2014 AMC 8 Problems/Problem 1

Problem

Harry and Terry are each told to calculate $8-(2+5)$. Harry gets the correct answer. Terry ignores the parentheses and calculates $8-2+5$. If Harry's answer is H and Terry's answer is T, what is $H-T$?

$\textbf{(A) }-10\qquad\textbf{(B) }-6\qquad\textbf{(C) }0\qquad\textbf{(D) }6\qquad \textbf{(E) }10$

Solution

We have $H=8-7=1$ and $T=8-2+5=11$. Clearly $1-11=-10$, so our answer is $\boxed{\textbf{(A)}-10}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by First Problem	Followed by Problem 2
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 2

Problem

Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad \textbf{(E)}\ 5$

Solution

The fewest amount of coins that can be used is 2 (a quarter and a dime). The greatest amount is 7, if he only uses nickels. Therefore we have $7-2=\boxed{\textbf{(E)}\ 5}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 1	Followed by Problem 3
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 3

Problem

Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?

$\textbf{(A) }240\qquad\textbf{(B) }250\qquad\textbf{(C) }260\qquad\textbf{(D) }270\qquad \textbf{(E) }280$

Solution

Isabella read $3\cdot 36+3\cdot 44$ pages in the first 6 days. Although this can be calculated directly, it is simpler to calculate it as $3\cdot (36+44)=3\cdot 80$, which gives that she read 240 pages. However, she read 10 more pages on the last day, for a total of $240+10=\boxed{\textbf{(B) }250}$ pages.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 2	Followed by Problem 4
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 4

Problem

The sum of two prime numbers is 85. What is the product of these two prime numbers?

$\textbf{(A) }85\qquad\textbf{(B) }91\qquad\textbf{(C) }115\qquad\textbf{(D) }133\qquad \textbf{(E) }166$

Solution

Since the two prime numbers sum to an odd number, one of them must be even. The only even prime number is 2. The other prime number is $85-2=83$, and the product of these two numbers is $83\cdot 2=\boxed{\textbf{(E) }166}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 3	Followed by Problem 5
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 5

Problem

Margie's car can go 32 miles on a gallon of gas, and gas currently costs $\$4$ per gallon. How many miles can Margie drive on $\$20$ worth of gas?

$\textbf{(A) } 64 \qquad\textbf{(B) } 128 \qquad\textbf{(C) } 160 \qquad\textbf{(D) } 320 \qquad\textbf{(E) } 640$

Solution

Margie can afford $20/4=5$ gallons of gas. She can go $32\cdot 5=\boxed{\textbf{(C) } 160}$ miles on this amount of gas.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 4	Followed by Problem 6
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 6

Problem

Six rectangles each with a common base width of 2 have lengths of $1, 4, 9, 16, 25$, and 36. What is the sum of the areas of the six rectangles?

$\textbf{(A) }91\qquad\textbf{(B) }93\qquad\textbf{(C) }162\qquad\textbf{(D) }182\qquad \textbf{(E) }202$

Solution

The sum of the areas is equal to $2*1+2*4+2*9+2*16+2*25+2*36$. This is simply equal to $2*(1+4+9+16+25+36)$, which is equal to $2*91$, which is equal to our final answer of $\boxed{\textbf{(D) }182}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 5	Followed by Problem 7
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 7

Problem

There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of number of girls to the number of boys in her class?

$\textbf{(A) }3:4\qquad\textbf{(B) }4:3\qquad\textbf{(C) }3:2\qquad\textbf{(D) }7:4\qquad \textbf{(E) }2:1$

Solution

We can set up an equation with x being the number of girls in the class. The number of boys in the class is equal to $x-4$. Since the total number of students is equal to 28, we get $x+x-4=28$. Solving this equation, we get $x=16$. There are $16-4=12$ boys in our class, and our answer is $16:12=\boxed{\textbf{(B) }4:3}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 6	Followed by Problem 8
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 8

Problem

Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\$\underline{1A2}$. What is the missing digit A of this 3-digit number?

$\textbf{(A) }0\qquad\textbf{(B) }1\qquad\textbf{(C) }2\qquad\textbf{(D) }3\qquad \textbf{(E) }4$

Solution

A number is divisible by 11 if the difference between the sum of the digits in the odd-numbered slots (e.g. the ones slot, the hundreds slot, etc.) and the sum in the even-numbered slots (e.g. the tens slot, the thousands slot) is a multiple of 11. So $1+2-A$ is equivalent to $0 \pmod{11}$. Clearly 3 , (A) cannot be equal to 11 or any multiple of 11 greater than that. So $3-A=0\longrightarrow A=\boxed{\textbf{(D) }3}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 7	Followed by Problem 9
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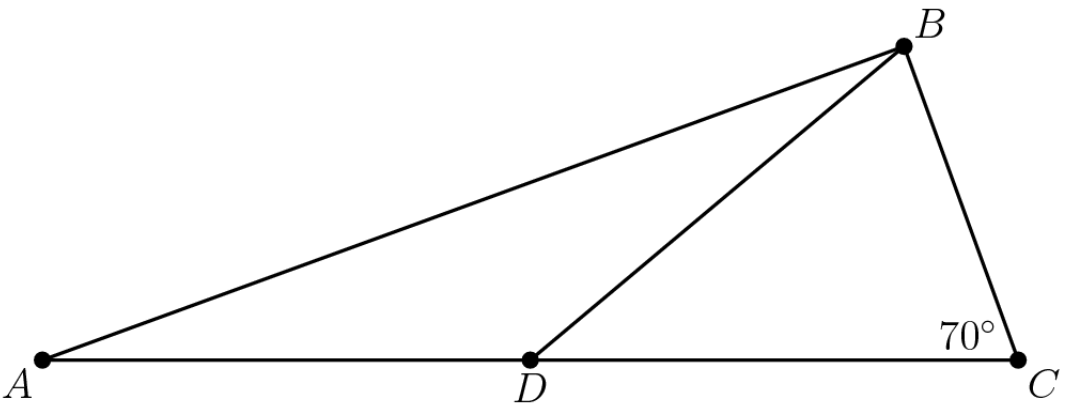
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 9

Problem

In $\triangle ABC$, D is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures 70°. What is the degree measure of $\angle ADB$?



$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 120 \qquad \textbf{(C)}\ 135 \qquad \textbf{(D)}\ 140 \qquad \textbf{(E)}\ 150$

Solution

BD = DC, so angle DBC = angle DCB = 70. Then CDB = 40. Since angle ADB and BDC are supplementary, $ADB = 180 - 40 = \boxed{\textbf{(D)}\ 140}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 8	Followed by Problem 10
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 10

Problem

The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?

$\textbf{(A)}\ 1979\qquad\textbf{(B)}\ 1980\qquad\textbf{(C)}\ 1981\qquad\textbf{(D)}\ 1982\qquad \textbf{(E)}\ 1983$

Solution

The seventh AMC 8 would have been given in 1991. If Samantha was 12 then, that means she was born 12 years ago. So therefore she was born in $\text{(A)}1979.$

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 9	Followed by Problem 11
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2014 AMC 8 Problems/Problem 11

Contents

Problem

Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks?

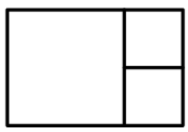
$\textbf{(A) }4\qquad\textbf{(B) }5\qquad\textbf{(C) }6\qquad\textbf{(D) }8\qquad\textbf{(E) }10$

Solution

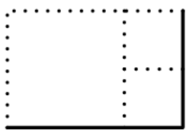
We can apply complementary counting and count the paths that DO go through the blocked intersection, which is $\dbinom{2}{1}\dbinom{3}{1}=6$. There are a total of $\dbinom{5}{2}=10$ paths, so there are $10-6=4$ paths possible. $\boxed{A}$ is the correct answer.

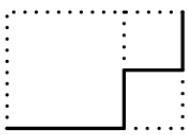
Solution 2

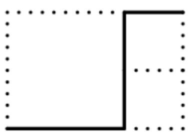
We can make a diagram of the roads available to Jack.

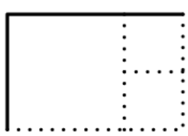


Then, we can simply list the possible routes.









There are 4 possible routes, so our answer is $\boxed{A}$.

2014 AMC 8 Problems/Problem 12

Contents

Problem

A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. readers were asked to match each celebrity with the correct baby pictures. What is the probability that a reader guessing at random will match all three correctly?

$\textbf{(A) }\frac{1}{9}\qquad\textbf{(B) }\frac{1}{6}\qquad\textbf{(C) }\frac{1}{4}\qquad\textbf{(D) }\frac{1}{3}\qquad\textbf{(E) }\frac{1}{2}$

Solution

Let's call the celebrities A, B, and C. There is a $\frac{1}{3}$ chance that celebrity A's picture will be selected, and a $\frac{1}{3}$ chance that his baby picture will be selected. That means there are two celebrities left. There is now a $\frac{1}{2}$ chance that celebrity B's picture will be selected, and another $\frac{1}{2}$ chance that his baby picture will be selected. This leaves a $\frac{1}{1}$ chance for the last celebrity, so the total probability is $\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{36}$. However, the order of the celebrities doesn't matter, so the final probability will be $3!\cdot\frac{1}{36}=\frac{1}{6}$ (B).

Solution 2

There is a $\frac{1}{3}$ chance that the reader will choose the correct baby picture for the first person. Next, the second person gives a $\frac{1}{2}$ chance, and the last person leaves only 1 choice. Thus, the probability is $\frac{1}{3\cdot 2}=\boxed{\text{(B) }\frac{1}{6}}$.

Solution 3

There are $3!$ ways assign the pictures to each of the celebrities. There is one favorable outcome where all of them are matched correctly, so the answer is $\boxed{\textbf{(B) }\frac{1}{6}}$.

See Also

2014 AMC 8 Problems/Problem 13

Problem

If n and m are integers and n^2+m^2 is even, which of the following is impossible?

$\textbf{(A)}$ n and m are even $\textbf{(B)}$ n and m are odd $\textbf{(C)}$ $n+m$ is even $\textbf{(D)}$ $n+m$ is odd
$\textbf{(E)}$ none of these are impossible

Solution

Since n^2+m^2 is even, either both n^2 and m^2 are even, or they are both odd. Therefore, n and m are either both even or both odd, since the square of an even number is even and the square of an odd number is odd. As a result, $n+m$ must be even. The answer, then, is $\boxed{D}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
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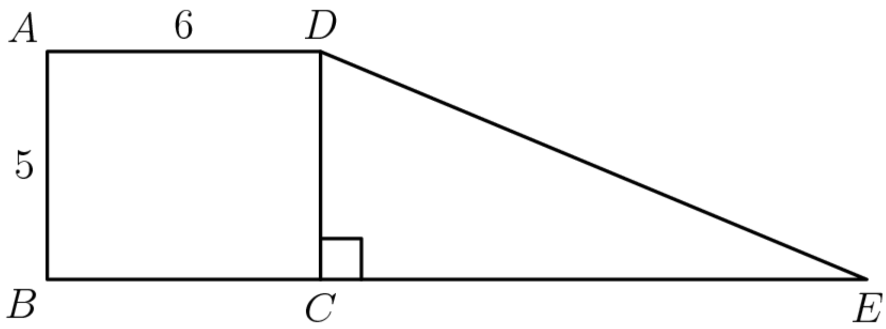
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 14

Problem

Rectangle $ABCD$ and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE?



$\textbf{(A) }12\qquad\textbf{(B) }13\qquad\textbf{(C) }14\qquad\textbf{(D) }15\qquad\textbf{(E) }16$

Solution

The area of $\triangle CDE$ is $\frac{DC\cdot CE}{2}$. The area of $ABCD$ is $AB\cdot AD=5\cdot 6=30$, which also must be equal to the area of $\triangle CDE$, which, since $DC=5$, must in turn equal $\frac{5\cdot CE}{2}$. Through transitivity, then, $\frac{5\cdot CE}{2}=30$, and $CE=12$. It should be apparent that $\triangle CDE$ is a $5-12-13$ triangle, so $DE=\boxed{13}$, or $\boxed{B}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
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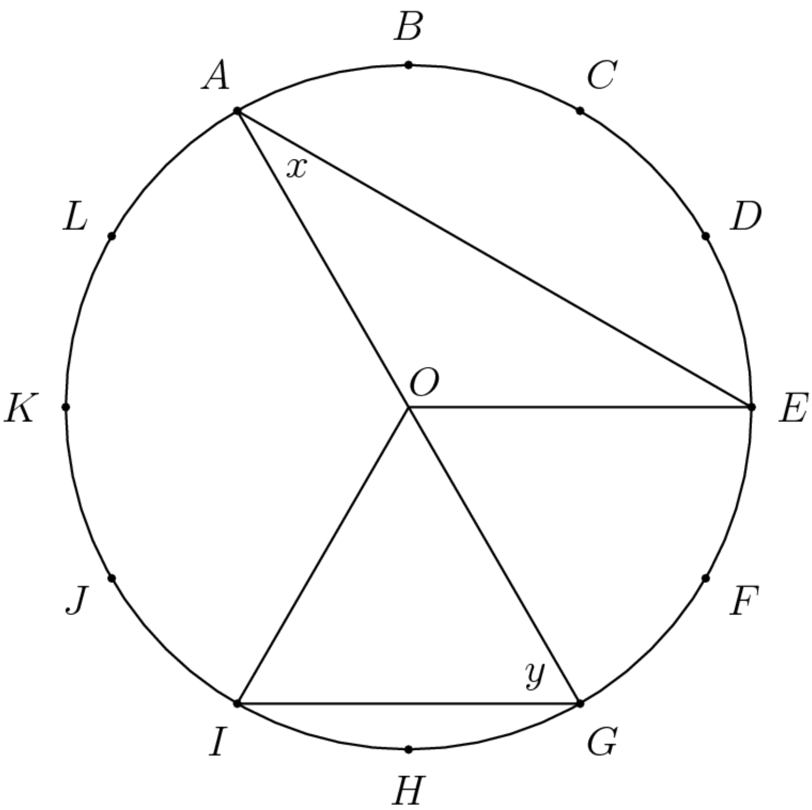
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 15

Problem

The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y?



$\textbf{(A)}\ 75 \qquad\textbf{(B)}\ 80 \qquad\textbf{(C)}\ 90 \qquad\textbf{(D)}\ 120 \qquad\textbf{(E)}\ 150$

Solution

For this problem, it is useful to know that the measure of an inscribed angle is half the measure of its corresponding central angle. Since each unit arc is $\frac{1}{12}$ of the circle's circumference, each unit central angle measures $\left(\frac{360}{12}\right)^{\circ} = 30^{\circ}$. Then, we know that the inscribed arc of $\angle x = 60^{\circ}$ so $m\angle x = 30^{\circ}$; and the inscribed arc of $\angle y = 120^{\circ}$ so $m\angle y = 60^{\circ}$. $m\angle x + m\angle y = 30 + 60 = \textbf{(C)}\ 90$

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 14	Followed by Problem 16
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 16

Problem

The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving the "Middle School Eight" teams?

$\textbf{(A) }60\qquad\textbf{(B) }88\qquad\textbf{(C) }96\qquad\textbf{(D) }144\qquad\textbf{(E) }160$

Solution

Within the conference, there are 8 teams, so there are $\binom{8}{2}=28$ pairings of teams, and each pair must play two games, for a total of $28\cdot 2=56$ games within the conference.

Each team also plays 4 games outside the conference, and there are 8 teams, so there are a total of $4\cdot 8=32$ games outside the conference.

Therefore, the total number of games is $56+32=\boxed{88}$, so (B) is our answer.

See Also

2014 AMC 8 Problems/Problem 17

Problem

George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first $\frac{1}{2}$ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last $\frac{1}{2}$ mile in order to arrive just as school begins today?

$\textbf{(A) }4\qquad\textbf{(B) }6\qquad\textbf{(C) }8\qquad\textbf{(D) }10\qquad \textbf{(E) }12$

Solution

Note that on a normal day, it takes him $1/3$ hour to get to school. However, today it took $\frac{1/2 \text{ mile}}{2 \text{ mph}}=1/4$ hour to walk the first $1/2$ mile. That means that he has $1/3 - 1/4 = 1/12$ hours left to get to school, and $1/2$ mile left to go. Therefore, his speed must be $\frac{1/2 \text{ mile}}{1/12 \text{ hour}}=\boxed{6 \text{ mph}}$, so (B) is the answer.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
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2014 AMC 8 Problems/Problem 18

Problem

Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely

$\textbf{(A)}$ all 4 are boys
$\textbf{(B)}$ all 4 are girls
$\textbf{(C)}$ 2 are girls and 2 are boys
$\textbf{(D)}$ 3 are of one gender and 1 is of the other gender
$\textbf{(E)}$ all of these outcomes are equally likely

Solution

We'll just start by breaking cases down. The probability of A occurring is $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$. The probability of B occurring is $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$.

The probability of C occurring is $\dbinom{4}{2} \cdot \left(\frac{1}{2}\right)^4 = \frac{3}{8}$, because we need to choose 2 of the 4 children to be girls.

For D, there are two possible cases, 3 girls and 1 boy or 3 boys and 1 girl. The probability of the first case is $\dbinom{4}{1} \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{4}$ because we need to choose 1 of the 4 children to be a boy. However, the second case has the same probability because we are choosing 1 of the 4 children to be a girl, so the total probability is $\frac{1}{4} \cdot 2 = \frac{1}{2}$.

So out of the four fractions, D is the largest. So our answer is $\boxed{\text{(D) 3 of one gender and 1 of the other}}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 19

Problem

A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?

$\textbf{(A) }\frac{5}{54}\qquad\textbf{(B) }\frac{1}{9}\qquad\textbf{(C) }\frac{5}{27}\qquad\textbf{(D) }\frac{2}{9}\qquad\textbf{(E) }\frac{1}{3}$

Solution

For the least possible surface area, we should have 1 cube in the center, and the other 5 with only 1 face exposed. This gives 5 square inches of white surface area. Since the cube has a surface area of 54 square inches, our answer is $\textbf{(A) }\frac{5}{54}$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 18	Followed by Problem 20
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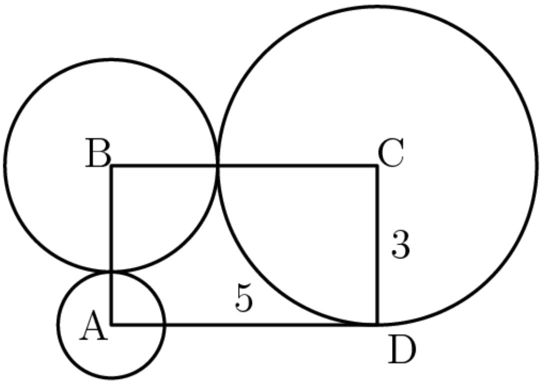
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 20

Problem

Rectangle $ABCD$ has sides $CD=3$ and $DA=5$. A circle of radius 1 is centered at A, a circle of radius 2 is centered at B, and a circle of radius 3 is centered at C. Which of the following is closest to the area of the region inside the rectangle but outside all three circles?



$\textbf{(A) }3.5\qquad\textbf{(B) }4.0\qquad\textbf{(C) }4.5\qquad\textbf{(D) }5.0\qquad\textbf{(E) }5.5$

Solution

The area in the rectangle but outside the circles is the area of the rectangle minus the area of all three of the quarter circles in the rectangle.

The area of the rectangle is $3\cdot5=15$. The area of all 3 quarter circles is $\frac{\pi}{4}+\frac{\pi(2)^2}{4}+\frac{\pi(3)^2}{4}=\frac{14\pi}{4}=\frac{7\pi}{2}$. Therefore the area in the rectangle but outside the circles is $15-\frac{7\pi}{2}$. π is approximately $\frac{22}{7}$, and substituting that in will give $15-11=\boxed{\text{(B)}\ 4.0}$

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 19	Followed by Problem 21
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All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 21

Problem

The 7-digit numbers $\underline{74A52B1}$ and $\underline{326AB4C}$ are each multiples of 3. Which of the following could be the value of C?

$\textbf{(A) }1\qquad\textbf{(B) }2\qquad\textbf{(C) }3\qquad\textbf{(D) }5\qquad \textbf{(E) }8$

Solution

The sum of a number's digits $\mod 3$ is congruent to the number $\pmod 3$. $74A52B1 \mod 3$ must be congruent to 0, since it is divisible by 3. Therefore, $7+4+A+5+2+B+1 \mod 3$ is also congruent to 0. $7+4+5+2+1 \equiv 1 \pmod 3$, so $A+B \equiv 2 \pmod 3$. As we know, $326AB4C \equiv 0 \pmod 3$, so $3+2+6+A+B+4+C = 15+A+B+C \equiv 0 \pmod 3$, and therefore $A+B+C \equiv 0 \pmod 3$. We can substitute 2 for $A+B$, so $2+C \equiv 0 \pmod 3$, and therefore $C \equiv 1 \pmod 3$. This means that C can be 1, 4, or 7, but the only one of those that is an answer choice is $\textbf{(A) }1$.

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 22

Problem

A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?

$\textbf{(A) } 1\qquad\textbf{(B) } 3\qquad\textbf{(C) } 5\qquad\textbf{(D) } 7\qquad\textbf{(E) } 9$

Solution

We can think of the number as $10a+b$, where a and b are digits. Since the number is equal to the product of the digits $(a \cdot b)$ plus the sum of the digits $(a+b)$, we can say that $10a+b=a \cdot b+a+b$. We can simplify this to $10a=a \cdot b+a$, and factor to $(10)a=(b+1)a$. Dividing by a, we have that $b+1=10$. Therefore, the units digit, b, is $\textbf{(E) } 9$

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 23

Problem

Three members of the Euclid Middle School girls' softball team had the following conversation.

Ashley: I just realized that our uniform numbers are all 2-digit primes.

Brittany : And the sum of your two uniform numbers is the date of my birthday earlier this month.

Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

Ashley: And the sum of your two uniform numbers is today's date.

What number does Caitlin wear?

$\textbf{(A) }11\qquad\textbf{(B) }13\qquad\textbf{(C) }17\qquad\textbf{(D) }19\qquad \textbf{(E) }23$

Solution

The maximum amount of days any given month can have is 31, and the smallest two digit primes are 11, 13, and 17. There are a few different sums that can be deduced from the following numbers, which are 24, 30, and 28, all of which represent the three days. Therefore, since Brittany says that the other two people's uniform numbers is earlier, so that means Caitlin and Ashley's numbers must add up to 24. Similarly, Caitlin says that the other two people's uniform numbers is later, so the sum must add up to 30. This leaves 28 as today's date. From this, Caitlin was referring to the uniform wearers 13 and 17, telling us that her number is 11, giving our solution as $\boxed{(A) = 11}$

See Also

2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 24

Problem

One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?

$\textbf{(A)}\ 2.5 \qquad \textbf{(B)}\ 3.0 \qquad \textbf{(C)}\ 3.5 \qquad \textbf{(D)}\ 4.0 \qquad \textbf{(E)}\ 4.5$

Solution

In order to maximize the median, we need to make the first half of the numbers as small as possible. Since there are 100 people, the median will be the average of the 50th and 51st largest amount of cans per person. To minimize the first 49, they would each have one can. Subtracting these 49 cans from the 252 cans gives us 203 cans left to divide among 51 people. Taking $\frac{203}{51}$ gives us 3 and a remainder of 50. Seeing this, the largest number of cans the $50th$ person could have is 3, which leaves 4 to the rest of the people. The average of 3 and 4 is 3.5. Thus our answer is $\boxed{\text{(C)}\ 3.5}$.

See Also

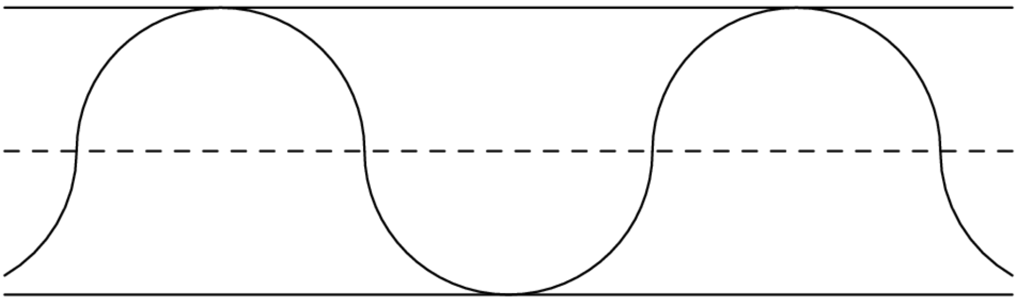
2014 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2014))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2014 AMC 8 Problems/Problem 25

Problem

A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

Note: 1 mile = 5280 feet



$\textbf{(A) }\frac{\pi}{11}\qquad\textbf{(B) }\frac{\pi}{10}\qquad\textbf{(C) }\frac{\pi}{5}\qquad\textbf{(D) }\frac{2\pi}{5}\qquad\textbf{(E) }\frac{2\pi}{3}$

Solution

There are two possible interpretations of the problem: that the road as a whole is 40 feet wide, or that each lane is 40 feet wide. Both interpretations will arrive at the same result. However, let us stick with the first interpretation for simplicity. Each lane must then be 20 feet wide, so Robert must be riding his bike in semicircles with radius 20 feet and diameter 40 feet. Since the road is 5280 feet long, over the whole mile, Robert rides $\frac{5280}{40}=132$ semicircles in total. Were the semicircles full circles, their circumference would be $2\pi\cdot 20=40\pi$ feet; as it is, the circumference of each is half that, or 20π feet. Therefore, over the stretch of highway, Robert rides a total of $132\cdot 20\pi=2640\pi$ feet, equivalent to $\frac{\pi}{2}$ miles. Robert rides at 5 miles per hour, so divide the $\frac{\pi}{2}$ miles by 5 mph to arrive at $\textbf{(B) }\frac{\pi}{10}$ hours.