2009 AMC 10A Problems

Contents

Problem 1

One can holds 12 ounces of soda. What is the minimum number of cans needed to provide a gallon (128 ounces) of soda?

(A) 7   (B) 8   (C) 9   (D) 10   (E) 11

Solution

Problem 2

Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes and quarters. Which of the following could not be the total value of the four coins, in cents?

(A) 15   (B) 25   (C) 35   (D) 45   (E) 55

Solution

Problem 3

Which of the following is equal to $1 + \dfrac{1}{1 + \frac{1}{1+1}}$?

(A) $\dfrac{5}{4}$ (B) $\dfrac{3}{2}$ (C) $\dfrac{5}{3}$ (D) 2   (E) 3

Solution

Problem 4

Eric plans to compete in a triathlon. He can average 2 miles per hour in the $\frac{1}{4}$-mile swim and 6 miles per hour in the 3-mile run. His goal is to finish the triathlon in 2 hours. To accomplish his goal what must his average speed in miles per hour, be for the 15-mile bicycle ride?

(A) $\dfrac{120}{11}$ (B) 11 (C) $\dfrac{56}{5}$ (D) $\dfrac{45}{4}$ (E) 12
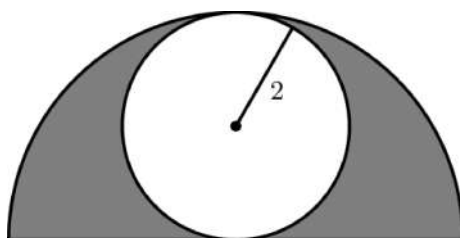
Solution

Problem 5

What is the sum of the digits of the square of $111,111,111$?

(A) 18 (B) 27 (C) 45 (D) 63 (E) 81

Solution

Problem 6

A circle of radius 2 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?

(A) $\dfrac{1}{2}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{2}{\pi}$ (D) $\dfrac{2}{3}$ (E) $\dfrac{3}{\pi}$

Solution

Problem 7

A carton contains milk that is 2% fat, an amount that is 40% less fat than the amount contained in a carton of whole milk. What is the percentage of fat in whole milk?

(A) $\dfrac{12}{5}$ (B) 3 (C) $\dfrac{10}{3}$ (D) 38 (E) 42

Solution

Problem 8

Three generations of the Wen family are going to the movies, two from each generation. The two members of the youngest generation receive a 50% discount as children. The two members of the oldest generation receive a 25% discount as senior citizens. The two members of the middle generation receive no discount. Grandfather Wen, whose senior ticket costs $\$6.00$, is paying for everyone. How many dollars must he pay?

(A) 34 (B) 36 (C) 42 (D) 46 (E) 48
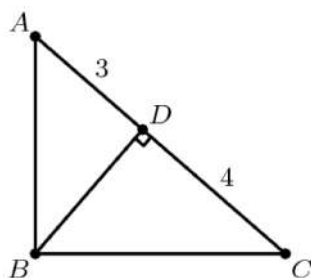
Solution

Problem 9

Positive integers a, b, and 2009, with $a < b < 2009$, form a geometric sequence with an integer ratio. What is a?

(A) 7 (B) 41 (C) 49 (D) 289 (E) 2009

Solution

Problem 10

Triangle ABC has a right angle at B. Point D is the foot of the altitude from B, $AD = 3$, and $DC = 4$. What is the area of $\triangle ABC$?

(A) $4\sqrt{3}$ (B) $7\sqrt{3}$ (C) 21 (D) $14\sqrt{3}$ (E) 42
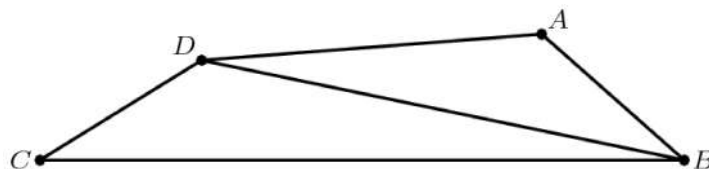
Solution

Problem 11

One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

(A) 8 (B) 27 (C) 64 (D) 125 (E) 216

Solution

Problem 12

In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15
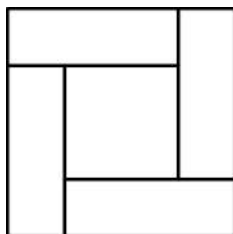
Solution

Problem 13

Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?

(A) $P^2 Q$ (B) $P^n Q^m$ (C) $P^n Q^{2m}$ (D) $P^{2m} Q^n$ (E) $P^{2n} Q^m$

Solution

Problem 14

Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?
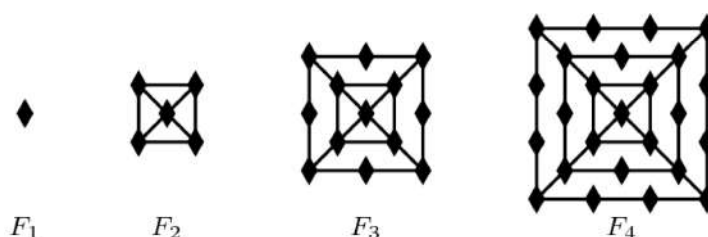
(A) 3 (B) $\sqrt{10}$ (C) $2 + \sqrt{2}$ (D) $2\sqrt{3}$ (E) 4

Solution

Problem 15

The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?

(A) 401 (B) 485 (C) 585 (D) 626 (E) 761

Solution

Problem 16

Let a, b, c, and d be real numbers with $|a - b| = 2$, $|b - c| = 3$, and $|c - d| = 4$. What is the sum of all possible values of $|a - d|$?

(A) 9 (B) 12 (C) 15 (D) 18 (E) 24

Solution

Problem 17

Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that EF is perpendicular to DB, and A and C lie on DE and DF, respectively. What is EF?

(A) 9 (B) 10 (C) $\dfrac{125}{12}$ (D) $\dfrac{103}{9}$ (E) 12

Solution

Problem 18

At Jefferson Summer Camp, 60% of the children play soccer, 30% of the children swim, and 40% of the soccer players swim. To the nearest whole percent, what percent of the non-swimmers play soccer?

(A) 30% (B) 40% (C) 49% (D) 51% (E) 70%

Solution

Problem 19

Circle A has radius 100. Circle B has an integer radius $r < 100$ and remains internally tangent to circle A as it rolls once around the circumference of circle A. The two circles have the same points of tangency at the beginning and end of circle B's trip. How many possible values can r have?

(A) 4   (B) 8   (C) 9   (D) 50   (E) 90
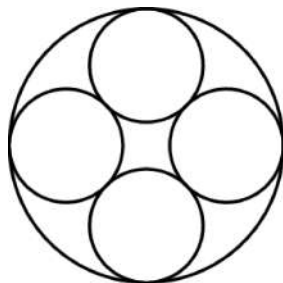
Solution

Problem 20

Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

(A) 20   (B) 30   (C) 55   (D) 65   (E) 80

Solution

Problem 21

Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle. In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?

(A) $3 - 2\sqrt{2}$ (B) $2 - \sqrt{2}$ (C) $4(3 - 2\sqrt{2})$ (D) $\frac{1}{2}(3 - \sqrt{2})$ (E) $2\sqrt{2} - 2$

Solution

Problem 22

Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

(A) $\frac{1}{9}$ (B) $\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{2}{11}$ (E) $\frac{1}{5}$

Solution

Problem 23

Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

(A) $\frac{9}{2}$ (B) $\frac{50}{11}$ (C) $\frac{21}{4}$ (D) $\frac{17}{3}$ (E) 6

Solution

Problem 24

Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?

(A) $\dfrac{1}{4}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{4}{7}$ (D) $\dfrac{5}{7}$ (E) $\dfrac{3}{4}$

Solution

Problem 25

For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Solution

See also

2009 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2009))	
Preceded by 2008 AMC 10B Problems	Followed by 2009 AMC 10B Problems
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AMC 10 Problems and Solutions	

- AMC 10
- AMC 10 Problems and Solutions
- 2009 AMC 10A
- Mathematics competition resources

Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2009_AMC_10A_Problems&oldid=73913"