2004 AMC 8 Problems/Problem 1

Problem

On a map, a 12-centimeter length represents 72 kilometers. How many kilometers does a 17-centimeter length represent?

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 102\qquad\textbf{(C)}\ 204\qquad\textbf{(D)}\ 864\qquad\textbf{(E)}\ 1224$

Solution

We set up the proportion $\frac{12 \text{cm}}{72 \text{km}}=\frac{17 \text{cm}}{x \text{km}}$. Thus $x=102 \Rightarrow \boxed{\textbf{(B)}\ 102}$

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 2

Contents

Problem

How many different four-digit numbers can be formed be rearranging the four digits in 2004?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 16\qquad\textbf{(D)}\ 24\qquad\textbf{(E)}\ 81$

Solution 1

Note that the four-digit number must start with either a 2 or a 4. The four-digit numbers that start with 2 are $2400, 2040$, and 2004. The four-digit numbers that start with 4 are $4200, 4020$, and 4002 which gives us a total of $\boxed{\textbf{(B)}\ 6}$.

Solution 2

There is only 2 choices for the first digit because you can't have 0 as the first digit because it wouldn't be a 4 digit number. Then there are 3 choices for the second and 2 for the third and 1 for the fourth. Then just like you would do for how many ways there are to arrange a word with two of the same letters, you do $\frac{2 \cdot 3 \cdot 2 \cdot 1}{2!}$. Which is $\frac{12}{2}$ which is simplified to $\boxed{\textbf{(B)}\ 6}$

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 3

Problem

Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they shared, how many meals should they have ordered to have just enough food for the 12 of them?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 18$

Solution

Set up the proportion $\frac{12\text{ meals}}{18\text{ people}}=\frac{x\text{ meals}}{12\text{ people}}$. Solving for x gives us $x=\boxed{\textbf{(A)}\ 8}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 4

Problem

Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

Lance, Sally, Joy, and Fred are chosen for the team. In how many ways can the three starters be chosen?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 6\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 10$

Solution

There are $\binom{4}{3}$ ways to choose three starters. Thus the answer is $\boxed{\textbf{(B)}\ 4}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 5

Contents

Problem

The losing team of each game is eliminated from the tournament. If sixteen teams compete, how many games will be played to determine the winner?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 16$

Solution 1

The remaining team will be the only undefeated one. The other $\boxed{\textbf{(D)}\ 15}$ teams must have lost a game before getting out, thus fifteen games yielding fifteen losers.

Solution 2

There will be 8 games the first round, 4 games the second round, 2 games the third round, and 1 game in the final round, giving us a total of $8+4+2+1=15$ games. $\boxed{\textbf{(D)}\ 15}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 6

Problem

After Sally takes 20 shots, she has made 55% of her shots. After she takes 5 more shots, she raises her percentage to 56%. How many of the last 5 shots did she make?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Sally made $0.55 * 20 = 11$ shots originally. Letting x be the number of shots she made, we have $\frac{11+x}{25} = 0.56$. Solving for x gives us $x = \boxed{\textbf{(C)}\ 3}$

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 7

Problem

An athlete's target heart rate, in beats per minute, is 80% of the theoretical maximum heart rate. The maximum heart rate is found by subtracting the athlete's age, in years, from 220. To the nearest whole number, what is the target heart rate of an athlete who is 26 years old?

$\textbf{(A)}\ 134 \qquad \textbf{(B)}\ 155 \qquad \textbf{(C)}\ 176 \qquad \textbf{(D)}\ 194 \qquad \textbf{(E)}\ 243$

Solution

The maximum heart rate is $220-26=194$ beats per minute. The target heart rate is then $0.8*194 \approx \boxed{\textbf{(B)}\ 155}$ beats per minute.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 8

Problem

Find the number of two-digit positive integers whose digits total 7.

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 7\qquad\textbf{(C)}\ 8\qquad\textbf{(D)}\ 9\qquad\textbf{(E)}\ 10$

Solution

The numbers are $16, 25, 34, 43, 52, 61, 70$ which gives us a total of $\boxed{\textbf{(B)}\ 7}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 9

Problem

The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

$\textbf{(A)}\ 55 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 57 \qquad \textbf{(D)}\ 58 \qquad \textbf{(E)}\ 59$

Solution

Let the 5 numbers be $a, b, c, d,$ and e. Thus $\frac{a+b+c+d+e}{5} = 54$ and $a+b+c+d+e = 270$. Since $\frac{a+b}{2} = 48$, $a+b = 96$. Substituting back into our original equation, we have $96 + c + d + e = 270$ and $c + d + e = 174$. Dividing by 3 gives the average of $\boxed{\textbf{(D)}\ 58}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 10

Problem

Handy Aaron helped a neighbor $1\frac{1}{4}$ hours on Monday, 50 minutes on Tuesday, from 8:20 to 10:45 on Wednesday morning, and a half-hour on Friday. He is paid $\$3$ per hour. How much did he earn for the week?

$\textbf{(A)}\ \$8 \qquad \textbf{(B)}\ \$9 \qquad \textbf{(C)}\ \$10 \qquad \textbf{(D)}\ \$12 \qquad \textbf{(E)}\ \$15$

Solution

Convert everything to minutes and add them together. On Monday he worked for $\frac{5}{4} \cdot 60 = 75$ minutes. On Tuesday he worked 50 minutes. On Wednesday he worked for 2 hours 25 minutes, or $2(60) + 25 = 145$ minutes. On Friday he worked 30 minutes. In total he worked for $75 + 50 + 145 + 30 = 300$ minutes, or $300/60 = 5$ hours and $5 \cdot 3 = \boxed{\textbf{(E)}\ \$15}$.

See Also

<table>
<tr><td colspan="2">2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))</td></tr>
<tr><td>Preceded by
Problem 9</td><td>Followed by
Problem 11</td></tr>
<tr><td colspan="2">1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25</td></tr>
<tr><td colspan="2">All AJHSME/AMC 8 Problems and Solutions</td></tr>
</table>

2004 AMC 8 Problems/Problem 11

Problem

The numbers $-2, 4, 6, 9$ and 12 are rearranged according to these rules:

1. The largest isn’t first, but it is in one of the first three places.
2. The smallest isn’t last, but it is in one of the last three places.
3. The median isn’t first or last.

What is the average of the first and last numbers?

$\textbf{(A)}\ 3.5 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6.5 \qquad \textbf{(D)}\ 7.5 \qquad \textbf{(E)}\ 8$

Solution

From rule 1, the largest number, 12, can be second or third. From rule 2, because there are five places, the smallest number -2 can either be third or fourth. The median, 6 can be second, third, or fourth. Because we know the middle three numbers, the first and last numbers are 4 and 9, disregarding their order. Their average is $(4+9)/2 = \boxed{\textbf{(C)}\ 6.5}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 12

Problem

Niki usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 9 hours, and during that time she has used it for 60 minutes. If she doesn't talk any more but leaves the phone on, how many more hours will the battery last?

$\textbf{(A)}\ 7 \qquad\textbf{(B)}\ 8 \qquad\textbf{(C)}\ 11 \qquad\textbf{(D)}\ 14 \qquad\textbf{(E)}\ 15$

Solution

When not being used, the cell phone uses up $1/24$ of its battery per hour. When being used, the cell phone uses up $1/3$ of its battery per hour. Since Niki's phone has been on for 9 hours, of those 8 simply on and 1 being used to talk, $8(1/24)+1(1/3) = 2/3$ of its battery has been used up. To drain the remaining $1/3$ the phone can last for $\frac{1/3}{1/24} = \boxed{\textbf{(B)}\ 8}$ more hours without being used.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 13

Problem

Amy, Bill and Celine are friends with different ages. Exactly one of the following statements is true.

I. Bill is the oldest.
II. Amy is not the oldest.
III. Celine is not the youngest.

Rank the friends from the oldest to the youngest.

$\textbf{(A)}\ \text{Bill, Amy, Celine} \qquad \textbf{(B)}\ \text{Amy, Bill, Celine} \qquad \textbf{(C)}\ \text{Celine, Amy, Bill}$
$\textbf{(D)}\ \text{Celine, Bill, Amy} \qquad \textbf{(E)}\ \text{Amy, Celine, Bill}$

Solution

If Bill is the oldest, then Amy is not the oldest, and both statements I and II are true, so statement I is not the true one.

If Amy is not the oldest, and we know Bill cannot be the oldest, then Celine is the oldest. This would mean she is not the youngest, and both statements II and III are true, so statement II is not the true one.

Therefore, statement III is the true statement, and both I and II are false. From this, Amy is the oldest, Celine is in the middle, and lastly Bill is the youngest. This order is $\boxed{\textbf{(E)}\ \text{Amy, Celine, Bill}}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 14

Contents

Problem

What is the area enclosed by the geoboard quadrilateral below?

$\textbf{(A)}\ 15\qquad\textbf{(B)}\ 18\frac{1}{2}\qquad\textbf{(C)}\ 22\frac{1}{2}\qquad\textbf{(D)}\ 27\qquad\textbf{(E)}\ 41$

Solution

Assign points to each of the four vertices and use the shoelace theorem to find the area. Letting the bottom left corner be $(0,0)$, counting the boxes, the points would be $(4,0)$, $(0,5)$, $(3,4)$, and $(10,10)$. Applying the Shoelace Theorem,

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 4 & 0 \\ 0 & 5 \\ 3 & 4 \\ 10 & 10 \end{vmatrix} = \frac{1}{2}|(20+30)-(15+40+40)| = \frac{1}{2}|50-95| = \boxed{\textbf{(C)}\ 22\frac{1}{2}}$$

Solution 2

Apply Pick's Theorem on the figure, and you will get

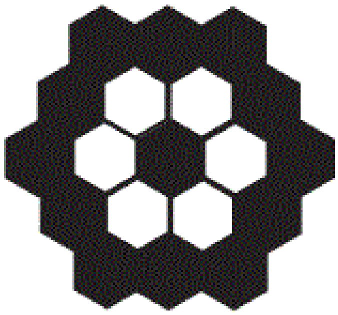
$$\boxed{\textbf{(C)}\ 22\frac{1}{2}}$$

See Also

2004 AMC 8 Problems/Problem 15

Problem

Thirteen black and six white hexagonal tiles were used to create the figure below. If a new figure is created by attaching a border of white tiles with the same size and shape as the others, what will be the difference between the total number of white tiles and the total number of black tiles in the new figure?



$\textbf{(A) }5\qquad\textbf{(B) }7\qquad\textbf{(C) }11\qquad\textbf{(D) }12\qquad\textbf{(E) }18$

Solution

The first ring around the middle tile has 6 tiles, and the second has 12. From this pattern, the third ring has 18 tiles. Of these, $6+18=24$ are white and $1+12=13$ are black, with a difference of $24-13=\boxed{\textbf{(C)}\ 11}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 16

Problem

Two 600 mL pitchers contain orange juice. One pitcher is $1/3$ full and the other pitcher is $2/5$ full. Water is added to fill each pitcher completely, then both pitchers are poured into one large container. What fraction of the mixture in the large container is orange juice?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{3}{16} \qquad \textbf{(C)}\ \frac{11}{30} \qquad \textbf{(D)}\ \frac{11}{19} \qquad \textbf{(E)}\ \frac{11}{15}$

Solution

The first pitcher contains $600 \cdot \frac{1}{3} = 200$ mL of orange juice. The second pitcher contains $600 \cdot \frac{2}{5} = 240$ mL of orange juice. In the large pitcher, there is a total of $200 + 240 = 440$ mL of orange juice and $600 + 600 = 1200$ mL of fluids, giving a fraction of $\frac{440}{1200} = \boxed{\textbf{(C)}\ \frac{11}{30}}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 17

Problem

Three friends have a total of 6 identical pencils, and each one has at least one pencil. In how many ways can this happen?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Solution

For each person to have at least one pencil, assign one of the pencil to each of the three friends so that you have 3 left. In partitioning the remaining 3 pencils into 3 distinct groups, use Ball-and-turn to find the number of possibilities is ${}_{3+3-1}C_{3-1} ={}_1 0C_2 = \boxed{\textbf{(D)}\ 10}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 18

Problem

Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?

$\textbf{(A)}\ \text{Alice} \qquad \textbf{(B)}\ \text{Ben} \qquad \textbf{(C)}\ \text{Cindy} \qquad \textbf{(D)}\ \text{Dave} \qquad \textbf{(E)}\ \text{Ellen}$

Solution

The only way to get Ben's score is with $1+3=4$. Cindy's score can be made of $3+4$ or $2+5$, but since Ben already hit the 3, Cindy hit $2+5=7$. Similar, Dave's darts were in the region $4+7=11$. Lastly, because there is no 7 left, $\boxed{\textbf{(A)}\ \text{Alice}}$ must have hit the regions $6+10=16$ and Ellen $8+9=17$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 19

Problem

A whole number larger than 2 leaves a remainder of 2 when divided by each of the numbers $3, 4, 5,$ and 6. The smallest such number lies between which two numbers?

$\textbf{(A)}\ 40\text{ and }49 \qquad \textbf{(B)}\ 60\text{ and }79 \qquad \textbf{(C)}\ 100\text{ and }129 \qquad \textbf{(D)}\ 210\text{ and }249 \qquad \textbf{(E)}\ 320\text{ and }369$

Solution

The smallest number divisible by $3, 4, 5,$ and 6, or their least common multiple, can be found to be 60. When 2 is added to a multiple of number, its remainder when divided by that number is 2. The number we are looking for is therefore 62, and between $\boxed{\textbf{(B)}\ 60\text{ and }79}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 20

Problem

Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

$\textbf{(A)}\ 12\qquad\textbf{(B)}\ 18\qquad\textbf{(C)}\ 24\qquad\textbf{(D)}\ 27\qquad\textbf{(E)}\ 36$

Solution 1

Working backwards, if $3/4$ of the chairs are taken and 6 are empty, then there are three times as many taken chairs as empty chairs, or $3\cdot6=18$. If x is the number of people in the room and $2/3$ are seated, then $\frac{2}{3}x=18$ and $x=\boxed{\mathrm{(D)}27}$.

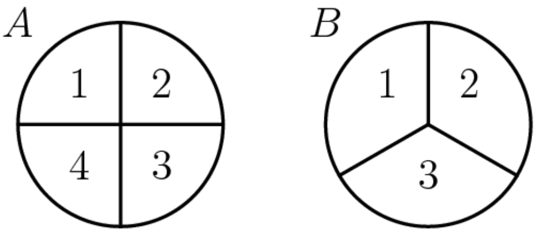
See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 21

Problem

Spinners A and B are spun. On each spinner, the arrow is equally likely to land on each number. What is the probability that the product of the two spinners' numbers is even?



$\textbf{(A)}\ \frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{3}{4}$

Solution

An even number comes from multiplying an even and even, even and odd, or odd and even. Since an odd number only comes from multiplying an odd and odd, there are less cases and it would be easier to find the probability of spinning two odd numbers from 1. Multiply the independent probabilities of each spinner getting an odd number together and subtract it from 1.

$$1-\frac{2}{4}\cdot\frac{2}{3} = 1-\frac{1}{3} = \boxed{\textbf{(D)}\ \frac{2}{3}}$$

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 22

Problem

At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men.

$\textbf{(A)}\ \frac{1}{3} \qquad \textbf{(B)}\ \frac{3}{8} \qquad \textbf{(C)}\ \frac{2}{5} \qquad \textbf{(D)}\ \frac{5}{12} \qquad \textbf{(E)}\ \frac{3}{5}$

Solution

Assume arbitrarily (and WLOG) there are 5 women in the room, of which $5 \cdot \frac{2}{5} = 2$ are single and $5-2=3$ are married. Each married woman came with her husband, so there are 3 married men in the room as well for a total of $5+3=8$ people. The fraction of the people that are married men is $\boxed{\textbf{(B)}\ \frac{3}{8}}$.

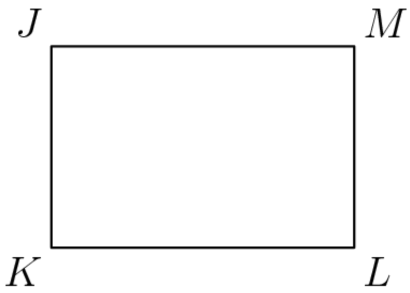
See Also

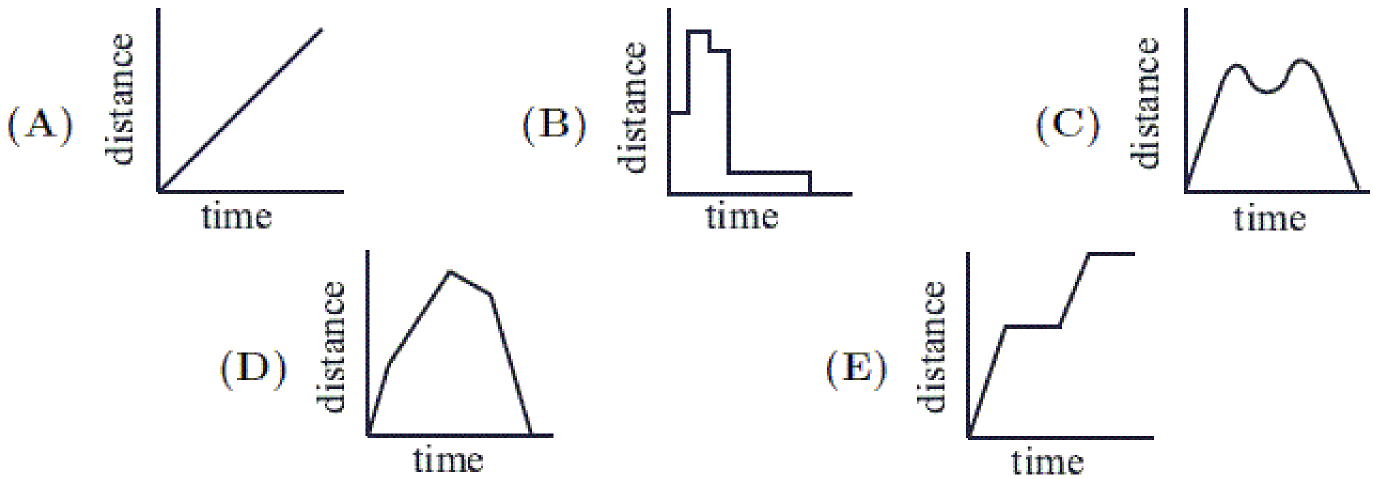
2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 23

Problem

Tess runs counterclockwise around rectangular block $JKLM$. She lives at corner J. Which graph could represent her straight-line distance from home?





Solution 1

For her distance to be represented as a constant horizontal line, Tess would have to be running in a circular shape with her home as the center. Since she is running around a rectangle, this is not possible, rulling out B and E with straight lines. Because JL is the diagonal of the rectangle, and L is at the middle distance around the perimeter, her maximum distance should be in the middle of her journey. The maximum in A is at the end, and C has two maximums, ruling both out. Thus the answer is $\boxed{\textbf{(D)}}$.

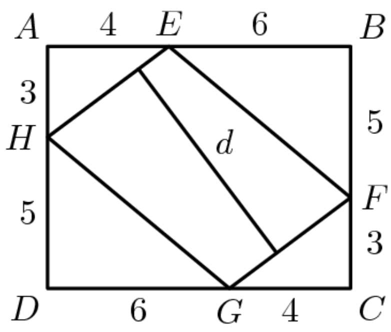
See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 24

Problem

In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



$\textbf{(A)}\ 6.8 \qquad \textbf{(B)}\ 7.1 \qquad \textbf{(C)}\ 7.6 \qquad \textbf{(D)}\ 7.8 \qquad \textbf{(E)}\ 8.1$

Solution

The area of the parallelogram can be found in two ways. The first is by taking rectangle $ABCD$ and subtracting the areas of the triangles cut out to create parallelogram $EFGH$. This is

$$(4+6)(5+3)-2\cdot\frac{1}{2}\cdot6\cdot5-2\cdot\frac{1}{2}\cdot3\cdot4=80-30-12=38$$

The second way is by multiplying the base of the parallelogram such as $\overline{FG}$ with its altitude d, which is perpendicular to both bases. $\triangle FGC$ is a $3-4-5$ triangle so $\overline{FG}=5$. Set these two representations of the area together.

$$5d=38\rightarrow d=\boxed{\textbf{(C)}\ 7.6}$$

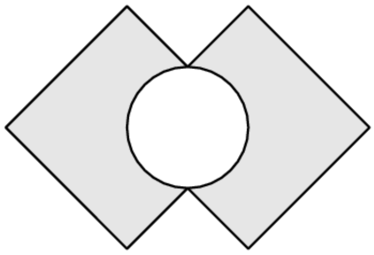
See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2004 AMC 8 Problems/Problem 25

Problem

Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



$\textbf{(A)}\ 16-4\pi \qquad \textbf{(B)}\ 16-2\pi \qquad \textbf{(C)}\ 28-4\pi \qquad \textbf{(D)}\ 28-2\pi \qquad \textbf{(E)}\ 32-2\pi$

Solution

If the circle was shaded in, the intersection of the two squares would be a smaller square with half the sidelength, 2. The area of this region would be the two larger squares minus the area of the intersection, the smaller square. This is $4^2+4^2-2^2=28$.

The diagonal of this smaller square created by connecting the two points of intersection of the squares is the diameter of the circle. This value can be found with Pythagorean or a $45^\circ-45^\circ-90^\circ$ circle to be $2\sqrt{2}$. The radius is half the diameter, $\sqrt{2}$. The area of the circle is $\pi r^2 = \pi(\sqrt{2})^2 = 2\pi$.

The area of the shaded region is the area of the circle minus the area of the two squares which is $\boxed{\textbf{(D)}\ 28-2\pi}$.

See Also

2004 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2004))	
Preceded by Problem 24	Followed by Last Question
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	