1. What is the value of

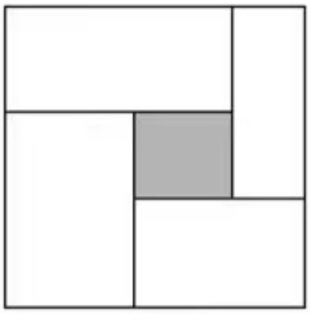
$$3+\cfrac{1}{3+\cfrac{1}{3+\frac{1}{3}}}\,?$$

(A) $\frac{31}{10}$ **(B)** $\frac{49}{15}$ **(C)** $\frac{33}{10}$ **(D)** $\frac{109}{33}$ **(E)** $\frac{15}{4}$

2. The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. Five rectangles, A, B, C, D, and E, are arranged in a square as shown below. These rectangles have dimensions 1×6, 2×4, 5×6, 2×7, and 2×3, respectively. (The figure is not drawn to scale.) Which of the five rectangles is the shaded one in the middle?



(A) A **(B)** B **(C)** C **(D)** D **(E)** E

4. The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n ?

(A) 3 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

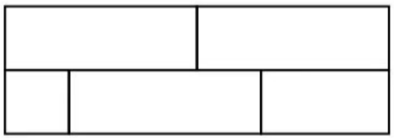
5. The *taxicab distance* between points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given by $|x_1 - x_2| + |y_1 - y_2|$. For how many points P with integer coordinates is the taxicab distance between P and the origin less than or equal to 20 ?

(A) 441 **(B)** 761 **(C)** 841 **(D)** 921 **(E)** 924

6. A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all possible values of X?

(A) 10 **(B)** 26 **(C)** 32 **(D)** 36 **(E)** 40

7. A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color—red, orange, yellow, blue, or green—so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?



(A) 120 **(B)** 270 **(C)** 360 **(D)** 540 **(E)** 720

8. The infinite product

$$\sqrt[3]{10} \cdot \sqrt[3]{\sqrt[3]{10}} \cdot \sqrt[3]{\sqrt[3]{\sqrt[3]{10}}} \cdots$$

evaluates to a real number. What is that number?

(A) $\sqrt{10}$ **(B)** $\sqrt[3]{100}$ **(C)** $\sqrt[4]{1000}$ **(D)** 10 **(E)** $10\sqrt[3]{10}$

9. On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternaters arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.

"Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.

"Are you an alternater?" The principal gave a piece of candy to each of the 15 children who answered yes.

"Are you a liar?" The principal gave a piece of candy to each of the 9 children who answered yes.

How many pieces of candy in all did the principal give to the children who always tell the truth?

(A) 7 **(B)** 12 **(C)** 21 **(D)** 27 **(E)** 31

10. How many ways are there to split the integers 1 through 14 into 7 pairs so that in each pair the greater number is at least 2 times the lesser number?

(A) 108 **(B)** 120 **(C)** 126 **(D)** 132 **(E)** 144

11. What is the product of all real numbers x such that the distance on the number line between $\log_6 x$ and $\log_6 9$ is twice the distance on the number line between $\log_6 10$ and 1?

(A) 10 **(B)** 18 **(C)** 25 **(D)** 36 **(E)** 81

12. Let M be the midpoint of $\overline{AB}$ in regular tetrahedron $ABCD$. What is $\cos(\angle CMD)$?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{\sqrt{3}}{2}$

13. Let $\mathcal{R}$ be the region in the complex plane consisting of all complex numbers z that can be written as the sum of complex numbers z_1 and z_2, where z_1 lies on the segment with endpoints 3 and $4i$, and z_2 has magnitude at most 1. What integer is closest to the area of $\mathcal{R}$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

14. What is the value of $(\log 5)^3 + (\log 20)^3 + (\log 8)(\log 0.25)$, where all logarithms have base 10?

(A) $\frac{3}{2}$ **(B)** $\frac{7}{4}$ **(C)** 2 **(D)** $\frac{9}{4}$ **(E)** 3

15. The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

(A) $\frac{24}{5}$ **(B)** $\frac{42}{5}$ **(C)** $\frac{81}{5}$ **(D)** 30 **(E)** 48

16. A *triangular number* is a positive integer that can be expressed in the form $t_n = 1 + 2 + 3 + \cdots + n$, for some positive integer n. The three smallest triangular numbers that are also perfect squares are $t_1 = 1 = 1^2$, $t_8 = 36 = 6^2$, and $t_{49} = 1225 = 35^2$. What is the sum of the digits of the fourth smallest triangular number that is also a perfect square?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 27

17. Suppose a is a real number such that the equation

$$a \cdot (\sin x + \sin(2x)) = \sin(3x)$$

has more than one solution in the interval $(0, \pi)$. The set of all such a can be written in the form $(p, q) \cup (q, r)$, where p, q, and r are real numbers with $p < q < r$. What is $p + q + r$?

(A) -4 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 4

18. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \ldots, T_n$ returns the point $(1, 0)$ back to itself?

(A) 359 **(B)** 360 **(C)** 719 **(D)** 720 **(E)** 721

19. Suppose that 13 cards numbered $1, 2, 3, \ldots, 13$ are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. In the example below, cards 1, 2, 3 are picked up on the first pass, 4 and 5 on the second pass, 6 on the third pass, 7, 8, 9, 10 on the fourth pass, and 11, 12, 13 on the fifth pass.

$$\boxed{7}\ \boxed{11}\ \boxed{8}\ \boxed{6}\ \boxed{4}\ \boxed{5}\ \boxed{9}\ \boxed{12}\ \boxed{1}\ \boxed{13}\ \boxed{10}\ \boxed{2}\ \boxed{3}$$

For how many of the 13! possible orderings of the cards will the 13 cards be picked up in exactly two passes?

(A) 4082 **(B)** 4095 **(C)** 4096 **(D)** 8178 **(E)** 8191

20. Isosceles trapezoid $ABCD$ has parallel sides $\overline{AD}$ and $\overline{BC}$, with $BC < AD$ and $AB = CD$. There is a point P in the plane such that $PA = 1$, $PB = 2$, $PC = 3$, and $PD = 4$. What is $\frac{BC}{AD}$?

(A) $\dfrac{1}{4}$ **(B)** $\dfrac{1}{3}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{2}{3}$ **(E)** $\dfrac{3}{4}$

21. Let $P(x) = x^{2022} + x^{1011} + 1$. Which of the following polynomials is a factor of $P(x)$?

(A) $x^2 - x + 1$ **(B)** $x^2 + x + 1$ **(C)** $x^4 + 1$ **(D)** $x^6 - x^3 + 1$ **(E)** $x^6 + x^3 + 1$

22. Let c be a real number, and let z_1 and z_2 be the two complex numbers satisfying the equation $z^2 - cz + 10 = 0$. Points z_1, z_2, $\frac{1}{z_1}$, and $\frac{1}{z_2}$ are the vertices of (convex) quadrilateral $\mathcal{Q}$ in the complex plane. When the area of $\mathcal{Q}$ obtains its maximum possible value, c is closest to which of the following?

(A) 4.5 **(B)** 5 **(C)** 5.5 **(D)** 6 **(E)** 6.5

23. Let h_n and k_n be the unique relatively prime positive integers such that

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \frac{h_n}{k_n}.$$

Let L_n denote the least common multiple of the numbers $1, 2, 3, \ldots, n$. For how many integers n with $1 \le n \le 22$ is $k_n < L_n$?

(A) 0 **(B)** 3 **(C)** 7 **(D)** 8 **(E)** 10

24. How many strings of length 5 formed from the digits $0, 1, 2, 3, 4$ are there such that for each $j \in \{1, 2, 3, 4\}$, at least j of the digits are less than j ? (For example, 02214 satisfies this condition because it contains at least 1 digit less than 1, at least 2 digits less than 2, at least 3 digits less than 3, and at least 4 digits less than 4. The string 23404 does not satisfy the condition because it does not contain at least 2 digits less than 2.)

(A) 500 **(B)** 625 **(C)** 1089 **(D)** 1199 **(E)** 1296

25. A circle with integer radius r is centered at (r, r). Distinct line segments of length c_i connect points $(0, a_i)$ to $(b_i, 0)$ for $1 \leq i \leq 14$ and are tangent to the circle, where a_i, b_i, and c_i are all positive integers and $c_1 \leq c_2 \leq \cdots \leq c_{14}$. What is the ratio $\frac{c_{14}}{c_1}$ for the least possible value of r ?

(A) $\frac{21}{5}$ **(B)** $\frac{85}{13}$ **(C)** 7 **(D)** $\frac{39}{5}$ **(E)** 17