2003 AMC 8 Problems/Problem 1

Problem

Jamie counted the number of edges of a cube, Jimmy counted the numbers of corners, and Judy counted the number of faces. They then added the three numbers. What was the resulting sum?

$\mathrm{(A)}\ 12 \qquad\mathrm{(B)}\ 16 \qquad\mathrm{(C)}\ 20 \qquad\mathrm{(D)}\ 22 \qquad\mathrm{(E)}\ 26$

Solution

On a cube, there are 12 edges, 8 corners, and 6 faces. Adding them up gets $12+8+6=\boxed{\mathrm{(E)}\ 26}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by First Question	Followed by Problem 2
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 2

Problem

Which of the following numbers has the smallest prime factor?

$\mathrm{(A)}\ 55 \qquad\mathrm{(B)}\ 57 \qquad\mathrm{(C)}\ 58 \qquad\mathrm{(D)}\ 59 \qquad\mathrm{(E)}\ 61$

Solution

The smallest prime factor is 2, and since 58 is the only multiple of 2, the answer is $\boxed{\mathrm{(C)}\ 58}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 1	Followed by Problem 3
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 3

Problem

A burger at Ricky C's weighs 120 grams, of which 30 grams are filler. What percent of the burger is not filler?

$\mathrm{(A)}\ 60\%\qquad\mathrm{(B)}\ 65\%\qquad\mathrm{(C)}\ 70\%\qquad\mathrm{(D)}\ 75\%\qquad\mathrm{(E)}\ 90\%$

Solution

There are 30 grams of filler, so there are $120-30=90$ grams that aren't filler. $\frac{90}{120}=\frac{3}{4}=\boxed{\mathrm{(D)}\ 75\%}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 2	Followed by Problem 4
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 4

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
  - 2.2 Solution 2
- 3 See Also

Problem

A group of children riding on bicycles and tricycles rode past Billy Bob's house. Billy Bob counted 7 children and 19 wheels. How many tricycles were there?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

Solution 1

If all the children were riding bicycles, there would be $2 \times 7 = 14$ wheels. Each tricycle adds an extra wheel and $19 - 14 = 5$ extra wheels are needed, so there are $\boxed{\textbf{(C)}\ 5}$ tricycles.

Solution 2

Setting up an equation, we have $a + b = 7$ children and $3a + 2b = 19$. Solving for the variables, we get, $a = \boxed{\textbf{(C)}\ 5}$ tricycles.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 3	Followed by Problem 5
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 5

Problem

If 20% of a number is 12, what is 30% of the same number?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 30$

Solution

20% of a number is equal to $\frac{1}{5}$ of that number. Let n=the number

$\frac{1}{5}n = 12$ Multiply both sides by 5

$n = 60$

30% of n is equal to $\frac{3}{10}n = \frac{3}{10} \cdot 60 = 3 \cdot 6 = \boxed{\textbf{(B)} 18}$

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 4	Followed by Problem 6
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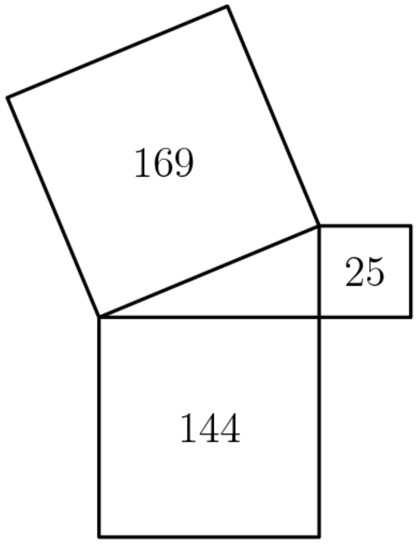
All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 6

Problem

Given the areas of the three squares in the figure, what is the area of the interior triangle?



$\mathrm{(A)}\ 13\qquad\mathrm{(B)}\ 30\qquad\mathrm{(C)}\ 60\qquad\mathrm{(D)}\ 300\qquad\mathrm{(E)}\ 1800$

Solution

The sides of the squares are $5, 12$ and 13 for the square with area $25, 144$ and 169, respectively. The legs of the interior triangle are 5 and 12, so the area is $\frac{5 \times 12}{2} = \boxed{\mathrm{(B)}\ 30}$

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 5	Followed by Problem 7
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 7

Contents

- 1 Problem
- 2 Solution
  - 2.1 Solution 1
  - 2.2 Solution 2
- 3 See Also

Problem

Blake and Jenny each took four 100-point tests. Blake averaged 78 on the four tests. Jenny scored 10 points higher than Blake on the first test, 10 points lower than him on the second test, and 20 points higher on both the third and fourth tests. What is the difference between Jenny's average and Blake's average on these four tests?

$\mathrm{(A)}\ 10 \qquad \mathrm{(B)}\ 15 \qquad \mathrm{(C)}\ 20 \qquad \mathrm{(D)}\ 25 \qquad \mathrm{(E)}\ 40$

Solution

Solution 1

Blake scored a total of $4 \times 78 = 312$ points. Jenny scored $10-10+20+20=40$ points higher than Blake, so her average is $\frac{312+40}{4}=88$. the difference is $88-78=\boxed{\mathrm{(A)}\ 10}$.

Solution 2

The total point difference between Blake's and Jenny's scores is $10-10+20+20=40$. The average of it is $\frac{40}{4} = \boxed{\mathrm{(A)}\ 10}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 6	Followed by Problem 8
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All AJHSME/AMC 8 Problems and Solutions	

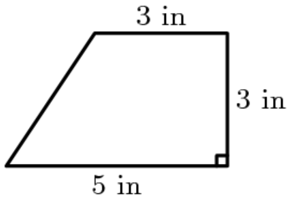
2003 AMC 8 Problems/Problem 8

Problem

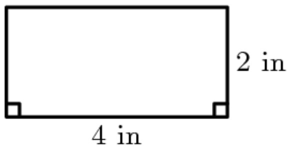
Bake Sale

Four friends, Art, Roger, Paul and Trisha, bake cookies, and all cookies have the same thickness. The shapes of the cookies differ, as shown.

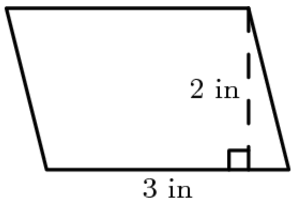
- Art's cookies are trapezoids.



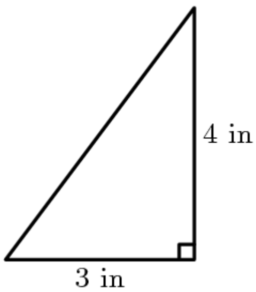
- Roger's cookies are rectangles.



- Paul's cookies are parallelograms.



- Trisha's cookies are triangles.



Each friend uses the same amount of dough, and Art makes exactly 12 cookies. Who gets the fewest cookies from one batch of cookie dough?

$\textbf{(A)}\ \text{Art}\qquad\textbf{(B)}\ \text{Roger}\qquad\textbf{(C)}\ \text{Paul}\qquad\textbf{(D)}\ \text{Trisha}\qquad\textbf{(E)}\ \text{There is a tie for fewest.}$

Solution

$\boxed{\textbf{(A)}\ \text{Art}}$ is the right answer because out of all the cookies, Art's had an area of 12 in^2, which was the greatest area out of all the cookies' areas. Roger's cookie had an area of 8 in^2, and both Paul and Trisha's cookies had an area of 6 in^2. This means Art makes less cookies, since his cookie area is the greatest. The answer is not that there is a tie between Paul and Trisha because they can make the most cookies with a given amount of cookie dough, not the least.

See Also

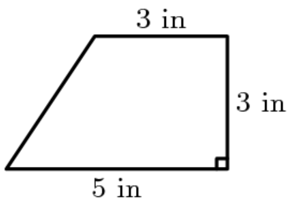
2003 AMC 8 Problems/Problem 9

Problem

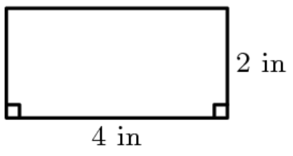
Problems 8, 9 and 10 use the data found in the accompanying paragraph and figures

Four friends, Art, Roger, Paul and Trisha, bake cookies, and all cookies have the same thickness. The shapes of the cookies differ, as shown.

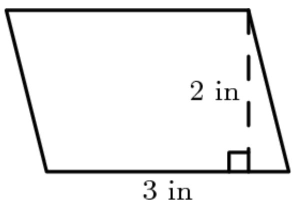
- Art's cookies are trapezoids:



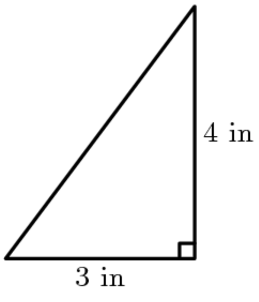
- Roger's cookies are rectangles:



- Paul's cookies are parallelograms:



- Trisha's cookies are triangles:



Each friend uses the same amount of dough, and Art makes exactly 12 cookies. Art's cookies sell for 60 cents each. To earn the same amount from a single batch, how much should one of Roger's cookies cost in cents?

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 75 \qquad \textbf{(E)}\ 90$

Solution

The area of one of Art's cookies is $3 \cdot 3 + \frac{2 \cdot 3}{2} = 9 + 3 = 12$. As he has 12 cookies in a batch, the amount of dough each person used is $12 \cdot 12 = 144$. Roger's cookies have an area of $\frac{144}{2 \cdot 4} = \frac{144}{8} = 18$ cookies in a batch. In total, the amount of money Art will earn is $12 \cdot 60 = 720$. Thus, the amount Roger would need to charge per cookie is $\dfrac{720}{18} = \boxed{\textbf{(C)}\ 40}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 8	Followed by Problem 10
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All AJHSME/AMC 8 Problems and Solutions	

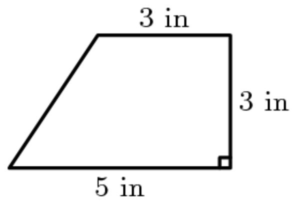
2003 AMC 8 Problems/Problem 10

Problem

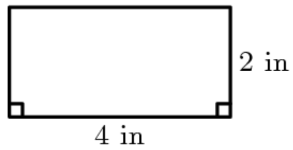
Problems 8, 9 and 10 use the data found in the accompanying paragraph and figures

Four friends, Art, Roger, Paul and Trisha, bake cookies, and all cookies have the same thickness. The shapes of the cookies differ, as shown.

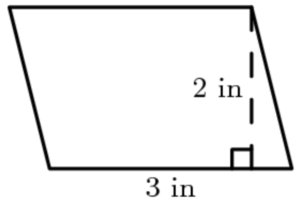
- Art's cookies are trapezoids:



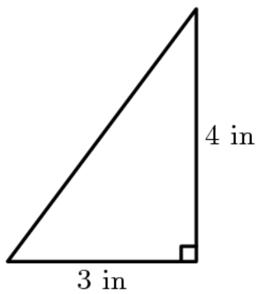
- Roger's cookies are rectangles:



- Paul's cookies are parallelograms:



- Trisha's cookies are triangles:



How many cookies will be in one batch of Trisha's cookies?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 24$

Solution

Art's cookies have areas of $3 \cdot 3 + \dfrac{2 \cdot 3}{2} = 9 + 3 = 12$. There are 12 cookies in one of Art's batches so everyone used $12 \cdot 12 = 144 \text{ in}^2$ of dough. Trisha's cookies have an area of $\dfrac{3 \cdot 4}{2} = 6$ so she has $\dfrac{144}{6} = \boxed{\textbf{(E)}\ 24}$ cookies per batch.

2003 AMC 8 Problems/Problem 11

Problem

Business is a little slow at Lou's Fine Shoes, so Lou decides to have a sale. On Friday, Lou increases all of Thursday's prices by 10 percent. Over the weekend, Lou advertises the sale: "Ten percent off the listed price. Sale starts Monday." How much does a pair of shoes cost on Monday that cost 40 dollars on Thursday?

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 39.60 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 40.40 \qquad \textbf{(E)}\ 44$

Solution

On Friday, the shoes would cost $40 \cdot 1.1 = 44$ dollars. Then on Monday, the shoes would cost $44 - \frac{44}{10} = 44 - 4.4 = \boxed{\textbf{(B)}\ 39.60}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 12

Problem

When a fair six-sided dice is tossed on a table top, the bottom face cannot be seen. What is the probability that the product of the faces than can be seen is divisible by 6?

$\textbf{(A)}\ 1/3 \qquad \textbf{(B)}\ 1/2 \qquad \textbf{(C)}\ 2/3 \qquad \textbf{(D)}\ 5/6 \qquad \textbf{(E)}\ 1$

Solution

All the possibilities where 6 is on any of the five sides is always divisible by six, and $1 \times 2 \times 3 \times 4 \times 5$ is divisible by 6 since $2 \times 3 = 6$. So, the answer is $\boxed{\textbf{(E)}\ 1}$ because the outcome is always divisible by 6.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
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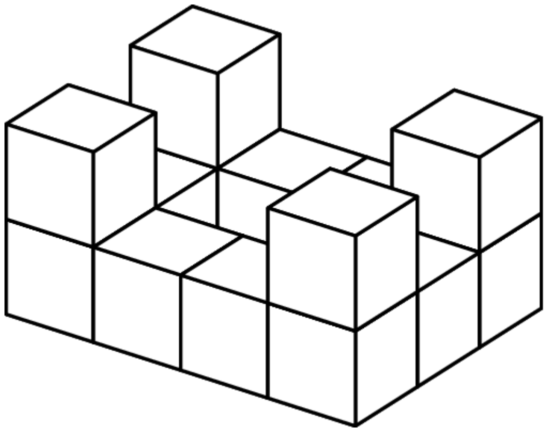
All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 13

Problem

Fourteen white cubes are put together to form the figure on the right. The complete surface of the figure, including the bottom, is painted red. The figure is then separated into individual cubes. How many of the individual cubes have exactly four red faces?



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Solution

This is the number cubes that are adjacent to another cube on two sides. The bottom corner cubes are connected on three sides, and the top corner cubes are connected on one. The number we are looking for is the number of middle cubes, which is $\boxed{\textbf{(B)}\ 6}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 14

Problem

In this addition problem, each letter stands for a different digit.

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

If T = 7 and the letter O represents an even number, what is the only possible value for W?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution

Since both T's are 7, then O has to equal 4, because 7 + 7 = 14. Then, F has to equal 1. To get R, we do 4 + 4 (since O = 4) to get R = 8. The value for W then has to be a number less than 5, otherwise it will change the value of O, and can't be a number that has already been used, like 4 or 1. The only other possibilities are 2 and 3. 2 doesn't work because it makes U = 4, which is what O already equals. So, the only possible value of W is 3 $\boxed{\text{D}}$

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
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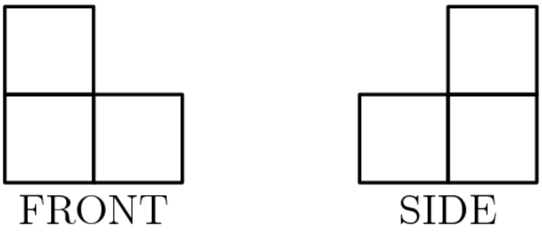
2003 AMC 8 Problems/Problem 15

Problem

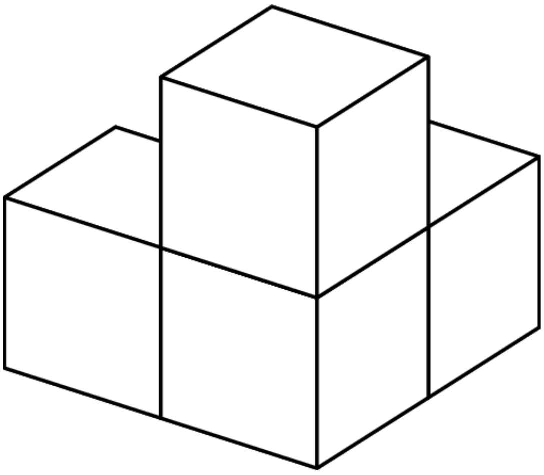
A figure is constructed from unit cubes. Each cube shares at least one face with another cube. What is the minimum number of cubes needed to build a figure with the front and side views shown?



$\textbf{(A)}\ 3 \qquad\textbf{(B)}\ 4 \qquad\textbf{(C)}\ 5 \qquad\textbf{(D)}\ 6 \qquad\textbf{(E)}\ 7$

Solution

In order to minimize the amount of cubes needed, we must match up as many squares of our given figures with each other to make different sides of the same cube. One example of the solution with $\boxed{\textbf{(B)}\ 4}$ cubes. Notice the corner cube cannot be removed for a figure of 3 cubes because each face of a cube must be touching another face.



See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 14	Followed by Problem 16
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 16

Problem

Ali, Bonnie, Carlo, and Dianna are going to drive together to a nearby theme park. The car they are using has 4 seats: 1 Driver seat, 1 front passenger seat, and 2 back passenger seat. Bonnie and Carlo are the only ones who know how to drive the car. How many possible seating arrangements are there?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 24$

Solution

There are only 2 people who can go in the driver's seat--Bonnie and Carlo. Any of the 3 remaining people can go in the front passenger seat. There are 2 people who can go in the first back passenger seat, and the remaining person must go in the last seat. Thus, there are $2 \cdot 3 \cdot 2$ or 12 ways. The answer is then $\boxed{\textbf{(D)}\ 12}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 15	Followed by Problem 17
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 17

Problem

The six children listed below are from two families of three siblings each. Each child has blue or brown eyes and black or blond hair. Children from the same family have at least one of these characteristics in common. Which two children are Jim's siblings?

Child	Eye Color	Hair Color
Benjamin	Blue	Black
Jim	Brown	Blonde
Nadeen	Brown	Black
Austin	Blue	Blonde
Tevyn	Blue	Black
Sue	Blue	Blonde

$\textbf{(A)}\ \text{Nadeen and Austin}\qquad\textbf{(B)}\ \text{Benjamin and Sue}\qquad\textbf{(C)}\ \text{Benjamin and Austin}\qquad\textbf{(D)}\ \text{Nadeen and Tevyn}$
$\textbf{(E)}\ \text{Austin and Sue}$

Solution

Jim has brown eyes and blonde hair. If you look for anybody who has brown eyes or blonde hair, you find that Nadeen, Austin, and Sue are Jim's possible siblings. However, the children have at least one common characteristics. Since Austin and Sue both have blonde hair, Nadeen is ruled out and therefore $\boxed{\textbf{(E)}\ \text{Austin and Sue}}$ are his siblings.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 16	Followed by Problem 18
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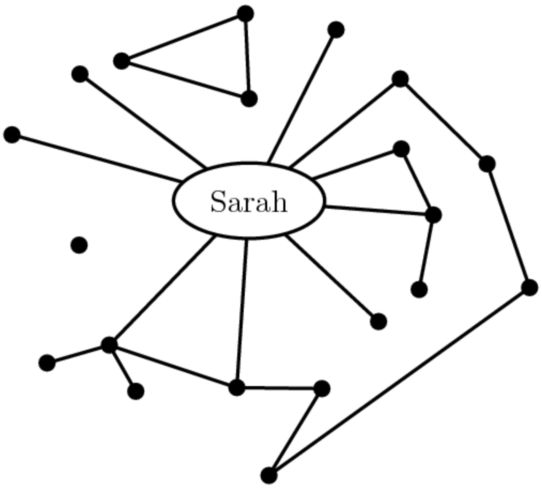
All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 18

Problem

Each of the twenty dots on the graph below represents one of Sarah's classmates. Classmates who are friends are connected with a line segment. For her birthday party, Sarah is inviting only the following: all of her friends and all of those classmates who are friends with at least one of her friends. How many classmates will not be invited to Sarah's party?



$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

There are 3 people who are friends with only each other who won't be invited, plus 1 person who has no friends, and 2 people who are friends of friends of friends who won't be invited. So the answer is $\boxed{\textbf{(D)}\ 6}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 17	Followed by Problem 19
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 19

Problem

How many integers between 1000 and 2000 have all three of the numbers 15, 20, and 25 as factors?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Find the least common multiple of $15, 20, 25$ by turning the numbers into their prime factorization.

$$15 = 3 * 5, 20 = 2^2 * 5, 25 = 5^2$$

Gather all necessary multiples $3, 2^2, 5^2$ when multiplied gets 300. The multiples of $300 - 300, 600, 900, 1200, 1500, 1800, 2100$. The number of multiples between 1000 and 2000 is $\boxed{\textbf{(C)}\ 3}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
Preceded by Problem 18	Followed by Problem 20
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All AJHSME/AMC 8 Problems and Solutions	

2003 AMC 8 Problems/Problem 20

Problem

What is the measure of the acute angle formed by the hands of the clock at 4:20 PM?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Solution

Imagine the clock as a circle. The minute hand will be at the 4 at 20 minutes past the hour. The central angle formed between 4 and 5 is 30 degrees (since it is 1/12 of a full circle, 360). By $4:20$, the hour hand would have moved $\frac{1}{3}$ way from 4 to 5 since $\frac{20}{60}$ is reducible to $\frac{1}{3}$. One third of the way from 4 to 5 is one third of 30 degrees, which is 10 degrees past the 4. Recall that the minute hand is at the 4, so the angle between them is $\boxed{10}$, and we are done.

See Also

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Preceded by Problem 19	Followed by Problem 21
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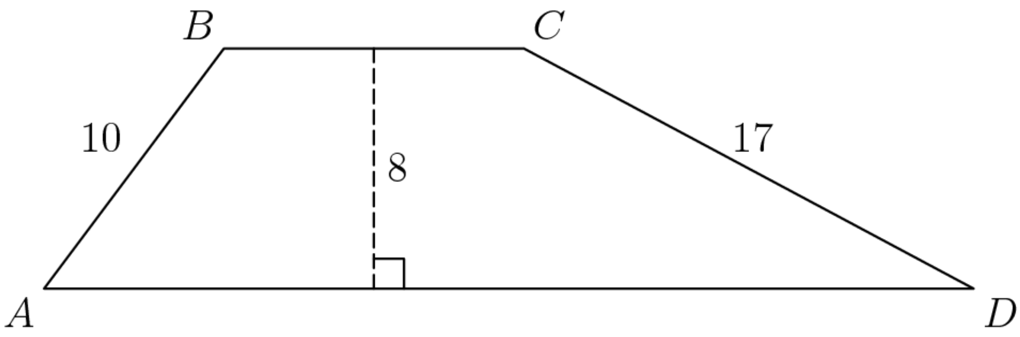
2003 AMC 8 Problems/Problem 21

Problem

The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC, in centimeters?



Solution

Using the formula for the area of a trapezoid, we have $164 = 8(\frac{BC+AD}{2})$. Thus $BC + AD = 41$. Drop perpendiculars from B to AD and from C to AD and let them hit AD at E and F respectively. Note that each of these perpendiculars has length 8. From the Pythagorean Theorem, $AE = 6$ and $DF = 15$ thus $AD = BC + 21$. Substituting back into our original equation we have $BC + BC + 21 = 41$ thus $BC = \boxed{\text{(B) } 10}$

See Also

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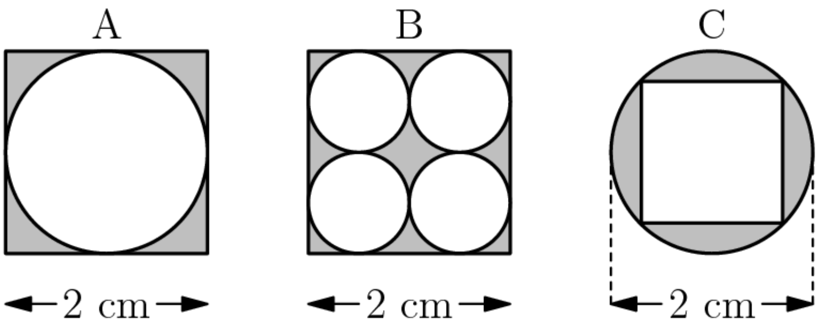
2003 AMC 8 Problems/Problem 22

Problem

The following figures are composed of squares and circles. Which figure has a shaded region with largest area?



$\textbf{(A)}\ \text{A only}\qquad\textbf{(B)}\ \text{B only}\qquad\textbf{(C)}\ \text{C only}\qquad\textbf{(D)}\ \text{both A and B}\qquad\textbf{(E)}\ \text{all are equal}$

Solution

First we have to find the area of the shaded region in each of the figures. In figure $\textbf{A}$ the area of the shaded region is the area of the circle subtracted from the area of the square. That is $2^2-1^2\pi=4-\pi$. In figure $\textbf{B}$ the area of the shaded region is the sum of the areas of the 4 circles subtracted from the area of the square. That is $2^2-4((\frac{1}{2})^2\pi)=4-4(\frac{\pi}{4})=4-\pi$. In figure $\textbf{C}$ the area of the shaded region is the area of the square subtracted from the area of the circle. The diameter of the circle and the diagonal of the square are equal to 2. We can easily find the area of the square using the area formula $\frac{d_1d_2}{2}$. So the area of the shaded region is $1^2\pi-\frac{2\cdot 2}{2}=\pi-2$. Clearly the largest area that we found among the three shaded regions is $\pi-2$. area so the answer is $\boxed{C}$

See Also

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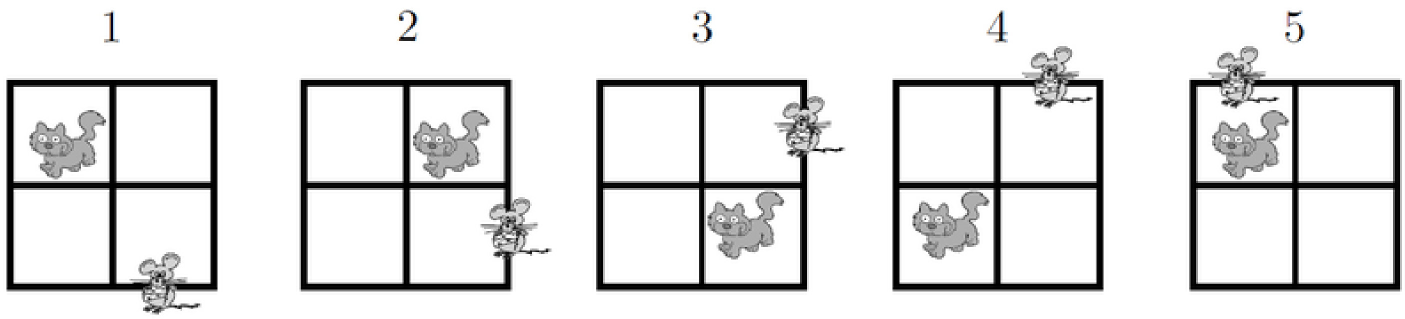
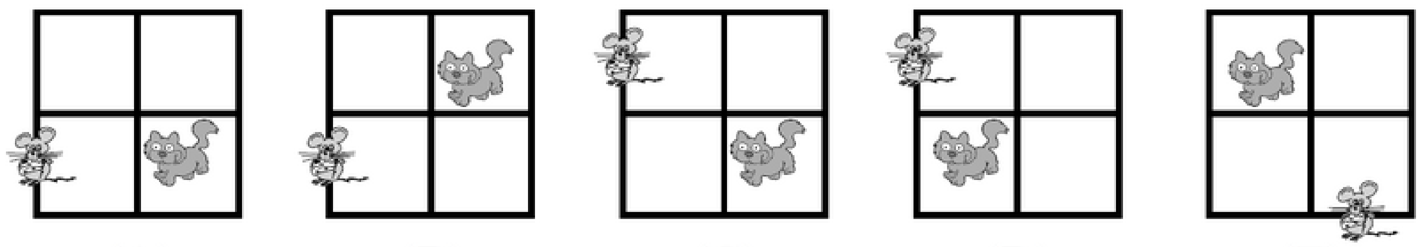
2003 AMC 8 Problems/Problem 23

Problem

In the pattern below, the cat (denoted as a cat in the figures below) moves clockwise through the four squares, and the mouse (denoted as a mouse in the figures below) moves counterclockwise through the eight exterior segments of the four squares.



If the pattern is continued, where would the cat and mouse be after the 247th move?



(A) (B) (C) (D) (E)

Solution

Break this problem into two parts: where the cat will be after the 247th move, and where the mouse will be.

The cat has four possible configurations which are repeated every four moves. 247 has a remainder of 3 when divided by 4. This corresponds to the position the cat has after the 3rd move, which is the bottom right corner.

Similarly, the mouse has eight possible configurations that repeat every eight moves. 247 has a remainder of 7 when divided by 8. This corresponds to the position the cat has after the 7th move, which can easily be found by writing two more steps to be the bottom edge on the left side of the grid.

The only configuration with the mouse in that position and the cat in the bottom right square is $\boxed{\textbf{(A)}}$.

See Also

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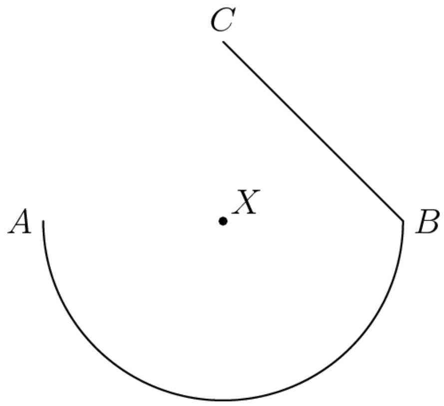
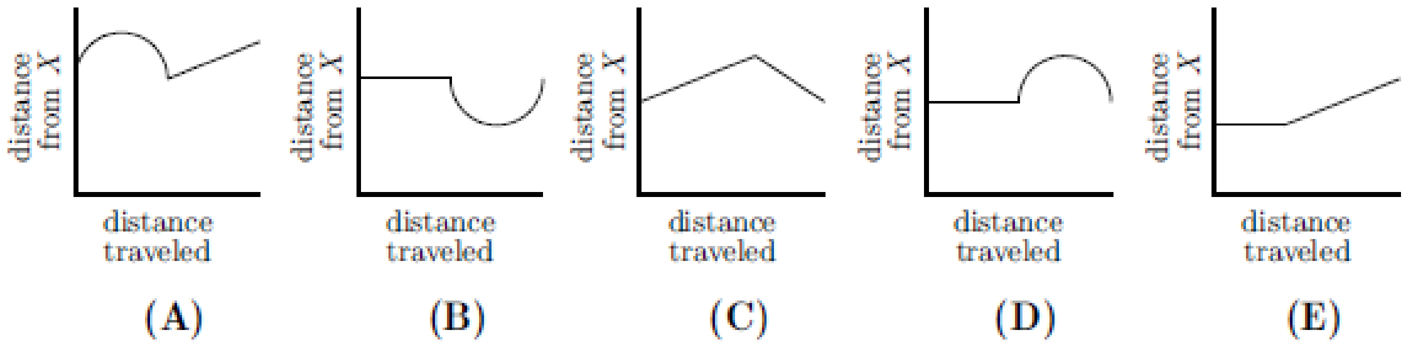
2003 AMC 8 Problems/Problem 24

Problem

A ship travels from point A to point B along a semicircular path, centered at Island X. Then it travels along a straight path from B to C. Which of these graphs best shows the ship's distance from Island X as it moves along its course?





Solution

The distance from X to any point on the semicircle will always be constant. On the graph, this will represent a straight line. The distance between X and line BC will not be constant though. We can easily prove that the distance between X and line BC will represent a semicircle (prove this by dividing $\triangle XCB$ into two congruent triangles using the perpendicular bisector from vertex X). Since the point on line BC and the perpendicular bisector from vertex X is the shortest distance between X and BC as well as the midpoint of line BC it will represent the shortest point on the semicircle in the graph as well as the midpoint of the semicircle. Using the information found, the answer choice that fits them all is $\boxed{\text{(B)}}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
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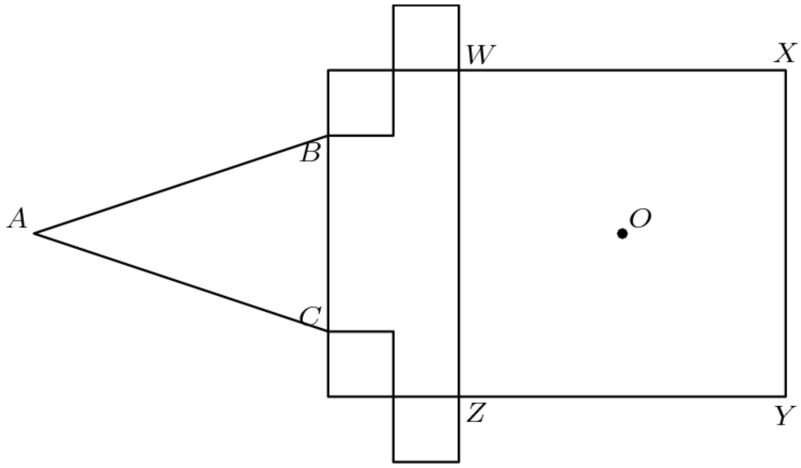
2003 AMC 8 Problems/Problem 25

Problem

In the figure, the area of square $WXYZ$ is 25 cm^2. The four smaller squares have sides 1 cm long, either parallel to or coinciding with the sides of the large square. In $\triangle ABC$, $AB = AC$, and when $\triangle ABC$ is folded over side $\overline{BC}$, point A coincides with O, the center of square $WXYZ$. What is the area of $\triangle ABC$, in square centimeters?



$\textbf{(A)}\ \frac{15}{4} \qquad \textbf{(B)}\ \frac{21}{4} \qquad \textbf{(C)}\ \frac{27}{4} \qquad \textbf{(D)}\ \frac{21}{2} \qquad \textbf{(E)}\ \frac{27}{2}$

Solution

The side lengths of square WXYZ must be 5 cm, since the area is $25\ cm^2$. First, you should determine the height of $\triangle ABC$. The distance from O to line WZ must be 2.5 cm, since line WX = 5 cm, and the distance from O to Z is half of that. The distance from line WZ to line BC must be 2, since the side lengths of the small squares are 1, and there are two squares from line WZ to line BC. So, the height of $\triangle ABC$ must be 4.5, which is 2.5 + 2. The length of BC can be determined by subtracting 2 from 5, since the length of WZ is 5, and the two squares in the corners give us 2 together. This gives us the base for $\triangle ABC$, which is 3. Then, we multiply 4.5 by 3 and divide by 2, to get an answer of $\boxed{\textbf{(C)}\ \frac{27}{4}}$.

See Also

2003 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2003))	
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