2018 AMC 12A Problems

2018 AMC 12A (Answer Key)
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Instructions 1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct. 2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer. 3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will *require* the use of a calculator). 4. Figures are not necessarily drawn to scale. 5. You will have **75 minutes** working time to complete the test.
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Contents

Problem 1

A large urn contains 100 balls, of which 36% are red and the rest are blue. How many of the blue balls must be removed so that the percentage of red balls in the urn will be 72%? (No red balls are to be removed.)

$\textbf{(A)}\ 28 \qquad\textbf{(B)}\ 32 \qquad\textbf{(C)}\ 36 \qquad\textbf{(D)}\ 50 \qquad\textbf{(E)}\ 64$

Solution

Problem 2

While exploring a cave, Carl comes across a collection of 5-pound rocks worth $\$14$ each, 4-pound rocks worth $\$11$ each, and 1-pound rocks worth $\$2$ each. There are at least 20 of each size. He can carry at most 18 pounds. What is the maximum value, in dollars, of the rocks he can carry out of the cave?

$\textbf{(A)}\ 48 \qquad\textbf{(B)}\ 49 \qquad\textbf{(C)}\ 50 \qquad\textbf{(D)}\ 51 \qquad\textbf{(E)}\ 52$

Solution

Problem 3

How many ways can a student schedule 3 mathematics courses -- algebra, geometry, and number theory -- in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

$\textbf{(A)}\ 3 \qquad\textbf{(B)}\ 6 \qquad\textbf{(C)}\ 12 \qquad\textbf{(D)}\ 18 \qquad\textbf{(E)}\ 24$

Solution

Problem 4

Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements were true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

(A) $(0,4)$ **(B)** $(4,5)$ **(C)** $(4,6)$ **(D)** $(5,6)$ **(E)** $(5,\infty)$

Solution

Problem 5

What is the sum of all possible values of k for which the polynomials x^2-3x+2 and x^2-5x+k have a root in common?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 10

Solution

Problem 6

For positive integers m and n such that $m+10<n+1$, both the mean and the median of the set $\{m, m+4, m+10, n+1, n+2, 2n\}$ are equal to n. What is $m+n$?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

Solution

Problem 7

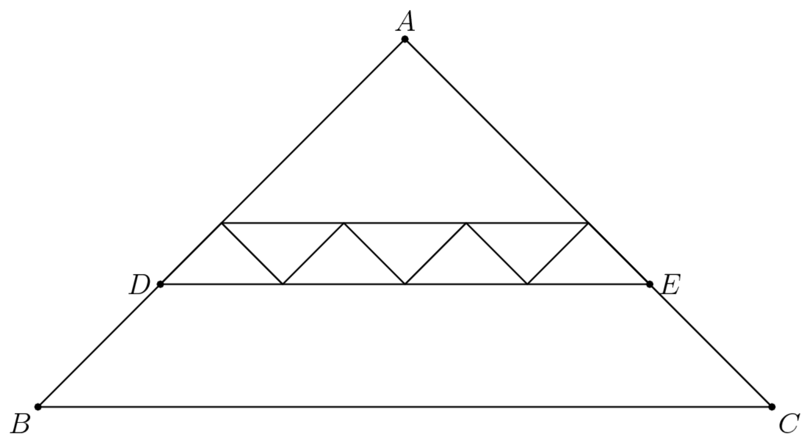
For how many (not necessarily positive) integer values of n is the value of $4000\cdot \left(\tfrac{2}{5}\right)^n$ an integer?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

Solution

Problem 8

All of the triangles in the diagram below are similar to iscoceles triangle ABC, in which $AB=AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?



(A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

Solution

Problem 9

Which of the following describes the largest subset of values of y within the closed interval $[0,\pi]$ for which

$$\sin(x+y)\leq \sin(x)+\sin(y)$$

for every x between 0 and π, inclusive?

(A) $y=0$ **(B)** $0\leq y\leq \frac{\pi}{4}$ **(C)** $0\leq y\leq \frac{\pi}{2}$ **(D)** $0\leq y\leq \frac{3\pi}{4}$ **(E)** $0\leq y\leq \pi$

Solution

Problem 10

How many ordered pairs of real numbers (x,y) satisfy the following system of equations?

$$x+3y=3$$

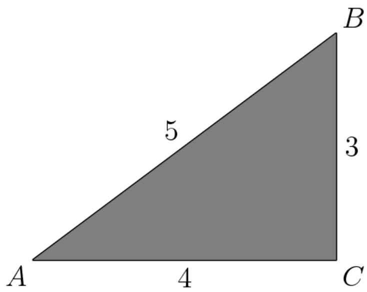
$$\big||x|-|y|\big|=1$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

Solution

Problem 11

A paper triangle with sides of lengths 3,4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



$\textbf{(A) } 1+\frac{1}{2}\sqrt{2} \qquad \textbf{(B) } \sqrt{3} \qquad \textbf{(C) } \frac{7}{4} \qquad \textbf{(D) } \frac{15}{8} \qquad \textbf{(E) } 2$

Solution

Problem 12

Let S be a set of 6 integers taken from $\{1,2,\dots,12\}$ with the property that if a and b are elements of S with $a<b$, then b is not a multiple of a. What is the least possible value of an element in $S?$

$\textbf{(A) } 2 \qquad \textbf{(B) } 3 \qquad \textbf{(C) } 4 \qquad \textbf{(D) } 5 \qquad \textbf{(E) } 7$

Solution

Problem 13

How many nonnegative integers can be written in the form

$$a_7\cdot3^7+a_6\cdot3^6+a_5\cdot3^5+a_4\cdot3^4+a_3\cdot3^3+a_2\cdot3^2+a_1\cdot3^1+a_0\cdot3^0,$$

where $a_i\in \{-1,0,1\}$ for $0\le i \le 7$?

$\textbf{(A) } 512 \qquad \textbf{(B) } 729 \qquad \textbf{(C) } 1094 \qquad \textbf{(D) } 3281 \qquad \textbf{(E) } 59,048$

Solution

Problem 14

The solutions to the equation $\log_{3x} 4 = \log_{2x} 8$, where x is a positive real number other than $\tfrac{1}{3}$ or $\tfrac{1}{2}$, can be written as $\tfrac{p}{q}$ where p and q are relatively prime positive integers. What is $p + q$?

$\textbf{(A) } 5 \qquad \textbf{(B) } 13 \qquad \textbf{(C) } 17 \qquad \textbf{(D) } 31 \qquad \textbf{(E) } 35$

Solution

Problem 15

A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called $\textit{symmetric}$ if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

$\textbf{(A) } 510 \qquad \textbf{(B) } 1022 \qquad \textbf{(C) } 8190 \qquad \textbf{(D) } 8192 \qquad \textbf{(E) } 65,534$

Solution

Problem 16

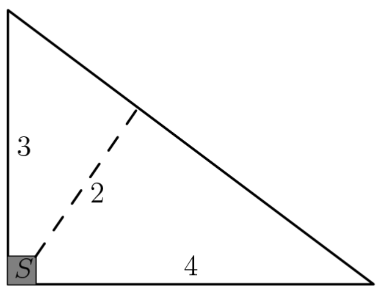
Which of the following describes the set of values of a for which the curves $x^2+y^2=a^2$ and $y=x^2-a$ in the real xy-plane intersect at exactly 3 points?

$\textbf{(A) } a=\frac{1}{4} \qquad \textbf{(B) } \frac{1}{4}< a< \frac{1}{2} \qquad \textbf{(C) } a>\frac{1}{4} \qquad \textbf{(D) } a=\frac{1}{2} \qquad \textbf{(E) } a>\frac{1}{2}$

Solution

Problem 17

Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



$\textbf{(A) } \frac{25}{27} \qquad \textbf{(B) } \frac{26}{27} \qquad \textbf{(C) } \frac{73}{75} \qquad \textbf{(D) } \frac{145}{147} \qquad \textbf{(E) } \frac{74}{75}$

Solution

Problem 18

Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

$\textbf{(A) } 60 \qquad \textbf{(B) } 65 \qquad \textbf{(C) } 70 \qquad \textbf{(D) } 75 \qquad \textbf{(E) } 80$

Solution

Problem 19

Let A be the set of positive integers that have no prime factors other than 2, 3, or 5. The infinite sum

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{12}+\frac{1}{15}+\frac{1}{16}+\frac{1}{18}+\frac{1}{20}+\cdots$$

of the reciprocals of the elements of A can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

$\textbf{(A) } 16 \qquad \textbf{(B) } 17 \qquad \textbf{(C) } 19 \qquad \textbf{(D) } 23 \qquad \textbf{(E) } 36$

Solution

Problem 20

Triangle ABC is an isosceles right triangle with $AB=AC=3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $AI>AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a+b+c$?

$\textbf{(A) } 9 \qquad \textbf{(B) } 10 \qquad \textbf{(C) } 11 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 13$

Solution

Problem 21

Which of the following polynomials has the greatest real root?

$\textbf{(A) } x^{19}+2018x^{11}+1 \qquad \textbf{(B) } x^{17}+2018x^{11}+1 \qquad \textbf{(C) } x^{19}+2018x^{13}+1 \qquad \textbf{(D) } x^{17}+2018x^{13}+1 \qquad \textbf{(E) } 2019x+201$

Solution

Problem 22

The solutions to the equations $z^2=4+4\sqrt{15}i$ and $z^2=2+2\sqrt 3i,$ where $i=\sqrt{-1},$ form the vertices of a parallelogram in the complex plane. The area of this parallelogram can be written in the form $p\sqrt q-r\sqrt s,$ where $p,q,r,$ and s are positive integers and neither q nor s is divisible by the square of any prime number. What is $p+q+r+s?$

$\textbf{(A) } 20 \qquad \textbf{(B) } 21 \qquad \textbf{(C) } 22 \qquad \textbf{(D) } 23 \qquad \textbf{(E) } 24$

Solution

Problem 23

In $\triangle PAT,$ $\angle P=36^{\circ},$ $\angle A=56^{\circ},$ and $PA=10.$ Points U and G lie on sides $\overline{TP}$ and $\overline{TA},$ respectively, so that $PU=AG=1.$ Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG},$ respectively. What is the degree measure of the acute angle formed by lines MN and $PA?$

$\textbf{(A) } 76 \qquad \textbf{(B) } 77 \qquad \textbf{(C) } 78 \qquad \textbf{(D) } 79 \qquad \textbf{(E) } 80$

Solution

Problem 24

Alice, Bob, and Carol play a game in which each of them chooses a real number between 0 and 1. The winner of the game is the one whose number is between the numbers chosen by the other two players. Alice announces that she will choose her number uniformly at random from all the numbers between 0 and 1, and Bob announces that he will choose his number uniformly at random from all the numbers between $\tfrac{1}{2}$ and $\tfrac{2}{3}.$ Armed with this information, what number should Carol choose to maximize her chance of winning?

$\textbf{(A) } \frac{1}{2} \qquad \textbf{(B) } \frac{13}{24} \qquad \textbf{(C) } \frac{7}{12} \qquad \textbf{(D) } \frac{5}{8} \qquad \textbf{(E) } \frac{2}{3}$

Solution

Problem 25

For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b, and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

$\textbf{(A) } 12 \qquad \textbf{(B) } 14 \qquad \textbf{(C) } 16 \qquad \textbf{(D) } 18 \qquad \textbf{(E) } 20$

Solution

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