# 2015 AMC12A

## Problem 1

What is the value of

$$(2^0 - 1 + 5^2 - 0)^{-1} \times 5?$$

下列算式的值是多少

$$(2^0 - 1 + 5^2 - 0)^{-1} \times 5?$$

$\textbf{(A)}\ -125 \qquad \textbf{(B)}\ -120 \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{5}{24} \qquad \textbf{(E)}\ 25$

## Problem 2

Two of the three sides of a triangle are 20 and 15. Which of the following numbers is not a possible perimeter of the triangle?

一个三角形的其中两条边长度是 20 和 15，下面哪个数字不可能成为三角形的周长?

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 57 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 67 \qquad \textbf{(E)}\ 72$

## Problem 3

Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

Patrick 先生给 15 个学生上数学课，他正批改考试试卷，发现当他把除了 Payton 外的所有其他学生的试卷都改好后，班级的平均分是 80，当他批改好 Payton 的试试卷后，班级的平均分变成了 81 分，问 Payton 这次考试得分是多少?

$\textbf{(A)}\ 81 \qquad \textbf{(B)}\ 85 \qquad \textbf{(C)}\ 91 \qquad \textbf{(D)}\ 94 \qquad \textbf{(E)}\ 95$

## Problem 4

The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

两个正数之和是它们之差的 5 倍，那么大的数比小的数的比值是多少？

$\textbf{(A)}\ \frac{5}{4} \qquad \textbf{(B)}\ \frac{3}{2} \qquad \textbf{(C)}\ \frac{9}{5} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{5}{2}$

## Problem 5

Amelia needs to estimate the quantity $\frac{a}{b}-c$, where $a, b,$ and $c$ are large positive integers. She rounds each of the integers so that the calculation will be easier to do mentally. In which of these situations will her answer necessarily be greater than the exact value of $\frac{a}{b}-c$?

Amelia 需要估算$\frac{a}{b}-c$的值，其中 *a*，*b* 和 *C* 都是很大的正整数，她需要对每个整数进行舍位（变小）或者进位（变大），这样算就更方便。在下面哪种情况她的结果会比$\frac{a}{b}-c$的准确值大？

（A） She rounds all three numbers.up. ｜ 她把三个数字都向上进位。
（B） She rounds *a* and *b* up, and she rounds *c* down. | 她把 *a* 和 *b* 向上进位，把 *c* 向下舍位。
（C） She rounds *a* and *c* up, and she rounds *b* down. | 她把 *a* 和 *c* 向上进位，把 *b* 向下舍位。
（D） She rounds *a* up, and she rounds *b* and *c* down. | 她把 *a* 向上进位，把 *b* 和 *c* 向下舍位。
（E） She rounds *c* up, and she rounds *a* and *b* down. | 她把 *c* 向上进位，把 *a* 和 *b* 向下舍位。

## Problem 6

Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2:1$?

两年前 Pete 的年龄是他表妹年龄的 3 倍，从那时起再往前推两年，Pete 的年龄是 Claire 年龄的 4 倍，多少年后他们年龄的比值将是$2:1$?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

## Problem 7

Two right circular cylinders have the same volume. The radius of the second cylinder is $10\%$ more than the radius of the first. What is the relationship between the heights of the two cylinders?

两个正圆柱体的体积相同，第二个圆柱体的半径比第一个的半径多 10％，问这两个圆柱体的高有什么关系？

（A）The second height is 10％ less than the first｜ 第二个的高比第一个高少 10％

（B）The first height is 10％ more than the second｜第一个的高比第二个高多 10％

（C）The second height is 21％ less than the first｜第二个的高比第一个高少 21％

（D）The first height is 21％ more than the second｜ 第一个的高比第二个高多 21％

（E）The second height is 80％ of the first｜第二个的高是第一个高的 80％

## Problem 8

The ratio of the length to the width of a rectangle is $4 : 3$. If the rectangle has diagonal of length $d$, then the area may be expressed as $kd^2$ for some constant $k$. What is $k$?

一个长方形的长和宽之比为$4 : 3$，如果长方形的对角线的长度是 $d$，那么它的面积可以写成 $kd^2$，这里 $k$ 是常数，那么 $k$ 是多少?

$\textbf{(A)}\ \frac{2}{7}\qquad\textbf{(B)}\ \frac{3}{7}\qquad\textbf{(C)}\ \frac{12}{25}\qquad\textbf{(D)}\ \frac{16}{25}\qquad\textbf{(E)}\ \frac{3}{4}$

## Problem 9

A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

一个盒子里装有 2 颗红色玻璃球，2 颗绿色玻璃球，2 颗黄色玻璃球。Carol 从盒子里随机挑选 2 颗玻璃球，然后 Claudia 从剩下的玻璃球中随机挑选 2 颗，最后 Cheryl 拿走最后剩下的两颗。问 Cheryl 拿走的是两颗颜色一样的玻璃球的概率是多少？

$\textbf{(A)}\ \frac{1}{10}\qquad\textbf{(B)}\ \frac{1}{6}\qquad\textbf{(C)}\ \frac{1}{5}\qquad\textbf{(D)}\ \frac{1}{3}\qquad\textbf{(E)}\ \frac{1}{2}$

## Problem 10

Integers $x$ and $y$ with $x > y > 0$ satisfy $x + y + xy = 80$. What is $x$?

整数 x 和 y 满足$x + y + xy = 80$且$x > y > 0$，则 x 是多少？

**(A)** 8 **(B)** 10 **(C)** 15 **(D)** 18 **(E)** 26

## Problem 11

On a sheet of paper, Isabella draws a circle of radius $2$, a circle of radius $3$, and all possible lines simultaneously tangent to both circles. Isabella notices that she has drawn exactly $k \geq 0$ lines. How many different values of $k$ are possible?

Isabella 在一张纸上画了一个半径为 2 的圆，一个半径为 3 的圆，以及这两个圆所有可能的公切线。Isabella 发现她恰好画了$k \geq 0$根直线。那么 k 有多少种不同的可能取值？

**(A)** 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

## Problem 12

The parabolas $y = ax^2 - 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area $12$. What is $a + b$?

抛物线$y = ax^2 - 2$和$y = 4 - bx^2$与坐标轴恰好相交于 4 个不同的点，且这 4 个点是一个面积为 12 的筝形的 4 个顶点。求$a + b$是多少?

**(A)** 1 **(B)** 1.5 **(C)** 2 **(D)** 2.5 **(E)** 3

## Problem 13

A league with 12 teams holds a round-robin tournament, with each team playing every other team exactly once. Games either end with one team victorious or else end in a draw. A team scores 2 points for every game it wins and 1 point for every game it draws. Which of the following is NOT a true statement about the list of 12 scores?

12 支队伍组成的联盟举行了一场循环赛，每支队伍都要和其他每支队伍打一场比赛。比赛的结果要么一胜一负，要么平局。每支队伍若胜一局则得 2 分，若平局则得 1 分。下面关于这 12 支队伍获得的 12 个分数的说法，哪个是错误的？

（A）There must be an even number of odd scores.丨一定有偶数个分数是奇数。

（B）There must be an even number of even scores.丨一定有偶数个分数是偶数。

（C）There cannot be two scores of 0.丨不可能有 2 个 0 分。

（D）The sum of the scores must be at least 100.丨所有分数之和一定至少是 100。

（E）The highest score must be at least 12.丨最高分一定至少是 12 分。

## Problem 14

What is the value of $a$ for which $\frac{1}{\log_2 a}+\frac{1}{\log_3 a}+\frac{1}{\log_4 a}=1$?

满足方程 $\frac{1}{\log_2 a}+\frac{1}{\log_3 a}+\frac{1}{\log_4 a}=1$ 的 $a$ 的值是多少？

**(A)** 9 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36

## Problem 15

What is the minimum number of digits to the right of the decimal point needed to express the fraction $\frac{123456789}{2^{26}\cdot 5^4}$ as a decimal?

小数点的右边至少需要有多少位才能把分数 $\frac{123456789}{2^{26}\cdot 5^4}$ 表达成小数？

**(A)** 4 **(B)** 22 **(C)** 26 **(D)** 30 **(E)** 104

## Problem 16

Tetrahedron $ABCD$ has $AB = 5, AC = 3, BC = 4, BD = 4, AD = 3,$ and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

四面体 *ABCD* 有 *AB*＝5，*AC*＝3，*BC*＝4，*BD*＝4，*AD*＝3，$CD = \frac{12}{5}\sqrt{2}$，那么这个四面体的体积为多少？

**(A)** $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

## Problem 17

Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

8 个人坐在一张圆桌旁，每个人都手持一枚硬币。8 个人一起扔硬币，扔到正面朝上的就站起来，扔到反面朝上的则仍然坐着，没有相邻的两个人同时站着的概率是多少？

**(A)** $\frac{47}{256}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{49}{256}$ **(D)** $\frac{25}{128}$ **(E)** $\frac{51}{256}$

## Problem 18

The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of $a$?

函数$f(x) = x^2 - ax + 2a$的零点都是整数。问 *a* 的所有可能值之和是多少？

**(A)** 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

## Problem 19

For some positive integers $p$, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter $p$, right angles at $B$ and $C$, $AB=2$, and $CD=AD$. How many different values of $p<2015$ are possible?

对于某些正整数 *p*，存在一个四边形 *ABCD*，使得它的边长都是正整数，周长是 *p*，直角顶点为 *B* 和 *C*，*AB*=2， *CD*=*AD*. 若$p<2015$，那么满足条件的不同的 *p* 值有多少个？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 61 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 63$

## Problem 20

Isosceles triangles $T$ and $T'$ are not congruent but have the same area and the same perimeter. The sides of $T$ have lengths of $5, 5,$ and $8$, while those of $T'$ have lengths of $a, a,$ and $b$. Which of the following numbers is closest to $b$?

等腰三角形 *T* 和 *T*′ 不是全等三角形，但是面积和周长都相等。T 的边长是 5，5，8，而 *T*′ 的边长是 *a*，*a*，*b*，下面哪个数字最接近 *b* 的值?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

## Problem 21

A circle of radius $r$ passes through both foci of, and exactly four points on, the ellipse with equation $x^2+16y^2=16$. The set of all possible values of $r$ is an interval $[a,b)$. What is $a+b$?

半径为 *r* 的圆通过方程为$x^2+16y^2=16$的椭圆的两个焦点，与椭圆恰好交于 4 个点，所有可能的 *r* 的取值是一个区间$[a,b)$，求$a+b$是多少？

$\textbf{(A)}\ 5\sqrt{2}+4 \qquad \textbf{(B)}\ \sqrt{17}+7 \qquad \textbf{(C)}\ 6\sqrt{2}+3 \qquad \textbf{(D)}\ \sqrt{15}+8 \qquad \textbf{(E)}\ 12$

## Problem 22

For each positive integer $n$, let $S(n)$ be the number of sequences of length $n$ consisting solely of the letters $A$ and $B$, with no more than three $A$s in a row and no more than three $B$s in a row. What is the remainder when $S(2015)$ is divided by 12?

*n* 是一个正整数， $S(n)$表示长度为 *n* 的满足以下要求的数列的个数：数列只由字母 *A* 和 *B* 组成，并且不允许有超过 3 个连续的 *A*，也不能有超过 3 个连续的 *B*。问$S(2015)$除以 12 所得的余数是多少？

**(A)** 0 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

## Problem 23

Let $S$ be a square of side length 1. Two points are chosen independently at random on the sides of $S$. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where $a, b,$ and $c$ are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

*S* 是边长为 1 的正方形，从 *S* 的边上随机选择 2 个点，这两个点的直线距离至少是$\frac{1}{2}$的概率是$\frac{a-b\pi}{c}$，其中 *a*，*b*，*c* 都是正整数，且 $\gcd(a,b,c)=1$，问$a+b+c$是多少？

**(A)** 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

## Problem 24

Rational numbers $a$ and $b$ are chosen at random among all rational numbers in the interval $[0, 2)$ that can be written as fractions $\frac{n}{d}$ where $n$ and $d$ are integers with $1 \leq d \leq 5$. What is the probability that

$$(\cos(a\pi) + i\sin(b\pi))^4$$

is a real number?

有理数 $a$ 和 $b$ 是从区间［0，2）内随机选择的两个有理数，均可以写成$\frac{n}{d}$的形式，其中 $n$ 和 $d$ 都是整数，且$1 \leq d \leq 5$，那么$(\cos(a\pi) + i\sin(b\pi))^4$是一个实数的概率是多少？

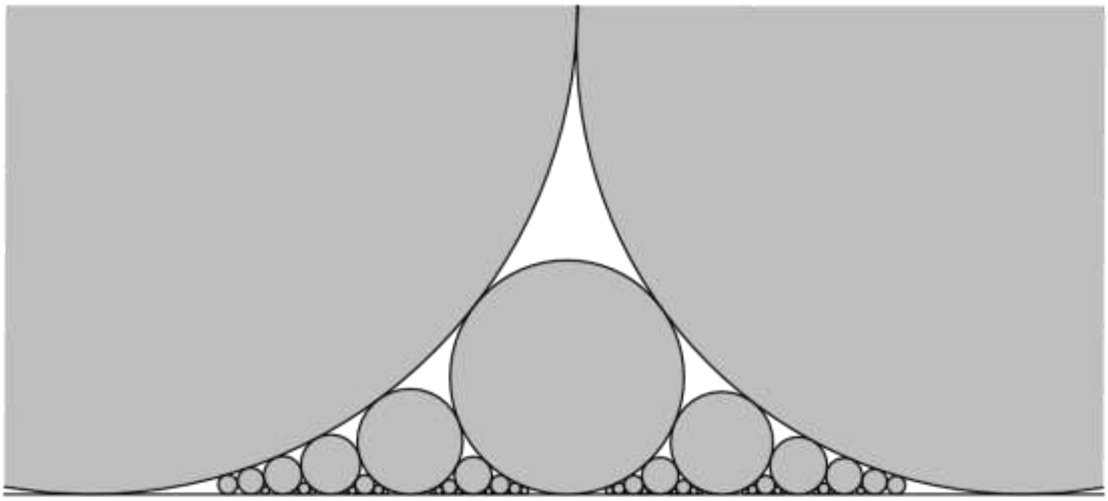
$\textbf{(A)}\ \frac{3}{50} \qquad \textbf{(B)}\ \frac{4}{25} \qquad \textbf{(C)}\ \frac{41}{200} \qquad \textbf{(D)}\ \frac{6}{25} \qquad \textbf{(E)}\ \frac{13}{50}$

## Problem 25

A collection of circles in the upper half-plane, all tangent to the $x$-axis, is constructed in layers as follows. Layer $L_0$ consists of two circles of radii $70^2$ and $73^2$ that are externally tangent. For $k \geq 1$, the circles in $\bigcup_{j=0}^{k-1} L_j$ are ordered according to their points of tangency with the $x$-axis. For every pair of consecutive circles in this order, a new circle is constructed externally tangent to each of the two circles in the pair. Layer $L_k$ consists of the $2^{k-1}$ circles constructed in this way. Let $S = \bigcup_{j=0}^{6} L_j$, and for every circle $C$ denote by $r(C)$ its radius. What is

$$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}?$$

在上半平面内，以如下方式画出若干层和 $x$ 轴相切的圆，第$L_0$层由两个半径分别$70^2$和$73^2$且互相外切的圆组成。当$k \geq 1$，集合$\bigcup_{j=0}^{k-1} L_j$内的所有的圆依照它们和 $x$ 轴的切点进行排序。对于这种排序下的任意连续 2 个圆，画出的新的圆和这 2 个圆都外切。第$L_k$层是由$2^{k-1}$个以这种方式画出来的圆组成。令$S = \bigcup_{j=0}^{6} L_j$，且用$r(C)$表示圆 $C$ 的半径，$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}$是多少？



**(A)** $\frac{286}{35}$ **(B)** $\frac{583}{70}$ **(C)** $\frac{715}{73}$ **(D)** $\frac{143}{14}$ **(E)** $\frac{1573}{146}$

# 2015 AMC 12A Answer Key

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| C | E | E | B | D | B | D | C | C | E | D | B | E |
| **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | |
| D | C | C | A | C | B | A | D | D | A | D | D | |