2013 AMC 12A Problems

2013 AMC 12A (Answer Key) Printable version: | AoPS Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2013) • PDF (http://www.artofproblemsolving.com/Forum/resources/files/usa/USA-AMC_12-AHSME-2013-44)
Instructions 1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct. 2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer. 3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator). 4. Figures are not necessarily drawn to scale. 5. You will have 75 minutes working time to complete the test.
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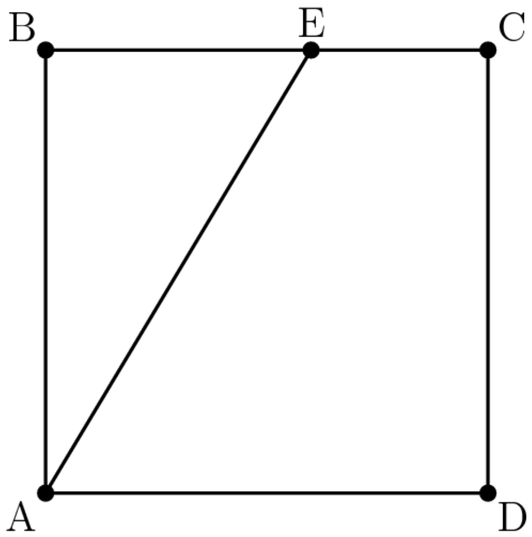
Contents

Problem 1

Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

$\textbf{(A)}\ 4 \qquad\textbf{(B)}\ 5 \qquad\textbf{(C)}\ 6 \qquad\textbf{(D)}\ 7 \qquad\textbf{(E)}\ 8$



Solution

Problem 2

A softball team played ten games, scoring $1,2,3,4,5,6,7,8,9$, and 10 runs. They lost by one run in exactly five games. In each of the other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

$\textbf{(A)}\ 35 \qquad\textbf{(B)}\ 40 \qquad\textbf{(C)}\ 45 \qquad\textbf{(D)}\ 50 \qquad\textbf{(E)}\ 55$

Solution

Problem 3

A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

$\textbf{(A)}\ 15 \qquad\textbf{(B)}\ 30 \qquad\textbf{(C)}\ 40 \qquad\textbf{(D)}\ 60 \qquad\textbf{(E)}\ 70$

Solution

Problem 4

What is the value of

$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

$\textbf{(A)}\ -1 \qquad\textbf{(B)}\ 1 \qquad\textbf{(C)}\ \frac{5}{3} \qquad\textbf{(D)}\ 2013 \qquad\textbf{(E)}\ 2^{4024}$

Solution

Problem 5

Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid $\$105$, Dorothy paid $\$125$, and Sammy paid $\$175$. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 35$

Solution

Problem 6

In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 36$

Solution

Problem 7

The sequence $S_1, S_2, S_3, \cdots, S_{10}$ has the property that every term beginning with the third is the sum of the previous two. That is,

$$S_n = S_{n-2} + S_{n-1} \text{ for } n \ge 3.$$

Suppose that $S_9 = 110$ and $S_7 = 42$. What is S_4?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 16$

Solution

Problem 8

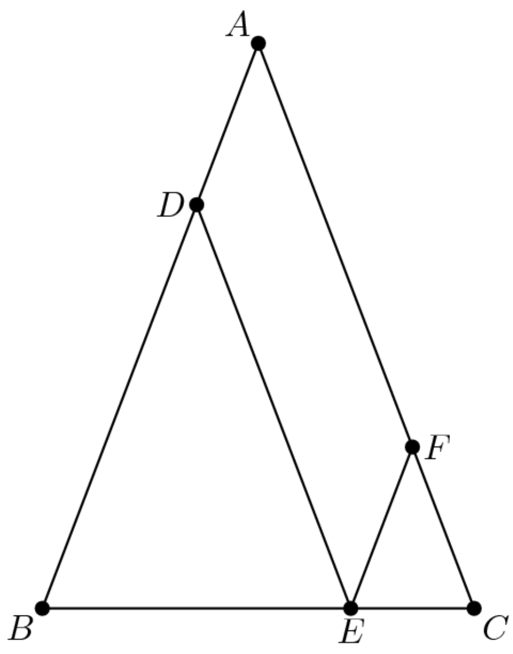
Given that x and y are distinct nonzero real numbers such that $x+\tfrac{2}{x} = y + \tfrac{2}{y}$, what is xy?

$\textbf{(A)}\ \frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 4$

Solution

Problem 9

In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points $D, E,$ and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



$\textbf{(A) } 48 \qquad \textbf{(B) } 52 \qquad \textbf{(C) } 56 \qquad \textbf{(D) } 60 \qquad \textbf{(E) } 72$

Solution

Problem 10

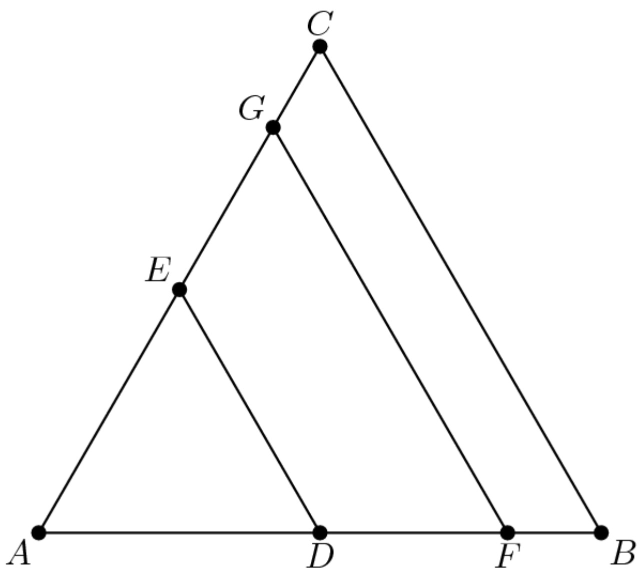
Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\cdots,$ with a and b different digits. What is the sum of the elements of S?

$\textbf{(A) } 11 \qquad \textbf{(B) } 44 \qquad \textbf{(C) } 110 \qquad \textbf{(D) } 143 \qquad \textbf{(E) } 155$

Solution

Problem 11

Triangle ABC is equilateral with $AB=1$. Points E and G are on $\overline{AC}$ and points D and F are on $\overline{AB}$ such that both $\overline{DE}$ and $\overline{FG}$ are parallel to $\overline{BC}$. Furthermore, triangle ADE and trapezoids $DFGE$ and $FBCG$ all have the same perimeter. What is $DE+FG$?



$\textbf{(A) } 1 \qquad \textbf{(B) } \frac{3}{2} \qquad \textbf{(C) } \frac{21}{13} \qquad \textbf{(D) } \frac{13}{8} \qquad \textbf{(E) } \frac{5}{3}$

Solution

Problem 12

The angles in a particular triangle are in arithmetic progression, and the side lengths are $4, 5, x$. The sum of the possible values of x equals $a+\sqrt{b}+\sqrt{c}$ where a, b, and c are positive integers. What is $a+b+c$?

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 38 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 44$

Solution

Problem 13

Let points $A = (0,0) ,\ B = (1,2),\ C = (3,3),$ and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p+q+r+s$?

$\textbf{(A)}\ 54 \qquad \textbf{(B)}\ 58 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 70 \qquad \textbf{(E)}\ 75$

Solution

Problem 14

The sequence

$\log_{12}{162}, \ \log_{12}{x}, \ \log_{12}{y}, \ \log_{12}{z}, \ \log_{12}{1250}$

is an arithmetic progression. What is x?

$\textbf{(A)}\ 125\sqrt{3} \qquad \textbf{(B)}\ 270 \qquad \textbf{(C)}\ 162\sqrt{5} \qquad \textbf{(D)}\ 434 \qquad \textbf{(E)}\ 225\sqrt{6}$

Solution

Problem 15

Rabbits Peter and Pauline have three offspring—Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

$\textbf{(A)}\ 96 \qquad \textbf{(B)}\ 108 \qquad \textbf{(C)}\ 156 \qquad \textbf{(D)}\ 204 \qquad \textbf{(E)}\ 372$

Solution

Problem 16

$A,\ B,\ C$ are three piles of rocks. The mean weight of the rocks in A is 40 pounds, the mean weight of the rocks in B is 50 pounds, the mean weight of the rocks in the combined piles A and B is 43 pounds, and the mean weight of the rocks in the combined piles A and C is 44 pounds. What is the greatest possible integer value for the mean in pounds of the rocks in the combined piles B and C?

$\textbf{(A)}\ 55 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 57 \qquad \textbf{(D)}\ 58 \qquad \textbf{(E)}\ 59$

Solution

Problem 17

A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^th pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

$\textbf{(A)}\ 720 \qquad \textbf{(B)}\ 1296 \qquad \textbf{(C)}\ 1728 \qquad \textbf{(D)}\ 1925 \qquad \textbf{(E)}\ 3850$

Solution

Problem 18

Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

$\textbf{(A)}\ \sqrt{2} \qquad \textbf{(B)}\ \frac{3}{2} \qquad \textbf{(C)}\ \frac{5}{3} \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 2$

Solution

Problem 19

In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 33 \qquad \textbf{(D)}\ 61 \qquad \textbf{(E)}\ 72$

Solution

Problem 20

Let S be the set $\{1, 2, 3, ..., 19\}$. For $a, b \in S$, define $a \succ b$ to mean that either $0 < a - b \le 9$ or $b - a > 9$. How many ordered triples (x, y, z) of elements of S have the property that $x \succ y$, $y \succ z$, and $z \succ x$?

$\textbf{(A)}\ 810 \qquad \textbf{(B)}\ 855 \qquad \textbf{(C)}\ 900 \qquad \textbf{(D)}\ 950 \qquad \textbf{(E)}\ 988$

Solution

Problem 21

Consider $A = \log(2013 + \log(2012 + \log(2011 + \log(\cdots + \log(3 + \log 2) \cdots))))$. Which of the following intervals contains A?

$\textbf{(A)}\ (\log 2016, \log 2017) \ \textbf{(B)}\ (\log 2017, \log 2018) \ \textbf{(C)}\ (\log 2018, \log 2019)$
$\textbf{(D)}\ (\log 2019, \log 2020) \ \textbf{(E)}\ (\log 2020, \log 2021)$

Solution

Problem 22

A palindrome is a nonnegative integer number that reads the same forwards and backwards when written in base 10 with no leading zeros. A 6-digit palindrome n is chosen uniformly at random. What is the probability that $\frac{n}{11}$ is also a palindrome?

$\textbf{(A)}\ \frac{8}{25} \qquad \textbf{(B)}\ \frac{33}{100} \qquad \textbf{(C)}\ \frac{7}{20} \qquad \textbf{(D)}\ \frac{9}{25} \qquad \textbf{(E)}\ \frac{11}{30}$

Solution

Problem 23

$ABCD$ is a square of side length $\sqrt{3}+1$. Point P is on $\overline{AC}$ such that $AP=\sqrt{2}$. The square region bounded by $ABCD$ is rotated 90° counterclockwise with center P, sweeping out a region whose area is $\frac{1}{c}(a\pi+b)$, where a, b, and c are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 19 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 23$

Solution

Problem 24

Three distinct segments are chosen at random among the segments whose end-points are the vertices of a regular 12-gon. What is the probability that the lengths of these three segments are the three side lengths of a triangle with positive area?

$\textbf{(A)}\ \frac{553}{715} \qquad \textbf{(B)}\ \frac{443}{572} \qquad \textbf{(C)}\ \frac{111}{143} \qquad \textbf{(D)}\ \frac{81}{104} \qquad \textbf{(E)}\ \frac{223}{286}$

Solution

Problem 25

Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = z^2 + iz + 1$. How many complex numbers z are there such that $\text{Im}(z) > 0$ and both the real and the imaginary parts of $f(z)$ are integers with absolute value at most 10?

$\textbf{(A)}\ 399 \qquad \textbf{(B)}\ 401 \qquad \textbf{(C)}\ 413 \qquad \textbf{(D)}\ 431 \qquad \textbf{(E)}\ 441$

Solution

See also

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