2005 AMC 12A Problems

Contents

Problem 1

Two is 10% of x and 20% of y. What is $x - y$?

$$(\mathrm{A})\ 1 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 5 \qquad (\mathrm{D})\ 10 \qquad (\mathrm{E})\ 20$$

Solution

Problem 2

The equations $2x + 7 = 3$ and $bx - 10 = -2$ have the same solution. What is the value of b?

$$(\mathrm{A})\ -8 \qquad (\mathrm{B})\ -4 \qquad (\mathrm{C})\ 2 \qquad (\mathrm{D})\ 4 \qquad (\mathrm{E})\ 8$$

Solution

Problem 3

A rectangle with diagonal length x is twice as long as it is wide. What is the area of the rectangle?

$$(\mathrm{A})\ \frac{1}{4}x^2 \qquad (\mathrm{B})\ \frac{2}{5}x^2 \qquad (\mathrm{C})\ \frac{1}{2}x^2 \qquad (\mathrm{D})\ x^2 \qquad (\mathrm{E})\ \frac{3}{2}x^2$$

Solution

Problem 4

A store normally sells windows at $\$100$ each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How much will they save if they purchase the windows together rather than separately?

$\mathrm{(A)}\ 100 \qquad \mathrm{(B)}\ 200 \qquad \mathrm{(C)}\ 300 \qquad \mathrm{(D)}\ 400 \qquad \mathrm{(E)}\ 500$

Solution

Problem 5

The average (mean) of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

$\mathrm{(A)}\ 23 \qquad \mathrm{(B)}\ 24 \qquad \mathrm{(C)}\ 25 \qquad \mathrm{(D)}\ 26 \qquad \mathrm{(E)}\ 27$

Solution

Problem 6

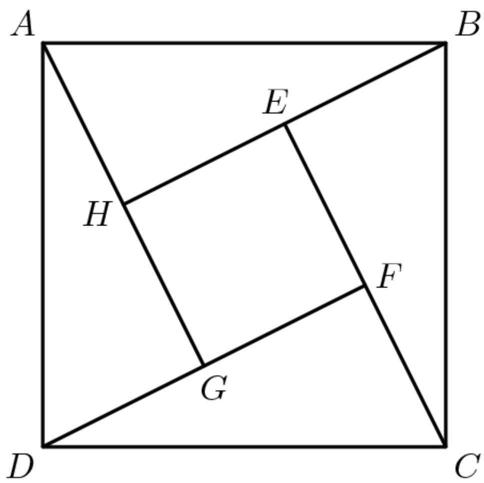
Josh and Mike live 13 miles apart. Yesterday, Josh started to ride his bicycle toward Mike's house. A little later Mike started to ride his bicycle toward Josh's house. When they met, Josh had ridden for twice the length of time as Mike and at four-fifths of Mike's rate. How many miles had Mike ridden when they met?

$\mathrm{(A)}\ 4 \qquad \mathrm{(B)}\ 5 \qquad \mathrm{(C)}\ 6 \qquad \mathrm{(D)}\ 7 \qquad \mathrm{(E)}\ 8$

Solution

Problem 7

Square $EFGH$ is inside the square $ABCD$ so that each side of $EFGH$ can be extended to pass through a vertex of $ABCD$. Square $ABCD$ has side length $\sqrt{50}$ and $BE = 1$. What is the area of the inner square $EFGH$?



$\mathrm{(A)}\ 25 \qquad \mathrm{(B)}\ 32 \qquad \mathrm{(C)}\ 36 \qquad \mathrm{(D)}\ 40 \qquad \mathrm{(E)}\ 42$

Solution

Problem 8

Let A, M, and C be digits with

$$(100A + 10M + C)(A + M + C) = 2005$$

What is A?

$(\mathrm{A})\ 1 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 3 \qquad (\mathrm{D})\ 4 \qquad (\mathrm{E})\ 5$

Solution

Problem 9

There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x. What is the sum of these values of a?

$(\mathrm{A})\ -16 \qquad (\mathrm{B})\ -8 \qquad (\mathrm{C})\ 0 \qquad (\mathrm{D})\ 8 \qquad (\mathrm{E})\ 20$

Solution

Problem 10

A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n?

$(\mathrm{A})\ 3 \qquad (\mathrm{B})\ 4 \qquad (\mathrm{C})\ 5 \qquad (\mathrm{D})\ 6 \qquad (\mathrm{E})\ 7$

Solution

Problem 11

How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

$(\mathrm{A})\ 41 \qquad (\mathrm{B})\ 42 \qquad (\mathrm{C})\ 43 \qquad (\mathrm{D})\ 44 \qquad (\mathrm{E})\ 45$

Solution

Problem 12

A line passes through $A\ (1,1)$ and $B\ (100,1000)$. How many other points with integer coordinates are on the line and strictly between A and B?

$(\mathrm{A})\ 0 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 3 \qquad (\mathrm{D})\ 8 \qquad (\mathrm{E})\ 9$

Solution

Problem 13

The regular 5-point star $ABCDE$ is drawn and in each vertex, there is a number. Each $A, B, C, D,$ and E are chosen such that all 5 of them came from set $\{3, 5, 6, 7, 9\}$. Each letter is a different number (so one possible way is $A = 3, B = 5, C = 6, D = 7, E = 9$). Let AB be the sum of the numbers on A and B, and so forth. If $AB, BC, CD, DE,$ and EA form an arithmetic sequence (not necessarily in increasing order), find the value of CD.

$(\mathrm{A})\ 9 \qquad (\mathrm{B})\ 10 \qquad (\mathrm{C})\ 11 \qquad (\mathrm{D})\ 12 \qquad (\mathrm{E})\ 13$

Solution

Problem 14

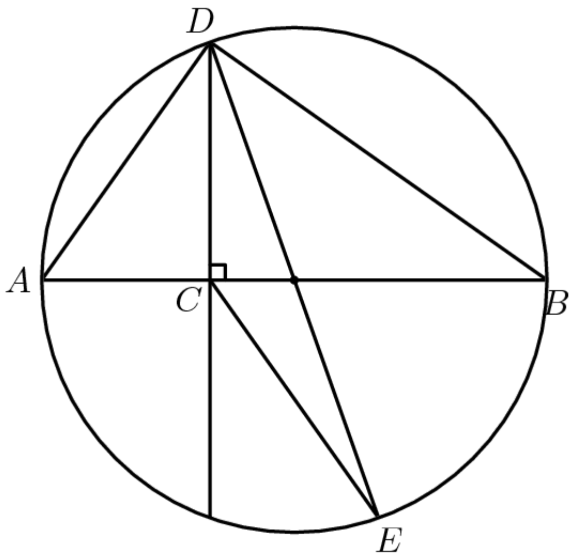
On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots?

$(\mathrm{A})\ \frac{5}{11} \qquad (\mathrm{B})\ \frac{10}{21} \qquad (\mathrm{C})\ \frac{1}{2} \qquad (\mathrm{D})\ \frac{11}{21} \qquad (\mathrm{E})\ \frac{6}{11}$

Solution

Problem 15

Let $\overline{AB}$ be a diameter of a circle and C be a point on $\overline{AB}$ with $2 \cdot AC = BC$. Let D and E be points on the circle such that $\overline{DC} \perp \overline{AB}$ and $\overline{DE}$ is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?

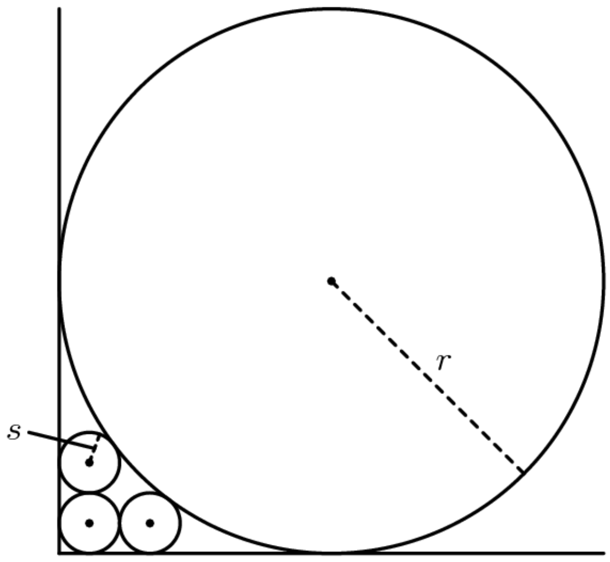


$(\mathrm{A})\ \frac{1}{6} \qquad (\mathrm{B})\ \frac{1}{4} \qquad (\mathrm{C})\ \frac{1}{3} \qquad (\mathrm{D})\ \frac{1}{2} \qquad (\mathrm{E})\ \frac{2}{3}$

Solution

Problem 16

Three circles of radius s are drawn in the first quadrant of the xy-plane. The first circle is tangent to both axes, the second is tangent to the first circle and the x-axis, and the third is tangent to the first circle and the y-axis. A circle of radius $r > s$ is tangent to both axes and to the second and third circles. What is r/s?



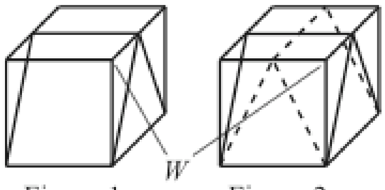
$(\mathrm{A})\ 5 \qquad (\mathrm{B})\ 6 \qquad (\mathrm{C})\ 8 \qquad (\mathrm{D})\ 9 \qquad (\mathrm{E})\ 10$

Solution

Problem 17

A unit cube is cut twice to form three triangular prisms, two of which are congruent, as shown in Figure 1. The cube is then cut in the same manner along the dashed lines shown in Figure 2. This creates nine pieces. What is the volume of the piece that contains vertex W?

(A) $\frac{1}{12}$ (B) $\frac{1}{9}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$



Solution

Problem 18

Call a number "prime-looking" if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?

(A) 100 (B) 102 (C) 104 (D) 106 (E) 108

Solution

Problem 19

A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled?

(A) 1404 (B) 1462 (C) 1604 (D) 1605 (E) 1804

Solution

Problem 20

For each x in $[0,1]$, define

$$\begin{cases} f(x) = 2x, & \text{if} \quad 0 \le x \le \frac{1}{2}; \\ f(x) = 2-2x, & \text{if} \quad \frac{1}{2} < x \le 1. \end{cases}$$

Let $f^{[2]}(x) = f(f(x))$, and $f^{[n+1]}(x) = f^{[n]}(f(x))$ for each integer $n \ge 2$. For how many values of x in $[0,1]$ is $f^{[2005]}(x) = \dfrac{1}{2}$?

(A) 0 (B) 2005 (C) 4010 (D) 2005^2 (E) 2^{2005}

Solution

Problem 21

How many ordered triples of integers (a,b,c), with $a \ge 2$, $b \ge 1$, and $c \ge 0$, satisfy both $\log_a b = c^{2005}$ and $a+b+c = 2005$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Solution

Problem 22

A rectangular box P is inscribed in a sphere of radius r. The surface area of P is 384, and the sum of the lengths of its 12 edges is 112. What is r?

(A)8 (B)10 (C)12 (D)14 (E)16

Solution

Problem 23

Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \ldots, 2^{25}\}$. What is the probability that $\log_a b$ is an integer?

$\mathrm{(A)}\frac{2}{25} \qquad \mathrm{(B)}\frac{31}{300} \qquad \mathrm{(C)}\frac{13}{100} \qquad \mathrm{(D)}\frac{7}{50} \qquad \mathrm{(E)}\frac{1}{2}$

Solution

Problem 24

Let $P(x) = (x-1)(x-2)(x-3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \cdot R(x)$?

$\mathrm{(A)}19 \qquad \mathrm{(B)}22 \qquad \mathrm{(C)}24 \qquad \mathrm{(D)}27 \qquad \mathrm{(E)}32$

Solution

Problem 25

Let S be the set of all points with coordinates (x, y, z), where x, y, and z are each chosen from the set $\{0, 1, 2\}$. How many equilateral triangles have all their vertices in S?

$\mathrm{(A)}72 \qquad \mathrm{(B)}76 \qquad \mathrm{(C)}80 \qquad \mathrm{(D)}84 \qquad \mathrm{(E)}88$

Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2005 AMC 12A
- 2005 AMC A Math Jam Transcript (http://www.artofproblemsolving.com/Community/AoPS_Y_MJ_Transcripts.php?mj_id=48)
- Mathematics competition resources