2005 AMC 12B Problems

Contents

Problem 1

A scout troop buys 1000 candy bars at a price of five for 2 dollars. They sell all the candy bars at the price of two for 1 dollar. What was their profit, in dollars?

$\mathrm{(A)}\ 100 \qquad \mathrm{(B)}\ 200 \qquad \mathrm{(C)}\ 300 \qquad \mathrm{(D)}\ 400 \qquad \mathrm{(E)}\ 500$

Solution

Problem 2

A positive number x has the property that $x\%$ of x is 4. What is x?

$\mathrm{(A)}\ 2 \qquad \mathrm{(B)}\ 4 \qquad \mathrm{(C)}\ 10 \qquad \mathrm{(D)}\ 20 \qquad \mathrm{(E)}\ 40$

Solution

Problem 3

Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

$\mathrm{(A)}\ \frac{1}{5} \qquad \mathrm{(B)}\ \frac{1}{3} \qquad \mathrm{(C)}\ \frac{2}{5} \qquad \mathrm{(D)}\ \frac{2}{3} \qquad \mathrm{(E)}\ \frac{4}{5}$

Solution

Problem 4

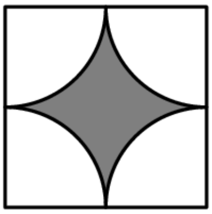
At the beginning of the school year, Lisa's goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the first 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

$\mathrm{(A) \ 1 } \qquad \mathrm{(B) \ 2 } \qquad \mathrm{(C) \ 3 } \qquad \mathrm{(D) \ 4 } \qquad \mathrm{(E) \ 5 }$

Solution

Problem 5

An 8-foot by 10-foot floor is tiled with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius $1/2$ foot centered at each corner of the tile. The remaining portion of the tile is shaded. How many square feet of the floor are shaded?



$\mathrm{(A) \ 80-20\pi } \qquad \mathrm{(B) \ 60-10\pi } \qquad \mathrm{(C) \ 80-10\pi } \qquad \mathrm{(D) \ 60+10\pi } \qquad \mathrm{(E) \ 80+10\pi }$

Solution

Problem 6

In $\triangle ABC$, we have $AC=BC=7$ and $AB=2$. Suppose that D is a point on line AB such that B lies between A and D and $CD=8$. What is BD?

$\mathrm{(A) \ 3 } \qquad \mathrm{(B) \ 2\sqrt{3} } \qquad \mathrm{(C) \ 4 } \qquad \mathrm{(D) \ 5 } \qquad \mathrm{(E) \ 4\sqrt{2} }$

Solution

Problem 7

What is the area enclosed by the graph of $|3x|+|4y|=12$?

$\mathrm{(A) \ 6 } \qquad \mathrm{(B) \ 12 } \qquad \mathrm{(C) \ 16 } \qquad \mathrm{(D) \ 24 } \qquad \mathrm{(E) \ 25 }$

Solution

Problem 8

For how many values of a is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$?

$\mathrm{(A) \ 0 } \qquad \mathrm{(B) \ 1 } \qquad \mathrm{(C) \ 2 } \qquad \mathrm{(D) \ 10 } \qquad \mathrm{(E) \ infinitely\ many }$

Solution

Problem 9

On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

$\mathrm{(A) \ 0 } \qquad \mathrm{(B) \ 1 } \qquad \mathrm{(C) \ 2 } \qquad \mathrm{(D) \ 4 } \qquad \mathrm{(E) \ 5 }$

Solution

Problem 10

The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous terms. What is the 2005^{th} term of the sequence?

$(\mathrm{A})\ 29 \qquad (\mathrm{B})\ 55 \qquad (\mathrm{C})\ 85 \qquad (\mathrm{D})\ 133 \qquad (\mathrm{E})\ 250$

Solution

Problem 11

An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is $\$20$ or more?

$(\mathrm{A})\ \frac{1}{4} \qquad (\mathrm{B})\ \frac{2}{7} \qquad (\mathrm{C})\ \frac{3}{7} \qquad (\mathrm{D})\ \frac{1}{2} \qquad (\mathrm{E})\ \frac{2}{3}$

Solution

Problem 12

The quadratic equation $x^2 + mx + n$ has roots twice those of $x^2 + px + m$, and none of $m, n,$ and p is zero. What is the value of n/p?

$(\mathrm{A})\ 1 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 4 \qquad (\mathrm{D})\ 8 \qquad (\mathrm{E})\ 16$

Solution

Problem 13

Suppose that $4^{x_1} = 5$, $5^{x_2} = 6$, $6^{x_3} = 7$, ... , $127^{x_{124}} = 128$. What is $x_1 x_2 ... x_{124}$?

$(\mathrm{A})\ 2 \qquad (\mathrm{B})\ \frac{5}{2} \qquad (\mathrm{C})\ 3 \qquad (\mathrm{D})\ \frac{7}{2} \qquad (\mathrm{E})\ 4$

Solution

Problem 14

A circle having center $(0, k)$, with $k > 6$, is tangent to the lines $y = x$, $y = -x$ and $y = 6$. What is the radius of this circle?

$(\mathrm{A})\ 6\sqrt{2} - 6 \qquad (\mathrm{B})\ 6 \qquad (\mathrm{C})\ 6\sqrt{2} \qquad (\mathrm{D})\ 12 \qquad (\mathrm{E})\ 6 + 6\sqrt{2}$

Solution

Problem 15

The sum of four two-digit numbers is 221. None of the eight digits is 0 and no two of them are the same. Which of the following is not included among the eight digits?

$(\mathrm{A})\ 1 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 3 \qquad (\mathrm{D})\ 4 \qquad (\mathrm{E})\ 5$

Solution

Problem 16

Eight spheres of radius 1, one per octant, are each tangent to the coordinate planes. What is the radius of the smallest sphere, centered at the origin, that contains these eight spheres?

$(A)\ \sqrt{2} \qquad (B)\ \sqrt{3} \qquad (C)\ 1 + \sqrt{2} \qquad (D)\ 1 + \sqrt{3} \qquad (E)\ 3$

Solution

Problem 17

How many distinct four-tuples (a,b,c,d) of rational numbers are there with

$a\cdot\log_{10}2+b\cdot\log_{10}3+c\cdot\log_{10}5+d\cdot\log_{10}7=2005$?

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ 17 \qquad \mathrm{(D)}\ 2004 \qquad \mathrm{(E)}\ \text{infinitely many}$

Solution

Problem 18

Let $A(2,2)$ and $B(7,7)$ be points in the plane. Define R as the region in the first quadrant consisting of those points C such that $\triangle ABC$ is an acute triangle. What is the closest integer to the area of the region R?

$\mathrm{(A)}\ 25 \qquad \mathrm{(B)}\ 39 \qquad \mathrm{(C)}\ 51 \qquad \mathrm{(D)}\ 60 \qquad \mathrm{(E)}\ 80$

Solution

Problem 19

Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2-y^2=m^2$ for some positive integer m. What is $x+y+m$?

$\mathrm{(A)}\ 88 \qquad \mathrm{(B)}\ 112 \qquad \mathrm{(C)}\ 116 \qquad \mathrm{(D)}\ 144 \qquad \mathrm{(E)}\ 154$

Solution

Problem 20

Let a,b,c,d,e,f,g and h be distinct elements in the set

$$\{-7,-5,-3,-2,2,4,6,13\}.$$

What is the minimum possible value of

$$(a+b+c+d)^2+(e+f+g+h)^2?$$

$\mathrm{(A)}\ 30 \qquad \mathrm{(B)}\ 32 \qquad \mathrm{(C)}\ 34 \qquad \mathrm{(D)}\ 40 \qquad \mathrm{(E)}\ 50$

Solution

Problem 21

A positive integer n has 60 divisors and $7n$ has 80 divisors. What is the greatest integer k such that 7^k divides n?

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ 2 \qquad \mathrm{(D)}\ 3 \qquad \mathrm{(E)}\ 4$

Solution

Problem 22

A sequence of complex numbers $z_0,z_1,z_2,\ldots$ is defined by the rule

$$z_{n+1} = \frac{iz_n}{\overline{z_n}},$$

where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose that $|z_0| = 1$ and $z_{2005} = 1$. How many possible values are there for z_0?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 2005 \qquad \textbf{(E)}\ 2^{2005}$

Solution

Problem 23

Let S be the set of ordered triples (x, y, z) of real numbers for which

$$\log_{10}(x + y) = z \text{ and } \log_{10}(x^2 + y^2) = z + 1.$$

There are real numbers a and b such that for all ordered triples (x, y, z) in S we have $x^3 + y^3 = a \cdot 10^{3z} + b \cdot 10^{2z}$. What is the value of $a + b$?

$\textbf{(A)}\ \frac{15}{2} \qquad \textbf{(B)}\ \frac{29}{2} \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ \frac{39}{2} \qquad \textbf{(E)}\ 24$

Solution

Problem 24

All three vertices of an equilateral triangle are on the parabola $y = x^2$, and one of its sides has a slope of 2. The x-coordinates of the three vertices have a sum of m/n, where m and n are relatively prime positive integers. What is the value of $m + n$?

$\mathrm{(A)}\ 14 \qquad \mathrm{(B)}\ 15 \qquad \mathrm{(C)}\ 16 \qquad \mathrm{(D)}\ 17 \qquad \mathrm{(E)}\ 18$

Solution

Problem 25

Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex?

$\mathrm{(A)}\ \frac{5}{256} \qquad \mathrm{(B)}\ \frac{21}{1024} \qquad \mathrm{(C)}\ \frac{11}{512} \qquad \mathrm{(D)}\ \frac{23}{1024} \qquad \mathrm{(E)}\ \frac{3}{128}$

Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2005 AMC 12B
- 2005 AMC B Math Jam Transcript (http://www.artofproblemsolving.com/Community/AoPS_Y_MJ_Transcripts.php?mj_id=49)
- Mathematics competition resources