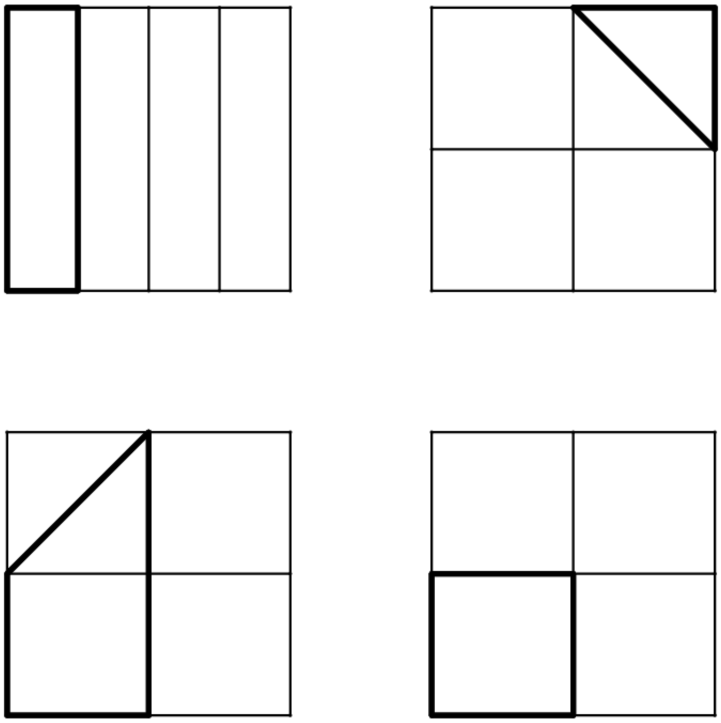
2011 AMC 8 Problems/Problem 7

Problem

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially bolded. What percent of the total area is partially bolded?



Solution

Assume that the area of each square is 1. Then, the area of the bolded region in the top left square is $\frac{1}{4}$. The area of the top right bolded region is $\frac{1}{8}$. The area of the bottom left bolded region is $\frac{3}{8}$. And the area of the bottom right bolded region is $\frac{1}{4}$. Add the four fractions: $\frac{1}{4}+\frac{1}{8}+\frac{3}{8}+\frac{1}{4}=1$. The four squares together have an area of 4, so the percentage bolded is $\frac{1}{4}\cdot 100=\boxed{\textbf{(C)}\ 25}$.

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 8

Problem

Bag A has three chips labeled 1, 3, and 5. Bag B has three chips labeled 2, 4, and 6. If one chip is drawn from each bag, how many different values are possible for the sum of the two numbers on the chips?

$\textbf{(A)}\ 4 \qquad\textbf{(B)}\ 5 \qquad\textbf{(C)}\ 6 \qquad\textbf{(D)}\ 7 \qquad\textbf{(E)}\ 9$

Solution

By adding a number from Bag A and a number from Bag B together, the values we can get are $3, 5, 7, 5, 7, 9, 7, 9, 11.$ Therefore the number of different values is $\boxed{\textbf{(B)}\ 5}$. You can also approach this problem by noticing the sums will make odd multiples from 3 to 11 inclusive.

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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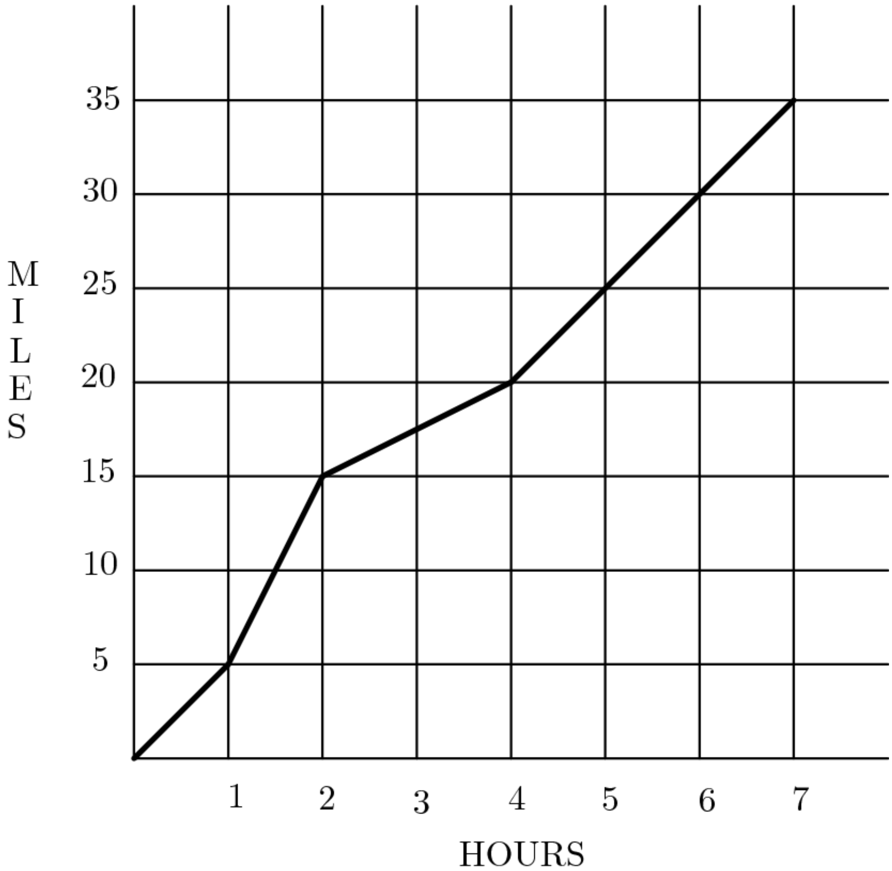
2011 AMC 8 Problems/Problem 9

Problem

Carmen takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of her ride. What is Carmen's average speed for her entire ride in miles per hour?



Solution

We observe the graph and see that the shape of the graph does not matter. We only want the total time it took Carmen and the total distance she traveled. Based on the graph, Carmen traveled 35 miles for 7 hours. Therefore, her average speed is $\boxed{\textbf{(E)}\ 5}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 10

Problem

The taxi fare in Gotham City is \$2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \$0.20 for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10?

$\textbf{(A)}\ 3.0\qquad\textbf{(B)}\ 3.25\qquad\textbf{(C)}\ 3.3\qquad\textbf{(D)}\ 3.5\qquad\textbf{(E)}\ 3.75$

Solution

Let x be the number of miles you ride. The number of miles you ride after the first half mile is $x-0.5.$ We can write this equation:

$$\begin{align*}10 &= 2.4 + 0.2 \times \frac{x-0.5}{0.1} + 2\\ 5.6 &= 2(x-0.5)\\ 2.8 &= x-0.5\\ x &= \boxed{\textbf{(C)}\ 3.3}\end{align*}$$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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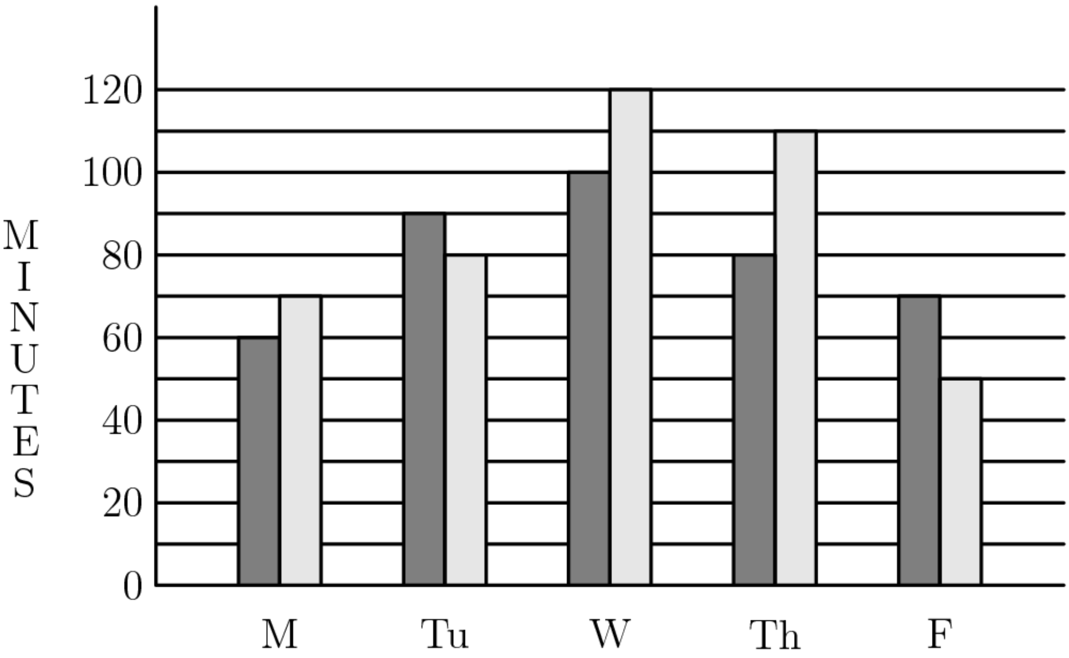
2011 AMC 8 Problems/Problem 11

Problem

The graph shows the number of minutes studied by both Asha (black bar) and Sasha (grey bar) in one week. On the average, how many more minutes per day did Sasha study than Asha?



$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 8\qquad\textbf{(C)}\ 9\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 12$

Solution

Average the differences between each day. We get $10, -10,\ 20,\ 30, -20$. We find the average of this list to get $\boxed{\textbf{(A)}\ 6}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 12

Contents

Problem

Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the probability that Angie and Carlos are seated opposite each other?

$\textbf{(A)}\ \frac14 \qquad\textbf{(B)}\ \frac13 \qquad\textbf{(C)}\ \frac12 \qquad\textbf{(D)}\ \frac23 \qquad\textbf{(E)}\ \frac34$

Solution 1

If we designate Angie to be on a certain side, then all placements of the other people can be considered unique. There are then $3!=6$ total seating arrangements. If Carlos is across from Angie, there are only $2!=2.$ ways to fill the remaining two seats. Then the probability Angie and Carlos are seated opposite each other is $\frac26=\boxed{\textbf{(B)}\ \frac13}$

Solution 2

If we seat Angie first, there would be only one out of three ways Carlos can sit across from Angie. So the final answer is $\boxed{\textbf{(B)}\ \frac13}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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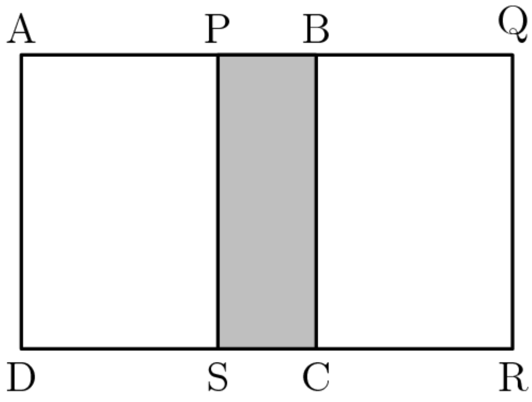
2011 AMC 8 Problems/Problem 13

Problem

Two congruent squares, $ABCD$ and $PQRS$, have side length 15. They overlap to form the 15 by 25 rectangle $AQRD$ shown. What percent of the area of rectangle $AQRD$ is shaded?



$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 25$

Solution

The overlap length is 5, so the shaded area is $5 \cdot 15 = 75$. The area of the whole shape is $25 \cdot 15 = 375$. The fraction $\frac{75}{375}$ reduces to $\frac{1}{5}$ or 20%. Therefore, the answer is $\boxed{\textbf{(C)}\ 20}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 14

Problem

There are 270 students at Colfax Middle School, where the ratio of boys to girls is $5 : 4$. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is $4 : 5$. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

$\textbf{(A)}\ \frac{7}{18} \qquad \textbf{(B)}\ \frac{7}{15} \qquad \textbf{(C)}\ \frac{22}{45} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{23}{45}$

Solution

At Colfax Middle School, there are $\frac{4}{9} \times 270 = 120$ girls. At Winthrop Middle School, there are $\frac{5}{9} \times 180 = 100$ girls. The ratio of girls to the total number of students is $\frac{120+100}{270+180} = \frac{220}{450} = \boxed{\textbf{(C)}\ \frac{22}{45}}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 15

Problem

How many digits are in the product $4^5 \cdot 5^{10}$?

$\textbf{(A)}\ 8 \qquad\textbf{(B)}\ 9 \qquad\textbf{(C)}\ 10 \qquad\textbf{(D)}\ 11 \qquad\textbf{(E)}\ 12$

Solution

$$4^5 \cdot 5^{10} = 2^{10} \cdot 5^{10} = 10^{10}.$$

That is one 1 followed by ten 0's, which is $\boxed{\textbf{(D)}\ 11}$ digits.

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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2011 AMC 8 Problems/Problem 16

Problem

Let A be the area of the triangle with sides of length $25, 25$, and 30. Let B be the area of the triangle with sides of length $25, 25,$ and 40. What is the relationship between A and B?

$\textbf{(A)}\ A = \dfrac{9}{16}B \qquad \textbf{(B)}\ A = \dfrac{3}{4}B \qquad \textbf{(C)}\ A = B \qquad \textbf{(D)}\ A = \dfrac{4}{3}B$
$\textbf{(E)}\ A = \dfrac{16}{9}B$

Solution

25-25-30

We can draw the altitude for the side with length 30. By HL Congruence, the two triangles formed are congruent. Thus the altitude splits the side with length 30 into two segments with length 15. By the Pythagorean Theorem, we have

$$15^2 + x^2 = 25^2$$

$$x^2 = 25^2 - 15^2$$

$$x^2 = (25+15)(25-15)$$

$$x^2 = 40 \cdot 10$$

$$x^2 = 400$$

$$x = \sqrt{400}$$

$$x = 20$$

Thus we have two 15-20-25 right triangles.

25-25-40

We can draw the altitude for the side with length 40. By HL Congruence, the two triangles formed are congruent. Thus the altitude splits the side with length 40 into two segments with length 20. From the 25-25-30 case, we know that the other side length is 15, so we have two 15-20-25 right triangles. Let the area of a 15-20-25 right triangle be x.

$$a = 2x$$

$$b = 2x$$

$$\boxed{\textbf{(C)}\ A = B}$$

See Also

2011 AMC 8 Problems/Problem 17

Problem

Let w, x, y, and z be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does $2w + 3x + 5y + 7z$ equal?

$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 27 \qquad \textbf{(D)}\ 35 \qquad \textbf{(E)}\ 56$

Solution

The prime factorization of 588 is $2^2 \cdot 3 \cdot 7^2$. We can see $w=2, x=1,$ and $z=2$. Because $5^0=1, y=0$.

$$2w+3x+5y+7z=4+3+0+14=\boxed{\textbf{(A)}\ 21}$$

See Also

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2011 AMC 8 Problems/Problem 18

Problem

A fair 6-sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{5}{12} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{7}{12} \qquad \textbf{(E)}\ \frac{5}{6}$

Solution

There are $6 \cdot 6 = 36$ ways to roll the two dice, and 6 of them result in two of the same number. Out of the remaining $36 - 6 = 30$ ways, the number of rolls where the first dice is greater than the second should be the same as the number of rolls where the second dice is greater than the first. In other words, there are $30/2 = 15$ ways the first roll can be greater than the second. The probability the first number is greater than or equal to the second number is

$$\frac{15+6}{36} = \frac{21}{36} = \boxed{\textbf{(D)}\ \frac{7}{12}}$$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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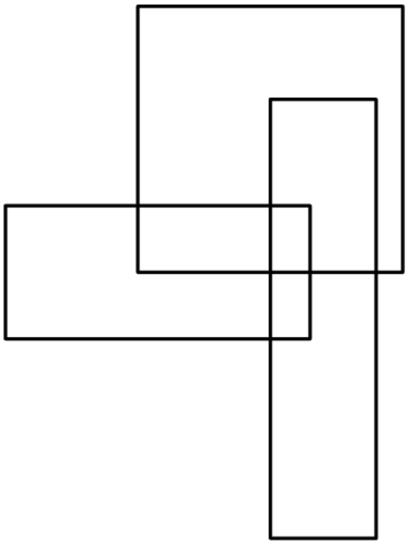
2011 AMC 8 Problems/Problem 19

Problem

How many rectangles are in this figure?



$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Solution

The figure can be divided into 7 sections. The number of rectangles with just one section is $3.$ The number of rectangles with two sections is $5.$ There are none with only three sections. The number of rectangles with four sections is $3.$ $3+5+3=\boxed{\textbf{(D)}\ 11}$

See Also

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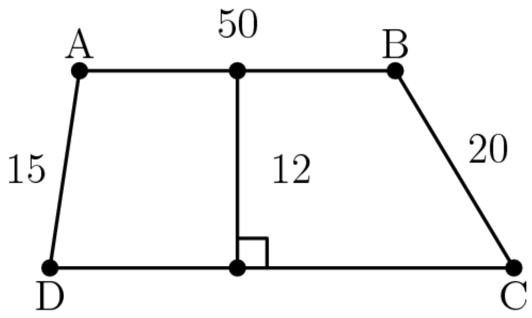
2011 AMC 8 Problems/Problem 20

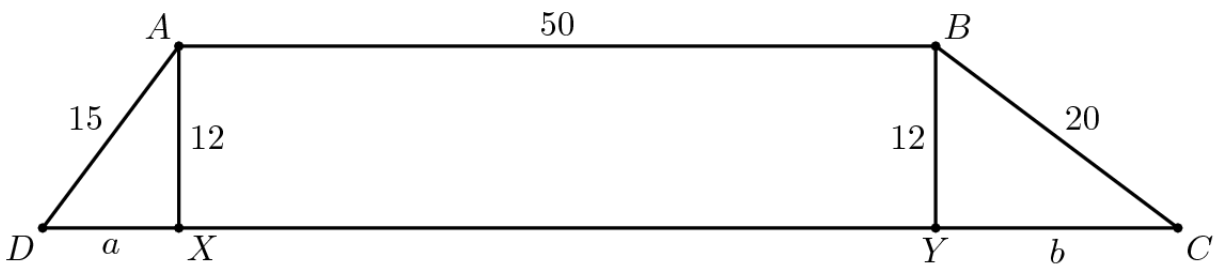
Problem

Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?



$\textbf{(A)}\ 600 \qquad \textbf{(B)}\ 650 \qquad \textbf{(C)}\ 700 \qquad \textbf{(D)}\ 750 \qquad \textbf{(E)}\ 800$

Solution



If you draw altitudes from A and B to CD, the trapezoid will be divided into two right triangles and a rectangle. You can find the values of a and b with the Pythagorean theorem.

$$a = \sqrt{15^2 - 12^2} = \sqrt{81} = 9$$

$$b = \sqrt{20^2 - 12^2} = \sqrt{256} = 16$$

$ABYX$ is a rectangle so $XY = AB = 50$.

$$CD = a + XY + b = 9 + 50 + 16 = 75$$

The area of the trapezoid is

$$12 \cdot \frac{(50 + 75)}{2} = 6(125) = \boxed{\textbf{(D)}\ 750}$$

See Also

2011 AMC 8 Problems/Problem 21

Problem

Students guess that Norb's age is $24, 28, 30, 32, 36, 38, 41, 44, 47$, and 49. Norb says, "At least half of you guessed too low, two of you are off by one, and my age is a prime number." How old is Norb?

$\textbf{(A)}\ 29 \qquad\textbf{(B)}\ 31 \qquad\textbf{(C)}\ 37 \qquad\textbf{(D)}\ 43 \qquad\textbf{(E)}\ 48$

Solution

If at least half the guesses are too low, then his age must be greater than $36.$

If two of the guesses are off by one, then his age is in between two guesses whose difference is $2.$ It could $31, 37,$ or $48,$ but because it is greater than 36 it can only be 37 or $48.$

Lastly, his age is a prime number so the answer must be $\boxed{\textbf{(C)}\ 37}$

See Also

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2011 AMC 8 Problems/Problem 22

Problem

What is the tens digit of 7^{2011}?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 1\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 7$

Solution

The first couple powers of 7 are $7, 49, 343, 2401, 16807.$ As you can see, the last two digits cycle after every 4 powers. $7^1 \pmod{100} \equiv 7^5 \pmod{100} \equiv 7^{2009} \pmod{100}.$ From there, we go two more powers. The last two digits are 43 so the tens digit is $\boxed{\textbf{(D)}\ 4}$

See Also

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2011 AMC 8 Problems/Problem 23

How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

$\textbf{(A) }24\qquad\textbf{(B) }48\qquad\textbf{(C) }60\qquad\textbf{(D) }84\qquad\textbf{(E) }108$

Solution

We can separate this into two cases. If an integer is a multiple of $5,$ the last digit must be either 0 or $5.$

Case 1: The last digit is $5.$ The leading digit can be $1,2,3,$ or $4.$ Because the second digit can be 0 but not the leading digit, there are also 4 choices. The third digit cannot be the leading digit or the second digit, so there are 3 choices. The number of integers is this case is $4\cdot4\cdot3\cdot1=48.$

Case 2: The last digit is $0.$ Because 5 is the largest digit, one of the remaining three digits must be $5.$ There are 3 ways to choose which digit should be $5.$ The remaining digits can be $1,2,3,$ or $4,$ but since they have to be different there are $4\cdot3$ ways to choose. The number of integers in this case is $1\cdot3\cdot4\cdot3=36.$

Therefore, the answer is $48+36=\boxed{\textbf{(D)}\ 84}$

See Also

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2011 AMC 8 Problems/Problem 24

In how many ways can 10001 be written as the sum of two primes?

$\textbf{(A) }0\qquad\textbf{(B) }1\qquad\textbf{(C) }2\qquad\textbf{(D) }3\qquad\textbf{(E) }4$

Solution

For the sum of two numbers to be odd, one must be odd and the other must be even, because All odd numbers are of the form $2n+1$ where n is an integer, and all even numbers are of the form $2m$ where m is an integer.

$$2n+1+2m=2m+2n+1=2(m+n)+1$$

and $m+n$ is an integer because m and n are both integers. The only even prime number is $2,$ so our only combination could be 2 and $9999.$ However, 9999 is clearly divisible by 3 so the number of ways 10001 can be written as the sum of two primes is $\boxed{\textbf{(A)}\ 0}$

See Also

2011 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2011))	
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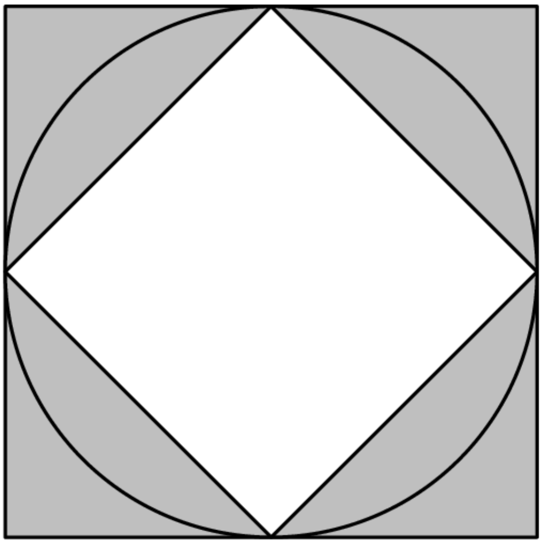
2011 AMC 8 Problems/Problem 25

Contents

Problem

A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the area between the two squares?



$\textbf{(A)}\ \frac{1}{2}\qquad\textbf{(B)}\ 1\qquad\textbf{(C)}\ \frac{3}{2}\qquad\textbf{(D)}\ 2\qquad\textbf{(E)}\ \frac{5}{2}$

Solution 1

The area of the smaller square is the one half of the product of its diagonals. Note that the distance from a corner of the smaller square to the center is equivalent to the circle's radius so the diagonal is equal to the diameter: $2*2*1/2=2.$

The circle's shaded area is the area of the smaller square subtracted from the area of the circle: $\pi - 2.$

If you draw the diagonals of the smaller square, you will see that the larger square is split 4 congruent half-shaded squares. The area between the squares is equal to the area of the smaller square: $2.$

Approximating π to $3.14,$ the ratio of the circle's shaded area to the area between the two squares is about

$$\frac{\pi-2}{2} \approx \frac{3.14-2}{2} = \frac{1.14}{2} \approx \boxed{\textbf{(A)}\ \frac{1}{2}}$$

Solution 2

For the ratio of the circle's shaded area to the area between the squares to be $1,$ they would have to be approximately the same size. For any ratio larger than that, the circle's shaded area must be greater. However, we can clearly see that the circle's shaded area is part of the area between the squares, and is approximately $\boxed{\textbf{(A)}\ \frac{1}{2}}$.

Note that this solution is not rigorous, because we still should show that the ratio is less than $\frac{3}{4}$.

See Also

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