2009 AMC 12A Problems

Contents

Problem 1

Kim's flight took off from Newark at 10:34 AM and landed in Miami at 1:18 PM. Both cities are in the same time zone. If her flight took h hours and m minutes, with $0 < m < 60$, what is $h + m$?

$\textbf{(A)}\ 46 \qquad \textbf{(B)}\ 47 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 53 \qquad \textbf{(E)}\ 54$

Solution

Problem 2

Which of the following is equal to $1 + \frac {1}{1 + \frac {1}{1 + 1}}$?

$\textbf{(A)}\ \frac {5}{4} \qquad \textbf{(B)}\ \frac {3}{2} \qquad \textbf{(C)}\ \frac {5}{3} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Solution

Problem 3

What number is one third of the way from $\frac14$ to $\frac34$?

$\textbf{(A)}\ \frac{1}{3}\qquad \textbf{(B)}\ \frac{5}{12}\qquad \textbf{(C)}\ \frac{1}{2}\qquad \textbf{(D)}\ \frac{7}{12}\qquad \textbf{(E)}\ \frac{2}{3}$

Solution

Problem 4

Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes, and quarters. Which of the following could not be the total value of the four coins, in cents?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 45 \qquad \textbf{(E)}\ 55$

Solution

Problem 5

One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 125 \qquad \textbf{(E)}\ 216$

Solution

Problem 6

Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m,n)?

$\textbf{(A)}\ P^2Q \qquad \textbf{(B)}\ P^nQ^m \qquad \textbf{(C)}\ P^nQ^{2m} \qquad \textbf{(D)}\ P^{2m}Q^n \qquad \textbf{(E)}\ P^{2n}Q^m$

Solution

Problem 7

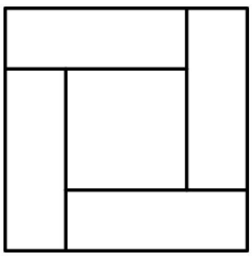
The first three terms of an arithmetic sequence are $2x - 3$, $5x - 11$, and $3x + 1$ respectively. The n th term of the sequence is 2009. What is n?

$\textbf{(A)}\ 255 \qquad \textbf{(B)}\ 502 \qquad \textbf{(C)}\ 1004 \qquad \textbf{(D)}\ 1506 \qquad \textbf{(E)}\ 8037$

Solution

Problem 8

Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?



$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ \sqrt{10} \qquad \textbf{(C)}\ 2 + \sqrt{2} \qquad \textbf{(D)}\ 2\sqrt{3} \qquad \textbf{(E)}\ 4$

Solution

Problem 9

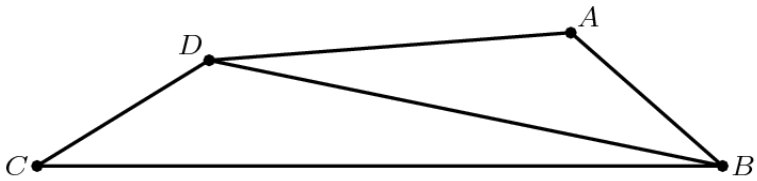
Suppose that $f(x+3) = 3x^2 + 7x + 4$ and $f(x) = ax^2 + bx + c$. What is $a+b+c$?

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Solution

Problem 10

In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?

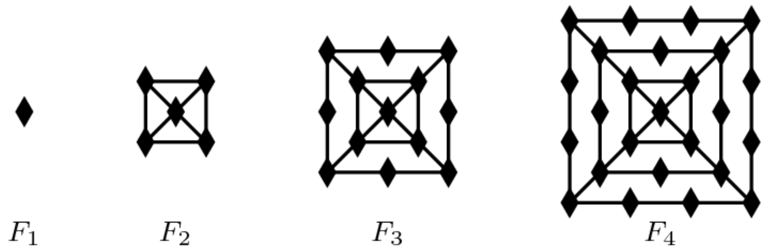


$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Solution

Problem 11

The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n\ge3$, F_n is constructed from $F_{n - 1}$ by surrounding it with a square and placing one more diamond on each side of the new square than $F_{n - 1}$ had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



$\textbf{(A)}\ 401 \qquad \textbf{(B)}\ 485 \qquad \textbf{(C)}\ 585 \qquad \textbf{(D)}\ 626 \qquad \textbf{(E)}\ 761$

Solution

Problem 12

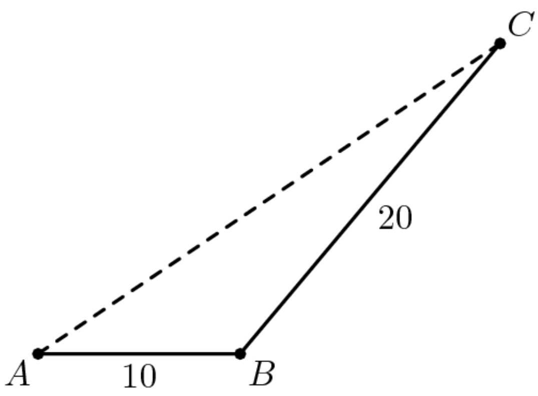
How many positive integers less than 1000 are 6 times the sum of their digits?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 12$

Solution

Problem 13

A ship sails 10 miles in a straight line from A to B, turns through an angle between 45° and 60°, and then sails another 20 miles to C. Let AC be measured in miles. Which of the following intervals contains AC^2?



$\textbf{(A) } [400, 500] \qquad \textbf{(B) } [500, 600] \qquad \textbf{(C) } [600, 700] \qquad \textbf{(D) } [700, 800] \textbf{ (E) } [800, 900]$

Solution

Problem 14

A triangle has vertices $(0, 0)$, $(1, 1)$, and $(6m, 0)$, and the line $y = mx$ divides the triangle into two triangles of equal area. What is the sum of all possible values of m?

$\textbf{(A) } -\frac{1}{3} \qquad \textbf{(B) } -\frac{1}{6} \qquad \textbf{(C) } \frac{1}{6} \qquad \textbf{(D) } \frac{1}{3} \qquad \textbf{(E) } \frac{1}{2}$

Solution

Problem 15

For what value of n is $i + 2i^2 + 3i^3 + \cdots + ni^n = 48 + 49i$?

Note: here $i = \sqrt{-1}$.

$\textbf{(A) } 24 \qquad \textbf{(B) } 48 \qquad \textbf{(C) } 49 \qquad \textbf{(D) } 97 \qquad \textbf{(E) } 98$

Solution

Problem 16

A circle with center C is tangent to the positive x and y-axes and externally tangent to the circle centered at $(3,0)$ with radius 1. What is the sum of all possible radii of the circle with center C?

$\textbf{(A) } 3 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 6 \qquad \textbf{(D) } 8 \qquad \textbf{(E) } 9$

Solution

Problem 17

Let $a + ar_1 + ar_1^2 + ar_1^3 + \cdots$ and $a + ar_2 + ar_2^2 + ar_2^3 + \cdots$ be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is r_1, and the sum of the second series is r_2. What is $r_1 + r_2$?

$\textbf{(A) } 0 \qquad \textbf{(B) } \frac{1}{2} \qquad \textbf{(C) } 1 \qquad \textbf{(D) } \frac{1+\sqrt{5}}{2} \qquad \textbf{(E) } 2$

Solution

Problem 18

For $k > 0$, let $I_k = 10\ldots 064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

$\textbf{(A) } 6 \qquad \textbf{(B) } 7 \qquad \textbf{(C) } 8 \qquad \textbf{(D) } 9 \qquad \textbf{(E) } 10$

Solution

Problem 19

Andrea inscribed a circle inside a regular pentagon, circumscribed a circle around the pentagon, and calculated the area of the region between the two circles. Bethany did the same with a regular heptagon (7 sides). The areas of the two regions were A and B, respectively. Each polygon had a side length of 2. Which of the following is true?

$\textbf{(A) } A = \frac{25}{49}B \qquad \textbf{(B) } A = \frac{5}{7}B \qquad \textbf{(C) } A = B \qquad \textbf{(D) } A = \frac{7}{5}B \qquad \textbf{(E) } A = \frac{49}{25}B$

Solution

Problem 20

Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

$\textbf{(A) } \frac{9}{2} \qquad \textbf{(B) } \frac{50}{11} \qquad \textbf{(C) } \frac{21}{4} \qquad \textbf{(D) } \frac{17}{3} \qquad \textbf{(E) } 6$

Solution

Problem 21

Let $p(x) = x^3 + ax^2 + bx + c$, where a, b, and c are complex numbers. Suppose that

$$p(2009 + 9002\pi i) = p(2009) = p(9002) = 0$$

What is the number of nonreal zeros of $x^{12} + ax^8 + bx^4 + c$?

$\textbf{(A) } 4 \qquad \textbf{(B) } 6 \qquad \textbf{(C) } 8 \qquad \textbf{(D) } 10 \qquad \textbf{(E) } 12$

Solution

Problem 22

A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into the two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a + b + c$?

$\textbf{(A) } 10 \qquad \textbf{(B) } 11 \qquad \textbf{(C) } 12 \qquad \textbf{(D) } 13 \qquad \textbf{(E) } 14$

Solution

Problem 23

Functions f and g are quadratic, $g(x) = -f(100 - x)$, and the graph of g contains the vertex of the graph of f. The four x-intercepts on the two graphs have x-coordinates x_1, x_2, x_3, and x_4, in increasing order, and $x_3 - x_2 = 150$. The value of $x_4 - x_1$ is $m + n\sqrt{p}$, where m, n, and p are positive integers, and p is not divisible by the square of any prime. What is $m + n + p$?

$\textbf{(A) } 602 \qquad \textbf{(B) } 652 \qquad \textbf{(C) } 702 \qquad \textbf{(D) } 752 \qquad \textbf{(E) } 802$

Solution

Problem 24

The tower function of twos is defined recursively as follows: $T(1) = 2$ and $T(n+1) = 2^{T(n)}$ for $n \geq 1$. Let $A = (T(2009))^{T(2009)}$ and $B = (T(2009))^A$. What is the largest integer k such that

$$\underbrace{\log_2 \log_2 \log_2 \ldots \log_2 B}_{k \text{ times}}$$

is defined?

$\textbf{(A)}\ 2009 \qquad \textbf{(B)}\ 2010 \qquad \textbf{(C)}\ 2011 \qquad \textbf{(D)}\ 2012 \qquad \textbf{(E)}\ 2013$

Solution

Problem 25

The first two terms of a sequence are $a_1 = 1$ and $a_2 = \frac{1}{\sqrt{3}}$. For $n \geq 1$,

$$a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}.$$

What is $|a_{2009}|$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 2 - \sqrt{3} \qquad \textbf{(C)}\ \frac{1}{\sqrt{3}} \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2 + \sqrt{3}$

Solution