2012 AMC 8 Problems/Problem 1

Contents

Problem

Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighborhood picnic?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 6\frac{2}{3} \qquad \textbf{(C)}\ 7\frac{1}{2} \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Solution

Solution 1

Since Rachelle uses 3 pounds of meat to make 8 hamburgers, she uses $\frac{3}{8}$ pounds of meat to make one hamburger. She'll need 24 times that amount of meat for 24 hamburgers, or $\frac{3}{8} \cdot 24 = \boxed{\textbf{(E)}\ 9}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 2

Problem

In the country of East Westmore, statisticians estimate there is a baby born every 8 hours and a death every day. To the nearest hundred, how many people are added to the population of East Westmore each year?

$\textbf{(A)}\ 600\qquad\textbf{(B)}\ 700\qquad\textbf{(C)}\ 800\qquad\textbf{(D)}\ 900\qquad\textbf{(E)}\ 1000$

Solution

There are $24 \text{ hours} \div 8 \text{ hours} = 3$ births and one death everyday in East Westmore. Therefore, the population increases by 2 people everyday. Thus, there are $2 \times 365 = 730$ people added to the population every year. Rounding, we find the answer is $\boxed{\textbf{(B)}\ 700}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
Preceded by Problem 1	Followed by Problem 3
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2012 AMC 8 Problems/Problem 3

Problem

On February 13 The Oshkosh Northwester listed the length of daylight as 10 hours and 24 minutes, the sunrise was $6:57\text{AM}$, and the sunset as $8:15\text{PM}$. The length of daylight and sunrise were correct, but the sunset was wrong. When did the sun really set?

$\textbf{(A)}\hspace{.05in}5:10\textsc{pm}\quad\textbf{(B)}\hspace{.05in}5:21\textsc{pm}\quad\textbf{(C)}\hspace{.05in}5:41\textsc{pm}\quad\textbf{(D)}\hspace{.05in}5:57\textsc{pm}\quad\textbf{(E)}\hspace{.05in}6:03\textsc{pm}$

Solution

The problem wants us to find the time of sunset and gives us the length of daylight and time of sunrise. So all we have to do is add the length of daylight to the time of sunrise to obtain the answer. Convert 10 hours and 24 minutes into $10:24$ in order to add easier.

Adding, we find that the time of sunset is
$6:57\textsc{am} + 10:24 \implies 17:21 \implies \boxed{\textbf{(B)}\ 5:21\textsc{pm}}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 4

Problem

Peter's family ordered a 12-slice pizza for dinner. Peter ate one slice and shared another slice equally with his brother Paul. What fraction of the pizza did Peter eat?

$\textbf{(A)}\ \frac{1}{24} \qquad \textbf{(B)}\ \frac{1}{12} \qquad \textbf{(C)}\ \frac{1}{8} \qquad \textbf{(D)}\ \frac{1}{6} \qquad \textbf{(E)}\ \frac{1}{4}$

Solution

Peter ate $1 + \frac{1}{2} = \frac{3}{2}$ slices. The pizza has 12 slices total. Taking the ratio of the amount of slices Peter ate to the amount of slices in the pizza, we find that Peter ate $\frac{\frac{3}{2}\text{ slices}}{12\text{ slices}} = \boxed{\textbf{(C)}\ \frac{1}{8}}$ of the pizza.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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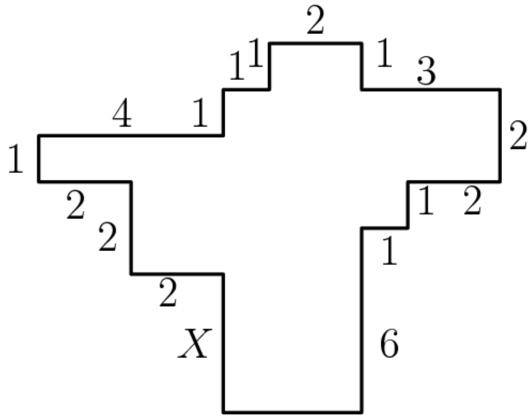
2012 AMC 8 Problems/Problem 5

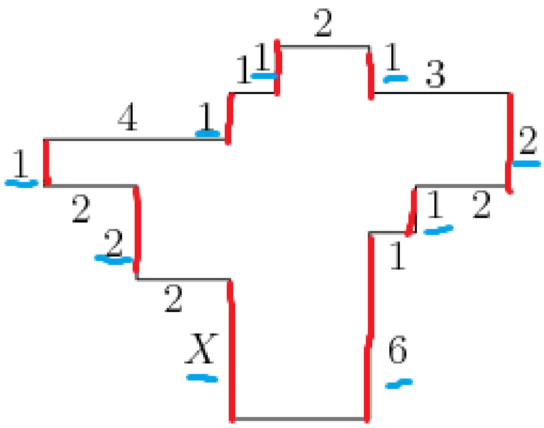
Problem

In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is , X in centimeters?



$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution



The figure is the same height on both sides, so the sum of the lengths contributing to the height on the left side will equal the sum of the lengths contributing to the height on the left side.

$1+1+1+2+X=1+2+1+6$
$5+X=10$
$X=5$

Thus, the answer is $\boxed{\textbf{(E)}\ 5}$.

See Also

2012 AMC 8 Problems/Problem 6

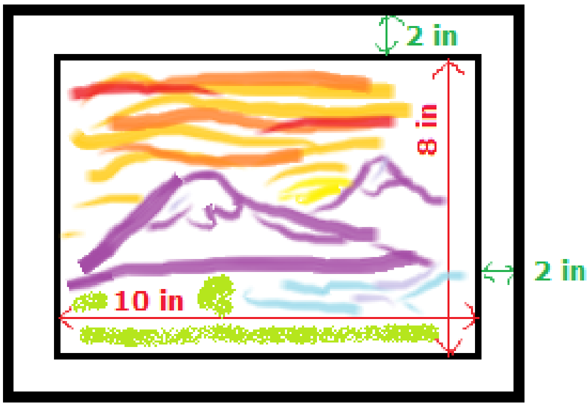
Problem

A rectangular photograph is placed in a frame that forms a border two inches wide on all sides of the photograph. The photograph measures 8 inches high and 10 inches wide. What is the area of the border, in square inches?

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 88$

Solution

First, we start with a sketch. It's always a good idea to start with a picture, although not as detailed as this one. (Tip: On the actual AMC 8 test, you should NOT color your picture in, even if it is beautiful)



In order to find the area of the frame, we need to subtract the area of the photograph from the area of the photograph and the frame together. The area of the photograph is $8 \times 10 = 80$ square inches. The height of the whole frame (including the photograph) would be $8 + 2 + 2 = 12$, and the width of the whole frame, $10 + 2 + 2 = 14$. Therefore, the area of the whole figure would be $12 \times 14 = 168$ square inches. Subtracting the area of the photograph from the area of both the frame and photograph, we find the answer to be $168 - 80 = \boxed{\textbf{(E)}\ 88}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 7

Problem

Isabella must take four 100-point tests in her math class. Her goal is to achieve an average grade of 95 on the tests. Her first two test scores were 97 and 91. After seeing her score on the third test, she realized she can still reach her goal. What is the lowest possible score she could have made on the third test?

$\textbf{(A)}\ 90 \qquad\textbf{(B)}\ 92 \qquad\textbf{(C)}\ 95 \qquad\textbf{(D)}\ 96 \qquad\textbf{(E)}\ 97$

Solution

Isabella wants an average grade of 95 on her 4 tests; this also means that she wants the sum of her test scores to be at least $95 \times 4 = 380$ (if she goes over this number, she'll be over her goal!). She's already taken two tests, which sum to $97 + 91 = 188$, which means she needs 192 more points to achieve her desired average. In order to minimize the score on the third test, we assume that Isabella will receive all 100 points on the fourth test. Therefore, the lowest Isabella could have scored on the third test would be $192 - 100 = \boxed{\textbf{(B)}\ 92}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 8

Contents

Problem

A shop advertises everything is "half price in today's sale." In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage off the original price?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 33 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 70$

Solution

Solution 1: With Algebra

Let the original price of an item be x.

First, everything is half-off, so the price is now $\frac{x}{2} = 0.5x$.

Next, the extra coupon applies 20% off on the sale price, so the price after this discount will be $100\% - 20\% = 80\%$ of what it was before. (Notice how this is not applied to the original price; if it were, the solution would be applying 50% + 20 % = 70% off the original price.)

$$80\% \cdot 0.5x = \frac{4}{5} \cdot 0.5x = 0.4x$$

The price of the item after all discounts have been applied is $0.4x = 40\% \cdot x$. However, we need to find the percentage off the original price, not the current percentage of the original price. We then subtract $40\%x$ from $100\%x$ (the original price of the item), to find the answer, $\boxed{\textbf{(D)}\ 60}$.

Solution 2: Fakesolving

Since the problem implies that the percentage off the original price will be the same for every item in the store, fakesolving is applicable here. Say we are buying an item worth 10 dollars, a convenient number to work with. First, it is clear that we'll get 50% off, which makes the price then 5 dollars. Taking 20% off of 5 dollars gives us 4 dollars. Therefore, we have saved a total of $\frac{10-4}{10} = \frac{6}{10} = \frac{60}{100} = \boxed{\textbf{(D)}\ 60}\%$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 9

Contents

Problem

The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

$\textbf{(A)}\hspace{.05in}61\qquad\textbf{(B)}\hspace{.05in}122\qquad\textbf{(C)}\hspace{.05in}139\qquad\textbf{(D)}\hspace{.05in}150\qquad\textbf{(E)}\hspace{.05in}161$

Solution

Solution 1: Algebra

Let the number of two-legged birds be x and the number of four-legged mammals be y. We can now use systems of equations to solve this problem.

Write two equations:

$$2x + 4y = 522$$

$$x + y = 200$$

Now multiply the latter equation by 2.

$$2x + 4y = 522$$

$$2x + 2y = 400$$

By subtracting the second equation from the first equation, we find that $2y = 122 \implies y = 61$. Since there were 200 heads, meaning that there were 200 animals, there were $200 - 61 = \boxed{\textbf{(C)}\ 139}$ two-legged birds.

Solution 2: Cheating the System

First, we "assume" there are 200 two-legged birds only, and 0 four-legged mammals. Of course, this poses a problem, as then there would only be $200 \cdot 2 = 400$ legs.

Now we have to do some swapping--for every two-legged bird we swap for a four-legged mammal, we gain 2 legs. For example, if we swapped one bird for one mammal, giving 199 birds and 1 mammal, there would be $400 + 1(2) = 402$ legs. If we swapped two birds for two mammals, there would be $400 + 2(2) = 404$ legs. If we swapped 50 birds for 50 mammals, there would be $400 + 50(2) = 500$ legs.

Notice that we must gain $522 - 400 = 122$ legs. This means we must swap out $122 \div 2 = 61$ birds. Therefore, there must be $200 - 61 = \boxed{\textbf{(C)}\ 139}$ birds. Checking our work, we find that $139 \cdot 2 + 61 \cdot 4 = 522$, and we are correct.

See Also

2012 AMC 8 Problems/Problem 10

Contents

Problem

How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 7\qquad\textbf{(C)}\ 8\qquad\textbf{(D)}\ 9\qquad\textbf{(E)}\ 12$

Solution 1

For this problem, all we need to do is find the amount of valid 4-digit numbers that can be made from the digits of 2012, since all of the valid 4-digit number will always be greater than 1000. The best way to solve this problem is by using casework.

There can be only two leading digits, namely 1 or 2.

When the leading digit is 1, you can make $\frac{3!}{2!1!} \implies 3$ such numbers.

When the leading digit is 2, you can make $3! \implies 6$ such numbers.

Summing the amounts of numbers, we find that there are $\boxed{\textbf{(D)}\ 9}$ such numbers.

Solution 2

Notice that the first digit cannot be 0, as the number is greater than 1000. Therefore, there are three digits that can be in the thousands.

The rest three digits of the number have no restrictions, and therefore there are $3! \implies 6$ for each leading digit.

Since the two 2's are indistinguishable, there are $\frac{3 \cdot 6}{2}$ such numbers $\implies \boxed{\textbf{(D)}\ 9}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 11

Contents

Problem

The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7, and x are all equal. What is the value of x?

$\textbf{(A)}\ 5\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 7\qquad\textbf{(D)}\ 11\qquad\textbf{(E)}\ 12$

Solution

Since there must be a unique mode, and 6 is already repeated twice, x cannot be any of the numbers already listed (3, 4, 5, 7). (If it were, the mode would not be unique.) So x must be 6, or a new number.

Solution 1: Guess & Check

We can eliminate answer choices $\textbf{(A)}\ 5$ and $\textbf{(C)}\ 7$, because of the above statement. Now we need to test the remaining answer choices.

Case 1: $x=6$

Mode: 6

Median: 6

Mean: $\frac{37}{7}$

Since the mean does not equal the median or mode, $\textbf{(B)}\ 6$ can also be eliminated.

Case 2: $x=11$

Mode: 6

Median: 6

Mean: 6

We are done with this problem, because we have found when $x=11$, the condition is satisfied. Therefore, the answer is $\boxed{\textbf{(D)}\ 11}$.

Solution 2: Algebra

Notice that the mean of this set of numbers, in terms of x, is:

$$\frac{3+4+5+6+6+7+x}{7}=\frac{31+x}{7}$$

Because we know that the mode must be 6 (it can't be any of the numbers already listed, as shown above, and no matter what x is, either 6 or a new number, it will not affect 6 being the mode), and we know that the mode must equal the mean, we can set the expression for the mean and 6 equal:

$$\frac{31+x}{7}=6$$
$$31+x=42$$
$$x=\boxed{\textbf{(D)}\ 11}$$

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 12

Problem

What is the units digit of 13^{2012}?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 9$

Solution

The problem wants us to find the units digit of 13^{2012}, therefore, we can eliminate the tens digit of 13, because the tens digit will not affect the final result. So our new expression is 3^{2012}. Now we need to look for a pattern in the units digit.

$3^1 \implies 3$

$3^2 \implies 9$

$3^3 \implies 7$

$3^4 \implies 1$

$3^5 \implies 3$

We observe that there is a pattern for the units digit which recurs every four powers of three. Using this pattern, we can subtract 1 from 2012 and divide by 4. The remainder is the power of three that we are looking for, plus one. 2011 divided by 4 leaves a remainder of 3, so the answer is the units digit of 3^{3+1}, or 3^4. Thus, we find that the units digit of 13^{2012} is $\boxed{\textbf{(A)}\ 1}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))
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2012 AMC 8 Problems/Problem 13

Problem

Jamar bought some pencils costing more than a penny each at the school bookstore and paid $\$1.43$. Sharona bought some of the same pencils and paid $\$1.87$. How many more pencils did Sharona buy than Jamar?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

We assume that the price of the pencils remains constant. Convert $\$1.43$ and $\$1.87$ to cents. Since the price of the pencils is more than one penny, we can find the price of one pencil (in cents) by taking the greatest common divisor of 143 and 187, which is 11. Therefore, Jamar bought $\frac{143}{11} \implies 13$ pencils and Sharona bought $\frac{187}{11} \implies 17$ pencils. Thus, Sharona bought $17-13=\boxed{\textbf{(C)}\ 4}$ more pencils than Jamar.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 14

Problem

In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference?

$\textbf{(A)}\hspace{.05in}6\qquad\textbf{(B)}\hspace{.05in}7\qquad\textbf{(C)}\hspace{.05in}8\qquad\textbf{(D)}\hspace{.05in}9\qquad\textbf{(E)}\hspace{.05in}10$

Solution

This problem is very similar to a handshake problem. We use the formula $\frac{n(n+1)}{2}$ to usually find the number of games played (or handshakes). Now we have to use the formula in reverse.

So we have the equation $\frac{n(n-1)}{2} = 21$. Solving, we find that the number of teams in the BIG N conference is $\boxed{\textbf{(B)}\ 7}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 15

Problem

The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?

$\textbf{(A)}\ 40\text{ and }50 \qquad \textbf{(B)}\ 51\text{ and }55 \qquad \textbf{(C)}\ 56\text{ and }60 \qquad \textbf{(D)}\ 61\text{ and }65 \qquad \textbf{(E)}\ 66\text{ and }99$

Solution

To find the answer to this problem, we need to find the least common multiple of 3, 4, 5, 6 and add 2 to the result. The least common multiple of the four numbers is 60, and by adding 2, we find that that such number is 62. Now we need to find the only given range that contains 62. The only such range is answer $\textbf{(D)}$, and so our final answer is $\boxed{\textbf{(D)}\ 61\text{ and }65}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 16

Contents

Problem

Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers?

$\textbf{(A)}\ 76531 \qquad \textbf{(B)}\ 86724 \qquad \textbf{(C)}\ 87431 \qquad \textbf{(D)}\ 96240 \qquad \textbf{(E)}\ 97403$

Solution 1

In order to maximize the sum of the numbers, the numbers must have their digits ordered in decreasing value. There are only two numbers from the answer choices with this property: 76531 and 87431. To determine the answer we will have to use estimation and the first two digits of the numbers.

For 76531 the number that would maximize the sum would start with 98. The first two digits of 76531 (when rounded) are 77. Adding 98 and 77, we find that the first three digits of the sum of the two numbers would be 175.

For 87431 the number that would maximize the sum would start with 96. The first two digits of 87431 (when rounded) are 87. Adding 96 and 87, we find that the first three digits of the sum of the two numbers would be 183.

From the estimations, we can say that the answer to this problem is $\boxed{\textbf{(C)}\ 87431}$.

Solution 2

In order to determine the largest number possible, we have to evenly distribute the digits when adding. The two numbers that show an example of this are 97531 and 86420. The digits can be interchangeable between numbers because we only care about the actual digits.

The first digit must be either 9 or 8. This immediately knocks out $\textbf{(A)}\ 76531$.

The second digit must be either 7 or 6. This doesn't cancel any choices.

The third digit must be either 5 or 4. This knocks out $\textbf{(B)}\ 86724$ and $\textbf{(D)}\ 96240$.

The fourth digit must be 3 or 2. This cancels out $\textbf{(E)}\ 97403$.

This leaves us with $\boxed{\textbf{(C)}\ 87431}$.

See Also

2012 AMC 8 Problems/Problem 17

Problem

A square with integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

The first answer choice $\textbf{(A)}\ 3$, can be eliminated since there must be 10 squares with integer side lengths. We then test the next largest sidelength which is 4. The square with area 16 can be partitioned into 8 squares with area 1 and two squares with area 4, which satisfies all the conditions of the problem. Therefore, the smallest possible value of the length of the side of the original square is $\boxed{\textbf{(B)}\ 4}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 18

Problem

What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?

$\textbf{(A)}\ 3127 \qquad \textbf{(B)}\ 3133 \qquad \textbf{(C)}\ 3137 \qquad \textbf{(D)}\ 3139 \qquad \textbf{(E)}\ 3149$

Solution

The problem states that the answer cannot be a perfect square or have prime factors less than 50. Therefore, the answer will be the product of at least two different primes greater than 50. The two smallest primes greater than 50 are 53 and 59. Multiplying these two primes, we obtain the number 3127, which is also the smallest number on the list of answer choices. So we are done, and the answer is $\boxed{\textbf{(A)}\ 3127}$.

See Also

2012 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2012))	
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2012 AMC 8 Problems/Problem 19

Problem

In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?

$\textbf{(A)}\hspace{.05in}6\qquad\textbf{(B)}\hspace{.05in}8\qquad\textbf{(C)}\hspace{.05in}9\qquad\textbf{(D)}\hspace{.05in}10\qquad\textbf{(E)}\hspace{.05in}12$

Solution

Let r be the number of red marbles, g be the number of green marbles, and b be the number of blue marbles.

If "all but 6 are red marbles", that means that the number of green marbles and the number of blue marbles amount to 6. Likewise, the number of red marbles and blue marbles amount to 8, and the number of red marbles and the number of green marbles amount to 4.

We have three equations:

$$g+b=6$$

$$r+b=8$$

$$r+g=4$$

We add all the equations to obtain a fourth equation:

$$2r+2g+2b=18$$

Now divide by 2 on both sides to find the total number of marbles:

$$r+g+b=9$$

Since the sum of the number of red marbles, green marbles, and blue marbles is the number of marbles in the jar, the total number of marbles in the jar is $\boxed{\textbf{(C)}\ 9}$.

Notice how we never knew how many of each color there were (there is 1 green marble, 5 blue marbles, and 3 red marbles).

See Also

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2012 AMC 8 Problems/Problem 20

Contents

Problem

What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?

$\textbf{(A)}\hspace{.05in}\frac{9}{23}<\frac{7}{21}<\frac{5}{19}\quad\textbf{(B)}\hspace{.05in}\frac{5}{19}<\frac{7}{21}<\frac{9}{23}\quad\textbf{(C)}\hspace{.05in}\frac{9}{23}<\frac{5}{19}<\frac{7}{21}$

$\textbf{(D)}\hspace{.05in}\frac{5}{19}<\frac{9}{23}<\frac{7}{21}\quad\textbf{(E)}\hspace{.05in}\frac{7}{21}<\frac{5}{19}<\frac{9}{23}$

Solution 1

The value of $\frac{7}{21}$ is $\frac{1}{3}$. Now we give all the fractions a common denominator.

$\frac{5}{19} \implies \frac{345}{1311}$

$\frac{1}{3} \implies \frac{437}{1311}$

$\frac{9}{23} \implies \frac{513}{1311}$

Ordering the fractions from least to greatest, we find that they are in the order listed. Therefore, our final answer is $\boxed{\textbf{(B)}\ \frac{5}{19}<\frac{7}{21}<\frac{9}{23}}$.

Solution 2

Instead of finding the LCD, we can subtract each fraction from 1 to get a common numerator. Thus,

$1-\frac{5}{19}=\frac{14}{19}$

$1-\frac{7}{21}=\frac{14}{21}$

$1-\frac{9}{23}=\frac{14}{23}$

All three fractions have common numerator 14. Now it is obvious the order of the fractions.
$\frac{14}{19}>\frac{14}{21}>\frac{14}{23} \implies \frac{5}{19}<\frac{7}{21}<\frac{9}{23}$. Therefore, our answer is $\boxed{\textbf{(B)}\ \frac{5}{19}<\frac{7}{21}<\frac{9}{23}}$.

2012 AMC 8 Problems/Problem 21

Problem

Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?

$\textbf{(A)}\hspace{.05in}5\sqrt2\qquad\textbf{(B)}\hspace{.05in}10\qquad\textbf{(C)}\hspace{.05in}10\sqrt2\qquad\textbf{(D)}\hspace{.05in}50\qquad\textbf{(E)}\hspace{.05in}50\sqrt2$

Solution

If Marla evenly distributes her 300 square feet of paint between the 6 faces, each face will get $300 \div 6 = 50$ square feet of paint. The surface area of one of the faces of the cube is $10^2 = 100$ square feet. Therefore, there will be $100 - 50 = \boxed{\textbf{(D)}\ 50}$ square feet of white on each side.

See Also

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2012 AMC 8 Problems/Problem 22

Problem

Let R be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R ?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 5\qquad\textbf{(C)}\ 6\qquad\textbf{(D)}\ 7\qquad\textbf{(E)}\ 8$

Solution

First, we find that the minimum value of the median of R will be 3.

We then experiment with sequences of numbers to determine other possible medians.

Median: 3

Sequence: $-2, -1, 0, 2, 3, 4, 6, 9, 14$

Median: 4

Sequence: $-1, 0, 2, 3, 4, 6, 9, 10, 14$

Median: 5

Sequence: $0, 2, 3, 4, 5, 6, 9, 10, 14$

Median: 6

Sequence: $0, 2, 3, 4, 6, 9, 10, 14, 15$

Median: 7

Sequence: $2, 3, 4, 6, 7, 8, 9, 10, 14$

Median: 8

Sequence: $2, 3, 4, 6, 8, 9, 10, 14, 15$

Median: 9

Sequence: $2, 3, 4, 6, 9, 14, 15, 16, 17$

Any number greater than 9 also cannot be a median of set R.

There are then $\boxed{\textbf{(D)}\ 7}$ possible medians of set R.

See Also

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2012 AMC 8 Problems/Problem 23

Contents

Problem

An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 4\sqrt3 \qquad \textbf{(E)}\ 6\sqrt3$

Solution 1

Let the perimeter of the equilateral triangle be $3s$. The side length of the equilateral triangle would then be s and the sidelength of the hexagon would be $\frac{s}{2}$.

A hexagon contains six equilateral triangles. One of these triangles would be similar to the large equilateral triangle in the ratio $1 : 4$, since the sidelength of the small equilateral triangle is half the sidelength of the large one. Thus, the area of one of the small equilateral triangles is 1. The area of the hexagon is then $1 \times 6 = \boxed{\textbf{(C)}\ 6}$.

Solution 2

Let the side length of the equilateral triangle be s and the side length of the hexagon be y. Since the perimeters are equal, we must have $3s = 6y$ which reduces to $s = 2y$. Substitute this value in to the area of an equilateral triangle to yield $\frac{(2y)^2\sqrt{3}}{4} = \frac{4y^2\sqrt{3}}{4}$.

Setting this equal to 4 gives us $\frac{4y^2\sqrt{3}}{4} = 4 \implies 4y^2\sqrt{3} = 16 \implies y^2\sqrt{3} = 4.$

Substitue $y^2\sqrt{3}$ into the area of a regular hexagon to yield $\frac{3(4)}{2} = 6.$

Therefore, our answer is $\boxed{\textbf{(C)}\ 6}$.

Solution 3

Let the side length of the triangle be s and the side length of the hexagon be t. As explained in Solution 1, $s = 2t$, or $t = \frac{s}{2}$. The area of the triangle is $\frac{s^2\sqrt{3}}{4} = 4$ and the area of the hexagon is $\frac{t^2\sqrt{3}}{4} \cdot 6 = \frac{3t^2\sqrt{3}}{2}$. Substituting $\frac{s}{2}$ in for t, we get

$$\frac{\frac{3s^2\sqrt{3}}{4}}{2} = \frac{3s^2\sqrt{3}}{8}.$$

$$\frac{s^2\sqrt{3}}{4} = 4 \implies \frac{s^2\sqrt{3}}{8} = 2 \implies \frac{3s^2\sqrt{3}}{8} = \boxed{\textbf{(C)}\ 6}.$$

Notes

The area of an equilateral triangle with side length s is $\frac{s^2\sqrt{3}}{4}$.

The area of a regular hexagon with side length s is $\frac{3s^2\sqrt{3}}{2}$.

See Also

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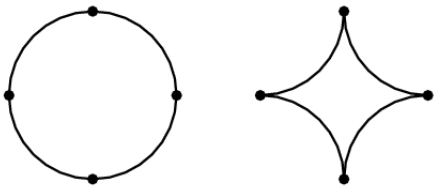
2012 AMC 8 Problems/Problem 24

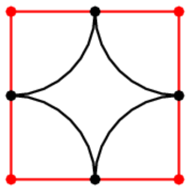
Problem

A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



$\textbf{(A)}\ \frac{4-\pi}{\pi}\qquad\textbf{(B)}\ \frac{1}{\pi}\qquad\textbf{(C)}\ \frac{\sqrt{2}}{\pi}\qquad\textbf{(D)}\ \frac{\pi-1}{\pi}\qquad\textbf{(E)}\ \frac{3}{\pi}$

Solution



Draw a square around the star figure. The sidelength of this square is 4, because the sidelength is the diameter of the circle. The square forms 4-quarter circles around the star figure. This is the equivalent of one large circle with radius 2, meaning that the total area of the quarter circles is 4π. The area of the square is 16. Thus, the area of the star figure is $16 - 4\pi$. The area of the circle is 4π. Taking the ratio of the two areas, we find the answer is $\boxed{\textbf{(A)}\ \frac{4-\pi}{\pi}}$.

See Also

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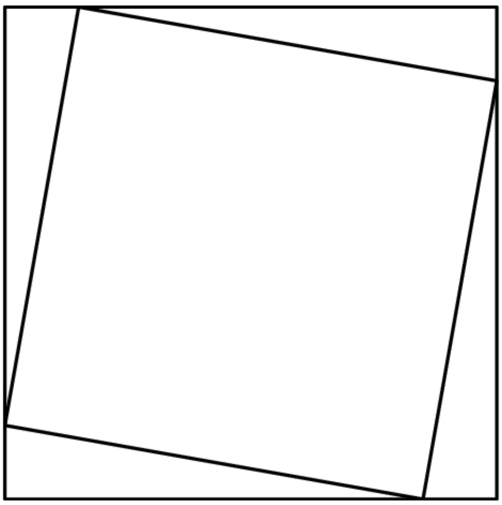
2012 AMC 8 Problems/Problem 25

Contents

Problem

A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length a, and the other of length b. What is the value of ab?



$\textbf{(A)}\hspace{.05in}\frac{1}{5}\qquad\textbf{(B)}\hspace{.05in}\frac{2}{5}\qquad\textbf{(C)}\hspace{.05in}\frac{1}{2}\qquad\textbf{(D)}\hspace{.05in}1\qquad\textbf{(E)}\hspace{.05in}4$

Solution 1

The total area of the four congruent triangles formed by the squares is $5-4=1$. Therefore, the area of one of these triangles is $\frac{1}{4}$. The height of one of these triangles is a and the base is b. Using the formula for area of the triangle, we have $\frac{ab}{2}=\frac{1}{4}$. Multiply by 2 on both sides to find that the value of ab is $\boxed{\textbf{(C)}\ \frac{1}{2}}$.

Solution 2

To solve this problem you could also use algebraic manipulation.

Since the area of the large square is 5, the sidelength is $\sqrt{5}$.

We then have the equation $a+b=\sqrt{5}$.

We also know that the side length of the smaller square is 2, since its area is 4. Then, the segment of length a and segment of length b form a right triangle whose hypotenuse would have length 2.

So our second equation is $\sqrt{a^2+b^2}=2$.

Square both equations.

$$a^2+2ab+b^2=5$$

$$a^2+b^2=4$$

Now, subtract, and obtain the equation $2ab=1$. We can deduce that the value of ab is $\boxed{\textbf{(C)}\ \frac{1}{2}}$.

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