2015 AMC12B

Problem 1

What is the value of $2-(-2)^{-2}$?

$2-(-2)^{-2}$的值是多少？

(A) -2 **(B)** $\frac{1}{16}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{9}{4}$ **(E)** 6

Problem 2

Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

Marie 连续做了 3 项耗时相同的任务，中间没有休息，她从下午 1:00 开始做第一个任务，在下午 2:40 完成第二个任务，她何时完成第三个任务？

(A) 3:10 PM **(B)** 3:30 PM **(C)** 4:00 PM **(D)** 4:10 PM **(E)** 4:30 PM

Problem 3

Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

Isaac 把一个整数写了 2 遍，另一个整数写了 3 遍。这 5 个数之和为 100，其中一个数是 28，另一个数是多少？

(A) 8 **(B)** 11 **(C)** 14 **(D)** 15 **(E)** 18

Problem 4

David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

David，Hikmet，Jack，Marta，Rand，Todd 和其他 6 人参加了一场 12 人的比赛。Rand 排在 Hikamet 前面 6 位，Marta 排在 Jaok 后面 1 位，David 排在 Hikmet 的后面 2 位，Jack 排在 Todd 后面 2 位，Todd 排在 Rand 后面 1 位。Marta 排名第 6，那么谁排名第 8?

(A) David **(B)** Hikmet **(C)** Jack **(D)** Rand **(E)** Todd

Problem 5

The Tigers beat the Sharks 2 out of the 3 times they played. They then played N more times, and the Sharks ended up winning at least 95% of all the games played. What is the minimum possible value for N?

老虎队在和鲨鱼队的 3 场比赛中赢了 2 场，然后他们又多打了 N 场比赛，最终鲨鱼队赢了比赛总场数的至少 95%，那么 N 的最小可能值是多少?

(A) 35 **(B)** 37 **(C)** 39 **(D)** 41 **(E)** 43

Problem 6

Back in 1930, Tillie had to memorize her multiplication facts from 0×0 to 12×12. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

在 1930 年，Tillie 需要记忆从0×0到12×12的所有乘法结果。她所用的乘法表的每行每列都用乘数标记，所得的乘积形成了表的主体，那么在这张表的主体中，有多少比例的数是奇数？结果保留两位小数。

(A) 0.21 **(B)** 0.25 **(C)** 0.46 **(D)** 0.50 **(E)** 0.75

Problem 7

A regular 15-gon has L lines of symmetry, and the smallest positive angle for which it has rotational symmetry is R degrees. What is $L+R$?

一个正十五边形有 L 条对称轴，且具有旋转对称的最小角度是 R 度，问 $L+R$ 是多少？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 54$

Problem 8

What is the value of $(625^{\log_5 2015})^{\frac{1}{4}}$?

$(625^{\log_5 2015})^{\frac{1}{4}}$的值是多少？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ \sqrt[4]{2015} \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 2015 \qquad \textbf{(E)}\ \sqrt[4]{5^{2015}}$

Problem 9

Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\tfrac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

Larry 和 Julius 轮流玩一种用球扔倒瓶子的游戏。Larry 先扔。赢家是第一个将瓶子扔倒的人。在每一回，每个选手扔倒瓶子的概率都是$\tfrac{1}{2}$，且相互独立。问 Larry 赢得游戏的概率是多少？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{3}{5} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{3}{4} \qquad \textbf{(E)}\ \frac{4}{5}$

Problem 10

How many noncongruent integer-sided triangles with positive area and perimeter less than 15 are neither equilateral, isosceles, nor right triangles?

有多少个不全等且边长为整数的三角形，满足面积为正，周长小于 15，并且既不是等边三角形，也不是等腰三角形或者直角三角形？

(A) 3　**(B)** 4　**(C)** 5　**(D)** 6　**(E)** 7

Problem 11

The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

直线$12x + 5y = 60$与坐标轴形成一个三角形，这个三角形的三条高的长度之和是多少？

(A) 20　**(B)** $\frac{360}{17}$　**(C)** $\frac{107}{5}$　**(D)** $\frac{43}{2}$　**(E)** $\frac{281}{13}$

Problem 12

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

a，b，c 是 3 个不同的 1 位数，方程$(x-a)(x-b)+(x-b)(x-c)=0$的根之和的最大值是多少？

(A) 15　**(B)** 15.5　**(C)** 16　**(D)**16.5　**(E)** 17

Problem 13

Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC = 70^\circ, \angle ADB = 40^\circ, AD = 4,$ and $BC = 6$. What is AC?

四边形 *ABCD* 内接在一个圆内，且满足$\angle BAC = 70^\circ, \angle ADB = 40^\circ, AD = 4,$$BC = 6$，问 AC 是多长?

(A) $3+\sqrt{5}$　**(B)** 6　**(C)** $\frac{9}{2}\sqrt{2}$　**(D)** $8-\sqrt{2}$　**(E)** 7

Problem 14

A circle of radius 2 is centered at A. An equilateral triangle with side 4 has a vertex at A. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

一个半径为 2 的圆，圆心在 *A* 点。一个边长为 4 的等边三角形的一个顶点是 *A* 点。同位于圆内但在三角形之外的区域面积和位于圆外但在三角形之内的区域面积的差是多少？

$\textbf{(A)}\ 8-\pi \qquad \textbf{(B)}\ \pi+2 \qquad \textbf{(C)}\ 2\pi-\frac{\sqrt{2}}{2} \qquad \textbf{(D)}\ 4(\pi-\sqrt{3}) \qquad \textbf{(E)}\ 2\pi-\frac{\sqrt{3}}{2}$

Problem 15

At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\frac{1}{6}$ chance of getting an A in English, and a $\frac{1}{4}$ chance of getting a B. In History, she has a $\frac{1}{4}$ chance of getting an A, and a $\frac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5?

在 Rachelle 的学校，一个 A 是 4 分，一个 B 是 3 分，一个 C 是 2 分，一个 D 是 1 分。她的四门课的 GPA 分数是通过将四门课的总分除以 4 算得的。她很确定，她会在数学和科学这两门课中得 A，在英语和历史这两门课的每一门都至少是 1 个 C，她认为英语获得 A 的概率是 $\frac{1}{6}$，获得 B 的概率是 $\frac{1}{4}$；而对于历史这门课，她获得 A 的概率是 $\frac{1}{4}$，获得 B 的概率是 $\frac{1}{3}$，而与她的英语成绩无关。问 Rachelle 的 GPA 至少是 3.5 分的概率是多少？

$\textbf{(A)}\ \frac{11}{72} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{3}{16} \qquad \textbf{(D)}\ \frac{11}{24} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 16

A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

以一个边长为 6 的正六边形的每条边为底，作出 6 个等腰三角形，且它们的腰长均为 8，再把这些等腰三角形折起来做成一个以六边形为底的棱锥。求此棱锥的体积是多少？

(A) 18 **(B)** 162 **(C)** $36\sqrt{21}$ **(D)** $18\sqrt{138}$ **(E)** $54\sqrt{21}$

Problem 17

An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n?

一枚非标准硬币正面朝上的概率为$\frac{1}{4}$。当被抛了 n 次，恰好 2 次正面朝上的概率和恰好 3 次正面朝上的概率相同。问 n 是多少？

(A) 5 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 13

Problem 18

For every composite positive integer n, define $r(n)$ to be the sum of the factors in the prime factorization of n. For example, $r(50) = 12$ because the prime factorization of 50 is 2×5^2, and $2 + 5 + 5 = 12$. What is the range of the function r, $\{r(n) : n \text{ is a composite positive integer}\}$?

对于每个正的合数，定义$r(n)$为 n 的质因数分解中所有因子之和。例如，$r(50) = 12$，因为 50 的质因数分解为2×5^2，并且$2 + 5 + 5 = 12$。那么函数 r 的值域是多少？｛$r(n)$：n 是一个正的合数｝.

（A）The set of positive integers | 所有正整数组成的集合

（B）The set of compositive positive integers | 所有正合数组成的築合

（C）The set of even positive integers | 所有正的偶数组成的集合

（D）The set of integers greater than 3 | 所有大于 3 的整数集

（E）The set of integers greater than 4 | 所有大于 4 的整数集

Problem 19

In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

在$\triangle ABC$中，$\angle C = 90^\circ$，$AB = 12$，在三角形的外部作出正方形 $ABXY$和 $ACWZ$，点 X，Y，Z和 W在同一个圆上，这个三角形的周长是多少？

(A) $12 + 9\sqrt{3}$ **(B)** $18 + 6\sqrt{3}$ **(C)** $12 + 12\sqrt{2}$ **(D)** 30 **(E)** 32

Problem 20

For every positive integer n, let $\text{mod}_5(n)$ be the remainder obtained when n is divided by 5. Define a function $f:\{0,1,2,3,\dots\}\times\{0,1,2,3,4\}\to\{0,1,2,3,4\}$ recursively as follows:

$$f(i,j)=\begin{cases}\text{mod}_5(j+1) & \text{if } i=0 \text{ and } 0\le j\le 4,\\ f(i-1,1) & \text{if } i\ge 1 \text{ and } j=0, \text{ and}\\ f(i-1,f(i,j-1)) & \text{if } i\ge 1 \text{ and } 1\le j\le 4.\end{cases}$$

What is $f(2015,2)$?

对于每个正整数 n， $\text{mod}_5(n)$表示 n 除以 5 所得余数，定义函数

$f:\{0,1,2,3,\dots\}\times\{0,1,2,3,4\}\to\{0,1,2,3,4\}$如下：

$$f(i,j)=\begin{cases}\text{mod}_5(j+1) & \text{if } i=0 \text{ and } 0\le j\le 4,\\ f(i-1,1) & \text{if } i\ge 1 \text{ and } j=0, \text{ and}\\ f(i-1,f(i,j-1)) & \text{if } i\ge 1 \text{ and } 1\le j\le 4.\end{cases}$$

则$f(2015,2)$是多少？

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 21

Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

Cozy 这只猫和 Dash 这条狗在有一定数目台阶的楼梯上爬楼梯，然而，Cozy 和 Dash 都是跳着爬楼梯，而不是一次一个台阶地走。Cozy 每跳一次就爬 2 个台阶（如果必要的话，他会仅仅跳过最后一个台阶）。Dash 每跳一次就爬 5 个台阶（如果最后剩下不足 5 个台阶，他就直接一步跳过去）。假设最终到达楼梯的顶部，Dash 跳的次数比 Cozy 少了 19 次，s 表示这个楼梯所有可能的台阶数的和，那么 s 的各个位上的数字之和是多少？

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

Problem 22

Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

6 张椅子均匀等距地绕着圆桌摆放，每张椅子坐 1 人。每个人都站起来，并且坐到新的椅子上去，新的椅子不能是这个人原来坐的那张椅子，也不能是他原来坐的椅子的左邻或者右邻的椅子。这样，每个人都各自坐到了一张新的椅子上。问一共有多少种坐法？

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Problem 23

A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

一个长方形的盒子大小是$a \times b \times c$，其中 *a*，*b*，*c* 都是整数，且$1 \leq a \leq b \leq c$，盒子的体积和表面积在数值上是相等的，那么可能的有序对（*a*，*b*，*c*）有多少个？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 26$

Problem 24

Four circles, no two of which are congruent, have centers at A, B, C, and D, and points P and Q lie on all four circles. The radius of circle A is $\tfrac{5}{8}$ times the radius of circle B, and the radius of circle C is $\tfrac{5}{8}$ times the radius of circle D. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let R be the midpoint of $\overline{PQ}$. What is $AR+BR+CR+DR$?

四个圆心分别为 *A*，*B*，*C*，*D* 的圆两两均不全等，点 *P* 和点 *Q* 都在这四个圆上。圆 *A* 的半径是圆 *B* 半径的$\frac{5}{8}$，圆 C 的半径是圆 D 半径的$\frac{5}{8}$，且 *AB*=*CD*=39， *PQ*=48，点 *R* 是$\overline{PQ}$的中点，那么 *AR*+*BR*+*CR* +*DR* 是多少？

$\textbf{(A)}\ 180 \qquad \textbf{(B)}\ 184 \qquad \textbf{(C)}\ 188 \qquad \textbf{(D)}\ 192 \qquad \textbf{(E)}\ 196$

Problem 25

A bee starts flying from point P_0. She flies 1 inch due east to point P_1. For $j \geq 1$, once the bee reaches point P_j, she turns 30° counterclockwise and then flies $j+1$ inches straight to point P_{j+1}. When the bee reaches P_{2015} she is exactly $a\sqrt{b} + c\sqrt{d}$ inches away from P_0, where a, b, c and d are positive integers and b and d are not divisible by the square of any prime. What is $a+b+c+d$?

一只蜜蜂从点P_0开始飞行。她先向东飞行 1 英寸到达点P_1，对于$j \geq 1$，一旦蜜蜂到达点P_j，她就逆时针转 30°，然后直线飞行$j+1$英寸到达点P_{j+1}，当蜜蜂到达点P_{2015}，她距离点P_0恰好$a\sqrt{b} + c\sqrt{d}$英寸，其中 *a*，*b*，*c* 和 *d* 都是正整数，且 *b* 和 *d* 都不能被任何质数的平方整除。问$a+b+c+d$等于多少？

(A) 2016 **(B)** 2024 **(C)** 2032 **(D)** 2040 **(E)** 2048

2015 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	A	B	B	A	D	D	C	C	E	D	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	D	C	D	D	C	B	D	D	B	D	B	