2014 AMC12B

Problem 1

Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

Leah 有 13 枚硬币，它们都是 1 分或者 5 分硬币。假设她比现在再多一枚 5 分硬币，那么她拥有的 5 分硬币和 1 分硬币的数目就一样多了，那么她现在所拥有的所有硬币总共价值多少？

$\textbf{(A)}\ 33 \qquad \textbf{(B)}\ 35 \qquad \textbf{(C)}\ 37 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 41$

Problem 2

Orvin went to the store with just enough money to buy 30 balloons. When he arrived he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

Orvin 带着足够多的钱去商店买 30 只气球，当他到达商店，发现商店正在对气球做促销降价活动：以原价买一只气球，那么第二只气球降价 $\frac{1}{3}$，问 Orvin 最多可以买到多少只气球？

$\textbf{(A)}\ 33 \qquad \textbf{(B)}\ 34 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 38 \qquad \textbf{(E)}\ 39$

Problem 3

Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?

Randy 在石子路上行驶了他总路程的前三分之一，接下来的 20 英里走的是石板路，最后剩下的总路程的五分之一走的是泥路。问 Randy 的总路程是多少英里？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ \frac{400}{11} \qquad \textbf{(C)}\ \frac{75}{2} \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ \frac{300}{7}$

Problem 4

Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

Susie 花钱买了 4 个玛芬蛋糕和 3 根香蕉，Calvin 花了 Susie 两倍的钱买了 2 个玛芬蛋糕和 16 根香蕉，问一个玛芬的价格是一根香蕉价格的几倍?

$\textbf{(A)}\ \frac{3}{2} \qquad \textbf{(B)}\ \frac{5}{3} \qquad \textbf{(C)}\ \frac{7}{4} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{13}{4}$

Problem 5

Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5 : 2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?

如图所示，Doug 用 8 块大小相同的玻璃做了一个正方形的窗户，每块玻璃的高和宽之比为 $5 : 2$，并且玻璃周围以及玻璃之间的窗框的宽度是 2 英寸，问正方形窗户的边长是多少英寸？

$\textbf{(A)}\ 26 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 34$

Problem 6

Ed and Ann both have lemonade with their lunch. Ed orders the regular size. Ann gets the large lemonade, which is 50% more than the regular. After both consume $\frac{3}{4}$ of their drinks, Ann gives Ed a third of what she has left, and 2 additional ounces. When they finish their lemonades they realize that they both drank the same amount. How many ounces of lemonade did they drink together?

Ed 和 Ann 吃午餐时都喝了柠檬水，Ed 点了中杯，Ann 点了大杯，大杯的量比中杯多 50％，当他俩都喝了各自总量的$\frac{3}{4}$，Ann 把她自己剩余量的三分之一再加 2 盎司给了 Ed，当他们都全部喝完后，发现他们喝的总量是一样的，问他俩总共喝了多少盎司的柠檬水？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 50$

Problem 7

For how many positive integers n is $\frac{n}{30-n}$ also a positive integer?

使得$\frac{n}{30-n}$是正整数的正整数 n 有多少个？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 8

In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

在如下图所示的加法中，A，B，C，D 都是不同的数字，则 D 总共有多少种不同的取值？

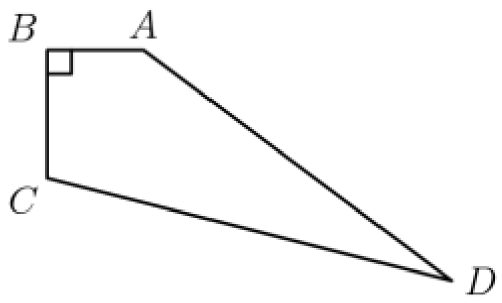
$$\begin{array}{rrrrrr} & A & B & B & C & B \\ + & B & C & A & D & A \\ \hline & D & B & D & D & D \end{array}$$

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 9

Convex quadrilateral $ABCD$ has $AB = 3$, $BC = 4$, $CD = 13$, $AD = 12$, and $\angle ABC = 90^\circ$, as shown. What is the area of the quadrilateral?

如图所示的凸四边形 *ABCD* 中 *AB*=3，*BC*=4，*CD*=13，*AD*=12， $\angle ABC = 90^\circ$，问这个四边形的面积是多少？



$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 58.5$

Problem 10

Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2+b^2+c^2?$.

Danica 开着她的新车在路上行驶了整数个小时，平均速度为 55 英里每小时。一开始，里程表的读数是 *abc* 英里，这里 *abc* 是一个三位整数，满足$a \geq 1$且$a+b+c \leq 7$，行程结束后，里程表读数为 *cba* 英里，问$a^2+b^2+c^2$是多少？

$\textbf{(A)}\ 26 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 37 \qquad \textbf{(E)}\ 41$

Problem 11

A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

由 11 个正整数形成的一组数，其平均值是 10，中位数是 9，唯一的众数是 8，问这组数里最大可能的整数是多少?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 31 \qquad \textbf{(D)}\ 33 \qquad \textbf{(E)}\ 35$

Problem 12

A set S consists of triangles whose sides have integer lengths less than 5, and no two elements of S are congruent or similar. What is the largest number of elements that S can have?

S 是由三角形组成的集合，*S* 中的三角形的边长都是小于 5 的整数，且 *S* 中不存在全等或者相似的两个三角形，问 *S* 中最多可以有多少个元素？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Problem 13

Real numbers a and b are chosen with $1 < a < b$ such that no triangles with positive area has side lengths $1, a,$ and b or $\frac{1}{b}, \frac{1}{a},$ and 1. What is the smallest possible value of b?

实数 *a*，*b* 满足$1 < a < b$，且以 1，*a*，*b* 或者$\frac{1}{b}, \frac{1}{a},$ 1 为三边都无法组成三角形，那么 *b* 的最小可能值是多少？

$\textbf{(A)}\ \frac{3+\sqrt{3}}{2} \qquad \textbf{(B)}\ \frac{5}{2} \qquad \textbf{(C)}\ \frac{3+\sqrt{5}}{2} \qquad \textbf{(D)}\ \frac{3+\sqrt{6}}{2} \qquad \textbf{(E)}\ 3$

Problem 14

A rectangular box has a total surface area of 94 square inches. The sum of the lengths of all its edges is 48 inches. What is the sum of the lengths in inches of all of its interior diagonals?

一个长方体盒子的表面积是 94 平方英寸，它的所有棱长之和是 48 英寸，那么它的所有在盒子内的对角线（不包括表面的对角线）长度之和是多少英寸？

$\textbf{(A)}\ 8\sqrt{3} \qquad \textbf{(B)}\ 10\sqrt{2} \qquad \textbf{(C)}\ 16\sqrt{3} \qquad \textbf{(D)}\ 20\sqrt{2} \qquad \textbf{(E)}\ 40\sqrt{2}$

Problem 15

When $p = \sum_{k=1}^{6} k \ln k$, the number e^p is an integer. What is the largest power of 2 that is a factor of e^p ?

p 定义为 $p = \sum_{k=1}^{6} k \ln k$，且数字$e^p$是个整数，问能够整除$e^p$的 2 的最大的幂是多少？

(A) 2^{12} **(B)** 2^{14} **(C)** 2^{16} **(D)** 2^{18} **(E)** 2^{20}

Problem 16

Let P be a cubic polynomial with $P(0) = k$, $P(1) = 2k$, and $P(-1) = 3k$. What is $P(2) + P(-2)$?

P 是一个三次多项式，且$P(0) = k$，$P(1) = 2k$，$P(-1) = 3k$，问$P(2) + P(-2)$是多少？

(A) 0 **(B)** k **(C)** $6k$ **(D)** $7k$ **(E)** $14k$

Problem 17

Let P be the parabola with equation $y = x^2$ and let $Q = (20, 14)$. There are real numbers r and s such that the line through Q with slope m does not intersect P if and only if $r < m < s$. What is $r + s$?

P 是一个方程为$y = x^2$的抛物线，点坐标为$Q = (20, 14)$，存在实数 r 和 s，满足当且仅当 $r < m < s$时，通过 Q 且斜率为 m 的直线和 P 不相交，那么 r +s 是多少？

(A) 1 **(B)** 26 **(C)** 40 **(D)** 52 **(E)** 80

Problem 18

The numbers 1, 2, 3, 4, 5, are to be arranged in a circle. An arrangement is bad if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

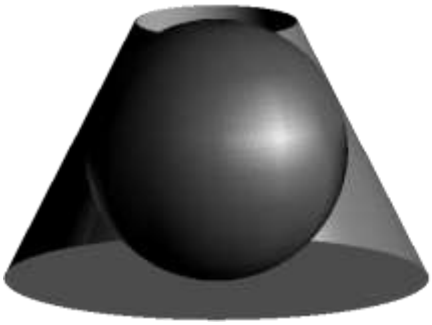
现在要将数字1, 2, 3, 4, 5排到一个圆上，有这样一个要求：对于 1 到 15 的每一个 *n* 来说，我们都能找到在圆上连续出现的 1 个或多个数，它们的和为 *n*. 若对于某一种排列这个要求无法满足，那么就称这种排列是坏的。如果一种排列可以通过旋转或者对称得到另外一种排列，那么这两种排列被认为是一样的，问一共有多少种不同的坏的排列？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 19

A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?

如图所示，一个球与一个被切掉顶部的正圆锥相内切，这个被切掉顶部的正圆椎的体积是球体积的 2 倍，那么这个被切掉顶部的正圆锥的下底面的半径和上底面的半径之比是多少？



$\text{(A)}\ \frac32 \quad \text{(B)}\ \frac{1+\sqrt5}2 \quad \text{(C)}\ \sqrt3 \quad \text{(D)}\ 2 \quad \text{(E)}\ \frac{3+\sqrt5}2$

Problem 20

For how many positive integers x is $\log_{10}(x-40) + \log_{10}(60-x) < 2$?

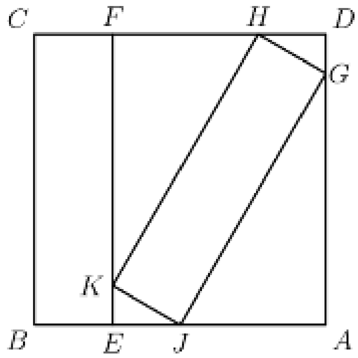
存在多少个正整数 *x*，满足$\log_{10}(x-40) + \log_{10}(60-x) < 2$？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 19 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ \text{infinitely many}$

Problem 21

In the figure, $ABCD$ is a square of side length 1. The rectangles $JKHG$ and $EBCF$ are congruent. What is BE?

如下图所示，*ABCD* 是个边长为 1 的正方形，矩形 *JKHG* 和矩形 *EBCF* 全等。求 *BE* 的长度是多少？



$\textbf{(A)}\ \frac{1}{2}(\sqrt{6}-2) \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ 2-\sqrt{3} \qquad \textbf{(D)}\ \frac{\sqrt{3}}{6} \qquad \textbf{(E)}\ 1-\frac{\sqrt{2}}{2}$

Problem 22

In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0<N<10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

在一个池塘里，共有 11 片百合花叶片排成一行，编号为 0 到 10。一只青蛙坐在叶片 1 上，当青蛙在叶片 *N*（$0<N<10$）上时，它就会以$\frac{N}{10}$的概率跳到叶片$N-1$上，或者以$1-\frac{N}{10}$的概率，跳到叶片$N+1$上。每一跳相互之间都是独立的，和前一跳无关。若青蛙跳到叶片 0 上，就会被一条在那里静静等待的蛇吃掉，若青蛙跳到叶片 10 上，它就会离开池塘，永不回来。那么青蛙能够逃走而不被吃掉的概率是多少？

$\textbf{(A)}\ \frac{32}{79} \qquad \textbf{(B)}\ \frac{161}{384} \qquad \textbf{(C)}\ \frac{63}{146} \qquad \textbf{(D)}\ \frac{7}{16} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 23

The number 2017 is prime. Let $S = \sum_{k=0}^{62} \binom{2014}{k}$. What is the remainder when S is divided by 2017?

已知 2017 是个质数，令 $S = \sum_{k=0}^{62} \binom{2014}{k}$，则 S 除以 2017 所得余数是多少？

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 684 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 1576 \qquad \textbf{(E)}\ 2016$

Problem 24

Let $ABCDE$ be a pentagon inscribed in a circle such that $AB = CD = 3$, $BC = DE = 10$, and $AE = 14$. The sum of the lengths of all diagonals of $ABCDE$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

ABCDE 是一个内接于圆的五边形，满足 *AB*=*CD*=3，*BC*=*DE*=10，*AE*=14，五边形 *ABCDE* 的所有对角线的长度之和是 $\frac{m}{n}$，其中 *m* 和 *n* 是互质的正整数，求 *m*+*n* 是多少？

$\textbf{(A)}\ 129 \qquad \textbf{(B)}\ 247 \qquad \textbf{(C)}\ 353 \qquad \textbf{(D)}\ 391 \qquad \textbf{(E)}\ 421$

Problem 25

Find the sum of all the positive solutions of $2\cos 2x \left(\cos 2x - \cos\left(\frac{2014\pi^2}{x}\right)\right) = \cos 4x - 1$

求以下方程所有正数解之和：$2\cos 2x \left(\cos 2x - \cos\left(\frac{2014\pi^2}{x}\right)\right) = \cos 4x - 1$

$\textbf{(A)}\ \pi \qquad \textbf{(B)}\ 810\pi \qquad \textbf{(C)}\ 1008\pi \qquad \textbf{(D)}\ 1080\pi \qquad \textbf{(E)}\ 1800\pi$

2014 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	C	E	B	A	D	D	C	B	D	E	B	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	C	E	E	B	E	B	C	C	C	D	D	