2002 AMC 8 Problems/Problem 1

Problem

A circle and two distinct lines are drawn on a sheet of paper. What is the largest possible number of points of intersection of these figures?

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad (C)\ 4 \qquad (D)\ 5 \qquad (E)\ 6$

Solution

The two lines can both intersect the circle twice, and can intersect each other once, so $2+2+1=\boxed{\text{(D)}\ 5}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by First Question	Followed by Problem 2
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All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 2

Problem

How many different combinations of \$5 bills and \$2 bills can be used to make a total of \$17? Order does not matter in this problem.

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 4 \qquad \text{(D)}\ 5 \qquad \text{(E)}\ 6$

Solution

You cannot use more than 4 $\$5$ bills, but if you use 3 $\$5$ bills, you can add another $\$2$ bill to make a combination. You can also use 1 $\$5$ bill and 6 $\$2$ bills to make another combination. There are no other possibilities, as making $\$17$ with 0 $\$5$ bills is impossible, so the answer is $\boxed{\text{(A)}\ 2}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 1	Followed by Problem 3
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All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 3

Problem

What is the smallest possible average of four distinct positive even integers?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Solution

In order to get the smallest possible average, we want the 4 even numbers to be as small as possible. The first 4 positive even numbers are 2, 4, 6, and 8. Their average is $\frac{2+4+6+8}{4} = \boxed{\text{(C)}\ 5}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 2	Followed by Problem 4
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All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 4

Problem

The year 2002 is a palindrome (a number that reads the same from left to right as it does from right to left). What is the product of the digits of the next year after 2002 that is a palindrome?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 25$

Solution

The palindrome right after 2002 is 2112. The product of the digits of 2112 is $\boxed{\textbf{(B)}\ 4}$.

See Also

2002 AMC 8 Problems/Problem 5

Problem

Carlos Montado was born on Saturday, November 9, 2002. On what day of the week will Carlos be 706 days old?

$\textbf{(A)}\ \text{Monday}\qquad\textbf{(B)}\ \text{Wednesday}\qquad\textbf{(C)}\ \text{Friday}\qquad\textbf{(D)}\ \text{Saturday}\qquad\textbf{(E)}\ \text{Sunday}$

Solution

Days of the week have a cycle that repeats every 7 days. Thus, after 100 cycles, or 700 days, it will be Saturday again. Six more days will make it $\text{Friday} \rightarrow \boxed{C}$

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 4	Followed by Problem 6
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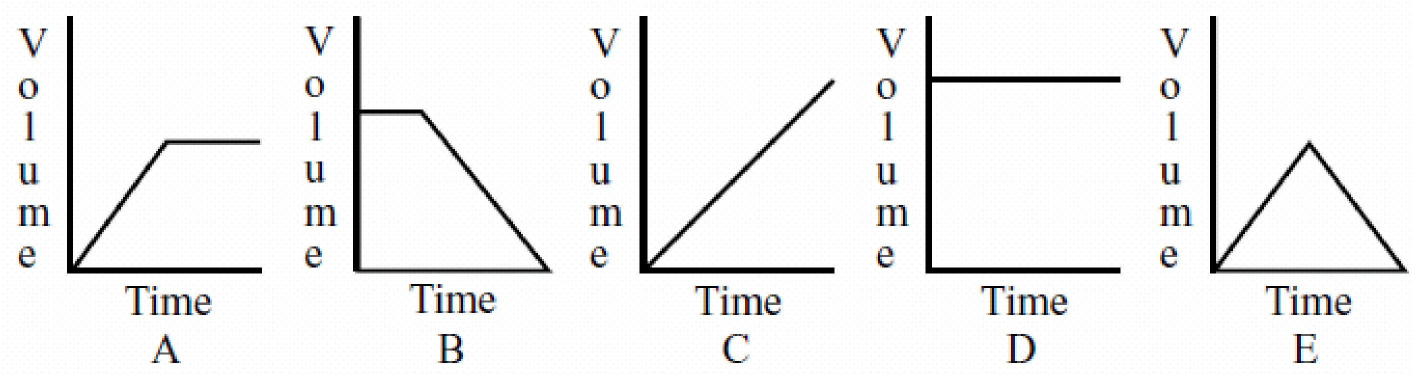
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 6

Problem

A birdbath is designed to overflow so that it will be self-cleaning. Water flows in at the rate of 20 milliliters per minute and drains at the rate of 18 milliliters per minute. One of these graphs shows the volume of water in the birdbath during the filling time and continuing into the overflow time. Which one is it?



$\text{(A)}\ \text{A} \qquad \text{(B)}\ \text{B} \qquad \text{(C)}\ \text{C} \qquad \text{(D)}\ \text{D} \qquad \text{(E)}\ \text{E}$

Solution

The change in the water volume has a net gain of $20-18=2$ millimeters per minute. The birdbath's volume increases at a constant rate until it reaches its maximum and starts overflowing to keep a constant volume. This is best represented by graph $\boxed{\text{(A)}\ A}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 5	Followed by Problem 7
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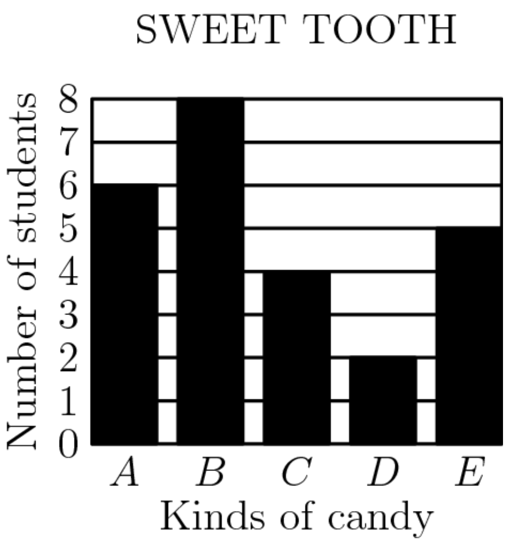
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 7

Problem

The students in Mrs. Sawyer's class were asked to do a taste test of five kinds of candy. Each student chose one kind of candy. A bar graph of their preferences is shown. What percent of her class chose candy E?



$\text{(A)}\ 5 \qquad \text{(B)}\ 12 \qquad \text{(C)}\ 15 \qquad \text{(D)}\ 16 \qquad \text{(E)}\ 20$

Solution

From the bar graph, we can see that 5 students chose candy E. There are $6+8+4+2+5=25$ total students in Mrs. Sawyers class. The percent that chose E is $\frac{5}{25}\cdot 100=\boxed{\text{(E)}\ 20}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 6	Followed by Problem 8
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Retrieved from "http://artofproblemsolving.com/wiki/index.php?title=2002_AMC_8_Problems/Problem_7&oldid=55868"

2002 AMC 8 Problems/Problem 8

Contents

Juan's Old Stamping Grounds

Problems 8,9 and 10 use the data found in the accompanying paragraph and table:

Juan organizes the stamps in his collection by country and by the decade in which they were issued. The prices he paid for them at a stamp shop were: Brazil and France, 6 cents each, Peru 4 cents each, and Spain 5 cents each. (Brazil and Peru are South American countries and France and Spain are in Europe.)

Number of Stamps by Decade

Country	50s	60s	70s	80s
Brazil	4	7	12	8
France	8	4	12	15
Peru	6	4	6	10
Spain	3	9	13	9

Juan's Stamp Collection

Problem

How many of his European stamps were issued in the '80s?

$\text{(A)}\ 9 \qquad \text{(B)}\ 15 \qquad \text{(C)}\ 18 \qquad \text{(D)}\ 24 \qquad \text{(E)}\ 42$

Solution

France and Spain are European countries. The number of '80s stamps from France is 15 and the number of '80s stamps from Spain is 9. The total number of stamps is $15+9=\boxed{\text{(D)}\ 24}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources)

Preceded by	Followed by
Problem 7	Problem 9

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All AJHSME/AMC 8 Problems and Solutions

2002 AMC 8 Problems/Problem 9

Contents

Juan's Old Stamping Grounds

Problems 8,9 and 10 use the data found in the accompanying paragraph and table:

Juan organizes the stamps in his collection by country and by the decade in which they were issued. The prices he paid for them at a stamp shop were: Brazil and France, 6 cents each, Peru 4 cents each, and Spain 5 cents each. (Brazil and Peru are South American countries and France and Spain are in Europe.)

Number of Stamps by Decade

Country	50s	60s	70s	80s
Brazil	4	7	12	8
France	8	4	12	15
Peru	6	4	6	10
Spain	3	9	13	9

Juan's Stamp Collection

Problem

His South American stamps issued before the '70s cost him

$\text{(A)}\ \$0.40 \qquad \text{(B)}\ \$1.06 \qquad \text{(C)}\ \$1.80 \qquad \text{(D)}\ \$2.38 \qquad \text{(E)}\ \$2.64$

Solution

The number of stamps from Brazil in the '50s and '60s is $4+7=11$, and they cost 6 cents each for a total of $11 \cdot \$0.06 = \$0.66$. The number of stamps from Peru in the '50s and '60s is $6+4=10$, and they cost 4 cents each for a total of $10 \cdot \$0.04 = \$0.40$. In total, he paid $0.66+0.40=\boxed{\text{(B)}\ \$1.06}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 8	Followed by Problem 10
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All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 10

Contents

Juan's Old Stamping Grounds

Problems 8,9 and 10 use the data found in the accompanying paragraph and table:

Juan organizes the stamps in his collection by country and by the decade in which they were issued. The prices he paid for them at a stamp shop were: Brazil and France, 6 cents each, Peru 4 cents each, and Spain 5 cents each. (Brazil and Peru are South American countries and France and Spain are in Europe.)

Number of Stamps by Decade

Country	50s	60s	70s	80s
Brazil	4	7	12	8
France	8	4	12	15
Peru	6	4	6	10
Spain	3	9	13	9

Juan's Stamp Collection

Problem

The average price of his '70s stamps is closest to

$\textbf{(A)}\ 3.5\text{ cents} \qquad \textbf{(B)}\ 4\text{ cents} \qquad \textbf{(C)}\ 4.5\text{ cents} \qquad \textbf{(D)}\ 5\text{ cents} \qquad \textbf{(E)}\ 5.5\text{ cents}$

Solution

The price of all the stamps in the '70s together over the total number of stamps is equal to the average price.

$$\frac{(12)(0.06)+(12)(0.06)+(6)(0.04)+(13)(0.05)}{12+12+6+13} = \frac{0.72+0.72+0.24+0.65}{43} = \frac{2.33}{43} \approx \boxed{\textbf{(E)}\ 5.5\text{ cents}}$$

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 9	Followed by Problem 11
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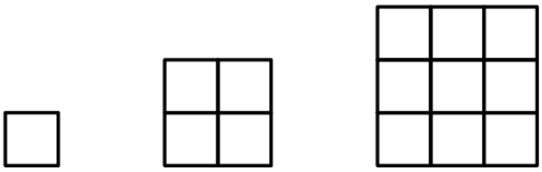
2002 AMC 8 Problems/Problem 11

Contents

Problem

A sequence of squares is made of identical square tiles. The edge of each square is one tile length longer than the edge of the previous square. The first three squares are shown. How many more tiles does the seventh square require than the sixth?



$\text{(A)}\ 11 \qquad \text{(B)}\ 12 \qquad \text{(C)}\ 13 \qquad \text{(D)}\ 14 \qquad \text{(E)}\ 15$

Solution

Solution 1

The first square has a sidelength of 1, the second square 2, and so on. The seventh square has 7 and is made of $7^2=49$ unit tiles. The sixth square has 6 and is made of $6^2=36$ unit tiles. The seventh square has $49-36=\boxed{\text{(C)}\ 13}$ more tiles than the sixth square.

Solution 2

The edge of each square is one tile length longer than the edge of the previous square, which means that each square has $2*$ edge length -1 more tiles than the previous square, because each square is just one edge added on the top and on the right to the previous square, with one overlapping tile. Then the seventh square has $2^27-1=13$ more tiles than the sixth square, which is $\boxed{\text{(C)}\ 13}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 10	Followed by Problem 12
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2002 AMC 8 Problems/Problem 12

Problem

A board game spinner is divided into three regions labeled A, B and C. The probability of the arrow stopping on region A is $\frac{1}{3}$ and on region B is $\frac{1}{2}$. The probability of the arrow stopping on region C is:

$\text{(A)}\ \frac{1}{12} \qquad \text{(B)}\ \frac{1}{6} \qquad \text{(C)}\ \frac{1}{5} \qquad \text{(D)}\ \frac{1}{3} \qquad \text{(E)}\ \frac{2}{5}$

Solution

Since the arrow must land in one of the three regions, the sum of the probabilities must be 1. Thus the answer is $1-\frac{1}{2}-\frac{1}{3}=\boxed{\text{(B)}\ \frac{1}{6}}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 11	Followed by Problem 13
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2002 AMC 8 Problems/Problem 13

Problem

For his birthday, Bert gets a box that holds 125 jellybeans when filled to capacity. A few weeks later, Carrie gets a larger box full of jellybeans. Her box is twice as high, twice as wide and twice as long as Bert's. Approximately, how many jellybeans did Carrie get?

$\text{(A)}\ 250 \qquad \text{(B)}\ 500 \qquad \text{(C)}\ 625 \qquad \text{(D)}\ 750 \qquad \text{(E)}\ 1000$

Solution

$1^3 = 12^3 = 88 * 125 = 1000 \boxed{\text{(E)}\ 1000}.$

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 12	Followed by Problem 14
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All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 14

Contents

Problem

A merchant offers a large group of items at 30% off. Later, the merchant takes 20% off these sale prices and claims that the final price of these items is 50% off the original price. The total discount is

$\text{(A)}\ 35\% \qquad \text{(B)}\ 44\% \qquad \text{(C)}\ 50\% \qquad \text{(D)}\ 56\% \qquad \text{(E)}\ 60\%$

Solution

Solution 1

Let's assume that each item is 100 dollars. First we take off 30% off of 100 dollars. $100 \cdot 0.7 = 70$

Next, we take off the extra 20% as asked by the problem. $70 \cdot 0.80 = 56$

So the final price of an item is \$56. We have to do $100 - 56$ because 56 was the final price and we wanted the discount.

$100 - 56 = 44$ so the final discount was $\boxed{\text{(B)}\ 44\%}$.

Solution 2

Assume the price was \$100. We can just do $100 \cdot 0.7 \cdot 0.8 = 56$ and then do $100 - 56 = \boxed{\text{(B)}\ 44}$. That is the discount percentage wise.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 13	Followed by Problem 15
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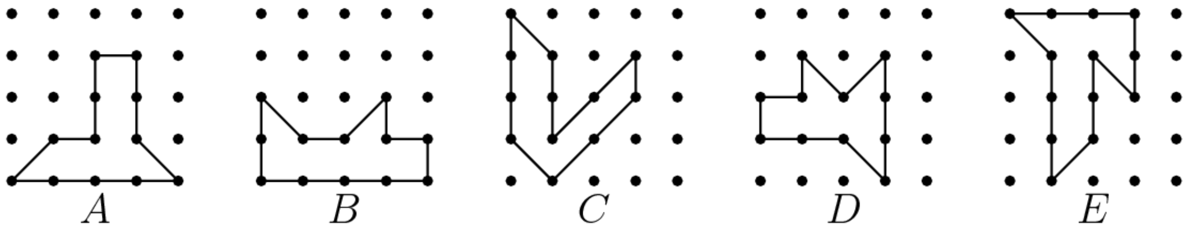
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 15

Problem

Which of the following polygons has the largest area?



Which of the following polygons has the largest area?

$\textbf{(A)}\text{A}\qquad\textbf{(B)}\ \text{B}\qquad\textbf{(C)}\ \text{C}\qquad\textbf{(D)}\ \text{D}\qquad\textbf{(E)}\ \text{E}$

Solution

Each polygon can be partitioned into unit squares and right triangles with sidelength 1. Count the number of boxes enclosed by each polygon, with the unit square being 1, and the triangle being being $.5$. A has 5, B has 5, C has 5, D has 4.5, and E has 5.5. Therefore, the polygon with the largest area is $\boxed{\textbf{(E)}\ \text{E}}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
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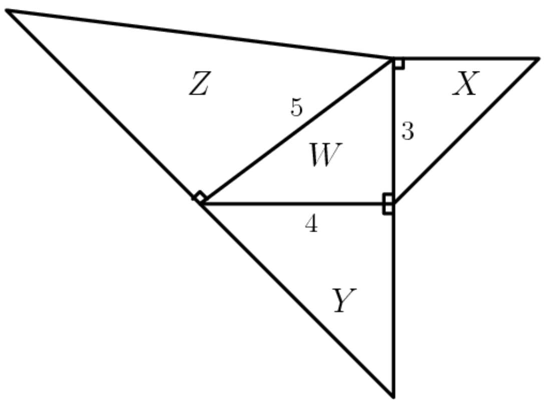
2002 AMC 8 Problems/Problem 16

Contents

Problem

Right isosceles triangles are constructed on the sides of a 3-4-5 right triangle, as shown. A capital letter represents the area of each triangle. Which one of the following is true?



$\textbf{(A)}\ X+Z=W+Y\qquad\textbf{(B)}\ W+X=Z\qquad\textbf{(C)}\ 3X+4Y=5Z$
$\textbf{(D)}\ X+W=\frac{1}{2}(Y+Z)\qquad\textbf{(E)}\ X+Y=Z$

Solution 1

The area of a right triangle can be found by using the legs of triangle as the base and height. In the three isosceles triangles, the length of their second leg is the same as the side that is connected to the $3-4-5$ triangle.

$$W=(3)(4)/2=6$$
$$X=(3)(3)/2=4.5$$
$$Y=(4)(4)/2=8$$
$$Z=(5)(5)/2=12.5$$

Plugging into the answer choices, the only that works is $\boxed{\textbf{(E)}\ X+Y=Z}$.

Solution 2

Looking at the diagram, we notice that three right isosceles triangles on one right triangle reminds us of the Pythagorean theorem, since each right isosceles triangle is actually half of a square. Each square's area represents a side length squared, so the squares on the legs of the right triangle adds to the square on the hypotenuse. This gives $2X+2Y=2Z$. Then, dividing by 2 we get $X+Y=Z$, which is one of the answer choices. Since there can only be one correct answer, and there is already one, we see that the answer must be $\boxed{\textbf{(E)}\ X+Y=Z}$.

2002 AMC 8 Problems/Problem 17

Contents

Problem

In a mathematics contest with ten problems, a student gains 5 points for a correct answer and loses 2 points for an incorrect answer. If Olivia answered every problem and her score was 29, how many correct answers did she have?

$\text{(A)}\ 5 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 7 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 9$

Solution 1

Let a be the number of problems she answers correctly and b be the number she answered incorrectly. Because she answers all of the questions $a+b=10$. Her score is equal to $5a-2b=29$. Use substitution.

$$\begin{aligned} b &= 10-a \\ 5a-2(10-a) &= 29 \\ 5a-20+2a &= 29 \\ 7a &= 49 \\ a &= \boxed{\text{(C)}\ 7} \end{aligned}$$

Solution 2

We can start with the full score, 50, and subtract not only 2 points for each correct answer but also the 5 points we gave her credit for. This expression is equivalent to her score, 29. Let x be the number of questions she answers correctly. Then, we will represent the number incorrect by $10-x$.

$$\begin{aligned} 50-7(10-x) &= 29 \\ 50-70+7x &= 29 \\ 7x &= 49 \\ x &= \boxed{\text{(C)}\ 7} \end{aligned}$$

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
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2002 AMC 8 Problems/Problem 18

Problem

Gage skated 1 hr 15 min each day for 5 days and 1 hr 30 min each day for 3 days. How long would he have to skate the ninth day in order to average 85 minutes of skating each day for the entire time?

$\text{(A)}\ 1\text{ hr}\qquad\text{(B)}\ 1\text{ hr }10\text{ min}\qquad\text{(C)}\ 1\text{ hr }20\text{ min}\qquad\text{(D)}\ 1\text{ hr }40\text{ min}\qquad\text{(E)}\ 2\text{ hr}$

Solution

Converting into minutes and adding, we get that she skated $75*5+90*3+x=375+270+x=645+x$ minutes total, where x is the amount she skated on day 9. Dividing by 9 to get the average, we get $\frac{645+x}{9}=85$. Solving for x,

$$645+x=765$$

$$x=120$$

Now we convert back into hours and minutes to get $\boxed{\text{(E)}\ 2\text{ hr}}$.

See Also

2002 AMC 8 Problems/Problem 19

Problem

How many whole numbers between 99 and 999 contain exactly one 0?

$\text{(A)}\ 72 \qquad \text{(B)}\ 90 \qquad \text{(C)}\ 144 \qquad \text{(D)}\ 162 \qquad \text{(E)}\ 180$

Solution

This list includes all the three digit whole numbers except 999. Because the hundreds digit cannot be 0, there are 2 ways to choose whether the tens digit or the ones digit is equal to 0. Then for the two remaining places, there are 9 ways to choose each digit. This gives a total of $(2)(9)(9) = \boxed{\text{(D)}\ 162}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
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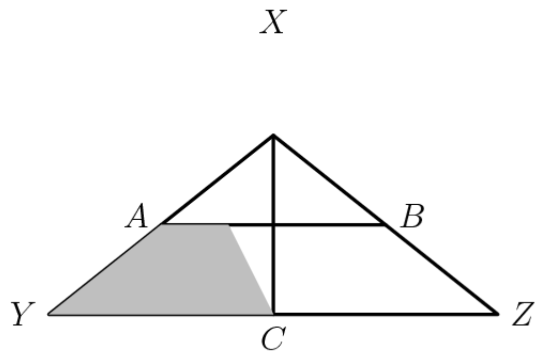
2002 AMC 8 Problems/Problem 20

Contents

Problem

The area of triangle XYZ is 8 square inches. Points A and B are midpoints of congruent segments $\overline{XY}$ and $\overline{XZ}$. Altitude $\overline{XC}$ bisects $\overline{YZ}$. What is the area (in square inches) of the shaded region?



$\textbf{(A)}\ 1\frac{1}{2}\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 2\frac{1}{2}\qquad\textbf{(D)}\ 3\qquad\textbf{(E)}\ 3\frac{1}{2}$

Solution 1

The shaded region is a right trapezoid. Assume WLOG that $YZ = 8$. Then because the area of $\triangle XYZ$ is equal to 8, the height of the triangle $XC = 2$. Because the line AB is a midsegment, the top base of the triangle is $\frac{1}{2}AB = \frac{1}{4}YZ = 2$. Also, AB divides XC in two, so the height of the trapezoid is $\frac{1}{2}2 = 1$. The bottom base is $\frac{1}{2}YZ = 4$. The area of the shaded region is $\frac{1}{2}(2+4)(1) = \boxed{\text{(D)}\ 3}$.

Solution 2

Since A and B are the midpoints of XY and XZ, respectively, $AY = AX = BX = BZ$. Draw segments AC and BC. Since $XY = XZ$, it means that X is on the perpendicular bisector of YZ. Then $YC = CZ$. AB is the line that connects the midpoints of two sides of a triangle together, which means that AB is parallel to and half in length of YZ. Then $AB = YC = CZ$. Since AB is parallel to YZ, and XY is the transversal, $\angle XAB = \angle AYC$. Similarly, $\angle XBA = \angle BZC$. Then, by SAS, $\triangle XAB = \triangle AYC = \triangle BZC$. Since corresponding parts of congruent triangles are congruent, $AC = BC = XA$. Using the fact that AB is parallel to YZ, $\angle ABC = \angle BCZ$ and $\angle BAC = \angle ACY$. Also, $\angle ABC = \angle BCZ = \angle ACY$ because $\triangle ABC$ is isosceles. Now $\triangle XAB = \triangle AYC = \triangle BZC = \triangle ABC$. Draw an altitude through each of them such that each triangle is split into two congruent right triangles. Now there are a total of 8 congruent small triangles, each with area 1. The shaded area has three of these triangles, so it has area 3.

Basically the proof is to show $\triangle XAB = \triangle AYC = \triangle BZC = \triangle ABC$. If you just look at the diagram you can easily see that the triangles are congruent and you would solve this a lot faster. Anyways, since those triangles are congruent, you can split each in half to find eight congruent triangles with area 1, and since the shaded region has three of these triangles, its area is $\boxed{(\text{D})\ 3}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
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2002 AMC 8 Problems/Problem 21

Problem

Harold tosses a coin four times. The probability that he gets at least as many heads as tails is

$\text{(A)}\ \frac{5}{16} \qquad \text{(B)}\ \frac{3}{8} \qquad \text{(C)}\ \frac{1}{2} \qquad \text{(D)}\ \frac{5}{8} \qquad \text{(E)}\ \frac{11}{16}$

Solution

Case 1: There are two heads, two tails. The number of ways to choose which two tosses are heads is ${}_4C_2 = 6$, and the other two must be tails.

Case 2: There are three heads, one tail. There are ${}_4C_1 = 4$ ways to choose which of the four tosses is a tail.

Case 3: There are four heads, no tails. This can only happen 1 way.

There are a total of $2^4 = 16$ possible configurations, giving a probability of $\frac{6+4+1}{16} = \boxed{\text{(E)}\ \frac{11}{16}}$.

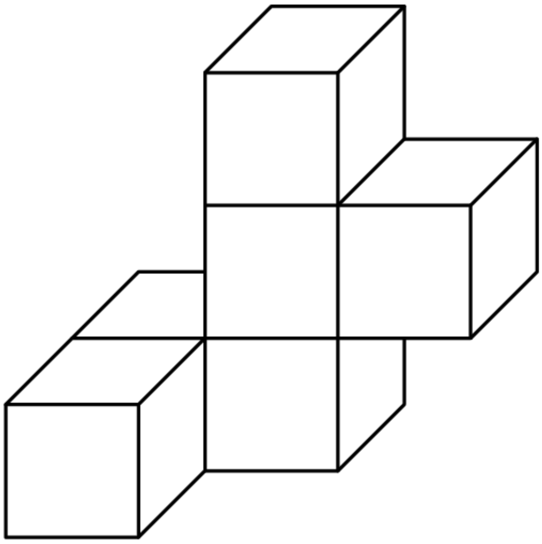
See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 22

Problem

Six cubes, each an inch on an edge, are fastened together, as shown. Find the total surface area in square inches. Include the top, bottom and sides.



$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 36$

Solution

Count the number of sides that are not exposed, where a cube is connected to another cube and subtract it from the total number of faces. There are 5 places with two adjacent cubes, covering 10 sides, and $(6)(6)=36$ faces. The exposed surface area is $36-10=\boxed{\text{(C)}\ 26}$.

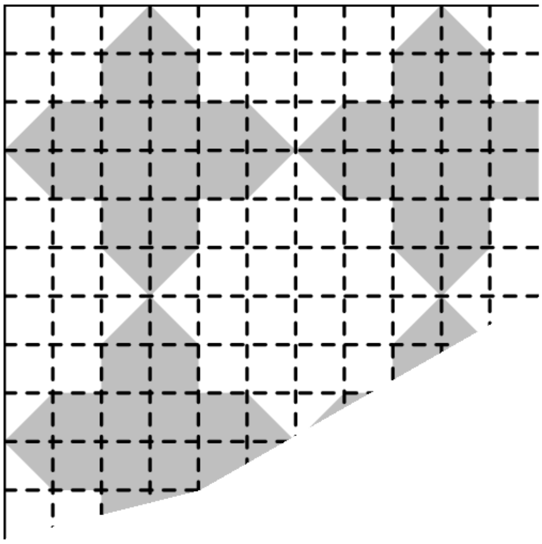
See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 23

Problem

A corner of a tiled floor is shown. If the entire floor is tiled in this way and each of the four corners looks like this one, then what fraction of the tiled floor is made of darker tiles?



$\textbf{(A)}\ \frac{1}{3}\qquad\textbf{(B)}\ \frac{4}{9}\qquad\textbf{(C)}\ \frac{1}{2}\qquad\textbf{(D)}\ \frac{5}{9}\qquad\textbf{(E)}\ \frac{5}{8}$

Solution

The same pattern is repeated for every 6×6 tile. Looking closer, there is also symmetry of the top 3×3 square, so the fraction of the entire floor in dark tiles is the same as the fraction in the square. Counting the tiles, there are 4 dark tiles, and 9 total tiles, giving a fraction of $\boxed{\text{(B)}\ \frac{4}{9}}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 24

Problem

Miki has a dozen oranges of the same size and a dozen pears of the same size. Miki uses her juicer to extract 8 ounces of pear juice from 3 pears and 8 ounces of orange juice from 2 oranges. She makes a pear-orange juice blend from an equal number of pears and oranges. What percent of the blend is pear juice?

$\text{(A)}\ 30\qquad\text{(B)}\ 40\qquad\text{(C)}\ 50\qquad\text{(D)}\ 60\qquad\text{(E)}\ 70$

Solution

A pear gives $8/3$ ounces of juice per pear. An orange gives $8/2=4$ ounces of juice per orange. If the pear-orange juice blend used one pear and one orange each, the percentage of pear juice would be

$$\frac{8/3}{8/3+4} \times 100 = \frac{8}{8+12} \times 100 = \boxed{\text{(B)}\ 40}$$

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2002 AMC 8 Problems/Problem 25

Problem

Loki, Moe, Nick and Ott are good friends. Ott had no money, but the others did. Moe gave Ott one-fifth of his money, Loki gave Ott one-fourth of his money and Nick gave Ott one-third of his money. Each gave Ott the same amount of money. What fractional part of the group's money does Ott now have?

$\text{(A)}\ \frac{1}{10} \qquad \text{(B)}\ \frac{1}{4} \qquad \text{(C)}\ \frac{1}{3} \qquad \text{(D)}\ \frac{2}{5} \qquad \text{(E)}\ \frac{1}{2}$

Solution

Since Ott gets equal amounts of money from each friend, we can say that he gets x dollars from each friend. This means that Moe has $5x$ dollars, Loki has $4x$ dollars, and Nick has $3x$ dollars. The total amount is $12x$ dollars, and since Ott gets $3x$ dollars total, $\frac{3x}{12x} = \frac{3}{12} = \boxed{\text{(B)}\ \frac{1}{4}}$.

See Also

2002 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2002))	
Preceded by Problem 24	Followed by Last Problem
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	