2008 AMC 12A Problems

Contents

Problem 1

A bakery owner turns on his doughnut machine at 8:30 AM. At 11:10 AM the machine has completed one third of the day's job. At what time will the doughnut machine complete the job?

$\mathrm{(A)}\ \text{1:50 PM} \qquad \mathrm{(B)}\ \text{3:00 PM} \qquad \mathrm{(C)}\ \text{3:30 PM} \qquad \mathrm{(D)}\ \text{4:30 PM} \qquad \mathrm{(E)}\ \text{5:50 PM}$

Solution

Problem 2

What is the reciprocal of $\frac{1}{2}+\frac{2}{3}$?

$\mathrm{(A)}\ \frac{6}{7} \qquad \mathrm{(B)}\ \frac{7}{6} \qquad \mathrm{(C)}\ \frac{5}{3} \qquad \mathrm{(D)}\ 3 \qquad \mathrm{(E)}\ \frac{7}{2}$

Solution

Problem 3

Suppose that $\tfrac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much as $\tfrac{1}{2}$ of 5 bananas?

$\mathrm{(A)}\ 2 \qquad \mathrm{(B)}\ \frac{5}{2} \qquad \mathrm{(C)}\ 3 \qquad \mathrm{(D)}\ \frac{7}{2} \qquad \mathrm{(E)}\ 4$

Solution

Problem 4

Which of the following is equal to the product

$$\frac{8}{4}\cdot\frac{12}{8}\cdot\frac{16}{12}\cdot\cdots\cdot\frac{4n+4}{4n}\cdot\cdots\cdot\frac{2008}{2004}?$$

(A) 251 (B) 502 (C) 1004 (D) 2008 (E) 4016

Solution

Problem 5

Suppose that

$$\frac{2x}{3} - \frac{x}{6}$$

is an integer. Which of the following statements must be true about x?

(A) It is negative. (B) It is even, but not necessarily a multiple of 3.
(C) It is a multiple of 3, but not necessarily even.
(D) It is a multiple of 6, but not necessarily a multiple of 12.
(E) It is a multiple of 12.

Solution

Problem 6

Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a $\$90$ rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves $\$15$ by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

(A) 750 (B) 900 (C) 1000 (D) 1050 (E) 1500

Solution

Problem 7

While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Solution

Problem 8

What is the volume of a cube whose surface area is twice that of a cube with volume 1?

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4 (E) 8

Solution

Problem 9

Older television screens have an aspect ratio of $4:3$. That is, the ratio of the width to the height is $4:3$. The aspect ratio of many movies is not $4:3$, so they are sometimes shown on a television screen by "letterboxing" - darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of $2:1$ and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?

(A) 2 (B) 2.25 (C) 2.5 (D) 2.7 (E) 3

Solution

Problem 10

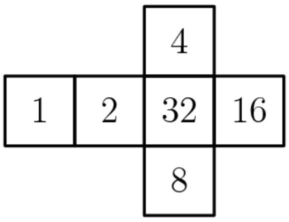
Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t?

(A) $\left(\frac{1}{5}+\frac{1}{7}\right)(t+1)=1$ (B) $\left(\frac{1}{5}+\frac{1}{7}\right)t+1=1$ (C) $\left(\frac{1}{5}+\frac{1}{7}\right)t=1$
(D) $\left(\frac{1}{5}+\frac{1}{7}\right)(t-1)=1$ (E) $(5+7)t=1$

Solution

Problem 11

Three cubes are each formed from the pattern shown. They are then stacked on a table one on top of another so that the 13 visible numbers have the greatest possible sum. What is that sum?



(A) 154 (B) 159 (C) 164 (D) 167 (E) 189

Solution

Problem 12

A function f has domain $[0,2]$ and range $[0,1]$. (The notation $[a,b]$ denotes $\{x : a \le x \le b\}$.) What are the domain and range, respectively, of the function g defined by $g(x)=1-f(x+1)$?

(A) $[-1,1],[-1,0]$ (B) $[-1,1],[0,1]$ **(C)** $[0,2],[-1,0]$ (D) $[1,3],[-1,0]$ (E) $[1,3],[0,1]$

Solution

Problem 13

Points A and B lie on a circle centered at O, and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

(A) $\frac{1}{16}$ (B) $\frac{1}{9}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

Solution

Problem 14

What is the area of the region defined by the inequality $|3x-18|+|2y+7|\le 3$?

(A) 3 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 5

Solution

Problem 15

Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Solution

Problem 16

The numbers $\log(a^3b^7)$, $\log(a^5b^{12})$, and $\log(a^8b^{15})$ are the first three terms of an arithmetic sequence, and the 12^{th} term of the sequence is $\log(b^n)$. What is n?

(A) 40 (B) 56 (C) 76 (D) 112 (E) 143

Solution

Problem 17

Let $a_1, a_2, \ldots$ be a sequence determined by the rule $a_n = a_{n-1}/2$ if a_{n-1} is even and $a_n = 3a_{n-1} + 1$ if a_{n-1} is odd. For how many positive integers $a_1 \leq 2008$ is it true that a_1 is less than each of a_2, a_3, and a_4?

$\mathrm{(A)}\ 250 \qquad \mathrm{(B)}\ 251 \qquad \mathrm{(C)}\ 501 \qquad \mathrm{(D)}\ 502 \qquad \mathrm{(E)}1004$

Solution

Problem 18

A triangle $\triangle ABC$ with sides 5, 6, 7 is placed in the three-dimensional plane with one vertex on the positive x axis, one on the positive y axis, and one on the positive z axis. Let O be the origin. What is the volume of $OABC$?

$\mathrm{(A)}\ \sqrt{85} \qquad \mathrm{(B)}\ \sqrt{90} \qquad \mathrm{(C)}\ \sqrt{95} \qquad \mathrm{(D)}\ 10 \qquad \mathrm{(E)}\ \sqrt{105}$

Solution

Problem 19

In the expansion of

$$\left(1 + x + x^2 + \cdots + x^{27}\right)\left(1 + x + x^2 + \cdots + x^{14}\right)^2,$$

what is the coefficient of x^{28}?

$\mathrm{(A)}\ 195 \qquad \mathrm{(B)}\ 196 \qquad \mathrm{(C)}\ 224 \qquad \mathrm{(D)}\ 378 \qquad \mathrm{(E)}\ 405$

Solution

Problem 20

Triangle ABC has $AC = 3$, $BC = 4$, and $AB = 5$. Point D is on $\overline{AB}$, and $\overline{CD}$ bisects the right angle. The inscribed circles of $\triangle ADC$ and $\triangle BCD$ have radii r_a and r_b, respectively. What is r_a/r_b?

$\mathrm{(A)}\ \frac{1}{28}\left(10 - \sqrt{2}\right) \qquad \mathrm{(B)}\ \frac{3}{56}\left(10 - \sqrt{2}\right) \qquad \mathrm{(C)}\ \frac{1}{14}\left(10 - \sqrt{2}\right) \qquad \mathrm{(D)}\ \frac{5}{56}\left(10 - \sqrt{2}\right)$
$\mathrm{(E)}\ \frac{3}{28}\left(10 - \sqrt{2}\right)$

Solution

Problem 21

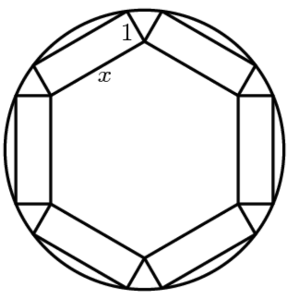
A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?

$\mathrm{(A)}\ 36 \qquad \mathrm{(B)}\ 40 \qquad \textbf{(C)}\ 44 \qquad \mathrm{(D)}\ 48 \qquad \mathrm{(E)}\ 52$

Solution

Problem 22

A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x. Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?



$\text{(A) } 2\sqrt{5}-\sqrt{3} \qquad \text{(B) } 3 \qquad \text{(C) } \frac{3\sqrt{7}-\sqrt{3}}{2} \qquad \text{(D) } 2\sqrt{3} \qquad \text{(E) } \frac{5+2\sqrt{3}}{2}$

Solution

Problem 23

The solutions of the equation $z^4+4z^3i-6z^2-4zi-i=0$ are the vertices of a convex polygon in the complex plane. What is the area of the polygon?

$\text{(A) } 2^{\frac{5}{8}} \qquad \text{(B) } 2^{\frac{3}{4}} \qquad \text{(C) } 2 \qquad \text{(D) } 2^{\frac{5}{4}} \qquad \text{(E) } 2^{\frac{3}{2}}$

Solution

Problem 24

Triangle ABC has $\angle C = 60^{\circ}$ and $BC = 4$. Point D is the midpoint of BC. What is the largest possible value of $\tan{\angle BAD}$?

$\text{(A) } \frac{\sqrt{3}}{6} \qquad \text{(B) } \frac{\sqrt{3}}{3} \qquad \text{(C) } \frac{\sqrt{3}}{2\sqrt{2}} \qquad \text{(D) } \frac{\sqrt{3}}{4\sqrt{2}-3} \qquad \text{(E) } 1$

Solution

Problem 25

A sequence (a_1,b_1), (a_2,b_2), (a_3,b_3), $\ldots$ of points in the coordinate plane satisfies

$(a_{n+1}, b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n)$ for $n = 1,2,3,\ldots$.

Suppose that $(a_{100},b_{100}) = (2,4)$. What is $a_1 + b_1$?

$\text{(A) } -\frac{1}{2^{97}} \qquad \text{(B) } -\frac{1}{2^{99}} \qquad \text{(C) } 0 \qquad \text{(D) } \frac{1}{2^{98}} \qquad \text{(E) } \frac{1}{2^{96}}$

Solution