2002 AMC 12B Problems

Contents

Problem 1

The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \ldots, 999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

$\text{(A)}\ 0 \qquad\text{(B)}\ 2 \qquad\text{(C)}\ 4 \qquad\text{(D)}\ 6 \qquad\text{(E)}\ 8$

Solution

Problem 2

What is the value of

$$(3x - 2)(4x + 1) - (3x - 2)4x + 1$$

when $x = 4$?

$\text{(A)}\ 0 \qquad\text{(B)}\ 1 \qquad\text{(C)}\ 10 \qquad\text{(D)}\ 11 \qquad\text{(E)}\ 12$

Solution

Problem 3

For how many positive integers n is $n^2 - 3n + 2$ a prime number?

$\text{(A)}\ \text{none} \qquad\text{(B)}\ \text{one} \qquad\text{(C)}\ \text{two} \qquad\text{(D)}\ \text{more than two, but finitely many} \qquad\text{(E)}\ \text{infinitely many}$

Solution

Problem 4

Let n be a positive integer such that $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is an integer. Which of the following statements is not true:

$\text{(A)}\ 2\ \text{divides}\ n \qquad \text{(B)}\ 3\ \text{divides}\ n \qquad \text{(C)}\ 6\ \text{divides}\ n \qquad \text{(D)}\ 7\ \text{divides}\ n \qquad \text{(E)}\ n > 84$

Solution

Problem 5

Let $v, w, x, y,$ and z be the degree measures of the five angles of a pentagon. Suppose that $v < w < x < y < z$ and $v, w, x, y,$ and z form an arithmetic sequence. Find the value of x.

$\text{(A)}\ 72 \qquad \text{(B)}\ 84 \qquad \text{(C)}\ 90 \qquad \text{(D)}\ 108 \qquad \text{(E)}\ 120$

Solution

Problem 6

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. Then the pair (a, b) is

$\text{(A)}\ (-2, 1) \qquad \text{(B)}\ (-1, 2) \qquad \text{(C)}\ (1, -2) \qquad \text{(D)}\ (2, -1) \qquad \text{(E)}\ (4, 4)$

Solution

Problem 7

The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

$\text{(A)}\ 50 \qquad \text{(B)}\ 77 \qquad \text{(C)}\ 110 \qquad \text{(D)}\ 149 \qquad \text{(E)}\ 194$

Solution

Problem 8

Suppose July of year N has five Mondays. Which of the following must occur five times in August of year N? (Note: Both months have 31 days.)

$\text{(A)}\ \text{Monday} \qquad \text{(B)}\ \text{Tuesday} \qquad \text{(C)}\ \text{Wednesday} \qquad \text{(D)}\ \text{Thursday} \qquad \text{(E)}\ \text{Friday}$

Solution

Problem 9

If a, b, c, d are positive real numbers such that a, b, c, d form an increasing arithmetic sequence and a, b, d form a geometric sequence, then $\frac{a}{d}$ is

$\text{(A)}\ \frac{1}{12} \qquad \text{(B)}\ \frac{1}{6} \qquad \text{(C)}\ \frac{1}{4} \qquad \text{(D)}\ \frac{1}{3} \qquad \text{(E)}\ \frac{1}{2}$

Solution

Problem 10

How many different integers can be expressed as the sum of three distinct members of the set $\{1, 4, 7, 10, 13, 16, 19\}$?

$\text{(A)}\ 13 \qquad \text{(B)}\ 16 \qquad \text{(C)}\ 24 \qquad \text{(D)}\ 30 \qquad \text{(E)}\ 35$

Solution

Problem 11

The positive integers $A, B, A-B,$ and $A+B$ are all prime numbers. The sum of these four primes is

$\text{(A)}\ \text{even} \qquad \text{(B)}\ \text{divisible by}\ 3 \qquad \text{(C)}\ \text{divisible by}\ 5 \qquad \text{(D)}\ \text{divisible by}\ 7 \qquad \text{(E)}\ \text{prime}$

Solution

Problem 12

For how many integers n is $\dfrac{n}{20-n}$ the square of an integer?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 10$

Solution

Problem 13

The sum of 18 consecutive positive integers is a perfect square. The smallest possible value of this sum is

$\textbf{(A)}\ 169 \qquad \textbf{(B)}\ 225 \qquad \textbf{(C)}\ 289 \qquad \textbf{(D)}\ 361 \qquad \textbf{(E)}\ 441$

Solution

Problem 14

Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 16$

Solution

Problem 15

How many four-digit numbers N have the property that the three-digit number obtained by removing the leftmost digit is one ninth of N?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Solution

Problem 16

Juan rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

$\textbf{(A)}\ \frac{1}{12} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{7}{12} \qquad \textbf{(E)}\ \frac{2}{3}$

Solution

Problem 17

Andy’s lawn has twice as much area as Beth’s lawn and three times as much area as Carlos’ lawn. Carlos’ lawn mower cuts half as fast as Beth’s mower and one third as fast as Andy’s mower. If they all start to mow their lawns at the same time, who will finish first?

$\textbf{(A)}\ \text{Andy} \qquad \textbf{(B)}\ \text{Beth} \qquad \textbf{(C)}\ \text{Carlos} \qquad \textbf{(D)}\ \text{Andy and Carlos tie for first.} \qquad \textbf{(E)}\ \text{All three tie.}$

Solution

Problem 18

A point P is randomly selected from the rectangular region with vertices $(0,0),(2,0),(2,1),(0,1)$. What is the probability that P is closer to the origin than it is to the point $(3,1)$?

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{4}{5} \qquad \textbf{(E)}\ 1$

Solution

Problem 19

If $a, b,$ and c are positive real numbers such that $a(b+c) = 152, b(c+a) = 162,$ and $c(a+b) = 170$, then abc is

$\text{(A)}\ 672 \qquad \text{(B)}\ 688 \qquad \text{(C)}\ 704 \qquad \text{(D)}\ 720 \qquad \text{(E)}\ 750$

Solution

Problem 20

Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

$\text{(A)}\ 24 \qquad \text{(B)}\ 26 \qquad \text{(C)}\ 28 \qquad \text{(D)}\ 30 \qquad \text{(E)}\ 32$

Solution

Problem 21

For all positive integers n less than 2002, let

$$a_n = \left\{ \begin{array}{lr} 11, & \text{if } n \text{ is divisible by } 13 \text{ and } 14;\\ 13, & \text{if } n \text{ is divisible by } 14 \text{ and } 11;\\ 14, & \text{if } n \text{ is divisible by } 11 \text{ and } 13;\\ 0, & \text{otherwise.} \end{array} \right.$$

Calculate $\displaystyle\sum_{n=1}^{2001} a_n$.

$\text{(A)}\ 448 \qquad \text{(B)}\ 486 \qquad \text{(C)}\ 1560 \qquad \text{(D)}\ 2001 \qquad \text{(E)}\ 2002$

Solution

Problem 22

For all integers n greater than 1, define $a_n = \dfrac{1}{\log_n 2002}$. Let $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $b - c$ equals

$\text{(A)}\ -2 \qquad \text{(B)}\ -1 \qquad \text{(C)}\ \frac{1}{2002} \qquad \text{(D)}\ \frac{1}{1001} \qquad \text{(E)}\ \frac{1}{2}$

Solution

Problem 23

In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side $\overline{BC}$ and the median from A to $\overline{BC}$ have the same length. What is BC?

$\text{(A)}\ \frac{1+\sqrt{2}}{2} \qquad \text{(B)}\ \frac{1+\sqrt{3}}{2} \qquad \text{(C)}\ \sqrt{2} \qquad \text{(D)}\ \frac{3}{2} \qquad \text{(E)}\ \sqrt{3}$

Solution

Problem 24

A convex quadrilateral $ABCD$ with area 2002 contains a point P in its interior such that $PA = 24, PB = 32, PC = 28, PD = 45$. Find the perimeter of $ABCD$.

$\text{(A)}\ 4\sqrt{2002} \qquad \text{(B)}\ 2\sqrt{8465} \qquad \text{(C)}\ 2(48+\sqrt{2002}) \qquad \text{(D)}\ 2\sqrt{8633} \qquad \text{(E)}\ 4(36+\sqrt{113})$

Solution

Problem 25

Let $f(x) = x^2 + 6x + 1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x) + f(y) \leq 0 \qquad \text{and} \qquad f(x) - f(y) \leq 0$$

The area of R is closest to $\text{(A) } 21 \qquad \text{(B) } 22 \qquad \text{(C) } 23 \qquad \text{(D) } 24 \qquad \text{(E) } 25$

Solution

See also

- AMC 12
- AMC 12 Problems and Solutions
- 2002 AMC 12A
- Mathematics competition resources

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