2005 AMC 12B Problems/Problem 1

The following problem is from both the 2005 AMC 12B #1 and 2005 AMC 10B #1, so both problems redirect to this page.

Problem

A scout troop buys 1000 candy bars at a price of five for 2 dollars. They sell all the candy bars at the price of two for 1 dollar. What was their profit, in dollars?

$\mathrm{(A) \ } 100\qquad \mathrm{(B) \ } 200\qquad \mathrm{(C) \ } 300\qquad \mathrm{(D) \ } 400\qquad \mathrm{(E) \ } 500$

Solution

$$\text{Expenses} = 1000 \cdot \frac{2}{5} = 400$$
$$\text{Revenue} = 1000 \cdot \frac{1}{2} = 500$$
$$\text{Profit} = \text{Revenue} - \text{Expenses} = 500 - 400 = \boxed{(A) 100}.$$

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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2005 AMC 12B Problems/Problem 2

The following problem is from both the 2005 AMC 12B #2 and 2005 AMC 10B #2, so both problems redirect to this page.

Problem

A positive number x has the property that $x\%$ of x is 4. What is x?

$(\mathrm{A})\ 2 \qquad (\mathrm{B})\ 4 \qquad (\mathrm{C})\ 10 \qquad (\mathrm{D})\ 20 \qquad (\mathrm{E})\ 40$

Solution

Since $x\%$ means $0.01x$, the statement $"x\% \text{ of } x \text{ is } 4"$ can be rewritten as $"0.01x \cdot x = 4"$:

$$0.01x \cdot x = 4 \Rightarrow x^2 = 400 \Rightarrow x = \boxed{(\mathrm{D})20}.$$

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
Preceded by Problem 1	Followed by Problem 3
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Category: Introductory Algebra Problems

2005 AMC 12B Problems/Problem 3

The following problem is from both the 2005 AMC 12B #3 and 2005 AMC 10B #5, so both problems redirect to this page.

Problem

Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

$\mathrm{(A)}\ \frac{1}{5} \qquad \mathrm{(B)}\ \frac{1}{3} \qquad \mathrm{(C)}\ \frac{2}{5} \qquad \mathrm{(D)}\ \frac{2}{3} \qquad \mathrm{(E)}\ \frac{4}{5}$

Solution

Let $m =$Brianna's money. We have $\frac{1}{5}m = \frac{1}{3}(\text{CDs}) \Rightarrow \frac{3}{5}m = (\text{CDs})$. Thus, the money left over is $m - \frac{3}{5}m = \frac{2}{5}m$, so the answer is $\boxed{(\text{C}) \frac{2}{5}}$.

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
Preceded by Problem 4	Followed by Problem 6
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2005 AMC 12B Problems/Problem 4

The following problem is from both the 2005 AMC 12B #4 and 2005 AMC 10B #6, so both problems redirect to this page.

Problem

At the beginning of the school year, Lisa's goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the first 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

$\mathrm{(A) \ } 1\qquad \mathrm{(B) \ } 2\qquad \mathrm{(C) \ } 3\qquad \mathrm{(D) \ } 4\qquad \mathrm{(E) \ } 5$

Solution

Lisa's goal was to get an A on $80\% \cdot 50 = 40$ quizzes. She already has A's on 22 quizzes, so she needs to get A's on $40-22=18$ more. There are $50-30=20$ quizzes left, so she can afford to get less than an A on $20-18=\boxed{\mathrm{(B)}2}$ of them.

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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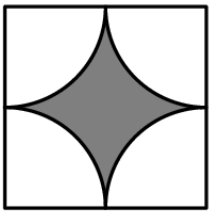
2005 AMC 12B Problems/Problem 5

The following problem is from both the 2005 AMC 12B #5 and 2005 AMC 10B #8, so both problems redirect to this page.

Problem

An 8-foot by 10-foot floor is tiles with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius $1/2$ foot centered at each corner of the tile. The remaining portion of the tile is shaded. How many square feet of the floor are shaded?



$\mathrm{(A)}\ 80-20\pi \qquad \mathrm{(B)}\ 60-10\pi \qquad \mathrm{(C)}\ 80-10\pi \qquad \mathrm{(D)}\ 60+10\pi \qquad \mathrm{(E)}\ 80+10\pi$

Solution

There are 80 tiles. Each tile has $[\text{square} - 4 \cdot (\text{quarter circle})]$ shaded. Thus:

$$\begin{aligned}\text{shaded area} &= 80\left(1 - 4\cdot\frac{1}{4}\cdot\pi\cdot\left(\frac{1}{2}\right)^2\right)\\ &= 80\left(1-\frac{1}{4}\pi\right)\\ &= \boxed{80-20\pi}.\end{aligned}$$

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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2005 AMC 12B Problems/Problem 6

The following problem is from both the 2005 AMC 12B #6 and 2005 AMC 10B #10, so both problems redirect to this page.

Problem

In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is BD?

$\mathrm{(A)}\ 3 \qquad \mathrm{(B)}\ 2\sqrt{3} \qquad \mathrm{(C)}\ 4 \qquad \mathrm{(D)}\ 5 \qquad \mathrm{(E)}\ 4\sqrt{2}$

Solution

Draw height CH. We have that $BH = 1$. From the Pythagorean Theorem, $CH = \sqrt{48}$. Since $CD = 8$, $HD = \sqrt{8^2 - 48} = \sqrt{16} = 4$, and $BD = HD - 1$, so $BD = \boxed{\mathrm{(A)} 3}$.

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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Category: Introductory Geometry Problems

2005 AMC 12B Problems/Problem 7

Problem

What is the area enclosed by the graph of $|3x| + |4y| = 12$?

$\mathrm{(A)}\ 6 \qquad \mathrm{(B)}\ 12 \qquad \mathrm{(C)}\ 16 \qquad \mathrm{(D)}\ 24 \qquad \mathrm{(E)}\ 25$

Solution

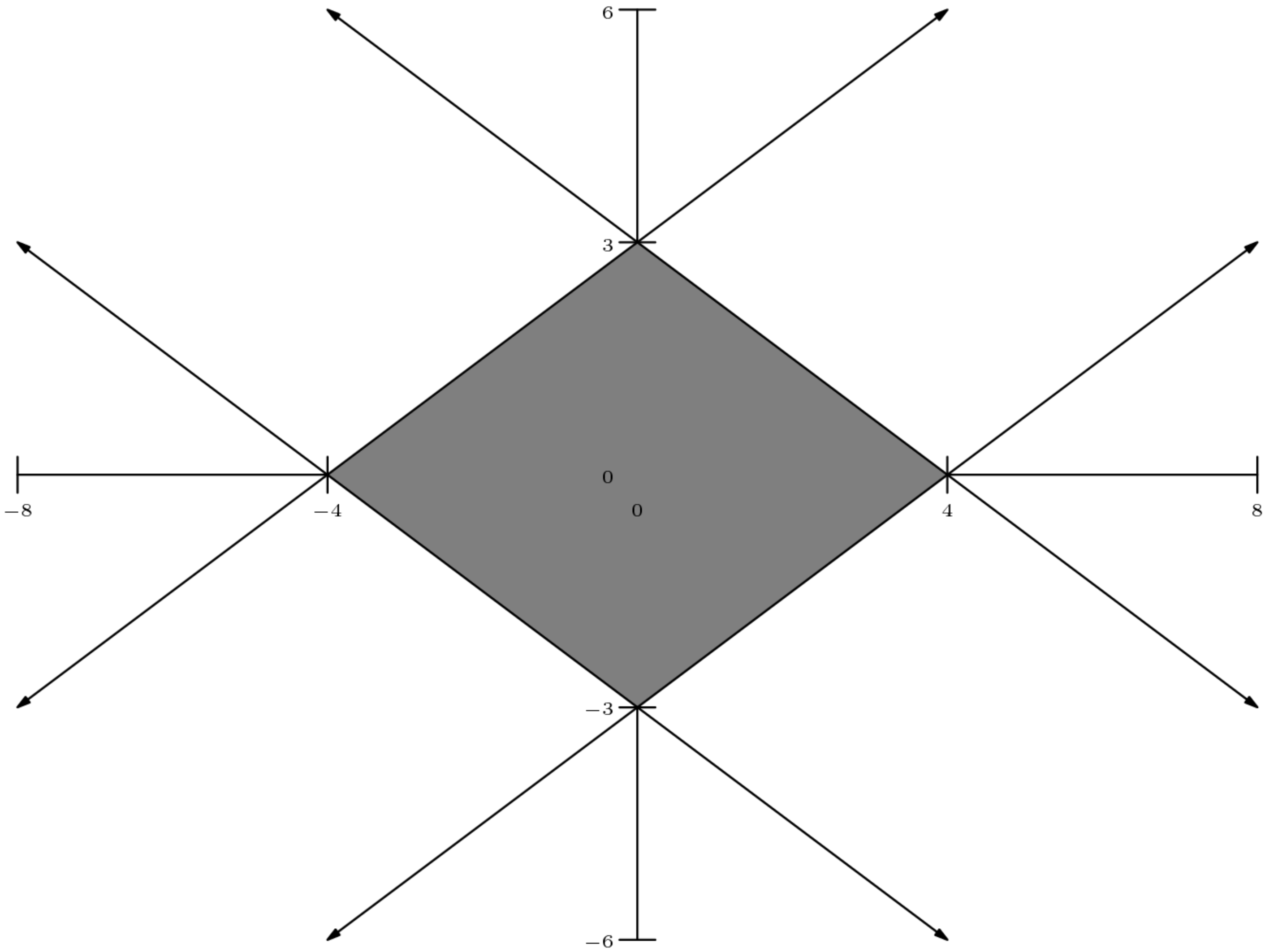
If we get rid of the absolute values, we are left with the following 4 equations (using the logic that if $|a| = b$, then a is either b or $-b$):

$$\begin{aligned} 3x + 4y &= 12 \\ -3x + 4y &= 12 \\ 3x - 4y &= 12 \\ -3x - 4y &= 12 \end{aligned}$$

We can then put these equations in slope-intercept form in order to graph them.

$$\begin{aligned} 3x + 4y = 12 &\implies y = -\frac{3}{4}x + 3 \\ -3x + 4y = 12 &\implies y = \frac{3}{4}x + 3 \\ 3x - 4y = 12 &\implies y = \frac{3}{4}x - 3 \\ -3x - 4y = 12 &\implies y = -\frac{3}{4}x - 3 \end{aligned}$$

Now you can graph the lines to find the shape of the graph:



We can easily see that it is a rhombus with diagonals of 6 and 8. The area is $\frac{1}{2}\times 6\times 8$, or $\boxed{\text{(D) } 24}$

See also

2005 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2005))	
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2005 AMC 12B Problems/Problem 8

Problem

For how many values of a is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$?

$\mathrm{(A)}\ 0\qquad\mathrm{(B)}\ 1\qquad\mathrm{(C)}\ 2\qquad\mathrm{(D)}\ 10\qquad\mathrm{(E)}\ \text{infinitely many}$

Solution

We see that the vertex of the quadratic function $y = x^2 + a^2$ is $(0,\ a^2)$. The y-intercept of the line $y = x + a$ is $(0,\ a)$. We want to find the values (if any) such that $a = a^2$. Solving for a, the only values that satisfy this are 0 and 1, so the answer is $\boxed{\mathrm{(C)}\ 2}$

See also

2005 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2005))	
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2005 AMC 12B Problems/Problem 9

The following problem is from both the 2005 AMC 12B #9 and 2005 AMC 10B #19, so both problems redirect to this page.

Problem

On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

$(\mathrm{A})\ 0 \qquad (\mathrm{B})\ 1 \qquad (\mathrm{C})\ 2 \qquad (\mathrm{D})\ 4 \qquad (\mathrm{E})\ 5$

Solution

To begin, we see that the remaining 30% of the students got 95 points. Assume that there are 20 students; we see that 2 students got 70 points, 5 students got 80 points, 4 students got 85 points, 3 students got 90 points, and 6 students got 95 points. The median is 85, since the 10^{th} and 11^{th} terms are both 85. The mean is $\frac{70\left(2\right)+80\left(5\right)+85\left(4\right)+90\left(3\right)+95\left(6\right)}{20}=\frac{1720}{20}=86$. The difference between the mean and median, therefore, is $\boxed{(\mathrm{B})\ 1}$.

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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2005 AMC 12B Problems/Problem 10

The following problem is from both the 2005 AMC 12B #10 and 2005 AMC 10B #11, so both problems redirect to this page.

Problem

The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous terms. What is the 2005^{th} term of the sequence?

$\mathrm{(A)}\ 29 \qquad \mathrm{(B)}\ 55 \qquad \mathrm{(C)}\ 85 \qquad \mathrm{(D)}\ 133 \qquad \mathrm{(E)}\ 250$

Solution

Performing this operation several times yields the results of 133 for the second term, 55 for the third term, and 250 for the fourth term. The sum of the cubes of the digits of 250 equal 133, a complete cycle. The cycle is... excluding the first term, the 2^{nd}, 3^{rd}, and 4^{th} terms will equal 133, 55, and 250, following the fourth term. Any term number that is equivalent to $1 \pmod 3$ will produce a result of 250. It just so happens that $2005 \equiv 1 \pmod 3$, which leads us to the answer of $\boxed{\mathrm{(E)}\ 250}$.

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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2005 AMC 12B Problems/Problem 11

> The following problem is from both the 2005 AMC 12B #11 and 2005 AMC 10B #15, so both problems redirect to this page.

Problem

An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is $\$20$ or more?

$\mathrm{(A)}\ \frac{1}{4}\qquad\mathrm{(B)}\ \frac{2}{5}\qquad\mathrm{(C)}\ \frac{3}{7}\qquad\mathrm{(D)}\ \frac{1}{2}\qquad\mathrm{(E)}\ \frac{2}{3}$

Solution

The only way to get a total of $\$20$ or more is if you pick a twenty and another bill, or if you pick both tens. There are a total of $\binom{8}{2}=\frac{8\times 7}{2\times 1}=28$ ways to choose 2 bills out of 8. There are 12 ways to choose a twenty and some other non-twenty bill. There is 1 way to choose both twenties, and also 1 way to choose both tens. Adding these up, we find that there are a total of 14 ways to attain a sum of 20 or greater, so there is a total probability of $\frac{14}{28}=\boxed{\mathrm{(D)}\ \frac{1}{2}}$.

See also

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2005 AMC 12B Problems/Problem 12

The following problem is from both the 2005 AMC 12B #12 and 2005 AMC 10B #16, so both problems redirect to this page.

Problem

The quadratic equation $x^2 + mx + n$ has roots twice those of $x^2 + px + m$, and none of $m, n,$ and p is zero. What is the value of n/p?

$\text{(A) } 1 \qquad \text{(B) } 2 \qquad \text{(C) } 4 \qquad \text{(D) } 8 \qquad \text{(E) } 16$

Solution

Let $x^2 + px + m = 0$ have roots a and b. Then

$$x^2 + px + m = (x-a)(x-b) = x^2 - (a+b)x + ab,$$

so $p = -(a+b)$ and $m = ab$. Also, $x^2 + mx + n = 0$ has roots $2a$ and $2b$, so

$$x^2 + mx + n = (x-2a)(x-2b) = x^2 - 2(a+b)x + 4ab,$$

and $m = -2(a+b)$ and $n = 4ab$. Thus $\frac{n}{p} = \frac{4ab}{-(a+b)} = \frac{4m}{\frac{m}{2}} = \boxed{\text{(D) } 8}$.

Indeed, consider the quadratics $x^2 + 8x + 16 = 0,\ x^2 + 16x + 64 = 0$.

See also

2005 AMC 10B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=43&year=2005))	
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- Vieta's Formulas

2005 AMC 12B Problems/Problem 13

Problem

Suppose that $4^{x1} = 5$, $5^{x2} = 6$, $6^{x3} = 7$, ... , $127^{x124} = 128$. What is $x_1 x_2 ... x_{124}$?

$\mathrm{(A)}\ 2\qquad \mathrm{(B)}\ \frac{5}{2}\qquad \mathrm{(C)}\ 3\qquad \mathrm{(D)}\ \frac{7}{2}\qquad \mathrm{(E)}\ 4$

Solution

We see that we can re-write $4^{x1} = 5$, $5^{x2} = 6$, $6^{x3} = 7$, ... , $127^{x124} = 128$ as $\left(...\left(\left(\left(4^{x1}\right)^{x2}\right)^{x3}\right)...\right)^{x124} = 128$ by using substitution. By using the properties of exponents, we know that $4^{x_1 x_2 ... x_{124}} = 128$.

$4^{x_1 x_2 ... x_{124}} = 128$
$2^{2x_1 x_2 ... x_{124}} = 2^7$
$2x_1 x_2 ... x_{124} = 7$
$x_1 x_2 ... x_{124} = \frac{7}{2}$

Therefore, the answer is $\boxed{\mathrm{(D)}\ \frac{7}{2}}$

See also

2005 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2005))	
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2005 AMC 12B Problems/Problem 14

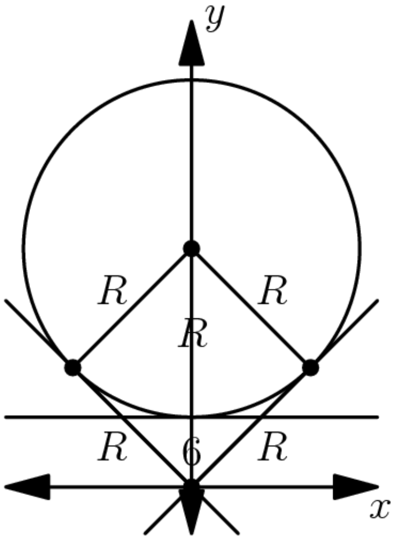
Problem

A circle having center $(0,k)$, with $k>6$, is tangent to the lines $y=x$, $y=-x$ and $y=6$. What is the radius of this circle?

$\text{(A)}\ 6\sqrt{2}-6 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 6\sqrt{2} \qquad \text{(D)}\ 12 \qquad \text{(E)}\ 6+6\sqrt{2}$

Solution

Let R be the radius of the circle. Draw the two radii that meet the points of tangency to the lines $y=\pm x$. We can see that a square is formed by the origin, two tangency points, and the center of the circle. The side lengths of this square are R and the diagonal is $k=R+6$. The diagonal of a square is $\sqrt{2}$ times the side length. Therefore, $R+6=R\sqrt{2} \Rightarrow R=\dfrac{6}{\sqrt{2}-1}=6+6\sqrt{2} \Rightarrow E$.



See also

2005 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2005))	
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2005 AMC 12B Problems/Problem 15

Contents

Problem

The sum of four two-digit numbers is 221. None of the eight digits is 0 and no two of them are the same. Which of the following is not included among the eight digits?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ 3 \qquad \mathrm{(D)}\ 4 \qquad \mathrm{(E)}\ 5$

Solution 1

221 can be written as the sum of eight two-digit numbers, let's say $\overline{ae}$, $\overline{bf}$, $\overline{cg}$, and $\overline{dh}$. Then $221 = 10(a+b+c+d)+(e+f+g+h)$. The last digit of 221 is 1, and $10(a+b+c+d)$ won't affect the units digits, so $(e+f+g+h)$ must end with 1. The smallest value $(e+f+g+h)$ can have is $(1+2+3+4)=10$, and the greatest value is $(6+7+8+9)=30$. Therefore, $(e+f+g+h)$ must equal 11 or 21.

Case 1: $(e+f+g+h)=11$

The only distinct positive integers that can add up to 11 is $(1+2+3+5)$. So, $a,b,c,$ and d must include four of the five numbers $(4,6,7,8,9)$. We have $10(a+b+c+d)=221-11=210$, or $a+b+c+d=21$. We can add all of $4+6+7+8+9=34$, and try subtracting one number to get to 21, but to no avail. Therefore, $(e+f+g+h)$ cannot add up to 11.

Case 2: $(e+f+g+h)=21$

Checking all the values for $e,f,g,$and h each individually may be time-consuming, instead of only having 1 solution like Case 1. We can try a different approach by looking at $(a+b+c+d)$ first. If $(e+f+g+h)=21$, $10(a+b+c+d)=221-21=200$, or $(a+b+c+d)=20$. That means $(a+b+c+d)+(e+f+g+h)=21+20=41$. We know $(1+2+3+4+5+6+7+8+9)=45$, so the missing digit is $45-41=\boxed{\mathrm{(D)}\ 4}$

Solution 2

Alternatively, we know that a number is congruent to the sum of its digits mod 9, so $221 \equiv 5 \equiv 1+2+3+4+5+6+7+8+9-d \equiv -d$, where d is some digit. Clearly, $\boxed{d=4}$.

See also

2005 AMC 12B Problems/Problem 16

Problem

Eight spheres of radius 1, one per octant, are each tangent to the coordinate planes. What is the radius of the smallest sphere, centered at the origin, that contains these eight spheres?

$(A)\ \sqrt{2} \qquad (B)\ \sqrt{3} \qquad (C)\ 1+\sqrt{2} \qquad (D)\ 1+\sqrt{3} \qquad (E)\ 3$

Solution

The eight spheres are formed by shifting spheres of radius 1 and center $(0, 0, 0)$ ± 1 in the x, y, z directions. Hence, the centers of the spheres are $(\pm 1, \pm 1, \pm 1)$. For a sphere centered at the origin to contain all eight spheres, its radius must be greater than or equal to the longest distance from the origin to one of these spheres. This length is the sum of the distance from $(\pm 1, \pm 1, \pm 1)$ to the origin and the radius of the spheres, or $\sqrt{3}+1$. To verify this is the longest length, we can see from the triangle inequality that the length from the origin to any other point on the spheres is strictly smaller. Thus, the answer is $\boxed{D}$.

See also

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Categories: Introductory Geometry Problems | 3D Geometry Problems

2005 AMC 12B Problems/Problem 17

Problem

How many distinct four-tuples (a, b, c, d) of rational numbers are there with

$$a \cdot \log_{10} 2 + b \cdot \log_{10} 3 + c \cdot \log_{10} 5 + d \cdot \log_{10} 7 = 2005?$$

$\text{(A)}\ 0 \qquad \text{(B)}\ 1 \qquad \text{(C)}\ 17 \qquad \text{(D)}\ 2004 \qquad \text{(E)}\ \text{infinitely many}$

Solution

Using the laws of logarithms, the given equation becomes

$$\log_{10} 2^a + \log_{10} 3^b + \log_{10} 5^c + \log_{10} 7^d = 2005$$

$$\Rightarrow \log_{10} 2^a \cdot 3^b \cdot 5^c \cdot 7^d = 2005$$

$$\Rightarrow 2^a \cdot 3^b \cdot 5^c \cdot 7^d = 10^{2005}$$

As a, b, c, d must all be rational, and there are no powers of 3 or 7 in 10^{2005}, $b = d = 0$. Then $2^a \cdot 5^c = 2^{2005} \cdot 5^{2005} \Rightarrow a = c = 2005$.

Only the four-tuple $(2005, 0, 2005, 0)$ satisfies the equation, so the answer is $\boxed{1} \Rightarrow \text{(B)}$.

See also

2005 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2005))	
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2005 AMC 12B Problems/Problem 18

Problem

Let $A(2,2)$ and $B(7,7)$ be points in the plane. Define R as the region in the first quadrant consisting of those points C such that $\triangle ABC$ is an acute triangle. What is the closest integer to the area of the region R?

$\text{(A)}\ 25 \qquad \text{(B)}\ 39 \qquad \text{(C)}\ 51 \qquad \text{(D)}\ 60 \qquad \text{(E)}\ 80$

Solution

For angle A and B to be acute, C must be between the two lines that are perpendicular to $\overline{AB}$ and contain points A and B. For angle C to be acute, first draw a $45-45-90$ triangle with $\overline{AB}$ as the hypotenuse. Note C cannot be inside this triangle's circumscribed circle or else $\angle C > 90^\circ$. Hence, the area of R is the area of the large triangle minus the area of the small triangle minus the area of the circle, which is $\frac{14^2}{2}-\frac{4^2}{2}-(\frac{5\sqrt{2}}{2})^2\pi = 98-8-\frac{25\pi}{2}$, which is approximately 51. The answer is $\boxed{C}$.

See also

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2005 AMC 12B Problems/Problem 19

Problem

Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

$\mathrm{(A)}\ 88 \qquad \mathrm{(B)}\ 112 \qquad \mathrm{(C)}\ 116 \qquad \mathrm{(D)}\ 144 \qquad \mathrm{(E)}\ 154$

Solution

let $x = 10a + b$, then $y = 10b + a$ where a and b are nonzero digits.

By difference of squares,

$$x^2 - y^2 = (x + y)(x - y)$$

$$= (10a + b + 10b + a)(10a + b - 10b - a)$$

$$= (11(a + b))(9(a - b))$$

For this product to be a square, the factor of 11 must be repeated in either $(a + b)$ or $(a - b)$, and given the constraints it has to be $(a + b) = 11$. The factor of 9 is already a square and can be ignored. Now $(a - b)$ must be another square, and since a cannot be 10 or greater then $(a - b)$ must equal 4 or 1. If $a - b = 4$ then $(a + b) + (a - b) = 11 + 4$, $2a = 15$, $a = 15/2$, which is not a digit. Hence the only possible value for $a - b$ is 1. Now we have $(a + b) + (a - b) = 11 + 1$, $2a = 12$, $a = 6$, then $b = 5$, $x = 65$, $y = 56$, $m = 33$, and $x + y + m = 154 \Rightarrow \boxed{E}$

See also

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2005 AMC 12B Problems/Problem 20

Problem

Let a, b, c, d, e, f, g and h be distinct elements in the set $\{-7, -5, -3, -2, 2, 4, 6, 13\}$.

What is the minimum possible value of $(a+b+c+d)^2 + (e+f+g+h)^2$?

$\mathrm{(A)}\ 30 \qquad \mathrm{(B)}\ 32 \qquad \mathrm{(C)}\ 34 \qquad \mathrm{(D)}\ 40 \qquad \mathrm{(E)}\ 50$

Solution

The sum of the set is $-7-5-3-2+2+4+6+13=8$, so if we could have the sum in each set of parenthesis be 4 then the minimum value would be $2(4^2)=32$. Considering the set of four terms containing 13, this sum could only be even if it had two or four odd terms. If it had all four odd terms then it would be $13-7-5-3=-2$, and with two odd terms then its minimum value is $13-7+2-2=6$, so we cannot achieve two sums of 4. The closest we could have to 4 and 4 is 3 and 5, which can be achieved through $13-7-5+2$ and $6-3-2+4$. So the minimum possible value is $3^2+5^2=34 \Rightarrow \boxed{C}$.

See also

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2005 AMC 12B Problems/Problem 21

Problem

A positive integer n has 60 divisors and $7n$ has 80 divisors. What is the greatest integer k such that 7^k divides n?

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ 2 \qquad \mathrm{(D)}\ 3 \qquad \mathrm{(E)}\ 4$

Solution

If n has 60 factors, then n is a product of $2 \times 2 \times 3 \times 5$ powers of (not necessarily distinct) primes. When multiplied by 7, the amount of factors of n increased by $\frac{80}{60} = \frac{4}{3}$, so there are 4 possible powers of 7 in the factorization of $7n$, and 3 possible powers of 7 in the factorization of n, which would be 7^0, 7^1, and 7^2. Therefore the highest power of 7 that could divide n is $2 \Rightarrow \boxed{C}$.

See also

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2005 AMC 12B Problems/Problem 22

Problem

A sequence of complex numbers $z_0, z_1, z_2, \ldots$ is defined by the rule

$$z_{n+1} = \frac{iz_n}{\overline{z_n}},$$

where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose that $|z_0| = 1$ and $z_{2005} = 1$. How many possible values are there for z_0?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ 4 \qquad \mathrm{(D)}\ 2005 \qquad \mathrm{(E)}\ 2^{2005}$

Solution

Since $|z_0| = 1$, let $z_0 = e^{i\theta_0}$, where θ_0 is an argument of z_0. I will prove by induction that $z_n = e^{i\theta_n}$, where $\theta_n = 2^n(\theta_0 + \frac{\pi}{2}) - \frac{\pi}{2}$.

Base Case: trivial

Inductive Step: Suppose the formula is correct for z_k, then

$$z_{k+1} = \frac{iz_k}{\overline{z_k}} = ie^{i\theta_k}e^{i\theta_k} = e^{i(2\theta_k + \pi/2)}$$

Since

$$2\theta_k + \frac{\pi}{2} = 2 \cdot 2^n(\theta_0 + \frac{\pi}{2}) - \pi + \frac{\pi}{2} = 2^{n+1}(\theta_0 + \frac{\pi}{2}) - \frac{\pi}{2} = \theta_{n+1}$$

the formula is proven

$z_{2005} = 1 \Rightarrow \theta_{2005} = 2k\pi$, where k is an integer. Therefore,

$$2^{2005}(\theta_0 + \frac{\pi}{2}) = (2k + \frac{1}{2})\pi$$

$$\theta_0 = \frac{k}{2^{2004}}\pi + \left(\frac{1}{2^{2006}} - \frac{1}{2}\right)\pi$$

The value of θ_0 only matters modulo 2π. Since $\frac{k + 2^{2005}}{2^{2004}}\pi \equiv \frac{k}{2^{2004}}\pi \mod 2\pi$, k only needs to take values from 0 to $2^{2005} - 1$, so the answer is $2^{2005} \Rightarrow \boxed{E}$

See Also

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2005 AMC 12B Problems/Problem 23

Contents

Problem

Let S be the set of ordered triples (x, y, z) of real numbers for which

$$\log_{10}(x+y) = z \text{ and } \log_{10}(x^2+y^2) = z+1.$$

There are real numbers a and b such that for all ordered triples $(x, y.z)$ in S we have $x^3+y^3 = a \cdot 10^{3z} + b \cdot 10^{2z}$. What is the value of $a+b$?

$\textbf{(A)}\ \frac{15}{2} \qquad \textbf{(B)}\ \frac{29}{2} \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ \frac{39}{2} \qquad \textbf{(E)}\ 24$

Solution 1

Let $x+y=s$ and $x^2+y^2=t$. Then, $\log(s)=z$ implies $\log(10s)=z+1=\log(t)$, so $t=10s$. Therefore, $x^3+y^3 = s * \frac{3t-s^2}{2} = s(15s - \frac{s^2}{2})$. Since $s=10^z$, we find that $x^3+y^3 = 15 \times 10^{2z} - (1/2) \times 10^{3z}$. Thus, $a+b = \frac{29}{2} \Rightarrow \boxed{B}$

Solution 2

First, remember that x^3+y^3 factors to $(x+y)(x^2-xy+y^2)$. By the givens, $x+y=10^z$ and $x^2+y^2=10^{z+1}$. These can be used to find xy:

$$(x+y)^2 = 10^{2z}$$

$$x^2+2xy+y^2 = 10^{2z}$$

$$2xy = 10^{2z} - 10^{z+1}$$

$$xy = \frac{10^{2z}-10^{z+1}}{2}$$

Therefore,

$$x^3+y^3 = a \cdot 10^{3z} + b \cdot 10^{2z} = 10^z\left(10^{z+1} - \frac{10^{2z}-10^{z+1}}{2}\right)$$

$$= 10^z\left(10^{z+1} - \frac{10^{2z}-10^{z+1}}{2}\right)$$

$$= 10^{2z+1} - \frac{10^{3z} - 10^{2z+1}}{2}$$

$$= -\frac{1}{2} \cdot 10^{3z} + \frac{3}{2} \cdot 10^{2z+1}$$

$$= -\frac{1}{2} \cdot 10^{3z} + 15 \cdot 10^{2z}.$$

It follows that $a = -\frac{1}{2}$ and $b = 15$, thus $a + b = \frac{29}{2}$.

See Also

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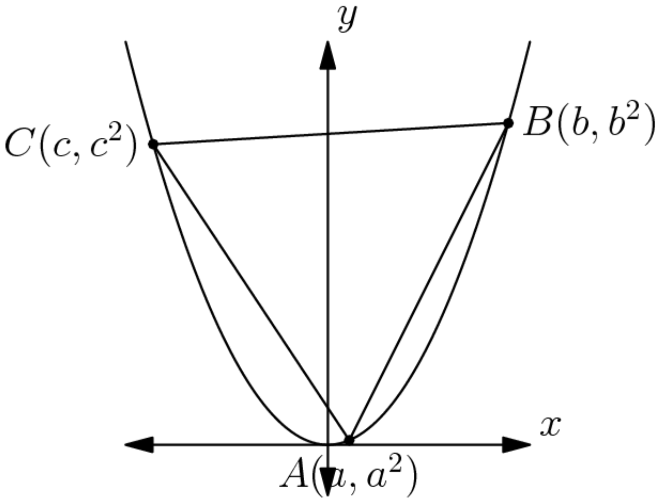
2005 AMC 12B Problems/Problem 24

Problem

All three vertices of an equilateral triangle are on the parabola $y = x^2$, and one of its sides has a slope of 2. The x-coordinates of the three vertices have a sum of m/n, where m and n are relatively prime positive integers. What is the value of $m + n$?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 18$

Solution



Using the slope formula and differences of squares, we find:

$a + b$ = the slope of AB,

$b + c$ = the slope of BC,

$a + c$ = the slope of AC.

So the value that we need to find is the sum of the slopes of the three sides of the triangle divided by 2. Without loss of generality, let AB be the side that has the smallest angle with the positive x-axis. Let J be an arbitrary point with the coordinates $(1, 0)$. Translate the triangle so A is at the origin. Then $tan(BOJ) = 2$. Since the slope of a line is equal to the tangent of the angle formed by the line and the positive x- axis, the answer is $\frac{tan(BOJ) + tan(BOJ + 60) + tan(BOJ - 60)}{2}$.

Using $tan(BOJ) = 2$, and the tangent addition formula, this simplifies to $\frac{3}{11}$, so the answer is $3 + 11 = \boxed{\textbf{(A)}\ 14}$

See also

2005 AMC 12B (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=44&year=2005))	
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2005 AMC 12B Problems/Problem 25

Problem

Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex?

$\mathrm{(A)}\ \frac{5}{256} \qquad\mathrm{(B)}\ \frac{21}{1024} \qquad\mathrm{(C)}\ \frac{11}{512} \qquad\mathrm{(D)}\ \frac{23}{1024} \qquad\mathrm{(E)}\ \frac{3}{128}$

Contents

Solution

Solution 1

We approach this problem by counting the number of ways ants can do their desired migration, and then multiple this number by the probability that each case occurs.

Let the octahedron be $ABCDEF$, with points B,C,D,E coplanar. Then the ant from A and the ant from F must move to plane $BCDE$. Suppose, without loss of generality, that the ant from A moved to point B. Then, we must consider three cases.

- Case 1: Ant from point F moved to point C

On the plane, points B and C are taken. The ant that moves to D can come from either E or C. The ant that moves to E can come from either B or D. Once these two ants are fixed, the other two ants must migrate to the "poles" of the octahedron, points A and F. Thus, there are two degrees of freedom in deciding which ant moves to D, two degrees of freedom in deciding which ant moves to E, and two degrees of freedom in deciding which ant moves to A. Hence, there are $2 \times 2 \times 2=8$ ways the ants can move to different points.

- Case 2: Ant from point F moved to point D

On the plane, points B and D are taken. The ant that moves to C must be from B or D, but the ant that moves to E must also be from B or D. The other two ants, originating from points C and E, must move to the poles. Therefore, there are two degrees of freedom in deciding which ant moves to C and two degrees of freedom in choosing which ant moves to A. Hence, there are $2 \times 2=4$ ways the ants can move to different points.

- Case 3: Ant from point F moved to point E

By symmetry to Case 1, there are 8 ways the ants can move to different points.

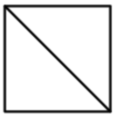
Given a point B, there is a total of $8+4+8=20$ ways the ants can move to different points. We oriented the square so that point B was defined as the point to which the ant from point A moved. Since the ant from point A can actually move to four different points, there is a total of $4\times 20=80$ ways the ants can move to different points.

Each ant acts independently, having four different points to choose from. Hence, each ant has probability $1/4$ of moving to the desired location. Since there are six ants, the probability of each case occuring is $\frac{1}{4^6}=\frac{1}{4096}$. Thus, the desired answer is $\frac{80}{4096}=\boxed{\frac{5}{256}}\Rightarrow \text{(A)}$.

Solution 2

Let $f(n)$ be the number of cycles of length n the can be walked among the vertices of an octahedron. For example, $f(3)$ would represent the number of ways in which an ant could navigate 2 vertices and then return back to the original spot. Since an ant cannot stay still, $f(1)=0$. We also easily see that $f(2)=1, f(3)=2$.

Now consider any four vertices of the octahedron. All four vertices will be connected by edges except for one pair. Let' s think of this as a square with one diagonal (from top left to bottom right).



Suppose an ant moved across this diagonal; then the ant at the other end can only move across the diagonal (which creates 2-cycle, bad) or it can move to another vertex, but then the ant at that vertex must move to the spot of the original ant (which creates 3-cycle, bad). Thus none of the ants can navigate the diagonal and can either shift clockwise or counterclockwise, and so $f(4)=2$.

For $f(6)$, consider an ant at the top of the octahedron. It has four choices. Afterwards, it can either travel directly to the bottom, and then it has 2 ways back up, or it can travel along the sides and then go to the bottom, of which simple counting gives us 6 ways back up. Hence, this totals $4\times(2+6)=32$.

Now, the number of possible ways is given by the sum of all possible cycles,

$$a\cdot f(2)\cdot f(2)\cdot f(2)+b\cdot f(2)\cdot f(4)+c\cdot f(3)\cdot f(3)+d\cdot f(6)$$

where the coefficients represent the number of ways we can configure these cycles. To find a, fix any face, there are 4 adjacent faces to select from to complete the cycle. From the four remaining faces there are only 2 ways to create cycles, hence $a=8$.

To find b, each cycle of 2 faces is distinguished by their common edge, and there are 12 edges, so $b=12$.

To find c, each three-cycle is distinguished by the vertex, and there are 8 edges. However, since the two three-cycles are indistinguishable, $c=8/2=4$.

Clearly $d=1$. Finally,

$$8(1)(1)(1)+12(1)(2)+4(2)(2)+(32)=80$$

Each bug has 4 possibilities to choose from, so the probability is $\frac{80}{4^6}=\frac{5}{256}$.

See also