The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7 Solutions

1. The value of $6 \times 2 - 3$ is
   (A) 9 (B) -6 (C) 12 (D) 15 (E) 10

Source: 2008 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: A

Solution:

Calculating, $6 \times 2 - 3 = 12 - 3 = 9$.

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2. The value of $1 + 0.01 + 0.0001$ is
   (A) 1.0011 (B) 1.011 (C) 1.1001 (D) 1.101 (E) 1.0101

Source: 2008 Gauss Grade 7 #2

Primary Topics: Number Sense

Secondary Topics: Decimals

Answer: E

Solution:

Calculating, $1 + 0.01 + 0.0001 = 1.01 + 0.0001 = 1.0100 + 0.0001 = 1.0101$.

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3. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ is equal to
   (A) 1 (B) $\frac{1}{64}$ (C) $\frac{3}{14}$ (D) $\frac{7}{8}$ (E) $\frac{3}{8}$

Source: 2008 Gauss Grade 7 #3

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: D

Solution:

Using a common denominator of 8, we have $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$.

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4. A regular polygon has perimeter 108 cm and each side has length 12 cm. How many sides does this polygon have?
   (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Source: 2008 Gauss Grade 7 #4

Primary Topics: Geometry and Measurement

Secondary Topics: Polygons | Perimeter

Answer: D

Solution:

Since the polygon has perimeter 108 cm and each side has length 12 cm, then the polygon has $108 \div 12 = 9$ sides.

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5. The smallest number in the set $\{3.2, 2.3, 3, 2.23, 3.22\}$ is
   (A) 3.2 (B) 2.3 (C) 3 (D) 2.23 (E) 3.22

Source: 2008 Gauss Grade 7 #5

Primary Topics: Number Sense

Secondary Topics: Decimals

Answer: D

Solution:

In the set, three of the numbers are greater than or equal to 3, and two of the numbers are less than 3.
The smallest number must be one of the numbers that is less than 3, that is, 2.3 or 2.23.
Of these two numbers, 2.23 is the smallest, so is the smallest number in the set.

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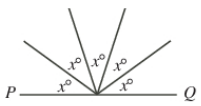
6. If PQ is a straight line, then the value of x is



   (A) 36 (B) 72 (C) 18 (D) 20 (E) 45

Source: 2008 Gauss Grade 7 #6

Primary Topics: Geometry and Measurement

Secondary Topics: Angles

Answer: A

Solution:

Since PQ is a straight line, then $x^\circ + x^\circ + x^\circ + x^\circ + x^\circ = 180^\circ$ or $5x = 180$ or $x = 36$.

7. Which of the following is a prime number?
(A) 20 (B) 21 (C) 23 (D) 25 (E) 27

Source: 2008 Gauss Grade 7 #7

Primary Topics: Number Sense

Secondary Topics: Prime Numbers

Answer: C

Solution:

20 is not a prime number, since it is divisible by 2.
21 is not a prime number, since it is divisible by 3.
25 is not a prime number, since it is divisible by 5.
27 is not a prime number, since it is divisible by 3.
23 is a prime number, since its only positive divisors are 1 and 23.

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8. Kayla went for a walk every day last week. Each day, she walked half as far as she did the day before. If she walked 8 kilometres on Monday last week, how many kilometres did she walk on Friday last week?
(A) 0.25 (B) 4 (C) 1 (D) 2 (E) 0.5

Source: 2008 Gauss Grade 7 #8

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Patterning/Sequences/Series

Answer: E

Solution:

Kayla walked 8 km on Monday.
On Tuesday, she walked $8 \div 2 = 4$ km.
On Wednesday, she walked $4 \div 2 = 2$ km.
On Thursday, she walked $2 \div 2 = 1$ km.
On Friday, she walked $1 \div 2 = 0.5$ km.

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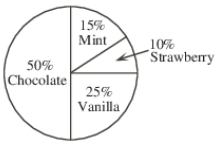
9. The circle graph shows the favourite ice cream flavours of those surveyed. What fraction of people surveyed selected either chocolate or strawberry as their favourite flavour of ice cream?



(A) $\frac{3}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{5}{8}$

Source: 2008 Gauss Grade 7 #9

Primary Topics: Data Analysis

Secondary Topics: Graphs | Fractions/Ratios

Answer: A

Solution:

Since 50% selected chocolate and 10% selected strawberry as their favourite flavour, then overall 50 chose chocolate or strawberry as their favourite flavour.
Now 60, so $\frac{3}{5}$ of the people surveyed selected chocolate or strawberry as their favourite flavour.

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10. Max sold glasses of lemonade for 25 cents each. He sold 41 glasses on Saturday and 53 glasses on Sunday. What were his total sales for these two days?
(A) \$23.50 (B) \$10.25 (C) \$13.25 (D) \$21.50 (E) \$24.25

Source: 2008 Gauss Grade 7 #10

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: A

Solution:

Since Max sold 41 glasses of lemonade on Saturday and 53 on Sunday, he sold $41 + 53 = 94$ glasses in total.
Since he charged 25 cents for each glass, then his total sales were $94 \times \$0.25 = \$23.50$.

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11. Chris bought two hockey sticks at the same price. He also bought a helmet for \$25. If Chris spent \$68 in total, how much did one hockey stick cost?
(A) \$9.00 (B) \$18.00 (C) \$21.50 (D) \$43.00 (E) \$41.50

Source: 2008 Gauss Grade 7 #11

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: C

Solution:

Since Chris spent \$68 in total and \$25 on the helmet, then he spent $\$68 - \$25 = \$43$ on the two hockey sticks.
Since the two sticks each cost the same amount, then this cost was $\$43 \div 2 = \$21.50$.

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12. In the chart, each number below the top row is the positive difference of the two numbers to the right and left in the row immediately above it. What is the value of x?

8	9	17	6	4	
	1	8	_	2	
		7	_	_	
			_	_	
			x		

(A) 1 (B) 2 (C) 3 (D) 4 (E) 0

Source: 2008 Gauss Grade 7 #12

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: B

Solution:

The number below and between 17 and 6 is $17 - 6 = 11$.
The number below and between 8 and 11 is $11 - 8 = 3$.
The number below and between 11 and 2 is $11 - 2 = 9$.
The number below and between 7 and 3 is $7 - 3 = 4$.
The number below and between 3 and 9 is $9 - 3 = 6$.

8	9	17	6	4
	1	8	11	2
		7	3	9
			4	6
			x	

Therefore, $x = 6 - 4 = 2$.

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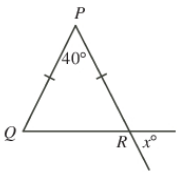
13. In the diagram, $\triangle PQR$ is isosceles. The value of x is



(A) 40 (B) 70 (C) 60 (D) 30 (E) 110

Source: 2008 Gauss Grade 7 #13

Primary Topics: Geometry and Measurement

Secondary Topics: Angles | Triangles

Answer: B

Solution:

Since $PQ = PR$, then $\angle PQR = \angle PRQ$.
Since the angles in a triangle add up to 180°, then $40° + \angle PQR + \angle PRQ = 180°$,
so $\angle PQR + \angle PRQ = 140°$.
Since $\angle PQR = \angle PRQ$, then $\angle PQR = \angle PRQ = 70°$.
Since the angle labelled as $x°$ is opposite $\angle PRQ$, then $x° = \angle PRQ = 70°$, so $x = 70$.

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14. Wesley is 15 and his sister Breenah is 7. The sum of their ages is 22. In how many years will the sum of their ages be double what it is now?
(A) 7 (B) 8 (C) 15 (D) 14 (E) 11

Source: 2008 Gauss Grade 7 #14

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: E

Solution:

The sum of Wesley's and Breenah's ages is 22.
After each year, each of their ages increases by 1, so the sum of their ages increases by 2.
For the sum to increase from 22 to $2 \times 22 = 44$, the sum must increase by 22, which will take $22 \div 2 = 11$ years.

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15. Using two transformations, the letter R is changed as shown: R → Я → ʁ.
Using the same two transformations, the letter L is changed as shown: L → ⅂ → Γ.
Using the same two transformations, the letter G is changed to
(A) G (B) Ↄ (C) Ɔ (D) Ɔ (E) Ↄ

Source: 2008 Gauss Grade 7 #15

Primary Topics: Geometry and Measurement

Secondary Topics: Transformations

Answer: D

Solution:

The first transformation is a 180° rotation of the letter, which gives G → Ɔ.
The second transformation is a reflection across a vertical axis, which gives G → Ɔ.

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16. In the diagram, each small square in the grid is the same size. What percent of the grid is shaded?

(A) 84 (B) 80 (C) 90 (D) 75 (E) 66

Source: 2008 Gauss Grade 7 #16

Primary Topics: Geometry and Measurement

Secondary Topics: Percentages | Area

Answer: D

Solution:

In the diagram, the length of one side of the large square is equal to eight side lengths of the smaller squares, so the large square consists of $8 \times 8 = 64$ small squares.
Of these 64 small squares, 48 are shaded. (We can obtain this number by counting the 48-shaded squares or by counting the 16 unshaded squares.)
As a percentage, this fraction equals $\frac{48}{64} \times 100\% = \frac{3}{4} \times 100\% = 75\%$.

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17. The length of a rectangle is 6 more than twice its width. If the perimeter of the rectangle is 120, what is its width?
(A) 8 (B) 18 (C) 27 (D) 38 (E) 22

Source: 2008 Gauss Grade 7 #17

Primary Topics: Geometry and Measurement

Secondary Topics: Perimeter | Equations Solving

Answer: B

Solution:

Solution 1
Since the perimeter of a rectangle equals twice the length plus twice the width, then the length plus the width equals $120 \div 2 = 60$.
Since the length equals twice the width plus 6, then twice the width plus the width equals $60 - 6 = 54$.
In other words, three times the width equals 54, so the width equals $54 \div 3 = 18$.

Solution 2
Let the width of the rectangle be w.
Then the length of the rectangle is $2w + 6$.
Since the perimeter is 120, then

$$2w + 2(2w + 6) = 1202w + 4w + 12 = 1206w = 108w = 18$$

so the width is 18.

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18. Rishi got the following marks on four math tests: 71, 77, 80, and 87. He will write one more math test. Each test is worth the same amount and all marks are between 0 and 100. Which of the following is a possible average for his five math tests?
(A) 88 (B) 62 (C) 82 (D) 84 (E) 86

Source: 2008 Gauss Grade 7 #18

Primary Topics: Data Analysis

Secondary Topics: Averages

Answer: C

Solution:

The sum of Rishi's marks so far is $71 + 77 + 80 + 87 = 315$.
Since Rishi's mark on his next test is between 0 and 100, the sum of his marks will be between $315 + 0 = 315$ and $315 + 100 = 415$ after his next test.
Since his average equals the sum of his marks divided by the number of marks, then his average will be between $\frac{315}{5} = 63$ and $\frac{415}{5} = 83$.
Of the given choices, the only one in this range is 82.

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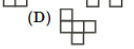
19. A 4×4 square grid can be entirely covered by three non-overlapping pieces made from 1×1 squares. If the first two pieces are  and , the third piece is
(A)  (B)  (C) (D) (E)

Source: 2008 Gauss Grade 7 #19

Primary Topics: Geometry and Measurement

Secondary Topics: Transformations

Answer: C

Solution:

After some experimentation, the only way in which the two given pieces can be put together to stay within a 4×4 grid and so that one of the given choices can fit together with them is to rotate the second piece by 90° clockwise, and combine to obtain

Therefore, the missing piece is

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20. The product of three *different* positive integers is 72. What is the smallest possible sum of these integers?
(A) 13 (B) 14 (C) 15 (D) 17 (E) 12

Source: 2008 Gauss Grade 7 #20

Primary Topics: Number Sense

Secondary Topics: Factoring

Answer: A

Solution:

The possible ways of writing 72 as the product of three different positive integers are: $1 \times 2 \times 36$; $1 \times 3 \times 24$; $1 \times 4 \times 18$; $1 \times 6 \times 12$; $1 \times 8 \times 9$; $2 \times 3 \times 12$; $2 \times 4 \times 9$; $3 \times 4 \times 6$.
(We can find all of these possibilities systematically by starting with the smallest possible first number and working through the possible second numbers, then go to the next possible smallest first number and continue.)
The sums of these sets of three numbers are 39, 28, 23, 19, 18, 17, 15, 13, so the smallest possible sum is 13.

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21. Andrea has finished the third day of a six day canoe trip. If she has completed $\frac{3}{7}$ of the trip's total distance of 168 km, how many km per day must she average for the remainder of her trip?
(A) 29 (B) 24 (C) 27 (D) 32 (E) 26

Source: 2008 Gauss Grade 7 #21

Primary Topics: Algebra and Equations

Secondary Topics: Rates | Fractions/Ratios

Answer: D

Solution:

Since Andrea has completed $\frac{3}{7}$ of the total 168 km, then she has completed $\frac{3}{7} \times 168$ km or $3 \times 24 = 72$ km.
This means that she has $168 - 72 = 96$ km remaining.
To complete the 96 km in her 3 remaining days, she must average $\frac{96}{3} = 32$ km per day.

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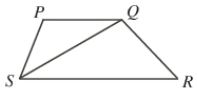
22. In the diagram, $PQRS$ is a trapezoid with an area of 12. RS is twice the length of PQ. The area of $\triangle PQS$ is



(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

Source: 2008 Gauss Grade 7 #22

Primary Topics: Geometry and Measurement

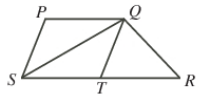
Secondary Topics: Area | Quadrilaterals

Answer: B

Solution:

Solution 1
Since PQ is parallel to SR, then the height of $\triangle PQS$ (considering PQ as the base) and the height of $\triangle SRQ$ (considering SR as the base) are the same (that is, the vertical distance between PQ and SR).
Since SR is twice the length of PQ and the heights are the same, then the area of $\triangle SRQ$ is twice the area of $\triangle PQS$.
In other words, the area of $\triangle PQS$ is $\frac{1}{3}$ of the total area of the trapezoid, or $\frac{1}{3} \times 12 = 4$.

Solution 2
Draw a line from Q to T, the midpoint of SR.



Since $SR = 2(PQ)$ and T is the midpoint of SR, then $PQ = ST = TR$.
We consider PQ, ST and TR as the bases of $\triangle PQS$, $\triangle STQ$ and $\triangle TRQ$, respectively.
Using these three segments as the bases, each of these triangles has the same height, since PQ is parallel to SR.
Since $PQ = ST = TR$ and these triangles have the same height, then the three triangles each have the same area.
The trapezoid is thus cut into three triangles of equal area.
Therefore, the area of $\triangle PQS$ is one-third of the area of entire trapezoid, or $\frac{1}{3} \times 12 = 4$.

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23. There are 24 ways in which Beverly, Dianne, Ethan, and Jamaal can arrange themselves to sit in a row of four seats. In how many ways can Beverly, Dianne, Ethan, and Jamaal arrange themselves in a row of four seats so that Ethan *does not* sit beside Dianne?
(A) 18 (B) 12 (C) 21 (D) 6 (E) 15

Source: 2008 Gauss Grade 7 #23

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: B

Solution:

Since Ethan does not sit next to Dianne, the four must arrange themselves in one of the configurations:
D _ E _ D _ _ E _ D _ E
E _ D _ E _ _ D _ E _ D
For each of these six configurations, there are two ways for Beverly and Jamaal to sit (either with Beverly on the left or with Jamaal on the left).
Therefore, there are $6 \times 2 = 12$ possible ways that the four can sit. (Try listing them out!)

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24. A star is made by overlapping two identical equilateral triangles, as shown. The entire star has an area of 36. What is the area of the shaded region?

(A) 24 (B) 18 (C) 27 (D) 33 (E) 30

Source: 2008 Gauss Grade 7 #24

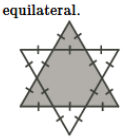
Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Area

Answer: C

Solution:

Since the two large triangles are equilateral, then each of their three angles equals $60°$.
Therefore, each of 6 small triangles in the star has an angle of $60°$ between the two equal sides.
But each of these 6 small triangles is isosceles so each of the remaining two angles must equal $\frac{1}{2}(180° - 60°)$ or $60°$.
Therefore, each of the small triangles is equilateral.



This shows us that the inner hexagon has all sides equal, and also that each angle is $180° - 60°$ or $120°$, so the hexagon is regular.
Next, we draw the three diagonals of the hexagon that pass through its centre (this is possible because of the symmetry of the hexagon).

Also, because of symmetry, each of the angles of the hexagon is split in half, to get $120° \div 2 = 60°$.
Therefore, each of the 6 new small triangles has two $60°$ angles, and so must have its third angle equal to $60°$ as well. Thus, each of the 6 new small triangles is equilateral.
So all 12 small triangles are equilateral. Since each has one side length marked by a single slash, then these 12 small triangles are all identical.
Since the total area of the star is 36, then the area of each small triangle is $36 \div 12 = 3$.
Since the shaded area is made up of 9 of these small triangles, its area is $9 \times 3 = 27$.

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25. The sum of all the digits of the integers from 98 to 101 is

$$9 + 8 + 9 + 9 + 1 + 0 + 0 + 1 + 0 + 1 = 38$$

The sum of all of the digits of the integers from 1 to 2008 is
(A) 30 054 (B) 27 018 (C) 28 036 (D) 30 036 (E) 28 054

Source: 2008 Gauss Grade 7 #25

Primary Topics: Counting and Probability

Secondary Topics: Digits | Counting

Answer: E

Solution:

First we look at the integers from 2000 to 2008.
Since we can ignore the 0s when adding up the digits, the sum of all of the digits of these integers is

$$2 + (2 + 1) + (2 + 2) + (2 + 3) + (2 + 4) + (2 + 5) + (2 + 6) + (2 + 7) + (2 + 8) = 54$$

Next, we look at the integers from 1 to 1999.
Again, since we can ignore digits of 0, we consider these numbers as 0001 to 1999, and in fact as the integers from 0000 to 1999, including 0000 to make 2000 integers in total.
Of these 2000 integers, 200 have a units digit of 0, 200 have a units digit of 1, and so on.
(One integer out of every 10 has a units digit of 0, and so on.)
Therefore, the sum of the units digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 200 + 400 + 600 + 800 + 1000 + 1200 + 1400 + 1600 + 1800 = 9000$$

Of these 2000 integers, 200 have a tens digit of 0, 200 have a tens digit of 1, and so on.
(Ten integers out of every 100 have a tens digit of 0, and so on.)
Therefore, the sum of the tens digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 9000$$

Of these 2000 integers, 200 have a hundreds digit of 0 (that is, 0000 to 0099 and 1000 to 1099), 200 have a hundreds digit of 1, and so on.
(One hundred integers out of every 1000 have a hundreds digit of 0, and so on.)
Therefore, the sum of the hundreds digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 9000$$

Of these 2000 integers, 1000 have a thousands digit of 0 and 1000 have a thousands digits of 1.
Therefore, the sum of the thousands digits of these integers is

$$1000(0) + 1000(1) = 1000$$

Overall, the sum of all of the digits of these integers is $54 + 9000 + 9000 + 9000 + 1000 = 28\,054$.
