

2023 年 AMC8 试卷

2023 American Mathematics Competitions 8

◇ Problem 1

What is the value of below expression?

下面表达式的值是多少?

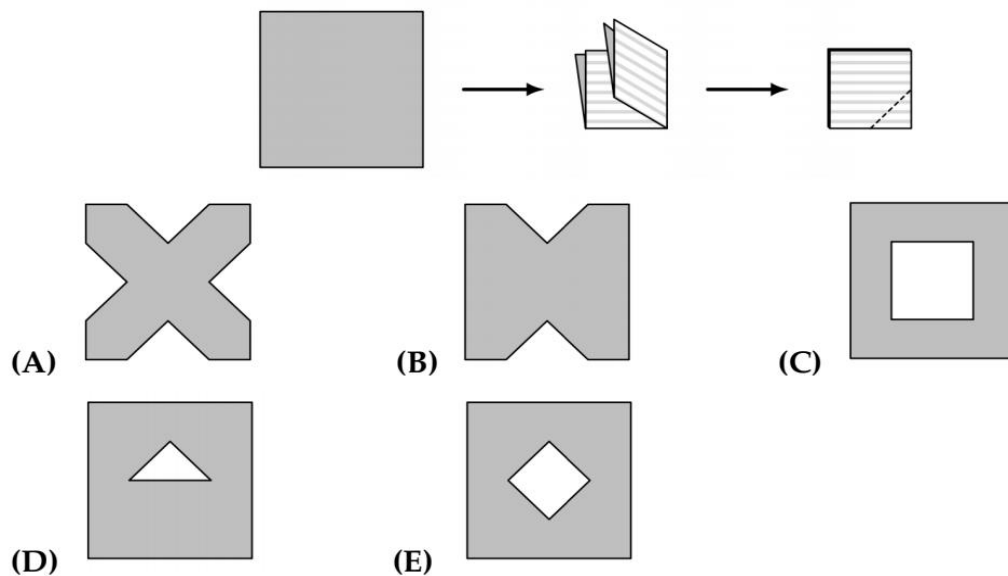
$$(8 \times 4 + 2) - (8 + 4 \times 2)$$

- (A) 0 (B) 6 (C) 10 (D) 18 (E) 24

◇ Problem 2

A square piece of paper is folded twice into four equal quarters, as shown below, then cut along the dashed line. When unfolded, the paper will match which of the following figures?

如下图所示，将一张正方形纸对折两次，变成四等分，然后沿虚线裁剪。问这张纸展开后会是什么图形？



◇ Problem 3

Wind chill is a measure of how cold people feel when exposed to wind outside. A good estimate for wind chill can be found using this calculation:

$$(\text{wind chill}) = (\text{air temperature}) - 0.7 \times (\text{wind speed}),$$

where temperature is measured in degrees Fahrenheit ($^{\circ}\text{F}$) and wind speed is measured in miles per hour (mph). Suppose the air temperature is 36°F and the wind speed is 18 mph. Which of the following is closest to the approximate wind chill?

风寒指数 是对人们暴露在室外风中时寒冷感觉的度量。通过下面的计算可以对风寒指数做出很好的估计：

$$(\text{风寒指数}) = (\text{气温}) - 0.7 \times (\text{风速}),$$

其中温度以华氏度 ($^{\circ}\text{F}$) 为单位，风速以英里每小时 (mph) 为单位。假设气温为 36°F ，风速为 18mph。问以下哪个选项最接近风寒指数？

- (A) 18 (B) 23 (C) 28 (D) 32 (E) 35

◇ Problem 4

The numbers from 1 to 49 are arranged in a spiral pattern on a square grid, beginning at the center. The first few numbers have been entered into the grid below. Consider the four numbers that will appear in the shaded squares, on the same diagonal as the number 7. How many of these four numbers are prime?

在正方形方格表上，数 1 到 49 从中心开始，按螺旋式方式排列。前面几个数已经填入到下面的方格表中。考虑与数 7 出现在同一个对角线上的阴影方格中的四个数。问这四个数中有几个质数？

		5	4	3		
		6	1	2		
		7				

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

◇ Problem 5

A lake contains 250 trout, along with a variety of other fish. When a marine biologist catches and releases a sample of 180 fish from the lake, 30 are identified as trout. Assume that the ratio of trout to the total number of fish is the same in both the sample and the lake. How many fish are there in the lake?

湖里有 250 条鳟鱼，以及其他一些鱼类。当一位海洋生物学家从湖中捕获并释放 180 条鱼的样本时，他发现其中 30 条鱼是鳟鱼。假设在样本中鳟鱼所占的比例和湖里鳟鱼在全部鱼类中所占的比例相同。问湖里有多少条鱼？

- (A) 1250 (B) 1500 (C) 1750 (D) 1800 (E) 2000

◇ Problem 6

The digits 2, 0, 2, and 3 are placed in the expression below, one digit per box. What is the maximum possible value of the expression?

将数字 2, 0, 2, 3 填入下面的表达式中，每个方框内填一个数字。问表达式的最大可能值是多少？

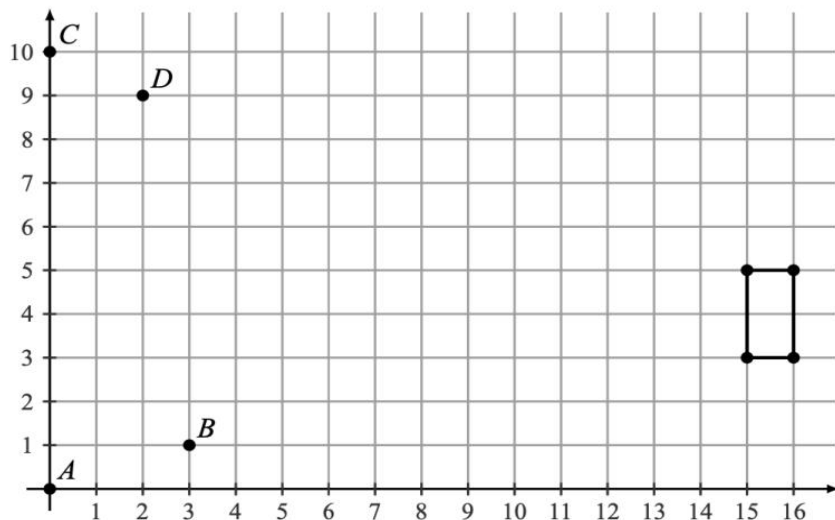
$$\boxed{}^{\boxed{}} \times \boxed{}^{\boxed{}}$$

- (A) 0 (B) 8 (C) 9 (D) 16 (E) 18

◇ Problem 7

A rectangle, with sides parallel to the x -axis and y -axis, has opposite vertices located at $(15,3)$ and $(16,5)$. A line is drawn through points $A(0,0)$ and $B(3,1)$. Another line is drawn through points $C(0,10)$ and $D(2,9)$. How many points on the rectangle lie on at least one of the two lines?

有一个矩形，它的边平行于 x 轴和 y 轴，两个相对的顶点是 $(15,3)$ 和 $(16,5)$ 。通过点 $A(0,0)$ 和 $B(3,1)$ 绘制一条直线。通过点 $C(0,10)$ 和 $D(2,9)$ 绘制另一条直线。问矩形的边界上有多少个点位于两条直线中的至少一条上？



- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

◇ Problem 8

Lola, Lolo, Tiya, and Tiyo participated in a ping pong tournament. Each player competed against each of the other three players exactly twice. Shown below are the win-loss records for the players. The numbers 1 and 0 represent a win or loss, respectively. For example, Lola won five matches and lost the fourth match. What was Tiyo's win-loss record?

Lola, Lolo, Tiya, Tiyo 参加一场乒乓球锦标赛。每名选手与其他三名选手中的每个人比赛恰好两次。下面显示的是选手的胜负记录。数字 1 和 0 分别代表赢或输。例如 Lola 一共赢了五场比赛，第四场比赛输了。问 Tiyo 的胜负记录是怎样的？

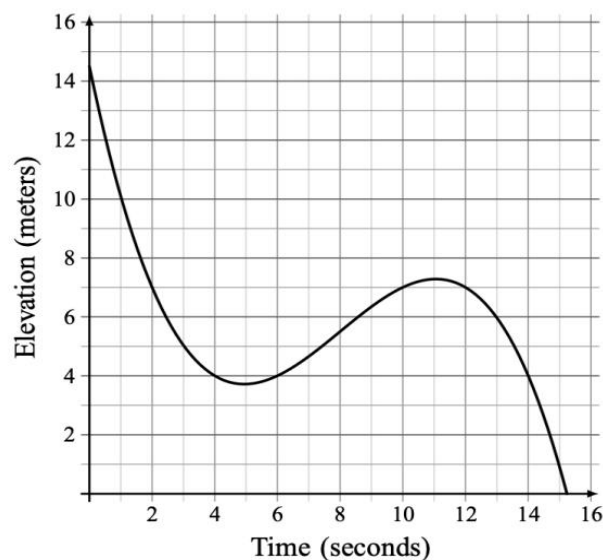
Player	Result
Lola	111011
Lolo	101010
Tiya	010100
Tiyo	??????

- (A) 000101 (B) 001001 (C) 010000 (D) 010101 (E) 011000

◇ Problem 9

Malaika is skiing on a mountain. The graph below shows her elevation, in meters, above the base of the mountain as she skis along a trail. In total, how many seconds does she spend at an elevation between 4 and 7 meters?

Malaika 正在山上滑雪。下图显示了她沿着小径滑雪时以山脚为基准的海拔高度（单位为米）。问她在 4 米到 7 米之间的高度共停留了多少秒？



elevation | 海拔高度, meter | 米, time | 时间, second | 秒

- (A) 6 (B) 8 (C) 10 (D) 12 (E) 14

◇ Problem 10

Harold made a plum pie to take on a picnic. He was able to eat only $\frac{1}{4}$ of the pie, and he left the rest for his friends. A moose came by and ate $\frac{1}{3}$ of what Harold left behind. After that, a porcupine ate $\frac{1}{3}$ of what the moose left behind. How much of the original pie still remained after the porcupine left?

Harold 做了一个李子派去野餐。他只吃掉了这个派的 $\frac{1}{4}$ ，并把剩下的派留给了他的朋友们。一只驼鹿经过并吃掉了 Harold 剩下的派的 $\frac{1}{3}$ 。在那之后，一只豪猪吃了驼鹿剩下的派的 $\frac{1}{3}$ 。问豪猪走后，原来的派还剩下多少？

- (A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{5}{12}$

◇ Problem 11

NASA's Perseverance Rover was launched on July 30, 2020. After traveling 292,526,838 miles, it landed on Mars in Jezero Crater about 6.5 months later. Which of the following is closest to the Rover's average interplanetary speed in miles per hour?

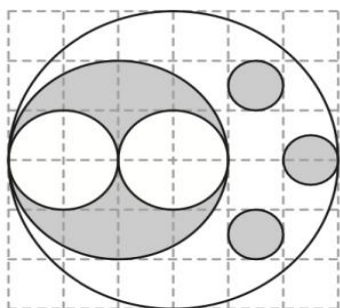
美国国家航空航天局的毅力漫游者号火星探测器于2020年7月30日发射。在飞行了292,526,838英里后，于大约6.5个月后登陆火星，降落在耶泽罗陨石坑。以英里每小时为单位，以下哪个选项最接近毅力漫游者号的平均星际行进速度？

- (A) 6,000 (B) 12,000 (C) 60,000 (D) 120,000 (E) 600,000

◇ Problem 12

The figure below shows a large white circle with a number of smaller white and shaded circles in its interior. What fraction of the interior of the large white circle is shaded?

如图所示，在一个大的白色圆圈内部有许多较小的白色和灰色圆圈。问灰色部分在大的白色圆圈内部所占的比例是多少？



- (A) $\frac{1}{4}$ (B) $\frac{11}{36}$ (C) $\frac{1}{3}$ (D) $\frac{19}{36}$ (E) $\frac{5}{9}$

◇ Problem 13

Along the route of a bicycle race, 7 water stations are evenly spaced between the start and finish lines, as shown in the figure below. There are also 2 repair stations evenly spaced between the start and finish lines. The 3rd water station is located 2 miles after the 1st repair station. How long is the race in miles?

如下图所示，在自行车比赛路线的起点和终点之间均匀分布着 7 个饮用水补给站。另外，在起点和终点之间还均匀分布着 2 个维修站。第三个饮用水补给站在第一个维修站后 2 英里处。问比赛的全程是多少英里？



start | 起始, finish | 结束, water | 水

- (A) 8 (B) 16 (C) 24 (D) 48 (E) 96

◇ Problem 14

Nicolas is planning to send a package to his friend Anton, who is a stamp collector. To pay for the postage, Nicolas would like to cover the package with a large number of stamps. Suppose he has a collection of 5-cent, 10-cent, and 25-cent stamps, with exactly 20 of each type. What is the greatest number of stamps Nicolas can use to make exactly \$7.10 in postage?

(Note: The amount \$7.10 corresponds to 7 dollars and 10 cents. One dollar is worth 100 cents.)

Nicolas 计划将一个包裹寄给爱好集邮的朋友 Anton。为了支付邮资，Nicolas 准备在包裹上贴大量的邮票。假设他有 5 美分，10 美分，25 美分的邮票，每种邮票恰好各有 20 张。Nicolas 最多可以使用多少张邮票来恰好支付 \$7.10 的邮资？

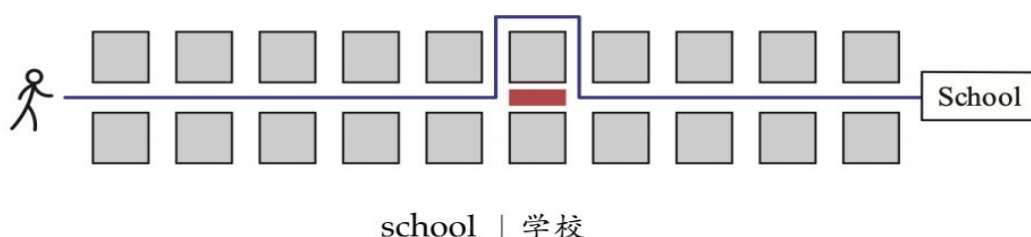
(注意：金额 \$7.10 是指 7 美元 10 美分。1 美元是 100 美分。)

- (A) 45 (B) 46 (C) 51 (D) 54 (E) 55

✧ Problem 15

Viswam walks half a mile to get to school each day. His route consists of 10 city blocks of equal length and he takes one minute to walk each block. Today, after walking 5 blocks, Viswam discovers that he has to make a detour, walking 3 blocks of equal length instead of 1 block to reach the next corner. From the time he starts his detour, at what speed, in miles per hour, must Viswam walk in order to arrive at school at his usual time?

Viswam 每天要步行半英里去上学。他上学的路线由 10 个等长的城市街区组成，每走一个街区需要一分钟。今天，在走了 5 个街区后，Viswam 发现他必须绕路，要走 3 个等长的街区而不是 1 个街区才能到达下一个交叉路口。从他绕行开始，Viswam 必须以每小时多少英里的速度步行才能在与平时同样的时间到达学校？



- (A) 4 (B) 4.2 (C) 4.5 (D) 4.8 (E) 5

✧ Problem 16

The letters P, Q, and R are entered into a 20×20 table according to the pattern shown below. How many Ps, Qs, and Rs will appear in the completed table?

字母 P, Q, R 按如下图所示的规律填入到 20×20 的方格表中。问填好的方格表中将出现多少个 P, 多少个 Q, 多少个 R?

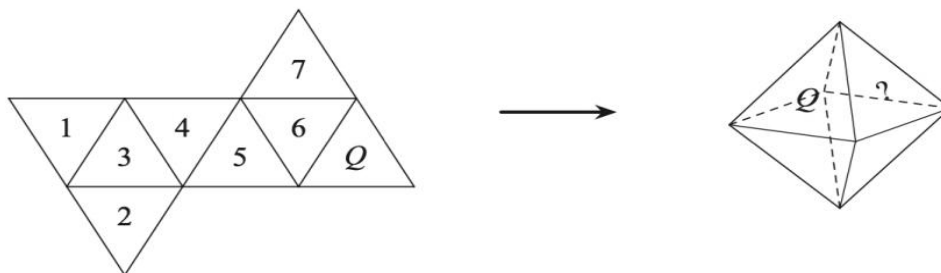
⋮	⋮	⋮	⋮	⋮	⋮
Q	R	P	Q	R	...
P	Q	R	P	Q	...
R	P	Q	R	P	...
Q	R	P	Q	R	...
P	Q	R	P	Q	...

- (A) 132 Ps, 134 Qs, 134 Rs | 132个 P, 134个 Q, 134个 R
 (B) 133 Ps, 133 Qs, 134 Rs | 133个 P, 133个 Q, 134个 R
 (C) 133 Ps, 134 Qs, 133 Rs | 133个 P, 134个 Q, 133个 R
 (D) 134 Ps, 132 Qs, 134 Rs | 134个 P, 132个 Q, 134个 R
 (E) 134 Ps, 133 Qs, 133 Rs | 134个 P, 133个 Q, 133个 R

◇ Problem 17

A regular octahedron has eight equilateral triangle faces with four faces meeting at each vertex. Jun will make the regular octahedron shown on the right by folding the piece of paper shown on the left. Which numbered face will end up to the right of Q ?

正八面体有八个面，都是等边三角形，在每个顶点处有四个面相交。如图所示，Jun 把左边的纸片折成了右边的正八面体。问出现在 Q 的右边的那个面上的数是几？



- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

◇ Problem 18

Greta Grasshopper sits on a long line of lily pads in a pond. From any lily pad, Greta can jump 5 pads to the right or 3 pads to the left. What is the fewest number of jumps Greta must make to reach the lily pad located 2023 pads to the right of her starting position?

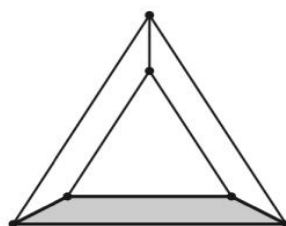
蚱蜢 Greta 坐在池塘里一排长长的睡莲叶上。从任何睡莲叶出发，Greta 都可以向右跳 5 个叶片或者向左跳 3 个叶片。问 Greta 最少跳多少次才能到达位于她起始位置右侧的第 2023 片睡莲叶上？

- (A) 405 (B) 407 (C) 409 (D) 411 (E) 413

◇ Problem 19

An equilateral triangle is placed inside a larger equilateral triangle so that the region between them can be divided into three congruent trapezoids, as shown below. The side length of the inner triangle is $\frac{2}{3}$ the side length of the larger triangle. What is the ratio of the area of one trapezoid to the area of the inner triangle?

如图所示，一个等边三角形在另一个更大的等边三角形内部，它们之间的区域可以分成三个全等的梯形。内部三角形的边长是大三角形边长的 $\frac{2}{3}$ 。问一个梯形的面积与内部三角形的面积之比是多少？



- (A) 1 : 3 (B) 3 : 8 (C) 5 : 12 (D) 7 : 16 (E) 4 : 9

◇ Problem 20

Two integers are inserted into the list 3, 3, 8, 11, 28 to double its range. The mode and median remain unchanged. What is the maximum possible sum of the two additional numbers?

将两个整数加入数据列表 3, 3, 8, 11, 28 使得它的极差加倍, 而众数和中位数保持不变. 问所添加的两个数之和的最大可能值是多少?

- (A) 56 (B) 57 (C) 58 (D) 60 (E) 61

◇ Problem 21

Alina writes the numbers $1, 2, \dots, 9$ on separate cards, one number per card. She wishes to divide the cards into 3 groups of 3 cards so that the sum of the numbers in each group will be the same. In how many ways can this be done?

Alina 将数 1, 2, ..., 9 写在不同的卡片上, 每张卡片上有一个数. 她希望将卡片分成 3 组, 每组 3 张卡片, 使得每组中的各数之和相同. 问有多少种这样的分组方法?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

◇ Problem 22

In a sequence of positive integers, each term after the second is the product of the previous two terms. The sixth term in the sequence is 4000. What is the first term?

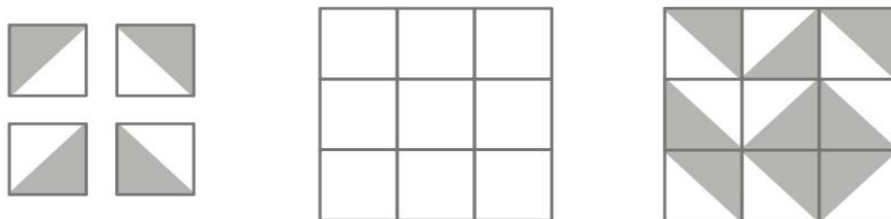
在一个正整数数列中, 第二项之后的每一项都是前面两项的乘积. 数列中的第六项是 4000. 问第一项是多少?

- (A) 1 (B) 2 (C) 4 (D) 5 (E) 10

◇ Problem 23

Each square in a 3×3 grid is randomly filled with one of the 4 gray-and-white tiles shown below on the left. What is the probability that the tiling will contain a large gray diamond in one of the smaller 2×2 grids? An example of such a tiling is on the right.

将 3×3 方格表中的每个小方格随机的用如下图左侧所示的 4 个灰白双色方块之一嵌入。有一个大灰色菱形将出现在某个 2×2 子方格表中的概率是多少？一个这样的镶嵌方案的例子如下图右侧所示。

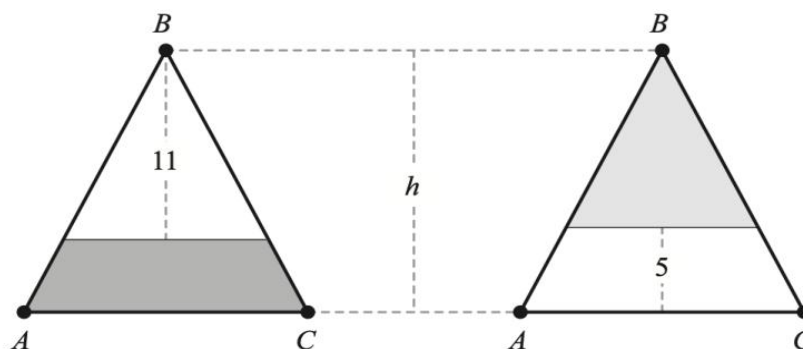


- (A) $\frac{1}{1024}$ (B) $\frac{1}{256}$ (C) $\frac{1}{64}$ (D) $\frac{1}{16}$ (E) $\frac{1}{4}$

◇ Problem 24

Isosceles triangle ABC has equal side lengths AB and BC . In the figures below, segments are drawn parallel to \overline{AC} so that the shaded portions of $\triangle ABC$ have the same area. The heights of the two unshaded portions are 11 and 5 units, respectively. What is the height h of $\triangle ABC$?

在等腰三角形 ABC 中，边 AB 和 BC 的长度相等。在下图中，由平行于 \overline{AC} 的线段形成的 $\triangle ABC$ 中的两个阴影部分具有相同的面积。两个空白部分的高度分别为 11 和 5。问 $\triangle ABC$ 的高 h 是多少？



- (A) 14.6 (B) 14.8 (C) 15 (D) 15.2 (E) 15.4

◇ Problem 25

Fifteen integers $a_1, a_2, a_3, \dots, a_{15}$ are arranged in order on a number line. The integers are equally spaced and have the property that

$$1 \leq a_1 \leq 10, \quad 13 \leq a_2 \leq 20, \quad \text{and} \quad 241 \leq a_{15} \leq 250.$$

What is the sum of the digits of a_{14} ?

十五个整数 $a_1, a_2, a_3, \dots, a_{15}$ 依次排列在数轴上. 这些整数呈等距排列, 并且具有以下特性

$$1 \leq a_1 \leq 10, \quad 13 \leq a_2 \leq 20, \quad 241 \leq a_{15} \leq 250.$$

问 a_{14} 的各位数字之和是多少?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12