2015 AMC 10A

Problem 1

What is the value of $(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?

$(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?的值是多少？

(A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

Problem 2

A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges total. How many square tiles are there in the box?

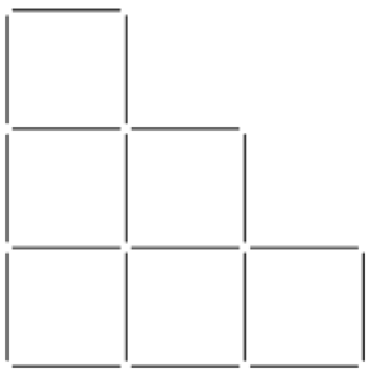
一个盒子装有一堆三角形形状和正方形形状的瓷砖，盒子里总共有 25 块瓷砖，含有 84 条边。那么盒子里有多少块正方形瓷砖?

(A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

Problem 3

Ann made a 3-step staircase using 18 toothpicks as shown in the figure. How many toothpicks does she need to add to complete a 5-step staircase?

Ann 用 18 根牙签做了三阶楼梯，如下图所示，她还需要添加多少根牙签就可以做成五阶楼梯？



(A) 9 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

Problem 4

Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

Pablo，Sofia 和 Mia 在一个聚会上得到了一些糖果蛋。Pablo 的糖果蛋的数量是 Sofia 的 3 倍，Sofia 的蛋的数量是 Mia 的 2 倍。Pablo 决定拿出一些蛋给 Sofia 和 Mia 这样三个人拥有的蛋的数目就一样多了。那么 Pablo 需要拿出他现有蛋个数的几分之几给 Sofia?

$\textbf{(A)}\ \frac{1}{12}\qquad\textbf{(B)}\ \frac{1}{6}\qquad\textbf{(C)}\ \frac{1}{4}\qquad\textbf{(D)}\ \frac{1}{3}\qquad\textbf{(E)}\ \frac{1}{2}$

Problem 5

Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the test average became 81. What was Payton's score on the test?

Patrick 先生给 15 个学生上数学课。他正批改考试试卷，发现当他把除了 Payton 外的所有其他学生的试卷都改好后，班级的平均分是 80，当他批改好 Payton 的考试试卷后，班级的平均分变成了 81 分。那么 Payton 这次考试得分是多少?

$\textbf{(A)}\ 81\qquad\textbf{(B)}\ 85\qquad\textbf{(C)}\ 91\qquad\textbf{(D)}\ 94\qquad\textbf{(E)}\ 95$

Problem 6

The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller number?

两个正数之和是它们之差的 5 倍。那么较大数与较小数的比值是多少?

$\textbf{(A)}\ \frac{5}{4}\qquad\textbf{(B)}\ \frac{3}{2}\qquad\textbf{(C)}\ \frac{9}{5}\qquad\textbf{(D)}\ 2\qquad\textbf{(E)}\ \frac{5}{2}$

Problem 7

How many terms are there in the arithmetic sequence $13, 16, 19, \dots, 70, 73$?

等差数列 13，16，19，…，70，73 有多少项?

$\textbf{(A)}\ 20\qquad\textbf{(B)}\ 21\qquad\textbf{(C)}\ 24\qquad\textbf{(D)}\ 60\qquad\textbf{(E)}\ 61$

Problem 8

Two years ago Pete was three times as old as his cousin Claire. 2 years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2:1$?

两年前 Pete 的年龄是他表妹 Claire 年龄的 3 倍。从那时候再往前推两年，Pete 的年龄是 Claire 年龄的 4 倍。多少年后他们年龄的比值将是 2:1?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 9

Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

2 个正圆柱体的体积相同，第二个圆柱体的半径比第一个的半径多 10%，这 2 个圆柱体的高有什么关系？

（A）The second height is 10％ less than the first丨第二个的高比第一个高少 10％

（B）The first height is 10％ more than the second. 丨第一个的高比第二个高多 10％

（C）The second height is 21％ less than the first. 丨第二个的高比第一个高少 21％

（D）The first height is 21％ more than the second丨第一个的高比第二个高多 21％

（E）The second height is 80％ of the first. 丨第二个的高是第一个高的 80％

Problem 10

How many rearrangements of $abcd$ are there in which no two adjacent letters are also adjacent letters in the alphabet? For example, no such rearrangements could include either ab or ba.

有多少种 *abcd* 的排列，满足不存在两个相邻的字母在字母表里也相邻？例如，这样的排列里不应该包含 *ab* 或者 *ba*.

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 11

The ratio of the length to the width of a rectangle is $4:3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

一个长方形的长和宽之比为 4：3。如果长方形的对角线的长度是 *d*，那么它的面积可以写成 kd^2，这里 *k* 是常数。则 *k* 是多少?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

Problem 12

Points $(\sqrt{\pi}, a)$ and $(\sqrt{\pi}, b)$ are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a - b|$?

点$(\sqrt{\pi}, a)$和$(\sqrt{\pi}, b)$是方程$y^2 + x^4 = 2x^2y + 1$的图像上的两个不同的点。$|a - b|$的值是多少？

(A) 1 **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\sqrt{1+\pi}$ **(E)** $1+\sqrt{\pi}$

Problem 13

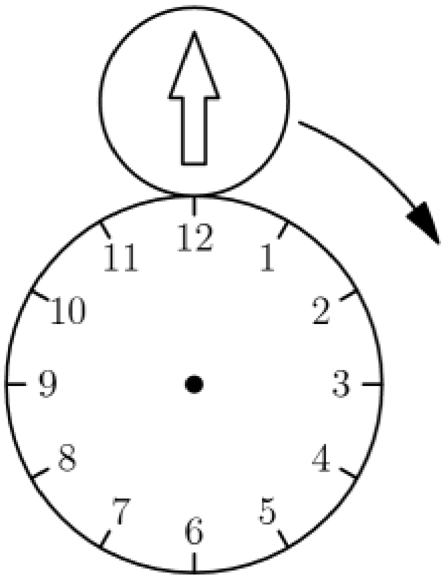
Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

Claudia 有 12 枚硬币，它们都是 5 分或者 10 分的硬币。拿 1 枚或者多枚硬币进行组合，可以得到 17 个不同的面值。那么 Claudia 的 10 分的硬币有多少个?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Problem 14

The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



下面这张图展示了一个半径为 20cm 的钟的圆盘面，还有一个半径为 10cm 的圆盘，和钟的圆盘面外切在 12 点的位置。这个圆盘上画了一个箭头，一开始竖直向上指.让圆盘绕着钟的圆盘面顺时针转动。当转到钟的圆盘面的哪个点，箭头会再次竖直向上指？

(A) $2o'clock$ **(B)** $3o'clock$ **(C)** $4o'clock$ **(D)** $6o'clock$ **(E)** $8o'clock$

Problem 15

Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

考虑分数$\frac{x}{y}$,所组成的集合，这里 *x* 和 *y* 是互质的正整数，这里面有多少个分数具有这个性质：当把分子和分母都增加 1，分数值增加 10%？

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

Problem 16

If $y+4=(x-2)^2, x+4=(y-2)^2$, and $x \neq y$, what is the value of x^2+y^2?

如果$y+4=(x-2)^2, x+4=(y-2)^2$，且 $x\neq y$，那么x^2+y^2的值是多少？

$\textbf{(A) } 10 \qquad \textbf{(B) } 15 \qquad \textbf{(C) } 20 \qquad \textbf{(D) } 25 \qquad \textbf{(E) } 30$

Problem 17

A line that passes through the origin intersects both the line $x=1$ and the line $y=1+\frac{\sqrt{3}}{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

一条通过原点的直线和直线$x=1$，$y=1+\frac{\sqrt{3}}{3}x$相交。这 3 条线形成一个等边三角形。这个三角形的周长是多少？

$\textbf{(A) } 2\sqrt{6} \qquad \textbf{(B) } 2+2\sqrt{3} \qquad \textbf{(C) } 6 \qquad \textbf{(D) } 3+2\sqrt{3} \qquad \textbf{(E) } 6+\frac{\sqrt{3}}{3}$

Problem 18

Hexadecimal (base-16) numbers are written using numeric digits 0 through 9 as well as the letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n?

十六进制的数的表示使用了数字 0 到 9，和字母 *A* 到 *F* 来代表 10 到 15。在前 1000 个正整数中，有 *n* 个数它们的十六进制表示只含有数字而不含有字母，那么 *n* 的各个位上数字之和是多少？

$\textbf{(A) } 17 \qquad \textbf{(B) } 18 \qquad \textbf{(C) } 19 \qquad \textbf{(D) } 20 \qquad \textbf{(E) } 21$

Problem 19

The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

等腰直角三角形 *ABC* 的直角顶点在 *C*，面积为 12.5。三等分$\angle ACB$的射线交 *AB* 于 *D* 和 *E*。则$\triangle CDE$的面积是多少？

$\textbf{(A) }\frac{5\sqrt{2}}{3}\qquad\textbf{(B) }\frac{50\sqrt{3}-75}{4}\qquad\textbf{(C) }\frac{15\sqrt{3}}{8}\qquad\textbf{(D) }\frac{50-25\sqrt{3}}{2}\qquad\textbf{(E) }\frac{25}{6}$

Problem 20

A rectangle with positive integer side lengths in cm has area A cm^2 and perimeter P cm. Which of the following numbers cannot equal $A+P$?

一个边长为正整数的长方形，面积为A cm^2，周长为P cm。下面哪个不可能等于 A+P?

$\textbf{(A) }100\qquad\textbf{(B) }102\qquad\textbf{(C) }104\qquad\textbf{(D) }106\qquad\textbf{(E) }108$

NOTE:

As it originally appeared in the AMC 10, this problem was stated incorrectly and had no answer; it has been modified here to be solvable. This is the original question:

正如它最初出现在 AMC 10 中一样，这个问题被错误地陈述并且没有答案。它已在此处修改。 这是原始问题：

A rectangle with side lengths in cm has an area of integer A cm^2 and a perimeter of integer P cm. Which of the following numbers cannot equal $A+P$?

$\textbf{(A) }100\qquad\textbf{(B) }102\qquad\textbf{(C) }104\qquad\textbf{(D) }106\qquad\textbf{(E) }108$

Problem 21

Tetrahedron $ABCD$ has $AB=5$, $AC=3$, $BC=4$, $BD=4$, $AD=3$, and $CD=\tfrac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

四面体 *ABCD* 有 *AB*=5，*AC*=3，*BC*=4，*BD*=4，*AD*=3，$CD=\tfrac{12}{5}\sqrt{2}$。这个四面体的体积为多少？

$\textbf{(A) }3\sqrt{2}\qquad\textbf{(B) }2\sqrt{5}\qquad\textbf{(C) }\frac{24}{5}\qquad\textbf{(D) }3\sqrt{3}\qquad\textbf{(E) }\frac{24}{5}\sqrt{2}$

Problem 22

Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

八个人坐在一张圆桌旁，每个人都手持一枚硬币。8 个人一起扔硬币，扔到正面朝上的则站起来，扔到反面朝上的仍然坐着。没有相邻的两个人同时站着的概率是多少？

$\textbf{(A)}\dfrac{47}{256} \qquad \textbf{(B)}\dfrac{3}{16} \qquad \textbf{(C)}\ \dfrac{49}{256} \qquad \textbf{(D)}\ \dfrac{25}{128} \qquad \textbf{(E)}\ \dfrac{51}{256}$

Problem 23

The zeroes of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

函数$f(x) = x^2 - ax + 2a$的零点都是整数，那么 a 的所有可能值之和为多少？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 18$

Problem 24

For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

对于某些正整数 p，存在一个四边形 $ABCD$，使得它的边长都是正整数，周长是 p，直角顶点为 B 和 C，AB=2 ，CD=AD。若 p<2015，满足条件的 p 值有多少个？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 61 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 63$

Problem 25

Let S be a square of side length 1. Two points are chosen at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers with $\gcd(a,b,c)=1$. What is $a+b+c$?

S 是边长为 1 的正方形。从 *S* 的边上随机选择 2 个点。这两个点的直线距离至少是$\frac{1}{2}$ 的概率是$\frac{a-b\pi}{c}$，其中 *a*，*b*，*c* 都是正整数，$\gcd(a,b,c)=1$。那么$a+b+c$的值是多少？

$\textbf{(A)}\ 59 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 61 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 63$

2015 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	D	D	B	E	B	B	B	D	C	C	C	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	B	B	D	E	D	B	C	A	C	B	A	

(Note:Problem20 ,this problem was originally stated incorrectly, and all contestants received full credit regardless of their answer)