2003 AMC 12B Problems

Contents

Problem 1

Which of the following is the same as

$$\frac{2-4+6-8+10-12+14}{3-6+9-12+15-18+21}?$$

$(\mathrm{A})\ -1 \qquad (\mathrm{B})\ -\frac{2}{3} \qquad (\mathrm{C})\ \frac{2}{3} \qquad (\mathrm{D})\ 1 \qquad (\mathrm{E})\ \frac{14}{3}$

Solution

Problem 2

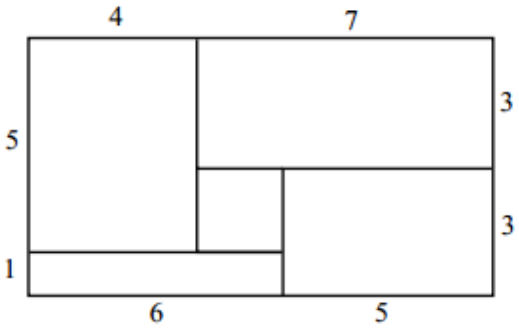
Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs 1 dollar more than a pink pill, and Al's pills cost a total of 546 dollars for the two weeks. How much does one green pill cost?

$(\mathrm{A})\ 7 \qquad (\mathrm{B})\ 14 \qquad (\mathrm{C})\ 19 \qquad (\mathrm{D})\ 20 \qquad (\mathrm{E})\ 39$

Solution

Problem 3

Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the figure. She plants one flower per square foot in each region. Asters cost $1 each, begonias $1.50 each, cannas $2 each, dahlias $2.50 each, and Easter lilies $3 each. What is the least possible cost, in dollars, for her garden?



(A) 108 (B) 115 (C) 132 (D) 144 (E) 156

Solution

Problem 4

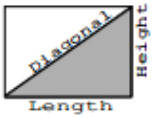
Moe uses a mower to cut his rectangular 90-foot by 150-foot lawn. The swath he cuts is 28 inches wide, but he overlaps each cut by 4 inches to make sure that no grass is missed. he walks at the rate of 5000 feet per hour while pushing the mower. Which of the following is closest to the number of hours it will take Moe to mow his lawn?

(A) 0.75 (B) 0.8 (C) 1.35 (D) 1.5 (E) 3

Solution

Problem 5

Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is 4 : 3. The horizontal length of a "27-inch" television screen is closest, in inches, to which of the following?



(A) 20 (B) 20.5 (C) 21 (D) 21.5 (E) 22

Solution

Problem 6

The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

(A) $-\sqrt{3}$ (B) $\frac{-2\sqrt{3}}{3}$ (C) $\frac{-\sqrt{3}}{3}$ (D) $\sqrt{3}$ (E) 3

Solution

Problem 7

Penniless Pete's piggy bank has no pennies in it, but it has 100 coins, all nickels, dimes, and quarters, whose total value is $8.35. It does not necessarily contain coins of all three types. What is the difference between the largest and smallest number of dimes that could be in the bank?

(A) 0 (B) 13 (C) 37 (D) 64 (E) 83

Solution

Problem 8

Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

(A) 3 (B) 4 (C) 6 (D) 9 (E) 10

Solution

Problem 9

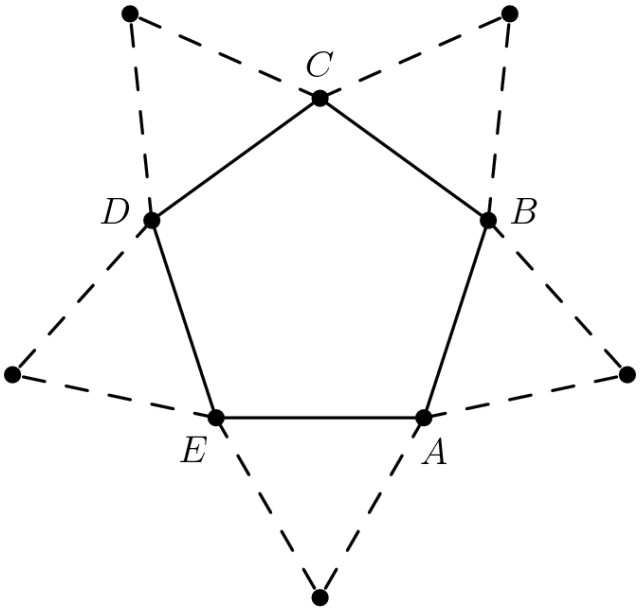
Let f be a linear function for which $f(6) - f(2) = 12$. What is $f(12) - f(2)$?

(A) 12 (B) 18 (C) 24 (D) 30 (E) 36

Solution

Problem 10

Several figures can be made by attaching two equilateral triangles to the regular pentagon ABCDE in two of the five positions shown. How many non-congruent figures can be constructed in this way?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Solution

Problem 11

Cassandra sets her watch to the correct time at noon. At the actual time of 1:00 PM, she notices that her watch reads 12:57 and 36 seconds. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch first reads 10:00 PM?

(A) 10:22 PM and 24 seconds (B) 10:24 PM (C) 10:25 PM (D) 10:27 PM (E) 10:30 PM

Solution

Problem 12

What is the largest integer that is a divisor of $(n+1)(n+3)(n+5)(n+7)(n+9)$ for all positive even integers n?

(A) 3 (B) 5 (C) 11 (D) 15 (E) 165

Solution

Problem 13

An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius?

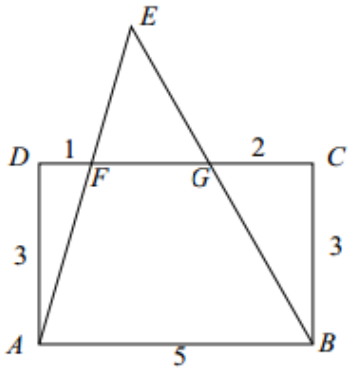
(A) 2 : 1 (B) 3 : 1 (C) 4 : 1 (D) 16 : 3 (E) 6 : 1

Solution

Problem 14

In rectangle ABCD, AB = 5 and BC = 3. Points F and G are on CD so that DF = 1 and GC = 2. Lines AF and BG intersect at E.

Find the area of $\triangle AEB$.

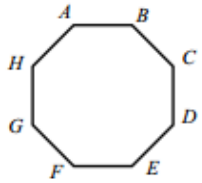


(A) 10 (B) $\frac{21}{2}$ (C) 12 (D) $\frac{25}{2}$ (E) 15

Solution

Problem 15

A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?

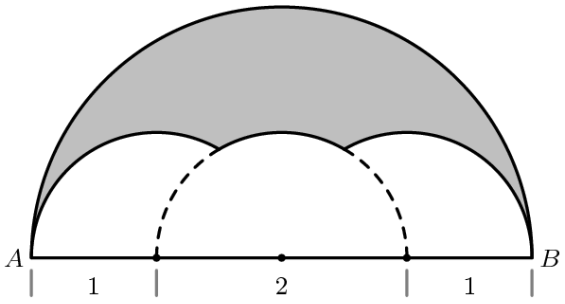


(A) $1-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\sqrt{2}-1$ (D) $\frac{1}{2}$ (E) $\frac{1+\sqrt{2}}{4}$

Solution

Problem 16

Three semicircles of radius 1 are constructed on diameter AB of a semicircle of radius 2. The centers of the small semicircles divide AB into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?



(A) $\pi-\sqrt{3}$ **(B)** $\pi-\sqrt{2}$ **(C)** $\frac{\pi+\sqrt{2}}{2}$ **(D)** $\frac{\pi+\sqrt{3}}{2}$ **(E)** $\frac{7}{6}\pi-\frac{\sqrt{3}}{2}$

Solution

Problem 17

If $\log(xy^3)=1$ and $\log(x^2y)=1$, what is $\log(xy)$?

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{3}{5}$ (E) 1

Solution

Problem 18

Let x and y be positive integers such that $7x^5 = 11y^{13}$. The minimum possible value of x has a prime factorization $a^c b^d$. What is $a + b + c + d$?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 33 \qquad \textbf{(E)}\ 34$

Solution (http://www.artofproblemsolving.com/wiki/index.php?title=2003_AMC_12B_Problems/Problem_18)

Problem 19

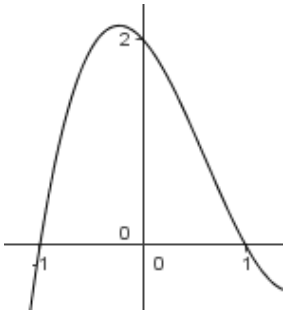
Let S be the set of permutations of the sequence $1, 2, 3, 4, 5$ for which the first term is not 1. A permutation is chosen randomly from S. The probability that the second term is 2, in lowest terms, is a/b. What is $a + b$?

$(A)\ 5 \qquad (B)\ 6 \qquad (C)\ 11 \qquad (D)\ 16 \qquad (E)\ 19$

Solution

Problem 20

Part of the graph of $f(x) = ax^3 + bx^2 + cx + d$ is shown. What is b?



$(A)\ -4 \qquad (B)\ -2 \qquad (C)\ 0 \qquad (D)\ 2 \qquad (E)\ 4$

Solution

Problem 21

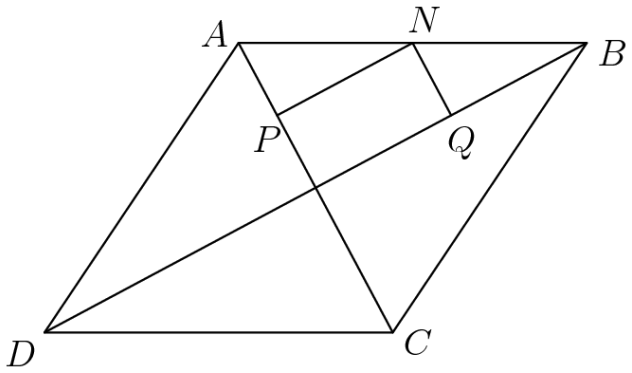
An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C. What is the probability that $AC < 7$?

$(A)\ \frac{1}{6} \qquad (B)\ \frac{1}{5} \qquad (C)\ \frac{1}{4} \qquad (D)\ \frac{1}{3} \qquad (E)\ \frac{1}{2}$

Solution

Problem 22

Let $ABCD$ be a rhombus with $AC = 16$ and $BD = 30$. Let N be a point on $\overline{AB}$, and let P and Q be the feet of the perpendiculars from N to $\overline{AC}$ and $\overline{BD}$, respectively. Which of the following is closest to the minimum possible value of PQ?



$(A)\ 6.5 \qquad (B)\ 6.75 \qquad (C)\ 7 \qquad (D)\ 7.25 \qquad (E)\ 7.5$

Solution

Problem 23

The number of x-intercepts on the graph of $y = \sin(1/x)$ in the interval $(0.0001, 0.001)$ is closest to

$(\mathrm{A})\ 2900 \qquad (\mathrm{B})\ 3000 \qquad (\mathrm{C})\ 3100 \qquad (\mathrm{D})\ 3200 \qquad (\mathrm{E})\ 3300$

Solution

Problem 24

Positive integers $a, b,$ and c are chosen so that $a < b < c,$ and the system of equations

$$2x + y = 2003 \text{ and } \quad y = |x - a| + |x - b| + |x - c|$$

has exactly one solution. What is the minimum value of c?

$(\mathrm{A})\ 668 \qquad (\mathrm{B})\ 669 \qquad (\mathrm{C})\ 1002 \qquad (\mathrm{D})\ 2003 \qquad (\mathrm{E})\ 2004$

Solution

Problem 25

Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distance between the points are less than the radius of the circle?

$(\mathrm{A})\ \frac{1}{36} \qquad (\mathrm{B})\ \frac{1}{24} \qquad (\mathrm{C})\ \frac{1}{18} \qquad (\mathrm{D})\ \frac{1}{12} \qquad (\mathrm{E})\ \frac{1}{9}$

Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2003 AMC 12B
- Mathematics competition resources

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