2006 AMC 8 Problems/Problem 1

Problem

Mindy made three purchases for $\$1.98$ dollars, $\$5.04$ dollars, and $\$9.89$ dollars. What was her total, to the nearest dollar?

$\textbf{(A)}\ 10\qquad\textbf{(B)}\ 15\qquad\textbf{(C)}\ 16\qquad\textbf{(D)}\ 17\qquad\textbf{(E)}\ 18$

Solution

The three prices round to $\$2$, $\$5$, and $\$10$, which has a sum of $\boxed{\textbf{(D)}\ 17}$

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 2

Problem

On the AMC 8 contest Billy answers 13 questions correctly, answers 7 questions incorrectly and doesn't answer the last 5. What is his score?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 13\qquad\textbf{(D)}\ 19\qquad\textbf{(E)}\ 26$

Solution

As the AMC 8 only rewards 1 point for each correct answer, everything is irrelevant except the number Billy answered correctly, $\boxed{\textbf{(C)}\ 13}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 3

Problem

Elisa swims laps in the pool. When she first started, she completed 10 laps in 25 minutes. Now, she can finish 12 laps in 24 minutes. By how many minutes has she improved her lap time?

$\textbf{(A)}\ \frac{1}{2}\qquad\textbf{(B)}\ \frac{3}{4}\qquad\textbf{(C)}\ 1\qquad\textbf{(D)}\ 2\qquad\textbf{(E)}\ 3$

Solution

When Elisa started, she finished a lap in $\frac{25}{10}=2.5$ minutes. Now, she finishes a lap is $\frac{24}{12}=2$ minutes. The difference is $2.5-2=\boxed{\textbf{(A)}\ \frac{1}{2}}$.

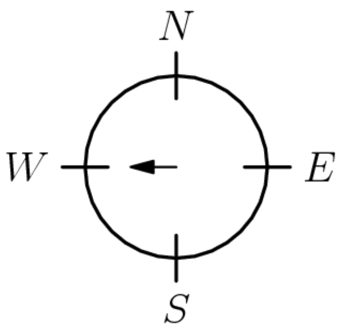
See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 4

Problem

Initially, a spinner points west. Chenille moves it clockwise $2\frac{1}{4}$ revolutions and then counterclockwise $3\frac{3}{4}$ revolutions. In what direction does the spinner point after the two moves?



$\textbf{(A)}\ \text{north} \qquad \textbf{(B)}\ \text{east} \qquad \textbf{(C)}\ \text{south} \qquad \textbf{(D)}\ \text{west} \qquad \textbf{(E)}\ \text{northwest}$

Solution

If the spinner goes clockwise $2\frac{1}{4}$ revolutions and then counterclockwise $3\frac{3}{4}$ revolutions, it ultimately goes counterclockwise $1\frac{1}{2}$ which brings the spinner pointing $\boxed{\textbf{(B)}\ \text{east}}$.

See Also

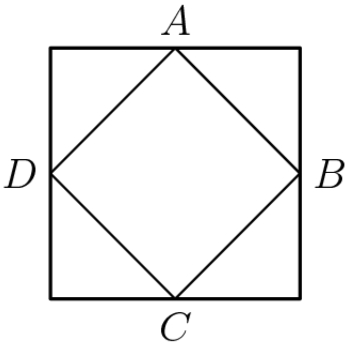
2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 5

Contents

Problem

Points A, B, C and D are midpoints of the sides of the larger square. If the larger square has area 60, what is the area of the smaller square?



$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 40$

Solution

Solution 1

Drawing segments AC and BD, the number of triangles outside square $ABCD$ is the same as the number of triangles inside the square. Thus areas must be equal so the area of $ABCD$ is half the area of the larger square which is $\frac{60}{2} = \boxed{\textbf{(D)}\ 30}$.

Solution 2

If the side length of the larger square is x, the side length of the smaller square is $\frac{\sqrt{2}\cdot x}{2}$. Therefore the area of the smaller square is $\frac{x^2}{2}$, half of the larger square's area, x^2.

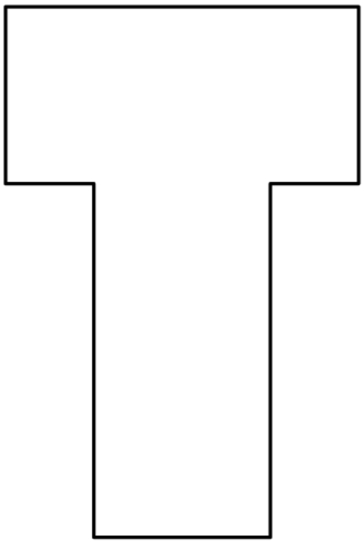
Thus, the area of the smaller square in the picture is $\frac{60}{2} = \boxed{\textbf{(D)}\ 30}$.

See Also

2006 AMC 8 Problems/Problem 6

Problem

The letter T is formed by placing two 2×4 inch rectangles next to each other, as shown. What is the perimeter of the T, in inches?



$\textbf{(A)}\ 12\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 20\qquad\textbf{(D)}\ 22\qquad\textbf{(E)}\ 24$

Solution

If the two rectangles were seperate, the perimeter would be $2(2(2+4)=24$. It easy to see that their connection erases 2 from each of the rectangles, so the final perimeter is $24-2\times2=\boxed{\textbf{(C)}\ 20}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 7

Problem

Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest radius.

$\textbf{(A)}\ X, Y, Z \qquad \textbf{(B)}\ Z, X, Y \qquad \textbf{(C)}\ Y, X, Z \qquad \textbf{(D)}\ Z, Y, X \qquad \textbf{(E)}\ X, Z, Y$

Solution

Using the formulas of circles, $C = 2\pi r$ and $A = \pi r^2$, we find that circle Y has a radius of 4 and circle Z has a radius of 3. Thus, the order from smallest to largest radius is $\boxed{\textbf{(B)}\ Z, X, Y}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 8

Problem

The table shows some of the results of a survey by radiostation KAMC. What percentage of the males surveyed listen to the station?

	Listen	Don't Listen	Total
Males	?	26	?
Females	58	?	96
Total	136	64	200

$\textbf{(A)}\ 39 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 52 \qquad \textbf{(D)}\ 55 \qquad \textbf{(E)}\ 75$

Solution

Filling out the chart, it becomes

	Listen	Don't Listen	Total
Males	78	26	104
Females	58	38	96
Total	136	64	200

Thus, the percentage of males surveyed that listen to the station is $100 \cdot \frac{78}{104}\% = \boxed{\textbf{(E)}\ 75\%}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 9

Problem

What is the product of $\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\cdots\times\frac{2006}{2005}$?

$\textbf{(A)}\ 1\qquad \textbf{(B)}\ 1002\qquad \textbf{(C)}\ 1003\qquad \textbf{(D)}\ 2005\qquad \textbf{(E)}\ 2006$

Solution

By telescoping, it's easy to see the sum becomes $\frac{2006}{2}=\boxed{\textbf{(C)}\ 1003}$.

See Also

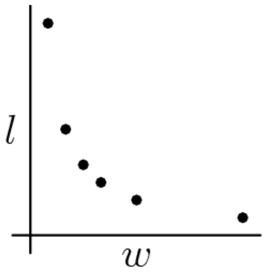
2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 10

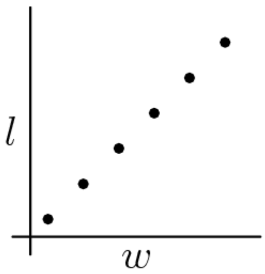
Problem

Jorge's teacher asks him to plot all the ordered pairs $(w.l)$ of positive integers for which w is the width and l is the length of a rectangle with area 12. What should his graph look like?

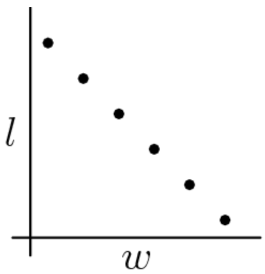
$\textbf{(A)}$



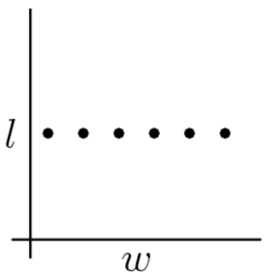
$\textbf{(B)}$



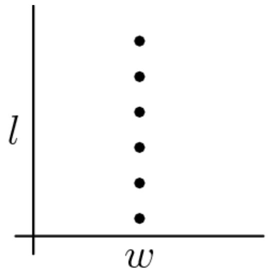
$\textbf{(C)}$



$\textbf{(D)}$



$\textbf{(E)}$



Solution

The length of the rectangle will relate invertly to the width, specifically using the theorem $l = \frac{12}{w}$. The only graph that could represent a inverted relationship is $\boxed{\textbf{(A)}}$. (The rest are linear graphs that represent direct relationships)

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 11

Problem

How many two-digit numbers have digits whose sum is a perfect square?

$\textbf{(A)}\ 13\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 17\qquad\textbf{(D)}\ 18\qquad\textbf{(E)}\ 19$

Solution

There is 1 integer whose digits sum to 1: 10.

There are 4 integers whose digits sum to 4: $13, 22, 31, \text{and } 40$.

There are 9 integers whose digits sum to 9: $18, 27, 36, 45, 54, 63, 72, 81, \text{and } 90$.

There are 3 integers whose digits sum to 16: $79, 88, \text{and } 97$.

Two digits cannot sum to 25 or any greater square since the greatest sum of digits of a two-digit number is $9 + 9 = 18$.

Thus, the answer is $1 + 4 + 9 + 3 = \boxed{\textbf{(C)} 17}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 12

Problem

Antonette gets 70% on a 10-problem test, 80% on a 20-problem test and 90% on a 30-problem test. If the three tests are combined into one 60-problem test, which percent is closest to her overall score?

$\textbf{(A)}\ 40\qquad\textbf{(B)}\ 77\qquad\textbf{(C)}\ 80\qquad\textbf{(D)}\ 83\qquad\textbf{(E)}\ 87$

Solution

$70\% \cdot 10 = 7$

$80\% \cdot 20 = 16$

$90\% \cdot 30 = 27$

Adding them up gets $7+16+27=50$. The overall percentage correct would be $\frac{50}{60}=\frac{5}{6}=5\cdot 16.\overline{6}=83.\overline{3}\approx\boxed{\textbf{(D)}\ 83}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 13

Problem

Cassie leaves Escanaba at 8:30 AM heading for Marquette on her bike. She bikes at a uniform rate of 12 miles per hour. Brian leaves Marquette at 9:00 AM heading for Escanaba on his bike. He bikes at a uniform rate of 16 miles per hour. They both bike on the same 62-mile route between Escanaba and Marquette. At what time in the morning do they meet?

$\textbf{(A)}\ 10:00\qquad\textbf{(B)}\ 10:15\qquad\textbf{(C)}\ 10:30\qquad\textbf{(D)}\ 11:00\qquad\textbf{(E)}\ 11:30$

Solution

If Cassie leaves $\frac{1}{2}$ an hour earlier then Brian, when Brian starts, the distance between them will be $62-\frac{12}{2}=56$. Every hour, they will get $12+16=28$ miles closer. $\frac{56}{28}=2$, so 2 hours from 9:00 AM is when they meet, which is $\boxed{\textbf{(D)}\ 11:00}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 14

Problem

Problems 14, 15 and 16 involve Mrs. Reed's English assignment.

A Novel Assignment

The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

If Bob and Chandra both read the whole book, Bob will spend how many more seconds reading than Chandra?

$\textbf{(A)}\ 7,600 \qquad \textbf{(B)}\ 11,400 \qquad \textbf{(C)}\ 12,500 \qquad \textbf{(D)}\ 15,200 \qquad \textbf{(E)}\ 22,800$

Solution

The information is the same for Problems 14,15, and 16. Therefore, we shall only use the information we need. All we need for this problem is that there's 760 pages, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds. A lot of people will find how long it takes Bob to read the book, how long it takes Chandra to read the book, and then find the seconds. However, if we just set up the expression, we can find an easier way.

$$760 \cdot 45 - 760 \cdot 30 = 760(45-30) = 760(15) = \boxed{\textbf{(B)} 11,400}$$

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 15

Contents

Problem

Problems 14, 15 and 16 involve Mrs. Reed's English assignment.

A Novel Assignment

The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

Chandra and Bob, who each have a copy of the book, decide that they can save time by "team reading" the novel. In this scheme, Chandra will read from page 1 to a certain page and Bob will read from the next page through page 760, finishing the book. When they are through they will tell each other about the part they read. What is the last page that Chandra should read so that she and Bob spend the same amount of time reading the novel?

$\textbf{(A)}\ 425 \qquad \textbf{(B)}\ 444 \qquad \textbf{(C)}\ 456 \qquad \textbf{(D)}\ 484 \qquad \textbf{(E)}\ 506$

Solution 1

Same as the previous problem, we only use the information we need. Note that it's not just Chandra reads half of it and Bob reads the rest since they have different reading rates. In this case, we set up an equation and solve.

Let x be the number of pages that Chandra reads.

$30x = 45(760 - x)$ Distribute the 45

$30x = 45(760) - 45x$ Add $45x$ to both sides

$75x = 45(760)$ Divide both sides by 15 to make it easier to solve

$5x = 3(760)$ Divide both sides by 5

$x = 3(152) = \boxed{\textbf{(C)} 456}$

Solution 2

Bob and Chandra read at a rate of $30 : 45$ seconds per page, respectively. Simplifying that gets us Bob reads 2 pages for every 3 pages that Chandra reads. Therefore Chandra should read $\frac{3}{2+3} = \frac{3}{5}$ of the book. $\frac{3}{5} \cdot 760$=$\boxed{\textbf{(C)} 456}

See Also

2006 AMC 8 Problems/Problem 16

Problem

Problems 14, 15 and 16 involve Mrs. Reed's English assignment.

A Novel Assignment

The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

Before Chandra and Bob start reading, Alice says she would like to team read with them. If they divide the book into three sections so that each reads for the same length of time, how many seconds will each have to read?

$\textbf{(A)}\ 6400 \qquad \textbf{(B)}\ 6600 \qquad \textbf{(C)}\ 6800 \qquad \textbf{(D)}\ 7000 \qquad \textbf{(E)}\ 7200$

Solution

The amount of pages Bob, Chandra, and Alice would read is in the ratio 4:6:9. Therefore, Bob, Chandra, and Alice read 160, 240, and 360 pages respectively. They would also be reading in the same amount of time because the ratio of pages read was based on the time it takes each of them to read a page. Therefore, the amount of seconds each person reads is $160 \cdot 45 = \boxed{\textbf{(E)}\ 7200}$.

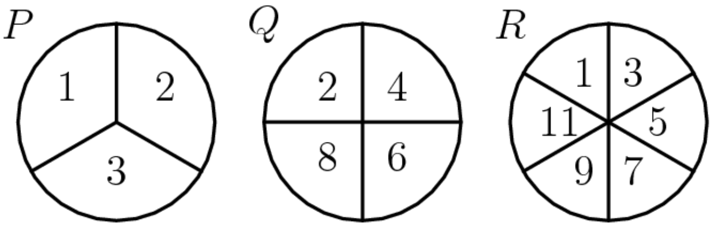
See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 17

Problem

Jeff rotates spinners P, Q and R and adds the resulting numbers. What is the probability that his sum is an odd number?



$\textbf{(A)}\ \frac{1}{4}\qquad\textbf{(B)}\ \frac{1}{3}\qquad\textbf{(C)}\ \frac{1}{2}\qquad\textbf{(D)}\ \frac{2}{3}\qquad\textbf{(E)}\ \frac{3}{4}$

Solution

In order for Jeff to have an odd number sum, the numbers must either be Odd + Odd + Odd or Even + Even + Odd. We easily notice that we cannot obtain Odd + Odd + Odd because spinner Q contains only even numbers. Therefore we must work with Even + Even + Odd and spinner Q will give us one of our even numbers. We also see that spinner R only contains odd, so spinner R must give us our odd nmber. We still need one even number from spinner P. There is only 1 even number: 2. Since spinning the required numbers are automatic on the other spinners, we only have to find the probability of spinning a 2 in spinner P, which clearly is $\boxed{\textbf{(B)}\ \frac{1}{3}}$

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 18

Problem

A cube with 3-inch edges is made using 27 cubes with 1-inch edges. Nineteen of the smaller cubes are white and eight are black. If the eight black cubes are placed at the corners of the larger cube, what fraction of the surface area of the larger cube is white? $\textbf{(A)}\ \frac{1}{9}\qquad\textbf{(B)}\ \frac{1}{4}\qquad\textbf{(C)}\ \frac{4}{9}\qquad\textbf{(D)}\ \frac{5}{9}\qquad\textbf{(E)}\ \frac{19}{27}$

Solution

The surface area of the cube is $6(3)(3) = 54$. Each of the eight black cubes has 3 faces on the outside, making $3(8) = 24$ black faces. Therefore there are $54-24 = 30$ white faces. To find the probability, we evaluate $\frac{30}{54} = \boxed{\textbf{(D)}\ \frac{5}{9}}$.

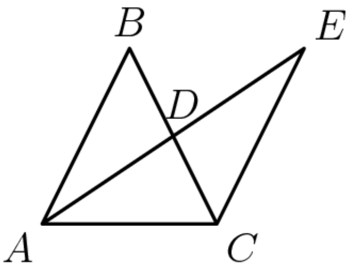
See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 19

Problem

Triangle ABC is an isosceles triangle with $\overline{AB}=\overline{BC}$. Point D is the midpoint of both $\overline{BC}$ and $\overline{AE}$, and $\overline{CE}$ is 11 units long. Triangle ABD is congruent to triangle ECD. What is the length of $\overline{BD}$?



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 4.5 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 5.5 \qquad \textbf{(E)}\ 6$

Solution

Since triangle ABD is congruent to triangle ECD and $\overline{CE}=11$, $\overline{AB}=11$. Since $\overline{AB}=\overline{BC}$, $\overline{BC}=11$. Because point D is the midpoint of $\overline{BC}$, $\overline{BD}=\frac{\overline{BC}}{2}=\frac{11}{2}=\boxed{\textbf{(D)}\ 5.5}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 20

Problem

A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?

$\textbf{(A) }0\qquad\textbf{(B) }1\qquad\textbf{(C) }2\qquad\textbf{(D) }3\qquad\textbf{(E) }4$

Solution

Since there are 6 players, a total of $\dfrac{6(6-1)}{2} = 15$ games are played. So far, $4+3+2+2+2 = 13$ games finished (one person won from each game), so Monica needs to win $15-13 = \boxed{\textbf{(C)}\ 2}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 21

Problem

An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. The aquarium is filled with water to a depth of 37 cm. A rock with volume 1000cm^3 is then placed in the aquarium and completely submerged. By how many centimeters does the water level rise?

$\textbf{(A)}\ 0.25\qquad\textbf{(B)}\ 0.5\qquad\textbf{(C)}\ 1\qquad\textbf{(D)}\ 1.25\qquad\textbf{(E)}\ 2.5$

Solution

The water level will rise 1cm for every $100 \cdot 40 = 4000 \text{cm}^2$. Since 1000 is $\frac{1}{4}$ of 4000, the water will rise $\frac{1}{4} \cdot 1 = \boxed{\textbf{(A)}\ 0.25}$

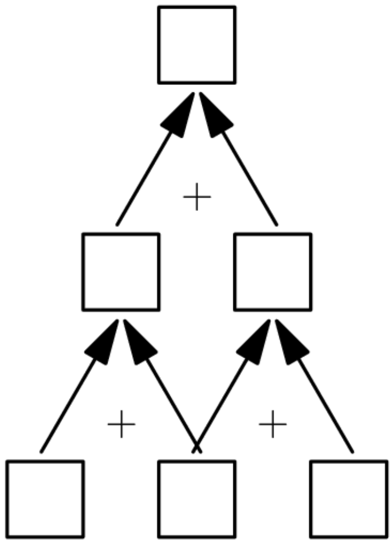
See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 22

Problem

Three different one-digit positive integers are placed in the bottom row of cells. Numbers in adjacent cells are added and the sum is placed in the cell above them. In the second row, continue the same process to obtain a number in the top cell. What is the difference between the largest and smallest numbers possible in the top cell?



$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 35$

Solution

If the lower cells contain A, B and C, then the second row will contain $A + B$ and $B + C$, and the top cell will contain $A + 2B + C$. To obtain the smallest sum, place 1 in the center cell and 2 and 3 in the outer ones. The top number will be 7. For the largest sum, place 9 in the center cell and 7 and 8 in the outer ones. This top number will be 33. The difference is $33 - 7 = \boxed{\textbf{(D)}\ 26}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 21	Followed by Problem 23
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2006 AMC 8 Problems/Problem 23

Contents

Problem

A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 5$

Solution

Solution 1

The counting numbers that leave a remainder of 4 when divided by 6 are $4, 10, 16, 22, 28, 34, \cdots$ The counting numbers that leave a remainder of 3 when divided by 5 are $3, 8, 13, 18, 23, 28, 33, \cdots$ So 28 is the smallest possible number of coins that meets both conditions. Because $4 \cdot 7 = 28$, there are $\boxed{\textbf{(A)}\ 0}$ coins left when they are divided among seven people.

Solution 2

If there were two more coins in the box, the number of coins would be divisible by both 6 and 5. The smallest number that is divisible by 6 and 5 is 30, so the smallest possible number of coins in the box is 28 and the remainder when divided by 7 is $\boxed{\textbf{(A)}\ 0}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 22	Followed by Problem 24
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All AJHSME/AMC 8 Problems and Solutions	

2006 AMC 8 Problems/Problem 24

Problem

In the multiplication problem below A, B, C, D and are different digits. What is $A+B$?

$$\begin{array}{cccc}& A & B & A\\ \times & & C & D\\ \hline C & D & C & D\\ \end{array}$$

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 9$

Solution

$CDCD = CD \cdot 101$, so $ABA = 101$. Therefore, $A = 1$ and $B = 0$, so $A+B=1+0=\boxed{\textbf{(A)}\ 1}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 23	Followed by Problem 25
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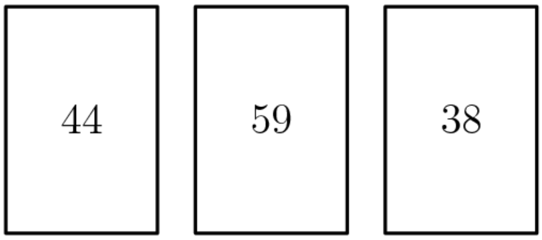
2006 AMC 8 Problems/Problem 25

Problem

Barry wrote 6 different numbers, one on each side of 3 cards, and laid the cards on a table, as shown. The sums of the two numbers on each of the three cards are equal. The three numbers on the hidden sides are prime numbers. What is the average of the hidden prime numbers?



$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Solution

Notice that 44 and 38 are both even, while 59 is odd. If any odd prime is added to 59, an even number will be obtained. However, the only way to obtain this even number would be to add another even number to 44, and a different one to 38. Since there is only one even prime (2), the middle card's hidden number cannot be an odd prime, and so must be even. Therefore, the middle card's hidden number must be 2, so the constant sum is $59+2=61$. Thus, the first card's hidden number is $61-44=17$, and the last card's hidden number is $61-38=23$.

Since the sum of the hidden primes is $2+17+23=42$, the average of the primes is $\dfrac{42}{3} = \boxed{\textbf{(B)} 14}$.

See Also

2006 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2006))	
Preceded by Problem 24	Followed by Last Problem
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