2005 AMC 8 Problems/Problem 1

Problem

Connie multiplies a number by 2 and gets 60 as her answer. However, she should have divided the number by 2 to get the correct answer. What is the correct answer?

$\textbf{(A)}\ 7.5\qquad\textbf{(B)}\ 15\qquad\textbf{(C)}\ 30\qquad\textbf{(D)}\ 120\qquad\textbf{(E)}\ 240$

Solution

If x is the number, then $2x=60$ and $x=30$. Dividing the number by 2 yields $30/2=\boxed{\textbf{(B)}\ 15}$.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 2

Problem

Karl bought five folders from Pay-A-Lot at a cost of $\$2.50$ each. Pay-A-Lot had a 20%-off sale the following day. How much could Karl have saved on the purchase by waiting a day?

$\textbf{(A)}\ \$1.00 \qquad \textbf{(B)}\ \$2.00 \qquad \textbf{(C)}\ \$2.50 \qquad \textbf{(D)}\ \$2.75 \qquad \textbf{(E)}\ \$5.00$

Solution

Karl paid $5 \cdot 2.50 = \$12.50$. 20% of this cost that he saved is $12.50 \cdot .2 = \boxed{\textbf{(C)}\ \$2.50}$.

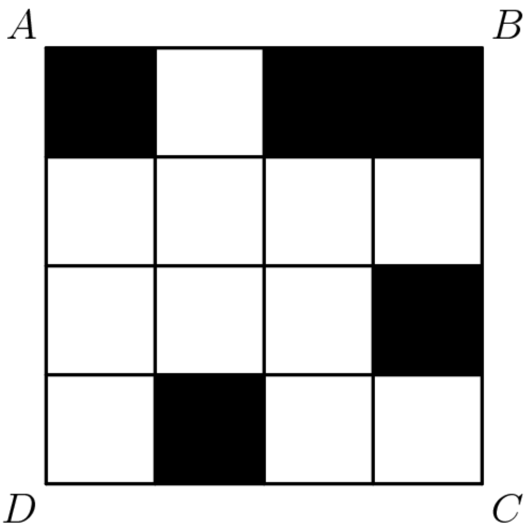
See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 3

Problem

What is the minimum number of small squares that must be colored black so that a line of symmetry lies on the diagonal $\overline{BD}$ of square $ABCD$?



$\textbf{(A) }1\qquad\textbf{(B) }2\qquad\textbf{(C) }3\qquad\textbf{(D) }4\qquad\textbf{(E) }5$

Solution

Rotating square $ABCD$ counterclockwise 45° so that the line of symmetry BD is a vertical line makes it easier to see that $\boxed{\textbf{(D)}\ 4}$ squares need to be colored to match its corresponding square.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 4

Problem

A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 64$

Solution

The perimeter of the triangle is $6.1+8.2+9.7=24$ cm. A square's perimeter is four times its sidelength, since all its sidelengths are equal. If the square's perimeter is 24, the sidelength is $24/4=6$, and the area is $6^2=\boxed{\textbf{(C)}\ 36}$.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 5

Problem

Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

$\textbf{(A)}\ 4\qquad\textbf{(B)}\ 5\qquad\textbf{(C)}\ 6\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 15$

Solution

Start by buying the largest packs first. After three 24-packs, $90-3(24)=18$ cans are left. After one 12-pack, $18-12=6$ cans are left. Then buy one more 6-pack. The total number of packs is $3+1+1=\boxed{\textbf{(B)}\ 5}$.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 6

Problem

Suppose d is a digit. For how many values of d is $2.00d5 > 2.005$?

$\textbf{(A)}\ 0\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 5\qquad\textbf{(D)}\ 6\qquad\textbf{(E)}\ 10$

Solution

We see that 2.0055 works but 2.0045 does not. The digit d can be from 5 through 9, which is $\boxed{\textbf{(C)}\ 5}$ values.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 7

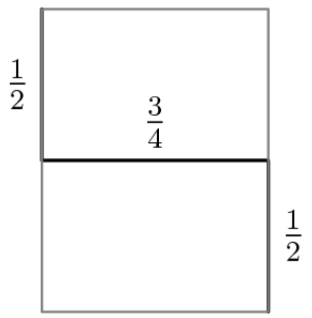
Problem

Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 1\frac{1}{4} \qquad \textbf{(C)}\ 1\frac{1}{2} \qquad \textbf{(D)}\ 1\frac{3}{4} \qquad \textbf{(E)}\ 2$

Solution

Draw a picture.



Find the length of the diagonal of the rectangle to find the length of the direct line to the starting time using Pythagorean Theorem.

$$\sqrt{(\frac{1}{2}+\frac{1}{2})^2+(\frac{3}{4})^2} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4} = \boxed{\textbf{(B)}\ 1\frac{1}{4}}$$

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 8

Problem

Suppose m and n are positive odd integers. Which of the following must also be an odd integer?

$\textbf{(A)}\ m+3n \qquad \textbf{(B)}\ 3m-n \qquad \textbf{(C)}\ 3m^2 + 3n^2 \qquad \textbf{(D)}\ (nm + 3)^2 \qquad \textbf{(E)}\ 3mn$

Solution

Assume WLOG that m and n are both 1. Plugging into each of the choices, we get $4, 2, 6, 16,$ and 3. The only odd integer is $\boxed{\textbf{(E)}\ 3mn}$.

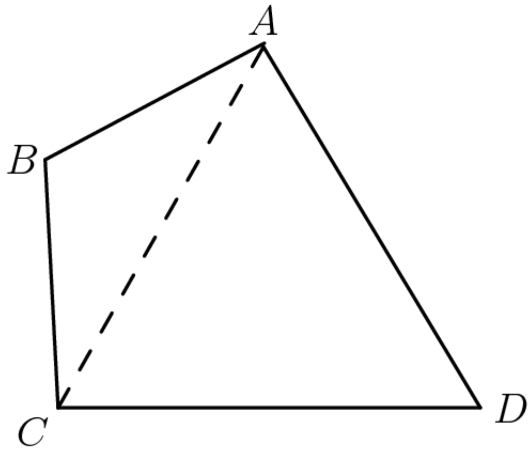
See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 9

Problem

In quadrilateral $ABCD$, sides $\overline{AB}$ and $\overline{BC}$ both have length 10, sides $\overline{CD}$ and $\overline{DA}$ both have length 17, and the measure of angle ADC is 60°. What is the length of diagonal $\overline{AC}$?



$\mathrm{(A)}\ 13.5\qquad\mathrm{(B)}\ 14\qquad\mathrm{(C)}\ 15.5\qquad\mathrm{(D)}\ 17\qquad\mathrm{(E)}\ 18.5$

Solution

Because $\overline{AD} = \overline{CD}$, $\triangle ADC$ is an equilateral triangle with $\angle DAC = \angle DCA$. Angles in a triangle add up to 180°, and since $\angle ADC = 60^\circ$, the other two angles are also 60°, and $\triangle ADC$ is an equilateral triangle. Therefore $\overline{AC} = \overline{DA} = \boxed{\textbf{(D)}\ 17}$.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 10

Problem

Joe had walked half way from home to school when he realized he was late. He ran the rest of the way to school. He ran 3 times as fast as he walked. Joe took 6 minutes to walk half way to school. How many minutes did it take Joe to get from home to school?

$\textbf{(A)}\ 7\qquad\textbf{(B)}\ 7.3\qquad\textbf{(C)}\ 7.7\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 8.3$

Solution

Use the equation $d = rt$ where d is the distance, r is the rate, and t is the time. The distances he ran and walked are equal, so $r_r t_r = r_w t_w$. Because he runs three times faster than he walks, $r_r = 3r_w$. We want to find the time he ran, $t_r = \frac{r_w t_w}{t_r} = \frac{(r_w)(6)}{3r_w} = 2$ minutes. He traveled for a total of $6+2 = \boxed{\textbf{(D)}\ 8}$ minutes.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 11

Problem

The sales tax rate in Bergville is 6%. During a sale at the Bergville Coat Closet, the price of a coat is discounted 20% from its $90.00 price. Two clerks, Jack and Jill, calculate the bill independently. Jack rings up $90.00 and adds 6% sales tax, then subtracts 20% from this total. Jill rings up $90.00, subtracts 20% of the price, then adds 6% of the discounted price for sales tax. What is Jack's total minus Jill's total?

$\textbf{(A)}\ \$1.06 \qquad \textbf{(B)}\ \$0.53 \qquad \textbf{(C)}\ \$0 \qquad \textbf{(D)}\ \$0.53 \qquad \textbf{(E)}\ \$1.06$

Solution

The price Jacks rings up is $(90.00)(1.06)(0.80)$. The price Jill rings up is $(90.00)(0.80)(1.06)$. By the commutative property of multiplication, these quantities are the same, and the difference is $\boxed{\textbf{(C)}\ \$0}$.

See Also

2005 AMC 8 Problems/Problem 12

Problem

Big Al, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big Al eat on May 5?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 22 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 34$

Solution

There are 5 days from May 1 to May 5. The number of bananas he eats each day is an arithmetic sequence. He eats n bananas on May 5, and $n-4(6)=n-24$ bananas on May 1. The sum of this arithmetic sequence is equal to 100.

$$\begin{aligned} \frac{n+n-24}{2} \cdot 5 &= 100 \\ n-12 &= 20 \\ n &= \boxed{\textbf{(D)}\ 32} \end{aligned}$$

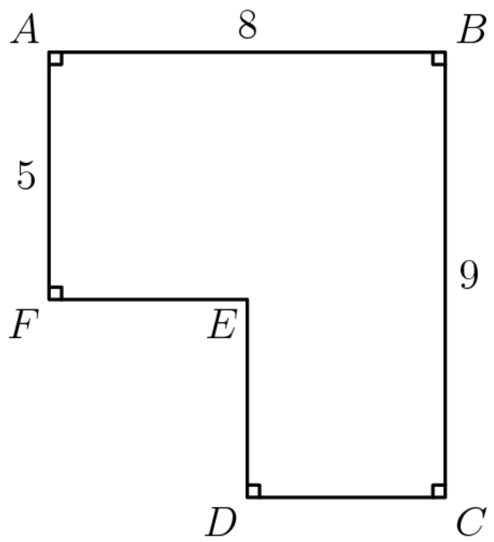
See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 13

Problem

The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $DE + EF$?



$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Solution

Notice that $AF + DE = BC$, so $DE = 4$. Let O be the intersection of the extensions of AF and DC, which makes rectangle $ABCO$. The area of the polygon is the area of $FEDO$ subtracted from the area of $ABCO$.

$$\text{Area} = 52 = 8 \cdot 9 - EF \cdot 4$$

Solving for the unknown, $EF = 5$, therefore $DE + EF = 4 + 5 = \boxed{\textbf{(C)}\ 9}$.

See Also

2005 AMC 8 Problems/Problem 14

Problem

The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 96 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 108 \qquad \textbf{(E)}\ 192$

Solution

Within each division, there are ${}_6C_2 = 15$ pairings, and each of these games happens twice. The same goes for the other division so that there are $4(15) = 60$ games within their own divisions. The number games between the two divisions is $(6)(6) = 36$. Together there are $60 + 36 = \boxed{\textbf{(B)}\ 96}$ conference games.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 15

Problem

How many different isosceles triangles have integer side lengths and perimeter 23?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 11$

Solution

Let b be the base of the isosceles triangles, and let a be the lengths of the other legs. From this, $2a+b=23$ and $b=23-2a$. From triangle inequality, $2a>b$, then plug in the value from the previous equation to get $2a>23-2a$ or $a>5.75$. The maximum value of a occurs when $b=1$, in which from the first equation $a=11$. Thus, a can have integer side lengths from 6 to 11, and there are $\boxed{\textbf{(C)}\ 6}$ triangles.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 16

Problem

A five-legged Martian has a drawer full of socks, each of which is red, white or blue, and there are at least five socks of each color. The Martian pulls out one sock at a time without looking. How many socks must the Martian remove from the drawer to be certain there will be 5 socks of the same color?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 15$

Solution

The Martian can pull out 12 socks, 4 of each color, without having 5 of the same kind yet. However, the next one he pulls out must be the fifth of one of the colors so he must remove $\boxed{\textbf{(D)}\ 13}$ socks.

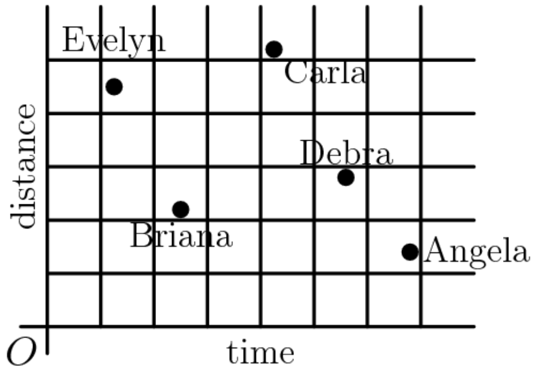
See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 17

Problem

The results of a cross-country team's training run are graphed below. Which student has the greatest average speed?



$\textbf{(A)}\ \text{Angela}\qquad\textbf{(B)}\ \text{Briana}\qquad\textbf{(C)}\ \text{Carla}\qquad\textbf{(D)}\ \text{Debra}\qquad\textbf{(E)}\ \text{Evelyn}$

Solution

Average speed is distance over time, or the slope of the line through the point and the origin. $\boxed{\textbf{(E)}\ \text{Evelyn}}$ has the steepest line, and runs the greatest distance for the shortest amount of time.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 18

Problem

How many three-digit numbers are divisible by 13?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 67 \qquad \textbf{(C)}\ 69 \qquad \textbf{(D)}\ 76 \qquad \textbf{(E)}\ 77$

Solution

Let k be any positive integer so that $13k$ is a multiple of 13. For the smallest three-digit number, $13k > 100$ and $k > \frac{100}{13} \approx 7.7$. For the greatest three-digit number, $13k < 999$ and $k < \frac{999}{13} \approx 76.8$. The number k can range from 8 to 76 so there are $\boxed{\textbf{(C)}\ 69}$ three-digit numbers.

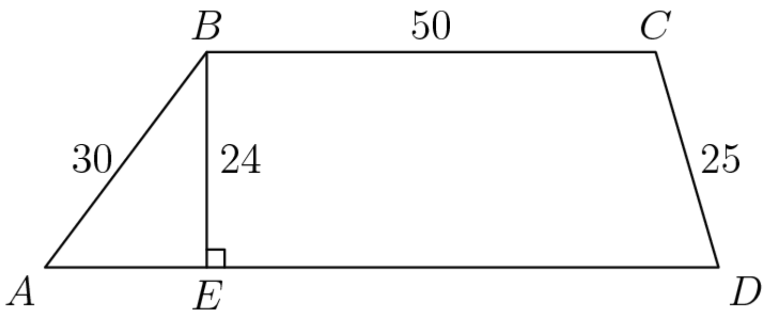
See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 19

Problem

What is the perimeter of trapezoid $ABCD$?



$\textbf{(A)}\ 180 \qquad\textbf{(B)}\ 188 \qquad\textbf{(C)}\ 196 \qquad\textbf{(D)}\ 200 \qquad\textbf{(E)}\ 204$

Solution

Draw altitudes from B and C to base AD to create a rectangle and two right triangles. The side opposite BC is equal to 50. The bases of the right triangles can be found using Pythagorean or special triangles to be 18 and 7. Add it together to get $AD = 18 + 50 + 7 = 75$. The perimeter is $75 + 30 + 50 + 25 = \boxed{\textbf{(A)}\ 180}$.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 20

Problem

Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 24$

Solution

Alice moves $5k$ steps and Bob moves $9k$ steps, where k is the turn they are on. Alice and Bob coincide when the number of steps they move collectively, $14k$, is a multiple of 12. Since 14 has a factor 2, k must have a factor of 6. The smallest number of turns that is a multiple of 6 is $\boxed{\textbf{(A)}\ 6}$.

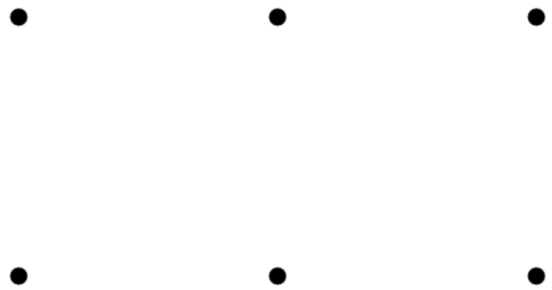
See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 21

Problem

How many distinct triangles can be drawn using three of the dots below as vertices?



$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Solution

The number of ways to choose three points to make a triangle is $\binom{6}{3} = 20$. However, two* of these are a straight line so we subtract 2 to get $\boxed{\textbf{(C)}\ 18}$.

- Note: We are assuming that there are no degenerate triangles in this problem, and that is why we subtract two.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 22

Problem

A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.

$\textbf{(A)}\ \text{SML} \qquad \textbf{(B)}\ \text{LMS} \qquad \textbf{(C)}\ \text{MSL} \qquad \textbf{(D)}\ \text{LSM} \qquad \textbf{(E)}\ \text{MLS}$

Solution

Suppose the small size costs $\$1$ and the large size has 10 oz. The medium size then costs $\$1.50$ and has 8 oz. The small size has 5 oz and the large size costs $\$1.95$. The small, medium, and large size cost respectively, $0.200, 0.188, 0.195$ dollars per oz. The sizes from best to worst buy are $\boxed{\textbf{(E)}\ \text{MLS}}$.

See Also

2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
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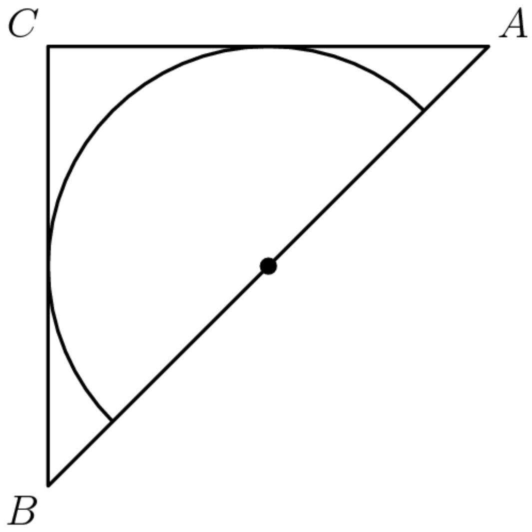
2005 AMC 8 Problems/Problem 23

Contents

Problem

Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse $\overline{AB}$ and is tangent to sides $\overline{AC}$ and $\overline{BC}$. What is the area of triangle ABC?



$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 3\pi \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 4\pi$

Solution

The semi circle has an area of $\pi r^2/2 = 2\pi$ and a radius of 2.

Because this is an isosceles right triangle, the center is the midpoint of the hypotenuse. Radii drawn to the tangent points of the semicircle and the radii also divide the legs into two equal segments. They also create a square in the top left corner. From this, we can conclude the legs of the triangle are twice the length of the radii, 4. The area of the triangle is $(4)(4)/2 = \boxed{\textbf{(B)}\ 8}$.

Easier More Logical Solution

We see half a square so first let's create a square. Once we have a square, we will have a full circle. This circle has a diameter of 4 which will be the side of the square. The area would be 4*4 = 16. Divide 16 by 2 to get he original shape and you get 8.

See Also

2005 AMC 8 Problems/Problem 24

Problem

A certain calculator has only two keys [+1] and [x2]. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed [+1], it would display "10." If you then pressed [x2], it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "200"?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Solution

It is easier to work backwards from 200, and have keys that display [-1] and [x0.5]. Use the second key when the number is even, and the first key when the number is odd until you get one. We get:

$$200, 100, 50, 25, 24, 12, 6, 3, 2, 1$$

This took $\boxed{\textbf{(B)}\ 9}$ keystrokes.

See Also

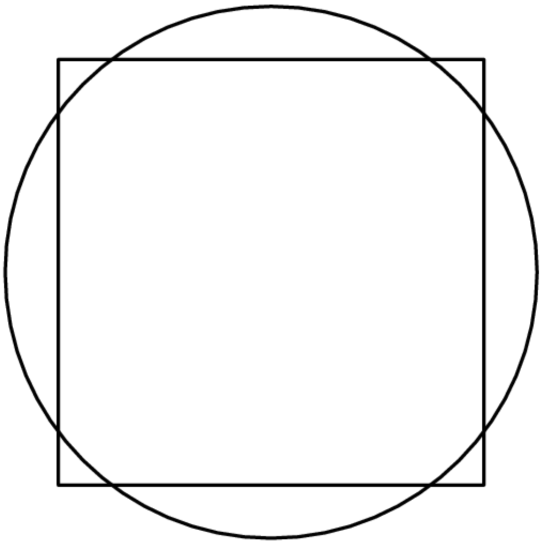
2005 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2005))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2005 AMC 8 Problems/Problem 25

Contents

Problem

A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?



$\textbf{(A)}\ \frac{2}{\sqrt{\pi}} \qquad \textbf{(B)}\ \frac{1+\sqrt{2}}{2} \qquad \textbf{(C)}\ \frac{3}{2} \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ \sqrt{\pi}$

Solution

Let the region within the circle and square be a. In other words, it is the intersection of the area of circle and square. Let r be the radius. We know that the area of the circle minus a is equal to the area of the square, minus a .

We get:

$$\pi r^2 - a = 4 - a$$

$$r^2 = \frac{4}{\pi}$$

$$r = \frac{2}{\sqrt{\pi}}$$

So the answer is $\boxed{\textbf{(A)}\ \frac{2}{\sqrt{\pi}}}$.

Solution 2

We realize that since the areas of the regions outside of the circle and the square are equal to each other, the area of the circle must be equal to the area of the square.

$$\pi r^2 = 4$$

$$r^2 = \frac{4}{\pi}$$

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So the answer is $\boxed{\textbf{(A) } \frac{2}{\sqrt{\pi}}}$.

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We realize that since the areas of the regions outside of the circle and the square are equal to each other, the area of the circle must be equal to the area of the square.

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So the answer is $\boxed{\textbf{(A) } \frac{2}{\sqrt{\pi}}}$.

Solution 3

We realize that since the areas of the regions outside of the circle and the square are equal to each other, the area of the circle must be equal to the area of the square.

$$r^2 = \frac{4}{\pi}$$

$$r = \frac{2}{\sqrt{\pi}}$$

So the answer is $\boxed{\textbf{(A) } \frac{2}{\sqrt{\pi}}}$.

See Also