2021Spring AMC10B

Problem 1

How many integer values of x satisfy $|x| < 3\pi$?

有多少个整数值满足$|x| < 3\pi$?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

Problem 2

What is the value of $\sqrt{\left(3-2\sqrt{3}\right)^2}+\sqrt{\left(3+2\sqrt{3}\right)^2}$?

算式$\sqrt{\left(3-2\sqrt{3}\right)^2}+\sqrt{\left(3+2\sqrt{3}\right)^2}$的值是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4\sqrt{3}-6 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 4\sqrt{3} \qquad \textbf{(E)}\ 4\sqrt{3}+6$

Problem 3

In an after-school program for juniors and seniors, there is a debate team with an equal number of students from each class on the team. Among the 28 students in the program, 25% of the juniors and 10% of the seniors are on the debate team. How many juniors are in the program?

在面向高二和高三学生的某课外课程中，有一个辩论队，每个年级参加辩论队的人数相同。已知在参加这个课程的 28 名同学中，有 25％的高二学生和 10％的高三学生参加了辩论队。问这个课程中有多少名高二的学生？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 20$

Problem 4

At a math contest, 57 students are wearing blue shirts, and another 75 students are wearing yellow shirts. The 132 students are assigned into 66 pairs. In exactly 23 of these pairs, both students are wearing blue shirts. In how many pairs are both students wearing yellow shirts?

在一次数学竞赛中，57 名学生穿着蓝色衬衫，另 75 外名学生穿着黄色衬衫。132 名学生被分成了 66 对。这其中恰好有 23 对，每对的两名学生都穿着蓝色衬衫。问两名学生都穿着黄色衬衫的对有多少个？

$\textbf{(A)}\ 23 \qquad\textbf{(B)}\ 32 \qquad\textbf{(C)}\ 37 \qquad\textbf{(D)}\ 41 \qquad\textbf{(E)}\ 64$

Problem 5

The ages of Jonie's four cousins are distinct single-digit positive integers. Two of the cousins' ages multiplied together give 24, while the other two multiply to 30. What is the sum of the ages of Jonie's four cousins?

Jonie 的四个表兄弟的年龄是不同的一位正整数。两个表兄弟的年龄相乘得到 24，而另外两个的年龄相乘得到 30。问 Jonie 的四个表兄弟的年龄总和是多少？

$\textbf{(A)}\ 21 \qquad\textbf{(B)}\ 22 \qquad\textbf{(C)}\ 23 \qquad\textbf{(D)}\ 24 \qquad\textbf{(E)}\ 25$

Problem 6

Ms. Blackwell gives an exam to two classes. The mean of the scores of the students in the morning class is 84, and the afternoon class's mean score is 70. The ratio of the number of students in the morning class to the number of students in the afternoon class is $\frac{3}{4}$. What is the mean of the scores of all the students?

Blackwell 女士在两个班进行考试。上午班学生的平均分是 84，而下午班的平均分是 70。上午班学生人数与下午班学生人数之比是$\frac{3}{4}$。问所有学生的平均分是多少？

$\textbf{(A)}\ 74 \qquad\textbf{(B)}\ 75 \qquad\textbf{(C)}\ 76 \qquad\textbf{(D)}\ 77 \qquad\textbf{(E)}\ 78$

Problem 7

In a plane, four circles with radii $1, 3, 5,$ and 7 are tangent to line ℓ at the same point $A,$ but they may be on either side of ℓ. Region S consists of all the points that lie inside exactly one of the four circles. What is the maximum possible area of region S?

在平面上，四个半径分别为 1，3，5，7 的圆都与直线ℓ相切于同一点 A，但每个圆可能在ℓ的任意一侧。区域 S 由恰好位于四个圆中一个内部的所有点组成。问区域 S 的最大可能面积是多少？

(A) 24π **(B)** 32π **(C)** 64π **(D)** 65π **(E)** 84π

Problem 8

Mr. Zhou places all the integers from 1 to 225 into a 15 by 15 grid. He places 1 in the middle square (eighth row and eighth column) and places other numbers one by one clockwise, as shown in part in the diagram below. What is the sum of the greatest number and the least number that appear in the second row from the top?

周先生把从 1 到 225 的所有整数放入一个 15 x 15 的方格表中。他把 1 放在正中间的方格（第八行第八列）里，然后把其他数按如图所示的方式一个接一个地顺时针放置。从顶部数第二行出现的最大数与最小数之和是多少？

$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	21	22	23	24	25	$\cdots$
$\cdots$	20	7	8	9	10	$\cdots$
$\cdots$	19	6	1	2	11	$\cdots$
$\cdots$	18	5	4	3	12	$\cdots$
$\cdots$	17	16	15	14	13	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

(A) 367 **(B)** 368 **(C)** 369 **(D)** 379 **(E)** 380

Problem 9

The point $P(a,b)$ in the xy-plane is first rotated counterclockwise by 90° around the point $(1,5)$ and then reflected about the line $y=-x$. The image of P after these two transformations is at $(-6,3)$. What is $b-a$?

xy 坐标平面中的点 *P*（*a*，*b*）首先绕着点（1，5）逆时针旋转 90°，然后沿直线 y＝－x 反射。经过这两次变换后 *P* 的影像是点（－6，3）。问 *b* - *a* 是多少？

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

Problem 10

An inverted cone with base radius $12 \mathrm{cm}$ and height $18 \mathrm{cm}$ is full of water. The water is poured into a tall cylinder whose horizontal base has a radius of $24 \mathrm{cm}$. What is the height in centimeters of the water in the cylinder?

一个底部半径为 12 厘米，高度为 18 厘米的倒置圆锥体中充满了水。将水倒入一个水平底半径为 24 厘部的高圆柱体中。问圆柱体中水的高是多少厘度？

(A) 1.5 **(B)** 3 **(C)** 4 **(D)** 4.5 **(E)** 6

Problem 11

Grandma has just finished baking a large rectangular pan of brownies. She is planning to make rectangular pieces of equal size and shape, with straight cuts parallel to the sides of the pan. Each cut must be made entirely across the pan. Grandma wants to make the same number of interior pieces as pieces along the perimeter of the pan. What is the greatest possible number of brownies she can produce?

奶奶刚在大长方形烤盘中烤好了巧克力蛋糕。她计划把蛋糕切成大小和形状相同的长方形块，切的时候要与盘的边缘平行。每刀切的时候都贯穿整个烤盘。奶奶希望烤盘内部的块数和与盘边相接触的块数一样多。问她最多能切出多少块巧克力蛋糕？

(A) 24 **(B)** 30 **(C)** 48 **(D)** 60 **(E)** 64

Problem 12

Let $N = 34 \cdot 34 \cdot 63 \cdot 270$. What is the ratio of the sum of the odd divisors of N to the sum of the even divisors of N?

令$N = 34 \cdot 34 \cdot 63 \cdot 270$。N 的奇约数之和与 *N* 的偶约数之和的比值是多少？

$\textbf{(A)}\ 1:16 \qquad \textbf{(B)}\ 1:15 \qquad \textbf{(C)}\ 1:14 \qquad \textbf{(D)}\ 1:8 \qquad \textbf{(E)}\ 1:3$

Problem 13

Let n be a positive integer and d be a digit such that the value of the numeral $\underline{32d}$ in base n equals 263, and the value of the numeral $\underline{324}$ in base n equals the value of the numeral $\underline{11d1}$ in base six. What is $n + d?$

令 *n* 是正整数，*d* 是一个数字，使得以 *n* 为底的数$\underline{32d}$的值等于 263，并且以 *n* 为底的数$\underline{324}$的值等于以六为底的数$\underline{11d1}$的值。问 *n*+*d* 是多少？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 16$

Problem 14

Three equally spaced parallel lines intersect a circle, creating three chords of lengths $38, 38,$ and 34. What is the distance between two adjacent parallel lines?

三条等间距的平行线与一个圆相交，形成三条长度分别为 38，38 和 34 的弦。问相邻的两条平行线之间的距离是多少？

$\textbf{(A)}\ 5\frac{1}{2} \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 6\frac{1}{2} \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 7\frac{1}{2}$

Problem 15

The real number x satisfies the equation $x+\frac{1}{x} = \sqrt{5}$. What is the value of $x^{11}-7x^{7}+x^3?$

实数 *x* 满足方程$x+\frac{1}{x} = \sqrt{5}$，问$x^{11}-7x^{7}+x^3$的值是多少？

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \sqrt{5}$

Problem 16

Call a positive integer an uphill integer if every digit is strictly greater than the previous digit. For example, 1357, 89, and 5 are all uphill integers, but 32, 1240, and 466 are not. How many uphill integers are divisible by 15?

如果一个正整数的每位数字都严格大于前一位数字，则称它为“向上”的整数。例如，1357，89 和 5 都是向上的整数，但 32，1240 和 466 则不是。问有多少个向上的整数可以被 15 整除？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 17

Ravon, Oscar, Aditi, Tyrone, and Kim play a card game. Each person is given 2 cards out of a set of 10 cards numbered $1, 2, 3, \dots, 10.$ The score of a player is the sum of the numbers of their cards. The scores of the players are as follows: Ravon--$11,$ Oscar--$4,$ Aditi--$7,$ Tyrone--$16,$ Kim--$17.$ Which of the following statements is true?

Ravon，Oscar，Aditi，Tyrone 和 Kim 玩纸牌游戏。每人从一套标有 1 数，2，3，…，10 的 10 张牌中分得 2 张牌。玩家的得分是他们手中的牌上所标数的和。各玩家的得分如下：Ravon-11，Oscar-4，Aditi-7，Tyrone-16，Kim-17。问以下哪个论断是正确的？

$\textbf{(A)}$ Ravon was given card 3.

$\textbf{(B)}$ Aditi was given card 3.

$\textbf{(C)}$ Ravon was given card 4.

$\textbf{(D)}$ Aditi was given card 4.

$\textbf{(E)}$ Tyrone was given card 7.

Problem 18

A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number?

不断抛掷一个均匀的 6 面骰子，直到出现一个奇数为止。在第一次出现奇数之前，每个偶数至少出现一次的概率是多少？

$\textbf{(A)}\ \frac{1}{120} \qquad \textbf{(B)}\ \frac{1}{32} \qquad \textbf{(C)}\ \frac{1}{20} \qquad \textbf{(D)}\ \frac{3}{20} \qquad \textbf{(E)}\ \frac{1}{6}$

Problem 19

Suppose that S is a finite set of positive integers. If the greatest integer in S is removed from S, then the average value (arithmetic mean) of the integers remaining is 32. If the least integer in S is also removed, then the average value of the integers remaining is 35. If the greatest integer is then returned to the set, the average value of the integers rises to 40. The greatest integer in the original set S is 72 greater than the least integer in S. What is the average value of all the integers in the set S?

假设 S 是正整数的有限集合。如果把 *S* 中最大的整数从 *S* 中移除，则其余的整数的（算术）平均值为 32。如果把 *S* 中的最小整数也移除，则其余的整数的平均值为 35。如果又把最大的整数加回到集合中，则整数的平均值上升到 40。原来集合 *S* 中最大的整数比 *S* 中最小的整数大 72。问集合 *S* 中所有整数的平均值是多少？

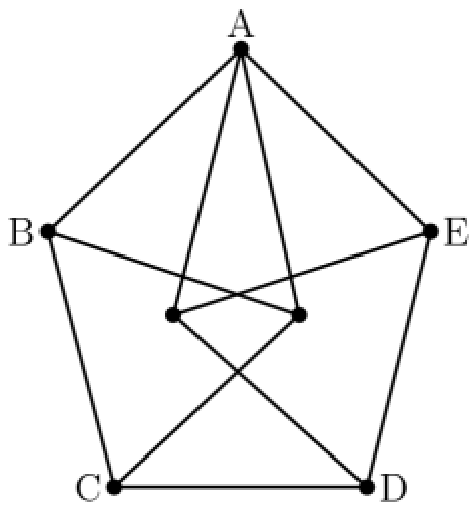
$\textbf{(A)}\ 36.2 \qquad \textbf{(B)}\ 36.4 \qquad \textbf{(C)}\ 36.6 \qquad \textbf{(D)}\ 36.8 \qquad \textbf{(E)}\ 37$

Problem 20

The figure is constructed from 11 line segments, each of which has length 2. The area of pentagon $ABCDE$ can be written is $\sqrt{m} + \sqrt{n}$, where m and n are positive integers. What is $m + n?$

上图由 11 条线段构成，每条线段的长度都是 2。五边形 *ABCDE* 的面积可以写成$\sqrt{m} + \sqrt{n}$

其中 *m* 和 *n* 是正整数。问 *m*+*n* 是多少？

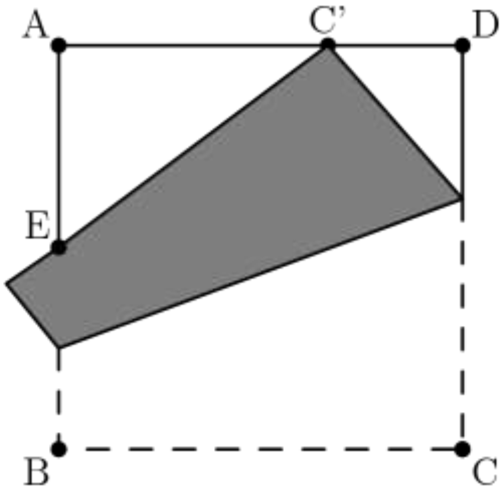


$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 24$

Problem 21

A square piece of paper has side length 1 and vertices $A, B, C,$ and D in that order. As shown in the figure, the paper is folded so that vertex C meets edge $\overline{AD}$ at point C', and edge $\overline{BC}$ intersects edge $\overline{AB}$ at point E. Suppose that $C'D = \frac{1}{3}$. What is the perimeter of triangle $\triangle AEC'$?

一张正方形纸的边长为 1，顶点依次为 A，B，C 和 D。如图所示，将纸张折叠，使得顶点 C 落在边$\overline{AD}$上的点 C'，边$\overline{BC}$与边$\overline{AB}$相交于点 E。假设$C'D = \frac{1}{3}$。$\triangle AEC'$?的周长是多少？



$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 1+\frac{2}{3}\sqrt{3} \qquad \textbf{(C)}\ \sqrt{136} \qquad \textbf{(D)}\ 1+\frac{3}{4}\sqrt{3} \qquad \textbf{(E)}\ \frac{7}{3}$

Problem 22

Ang, Ben, and Jasmin each have 5 blocks, colored red, blue, yellow, white, and green; and there are 5 empty boxes. Each of the people randomly and independently of the other two people places one of their blocks into each box. The probability that at least one box receives 3 blocks all of the same color is $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n?$

Ang，Ben 和 Jasmin 每人有 5 块积木，颜色分别是红色，蓝色，黄色，白色和绿色；还有 5 个空盒子。每人随机地并独立于另外两人将积木放入盒中，每个盒子里放一块积木。至少有一个盒子里所放的 3 块积木是相同颜色的概率是$\frac{m}{n}$，其中 m 和 n 是互质的正整数。问 $m+n$ 的值是多少？

$\textbf{(A)}\ 47 \qquad \textbf{(B)}\ 94 \qquad \textbf{(C)}\ 227 \qquad \textbf{(D)}\ 471 \qquad \textbf{(E)}\ 542$

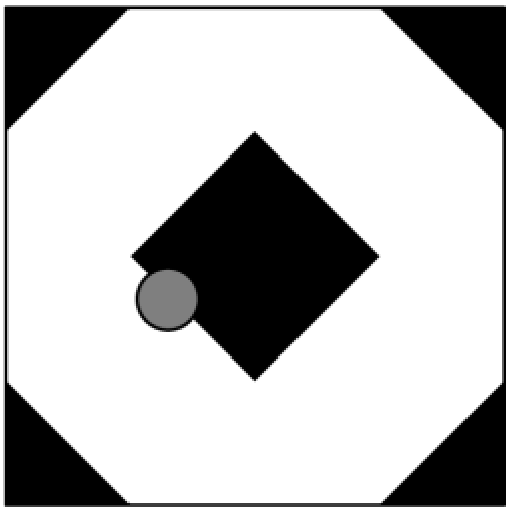
Problem 23

A square with side length 8 is colored white except for 4 black isosceles right triangular regions with legs of length 2 in each corner of the square and a black diamond with side length $2\sqrt{2}$ in the center of the square, as shown in the diagram. A circular coin with diameter 1 is dropped onto the square and lands in a random location where the coin is completely contained within the square. The probability that the coin will cover part of the black region of the square can be written as

$$\frac{1}{196}\left(a+b\sqrt{2}+\pi\right)$$

, where a and b are positive integers. What is $a+b$?

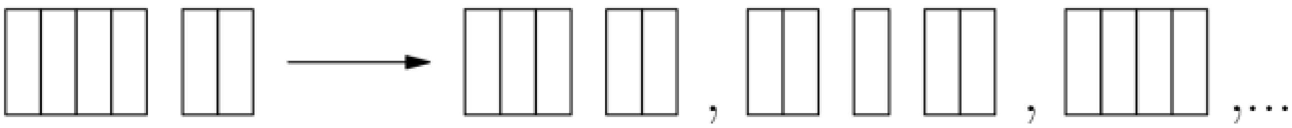
如图所示，在一个边长为 8 的正方形中，除去角落处的 4 个直角边长为 2 的黑色等腰直角三角形区域，和中心处一个边长为 2 的黑色钻石型区域，其余部分为白色。一枚直径为 1 的圆形硬币随机地掉落在正方形上，硬币完全处于正方形区域中。硬币会覆盖正方形部分黑色区域的概率可以写成 $\frac{1}{196}\left(a+b\sqrt{2}+\pi\right)$，其中 a 和 b 是正整数。问 $a+b$ 是多少？



$\textbf{(A)}\ 64 \qquad \textbf{(B)}\ 66 \qquad \textbf{(C)}\ 68 \qquad \textbf{(D)}\ 70 \qquad \textbf{(E)}\ 72$

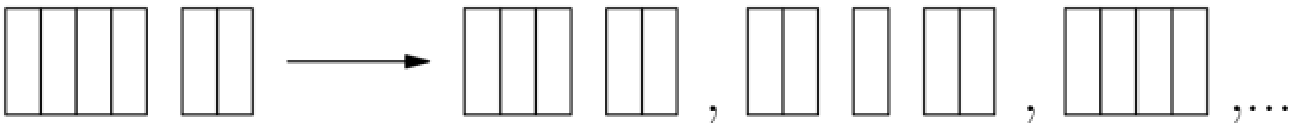
Problem 24

Arjun and Beth play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. For example, a set of walls of sizes 4 and 2 can be changed into any of the following by one move: $(3,2),(2,1,2),(4),(4,1),(2,2)$, or $(1,1,2)$.



Arjun plays first, and the player who removes the last brick wins. For which starting configuration is there a strategy that guarantees a win for Beth?

Arjun 和 Beth 玩一个游戏，他们轮流从一个由砖块组成，可能包括空隙的“墙”上移除一块砖或两块相邻的砖。这些墙的高度都和砖的高度一样。例如，一个由 4 块砖和 2 块砖组成的墙可以通过一次操作变为以下的一种构型：（3，2），（2，1，2），（4），（4，1），（2，2）或者（1，1，2）。



Arjun 首先开始，谁取走最后一块砖谁将获胜。对于哪种起始的构型，Beth 可以有必胜策略？

(A) $(6,1,1)$ **(B)** $(6,2,1)$ **(C)** $(6,2,2)$ **(D)** $(6,3,1)$ **(E)** $(6,3,2)$

Problem 25

Let S be the set of lattice points in the coordinate plane, both of whose coordinates are integers between 1 and 30, inclusive. Exactly 300 points in S lie on or below a line with equation $y=mx$. The possible values of m lie in an interval of length $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$?

设 S 是坐标平面上横纵坐标都是从 1 到 30 之间（包括 1 和 30）的整数的格点组成的集合。在 S 中恰好有 300 个点在解析式是$y=mx$的直线上或者位于该直线的下方。m 的可能值构成长度为$\frac{a}{b}$的区间，其中 a 和 b 是互质的正整数。问 $a+b$ 是多少？

(A) 31 **(B)** 47 **(C)** 62 **(D)** 72 **(E)** 85

2021Spring AMC10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	D	C	B	B	C	D	A	D	A	D	C	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	B	C	C	C	D	D	A	D	C	B	E	