2011 AMC 12B Problems

2011 AMC 12B (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have 75 minutes working time to complete the test.

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Contents

Problem 1

What is

$$\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}?$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{147}{60} \qquad \textbf{(E)}\ \frac{43}{3}$

Solution

Problem 2

Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 85 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 95$

Solution

Problem 3

LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

$\textbf{(A)}\ \frac{A+B}{2} \qquad \textbf{(B)}\ \frac{A-B}{2} \qquad \textbf{(C)}\ \frac{B-A}{2} \qquad \textbf{(D)}\ B-A \qquad \textbf{(E)}\ A+B$

Solution

Problem 4

In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

$\textbf{(A)}\ 116 \qquad \textbf{(B)}\ 161 \qquad \textbf{(C)}\ 204 \qquad \textbf{(D)}\ 214 \qquad \textbf{(E)}\ 224$

Solution

Problem 5

Let N be the second smallest positive integer that is divisible by every positive integer less than 7. What is the sum of the digits of N?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 9$

Solution

Problem 6

Two tangents to a circle are drawn from a point A. The points of contact B and C divide the circle into arcs with lengths in the ratio $2 : 3$. What is the degree measure of $\angle BAC$?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 60$

Solution

Problem 7

Let x and y be two-digit positive integers with mean 60. What is the maximum value of the ratio $\frac{x}{y}$?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ \frac{33}{7} \qquad \textbf{(C)}\ \frac{39}{7} \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ \frac{99}{10}$

Solution

Problem 8

Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

$\textbf{(A)}\ \frac{\pi}{3} \qquad \textbf{(B)}\ \frac{2\pi}{3} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{4\pi}{3} \qquad \textbf{(E)}\ \frac{5\pi}{3}$

Solution

Problem 9

Two real numbers are selected independently and at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{2}{3}$

Solution

Problem 10

Rectangle $ABCD$ has $AB=6$ and $BC=3$. Point M is chosen on side AB so that $\angle AMD=\angle CMD$. What is the degree measure of $\angle AMD$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Solution

Problem 11

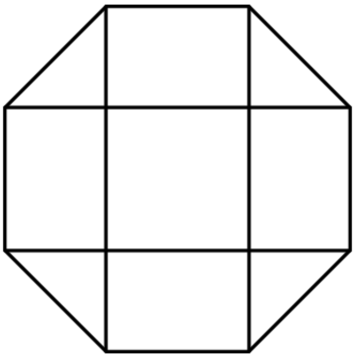
A frog located at (x,y), with both x and y integers, makes successive jumps of length 5 and always lands on points with integer coordinates. Suppose that the frog starts at $(0,0)$ and ends at $(1,0)$. What is the smallest possible number of jumps the frog makes?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

Problem 12

A dart board is a regular octagon divided into regions as shown below. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2-\sqrt{2}$

Solution

Problem 13

Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6$ and 9. What is the sum of the possible values of w?

(A) 16 **(B)** 31 **(C)** 48 **(D)** 62 **(E)** 93

Solution

Problem 14

A segment through the focus F of a parabola with vertex V is perpendicular to $\overline{FV}$ and intersects the parabola in points A and B. What is $\cos\left(\angle AVB\right)$?

(A) $-\frac{3\sqrt{5}}{7}$ **(B)** $-\frac{2\sqrt{5}}{5}$ **(C)** $-\frac{4}{5}$ **(D)** $-\frac{3}{5}$ **(E)** $-\frac{1}{2}$

Solution

Problem 15

How many positive two-digit integers are factors of $2^{24}-1$?

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

Solution

Problem 16

Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside of the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1+\frac{\sqrt{3}}{3}$ **(E)** 2

Solution

Problem 17

Let $f(x) = 10^{10x}, g(x) = \log_{10}\left(\frac{x}{10}\right), h_1(x) = g(f(x))$, and $h_n(x) = h_1(h_{n-1}(x))$ for integers $n \geq 2$. What is the sum of the digits of $h_{2011}(1)$?

(A) 16081 **(B)** 16089 **(C)** 18089 **(D)** 18098 **(E)** 18099

Solution

Problem 18

A pyramid has a square base with side of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

$\textbf{(A)}\ 5\sqrt{2} - 7 \qquad \textbf{(B)}\ 7 - 4\sqrt{3} \qquad \textbf{(C)}\ \frac{2\sqrt{2}}{27} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{9} \qquad \textbf{(E)}\ \frac{\sqrt{3}}{9}$

Solution

Problem 19

A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \le 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

$\textbf{(A)}\ \frac{51}{101} \qquad \textbf{(B)}\ \frac{50}{99} \qquad \textbf{(C)}\ \frac{51}{100} \qquad \textbf{(D)}\ \frac{52}{101} \qquad \textbf{(E)}\ \frac{13}{25}$

Solution

Problem 20

Triangle ABC has $AB = 13, BC = 14$, and $AC = 15$. The points D, E, and F are the midpoints of $\overline{AB}, \overline{BC}$, and $\overline{AC}$ respectively. Let $X \neq E$ be the intersection of the circumcircles of ΔBDE and ΔCEF. What is $XA + XB + XC$?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 14\sqrt{3} \qquad \textbf{(C)}\ \frac{195}{8} \qquad \textbf{(D)}\ \frac{129\sqrt{7}}{14} \qquad \textbf{(E)}\ \frac{69\sqrt{2}}{4}$

Solution

Problem 21

The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is obtained by reversing the digits of the arithmetic mean. What is $|x - y|$?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 54 \qquad \textbf{(D)}\ 66 \qquad \textbf{(E)}\ 70$

Solution

Problem 22

Let T_1 be a triangle with sides $2011, 2012$, and 2013. For $n \ge 1$, if $T_n = \Delta ABC$ and D, E, and F are the points of tangency of the incircle of ΔABC to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

$\textbf{(A)}\ \frac{1509}{8} \qquad \textbf{(B)}\ \frac{1509}{32} \qquad \textbf{(C)}\ \frac{1509}{64} \qquad \textbf{(D)}\ \frac{1509}{128} \qquad \textbf{(E)}\ \frac{1509}{256}$

Solution

Problem 23

A bug travels in the coordinate plane, moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 20. How many points with integer coordinates lie on at least one of these paths?

$\textbf{(A)}\ 161 \qquad \textbf{(B)}\ 185 \qquad \textbf{(C)}\ 195 \qquad \textbf{(D)}\ 227 \qquad \textbf{(E)}\ 255$

Solution

Problem 24

Let $P(z) = z^8 + \left(4\sqrt{3} + 6\right) z^4 - \left(4\sqrt{3} + 7\right)$. What is the minimum perimeter among all the 8-sided polygons in the complex plane whose vertices are precisely the zeros of $P(z)$?

$\textbf{(A)}\ 4\sqrt{3} + 4 \qquad \textbf{(B)}\ 8\sqrt{2} \qquad \textbf{(C)}\ 3\sqrt{2} + 3\sqrt{6} \qquad \textbf{(D)}\ 4\sqrt{2} + 4\sqrt{3} \qquad \textbf{(E)}\ 4\sqrt{3} + 6$

Solution

Problem 25

For every m and k integers with k odd, denote by $\left[\frac{m}{k}\right]$ the integer closest to $\frac{m}{k}$. For every odd integer k, let $P(k)$ be the probability that

$$\left[\frac{n}{k}\right] + \left[\frac{100 - n}{k}\right] = \left[\frac{100}{k}\right]$$

for an integer n randomly chosen from the interval $1 \leq n \leq 99!$. What is the minimum possible value of $P(k)$ over the odd integers k in the interval $1 \leq k \leq 99$?

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{50}{99} \qquad \textbf{(C)}\ \frac{44}{87} \qquad \textbf{(D)}\ \frac{34}{67} \qquad \textbf{(E)}\ \frac{7}{13}$

Solution

See also

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