2019 AMC 10B

Problem 1

Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the first container to the volume of the second container?

Alicia 有两个容器，第一个里面的水是$\frac{5}{6}$满的，第二个是空的；她将第一个容器中的所有水倒入第二个容器中，此时第二个容器中的水是$\frac{3}{4}$满的， 较小容器的容积与较大容器的容积之比是多少？

$\textbf{(A)}\ \frac{5}{8} \qquad \textbf{(B)}\ \frac{4}{5} \qquad \textbf{(C)}\ \frac{7}{8} \qquad \textbf{(D)}\ \frac{9}{10} \qquad \textbf{(E)}\ \frac{11}{12}$

Problem 2

Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

考虑论断：“如果 n 不是质数，那么 $n-2$ 就是质数。” 以下的哪个 n 值是此论断的反例？

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 19 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 27$

Problem 3

In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument?

在一所有 500 名学生的高中，40%的高三学生会演奏乐器，而 30%的非高三学生不会演奏乐器。整体而言，46.8%的学生不会演奏乐器。有多少非高三学生会演奏乐器？

$\textbf{(A)}\ 66 \qquad \textbf{(B)}\ 154 \qquad \textbf{(C)}\ 186 \qquad \textbf{(D)}\ 220 \qquad \textbf{(E)}\ 266$

Problem 4

All lines with equation $ax + by = c$ such that a, b, c form an arithmetic progression pass through a common point. What are the coordinates of that point?

a，*b*，*c* 是等差列的，且方程 *ax* + *by* = *c* 表示的直线都通过一公共点。那个点的坐标是什么？

(A) $(-1,2)$ **(B)** $(0,1)$ **(C)** $(1,-2)$ **(D)** $(1,0)$ **(E)** $(1,2)$

Problem 5

Triangle ABC lies in the first quadrant. Points A, B, and C are reflected across the line $y = x$ to points A', B', and C', respectively. Assume that none of the vertices of the triangle lie on the line $y = x$. Which of the following statements is not always true?

三角形 *ABC* 位于第一象限。点 *A*，*B* 和 *C* 沿直线 *y* = *x* 反射分别得到点 A', B', 和 C'。假设三角形的顶点都不在 *y* = *x* 这条在线。以下哪个陈述不是一定正确？

(A) Triangle $A'B'C'$ lies in the first quadrant | 三角形$A'B'C'$在第一象限

(B) Triangles ABC and $A'B'C'$ have the same area | 三角形 *ABC* 和 $A'B'C'$的面积相同

(C) The slope of line AA' is -1 | 直线AA'的斜率是-1

(D) The slopes of lines AA' and CC' are the same | 直AA'和CC'的斜率相同

(E) Lines AB and $A'B'$ are perpendicular to each other | 直线 *AB* 和 $A'B'$互相垂直

Problem 6

There is a real n such that $(n+1)! + (n+2)! = n! \cdot 440$. What is the sum of the digits of n?

正整数 *n* 满足等式$(n+1)! + (n+2)! = n! \cdot 440$。 *n* 的各个数位上的数字之和是多少？

(A) 3 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 12

Problem 7

Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n?

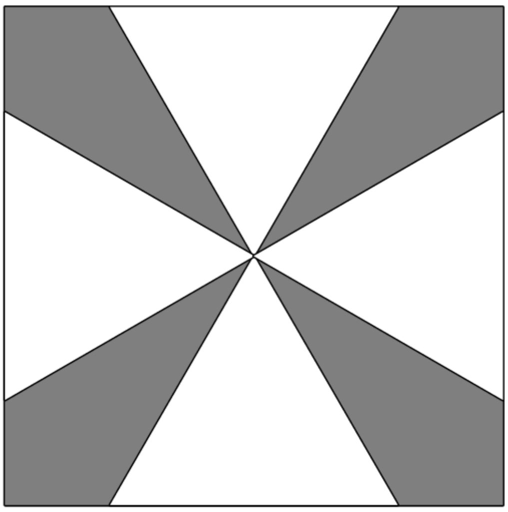
商店里每一块糖的售价都是整数分钱。Casper 有足够的钱购买 12 块红色糖果，14 块绿色糖果，15 块蓝色糖果或者 n 块紫色糖果。一块紫色糖果售价 20 美分。 n 的最小可能价值是多少？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 28$

Problem 8

The figure below shows a square and four equilateral triangles, with each triangle having a side lying on a side of the square, such that each triangle has side length 2 and the third vertices of the triangles meet at the center of the square. The region inside the square but outside the triangles is shaded. What is the area of the shaded region?

下图显示了正方形和四个等边三角形，每个三角形有一条边在正方形的一条边上，使得每个三角形的边长为 2，并且各三角形的第三顶点在正方形的中心相交。在正方形内但在三角形外的区域用阴影表示。问阴影区域的面积是多少？



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 12 - 4\sqrt{3} \qquad \textbf{(C)}\ 3\sqrt{3} \qquad \textbf{(D)}\ 4\sqrt{3} \qquad \textbf{(E)}\ 16 - 4\sqrt{3}$

Problem 9

The function f is defined by $f(x) = \lfloor |x| \rfloor - |\lfloor x \rfloor|$ for all real numbers x, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r. What is the range of f?

对任何实数 x，函数 f 定义为 $f(x) = \lfloor |x| \rfloor - |\lfloor x \rfloor|$，其中 $\lfloor r \rfloor$ 表示小于等于实数 r 的最大整数. 问 f 的取值范围是多少?

(A) $\{-1, 0\}$

(B) The set of nonpositive integers | 非正整数集

(C) $\{-1, 0, 1\}$

(D) $\{0\}$

(E) The set of nonnegative integers | 非负整数集

Problem 10

In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

在给定的平面中，点 A 和 B 相距 10 个单位长度。平面上有多少个点 C，使得 $\triangle ABC$ 的周长是 50 单位长度，且 $\triangle ABC$ 的面积是 100 平方单位？

(A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** infinitely many

Problem 11

Two jars each contain the same number of marbles, and every marble is either blue or green. In Jar 1 the ratio of blue to green marbles is $9:1$, and the ratio of blue to green marbles in Jar 2 is $8:1$. There are 95 green marbles in all. How many more blue marbles are in Jar 1 than in Jar 2?

两个罐子中的每个都包含相同数量的玻璃球，每个玻璃球都是蓝色或绿色的。在第 1 个 罐子中，蓝色与绿色玻璃球的比例为 9：1，而在第 2 个罐子中，蓝色与绿色玻璃球的比例为 8：1。总共有 95 个的绿色玻璃球。那么第 1 个罐子中的蓝色玻璃球比第 2 个罐子的多多少个？

(A) 5 **(B)** 10 **(C)** 25 **(D)** 45 **(E)** 50

Problem 12

What is the greatest possible sum of the digits in the base-seven representation of a positive integer less than 2019?

一个小于 2019 的正整数，它的七进制表示中的各个数位上的数字之和的最大可能值是多少？

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 27$

Problem 13

What is the sum of all real numbers x for which the median of the numbers $4,6,8,17,$ and x is equal to the mean of those five numbers?

实数 x 满足与 4，6，8，17 这五个数的中位数等于这五个数的平均值，问所有满足条件的实数 x 的总和是多少？

$\textbf{(A)}\ -5 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ \frac{15}{4} \qquad \textbf{(E)}\ \frac{35}{4}$

Problem 14

The base-ten representation for 19! is $121,6T5,100,40M,832,H00$, where T, M, and H denote digits that are not given. What is $T+M+H$?

在十进制表示下，19!是 121, 6T5, 100, 40M, 832, H00，其中 T，M，H 所表示的数字没有给出。问 $T+M+H$ 是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 17$

Problem 15

Right triangles T_1 and T_2 have areas 1 and 2, respectively. A side of T_1 is congruent to a side of T_2, and a different side of T_1 is congruent to a different side of T_2. What is the square of the product of the other (third) sides of T_1 and T_2?

直角三角形 T_1 和 T_2 的面积分别为 1 和 2， T_1 的一边与 T_2 的一边相等，T_1 的另一边与 T_2 的另一边相等。T_1 和 T_2 的其它（第三）边长度乘积的平方是多少？

$\textbf{(A)}\ \frac{28}{3} \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ \frac{32}{3} \qquad \textbf{(D)}\ \frac{34}{3} \qquad \textbf{(E)}\ 12$

Problem 16

In $\triangle ABC$ with a right angle at C, point D lies in the interior of $\overline{AB}$ and point E lies in the interior of $\overline{BC}$ so that $AC = CD, DE = EB,$ and the ratio $AC : DE = 4 : 3$. What is the ratio $AD : DB$?

在$\triangle ABC$中 C为直角顶点，点 D在$\overline{AB}$的内部， E在$\overline{BC}$的内部， 使得 $AC = CD$, $DE = EB$, 并且有比例关系 $AC : DE = 4 : 3$。 问比例 $AD : DB$ 是多少？

(A) $2:3$ **(B)** $2:\sqrt{5}$ **(C)** $1:1$ **(D)** $3:\sqrt{5}$ **(E)** $3:2$

Problem 17

A red ball and a green ball are randomly and independently tossed into bins numbered with positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

一个红色球和一个绿色球被随机且独立地扔进用正整数编号的篚里，对于每个球来说，扔进编号为 k 的篚里的概率是2^{-k}，k = 1，2，3，…。红色球被扔进比绿色球编号更高的篚里的概率是多少？

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{3}{7}$

Problem 18

Henry decides one morning to do a workout, and he walks $\frac{3}{4}$ of the way from his home to his gym. The gym is 2 kilometers away from Henry's home. At that point, he changes his mind and walks $\frac{3}{4}$ of the way from where he is back toward home. When he reaches that point, he changes his mind again and walks $\frac{3}{4}$ of the distance from there back toward the gym. If Henry keeps changing his mind when he has walked $\frac{3}{4}$ of the distance toward either the gym or home from the point where he last changed his mind, he will get very close to walking back and forth between a point A kilometers from home and a point B kilometers from home. What is $|A-B|$?

一天早上 Henry 想早起锻炼，他步行了从家到体育馆距离的$\frac{3}{4}$。他家到体育馆距离是 2km。在此时，Henry 改变了主意，掉头回家，并走了该处到家距离的$\frac{3}{4}$。当他到达那一点时，他再次改变主意，并行走该处到体育馆距离的$\frac{3}{4}$。如果 Henry 总是这样在刚到达这点时候改变主意朝体育馆或者家走$\frac{3}{4}$距离，那么他将非常接近于在离家 A km 和离家 B km 的两个点之间移动，问$|A-B|$是多少？

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 1\frac{1}{5} \qquad \textbf{(D)}\ 1\frac{1}{4} \qquad \textbf{(E)}\ 1\frac{1}{2}$

Problem 19

Let S be the set of all positive integer divisors of $100,000.$ How many numbers are the product of two distinct elements of S?

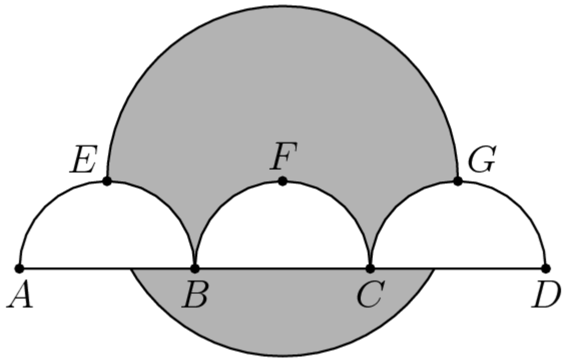
设 S 是 100, 000 的所有正约数构成的集合。 问有多少数是 S 中两个不同元素的乘积？

$\textbf{(A)}\ 98 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 117 \qquad \textbf{(D)}\ 119 \qquad \textbf{(E)}\ 121$

Problem 20

As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB=BC=CD=2.$ Three semicircles of radius $1,$ $\overset{\frown}{\text{AEB}},\overset{\frown}{\text{BFC}},$ and $\overset{\frown}{\text{CGD}},$ have their diameters on $\overline{AD},$ and are tangent to line EG at $E,F,$ and $G,$ respectively. A circle of radius 2 has its center on $F.$ The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form $\frac{a}{b}\cdot\pi-\sqrt{c}+d,$ where $a,b,c,$ and d are positive integers and a and b are relatively prime. What is $a+b+c+d$?

如图所示， 线段$\overline{AD}$被点 B 和 C 三等分，使得 $AB = BC = CD = 2$。 三个半径为 1 的半圆 $\overset{\frown}{\text{AEB}},\overset{\frown}{\text{BFC}},$和 $\overset{\frown}{\text{CGD}},$的直径都在 AD 上，位于由直线 AD 确定的同一半平面内，分别与直线线 EG 相切于 E、F 和 G。半径为 2 的圆的圆心在 F。 在圆内但在三个半圆外的区域面积在图中用阴影表示，其面积可以用如下形式表示$\frac{a}{b}\cdot\pi-\sqrt{c}+d,$其中 a、b、c 和 d 是正整数，并且 a 和 b 是互质的。 问 $a + b + c + d$ 是多少？



$\textbf{(A) }13\qquad\textbf{(B) }14\qquad\textbf{(C) }15\qquad\textbf{(D) }16\qquad\textbf{(E) }17$

Problem 21

Debra flips a fair coin repeatedly, keeping track of how many heads and how many tails she has seen in total, until she gets either two heads in a row or two tails in a row, at which point she stops flipping. What is the probability that she gets two heads in a row but she sees a second tail before she sees a second head?

Debra 反复翻转一枚标准的硬币，并持续记录她总共看到了多少次正面和多少次反面，直到她连续看到两次正面或两次反面后停止。 她连续看到两个正面，但是她第二次看到反面是在第二次看到正面之前的概率是多少？

$\textbf{(A) }\frac{1}{36}\qquad\textbf{(B) }\frac{1}{24}\qquad\textbf{(C) }\frac{1}{18}\qquad\textbf{(D) }\frac{1}{12}\qquad\textbf{(E) }\frac{1}{6}$

Problem 22

Raashan, Sylvia, and Ted play the following game. Each starts with $\$1$. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives $\$1$ to that player. What is the probability that after the bell has rung 2019 times, each player will have $\$1$? (For example, Raashan and Ted may each decide to give $\$1$ to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have $\$0$, Sylvia will have $\$2$, and Ted will have $\$1$, and that is the end of the first round of play. In the second round Rashaan has no money to give, but Sylvia and Ted might choose each other to give their $\$1$ to, and the holdings will be the same at the end of the second round.)

Raashan，Sylvia 和 Ted 玩下面的游戏。 每个人从$\$1$开始。 闹钟每 15 秒钟响铃一次，此时每个有钱的玩家同时独立随机地选择另外两个玩家中的一人，并给那个玩家$\$1$。铃声响了 2019 次后，每个玩家都有$\$1$的概率是多少？（例如，Raashan 和 Ted 可能会各自决定给 Silvia 1 美元，Silvia 可能会决定给 Ted $ 1 美元，这时 Raashan 会有$\$0$，Silvia 会有$\$2$，Ted 会有$\$1$，这就是游戏第一轮结束时的状态。 在第二轮，Raashan 没有钱，但是 Sylvia 和 Ted 可以选择互相给对方$\$1$，在游戏第二轮结束时，他们的财产状况和之前一样。）

$\textbf{(A)}\ \frac{1}{7} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{2}{3}$

Problem 23

Points $A(6, 13)$ and $B(12, 11)$ lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

点 $A(6, 13)$和 $B(12, 11)$位于平面内的圆ω上。 假设在 A 和 B 处的切线相交于 x 轴上的一点。那么ω的面积是多少？

$\textbf{(A)}\ \frac{83\pi}{8} \qquad \textbf{(B)}\ \frac{21\pi}{2} \qquad \textbf{(C)}\ \frac{85\pi}{8} \qquad \textbf{(D)}\ \frac{43\pi}{4} \qquad \textbf{(E)}\ \frac{87\pi}{8}$

Problem 24

Define a sequence recursively by $x_0 = 5$ and $x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$ for all nonnegative integers n. Let m be the least positive integer such that $x_m \leq 4 + \frac{1}{2^{20}}$. In which of the following intervals does m lie?

定义一个数列通过初始值$x_0 = 5$与通项公式$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$对于所有非负整数$n$. 令$m$是最小的正整数使得$x_m \leq 4 + \frac{1}{2^{20}}$.求$m$在哪个区间？

$\textbf{(A)}\ [9,26] \qquad \textbf{(B)}\ [27,80] \qquad \textbf{(C)}\ [81,242] \qquad \textbf{(D)}\ [243,728] \qquad \textbf{(E)}\ [729,\infty)$

Problem 25

How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

有多少个长度为 19 的由 0 和 1 组成的序列，以 0 开头，以 0 结尾，不包含连续两个 0，也不包含连续三个 1？

$\textbf{(A)}\ 55 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 65 \qquad \textbf{(D)}\ 70 \qquad \textbf{(E)}\ 75$

2019 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	E	B	A	E	C	B	B	A	A	A	C	A
14	15	16	17	18	19	20	21	22	23	24	25	
C	A	A	C	C	C	E	B	B	C	C	C	