2007 AMC 12A Problems

Contents

Problem 1

One ticket to a show costs 20 at full price. Susan buys 4 tickets using a coupon that gives her a 25% discount. Pam buys 5 tickets using a coupon that gives her a 30% discount. How many more dollars does Pam pay than Susan?

$\mathrm{(A)}\ 2\qquad \mathrm{(B)}\ 5\qquad \mathrm{(C)}\ 10\qquad \mathrm{(D)}\ 15\qquad \mathrm{(E)}\ 20$

Solution

Problem 2

An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

$\mathrm{(A)}\ 0.5\qquad \mathrm{(B)}\ 1\qquad \mathrm{(C)}\ 1.5\qquad \mathrm{(D)}\ 2\qquad \mathrm{(E)}\ 2.5$

Solution

Problem 3

The larger of two consecutive odd integers is three times the smaller. What is their sum?

$\mathrm{(A)}\ 4\qquad \mathrm{(B)}\ 8\qquad \mathrm{(C)}\ 12\qquad \mathrm{(D)}\ 16\qquad \mathrm{(E)}\ 20$

Solution

Problem 4

Kate rode her bicycle for 30 minutes at a speed of 16 mph, then walked for 90 minutes at a speed of 4 mph. What was her overall average speed in miles per hour?

$(A)\ 7 \qquad (B)\ 9 \qquad (C)\ 10 \qquad (D)\ 12 \qquad (E)\ 14$

Solution

Problem 5

Last year Mr. Jon Q. Public received an inheritance. He paid 20% in federal taxes on the inheritance, and paid 10% of what he had left in state taxes. He paid a total of $\$10\,500$ for both taxes. How many dollars was his inheritance?

$(A)\ 30\,000 \qquad (B)\ 32\,500 \qquad (C)\ 35\,000 \qquad (D)\ 37\,500 \qquad (E)\ 40\,000$

Solution

Problem 6

Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside triangle ABC, angle ABC measures 40 degrees, and angle ADC measures 140 degrees. What is the degree measure of angle BAD?

$(A)\ 20 \qquad (B)\ 30 \qquad (C)\ 40 \qquad (D)\ 50 \qquad (E)\ 60$

Solution

Problem 7

Let a, b, c, d, and e be five consecutive terms in an arithmetic sequence, and suppose that $a + b + c + d + e = 30$. Which of a, b, c, d, or e can be found?

$(A)\ a \qquad (B)\ b \qquad (C)\ c \qquad (D)\ d \qquad (E)\ e$

Solution

Problem 8

A star-polygon is drawn on a clock face by drawing a chord from each number to the fifth number counted clockwise from that number. That is, chords are drawn from 12 to 5, from 5 to 10, from 10 to 3, and so on, ending back at 12. What is the degree measure of the angle at each vertex in the star polygon?

$(A)\ 20 \qquad (B)\ 24 \qquad (C)\ 30 \qquad (D)\ 36 \qquad (E)\ 60$

Solution

Problem 9

Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

$(A)\ \frac{2}{3} \qquad (B)\ \frac{3}{4} \qquad (C)\ \frac{4}{5} \qquad (D)\ \frac{5}{6} \qquad (E)\ \frac{7}{8}$

Solution

Problem 10

A triangle with side lengths in the ratio $3 : 4 : 5$ is inscribed in a circle with radius 3. What is the area of the triangle?

$\text{(A) } 8.64 \qquad \text{(B) } 12 \qquad \text{(C) } 5\pi \qquad \text{(D) } 17.28 \qquad \text{(E) } 18$

Solution

Problem 11

A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with the terms 247, 475, and 756 and end with the term 824. Let S be the sum of all the terms in the sequence. What is the largest prime factor that always divides S?

$\text{(A) } 3 \qquad \text{(B) } 7 \qquad \text{(C) } 13 \qquad \text{(D) } 37 \qquad \text{(E) } 43$

Solution

Problem 12

Integers $a, b, c,$ and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad - bc$ is even?

$\text{(A) } \frac{3}{8} \qquad \text{(B) } \frac{7}{16} \qquad \text{(C) } \frac{1}{2} \qquad \text{(D) } \frac{9}{16} \qquad \text{(E) } \frac{5}{8}$

Solution

Problem 13

A piece of cheese is located at $(12, 10)$ in a coordinate plane. A mouse is at $(4, -2)$ and is running up the line $y = -5x + 18$. At the point (a, b) the mouse starts getting farther from the cheese rather than closer to it. What is $a + b$?

$\text{(A) } 6 \qquad \text{(B) } 10 \qquad \text{(C) } 14 \qquad \text{(D) } 18 \qquad \text{(E) } 22$

Solution

Problem 14

Let a, b, c, d, and e be distinct integers such that

$$(6 - a)(6 - b)(6 - c)(6 - d)(6 - e) = 45$$

What is $a + b + c + d + e$?

$\text{(A) } 5 \qquad \text{(B) } 17 \qquad \text{(C) } 25 \qquad \text{(D) } 27 \qquad \text{(E) } 30$

Solution

Problem 15

The set $\{3, 6, 9, 10\}$ is augmented by a fifth element n, not equal to any of the other four. The median of the resulting set is equal to its mean. What is the sum of all possible values of n?

$\text{(A) } 7 \qquad \text{(B) } 9 \qquad \text{(C) } 19 \qquad \text{(D) } 24 \qquad \text{(E) } 26$

Solution

Problem 16

How many three-digit numbers are composed of three distinct digits such that one digit is the average of the other two?

$\textbf{(A)}\ 96 \qquad \textbf{(B)}\ 104 \qquad \textbf{(C)}\ 112 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 256$

Solution

Problem 17

Suppose that $\sin a + \sin b = \sqrt{\frac{5}{3}}$ and $\cos a + \cos b = 1$. What is $\cos (a - b)$?

$\textbf{(A)}\ \sqrt{\frac{5}{3}} - 1 \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ 1$

Solution

Problem 18

The polynomial $f(x) = x^{4} + ax^{3} + bx^{2} + cx + d$ has real coefficients, and $f(2i) = f(2 + i) = 0$. What is $a + b + c + d$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 16$

Solution

Problem 19

Triangles ABC and ADE have areas 2007 and $7002,$ respectively, with $B = (0,0),$ $C = (223,0),$ $D = (680,380),$ and $E = (689,389).$ What is the sum of all possible x-coordinates of A?

$\textbf{(A)}\ 282 \qquad \textbf{(B)}\ 300 \qquad \textbf{(C)}\ 600 \qquad \textbf{(D)}\ 900 \qquad \textbf{(E)}\ 1200$

Solution

Problem 20

Corners are sliced off a unit cube so that the six faces each become regular octagons. What is the total volume of the removed tetrahedra?

$\textbf{(A)}\ \frac{5\sqrt{2} - 7}{3} \qquad \textbf{(B)}\ \frac{10 - 7\sqrt{2}}{3} \qquad \textbf{(C)}\ \frac{3 - 2\sqrt{2}}{3} \qquad \textbf{(D)}\ \frac{8\sqrt{2} - 11}{3} \qquad \textbf{(E)}\ \frac{6 - 4\sqrt{2}}{3}$

Solution

Problem 21

The sum of the zeros, the product of the zeros, and the sum of the coefficients of the function $f(x) = ax^{2} + bx + c$ are equal. Their common value must also be which of the following?

$\textbf{(A)}\ \text{the coefficient of }x^{2}\qquad \textbf{(B)}\ \text{the coefficient of }x$
$\textbf{(C)}\ \text{the y-intercept of the graph of }y = f(x)$
$\textbf{(D)}\ \text{one of the x-intercepts of the graph of }y = f(x)$
$\textbf{(E)}\ \text{the mean of the x-intercepts of the graph of }y = f(x)$

Solution

Problem 22

For each positive integer n, let $S(n)$ denote the sum of the digits of n. For how many values of n is $n + S(n) + S(S(n)) = 2007?$

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ 3 \qquad \mathrm{(D)}\ 4 \qquad \mathrm{(E)}\ 5$

Solution

Problem 23

Square $ABCD$ has area $36,$ and $\overline{AB}$ is parallel to the x-axis. Vertices $A,$ $B,$ and C are on the graphs of $y = \log_{a}x,$ $y = 2\log_{a}x,$ and $y = 3\log_{a}x,$ respectively. What is $a?$

$\mathrm{(A)}\ \sqrt[6]{3} \qquad \mathrm{(B)}\ \sqrt{3} \qquad \mathrm{(C)}\ \sqrt[3]{6} \qquad \mathrm{(D)}\ \sqrt{6} \qquad \mathrm{(E)}\ 6$

Solution

Problem 24

For each integer $n > 1$, let $F(n)$ be the number of solutions to the equation $\sin x = \sin (nx)$ on the interval $[0,\pi]$. What is $\sum_{n=2}^{2007} F(n)$?

$\mathrm{(A)}\ 2014524 \ \mathrm{(B)}\ 2015028 \ \mathrm{(C)}\ 2015033 \ \mathrm{(D)}\ 2016532 \ \mathrm{(E)}\ 2017033$

Solution

Problem 25

Call a set of integers spacy if it contains no more than one out of any three consecutive integers. How many subsets of $\{1,2,3,\ldots,12\},$ including the empty set, are spacy?

$\mathrm{(A)}\ 121 \qquad \mathrm{(B)}\ 123 \qquad \mathrm{(C)}\ 125 \qquad \mathrm{(D)}\ 127 \qquad \mathrm{(E)}\ 129$

Solution

See also

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