2012 AMC12A

Problem 1

A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

一个虫子从-2这个点开始沿着数轴爬行，它先爬到-6，然后掉头再爬到5，那么这只虫子总共爬了多长？

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Problem 2

Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

Cagney 每 20 秒可以给一个蛋糕上霜，Lacey 每 30 秒可以给一个蛋糕上霜，若他俩合作，那么 5 分钟内可以给多少个蛋糕上霜？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 30$

Problem 3

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box with twice the height, three times the width, and the same length as the first box can hold n grams of clay. What is n?

一个 2 厘米高，3 厘米宽，5 厘米长的盒子能够装 40 克的泥土。第二个盒子高度和宽度分别是前一个盒子的 2 倍和 3 倍，长度和前一个盒子一样长，可以装 n 克的泥土，问 n 是多少？

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 160 \qquad \textbf{(C)}\ 200 \qquad \textbf{(D)}\ 240 \qquad \textbf{(E)}\ 280$

Problem 4

In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

一包玻璃球有$\frac{3}{5}$是蓝色的，剩下的都是红色的，如果红色玻璃球的数量翻倍而蓝色的个数保持不变，那么最后总数的几分之几将是红色？

$\textbf{(A)}\ \frac{2}{5} \qquad \textbf{(B)}\ \frac{3}{7} \qquad \textbf{(C)}\ \frac{4}{7} \qquad \textbf{(D)}\ \frac{3}{5} \qquad \textbf{(E)}\ \frac{4}{5}$

Problem 5

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

一种水果沙拉由蓝莓、覆盆子、葡萄和樱桃组成，这种水果沙拉总共有 280 个水果，其中覆盆子的个数是蓝莓的 2 倍，葡萄是樱桃的 3 倍，樱桃是覆盆子的 4 倍，问这种水果沙拉里有多少个樱桃?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 96$

Problem 6

The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

三个整数两两求和结果分别为 12，17 和 19，问中间那个数是多少?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 7

Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

玛丽把一个圆分成了 12 个扇形，这些扇形的圆心角的度数都是整数且形成一个等差数列，那么最小的那个扇形角度最小可能是多少度？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Problem 8

An *iterative average* of the numbers 1, 2, 3, 4, and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

数字 1，2，3，4，5 的迭代平均值由以下方式计算，把这 5 个数字以某种顺序排列，先计算前 2 个数的平均值，再算出这个平均值和第三个数的平均值，然后再算出新得到的平均值和第四个数的平均值，最后算出这个平均值和第五个数的平均值。问用这种方法最后得到的数的最大值和最小值之差是多少？

$\textbf{(A)}\ \frac{31}{16}\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ \frac{17}{8}\qquad\textbf{(D)}\ 3\qquad\textbf{(E)}\ \frac{65}{16}$

Problem 9

A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not by 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

如果某一年份能被 400 整除（例如 2000 年）或能被 4 整除但不能被 100 整除（例如 2012 年），那么这一年被称为闰年，小说家查尔斯狄更斯的 200 周年纪念日是 2012 年 2 月 7 日星期二，问狄根思是星期几出生的?

$\textbf{(A)}\ \text{Friday}\qquad\textbf{(B)}\ \text{Saturday}\qquad\textbf{(C)}\ \text{Sunday}\qquad\textbf{(D)}\ \text{Monday}\qquad\textbf{(E)}\ \text{Tuesday}$

Problem 10

A triangle has area 30, one side of length 10, and the median to that side of length 9. Let θ be the acute angle formed by that side and the median. What is $\sin{\theta}$?

一个三角形的面积是 30，其中一条边长是 10，并且这条边上的中线长度是 9，令θ表示这条边与这条中线形成的锐角，那么$\sin\theta$是多少？

$\textbf{(A)}\ \frac{3}{10}\qquad\textbf{(B)}\ \frac{1}{3}\qquad\textbf{(C)}\ \frac{9}{20}\qquad\textbf{(D)}\ \frac{2}{3}\qquad\textbf{(E)}\ \frac{9}{10}$

Problem 11

Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

Alox，Mel 和 Chelsen 玩一个有 6 个回合的游戏，在每个回合都有一个赢家，并且每个回合的结果都是独立的。对于每个回合，Alex 赢的概率是 $\frac{1}{2}$，Mel 赢的概率是 Chelsea 的 2 倍，问 Alex 赢得 3 个回合，Mel 赢得 2 个回合，Chelsca 赢得 1 个回合的概率是多少？

$\textbf{(A)}\ \frac{5}{72} \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ 1$

Problem 12

A square region $ABCD$ is externally tangent to the circle with equation $x^2+y^2=1$ at the point $(0,1)$ on the side CD. Vertices A and B are on the circle with equation $x^2+y^2=4$. What is the side length of this square?

正方形 *ABCD* 的边 *CD* 和方程为$x^2+y^2=1$的圆外切于点（0，1），顶点 A 和 B 在方程为$x^2+y^2=4$的圆上，问这个正方形的边长是多少？

$\textbf{(A)}\ \frac{\sqrt{10}+5}{10} \qquad \textbf{(B)}\ \frac{2\sqrt{5}}{5} \qquad \textbf{(C)}\ \frac{2\sqrt{2}}{3} \qquad \textbf{(D)}\ \frac{2\sqrt{19}-4}{5} \qquad \textbf{(E)}\ \frac{9-\sqrt{17}}{5}$

Problem 13

Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00\ \text{AM}$, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00\ \text{PM}$. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12\ \text{PM}$. On Wednesday Paula worked by herself and finished the house by working until $7:12\ \text{PM}$. How long, in minutes, was each day's lunch break?

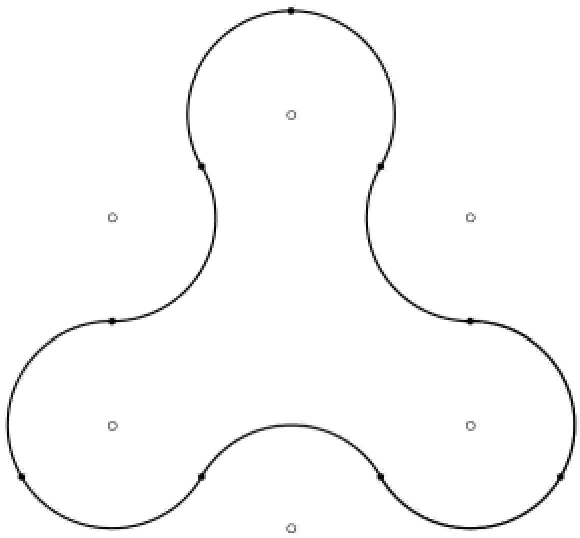
油漆匠 Paula 和她的两个助理每个人都以恒定且不同的速度粉刷油漆，他们总是在早上 8:00 开始工作，并且三个人中午吃饭的时长都相同。周一这一天这三人粉刷了房子的 50%，在下午 4:00 下班；周二 Paula 不在，两个助手总共仅粉刷了房子的 24%，下午 2:12 就下班了；周三 Paula 自己一个人工作，一直涂到晚上 7:12，把整个房子全部涂完。问每天的午餐时长是多少分钟？

(A) 30　　**(B)** 36　　**(C)** 42　　**(D)** 48　　**(E)** 60

Problem 14

The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?

图中所示的闭合曲线由 9 段全等的圆弧组成，每段圆弧长度均为 $\frac{2\pi}{3}$，且这 9 段圆弧所对应的圆心是一个边长为 2 的正六边形的顶点，问这段闭合曲线所包围的面积是多少?



(A) $2\pi+6$　　**(B)** $2\pi+4\sqrt{3}$　　**(C)** $3\pi+4$　　**(D)** $2\pi+3\sqrt{3}+2$　　**(E)** $\pi+6\sqrt{3}$

Problem 15

A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

一个3×3的正方形被分成 9 个单位正方形，每个单位正方形被涂成白色或者黑色，颜色的选择是等可能且是随机独立的，然后把整个大的正方形绕着它的中心顺时针旋转 90°，如果白色正方形被转到了之前被黑色正方形占用的位置，那么这个白色正方形就被涂成黑色．所有其他正方形的颜色不变，问整个3×3的正方形现在是全黑的概率是多少？

$\textbf{(A)}\ \frac{49}{512} \qquad \textbf{(B)}\ \frac{7}{64} \qquad \textbf{(C)}\ \frac{121}{1024} \qquad \textbf{(D)}\ \frac{81}{512} \qquad \textbf{(E)}\ \frac{9}{32}$

Problem 16

Circle C_1 has its center O lying on circle C_2. The two circles meet at X and Y. Point Z in the exterior of C_1 lies on circle C_2 and $XZ = 13$, $OZ = 11$, and $YZ = 7$. What is the radius of circle C_1?

圆C_1的圆心O在圆C_2上，这两个圆交于点X和Y，点Z在圆C_2上且在圆C_1之外，满足 XZ=13，OZ=11，且 YZ=7. 求圆C_1的半径是多少?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ \sqrt{26} \qquad \textbf{(C)}\ 3\sqrt{3} \qquad \textbf{(D)}\ 2\sqrt{7} \qquad \textbf{(E)}\ \sqrt{30}$

Problem 17

Let S be a subset of $\{1,2,3,\dots,30\}$ with the property that no pair of distinct elements in S has a sum divisible by 5. What is the largest possible size of S?

S是集合$\{1,2,3,\dots,30\}$的一个子集，它需要满足的性质是：S中不存在一对不同的完素，它们的和可以被 5 整除，问 S中最多可以有多少个元素？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 13 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 18$

Problem 18

Triangle ABC has $AB = 27$, $BC = 25$, and $CA = 26$. Let I denote the intersection of the internal angle bisectors of $\triangle ABC$. What is BI?

在三角形 ABC 中， AB=27，BC=25，CA= 26. 令点 I 表示$\triangle ABC$的 3 条内角平分线的交点，那么 BI 是多少？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 5+\sqrt{26}+3\sqrt{3} \qquad \textbf{(C)}\ 3\sqrt{26} \qquad \textbf{(D)}\ \frac{2}{3}\sqrt{546} \qquad \textbf{(E)}\ 9\sqrt{3}$

Problem 19

Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

Adam，Benin，Chiang，Deshawn，Esther 和 Fiona 都有网络账号. 他们中的一些人（但不是所有）相互之间是网络朋友，并且他们都没有在这一组人之外的网络朋友，他们每个人都有相同数目的网络朋友，问他们一共可能有多少种不同的方式成为朋友？

$\textbf{(A)}\ 60 \qquad \textbf{(B)}\ 170 \qquad \textbf{(C)}\ 290 \qquad \textbf{(D)}\ 320 \qquad \textbf{(E)}\ 660$

Problem 20

Consider the polynomial

$$P(x) = \prod_{k=0}^{10}(x^{2^k}+2^k) = (x+1)(x^2+2)(x^4+4)\cdots(x^{1024}+1024)$$

The coefficient of x^{2012} is equal to 2^a. What is a?

考虑多项式

$$P(x) = \prod_{k=0}^{10}(x^{2^k}+2^k) = (x+1)(x^2+2)(x^4+4)\cdots(x^{1024}+1024)$$

其中x^{2012}的系数是2^a，问a是多少？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 24$

Problem 21

Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$

$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$$

What is a?

a，b和c是正整数，且$a \geq b \geq c$，满足
$a^2 - b^2 - c^2 + ab = 2011$，$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$，
求a是多少？

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

Problem 22

Distinct planes $p_1, p_2, \ldots, p_k$ intersect the interior of a cube Q. Let S be the union of the faces of Q and let $P = \bigcup_{j=1}^{k} p_j$. The intersection of P and S consists of the union of all segments joining the midpoints of every pair of edges belonging to the same face of Q. What is the difference between the maximum and minimum possible values of k?

$p_1, p_2, \ldots, p_k$是 k 个与正方体 Q的内部空间相交的不同的平面，集合 S是 Q的各个面组成的集合，并且令 $P = \bigcup_{j=1}^{k} p_j$，集合 P和 S的交集是：Q的同一平面任意两条边的中点的连线组成的集合。问 k的最大可能值和最小可能值的差是多少？

(A) 8 **(B)** 12 **(C)** 20 **(D)** 23 **(E)** 24

Problem 23

Let S be the square one of whose diagonals has endpoints $(0.1, 0.7)$ and $(-0.1, -0.7)$. A point $v = (x, y)$ is chosen uniformly at random over all pairs of real numbers x and y such that $0 \leq x \leq 2012$ and $0 \leq y \leq 2012$. Let $T(v)$ be a translated copy of S centered at v. What is the probability that the square region determined by $T(v)$ contains exactly two points with integer coordinates in its interior?

S是一个正方形，其中一条对角线的两个端点的坐标是（0.1，0.7）和（−0.1，−0.7），已知实数 x 和 y，满足 0 ≤ x ≤ 2012 和 0 ≤y≤ 2012，从此区间内随机选择 x 和 y，组成一个点 v（x，y），$T(v)$表示 S 平移后使得中心和 v 重合后的正方形。问由$T(v)$所确定的正方形区域内部包含恰好两个整数点（坐标都为整数的点）的概率是多少？

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{7}{50} \qquad \textbf{(C)}\ \frac{4}{25} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{8}{25}$

Problem 24

Let $\{a_k\}_{k=1}^{2011}$ be the sequence of real numbers defined

by $a_1 = 0.201, a_2 = (0.2011)^{a_1}, a_3 = (0.20101)^{a_2}, a_4 = (0.201011)^{a_3}$, and in general,

$$a_k = \begin{cases} (0.\underbrace{20101\cdots 0101}_{k+2 \text{ digits}})^{a_{k-1}} & \text{if } k \text{ is odd,} \\ (0.\underbrace{20101\cdots 01011}_{k+2 \text{ digits}})^{a_{k-1}} & \text{if } k \text{ is even.} \end{cases}$$

Rearranging the numbers in the sequence $\{a_k\}_{k=1}^{2011}$ in decreasing order produces a new sequence $\{b_k\}_{k=1}^{2011}$. What is the sum of all integers k, $1 \le k \le 2011$, such that $a_k = b_k$?

令$\{a_k\}_{k=1}^{2011}$为实数数列，满足$a_1 = 0.201$,$a_2 = (0.2011)^{a_1}$,$a_3 = (0.20101)^{a_2}$,

$a_4 = (0.201011)^{a_3}$，且一般有，

$$a_k = \begin{cases} (0.\underbrace{20101\cdots 0101}_{k+2 \text{ 个数字}})^{a_{k-1}} & \text{如果k是奇数} \\ (0.\underbrace{20101\cdots 01011}_{k+2 \text{ 个数字}})^{a_{k-1}} & \text{如果k是偶数} \end{cases}$$

把数列$\{a_k\}_{k=1}^{2011}$中的所有项重新按照递减的顺序排序，得到一个新的数列$\{b_k\}_{k=1}^{2011}$，那么使得$a_k = b_k$的所有整数k且满足$1 \le k \le 2011$的和是多少？

$\textbf{(A)}\ 671 \qquad \textbf{(B)}\ 1006 \qquad \textbf{(C)}\ 1341 \qquad \textbf{(D)}\ 2011 \qquad \textbf{(E)}\ 2012$

Problem 25

Let $f(x)=|2\{x\}-1|$ where $\{x\}$ denotes the fractional part of x. The number n is the smallest positive integer such that the equation $nf(xf(x))=x$ has at least 2012 real solutions. What is n? Note: the fractional part of x is a real number $y=\{x\}$ such that $0\le y<1$ and $x-y$ is an integer.

令$f(x)=|2\{x\}-1|$,其中$\{x\}$表示x的分数部分,那么使得方程$nf(xf(x))=x$ 有至少 2012 个实数解的最小正整数n是多少？（注意：x的分数部分是这样一个实数$y=\{x\}$，满足$0\le y<1$，且$x-y$是一个整数。）

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 64$

2012 AMC 12A Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
E	D	D	C	D	D	C	C	A	D	B	D	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	A	E	B	A	B	B	E	C	C	C	C	