2010 AMC 8 Problems/Problem 1

Problem

At Euclid Middle School the mathematics teachers are Miss Germain, Mr. Newton, and Mrs. Young. There are 11 students in Mrs. Germain's class, 8 students in Mr. Newton's class, and 9 students in Mrs. Young's class taking the AMC 8 this year. How many mathematics students at Euclid Middle School are taking the contest?

$\textbf{(A)}\ 26 \qquad\textbf{(B)}\ 27\qquad\textbf{(C)}\ 28\qquad\textbf{(D)}\ 29\qquad\textbf{(E)}\ 30$

Solution

Given that these are the only math teachers at Euclid Middle School and we are told how many from each class are taking the AMC 8, we simply add the three numbers to find the total. $11+8+9=\boxed{\textbf{(C)}\ 28}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by First Problem	Followed by Problem 2
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 2

Problem

If $a@b = \dfrac{a \times b}{a+b}$ for a, b positive integers, then what is $5@10$?

$\textbf{(A)}\ \frac{3}{10} \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{10}{3} \qquad \textbf{(E)}\ 50$

Solution

Substitute $a = 5$ and $b = 10$ into the expression for $a@b$ to get:

$$5@10 = \frac{5 \times 10}{5+10} = \frac{50}{15} = \frac{10}{3}$$

Thus, answer choice $\boxed{\textbf{(D)}\ \frac{10}{3}}$ is correct.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 1	Followed by Problem 3
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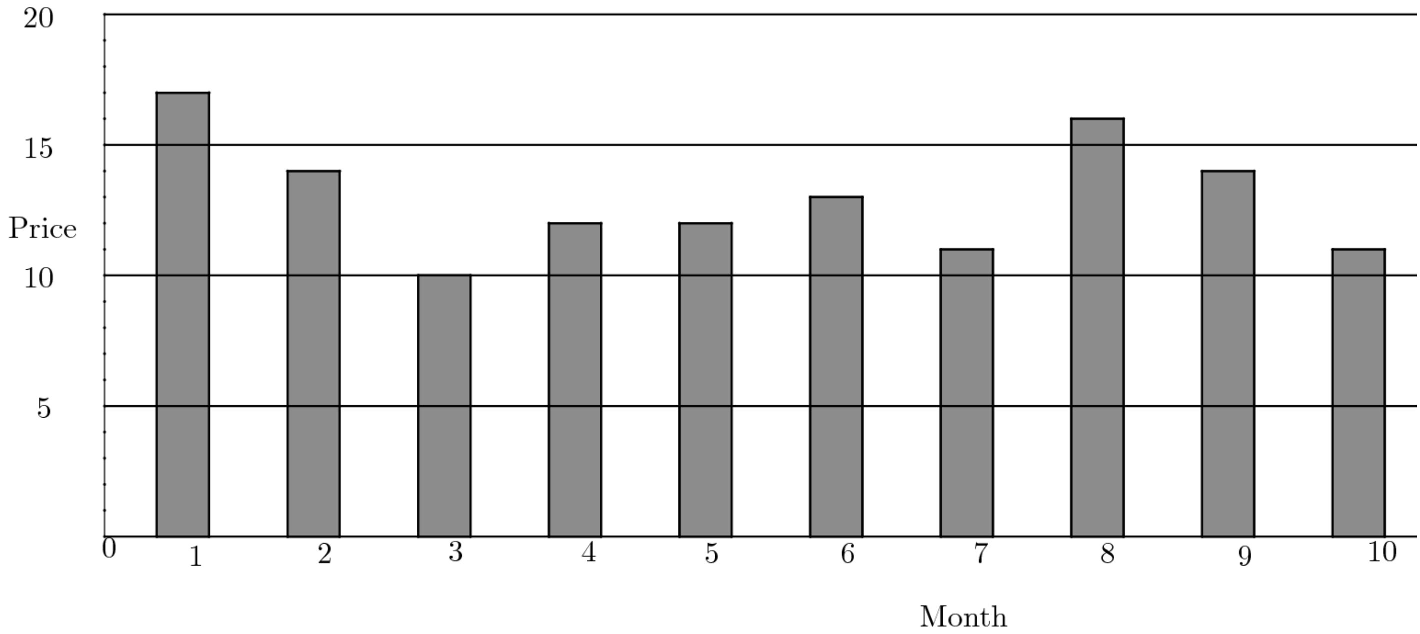
All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 3

Problem

The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price more than the lowest price?



$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 62 \qquad \textbf{(C)}\ 70 \qquad \textbf{(D)}\ 89 \qquad \textbf{(E)}\ 100$

Solution

The highest price was in Month 1, which was \$17. The lowest price was in Month 3, which was \$10. 17 is $\frac{17}{10} \cdot 100 = 170\%$ of 10, and is $170 - 100 = 70\%$ more than 10. Therefore, the answer is $\boxed{\textbf{(C)}\ 70}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 2	Followed by Problem 4
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 4

Problem

What is the sum of the mean, median, and mode of the numbers $2, 3, 0, 3, 1, 4, 0, 3$?

$\textbf{(A)}\ 6.5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 7.5 \qquad \textbf{(D)}\ 8.5 \qquad \textbf{(E)}\ 9$

Solution

Putting the numbers in numerical order we get the list $0, 0, 1, 2, 3, 3, 3, 4.$ The mode is $3.$ The median is $\frac{2+3}{2} = 2.5.$ The average is $\frac{0+0+1+2+3+3+3+4}{8} = \frac{16}{8} = 2.$ The sum of all three is $3 + 2.5 + 2 = \boxed{\textbf{(C)}\ 7.5}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 3	Followed by Problem 5
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 5

Problem

Alice needs to replace a light bulb located 10 centimeters below the ceiling in her kitchen. The ceiling is 2.4 meters above the floor. Alice is 1.5 meters tall and can reach 46 centimeters above the top of her head. Standing on a stool, she can just reach the light bulb. What is the height of the stool, in centimeters?

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 34 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 38 \qquad \textbf{(E)}\ 40$

Solution

Convert everything to the same unit. Since the answer is in centimeters, change meters to centimeters by moving the decimal place two places to the right.

The ceiling is 240 centimeters above the floor. The combined height of Alice and the light bulb when she reaches for it is $10 + 150 + 46 = 206$ centimeters. That means the stool's height needs to be $240 - 206 = \boxed{\textbf{(B)}\ 34}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 4	Followed by Problem 6
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 6

Problem

Which of the following figures has the greatest number of lines of symmetry?

$\textbf{(A)}$ equilateral triangle

$\textbf{(B)}$ non-square rhombus

$\textbf{(C)}$ non-square rectangle

$\textbf{(D)}$ isosceles trapezoid

$\textbf{(E)}$ square

Solution

An equilateral triangle has 3 lines of symmetry. A non-square rhombus has 2 lines of symmetry. A non-square rectangle has 2 lines of symmetry. An isosceles trapezoid has 1 line of symmetry. A square has 8 lines of symmetry.

Therefore, the answer is $\boxed{\textbf{(E)}\ \text{square}}$.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 5	Followed by Problem 7
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 7

Problem

Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Freddie would need so he could pay any amount of money less than a dollar?

$\textbf{(A)}\ 6 \qquad\textbf{(B)}\ 10\qquad\textbf{(C)}\ 15\qquad\textbf{(D)}\ 25\qquad\textbf{(E)}\ 99$

Solution

It would have a total of 10 coins: 4 pennies, 1 nickel, 2 dimes, and 3 quarters, so the answer is $\boxed{\textbf{(B)}\ 10}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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2010 AMC 8 Problems/Problem 8

Problem

As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction $1/2$ mile in front of her. After she passes him, she can see him in her rear mirror until he is $1/2$ mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson?

$\textbf{(A)}\ 6 \qquad\textbf{(B)}\ 8 \qquad\textbf{(C)}\ 12 \qquad\textbf{(D)}\ 15 \qquad\textbf{(E)}\ 16$

Solution

Because they are skating in the same direction, Emily is skating relative to Emerson $12 - 8 = 4$ mph. Now we can look at it as if Emerson is not moving at all[on his skateboard] and Emily is riding at 4 mph. It takes her

$$\frac{1}{2}\text{ mile}\cdot\frac{1\text{ hour}}{4\text{ miles}} = \frac{1}{8}\text{ hour}$$

to skate the $1/2$ mile to reach him, and then the same amount of time to be $1/2$ mile ahead of him. This totals to

$$2\cdot\frac{1}{8}\text{ hour}\cdot\frac{60\text{ minutes}}{1\text{ hour}} = \boxed{\textbf{(D)}\ 15}\text{ minutes}$$

Last edited on Nov 11, 2014, 10:23 am.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 9

Problem

Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

$\textbf{(A)}\ 64 \qquad\textbf{(B)}\ 75\qquad\textbf{(C)}\ 80\qquad\textbf{(D)}\ 84\qquad\textbf{(E)}\ 86$

Solution

Ryan answered $(0.8)(25)=20$ problems correct on the first test, $(0.9)(40)=36$ on the second, and $(0.7)(10)=7$ on the third. This amounts to a total of $20+36+7=63$ problems correct. The total number of problems is $25+40+10=75.$ Therefore, the percentage is $\dfrac{63}{75} \rightarrow \boxed{\textbf{(D)}\ 84}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 10

Problem 9

Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed. If a total of 24 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{5}{6} \qquad \textbf{(E)}\ \frac{7}{8}$

Solution

The pepperoni circles' diameter is 2, since $\frac{12}{6}=2$. From that we see that the area of the 24 circles of pepperoni is $\left(\frac{2}{2}\right)^2(24\pi)=24\pi$. The large pizza's area is $6^2\pi$. Therefore, the ratio is $\frac{24\pi}{36\pi}=\boxed{\textbf{(B)}\ \frac{2}{3}}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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2010 AMC 8 Problems/Problem 11

Contents

Problem

The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

$\textbf{(A)}\ 48 \qquad\textbf{(B)}\ 64 \qquad\textbf{(C)}\ 80 \qquad\textbf{(D)}\ 96\qquad\textbf{(E)}\ 112$

Solution

Let the height of the taller tree be h and let the height of the smaller tree be $h-16$. Since the ratio of the smaller tree to the larger tree is $\frac{3}{4}$, we have $\frac{h-16}{h}=\frac{3}{4}$. Solving for h gives us $h=64 \Rightarrow \boxed{\textbf{(B)}\ 64}$

Solution 2

To answer this problem, you have to make it so that we have the same proportion as 3:4, but the difference between them is 16. Since the two numbers are consecutive, if we multiply both of them by 16, we would get a difference of 16 between them. So, it would be 48:64 and since we need to find the height of the taller tree, we get B(64)

See Also

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Caption

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 12

Contents

Problem

Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed so that 75% of the remaining balls are red?

$\textbf{(A)}\ 25\qquad\textbf{(B)}\ 50\qquad\textbf{(C)}\ 75\qquad\textbf{(D)}\ 100\qquad\textbf{(E)}\ 150$

Solution 1

Since 80 percent of the 500 balls are red, there are 400 red balls. Therefore, there must be 100 blue balls. For the 100 blue balls to be 25% or $\frac{1}{4}$ of the bag, there must be 400 balls in the bag so 100 red balls must be removed. The answer is $\boxed{\textbf{(D)}\ 100}$.

Solution 2

We could also set up a proportion. Since we know there are 400 red balls, we let the amount of red balls removed be x, so $\frac{400-x}{500-x}=\frac{3}{4}$. Cross-multiplying gives us $1600-4x=1500-3x \implies x=100$, so our answer is $\boxed{\textbf{(D)}\ 100}$.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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2010 AMC 8 Problems/Problem 13

Contents

Problem

The lengths of the sides of a triangle in inches are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Solution 1

Let n, $n+1$, and $n+2$ be the lengths of the sides of the triangle. Then the perimeter of the triangle is $n+(n+1)+(n+2)=3n+3$. Using the fact that the length of the smallest side is 30% of the perimeter, it follows that:

$n=0.3(3n+3)\Rightarrow n=0.9n+0.9\Rightarrow 0.1n=0.9\Rightarrow n=9$. The longest side is then $n+2=11$. Thus, answer choice $\boxed{\textbf{(E)}\ 11}$ is correct.

Solution 2

Since the length of the shortest side is a whole number and is equal to $\dfrac{3}{10}$ of the perimeter, it follows that the perimeter must be a multiple of 10. Adding the two previous integers to each answer choice, we see that $11+10+9=30$. Thus, answer choice $\boxed{\textbf{(E)}\ 11}$ is correct.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 14

Problem

What is the sum of the prime factors of 2010?

$\textbf{(A)}\ 67\qquad\textbf{(B)}\ 75\qquad\textbf{(C)}\ 77\qquad\textbf{(D)}\ 201\qquad\textbf{(E)}\ 210$

Solution

First, we must find the prime factorization of 2010. $2010=2\cdot3\cdot5\cdot67$ We add the factors up to get $\boxed{\textbf{(C)}\ 77}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 15

Problem

A jar contains 5 different colors of gumdrops. 30% are blue, 20% are brown, 15% are red, 10% are yellow, and other 30 gumdrops are green. If half of the blue gumdrops are replaced with brown gumdrops, how many gumdrops will be brown?

$\textbf{(A)}\ 35\qquad\textbf{(B)}\ 36\qquad\textbf{(C)}\ 42\qquad\textbf{(D)}\ 48\qquad\textbf{(E)}\ 64$

Solution

We do $100-30-20-15-10$ to find the percent of gumdrops that are green. We find that 25% of the gumdrops are green. That means there are 120 gumdrops. If we replace half of the blue gumdrops with brown gumdrops, then 15% of the jar's gumdrops are brown. $\frac{35}{100} \cdot 120 = 42 \Rightarrow \boxed{\textbf{(C)}\ 42}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 16

Problem

A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle? $\textbf{(A)}\ \frac{\sqrt{\pi}}{2} \qquad \textbf{(B)}\ \sqrt{\pi} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ 2\pi \qquad \textbf{(E)}\ \pi^2$

Solution

Let the side length of the square be s, and let the radius of the circle be r. Thus we have $s^2=r^2\pi$. Dividing each side by r^2, we get $s^2/r^2=\pi$. Since $(s/r)^2=s^2/r^2$, we have
$s/r=\sqrt{\pi}\Rightarrow\boxed{\textbf{(B)}\ \sqrt{\pi}}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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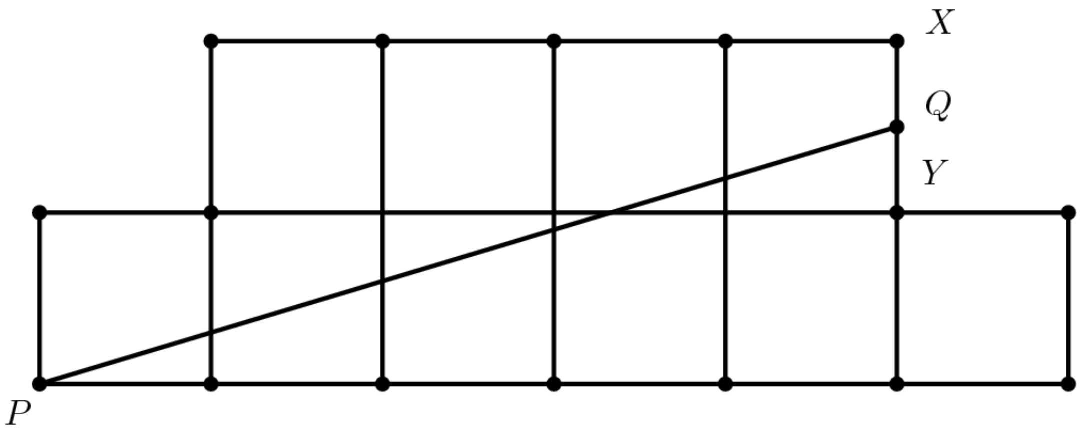
All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 17

Problem

The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\dfrac{XQ}{QY}$?



$\textbf{(A)}\ \frac{2}{5}\qquad\textbf{(B)}\ \frac{1}{2}\qquad\textbf{(C)}\ \frac{3}{5}\qquad\textbf{(D)}\ \frac{2}{3}\qquad\textbf{(E)}\ \frac{3}{4}$

Solution

We see that half the area of the octagon is 5. We see that the triangle area is $5-1=4$. That means that $\frac{5h}{2}=4\rightarrow h=\frac{8}{5}$.

$$\text{QY}=\frac{8}{5}-1=\frac{3}{5}$$

Meaning, $\frac{\frac{2}{5}}{\frac{3}{5}}=\boxed{\textbf{(D)}\ \frac{2}{3}}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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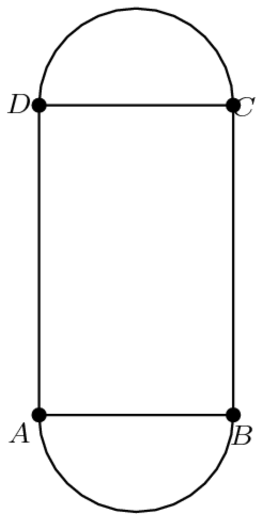
2010 AMC 8 Problems/Problem 18

Problem

A decorative window is made up of a rectangle with semicircles at either end. The ratio of AD to AB is $3:2$. And AB is 30 inches. What is the ratio of the area of the rectangle to the combined area of the semicircles?



$\textbf{(A)}\ 2:3\qquad\textbf{(B)}\ 3:2\qquad\textbf{(C)}\ 6:\pi\qquad\textbf{(D)}\ 9:\pi\qquad\textbf{(E)}\ 30:\pi$

Solution

We can set a proportion:

$$\frac{AD}{AB}=\frac{3}{2}$$

We substitute AB with 30 and solve for AD.

$$\frac{AD}{30}=\frac{3}{2}$$

$$AD=45$$

We calculate the combined area of semicircle by putting together semicircle AB and CD to get a circle with radius 15. Thus, the area is 225π. The area of the rectangle is $30\cdot 45=1350$. We calculate the ratio:

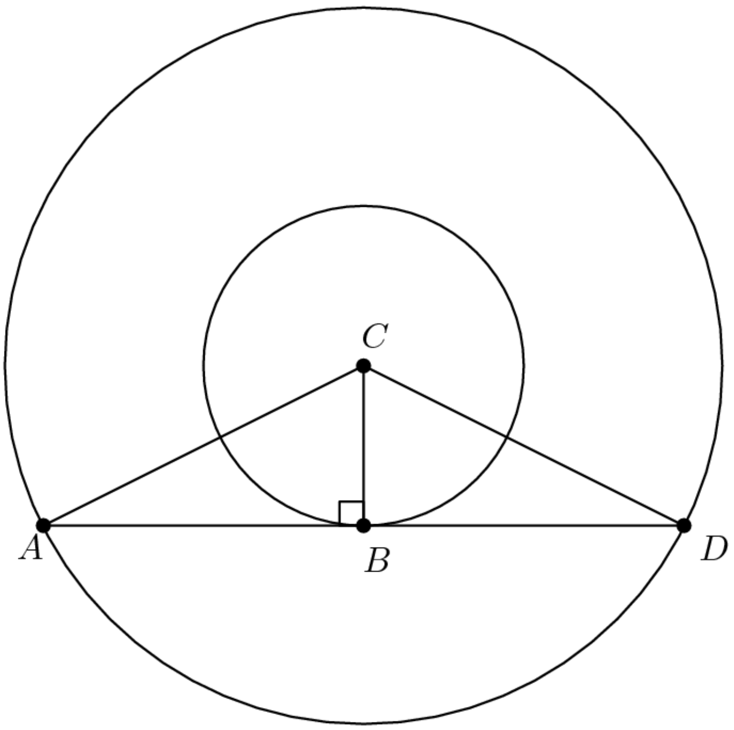
$$\frac{1350}{225\pi}=\frac{6}{\pi}\Rightarrow\boxed{\textbf{(C)}\ 6:\pi}$$

See Also

2010 AMC 8 Problems/Problem 19

Problem

The two circles pictured have the same center C. Chord $\overline{AD}$ is tangent to the inner circle at B, AC is 10, and chord $\overline{AD}$ has length 16. What is the area between the two circles?



$\textbf{(A)}\ 36\pi \qquad \textbf{(B)}\ 49\pi \qquad \textbf{(C)}\ 64\pi \qquad \textbf{(D)}\ 81\pi \qquad \textbf{(E)}\ 100\pi$

Solution

Since $\triangle ACD$ is isosceles, CB bisects AD. Thus $AB = BD = 8$. From the Pythagorean Theorem, $CB = 6$. Thus the area between the two circles is $100\pi - 36\pi = 64\pi$ $\boxed{\textbf{(C)}\ 64\pi}$

Note: The length AC is necessary information, as this tells us the radius of the larger circle. The area of the annulus is $\pi(AC^2 - BC^2) = \pi AB^2 = 64\pi$.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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2010 AMC 8 Problems/Problem 20

Problem 20

In a room, $2/5$ of the people are wearing gloves, and $3/4$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 20$

Solution

Let x be the number of people wearing both a hat and a glove. Since the number of people wearing a hat or a glove must be whole numbers, the number of people in the room must be a multiple of $LCM(4,5)=20$. Since we are trying to find the minimum x, we must use the smallest possible value for the number of people in the room. Similarly, we can assume that there are no people present who are wearing neither of the two items since this would unnecessarily increase the number of people in the room. Thus, we can say that there are 20 people in the room, all of which are wearing at least a hat or a glove.

It follows that there are $\frac{2}{5}\cdot 20=8$ people wearing gloves and $\frac{3}{4}\cdot 20=15$ people wearing hats. Then by applying the Principle of Inclusion Exclusion (PIE), the total number of people in the room wearing either a hat or a glove or both is $8+15-x=23-x$. Since we know that this equals 20, it follows that $23-x=20$, which implies that $x=3$. Thus, $\boxed{\textbf{(A)}\ 3}$ is the correct answer.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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2010 AMC 8 Problems/Problem 21

Problem

Hui is an avid reader. She bought a copy of the best seller Math is Beautiful. On the first day, Hui read $1/5$ of the pages plus 12 more, and on the second day she read $1/4$ of the remaining pages plus 15 pages. On the third day she read $1/3$ of the remaining pages plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 180 \qquad \textbf{(C)}\ 240 \qquad \textbf{(D)}\ 300 \qquad \textbf{(E)}\ 360$

Solution

Let x be the number of pages in the book. After the first day, Hui had $\frac{4x}{5}-12$ pages left to read. After the second, she had $(\frac{3}{4})(\frac{4x}{5}-12)-15 = \frac{3x}{5}-24$ left. After the third, she had $(\frac{2}{3})(\frac{3x}{5}-24)-18=\frac{2x}{5}-34$ left. This is equivalent to $62.$

$$\begin{aligned} \frac{2x}{5}-34 &= 62 \\ 2x-170 &= 310 \\ 2x &= 480 \\ x &= \boxed{\textbf{(C)}\ 240} \end{aligned}$$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
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2010 AMC 8 Problems/Problem 22

Contents

Problem

The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 6\qquad\textbf{(E)}\ 8$

Solution 1

Let the hundreds, tens, and units digits of the original three-digit number be a, b, and c, respectively. We are given that $a=c+2$. The original three-digit number is equal to $100a+10b+c=100(c+2)+10b+c=101c+10b+200$. The hundreds, tens, and units digits of the reversed three-digit number are c, b, and a, respectively. This number is equal to $100c+10b+a=100c+10b+(c+2)=101c+10b+2$. Subtracting this expression from the expression for the original number, we get $(101c+10b+200)-(101c+10b+2)=198$. Thus, the units digit in the final result is $\boxed{\textbf{(E)}\ 8}$

Solution 2

The result must hold for any three-digit number with hundreds digit being 2 more than the units digit. 301 is such a number. Evaluating, we get $301-103=198$. Thus, the units digit in the final result is $\boxed{\textbf{(E)}\ 8}$

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2010 AMC 8 Problems/Problem 24

Problem

What is the correct ordering of the three numbers, 10^8, 5^{12}, and 2^{24}?

$\textbf{(A)}\ 2^{24}<10^8<5^{12}$
$\textbf{(B)}\ 2^{24}<5^{12}<10^8$
$\textbf{(C)}\ 5^{12}<2^{24}<10^8$
$\textbf{(D)}\ 10^8<5^{12}<2^{24}$
$\textbf{(E)}\ 10^8<2^{24}<5^{12}$

Solution

Since all of the exponents are multiples of four, we can simplify the problem by taking the fourth root of each number. Evaluating we get $10^2=100$, $5^3=125$, and $2^6=64$. Since $64<100<125$, it follows that $\boxed{\textbf{(A)}\ 2^{24}<10^8<5^{12}}$ is the correct answer.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
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2010 AMC 8 Problems/Problem 25

Contents

Problem

Everyday at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

$\textbf{(A)}\ 13 \qquad\textbf{(B)}\ 18\qquad\textbf{(C)}\ 20\qquad\textbf{(D)}\ 22\qquad\textbf{(E)}\ 24$

Solution

We will systematically consider all of the possibilities. A valid climb can be thought of as a sequence of some or all of the numbers 1, 2, and 3, in which the sum of the sequence adds to 6. Since there is only one way to create a sequence which contains all $1s$, all $2s$, or all $3s$, there are three possible sequences which only contain one number. If we attempt to create sequences which contain one 2 and the rest $1s$, the sequence will contain one 2 and four $1s$. We can place the 2 in either the first, second, third, fourth, or fifth position, giving a total of five possibilities. If we attempt to create sequences which contain one 3 and the rest $1s$, the sequence will contain one 3 and three $1s$. We can place the 3 in either the first, second, third, or fourth position, giving a total of four possibilities. For sequences which contain exactly two $2s$ and the rest $1s$, the sequence will contain two $2s$ and two $1s$. The two $2s$ could be next to each other, separated by one 1 in between, or separated by two $1s$ in between. We can place the two $2s$ next to each other in three ways, separated by one 1 in two ways, and separated by two $1s$ in only one way. This gives us a total of six ways to create a sequence which contains two $2s$ and two $1s$. Note that we cannot have a sequence of only $2s$ and $3s$ since the sum will either be 5 or greater than 6. We now only need to consider the case where we use all three numbers in the sequence. Since all three numbers add to 6, the number of permutations of the three numbers is $3!=6$. Adding up the number of sequences above, we get: $3+5+4+6+6=24$. Thus, answer choice $\boxed{\textbf{(E)}\ 24}$ is correct.

Solution 2

A recursive approach is quick and easy. The number of ways to climb one stair is 1. There are 2 ways to climb two stairs: $1,1$ or 2. For 3 stairs, there are 4 ways: $(1,1,1)$ $(1,2)$ $(2,1)$ (3)

For four stairs, consider what step they came from to land on the fourth stair. They could have hopped straight from the 1st, done a double from #2, or used a single step from #3. The ways to get to each of these steps are $1+2+4=7$ ways to get to step 4. The pattern can then be extended: 4 steps: $1+2+4=7$ ways. 5 steps: $2+4+7=13$ ways. 6 steps: $4+7+13=24$ ways.

Thus, there are $\boxed{\textbf{(E)}\ 24}$ ways to get to step 6.

See Also

2010 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2010))	
Preceded by Problem 24	Followed by Last Problem
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