2019 AMC 12B

Problem 1

Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the first container to the volume of the second container?

Alicia 有两个容器。 第一个里面的水是 $\frac{5}{6}$ 满的，第二个是空的。 她将第一个容器中的所有水倒入第二个容器中，此时第二个容器中的水是 $\frac{3}{4}$ 满的。 较小容器的容积与较大容器的容积之比是多少？

$\textbf{(A) } \frac{5}{8} \qquad \textbf{(B) } \frac{4}{5} \qquad \textbf{(C) } \frac{7}{8} \qquad \textbf{(D) } \frac{9}{10} \qquad \textbf{(E) } \frac{11}{12}$

Problem 2

Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

考虑论断：“如果 n 不是质数，那么 $n-2$ 就是质数。” 以下的哪个 n 值是此论断的反例？

$\textbf{(A) } 11 \qquad \textbf{(B) } 15 \qquad \textbf{(C) } 19 \qquad \textbf{(D) } 21 \qquad \textbf{(E) } 27$

Problem 3

Which one of the following rigid transformations (isometries) maps the line segment $\overline{AB}$ onto the line segment $\overline{A'B'}$ so that the image of $A(-2,1)$ is $A'(2,-1)$ and the image of $B(-1,4)$ is $B'(1,-4)$?

下列哪个刚性变换（等距变换）把线段 $\overline{AB}$ 映像到线段 $\overline{A'B'}$，并且使得 $A(-2,1)$ 的映像是 $A'(2,-1)$， $B(-1,4)$ 的映像是 $B'(1,-4)$？

(A) reflection in the y-axis | 关于 y 轴的反射

(B) counterclockwise rotation around the origin by 90° | 绕原点逆时针旋转 90°

(C) translation by 3 units to the right and 5 units down | 向右平移 3 个单位再向下平移 5 个单位

(D) reflection in the x-axis | 关于 x 轴的反射

(E) clockwise rotation about the origin by 180° | 绕原点顺时针旋转 180°

Problem 4

A positive integer n satisfies the equation $(n+1)! + (n+2)! = 440 \cdot n!$. What is the sum of the digits of n?

正整数 n 满足等式 $(n+1)!+(n+2)!=440\cdot n!$. n 的数字总和是多少？

(A) 2 **(B)** 5 **(C)** 10 **(D)** 12 **(E)** 15

Problem 5

Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n?

商店里每一块糖的售价都是整分钱。Casper 有足够的钱购买 12 块红色糖果，14 块绿色糖果，15 块蓝色糖果，或者 n 块紫色糖果。一块紫色糖果售价 20 美分。 n 的最小可能价值是多少？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 28$

Problem 6

In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

在给定的平面中， 点 A 和 B 相距 10 个单位。平面上有多少个点 C， 使得$\triangle ABC$的周长是 50 单位，$\triangle ABC$的面积是 100 平方单位？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ \text{infinitely many}$

Problem 7

What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17,$ and x is equal to the mean of those five numbers?

实数 x 满足 4，6，8，17 和 x 的中位数等于这五个数的平均值，问所有这样的实数 x 的总和是多少？

$\textbf{(A)}\ -5 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ \frac{15}{4} \qquad \textbf{(E)}\ \frac{35}{4}$

Problem 8

Let $f(x)=x^2(1-x)^2$. What is the value of the sum

$$f\left(\frac{1}{2019}\right)-f\left(\frac{2}{2019}\right)+f\left(\frac{3}{2019}\right)-f\left(\frac{4}{2019}\right)+\cdots+f\left(\frac{2017}{2019}\right)-f\left(\frac{2018}{2019}\right)?$$

设 $f(x)=x^2(1-x)^2$． 下面和式的值是多少？

$$f\left(\frac{1}{2019}\right)-f\left(\frac{2}{2019}\right)+f\left(\frac{3}{2019}\right)-f\left(\frac{4}{2019}\right)+\mathrm{L}\ +f\left(\frac{2017}{2019}\right)-f\left(\frac{2018}{2019}\right)$$

(A) 0 **(B)** $\frac{1}{2019^4}$ **(C)** $\frac{2018^2}{2019^4}$ **(D)** $\frac{2020^2}{2019^4}$ **(E)** 1

Problem 9

For how many integral values of x can a triangle of positive area be formed having side lengths $\log_2 x, \log_4 x, 3$?

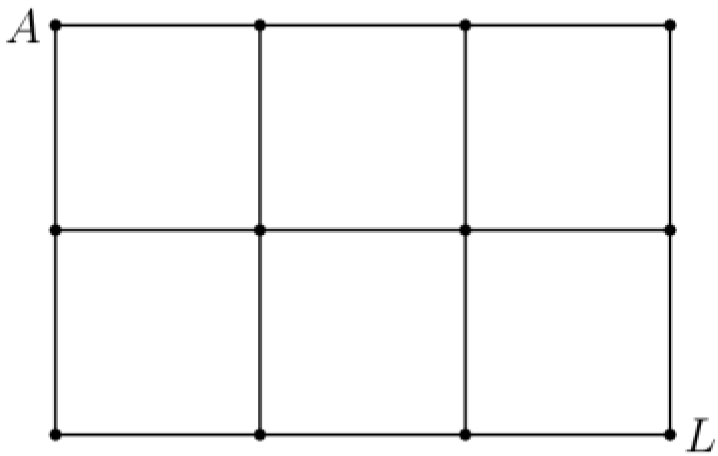
有多少个正整数值 x 使得可以组成一个面积为正的三角形，其三边长分别为 $\log_2 x, \log_4 x$ 和 3？

(A) 57 **(B)** 59 **(C)** 61 **(D)** 62 **(E)** 63

Problem 10

The figure below is a map showing 12 cities and 17 roads connecting certain pairs of cities. Paula wishes to travel along exactly 13 of those roads, starting at city A and ending at city L, without traveling along any portion of a road more than once. (Paula is allowed to visit a city more than once.) How many different routes can Paula take?

下图是一个包括 12 个城市和连接城市对的 17 条道路的地图。 Paula 希望游历其中的 13 条道路，从 A 城市出发，到 L 城市结束，每条路最多经过一次。（Paula 允许经过一个城市多于一次。）Paula 可以有多少条不同的路线？



(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 11

How many unordered pairs of edges of a given cube determine a plane?

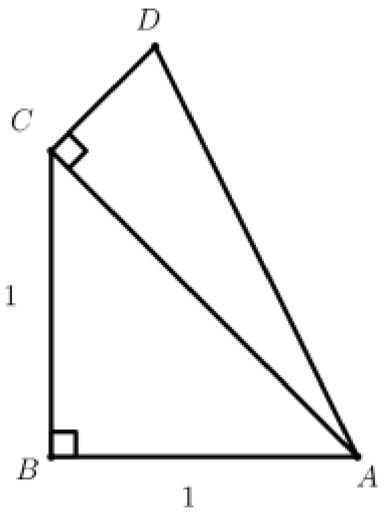
给定立方体中可以决定一个平面的两条棱的无序对共有多少个？

(A) 12 **(B)** 28 **(C)** 36 **(D)** 42 **(E)** 66

Problem 12

Right triangle ACD with right angle at C is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle ABC with leg length 1, as shown, so that the two triangles have equal perimeters. What is $\sin(2\angle BAD)$?

如果所示，沿边长为 1 的等腰直角三角形 *ABC* 的斜边 *AC* 向外做以 *C* 为直角顶点的直角三角形 *ACD*，使得两个直角三角形具有相同的周长。问 $\sin(2\angle BAD)$ 是多少？



(A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{7}{9}$ **(E)** $\frac{\sqrt{3}}{2}$

Problem 13

A red ball and a green ball are randomly and independently tossed into bins numbered with positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

一个红色球和一个绿色球被随机和独立地扔进用正整数编号的筐里，对于每个球来说，扔进编号为 k 的筐里的概率是 2^{-k}，$k=1,2,3,\ldots$。红色球被扔进比绿色球编号更高的筐里的概率是多少？

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{3}{7}$

Problem 14

Let S be the set of all positive integer divisors of $100,000.$ How many numbers are the product of two distinct elements of S?

设 S 是 100, 000 的所有正约数构成的集合。问有多少数是 S 中两个不同元素的乘积？

$\textbf{(A) }98\qquad\textbf{(B) }100\qquad\textbf{(C) }117\qquad\textbf{(D) }119\qquad\textbf{(E) }121$

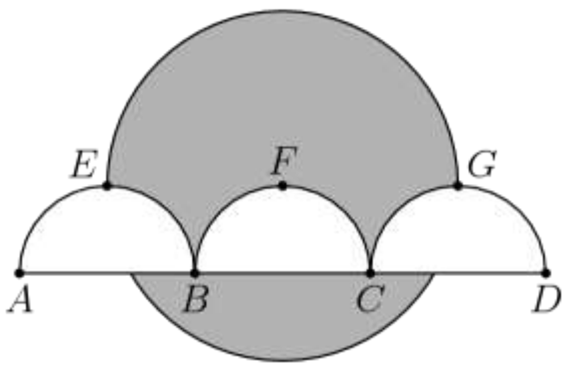
Problem 15

As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB=BC=CD=2.$ Three semicircles of radius $1,$ $\overarc{AEB},\overarc{BFC},$ and $\overarc{CGD},$ have their diameters on $\overline{AD},$ and are tangent to line EG at $E,F,$ and $G,$ respectively. A circle of radius 2 has its center on $F.$ The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form $\frac{a}{b}\cdot\pi-\sqrt{c}+d,$ where $a,b,c,$ and d are positive integers and a and b are relatively prime. What is $a+b+c+d$?

如图所示，线段 $\overline{AD}$ 被点 B 和 C 三等分，使得 $AB = BC = CD = 2$。三个半径为 1 的半圆 $\overarc{AEB},\overarc{BFC},$和$\overarc{CGD},$的直径都在 AD 上，位于由直线$\overline{AD}$确定的同一平面内，分别与直线 EG 相切于 E，F 和 G。半径为 2 的圆的圆心在 F。在圆内但在三个半圆外的区域面积，在图中用阴影表示，其面积可以用如下形式表示

$$\frac{a}{b}\cdot\pi-\sqrt{c}+d$$

其中 a、b、c 和 d 是正整数，并且 a 和 b 是互质的。问 $a+b+c+d$ 是多少？



$\textbf{(A) }13\qquad\textbf{(B) }14\qquad\textbf{(C) }15\qquad\textbf{(D) }16\qquad\textbf{(E) }17$

Problem 16

There are lily pads in a row numbered 0 to 11, in that order. There are predators on lily pads 3 and 6, and a morsel of food on lily pad 10. Fiona the frog starts on pad 0, and from any given lily pad, has a $\frac{1}{2}$ chance to hop to the next pad, and an equal chance to jump 2 pads. What is the probability that Fiona reaches pad 10 without landing on either pad 3 or pad 6?

荷叶从 0 到 11 编号在池塘中排成一行。青蛙 Fiona 坐在 0 号荷叶上，食物在 10 号荷叶上，而青蛙的天敌在 3 号和 6 号荷叶上。每个单位时间，青蛙跳向下一个编号更大的荷叶，或者这片荷叶的下一片，概率各为$\frac{1}{2}$，并且独立于之前的跳跃。那么 Fiona 躲过 3 号和 6 号荷叶，并且跳到 10 号荷叶的概率是多少？

$\textbf{(A) } \frac{15}{256} \qquad \textbf{(B) } \frac{1}{16} \qquad \textbf{(C) } \frac{15}{128} \qquad \textbf{(D) } \frac{1}{8} \qquad \textbf{(E) } \frac{1}{4}$

Problem 17

How many nonzero complex numbers z have the property that $0, z,$ and $z^3,$ when represented by points in the complex plane, are the three distinct vertices of an equilateral triangle?

有多少个非零复数 z 具有性质：当把 0， z 和 z^3 表示成复平面上的点时，他们构成一个等边三角形的三个不同顶点？

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } \text{infinitely many}$

Problem 18

Square pyramid $ABCDE$ has base $ABCD$, which measures 3 cm on a side, and altitude $\overline{AE}$ perpendicular to the base, which measures 6 cm. Point P lies on $\overline{BE}$, one third of the way from B to E; point Q lies on $\overline{DE}$, one third of the way from D to E; and point R lies on $\overline{CE}$, two thirds of the way from C to E. What is the area, in square centimeters, of $\triangle PQR$?

四棱锥 *ABCDE* 的底面是正方形 *ABCD*， 每条边长 3 cm， 高$\overline{AE}$垂直于底面， 长度是 6 cm。点 *P* 位于$\overline{BE}$,上，是从 *B* 到 *E* 的第一个三等分点；点 *Q* 位于$\overline{DE}$,上，是从 *D* 到 *E* 的第一个三等分点；并且 *R* 位于$\overline{CE}$,上，从 *C* 到 *E* 的第二个三等分点。问$\triangle PQR$?的面积是多少平方厘米？

$\textbf{(A) } \frac{3\sqrt{2}}{2} \qquad \textbf{(B) } \frac{3\sqrt{3}}{2} \qquad \textbf{(C) } 2\sqrt{2} \qquad \textbf{(D) } 2\sqrt{3} \qquad \textbf{(E) } 3\sqrt{2}$

Problem 19

Raashan, Sylvia, and Ted play the following game. Each starts with $\$1$. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives $\$1$ to that player. What is the probability that after the bell has rung 2019 times, each player will have $\$1$? (For example, Raashan and Ted may each decide to give $\$1$ to Sylvia, and Sylvia may decide to give her her dollar to Ted, at which point Raashan will have $\$0$, Sylvia will have $\$2$, and Ted will have $\$1$, and that is the end of the first round of play. In the second round Rashaan has no money to give, but Sylvia and Ted might choose each other to give their $\$1$ to, and the holdings will be the same at the end of the second round.)

Raashan、Sylvia 和 Ted 玩下面的游戏。 每个人从 1 美元开始。 闹钟每 15 秒钟响铃一次，此时每个有钱的玩家同时独立随机地选择另外两个玩家中的一人，并给那个玩家 1 美元。铃声响了 2019 次后，每个玩家都有 1 美元的概率是多少？ （例如，Raashan 和 Ted 可能会各自决定给 Silvia 1 美元，Silvia 可能会决定给 Ted 1 美元，这时 Raashan 会有 0 美元，Silvia 会有 2 美元，Ted 会有 1 美元，这就是游戏第一轮结束时的状态。在第二轮，Raashan 没有钱，但是 Sylvia 和 Ted 可以选择互相给对方 1 美元，在游戏第二轮结束时，他们的财产状况和之前一样。）

$\textbf{(A) } \frac{1}{7} \qquad \textbf{(B) } \frac{1}{4} \qquad \textbf{(C) } \frac{1}{3} \qquad \textbf{(D) } \frac{1}{2} \qquad \textbf{(E) } \frac{2}{3}$

Problem 20

Points $A(6,13)$ and $B(12,11)$ lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

点 $A(6,\ 13)$ 和 $B(12,\ 11)$ 位于平面内的圆上。假设在 A 和 B 处的切线相交于 x 轴上的一点。那么 ω 的面积是多少？

(A) $\frac{83\pi}{8}$ **(B)** $\frac{21\pi}{2}$ **(C)** $\frac{85\pi}{8}$ **(D)** $\frac{43\pi}{4}$ **(E)** $\frac{87\pi}{8}$

Problem 21

How many quadratic polynomials with real coefficients are there such that the set of roots equals the set of coefficients? (For clarification: If the polynomial is $ax^2+bx+c, a\neq 0,$ and the roots are r and s, then the requirement is that $\{a,b,c\}=\{r,s\}$.)

有多少个实系数二次多项式使得它的根构成的集合与它的系数构成的集合相同？（明确的说，如果多项式是 $ax^2+bx+c, a\neq 0,$，并且它的根是 r 和 s，那么要求是 $\{a,b,c\}=\{r,s\}$。）

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** infinitely many

Problem 22

Define a sequence recursively by $x_0=5$ and $x_{n+1}=\frac{x_n^2+5x_n+4}{x_n+6}$ for all nonnegative integers n. Let m be the least positive integer such that $x_m\leq 4+\frac{1}{2^{20}}$. In which of the following intervals does m lie?

定义递归数列如下：$x_0=5$，并且对于所有非负整数 n，$x_{n+1}=\frac{x_n^2+5x_n+4}{x_n+6}$

设 m 是最小的正整数，使得 $x_m\leq 4+\frac{1}{2^{20}}$。那么 m 在下述哪个区间中？

(A) $[9,26]$ **(B)** $[27,80]$ **(C)** $[81,242]$ **(D)** $[243,728]$ **(E)** $[729,\infty]$

Problem 23

How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

有多少个长度为 19 的由 0 和 1 组成的序列，以 0 开头，以 0 结尾，不包含连续两个 0，也不包含连续三个 1？

$\textbf{(A) } 55 \qquad \textbf{(B) } 60 \qquad \textbf{(C) } 65 \qquad \textbf{(D) } 70 \qquad \textbf{(E) } 75$

Problem 24

Let $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}.$ Let S denote all points in the complex plane of the form $a + b\omega + c\omega^2$, where $0 \leq a \leq 1, 0 \leq b \leq 1,$ and $0 \leq c \leq 1.$ What is the area of S?

设 $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$，用 S 表示所有复平面上具有形式 $a + b\omega + c\omega^2$ 的点集，其中 $0 \leq a \leq 1$，$0 \leq b \leq 1$，$0 \leq c \leq 1$。问 S 的面积是多少?

$\textbf{(A) } \frac{1}{2}\sqrt{3} \qquad \textbf{(B) } \frac{3}{4}\sqrt{3} \qquad \textbf{(C) } \frac{3}{2}\sqrt{3} \qquad \textbf{(D) } \frac{1}{2}\pi\sqrt{3} \qquad \textbf{(E) } \pi$

Problem 25

Let $ABCD$ be a convex quadrilateral with $BC = 2$ and $CD = 6.$ Suppose that the centroids of $\triangle ABC, \triangle BCD,$ and $\triangle ACD$ form the vertices of an equilateral triangle. What is the maximum possible value of the area of $ABCD$?

设 $ABCD$ 是一个凸四边形，$BC = 2$，并且 $CD = 6$。假设$\triangle ABC, \triangle BCD,$和$\triangle ACD$的重心组成一个等边三角形。问 $ABCD$ 最大可能的面积是多少?

$\textbf{(A) } 27 \qquad \textbf{(B) } 16\sqrt{3} \qquad \textbf{(C) } 12 + 10\sqrt{3} \qquad \textbf{(D) } 9 + 12\sqrt{3} \qquad \textbf{(E) } 30$

2019 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	E	E	C	B	A	A	A	B	E	D	D	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	E	A	D	C	B	C	B	C	C	C	C	