2008 AMC 12B Problems

Contents

Problem 1

A basketball player made 5 baskets during a game. Each basket was worth either 2 or 3 points. How many different numbers could represent the total points scored by the player?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

(Solution)

Problem 2

A 4×4 block of calendar dates is shown. The order of the numbers in the second row is to be reversed. Then the order of the numbers in the fourth row is to be reversed. Finally, the numbers on each diagonal are to be added. What will be the positive difference between the two diagonal sums?

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 10$

(Solution)

Problem 3

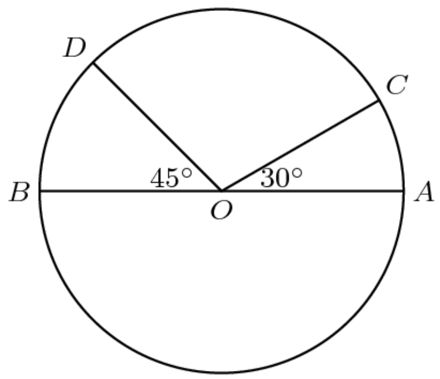
A semipro baseball league has teams with 21 players each. League rules state that a player must be paid at least $15,000$ dollars, and that the total of all players' salaries for each team cannot exceed $700,000$ dollars. What is the maximum possible salary, in dollars, for a single player?

$\textbf{(A) } 270,000 \qquad \textbf{(B) } 385,000 \qquad \textbf{(C) } 400,000 \qquad \textbf{(D) } 430,000 \qquad \textbf{(E) } 700,000$

(Solution)

Problem 4

On circle O, points C and D are on the same side of diameter $\overline{AB}$, $\angle AOC = 30^\circ$, and $\angle DOB = 45^\circ$. What is the ratio of the area of the smaller sector COD to the area of the circle?



$\textbf{(A) } \frac{2}{9} \qquad \textbf{(B) } \frac{1}{4} \qquad \textbf{(C) } \frac{5}{18} \qquad \textbf{(D) } \frac{7}{24} \qquad \textbf{(E) } \frac{3}{10}$

(Solution)

Problem 5

A class collects 50 dollars to buy flowers for a classmate who is in the hospital. Roses cost 3 dollars each, and carnations cost 2 dollars each. No other flowers are to be used. How many different bouquets could be purchased for exactly 50 dollars?

$\textbf{(A) } 1 \qquad \textbf{(B) } 7 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 16 \qquad \textbf{(E) } 17$

(Solution)

Problem 6

Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to 00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year?

$\textbf{(A) } 2500 \qquad \textbf{(B) } 3000 \qquad \textbf{(C) } 3500 \qquad \textbf{(D) } 4000 \qquad \textbf{(E) } 4500$

(Solution)

Problem 7

For real numbers a and b, define $a\$b = (a-b)^2$. What is $(x-y)^2\$(y-x)^2$?

$\textbf{(A) } 0 \qquad \textbf{(B) } x^2+y^2 \qquad \textbf{(C) } 2x^2 \qquad \textbf{(D) } 2y^2 \qquad \textbf{(E) } 4xy$

(Solution)

Problem 8

Points B and C lie on $\overline{AD}$. The length of $\overline{AB}$ is 4 times the length of $\overline{BD}$, and the length of $\overline{AC}$ is 9 times the length of $\overline{CD}$. The length of $\overline{BC}$ is what fraction of the length of $\overline{AD}$?

$\textbf{(A)}\ \frac{1}{36} \qquad \textbf{(B)}\ \frac{1}{13} \qquad \textbf{(C)}\ \frac{1}{10} \qquad \textbf{(D)}\ \frac{5}{36} \qquad \textbf{(E)}\ \frac{1}{5}$

(Solution)

Problem 9

Points A and B are on a circle of radius 5 and $AB = 6$. Point C is the midpoint of the minor arc AB. What is the length of the line segment AC?

$\textbf{(A)}\ \sqrt{10} \qquad \textbf{(B)}\ \frac{7}{2} \qquad \textbf{(C)}\ \sqrt{14} \qquad \textbf{(D)}\ \sqrt{15} \qquad \textbf{(E)}\ 4$

(Solution)

Problem 10

Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?

$\textbf{(A)}\ 500 \qquad \textbf{(B)}\ 900 \qquad \textbf{(C)}\ 950 \qquad \textbf{(D)}\ 1000 \qquad \textbf{(E)}\ 1900$

(Solution)

Problem 11

A cone-shaped mountain has its base on the ocean floor and has a height of 8000 feet. The top $\frac{1}{8}$ of the volume of the mountain is above water. What is the depth of the ocean at the base of the mountain in feet?

$\textbf{(A)}\ 4000 \qquad \textbf{(B)}\ 2000(4-\sqrt{2}) \qquad \textbf{(C)}\ 6000 \qquad \textbf{(D)}\ 6400 \qquad \textbf{(E)}\ 7000$

(Solution)

Problem 12

For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?

$\textbf{(A)}\ 2008 \qquad \textbf{(B)}\ 4015 \qquad \textbf{(C)}\ 4016 \qquad \textbf{(D)}\ 4030056 \qquad \textbf{(E)}\ 4032064$

(Solution)

Problem 13

Vertex E of equilateral triangle $\triangle ABE$ is in the interior of unit square $ABCD$. Let R be the region consisting of all points inside $ABCD$ and outside $\triangle ABE$ whose distance from $\overline{AD}$ is between $\frac{1}{3}$ and $\frac{2}{3}$. What is the area of R?

$\textbf{(A)}\ \frac{12-5\sqrt{3}}{72} \qquad \textbf{(B)}\ \frac{12-5\sqrt{3}}{36} \qquad \textbf{(C)}\ \frac{\sqrt{3}}{18} \qquad \textbf{(D)}\ \frac{3-\sqrt{3}}{9} \qquad \textbf{(E)}\ \frac{\sqrt{3}}{12}$

(Solution)

Problem 14

A circle has a radius of $\log_{10}(a^2)$ and a circumference of $\log_{10}(b^4)$. What is $\log_a b$?

$\textbf{(A)}\ \frac{1}{4\pi} \qquad \textbf{(B)}\ \frac{1}{\pi} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ 2\pi \qquad \textbf{(E)}\ 10^{2\pi}$

(Solution)

Problem 15

On each side of a unit square, an equilateral triangle of side length 1 is constructed. On each new side of each equilateral triangle, another equilateral triangle of side length 1 is constructed. The interiors of the square and the 12 triangles have no points in common. Let R be the region formed by the union of the square and all the triangles, and S be the smallest convex polygon that contains R. What is the area of the region that is inside S but outside R?

$\textbf{(A)}\ \frac{1}{4} \qquad \textbf{(B)}\ \frac{\sqrt{2}}{4} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 2\sqrt{3}$

(Solution)

Problem 16

A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a,b)?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

(Solution)

Problem 17

Let A, B and C be three distinct points on the graph of $y=x^2$ such that line AB is parallel to the x-axis and $\triangle ABC$ is a right triangle with area 2008. What is the sum of the digits of the y-coordinate of C?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

(Solution)

Problem 18

A pyramid has a square base $ABCD$ and vertex E. The area of square $ABCD$ is 196, and the areas of $\triangle ABE$ and $\triangle CDE$ are 105 and 91, respectively. What is the volume of the pyramid?

$\textbf{(A)}\ 392 \qquad \textbf{(B)}\ 196\sqrt{6} \qquad \textbf{(C)}\ 392\sqrt{2} \qquad \textbf{(D)}\ 392\sqrt{3} \qquad \textbf{(E)}\ 784$

(Solution)

Problem 19

A function f is defined by $f(z) = (4 + i) z^2 + \alpha z + \gamma$ for all complex numbers z, where α and γ are complex numbers and $i^2 = -1$. Suppose that $f(1)$ and $f(i)$ are both real. What is the smallest possible value of $|\alpha| + |\gamma|$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \sqrt{2} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 2\sqrt{2} \qquad \textbf{(E)}\ 4$

(Solution)

Problem 20

Michael walks at the rate of 5 feet per second on a long straight path. Trash pails are located every 200 feet along the path. A garbage truck travels at 10 feet per second in the same direction as Michael and stops for 30 seconds at each pail. As Michael passes a pail, he notices the truck ahead of him just leaving the next pail. How many times will Michael and the truck meet?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

(Solution)

Problem 21

Two circles of radius 1 are to be constructed as follows. The center of circle A is chosen uniformly and at random from the line segment joining $(0,0)$ and $(2,0)$. The center of circle B is chosen uniformly and at random, and independently of the first choice, from the line segment joining $(0,1)$ to $(2,1)$. What is the probability that circles A and B intersect?

$\textbf{(A)}\ \frac{2+\sqrt{2}}{4} \qquad \textbf{(B)}\ \frac{3\sqrt{3}+2}{8} \qquad \textbf{(C)}\ \frac{2\sqrt{2}-1}{2} \qquad \textbf{(D)}\ \frac{2+\sqrt{3}}{4} \qquad \textbf{(E)}\ \frac{4\sqrt{3}-3}{4}$

(Solution)

Problem 22

A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

$\textbf{(A)}\ \frac{11}{20} \qquad \textbf{(B)}\ \frac{4}{7} \qquad \textbf{(C)}\ \frac{81}{140} \qquad \textbf{(D)}\ \frac{3}{5} \qquad \textbf{(E)}\ \frac{17}{28}$

(Solution)

Problem 23

The sum of the base-10 logarithms of the divisors of 10^n is 792. What is n?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

(Solution)

Problem 24

Let $A_0=(0,0)$. Distinct points $A_1,A_2,\dots$ lie on the x-axis, and distinct points $B_1,B_2,\dots$ lie on the graph of $y=\sqrt{x}$. For every positive integer n, $A_{n-1}B_nA_n$ is an equilateral triangle. What is the least n for which the length $A_0A_n\geq100$?

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 21$

(Solution)

Problem 25

Let $ABCD$ be a trapezoid with $AB||CD$, $AB=11$, $BC=5$, $CD=19$, and $DA=7$. Bisectors of $\angle A$ and $\angle D$ meet at P, and bisectors of $\angle B$ and $\angle C$ meet at Q. What is the area of hexagon $ABQCDP$?

$\textbf{(A)}\ 28\sqrt{3} \qquad \textbf{(B)}\ 30\sqrt{3} \qquad \textbf{(C)}\ 32\sqrt{3} \qquad \textbf{(D)}\ 35\sqrt{3} \qquad \textbf{(E)}\ 36\sqrt{3}$

(Solution)