2004 AMC 12B Problems

Contents

Problem 1

At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?

$(\mathrm{A})3 \qquad (\mathrm{B})6 \qquad (\mathrm{C})9 \qquad (\mathrm{D})12 \qquad (\mathrm{E})15$

Solution

Problem 2

In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

$(\mathrm{A})5 \qquad (\mathrm{B})6 \qquad (\mathrm{C})8 \qquad (\mathrm{D})9 \qquad (\mathrm{E})10$

Solution

Problem 3

If x and y are positive integers for which $2^x 3^y = 1296$, what is the value of $x + y$?

$(\mathrm{A})8 \qquad (\mathrm{B})9 \qquad (\mathrm{C})10 \qquad (\mathrm{D})11 \qquad (\mathrm{E})12$

Solution

Problem 4

An integer x, with $10 \leq x \leq 99$, is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is a 7?

$\text{(A)}\frac{1}{9} \qquad \text{(B)}\frac{1}{5} \qquad \text{(C)}\frac{19}{90} \qquad \text{(D)}\frac{2}{9} \qquad \text{(E)}\frac{1}{3}$

Solution

Problem 5

On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

$\text{(A)}5 \qquad \text{(B)}6 \qquad \text{(C)}7 \qquad \text{(D)}8 \qquad \text{(E)}9$

Solution

Problem 6

Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and downtown Minneapolis?

$\text{(A)}13 \qquad \text{(B)}14 \qquad \text{(C)}15 \qquad \text{(D)}16 \qquad \text{(E)}17$

Solution

Problem 7

A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?

$\text{(A)}200+25\pi \quad \text{(B)}100+75\pi \quad \text{(C)}75+100\pi \quad \text{(D)}100+100\pi \quad \text{(E)}100+125\pi$

Solution

Problem 8

A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?

$\text{(A)}5 \qquad \text{(B)}8 \qquad \text{(C)}9 \qquad \text{(D)}10 \qquad \text{(E)}11$

Solution

Problem 9

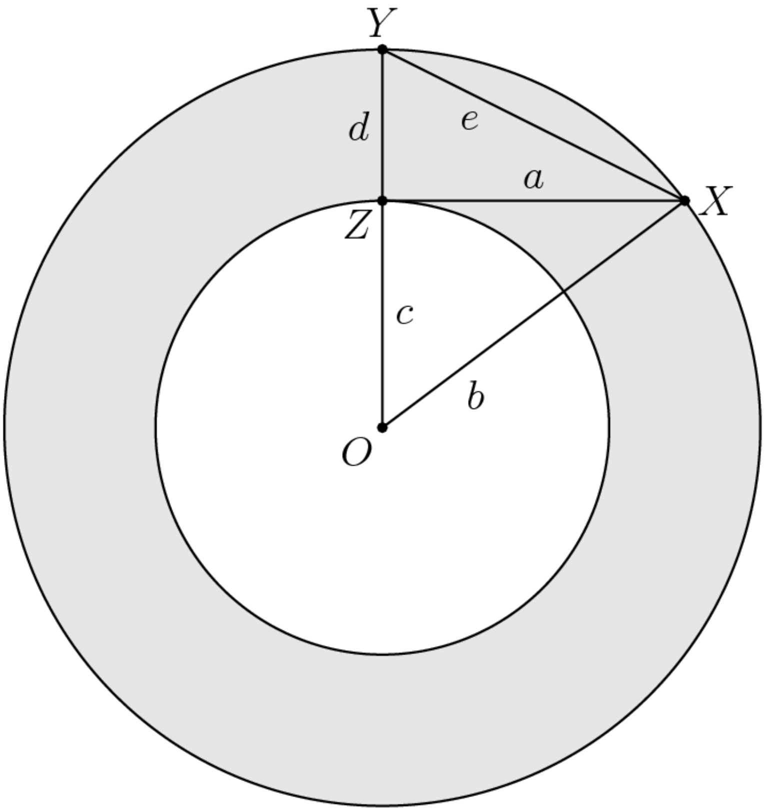
The point $(-3,2)$ is rotated 90° clockwise around the origin to point B. Point B is then reflected over the line $x=y$ to point C. What are the coordinates of C?

$\text{(A)}\ (-3,-2) \qquad \text{(B)}\ (-2,-3) \qquad \text{(C)}\ (2,-3) \qquad \text{(D)}\ (2,3) \qquad \text{(E)}\ (3,2)$

Solution

Problem 10

An annulus is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b>c$. Let OX be a radius of the larger circle, let XZ be tangent to the smaller circle at Z, and let OY be the radius of the larger circle that contains Z. Let $a=XZ$, $d=YZ$, and $e=XY$. What is the area of the annulus?



$\text{(A) } \pi a^2 \qquad \text{(B) } \pi b^2 \qquad \text{(C) } \pi c^2 \qquad \text{(D) } \pi d^2 \qquad \text{(E) } \pi e^2$

Solution

Problem 11

All the students in an algebra class took a 100-point test. Five students scored 100, each student scored at least 60, and the mean score was 76. What is the smallest possible number of students in the class?

$\text{(A) } 10 \qquad \text{(B) } 11 \qquad \text{(C) } 12 \qquad \text{(D) } 13 \qquad \text{(E) } 14$

Solution

Problem 12

In the sequence $2001, 2002, 2003, \ldots$, each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

$\text{(A) } -2004 \qquad \text{(B) } -2 \qquad \text{(C) } 0 \qquad \text{(D) } 4003 \qquad \text{(E) } 6007$

Solution

Problem 13

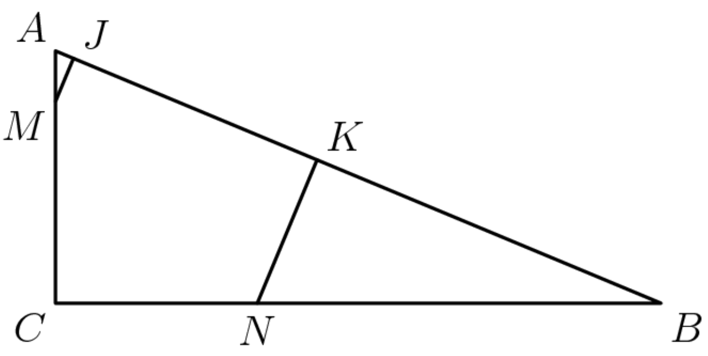
If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$ with a and b real, what is the value of $a + b$?

$\text{(A) } -2 \qquad \text{(B) } -1 \qquad \text{(C) } 0 \qquad \text{(D) } 1 \qquad \text{(E) } 2$

Solution

Problem 14

In $\triangle ABC$, $AB = 13$, $AC = 5$, and $BC = 12$. Points M and N lie on AC and BC, respectively, with $CM = CN = 4$. Points J and K are on AB so that MJ and NK are perpendicular to AB. What is the area of pentagon $CMJKN$?



$\textbf{(A) } 15 \qquad \textbf{(B) } \frac{81}{5} \qquad \textbf{(C) } \frac{205}{12} \qquad \textbf{(D) } \frac{240}{13} \qquad \textbf{(E) } 20$

Solution

Problem 15

The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?

$\textbf{(A) } 9 \qquad \textbf{(B) } 18 \qquad \textbf{(C) } 27 \qquad \textbf{(D) } 36 \qquad \textbf{(E) } 45$

Solution

Problem 16

A function f is defined by $f(z) = i\overline{z}$, where $i = \sqrt{-1}$ and $\overline{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 8$

Solution

Problem 17

For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots. If the sum of the base-2 logarithms of the roots is 5, what is the value of a?

$\textbf{(A) } -256 \qquad \textbf{(B) } -64 \qquad \textbf{(C) } -8 \qquad \textbf{(D) } 64 \qquad \textbf{(E) } 256$

Solution

Problem 18

Points A and B are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of AB. What is the length of AB?

$\textbf{(A) } 2\sqrt{5} \qquad \textbf{(B) } 5 + \frac{\sqrt{2}}{2} \qquad \textbf{(C) } 5 + \sqrt{2} \qquad \textbf{(D) } 7 \qquad \textbf{(E) } 5\sqrt{2}$

Solution

Problem 19

A truncated cone has horizontal bases with radii 18 and 2. A sphere is tangent to the top, bottom, and lateral surface of the truncated cone. What is the radius of the sphere?

$\mathrm{(A)}\ 6 \qquad \mathrm{(B)}\ 4\sqrt{5} \qquad \mathrm{(C)}\ 9 \qquad \mathrm{(D)}\ 10 \qquad \mathrm{(E)}\ 6\sqrt{3}$

Solution

Problem 20

Each face of a cube is painted either red or blue, each with probability $1/2$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

$\textbf{(A)}\ \frac{1}{4} \qquad \textbf{(B)}\ \frac{5}{16} \qquad \textbf{(C)}\ \frac{3}{8} \qquad \textbf{(D)}\ \frac{7}{16} \qquad \textbf{(E)}\ \frac{1}{2}$

Solution

Problem 21

The graph of $2x^2 + xy + 3y^2 - 11x - 20y + 40 = 0$ is an ellipse in the first quadrant of the xy-plane. Let a and b be the maximum and minimum values of $\frac{y}{x}$ over all points (x,y) on the ellipse. What is the value of $a+b$?

$\mathrm{(A)}\ 3 \qquad \mathrm{(B)}\ \sqrt{10} \qquad \mathrm{(C)}\ \frac{7}{2} \qquad \mathrm{(D)}\ \frac{9}{2} \qquad \mathrm{(E)}\ 2\sqrt{14}$

Solution

Problem 22

The square

50	b	c
d	e	f
g	h	2

is a multiplicative magic square. That is, the product of the numbers in each row, column, and diagonal is the same. If all the entries are positive integers, what is the sum of the possible values of g?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 136$

Solution

Problem 23

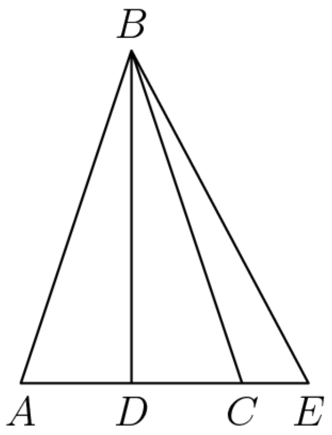
The polynomial $x^3 - 2004x^2 + mx + n$ has integer coefficients and three distinct positive zeros. Exactly one of these is an integer, and it is the sum of the other two. How many values of n are possible?

$\mathrm{(A)}\ 250,000 \qquad \mathrm{(B)}\ 250,250 \qquad \mathrm{(C)}\ 250,500 \qquad \mathrm{(D)}\ 250,750 \qquad \mathrm{(E)}\ 251,000$

Solution

Problem 24

In $\triangle ABC$, $AB = BC$, and $\overline{BD}$ is an altitude. Point E is on the extension of $\overline{AC}$ such that $BE = 10$. The values of $\tan \angle CBE$, $\tan \angle DBE$, and $\tan \angle ABE$ form a geometric progression, and the values of $\cot \angle DBE, \cot \angle CBE, \cot \angle DBC$ form an arithmetic progression. What is the area of $\triangle ABC$?



$\mathrm{(A)}\ 16 \qquad \mathrm{(B)}\ \frac{50}{3} \qquad \mathrm{(C)}\ 10\sqrt{3} \qquad \mathrm{(D)}\ 8\sqrt{5} \qquad \mathrm{(E)}\ 18$

Solution

Problem 25

Given that 2^{2004} is a 604-digit number whose first digit is 1, how many elements of the set $S = \{2^0, 2^1, 2^2, \ldots, 2^{2003}\}$ have a first digit of 4?

$\mathrm{(A)}\ 194 \qquad \mathrm{(B)}\ 195 \qquad \mathrm{(C)}\ 196 \qquad \mathrm{(D)}\ 197 \qquad \mathrm{(E)}\ 198$

Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2004 AMC 12B
- 2004 AMC B Math Jam Transcript (http://www.artofproblemsolving.com/Community/AoPS_Y_MJ_Transcripts.php?mj_id=28)
- Mathematics competition resources