2000 AMC 12 Problems

Contents

Problem 1

In the year 2001, the United States will host the International Mathematical Olympiad. Let $I, M,$ and O be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What is the largest possible value of the sum $I + M + O$?

$\textbf{(A)}\ 23 \qquad \textbf{(B)}\ 55 \qquad \textbf{(C)}\ 99 \qquad \textbf{(D)}\ 111 \qquad \textbf{(E)}\ 671$

Solution

Problem 2

$2000(2000^{2000}) =$

$\textbf{(A)}\ 2000^{2001} \qquad \textbf{(B)}\ 4000^{2000} \qquad \textbf{(C)}\ 2000^{4000} \qquad \textbf{(D)}\ 4,000,000^{2000} \qquad \textbf{(E)}\ 2000^{4,000,000}$

Solution

Problem 3

Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

$\textbf{(A)}\ 40 \qquad \textbf{(B)}\ 50 \qquad \textbf{(C)}\ 55 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Solution

Problem 4

The Fibonacci sequence $1,1,2,3,5,8,13,21,\ldots$ starts with two 1's, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 9$

Solution

Problem 5

If $|x-2|=p$, where $x<2$, then $x-p=$

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 2-2p \qquad \textbf{(D)}\ 2p-2 \qquad \textbf{(E)}\ |2p-2|$

Solution

Problem 6

Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 119 \qquad \textbf{(D)}\ 180 \qquad \textbf{(E)}\ 231$

Solution

Problem 7

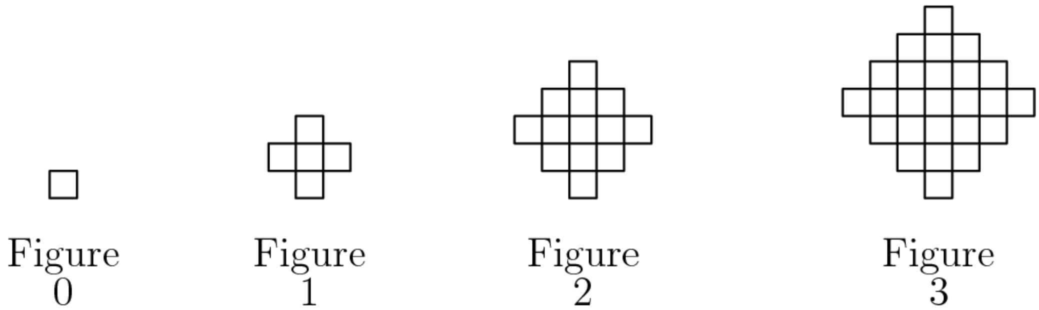
How many positive integers b have the property that $\log_b 729$ is a positive integer?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Solution

Problem 8

Figures 0, 1, 2, and 3 consist of 1, 5, 13, and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?



$\textbf{(A)}\ 10401 \qquad \textbf{(B)}\ 19801 \qquad \textbf{(C)}\ 20201 \qquad \textbf{(D)}\ 39801 \qquad \textbf{(E)}\ 40801$

Solution

Problem 9

Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were $71,76,80,82,$ and 91. What was the last score Mrs. Walters entered?

$\textbf{(A)}\ 71 \qquad \textbf{(B)}\ 76 \qquad \textbf{(C)}\ 80 \qquad \textbf{(D)}\ 82 \qquad \textbf{(E)}\ 91$

Solution

Problem 10

The point $P=(1,2,3)$ is reflected in the xy-plane, then its image Q is rotated 180° about the x-axis to produce R, and finally, R is translated 5 units in the positive-y direction to produce S. What are the coordinates of S?

$\textbf{(A) } (1,7,-3) \qquad \textbf{(B) } (-1,7,-3) \qquad \textbf{(C) } (-1,-2,8) \qquad \textbf{(D) } (-1,3,3) \qquad \textbf{(E) } (1,3,3)$

Solution

Problem 11

Two non-zero real numbers, a and b, satisfy $ab=a-b$. Which of the following is a possible value of $\frac{a}{b}+\frac{b}{a}-ab$?

$\textbf{(A) } -2 \qquad \textbf{(B) } \frac{-1}{2} \qquad \textbf{(C) } \frac{1}{3} \qquad \textbf{(D) } \frac{1}{2} \qquad \textbf{(E) } 2$

Solution

Problem 12

Let $A, M,$ and C be nonnegative integers such that $A+M+C=12$. What is the maximum value of $A\cdot M\cdot C+A\cdot M+M\cdot C+A\cdot C$?

$\textbf{(A) } 62 \qquad \textbf{(B) } 72 \qquad \textbf{(C) } 92 \qquad \textbf{(D) } 102 \qquad \textbf{(E) } 112$

Solution

Problem 13

One morning each member of Angela’ s family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

$\textbf{(A) } 3 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 5 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 7$

Solution

Problem 14

When the mean, median, and mode of the list

$$10,2,5,2,4,2,x$$

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

$\textbf{(A) } 3 \qquad \textbf{(B) } 6 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 17 \qquad \textbf{(E) } 20$

Solution

Problem 15

Let f be a function for which $f(x/3)=x^2+x+1$. Find the sum of all values of z for which $f(3z)=7$.

$\textbf{(A) } -1/3 \qquad \textbf{(B) } -1/9 \qquad \textbf{(C) } 0 \qquad \textbf{(D) } 5/9 \qquad \textbf{(E) } 5/3$

Solution

Problem 16

A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered $1,2,\ldots,17$, the second row $18,19,\ldots,34$, and so on down the board. If the board is renumbered so that the left column, top to bottom, is $1,2,\ldots,13$, the second column

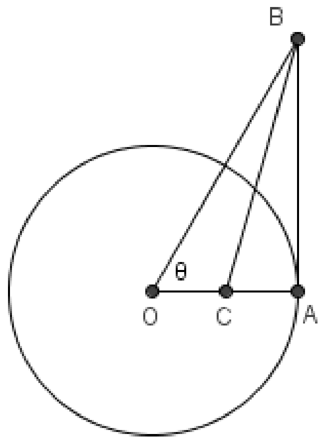
$14, 15, \ldots, 26$ and so on across the board, some squares have the same numbers in both numbering systems. Find the sum of the numbers in these squares (under either system).

$\textbf{(A)}\ 222 \qquad \textbf{(B)}\ 333 \qquad \textbf{(C)}\ 444 \qquad \textbf{(D)}\ 555 \qquad \textbf{(E)}\ 666$

Solution

Problem 17

A circle centered at O has radius 1 and contains the point A. The segment AB is tangent to the circle at A and $\angle AOB = \theta$. If point C lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then $OC =$



$\textbf{(A)}\ \sec^2 \theta - \tan \theta \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\cos^2 \theta}{1 + \sin \theta} \qquad \textbf{(D)}\ \frac{1}{1 + \sin \theta} \qquad \textbf{(E)}\ \frac{\sin \theta}{\cos^2 \theta}$

Solution

Problem 18

In year N, the 300th day of the year is a Tuesday. In year $N + 1$, the 200th day is also a Tuesday. On what day of the week did the 100th day of year $N - 1$ occur?

$\textbf{(A)}\ \text{Thursday} \qquad \textbf{(B)}\ \text{Friday} \qquad \textbf{(C)}\ \text{Saturday} \qquad \textbf{(D)}\ \text{Sunday} \qquad \textbf{(E)}\ \text{Monday}$

Solution

Problem 19

In triangle ABC, $AB = 13$, $BC = 14$, $AC = 15$. Let D denote the midpoint of $\overline{BC}$ and let E denote the intersection of $\overline{BC}$ with the bisector of angle BAC. Which of the following is closest to the area of the triangle ADE?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 2.5 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 3.5 \qquad \textbf{(E)}\ 4$

Solution

Problem 20

If x, y, and z are positive numbers satisfying $x + \frac{1}{y} = 4, y + \frac{1}{z} = 1$, and $z + \frac{1}{x} = \frac{7}{3}$, then what is the value of xyz ?

$\textbf{(A)}\ 2/3 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 4/3 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 7/3$

Solution

Problem 21

Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

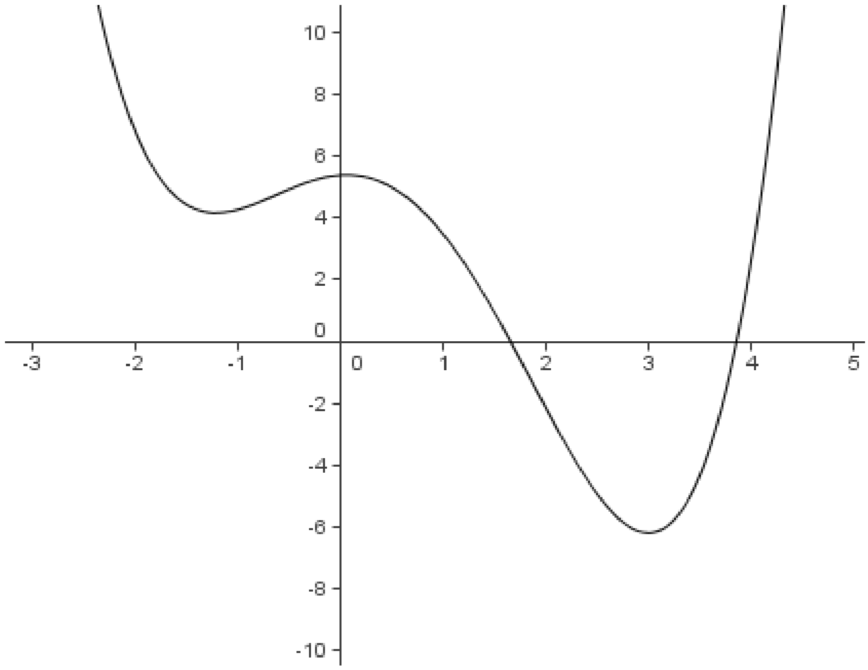
$\textbf{(A)}\ \dfrac{1}{2m+1} \qquad \textbf{(B)}\ m \qquad \textbf{(C)}\ 1-m \qquad \textbf{(D)}\ \dfrac{1}{4m} \qquad \textbf{(E)}\ \dfrac{1}{8m^2}$

Solution

Problem 22

The graph below shows a portion of the curve defined by the quartic polynomial $P(x)=x^4+ax^3+bx^2+cx+d$. Which of the following is the smallest?

$\textbf{(A)}\ P(-1)$
$\textbf{(B)}$ The product of the zeros of P
$\textbf{(C)}$ The product of the non-real zeros of P
$\textbf{(D)}$ The sum of the coefficients of P
$\textbf{(E)}$ The sum of the real zeros of P



Solution

Problem 23

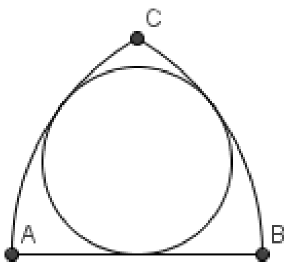
Professor Gamble buys a lottery ticket, which requires that he pick six different integers from 1 through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property— the sum of the base-ten logarithms is an integer. What is the probability that Professor Gamble holds the winning ticket?

$\textbf{(A)}\ 1/5 \qquad \textbf{(B)}\ 1/4 \qquad \textbf{(C)}\ 1/3 \qquad \textbf{(D)}\ 1/2 \qquad \textbf{(E)}\ 1$

Solution

Problem 24

If circular arcs AC and BC have centers at B and A, respectively, then there exists a circle tangent to both $\overarc{AC}$ and $\overarc{BC}$, and to $\overline{AB}$. If the length of $\overarc{BC}$ is 12, then the circumference of the circle is



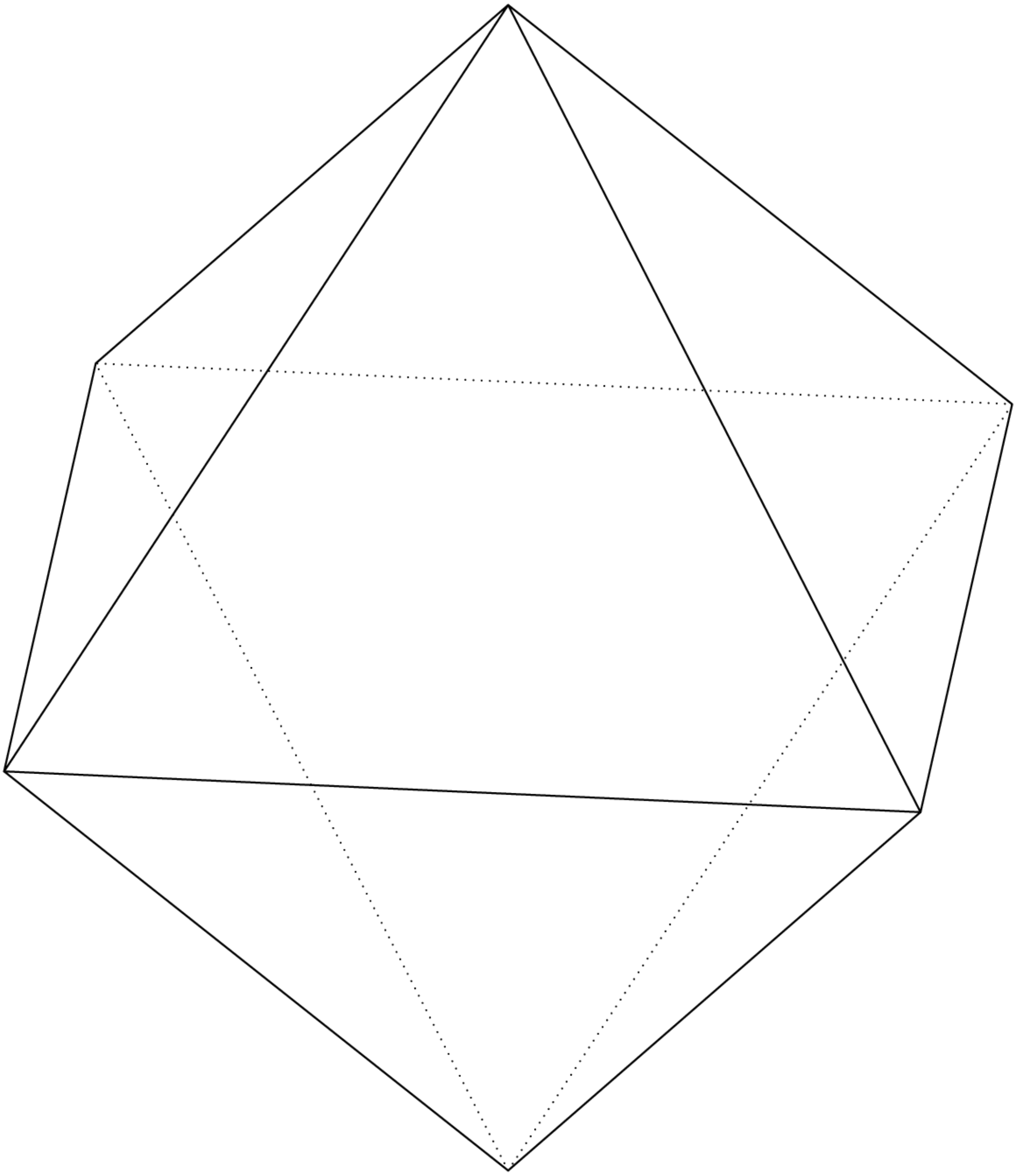
$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 28$

Solution

Problem 25

Eight congruent equilateral triangles, each of a different color, are used to construct a regular octahedron. How many distinguishable ways are there to construct the octahedron? (Two colored octahedrons are distinguishable if neither can be rotated to look just like the other.)

$\textbf{(A)}\ 210 \qquad \textbf{(B)}\ 560 \qquad \textbf{(C)}\ 840 \qquad \textbf{(D)}\ 1260 \qquad \textbf{(E)}\ 1680$



Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2000 AMC 12
- Mathematics competition resources