2012 AMC 12B Problems

2012 AMC 12B (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have 75 minutes working time to complete the test.

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Contents

Problem 1

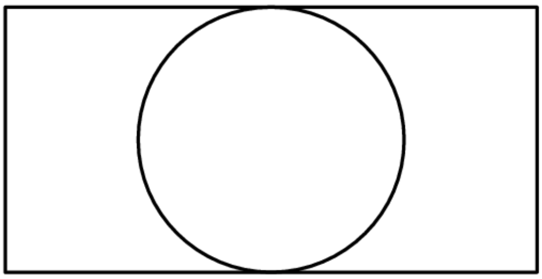
Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

$\textbf{(A)}\ 48\qquad\textbf{(B)}\ 56\qquad\textbf{(C)}\ 64\qquad\textbf{(D)}\ 72\qquad\textbf{(E)}\ 80$

Solution

Problem 2

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?



$\textbf{(A)}\ 50\qquad\textbf{(B)}\ 100\qquad\textbf{(C)}\ 125\qquad\textbf{(D)}\ 150\qquad\textbf{(E)}\ 200$

Solution

Problem 3

For a science project, Sammy observed a chipmunk and squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

$\textbf{(A)}\ 30\qquad\textbf{(B)}\ 36\qquad\textbf{(C)}\ 42\qquad\textbf{(D)}\ 48\qquad\textbf{(E)}\ 54$

Solution

Problem 4

Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Etienne has 400 euros, by what percent is the value of Etienne's money greater that the value of Diana's money?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 6.5\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 13$

Solution

Problem 5

Two integers have a sum of 26. when two more integers are added to the first two, the sum is 41. Finally, when two more integers are added to the sum of the previous 4 integers, the sum is 57. What is the minimum number of even integers among the 6 integers?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Solution

Problem 6

In order to estimate the value of $x-y$ where x and y are real numbers with $x>y>0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

$\textbf{(A)}$ Her estimate is larger than $x-y$

$\textbf{(B)}$ Her estimate is smaller than $x-y$

$\textbf{(C)}$ Her estimate equals $x-y$

$\textbf{(D)}$ Her estimate equals $y-x$

$\textbf{(E)}$ Her estimate is 0

Solution

Problem 7

Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

Note: 1 foot is equal to 12 inches.

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 18.5 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 20.5 \qquad \textbf{(E)}\ 22.5$

Solution

Problem 8

A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

$\textbf{(A)}\ 729 \qquad \textbf{(B)}\ 972 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 2187 \qquad \textbf{(E)}\ 2304$

Solution

Problem 9

It takes Clea 60 seconds to walk down an escalator when it is not moving, and 24 seconds when it is moving. How many seconds would it take Clea to ride the escalator down when she is not walking?

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 52$

Solution

Problem 10

What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x-4)^2 + 9y^2 = 81$?

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 37.5 \qquad \textbf{(E)}\ 42$

Solution

Problem 11

In the equation below, A and B are consecutive positive integers, and A, B, and $A+B$ represent number bases:

$$132_A+43_B=69_{A+B}.$$

What is $A+B$?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 17$

Solution

Problem 12

How many sequences of zeros and ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

$\textbf{(A)}\ 190\qquad\textbf{(B)}\ 192\qquad\textbf{(C)}\ 211\qquad\textbf{(D)}\ 380\qquad\textbf{(E)}\ 382$

Solution

Problem 13

Two parabolas have equations $y= x^2 + ax +b$ and $y= x^2 + cx +d$, where $a, b, c,$ and d are integers, each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas will have a least one point in common?

$\textbf{(A)}\ \frac{1}{2}\qquad\textbf{(B)}\ \frac{25}{36}\qquad\textbf{(C)}\ \frac{5}{6}\qquad\textbf{(D)}\ \frac{31}{36}\qquad\textbf{(E)}\ 1$

Solution

Problem 14

Bernardo and Silvia play the following game. An integer between 0 and 999 inclusive is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she addes 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

$\textbf{(A)}\ 7\qquad\textbf{(B)}\ 8\qquad\textbf{(C)}\ 9\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 11$

Solution

Problem 15

Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

$\textbf{(A)}\ \frac{1}{8}\qquad\textbf{(B)}\ \frac{1}{4}\qquad\textbf{(C)}\ \frac{\sqrt{10}}{10}\qquad\textbf{(D)}\ \frac{\sqrt{5}}{6}\qquad\textbf{(E)}\ \frac{\sqrt{5}}{5}$

Solution

Problem 16

Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those girls but disliked by the third. In how many different ways is this possible?

$\textbf{(A)}\ 108\qquad\textbf{(B)}\ 132\qquad\textbf{(C)}\ 671\qquad\textbf{(D)}\ 846\qquad\textbf{(E)}\ 1105$

Solution

Problem 17

Square $PQRS$ lies in the first quadrant. Points $(3,0), (5,0), (7,0),$ and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 6.2\qquad\textbf{(C)}\ 6.4\qquad\textbf{(D)}\ 6.6\qquad\textbf{(E)}\ 6.8$

Solution

Problem 18

Let $(a_1,a_2, \dots ,a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either a_i+1 or a_i-1 or both appear somewhere before a_i in the list. How many such lists are there?

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 512 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 181,440 \qquad \textbf{(E)}\ 362,880$

Solution

Problem 19

A unit cube has vertices $P_1,P_2,P_3,P_4,P_1',P_2',P_3',$ and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1\le i\le 4,$ vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments P_1P_2, P_1P_3, P_1P_4, $P_1'P_2'$, $P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?

$\textbf{(A)}\ \frac{3\sqrt{2}}{4}\qquad\textbf{(B)}\ \frac{7\sqrt{6}}{16}\qquad\textbf{(C)}\ \frac{\sqrt{5}}{2}\qquad\textbf{(D)}\ \frac{2\sqrt{3}}{3}\qquad\textbf{(E)}\ \frac{\sqrt{6}}{2}$

Solution

Problem 20

A trapezoid has side lengths 3, 5, 7, and 11. The sums of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1}+r_2\sqrt{n_2}+r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of any prime. What is the greatest integer less than or equal to $r_1+r_2+r_3+n_1+n_2$?

$\textbf{(A)}\ 57\qquad\textbf{(B)}\ 59\qquad\textbf{(C)}\ 61\qquad\textbf{(D)}\ 63\qquad\textbf{(E)}\ 65$

Solution

Problem 21

Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB=40$, and $EF=41(\sqrt{3}-1)$. What is the side-length of the square?

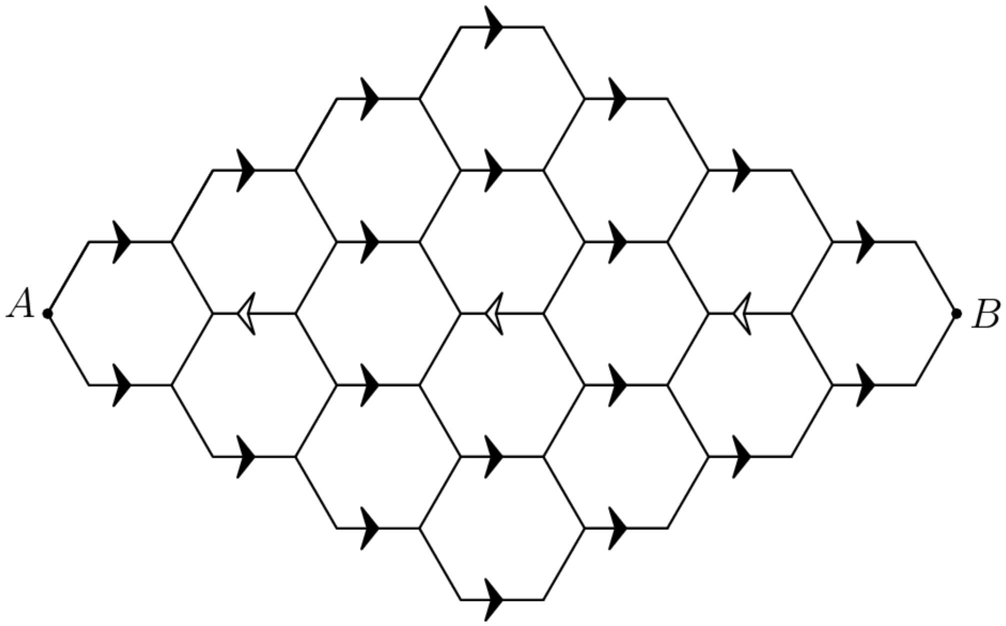
$\textbf{(A)}\ 29\sqrt{3} \qquad\textbf{(B)}\ \frac{21}{2}\sqrt{2}+\frac{41}{2}\sqrt{3} \qquad\textbf{(C)}\ 20\sqrt{3}+16$

$\textbf{(D)}\ 20\sqrt{2}+13\sqrt{3} \qquad\textbf{(E)}\ 21\sqrt{6}$

Solution

Problem 22

A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

Solution

Problem 23

Consider all polynomials of a complex variable, $P(z) = 4z^4 + az^3 + bz^2 + cz + d$, where $a, b, c,$ and d are integers, $0 \le d \le c \le b \le a \le 4$, and the polynomial has a zero z_0 with $|z_0| = 1.$ What is the sum of all values $P(1)$ over all the polynomials with these properties?

(A) 84 **(B)** 92 **(C)** 100 **(D)** 108 **(E)** 120

Solution

Problem 24

Define the function f_1 on the positive integers by setting $f_1(1) = 1$ and if $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ is the prime factorization of $n > 1$, then

$$f_1(n) = (p_1 + 1)^{e_1 - 1}(p_2 + 1)^{e_2 - 1} \cdots (p_k + 1)^{e_k - 1}.$$

For every $m \ge 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many N in the range $1 \le N \le 400$ is the sequence $(f_1(N), f_2(N), f_3(N), \dots)$ unbounded?

Note: A sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

Solution

Problem 25

Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}, \text{ and } (x, y) \ne (0, 0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t = \triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t) = \tan(\angle CBA)$. What is

$$\prod_{t \in T} f(t)?$$

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{625}{144} \qquad \textbf{(C)}\ \frac{125}{24} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{625}{24}$

Solution

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