2011 AMC 10B Problems

Contents

Problem 1

What is

$$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{147}{60} \qquad \textbf{(E)}\ \frac{43}{3}$

Solution

Problem 2

Josanna's test scores to date are $90, 80, 70, 60,$ and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 85 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 95$

Solution

Problem 3

At a store, when a length is reported as x inches that means the length is at least $x-0.5$ inches and at most $x+0.5$ inches. Suppose the dimensions of a rectangular tile are reported as 2 inches by 3 inches. In square inches, what is the minimum area for the rectangle?

$\textbf{(A)}\ 3.75 \qquad \textbf{(B)}\ 4.5 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8.75$

Solution

Problem 4

LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip, it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

$\textbf{(A)}\ \frac{A+B}{2} \qquad \textbf{(B)}\ \frac{A-B}{2} \qquad \textbf{(C)}\ \frac{B-A}{2} \qquad \textbf{(D)}\ B-A \qquad \textbf{(E)}\ A+B$

Solution

Problem 5

In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

$\textbf{(A)}\ 116 \qquad \textbf{(B)}\ 161 \qquad \textbf{(C)}\ 204 \qquad \textbf{(D)}\ 214 \qquad \textbf{(E)}\ 224$

Solution

Problem 6

On Halloween Casper ate $1/3$ of his candies and then gave 2 candies to his brother. The next day he ate $1/3$ of his remaining candies and then gave 4 candies to his sister. On the third day he ate his final 8 candies. How many candies did Casper have at the beginning?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 39 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 66$

Solution

Problem 7

The sum of two angles of a triangle is $6/5$ of a right angle, and one of these two angles is 30° larger than the other. What is the degree measure of the largest angle in the triangle?

$\textbf{(A)}\ 69 \qquad \textbf{(B)}\ 72 \qquad \textbf{(C)}\ 90 \qquad \textbf{(D)}\ 102 \qquad \textbf{(E)}\ 108$

Solution

Problem 8

At a certain beach if it is at least $80^{\circ} F$ and sunny, then the beach will be crowded. On June 10 the beach was not crowded. What can be concluded about the weather conditions on June 10?

$\textbf{(A)}$ The temperature was cooler than 80°F and it was not sunny.

$\textbf{(B)}$ The temperature was cooler than 80°F or it was not sunny.

$\textbf{(C)}$ If the temperature was at least 80°F, then it was sunny.

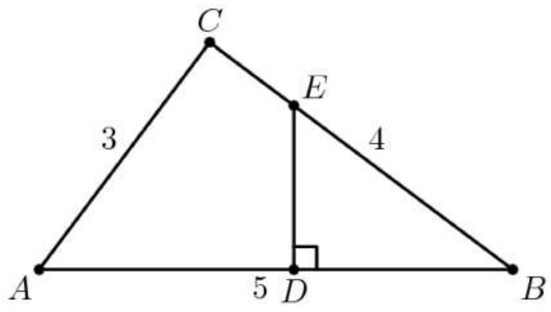
$\textbf{(D)}$ If the temperature was cooler than 80°F, then it was sunny.

$\textbf{(E)}$ If the temperature was cooler than 80°F, then it was not sunny.

Solution

Problem 9

The area of $\triangle EBD$ is one third of the area of $3-4-5$ $\triangle ABC$. Segment DE is perpendicular to segment AB. What is BD?



$\textbf{(A)}\ \frac{4}{3} \qquad \textbf{(B)}\ \sqrt{5} \qquad \textbf{(C)}\ \frac{9}{4} \qquad \textbf{(D)}\ \frac{4\sqrt{3}}{3} \qquad \textbf{(E)}\ \frac{5}{2}$

Solution

Problem 10

Consider the set of numbers $\{1, 10, 10^2, 10^3, \ldots, 10^{10}\}$. The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)} 101$

Solution

Problem 11

There are 52 people in a room. What is the largest value of n such that the statement "At least n people in this room have birthdays falling in the same month" is always true?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 12$

Solution

Problem 12

Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has a width of 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

$\textbf{(A)}\ \frac{\pi}{3} \qquad \textbf{(B)}\ \frac{2\pi}{3} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{4\pi}{3} \qquad \textbf{(E)}\ \frac{5\pi}{3}$

Solution

Problem 13

Two real numbers are selected independently at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{2}{3}$

Solution

Problem 14

A rectangular parking lot has a diagonal of 25 meters and an area of 168 square meters. In meters, what is the perimeter of the parking lot?

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 58 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 68 \qquad \textbf{(E)}\ 70$

Solution

Problem 15

Let $@$ denote the "averaged with" operation: $a@b = (a+b)/2$. Which of the following distributive laws hold for all numbers $x, y,$ and z?

$$\text{I. x @ (y + z) = (x @ y) + (x @ z)}$$

$$\text{II. x + (y @ z) = (x + y) @ (x + z)}$$

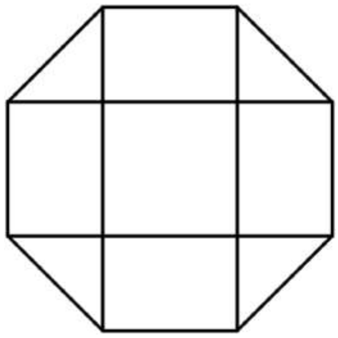
$$\text{III. x @ (y @ z) = (x @ y) @ (x @ z)}$$

$\textbf{(A)}\ \text{I only} \qquad \textbf{(B)}\ \text{II only} \qquad \textbf{(C)}\ \text{III only} \qquad \textbf{(D)}\ \text{I and III only} \qquad \textbf{(E)}\ \text{II and III only}$

Solution

Problem 16

A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is probability that the dart lands within the center square?

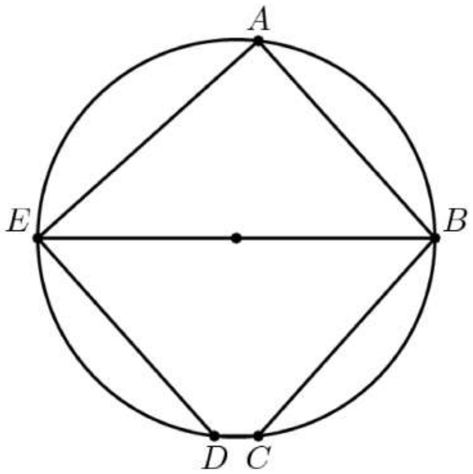


$\textbf{(A)}\ \frac{\sqrt{2} - 1}{2} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{2 - \sqrt{2}}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{4} \qquad \textbf{(E)}\ 2 - \sqrt{2}$

Solution

Problem 17

In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4:5$. What is the degree measure of angle BCD?



$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 125 \qquad \textbf{(C)}\ 130 \qquad \textbf{(D)}\ 135 \qquad \textbf{(E)}\ 140$

Solution

Problem 18

Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

$\textbf{(A) } 15 \qquad \textbf{(B) } 30 \qquad \textbf{(C) } 45 \qquad \textbf{(D) } 60 \qquad \textbf{(E) } 75$

Solution

Problem 19

What is the product of all the roots of the equation

$$\sqrt{5|x|+8} = \sqrt{x^2-16}.$$

$\textbf{(A) } -64 \qquad \textbf{(B) } -24 \qquad \textbf{(C) } -9 \qquad \textbf{(D) } 24 \qquad \textbf{(E) } 576$

Solution

Problem 20

Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

$\textbf{(A) } \frac{\sqrt{3}}{3} \qquad \textbf{(B) } \frac{\sqrt{3}}{2} \qquad \textbf{(C) } \frac{2\sqrt{3}}{3} \qquad \textbf{(D) } 1+\frac{\sqrt{3}}{3} \qquad \textbf{(E) } 2$

Solution

Problem 21

Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

$\textbf{(A) } 16 \qquad \textbf{(B) } 31 \qquad \textbf{(C) } 48 \qquad \textbf{(D) } 62 \qquad \textbf{(E) } 93$

Solution

Problem 22

A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

$\textbf{(A) } 5\sqrt{2}-7 \qquad \textbf{(B) } 7-4\sqrt{3} \qquad \textbf{(C) } \frac{2\sqrt{2}}{27} \qquad \textbf{(D) } \frac{\sqrt{2}}{9} \qquad \textbf{(E) } \frac{\sqrt{3}}{9}$

Solution

Problem 23

What is the hundreds digit of 2011^{2011}?

$\textbf{(A) } 1 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 5 \qquad \textbf{(D) } 6 \qquad \textbf{(E) } 9$

Solution

Problem 24

A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \le 100$ for all m such that $1/2 < m < a$. What is the maximum possible value of a?

$\textbf{(A) } \frac{51}{101} \qquad \textbf{(B) } \frac{50}{99} \qquad \textbf{(C) } \frac{51}{100} \qquad \textbf{(D) } \frac{52}{101} \qquad \textbf{(E) } \frac{13}{25}$

Solution

Problem 25

Let T_1 be a triangle with sides $2011, 2012,$ and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and $D, E,$ and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC and $AC,$ respectively, then T_{n+1} is a triangle with side lengths $AD, BE,$ and $CF,$ if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

$\textbf{(A)}\ \frac{1509}{8} \qquad \textbf{(B)}\ \frac{1509}{32} \qquad \textbf{(C)}\ \frac{1509}{64} \qquad \textbf{(D)}\ \frac{1509}{128} \qquad \textbf{(E)}\ \frac{1509}{256}$

Solution

See also

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