2013 AMC 12B Problems

2013 AMC 12B (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have 75 minutes working time to complete the test.

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Problem 1

On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was 3°. In degrees, what was the low temperature in Lincoln that day?

$\textbf{(A) } -13 \qquad \textbf{(B) } -8 \qquad \textbf{(C) } -5 \qquad \textbf{(D) } -3 \qquad \textbf{(E) } 11$

Solution

Problem 2

Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

$\textbf{(A) } 600 \qquad \textbf{(B) } 800 \qquad \textbf{(C) } 1000 \qquad \textbf{(D) } 1200 \qquad \textbf{(E) } 1400$

Solution

Problem 3

When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n?

$\textbf{(A) } 146 \qquad \textbf{(B) } 147 \qquad \textbf{(C) } 148 \qquad \textbf{(D) } 149 \qquad \textbf{(E) } 150$

Solution

Problem 4

Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?

$\textbf{(A) } 10 \qquad \textbf{(B) } 16 \qquad \textbf{(C) } 25 \qquad \textbf{(D) } 30 \qquad \textbf{(E) } 40$

Solution

Problem 5

The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

$\textbf{(A) } 22 \qquad \textbf{(B) } 23.25 \qquad \textbf{(C) } 24.75 \qquad \textbf{(D) } 26.25 \qquad \textbf{(E) } 28$

Solution

Problem 6

Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x+y$?

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 8$

Solution

Problem 7

Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "$1, 2$". Jo then says "$1, 2, 3$", and so on. What is the 53^{rd} number said?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Solution

Problem 8

Line l_1 has equation $3x - 2y = 1$ and goes through $A = (-1, -2)$. Line l_2 has equation $y = 1$ and meets line l_1 at point B. Line l_3 has positive slope, goes through point A, and meets l_2 at point C. The area of $\triangle ABC$ is 3. What is the slope of l_3?

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{3}{4} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{4}{3} \qquad \textbf{(E)}\ \frac{3}{2}$

Solution

Problem 9

What is the sum of the exponents of the prime factors of the square root of the largest perfect square that divides $12!$?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Solution

Problem 10

Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

$\textbf{(A)}\ 62 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 83 \qquad \textbf{(D)}\ 102 \qquad \textbf{(E)}\ 103$

Solution

Problem 11

Two bees start at the same spot and fly at the same rate in the following directions. Bee A travels 1 foot north, then 1 foot east, then 1 foot upwards, and then continues to repeat this pattern. Bee B travels 1 foot south, then 1 foot west, and then continues to repeat this pattern. In what directions are the bees traveling when they are exactly 10 feet away from each other?

$\textbf{(A)}\ A\text{ east, }B\text{ west}$

$\textbf{(B)}\ A\text{ north, }B\text{ south}$

$\textbf{(C)}\ A\text{ north, }B\text{ west}$

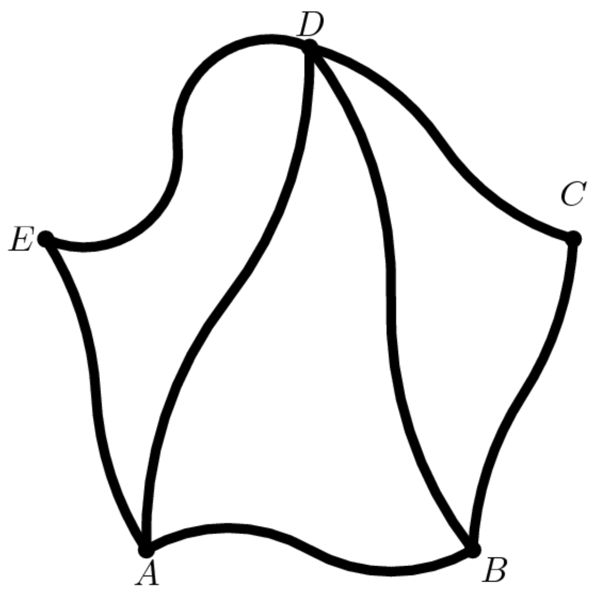
$\textbf{(D)}\ A\text{ up, }B\text{ south}$

$\textbf{(E)}\ A\text{ up, }B\text{ west}$

Solution

Problem 12

Cities A, B, C, D, and E are connected by roads $\widetilde{AB}$, $\widetilde{AD}$, $\widetilde{AE}$, $\widetilde{BC}$, $\widetilde{BD}$, $\widetilde{CD}$, and $\widetilde{DE}$. How many different routes are there from A to B that use each road exactly once? (Such a route will necessarily visit some cities more than once.)



$\textbf{(A) } 7 \qquad \textbf{(B) } 9 \qquad \textbf{(C) } 12 \qquad \textbf{(D) } 16 \qquad \textbf{(E) } 18$

Solution

Problem 13

The internal angles of quadrilateral $ABCD$ form an arithmetic progression. Triangles ABD and DCB are similar with $\angle DBA = \angle DCB$ and $\angle ADB = \angle CBD$. Moreover, the angles in each of these two triangles also form an arithmetic progression. In degrees, what is the largest possible sum of the two largest angles of $ABCD$?

$\textbf{(A) } 210 \qquad \textbf{(B) } 220 \qquad \textbf{(C) } 230 \qquad \textbf{(D) } 240 \qquad \textbf{(E) } 250$

Solution

Problem 14

Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N ?

$\textbf{(A) } 55 \qquad \textbf{(B) } 89 \qquad \textbf{(C) } 104 \qquad \textbf{(D) } 144 \qquad \textbf{(E) } 273$

Solution

Problem 15

The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!...a_m!}{b_1!b_2!...b_n!},$$

where $a_1 \ge a_2 \ge ... \ge a_m$ and $b_1 \ge b_2 \ge ... \ge b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 5$

Solution

Problem 16

Let $ABCDE$ be an equiangular convex pentagon of perimeter 1. The pairwise intersections of the lines that extend the sides of the pentagon determine a five-pointed star polygon. Let s be the perimeter of this star. What is the difference between the maximum and the minimum possible values of s?

$\textbf{(A) } 0 \qquad \textbf{(B) } \frac{1}{2} \qquad \textbf{(C) } \frac{\sqrt{5}-1}{2} \qquad \textbf{(D) } \frac{\sqrt{5}+1}{2} \qquad \textbf{(E) } \sqrt{5}$

Solution

Problem 17

Let $a, b,$ and c be real numbers such that

$$a+b+c=2, \text{ and}$$

$$a^2+b^2+c^2=12$$

What is the difference between the maximum and minimum possible values of c?

$\textbf{(A) } 2 \qquad \textbf{(B) } \frac{10}{3} \qquad \textbf{(C) } 4 \qquad \textbf{(D) } \frac{16}{3} \qquad \textbf{(E) } \frac{20}{3}$

Solution

Problem 18

Barbara and Jenna play the following game, in which they take turns. A number of coins lie on a table. When it is Barbara' s turn, she must remove 2 or 4 coins, unless only one coin remains, in which case she loses her turn. What it is Jenna' s turn, she must remove 1 or 3 coins. A coin flip determines who goes first. Whoever removes the last coin wins the game. Assume both players use their best strategy. Who will win when the game starts with 2013 coins and when the game starts with 2014 coins?

$\textbf{(A)}$ Barbara will win with 2013 coins and Jenna will win with 2014 coins.

$\textbf{(B)}$ Jenna will win with 2013 coins, and whoever goes first will win with 2014 coins.

$\textbf{(C)}$ Barbara will win with 2013 coins, and whoever goes second will win with 2014 coins.

$\textbf{(D)}$ Jenna will win with 2013 coins, and Barbara will win with 2014 coins.

$\textbf{(E)}$ Whoever goes first will win with 2013 coins, and whoever goes second will win with 2014 coins.

Solution

Problem 19

In triangle ABC, $AB=13$, $BC=14$, and $CA=15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD}\perp\overline{BC}$, $\overline{DE}\perp\overline{AC}$, and $\overline{AF}\perp\overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

$\textbf{(A) } 18 \qquad \textbf{(B) } 21 \qquad \textbf{(C) } 24 \qquad \textbf{(D) } 27 \qquad \textbf{(E) } 30$

Solution

Problem 20

For $135^\circ < x < 180^\circ$, points $P=(\cos x, \cos^2 x), Q=(\cot x, \cot^2 x), R=(\sin x, \sin^2 x)$ and $S=(\tan x, \tan^2 x)$ are the vertices of a trapezoid. What is $\sin(2x)$?

$\textbf{(A) } 2-2\sqrt{2} \qquad \textbf{(B)} 3\sqrt{3}-6 \qquad \textbf{(C) } 3\sqrt{2}-5 \qquad \textbf{(D) } -\frac{3}{4} \qquad \textbf{(E) } 1-\sqrt{3}$

Solution

Problem 21

Consider the set of 30 parabolas defined as follows: all parabolas have as focus the point (0,0) and the directrix lines have the form $y = ax + b$ with a and b integers such that $a \in \{-2, -1, 0, 1, 2\}$ and $b \in \{-3, -2, -1, 1, 2, 3\}$. No three of these parabolas have a common point. How many points in the plane are on two of these parabolas?

$\textbf{(A)}\ 720 \qquad \textbf{(B)}\ 760 \qquad \textbf{(C)}\ 810 \qquad \textbf{(D)}\ 840 \qquad \textbf{(E)}\ 870$

Solution

Problem 22

Let $m > 1$ and $n > 1$ be integers. Suppose that the product of the solutions for x of the equation

$$8(\log_n x)(\log_m x) - 7\log_n x - 6\log_m x - 2013 = 0$$

is the smallest possible integer. What is $m + n$?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 272$

Solution

Problem 23

Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers 10,444 and 3,245, and LeRoy obtains the sum $S = 13,689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Solution

Problem 24

Let ABC be a triangle where M is the midpoint of $\overline{AC}$, and $\overline{CN}$ is the angle bisector of $\angle ACB$ with N on $\overline{AB}$. Let X be the intersection of the median $\overline{BM}$ and the bisector $\overline{CN}$. In addition $\triangle BXN$ is equilateral with $AC = 2$. What is BN^2?

$\textbf{(A)}\ \frac{10-6\sqrt{2}}{7} \qquad \textbf{(B)}\ \frac{2}{9} \qquad \textbf{(C)}\ \frac{5\sqrt{2}-3\sqrt{3}}{8} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{6} \qquad \textbf{(E)}\ \frac{3\sqrt{3}-4}{5}$

Solution

Problem 25

Let G be the set of polynomials of the form

$$P(z) = z^n + c_{n-1}z^{n-1} + \cdots + c_2z^2 + c_1z + 50,$$

where $c_1, c_2, \cdots, c_{n-1}$ are integers and $P(z)$ has distinct roots of the form $a + ib$ with a and b integers. How many polynomials are in G?

$\textbf{(A)}\ 288 \qquad \textbf{(B)}\ 528 \qquad \textbf{(C)}\ 576 \qquad \textbf{(D)}\ 992 \qquad \textbf{(E)}\ 1056$

Solution

See also