

# Prealgebra 1

## Lesson 1

### Arithmetic Rules, Day 1

- Rigorous rules for addition and multiplication
- Longer, more complicated sums
- Gauss's clever summation technique

## Lesson 2

### Arithmetic Rules, Day 2

- Reciprocals
- Rigorous definition of division to set the stage for fractions

## Lesson 3

### Exponents and Exponent Laws

- Squares and higher powers
- Products and quotients of powers
- Power of a power

## Lesson 4

### Non-positive Exponents

- Zero as an exponent
- Negative exponents
- Extension of exponent laws to non-positive exponents

## Lesson 5

### Multiples and Divisibility

- Principles of multiples
- Divisibility tests for 2, 3, 4, 5, 6, 8, 9

## Lesson 6

### Primes and Prime factorization

- Primes and composites
- Prime factorization
- Problem-solving with prime factorizations
- Infinitude of primes

## Lesson 7

## **Least Common Multiple (LCM) and Greatest Common Divisor (GCD)**

- Common multiples and common divisors
- Definition of LCM and GCD
- Application of prime factorizations to LCM and GCD

## **Lesson 8**

### **Fractions, Day 1**

- Formal definition of fractions
- Fraction multiplication and division
- Fraction simplification

## **Lesson 9**

### **Fractions, Day 2**

- Fraction comparison
- Fraction addition and subtraction
- Word problems with fractions

## **Lesson 10**

### **Linear Equations**

- Expressions and Equations
- Expression simplification
- One- and two-step linear equations
- Basic word problems

## **Lesson 11**

### **Advanced Linear Equations and Word Problems**

- Complicated linear equations
- More complex word problems
- Linear equations with no solution or with infinitely many solutions

## **Lesson 12**

### **Inequalities**

- Principles of inequalities
- Inequalities on the number line
- Linear inequalities
- Word problems with inequalities

## **Lesson 13**

## **Decimal Arithmetic**

- Definition of decimals
- Decimal addition, subtraction, multiplication, and division
- Decimal comparison

## **Lesson 14**

### **Decimals and Fractions**

- Decimal approximation
- Conversion between decimals and fractions
- Rational numbers and their decimal representation

## **Lesson 15**

### **Ratio and Proportion**

- Definition of ratio and proportion
- Proportional thinking
- Part-to-part and part-to-whole problem-solving
- Word problems with ratios and proportions

## **Lesson 16**

### **Conversion, Speed, and Other Rates**

- Units and conversion factors
- Relationship between speed, distance, and time
- Joint work, relative speed, and average speed

# Prealgebra 2

## Lesson 1

### Introduction to Percents

- Definition of percent
- Relationships among percents, fractions, and decimals
- Percents of numbers
- Basic percent word problems

## Lesson 2

### Percent Increase and Decrease

- More percent word problems
- Percentage increase and decrease

## Lesson 3

### Squares and Square Roots

- Definition of square root
- Equations with square roots
- Non-integer square roots

## Lesson 4

### Arithmetic with Square Roots

- Products and quotients of square roots
- Simplification of square roots
- Sums and differences of square roots

## Lesson 5

### Angles and Parallel Lines

- Angle measurement
- Parallel lines

## Lesson 6

### Angles in Polygons

- Angles in a triangle
- Angles in other polygons

## Lesson 7

### Perimeter and Area

- Segments and perimeter
- Rectangle and right triangle area

## Lesson 8

### More Triangles and Circles

- Triangle area
- Circumference of a circle
- Area of a circle
- Unusual areas

## Lesson 9

### Pythagorean Theorem

- Pythagorean Theorem fundamentals
- Pythagorean triples

## Lesson 10

### Special Triangles

- Challenging Pythagorean Theorem problems
- 30-60-90 and 45-45-90 triangles

## Lesson 11

### Quadrilaterals

- Types of quadrilaterals
- Quadrilateral area

## Lesson 12

### Basic Statistics

- Average (mean)
- Averages as a balancing act
- Median, mode, and range

## Lesson 13

### Statistics, Graphs, and Charts

- Limits of basic statistics
- Types of graphs and charts

## Lesson 14

### Counting as Arithmetic

- Numbers in lists
- Venn diagrams
- Multiplication principle
- Casework

## Lesson 15

### Counting and Probability

- Pairs
- Introduction to probability

## Lesson 16

### Problem-Solving Strategies

- Find a pattern
- Make a list
- Draw a picture
- Work backwards

# Introduction to Algebra A

## Lesson 1

### Follow the Rules

- Rules of arithmetic
- Distribution and factoring with numbers
- Exponent rules

## Lesson 2

### Fractional Exponents, Radicals, and Variables

- Fractional exponents and radicals
- Variables in expressions
- Simplifying expressions

## Lesson 3

### Variables and Expressions

- Distribution and factoring with variables
- Expressions with fractions
- Expressions with radicals

## Lesson 4

### Linear Equations

- Solving linear equations
- Word problems

## Lesson 5

### More Variables

- Evaluating expressions with multiple variables
- Manipulating expressions

## Lesson 6

### Linear Equations with Multiple Variables

- Basic manipulations
- Substitution and Elimination
- Word Problems

## Lesson 7

### Ratio and Percent

- Ratios
- Parts and wholes
- Percentages and arithmetic with percents

## Lesson 8

### More Ratios and Proportion

- Working with units
- Direct and inverse proportion
- Joint proportion

## Lesson 9

### Common Errors and Challenging Problems

- Exponents do not distribute over addition
- Extraneous solutions and dividing by zero
- Difficult problems

## Lesson 10

### Graphing Lines (Part 1)

- Cartesian plane
- Distance between two points
- Graphs of equations
- Going from lines to equations

## Lesson 11

### Graphing Lines (Part 2) and Introduction to Inequalities

- Perpendicular and parallel lines
- How graphing relates to systems of equations
- Inequality manipulation and interval notation

## Lesson 12

### Graphing Inequalities

- Two variable inequalities
- Graphing to solve inequalities
- Optimization and linear programming

## Lesson 13

### Quadratic Equations (Part 1)

- Distributing



- Factoring

## Lesson 14

### Special Factorizations

- Vieta's formulas
- Difference of squares
- Sum and difference of cubes

## Lesson 15

### Simon's Favorite Factoring Trick and Complex Numbers

- Completing the rectangle
- Introduction to complex numbers and arithmetic

## Lesson 16

### Quadratic Equations (Part 2)

- Completing the square
- The quadratic formula

# Introduction to Algebra B

## Lesson 1

### Factorization

- Factoring quadratics
- Sums and products of roots of a quadratic

## Lesson 2

### Quadratics and Complex Numbers

- Simon's favorite factoring trick
- Introduction to complex numbers

## Lesson 3

### Completing the Square and the Quadratic Formula

- Completing the square
- Proving the quadratic formula

## Lesson 4

### Graphing Quadratics

- Parabolas
- Circles

## Lesson 5

### Quadratic Inequalities

- Quadratic inequalities
- Beyond quadratics

## Lesson 6

### Optimizing Quadratics and AM-GM Inequality

- Quadratic optimization
- The AM-GM inequality

## Lesson 7

### Functions

- What is a function?
- Adding, subtracting, multiplying, and dividing functions

## Lesson 8

### Composition and Inverses

- Function composition
- Inverse functions

## Lesson 9

### Graphing Functions

- Transformations
- Graphs of inverse functions

## Lesson 10

### Polynomials

- What is a polynomial?
- Adding and multiplying polynomials

## Lesson 11

### Exponential Functions

- Compound interest
- Logarithms

## Lesson 12

### Special Functions Part 1

- Radicals
- Absolute value

## Lesson 13

### Special Functions Part 2

- Floor and ceiling
- Rational functions
- Piecewise-defined functions

## Lesson 14

### Sequences & Series Part 1

- Arithmetic sequences
- Arithmetic series
- Geometric sequences

## Lesson 15

## Sequences & Series Part 2

- Geometric series
- Telescoping series

## Lesson 16

### Challenging Problems Day

- Self-similarity
- Symmetry

# Introduction to Geometry

## Lessons

1	Angles
2	Triangle Angles and Congruent Triangles
3	Isosceles & Equilateral Triangles, Perimeter, and Area
4	Similar Triangles
5	Similar Triangles and Right Triangles
6	More Right Triangles!
7	Special Parts of a Triangle
8	Special Parts of a Triangle, Continued!
9	Quadrilaterals
10	More Quadrilaterals
11	Polygons
12	Geometric Inequalities
13	Introduction to Circles
14	Circles and Angles
15	Tangents
16	Power of a Point
17	3D Geometry Part 1

18	3D Geometry Part 2
19	Transformations
20	Analytic Geometry
21	More Analytic Geometry
22	Basic Trigonometry
23	Problems!
24	More Problems!

# Introduction to Counting & Probability

## Lesson 1

### Lists, Venn Diagrams, Addition, Multiplication

- Counting Lists of Numbers
- Counting with Addition and Subtraction
- Permutations

## Lesson 2

### Casework, Constructions, and Restriction

- Casework
- Complementary Counting
- Constructive Counting
- Counting with Restrictions

## Lesson 3

### Overcounting and Combinations

- Permutations with Repeated Elements
- Counting Pairs of Items
- Counting with Symmetries
- Combinations
- Combinatorial Identities

## Lesson 4

### Combinations and Distinguishability

- Paths on a Grid
- Distinguishability

## Lesson 5

### Challenging Problems Day

- Applications of Lessons 1-4

## Lesson 6

### Introduction to Probability

- Definition of Probability
- Counting Techniques in Probability

## Lesson 7

## **Probability and Arithmetic**

- Probability and Addition
- Complementary Probability
- Probability and Multiplication
- Probability and Dependent Events

### **Lesson 8**

#### **Geometric Probability, Think About It!, and Expected Value**

- Using Symmetry in Problem-Solving
- Probability Using Lengths
- Probability Using Areas
- Definition of Expected Value
- Problem-Solving with Expected Value

### **Lesson 9**

#### **Pascal's Triangle and Identities**

- Constructing Pascal's Triangle
- Pascal's Triangle as Combinations
- More Combinatorial Identities

### **Lesson 10**

#### **Distributions and The Hockey Stick Identity**

- Distributions
- Sticks and Stones
- The Hockey Stick Identity

### **Lesson 11**

#### **The Binomial Theorem**

- Proving the Binomial Theorem
- Applying the Binomial Theorem to Problems
- The Binomial Theorem in Identities

### **Lesson 12**

#### **Challenging Problems Day 2**

- Applications of Lessons 6-11



# Introduction to Number Theory

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|----|---|
| 1  | Integers, Fractions, Decimals, and Number Bases                 |
| 2  | Base Number Arithmetic  |
| 3  | Multiples, Divisors, and Prime Numbers                          |
| 4  | Common Factors, Common Multiples, Euclidean Algorithm           |
| 5  | Divisor Problems, More with the Euclidean Algorithm             |
| 6  | Factorials, Special Integers, Algebra with Integers             |
| 7  | Units Digit, Introduction to Modular Arithmetic                 |
| 8  | Calculations with Modular Arithmetic                            |
| 9  | Divisibility Rules and Multiplicative Inverses                  |
| 10 | Multiplicative Inverses, Solving Linear Congruences             |
| 11 | Systems of Linear Congruences and the Chinese Remainder Theorem |
| 12 | Number Sense and Applications of Number Theory                  |