2015 AMC 8 Problems/Problem 1

How many square yards of carpet are required to cover a rectangular floor that is 12 feet long and 9 feet wide? (There are 3 feet in a yard.)

$\textbf{(A) }12\qquad\textbf{(B) }36\qquad\textbf{(C) }108\qquad\textbf{(D) }324\qquad \textbf{(E) }972$

Solution

First, we multiply $12\cdot9$ to get that you need 108 square feet of carpet you need to cover. Since there are 9 square feet in a square yard, you divide 108 by 9 to get 12 square yards, so our answer is $\boxed{\textbf{(A) }12}$.

Solution 2

Since there are 3 feet in a yard, we divide 9 by 3 to get 3, and 12 by 3 to get 4. To find the area of the carpet, we then multiply these two values together to get $\boxed{\textbf{(A) }12}$.

See Also

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Placement:Easy Geometry

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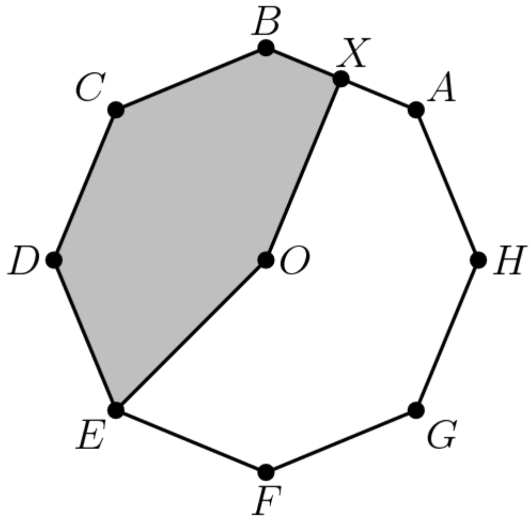
2015 AMC 8 Problems/Problem 2

Point O is the center of the regular octagon $ABCDEFGH$, and X is the midpoint of the side $\overline{AB}$. What fraction of the area of the octagon is shaded?

$\textbf{(A) }\frac{11}{32} \quad\textbf{(B) }\frac{3}{8} \quad\textbf{(C) }\frac{13}{32} \quad\textbf{(D) }\frac{7}{16}\quad \textbf{(E) }\frac{15}{32}$

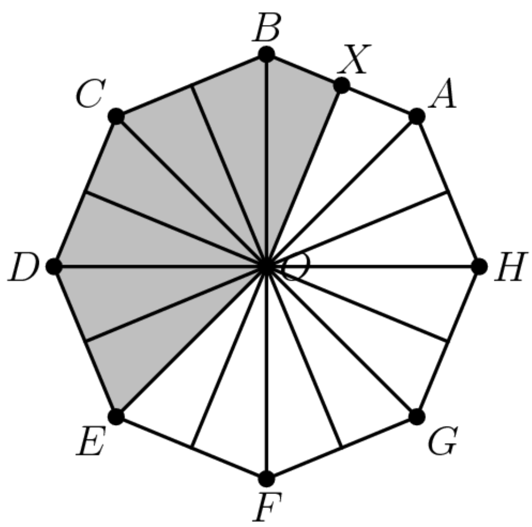


Contents

Solution 1

Since octagon $ABCDEFGH$ is a regular octagon, it is split into 8 equal parts, such as triangles $\triangle ABO, \triangle BCO, \triangle CDO$, etc. These parts, since they are all equal, are $\frac{1}{8}$ of the octagon each. The shaded region consists of 3 of these equal parts plus half of another, so the fraction of the octagon that is shaded is $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}=\boxed{\textbf{(D) }\frac{7}{16}}$.

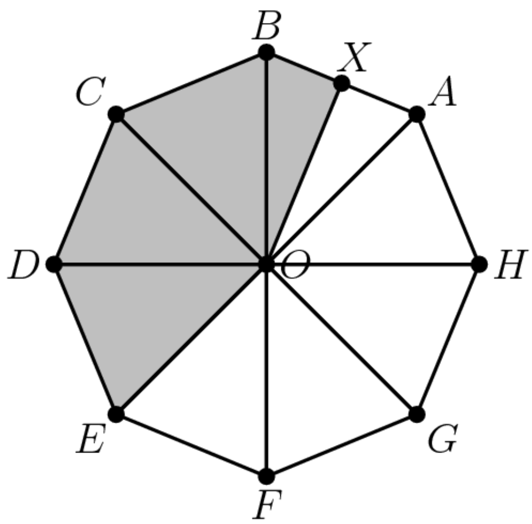
Solution 2



The octagon has been divided up into 16 identical triangles (and thus they each have equal area). Since the shaded region occupies 7 out of the 16 total triangles, the answer is $\boxed{\textbf{(D)}\ \frac{7}{16}}$.

Solution 3

For starters what I find helpful is to divide the whole octagon up into triangles as shown here:



Now it is just a matter of counting the larger triangles remember that $\triangle BOX$ and $\triangle XOA$ are not full triangles and are only half for these purposes. We count it up and we get a total of $\frac{3.5}{8}$ of the shape shaded. We then simplify it to get our answer of $\boxed{\textbf{(D)}\ \frac{7}{16}}$.

See Also

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2015 AMC 8 Problems/Problem 3

Jack and Jill are going swimming at a pool that is one mile from their house. They leave home simultaneously. Jill rides her bicycle to the pool at a constant speed of 10 miles per hour. Jack walks to the pool at a constant speed of 4 miles per hour. How many minutes before Jack does Jill arrive?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Solution

Using $d=rt$, we can set up an equation for when Jill arrives at the swimming pool:

$$1=10t$$

Solving for t, we get that Jill gets to the pool in $\frac{1}{10}$ of an hour, which is 6 minutes. Doing the same for Jack, we get that

Jack arrives at the pool in $\frac{1}{4}$ of an hour, which in turn is 15 minutes. Thus, Jill has to wait

$$15-6=\boxed{\textbf{(D)}\ 9}$$

minutes for Jack to arrive at the pool.

See Also

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2015 AMC 8 Problems/Problem 4

The Centerville Middle School chess team consists of two boys and three girls. A photographer wants to take a picture of the team to appear in the local newspaper. She decides to have them sit in a row with a boy at each end and the three girls in the middle. How many such arrangements are possible?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 12$

Solution

There are 2 ways to order the boys on the end, and there are $3! = 6$ ways to order the girls in the middle. We get the answer to be $2 \cdot 6 = \boxed{\textbf{(E)}\ 12}$.

See Also

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2015 AMC 8 Problems/Problem 5

Billy's basketball team scored the following points over the course of the first 11 games of the season:

$$42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73$$

If his team scores 40 in the 12th game, which of the following statistics will show an increase?

$\textbf{(A) }\text{range}\qquad\textbf{(B) }\text{median}\qquad\textbf{(C) }\text{mean}\qquad\textbf{(D) }\text{mode}\qquad\textbf{(E) }\text{mid-range}$

Solution

When they score a 40 on the next game, the range increases from $73-42=31$ to $73-40=33$. This means the $\boxed{\textbf{(A)}\ \text{range}}$ increased.

Solution 2

Because 40 is less than the score of every game they've played so far, the measures of center will never rise. Only measures of spread, such as the $\boxed{\textbf{(A)}\ \text{range}}$, may increase.

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2015 AMC 8 Problems/Problem 6

In $\triangle ABC$, $AB = BC = 29$, and $AC = 42$. What is the area of $\triangle ABC$?

$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 420 \qquad \textbf{(C)}\ 500 \qquad \textbf{(D)}\ 609 \qquad \textbf{(E)}\ 701$

Solution 1

We know the semi-perimeter of $\triangle ABC$ is $\dfrac{29+29+42}{2} = 50$. Next, we use Heron's Formula to find that the area of the triangle is just $\sqrt{50(50-29)^2(50-42)} = \sqrt{50 \cdot 21^2 \cdot 8} = \boxed{\textbf{(B)}\ 420}$.

Solution 2

Splitting the isosceles triangle in half, we get a right triangle with hypotenuse 29 and leg 21. Using the Pythagorean Theorem , we know the height is $\sqrt{29^2 - 21^2} = 20$. Now that we know the height, the area is $\dfrac{(20)(42)}{2} = \boxed{\textbf{(B)}\ 420}$.

See Also

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2015 AMC 8 Problems/Problem 7

Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?

$\textbf{(A) }\frac{1}{9}\qquad\textbf{(B) }\frac{2}{9}\qquad\textbf{(C) }\frac{4}{9}\qquad\textbf{(D) }\frac{1}{2}\qquad \textbf{(E) }\frac{5}{9}$

Solution

We can instead find the probability that their product is odd, and subtract this from 1. In order to get an odd product, we have to draw an odd number from each box. We have a $\frac{2}{3}$ probability of drawing an odd number from one box, so there is a $\left(\frac{2}{3}\right)^2=\frac{4}{9}$ probability of having an odd product. Thus, there is a $1-\frac{4}{9}=\boxed{\textbf{(E) }\frac{5}{9}}$ probability of having an even product.

Solution 2

You can also make this problem into a spinner problem. You have the first spinner with 3 equally divided sections, $1, 2$ and 3. You make a second spinner that is identical to the first, with 3 equal sections of 1, 2, and 3. If the first spinner lands on 1, to be even, it must land on two. You write down the first combination of numbers $(1,2)$. Next, if the spinner lands on 2, it can land on any number on the second spinner. We now have the combinations of $(1,2), (2,1), (2,2), (2,3)$. Finally, if the first spinner ends on 3, we

have $(3,2)$. Since there are $3*3=9$ possible combinations, and we have 5 evens, the final answer is

$\boxed{\textbf{(E) }\frac{5}{9}}$.

See Also

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2015 AMC 8 Problems/Problem 8

What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?

$\textbf{(A) }24\qquad\textbf{(B) }29\qquad\textbf{(C) }43\qquad\textbf{(D) }48\qquad \textbf{(E) }57$

Solution

We know from the triangle inequality that the last side, s, fulfills $s<5+19=24$. Adding $5+19$ to both sides of the inequality, we get $s+5+19<48$, and because $s+5+19$ is the perimeter of our triangle, $\boxed{\textbf{(D) }48}$ is our answer.

See Also

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2015 AMC 8 Problems/Problem 9

On her first day of work, Janabel sold one widget. On day two, she sold three widgets. On day three, she sold five widgets, and on each succeeding day, she sold two more widgets than she had sold on the previous day. How many widgets in total had Janabel sold after working 20 days?

$\textbf{(A)}\ 39 \qquad\textbf{(B)}\ 40 \qquad\textbf{(C)}\ 210 \qquad\textbf{(D)}\ 400 \qquad\textbf{(E)}\ 401$

Solution 1

The sum of $1, 3, 5, ...39$ is $\frac{(1+39)(20)}{2} = \boxed{\textbf{(D)}\ 400}$

Solution 2

The sum is just the sum of the first 20 odd integers, which is $20^2 = \boxed{\textbf{(D)}\ 400}$

See Also

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2015 AMC 8 Problems/Problem 10

How many integers between 1000 and 9999 have four distinct digits?

$\textbf{(A) }3024\qquad\textbf{(B) }4536\qquad\textbf{(C) }5040\qquad\textbf{(D) }6480\qquad \textbf{(E) }6561$

Solution 1

The question can be rephrased to "How many four-digit positive integers have four distinct digits?", since numbers between 1000 and 9999 are four-digit integers. There are 9 choices for the first number, since it cannot be 0, there are only 9 choices left for the second number since it must differ from the first, 8 choices for the third number, since it must differ from the first two, and 7 choices for the fourth number, since it must differ from all three. This means there are $9 \times 9 \times 8 \times 7 = \boxed{\textbf{(B) }4536}$ integers between 1000 and 9999 with four distinct digits.

See Also

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2015 AMC 8 Problems/Problem 11

In the small country of Mathland, all automobile license plates have four symbols. The first must be a vowel (A, E, I, O, or U), the second and third must be two different letters among the 21 non-vowels, and the fourth must be a digit (0 through 9). If the symbols are chosen at random subject to these conditions, what is the probability that the plate will read "AMC8"?

$\textbf{(A) } \frac{1}{22,050} \qquad \textbf{(B) } \frac{1}{21,000}\qquad \textbf{(C) } \frac{1}{10,500}\qquad \textbf{(D) } \frac{1}{2,100} \qquad \textbf{(E) } \frac{1}{1,050}$

Solution 1

There is one favorable case, which is the license plate says "AMC8". We must now find how many total cases there are. There are 5 choices for the first letter (since it must be a vowel), 21 choices for the second letter (since it must be of 21 consonants), 20 choices for the third letter (since it must differ from the second letter), and 10 choices for the number. This leads to $5 \cdot 21 \cdot 20 \cdot 10 = 21000$ total possible license plates. That means the probability of a license plate saying "AMC8" is $\boxed{\textbf{(B) } \frac{1}{21,000}}$.

Solution 2

The probability of choosing A as the first letter is $\frac{1}{5}$. The probability of choosing M next is $\frac{1}{21}$. The probability of choosing C as the third letter is $\frac{1}{20}$ (since there are 20 other consonants to choose from other then M). The probability of having 8 as the last number is $\frac{1}{10}$. We multiply all these to obtain

$$\frac{1}{5}\cdot\frac{1}{21}\cdot\frac{1}{20}\cdot\frac{1}{10} = \frac{1}{5\times 21\times 20\times 10} = \frac{1}{21\times 100\times 10} = \boxed{\textbf{(B) } \frac{1}{21,000}}$$

See Also

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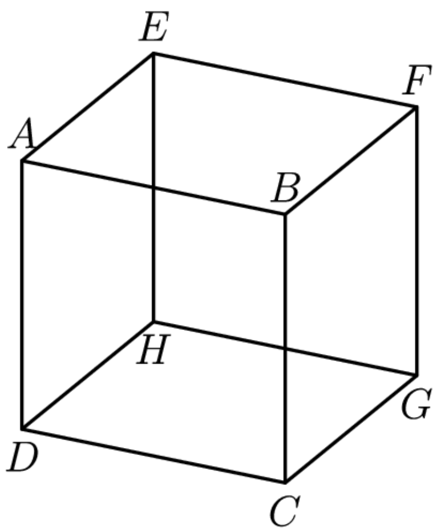
2015 AMC 8 Problems/Problem 12

How many pairs of parallel edges, such as $\overline{AB}$ and $\overline{GH}$ or $\overline{EH}$ and $\overline{FG}$, does a cube have?

$\textbf{(A) }6 \quad\textbf{(B) }12 \quad\textbf{(C) }18 \quad\textbf{(D) }24 \quad\textbf{(E) }36$



Solution 1

We first count the number of pairs of parallel lines that are in the same direction as $\overline{AB}$. The pairs of parallel lines are $\overline{AB}$ and $\overline{EF}$, $\overline{CD}$ and $\overline{GH}$, $\overline{AB}$ and $\overline{CD}$, $\overline{EF}$ and $\overline{GH}$, $\overline{AB}$ and $\overline{GH}$, and $\overline{CD}$ and $\overline{EF}$. These are 6 pairs total. We can do the same for the lines in the same direction as $\overline{AE}$ and $\overline{AD}$. This means there are $6\cdot 3=\boxed{\textbf{(C) } 18}$ total pairs of parallel lines.

Solution 2

Pick a random edge. Given another edge, the probability that it is parallel to this edge is $\frac{3}{12-1}=\frac{3}{11}$. Keep in mind we already used one edge. There are 12 edges so $\binom{12}{2}=66$ pairs. So our answer is $\frac{3}{11}\times 66=\boxed{\textbf{(C) } 18}$.

See Also

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2015 AMC 8 Problems/Problem 13

How many subsets of two elements can be removed from the set $\{1,2,3,4,5,6,7,8,9,10,11\}$ so that the mean (average) of the remaining numbers is 6?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

Since there will be 9 elements after removal, and their mean is 6, we know their sum is 54. We also know that the sum of the set pre-removal is 66. Thus, the sum of the 2 elements removed is $66-54=12$. There are only $\boxed{\textbf{(D)}\ 5}$ subsets of 2 elements that sum to 12:

$\{1,11\},\{2,10\},\{3,9\},\{4,8\},\{5,7\}$.

See Also

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2015 AMC 8 Problems/Problem 14

Which of the following integers cannot be written as the sum of four consecutive odd integers?

$\textbf{(A)}\ 16 \quad \textbf{(B)}\ 40 \quad \textbf{(C)}\ 72 \quad \textbf{(D)}\ 100 \quad \textbf{(E)}\ 200$

Contents

Solution 1

Let our 4 numbers be $n, n+2, n+4, n+6$, where n is odd. Then our sum is $4n+12$. The only answer choice that cannot be written as $4n+12$, where n is odd, is $\boxed{\textbf{(D)}\ 100}$.

Solution 2

If the four consecutive odd integers are $2n-3,\ 2n-1,\ 2n+1$ and $2n+3$ then the sum is $8n$. All the integers are divisible by 8 except $\boxed{\textbf{(D)}\ 100}$.

Solution 3

If the four consecutive odd integers are $a,\ a+2,\ a+4$ and $a+6$, the sum is $4a+12$, and $4a+12$ divided by 4 gives $a+3$. This means that $a+3$ must be even. The only integer that does not give an even integer when divided by 4 is 100, so the answer is $\boxed{\textbf{(D)}\ 100}$.

Solution 4

From Solution 1, we have the sum of the 4 numbers to be equal to $4n+12$. Taking mod 8 gives us $4n+4 \equiv b \pmod{8}$ for some residue b and for some odd integer n. Since $n \equiv 1 \pmod{2}$, we can express it as the equation $n = 2a+1$ for some integer a. Multiplying 4 to each side of the equation yields $4n = 8a+4$, and taking mod 8 gets us $4n \equiv 4 \pmod{8}$, so $b = 4$. All the answer choices except choice D is a multiple of 8, and since 100 satisfies all the conditions the answer is $\boxed{\textbf{(D)}\ 100}$.

See Also

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2015 AMC 8 Problems/Problem 15

At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

$\textbf{(A) }49\qquad\textbf{(B) }70\qquad\textbf{(C) }79\qquad\textbf{(D) }99\qquad\textbf{(E) }149$

Solution 1

We can see that this is a Venn Diagram Problem.

First, we analyze the information given. There are 198 students. Let's use A as the first issue and B as the second issue.

149 students were for the A, and 119 students were for B. There were also 29 students against both A and B.

Solving this without a Venn Diagram, we subtract 29 away from the total, 198. Out of the remaining 169 , we have 149 people for A and

119 people for B. We add this up to get 268 . Since that is more than what we need, we subtract 169 from 268 to get

$\boxed{\textbf{(D) }99}$

Solution 2

There are 198 people. We know that 29 people voted against both the first issue and the second issue. That leaves us with 169 people that voted for at least one of them. If 119 people voted for both of them, then that would leave 20 people out of the vote, because 149 is less than 198 people. 198-149 is 20, so to make it even, we have to take 20 away from the 119 people, which leaves us with $\boxed{\textbf{(D) }99}$

See Also

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2015 AMC 8 Problems/Problem 16

In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

$\textbf{(A) } \frac{2}{15} \qquad \textbf{(B) } \frac{4}{11} \qquad \textbf{(C) } \frac{11}{30} \qquad \textbf{(D) } \frac{3}{8} \qquad \textbf{(E) } \frac{11}{15}$

Solution 1

Let the number of sixth graders be s, and the number of ninth graders be n. Thus, $\frac{n}{3}=\frac{2s}{5}$, which simplifies to $n=\frac{6s}{5}$. Since we are trying to find the value of $\frac{\frac{n}{3}+\frac{2s}{5}}{n+s}$, we can just substitute n for $\frac{6s}{5}$ into the equation. We then get a value of $\frac{\frac{\frac{6s}{5}}{3}+\frac{2s}{5}}{\frac{6s}{5}+s}=\frac{\frac{6s+6s}{15}}{\frac{11s}{5}}=\frac{\frac{4s}{5}}{\frac{11s}{5}}=\boxed{\textbf{(B) }\frac{4}{11}}$

Solution 2

We see that the minimum number of ninth graders is 6, because if there are 3 then there is 1 ninth grader with a buddy, which would mean 2.5 sixth graders with a buddy, and that's impossible. With 6 ninth graders, 2 of them are in the buddy program, so there $\frac{2}{\frac{2}{5}}=5$ sixth graders total, two of whom have a buddy. Thus, the desired fraction is $\frac{2+2}{5+6}=\boxed{\textbf{(B) }\frac{4}{11}}$.

See Also

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2015 AMC 8 Problems/Problem 17

Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?

$\textbf{(A) }4 \qquad \textbf{(B) }6 \qquad \textbf{(C) }8 \qquad \textbf{(D) }9 \qquad \textbf{(E) }12$

Contents

Solution 1

So $\frac{d}{v}=\frac{1}{3}$ and $\frac{d}{v+18}=\frac{1}{5}$.

This gives $d=\frac{1}{5}v+3.6=\frac{1}{3}v$, which gives $v=27$, which then gives $d=\boxed{\textbf{(D)}~9}$

Solution 2

$d=rt$, d is obviously constant

$\frac{1}{3} \times r = \frac{1}{5} \times (r+18)$

$\frac{r}{3} = \frac{r}{5} + \frac{18}{5}$

$\frac{2r}{15} = \frac{18}{5}$

$10r=270$ so $r=27$, plug into the first one and it's $\boxed{\textbf{(D)}~9}$ miles to school

Solution 3

We set up an equation in terms of d the distance and x the speed In miles per hour. We have
$d = \left(\frac{1}{3}\right)(x) = \left(\frac{1}{5}\right)(x+18)$

$$d=(5)(x)=(3)(x+18)$$

$$5x=3x+54$$

$$2x=54$$

$$x=27$$

So $d=\frac{27}{3}=\boxed{\textbf{(D)}~9}$

2015 AMC 8 Problems/Problem 18

An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, $2, 5, 8, 11, 14$ is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 42$

1				25
		X		
17				81

Contents

Solution 1

We begin filling in the table. The top row has a first term 1 and a fifth term 25, so we have the common difference is $\frac{25-1}{4} = 6$. This means we can fill in the first row of the table:

1	7	13	19	25
		X		
17				81

The fifth row has a first term of 17 and a fifth term of 81, so the common difference is $\frac{81-17}{4} = 16$. We can fill in the fifth row of the table as shown:

1	7	13	19	25
		X		
17	33	49	65	81

We must find the third term of the arithmetic sequence with a first term of 13 and a fifth term of 49. The common difference of this sequence is $\dfrac{49-13}{4}=9$, so the third term is $13+2\cdot 9=\boxed{\textbf{(B)}\ 31}$.

Solution 2

The middle term of the first row is $\dfrac{25+1}{2}=13$, since the middle number is just the average in an arithmetic sequence. Similarly, the middle of the bottom row is $\dfrac{17+81}{2}=49$. Applying this again for the middle column, the answer is $\dfrac{49+13}{2}=\boxed{\textbf{(B)}\ 31}$.

Solution 3

The value of X is simply the average of the average values of both diagonals that contain X. This is

$$\frac{\frac{1+81}{2}+\frac{17+25}{2}}{2}=\frac{\frac{82}{2}+\frac{42}{2}}{2}=\frac{41+21}{2}=\boxed{\textbf{(B)}\ 31}$$

See Also

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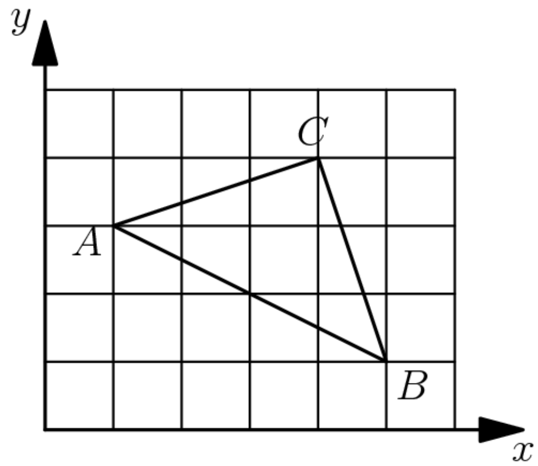
2015 AMC 8 Problems/Problem 19

A triangle with vertices as $A=(1,3)$, $B=(5,1)$, and $C=(4,4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?

$\textbf{(A) }\frac{1}{6} \qquad \textbf{(B) }\frac{1}{5} \qquad \textbf{(C) }\frac{1}{4} \qquad \textbf{(D) }\frac{1}{3} \qquad \textbf{(E) }\frac{1}{2}$



Contents

Solution 1

The area of $\triangle ABC$ is equal to half the product of its base and height. By the Pythagorean Theorem, we find its height is $\sqrt{1^2+2^2}=\sqrt{5}$, and its base is $\sqrt{2^2+4^2}=\sqrt{20}$. We multiply these and divide by 2 to find the of the triangle is $\frac{\sqrt{5\cdot20}}{2}=\frac{\sqrt{100}}{2}=\frac{10}{2}=5$. Since the grid has an area of 30, the fraction of the grid covered by the triangle is $\frac{5}{30}=\boxed{\textbf{(A) }\frac{1}{6}}$.

Solution 2

Note angle $\angle ACB$ is right, thus the area is $\sqrt{1^2+3^2}\times\sqrt{1^2+3^2}\times\frac{1}{2}=10\times\frac{1}{2}=5$ thus the fraction of the total is $\frac{5}{30}=\boxed{\textbf{(A) }\frac{1}{6}}$

Solution 3

By the Shoelace theorem, the area of
$$\triangle ABC = |\frac{1}{2}(15+4+4-1-20-12)| = |\frac{1}{2}(-10)| = 5.$$

This means the fraction of the total area is $\frac{5}{30} = \boxed{\textbf{(A)}\ \frac{1}{6}}$

Solution 4

The smallest rectangle that follows the grid lines and completely encloses $\triangle ABC$ has an area of 12, where $\triangle ABC$ splits the rectangle into four triangles. The area of $\triangle ABC$ is therefore $12 - (\frac{4\cdot 2}{2} + \frac{3\cdot 1}{2} + \frac{3\cdot 1}{2}) = 12 - (4 + \frac{3}{2} + \frac{3}{2}) = 12 - 7 = 5$. That means that $\triangle ABC$ takes up $\frac{5}{30} = \boxed{\textbf{(A)}\ \frac{1}{6}}$ of the grid.

Solution 5

Using Pick's Theorem, the area of the triangle is $4 + \frac{4}{2} - 1 = 5$. Therefore, the triangle takes up $\frac{5}{30} = \boxed{\textbf{(A)}\ \frac{1}{6}}$ of the grid.

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2015 AMC 8 Problems/Problem 20

Ralph went to the store and bought 12 pairs of socks for a total of $24. Some of the socks he bought cost $1 a pair, some of the socks he bought cost $3 a pair, and some of the socks he bought cost $4 a pair. If he bought at least one pair of each type, how many pairs of $1 socks did Ralph buy?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Solution 1

So let there be x pairs of $\$1$ socks, y pairs of $\$3$ socks, z pairs of $\$4$ socks.

We have $x + y + z = 12$, $x + 3y + 4z = 24$, and $x, y, z \geq 1$.

Now we subtract to find $2y + 3z = 12$, and $y, z \geq 1$. It follows that y is a multiple of 3 and $2y$ is a multiple of 6, so since $0 < 2y < 12$, we must have $2y = 6$.

Therefore, $y = 3$, and it follows that $z = 2$. Now $x = 12 - y - z = 12 - 3 - 2 = \boxed{\textbf{(D)}\ 7}$, as desired.

Solution 2

Since the total cost of the socks was $\$24$ and Ralph bought 12 pairs, the average cost of each pair of socks is $\dfrac{\$24}{12} = \$2$.

There are two ways to make packages of socks that average to $\$2$. You can have:

- Two $\$1$ pairs and one $\$4$ pair (package adds up to $\$6$)
- One $\$1$ pair and one $\$3$ pair (package adds up to $\$4$)

So now we need to solve

$$6a + 4b = 24,$$

where a is the number of $\$6$ packages and b is the number of $\$4$ packages. We see our only solution (that has at least one of each pair of sock) is $a = 2, b = 3$, which yields the answer of $2 \times 2 + 3 \times 1 = \boxed{\textbf{(D)}\ 7}$.

See Also

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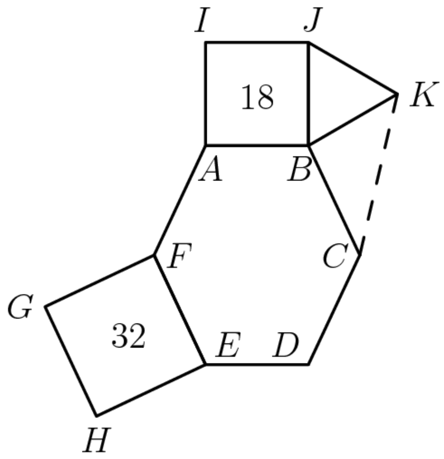
2015 AMC 8 Problems/Problem 21

In the given figure hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE=BC$. What is the area of $\triangle KBC$?

$\textbf{(A) }6\sqrt{2}\quad\textbf{(B) }9\quad\textbf{(C) }12\quad\textbf{(D) }9\sqrt{2}\quad\textbf{(E) }32$.



Solution

Clearly, since $\overline{FE}$ is a side of a square with area 32, $\overline{FE} = \sqrt{32} = 4 \sqrt{2}$. Now, since $\overline{FE} = \overline{BC}$, we have $\overline{BC} = 4 \sqrt{2}$.

Now, $\overline{JB}$ is a side of a square with area 18, so $\overline{JB} = \sqrt{18} = 3 \sqrt{2}$. Since ΔJBK is equilateral, $\overline{BK} = 3 \sqrt{2}$.

Lastly, ΔKBC is a right triangle. We see that $\angle JBA + \angle ABC + \angle CBK + \angle KBJ = 360^{\circ} \rightarrow 90^{\circ} + 120^{\circ} + \angle CBK + 60^{\circ} = 360^{\circ} \rightarrow \angle CBK = 90^{\circ}$, so ΔKBC is a right triangle with legs $3 \sqrt{2}$ and $4 \sqrt{2}$. Now, its area is $\dfrac{3 \sqrt{2} \cdot 4 \sqrt{2}}{2} = \dfrac{24}{2} = \boxed{\textbf{(C) }12}$.

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2015 AMC 8 Problems/Problem 22

On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

$\textbf{(A) } 21 \qquad \textbf{(B) } 30 \qquad \textbf{(C) } 60 \qquad \textbf{(D) } 90 \qquad \textbf{(E) } 1080$

Solution

As we read through this text, we find that the given information means that the number of students in the group has 12 factors, since each arrangement is a factor. The smallest integer with 12 factors is $2^2 \cdot 3 \cdot 5 = \boxed{\textbf{(C)}\ 60}$.

See Also

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2015 AMC 8 Problems/Problem 23

Tom has twelve slips of paper which he wants to put into five cups labeled A, B, C, D, E. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from A to E. The numbers on the papers are $2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3.5, 4,$ and 4.5. If a slip with 2 goes into cup E and a slip with 3 goes into cup B, then the slip with 3.5 must go into what cup?

$\textbf{(A) } A \qquad \textbf{(B) } B \qquad \textbf{(C) } C \qquad \textbf{(D) } D \qquad \textbf{(E) } E$

Solution

The numbers have a sum of $6+5+12+4+8=35$, which averages to 7, which means A, B, C, D, E will have values $5, 6, 7, 8, 9$, respectively. Now it's process of elimination: Cup A will have a sum of 5, so putting a 3.5 slip in the cup will leave $5-3.5=1.5$; However, all of our slips are bigger than 1.5, so this is impossible. Cup B has a sum of 6, but we are told that it already has a 3 slip, leaving $6-3=3$, which is too small for the 3.5 slip. Cup C is a little bit trickier, but still manageable. It must have a value of 7, so adding the 3.5 slip leaves room for $7-3.5=3.5$. This looks good at first, as we do have slips smaller than that, but upon closer inspection, we see that no slip fits exactly, and the smallest sum of two slips is $2+2=4$, which is too big, so this case is also impossible. Cup E has a sum of 9, but we are told it already has a 2 slip, so we are left with $9-2=7$, which is identical to the Cup C case, and thus also impossible. With all other choices removed, we are left with the answer: Cup $\boxed{\textbf{(D) } D}$

See Also

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2015 AMC 8 Problems/Problem 24

A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with $N > 2M$ and $M > 4$. Each team plays a 76 game schedule. How many games does a team play within its own division?

$\textbf{(A) } 36 \qquad \textbf{(B) } 48 \qquad \textbf{(C) } 54 \qquad \textbf{(D) } 60 \qquad \textbf{(E) } 72$

Solution 1

On one team they play $\binom{3}{2}N$ games in their division and $4(M)$ games in the other. This gives

$3N + 4M = 76$

Since $M > 4$ we start by trying $M = 5$. This doesn't work because 56 is not divisible by 3.

Next $M = 6$, does not work because 52 is not divisible by 3

We try $M = 7$ this does work giving $N = 16, \ M = 7$ and thus $3 \times 16 = \boxed{\textbf{(B) } 48}$ games in their division.

Solution 2

$76 = 3N + 4M > 10M$, giving $M \leq 7$. Since $M > 4$, we have $M = 5, 6, 7$ Since $4M$ is $1 \pmod 3$, we must have M equal to $1 \pmod 3$, so $M = 7$.

This gives $3N = 48$, as desired. The answer is $\boxed{\textbf{(B) } 48}$.

See Also

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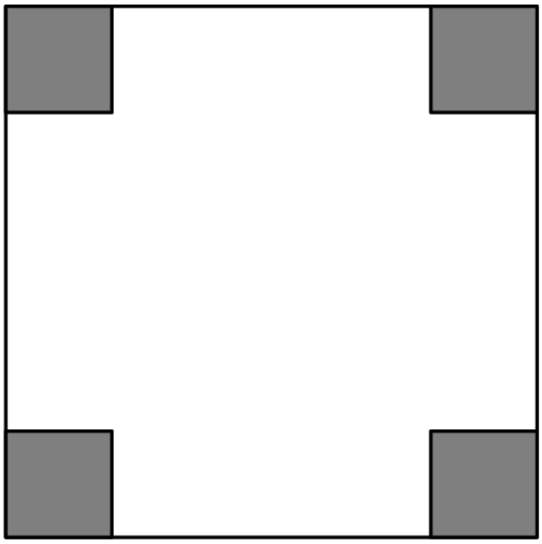
2015 AMC 8 Problems/Problem 25

One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?

$\textbf{(A) } 9\qquad \textbf{(B) } 12\frac{1}{2}\qquad \textbf{(C) } 15\qquad \textbf{(D) } 15\frac{1}{2}\qquad \textbf{(E) } 17$



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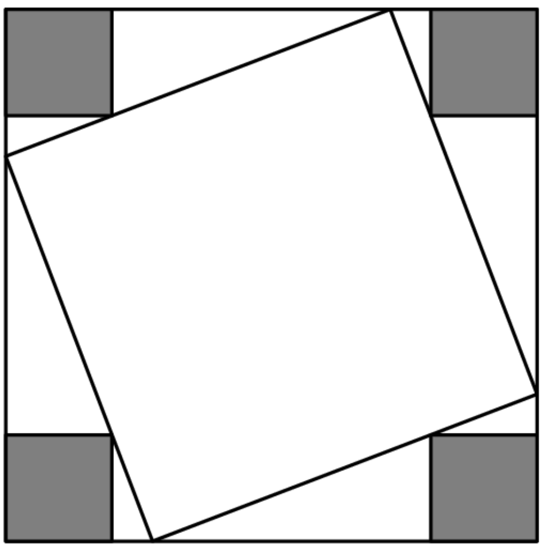
Solution 1

We can draw a diagram as shown.



Let us focus on the big triangles taking up the rest of the space. The triangles on top of the unit square between the inscribed square, are similiar to the 4 big triangles by $AA.$ Let the height of a big triangle be x then $\frac{x}{x-1} = \frac{5-x}{1}$.

$$x = -x^2 + 6x - 5$$

$$x^2 - 5x + 5 = 0$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - (4)(1)(5)}}{2}$$

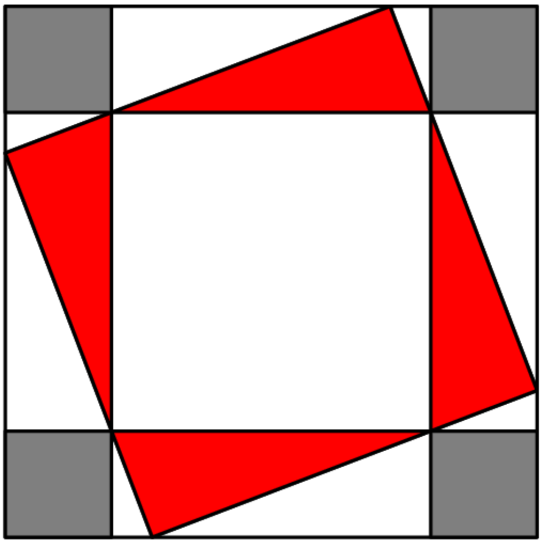
$$x = \frac{5 \pm \sqrt{5}}{2}$$

Thus $x = \frac{5-\sqrt{5}}{2}$, because by symmetry, $x < \frac{5}{2}$.

This means the area of each triangle is $\frac{5-\sqrt{5}}{2} * (5 - \frac{5-\sqrt{5}}{2}) * \frac{1}{2} = \frac{5}{2}$ This the area of the square is $25 - (4 * \frac{5}{2}) = \boxed{\textbf{(C) } 15}$

Solution 2

We draw a square as shown:



We wish to find the area of the square. The area of the larger square is composed of the smaller square and the four red triangles. The red triangles have base 3 and height 1, so the combined area of the four triangles is $4 \cdot \frac{3}{2} = 6$. The area of the smaller square is 9. We add these to see that the area of the large square is $9 + 6 = \boxed{\textbf{(C) } 15}$.

Solution 3

Let us find the area of the triangles and the unit squares: on each side, there are two triangles. They both have one leg of length 1, and let's label the other legs x for one of the triangles and y for the other. Note that $x + y = 3$. The area of each of the triangles is $\frac{x}{2}$ and $\frac{y}{2}$, and there are 4 of each. So now we need to find $4\left(\frac{x}{2}\right) + 4\left(\frac{y}{2}\right)$.

$(4)\frac{x}{2} + (4)\frac{y}{2} \Rightarrow \quad 4\left(\frac{x}{2} + \frac{y}{2}\right) \Rightarrow \quad 4\left(\frac{x+y}{2}\right)$ Remember that $x + y = 3$, so substituting this in we find that the area of all of the triangles is $4\left(\frac{3}{2}\right) = 6$. The area of the 4 unit squares is 4, so the area of the square we need is $25 - (4+6) = \boxed{\textbf{(C) } 15}$

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