2009 AMC 12B Problems

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Problem 1

Each morning of her five-day workweek, Jane bought either a 50-cent muffin or a 75-cent bagel. Her total cost for the week was a whole number of dollars. How many bagels did she buy?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution

Problem 2

Paula the painter had just enough paint for 30 identically sized rooms. Unfortunately, on the way to work, three cans of paint fell off her truck, so she had only enough paint for 25 rooms. How many cans of paint did she use for the 25 rooms?

$\textbf{(A)}\ 10\qquad\textbf{(B)}\ 12\qquad\textbf{(C)}\ 15\qquad\textbf{(D)}\ 18\qquad\textbf{(E)}\ 25$

Solution

Problem 3

Twenty percent off 60 is one-third more than what number?

$\textbf{(A)}\ 16\qquad\textbf{(B)}\ 30\qquad\textbf{(C)}\ 32\qquad\textbf{(D)}\ 36\qquad\textbf{(E)}\ 48$

Solution

Problem 4

A rectangular yard contains two flower beds in the shape of congruent isosceles right triangles. The remainder of the yard has a trapezoidal shape, as shown. The parallel sides of the trapezoid have lengths 15 and 25 meters. What fraction of the yard is occupied by the flower beds?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{1}{3}$

Solution

Problem 5

Kiana has two older twin brothers. The product of their ages is 128. What is the sum of their three ages?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 24$

Solution

Problem 6

By inserting parentheses, it is possible to give the expression

$$2\times3 + 4\times5$$

several values. How many different values can be obtained?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

Problem 7

In a certain year the price of gasoline rose by 20% during January, fell by 20% during February, rose by 25% during March, and fell by $x\%$ during April. The price of gasoline at the end of April was the same as it had been at the beginning of January. To the nearest integer, what is x?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 35$

Solution

Problem 8

When a bucket is two-thirds full of water, the bucket and water weigh a kilograms. When the bucket is one-half full of water the total weight is b kilograms. In terms of a and b, what is the total weight in kilograms when the bucket is full of water?

$\textbf{(A)}\ \frac23a + \frac13b \qquad \textbf{(B)}\ \frac32a - \frac12b \qquad \textbf{(C)}\ \frac32a + b \qquad \textbf{(D)}\ \frac32a + 2b \qquad \textbf{(E)}\ 3a - 2b$

Solution

Problem 9

Triangle ABC has vertices $A = (3,0)$, $B = (0,3)$, and C, where C is on the line $x + y = 7$. What is the area of $\triangle ABC$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 14$

Solution

Problem 10

A particular 12-hour digital clock displays the hour and minute of a day. Unfortunately, whenever it is supposed to display a 1, it mistakenly displays a 9. For example, when it is 1:16 PM the clock incorrectly shows 9:96 PM. What fraction of the day will the clock show the correct time?

$\textbf{(A)}\ \frac{1}{2}\qquad\textbf{(B)}\ \frac{5}{8}\qquad\textbf{(C)}\ \frac{3}{4}\qquad\textbf{(D)}\ \frac{5}{6}\qquad\textbf{(E)}\ \frac{9}{10}$

Solution

Problem 11

On Monday, Millie puts a quart of seeds, 25% of which are millet, into a bird feeder. On each successive day she adds another quart of the same mix of seeds without removing any seeds that are left. Each day the birds eat only 25% of the millet in the feeder, but they eat all of the other seeds. On which day, just after Millie has placed the seeds, will the birds find that more than half the seeds in the feeder are millet?

$\textbf{(A)}\ \text{Tuesday}\qquad\textbf{(B)}\ \text{Wednesday}\qquad\textbf{(C)}\ \text{Thursday}\qquad\textbf{(D)}\ \text{Friday}\qquad\textbf{(E)}\ \text{Saturday}$

Solution

Problem 12

The fifth and eighth terms of a geometric sequence of real numbers are $7!$ and $8!$ respectively. What is the first term?

$\textbf{(A)}\ 60\qquad\textbf{(B)}\ 75\qquad\textbf{(C)}\ 120\qquad\textbf{(D)}\ 225\qquad\textbf{(E)}\ 315$

Solution

Problem 13

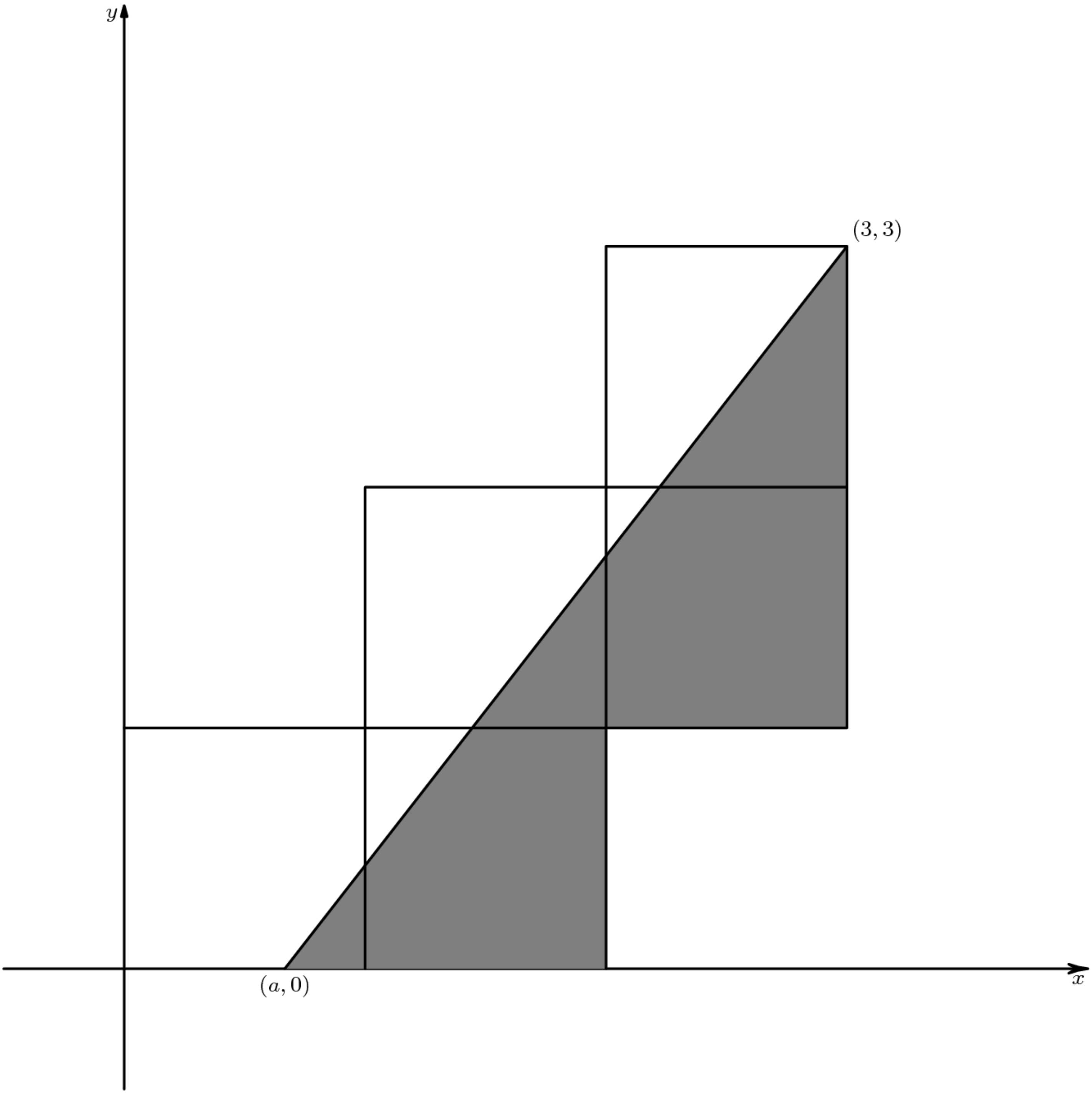
Triangle ABC has $AB=13$ and $AC=15$, and the altitude to $\overline{BC}$ has length 12. What is the sum of the two possible values of BC?

$\textbf{(A)}\ 15\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 17\qquad\textbf{(D)}\ 18\qquad\textbf{(E)}\ 19$

Solution

Problem 14

Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin. The slanted line, extending from $(a,0)$ to $(3,3)$, divides the entire region into two regions of equal area. What is a?



(A) $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

Solution

Problem 15

Assume $0 < r < 3$. Below are five equations for x. Which equation has the largest solution x?

(A) $3(1+r)^x = 7$ **(B)** $3(1+r/10)^x = 7$ **(C)** $3(1+2r)^x = 7$
(D) $3(1+\sqrt{r})^x = 7$ **(E)** $3(1+1/r)^x = 7$

Solution

Problem 16

Trapezoid $ABCD$ has $AD||BC$, $BD = 1$, $\angle DBA = 23^{\circ}$, and $\angle BDC = 46^{\circ}$. The ratio $BC : AD$ is $9 : 5$. What is CD?

$\textbf{(A)}\ \frac{7}{9} \qquad \textbf{(B)}\ \frac{4}{5} \qquad \textbf{(C)}\ \frac{13}{15} \qquad \textbf{(D)}\ \frac{8}{9} \qquad \textbf{(E)}\ \frac{14}{15}$

Solution

Problem 17

Each face of a cube is given a single narrow stripe painted from the center of one edge to the center of its opposite edge. The choice of the edge pairing is made at random and independently for each face. What is the probability that there is a continuous stripe encircling the cube?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{3}{16} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{3}{8} \qquad \textbf{(E)}\ \frac{1}{2}$

Solution

Problem 18

Rachel and Robert run on a circular track. Rachel runs counterclockwise and completes a lap every 90 seconds, and Robert runs clockwise and completes a lap every 80 seconds. Both start from the start line at the same time. At some random time between 10 minutes and 11 minutes after they begin to run, a photographer standing inside the track takes a picture that shows one-fourth of the track, centered on the starting line. What is the probability that both Rachel and Robert are in the picture?

$\textbf{(A)}\ \frac{1}{16} \qquad \textbf{(B)}\ \frac{1}{8} \qquad \textbf{(C)}\ \frac{3}{16} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{5}{16}$

Solution

Problem 19

For each positive integer n, let $f(n) = n^4 - 360n^2 + 400$. What is the sum of all values of $f(n)$ that are prime numbers?

$\textbf{(A)}\ 794 \qquad \textbf{(B)}\ 796 \qquad \textbf{(C)}\ 798 \qquad \textbf{(D)}\ 800 \qquad \textbf{(E)}\ 802$

Solution

Problem 20

A convex polyhedron Q has vertices $V_1, V_2, \ldots, V_n$, and 100 edges. The polyhedron is cut by planes $P_1, P_2, \ldots, P_n$ in such a way that plane P_k cuts only those edges that meet at vertex V_k. In addition, no two planes intersect inside or on Q. The cuts produce n pyramids and a new polyhedron R. How many edges does R have?

$\textbf{(A)}\ 200 \qquad \textbf{(B)}\ 2n \qquad \textbf{(C)}\ 300 \qquad \textbf{(D)}\ 400 \qquad \textbf{(E)}\ 4n$

Solution

Problem 21

Ten women sit in 10 seats in a line. All of the 10 get up and then reseat themselves using all 10 seats, each sitting in the seat she was in before or a seat next to the one she occupied before. In how many ways can the women be reseated?

$\textbf{(A)}\ 89 \qquad \textbf{(B)}\ 90 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 2^{10} \qquad \textbf{(E)}\ 2^2 3^8$

Solution

Problem 22

Parallelogram $ABCD$ has area $1{,}000{,}000$. Vertex A is at $(0,0)$ and all other vertices are in the first quadrant. Vertices B and D are lattice points on the lines $y = x$ and $y = kx$ for some integer $k > 1$, respectively. How many such parallelograms are there?

$\textbf{(A) } 49 \qquad \textbf{(B) } 720 \qquad \textbf{(C) } 784 \qquad \textbf{(D) } 2009 \qquad \textbf{(E) } 2048$

Solution

Problem 23

A region S in the complex plane is defined by

$$S = \{x + iy : -1 \le x \le 1, -1 \le y \le 1\}.$$

A complex number $z = x + iy$ is chosen uniformly at random from S. What is the probability that $\left(\frac34 + \frac34 i\right)z$ is also in S?

$\textbf{(A) } \frac12 \qquad \textbf{(B) } \frac23 \qquad \textbf{(C) } \frac34 \qquad \textbf{(D) } \frac79 \qquad \textbf{(E) } \frac78$

Solution

Problem 24

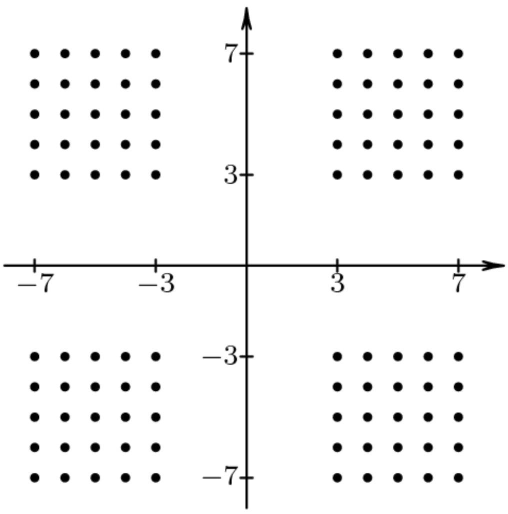
For how many values of x in $[0, \pi]$ is $\sin^{-1}(\sin 6x) = \cos^{-1}(\cos x)$? Note: The functions $\sin^{-1} = \arcsin$ and $\cos^{-1} = \arccos$ denote inverse trigonometric functions.

$\textbf{(A) } 3 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 5 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 7$

Solution

Problem 25

The set G is defined by the points (x, y) with integer coordinates, $3 \le |x| \le 7$, $3 \le |y| \le 7$. How many squares of side at least 6 have their four vertices in G?



$\textbf{(A) } 125 \qquad \textbf{(B) } 150 \qquad \textbf{(C) } 175 \qquad \textbf{(D) } 200 \qquad \textbf{(E) } 225$

Solution