2001 AMC 8 Problems/Problem 1

Problem

Casey's shop class is making a golf trophy. He has to paint 300 dimples on a golf ball. If it takes him 2 seconds to paint one dimple, how many minutes will he need to do his job?

$\text{(A)}\ 4 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 8 \qquad \text{(D)}\ 10 \qquad \text{(E)}\ 12$

Solution

It will take him $300 \cdot 2 = 600$ seconds to paint all the dimples.

This is equivilant to $\dfrac{600}{60} = 10$ minutes $\Rightarrow \boxed{D}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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Category: Introductory Algebra Problems

2001 AMC 8 Problems/Problem 2

Contents

Problem

I'm thinking of two whole numbers. Their product is 24 and their sum is 11. What is the larger number?

$\text{(A)}\ 3 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 12$

Solution 1

Let the numbers be x and y. Then we have $x+y=11$ and $xy=24$. Solving for x in the first equation yields $x=11-y$, and substituting this into the second equation gives $(11-y)(y)=24$. Simplifying this gives $-y^2+11y=24$, or $y^2-11y+24=0$. This factors as $(y-3)(y-8)=0$, so $y=3$ or $y=8$, and the corresponding x values are $x=8$ and $x=3$. These are essentially the same answer: one number is 3 and one number is 8, so the largest number is $8, \boxed{\text{D}}$.

Solution 2

Use the answers to attempt to "reverse engineer" an appropriate pair of numbers. Looking at option A, guess that one of the numbers is 3. If the sum of two numbers is 11 and one is 3, then other must be $11-3=8$. The product of those numbers is $3\cdot 8=24$, which is the second condition of the problem, so our number are 3 and 8.

However, 3 is the smaller of the two numbers, so the answer is 8 or $\boxed{D}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 3

Problem

Granny Smith has \$63. Elberta has \$2 more than Anjou and Anjou has one-third as much as Granny Smith. How many dollars does Elberta have?

$\text{(A)}\ 17 \qquad \text{(B)}\ 18 \qquad \text{(C)}\ 19 \qquad \text{(D)}\ 21 \qquad \text{(E)}\ 23$

Solution

Since Anjou has $\frac{1}{3}$ the amount of money as Granny Smith and Granny Smith has $\$63$, Anjou has $\frac{1}{3} \times 63 = 21$ dollars. Elberta has $\$2$ more than this, so she has $\$23$, or $\boxed{\text{E}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 2	Followed by Problem 4
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2001 AMC 8 Problems/Problem 4

Problem

The digits 1, 2, 3, 4 and 9 are each used once to form the smallest possible even five-digit number. The digit in the tens place is

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 9$

Solution

Since the number is even, the last digit must be 2 or 4. To make the smallest possible number, the ten-thousands digit must be as small as possible, so the ten-thousands digit is 1. Simillarly, the thousands digit has second priority, so it must also be as small as possible once the ten-thousands digit is decided, so the thousands digit is 2. Similarly, the hundreds digit needs to be the next smallest number, so it is 3. However, for the tens digit, we can't use 4, since we already used 2 and the number must be even, so the units digit must be 4 and the tens digit is $9, \boxed{\text{E}}$ (The number is 12394.)

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 3	Followed by Problem 5
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2001 AMC 8 Problems/Problem 5

Problem

On a dark and stormy night Snoopy suddenly saw a flash of lightning. Ten seconds later he heard the sound of thunder. The speed of sound is 1088 feet per second and one mile is 5280 feet. Estimate, to the nearest half-mile, how far Snoopy was from the flash of lightning.

$\text{(A)}\ 1 \qquad \text{(B)}\ 1\frac{1}{2} \qquad \text{(C)}\ 2 \qquad \text{(D)}\ 2\frac{1}{2} \qquad \text{(E)}\ 3$

Solution

During those 10 seconds, the sound traveled $1088 \times 10 = 10880$ feet from the lightning to Snoopy. This is equivalent to $\frac{10880}{5280} \approx 2$ miles, $\boxed{\text{C}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 4	Followed by Problem 6
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2001 AMC 8 Problems/Problem 6

Problem

Six trees are equally spaced along one side of a straight road. The distance from the first tree to the fourth is 60 feet. What is the distance in feet between the first and last trees?

$\text{(A)}\ 90 \qquad \text{(B)}\ 100 \qquad \text{(C)}\ 105 \qquad \text{(D)}\ 120 \qquad \text{(E)}\ 140$

Solution

There are 3 spaces between the 1st and 4th trees, so each of these spaces has $\frac{60}{3} = 20$ feet. Between the first and last trees there are 5 spaces, so the distance between them is $20 \times 5 = 100$ feet, $\boxed{\text{B}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 5	Followed by Problem 7
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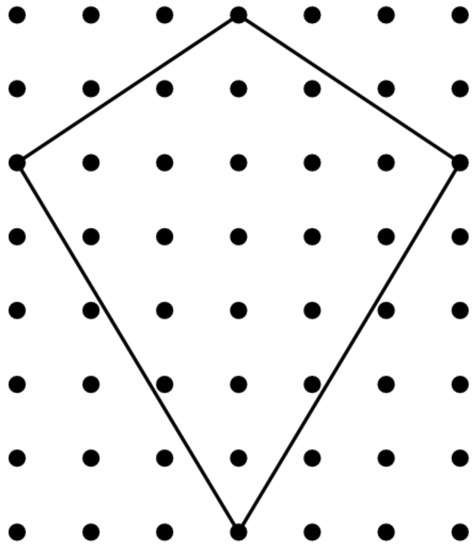
2001 AMC 8 Problems/Problem 7

To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



Contents

Problem

What is the number of square inches in the area of the small kite?

$\text{(A)}\ 21 \qquad \text{(B)}\ 22 \qquad \text{(C)}\ 23 \qquad \text{(D)}\ 24 \qquad \text{(E)}\ 25$

Solution 1

The area of a kite is half the product of its diagonals. The diagonals have lengths of 6 and 7, so the area is $\frac{(6)(7)}{2} = 21, \boxed{\text{A}}$.

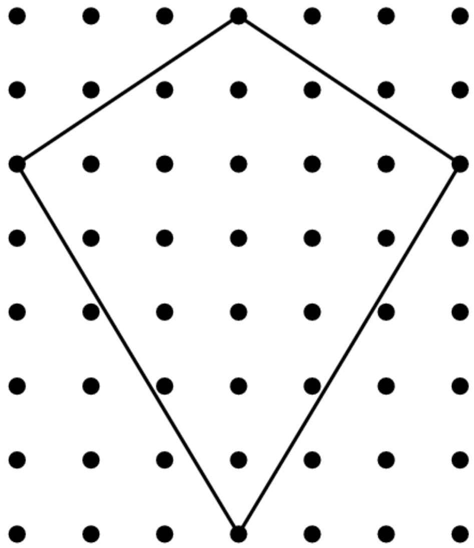
Solution 2

Drawing in the diagonals of the kite will form four right triangles on the "inside" part of the grid. Drawing in the border of the 7 by 6 grid will form four right triangles on the "outside" part of the grid. Since each right triangle on the inside can be paired with a congruent right triangle that is on the outside, the area of the kite is half the total area of the grid, or $\frac{(6)(7)}{2} = 21, \boxed{\text{A}}$.

See Also

2001 AMC 8 Problems/Problem 8

To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



Problem

Genevieve puts bracing on her large kite in the form of a cross connecting opposite corners of the kite. How many inches of bracing material does she need?

$\text{(A)}\ 30 \qquad \text{(B)}\ 32 \qquad \text{(C)}\ 35 \qquad \text{(D)}\ 38 \qquad \text{(E)}\ 39$

Solution

Each diagonal of the large kite is 3 times the length of the corresponding diagonal of the short kite since it was made with a grid 3 times as long in each direction. The diagonals of the small kite are 6 and 7, so the diagonals of the large kite are 18 and 21, and the amount of bracing Genevieve needs is the sum of these lengths, which is $39, \boxed{\text{E}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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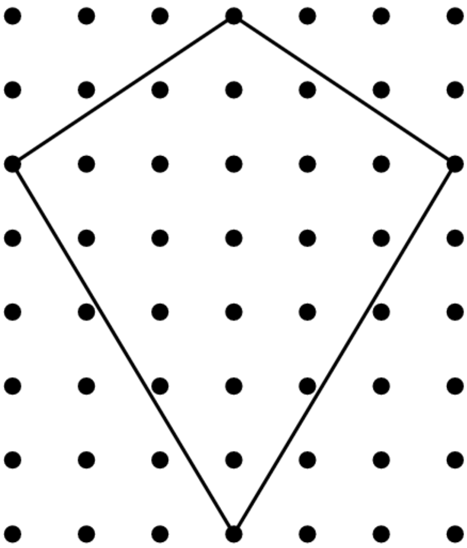
2001 AMC 8 Problems/Problem 9

To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



Problem

The large kite is covered with gold foil. The foil is cut from a rectangular piece that just covers the entire grid. How many square inches of waste material are cut off from the four corners?

$\textbf{(A)}\ 63 \qquad \textbf{(B)}\ 72 \qquad \textbf{(C)}\ 180 \qquad \textbf{(D)}\ 189 \qquad \textbf{(E)}\ 264$

Solution

The large grid has dimensions three times that of the small grid, so its dimensions are $3(6) \times 3(7)$, or 18×21, so the area is $(18)(21) = 378$. The area of the kite is half the product of its diagonals, and the diagonals are the dimensions of the rectangle, so the area of the kite is $\frac{(18)(21)}{2} = 189$. Thus, the area of the remaining gold is $378 - 189 = 189, \boxed{\text{D}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 10

Problem

A collector offers to buy state quarters for 2000% of their face value. At that rate how much will Bryden get for his four state quarters?

$\text{(A)}\ 20\text{ dollars} \qquad \text{(B)}\ 50\text{ dollars} \qquad \text{(C)}\ 200\text{ dollars} \qquad \text{(D)}\ 500\text{ dollars} \qquad \text{(E)}\ 2000\text{ dollars}$

Solution

2000% is equivalent to $20 \times 100\%$. Therefore, 2000% of a number is the same as 20 times that number. 4 quarters is 1 dollar, so Bryden will get $20 \times 1 = 20$ dollars, $\boxed{\text{A}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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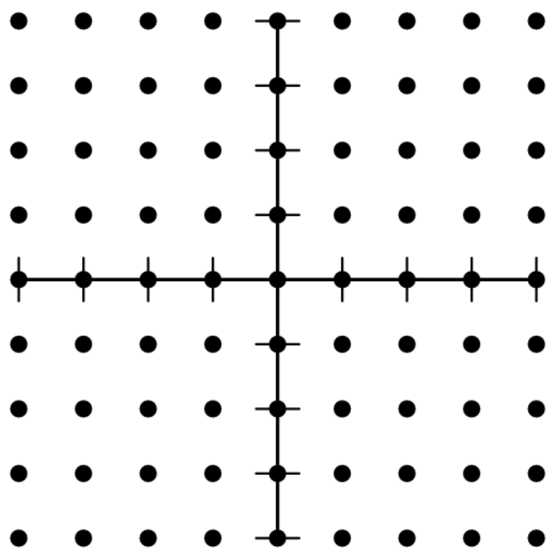
2001 AMC 8 Problems/Problem 11

Contents

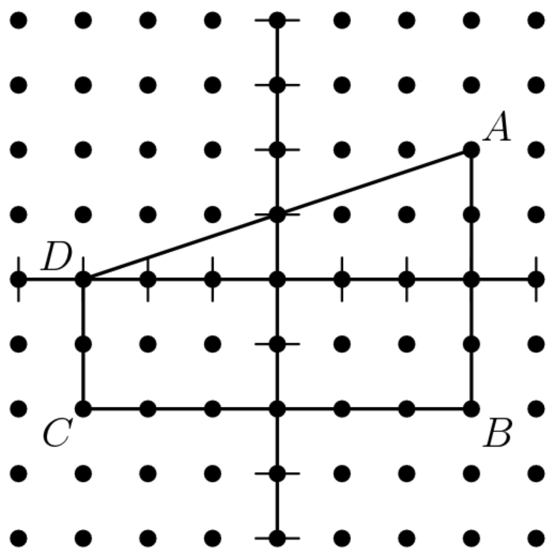
Problem

Points A, B, C and D have these coordinates: $A(3,2)$, $B(3,-2)$, $C(-3,-2)$ and $D(-3,0)$. The area of quadrilateral $ABCD$ is



$\text{(A)}\ 12 \qquad \text{(B)}\ 15 \qquad \text{(C)}\ 18 \qquad \text{(D)}\ 21 \qquad \text{(E)}\ 24$

Solution 1



This quadrilateral is a trapezoid, because $AB \parallel CD$ but BC is not parallel to AD. The area of a trapezoid is the product of its height and its median, where the median is the average of the side lengths of the bases. The two bases are AB and CD, which have lengths 2 and 4, respectively, so the length of the median is $\frac{2+4}{2} = 3$. CB is perpendicular to the bases, so it is the height, and has length 6. Therefore, the area of the trapezoid is $(3)(6) = 18, \boxed{\text{C}}$

Solution 2

Using the diagram above, the figure can be divided along the x-axis into two familiar regions that do not overlap: a right triangle and a rectangle. Since the areas do not overlap, the area of the entire trapezoid is the sum of the area of the triangle and the area of the rectangle.

$$A_{trap} = A_{tri} + A_{rect}$$

$$A_{trap} = \frac{1}{2}bh + lw$$

$$A_{trap} = \frac{1}{2} \cdot 6 \cdot 2 + 6 \cdot 2$$

$$A_{trap} = 6 + 12 = 18 \rightarrow \boxed{C}$$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 12

Problem

If $a \otimes b = \dfrac{a+b}{a-b}$, then $(6 \otimes 4) \otimes 3 =$

$\text{(A)}\ 4 \qquad \text{(B)}\ 13 \qquad \text{(C)}\ 15 \qquad \text{(D)}\ 30 \qquad \text{(E)}\ 72$

Solution

$6 \otimes 4 = \dfrac{6+4}{6-4} = 5.\ 5 \otimes 3 = \dfrac{5+3}{5-3} = 4, \boxed{\text{A}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 13

Problem

Of the 36 students in Richelle's class, 12 prefer chocolate pie, 8 prefer apple, and 6 prefer blueberry. Half of the remaining students prefer cherry pie and half prefer lemon. For Richelle's pie graph showing this data, how many degrees should she use for cherry pie?

$\text{(A)}\ 10 \qquad \text{(B)}\ 20 \qquad \text{(C)}\ 30 \qquad \text{(D)}\ 50 \qquad \text{(E)}\ 72$

Solution

There are 36 students in the class: 12 prefer chocolate pie, 8 prefer apple pie, and 6 prefer blueberry pie. Therefore, $36-12-8-6=10$ students prefer cherry pie or lemon pie. Half of these prefer each, so 5 students prefer cherry pie. This means that $\frac{5}{36}$ of the students prefer cherry pie, so $\frac{5}{36}$ of the full 360° should be used for cherry pie. This is $(\frac{5}{36})(360^\circ)=50^\circ, \boxed{\text{D}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 14

Problem

Tyler has entered a buffet line in which he chooses one kind of meat, two different vegetables and one dessert. If the order of food items is not important, how many different meals might he choose?

- Meat: beef, chicken, pork
- Vegetables: baked beans, corn, potatoes, tomatoes
- Dessert: brownies, chocolate cake, chocolate pudding, ice cream

$\text{(A)}\ 4 \qquad \text{(B)}\ 24 \qquad \text{(C)}\ 72 \qquad \text{(D)}\ 80 \qquad \text{(E)}\ 144$

Solution

There are 3 possibilities for the meat and 4 possibilities for the dessert, for a total of $4 \times 3 = 12$ possibilities for the meat and the dessert. There are 4 possibilities for the first vegetable and 3 possibilities for the second, but order doesn't matter, so we overcounted by a factor of 2. For example, we counted 'baked beans and corn' and 'corn and baked beans' as 2 different possibilities, so the total possibilites for the two vegetables is $\frac{4 \times 3}{2} = 6$, and the total number of possibilites is $12 \times 6 = 72, \boxed{\text{C}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 15

Problem

Homer began peeling a pile of 44 potatoes at the rate of 3 potatoes per minute. Four minutes later Christen joined him and peeled at the rate of 5 potatoes per minute. When they finished, how many potatoes had Christen peeled?

$\text{(A)}\ 20 \qquad \text{(B)}\ 24 \qquad \text{(C)}\ 32 \qquad \text{(D)}\ 33 \qquad \text{(E)}\ 40$

Solution

After the 4 minutes of Homer peeling alone, he had peeled $4\times 3=12$ potatoes. This means that there are $44-12=32$ potatoes left. Once Christen joins him, the two are peeling potatoes at a rate of $3+5=8$ potatoes per minute. So, they finish peeling after another $\frac{32}{8}=4$ minutes. In these 4 minutes, Christen peeled $4\times 5=20$ potatoes, $\boxed{\text{A}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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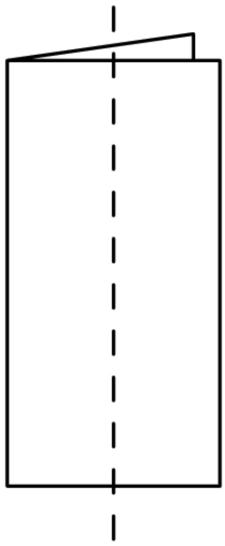
2001 AMC 8 Problems/Problem 16

Problem

A square piece of paper, 4 inches on a side, is folded in half vertically. Both layers are then cut in half parallel to the fold. Three new rectangles are formed, a large one and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?



$\text{(A)}\ \frac{1}{3} \qquad \text{(B)}\ \frac{1}{2} \qquad \text{(C)}\ \frac{3}{4} \qquad \text{(D)}\ \frac{4}{5} \qquad \text{(E)}\ \frac{5}{6}$

Solution

The smaller rectangles each have the same height as the original square, but have $\frac{1}{4}$ the length, since the paper is folded in half and then cut in half the same way. The larger rectangle has the same height as the original square but has $\frac{1}{2}$ the length, since the paper is cut in half after the fold but the fold retains both sides of the larger rectangle. Therefore, the smaller rectangles have dimensions 4×1 and the larger rectangle has dimensions 4×2. The ratio of their perimeters is $\frac{2(4+1)}{2(4+2)} = \frac{5}{6}, \boxed{\text{E}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 17

Problem

For the game show Who Wants To Be A Millionaire?, the dollar values of each question are shown in the following table (where K = 1000).

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Value	100	200	300	500	1K	2K	4K	8K	16K	32K	64K	125K	250K	500K	1000K

Between which two questions is the percent increase of the value the smallest?

$\text{(A)}\ \text{From 1 to 2} \qquad \text{(B)}\ \text{From 2 to 3} \qquad \text{(C)}\ \text{From 3 to 4} \qquad \text{(D)}\ \text{From 11 to 12} \qquad \text{(E)}\ \text{From 14 to 15}$

Solution

Notice that in two of the increases, the dollar amount doubles. The increases in which this is not true is 2 to 3, 3 to 4, and 11 to 12. We can disregard 11 to 12 since that increase is almost 2 times. The increase from 2 to 3 is $\frac{300-200}{200}=\frac{1}{2}$, so it's only multiplied by a factor of 1.5. The increase from 3 to 4 is $\frac{500-300}{300}=\frac{2}{3}$, so it's only multiplied by a factor of $1.\overline{6}$. Therefore, the smallest percent increase is $\text{From 2 to 3}, \boxed{\text{B}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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2001 AMC 8 Problems/Problem 18

Problem

Two dice are thrown. What is the probability that the product of the two numbers is a multiple of 5?

$\text{(A)}\ \frac{1}{36} \qquad \text{(B)}\ \frac{1}{18} \qquad \text{(C)}\ \frac{1}{6} \qquad \text{(D)}\ \frac{11}{36} \qquad \text{(E)}\ \frac{1}{3}$

Solution

This is equivalent to asking for the probability that at least one of the numbers is a multiple of 5, since if one of the numbers is a multiple of 5, then the product with it and another integer is also a multiple of 5, and if a number is a multiple of 5, then since 5 is prime, one of the factors must also have a factor of 5, and 5 is the only multiple of 5 on a die, so one of the numbers rolled must be a 5. To find the probability of rolling at least one 5, we can find the probability of not rolling a 5 and subtract that from 1, since you either roll a 5 or not roll a 5. The probability of not rolling a 5 on either dice is $\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)=\frac{25}{36}$. Therefore, the probability of rolling at least one five, and thus rolling two numbers whose product is a multiple of 5, is $1-\frac{25}{36}=\frac{11}{36}, \boxed{\text{D}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
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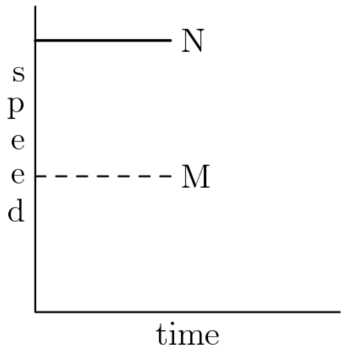
2001 AMC 8 Problems/Problem 19

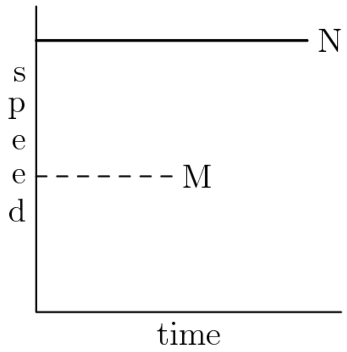
Problem

Car M traveled at a constant speed for a given time. This is shown by the dashed line. Car N traveled at twice the speed for the same distance. If Car N's speed and time are shown as solid line, which graph illustrates this?

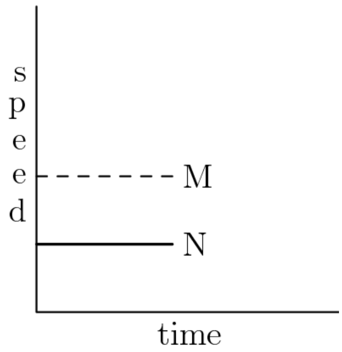
(A)



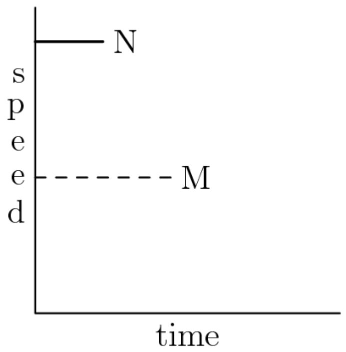
(B)



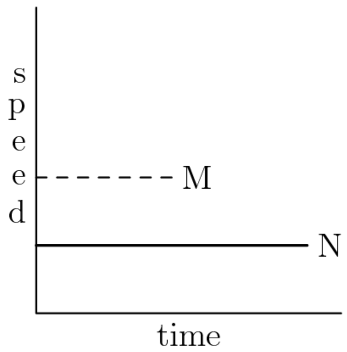
(C)



(D)



(E)



Solution

Since car N has twice the speed, it must be twice as high on the speed axis. Also, since cars M and N travel at the same distance but car N has twice the speed, car N must take twice the time. Therefore, line N must be half the size of line M. Since the speeds are constant, both lines are horizontal. Reviewing the graphs, we see that the only one satisfying these conditions is graph $\boxed{D}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 18	Followed by Problem 20
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All AJHSME/AMC 8 Problems and Solutions	

2001 AMC 8 Problems/Problem 20

Problem

Kaleana shows her test score to Quay, Marty and Shana, but the others keep theirs hidden. Quay thinks, "At least two of us have the same score." Marty thinks, "I didn't get the lowest score." Shana thinks, "I didn't get the highest score." List the scores from lowest to highest for Marty (M), Quay (Q) and Shana (S).

$\text{(A)}\ \text{S,Q,M} \qquad \text{(B)}\ \text{Q,M,S} \qquad \text{(C)}\ \text{Q,S,M} \qquad \text{(D)}\ \text{M,S,Q} \qquad \text{(E)}\ \text{S,M,Q}$

Solution

Since the only other score Quay knows is Kaleana's, and he knows that two of them have the same score, Quay and Kaleana must have the same score, and $K=Q$. Marty knows that he didn't get the lowest score, and the only other score he knows is Kaleana's, so Marty must know that Kaleana must have a lower score than him, and $M>K$. Finally, Shana knows that she didn't get the highest score, and the only other score she knows is Kaleana's, so Shana must know that Kaleana must have a higher score than her, and $S<K$. Putting these together and substituting Q for K, we have $S<Q<M$, and from least to greatest this is S, Q, M, therefore the answer is $\boxed{\text{A}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 19	Followed by Problem 21
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2001 AMC 8 Problems/Problem 21

Problem

The mean of a set of five different positive integers is 15. The median is 18. The maximum possible value of the largest of these five integers is

$\text{(A)}\ 19 \qquad \text{(B)}\ 24 \qquad \text{(C)}\ 32 \qquad \text{(D)}\ 35 \qquad \text{(E)}\ 40$

Solution

Since there is an odd number of terms, the median is the number in the middle, specifically, the third largest number is 18, and there are 2 numbers less than 18 and 2 numbers greater than 18. The sum of these integers is $5(15) = 75$, since the mean is 15. To make the largest possible number with a given sum, the other numbers must be as small as possible. The two numbers less than 18 must be positive and distinct, so the smallest possible numbers for these are 1 and 2. One of the numbers also needs to be as small as possible, so it must be 19. This means that the remaining number, the maximum possible value for a number in the set, is $75-1-2-18-19=35, \boxed{\text{(D)}\ 35}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 20	Followed by Problem 22
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2001 AMC 8 Problems/Problem 22

Problem

On a twenty-question test, each correct answer is worth 5 points, each unanswered question is worth 1 point and each incorrect answer is worth 0 points. Which of the following scores is NOT possible?

$\text{(A)}\ 90 \qquad \text{(B)}\ 91 \qquad \text{(C)}\ 92 \qquad \text{(D)}\ 95 \qquad \text{(E)}\ 97$

Solution

The highest possible score is if you get every answer right, to get $5(20)=100$. The second highest possible score is if you get 19 questions right and leave the remaining one blank, to get a $5(19)+1(1)=96$. Therefore, no score between 96 and 100, exclusive, is possible, so 97 is not possible, $\boxed{\text{E}}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 21	Followed by Problem 23
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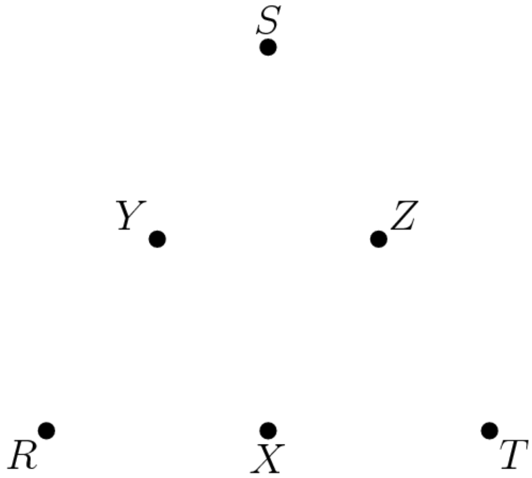
2001 AMC 8 Problems/Problem 23

Problem

Points R, S and T are vertices of an equilateral triangle, and points X, Y and Z are midpoints of its sides. How many noncongruent triangles can be drawn using any three of these six points as vertices?



$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 20$

Solution

There are 6 points in the figure, and 3 of them are needed to form a triangle, so there are $\binom{6}{3} = 20$ possible triples of 3 of the 6 points. However, some of these created congruent triangles, and some don't even make triangles at all.

Case 1: Triangles congruent to $\triangle RST$ There is obviously only 1 of these: $\triangle RST$ itself.

Case 2: Triangles congruent to $\triangle SYZ$ There are 4 of these: $\triangle SYZ, \triangle RXY, \triangle TXZ,$ and $\triangle XYZ$.

Case 3: Triangles congruent to $\triangle RSX$ There are 6 of these: $\triangle RSX, \triangle TSX, \triangle STY, \triangle RTY, \triangle RSZ,$ and $\triangle RTZ$.

Case 4: Triangles congruent to $\triangle SYX$ There are again 6 of these: $\triangle SYX, \triangle SZX, \triangle TYZ, \triangle TYX, \triangle RXZ,$ and $\triangle RYZ$.

However, if we add these up, we accounted for only $1+4+6+6=17$ of the 20 possible triplets. We see that the remaining triplets don't even form triangles; they are $SYR, RXT,$ and TZS. Adding these 3 into the total yields for all of the possible triplets, so we see that there are only 4 possible non-congruent, non-degenerate triangles, $\boxed{\text{D}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 22	Followed by Problem 24
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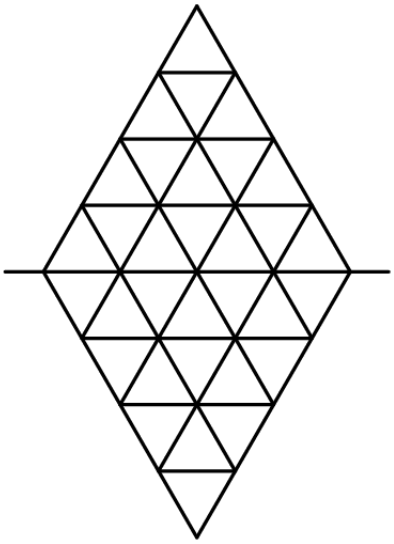
2001 AMC 8 Problems/Problem 24

Problem

Each half of this figure is composed of 3 red triangles, 5 blue triangles and 8 white triangles. When the upper half is folded down over the centerline, 2 pairs of red triangles coincide, as do 3 pairs of blue triangles. There are 2 red-white pairs. How many white pairs coincide?



$(A)\ 4 \qquad (B)\ 5 \qquad (C)\ 6 \qquad (D)\ 7 \qquad (E)\ 9$

Solution

Each half has 3 red triangles, 5 blue triangles, and 8 white triangles. There are also 2 pairs of red triangles, so 2 red triangles on each side are used, leaving 1 red triangle, 5 blue triangles, and 8 white triangles remaining on each half. Also, there are 3 pairs of blue triangles, using 3 blue triangles on each side, so there is 1 red triangle, 2 blue triangles, and 8 white triangles remaining on each half. Also, we have 2 red-white pairs. This obviously can't use 2 red triangles on one side, since there is only 1 on each side, so we must use 1 red triangle and 1 white triangle per side, leaving 2 blue triangles and 7 white triangles apiece. The remaining blue triangles cannot be matched with other blue triangles since that would mean there were more than 3 blue pairs, so the remaining blue triangles must be paired with white triangles, yielding 4 blue-white pairs, one for each of the remaining blue triangles. This uses 2 blue triangles and 2 white triangles on each side, leaving 5 white triangles apiece, which must be paired with each other, so there are 5 white-white pairs, $\boxed{B}$.

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 23	Followed by Problem 25
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2001 AMC 8 Problems/Problem 25

Problem

There are 24 four-digit whole numbers that use each of the four digits 2, 4, 5 and 7 exactly once. Only one of these four-digit numbers is a multiple of another one. Which of the following is it?

$\textbf{(A)}\ 5724 \qquad \textbf{(B)}\ 7245 \qquad \textbf{(C)}\ 7254 \qquad \textbf{(D)}\ 7425 \qquad \textbf{(E)}\ 7542$

Solution

We begin by narrowing down the possibilities. If the larger number were twice the smaller number, then the smallest possibility for the larger number is $2457 \times 2 = 4914$, since 2457 is the smallest number in the set. The largest possibility would have to be twice the largest number in the set such that when it is multiplied by 2, it is less than or equal to 7542, the largest number in the set. This happens to be $2754 \times 2 = 5508$. Therefore, the number would have to be between 4914 and 5508, and also even. The only even numbers in the set and in this range are 5472 and 5274. A quick check reveals that neither of these numbers is twice a number in the set. The number can't be quadruple or more another number in the set since $2457 \times 4 = 9828$, well past the range of the set. Therefore, the number must be triple another number in the set. The least possibility is $2457 \times 3 = 7371$ and the greatest is $2475 \times 3 = 7425$, since any higher number in the set multiplied by 3 would be out of the range of the set. Reviewing, we find that the upper bound does in fact work, so the multiple is $2475 \times 3 = 7425, \boxed{\text{D}}$

Or, since the greatest number possible divided by the smallest number possible is slightly greater than 3, divide all of the choices by 2, then 3 and see if the resulting answer contains 2,4,5 and 7. Doing so, you find that $7425/3 = 2475, \boxed{\text{D}}$

See Also

2001 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2001))	
Preceded by Problem 24	Followed by Last Question
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	