

2016 AMC 12B Problems

2016 AMC 12B (Answer Key) Printable version: AoPS Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2016) • PDF (http://www.artofproblemsolving.com/Forum/resources/files/usa/USA-AMC_12-AHSME-2016-44)
Instructions <ol style="list-style-type: none">1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator).4. Figures are not necessarily drawn to scale.5. You will have 75 minutes working time to complete the test.
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Problem 1

What is the value of $\frac{2a^{-1} + \frac{a^{-1}}{2}}{a}$ when $a = \frac{1}{2}$?

- (A) 1 (B) 2 (C) $\frac{5}{2}$ (D) 10 (E) 20

Solution

Problem 2

The harmonic mean of two numbers can be calculated as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?

- (A) 2 (B) 45 (C) 504 (D) 1008 (E) 2015

Solution

Problem 3

Let $x = -2016$. What is the value of $\left| |x| - x - |x| \right| - x$?

- (A) -2016 (B) 0 (C) 2016 (D) 4032 (E) 6048

Solution

Problem 4

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

- (A) 75 (B) 90 (C) 135 (D) 150 (E) 270

Solution

Problem 5

The War of 1812 started with a declaration of war on Thursday, June 18, 1812. The peace treaty to end the war was signed 919 days later, on December 24, 1814. On what day of the week was the treaty signed?

- (A) Friday (B) Saturday (C) Sunday (D) Monday (E) Tuesday

Solution

Problem 6

All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and \overline{BC} parallel to the x -axis. The area of the triangle is 64. What is the length of \overline{BC} ?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 16

Solution

Problem 7

Josh writes the numbers $1, 2, 3, \dots, 99, 100$. He marks out 1 , skips the next number (2), marks out 3 , and continues skipping and marking out the next number to the end of the list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6 , skips 8 , marks out 10 , and so on to the end. Josh continues in this manner until only one number remains. What is that number?

- (A) 13 (B) 32 (C) 56 (D) 64 (E) 96

Solution

Problem 8

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

- (A) 14.0 (B) 16.0 (C) 20.0 (D) 33.3 (E) 55.6

Solution

Problem 9

Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

- (A) 256 (B) 336 (C) 384 (D) 448 (E) 512

Solution

Problem 10

A quadrilateral has vertices $P(a, b)$, $Q(b, a)$, $R(-a, -b)$, and $S(-b, -a)$, where a and b are integers with $a > b > 0$. The area of $PQRS$ is 16 . What is $a + b$?

- (A) 4 (B) 5 (C) 6 (D) 12 (E) 13

Solution

Problem 11

How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$ and the line $x = 5.1$?

- (A) 30 (B) 41 (C) 45 (D) 50 (E) 57

Solution

Problem 12

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers of consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18 . What is the number in the center?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Solution

Problem 13

Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to

the airplane's altitude, in miles?

- (A) 3.5 (B) 4 (C) 4.5 (D) 5 (E) 5.5

Solution

Problem 14

The sum of an infinite geometric series is a positive number S , and the second term in the series is $\mathbf{1}$. What is the smallest possible value of S ?

- (A) $\frac{1 + \sqrt{5}}{2}$ (B) 2 (C) $\sqrt{5}$ (D) 3 (E) 4

Solution

Problem 15

All the numbers $\mathbf{2, 3, 4, 5, 6, 7}$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

- (A) 312 (B) 343 (C) 625 (D) 729 (E) 1680

Solution

Problem 16

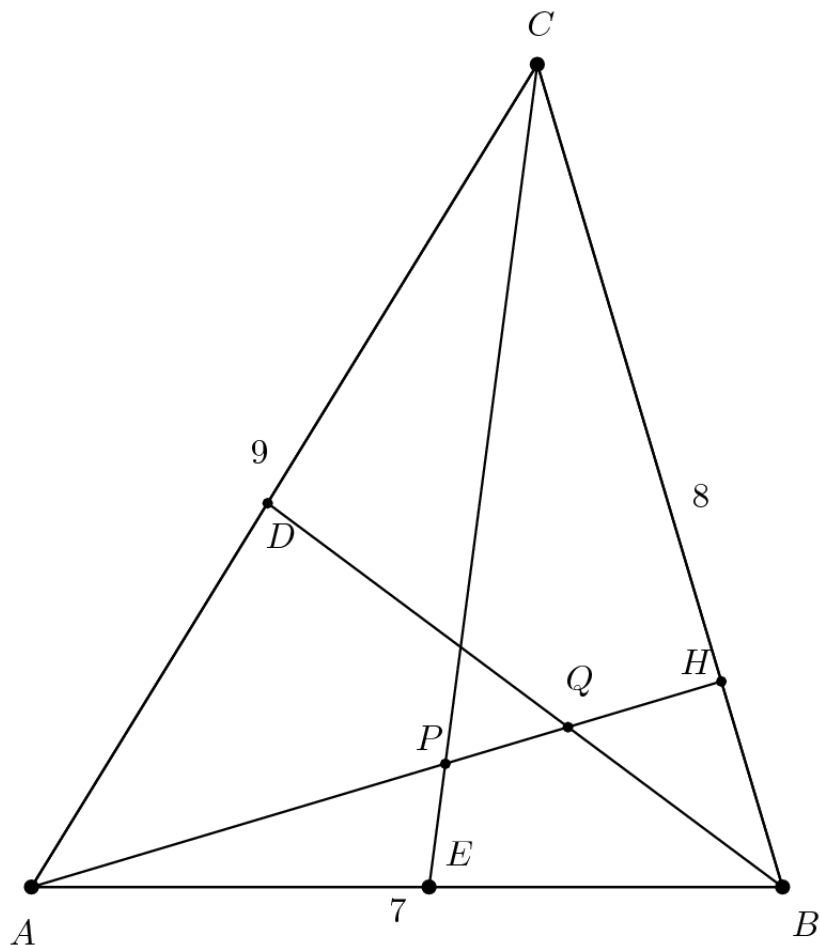
In how many ways can $\mathbf{345}$ be written as the sum of an increasing sequence of two or more consecutive positive integers?

- (A) 1 (B) 3 (C) 5 (D) 6 (E) 7

Solution

Problem 17

In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and \overline{AH} is an altitude. Points D and E lie on sides \overline{AC} and \overline{AB} , respectively, so that \overline{BD} and \overline{CE} are angle bisectors, intersecting \overline{AH} at Q and P , respectively. What is PQ ?



- (A) 1 (B) $\frac{5}{8}\sqrt{3}$ (C) $\frac{4}{5}\sqrt{2}$ (D) $\frac{8}{15}\sqrt{5}$ (E) $\frac{6}{5}$

Solution

Problem 18

What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

- (A) $\pi + \sqrt{2}$ (B) $\pi + 2$ (C) $\pi + 2\sqrt{2}$ (D) $2\pi + \sqrt{2}$ (E) $2\pi + 2\sqrt{2}$

Solution

Problem 19

Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

- (A) $\frac{1}{8}$ (B) $\frac{1}{7}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

Solution

Problem 20

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B , B beat C , and C beat A ?

- (A) 385 (B) 665 (C) 945 (D) 1140 (E) 1330

Solution

Problem 21

Let $ABCD$ be a unit square. Let Q_1 be the midpoint of \overline{CD} . For $i = 1, 2, \dots$, let P_i be the intersection of $\overline{AQ_i}$ and \overline{BD} , and let Q_{i+1} be the foot of the perpendicular from P_i to \overline{CD} . What is

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i?$$

- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 1

Solution

Problem 22

For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period of 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. In which interval does n lie?

- (A) [1, 200] (B) [201, 400] (C) [401, 600] (D) [601, 800] (E) [801, 999]

Solution

Problem 23

What is the volume of the region in three-dimensional space defined by the inequalities

$$|x| + |y| + |z| \leq 1 \text{ and } |x| + |y| + |z - 1| \leq 1$$

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 1

Solution

Problem 24

There are exactly 77,000 ordered quadruplets (a, b, c, d) such that $\gcd(a, b, c, d) = 77$ and $\text{lcm}(a, b, c, d) = n$. What is the smallest possible value for n ?

- (A) 13,860 (B) 20,790 (C) 21,560 (D) 27,720 (E) 41,580

Solution

Problem 25

The sequence (a_n) is defined recursively by $a_0 = 1$, $a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \geq 2$. What is the smallest positive integer k such that the product $a_1a_2 \cdots a_k$ is an integer?

- (A) 17 (B) 18 (C) 19 (D) 20 (E) 21

Solution

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