2009 AMC 8 Problems/Problem 1

Problem

Bridget bought a bag of apples at the grocery store. She gave half of the apples to Ann. Then she gave Cassie 3 apples, keeping 4 apples for herself. How many apples did Bridget buy?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 14$

Solution

Let us work backwards. We know that Cassie had 4 apples for herself at the end, and we know she gave away 3 apples before. Therefore, she had 7 apples before giving half of her original amount of apples to someone else. Since half of the amount of original apples is equal to seven, then the original amount of apples Bridget had is $7 \cdot 2$, giving us the answer $\boxed{\textbf{(E)}\ 14}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by First Problem	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 2

Problem

On average, for every 4 sports cars sold at the local dealership, 7 sedans are sold. The dealership predicts that it will sell 28 sports cars next month. How many sedans does it expect to sell?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 49 \qquad \textbf{(E)}\ 112$

Solution

This means the ratio is 4:7. If the ratio now is 28:x, then that means $x = \boxed{\textbf{(D)}\ 49}$.

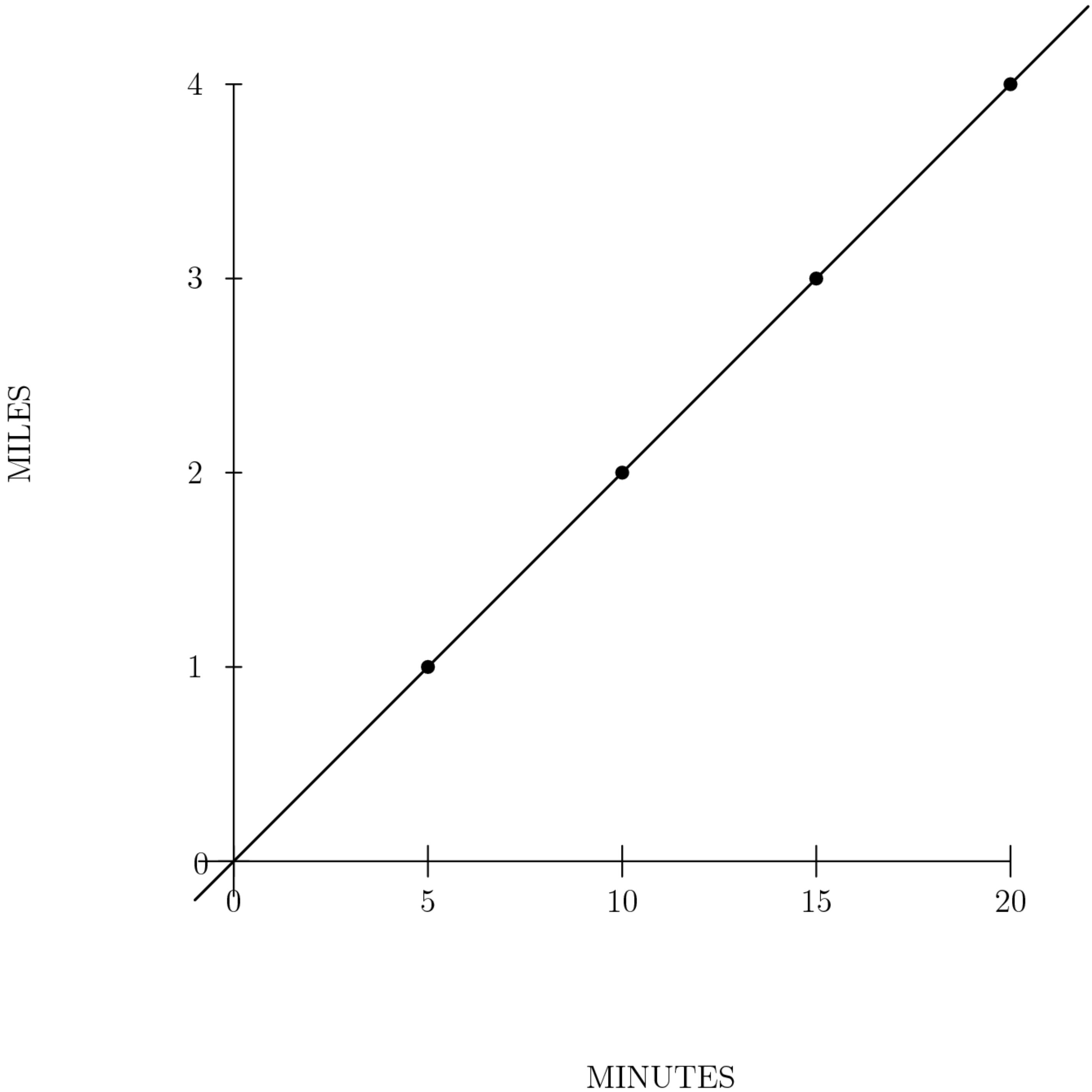
See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 3

Problem

The graph shows the constant rate at which Suzanna rides her bike. If she rides a total of a half an hour at the same speed, how many miles would she have ridden?



$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 5.5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 6.5 \qquad \textbf{(E)}\ 7$

Solution

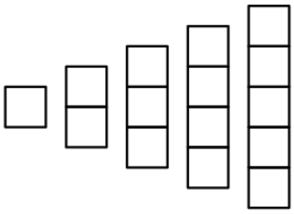
Suzanna's speed is $\frac{1}{5}$. This means she runs $\frac{1}{5}\cdot 30=\boxed{\textbf{(C)}\ 6}$

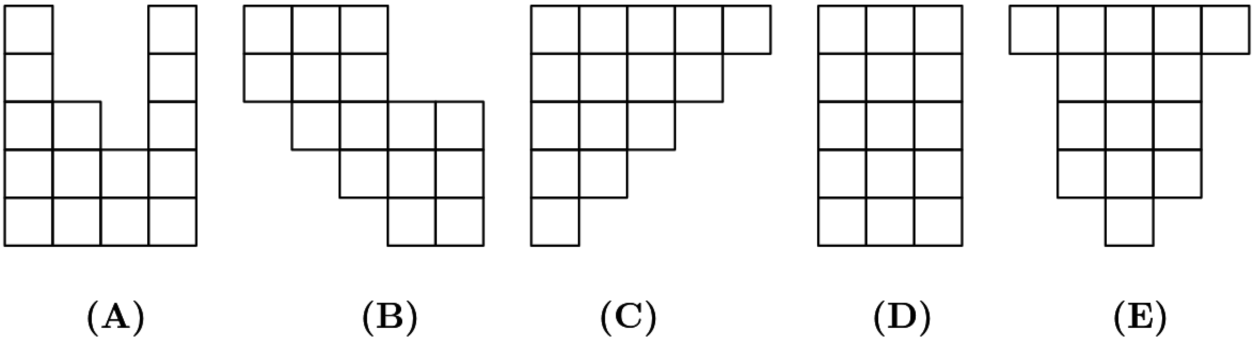
See Also

2009 AMC 8 Problems/Problem 4

Problem

The five pieces shown below can be arranged to form four of the five figures shown in the choices. Which figure cannot be formed?





Solution

The answer is $\boxed{\textbf{(B)}}$ because the longest piece can't fit in the figure.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 5

Problem

A sequence of numbers starts with 1, 2, and 3. The fourth number of the sequence is the sum of the previous three numbers in the sequence: $1+2+3=6$. In the same way, every number after the fourth is the sum of the previous three numbers. What is the eighth number in the sequence?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 37 \qquad \textbf{(D)}\ 68 \qquad \textbf{(E)}\ 99$

Solution

List them out, adding the three previous numbers to get the next number,

$$1,2,3,6,11,20,37,\boxed{\textbf{(D)}\ 68}$$

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 6

Problem 6

Steve's empty swimming pool will hold $24,000$ gallons of water when full. It will be filled by 4 hoses, each of which supplies 2.5 gallons of water per minute. How many hours will it take to fill Steve's pool?

$\textbf{(A)}\ 40 \qquad \textbf{(B)}\ 42 \qquad \textbf{(C)}\ 44 \qquad \textbf{(D)}\ 46 \qquad \textbf{(E)}\ 48$

Solution

Each of the four hoses hose fills $24,000/4=6,000$ gallons of water. At the rate it goes at it will take $6,000/2.5=2400$ minutes, or $\boxed{\textbf{(A)}\ 40}$ hours.

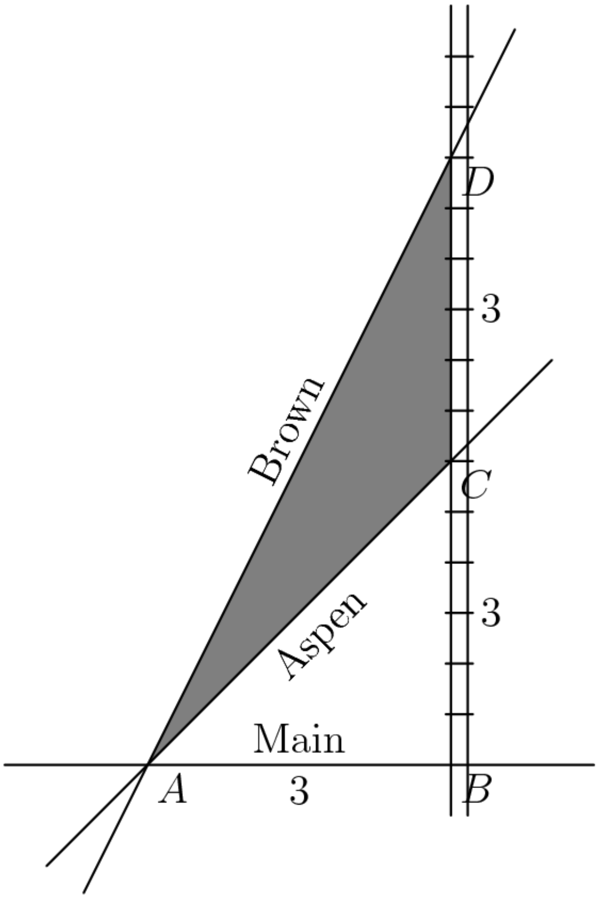
See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 7

Problem

The triangular plot of ACD lies between Aspen Road, Brown Road and a railroad. Main Street runs east and west, and the railroad runs north and south. The numbers in the diagram indicate distances in miles. The width of the railroad track can be ignored. How many square miles are in the plot of land ACD?



$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4.5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 9$

Solution

The area of a triangle is $\frac{1}{2}bh$. If we let CD be the base of the triangle, then the height is AB, and the area is $\frac{1}{2} \cdot 3 \cdot 3 = \boxed{\textbf{(C)}\ 4.5}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 8

Problem

The length of a rectangle is increased by 10% percent and the width is decreased by 10% percent. What percent of the old area is the new area?

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 99 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 101 \qquad \textbf{(E)}\ 110$

Solution

In a rectangle with dimensions 10×10, the new rectangle would have dimensions 11×9. The ratio of the new area to the old area is $99/100 = \boxed{\textbf{(B)}\ 99}$.

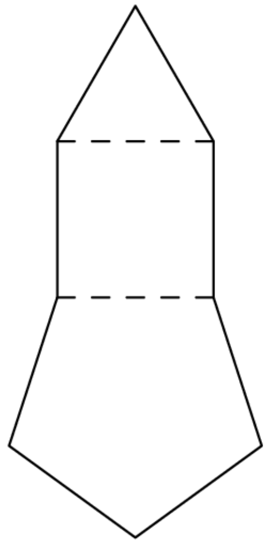
See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 9

Problem

Construct a square on one side of an equilateral triangle. One on non-adjacent side of the square, construct a regular pentagon, as shown. One a non-adjacent side of the pentagon, construct a hexagon. Continue to construct regular polygons in the same way, until you construct an octagon. How many sides does the resulting polygon have?



$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 23 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 29$

Solution

Of the six shapes used to create the polygon, the triangle and octagon are adjacent to the others on one side, and the others are adjacent on two sides. In the triangle and octagon $3+8-2(1)=9$ sides are on the outside of the final polygon. In the other shapes $4+5+6+7-4(2)=14$ sides are on the outside. The resulting polygon has $9+14=\boxed{\textbf{(B)}\ 23}$ sides.

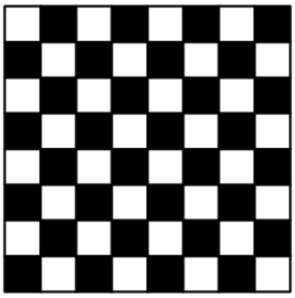
See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 10

Problem

On a checkerboard composed of 64 unit squares, what is the probability that a randomly chosen unit square does not touch the outer edge of the board?



$\textbf{(A)}\ \frac{1}{16} \qquad \textbf{(B)}\ \frac{7}{16} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{9}{16} \qquad \textbf{(E)}\ \frac{49}{64}$

Solution

There are $8^2=64$ total squares. There are $(8-1)(4)=28$ unit squares on the perimeter and therefore $64-28=36$ NOT on the perimeter. The probability of choosing one of those squares is $\frac{36}{64} = \boxed{\textbf{(D)}\ \frac{9}{16}}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 11

Problem

The Amaco Middle School bookstore sells pencils costing a whole number of cents. Some seventh graders each bought a pencil, paying a total of 1.43 dollars. Some of the 30 sixth graders each bought a pencil, and they paid a total of 1.95 dollars. How many more sixth graders than seventh graders bought a pencil?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Because the pencil costs a whole number of cents, the cost must be a factor of both 143 and 195. They can be factored into $11 \cdot 13$ and $3 \cdot 5 \cdot 13$. The common factor cannot be 1 or there would have to be more than 30 sixth graders, so the pencil costs 13 cents. The difference in costs that the sixth and seventh graders paid is $195 - 143 = 52$ cents, which is equal to $52/13 = \boxed{\textbf{(D)}\ 4}$ pencils.

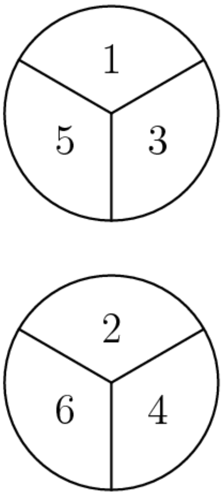
See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 12

Problem

The two spinners shown are spun once and each lands on one of the numbered sectors. What is the probability that the sum of the numbers in the two sectors is prime?



$\textbf{(A)}\ \frac{1}{2}\qquad\textbf{(B)}\ \frac{2}{3}\qquad\textbf{(C)}\ \frac{3}{4}\qquad\textbf{(D)}\ \frac{7}{9}\qquad\textbf{(E)}\ \frac{5}{6}$

Solution

The possible sums are

	1	3	5
2	3	5	7
4	5	7	9
6	7	9	11

Only 9 is not prime, so there are 7 prime numbers and 9 total numbers for a probability of $\boxed{\textbf{(D)}\ \frac{7}{9}}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 13

Problem

A three-digit integer contains one of each of the digits 1, 3, and 5. What is the probability that the integer is divisible by 5?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{5}{6}$

Solution

The three digit numbers are $135, 153, 351, 315, 513, 531$. The numbers that end in 5 are divisible are 5, and the probability of choosing those numbers is $\boxed{\textbf{(B)}\ \frac{1}{3}}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 14

Problem

Austin and Temple are 50 miles apart along Interstate 35. Bonnie drove from Austin to her daughter's house in Temple, averaging 60 miles per hour. Leaving the car with her daughter, Bonnie rode a bus back to Austin along the same route and averaged 40 miles per hour on the return trip. What was the average speed for the round trip, in miles per hour?

$\textbf{(A)}\ 46 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 52 \qquad \textbf{(E)}\ 54$

Solution

The way to Temple took $\frac{50}{60}=\frac{5}{6}$ hours, and the way back took $\frac{50}{40}=\frac{5}{4}$ for a total of $\frac{5}{6}+\frac{5}{4}=\frac{25}{12}$ hours. The trip is $50 \cdot 2=100$ miles. The average speed is $\frac{100}{25/12}=\boxed{\textbf{(B)}\ 48}$ miles per hour.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 15

Problem

A recipe that makes 5 servings of hot chocolate requires 2 squares of chocolate, $\frac{1}{4}$ cup sugar, 1 cup water and 4 cups milk. Jordan has 5 squares of chocolate, 2 cups of sugar, lots of water and 7 cups of milk. If she maintains the same ratio of ingredients, what is the greatest number of servings of hot chocolate she can make?

$$\textbf{(A)}\ 5\frac{1}{8} \qquad \textbf{(B)}\ 6\frac{1}{4} \qquad \textbf{(C)}\ 7\frac{1}{2} \qquad \textbf{(D)}\ 8\frac{3}{4} \qquad \textbf{(E)}\ 9\frac{7}{8}$$

Solution

Asumming excesses of the other ingredients, the chocolate can make $\frac{5}{2}\cdot 5 = 12.5$ servings, the sugar can make $\frac{2}{1/4}\cdot 5 = 40$ servings, the water can make unlimited servings, and the milk can make $\frac{7}{4}\cdot 5 = 8.75$ servings. Limited by the amount of milk, Jordan can make at most $\boxed{\textbf{(D)}\ 8\frac{3}{4}}$ servings.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 16

Problem

How many 3-digit positive integers have digits whose product equals 24?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 24$

Solution

With the digits listed from least to greatest, the 3-digit integers are $138, 146, 226, 234$. 226 can be arranged in $\frac{3!}{2!} = 3$ ways, and the other three can be arranged in $3! = 6$ ways. There are $3 + 6(3) = \boxed{\textbf{(D)}\ 21}$ 3-digit positive integers.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 15	Followed by Problem 17
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 17

Problem

The positive integers x and y are the two smallest positive integers for which the product of 360 and x is a square and the product of 360 and y is a cube. What is the sum of x and y?

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 85 \qquad \textbf{(C)}\ 115 \qquad \textbf{(D)}\ 165 \qquad \textbf{(E)}\ 610$

Solution

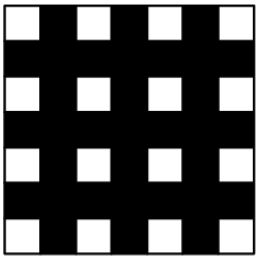
The prime factorization of $360 = 2^3 * 3^2 * 5$. If a number is a perfect square, all of the exponents in its prime factorization must be even. Thus we need to multiply by a 2 and a 5, for a product of 10, which is x. Similarly, y can be found by making all the exponents divisible by 3, so $y = 3 * 5^2 = 75$. Thus $x+y = \boxed{\textbf{(B)}\ 85}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 18

The diagram represents a 7-foot-by-7-foot floor that is tiled with 1-square-foot black tiles and white tiles. Notice that the corners have white tiles. If a 15-foot-by-15-foot floor is to be tiled in the same manner, how many white tiles will be needed?



$\textbf{(A)}\ 49 \qquad \textbf{(B)}\ 57 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 96 \qquad \textbf{(E)}\ 126$

Solution

In a 1-foot-by-1-foot floor, there is 1 white tile. In a 3-by-3, there are 4. Continuing on, you can deduce the n^{th} positive odd integer floor has n^2 white tiles. 15 is the 8^{th} odd integer, so there are $\boxed{\textbf{(C)}\ 64}$ white tiles.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 19

Problem

Two angles of an isosceles triangle measure 70° and x°. What is the sum of the three possible values of x?

$\textbf{(A)}\ 95 \qquad \textbf{(B)}\ 125 \qquad \textbf{(C)}\ 140 \qquad \textbf{(D)}\ 165 \qquad \textbf{(E)}\ 180$

Solution

There are 3 cases: where x° is a base angle with the 70° as the other angle, where x° is a base angle with 70° as the vertex angle, and where x° is the vertex angle with 70° as a base angle.

Case 1: x° is a base angle with the 70° as the other angle: Here, $x = 70$, since base angles are congruent.

Case 2: x° is a base angle with 70° as the vertex angle: Here, the 2 base angles are both x°, so we can use the equation $2x + 70 = 180$, which simplifies to $x = 55$.

Case 3: x° is the vertex angle with 70° as a base angle: Here, both base angles are 70°, since base angles are congruent. Thus, we can use the equation $x + 140 = 180$, which simplifies to $x = 40$.

Adding up all the cases, we get $70 + 55 + 40 = 165$, so the answer is $\boxed{\textbf{(D)}\ 165}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 20

Problem

How many non-congruent triangles have vertices at three of the eight points in the array shown below?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Solution

Assume the base of the triangle is on the bottom four points, because a congruent triangle can be made by reflecting the base on the top four points. For a triangle with a base of length 1, there are 3 triangles. For a triangle with a base of length 2, there are 3 triangles. For length 3, there are 2. In total, the number of non-congruent triangles is $3+3+2=\boxed{\textbf{(D)}\ 8}$.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 21

Problem

Andy and Bethany have a rectangular array of numbers greater than zero with 40 rows and 75 columns. Andy adds the numbers in each row. The average of his 40 sums is A. Bethany adds the numbers in each column. The average of her 75 sums is B. Using only the answer choices given, What is the value of $\frac{A}{B}$?

$\textbf{(A)}\ \frac{64}{225} \qquad \textbf{(B)}\ \frac{8}{15} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{15}{8} \qquad \textbf{(E)}\ \frac{225}{64}$

Solution

Note that $40A = 75B = \text{sum of the numbers in the array}$. This means that $\frac{A}{B} = \frac{15}{8} \textbf{(D)}$ using basic algebraic manipulation.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 22

Problem

How many whole numbers between 1 and 1000 do not contain the digit 1?

$\textbf{(A)}\ 512 \qquad \textbf{(B)}\ 648 \qquad \textbf{(C)}\ 720 \qquad \textbf{(D)}\ 728 \qquad \textbf{(E)}\ 800$

Solution

Note that this is the same as finding how many numbers with up to three digits do not contain 1.

Since there are 10 total possible digits, and only one of them is not allowed (1), each digit has its choice of 9 digits, for a total of $9*9*9=729$ such numbers. However, we over counted by one; 0 is not between 1 and 1000, so there are $\boxed{\textbf{(D)}\ 728}$ numbers.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 23

Problem

On the last day of school, Mrs. Wonderful gave jelly beans to her class. She gave each boy as many jelly beans as there were boys in the class. She gave each girl as many jelly beans as there were girls in the class. She brought 400 jelly beans, and when she finished, she had six jelly beans left. There were two more boys than girls in her class. How many students were in her class?

$\textbf{(A)}\ 26 \qquad \textbf{(B)}\ 28 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 34$

Solution

If there are x girls, then there are $x+2$ boys. She gave each girl x jellybeans and each boy $x+2$ jellybeans, for a total of $x^2 + (x+2)^2$ jellybeans. She gave away $400-6=394$ jellybeans.

$$\begin{aligned} x^2 + (x+2)^2 &= 394 \\ x^2 + x^2 + 4x + 4 &= 394 \\ 2x^2 + 4x - 390 &= 0 \\ x^2 + 2x - 195 &= 0 \\ (x+15)(x-13) &= 0 \end{aligned}$$

Because $x=-15, 13$, there are 13 girls, 15 boys, and $13+15=\boxed{\textbf{(B)}\ 28}$ students.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 22	Followed by Problem 24
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2009 AMC 8 Problems/Problem 24

Problem

The letters A, B, C and D represent digits. If $\begin{array}{ccc}&A&B\\ +&C&A\\ \hline &D&A\end{array}$ and $\begin{array}{ccc}&A&B\\ -&C&A\\ \hline &&A\end{array}$, what digit does D represent?

$\textbf{(A)}\ 5\qquad\textbf{(B)}\ 6\qquad\textbf{(C)}\ 7\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 9$

Solution

Because $B+A=A$, B must be 0 Next, because $B-A=A\implies 0-A=A,$ we get $A=5$

Now we can rewrite this as $\begin{array}{ccc}&5&0\\ +&C&5\\ \hline &D&5\end{array}$. Therefore, $D=5+C$

Finally, $A-1-C=0\implies A=C+1\implies C=4,$ So we have $D=5+C\implies D=5+4=\boxed{\textbf{(E)}\ 9}$

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 23	Followed by Problem 25
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

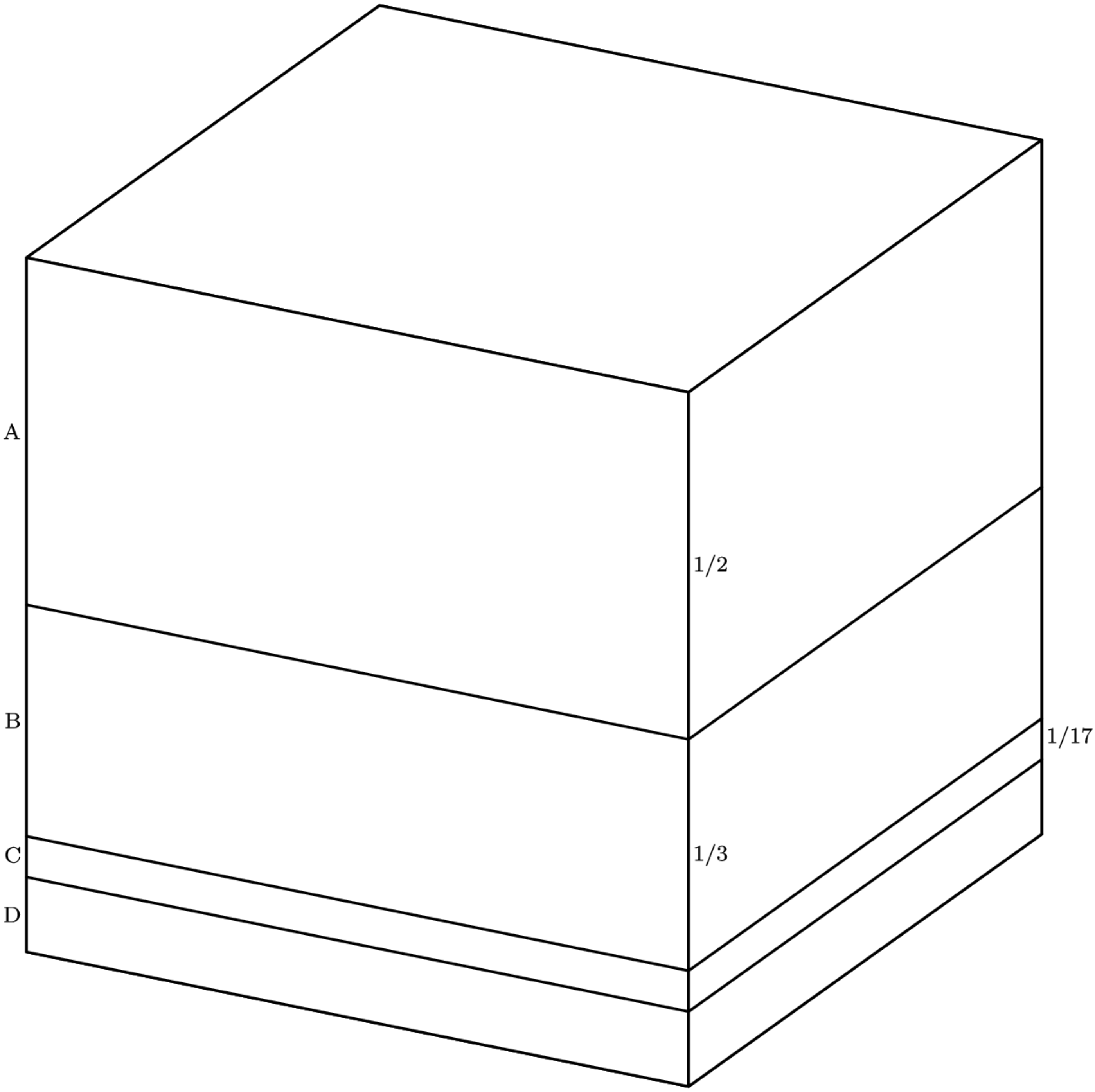
2009 AMC 8 Problems/Problem 25

Contents

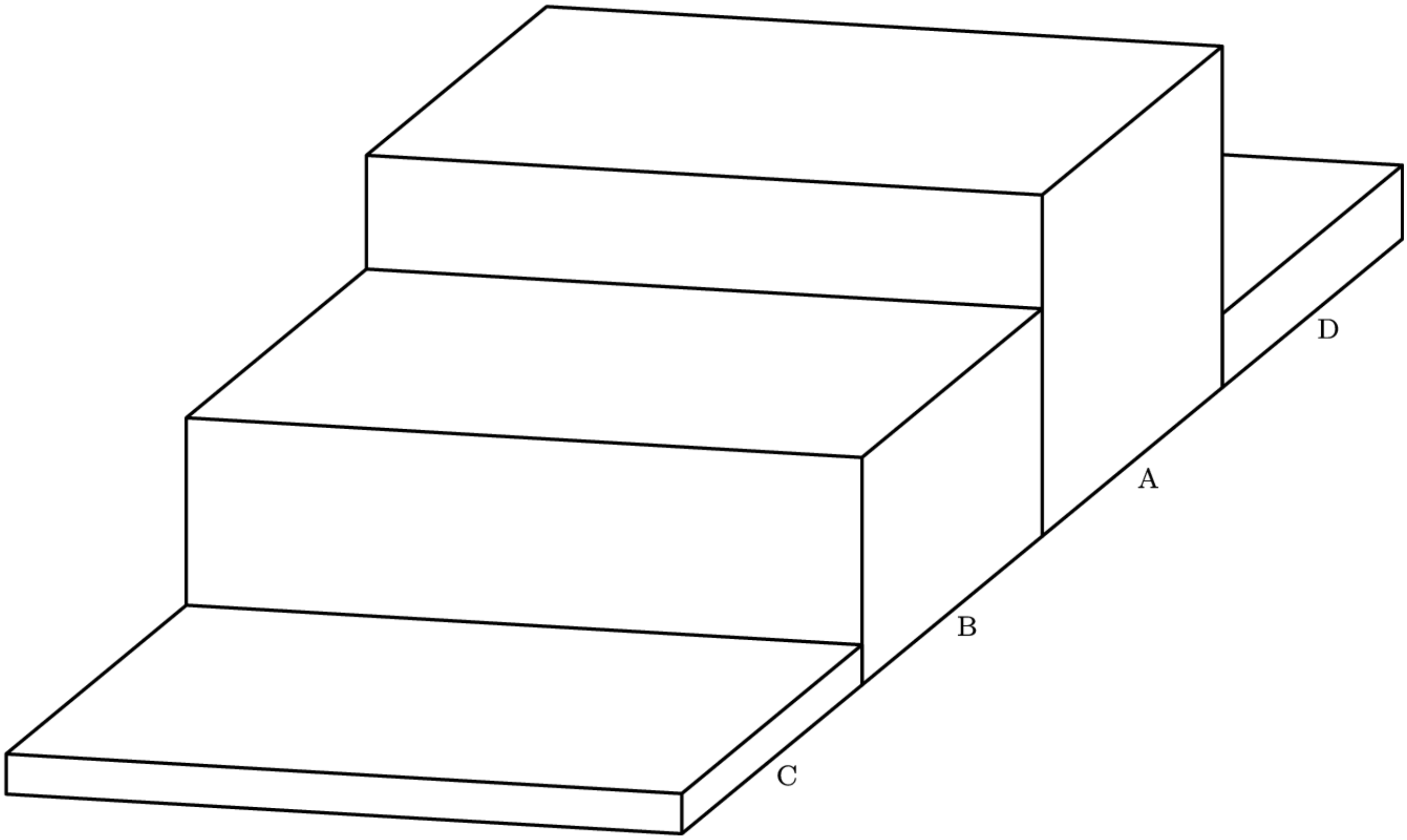
- 1 Problem
- 2 Solution 1
- 3 Solution 2
- 4 See Also

Problem

A one-cubic-foot cube is cut into four pieces by three cuts parallel to the top face of the cube. The first cube is $\frac{1}{2}$ foot from the top face. The second cut is $\frac{1}{3}$ foot below the first cut, and the third cut is $\frac{1}{17}$ foot below the second cut. From the top to the bottom the pieces are labeled A, B, C, and D. The pieces are then glued together end to end as shown in the second diagram. What is the total surface area of this solid in square feet?



A
B
C
D
1/2
1/3
1/17



$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ \frac{419}{51} \qquad \textbf{(D)}\ \frac{158}{17} \qquad \textbf{(E)}\ 11$

Solution 1

The tops of A, B, C, and D in the figure formed has sum $1+1+1+1=4$ as do the bottoms. Thus, the total so far is 8. Now, one of the sides has area one, since it combines all of the heights of A, B, C, and D, which is 1. The other side also satisfies this. Thus the total area now is 10. From the front, the surface area is half, because if you looked at it straight from the front it would look exactly like A, with surface area half. From the back it is the same thing. Thus, the total is $10+\frac{1}{2}+\frac{1}{2}=11$, or $\boxed{\textbf{(E)}\ 11}$.

Solution 2

The top parts and the bottom parts sum to 8. The sides add on another 2. Therefore, the only logical answer would be $\boxed{\textbf{(E)}\ 11}$ since it is the only answer greater than 10.

See Also

2009 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2009))	
Preceded by Problem 24	Followed by Last Problem
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	