2004 AMC 12B Problems/Problem 1

Problem

At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?

$\mathrm{(A)}\ 3 \qquad\mathrm{(B)}\ 6 \qquad\mathrm{(C)}\ 9 \qquad\mathrm{(D)}\ 12 \qquad\mathrm{(E)}\ 15$

Solution

Each day Jenny makes half as many free throws as she does at the next practice. Hence on the fourth day she made $\frac{1}{2} \cdot 48 = 24$ free throws, on the third 12, on the second 6, and on the first $3 \Rightarrow \mathrm{(A)}$.

Because there are five days, or four transformations between days (day 1 $\longrightarrow$ day 2 $\longrightarrow$ day 3 $\longrightarrow$ day 4 $\longrightarrow$ day 5), she makes $48 \cdot \frac{1}{2^4} = \boxed{\mathrm{(A)}\ 3}$

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by First Question	Followed by Problem 2
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Category: Introductory Algebra Problems

2004 AMC 12B Problems/Problem 2

The following problem is from both the 2004 AMC 12B #2 and 2004 AMC 10B #5, so both problems redirect to this page.

Problem 2

In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

$\mathrm{(A)}\ 5\qquad\mathrm{(B)}\ 6\qquad\mathrm{(C)}\ 8\qquad\mathrm{(D)}\ 9\qquad\mathrm{(E)}\ 10$

Solution

If $a=0$ or $c=0$, the expression evaluates to $-d<0$.
If $b=0$, the expression evaluates to $c-d\le 2$.
Case $d=0$ remains. In that case, we want to maximize $c \cdot a^b$ where $\{a,b,c\}=\{1,2,3\}$. Trying out the six possibilities we get that the best one is $(a,b,c)=(3,2,1)$, where $c\cdot a^b = 1\cdot 3^2 = \boxed{\mathrm{(D)}\ 9}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 1	Followed by Problem 3
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Preceded by Problem 4	Followed by Problem 6
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2004 AMC 12B Problems/Problem 3

Problem

If x and y are positive integers for which $2^x3^y = 1296$, what is the value of $x+y$?

$\mathrm{(A)}\ 8\qquad\mathrm{(B)}\ 9\qquad\mathrm{(C)}\ 10\qquad\mathrm{(D)}\ 11\qquad\mathrm{(E)}\ 12$

Solution

$1296 = 2^4 3^4$ and $4+4=\boxed{8}\Longrightarrow\mathrm{(A)}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 2	Followed by Problem 4
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2004 AMC 12B Problems/Problem 4

Problem

An integer x, with $10 \leq x \leq 99$, is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is a 7?

$\mathrm{(A)}\ \frac{1}{9} \qquad\mathrm{(B)}\ \frac{1}{5} \qquad\mathrm{(C)}\ \frac{19}{90} \qquad\mathrm{(D)}\ \frac{2}{9} \qquad\mathrm{(E)}\ \frac{1}{3}$

Solution

The digit 7 can be either the tens digit ($70, 71, \dots, 79$: 10 possibilities), or the ones digit ($17, 27, \dots, 97$: 9 possibilities), but we counted the number 77 twice. This means that out of the 90 two-digit numbers, $10 + 9 - 1 = 18$ have at least one digit equal to 7. Therefore the probability is $\frac{18}{90} = \boxed{\frac{1}{5}} \Longrightarrow \mathrm{(B)}$.

By complementary counting, we count the numbers that do not contain a 7, then subtract from the total. There is a $\frac{8}{9} \cdot \frac{9}{10}$ probability of choosing a number that does NOT contain a 7. Subtract this from 1 and simplify yields $1 - \frac{8}{9} \cdot \frac{9}{10} = \frac{90}{90} - \frac{72}{90} = \frac{18}{90} = \frac{1}{5}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 3	Followed by Problem 5
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2004 AMC 12B Problems/Problem 5

The following problem is from both the 2004 AMC 12B #5 and 2004 AMC 10B #7, so both problems redirect to this page.

Contents

Problem

On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

$\text{(A)}\ 5\qquad\text{(B)}\ 6\qquad\text{(C)}\ 7\qquad\text{(D)}\ 8\qquad\text{(E)}\ 9$

Solution 1

Isabella had $60+d$ Canadian dollars. Setting up an equation we get $d=\frac{7}{10}\cdot(60+d)$, which solves to $d=140$, and the sum of digits of d is $\boxed{\text{(A)}\ 5}$.

Solution 2

Each time Isabella exchanges 7 U.S. dollars, she gets 7 Canadian dollars and 3 Canadian dollars extra. Isabella received a total of 60 Canadian dollars extra, therefore she exchanged 7 U.S. dollars $\frac{60}{3}=20$ times. Thus $d=7\cdot 20=\boxed{\text{(A)}\ 5}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 4	Followed by Problem 6
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2004 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
Preceded by Problem 6	Followed by Problem 8
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2004 AMC 12B Problems/Problem 6

The following problem is from both the 2004 AMC 12B #6 and 2004 AMC 10B #8, so both problems redirect to this page.

Problem

Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and downtown Minneapolis?

$\mathrm{(A)}\ 13\qquad\mathrm{(B)}\ 14\qquad\mathrm{(C)}\ 15\qquad\mathrm{(D)}\ 16\qquad\mathrm{(E)}\ 17$

Solution

The directions "southwest" and "southeast" are orthogonal. Thus the described situation is a right triangle with legs 8 miles and 10 miles long. The hypotenuse length is $\sqrt{8^2+10^2}\approx 12.8$, and thus the answer is $\boxed{\mathrm{(A)}\ 13}$.

Without a calculator one can note that $8^2+10^2=164<169=13^2\Rightarrow\boxed{\mathrm{(A)}\ 13}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 5	Followed by Problem 7
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2004 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
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2004 AMC 12B Problems/Problem 7

The following problem is from both the 2004 AMC 12B #7 and 2004 AMC 10B #9, so both problems redirect to this page.

Problem

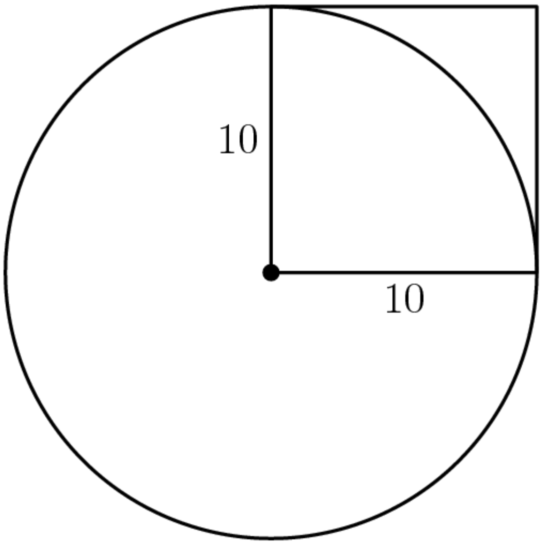
A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?

$\mathrm{(A)\ }200+25\pi\quad\mathrm{(B)\ }100+75\pi\quad\mathrm{(C)\ }75+100\pi\quad\mathrm{(D)\ }100+100\pi\quad\mathrm{(E)\ }100+125\pi$

Solution

The area of the circle is $S_{\bigcirc}=100\pi$; the area of the square is $S_{\square}=100$.

Exactly $\frac{1}{4}$ of the circle lies inside the square. Thus the total area is $\frac{3}{4}S_{\bigcirc}+S_{\square}=\boxed{\mathrm{(B)\ }100+75\pi}$.



See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 6	Followed by Problem 8
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2004 AMC 12B Problems/Problem 8

> The following problem is from both the 2004 AMC 12B #8 and 2004 AMC 10B #10, so both problems redirect to this page.

Problem

A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?

$\mathrm{(A)}\ 5\qquad\mathrm{(B)}\ 8\qquad\mathrm{(C)}\ 9\qquad\mathrm{(D)}\ 10\qquad\mathrm{(E)}\ 11$

Solution

The sum of the first n odd numbers is n^2. As in our case $n^2=100$, we have $n=\boxed{\mathrm{(D)}\ 10}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 7	Followed by Problem 9
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2004 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
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2004 AMC 12B Problems/Problem 9

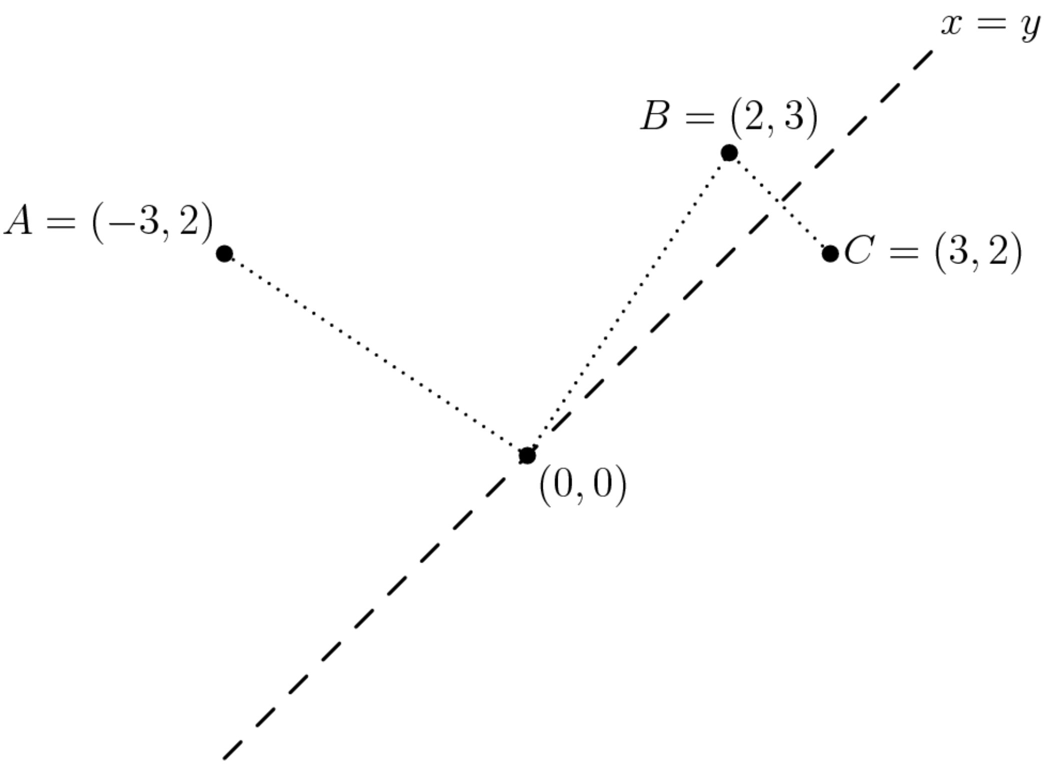
Problem

The point $(-3,2)$ is rotated 90° clockwise around the origin to point B. Point B is then reflected over the line $x=y$ to point C. What are the coordinates of C?

$\mathrm{(A)}\ (-3,-2) \qquad \mathrm{(B)}\ (-2,-3) \qquad \mathrm{(C)}\ (2,-3) \qquad \mathrm{(D)}\ (2,3) \qquad \mathrm{(E)}\ (3,2)$

Solution

The entire situation is in the picture below. The correct answer is $\boxed{\mathrm{(E)}\ (3,2)}$.



See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 8	Followed by Problem 10
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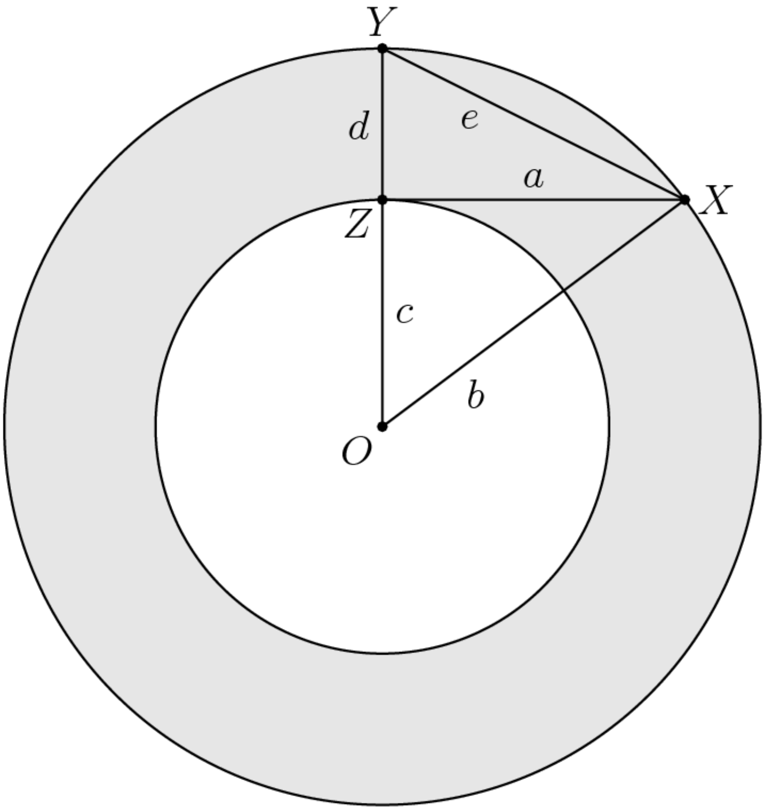
2004 AMC 12B Problems/Problem 10

The following problem is from both the 2004 AMC 12B #10 and 2004 AMC 10B #12, so both problems redirect to this page.

Problem

An annulus is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b > c$. Let OX be a radius of the larger circle, let XZ be tangent to the smaller circle at Z, and let OY be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?



$\mathrm{(A)}\ \pi a^2 \qquad \mathrm{(B)}\ \pi b^2 \qquad \mathrm{(C)}\ \pi c^2 \qquad \mathrm{(D)}\ \pi d^2 \qquad \mathrm{(E)}\ \pi e^2$

Solution

The area of the large circle is πb^2, the area of the small one is πc^2, hence the shaded area is $\pi(b^2 - c^2)$.

From the Pythagorean Theorem for the right triangle OXZ we have $a^2 + c^2 = b^2$, hence $b^2 - c^2 = a^2$ and thus the shaded area is $\boxed{\mathrm{(A)}\ \pi a^2}$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 9	Followed by Problem 11
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2004 AMC 12B Problems/Problem 11

Problem

All the students in an algebra class took a 100-point test. Five students scored 100, each student scored at least 60, and the mean score was 76. What is the smallest possible number of students in the class?

$\mathrm{(A)}\ 10 \qquad \mathrm{(B)}\ 11 \qquad \mathrm{(C)}\ 12 \qquad \mathrm{(D)}\ 13 \qquad \mathrm{(E)}\ 14$

Solution

Let the number of students be $n\geq 5$. Then the sum of their scores is at least $5\cdot 100 + (n-5)\cdot 60$. At the same time, we need to achieve the mean 76, which is equivalent to achieving the sum $76n$.

Hence we get a necessary condition on n: we must have $5\cdot 100 + (n-5)\cdot 60 \leq 76n$. This can be simplified to $200 \leq 16n$. The smallest integer n for which this is true is $n=13$.

To finish our solution, we now need to find one way how 13 students could have scored on the test. We have $13\cdot 76 = 988$ points to divide among them. The five 100s make 500, hence we must divide the remaining 488 points among the other 8 students. This can be done e.g. by giving 61 points to each of them.

Hence the smallest possible number of students is $\boxed{\mathrm{(D)}\ 13}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 12

The following problem is from both the 2004 AMC 12B #12 and 2004 AMC 10B #19, so both problems redirect to this page.

Contents

Problem

In the sequence $2001, 2002, 2003, \ldots$, each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

$\mathrm{(A)}\ -2004 \qquad \mathrm{(B)}\ -2 \qquad \mathrm{(C)}\ 0 \qquad \mathrm{(D)}\ 4003 \qquad \mathrm{(E)}\ 6007$

Solution

Solution 1

We already know that $a_1 = 2001$, $a_2 = 2002$, $a_3 = 2003$, and $a_4 = 2000$. Let's compute the next few terms to get the idea how the sequence behaves. We get $a_5 = 2002 + 2003 - 2000 = 2005$, $a_6 = 2003 + 2000 - 2005 = 1998$, $a_7 = 2000 + 2005 - 1998 = 2007$, and so on.

We can now discover the following pattern: $a_{2k+1} = 1999 + 2k$ and $a_{2k} = 2004 - 2k$. This is easily proved by induction. It follows that $a_{2004} = a_{2\cdot 1002} = 2004 - 2\cdot 1002 = \boxed{0}$.

Solution 2

Note that the recurrence $a_n + a_{n+1} - a_{n+2} \quad = \quad a_{n+3}$ can be rewritten as $a_n + a_{n+1} \quad = \quad a_{n+2} + a_{n+3}$.

Hence we get that $a_1 + a_2 \quad = \quad a_3 + a_4 \quad = \quad a_5 + a_6 \quad = \cdots$ and also $a_2 + a_3 \quad = \quad a_4 + a_5 \quad = \quad a_6 + a_7 \quad = \cdots$

From the values given in the problem statement we see that $a_3 = a_1 + 2$.

From $a_1 + a_2 = a_3 + a_4$ we get that $a_4 = a_2 - 2$.

From $a_2 + a_3 = a_4 + a_5$ we get that $a_5 = a_3 + 2$.

Following this pattern, we get $a_{2004} = a_{2002} - 2 = a_{2000} - 4 = \cdots = a_2 - 2002 = \boxed{0}$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 13

Problem

If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$ with a and b real, what is the value of $a + b$?

$\mathrm{(A)}\ -2 \qquad \mathrm{(B)}\ -1 \qquad \mathrm{(C)}\ 0 \qquad \mathrm{(D)}\ 1 \qquad \mathrm{(E)}\ 2$

Solution

By the definition of an inverse function, $x = f(f^{-1}(x)) = a(bx + a) + b = abx + a^2 + b$. By comparing coefficients, we have $ab = 1 \Longrightarrow b = \frac{1}{a}$ and $a^2 + b = a^2 + \frac{1}{a} = 0$. Simplifying,

$$a^3 + 1 = 0$$

and $a = b = -1$. Thus $a + b = -2 \Rightarrow \mathrm{(A)}$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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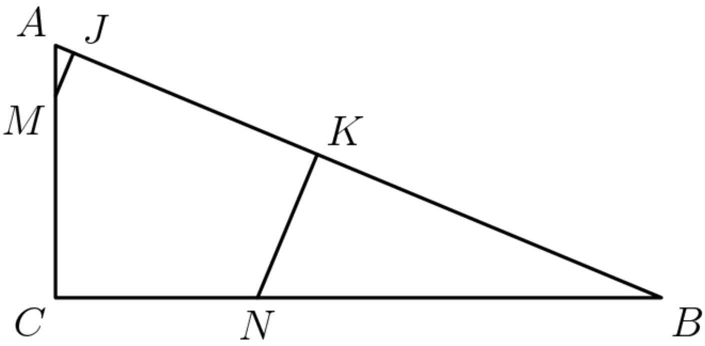
Category: Introductory Algebra Problems

2004 AMC 12B Problems/Problem 14

Contents

Problem

In $\triangle ABC$, $AB = 13$, $AC = 5$, and $BC = 12$. Points M and N lie on AC and BC, respectively, with $CM = CN = 4$. Points J and K are on AB so that MJ and NK are perpendicular to AB. What is the area of pentagon $CMJKN$?



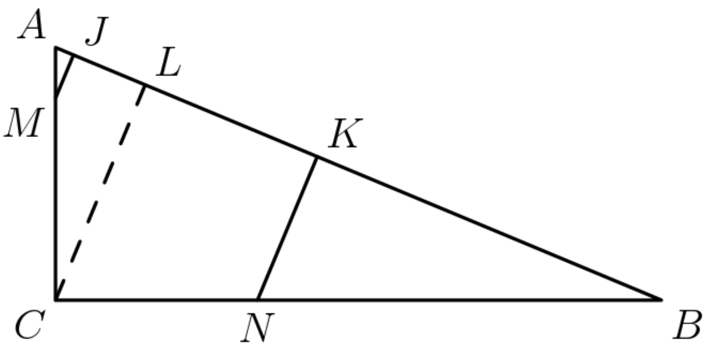
$\mathrm{(A)}\ 15 \qquad\mathrm{(B)}\ \frac{81}{5} \qquad\mathrm{(C)}\ \frac{205}{12} \qquad\mathrm{(D)}\ \frac{240}{13} \qquad\mathrm{(E)}\ 20$

Solution

Solution 1

The triangle ABC is clearly a right triangle, its area is $\frac{5\cdot 12}{2}=30$. If we knew the areas of triangles AMJ and BNK, we could subtract them to get the area of the pentagon.

Draw the height CL from C onto AB. As $AB=13$ and the area is 30, we get $CL=\frac{60}{13}$. The situation is shown in the picture below:



Now note that the triangles ABC, AMJ, ACL, CBL and NBK all have the same angles and therefore they are similar. We already know some of their sides, and we will use this information to compute their areas. Note that if two polygons are similar with ratio k, their areas have ratio k^2. We will use this fact repeatedly. Below we will use $[XYZ]$ to denote the area of the triangle XYZ.

We have $\frac{CL}{BC} = \frac{60/13}{12} = \frac{5}{13}$, hence $[ACL] = \frac{25[ABC]}{169} = \frac{750}{169}$.

Also, $\frac{CL}{AC} = \frac{60/13}{5} = \frac{12}{13}$, hence $[CBL] = \frac{144[ABC]}{169} = \frac{4320}{169}$.

Now for the smaller triangles:

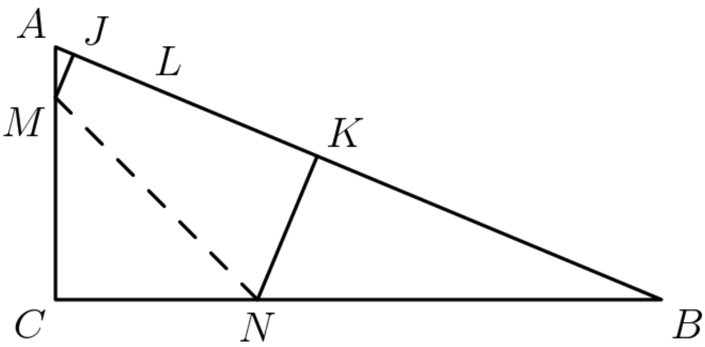
We know that $\frac{AM}{AC} = \frac{1}{5}$, hence $[AMJ] = \frac{[ACL]}{25} = \frac{30}{169}$.

Similarly, $\frac{BN}{BC} = \frac{8}{12} = \frac{2}{3}$, hence $[NBK] = \frac{4[CBL]}{9} = \frac{1920}{169}$.

Finally, the area of the pentagon is $30 - \frac{30}{169} - \frac{1920}{169} = \boxed{\frac{240}{13}}$.

Solution 2

Split the pentagon along a different diagonal as follows:



The area of the pentagon is then the sum of the areas of the resulting right triangle and trapezoid. As before, triangles ABC, AMJ, and NBK are all similar.

Since $BN = 12 - 4 = 8$, $NK = \frac{5}{13}(8) = \frac{40}{13}$ and $BK = \frac{12}{13}(8) = \frac{96}{13}$. Since $AM = 5 - 4 = 1$, $JM = \frac{12}{13}$ and $AJ = \frac{5}{13}$.

The trapezoid's height is therefore $13 - \frac{5}{13} - \frac{96}{13} = \frac{68}{13}$, and its area is $\frac{1}{2}\left(\frac{68}{13}\right)\left(\frac{12}{13} + \frac{40}{13}\right) = \frac{34}{13}(4) = \frac{136}{13}$.

Triangle MCN has area $\frac{1}{2}(4)(4) = 8$, and the total area is $\frac{104 + 136}{13} = \boxed{\frac{240}{13}}$.

Solution 3

Because triangle ABC, triangle NBK, and triangle AMJ are similar right triangles whose hypotenuses are in the ratio 13 : 8 : 1, their areas are in the ratio 169 : 64 : 1. The area of triangle ABC is 1/2 (12)(5) = 30, so the areas of triangle NBK and triangle AMJ are (64/169) (30) and (1/169)(30), respectively. Thus the area of pentagon CMJKN is (1 − 64/169 - 1/169)(30)= $\boxed{\textbf{(D)} 240/13}$

Credit to
http://billingswest.billings.k12.mt.us/math/AMC%201012/AMC%2012%20work%20sheets/2004%20AMC%2012B%20ws-15.pdf
for Solution 3.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 15

The following problem is from both the 2004 AMC 12B #15 and 2004 AMC 10B #17, so both problems redirect to this page.

Contents

Problem

The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?

$\mathrm{(A) \ } 9 \qquad \mathrm{(B) \ } 18 \qquad \mathrm{(C) \ } 27 \qquad \mathrm{(D) \ } 36\qquad \mathrm{(E) \ } 45$

Solution 1

If Jack's current age is $\overline{ab}=10a+b$, then Bill's current age is $\overline{ba}=10b+a$.

In five years, Jack's age will be $10a+b+5$ and Bill's age will be $10b+a+5$.

We are given that $10a+b+5=2(10b+a+5)$. Thus $8a = 19b+5$.

For $b=1$ we get $a=3$. For $b=2$ and $b=3$ the value $\frac{19b+5}{8}$ is not an integer, and for $b\geq 4$ it is more than 9. Thus the only solution is $(a,b)=(3,1)$, and the difference in ages is $31-13=\boxed{\mathrm{(B)\ }18}$.

Solution 2

Age difference does not change in time. Thus in five years Bill's age will be equal to their age difference.

The age difference is $(10a+b)-(10b+a)=9(a-b)$, hence it is a multiple of 9. Thus Bill's current age modulo 9 must be 4.

Thus Bill's age is in the set $\{13,22,31,40,49,58,67,76,85,94\}$.

As Jack is older, we only need to consider the cases where the tens digit of Bill's age is smaller than the ones digit. This leaves us with the options $\{13,49,58,67\}$.

Checking each of them, we see that only 13 works, and gives the solution $31-13=\boxed{\mathrm{(B)\ }18}$.

See also

2004 AMC 12B Problems/Problem 16

Problem

A function f is defined by $f(z) = i\overline{z}$, where $i = \sqrt{-1}$ and $\overline{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?

$\mathrm{(A)}\ 0\qquad\mathrm{(B)}\ 1\qquad\mathrm{(C)}\ 2\qquad\mathrm{(D)}\ 4\qquad\mathrm{(E)}\ 8$

Solution

Let $z = a + bi$, so $\overline{z} = a - bi$. By definition, $z = a + bi = f(z) = i(a - bi) = b + ai$, which implies that all solutions to $f(z) = z$ lie on the line $y = x$ on the complex plane. The graph of $|z| = 5$ is a circle centered at the origin, and there are $2 \Rightarrow \mathrm{(C)}$ intersections.

See also

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Category: Introductory Algebra Problems

2004 AMC 12B Problems/Problem 17

Problem

For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots. If the sum of the base-2 logarithms of the roots is 5, what is the value of a?

$\mathrm{(A)}\ -256 \qquad \mathrm{(B)}\ -64 \qquad \mathrm{(C)}\ -8 \qquad \mathrm{(D)}\ 64 \qquad \mathrm{(E)}\ 256$

Solution

Let the three roots be x_1, x_2, x_3.

$$\log_2 x_1 + \log_2 x_2 + \log_2 x_3 = \log_2 x_1x_2x_3 = 5 \Longrightarrow x_1x_2x_3 = 32$$

By Vieta's formulas,

$$8(x-x_1)(x-x_2)(x-x_3) = 8x^3 + 4ax^2 + 2bx + a$$

gives us that $a = -8x_1x_2x_3 = -256 \Rightarrow \mathrm{(A)}$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 18

Contents

Problem

Points A and B are on the parabola $y=4x^2+7x-1$, and the origin is the midpoint of AB. What is the length of AB?

$$\mathrm{(A)}\ 2\sqrt5 \qquad \mathrm{(B)}\ 5+\frac{\sqrt2}{2} \qquad \mathrm{(C)}\ 5+\sqrt2 \qquad \mathrm{(D)}\ 7 \qquad \mathrm{(E)}\ 5\sqrt2$$

Solution

Let the coordinates of A be (x_A,y_A). As A lies on the parabola, we have $y_A=4x_A^2+7x_A-1$. As the origin is the midpoint of AB, the coordinates of B are $(-x_A,-y_A)$. We need to choose x_A so that B will lie on the parabola as well. In other words, we need $-y_A = 4(-x_A)^2 + 7(-x_A) - 1$.

Substituting for y_A, we get: $-4x_A^2 - 7x_A + 1 = 4(-x_A)^2 + 7(-x_A) - 1$.

This simplifies to $8x_A^2 - 2 = 0$, which solves to $x_A = \pm 1/2$. Both roots lead to the same pair of points: $(1/2,7/2)$ and $(-1/2,-7/2)$. Their distance is $\sqrt{1^2+7^2} = \sqrt{50} = \boxed{5\sqrt2}$.

Alternate Solution

Let the coordinates of A and B be (x_A,y_A) and (x_B,y_B), respectively. Since the median of the points lies on the origin, $x_A+x_B=y_A+y_B=0$ and expanding y_A+y_B, we find:

$$4x_A^2+7x_A-1+4x_B^2+7x_B-1=0$$

$$4(x_A^2+x_B^2)+7(x_A+x_B)=2$$

$$x_A^2+x_B^2=\frac{1}{2}.$$

It also follows that $(x_A+x_B)^2=0$. Expanding this, we find:

$$x_A^2+2x_Ax_B+x_B^2=0$$

$$\frac{1}{2}+2x_Ax_B=0$$

$$x_Ax_B=-\frac{1}{4}.$$

To find the distance between the points, $\sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$ must be found. Expanding $y_A - y_B$:

$$y_A - y_B = 4x_A^2 + 7x_A - 1 - 4x_B^2 - 7x_B + 1$$

$$= 4(x_A^2 - x_B^2) + 7(x_A - x_B)$$

$$= 4(x_A + x_B)(x_A - x_B) + 7(x_A - x_B)$$

$$= 7(x_A - x_B)$$

we find the distance to be $\sqrt{50(x_A - x_B)^2}$. Expanding this yields $5\sqrt{2(x_A^2 + x_B^2 - 2x_Ax_B)} = \boxed{5\sqrt{2}}$.

See Also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 19

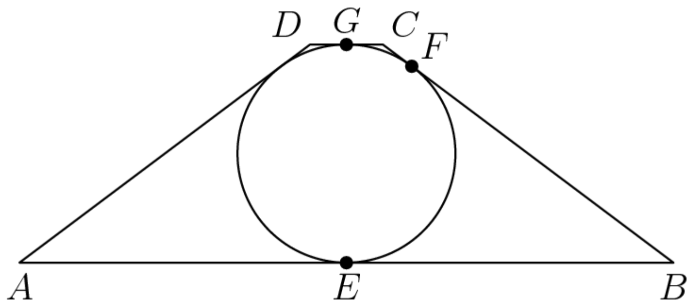
Problem

A truncated cone has horizontal bases with radii 18 and 2. A sphere is tangent to the top, bottom, and lateral surface of the truncated cone. What is the radius of the sphere?

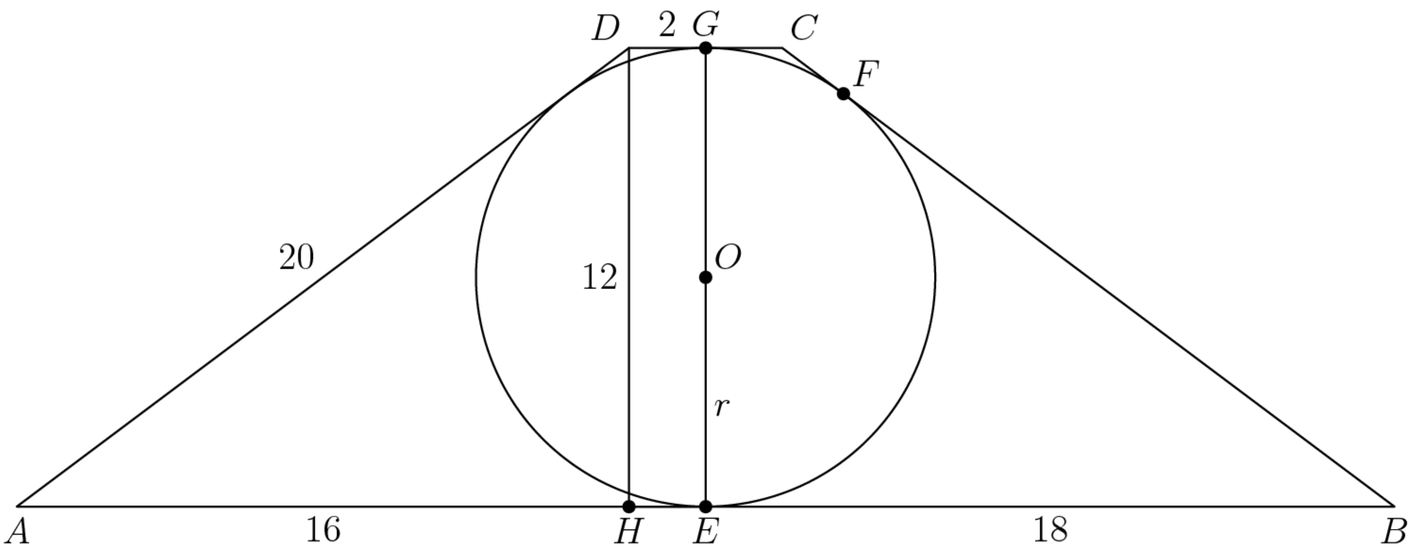
$\mathrm{(A)}\ 6 \qquad\mathrm{(B)}\ 4\sqrt{5} \qquad\mathrm{(C)}\ 9 \qquad\mathrm{(D)}\ 10 \qquad\mathrm{(E)}\ 6\sqrt{3}$

Solution

Consider a trapezoidal (label it $ABCD$ as follows) cross-section of the truncate cone along a diameter of the bases:



Above, $E, F,$ and G are points of tangency. By the Two Tangent Theorem, $BF = BE = 18$ and $CF = CG = 2$, so $BC = 20$. We draw H such that it is the foot of the altitude $\overline{HD}$ to $\overline{AB}$:



By the Pythagorean Theorem,

$$r = \frac{DH}{2} = \frac{\sqrt{20^2 - 16^2}}{2} = \boxed{6} \Rightarrow \mathrm{(A)}.$$

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 20

Problem

Each face of a cube is painted either red or blue, each with probability $1/2$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

$\textbf{(A)}\ \frac14 \qquad \textbf{(B)}\ \frac5{16} \qquad \textbf{(C)}\ \frac38 \qquad \textbf{(D)}\ \frac7{16} \qquad \textbf{(E)}\ \frac12$

Solution

There are 2^6 possible colorings of the cube. Consider the color that appears with greater frequency. The property obviously holds true if 5 or 6 of the faces are colored the same, which for each color can happen in $6+1=7$ ways. If 4 of the faces are colored the same, there are 3 possible cubes (corresponding to the 3 possible ways to pick pairs of opposite faces for the other color). If 3 of the faces are colored the same, the property obviously cannot be satisfied. Thus, there are a total of $2(7+3) = 20$ ways for this to occur, and the desired probability is $\frac{20}{2^6} = \frac{5}{16}\ \mathrm{(B)}$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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Category: Introductory Combinatorics Problems

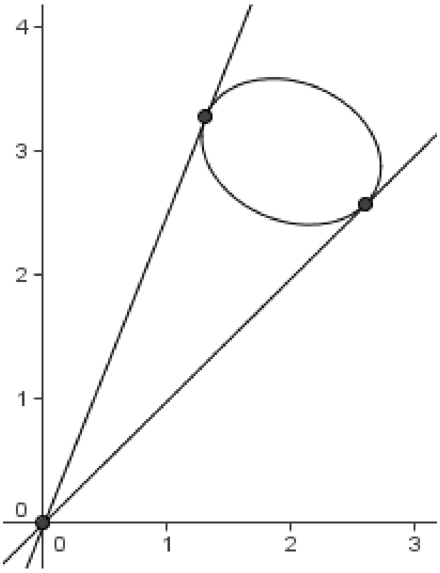
2004 AMC 12B Problems/Problem 21

Problem

The graph of $2x^2 + xy + 3y^2 - 11x - 20y + 40 = 0$ is an ellipse in the first quadrant of the xy-plane. Let a and b be the maximum and minimum values of $\frac{y}{x}$ over all points (x, y) on the ellipse. What is the value of $a + b$?

$\mathrm{(A)}\ 3 \qquad \mathrm{(B)}\ \sqrt{10} \qquad \mathrm{(C)}\ \frac{7}{2} \qquad \mathrm{(D)}\ \frac{9}{2} \qquad \mathrm{(E)}\ 2\sqrt{14}$

Solution



$\frac{y}{x}$ represents the slope of a line passing through the origin. It follows that since a line $y = mx$ intersects the ellipse at either $0, 1,$ or 2 points, the minimum and maximum are given when the line $y = mx$ is a tangent, with only one point of intersection. Substituting,

$$2x^2 + x(mx) + 3(mx)^2 - 11x - 20(mx) + 40 = 0$$

Rearranging by the degree of x,

$$(3m^2 + m + 2)x^2 - (20m + 11)x + 40 = 0$$

Since the line $y = mx$, we want the discriminant,

$$(20m + 11)^2 - 4 \cdot 40 \cdot (3m^2 + m + 2) = -80m^2 + 280m - 199$$

to be equal to 0. We want $a + b$, which is the sum of the roots of the above quadratic. By Vieta's formulas, that is $\frac{280}{80} = \frac{7}{2} \Rightarrow \mathrm{(C)}$.

See also

2004 AMC 12B Problems/Problem 22

Contents

Problem

The square

$$\begin{array}{|c|c|c|}\hline 50 & b & c \\ \hline d & e & f \\ \hline g & h & 2 \\ \hline \end{array}$$

is a multiplicative magic square. That is, the product of the numbers in each row, column, and diagonal is the same. If all the entries are positive integers, what is the sum of the possible values of g?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 136$

Solution A

If the power of a prime p^n other than $2,5$ divides g, then from $50 \cdot 2e = 50dg$ it follows that $p^n|e$, but then considering the product of the diagonals, $p^{2n}|gec$ but $p^{2n} \nmid 100e$, contradiction. So the only prime factors of g are 2 and 5.

It suffices now to consider the two magic squares comprised of the powers of 2 and 5 of the corresponding terms. These satisfy the normal requirement that the sums of rows, columns, and diagonals are the same, owing to our rules of exponents; additionally, all terms are non-negative.

The powers of 2:

$$\begin{array}{|c|c|c|}\hline 1 & b & c \\ \hline d & e & f \\ \hline g & h & 1 \\ \hline \end{array}$$

So $1 + 1 + e = g + e + c \Longrightarrow g = 2 - c$, so $g = 0, 1, 2$. Indeed, we have the magic squares

$$\begin{array}{|c|c|c|}\hline 1 & 0 & 2 \\ \hline 2 & 1 & 0 \\ \hline 0 & 2 & 1 \\ \hline \end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 0 \\ \hline 0 & 1 & 2 \\ \hline 2 & 0 & 1 \\ \hline \end{array},$$

The powers of 5:

$$\begin{array}{|c|c|c|}\hline 2 & b & c \\ \hline d & e & f \\ \hline g & h & 0 \\ \hline \end{array}$$

Again, we get $2 + e = g + e + c \Longrightarrow g = 0,1,2$. However, if we let $g = 2, c = 0$, then $e = d + e + f \Longrightarrow d = f = 0$, which obviously gives us a contradiction, and similarly for $g = 0, c = 2$. For $g = 1$, we get

2	0	1
0	1	2
1	2	0

In conclusion, g can be $2^0 \cdot 5^1, 2^1 \cdot 5^1, 2^2 \cdot 5^1$, and their sum is $\boxed{(\mathbf{C})35}$.

Solution B

All the unknown entries can be expressed in terms of b. Since 100e = beh = ceg = def, it follows that h = 100/b, g = 100/c, and f = 100/d. Comparing rows 1 and 3 then gives 50bc = 2 * 100/b * 100/c, from which c = 20/b. Comparing columns 1 and 3 gives 50d*100/c= 2c*100/d, from which d = c/5 = 4/b. Finally, f = 25b, g = 5b, and e = 10. All the entries are positive integers if and only if b = 1, 2, or 4. The corresponding values for g are 5, 10, and 20, and their sum is $\boxed{(\mathbf{C})35}$.

Credit to Solution B goes to http://billingswest.billings.k12.mt.us/math/AMC%201012/AMC%2012%20work%20sheets/2004%20AMC%2012B%20ws-15.pdf, a page with a play-by-play explanation of the solutions to this test's problems.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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Category: Intermediate Algebra Problems

2004 AMC 12B Problems/Problem 23

Problem

The polynomial $x^3 - 2004x^2 + mx + n$ has integer coefficients and three distinct positive zeros. Exactly one of these is an integer, and it is the sum of the other two. How many values of n are possible?

$\mathrm{(A)}\ 250{,}000 \qquad \mathrm{(B)}\ 250{,}250 \qquad \mathrm{(C)}\ 250{,}500 \qquad \mathrm{(D)}\ 250{,}750 \qquad \mathrm{(E)}\ 251{,}000$

Solution

Let the roots be $r, s, r+s$, and let $t = rs$. Then

$$(x-r)(x-s)(x-(r+s))$$
$$= x^3 - (r+s+r+s)x^2 + (rs + r(r+s) + s(r+s))x - rs(r+s) = 0$$

and by matching coefficients, $2(r+s) = 2004 \Longrightarrow r+s = 1002$. Then our polynomial looks like

$$x^3 - 2004x^2 + (t + 1002^2)x - 1002t = 0$$

and we need the number of possible products $t = rs = r(1002-r)$.

Since $r > 0$ and $t > 0$, it follows that $0 < t = r(1002-r) < 501^2 = 251001$, with the endpoints not achievable because the roots must be distinct. Because r cannot be an integer, there are $251000 - 500 = 250{,}500\ \mathrm{(C)}$ possible values of $n = -1002t$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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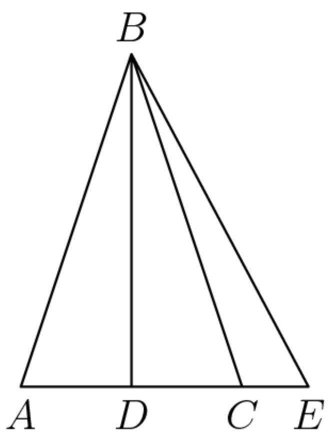
Category: Intermediate Algebra Problems

2004 AMC 12B Problems/Problem 24

Problem

In $\triangle ABC$, $AB = BC$, and $\overline{BD}$ is an altitude. Point E is on the extension of $\overline{AC}$ such that $BE = 10$. The values of $\tan \angle CBE$, $\tan \angle DBE$, and $\tan \angle ABE$ form a geometric progression, and the values of $\cot \angle DBE, \cot \angle CBE, \cot \angle DBC$ form an arithmetic progression. What is the area of $\triangle ABC$?



$\text{(A)}\ 16 \qquad \text{(B)}\ \frac{50}{3} \qquad \text{(C)}\ 10\sqrt{3} \qquad \text{(D)}\ 8\sqrt{5} \qquad \text{(E)}\ 18$

Solution

Let $\alpha = DBC$. Then the first condition tells us that

$$\tan^2 DBE = \tan(DBE-\alpha)\tan(DBE+\alpha) = \frac{\tan^2 DBE - \tan^2 \alpha}{1 - \tan^2 DBE \tan^2 \alpha},$$

and multiplying out gives us $(\tan^4 DBE - 1)\tan^2 \alpha = 0$. Since $\tan \alpha \neq 0$, we have $\tan^4 DBE = 1 \implies \angle DBE = 45^\circ$.

The second condition tells us that $2\cot(45-\alpha) = 1 + \cot \alpha$. Expanding, we have $1 + \cot \alpha = 2\left[\frac{\cot \alpha + 1}{\cot \alpha - 1}\right] \implies (\cot \alpha - 3)(\cot \alpha + 1) = 0$. Evidently $\cot \alpha \neq -1$, so we get $\cot \alpha = 3$.

Now $BD = 5\sqrt{2}$ and $AC = \frac{2BD}{\cot \alpha} = \frac{10\sqrt{2}}{3}$. Thus, $[ABC] = \frac{1}{2} \cdot 5\sqrt{2} \cdot \frac{10\sqrt{2}}{3} = \frac{50}{3}\ \text{(B)}$.

See also

2004 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12B Problems/Problem 25

Contents

Problem

Given that 2^{2004} is a 604-digit number whose first digit is 1, how many elements of the set $S = \{2^0, 2^1, 2^2, \ldots, 2^{2003}\}$ have a first digit of 4?

$\mathrm{(A)}\ 194 \qquad \mathrm{(B)}\ 195 \qquad \mathrm{(C)}\ 196 \qquad \mathrm{(D)}\ 197 \qquad \mathrm{(E)}\ 198$

Solution 1

Given n digits, there must be exactly one power of 2 with n digits such that the first digit is 1. Thus S contains 603 elements with a first digit of 1. For each number in the form of 2^k such that its first digit is 1, then 2^{k+1} must either have a first digit of 2 or 3, and 2^{k+2} must have a first digit of $4, 5, 6, 7$. Thus there are also 603 numbers with first digit $\{2, 3\}$ and 603 numbers with first digit $\{4, 5, 6, 7\}$. By using complementary counting, there are $2004 - 3 \times 603 = 195$ elements of S with a first digit of $\{8, 9\}$. Now, 2^k has a first digit of $\{8, 9\}$ if and only if the first digit of 2^{k-1} is 4, so there are $\boxed{195} \Rightarrow \mathrm{(B)}$ elements of S with a first digit of 4.

Solution 2

We can make the following chart for the possible loops of leading digits:

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow 1$$

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 9 \rightarrow 1$$

$$1 \rightarrow 2 \rightarrow 5 \rightarrow 1$$

$$1 \rightarrow 3 \rightarrow 6 \rightarrow 1$$

$$1 \rightarrow 3 \rightarrow 7 \rightarrow 1$$

Thus each loop from $1 \rightarrow 1$ can either have 3 or 4 numbers. Let there be x of the sequences of 3 numbers, and let there be y of the sequences of 4 numbers. We note that a 4 appears only in the loops of 4, and also we are given that 2^{2004} has 604 digits.

$$3x + 4y = 2004$$

$$x + y = 603$$

Solving gives $x = 408$ and $y = 195$, thus the answer is (B).

See also