2021Spring AMC10A

Problem 1

What is the value of

$$(2^2-2)-(3^2-3)+(4^2-4)?$$

算式的值是多少?

$$(2^2-2)-(3^2-3)+(4^2-4)?$$

(A) 1 **(B)** 2 **(C)** 5 **(D)** 8 **(E)** 12

Problem 2

Portia's high school has 3 times as many students as Lara's high school. The two high schools have a total of 2600 students. How many students does Portia's high school have?

Portia 高中的学生人数是 Lara 高中的 3 倍。这两所高中总共有 2600 名学生。问 Portia 高中有多少名学生

(A) 600 **(B)** 650 **(C)** 1950 **(D)** 2000 **(E)** 2050

Problem 3

The sum of two natural numbers is $17{,}402$. One of the two numbers is divisible by 10. If the units digit of that number is erased, the other number is obtained. What is the difference of these two numbers?

两个自然数之和是 17，402。这两个数中的一个可以被 10 整除。如果去掉该数的个位数字则得到另外一个数。问这两个数的差是多少？

(A) 10,272 **(B)** 11,700 **(C)** 13,362 **(D)** 14,238 **(E)** 15,426

Problem 4

A cart rolls down a hill, travelling 5 inches the first second and accelerating so that during each successive 1-second time interval, it travels 7 inches more than during the previous 1-second interval. The cart takes 30 seconds to reach the bottom of the hill. How far, in inches, does it travel?

一辆小车冲下山坡，它第一秒移动了 5 英寸，并且速度不断加快，在每个连续的 1 秒时间间隔内，它都比前 1 秒多移动 7 英寸。小车用了 30 秒到达山脚。问它一共行进了多少英寸？

(A) 215 **(B)** 360 **(C)** 2992 **(D)** 3195 **(E)** 3242

Problem 5

The quiz scores of a class with $k > 12$ students have a mean of 8. The mean of a collection of 12 of these quiz scores is 14. What is the mean of the remaining quiz scores in terms of k?

一个共有 k>12 名学生的班级进行测验的平均分为 8 分。其中 12 名学生的测验平均分是 14 分。问其余学生的测验平均分如何用 k 来表示？

$\textbf{(A)}\ \frac{14-8}{k-12} \qquad \textbf{(B)}\ \frac{8k-168}{k-12} \qquad \textbf{(C)}\ \frac{14}{12}-\frac{8}{k} \qquad \textbf{(D)}\ \frac{14(k-12)}{k^2} \qquad \textbf{(E)}\ \frac{14(k-12)}{8k}$

Problem 6

Chantal and Jean start hiking from a trailhead toward a fire tower. Jean is wearing a heavy backpack and walks slower. Chantal starts walking at 4 miles per hour. Halfway to the tower, the trail becomes really steep, and Chantal slows down to 2 miles per hour. After reaching the tower, she immediately turns around and descends the steep part of the trail at 3 miles per hour. She meets Jean at the halfway point. What was Jean's average speed, in miles per hour, until they meet?

Chantal 和 Jean 从山路的起点开始向消防塔徒步旅行。Jean 背着一个沉重的背包，走得较慢。Chantal 开始以每小时 4 英里的速度行走。走到路程的一半，山路变得非常陡峭，Chantal 的速度减慢到每小时 2 英里。到达塔后，她立即掉头，以每小时 3 英里的速度沿着陡峭的山路向下走。她在路程的一半处遇见了 Jean。从出发到他们相遇，Jean 的平均速度是每小时多少英里？

$\textbf{(A)}\ \frac{12}{13} \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ \frac{13}{12} \qquad \textbf{(D)}\ \frac{24}{13} \qquad \textbf{(E)}\ 2$

Problem 7

Tom has a collection of 13 snakes, 4 of which are purple and 5 of which are happy. He observes that all of his happy snakes can add,

none of his purple snakes can subtract, and

all of his snakes that can't subtract also can't add.

Which of these conclusions can be drawn about Tom's snakes?

汤姆有 13 条蛇，其中 4 条是紫色的，5 条是快乐的。他观察发现

- 他的所有快乐的蛇都能做加法，
- 他的紫色的蛇不会做减法，而且
- 他所有不会做减法的蛇也不会做加法。

关于汤姆的蛇，可以得出以下哪个结论？

$\textbf{(A)}$ Purple snakes can add. | 紫色的蛇可以做加法。

$\textbf{(B)}$ Purple snakes are happy. | 紫色的蛇是快乐的。

$\textbf{(C)}$ Snakes that can add are purple. | 能做加法的蛇是紫色的。

$\textbf{(D)}$ Happy snakes are not purple. | 快乐的蛇不是紫色的。

$\textbf{(E)}$ Happy snakes can't subtract. | 快乐的蛇不会做减法。

Problem 8

When a student multiplied the number 66 by the repeating decimal, $\underline{1}.\underline{a}\ \underline{b}\ \underline{a}\ \underline{b}\ldots = \underline{1}.\overline{\underline{a}\ \underline{b}}$, where a and b are digits, he did not notice the notation and just multiplied 66 times $\underline{1}.\underline{a}\ \underline{b}$. Later he found that his answer is 0.5 less than the correct answer. What is the 2-digit number $\underline{a}\ \underline{b}$?

一名学生在用 66 乘以如下的循环小数$\underline{1}.\underline{a}\ \underline{b}\ \underline{a}\ \underline{b}\ldots = \underline{1}.\overline{\underline{a}\ \underline{b}}$,时，其中 *a* 和 *b* 是数字，他没有注意到循环小数标识，而只是做了 66 乘以$\underline{1}.\underline{a}\ \underline{b}$。后来他发现他的答案比正确答案小 0.5.问 2 位整数$\underline{a}\ \underline{b}$是多少?

(A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

Problem 9

What is the least possible value of $(xy-1)^2+(x+y)^2$ for real numbers x and y?

对于实数 *x* 和 *y*，$(xy-1)^2+(x+y)^2$的最小可能值是多少？

(A) 0 **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** 2

Problem 10

Which of the following is equivalent to

$$(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})?$$

算式与下面那个表达式相等

$$(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})?$$

(A) $3^{127}+2^{127}$ **(B)** $3^{127}+2^{127}+2\cdot3^{63}+3\cdot2^{63}$ **(C)** $3^{128}-2^{128}$

(D) $3^{128}+2^{128}$ **(E)** 5^{127}

Problem 11

For which of the following integers b is the base-b number $2021_b - 221_b$ not divisible by 3?

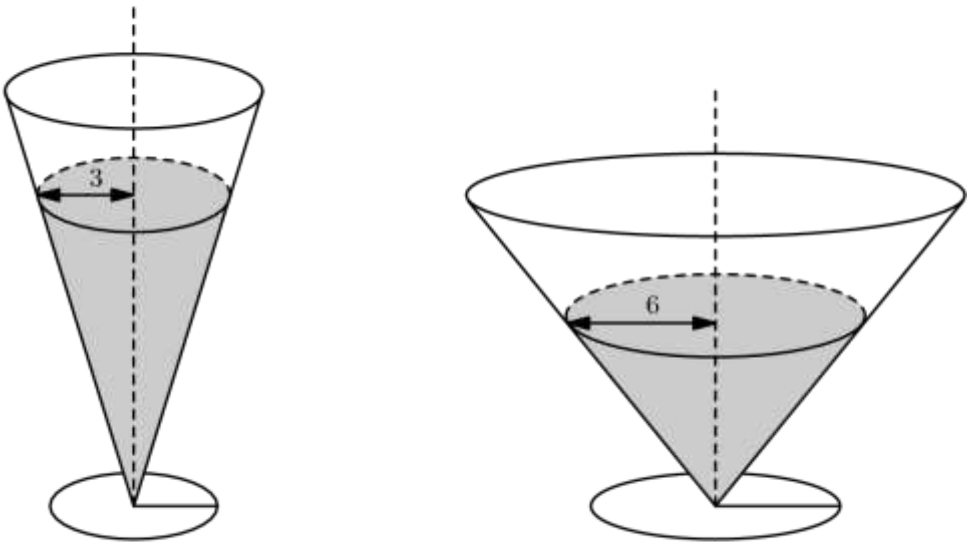
选择下面哪个整数 *b* 为基数，可以使得 *b* 进制数$2021_b - 221_b$不能被 3 整除？

(A) 3 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 8

Problem 12

Two right circular cones with vertices facing down as shown in the figure below contain the same amount of liquid. The radii of the tops of the liquid surfaces are 3 cm and 6 cm. Into each cone is dropped a spherical marble of radius 1 cm, which sinks to the bottom and is completely submerged without spilling any liquid. What is the ratio of the rise of the liquid level in the narrow cone to the rise of the liquid level in the wide cone?

如图所示，两个顶点朝下的正圆锥包含相同量的液体。液体顶部表面的半径分别为 3 厘米和 6 厘米。在每个圆锥体中放入一个半径为 1 厘米的球形弹子，它沉入底部，完全浸没，没有任何液体溢出。问窄圆锥内液面上升的高度与宽圆锥内液面上升的高度之比是多少



(A) 1 : 1 **(B)** 47 : 43 **(C)** 2 : 1 **(D)** 40 : 13 **(E)** 4 : 1

Problem 13

What is the volume of tetrahedron $ABCD$ with edge lengths $AB=2$, $AC=3$, $AD=4$, $BC=\sqrt{13}$, $BD=2\sqrt{5}$, and $CD=5$?

在四面体 *ABCD* 中，各边长为

$AB=2$, $AC=3$, $AD=4$, $BC=\sqrt{13}$, $BD=2\sqrt{5}$, and $CD=5$?,

问它的体积是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 2\sqrt{3} \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 3\sqrt{3} \qquad \textbf{(E)}\ 6$

Problem 14

All the roots of the polynomial $z^6-10z^5+Az^4+Bz^3+Cz^2+Dz+16$ are positive integers, possibly repeated. What is the value of B?

多项式$z^6-10z^5+Az^4+Bz^3+Cz^2+Dz+16$的根都是正整数，有可能重复。问 *B* 的取值是多少？

$\textbf{(A)}\ -88 \qquad \textbf{(B)}\ -80 \qquad \textbf{(C)}\ -64 \qquad \textbf{(D)}\ -41 \qquad \textbf{(E)}\ -40$

Problem 15

Values for $A, B, C,$ and D are to be selected from $\{1, 2, 3, 4, 5, 6\}$ without replacement (i.e. no two letters have the same value). How many ways are there to make such choices so that the two curves $y=Ax^2+B$ and $y=Cx^2+D$ intersect? (The order in which the curves are listed does not matter; for example, the choices $A=3, B=2, C=4, D=1$ is considered the same as the choices $A=4, B=1, C=3, D=2.$)

A，*B*，*C* 和 *D* 的值从 {1，2，3，4，5，6} 中不重复地选取（即，没有两个字母的取值相同）。使得两条曲线$y=Ax^2+B$和$y=Cx^2+D$相交的不同取值方式有多少种？（不考虑曲线列出的顺序；例如，$A=3, B=2, C=4, D=1$与$A=4, B=1, C=3, D=2.$被认为是相同的。）

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 90 \qquad \textbf{(D)}\ 180 \qquad \textbf{(E)}\ 360$

Problem 16

In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots, 200, 200, \ldots, 200$

What is the median of the numbers in this list?

在下面的数据列表中，对于 1≤*n*≤200，整数 *n* 出现了 n 次。

$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots, 200, 200, \ldots, 200$

问这组数据列表的中位数是多少？

$\textbf{(A)}\ 100.5 \qquad \textbf{(B)}\ 134 \qquad \textbf{(C)}\ 142 \qquad \textbf{(D)}\ 150.5 \qquad \textbf{(E)}\ 167$

Problem 17

Trapezoid $ABCD$ has $\overline{AB} \parallel \overline{CD}$, $BC = CD = 43$, and $\overline{AD} \perp \overline{BD}$. Let O be the intersection of the diagonals $\overline{AC}$ and $\overline{BD}$, and let P be the midpoint of $\overline{BD}$. Given that $OP = 11$, the length AD can be written in the form $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. What is $m + n$?

在梯形 *ABCD* 中，$\overline{AB} \parallel \overline{CD}$，$BC = CD = 43$，并且$\overline{AD} \perp \overline{BD}$。设 *O* 是对角线$\overline{AC}$和$\overline{BD}$的交点，*P* 是$\overline{BD}$的中点。已知 *OP*＝11，*AD* 的长度可以表示成$m\sqrt{n}$，其中 *m* 和 *n* 是正整数，并且 *n* 不能被任何质数的平方所整除。问 *m*＋*n* 的值是多少？

$\textbf{(A)}\ 65 \qquad \textbf{(B)}\ 132 \qquad \textbf{(C)}\ 157 \qquad \textbf{(D)}\ 194 \qquad \textbf{(E)}\ 215$

Problem 18

Let f be a function defined on the set of positive rational numbers with the property that $f(a\cdot b)=f(a)+f(b)$ for all positive rational numbers a and b. Furthermore, suppose that f also has the property that $f(p)=p$ for every prime number p. For which of the following numbers x is $f(x)<0$?

令 *f* 是一个定义在正有理数集合上的函数，它具有性质：对于所有的正有理数 *a* 和 *b*，$f(a\cdot b)=f(a)+f(b)$。假设 *f* 还具有性质：对于每一个质数 *p*，$f(p)=p$。问以下哪个数 *x*，满足$f(x)<0$?

$\textbf{(A)}\ \frac{17}{32} \qquad \textbf{(B)}\ \frac{11}{16} \qquad \textbf{(C)}\ \frac{7}{9} \qquad \textbf{(D)}\ \frac{7}{6} \qquad \textbf{(E)}\ \frac{25}{11}$

Problem 19

The area of the region bounded by the graph of $x^2+y^2=3|x-y|+3|x+y|$ is $m+n\pi$, where m and n are integers. What is $m+n$?

由$x^2+y^2=3|x-y|+3|x+y|$的图像所界定的图形的面积是$m+n\pi$，其中 *m* 和 *n* 是整数。问 *m*+*n* 是多少？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 45 \qquad \textbf{(E)}\ 54$

Problem 20

In how many ways can the sequence $1,2,3,4,5$ be rearranged so that no three consecutive terms are increasing and no three consecutive terms are decreasing?

数列 1，2，3，4，5 有多少种重新排列的方式，使得没有连续三项是递增的，也没有连续三项是递减的？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 44$

Problem 21

Let $ABCDEF$ be an equiangular hexagon. The lines $AB, CD,$ and EF determine a triangle with area $192\sqrt{3}$, and the lines $BC, DE,$ and FA determine a triangle with area $324\sqrt{3}$. The perimeter of hexagon $ABCDEF$ can be expressed as $m+n\sqrt{p}$, where $m, n,$ and p are positive integers and p is not divisible by the square of any prime. What is $m+n+p$?

设 *ABCDEF* 是等角六边形。由 *AB*，*CD* 和 *EF* 所组成的三角形面积是$192\sqrt{3}$，并且由 *BC*，*DE* 和 *FA* 所组成的三角形的面积是$324\sqrt{3}$。六边形 *ABCDEF* 的周长可用$m+n\sqrt{p}$表达，其中 *m*，*n* 和 *p* 是正整数，并且 *p* 不能被任何质数的平方所整除。问$m+n+p$的值是多少？

(A) 47 **(B)** 52 **(C)** 55 **(D)** 58 **(E)** 63

Problem 22

Hiram's algebra notes are 50 pages long and are printed on 25 sheets of paper; the first sheet contains pages 1 and 2, the second sheet contains pages 3 and 4, and so on. One day he leaves his notes on the table before leaving for lunch, and his roommate decides to borrow some pages from the middle of the notes. When Hiram comes back, he discovers that his roommate has taken a consecutive set of sheets from the notes and that the average (mean) of the page numbers on all remaining sheets is exactly 19. How many sheets were borrowed?

Hiram 的代数笔记有 50 页，打印在 25 张纸上；第一张纸包括第 1 和第 2 页，第二张纸包括第 3 和第 4 页，依此类推。有一天，他去午餐前把笔记放在桌子上，室友决定从笔记中间借几页。当 Hiram 回来时，他发现他的室友从笔记中拿走了连续的若干张纸，并且所有剩余纸张上页码的平均值正好是 19。问有多少张纸被借走了？

(A) 10 **(B)** 13 **(C)** 15 **(D)** 17 **(E)** 20

Problem 23

Frieda the frog begins a sequence of hops on a 3×3 grid of squares, moving one square on each hop and choosing at random the direction of each hop-up, down, left, or right. She does not hop diagonally. When the direction of a hop would take Frieda off the grid, she "wraps around" and jumps to the opposite edge. For example if Frieda begins in the center square and makes two hops "up", the first hop would place her in the top row middle square, and the second hop would cause Frieda to jump to the opposite edge, landing in the bottom row middle square. Suppose Frieda starts from the center square, makes at most four hops at random, and stops hopping if she lands on a corner square. What is the probability that she reaches a corner square on one of the four hops?

青蛙 Frieda 在一个 3 x 3 的方格表上开始一系列跳跃，每次跳跃都随机选择一个方向－向上，向下，向左或向右，从一个方格移动到旁边的方格。她不能斜着跳。当跳跃的方向会使得 Frieda 离开方格表时，她会“绕个圈”，跳到相对的另一边。例如，如果 Frieda 从中心方格开始，向上跳跃两次，第一次跳跃后她将位于最上面一行的中间方格，第二次跳跃将使得 Frieda 跳到相对的边，落在最下面一行的中间方格。假设 Frieda 从中心方格出发，最多随机跳跃四次，并且当到达角落方格时就停止跳跃。问她在四次跳跃中到达角落方格的概率是多少？

$\textbf{(A)}\ \frac{9}{16} \qquad \textbf{(B)}\ \frac{5}{8} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{25}{32} \qquad \textbf{(E)}\ \frac{13}{16}$

Problem 24

The interior of a quadrilateral is bounded by the graphs of $(x+ay)^2 = 4a^2$ and $(ax-y)^2 = a^2$, where a is a positive real number. What is the area of this region in terms of a, valid for all $a > 0$?

设 *a* 是正实数，考虑由$(x+ay)^2 = 4a^2$和$(ax-y)^2 = a^2$组成的四边形的内部。对所有的 *a* >0 而言，这个区域的面积怎样用 *a* 来表示？

$\textbf{(A)}\ \frac{8a^2}{(a+1)^2} \qquad \textbf{(B)}\ \frac{4a}{a+1} \qquad \textbf{(C)}\ \frac{8a}{a+1} \qquad \textbf{(D)}\ \frac{8a^2}{a^2+1} \qquad \textbf{(E)}\ \frac{8a}{a^2+1}$

Problem 25

How many ways are there to place 3 indistinguishable red chips, 3 indistinguishable blue chips, and 3 indistinguishable green chips in the squares of a 3×3 grid so that no two chips of the same color are directly adjacent to each other, either vertically or horizontally?

将 3 枚不可区分的红色筹码，3 枚不可区分的蓝色筹码和 3 枚不可区分的绿色筹码分别放在 3 x 3 方格表的各个小方格中，使得无论是垂直方向还是水平方向，都没有两个相同颜色的筹码相邻，问共有多少种放法？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 36$

2021Spring AMC10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	C	D	D	B	A	D	E	D	C	E	E	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
A	C	C	D	E	E	D	C	B	D	D	E	