2017 AMC 10A Problems

2017 AMC 10A (Answer Key) Printable version: | AoPS Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017) • PDF (http://www.artofproblemsolving.com/Forum/resources/files/usa/USA-AMC_10-2017-43)
Instructions 1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct. 2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer. 3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the SAT if before 2006. No problems on the test will require the use of a calculator). 4. Figures are not necessarily drawn to scale. 5. You will have 75 minutes working time to complete the test.
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

Contents

Problem 1

What is the value of $(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)$?

$\textbf{(A)}\ 70 \qquad \textbf{(B)}\ 97 \qquad \textbf{(C)}\ 127 \qquad \textbf{(D)}\ 159 \qquad \textbf{(E)}\ 729$

Solution

Problem 2

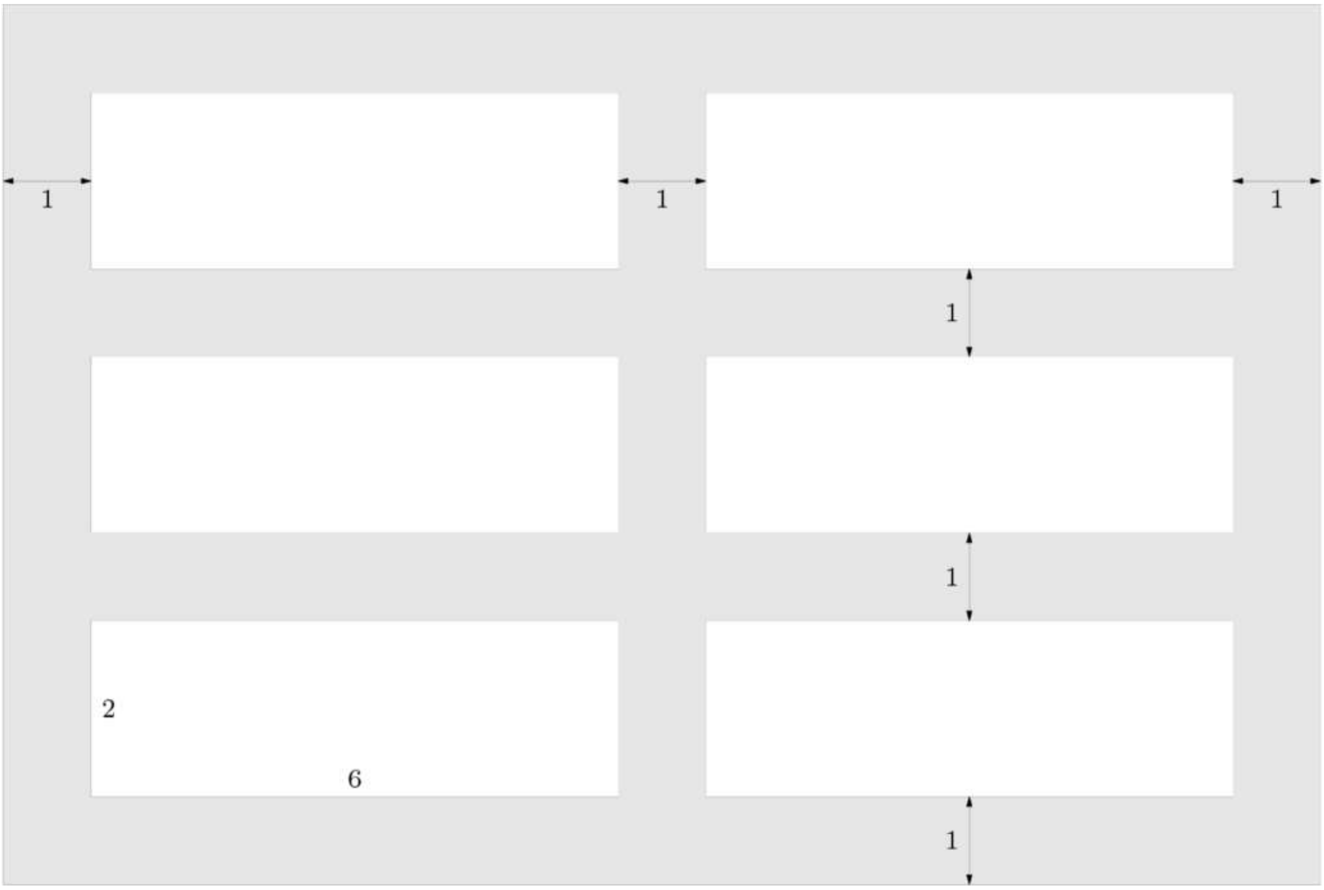
Pablo buys popsicles for his friends. The store sells single popsicles for $\$1$ each, 3-popsicle boxes for $\$2$ each, and 5-popsicle boxes for $\$3$. What is the greatest number of popsicles that Pablo can buy with $\$8$?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 15$

Solution

Problem 3

Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and also surrounded by 1-foot-wide walkways, as shown on the diagram. What is the total area of the walkways, in square feet?



$\textbf{(A)}\ 72 \qquad \textbf{(B)}\ 78 \qquad \textbf{(C)}\ 90 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 150$

Solution

Problem 4

Mia is “helping” her mom pick up 30 toys that are strewn on the floor. Mia’s mom manages to put 3 toys into the toy box every 30 seconds, but each time immediately after those 30 seconds have elapsed, Mia takes 2 toys out of the box. How much time, in minutes, will it take Mia and her mom to put all 30 toys into the box for the first time?

$\textbf{(A)}\ 13.5 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 14.5 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 15.5$

Solution

Problem 5

The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 12$

Solution

Problem 6

Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which of of these statements necessarily follows logically?

$\textbf{(A)}$ If Lewis did not receive an A, then he got all of the multiple choice questions wrong.
$\textbf{(B)}$ If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.
$\textbf{(C)}$ If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.
$\textbf{(D)}$ If Lewis received an A, then he got all of the multiple choice questions right.
$\textbf{(E)}$ If Lewis received an A, then he got at least one of the multiple choice questions right.

Solution

Problem 7

Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

$\textbf{(A)}\ 30\%\qquad\textbf{(B)}\ 40\%\qquad\textbf{(C)}\ 50\%\qquad\textbf{(D)}\ 60\%\qquad\textbf{(E)}\ 70\%$

Solution

Problem 8

At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other a hug, and people who do not know each other shake hands. How many handshakes occur?

$\textbf{(A)}\ 240\qquad\textbf{(B)}\ 245\qquad\textbf{(C)}\ 290\qquad\textbf{(D)}\ 480\qquad\textbf{(E)}\ 490$

Solution

Problem 9

Minnie rides on a flat road at 20 kilometers per hour (kph), downhill at 30 kph, and uphill at 5 kph. Penny rides on a flat road at 30 kph, downhill at 40 kph, and uphill at 10 kph. Minnie goes from town A to town B, a distance of 10 km all uphill, then from town B to town C, a distance of 15 km all downhill, and then back to town A, a distance of 20 km on the flat. Penny goes the other way around using the same route. How many more minutes does it take Minnie to complete the 45-km ride than it takes Penny?

$\textbf{(A)}\ 45\qquad\textbf{(B)}\ 60\qquad\textbf{(C)}\ 65\qquad\textbf{(D)}\ 90\qquad\textbf{(E)}\ 95$

Solution

Problem 10

Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

$\textbf{(A)}\ 16\qquad\textbf{(B)}\ 17\qquad\textbf{(C)}\ 18\qquad\textbf{(D)}\ 19\qquad\textbf{(E)}\ 20$

Solution

Problem 11

The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 12\qquad\textbf{(C)}\ 18\qquad\textbf{(D)}\ 20\qquad\textbf{(E)}\ 24$

Solution

Problem 12

Let S be a set of points (x, y) in the coordinate plane such that two of the three quantities $3,\ x+2,$ and $y-4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description for S?

$\textbf{(A)}$ a single point $\qquad\textbf{(B)}$ two intersecting lines
$\textbf{(C)}$ three lines whose pairwise intersections are three distinct points
$\textbf{(D)}$ a triangle $\qquad\textbf{(E)}$ three rays with a common endpoint

Solution

Problem 13

Define a sequence recursively by $F_0=0,\ F_1=1,$ and $F_n =$ the remainder when $F_{n-1}+F_{n-2}$ is divided by $3,$ for all $n\geq 2.$ Thus the sequence starts $0,1,1,2,0,2,\ldots$ What is $F_{2017}+F_{2018}+F_{2019}+F_{2020}+F_{2021}+F_{2022}+F_{2023}+F_{2024}?$

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 7\qquad\textbf{(C)}\ 8\qquad\textbf{(D)}\ 9\qquad\textbf{(E)}\ 10$

Solution

Problem 14

Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was A dollars. The cost of his movie ticket was 20% of the difference between A and the cost of his soda, while the cost of his soda was 5% of the difference between A and the cost of his movie ticket. To the nearest whole percent, what fraction of A did Roger pay for his movie ticket and soda?

$\textbf{(A)}\ 9\%\qquad\textbf{(B)}\ 19\%\qquad\textbf{(C)}\ 22\%\qquad\textbf{(D)}\ 23\%\qquad\textbf{(E)}\ 25\%$

Solution

Problem 15

Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloé's number?

$\textbf{(A)}\ \frac{1}{2}\qquad\textbf{(B)}\ \frac{2}{3}\qquad\textbf{(C)}\ \frac{3}{4}\qquad\textbf{(D)}\ \frac{5}{6}\qquad\textbf{(E)}\ \frac{7}{8}$

Solution

Problem 16

There are 10 horses, named Horse 1, Horse 2, $\ldots$, Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 3\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 5\qquad\textbf{(E)}\ 6$

Solution

Problem 17

Distinct points P, Q, R, S lie on the circle $x^{2}+y^{2}=25$ and have integer coordinates. The distances PQ and RS are irrational numbers. What is the greatest possible value of the ratio $\frac{PQ}{RS}$?

$\textbf{(A)}\ 3\qquad\textbf{(B)}\ 5\qquad\textbf{(C)}\ 3\sqrt{5}\qquad\textbf{(D)}\ 7\qquad\textbf{(E)}\ 5\sqrt{2}$

Solution

Problem 18

Amelia has a coin that lands heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. The probability that Amelia wins is $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $q-p$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Problem 19

Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 40$

Solution

Problem 20

Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 1239 \qquad \textbf{(E)}\ 1265$

Solution

Problem 21

A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

$\textbf{(A)}\ \frac{12}{13} \qquad \textbf{(B)}\ \frac{35}{37} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{37}{35} \qquad \textbf{(E)}\ \frac{13}{12}$

Solution

Problem 22

Sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle ABC are tangent to a circle at points B and C respectively. What fraction of the area of $\triangle ABC$ lies outside the circle?

$\textbf{(A)}\ \frac{4\sqrt{3}\pi}{27}-\frac{1}{3} \qquad \textbf{(B)}\ \frac{\sqrt{3}}{2}-\frac{\pi}{8} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \sqrt{3}-\frac{2\sqrt{3}\pi}{9} \qquad \textbf{(E)}\ \frac{4}{3}-\frac{4\sqrt{3}\pi}{27}$

Solution

Problem 23

How many triangles with positive area have all their vertices at points (i,j) in the coordinate plane, where i and j are integers between 1 and 5, inclusive?

$\textbf{(A)}\ 2128 \qquad \textbf{(B)}\ 2148 \qquad \textbf{(C)}\ 2160 \qquad \textbf{(D)}\ 2200 \qquad \textbf{(E)}\ 2300$

Solution

Problem 24

For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

$\textbf{(A)}\ -9009 \qquad\textbf{(B)}\ -8008 \qquad\textbf{(C)}\ -7007 \qquad\textbf{(D)}\ -6006 \qquad\textbf{(E)}\ -5005$

Solution

Problem 25

How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and $999?$ For example, both 121 and 211 have this property.

$\textbf{(A)}\ 226 \qquad\textbf{(B)}\ 243 \qquad\textbf{(C)}\ 270 \qquad\textbf{(D)}\ 469 \qquad\textbf{(E)}\ 486$

Solution

See also

<table>
<tr><td colspan="2">2017 AMC 10A (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2017))</td></tr>
<tr><td>Preceded by
2016 AMC 10B Problems</td><td>Followed by
2017 AMC 10B Problems</td></tr>
<tr><td colspan="2">1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25</td></tr>
<tr><td colspan="2">All AMC 10 Problems and Solutions</td></tr>
</table>

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