2014 AMC12A

Problem 1

What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?

下列式子的值是多少

$$10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}?$$

(A) 3 **(B)** 8 **(C)** $\frac{25}{2}$ **(D)** $\frac{170}{3}$ **(E)** 170

Problem 2

At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is 24.50. How much would 8 adult tickets and 6 child tickets cost?

在剧院，儿童票半价，已知 5 张成人票和 4 张儿童票的总价是 24.5 元，问 8 张成人票和 6 张儿童票需要多少元？

(A) 35 **(B)** 38.50 **(C)** 40 **(D)** 42 **(E)** 42.50

Problem 3

Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

当 Ralph 走过 Jane 街时，她经过了一排 4 座楼房，每座楼房都涂有不同的颜色。她先经过桔色的房子，再经过红色房子，她先经过蓝色房子，再经过黄色房子，蓝色房子不和黄色房子相邻，这 4 个涂色的房子有多少种可能的排列方法？

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Problem 4

Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

假设 *a* 头牛 *c* 天内产出 *b* 加仑的牛奶，以这样的速度计算，*d* 头牛在 c 天内会产出多少加仑的牛奶？

$\textbf{(A)}\ \frac{bde}{ac} \qquad \textbf{(B)}\ \frac{ac}{bde} \qquad \textbf{(C)}\ \frac{abde}{c} \qquad \textbf{(D)}\ \frac{bcde}{a} \qquad \textbf{(E)}\ \frac{abc}{de}$

Problem 5

On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz?

在一次代数测试中，10％的学生得了 70 分，35％的学生得了 80 分，30％的学生得了 90 分，剩下的得了 100 分，问这次考试学生分数的平均数和中位数的差是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 6

The difference between a two-digit number and the number obtained by reversing its digits is 5 times the sum of the digits of either number. What is the sum of the two digit number and its reverse?

一个两位数与将这两位上的数字交换位置所得的新的两位数，它们的差是新旧两位数其中之一的各个位上数字之和的 5 倍，问原来的两位数和各位交换数字后所得新的两位数之和是多少?

$\textbf{(A)}\ 44 \qquad \textbf{(B)}\ 55 \qquad \textbf{(C)}\ 77 \qquad \textbf{(D)}\ 99 \qquad \textbf{(E)}\ 110$

Problem 7

The first three terms of a geometric progression are $\sqrt{3}$, $\sqrt[3]{3}$, and $\sqrt[6]{3}$. What is the fourth term?

一个等比数列的前 3 项是$\sqrt{3}$，$\sqrt[3]{3}$和$\sqrt[6]{3}$，问第 4 项是多少?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \sqrt[7]{3} \qquad \textbf{(C)}\ \sqrt[8]{3} \qquad \textbf{(D)}\ \sqrt[9]{3} \qquad \textbf{(E)}\ \sqrt[10]{3}$

Problem 8

A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least 50

Coupon 2: 20 dollars off the listed price if the listed price is at least 100

Coupon 3: 18% off the amount by which the listed price exceeds 100

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

一位想要买家电的顾客有 3 张优惠券，但只能用其中 1 张：

优惠券 1:如果标价最低是 50 元，那么标价降低 10%

优惠券 2:如果标价最低是 100 元，那么标价降低 20 美元

优惠券 3:标价超过 100 元的部分，降低 18%

对于下面哪个标价，使用优惠券 1 会比使用优惠券 2 或 3 都划算？

$\textbf{(A)}\ 179.95 \qquad \textbf{(B)}\ 199.95 \qquad \textbf{(C)}\ 219.95 \qquad \textbf{(D)}\ 239.95 \qquad \textbf{(E)}\ 259.95$

Problem 9

Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

以 *a* 开头的 5 个连续的正整数的平均值是 *b*，那么以 *b* 开头的 5 个连续的整数的平均值是多少？

$\textbf{(A)}\ a+3 \qquad \textbf{(B)}\ a+4 \qquad \textbf{(C)}\ a+5 \qquad \textbf{(D)}\ a+6 \qquad \textbf{(E)}\ a+7$

Problem 10

Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle. What is the length of one of the two congruent sides of one of the isosceles triangles?

以边长为 1 的等边三角形的三条边分别为底，作出 3 个全等的等腰三角形，这三个等腰三角形的面积之和与这个等边三角形的面积相等，那么其中一个等腰三角形的一条腰长是多少？

$\textbf{(A)}\ \frac{\sqrt{3}}{4} \qquad \textbf{(B)}\ \frac{\sqrt{3}}{3} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(E)}\ \frac{\sqrt{3}}{2}$

Problem 11

David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

David 从他家开车到机场去坐飞机，他在第一小时开了 35 英里，但是意识到如果继续按照这个速度开车，会迟到 1 小时，剩下的路程他把车速增加了 15 英里每小时，结果提前了 30 分钟到达，问从他家到机场有多少英里？

$\textbf{(A)}\ 140 \qquad \textbf{(B)}\ 175 \qquad \textbf{(C)}\ 210 \qquad \textbf{(D)}\ 245 \qquad \textbf{(E)}\ 280$

Problem 12

Two circles intersect at points A and B. The minor arcs AB measure 30° on one circle and 60° on the other circle. What is the ratio of the area of the larger circle to the area of the smaller circle?

两个圆交于点 *A* 和点 *B*，劣弧 *AB* 在其中一个圆上的度数是 30°，在另一个圆上的度数是 60°，问大圆和小圆的面积之比是多少?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 1+\sqrt{3} \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 2+\sqrt{3} \qquad \textbf{(E)}\ 4$

Problem 13

A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

一家高档的旅馆有 5 个房间，每个房间的颜色装饰都不同。一天 5 个朋友来这里过夜，那天晚上没有其他客人。这 5 个朋友可以以任何他们希望的方式组合入住，但每个房间不得超过 2 人。问他们有多少种可能的入住方式？

$\textbf{(A)}\ 2100 \qquad \textbf{(B)}\ 2220 \qquad \textbf{(C)}\ 3000 \qquad \textbf{(D)}\ 3120 \qquad \textbf{(E)}\ 3125$

Problem 14

Let $a < b < c$ be three integers such that a, b, c is an arithmetic progression and a, c, b is a geometric progression. What is the smallest possible value of c?

令$a < b < c$，且都是整数，满足 a, b, c 成等比数列，问 c 可能的最小值是多少？

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 6$

Problem 15

A five-digit palindrome is a positive integer with respective digits $abcba$, where a is non-zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S?

1 个五位的回环数是一个正整数且各位数字是 *abcba*，其中最高位 *a* 不为 0，令 *S* 为所有五位回环数之和。问 *S* 的各个位上数字之和是多少？

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 27 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 45$

Problem 16

The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

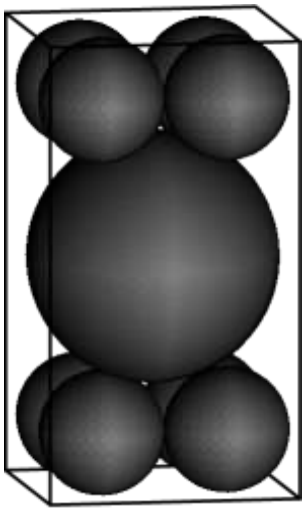
$(8)(888\ldots8)$的乘积是一个各个位上数字之和为 1000 的整数，其中第二个乘数一共有 k 位. 问 k 是多少？

$\textbf{(A)}\ 901 \qquad \textbf{(B)}\ 911 \qquad \textbf{(C)}\ 919 \qquad \textbf{(D)}\ 991 \qquad \textbf{(E)}\ 999$

Problem 17

A $4 \times 4 \times h$ rectangular box contains a sphere of radius 2 and eight smaller spheres of radius 1. The smaller spheres are each tangent to three sides of the box, and the larger sphere is tangent to each of the smaller spheres. What is h?

一个$4 \times 4 \times h$的长方体盒子装着 1 个半径为 2 的大球和 8 个半径为 1 的小球。每个小球都和盒子的三个面相切，并且大球和每个小球都外切。问 *h* 是多少？



(A) $2+2\sqrt{7}$ **(B)** $3+2\sqrt{5}$ **(C)** $4+2\sqrt{7}$ **(D)** $4\sqrt{5}$ **(E)** $4\sqrt{7}$

Problem 18

The domain of the function $f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}} x))))$ is an interval of length $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

$f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}} x))))$的定义域是一个长度为$\frac{m}{n}$的区间，其中 *m* 和 *n* 是互质的正整数。那么 *m*+*n* 是多少？

(A) 19 **(B)** 31 **(C)** 271 **(D)** 319 **(E)** 511

Problem 19

There are exactly N distinct rational numbers k such that $|k| < 200$ and

$$5x^2 + kx + 12 = 0$$

has at least one integer solution for x. What is N?

存在恰好 N 个不同的有理数 k，满足：$|k| < 200$，且关于 x 的方程$5x^2 + kx + 12 = 0$至少有 1 个整数根，那么 N是多少？

(A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 78

Problem 20

In $\triangle BAC$, $\angle BAC = 40^\circ$, $AB = 10$, and $AC = 6$. Points D and E lie on $\overline{AB}$ and $\overline{AC}$ respectively. What is the minimum possible value of $BE + DE + CD$?

在$\triangle BAC$中，$\angle BAC = 40^\circ$，$AB = 10$，$AC = 6$，点D和点E分别在线段$\overline{AB}$和$\overline{AC}$上，那么$BE + DE + CD$的最小可能值是多少？

(A) $6\sqrt{3} + 3$ **(B)** $\frac{27}{2}$ **(C)** $8\sqrt{3}$ **(D)** 14 **(E)** $3\sqrt{3} + 9$

Problem 21

For every real number x, let $\lfloor x \rfloor$ denote the greatest integer not exceeding x, and let

$$f(x) = \lfloor x \rfloor (2014^{x - \lfloor x \rfloor} - 1).$$

The set of all numbers x such that $1 \le x < 2014$ and $f(x) \le 1$ is a union of disjoint intervals. What is the sum of the lengths of those intervals?

对于任何实数 x，定义$\lfloor x \rfloor$为不超过 x 的最大整数，且函数$f(x) = \lfloor x \rfloor (2014^{x - \lfloor x \rfloor} - 1)$满足$1 \le x < 2014$，$f(x) \le 1$的所有 x 组成的集合是一些不连续的区间的并集。问这些区间的长度之和是多少？

(A) 1 **(B)** $\frac{\log 2015}{\log 2014}$ **(C)** $\frac{\log 2014}{\log 2013}$ **(D)** $\frac{2014}{2013}$ **(E)** $2014^{\frac{1}{2014}}$

Problem 22

The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m, n) are there such that $1 \leq m \leq 2012$ and$5^n < 2^m < 2^{m+2} < 5^{n+1}$?

数字5^{867}位于2^{2013}和2^{2014}之间，有多少对整数（m，n），满足$1 \leq m \leq 2012$且$5^n < 2^m < 2^{m+2} < 5^{n+1}$?

(A) 278 **(B)** 279 **(C)** 280 **(D)** 281 **(E)** 282

Problem 23

The fraction

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\ldots b_2b_1b_0},$$

where n is the length of the period of the repeating decimal expansion. What is the sum $b_0 + b_1 + \cdots + b_{n-1}$?

分数

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\ldots b_2b_1b_0},$$

这里n表示这个循环小数的循环周期的长度。问$b_0 + b_1 + \cdots + b_{n-1}$是多少？

(A) 874 **(B)** 883 **(C)** 887 **(D)** 891 **(E)** 892

Problem 24

Let $f_0(x) = x + |x - 100| - |x + 100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?

函数$f_0(x) = x + |x - 100| - |x + 100|$，且对于$n \geq 1$，有$f_n(x) = |f_{n-1}(x)| - 1$，问存在多少个$x$的值，使得$f_{100}(x) = 0$?

(A) 299 **(B)** 300 **(C)** 301 **(D)** 302 **(E)** 303

Problem 25

The parabola P has focus $(0,0)$ and goes through the points $(4,3)$ and $(-4,-3)$. For how many points $(x,y) \in P$ with integer coordinates is it true that $|4x+3y| \leq 1000$?

抛物线 *P* 的焦点为（0，0），且通过点（4，3）和（－4，－3），有多少个点$(x,y) \in P$，且 *x*，*y* 均为整数，满足$|4x+3y| \leq 1000$?

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

2014 AMC 12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	B	A	C	D	A	C	B	B	C	D	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	B	D	A	C	E	D	A	B	B	C	B	