2013 AMC 8 Problems/Problem 1

Problem

Danica wants to arrange her model cars in rows with exactly 6 cars in each row. She now has 23 model cars. What is the smallest number of additional cars she must buy in order to be able to arrange all her cars this way?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

In order to have her model cars in perfect, complete rows of 6, Danica must have a number of cars that is a multiple of 6. The smallest multiple of 6 which is larger than 23 is 24, so she'll need to buy $\boxed{\textbf{(A)}\ 1}$ more model car.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by First Problem	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 2

Problem

A sign at the fish market says, "50% off, today only: half-pound packages for just \$3 per package." What is the regular price for a full pound of fish, in dollars?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Solution

The 50% off price of half a pound of fish is \$3, so the 100%, or the regular price, of a half pound of fish is \$6. Consequently, if half a pound of fish costs \$6, then a whole pound of fish is $\boxed{\textbf{(D)}\ 12}$ dollars.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 3

Problem

What is the value of $4 \cdot (-1+2-3+4-5+6-7+\cdots+1000)$?

$\textbf{(A)}\ -10 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 500 \qquad \textbf{(E)}\ 2000$

Solution

Notice that we can pair up every two numbers to make a sum of 1:

$$\begin{aligned}(-1+2-3+4-\cdots+1000) &= ((-1+2)+(-3+4)+\cdots+(-999+1000)) \\ &= (1+1+\cdots+1) \\ &= 500\end{aligned}$$

Therefore, the answer is $4 \cdot 500 = \boxed{\textbf{(E)}\ 2000}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 4

Problem

Eight friends ate at a restaurant and agreed to share the bill equally. Because Judi forgot her money, each of her seven friends paid an extra \$2.50 to cover her portion of the total bill. What was the total bill?

$\textbf{(A)}\ \$120 \qquad \textbf{(B)}\ \$128 \qquad \textbf{(C)}\ \$140 \qquad \textbf{(D)}\ \$144 \qquad \textbf{(E)}\ \$160$

Solution

Each of her seven friends paid $\$2.50$ to cover Judi's portion. Therefore, Judi's portion must be $\$2.50 \cdot 7 = \$17.50$. Since each friend was supposed to pay the same amount, each friend was supposed to pay $\frac{1}{8}$ of the total bill; the total bill must be $8 \cdot 7 \cdot \$2.50 = \boxed{\textbf{(C)}\ \$140}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 5

Problem

Hammie is in the 6^{th} grade and weighs 106 pounds. His quadruplet sisters are tiny babies and weigh 5, 5, 6, and 8 pounds. Which is greater, the average (mean) weight of these five children or the median weight, and by how many pounds?

$\textbf{(A)}\ \text{median, by 60} \qquad \textbf{(B)}\ \text{median, by 20} \qquad \textbf{(C)}\ \text{average, by 5} \qquad \textbf{(D)}\ \text{average, by 15} \qquad \textbf{(E)}\ \text{average, by 2}$

Solution

The median here is obviously less than the mean, so options (A) and (B) are out.

Lining up the numbers (5, 5, 6, 8, 106), we see that the median weight is 6 pounds.

The average weight of the five kids is $\dfrac{5+5+6+8+106}{5} = \dfrac{130}{5} = 26$.

Therefore, the average weight is bigger, by $26-6=20$ pounds, making the answer $\boxed{\textbf{(E)}\ \text{average, by 20}}$.

See Also

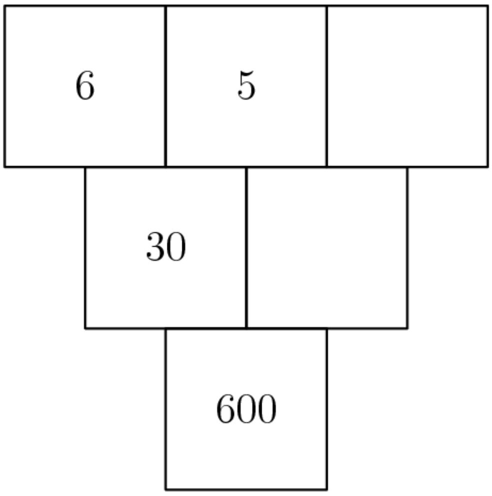
2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 6

Contents

Problem

The number in each box below is the product of the numbers in the two boxes that touch it in the row above. For example, $30 = 6 \times 5$. What is the missing number in the top row?

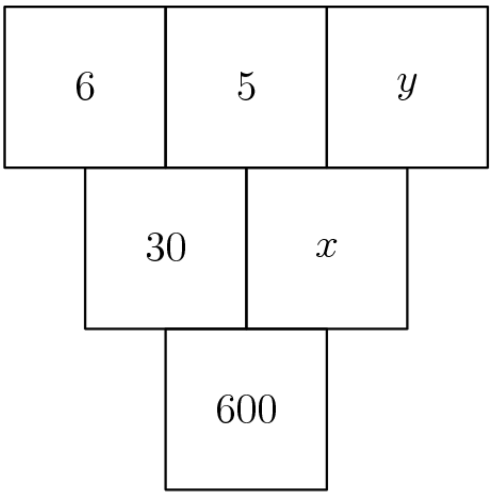


$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

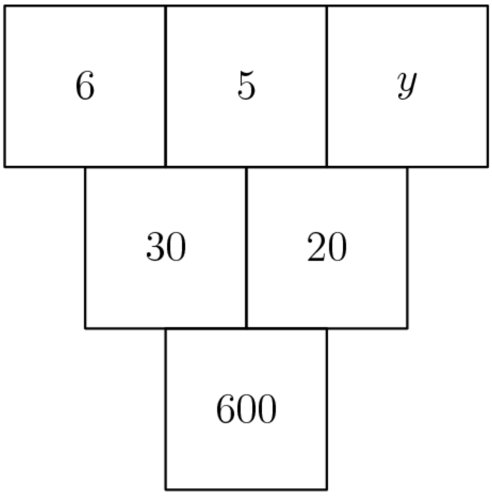
Solution

Solution 1: Working Backwards

Let the value in the empty box in the middle row be x, and the value in the empty box in the top row be y. y is the answer we're looking for.



We see that $600 = 30x$, making $x = 20$.

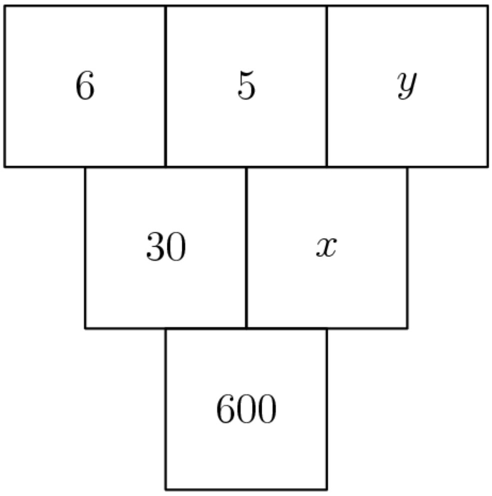


It follows that $20 = 5y$, so $y = \boxed{\textbf{(C) } 4}$.

Solution 2: Jumping Back to the Start

Another way to do this problem is to realize what makes up the bottommost number. This method doesn't work quite as well for this problem, but in a larger tree, it might be faster. (In this case, Solution 1 would be faster since there's only two missing numbers.)

Again, let the value in the empty box in the middle row be x, and the value in the empty box in the top row be y. y is the answer we're looking for.



We can write some equations:

$$600 = 30x$$
$$30 = 6 \cdot 5$$
$$x = 5y$$

Now we can substitute into the first equation using the two others:

$$600 = (6 \cdot 5)(5y)$$
$$600 = 6 \cdot 5 \cdot 5 \cdot y$$
$$600 = 6 \cdot 25 \cdot y$$
$$600 = 150y$$
$$\boxed{\textbf{(C) } 4} = y$$

See Also

2013 AMC 8 Problems/Problem 7

Contents

Problem

Trey and his mom stopped at a railroad crossing to let a train pass. As the train began to pass, Trey counted 6 cars in the first 10 seconds. It took the train 2 minutes and 45 seconds to clear the crossing at a constant speed. Which of the following was the most likely number of cars in the train?

$\textbf{(A)}\ 60 \qquad \textbf{(B)}\ 80 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 140$

Solution 1

If Trey saw $\frac{6 \text{ cars}}{10 \text{ seconds}}$, then he saw $\frac{3 \text{ cars}}{5 \text{ seconds}}$.

2 minutes and 45 seconds can also be expressed as $2 \cdot 60 + 45 = 165$ seconds.

Trey's rate of seeing cars, $\frac{3 \text{ cars}}{5 \text{ seconds}}$, can be multiplied by $165 \div 5 = 33$ on the top and bottom (and preserve the same rate):

$\frac{3 \cdot 33 \text{ cars}}{5 \cdot 33 \text{ seconds}} = \frac{99 \text{ cars}}{165 \text{ seconds}}$. It follows that the most likely number of cars is $\boxed{(C)}\ \textbf{100}$.

Solution 2

2 minutes and 45 seconds is equal to $120 + 45 = 165 \text{ seconds}$.

Since Trey probably counts around 6 cars every 10 seconds, there are $\left\lfloor \frac{165}{10} \right\rfloor = 16$ groups of 6 cars that Trey most likely counts. Since $16 \times 6 = 96 \text{ cars}$, the closest answer choice is $\textbf{(C)}\ 100$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 8

Problem

A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{3}{8} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{3}{4}$

Solution

First, there are $2^3 = 8$ ways to flip the coins, in order.

The ways to get two consecutive heads are HHT and THH.

The way to get three consecutive heads is HHH.

Therefore, the probability of flipping at least two consecutive heads is $\boxed{\textbf{(C)}\ \frac{3}{8}}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 9

Contents

Problem

The Incredible Hulk can double the distance he jumps with each succeeding jump. If his first jump is 1 meter, the second jump is 2 meters, the third jump is 4 meters, and so on, then on which jump will he first be able to jump more than 1 kilometer?

$\textbf{(A)}\ 9^\text{th} \qquad \textbf{(B)}\ 10^\text{th} \qquad \textbf{(C)}\ 11^\text{th} \qquad \textbf{(D)}\ 12^\text{th} \qquad \textbf{(E)}\ 13^\text{th}$

Solution

This is a geometric sequence in which the common ratio is 2. To find the jump that would be over a 1000 meters, we note that $2^{10} = 1024$.

However, because the first term is $2^0 = 1$ and not $2^1 = 2$, the solution to the problem is $10 - 0 + 1 = \boxed{\textbf{(C)}\ 11^\text{th}}$

Solution 2

We can also solve this problem by listing out how far the Hulk jumps on each jump: on the first jump, he goes 1 meter, the second jump 2 meters, and so on. Listing out these numbers, we get:

$$1, 2, 4, 8, 16, 32, 64, 128, 256, 512, \boxed{\textbf{1024}}$$

On the 11th jump, the Hulk jumps 1024 meters > 1000 meters (1 kilometer), so our answer is the 11th jump, or $\boxed{\textbf{(C)}}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 10

Contents

Problem

What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?

$\textbf{(A)}\ 110 \qquad \textbf{(B)}\ 165 \qquad \textbf{(C)}\ 330 \qquad \textbf{(D)}\ 625 \qquad \textbf{(E)}\ 660$

Solution 1

To find either the LCM or the GCF of two numbers, always prime factorize first.

The prime factorization of $180 = 3^2 \times 5 \times 2^2$.

The prime factorization of $594 = 3^3 \times 11 \times 2$.

Then, to find the LCM, we have to find the greatest power of all the numbers there are; if one number is one but not the other, use it (this is $3^3, 5, 11, 2^2$). Multiply all of these to get 5940.

For the GCF of 180 and 594, use the least power of all of the numbers that are in both factorizations and multiply. $3^2 \times 2$ = 18.

Thus the answer = $\frac{5940}{18}$ = $\boxed{\textbf{(C)}\ 330}$.

Similar Solution

We start off with a similar approach as the original solution. From the prime factorizations, the GCF is 18.

It is a well known fact that $\gcd(m,n) \times \text{lcm}(m,n) = |mn|$. So we have, $18 \times \text{lcm}(180, 594) = 594 \times 180$.

Dividing by 18 yields $\text{lcm}(180, 594) = 594 \times 10 = 5940$.

Therefore, $\frac{\text{lcm}(180, 594)}{\text{gcf}(180, 594)} = \frac{5940}{18} = \boxed{\textbf{(C)}\ 330}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 11

Problem

Ted's grandfather used his treadmill on 3 days this week. He went 2 miles each day. On Monday he jogged at a speed of 5 miles per hour. He walked at the rate of 3 miles per hour on Wednesday and at 4 miles per hour on Friday. If Grandfather had always walked at 4 miles per hour, he would have spent less time on the treadmill. How many minutes less?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

We use that fact that $d=rt$. Let d= distance, r= rate or speed, and t=time. In this case, let x represent the time.

On Monday, he was at a rate of 5 m.p.h. So, $5x = 2 \text{ miles} \implies x = \frac{2}{5} \text{ hours}$.

For Wednesday, he walked at a rate of 3 m.p.h. Therefore, $3x = 2 \text{ miles} \implies x = \frac{2}{3} \text{ hours}$.

On Friday, he walked at a rate of 4 m.p.h. So, $4x = 2 \text{ miles} \implies x = \frac{2}{4} \text{hours}$.

Adding up the hours yields $\frac{2}{5} \text{ hours}$ + $\frac{2}{3} \text{ hours}$ + $\frac{2}{4} \text{ hours}$ = $\frac{94}{60} \text{ hours}$.

We now find the amount of time Grandfather would have taken if he walked at 4 m.p.h per day. Set up the equation, $4x = 2 \text{ miles} \times 3 \text{ days} \implies x = \frac{3}{2} \text{ hours}$.

To find the amount of time saved, subtract the two amounts: $\frac{94}{60} \text{ hours}$ - $\frac{3}{2} \text{ hours}$ = $\frac{4}{60} \text{ hours}$. To convert this to minutes, we multiply by 60.

Thus, the solution to this problem is $\frac{4}{60} \times 60 = \boxed{\textbf{(D)}\ 4}$

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 10	Followed by Problem 12
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 12

Problem

At the 2013 Winnebago County Fair a vendor is offering a "fair special" on sandals. If you buy one pair of sandals at the regular price of $50, you get a second pair at a 40% discount, and a third pair at half the regular price. Javier took advantage of the "fair special" to buy three pairs of sandals. What percentage of the $150 regular price did he save?

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 33 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 45$

Solution

First, find the amount of money one will pay for three sandals without the discount. We have $\$50 \times 3 \text{ sandals} = \$150$.

Then, find the amount of money using the discount: $50 + 0.6 \times 50 + \frac{1}{2} \times 50 = \$105$.

Finding the percentage yields $\frac{105}{150} = 70\%$.

To find the percent saved, we have $100\% - 70\% = \boxed{\textbf{(B)}\ 30}$

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 13

Problem

When Clara totaled her scores, she inadvertently reversed the units digit and the tens digit of one score. By which of the following might her incorrect sum have differed from the correct one?

$\textbf{(A)}\ 45 \qquad \textbf{(B)}\ 46 \qquad \textbf{(C)}\ 47 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 49$

Solution

Let the two digits be a and b.

The correct score was $10a+b$. Clara misinterpreted it as $10b+a$. The difference between the two is $|9a-9b|$ which factors into $|9(a-b)|$. Therefore, since the difference is a multiple of 9, the only answer choice that is a multiple of 9 is $\boxed{\textbf{(A)}\ 45}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 12	Followed by Problem 14
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 14

Problem

Abe holds 1 green and 1 red jelly bean in his hand. Bea holds 1 green, 1 yellow, and 2 red jelly beans in her hand. Each randomly picks a jelly bean to show the other. What is the probability that the colors match?

$\textbf{(A)}\ \frac14 \qquad \textbf{(B)}\ \frac13 \qquad \textbf{(C)}\ \frac38 \qquad \textbf{(D)}\ \frac12 \qquad \textbf{(E)}\ \frac23$

Solution

The probability that both show a green bean is $\frac{1}{2}\cdot\frac{1}{4}=\frac{1}{8}$. The probability that both show a red bean is $\frac{1}{2}\cdot\frac{2}{4}=\frac{1}{4}$. Therefore the probability is $\frac{1}{4}+\frac{1}{8}=\boxed{\textbf{(C)}\ \frac{3}{8}}$

See Also

2013 AMC 8 (Problems • Answer Key • Resources)	
Preceded by Problem 13	Followed by Problem 15
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 15

Problem

If $3^p + 3^4 = 90$, $2^r + 44 = 76$, and $5^3 + 6^s = 1421$, what is the product of p, r, and s?

$\textbf{(A)}\ 27 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 70 \qquad \textbf{(E)}\ 90$

Solution

$3^p + 3^4 = 90$
$3^p + 81 = 90$
$3^p = 9$

Therefore, $p = 2$.

$2^r + 44 = 76$
$2^r = 32$

Therefore, $r = 5$.

$5^3 + 6^s = 1421$
$125 + 6^s = 1421$
$6^s = 1296$

To most people, it would not be immediately evident that $6^4 = 1296$, so we can multiply 6's until we get the desired number:

$6 \cdot 6 = 36$

$6 \cdot 36 = 216$

$6 \cdot 216 = 1296 = 6^4$, so $s = 4$.

Therefore the answer is $2 \cdot 5 \cdot 4 = \boxed{\textbf{(B)}\ 40}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 14	Followed by Problem 16
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 16

Contents

Problem

A number of students from Fibonacci Middle School are taking part in a community service project. The ratio of 8^{th}-graders to 6^{th}-graders is $5:3$, and the the ratio of 8^{th}-graders to 7^{th}-graders is $8:5$. What is the smallest number of students that could be participating in the project?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 55 \qquad \textbf{(D)}\ 79 \qquad \textbf{(E)}\ 89$

Solution

Solution 1: Algebra

We multiply the first ratio by 8 on both sides, and the second ratio by 5 to get the same number for 8th graders, in order that we can put the two ratios together:

$$5:3 = 5(8):3(8) = 40:24$$

$$8:5 = 8(5):5(5) = 40:25$$

Therefore, the ratio of 8th graders to 7th graders to 6th graders is $40:25:24$. Since the ratio is in lowest terms, the smallest number of students participating in the project is

$40+25+24 = \boxed{\textbf{(E)}\ 89}$.

Solution 2: Fakesolving

The number of 8th graders has to be a multiple of 8 and 5, so assume it is 40 (the smallest possibility). Then there are $40*\frac{3}{5}=24$ 6th graders and $40*\frac{5}{8}=25$ 7th graders. The numbers of students is

$40+24+25=\boxed{\textbf{(E)}\ 89}$

See Also

2013 AMC 8 (Problems • Answer Key • Resources)

Preceded by Problem 15 | Followed by Problem 17

1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25

All AJHSME/AMC 8 Problems and Solutions

2013 AMC 8 Problems/Problem 17

Contents

Problem

The sum of six consecutive positive integers is 2013. What is the largest of these six integers?

$\textbf{(A)}\ 335 \qquad \textbf{(B)}\ 338 \qquad \textbf{(C)}\ 340 \qquad \textbf{(D)}\ 345 \qquad \textbf{(E)}\ 350$

Solution 1

The mean of these numbers is $\frac{\frac{2013}{3}}{2}=\frac{671}{2}=335.5$. Therefore the numbers are $333, 334, 335, 336, 337, 338$, so the answer is $\boxed{\textbf{(B)}\ 338}$

Solution 2

Let the 4^{th} number be x. Then our desired number is $x+2$.

Our integers are $x-3,x-2,x-1,x,x+1,x+2$, so we have that $6x-3 = 2013 \implies x=\frac{2016}{6} = 336 \implies x+2 = \boxed{\textbf{(B)}\ 338}$.

Solution 3

Let the first term be x. Our integers are $x,x+1,x+2,x+3,x+4,x+5$. We have, $6x+15 = 2013 \implies x=333 \implies x+5 = \boxed{\textbf{(B)}\ 338}$

See Also

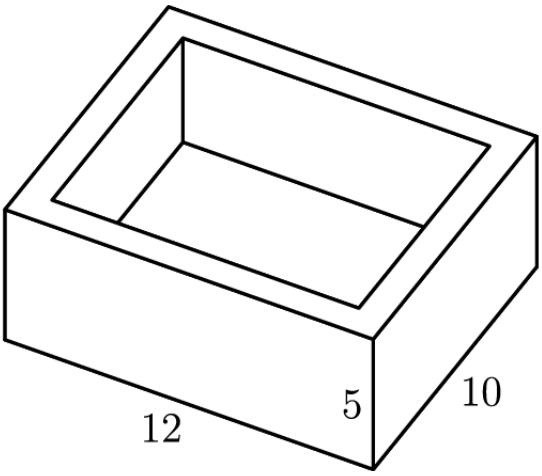
2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 16	Followed by Problem 18
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 18

Contents

Problem

Isabella uses one-foot cubical blocks to build a rectangular fort that is 12 feet long, 10 feet wide, and 5 feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain?



$\textbf{(A)}\ 204 \qquad \textbf{(B)}\ 280 \qquad \textbf{(C)}\ 320 \qquad \textbf{(D)}\ 340 \qquad \textbf{(E)}\ 600$

Solution 1

There are $10 \cdot 12 = 120$ cubes on the base of the box. Then, for each of the 4 layers above the bottom (as since each cube is 1 foot by 1 foot by 1 foot and the box is 5 feet tall, there are 4 feet left), there are $9 + 11 + 9 + 11 = 40$ cubes. Hence, the answer is $120 + 4 \cdot 40 = \boxed{\textbf{(B)}\ 280}$.

Solution 2

We can just calculate the volume of the prism that was cut out of the original $12 \times 10 \times 5$ box. Each interior side of the fort will be 2 feet shorter than each side of the outside. Since the floor is 1 foot, the height will be 4 feet. So the volume of the interior box is $10 \times 8 \times 4 = 320 \text{ ft}^3$.

The volume of the original box is $12 \times 10 \times 5 = 600 \text{ ft}^3$. Therefore, the number of blocks contained in the fort is $600 - 320 = \boxed{\textbf{(B)}\ 280}$.

See Also

2013 AMC 8 Problems/Problem 19

Problem

Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from highest to lowest?

$\textbf{(A)}\ \text{Hannah, Cassie, Bridget} \qquad \textbf{(B)}\ \text{Hannah, Bridget, Cassie}$
$\textbf{(C)}\ \text{Cassie, Bridget, Hannah} \qquad \textbf{(D)}\ \text{Cassie, Hannah, Bridget}$
$\textbf{(E)}\ \text{Bridget, Cassie, Hannah}$

Solution

If Hannah did better than Cassie, there would be no way she could know for sure that she didn't get the lowest score in the class. Therefore, Hannah did worse than Cassie. Similarly, if Hannah did worse than Bridget, there is no way Bridget could have known that she didn't get the highest in the class. Therefore, Hannah did better than Bridget, so our order is $\boxed{\textbf{(D)}\ \text{Cassie, Hannah, Bridget}}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 18	Followed by Problem 20
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

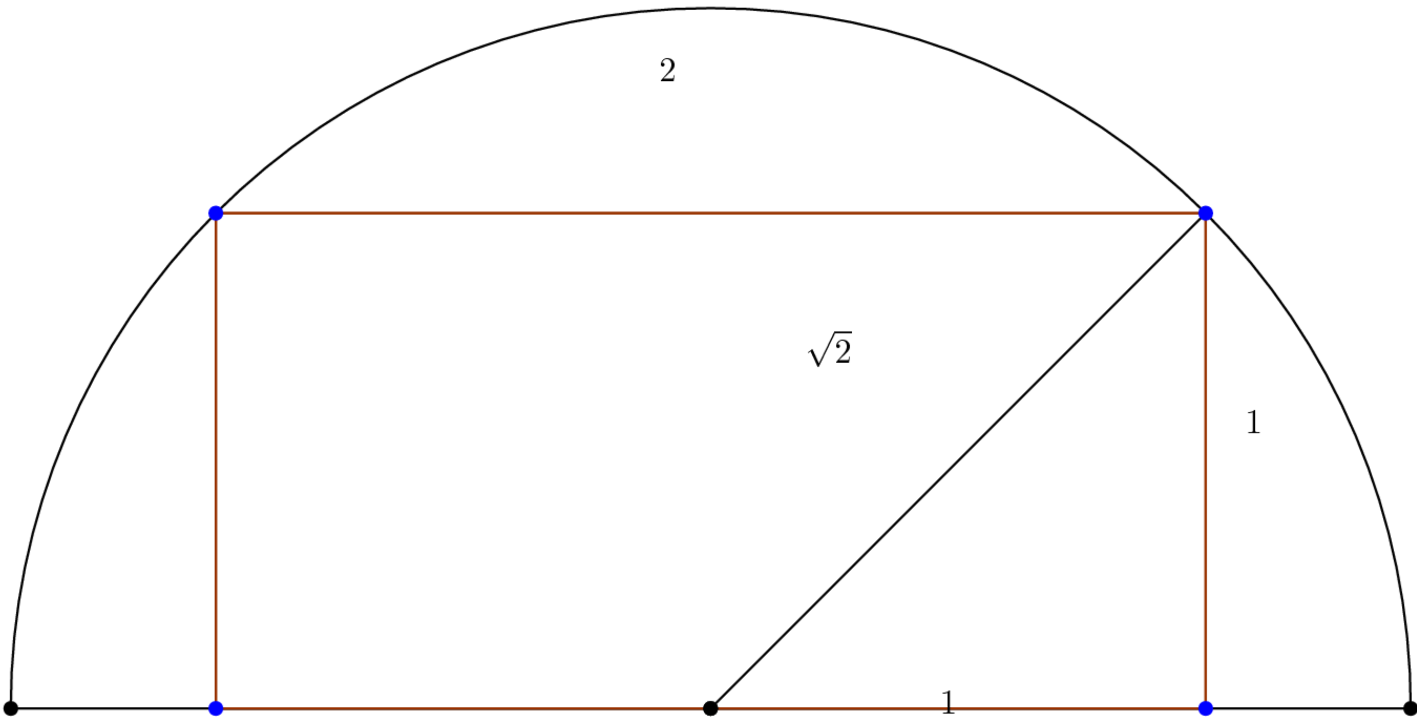
2013 AMC 8 Problems/Problem 20

Problem

A 1×2 rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle?

$\textbf{(A)}\ \frac{\pi}{2} \qquad \textbf{(B)}\ \frac{2\pi}{3} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{4\pi}{3} \qquad \textbf{(E)}\ \frac{5\pi}{3}$

Solution



A semicircle has symmetry, so the center is exactly at the midpoint of the 2 side on the rectangle, making the radius, by the Pythagorean Theorem, $\sqrt{1^2+1^2}=\sqrt{2}$. The area is $\frac{2\pi}{2}=\boxed{\textbf{(C)}\ \pi}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 19	Followed by Problem 21
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

Thank You for reading these answers by the followers of AoPS

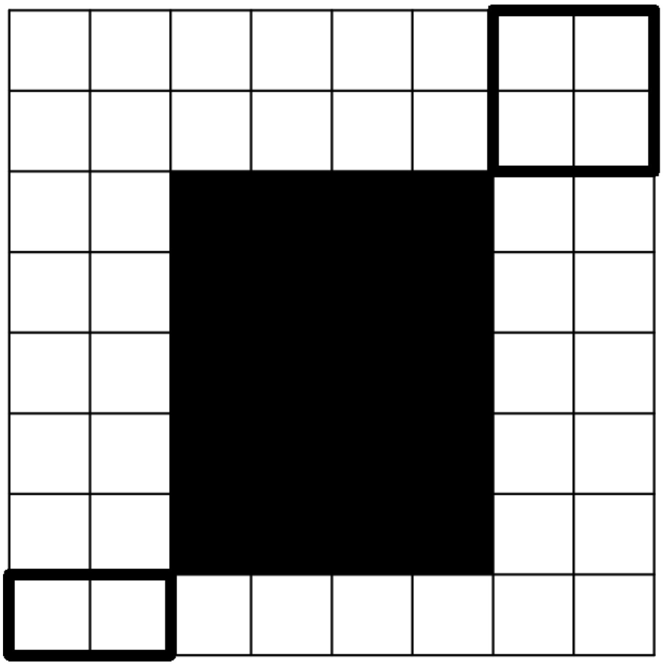
2013 AMC 8 Problems/Problem 21

Problem

Samantha lives 2 blocks west and 1 block south of the southwest corner of City Park. Her school is 2 blocks east and 2 blocks north of the northeast corner of City Park. On school days she bikes on streets to the southwest corner of City Park, then takes a diagonal path through the park to the northeast corner, and then bikes on streets to school. If her route is as short as possible, how many different routes can she take?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 18$

Solution



Using combinations, we get that the number of ways to get from Samantha's house to City Park is $\binom{3}{1} = 3$, and the number of ways to get from City Park to school is $\binom{4}{2} = 6$. Since there's one way to go through City Park (just walking straight through), the number of different ways to go from Samantha's house to City Park to school $3 \cdot 6 = \boxed{\textbf{(E)}\ 18}$.

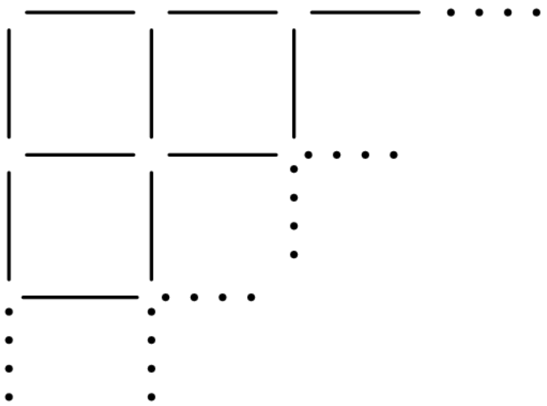
See Also

2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 20	Followed by Problem 22
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 22

Problem

Toothpicks are used to make a grid that is 60 toothpicks long and 32 toothpicks wide. How many toothpicks are used altogether?



$\textbf{(A)}\ 1920 \qquad \textbf{(B)}\ 1952 \qquad \textbf{(C)}\ 1980 \qquad \textbf{(D)}\ 2013 \qquad \textbf{(E)}\ 3932$

Solution

There are 61 vertical columns with a length of 32 toothpicks, and there are 33 horizontal rows with a length of 60 toothpicks. An effective way to verify this is to try a small case, i.e. a 2×3 grid of toothpicks. Thus, our answer is $61 \cdot 32 + 33 \cdot 60 = \boxed{\textbf{(E)}\ 3932}$.

See Also

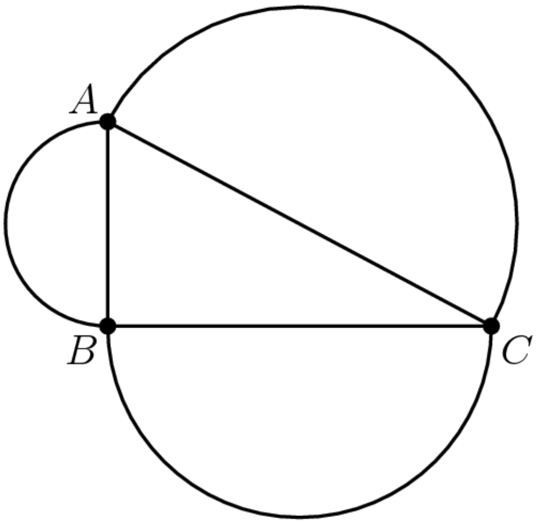
2013 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2013))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2013 AMC 8 Problems/Problem 23

Contents

Problem

Angle ABC of $\triangle ABC$ is a right angle. The sides of $\triangle ABC$ are the diameters of semicircles as shown. The area of the semicircle on $\overline{AB}$ equals 8π, and the arc of the semicircle on $\overline{AC}$ has length 8.5π. What is the radius of the semicircle on $\overline{BC}$?



$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 7.5 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 8.5 \qquad \textbf{(E)}\ 9$

Solution 1

If the semicircle on AB were a full circle, the area would be 16π. Therefore the diameter of the first circle is 8. The arc of the largest semicircle would normally have a complete diameter of 17. The Pythagorean theorem says that the other side has length 15, so the radius is $\boxed{\textbf{(B)}\ 7.5}$.

Solution 2

We go as in Solution 1, finding the diameter of the circle on AC and AB. Then, an extended version of the theorem says that the sum of the semicircles on the left is equal to the biggest one, so the area of the largest is $\frac{289\pi}{8}$, and the middle one is $\frac{289\pi}{8} - \frac{64\pi}{8} = \frac{225\pi}{8}$, so the radius is $\frac{15}{2} = \boxed{\textbf{(B)}\ 7.5}$.

See Also

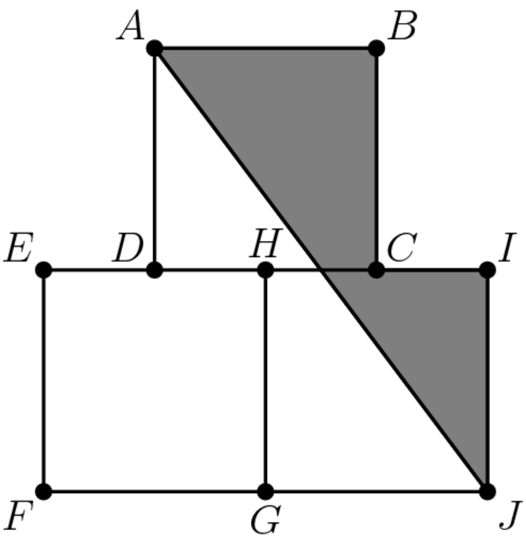
2013 AMC 8 Problems/Problem 24

Contents

Problem

Squares $ABCD$, $EFGH$, and $GHIJ$ are equal in area. Points C and D are the midpoints of sides IH and HE, respectively. What is the ratio of the area of the shaded pentagon $AJICB$ to the sum of the areas of the three squares?

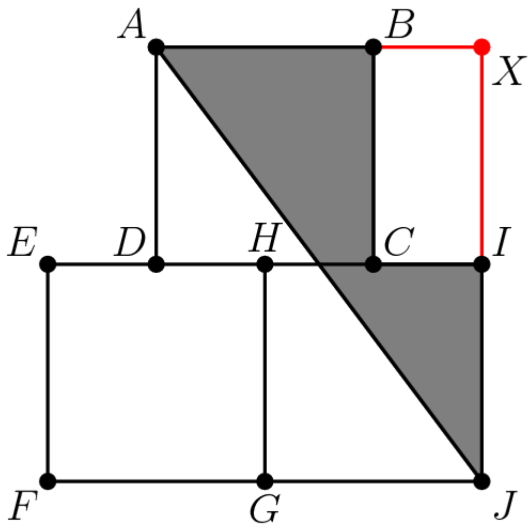
$\textbf{(A)}\hspace{.05in}\frac{1}{4}\qquad\textbf{(B)}\hspace{.05in}\frac{7}{24}\qquad\textbf{(C)}\hspace{.05in}\frac{1}{3}\qquad\textbf{(D)}\hspace{.05in}\frac{3}{8}\qquad\textbf{(E)}\hspace{.05in}\frac{5}{12}$



Easiest Solution

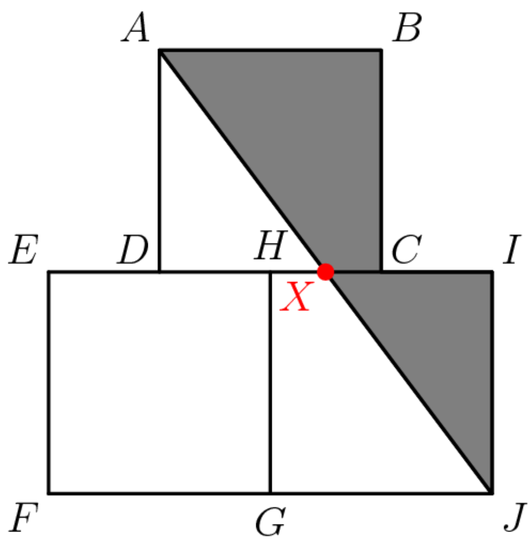
We can obviously see that the pentagon is made of two congruent triangles. We can fit one triangle into the gap in the upper square. Therefore, the answer is just $\frac{1}{3} \implies \boxed{C}$

Solution 1



First let $s = 2$ (where s is the side length of the squares) for simplicity. We can extend $\overline{IJ}$ until it hits the extension of $\overline{AB}$. Call this point X. The area of triangle AXJ then is $\frac{3 \cdot 4}{2}$ The area of rectangle $BXIC$ is $2 \cdot 1 = 2$. Thus, our desired area is $6 - 2 = 4$. Now, the ratio of the shaded area to the combined area of the three squares is $\frac{4}{3 \cdot 2^2} = \boxed{\textbf{(C)}\ \frac{1}{3}}$.

Solution 2



Let the side length of each square be 1.

Let the intersection of AJ and EI be X.

Since $[ABCD] = [GHIJ]$, $AD = IJ$. Since $\angle IXJ$ and $\angle AXD$ are vertical angles, they are congruent. We also have $\angle JIH \cong \angle ADC$ by definition.

So we have $\triangle ADX \cong \triangle JIX$ by AAS congruence. Therefore, $DX = JX$.

Since C and D are midpoints of sides, $DH = CJ = \frac{1}{2}$. This combined with $DX = JX$ yields $HX = CX = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

The area of trapezoid $ABCX$ is $\frac{1}{2}(AB+CX)(BC)=\frac{1}{2}\times\frac{5}{4}\times 1=\frac{5}{8}$.

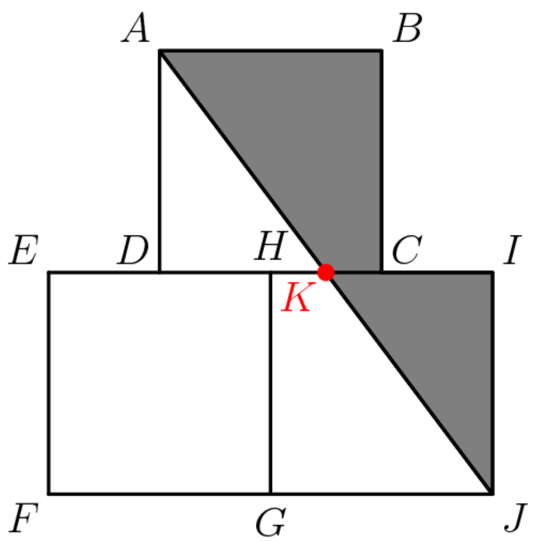
The area of triangle JIX is $\frac{1}{2}\times XJ\times IJ=\frac{1}{2}\times\frac{3}{4}\times 1=\frac{3}{8}$.

So the area of the pentagon $AJICB$ is $\frac{3}{8}+\frac{5}{8}=1$.

The area of the 3 squares is $1\times 3=3$.

Therefore, $\frac{[AJICB]}{[ABCIJFED]}=\boxed{\textbf{(C)}\ \frac{1}{3}}$.

Solution 3



Let the intersection of AJ and EI be K.

Now we have $\triangle ADK$ and $\triangle KIJ$.

Because both triangles has a side on congruent squares therefore $AD\cong IJ$.

Because $\angle AKD$ and $\angle JKI$ are vertical angles $\angle AKD\cong\angle JKI$.

Also both $\angle ADK$ and $\angle JIK$ are right angles so $\angle ADK\cong\angle JIK$.

Therefore by AAS(Angle, Angle, Side) $\triangle ADK\cong\triangle KIJ$.

Then translating/rotating the shaded $\triangle JIK$ into the position of $\triangle ADK$

So the shaded area now completely covers the square $ABCD$

Set the area of a square as x

Therefore, $\frac{x}{3x}=\boxed{\textbf{(C)}\ \frac{1}{3}}$.

See Also

2013 AMC 8 (Problems • Answer Key • Resources)

Preceded by Problem 23 | Followed by Problem 25

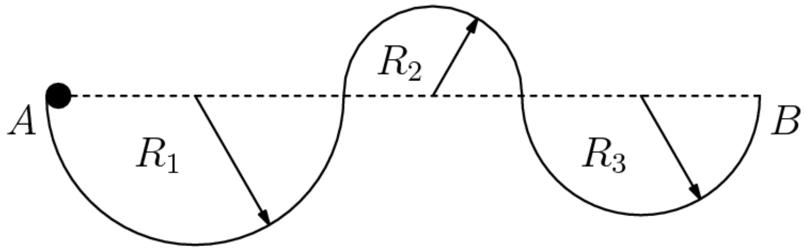
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19

2013 AMC 8 Problems/Problem 25

Contents

Problem

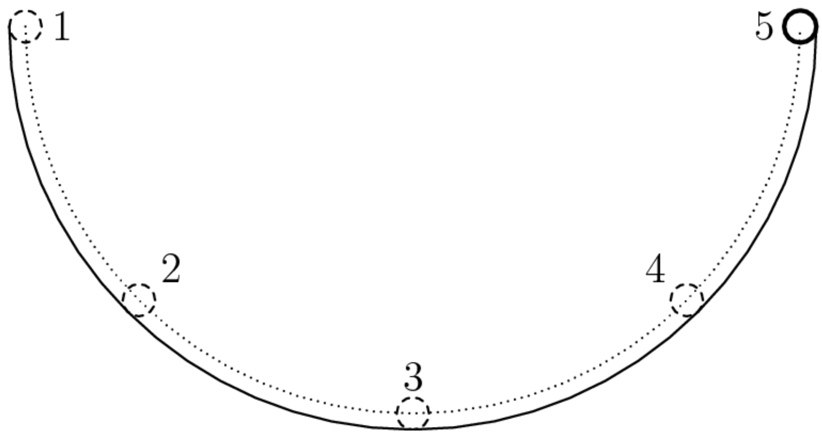
A ball with diameter 4 inches starts at point A to roll along the track shown. The track is comprised of 3 semicircular arcs whose radii are $R_1 = 100$ inches, $R_2 = 60$ inches, and $R_3 = 80$ inches, respectively. The ball always remains in contact with the track and does not slip. What is the distance the center of the ball travels over the course from A to B?



$\textbf{(A)}\ 238\pi \qquad \textbf{(B)}\ 240\pi \qquad \textbf{(C)}\ 260\pi \qquad \textbf{(D)}\ 280\pi \qquad \textbf{(E)}\ 500\pi$

Solution 1

The radius of the ball is 2 inches. If you think about the ball rolling or draw a path for the ball (see figure below), you see that in A and C it loses $2\pi * 2/2 = 2\pi$ inches, and it gains 2π inches on B.



So, the departure from the length of the track means that the answer is $\frac{200+120+160}{2} * \pi + (-2-2+2) * \pi = 240\pi - 2\pi = \boxed{\textbf{(A)}\ 238\pi}$.

Solution 2

The total length of all of the arcs is $100\pi + 80\pi + 60\pi = 240\pi$. Since we want the path from the center, the actual distance will be shorter. Therefore, the only answer choice less than 240π is $\boxed{\textbf{(A)}\ 238\pi}$. This solution may be invalid on other problems because the actual distance can be longer if the path the center travels is on the outside of the curve, as it is in the middle bump. In this problem it works though.