2002 AMC 12A Problems

Contents

Problem 1

Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$

$\text{(A) } \frac{7}{2} \qquad \text{(B) } 4 \qquad \text{(C) } 5 \qquad \text{(D) } 7 \qquad \text{(E) } 13$

Solution

Problem 2

Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

$\text{(A) } 15 \qquad \text{(B) } 34 \qquad \text{(C) } 43 \qquad \text{(D) } 51 \qquad \text{(E) } 138$

Solution

Problem 3

According to the standard convention for exponentiation,

$$2^{2^{2^{2}}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65536.$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Solution

Problem 4

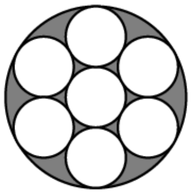
Find the degree measure of an angle whose complement is 25% of its supplement.

(A) 48 (B) 60 (C) 75 (D) 120 (E) 150

Solution

Problem 5

Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



(A) π (B) 1.5π (C) 2π (D) 3π (E) 3.5π

Solution

Problem 6

For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \le m + n$?

(A) 4 (B) 6 (C) 9 (D) 12 (E) infinitely many

Solution

Problem 7

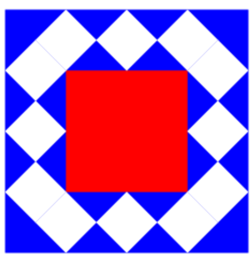
A 45° arc of circle A is equal in length to a 30° arc of circle B. What is the ratio of circle A's area and circle B's area?

(A) 4/9 (B) 2/3 (C) 5/6 (D) 3/2 (E) 9/4

Solution

Problem 8

Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



$\text{(A) } B=W \qquad \text{(B) } W=R \qquad \text{(C) } B=R \qquad \text{(D) } 3B=2R \qquad \text{(E) } 2R=W$

Solution

Problem 9

Jamal wants to save 30 files onto disks, each with 1.44 MB space. 3 of the files take up 0.8 MB, 12 of the files take up 0.7 MB, and the rest take up 0.4 MB. It is not possible to split a file onto 2 different disks. What is the smallest number of disks needed to store all 30 files?

$\text{(A) } 12 \qquad \text{(B) } 13 \qquad \text{(C) } 14 \qquad \text{(D) } 15 \qquad \text{(E)} 16$

Solution

Problem 10

Sarah places four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then pours half the coffee from the first cup to the second and, after stirring thoroughly, pours half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

$\text{(A) } \frac{1}{4} \qquad \text{(B) } \frac{1}{3} \qquad \text{(C) } \frac{3}{8} \qquad \text{(D) } \frac{2}{5} \qquad \text{(E) } \frac{1}{2}$

Solution

Problem 11

Mr. Earl E. Bird gets up every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

$\text{(A) } 45 \qquad \text{(B) } 48 \qquad \text{(C) } 50 \qquad \text{(D) } 55 \qquad \text{(E)} 58$

Solution

Problem 12

Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

$\text{(A) } 0 \qquad \text{(B) } 1 \qquad \text{(C) } 2 \qquad \text{(D) } 4 \qquad \text{(E) more than } 4$

Solution

Problem 13

Two different positive numbers a and b each differ from their reciprocals by 1. What is $a+b$?

$\text{(A) } 1 \qquad \text{(B) } 2 \qquad \text{(C) } \sqrt 5 \qquad \text{(D) } \sqrt 6 \qquad \text{(E) } 3$

Solution

Problem 14

For all positive integers n, let $f(n)=\log_{2002} n^2$. Let $N=f(11)+f(13)+f(14)$. Which of the following relations is true?

$\text{(A) } N<1 \qquad \text{(B) } N=1 \qquad \text{(C) } 1<N<2 \qquad \text{(D) } N=2 \qquad \text{(E) } N>2$

Solution

Problem 15

The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

Solution

Problem 16

Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

(A) 2/5 (B) 9/20 (C) 1/2 (D) 11/20 (E) 24/25

Solution

Problem 17

Several sets of prime numbers, such as $\{7, 83, 421, 659\}$ use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

(A) 193 (B) 207 (C) 225 (D) 252 (E) 447

Solution

Problem 18

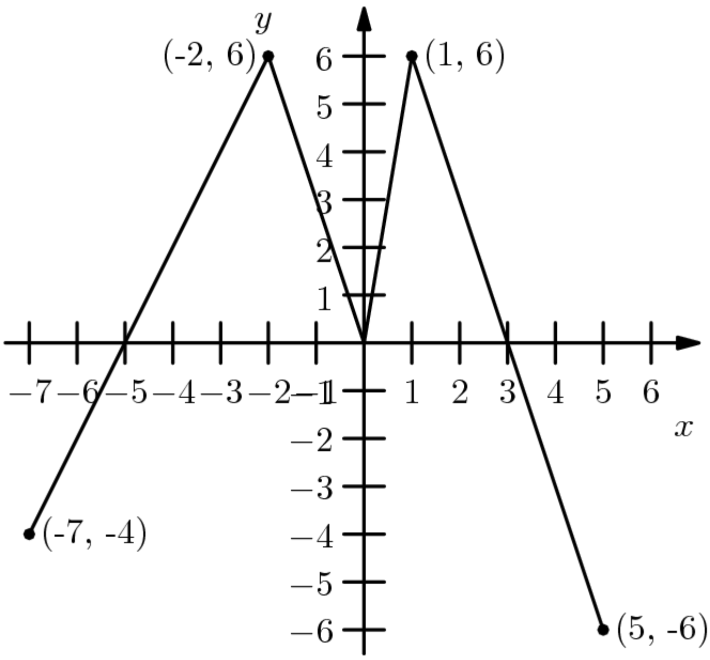
Let C_1 and C_2 be circles defined by $(x-10)^2 + y^2 = 36$ and $(x+15)^2 + y^2 = 81$ respectively. What is the length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q?

(A) 15 (B) 18 (C) 20 (D) 21 (E) 24

Solution

Problem 19

The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 6$ have?



(A) 2 (B) 4 (C) 5 (D) 6 (E) 7

Solution

Problem 20

Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

$\text{(A)}\ 3 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 5 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 9$

Solution

Problem 21

Consider the sequence of numbers: $4,7,1,8,9,7,6,\dots$ For $n>2$, the n-th term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. The smallest value of n for which $S_n>10,000$ is:

$\text{(A)}\ 1992 \qquad \text{(B)}\ 1999 \qquad \text{(C)}\ 2001 \qquad \text{(D)}\ 2002 \qquad \text{(E)}\ 2004$

Solution

Problem 22

Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60^\circ$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?

$\textbf{(A)}\ \frac{2-\sqrt{2}}{2} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{3-\sqrt{3}}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{5-\sqrt{5}}{5}$

Solution

Problem 23

In triangle ABC, side AC and the perpendicular bisector of BC meet in point D, and BD bisects $\angle ABC$. If $AD=9$ and $DC=7$, what is the area of triangle ABD?

$\text{(A)}\ 14 \qquad \text{(B)}\ 21 \qquad \text{(C)}\ 28 \qquad \text{(D)}\ 14\sqrt{5} \qquad \text{(E)}\ 28\sqrt{5}$

Solution

Problem 24

Find the number of ordered pairs of real numbers (a,b) such that $(a+bi)^{2002} = a-bi$.

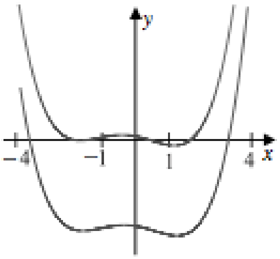
$\text{(A)}\ 1001 \qquad \text{(B)}\ 1002 \qquad \text{(C)}\ 2001 \qquad \text{(D)}\ 2002 \qquad \text{(E)}\ 2004$

Solution

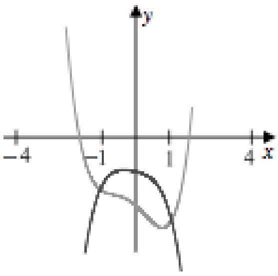
Problem 25

The nonzero coefficients of a polynomial P with real coefficients are all replaced by their mean to form a polynomial Q. Which of the following could be a graph of $y=P(x)$ and $y=Q(x)$ over the interval $-4\leq x \leq 4$?

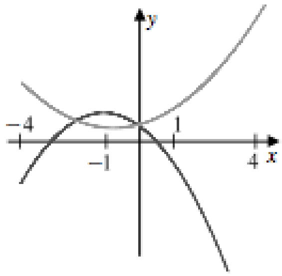
(A)



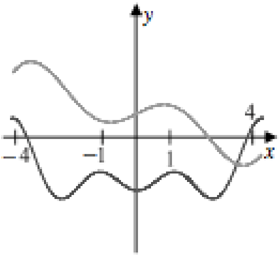
(B)



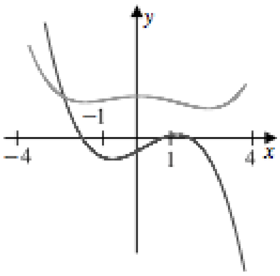
(C)



(D)



(E)



Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2002 AMC 12A
- Mathematics competition resources