2011 AMC 12A Problems

2011 AMC 12A (Answer Key)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the test if before 2006. No problems on the test will require the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have 75 minutes working time to complete the test.

1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

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Problem 1

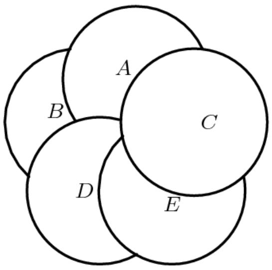
A cell phone plan costs 20 dollars each month, plus 5 cents per text message sent, plus 10 cents for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

$\textbf{(A)}\ 24.00 \qquad \textbf{(B)}\ 24.50 \qquad \textbf{(C)}\ 25.50 \qquad \textbf{(D)}\ 28.00 \qquad \textbf{(E)}\ 30.00$

Solution

Problem 2

There are 5 coins placed flat on a table according to the figure. What is the order of the coins from top to bottom?



$\textbf{(A)}\ (C, A, E, D, B) \qquad \textbf{(B)}\ (C, A, D, E, B) \qquad \textbf{(C)}\ (C, D, E, A, B) \qquad \textbf{(D)}\ (C, E, A, D, B) \qquad \textbf{(E)}\ (C, E, D, A, B$

Solution

Problem 3

A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Solution

Problem 4

At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ \frac{37}{3} \qquad \textbf{(C)}\ \frac{88}{7} \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 14$

Solution

Problem 5

Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 60$

Solution

Problem 6

The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Solution

Problem 7

A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was 17.71. What was the cost of a pencil in cents?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 77$

Solution

Problem 8

In the eight term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A+H$?

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 43$

Solution

Problem 9

At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her siblings and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and with half the twins. How many handshakes took place?

$\textbf{(A)}\ 324 \qquad \textbf{(B)}\ 441 \qquad \textbf{(C)}\ 630 \qquad \textbf{(D)}\ 648 \qquad \textbf{(E)}\ 882$

Solution

Problem 10

A pair of standard 6-sided dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

$\textbf{(A)}\ \frac{1}{36} \qquad \textbf{(B)}\ \frac{1}{12} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{5}{18}$

Solution

Problem 11

Circles $A, B,$ and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}.$ What is the area inside circle C but outside circle A and circle $B?$

$\textbf{(A)}\ 3 - \frac{\pi}{2} \qquad \textbf{(B)}\ \frac{\pi}{2} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{3\pi}{4} \qquad \textbf{(E)}\ 1+\frac{\pi}{2}$

Solution

Problem 12

A power boat and a raft both left dock A on a river and headed downstream. The raft drifted at the speed of the river current. The power boat maintained a constant speed with respect to the river. The power boat reached dock B downriver, then immediately turned and traveled back upriver. It eventually met the raft on the river 9 hours after leaving dock $A.$ How many hours did it take the power boat to go from A to $B?$

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 3.5 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 4.5 \qquad \textbf{(E)}\ 5$

Solution

Problem 13

Triangle ABC has side-lengths $AB = 12, BC = 24,$ and $AC = 18.$ The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at $N.$ What is the perimeter of $\triangle AMN?$

$\textbf{(A)}\ 27 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 33 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 42$

Solution

Problem 14

Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a,b) lies above the parabola $y=ax^2-bx$?

$\textbf{(A)}\ \frac{11}{81} \qquad \textbf{(B)}\ \frac{13}{81} \qquad \textbf{(C)}\ \frac{5}{27} \qquad \textbf{(D)}\ \frac{17}{81} \qquad \textbf{(E)}\ \frac{19}{81}$

Solution

Problem 15

The circular base of a hemisphere of radius 2 rests on the base of a square pyramid of height 6. The hemisphere is tangent to the other four faces of the pyramid. What is the edge-length of the base of the pyramid?

$\textbf{(A)}\ 3\sqrt{2} \qquad \textbf{(B)}\ \frac{13}{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{13}{2}$

Solution

Problem 16

Each vertex of convex polygon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

$\textbf{(A)}\ 2520 \qquad \textbf{(B)}\ 2880 \qquad \textbf{(C)}\ 3120 \qquad \textbf{(D)}\ 3250 \qquad \textbf{(E)}\ 3750$

Solution

Problem 17

Circles with radii 1, 2, and 3 are mutually externally tangent. What is the area of the triangle determined by the points of tangency?

$\textbf{(A)}\ \frac{3}{5} \qquad \textbf{(B)}\ \frac{4}{5} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{6}{5} \qquad \textbf{(E)}\ \frac{4}{3}$

Solution

Problem 18

Suppose that $|x+y|+|x-y|=2.$ What is the maximum possible value of x^2-6x+y^2?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Solution

Problem 19

At a competition with N players, the number of players given elite status is equal to $2^{1+\lfloor \log_2 (N-1) \rfloor} - N$. Suppose that 19 players are given elite status. What is the sum of the two smallest possible values of N?

$\textbf{(A)}\ 38 \qquad \textbf{(B)}\ 90 \qquad \textbf{(C)}\ 154 \qquad \textbf{(D)}\ 406 \qquad \textbf{(E)}\ 1024$

Solution

Problem 20

Let $f(x) = ax^2 + bx + c$, where a, b, and c are integers. Suppose that $f(1) = 0$, $50 < f(7) < 60$, $70 < f(8) < 80$, $5000k < f(100) < 5000(k+1)$ for some integer k. What is k?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Problem 21

Let $f_1(x) = \sqrt{1-x}$, and for integers $n \geq 2$, let $f_n(x) = f_{n-1}(\sqrt{n^2 - x})$. If N is the largest value of n for which the domain of f_n is nonempty, the domain of f_N is $[c]$. What is $N+c$?

$\textbf{(A)}\ -226 \qquad \textbf{(B)}\ -144 \qquad \textbf{(C)}\ -20 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 144$

Solution

Problem 22

Let R be a square region and $n \ge 4$ an integer. A point X in the interior of R is called n-ray partitional if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

$\textbf{(A)}\ 1500 \qquad \textbf{(B)}\ 1560 \qquad \textbf{(C)}\ 2320 \qquad \textbf{(D)}\ 2480 \qquad \textbf{(E)}\ 2500$

Solution

Problem 23

Let $f(z) = \frac{z+a}{z+b}$ and $g(z) = f(f(z))$, where a and b are complex numbers. Suppose that $\left| a \right| = 1$ and $g(g(z)) = z$ for all z for which $g(g(z))$ is defined. What is the difference between the largest and smallest possible values of $\left| b \right|$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \sqrt{2}-1 \qquad \textbf{(C)}\ \sqrt{3}-1 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2$

Solution

Problem 24

Consider all quadrilaterals $ABCD$ such that $AB=14$, $BC=9$, $CD=7$, and $DA=12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

$\textbf{(A)}\ \sqrt{15} \qquad \textbf{(B)}\ \sqrt{21} \qquad \textbf{(C)}\ 2\sqrt{6} \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 2\sqrt{7}$

Solution

Problem 25

Triangle ABC has $\angle BAC = 60^{\circ}$, $\angle CBA \leq 90^{\circ}$, $BC=1$, and $AC \geq AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

$\textbf{(A)}\ 60^{\circ} \qquad \textbf{(B)}\ 72^{\circ} \qquad \textbf{(C)}\ 75^{\circ} \qquad \textbf{(D)}\ 80^{\circ} \qquad \textbf{(E)}\ 90^{\circ}$

Solution

See also

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