2013 AMC 10B

Problem 1

What is $\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}$?

$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}$的值是多少？

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{49}{20} \qquad \textbf{(E)}\ \frac{43}{3}$

Problem 2

Mr. Green measures his rectangular garden by walking two of the sides and finding that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

格林先生通过用自己的步长来测量他的矩形花园，发现大小是 15 步 x 20 步。格林先生的每一步是 2 英尺长。他期望他的花园每平方英尺的土地可以产出半磅的土豆。那么格林先生期望他的花园总共可以产出多少磅的土豆？

$\textbf{(A)}\ 600 \qquad \textbf{(B)}\ 800 \qquad \textbf{(C)}\ 1000 \qquad \textbf{(D)}\ 1200 \qquad \textbf{(E)}\ 1400$

Problem 3

On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and the low temperatures was 3°. In degrees, what was the low temperature in Lincoln that day?

内布拉斯加州林肯市在一月份的某天，高温比低温高 16 度，且高温和低温的平均值是 3 度，那么林肯市那天的低温是多少度？

$\textbf{(A)}\ -13 \qquad \textbf{(B)}\ -8 \qquad \textbf{(C)}\ -5 \qquad \textbf{(D)}\ -3 \qquad \textbf{(E)}\ 11$

Problem 4

When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n?

当从 3 数到 201，数到的第 51 个数是 53。当从 201 倒数到 3，数到的第 *n* 个数是 53。问 *n* 是多少?

$\textbf{(A)}\ 146 \qquad \textbf{(B)}\ 147 \qquad \textbf{(C)}\ 148 \qquad \textbf{(D)}\ 149 \qquad \textbf{(E)}\ 150$

Problem 5

Positive integers a and b are each less than 6. What is the smallest possible value for $2 \cdot a - a \cdot b$?

正整数 *a* 和 *b* 都比 6 小。$2 \cdot a - a \cdot b$的最小可能值是多少？

$\textbf{(A)}\ -20 \qquad \textbf{(B)}\ -15 \qquad \textbf{(C)}\ -10 \qquad \textbf{(D)}\ 0 \qquad \textbf{(E)}\ 2$

Problem 6

The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

33 个五年级学生的平均年龄是 11 岁。他们的 55 个父母的平均年龄是 33 岁。问所有这些父母和五年级学生的平均年龄是多少岁?

$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 23.25 \qquad \textbf{(C)}\ 24.75 \qquad \textbf{(D)}\ 26.25 \qquad \textbf{(E)}\ 28$

Problem 7

Six points are equally spaced around a circle of radius 1. Three of these points are the vertices of a triangle that is neither equilateral nor isosceles. What is the area of this triangle?

6 个点等间隔地分布在一个半径为 1 的圆上，其中 3 个点是一个三角形的三个顶点，而这个三角形既不是等边三角形，也不是等腰三角形，问这个三角形的面积是多少？

$\textbf{(A)}\ \frac{\sqrt{3}}{3} \qquad \textbf{(B)}\ \frac{\sqrt{3}}{2} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \sqrt{2} \qquad \textbf{(E)}\ 2$

Problem 8

Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?

Ray 的汽车每加仑汽油可以行驶 40 英里，Tom 的汽车每加仑汽油可以行驶 10 英里。Ray 和 Tom 各自行驶了相同的英里数，问这两辆车平均每加仑汽油总共行驶了多少英里？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 40$

Problem 9

Three positive integers are each greater than 1, have a product of 27000, and are pairwise relatively prime. What is their sum?

三个正整数都大于 1，乘积为 27000，且两两互质。那么这三个数之和是多少?

$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 137 \qquad \textbf{(C)}\ 156 \qquad \textbf{(D)}\ 160 \qquad \textbf{(E)}\ 165$

Problem 10

A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

一支篮球队队员在二分球上有 50％的成功率，三分球上有 40％的成功率，最后总得分是 54 分。他们在二分球上尝试的次数比三分球多 50％。问他们尝试了多少次三分球?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 30$

Problem 11

Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x+y$?

实数 *x* 和 *y* 满足方程$x^2 + y^2 = 10x - 6y - 34$。$x+y$的值是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 12

Let S be the set of sides and diagonals of a regular pentagon. A pair of elements of S are selected at random without replacement. What is the probability that the two chosen segments have the same length?

S 是一个正五边形的所有边和对角线所组成的集合，从 *S* 中随机选择两个元素，且不放回。问选择的两根线段有同样长度的概率是多少？

$\textbf{(A)}\ \frac{2}{5} \qquad \textbf{(B)}\ \frac{4}{9} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{4}{5}$

Problem 13

Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "$1, 2$" . Jo then says "$1, 2, 3$" , and so on. What is the 53rd number said?

Jo 和 Blair 轮流从 1 数到某个数，数到的最后一个数比另外一个人数到的最后一个数大 1。Jo 首先开始，数”1”， 所以 Blair 接着数”1，2”， Jo 然后数”1，2，3”， 以此类推。问说出的第 53 个数是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 14

Define $a\clubsuit b = a^2b - ab^2$. Which of the following describes the set of points (x, y) for which $x\clubsuit y = y\clubsuit x$?

定义$a\clubsuit b = a^2b - ab^2$，下面哪个描述了满足方程$x\clubsuit y = y\clubsuit x$的点$(x, y)$组成的集合？

（A）a finite set of points | 有限个点所组成的集合

（B）one line | 一条直线

（C）two parallel lines | 两条平行线

（D）two intersecting lines | 两条相交的直线

（E）three lines | 三条直线

Problem 15

A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{a}{b}$?

一根电线被切成两段，一段长度为 a，另一段长度为 b。长度为 a 的那段被弯折成一个正三角形，而长度为 b 的那段被弯折成一个正六边形。三角形和正六边形的面积相等。$\frac{a}{b}$是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{\sqrt{6}}{2} \qquad \textbf{(C)}\ \sqrt{3} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{3\sqrt{2}}{2}$

Problem 16

In triangle ABC, medians AD and CE intersect at P, $PE=1.5$, $PD=2$, and $DE=2.5$. What is the area of $AEDC$?

三角形 ABC 中，中线 AD 和 CE 交于点 P，PE＝1.5，PD=2，DE=2.5。问 $AEDC$ 的面积是多少？

$\textbf{(A)}13 \qquad \textbf{(B)}13.5 \qquad \textbf{(C)}14 \qquad \textbf{(D)}14.5 \qquad \textbf{(E)}15$

Problem 17

Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

Alex 有 75 张红色礼券和 75 张蓝色礼券。Alex 可以在某个报刊亭用 2 张红色的礼券兑换一张银色礼券和一张蓝色礼券。而在另一个报刊亭他可以用 3 张蓝色礼券兑换一张银色礼券和一张红色礼券。Alex 一直持续地兑换礼券直到无法再继续兑换。问 Alex 最后将有多少张银色礼券？

$\textbf{(A)}62 \qquad \textbf{(B)}82 \qquad \textbf{(C)}83 \qquad \textbf{(D)}102 \qquad \textbf{(E)}103$

Problem 18

The number 2013 has the property that its units digit is the sum of its other digits, that is $2+0+1=3$. How many integers less than 2013 but greater than 1000 share this property?

数字 2013 有这样一个性质：它的个位数是其他位上的数之和，即 2+0+1=3。大于 1000 小于 2013 的所有整数中，有多少个数有这样的性质？

$\textbf{(A)}\ 33 \qquad \textbf{(B)}\ 34 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 46 \qquad \textbf{(E)}\ 58$

Problem 19

The real numbers c, b, a form an arithmetic sequence with $a \geq b \geq c \geq 0$. The quadratic ax^2+bx+c has exactly one root. What is this root?

实数 *c*，*b*，*a* 成等差数列，且 *a*⩾*b*⩾*c*⩾0。一元二次方程ax^2+bx+c恰好只有 1 个根。问这个根是多少?

$\textbf{(A)}\ -7-4\sqrt{3} \qquad \textbf{(B)}\ -2-\sqrt{3} \qquad \textbf{(C)}\ -1 \qquad \textbf{(D)}\ -2+\sqrt{3} \qquad \textbf{(E)}\ -7+4\sqrt{3}$

Problem 20

The number 2013 is expressed in the form

$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!}$, where $a_1 \geq a_2 \geq \cdots \geq a_m$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ are positive integers and a_1+b_1 is as small as possible. What is $|a_1-b_1|$?

数字 2013 被写成如下形式：$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!}$，其中$a_1 \geq a_2 \geq \cdots \geq a_m$和 b1⩾b2 ⩾•••⩾bn 都是正整数，并且$a_1+b_1$取尽可能小的值，问$|a_1-b_1|$是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 21

Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N?

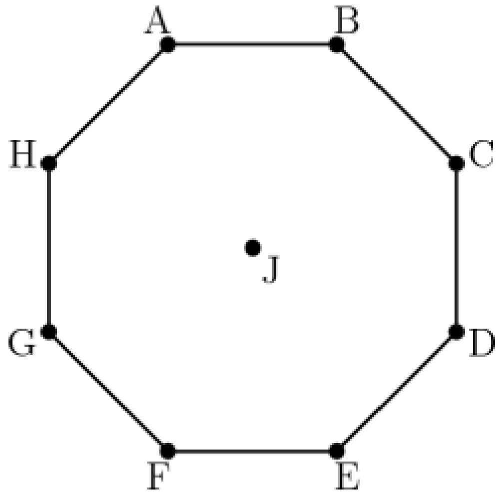
两个非递减的非负整数数列的第一项不同，这两个数列都有这样一个性质：从第三项开始，每一项都是前面两项之和，且这两个数列的第七项都是 *N*。问 *N* 的最小可能值是多少？

$\textbf{(A)}\ 55 \qquad \textbf{(B)}\ 89 \qquad \textbf{(C)}\ 104 \qquad \textbf{(D)}\ 144 \qquad \textbf{(E)}\ 273$

Problem 22

The regular octagon $ABCDEFGH$ has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are all equal. In how many ways can this be done?

正八边形 *ABCDEFGH* 的中心是 J。数字 1 到 9 将被分配到这个八边形的每个顶点和中心，使得直线 *AJE*，*BJF*，*CJG* 和 *DJH* 上的三个数字之和都相等。一共有多少种分配方法？



$\textbf{(A)}\ 384 \qquad \textbf{(B)}\ 576 \qquad \textbf{(C)}\ 1152 \qquad \textbf{(D)}\ 1680 \qquad \textbf{(E)}\ 3456$

Problem 23

In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

三角形 *ABC* 中， *AB*=13，*BC*=14，*CA*＝15。点 *D*，*E*，*F* 分别在线段$\overline{BC}$，$\overline{CA}$，$\overline{DE}$上，满足$\overline{AD} \perp \overline{BC}$，$\overline{DE} \perp \overline{AC}$，$\overline{AF} \perp \overline{BF}$。线段$\overline{DF}$的长度可以写成$\frac{m}{n}$，其中 *m* 和 *n* 是互质的正整数。问 *m*＋*n* 是多少？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 30$

Problem 24

A positive integer n is nice if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \dots, 2019\}$ are nice?

如果存在一个正整数 *m*，它恰好有 4 个正整数因子（包括 1 和 *m*）且这 4 个因子之和是 *n*，那么 *n* 就被称为是好的。问集合｛2010，2011，2012，…，2019｝中有多少个数是好的？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 25

Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers $10,444$ and $3,245$, and LeRoy obtains the sum $S = 13,689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

Bernardo 选择了一个三位正整数，并把它的五进制和六进制表示写在黑板上。之后 LeRoy 看到了 Bernardo 写的这两个数，他把这两个数当成了十进制的数，把它们相加，得到和为 *S*。例如，若 *N*＝749，Bernardo 就在黑板上写下数字 10，444 和 3，245，LeRoy 得到两数之和 *S*＝13，689。问有多少个这样的 *N*，使得 *S* 的最右边的两位数字和 2*N* 最右边的两位数字依照从左到右的顺序分别相等？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

2013 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	A	C	D	B	C	B	B	D	C	B	B	E
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	B	B	E	D	D	B	C	C	B	A	E	