2002 AMC 12A Problems/Problem 1

The following problem is from both the 2002 AMC 12A #1 and 2002 AMC 10A #10, so both problems redirect to this page.

Contents

Problem

Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$

$\mathrm{(A) \ } \frac{7}{2}\qquad \mathrm{(B) \ } 4\qquad \mathrm{(C) \ } 5\qquad \mathrm{(D) \ } 7\qquad \mathrm{(E) \ } 13$

Solution

Solution 1

We expand to get $2x^2-8x+3x-12+2x^2-12x+3x-18=0$ which is $4x^2-14x-30=0$ after combining like terms. Using the quadratic part of Vieta's Formulas, we find the sum of the roots is $\frac{14}{4}=\boxed{\text{(A) }7/2}$.

Solution 2

Combine terms to get $(2x+3)\cdot\Big((x-4)+(x-6)\Big)=(2x+3)(2x-10)=0$, hence the roots are $-\frac{3}{2}$ and 5, thus our answer is $-\frac{3}{2}+5=\boxed{\text{(A) }7/2}$.

See also

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2002 AMC 12A Problems/Problem 2

The following problem is from both the 2002 AMC 12A #2 and 2002 AMC 10A #6, so both problems redirect to this page.

Problem

Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

$\text{(A) } 15 \qquad \text{(B) } 34 \qquad \text{(C) } 43 \qquad \text{(D) } 51 \qquad \text{(E) } 138$

Solution

We work backwards; the number that Cindy started with is $3(43)+9=138$. Now, the correct result is $\frac{138-3}{9}=\frac{135}{9}=15$. Our answer is $\boxed{\text{(A) } 15}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
Preceded by Problem 1	Followed by Problem 3
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2002 AMC 12A Problems/Problem 3

> The following problem is from both the 2002 AMC 12A #3 and 2002 AMC 10A #3, so both problems redirect to this page.

Problem

According to the standard convention for exponentiation,

$$2^{2^{2^{2}}} = 2^{(2^{(2^2)})} = 2^{16} = 65536.$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

$\mathrm{(A) \ } 0\qquad \mathrm{(B) \ } 1\qquad \mathrm{(C) \ } 2\qquad \mathrm{(D) \ } 3\qquad \mathrm{(E) \ } 4$

Solution

The best way to solve this problem is by simple brute force.

It is convenient to drop the usual way how exponentiation is denoted, and to write the formula as $2\uparrow 2\uparrow 2\uparrow 2$, where $\uparrow$ denotes exponentiation. We are now examining all ways to add parentheses to this expression. There are 5 ways to do so:

1. $2\uparrow (2\uparrow (2\uparrow 2))$
2. $2\uparrow ((2\uparrow 2)\uparrow 2)$
3. $((2\uparrow 2)\uparrow 2)\uparrow 2$
4. $(2\uparrow (2\uparrow 2))\uparrow 2$
5. $(2\uparrow 2)\uparrow (2\uparrow 2)$

We can note that $2\uparrow (2\uparrow 2) = (2\uparrow 2)\uparrow 2 = 16$. Therefore options 1 and 2 are equal, and options 3 and 4 are equal. Option 1 is the one given in the problem statement. Thus we only need to evaluate options 3 and 5.

$((2\uparrow 2)\uparrow 2)\uparrow 2 = 16\uparrow 2 = 256$

$(2\uparrow 2)\uparrow (2\uparrow 2) = 4\uparrow 4 = 256$

Thus the only other result is 256, and our answer is $\boxed{\mathrm{(B)}\ 1}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 4

Problem

Find the degree measure of an angle whose complement is 25% of its supplement.

$\mathrm{(A)}\ 48 \qquad \mathrm{(B)}\ 60 \qquad \mathrm{(C)}\ 75 \qquad \mathrm{(D)}\ 120 \qquad \mathrm{(E)}\ 150$

Solution

We can create an equation for the question, $4(90-x)=(180-x)$

$360-4x=180-x$

$3x=180$

After simplifying, we get $x=60 \Rightarrow \mathrm{(B)}$

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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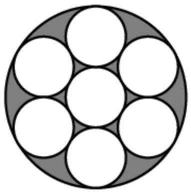
2002 AMC 12A Problems/Problem 5

The following problem is from both the 2002 AMC 12A #5 and 2002 AMC 10A #5, so both problems redirect to this page.

Problem

Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



$\mathrm{(A)}\ \pi \qquad \mathrm{(B)}\ 1.5\pi \qquad \mathrm{(C)}\ 2\pi \qquad \mathrm{(D)}\ 3\pi \qquad \mathrm{(E)}\ 3.5\pi$

Solution

The outer circle has radius $1+1+1=3$, and thus area 9π. The little circles have area π each; since there are 7, their total area is 7π. Thus, our answer is $9\pi-7\pi=\boxed{2\pi\Rightarrow\text{(C)}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 6

The following problem is from both the 2002 AMC 12A #6 and 2002 AMC 10A #4, so both problems redirect to this page.

Contents

Problem

For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \le m + n$?

$\textbf{(A) } 4\qquad \textbf{(B) } 6\qquad \textbf{(C) } 9\qquad \textbf{(D) } 12\qquad \textbf{(E) }$ infinitely many

Solution

Solution 1

For any m we can pick $n = 1$, we get $m \cdot 1 \le m + 1$, therefore the answer is $\boxed{\text{(E) infinitely many}}$.

Solution 2

Another solution, slightly similar to this first one would be using Simon's Favorite Factoring Trick.

$$(m-1)(n-1) \le 1$$

Let $n = 1$, then

$$0 \le 1$$

This means that there are infinitely many numbers m that can satisfy the inequality. So the answer is $\boxed{\text{(E) infinitely many}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 7

The following problem is from both the 2002 AMC 12A #7 and 2002 AMC 10A #7, so both problems redirect to this page.

Problem

A 45° arc of circle A is equal in length to a 30° arc of circle B. What is the ratio of circle A's area and circle B's area?

$\mathrm{(A)}\ 4/9 \qquad \mathrm{(B)}\ 2/3 \qquad \mathrm{(C)}\ 5/6 \qquad \mathrm{(D)}\ 3/2 \qquad \mathrm{(E)}\ 9/4$

Solution

Let r_1 and r_2 be the radii of circles A and B, respectively.

It is well known that in a circle with radius r, a subtended arc opposite an angle of θ degrees has length $\frac{\theta}{360} \cdot 2\pi r$.

Using that here, the arc of circle A has length $\frac{45}{360}\cdot2\pi r_1=\frac{r_1\pi}{4}$. The arc of circle B has length $\frac{30}{360}\cdot2\pi r_2=\frac{r_2\pi}{6}$. We know that they are equal, so $\frac{r_1\pi}{4}=\frac{r_2\pi}{6}$, so we multiply through and simplify to get $\frac{r_1}{r_2}=\frac{2}{3}$. As all circles are similar to one another, the ratio of the areas is just the square of the ratios of the radii, so our answer is $\boxed{\mathrm{(A)}\ 4/9}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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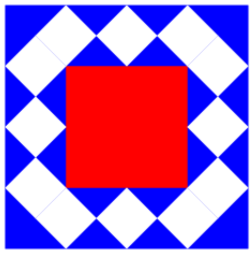
2002 AMC 12A Problems/Problem 8

The following problem is from both the 2002 AMC 12A #8 and 2002 AMC 10A #8, so both problems redirect to this page.

Problem

Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



$\text{(A)}\ B = W \qquad \text{(B)}\ W = R \qquad \text{(C)}\ B = R \qquad \text{(D)}\ 3B = 2R \qquad \text{(E)}\ 2R = W$

Solution

The blue that's touching the center red square makes up 8 triangles, or 4 squares. Each of the corners is 2 squares and each of the edges is 1, totaling 12 squares. There are 12 white squares, thus we have $\boxed{B=W \Rightarrow \text{(A)}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 9

The following problem is from both the 2002 AMC 12A #9 and 2002 AMC 10A #11, so both problems redirect to this page.

Problem

Jamal wants to save 30 files onto disks, each with 1.44 MB space. 3 of the files take up 0.8 MB, 12 of the files take up 0.7 MB, and the rest take up 0.4 MB. It is not possible to split a file onto 2 different disks. What is the smallest number of disks needed to store all 30 files?

$\text{(A)}\ 12 \qquad \text{(B)}\ 13 \qquad \text{(C)}\ 14 \qquad \text{(D)}\ 15 \qquad \text{(E)} 16$

Solution

A 0.8 MB file can either be on its own disk, or share it with a 0.4 MB. Clearly it is better to pick the second possibility. Thus we will have 3 disks, each with one 0.8 MB file and one 0.4 MB file.

We are left with 12 files of 0.7 MB each, and 12 files of 0.4 MB each. Their total size is $12\cdot (0.7 + 0.4) = 13.2$ MB. The total capacity of 9 disks is $9\cdot 1.44 = 12.96$ MB, hence we need at least 10 more disks. And we can easily verify that 10 disks are indeed enough: six of them will carry two 0.7 MB files each, and four will carry three 0.4 MB files each.

Thus our answer is $3+10=\boxed{\text{(B)}\ 13}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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Category: Intermediate Algebra Problems

2002 AMC 12A Problems/Problem 10

The following problem is from both the 2002 AMC 12A #10 and 2002 AMC 10A #17, so both problems redirect to this page.

Problem

Sarah places four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then pours half the coffee from the first cup to the second and, after stirring thoroughly, pours half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

$\text{(A)}\ \frac{1}{4}\qquad\text{(B)}\ \frac{1}{3}\qquad\text{(C)}\ \frac{3}{8}\qquad\text{(D)}\ \frac{2}{5}\qquad\text{(E)}\ \frac{1}{2}$

Solution

We will simulate the process in steps.

In the beginning, we have:

- 4 ounces of coffee in cup 1
- 4 ounces of cream in cup 2

In the first step we pour $4/2=2$ ounces of coffee from cup 1 to cup 2, getting:

- 2 ounces of coffee in cup 1
- 2 ounces of coffee and 4 ounces of cream in cup 2

In the second step we pour $2/2=1$ ounce of coffee and $4/2=2$ ounces of cream from cup 2 to cup 1, getting:

- $2+1=3$ ounces of coffee and $0+2=2$ ounces of cream in cup 1
- the rest in cup 2

Hence at the end we have $3+2=5$ ounces of liquid in cup 1, and out of these 2 ounces is cream. Thus the answer is $\boxed{\text{(D)}\ \frac{2}{5}}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 11

The following problem is from both the 2002 AMC 12A #11 and 2002 AMC 10A #12, so both problems redirect to this page.

Contents

Problem

Mr. Earl E. Bird gets up every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

$\text{(A)}\ 45 \qquad \text{(B)}\ 48 \qquad \text{(C)}\ 50 \qquad \text{(D)}\ 55 \qquad \text{(E)} 58$

Solution

Solution 1

Let the time he needs to get there in be t and the distance he travels be d. From the given equations, we know that $d=\left(t+\frac{1}{20}\right)40$ and $d=\left(t-\frac{1}{20}\right)60$. Setting the two equal, we have $40t+2=60t-3$ and we find $t=\frac{1}{4}$ of an hour. Substituting t back in, we find $d=12$. From $d=rt$, we find that r, and our answer, is $\boxed{\text{(B)}\ 48}$.

Solution 2

Since either time he arrives at is 3 minutes from the desired time, the answer is merely the harmonic mean of 40 and 60. The harmonic mean of a and b is $\frac{2}{\frac{1}{a}+\frac{1}{b}}=\frac{2ab}{a+b}$. In this case, a and b are 40 and 60, so our answer is $\frac{4800}{100}=48$, so $\boxed{\text{(B)}\ 48}$.

Solution 3

A more general form of the argument in Solution 2, with proof:

Let d be the distance to work, and let s be the correct average speed. Then the time needed to get to work is $t=\frac{d}{s}$.

We know that $t+\frac{3}{60}=\frac{d}{40}$ and $t-\frac{3}{60}=\frac{d}{60}$. Summing these two equations, we get: $2t=\frac{d}{40}+\frac{d}{60}$.

Substituting $t=\frac{d}{s}$ and dividing both sides by d, we get $\frac{2}{s}=\frac{1}{40}+\frac{1}{60}$, hence $s=\boxed{48}$.

(Note that this approach would work even if the time by which he is late was different from the time by which he is early in the other case - we would simply take a weighted sum in step two, and hence obtain a weighted harmonic mean in step three.)

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 12

The following problem is from both the 2002 AMC 12A #12 and 2002 AMC 10A #14, so both problems redirect to this page.

Problem

Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

$\text{(A) }0 \qquad \text{(B) }1 \qquad \text{(C) }2 \qquad \text{(D) }4 \qquad \text{(E) more than 4}$

Solution

Consider a general quadratic with the coefficient of x^2 being 1 and the roots being r and s. It can be factored as $(x-r)(x-s)$ which is just $x^2-(r+s)x+rs$. Thus, the sum of the roots is the negative of the coefficient of x and the product is the constant term. (In general, this leads to Vieta's Formulas).

We now have that the sum of the two roots is 63 while the product is k. Since both roots are primes, one must be 2, otherwise the sum would be even. That means the other root is 61 and the product must be 122. Hence, our answer is $\boxed{\text{(B) }1}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 13

Problem

Two different positive numbers a and b each differ from their reciprocals by 1. What is $a+b$?

$\text{(A) }1 \qquad \text{(B) }2 \qquad \text{(C) }\sqrt 5 \qquad \text{(D) }\sqrt 6 \qquad \text{(E) }3$

Solution

Each of the numbers a and b is a solution to $\left| x - \frac 1x \right| = 1$.

Hence it is either a solution to $x - \frac 1x = 1$, or to $\frac 1x - x = 1$. Then it must be a solution either to $x^2 - x - 1 = 0$, or to $x^2 + x - 1 = 0$.

There are in total four such values of x, namely $\frac{\pm 1 \pm \sqrt 5}2$.

Out of these, two are positive: $\frac{-1+\sqrt 5}2$ and $\frac{1+\sqrt 5}2$. We can easily check that both of them indeed have the required property, and their sum is $\frac{-1+\sqrt 5}2 + \frac{1+\sqrt 5}2 = \boxed{(C) \sqrt 5}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 14

Problem

For all positive integers n, let $f(n)=\log_{2002} n^2$. Let $N=f(11)+f(13)+f(14)$. Which of the following relations is true?

$\text{(A)}\ N<1 \qquad\text{(B)}\ N=1 \qquad\text{(C)}\ 1<N<2 \qquad\text{(D)}\ N=2 \qquad\text{(E)}\ N>2$

Solution

First, note that $2002 = 11 \cdot 13 \cdot 14$.

Using the fact that for any base we have $\log a + \log b = \log ab$, we get that $N=\log_{2002} (11^2 \cdot 13^2 \cdot 14^2) = \log_{2002} 2002^2 = \boxed{(D) N=2}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 15

The following problem is from both the 2002 AMC 12A #15 and 2002 AMC 10A #21, so both problems redirect to this page.

Contents

Problem

The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

$\text{(A) } 11 \qquad \text{(B) } 12 \qquad \text{(C) } 13 \qquad \text{(D) } 14 \qquad \text{(E) } 15$

Solution

As the unique mode is 8, there are at least two 8s.

As the range is 8 and one of the numbers is 8, the largest one can be at most 16.

If the largest one is 16, then the smallest one is 8, and thus the mean is strictly larger than 8, which is a contradiction.

If the largest one is 15, then the smallest one is 7. This means that we already know four of the values: 8, 8, 7, 15. Since the mean of all the numbers is 8, their sum must be 64. Thus the sum of the missing four numbers is $64-8-8-7-15=26$. But if 7 is the smallest number, then the sum of the missing numbers must be at least $4\cdot 7=28$, which is again a contradiction.

If the largest number is 14, we can easily find the solution $(6,6,6,8,8,8,8,14)$. Hence, our answer is $\boxed{\text{(D) } 14}$.

Note

The solution for 14 is, in fact, unique. As the median must be 8, this means that both the 4^{th} and the 5^{th} number, when ordered by size, must be 8s. This gives the partial solution $(6,a,b,8,8,c,d,14)$. For the mean to be 8 each missing variable must be replaced by the smallest allowed value.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 16

The following problem is from both the 2002 AMC 12A #16 and 2002 AMC 10A #24, so both problems redirect to this page.

Contents

Problem

Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, ..., 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

$\text{(A)}\ 2/5 \qquad \text{(B)}\ 9/20 \qquad \text{(C)}\ 1/2 \qquad \text{(D)}\ 11/20 \qquad \text{(E)}\ 24/25$

Solution

Solution 1

This is not too bad using casework.

Tina gets a sum of 3: This happens in only one way $(1,2)$ and Sergio can choose a number from 4 to 10, inclusive. There are 7 ways that Sergio gets a desirable number here.

Tina gets a sum of 4: This once again happens in only one way $(1,3)$. Sergio can choose a number from 5 to 10, so 6 ways here.

Tina gets a sum of 5: This can happen in two ways $(1,4)$ and $(2,3)$. Sergio can choose a number from 6 to 10, so $2\cdot 5=10$ ways here.

Tina gets a sum of 6: Two ways here $(1,5)$ and $(2,4)$. Sergio can choose a number from 7 to 10, so $2*4=8$ here.

Tina gets a sum of 7: Two ways here $(2,5)$ and $(3,4)$. Sergio can choose from 8 to 10, so $2\cdot 3=6$ ways here.

Tina gets a sum of 8: Only one way possible $(3,5)$. Sergio chooses 9 or 10, so 2 ways here.

Tina gets a sum of 9: Only one way $(4,5)$. Sergio must choose 10, so 1 way.

In all, there are $7+6+10+8+6+2+1=40$ ways. Tina chooses two distinct numbers in $\binom{5}{2}=10$ ways while Sergio chooses a number in 10 ways, so there are $10\cdot 10=100$ ways in all.

Since $\frac{40}{100}=\frac{2}{5}$, our answer is $\boxed{(\text{A})\frac{2}{5}}$.

Solution 2

We want to find the average of the smallest possible chance of Sergio winning and the largest possible chance of Sergio winning. This is because the probability decreases linearly. The largest possibility of Sergio winning if Tina chooses a 1 and a 2. The chances of Sergio winning is then $\frac{7}{10}$. The smallest possibility of Sergio winning is if Tina chooses a 4 and a 5. The chances of Sergio winning then is $\frac{1}{10}$. The average of $\frac{7}{10}$ and $\frac{1}{10}$ is $\boxed{(\text{A})\frac{2}{5}}$.

Solution 3

The expected value of a number randomly selected form the set $\{1, 2, 3, 4, 5\}$ is 3. Therefore, Tina's expected sum is $3 + 3 = 6$. The probability that Sergio selects a number larger than 6 from his set is $\boxed{\frac{2}{5}}$. This works because of symmetry.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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All AMC 10 Problems and Solutions	

Categories: Introductory Probability Problems | Introductory Combinatorics Problems

2002 AMC 12A Problems/Problem 17

Problem

Several sets of prime numbers, such as $\{7,83,421,659\}$ use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

$\text{(A) } 193 \qquad \text{(B) } 207 \qquad \text{(C) } 225 \qquad \text{(D) } 252 \qquad \text{(E) } 447$

Solution

Neither of the digits 4, 6, and 8 can be a units digit of a prime. Therefore the sum of the set is at least $40 + 60 + 80 + 1 + 2 + 3 + 5 + 7 + 9 = 207$.

We can indeed create a set of primes with this sum, for example the following set works: $\{41, 67, 89, 2, 3, 5\}$.

Thus the answer is $\boxed{(B) 207}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 18

Contents

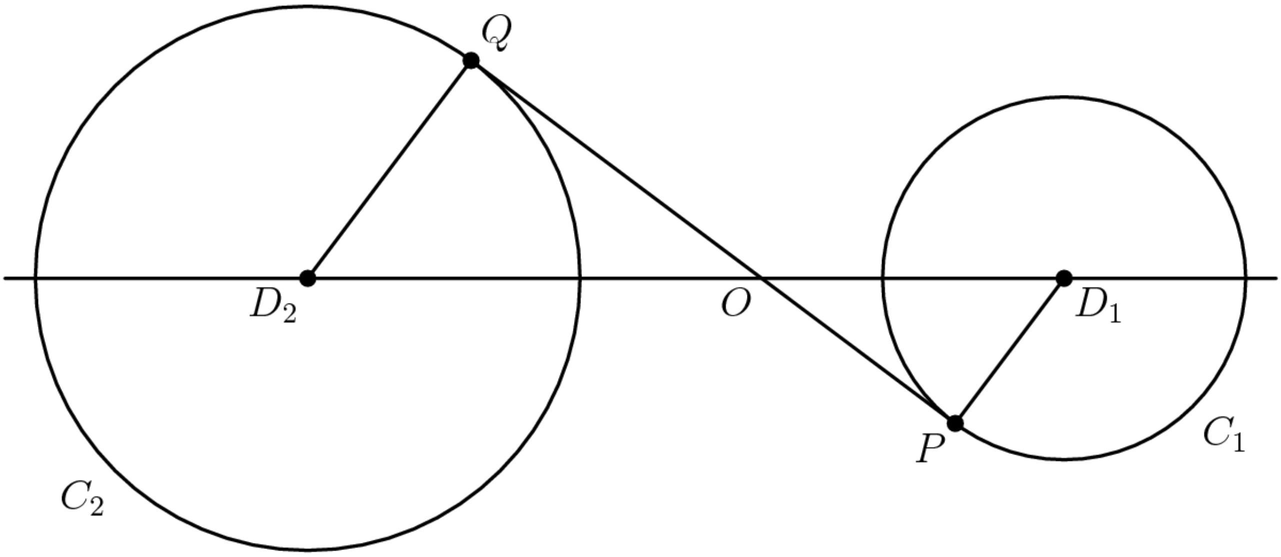
Problem

Let C_1 and C_2 be circles defined by $(x-10)^2+y^2=36$ and $(x+15)^2+y^2=81$ respectively. What is the length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q?

$\text{(A) } 15 \qquad \text{(B) } 18 \qquad \text{(C) } 20 \qquad \text{(D) } 21 \qquad \text{(E) } 24$

Solution 1

(C) First examine the formula $(x-10)^2+y^2=36$, for the circle C_1. Its center, D_1, is located at (10,0) and it has a radius of $\sqrt{36}$ = 6. The next circle, using the same pattern, has its center, D_2, at (-15,0) and has a radius of $\sqrt{81}$ = 9. So we can construct this diagram:



Line PQ is tangent to both circles, so it forms a right angle with the radii (6 and 9). This, as well as the two vertical angles near O, prove triangles S_2QO and S_1PO similar by AA, with a scale factor of 6:9, or 2:3. Next, we must subdivide the line D_2D_1 in a 2:3 ratio to get the length of the segments D_2O and D_1O. The total length is 10 − (-15), or 25, so applying the ratio, D_2O = 15 and D_1O = 10. These are the hypotenuses of the triangles. We already know the length of D_2Q and D_1P, 9 and 6 (they're radii). So in order to find PQ, we must find the length of the longer legs of the two triangles and add them.

$15^2-9^2=(15-9)(15+9)=6\times 24=144$

$\sqrt{144}=12$

$10^2-6^2=(10-6)(10+6)=4\times 16=64$

$\sqrt{64}=8$

Finally, the length of PQ is $12+8=\boxed{20}$, or C.

Solution 2

Using the above diagram, imagine that segment $\overline{QS_2}$ is shifted to the right to match up with $\overline{PS_1}$. Then shift $\overline{QP}$ downwards to make a right triangle. We know $\overline{S_2S_1} = 25$ from the given information and the newly created leg has length $\overline{QS_2} + \overline{PS_1} = 9 + 6 = 15$. Hence by Pythagorean theorem

$15^2 + \overline{QP}^2 = 25^2$.

$\overline{QP} = \boxed{20}$, or C.

See Also

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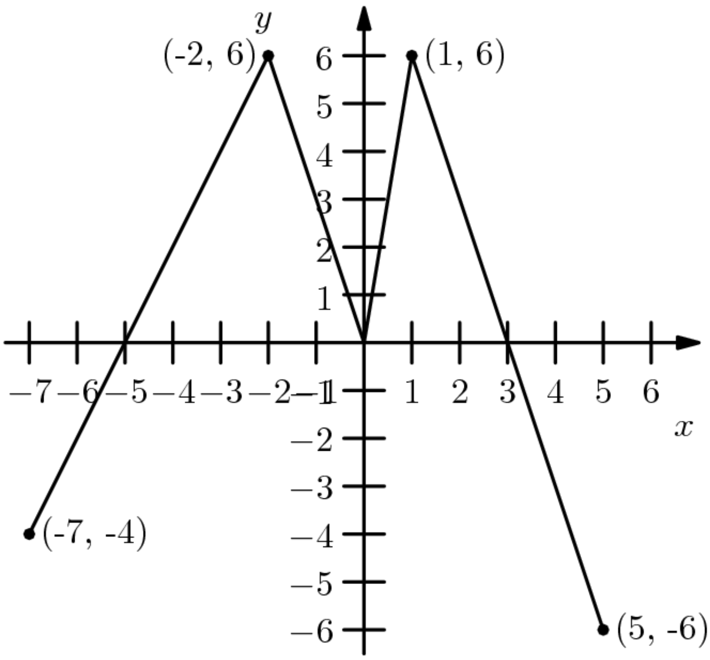
2002 AMC 12A Problems/Problem 19

Problem

The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 6$ have?



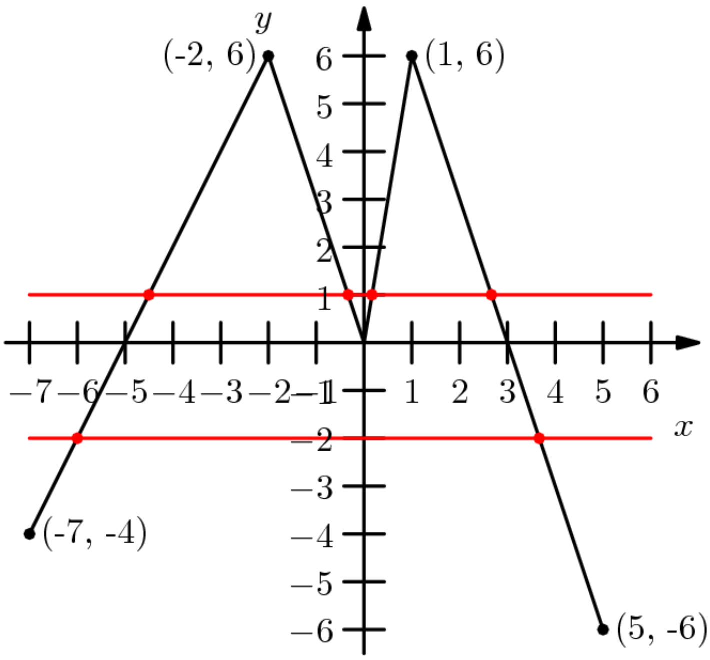
$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

First of all, note that the equation $f(t) = 6$ has two solutions: $t = -2$ and $t = 1$.

Given an x, let $f(x) = t$. Obviously, to have $f(f(x)) = 6$, we need to have $f(t) = 6$, and we already know when that happens. In other words, the solutions to $f(f(x)) = 6$ are precisely the solutions to ($f(x) = -2$ or $f(x) = 1$).

Without actually computing the exact values, it is obvious from the graph that the equation $f(x) = -2$ has two and $f(x) = 1$ has four different solutions, giving us a total of $2 + 4 = \boxed{(D) 6}$ solutions.



See Also

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Categories: Introductory Algebra Problems | Graphing Problems

2002 AMC 12A Problems/Problem 20

Problem

Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

$\text{(A) } 3 \qquad \text{(B) } 4 \qquad \text{(C) } 5 \qquad \text{(D) } 8 \qquad \text{(E) } 9$

Solution

The repeating decimal $0.\overline{ab}$ is equal to

$$\frac{10a+b}{100}+\frac{10a+b}{10000}+\cdots=(10a+b)\cdot\left(\frac{1}{10^2}+\frac{1}{10^4}+\cdots\right)=(10a+b)\cdot\frac{1}{99}=\frac{10a+b}{99}$$

When expressed in lowest terms, the denominator of this fraction will always be a divisor of the number $99=3\cdot3\cdot11$. This gives us the possibilities $\{1,3,9,11,33,99\}$. As a and b are not both nine and not both zero, the denominator 1 can not be achieved, leaving us with $\boxed{(C) 5}$ possible denominators.

(The other ones are achieved e.g. for ab equal to 33, 11, 9, 3, and 1, respectively.)

See Also

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2002 AMC 12A Problems/Problem 21

Problem

Consider the sequence of numbers: $4, 7, 1, 8, 9, 7, 6, \dots$ For $n > 2$, the n-th term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. The smallest value of n for which $S_n > 10,000$ is:

$\text{(A) } 1992 \qquad \text{(B) } 1999 \qquad \text{(C) } 2001 \qquad \text{(D) } 2002 \qquad \text{(E) } 2004$

Solution

The sequence is infinite. As there are only 100 pairs of digits, sooner or later a pair of consecutive digits will occur for the second time. As each next digit only depends on the previous two, from this point on the sequence will be periodic.

(Additionally, as every two consecutive digits uniquely determine the previous one as well, the first pair of digits that will occur twice must be the first pair $4, 7$.)

Hence it is a good idea to find the period. Writing down more terms of the sequence, we get:

$$4, 7, 1, 8, 9, 7, 6, 3, 9, 2, 1, 3, 4, 7, \dots$$

and we found the period. The length of the period is 12, and its sum is $4 + 7 + \cdots + 1 + 3 = 60$. Hence for each k we have $S_{12k} = 60k$.

We have $\lfloor 10000/60 \rfloor = 166$ and $166 \cdot 12 = 1992$, therefore $S_{1992} = 60 \cdot 166 = 9960$. The rest can now be computed by hand, we get $S_{1998} = 9960 + 4 + 7 + 1 + 8 + 9 + 7 = 9996$, and $S_{1999} = 9996 + 6 = 10002$, thus the answer is $\boxed{(B) 1999}$.

See Also

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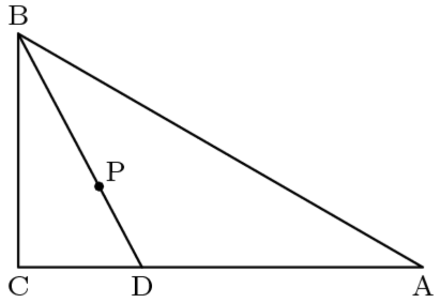
2002 AMC 12A Problems/Problem 22

Problem

Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60^\circ$, and $AB = 10$. Let P be randomly chosen inside ABC , and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?



$\textbf{(A)}\ \frac{2-\sqrt{2}}{2}\qquad\textbf{(B)}\ \frac{1}{3}\qquad\textbf{(C)}\ \frac{3-\sqrt{3}}{3}\qquad\textbf{(D)}\ \frac{1}{2}\qquad\textbf{(E)}\ \frac{5-\sqrt{5}}{5}$

Solution

Clearly $BC = 5$ and $AC = 5\sqrt{3}$. Choose a P' and get a corresponding D' such that $BD' = 5\sqrt{2}$ and $CD' = 5$. For $BD > 5\sqrt{2}$ we need $CD > 5$, creating an isoclese right triangle with hyptonuse $5\sqrt{2}$. Thus the point P may only lie in the triangle ABD'. The probability of it doing so is the ratio of areas of ABD' to ABC, or equivalently, the ratio of AD' to AC because the triangles have identical altitudes when taking AD' and AC as bases. This ratio is equal to $\frac{AC-CD'}{AC} = 1 - \frac{CD'}{AC} = 1 - \frac{5}{5\sqrt{3}} = 1 - \frac{\sqrt{3}}{3} = \frac{3-\sqrt{3}}{3}$. Thus the answer is $\boxed{C}$.

See Also

2002 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2002))	
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2002 AMC 12A Problems/Problem 23

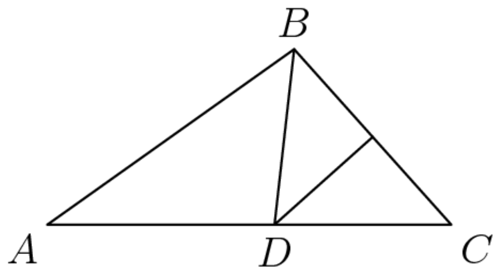
Problem

In triangle ABC , side AC and the perpendicular bisector of BC meet in point D, and BD bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

$\text{(A)}\ 14 \qquad \text{(B)}\ 21 \qquad \text{(C)}\ 28 \qquad \text{(D)}\ 14\sqrt{5} \qquad \text{(E)}\ 28\sqrt{5}$

Solution

Solution 1



Looking at the triangle BCD, we see that its perpendicular bisector reaches the vertex, therefore implying it is isosceles. Let $x = \angle C$, so that $B = 2x$ from given and the previous deducted. Then $\angle ABD = x, \angle ADB = 2x$ because any exterior angle of a triangle has a measure that is the sum of the two interior angles that are not adjacent to the exterior angle. That means $\triangle ABD$ and $\triangle ACB$ are similar, so $\frac{16}{AB} = \frac{AB}{9} \Longrightarrow AB = 12.$

Then by using Heron's Formula on ABD (with sides $12, 7, 9$), we have $[\triangle ABD] = \sqrt{14(2)(7)(5)} = 14\sqrt{5} \Longrightarrow \boxed{\text{D}}$.

Solution 2

Let M be the point of the perpendicular bisector on BC. By the perpendicular bisector theorem, $BD = DC = 7$ and $BM = MC$. Also, by the angle bisector theorem, $\frac{AB}{BC} = \frac{9}{7}$. Thus, let $AB = 9x$ and $BC = 7x$. In addition, $BM = 3.5x$.

Thus, $\cos \angle CBD = \frac{3.5x}{7} = \frac{x}{2}$. Additionally, using the Law of Cosines and the fact that $\angle CBD = \angle ABD$, $81 = 49 + 81x^2 - 2(9x)(7) \cos \angle CBD$

Substituting and simplifying, we get $x = 4/3$

Thus, $AB = 12$. We now know all sides of $\triangle ABD$. Using Heron's Formula on $\triangle ABD$, $\sqrt{(14)(2)(7)(5)} = 14\sqrt{5} \Longrightarrow \boxed{\text{D}}$

See Also

2002 AMC 12A Problems/Problem 24

Contents

Problem

Find the number of ordered pairs of real numbers (a,b) such that $(a+bi)^{2002}=a-bi$.

$\text{(A) }1001 \qquad \text{(B) }1002 \qquad \text{(C) }2001 \qquad \text{(D) }2002 \qquad \text{(E) }2004$

Solution

Let $s=\sqrt{a^2+b^2}$ be the magnitude of $a+bi$. Then the magnitude of $(a+bi)^{2002}$ is s^{2002}, while the magnitude of $a-bi$ is s. We get that $s^{2002}=s$, hence either $s=0$ or $s=1$.

For $s=0$ we get a single solution $(a,b)=(0,0)$.

Let's now assume that $s=1$. Multiply both sides by $a+bi$. The left hand side becomes $(a+bi)^{2003}$, the right hand side becomes $(a-bi)(a+bi)=a^2+b^2=1$. Hence the solutions for this case are precisely all the 2003rd complex roots of unity, and there are 2003 of those.

The total number of solutions is therefore $1+2003=\boxed{2004}$.

Solution 2

As in the other solution, split the problem into when $s=0$ and when $s=1$. When $s=1$ and $a+bi=\cos\theta+i\sin\theta$,

$$(a+bi)^{2002}=\cos(2002\theta)+i\sin(2002\theta)$$
$$=a-bi=\cos\theta-i\sin\theta=\cos(-\theta)+i\sin(-\theta)$$

so we must have $2002\theta=-\theta+2\pi k$ and hence $\theta=\frac{2\pi k}{2003}$. Since θ is restricted to $[0,2\pi)$, k can range from 0 to 2002 inclusive, which is $2002-0+1=2003$ values. Thus the total is $1+2003=\boxed{\textbf{(E) }2004}$.

See Also

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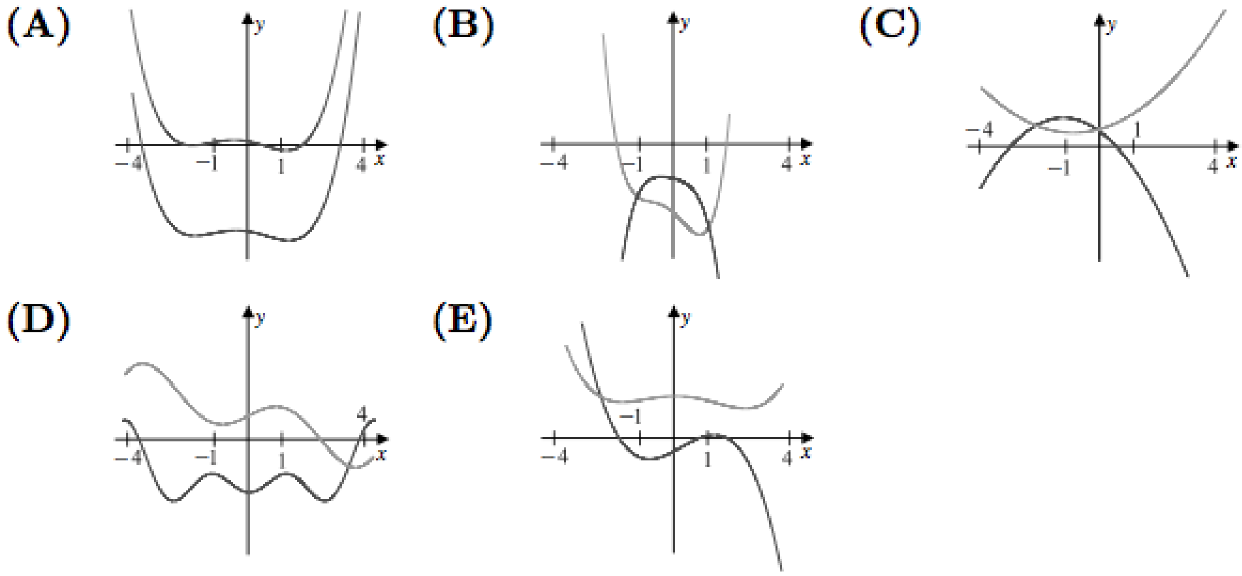
2002 AMC 12A Problems/Problem 25

Contents

Problem

The nonzero coefficients of a polynomial P with real coefficients are all replaced by their mean to form a polynomial Q. Which of the following could be a graph of $y = P(x)$ and $y = Q(x)$ over the interval $-4 \leq x \leq 4$?



Solution

The sum of the coefficients of P and of Q will be equal, so $P(1) = Q(1)$. The only answer choice with an intersection between the two graphs at $x = 1$ is (B). (The polynomials in the graph are $P(x) = 2x^4 - 3x^2 - 3x - 4$ and $Q(x) = -2x^4 - 2x^2 - 2x - 2$.)

Solution 2

We know every coefficient is equal, so we get $ax^n + ... + ax + a = 0$ which equals $x^n + ... + x + 1 = 0$. We see apparently that x cannot be positive, for it would yield a number greater than zero for $Q(x)$. We look at the zeros of the answer choices. A, C, D, and E have a positive zero, which eliminates them. B is the answer.

See Also