2007 AMC 12B Problems

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Problem 1

Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

$$\mathrm{(A)}\ 678 \qquad \mathrm{(B)}\ 768 \qquad \mathrm{(C)}\ 786 \qquad \mathrm{(D)}\ 867 \qquad \mathrm{(E)}\ 876$$

Solution

Problem 2

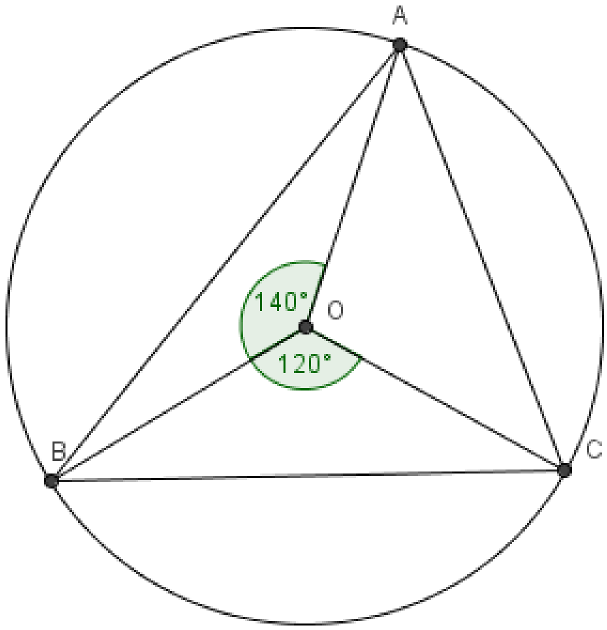
A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

$$(\mathrm{A})22 \qquad (\mathrm{B})24 \qquad (\mathrm{C})25 \qquad (\mathrm{D})26 \qquad (\mathrm{E})28$$

Solution

Problem 3

The point O is the center of the circle circumscribed about triangle ABC, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$, as shown. What is the degree measure of $\angle ABC$?



$(A)35 \qquad (B)40 \qquad (C)45 \qquad (D)50 \qquad (E)60$

Solution

Problem 4

At Frank's Fruit Market, 3 bananas cost as much as 2 apples, and 6 apples cost as much as 4 oranges. How many oranges cost as much as 18 bananas?

$(A)6 \qquad (B)8 \qquad (C)9 \qquad (D)12 \qquad (E)18$

Solution

Problem 5

The 2007 AMC 12 contests will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

$(A)13 \qquad (B)14 \qquad (C)15 \qquad (D)16 \qquad (E)17$

Solution

Problem 6

Triangle ABC has side lengths $AB = 5$, $BC = 6$, and $AC = 7$. Two bugs start simultaneously from A and crawl along the sides of the triangle in opposite directions at the same speed. They meet at point D. What is BD?

$(A)1 \qquad (B)2 \qquad (C)3 \qquad (D)4 \qquad (E)5$

Solution

Problem 7

All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90^\circ$. What is the degree measure of $\angle E$?

$(A)90 \qquad (B)108 \qquad (C)120 \qquad (D)144 \qquad (E)150$

Solution

Problem 8

Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N ?

$(\mathrm{A})2 \qquad (\mathrm{B})3 \qquad (\mathrm{C})4 \qquad (\mathrm{D})5 \qquad (\mathrm{E})6$

Solution

Problem 9

A function f has the property that $f(3x-1)=x^2+x+1$ for all real numbers x. What is $f(5)$?

$(\mathrm{A})7 \qquad (\mathrm{B})13 \qquad (\mathrm{C})31 \qquad (\mathrm{D})111 \qquad (\mathrm{E})211$

Solution

Problem 10

Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

$(\mathrm{A})4 \qquad (\mathrm{B})6 \qquad (\mathrm{C})8 \qquad (\mathrm{D})10 \qquad (\mathrm{E})12$

Solution

Problem 11

The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?

$(\mathrm{A})125 \qquad (\mathrm{B})144 \qquad (\mathrm{C})153 \qquad (\mathrm{D})173 \qquad (\mathrm{E})180$

Solution

Problem 12

A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

$(\mathrm{A})85 \qquad (\mathrm{B})88 \qquad (\mathrm{C})93 \qquad (\mathrm{D})94 \qquad (\mathrm{E})98$

Solution

Problem 13

A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

$(\mathrm{A})\frac{1}{63} \qquad (\mathrm{B})\frac{1}{21} \qquad (\mathrm{C})\frac{1}{10} \qquad (\mathrm{D})\frac{1}{7} \qquad (\mathrm{E})\frac{1}{3}$

Solution

Problem 14

Point P is inside equilateral $\triangle ABC$. Points Q, R, and S are the feet of the perpendiculars from P to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ=1$, $PR=2$, and $PS=3$, what is AB?

$(\mathrm{A})4 \qquad (\mathrm{B})3\sqrt{3} \qquad (\mathrm{C})6 \qquad (\mathrm{D})4\sqrt{3} \qquad (\mathrm{E})9$

Solution

Problem 15

The geometric series $a + ar + ar^2 \cdots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?

$(A)\frac{4}{3} \qquad (B)\frac{12}{7} \qquad (C)\frac{3}{2} \qquad (D)\frac{7}{3} \qquad (E)\frac{5}{2}$

Solution

Problem 16

Each face of a regular tetrahedron is painted either red, white, or blue. Two colorings are considered indistinguishable if two congruent tetrahedra with those colorings can be rotated so that their appearances are identical. How many distinguishable colorings are possible?

$(A)15 \qquad (B)18 \qquad (C)27 \qquad (D)54 \qquad (E)81$

Solution

Problem 17

If a is a nonzero integer and b is a positive number such that $ab^2 = \log_{10} b$, what is the median of the set $\{0, 1, a, b, 1/b\}$?

$(A)0 \qquad (B)1 \qquad (C)a \qquad (D)b \qquad (E)\frac{1}{b}$

Solution

Problem 18

Let a, b, and c be digits with $a \neq 0$. The three-digit integer abc lies one third of the way from the square of a positive integer to the square of the next larger integer. The integer acb lies two thirds of the way between the same two squares. What is $a + b + c$?

$(A)10 \qquad (B)13 \qquad (C)16 \qquad (D)18 \qquad (E)21$

Solution

Problem 19

Rhombus $ABCD$, with side length 6, is rolled to form a cylinder of volume 6 by taping $\overline{AB}$ to $\overline{DC}$. What is $\sin(\angle ABC)$?

$(A)\frac{\pi}{9} \qquad (B)\frac{1}{2} \qquad (C)\frac{\pi}{6} \qquad (D)\frac{\pi}{4} \qquad (E)\frac{\sqrt{3}}{2}$

Solution

Problem 20

The parallelogram bounded by the lines $y = ax + c$, $y = ax + d$, $y = bx + c$, and $y = bx + d$ has area 18. The parallelogram bounded by the lines $y = ax + c$, $y = ax - d$, $y = bx + c$, and $y = bx - d$ has area 72. Given that a, b, c, and d are positive integers, what is the smallest possible value of $a + b + c + d$?

$(A)13 \qquad (B)14 \qquad (C)15 \qquad (D)16 \qquad (E)17$

Solution

Problem 21

The first 2007 positive integers are each written in base 3. How many of these base-3 representations are palindromes? (A palindrome is a number that reads the same forward and backward.)

$\mathrm{(A)}100 \qquad \mathrm{(B)}101 \qquad \mathrm{(C)}102 \qquad \mathrm{(D)}103 \qquad \mathrm{(E)}104$

Solution

Problem 22

Two particles move along the edges of equilateral $\triangle ABC$ in the direction

$$A\Rightarrow B\Rightarrow C\Rightarrow A,$$

starting simultaneously and moving at the same speed. One starts at A, and the other starts at the midpoint of $\overline{BC}$. The midpoint of the line segment joining the two particles traces out a path that encloses a region R. What is the ratio of the area of R to the area of $\triangle ABC$?

$\mathrm{(A)}\frac{1}{16} \qquad \mathrm{(B)}\frac{1}{12} \qquad \mathrm{(C)}\frac{1}{9} \qquad \mathrm{(D)}\frac{1}{6} \qquad \mathrm{(E)}\frac{1}{4}$

Solution

Problem 23

How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?

$\mathrm{(A)}6 \qquad \mathrm{(B)}7 \qquad \mathrm{(C)}8 \qquad \mathrm{(D)}10 \qquad \mathrm{(E)}12$

Solution

Problem 24

How many pairs of positive integers (a,b) are there such that $gcd(a,b)=1$ and

$$\frac{a}{b}+\frac{14b}{9a}$$

is an integer?

$\mathrm{(A)}4 \qquad \mathrm{(B)}6 \qquad \mathrm{(C)}9 \qquad \mathrm{(D)}12 \qquad \mathrm{(E)}\text{infinitely many}$

Solution

Problem 25

Points A,B,C,D and E are located in 3-dimensional space with $AB=BC=CD=DE=EA=2$ and $\angle ABC=\angle CDE=\angle DEA=90^o$. The plane of $\triangle ABC$ is parallel to $\overline{DE}$. What is the area of $\triangle BDE$?

$\mathrm{(A)}\sqrt{2} \qquad \mathrm{(B)}\sqrt{3} \qquad \mathrm{(C)}2 \qquad \mathrm{(D)}\sqrt{5} \qquad \mathrm{(E)}\sqrt{6}$

Solution

See also