2004 AMC 12A Problems/Problem 1

The following problem is from both the 2004 AMC 12A #1 and 2004 AMC 10A #3, so both problems redirect to this page.

Problem

Alicia earns 20 dollars per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes?

$\mathrm{(A) \ } 0.0029 \qquad \mathrm{(B) \ } 0.029 \qquad \mathrm{(C) \ } 0.29 \qquad \mathrm{(D) \ } 2.9 \qquad \mathrm{(E) \ } 29$

Solution

20 dollars is the same as 2000 cents, and 1.45% of 2000 is $0.0145\times2000=29$ cents.
$\Rightarrow \boxed{\mathrm{(E)}\ 29}$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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Preceded by Problem 2	Followed by Problem 4
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2004 AMC 12A Problems/Problem 2

Problem

On the AMC 12, each correct answer is worth 6 points, each incorrect answer is worth 0 points, and each problem left unanswered is worth 2.5 points. If Charlyn leaves 8 of the 25 problems unanswered, how many of the remaining problems must she answer correctly in order to score at least 100?

$\mathrm{(A)}11 \qquad \mathrm{(B)}13 \qquad \mathrm{(C)}14 \qquad \mathrm{(D)}16 \qquad \mathrm{(E)}17$

Solution

She gets $8 * 2.5 = 20$ points for the problems she didn't answer. She must get $\left\lceil \frac{100-20}{6} \right\rceil = 14 \Rightarrow \mathrm{(C)}$ problems right to score at least 100.

See Also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 1	Followed by Problem 3
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2004 AMC 12A Problems/Problem 3

Problem

For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$?

$\mathrm{(A)}\ 33 \qquad \mathrm{(B)}\ 49 \qquad \mathrm{(C)}\ 50 \qquad \mathrm{(D)}\ 99 \qquad \mathrm{(E)}\ 100$

Solution

Every integer value of y leads to an integer solution for x Since y must be positive, $y \geq 1$

Also, $y = \dfrac{100 - x}{2}$ Since x must be positive, $y < 50$

$1 \leq y < 50$ This leaves 49 values for y, which mean there are 49 solutions to the equation $\Rightarrow \mathrm{(B)}$

See Also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 2	Followed by Problem 4
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2004 AMC 12A Problems/Problem 4

The following problem is from both the 2004 AMC 12A #4 and 2004 AMC 10A #6, so both problems redirect to this page.

Contents

Problem

Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and grand-daughters have no daughters?

$\mathrm{(A) \ } 22\qquad \mathrm{(B) \ } 23\qquad \mathrm{(C) \ } 24\qquad \mathrm{(D) \ } 25\qquad \mathrm{(E) \ } 26$

Solution

Since Bertha has 6 daughters, she has $30-6=24$ granddaughters, of which none have daughters. Of Bertha's daughters, $\frac{24}{6}=4$ have daughters, so $6-4=2$ do not have daughters. Therefore, of Bertha's daughters and granddaughters, $24+2=26$ do not have daughters. $\boxed{\mathrm{(E) \ } 26}$

Solution 2

Draw a tree diagram and see that the answer can be found in the sum of $6+6$ granddaughters, $5+5$ daughters, and 4 more daughters. Adding them together gives the answer of $\boxed{\mathrm{(E) \ } 26}$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 3	Followed by Problem 5
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2004 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
Preceded by Problem 5	Followed by Problem 7
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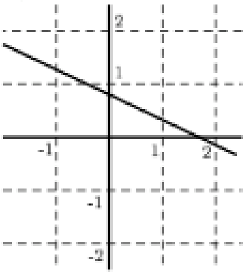
All AMC 10 Problems and Solutions	

2004 AMC 12A Problems/Problem 5

Problem

The graph of the line $y = mx + b$ is shown. Which of the following is true?



$(\mathrm{A}) mb < -1 \qquad (\mathrm{B}) -1 < mb < 0 \qquad (\mathrm{C}) mb = 0 \qquad (\mathrm{D})$
$0 < mb < 1 \qquad (\mathrm{E}) mb > 1$

Solution

It looks like it has a slope of $-\frac{1}{2}$ and is shifted $\frac{4}{5}$ up.

$$\frac{4}{5} \cdot \frac{-1}{2} = \frac{-4}{10} \Rightarrow (\mathrm{B})$$

See also

2004 AMC 12A Problems/Problem 6

Problem

Let $U = 2 \cdot 2004^{2005}$, $V = 2004^{2005}$, $W = 2003 \cdot 2004^{2004}$, $X = 2 \cdot 2004^{2004}$, $Y = 2004^{2004}$ and $Z = 2004^{2003}$. Which of the following is the largest?

$\mathrm{(A)}U-V \qquad \mathrm{(B)}V-W \qquad \mathrm{(C)}W-X \qquad \mathrm{(D)}X-Y \qquad \mathrm{(E)}Y-Z$

Solution

$$\begin{aligned} U-V &= 2004^{2005} \\ V-W &= 2004^{2004} \\ W-X &= 2001*2004^{2004} \\ X-Y &= 2004^{2004} \\ Y-Z &= 2003*2004^{2003} \end{aligned}$$

After comparison, $U-V$ is the largest. $\mathrm{(A)}$

See Also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 5	Followed by Problem 7
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2004 AMC 12A Problems/Problem 7

The following problem is from both the 2004 AMC 12A #7 and 2004 AMC 10A #8, so both problems redirect to this page.

Problem

A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

$\mathrm{(A) \ } 36 \qquad \mathrm{(B) \ } 37 \qquad \mathrm{(C) \ } 38 \qquad \mathrm{(D) \ } 39 \qquad \mathrm{(E) \ } 40$

Solution

Look at a set of 3 rounds, where the players have $x+1$, x, and $x-1$ tokens. Each of the players will gain two tokens from the others and give away 3 tokens, so overall, each player will lose 1 token. Therefore, after 12 sets of 3 rounds, or 36 rounds, the players will have 3, 2, and 1 tokens, respectively. After 1 more round, player A will give away his last 3 tokens and the game will end.
$\boxed{\mathrm{(B)}\ 37}$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
Preceded by Problem 6	Followed by Problem 8
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2004 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2004))	
Preceded by Problem 7	Followed by Problem 9
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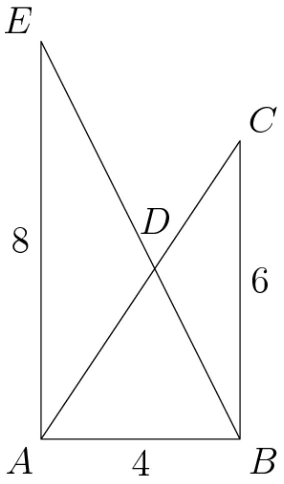
2004 AMC 12A Problems/Problem 8

The following problem is from both the 2004 AMC 12A #8 and 2004 AMC 10A #9, so both problems redirect to this page.

Problem

In the overlapping triangles $\triangle{ABC}$ and $\triangle{ABE}$ sharing common side AB, $\angle{EAB}$ and $\angle{ABC}$ are right angles, $AB=4$, $BC=6$, $AE=8$, and $\overline{AC}$ and $\overline{BE}$ intersect at D. What is the difference between the areas of $\triangle{ADE}$ and $\triangle{BDC}$?



$\mathrm{(A)}\ 2\qquad\mathrm{(B)}\ 4\qquad\mathrm{(C)}\ 5\qquad\mathrm{(D)}\ 8\qquad\mathrm{(E)}\ 9$

Contents

Solution 1

Since $AE \perp AB$ and $BC \perp AB$, $AE \parallel BC$. By alternate interior angles and $AA\sim$, we find that $\triangle ADE \sim \triangle CDB$, with side length ratio $\frac{4}{3}$. Their heights also have the same ratio, and since the two heights add up to 4, we have that $h_{ADE} = 4 \cdot \frac{4}{7} = \frac{16}{7}$ and $h_{CDB} = 3 \cdot \frac 47 = \frac{12}{7}$. Subtracting the areas, $\frac{1}{2} \cdot 8 \cdot \frac{16}{7} - \frac 12 \cdot 6 \cdot \frac{12}{7} = 4 \Rightarrow \boxed{\mathrm{(B)}\ 4}$.

Solution 2

Let $[X]$ represent the area of figure X. Note that $[\triangle BEA]=[\triangle ABD]+[\triangle ADE]$ and $[\triangle BCA]=[\triangle ABD]+[\triangle BDC]$.

$$[\triangle ADE]-[\triangle BDC]=[\triangle BEA]-[\triangle BCA]=\frac{1}{2}\times 8\times 4-\frac{1}{2}\times 6\times 4=16-12=4\Rightarrow \boxed{\mathrm{(B)}\ 4}$$

See also

- AoPS topic (http://www.artofproblemsolving.com/Forum/viewtopic.php?t=131320)

2004 AMC 12A Problems/Problem 9

The following problem is from both the 2004 AMC 12A #9 and 2004 AMC 10A #11, so both problems redirect to this page.

Problem

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

$\mathrm{(A) \ } 10 \qquad \mathrm{(B) \ } 25 \qquad \mathrm{(C) \ } 36 \qquad \mathrm{(D) \ } 50 \qquad \mathrm{(E) \ } 60$

Solution

When the diameter is increased by 25%, it is increased by $\dfrac{5}{4}$, so the area of the base is increased by $\left(\dfrac{5}{4}\right)^2 = \dfrac{25}{16}$.

To keep the volume the same, the height must be $\dfrac{1}{\frac{25}{16}} = \dfrac{16}{25}$ of the original height, which is a 36% reduction. $\boxed{\mathrm{(C)}\ 36}$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 10

Problem

The sum of 49 consecutive integers is 7^5. What is their median?

$\mathrm{(A)}\ 7 \qquad \mathrm{(B)}\ 7^2 \qquad \mathrm{(C)}\ 7^3 \qquad \mathrm{(D)}\ 7^4 \qquad \mathrm{(E)}\ 7^5$

Solution

The median of a sequence is the middle number of the sequence when the sequence is arranged in order. Since the integers are consecutive, the median is also the mean, so the median is $\frac{7^5}{49} = 7^3\ \mathrm{(C)}$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 11

The following problem is from both the 2004 AMC 12A #11 and 2004 AMC 10A #14, so both problems redirect to this page.

Contents

Problem

The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

$\text{(A)}\ 0\qquad\text{(B)}\ 1\qquad\text{(C)}\ 2\qquad\text{(D)}\ 3\qquad\text{(E)}\ 4$

Solution 1

Let the total value (in cents) of the coins Paula has originally be x, and the number of coins she has be n. Then $\frac{x}{n}=20\Longrightarrow x=20n$ and $\frac{x+25}{n+1}=21$. Substituting yields $20n+25=21(n+1),$ implying $n=4,\ x=80.$ It is easy to see that Paula has 3 quarters and 1 nickel, so she has $\boxed{\text{(A)}\ 0}$ dimes.

Solution 2

If the new coin was worth 20 cents, adding it would not change the mean. The additional 5 cents raise the mean by 1, thus the new number of coins must be 5. Therefore there were 4 coins worth a total of $4\times 20=80$ cents. As in the previous solution, we conclude that the only way to get 80 cents using 4 coins is $25+25+25+5$. $\boxed{\text{(A)}\ 0}$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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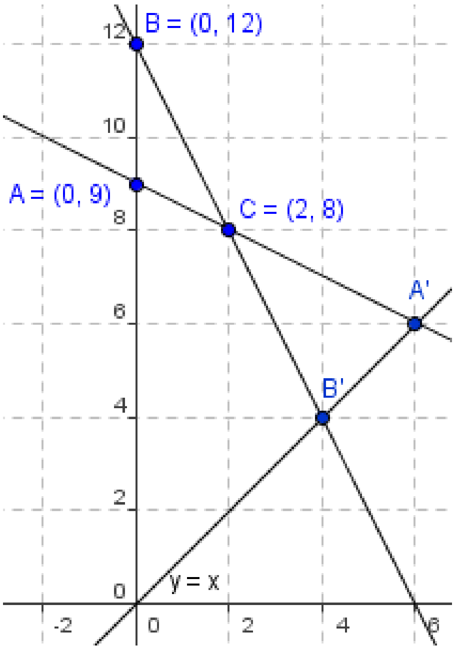
2004 AMC 12A Problems/Problem 12

Problem

Let $A = (0, 9)$ and $B = (0, 12)$. Points A' and B' are on the line $y = x$, and $\overline{AA'}$ and $\overline{BB'}$ intersect at $C = (2, 8)$. What is the length of $\overline{A'B'}$?

$(\mathrm{A}) 2 \qquad (\mathrm{B}) 2\sqrt{2} \qquad (\mathrm{C}) 3 \qquad (\mathrm{D}) 2 + \sqrt{2} \qquad (\mathrm{E}) 3\sqrt{2}$

Solution



The equation of $\overline{AA'}$ can be found using points A, C to be $y - 9 = \left(\frac{9-8}{0-2}\right)(x - 0) \Longrightarrow y = -\frac{1}{2}x + 9$. Similarily, $\overline{BB'}$ has the equation $y - 12 = \left(\frac{12-8}{0-2}\right)(x - 0) \Longrightarrow y = -2x + 12$. These two equations intersect the line $y = x$ at $(6, 6)$ and $(4, 4)$. Using the distance formula or $45 - 45 - 90$ right triangles, the answer is $2\sqrt{2}\ (\mathrm{B})$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 13

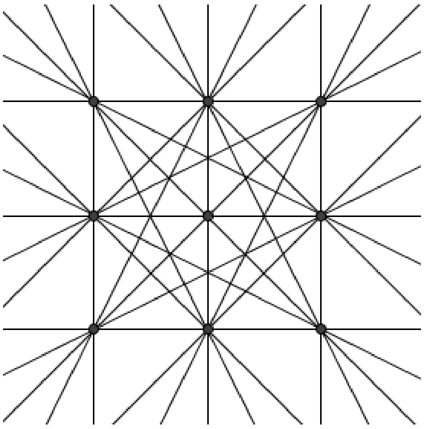
Problem

Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1, 0, or 1. How many distinct lines pass through at least two members of S?

$\mathrm{(A)}\ 8 \qquad \mathrm{(B)}\ 20 \qquad \mathrm{(C)}\ 24 \qquad \mathrm{(D)}\ 27 \qquad \mathrm{(E)}\ 36$

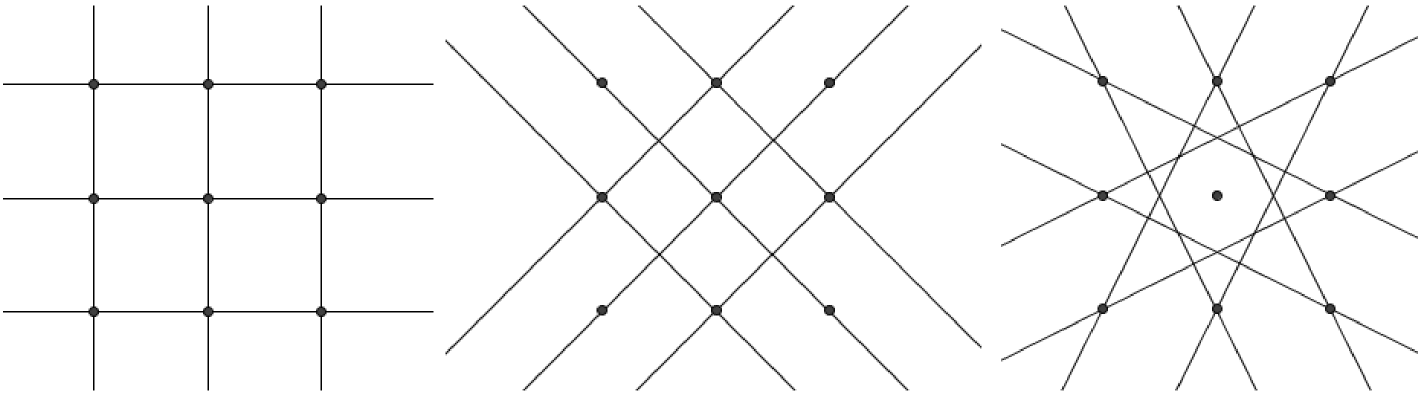
Contents

Solution



Solution 1

Let's count them by cases:



- Case 1: The line is horizontal or vertical, clearly $3 \cdot 2 = 6$.
- Case 2: The line has slope ± 1, with 2 through $(0, 0)$ and 4 additional ones one unit above or below those. These total 6.
- Case 3: The only remaining lines pass through two points, a vertex and a non-vertex point on the opposite side. Thus we have each vertex pairing up with two points on the two opposites sides, giving $4 \cdot 2 = 8$ lines.

These add up to $6 + 6 + 8 = 20\ \mathrm{(B)}$.

Solution 2

There are $\binom{9}{2} = 36$ ways to pick two points, but we've clearly overcounted all of the lines which pass through three points. In fact, each line which passes through three points will have been counted $\binom{3}{2} = 3$ times, so we have to subtract 2 for each of these lines. Quick counting yields 3 horizontal, 3 vertical, and 2 diagonal lines, so the answer is $36 - 2(3 + 3 + 2) = 20$ distinct lines.

See also

2004 AMC 12A Problems/Problem 14

The following problem is from both the 2004 AMC 12A #14 and 2004 AMC 10A #18, so both problems redirect to this page.

Contents

Problem

A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term in the geometric progression?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 4 \qquad \mathrm{(C)}\ 36 \qquad \mathrm{(D)}\ 49 \qquad \mathrm{(E)}\ 81$

Solution 1

Let d be the common difference. Then 9, $9+d+2=11+d$, $9+2d+20=29+2d$ are the terms of the geometric progression. Since the middle term is the geometric mean of the other two terms, $(11+d)^2 = 9(2d+29) \Longrightarrow d^2 + 4d - 140 = (d+14)(d-10) = 0$. The smallest possible value occurs when $d = -14$, and the third term is $2(-14) + 29 = 1 \Rightarrow \boxed{\mathrm{(A)}\ 1}$.

Solution 2

Let d be the common difference and r be the common ratio. Then the arithmetic sequence is 9, $9+d$, and $9+2d$. The geometric sequence (when expressed in terms of d) has the terms 9, $11+d$, and $29+2d$. Thus, we get the following equations:

$$9r = 11+d \Rightarrow d = 9r-11$$

$$9r^2 = 29+2d$$

Plugging in the first equation into the second, our equation becomes $9r^2 = 29 + 18r - 22 \Longrightarrow 9r^2 - 18r - 7 = 0$. By the quadratic formula, r can either be $-\frac{1}{3}$ or $\frac{7}{3}$. If r is $-\frac{1}{3}$, the third term (of the geometric sequence) would be 1, and if r is $\frac{7}{3}$, the third term would be 49. Clearly the minimum possible value for the third term of the geometric sequence is $\boxed{\mathrm{(A)}\ 1}$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 15

The following problem is from both the 2004 AMC 12A #15 and 2004 AMC 10A #17, so both problems redirect to this page.

Contents

Problem

Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

$\mathrm{(A)}\ 250 \qquad \mathrm{(B)}\ 300 \qquad \mathrm{(C)}\ 350 \qquad \mathrm{(D)}\ 400 \qquad \mathrm{(E)}\ 500$

Solution 1

Call the length of the race track x. When they meet at the first meeting point, Brenda has run 100 meters, while Sally has run $\frac{x}{2}-100$ meters. By the second meeting point, Sally has run 150 meters, while Brenda has run $x-150$ meters. Since they run at a constant speed, we can set up a proportion: $\frac{100}{x-150}=\frac{\frac{x}{2}-100}{150}$. Cross-multiplying, we get that $x=350 \Longrightarrow \boxed{\mathrm{(C)}\ 350}$.

Sidenote by carlos8:

Since they run at constant speeds, Brenda must've ran 200 meters to get to the second meeting point, therefore we can make an equation $200=x-150$, solving for x, gives us our answer 350.

Solution 2

The total distance the girls run between the start and the first meeting is one half of the track length. The total distance they run between the two meetings is the track length. As the girls run at constant speeds, the interval between the meetings is twice as long as the interval between the start and the first meeting. Thus between the meetings Brenda will run $2\times 100=200$ meters. Therefore the length of the track is $150+200=350$ meters $\Rightarrow \boxed{\mathrm{(C)}\ 350}$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 16

Problem

The set of all real numbers x for which

$$\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$$

is defined is $\{x|x > c\}$. What is the value of c?

$\mathrm{(A)} 0\qquad \mathrm{(B)} 2001^{2002}\qquad \mathrm{(C)} 2002^{2003}\qquad \mathrm{(D)} 2003^{2004}\qquad \mathrm{(E)} 2001^{2002^{2003}}$

Solution

We know that the domain of $\log_k n$, where k is a constant, is $n > 0$. So $\log_{2003}(\log_{2002}(\log_{2001} x)) > 0$. By the definition of logarithms, we then have $\log_{2002}(\log_{2001} x)) > 2003^0 = 1$. Then $\log_{2001} x > 2002^1 = 2002$ and $x > 2001^{2002}$ $\mathrm{(B)}$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 17

The following problem is from both the 2004 AMC 12A #17 and 2004 AMC 10A #24, so both problems redirect to this page.

Problem

Let f be a function with the following properties:

$$\begin{array}{ll}(i) & f(1)=1, \text{ and} \\ (ii) & f(2n)=n\times f(n), \text{ for any positive integer } n.\end{array}$$

What is the value of $f(2^{100})$?

$\text{(A)}\ 1 \qquad \text{(B)}\ 2^{99} \qquad \text{(C)}\ 2^{100} \qquad \text{(D)}\ 2^{4950} \qquad \text{(E)}\ 2^{9999}$

Solution

$$f(2^{100}) = f(2 \times 2^{99}) = 2^{99} \times f(2^{99}) = 2^{99} \cdot 2^{98} \times f(2^{98}) = \ldots$$
$$= 2^{99} 2^{98} \cdots 2^1 \cdot 1 \cdot f(1) = 2^{99+98+\ldots+2+1} = 2^{\frac{99(100)}{2}} = 2^{4950} \Rightarrow \text{(D)}.$$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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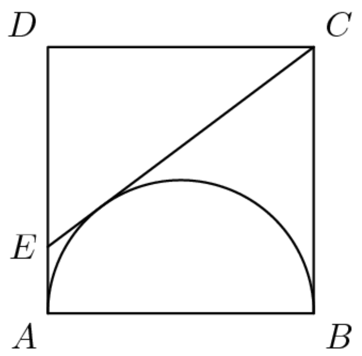
Category: Introductory Algebra Problems

2004 AMC 12A Problems/Problem 18

The following problem is from both the 2004 AMC 12A #18 and 2004 AMC 10A #22, so both problems redirect to this page.

Problem

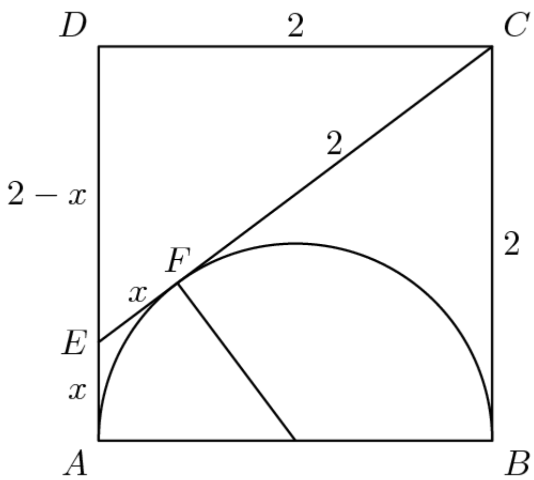
Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?



$\mathrm{(A) \ } \frac{2+\sqrt{5}}{2} \qquad \mathrm{(B) \ } \sqrt{5} \qquad \mathrm{(C) \ } \sqrt{6} \qquad \mathrm{(D) \ } \frac{5}{2} \qquad \mathrm{(E) \ } 5-\sqrt{5}$

Contents

Solution 1



Let the point of tangency be F. By the Two Tangent Theorem $BC = FC = 2$ and $AE = EF = x$. Thus $DE = 2 - x$. The Pythagorean Theorem on $\triangle CDE$ yields

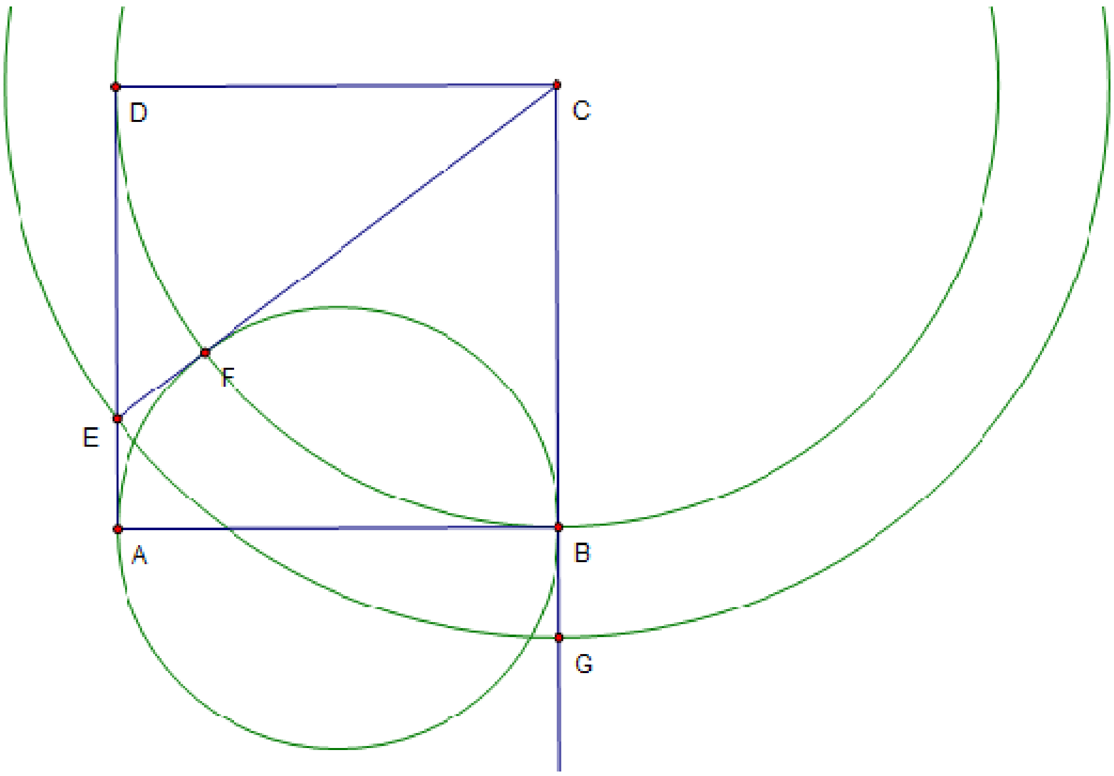
$$DE^2 + CD^2 = CE^2$$
$$(2-x)^2 + 2^2 = (2+x)^2$$
$$x^2 - 4x + 8 = x^2 + 4x + 4$$
$$x = \frac{1}{2}$$

Hence $CE = FC + x = \frac{5}{2} \Rightarrow \boxed{\text{(D) } \frac{5}{2}}$.

Solution 2

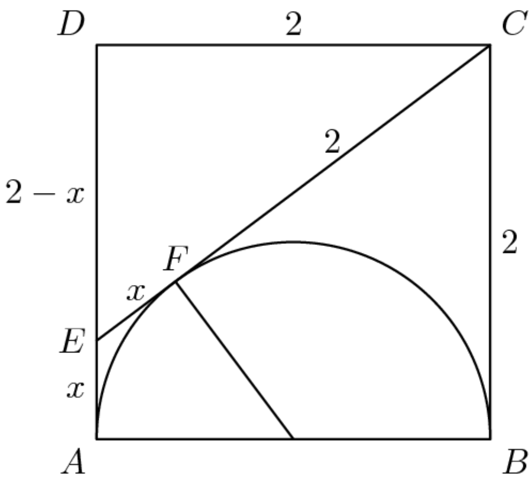
Call the point of tangency point F and the midpoint of AB as G. $CF = 2$ by the pythagorean theorem. Notice that $\angle EGF = \frac{180 - 2 \cdot \angle CGF}{2} = 90 - \angle CGF$. Thus, $EF = \cot \theta = \frac{1}{2}$. Adding, the answer is $\frac{5}{2}$.

Solution 3



Clearly, $EA = EF = BG$. Thus, the sides of right triangle CDE are in arithmetic progression. Thus it is similar to the triangle $3-4-5$ and since $DC = 2$, $CE = 5/2$.

Solution 4



Let us call the midpoint of side AB, point G. Since the semicircle has radius 1, we can do the Pythagorean theorem on sides GB, BC, GC. We get $GC = \sqrt{5}$. We then know that $CF = 2$ by Pythagorean theorem. Then by connecting EG, we get similar triangles EFG and GFC. Solving the ratios, we get $x = \frac{1}{2}$, so the answer is $\frac{5}{2} \Rightarrow \boxed{\text{(D) } \frac{5}{2}}$.

- Solution by $AOPS12142015$

See also

- AoPS topic (http://www.artofproblemsolving.com/Forum/viewtopic.php?t=131334)

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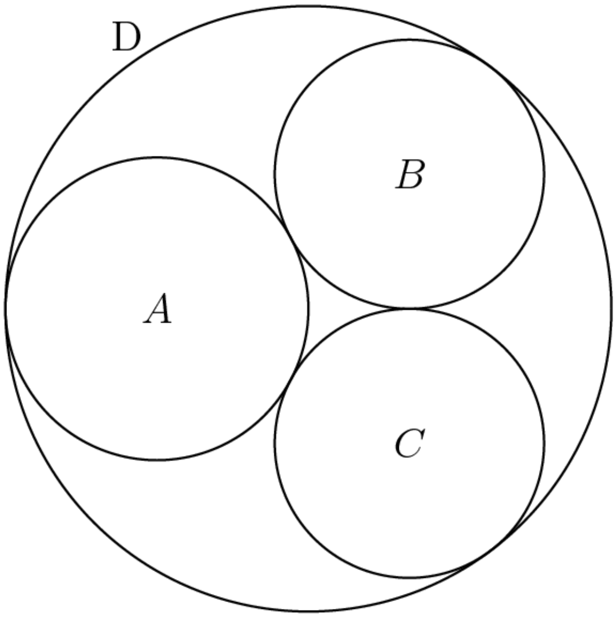
Category: Intermediate Geometry Problems

2004 AMC 12A Problems/Problem 19

Contents

Problem 19

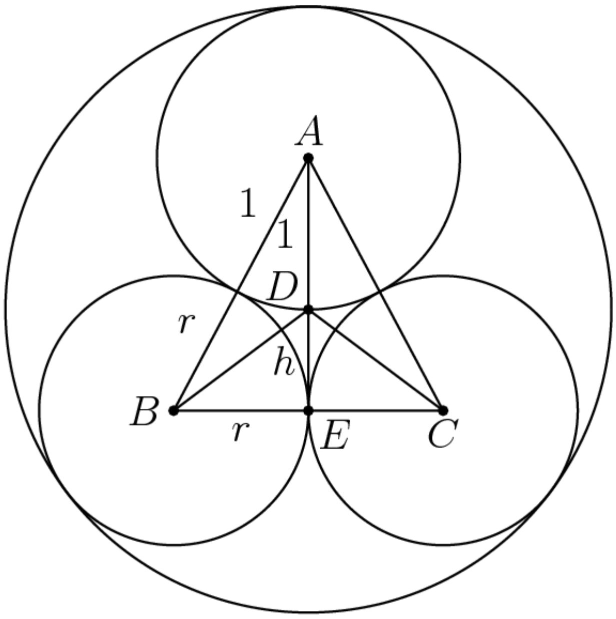
Circles A, B and C are externally tangent to each other, and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



$$\text{(A) } \frac{2}{3} \qquad \text{(B) } \frac{\sqrt{3}}{2} \qquad \text{(C) } \frac{7}{8} \qquad \text{(D) } \frac{8}{9} \qquad \text{(E) } \frac{1+\sqrt{3}}{3}$$

Solution

Solution 1



Note that $BD = 2 - r$ since D is the center of the larger circle of radius 2. Using the Pythagorean Theorem on $\triangle BDE$,

$$\begin{aligned} r^2 + h^2 &= (2-r)^2 \\ r^2 + h^2 &= 4 - 4r + r^2 \\ h^2 &= 4 - 4r \\ h &= 2\sqrt{1-r} \end{aligned}$$

Now using the Pythagorean Theorem on $\triangle BAE$,

$$\begin{aligned} r^2 + (h+1)^2 &= (r+1)^2 \\ r^2 + h^2 + 2h + 1 &= r^2 + 2r + 1 \\ h^2 + 2h &= 2r \end{aligned}$$

Substituting h,

$$\begin{aligned} (4-4r) + 4\sqrt{1-r} &= 2r \\ 4\sqrt{1-r} &= 6r - 4 \\ 16 - 16r &= 36r^2 - 48r + 16 \\ 0 &= 36r^2 - 32r \\ r &= \frac{32}{36} = \frac{8}{9} \implies \qquad \textbf{(D)} \end{aligned}$$

Solution 2

We can apply Descartes' Circle Formula.

The four circles have curvatures $-\dfrac{1}{2}, 1, \dfrac{1}{r},$ and $\dfrac{1}{r}$.

We have $2((-\frac{1}{2})^2 + 1^2 + \frac{1}{r^2} + \frac{1}{r^2}) = (-\frac{1}{2} + 1 + \frac{1}{r} + \frac{1}{r})^2$

Simplifying, we get $\frac{10}{4} + \frac{4}{r^2} = \frac{1}{4} + \frac{2}{r} + \frac{4}{r^2}$

$$\frac{2}{r} = \frac{9}{4}$$

$$r = \frac{8}{9} \Longrightarrow \qquad \textbf{(D)}$$

See Also

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Category: Introductory Geometry Problems

2004 AMC 12A Problems/Problem 20

Problem

Select numbers a and b between 0 and 1 independently and at random, and let c be their sum. Let A, B and C be the results when a, b and c, respectively, are rounded to the nearest integer. What is the probability that $A + B = C$?

$\textbf{(A)}\ \frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{3}{4}$

Contents

Solution

Solution 1

Casework:

1. $0 + 0 = 0$. The probability that $a < \frac{1}{2}$ and $b < \frac{1}{2}$ is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Notice that the sum $a + b$ ranges from 0 to 1 with a symmetric distribution across $a + b = c = \frac{1}{2}$, and we want $c < \frac{1}{2}$. Thus the chance is $\frac{\frac{1}{4}}{2} = \frac{1}{8}$.
2. $0 + 1 = 1$. The probability that $a < \frac{1}{2}$ and $b > \frac{1}{2}$ is $\frac{1}{4}$, but now $\frac{1}{2} < a + b = c < \frac{3}{2}$, which makes $C = 1$ automatically. Hence the chance is $\frac{1}{4}$.
3. $1 + 0 = 1$. This is the same as the previous case.
4. $1 + 1 = 2$. We recognize that this is equivalent to the first case.

Our answer is $2\left(\frac{1}{8} + \frac{1}{4}\right) = \frac{3}{4} \Rightarrow \textbf{(E)}$.

Solution 2

Use areas to deal with this continuous probability problem. Set up a unit square with values of a on x-axis and b on y-axis.

If $a + b < 1/2$ then this will work because $A = B = C = 0$. Similarly if $a + b > 3/2$ then this will work because in order for this to happen, a and b are each greater than $1/2$ making $A = B = 1$, and $C = 2$. Each of these triangles in the unit square has area of 1/8.

The only case left is when $C = 1$. Then each of A and B must be 1 and 0, in any order. These cut off squares of area 1/2 from the upper left and lower right corners of the unit square.

Then the area producing the desired result is 3/4. Since the area of the unit square is 1, the probability is $\frac{3}{4}$.

See also

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Category: Introductory Combinatorics Problems

2004 AMC 12A Problems/Problem 21

Problem

If $\sum_{n=0}^{\infty} \cos^{2n}\theta = 5$, what is the value of $\cos 2\theta$?

$\text{(A)}\frac{1}{5} \qquad \text{(B)}\frac{2}{5} \qquad \text{(C)}\frac{\sqrt{5}}{5} \qquad \text{(D)}\frac{3}{5} \qquad \text{(E)}\frac{4}{5}$

Solution

This is an infinite geometric series, which sums to $\frac{\cos^0\theta}{1-\cos^2\theta} = 5 \Longrightarrow 1 = 5 - 5\cos^2\theta \Longrightarrow \cos^2\theta = \frac{4}{5}$. Using the formula $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{4}{5}\right) - 1 = \frac{3}{5} \Rightarrow \text{(D)}$.

See also

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Category: Introductory Trigonometry Problems

2004 AMC 12A Problems/Problem 22

The following problem is from both the 2004 AMC 12A #22 and 2004 AMC 10A #25, so both problems redirect to this page.

Contents

Problem

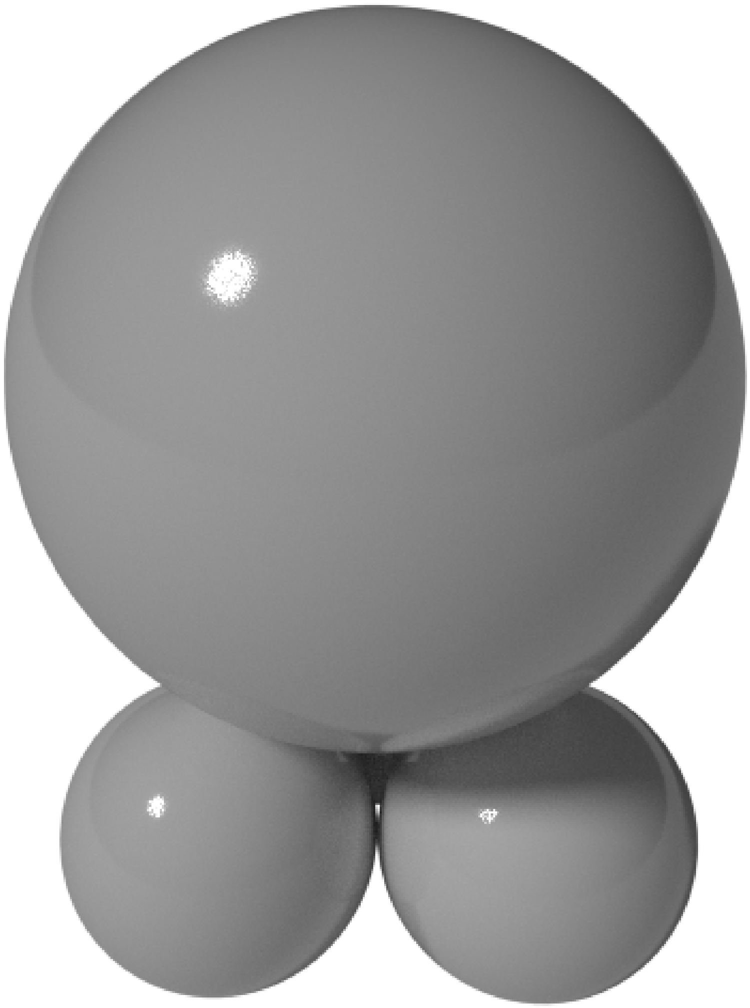
Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

$\text{(A)}\ 3 + \frac{\sqrt{30}}{2} \qquad \text{(B)}\ 3 + \frac{\sqrt{69}}{3} \qquad \text{(C)}\ 3 + \frac{\sqrt{123}}{4} \qquad \text{(D)}\ \frac{52}{9} \qquad \text{(E)}\ 3 + 2\sqrt{2}$

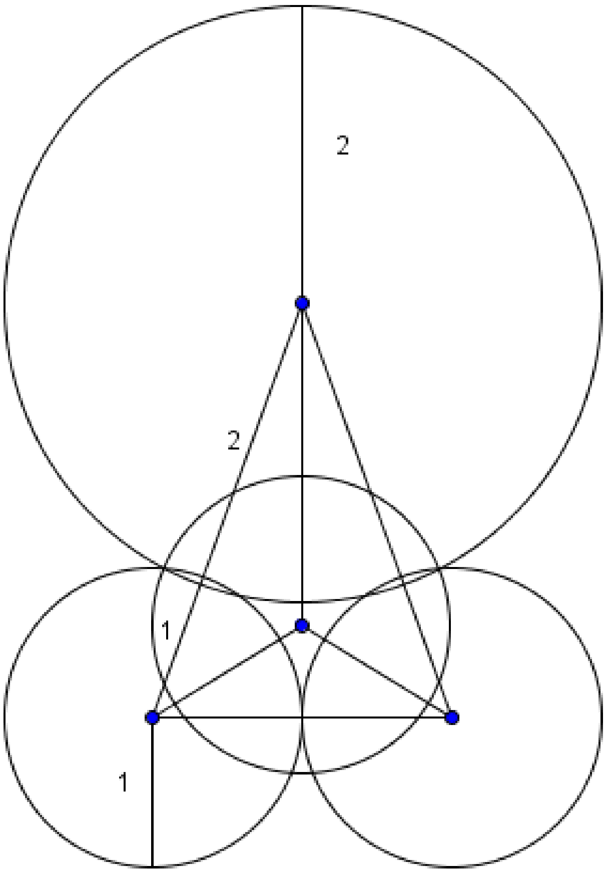
Solution 1

We draw the three spheres of radius 1:

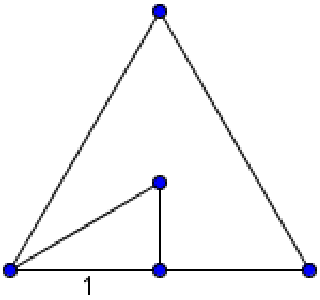
And then add the sphere of radius 2:

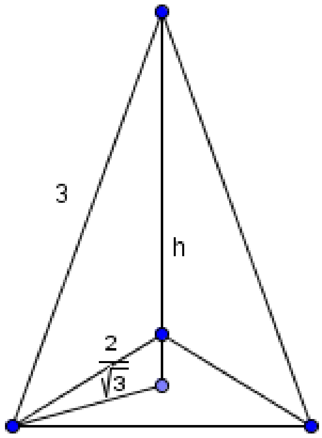


The height from the center of the bottom sphere to the plane is 1, and from the center of the top sphere to the tip is 2.



We now need the vertical height of the centers. If we connect the centers, we get a triangular pyramid with an equilateral triangle base. The distance from the vertex of the equilateral triangle to its centroid can be found by $30-60-90\triangle$s to be $\frac{2}{\sqrt{3}}$.





By the Pythagorean Theorem, we have $\left(\frac{2}{\sqrt{3}}\right)^2+h^2=3^2 \Longrightarrow h=\frac{\sqrt{69}}{3}$. Adding the heights up, we get $\frac{\sqrt{69}}{3}+1+2 \Rightarrow \boxed{\text{(B) } 3+\frac{\sqrt{69}}{3}}$

Solution 2

Connect the centers of the spheres. Note that the resulting prism is a tetrahedron with base lengths of 2 and side lengths of 3. Drop a height from the top of the tetrahedron to the centroid of its equilateral triangle base. Using the Pythagorean Theorem, it is easy to see that the circumradius of the base is $\frac{\sqrt{3}}{3}$. We can use PT again to find the height of the tetrahedron given its base's circumradius and it's leg lengths. Finally, we add the distance from the top of the tetrahedron to the top of the sphere of radius 2 and the distance from the bottom of the prism to the ground to get an answer of $3+\frac{\sqrt{69}}{3}$.

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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2004 AMC 12A Problems/Problem 23

Problem

A polynomial

$$P(x) = c_{2004}x^{2004} + c_{2003}x^{2003} + ... + c_1x + c_0$$

has real coefficients with $c_{2004} \neq 0$ and 2004 distinct complex zeroes $z_k = a_k + b_ki$, $1 \leq k \leq 2004$ with a_k and b_k real, $a_1 = b_1 = 0$, and

$$\sum_{k=1}^{2004} a_k = \sum_{k=1}^{2004} b_k.$$

Which of the following quantities can be a nonzero number?

$(\text{A}) c_0 \qquad (\text{B}) c_{2003} \qquad (\text{C}) b_2b_3...b_{2004} \qquad (\text{D}) \sum_{k=1}^{2004} a_k \qquad (\text{E}) \sum_{k=1}^{2004} c_k$

Solution

We have to evaluate the answer choices and use process of elimination:

- (A): We are given that $a_1 = b_1 = 0$, so $z_1 = 0$. If one of the roots is zero, then $P(0) = c_0 = 0$.
- (B): By Vieta's formulas, we know that $-\frac{c_{2003}}{c_{2004}}$ is the sum of all of the roots of $P(x)$. Since that is real, $\sum_{k=1}^{2004} b_k = 0 = \sum_{k=1}^{2004} a_k$, and $\frac{c_{2003}}{c_{2004}} = 0$, so $c_{2003} = 0$.
- (C): All of the coefficients are real. For sake of contradiction suppose none of $b_{2...2004}$ are zero. Then for each complex root z_i, its complex conjugate $\overline{z_i} = a_i - b_ik$ is also a root. So the roots should pair up, but we have an odd number of imaginary roots! (Remember that $b_1 = 0$.) This gives us the contradiction, and therefore the product is equal to zero.
- (D): We are given that $\sum_{k=1}^{2004} a_k = \sum_{k=1}^{2004} b_k$. Since the coefficients are real, it follows that if a root is complex, its conjugate is also a root; and the sum of the imaginary parts of complex conjugates is zero. Hence the RHS is zero.

There is, however, no reason to believe that $\boxed{\text{E}}$ should be zero (in fact, that quantity is $P(1)$, and there is no evidence that 1 is a root of $P(x)$).

See also

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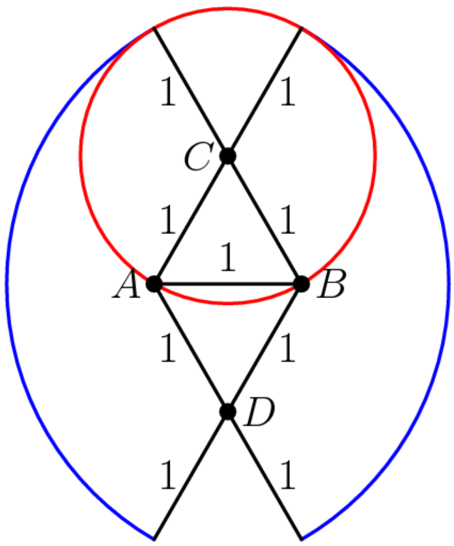
2004 AMC 12A Problems/Problem 24

Problem 24

A plane contains points A and B with $AB = 1$. Let S be the union of all disks of radius 1 in the plane that cover $\overline{AB}$. What is the area of S?

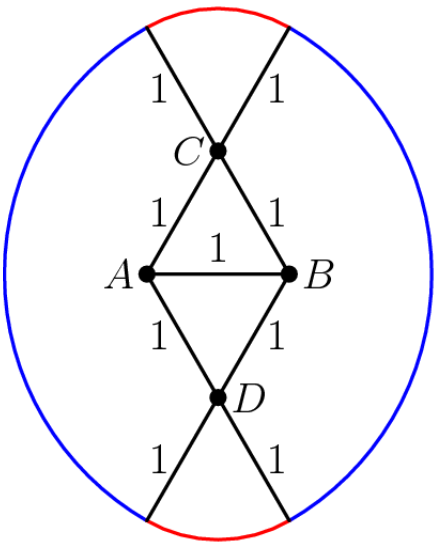
$(\mathrm{A}) 2\pi + \sqrt{3} \qquad (\mathrm{B}) \frac{8\pi}{3} \qquad (\mathrm{C}) 3\pi - \frac{\sqrt{3}}{2} \qquad (\mathrm{D}) \frac{10\pi}{3} - \sqrt{3} \qquad (\mathrm{E}) 4\pi - 2\sqrt{3}$

Solution



As the red circles move about segment AB, they cover the area we are looking for. On the left side, the circle must move around pivoted on B. On the right side, the circle must move pivoted on A However, at the top and bottom, the circle must lie on both A and B, giving us our upper and lower bounds.

This egg-like shape is S.



The area of the region can be found by dividing it into several sectors, namely

$$A = 2(\text{Blue Sector}) + 2(\text{Red Sector}) - 2(\text{Equilateral Triangle})$$
$$A = 2\left(\frac{120^\circ}{360^\circ} \cdot \pi(2)^2\right) + 2\left(\frac{60^\circ}{360^\circ} \cdot \pi(1)^2\right) - 2\left(\frac{(1)^2\sqrt{3}}{4}\right)$$
$$A = \frac{8\pi}{3} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$
$$A = 3\pi - \frac{\sqrt{3}}{2} \Longrightarrow \textbf{(C)}$$

See also

2004 AMC 12A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2004))	
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Category: Intermediate Geometry Problems

2004 AMC 12A Problems/Problem 25

Problem

For each integer $n \geq 4$, let a_n denote the base-n number $0.\overline{133}_n$. The product $a_4a_5\cdots a_{99}$ can be expressed as $\frac{m}{n!}$, where m and n are positive integers and n is as small as possible. What is the value of m?

$\mathrm{(A)}98 \qquad \mathrm{(B)}101 \qquad \mathrm{(C)}132 \qquad \mathrm{(D)}798 \qquad \mathrm{(E)}962$

Solution

This is an infinite geometric series with common ratio $\frac{1}{x^3}$ and initial term $x^{-1} + 3x^{-2} + 3x^{-3}$, so $a_x = \left(\frac{1}{x} + \frac{3}{x^2} + \frac{3}{x^3}\right)\left(\frac{1}{1-\frac{1}{x^3}}\right) = \frac{x^2+3x+3}{x^3} \cdot \frac{x^3}{x^3-1} = \frac{x^2+3x+3}{x^3-1}$
$= \frac{(x+1)^3-1}{x(x^3-1)}$.

Alternatively, we could have used the algebraic manipulation for repeating decimals,

$$\begin{aligned} a_x &= \frac{1}{x} + \frac{3}{x^2} + \frac{3}{x^3} + \frac{1}{x^4} + \frac{3}{x^5} + \frac{3}{x^6} + \cdots \\ a_x \cdot x^3 &= x^2 + 3x + 3 + a_x \\ a_x(x^3-1) &= x^2 + 3x + 3 \\ a_x &= \frac{x^2+3x+3}{x^3-1} = \frac{(x+1)^3-1}{x(x^3-1)} \end{aligned}$$

Telescoping,

$$\begin{aligned} a_4a_5\cdots a_{99} &= \frac{(5^3-1)(6^3-1)\cdots(100^3-1)}{4\cdot5\cdot6\cdot\cdots\cdot99\cdot(4^3-1)(5^3-1)\cdots(99^3-1)} \\ a_4a_5\cdots a_{99} &= \frac{999999}{4\cdot5\cdot6\cdot\cdots\cdot99\cdot63} = \frac{13\cdot37\cdot33\cdot6}{99!} \end{aligned}$$

Some factors cancel, (after all, $13 \cdot 37 \cdot 33 \cdot 6$ isn't one of the answer choices)

$$\frac{13\cdot37\cdot33\cdot6}{99!} = \frac{13\cdot37\cdot2}{98!}$$

Since the only factor in the numerator that goes into 98 is 2, n is minimized. Therefore the answer is $13 \cdot 37 \cdot 2 = 962 \Rightarrow \mathrm{(E)}$.

See Also