The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7
Solutions

1. The value of $(4 \times 3) + 2$ is
   (A) 33 (B) 10 (C) 14 (D) 24 (E) 11

Source: 2014 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: C

Solution:

Evaluating, $(4 \times 3) + 2 = 12 + 2 = 14$.

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2. Which of the following numbers is closest to 100 on the number line?
   (A) 98 (B) 95 (C) 103 (D) 107 (E) 110

Source: 2014 Gauss Grade 7 #2

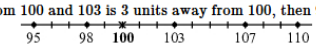
Primary Topics: Number Sense

Secondary Topics: Estimation

Answer: A

Solution:

Solution 1
We place each of the five answers and 100 on a number line.
Of the five answers given, the two closest numbers to 100 are 98 and 103.
Since 98 is 2 units away from 100 and 103 is 3 units away from 100, then 98 is closest to 100.



Solution 2
We calculate the positive difference between 100 and each of the five possible answers.
The number closest to 100 on the number line will produce the smallest of these positive differences.

Possible Answers	98	95	103	107	110
Positive Differences	$100 - 98 = 2$	$100 - 95 = 5$	$103 - 100 = 3$	$107 - 100 = 7$	$110 - 100 = 10$

Since the smallest positive difference is 2, then 98 is the closest to 100 on the number line.

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3. Five times a number equals one hundred. The number is
   (A) 50 (B) 10 (C) 15 (D) 25 (E) 20

Source: 2014 Gauss Grade 7 #3

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: E

Solution:

Since five times the number equals one hundred, then the number equals one hundred divided by five.
Therefore the number is $100 \div 5 = 20$.

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4. The spinner shown is divided into 6 sections of equal size. What is the probability of landing on a section that contains the letter P using this spinner?

   (A) $\frac{3}{6}$ (B) $\frac{4}{6}$ (C) $\frac{5}{6}$ (D) $\frac{2}{6}$ (E) $\frac{1}{6}$

Source: 2014 Gauss Grade 7 #4

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: D

Solution:

The spinner has 6 sections in total and 2 of these sections contain the letter Q.
Sections are equal to one another in size and thus they are each equally likely to be landed on.
Therefore, the probability of landing on a section that contains the letter Q is $\frac{2}{6}$.

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5. One scoop of fish food can feed 8 goldfish. How many goldfish can 4 scoops of fish food feed?
   (A) 12 (B) 16 (C) 8 (D) 64 (E) 32

Source: 2014 Gauss Grade 7 #5

Primary Topics: Algebra and Equations

Secondary Topics: Rates

Answer: E

Solution:

Each scoop of fish food can feed 8 goldfish.
Therefore, 4 scoops of fish food can feed $4 \times 8 = 32$ goldfish.

6. Which of these fractions is equivalent to $\frac{15}{25}$?
(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{3}{5}$ (D) $\frac{1}{2}$ (E) $\frac{5}{7}$

Source: 2014 Gauss Grade 7 #6

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: C

Solution:

Both the numerator and the denominator are divisible by 5.
Dividing, we get $\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$. Therefore, $\frac{15}{25}$ is equivalent to $\frac{3}{5}$.

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7. How many positive two-digit whole numbers are divisible by 7?
(A) 11 (B) 9 (C) 15 (D) 12 (E) 13

Source: 2014 Gauss Grade 7 #7

Primary Topics: Number Sense

Secondary Topics: Divisibility | Counting

Answer: E

Solution:

The largest two-digit number that is a multiple of 7 is $7 \times 14 = 98$.
Thus, there are 14 positive multiples of 7 that are less than 100.
However, this includes $7 \times 1 = 7$ which is not a two-digit number.
Therefore, there are $14 - 1 = 13$ positive two-digit numbers which are divisible by 7.
(Note that these 13 numbers are $14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91$, and 98.)
Since $7 \times 1 = 7$ is a single-digit number and $7 \times 2 = 14$, the smallest positive two-digit number that is divisible by 7 is 14.
Since $7 \times 15 = 105$ is a three-digit number and $7 \times 14 = 98$, the largest two-digit number that is divisible by 7 is 98.
The number of two-digit numbers that are divisible by 7 is equivalent to the number of multiples of 7 from (and including) 7×2 up to and including 7×14.
There are $14 - 2 + 1 = 13$ multiples of 7 from 14 to 98, so there are 13 two-digit numbers that are divisible by 7.

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8. If $9210 - 9124 = 210 - \square$, the value represented by the $\square$ is
(A) 296 (B) 210 (C) 186 (D) 124 (E) 24

Source: 2014 Gauss Grade 7 #8

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: D

Solution:

Solution 1
Evaluating the left side of the equation, we get $9210 - 9124 = 86$.
Therefore the right side of the equation, $210 - \square$, must also equal 86.
Since $210 - 124 = 86$, then the value represented by the $\square$ is 124.
Solution 2
Since $9210 - 9124 = (9000 + 210) - (9000 + 124) = 9000 - 9000 + 210 - 124 = 210 - 124$, then the value represented by the $\square$ is 124.

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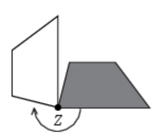
9. A clockwise rotation around point Z (that is, a rotation in the direction of the arrow) transforms the shaded quadrilateral to the unshaded quadrilateral. The angle of rotation is approximately



(A) $180°$ (B) $270°$ (C) $360°$ (D) $45°$ (E) $135°$

Source: 2014 Gauss Grade 7 #9

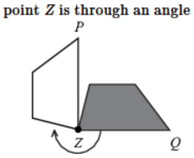
Primary Topics: Geometry and Measurement

Secondary Topics: Transformations | Angles

Answer: B

Solution:

The measure of $\angle PZQ$ formed by the bottom edge of the shaded quadrilateral, ZQ, and this same edge after the rotation, ZP, is approximately $90°$.
The transformation of the shaded quadrilateral to the unshaded quadrilateral is a clockwise rotation around point Z through an angle equal to the measure of reflex angle PZQ.
The measure of $\angle PZQ$ added to the measure of reflex $\angle PZQ$ is equal to the measure of one complete rotation, or $360°$.
Therefore, the measure of reflex angle PZQ is approximately $360° - 90°$ or $270°$.
Thus the clockwise rotation around point Z is through an angle of approximately $270°$.



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10. Which one of the following is equal to 17?
(A) $3-4\times5+6$ (B) $3\times4+5\div6$ (C) $3+4\times5-6$
(D) $3\div4+5-6$ (E) $3\times4\div5+6$

Source: 2014 Gauss Grade 7 #10
Primary Topics: Number Sense
Secondary Topics: Operations

Answer: C
Solution:
In the table below, each of the five expressions is evaluated using the correct order of operations.

Expression	Value
$(A)\ 3-4\times5+6$	$3-20+6=-17+6=-11$
$(B)\ 3\times4+5\div6$	$12+5\div6=12+\frac{5}{6}=12\frac{5}{6}$
$(C)\ 3+4\times5-6$	$3+20-6=23-6=17$
$(D)\ 3\div4+5-6$	$\frac{3}{4}+5-6=5\frac{3}{4}-6=\frac{23}{4}-\frac{24}{4}=-\frac{1}{4}$
$(E)\ 3\times4\div5+6$	$12\div5+6=\frac{12}{5}+6=2\frac{2}{5}+6=8\frac{2}{5}$

The only expression that is equal to 17 is $3+4\times5-6$, or (C).

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11. Consider the set $\{0.34, 0.304, 0.034, 0.43\}$. The sum of the smallest and largest numbers in the set is
(A) 0.77 (B) 0.734 (C) 0.077 (D) 0.464 (E) 0.338

Source: 2014 Gauss Grade 7 #11
Primary Topics: Number Sense
Secondary Topics: Decimals

Answer: D
Solution:
Since each of the numbers in the set is between 0 and 1, then the tenths digit of each number contributes more to its value than any of its other digits.
The largest tenths digit of the given numbers is 4, and so 0.43 is the largest number in the set.
The smallest tenths digit of the given numbers is 0, so 0.034 is the smallest number in the-set.
Therefore, the sum of the smallest number in the set and the largest number in the set is\linebreak $0.034+0.43=0.464$.

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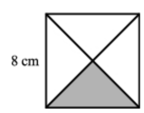
12. The diagonals have been drawn in the square shown. The area of the shaded region of the square is



(A) 4 cm^2 (B) 8 cm^2 (C) 16 cm^2 (D) 56 cm^2 (E) 64 cm^2

Source: 2014 Gauss Grade 7 #12
Primary Topics: Geometry and Measurement
Secondary Topics: Area | Triangles | Quadrilaterals

Answer: C
Solution:
The two diagonals of a square bisect one another (divide each other into two equal lengths) at the centre of the square.
Therefore, the two diagonals divide the square into four identical triangles.
One of these four triangles is the shaded region which has area equal to one quarter of the area of the square.
Since the area of the square is $8\times8=64$ cm^2, the area of the shaded region is $64\div4=16$ cm^2.

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13. In the special square shown, the sum of the three numbers in each column equals the sum of the three numbers in each row. The value of x is

13	8	
14	x	10
9		

(A) 3 (B) 4 (C) 5 (D) 6 (E) 12

Source: 2014 Gauss Grade 7 #13
Primary Topics: Algebra and Equations
Secondary Topics: Equations Solving

Answer: E
Solution:
The sum of the three numbers in the first column is $13+14+9=36$.
The sum of the numbers in each column and in each row in the square is the same and so the sum of the three numbers in the second row is also-36.
That is, $14+x+10=36$ or $x+24=36$, and so $x=36-24=12$.

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14. In the diagram shown, the number of rectangles of all sizes is

(A) 11 (B) 15 (C) 7 (D) 13 (E) 9

Source: 2014 Gauss Grade 7 #14

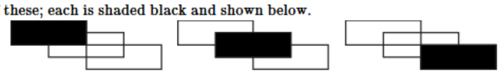
Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: A

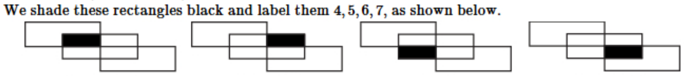
Solution:

We systematically count rectangles by searching for groups of rectangles that are of similar size.
The largest rectangles in the diagram are all roughly the same size and overlap in pairs.
There are 3 of these; each is shaded black and shown below.



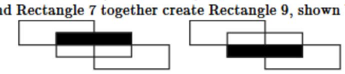
Rectangle 1 Rectangle 2 Rectangle 3

Rectangle 2 (shown above) consists of 4 small rectangles.
We shade these rectangles black and label them $4, 5, 6, 7$, as shown below.



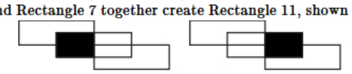
Rectangle 4 Rectangle 5 Rectangle 6 Rectangle 7

Together, Rectangle 4 and Rectangle 5 (shown above) create Rectangle 8, shown below.
Similarly, Rectangle 6 and Rectangle 7 together create Rectangle 9, shown below.



Rectangle 8 Rectangle 9

Finally, Rectangle 4 and Rectangle 6 (shown above) together create Rectangle 10, shown below.
Similarly, Rectangle 5 and Rectangle 7 together create Rectangle 11, shown below.



Rectangle 10 Rectangle 11

There are no other rectangles that can be formed.
In total, there are 11 rectangles in the given diagram.

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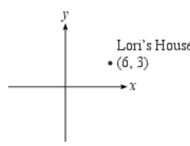
15. The diagram shows Lori's house located at $(6, 3)$. If Alex's house is located at $(-2, -4)$, what translation is needed to get from Lori's house to Alex's house?



(A) 4 units left, 1 unit up (B) 8 units right, 7 units up
(C) 4 units left, 1 unit down (D) 8 units left, 7 units down
(E) 7 units right, 8 units down

Source: 2014 Gauss Grade 7 #15

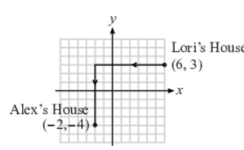
Primary Topics: Geometry and Measurement

Secondary Topics: Transformations | Coordinate Geometry

Answer: D

Solution:

The horizontal translation needed to get from Lori's house to Alex's house is the difference between the x-coordinate of Lori's house, 6, and the x-coordinate of Alex's house, --2, or $6 - (-2) = 6 + 2 = 8$.
The vertical translation needed to get from Lori's house to Alex's house is the difference between the y-coordinate of Lori's house, 3, and the y-coordinate of Alex's house, -4, or $3 - (-4) = 3 + 4 = 7$.
From Lori's house, Alex's house is left and down.
Therefore the translation needed to get from Lori's house to Alex's house is 8 units left and 7 units down.



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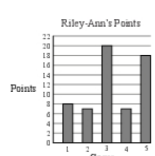
16. The graph shows points scored by Riley-Ann in her first five basketball games. The difference between the mean and the median of the number of points that she scored is



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Source: 2014 Gauss Grade 7 #16

Primary Topics: Data Analysis

Secondary Topics: Graphs | Averages

Answer: D

Solution:

Reading from the graph, Riley-Ann scored 8 points in Game 1, 7 points in Game 2, 20 points in Game 3, 7 points in Game 4, and 18 points in Game 5.
Therefore the mean number of points that she scored per game is $\frac{8+7+20+7+18}{5} = \frac{60}{5} = 12$.
Since the ordered list (smallest to largest) of the number of points scored per game is $7, 7, 8, 18, 20$, then the median is 8, the number in the middle of this ordered list.
The difference between the mean and the median of the number of points that Riley-Ann scored is $12 - 8 = 4$.

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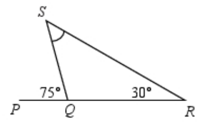
17. In the diagram shown, PQR is a straight line segment. The measure of $\angle QSR$ is



(A) 25° (B) 30° (C) 35° (D) 40° (E) 45°

Source: 2014 Gauss Grade 7 #17

Primary Topics: Geometry and Measurement

Secondary Topics: Angles | Triangles

Answer: E

Solution:

Solution 1
Since PQR is a straight line segment, then $\angle PQR = 180^\circ$.
Since $\angle SQP + \angle SQR = 180^\circ$, then $\angle SQR = 180^\circ - \angle SQP = 180^\circ - 75^\circ = 105^\circ$.
The three angles in a triangle add to 180°, so $\angle QSR + \angle SQR + \angle QRS = 180^\circ$, or
$\angle QSR = 180^\circ - \angle SQR - \angle QRS = 180^\circ - 105^\circ - 30^\circ = 45^\circ$.
Solution 2
The exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles of the triangle.
Since $\angle SQP$ is an exterior angle of $\triangle SQR$, and the two opposite interior angles are $\angle QSR$ and $\angle QRS$, then $\angle SQP = \angle QSR + \angle QRS$.
Thus, $75^\circ = \angle QSR + 30^\circ$ or $\angle QSR = 75^\circ - 30^\circ = 45^\circ$.

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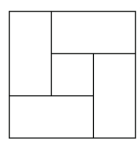
18. In the figure shown, the outer square has an area of 9 cm^2, the inner square has an area of 1cm ^{2}, and the four rectangles are identical. What is the perimeter of one of the four identical rectangles?



(A) 6 cm (B) 8 cm (C) 10 cm (D) 9 cm (E) 7 cm

Source: 2014 Gauss Grade 7 #18

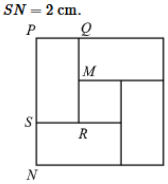
Primary Topics: Geometry and Measurement

Secondary Topics: Area | Perimeter

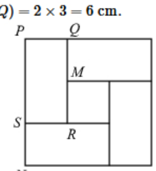
Answer: A

Solution:

Solution 1
The outer square has an area of 9 cm^2, so the sides of this outer square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
The inner square has an area of 1 cm^2, so the sides of this inner square have length 1 cm (since $1 \times 1 = 1$), and thus $MR = 1$ cm.
Since $PN = 3$ cm, then $PS + SN = 3$ cm and so $QR + SN = 3$ cm (since $QR = PS$).
But $QR = QM + MR$, so then $QM + MR + SN = 3$ cm or $QM + 1 + SN = 3$ cm (since $MR = 1$ cm).
From this last equation we get $QM + SN = 2$ cm.



Since each of QM and SN is the width of an identical rectangle, then $QM = SN = 1$ cm.
Using $PS + SN = 3$ cm, we get $PS + 1 = 3$ cm and so $PS = 2$ cm.
Since the rectangles are identical, then $SN = PQ = 1$ cm.
The perimeter of rectangle $PQRS$ is $2 \times (PS + PQ) = 2 \times (2 + 1) = 2 \times 3 = 6$ cm.
Solution 2
The outer square has an area of 9 cm^2, so the sides of this outer square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
Since $PN = 3$ cm, then $PS + SN = 3$ cm.
Since each of PQ and SN is the width of an identical rectangle, then $PQ = SN$ and so $PS + SN = PS + PQ = 3$ cm.
The perimeter of $PQRS$ is $2 \times (PS + PQ) = 2 \times 3 = 6$ cm.



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19. Sarah's hand length is 20 cm. She measures the dimensions of her rectangular floor to be 18 by 22 hand lengths. Which of the following is the closest to the area of the floor?
(A) 160 000 cm^2 (B) 80 000 cm^2 (C) 200 000 cm^2
(D) 16 000 cm^2 (E) 20 000 cm^2

Source: 2014 Gauss Grade 7 #19

Primary Topics: Geometry and Measurement

Secondary Topics: Estimation | Area

Answer: A

Solution:

The width of Sarah's rectangular floor is 18 hand lengths.
Since each hand length is 20 cm, then the width of the floor is $18 \times 20 = 360$ cm.
The length of Sarah's rectangular floor is 22 hand lengths.
Since each hand length is 20 cm, then the length of the floor is $22 \times 20 = 440$ cm.
Thus the area of the floor is $360 \times 440 = 158\,400$ cm^2.
Of the given answers, the closest to the area of the floor is $160\,000$ cm^2.

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20. The product of three consecutive odd numbers is 9177. What is the sum of the numbers?
(A) 51 (B) 57 (C) 60 (D) 63 (E) 69

Source: 2014 Gauss Grade 7 #20

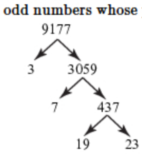
Primary Topics: Number Sense

Secondary Topics: Patterning/Sequences/Series

Answer: D

Solution:

Solution 1
Since $20 \times 20 \times 20 = 8000$ and $30 \times 30 \times 30 = 27\,000$, then we might guess that the three consecutive odd numbers whose product is 9177 are closer to 20 than they are to 30.
Using trial and error, we determine that $21 \times 23 \times 25 = 12\,075$, which is too large.
The next smallest set of three consecutive odd numbers is $19, 21, 23$ and the product of these three numbers is $19 \times 21 \times 23 = 9177$, as required.
Thus, the sum of the three consecutive odd numbers whose product is 9177 is $19 + 21 + 23 = 63$.
Solution 2
We begin by determining the prime numbers whose product is 9177.
(This is called the prime factorization of 9177.)
This prime factorization of 9177 is shown in the factor tree to the right. That is,
$9177 = 3 \times 3059 = 3 \times 7 \times 437 = 3 \times 7 \times 19 \times 23$. Since $3 \times 7 = 21$, then $9177 = 21 \times 19 \times 23$ and so the three consecutive numbers whose product is 9177 are $19, 21, 23$.
Thus, the sum of the three consecutive odd numbers whose product is 9177 is $19 + 21 + 23 = 63$.



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21. A bicycle at Store P costs \$200. The regular price of the same bicycle at Store Q is 15% more than it is at Store P. The bicycle is on sale at Store Q for 10% off of the regular price. What is the sale price of the bicycle at Store Q?
(A) \$230.00 (B) \$201.50 (C) \$199.00 (D) \$207.00 (E) \$210.00

Source: 2014 Gauss Grade 7 #21

Primary Topics: Algebra and Equations

Secondary Topics: Percentages | Decimals

Answer: D

Solution:

At Store Q, the bicycle's regular price is 15% more than the price at Store P, or 15% more than \$200.
Since 15% of 200 is $\frac{15}{100} \times 200 = 0.15 \times 200 = 30$, then 15% more than \$200 is \$200 + \$30 or \$230.
This bicycle is on sale at Store Q for 10% off of the regular price, \$230.
Since 10% of 230 is $\frac{10}{100} \times 230 = 0.10 \times 230 = 23$, then 10% off of \$230 is \$230 − \$23 or \$207.
The sale price of the bicycle at Store Q is \$207.

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22. Each face of a cube is painted with exactly one colour. What is the smallest number of colours needed to paint a cube so that no two faces that share an edge are the same colour?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Source: 2014 Gauss Grade 7 #22

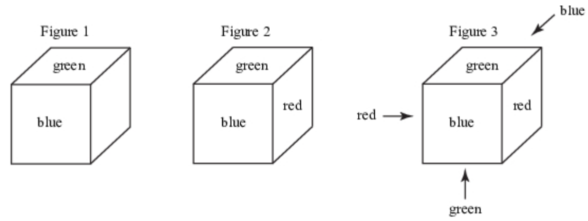
Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: B

Solution:

Assume the top face of the cube is coloured green.
Since the front face of the cube shares an edge with the top face, it cannot be coloured green. Thus, we need at least two colours.
Thus, we assume that the front face is coloured blue, as shown in Figure 1.
Since the right face shares an edge with the top face and with the front face, it cannot be coloured green or blue. Thus, we need at least three colours.
Thus, we assume that the right face is coloured red, as shown in Figure 2.
We have shown that at least 3 colours are needed. In fact, the cube can be coloured with exactly 3 colours by colouring the left face red, the back face blue, and the bottom face green (Figure 3).
In this way, the cube is coloured with exactly 3 colours and no two faces that share an edge are the same colour.
Therefore, 3 is the smallest number of colours needed to paint a cube so that no two faces that share an edge are the same colour.



23. Two standard six-sided dice are tossed. One die is red and the other die is blue. What is the probability that the number appearing on the red die is greater than the number appearing on the blue die?
(A) $\frac{18}{36}$ (B) $\frac{25}{36}$ (C) $\frac{15}{36}$ (D) $\frac{12}{36}$ (E) $\frac{17}{36}$

Source: 2014 Gauss Grade 7 #23

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: C

Solution:

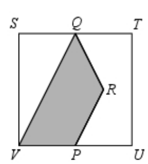
Solution 1
For each of the 6 possible outcomes that could appear on the red die, there are 6 possible outcomes that could appear on the blue die.
That is, the total number of possible outcomes when a standard six-sided red die and a standard six-sided blue die are rolled is $6 \times 6 = 36$.
These 36 outcomes are shown in the table below.
When a number that appears on the red die is greater than a number that appears on the blue die, a checkmark has been placed in the appropriate cell, corresponding to the intersection of the column and row.
For example, the table cell containing the double checkmark represents the outcome of a 4 appearing on the red die and a 2 appearing on the blue die.

	1	2	3	4	5	6
1		✓	✓	✓	✓	✓
2			✓	✓✓	✓	✓
3				✓	✓	✓
4					✓	✓
5						✓
6						

Of the 36 possible outcomes, $1+2+3+4+5$ or 15 have a number appearing on the red die that is larger than the number appearing on the blue die.
The probability that the number appearing on the red die is greater than the number appearing on the blue die is $\frac{15}{36}$. Solution 2
As in Solution 1, we determine the total number of possible outcomes to be 36.
Each of these 36 outcomes can be grouped into one of three possibilities; the number appearing on the red die is greater than the number appearing on the blue die, the number appearing on the red die is less than the number appearing on the blue die, or the numbers appearing on the two dice are equal.
There are 6 possible outcomes in which the numbers appearing on the two dice are equal (both numbers are 1, both numbers are 2, and so on).
Of the 36 total outcomes, this leaves $36 - 6 = 30$ outcomes in which either the number appearing on the red die is greater than the number appearing on the blue die, or the number appearing on the red die is less than the number appearing on the blue die.
These two possibilities are equally likely to happen (since both dice are identical except for colour), and so the number appearing on the red die will be greater than the number appearing on the blue die in half of the 30 outcomes, or 15 outcomes.
Thus, the probability that the number appearing on the red die is greater than the number appearing on the blue die is $\frac{15}{36}$.

24. In the diagram shown,
- $STUV$ is a square,
- Q and P are the midpoints of ST and UV,
- $PR = QR$, and
- VQ is parallel to PR.

What is the ratio of the shaded area to the unshaded area?



(A) $2:3$ (B) $3:5$ (C) $1:1$ (D) $7:9$ (E) $5:7$

Source: 2014 Gauss Grade 7 #24

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Fractions/Ratios

Answer: B

Solution:

We begin by joining Q to P.
Since Q and P are the midpoints of ST and UV, then QP is parallel to both SV and TU and rectangles $SQPV$ and $QTUP$ are identical.
In rectangle $SQPV$, VQ is a diagonal.
Similarly, since PR is parallel to VQ then PR extended to T is a diagonal of rectangle $QTUP$, as shown in Figure 1.
In Figure 2, we label points A, B, C, D, E, and F, the midpoints of SQ, QT, TU, UP, PV, and VS, respectively.
We join A to E, B to D and F to C, with FC intersecting QP at the centre of the square O, as shown.
Since $PR = QR$ and R lies on diagonal PT, then both FC and BD pass through R. (That is, R is the centre of $QTUP$.)

The line segments AE, QP, BD, and FC divide square $STUV$ into 8 identical rectangles.
In one of these rectangles, $QBRO$, diagonal QR divides the rectangle into 2 equal areas.
That is, the area of $\triangle QOR$ is half of the area of rectangle $QBRO$.
Similarly, the area of $\triangle POR$ is half of the area of rectangle $PORD$.

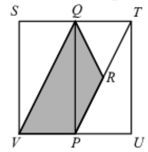


Figure 1

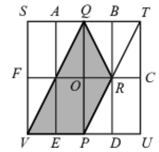


Figure 2

Rectangle $SQPV$ has area equal to 4 of the 8 identical rectangles.
Therefore, $\triangle QPV$ has area equal to 2 of the 8 identical rectangles (since diagonal VQ divides the area of $SQPV$ in half).
Thus the total shaded area, which is $\triangle QOR + \triangle POR + \triangle QPV$, is equivalent to the area of $\frac{1}{2} + \frac{1}{2} + 2$ or 3 of the identical rectangles.
Since square $STUV$ is divided into 8 of these identical rectangles, and the shaded area is equivalent to the area of 3 of these 8 rectangles, then the unshaded area occupies an area equal to that of the remaining $8 - 3$ or 5 rectangles.
Therefore, the ratio of the shaded area to the unshaded area is $3 : 5$.

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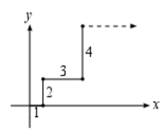
25. On a coordinate grid, Paul draws a line segment of length 1 from the origin to the right, stopping at $(1, 0)$. He then draws a line segment of length 2 up from this point, stopping at $(1, 2)$. He continues to draw line segments to the right and up, increasing the length of the line segment he draws by 1 each time. One of his line segments stops at the point $(529, 506)$. What is the endpoint of the next line segment that he draws?



(A) $(529, 552)$ (B) $(576, 506)$ (C) $(575, 506)$ (D) $(529, 576)$ (E) $(576, 552)$

Source: 2014 Gauss Grade 7 #25

Primary Topics: Geometry and Measurement

Secondary Topics: Coordinate Geometry | Patterning/Sequences/Series

Answer: A

Solution:

We begin by listing the first few line segment endpoints as determined by the number of segments that Paul has drawn.

Line Segment Number	Endpoint of the Line Segment
1	$(1, 0)$
2	$(1, 2)$
3	$(4, 2)$
4	$(4, 6)$
5	$(9, 6)$
6	$(9, 12)$
7	$(16, 12)$
8	$(16, 20)$

Since each new line segment is drawn either vertically or horizontally from the endpoint of the previous line segment, then only one of the x-coordinate or y-coordinate changes from one endpoint to the next endpoint.
Further, since the odd numbered line segments are horizontal, the x-coordinate of the endpoint of these segments changes from the previous endpoint while the y-coordinate remains the same.
Similarly, since the even numbered line segments are vertical, the y-coordinate of the endpoint of these segments changes from the previous endpoint while the x-coordinate remains the same.
We are given that one of the line segments ends at the point $(529, 506)$.
We will begin by determining the number of segments that must be drawn for the x-coordinate of the endpoint of the final line segment to be 529.
The first line segment is drawn horizontally from the origin, to the right, has length 1, and thus ends with an x-coordinate of 1.
The third line segment is drawn horizontally to the right, has length 3 and thus ends with an x-coordinate of $1 + 3 = 4$.
The fifth line segment is drawn horizontally to the right, has length 5 and thus ends with an x-coordinate of $1 + 3 + 5 = 9$.
To get an idea of how many line segments are required before the x-coordinate of the endpoint is 529, we begin by considering the first 21 line segments.
The endpoint of the 21st line segment has x-coordinate equal to $1 + 3 + 5 + \cdots + 17 + 19 + 21$.
To make this sum easier to determine, we rearrange the terms to get

$$(1 + 21) + (3 + 19) + (5 + 17) + (7 + 15) + (9 + 13) + 11 = 22 + 22 + 22 + 22 + 22 + 11 = 22 \times 5 + 11 = 121.$$

Since 121 is much smaller than 529, we continue to try larger numbers of line segments until we finally reach 45 line segments.
The endpoint of the 45th line segment has x-coordinate equal to $1 + 3 + 5 + \cdots + 41 + 43 + 45$.
To make this sum easier to determine, we rearrange the terms to get

$$(1 + 45) + (3 + 43) + (5 + 41) + \cdots + (19 + 27) + (21 + 25) + 23 = 46 \times 11 + 23 = 529.$$

Therefore, the 45th line segment ends with an x-coordinate of 529.

The next line segment (the 46th) will also have an endpoint with this same x-coordinate, 529, since even numbered line segments are vertical (thus, only the y-coordinate will change).
This tells us that $(529, 506)$ is the endpoint of either the 45th or the 46th line segment.
At this point we may confirm that 506 is the y-coordinate of the endpoint of the 45th line segment (and hence the 44th line segment as well).
The second line segment is drawn vertically from the x-axis, upward, has length-2, and thus ends with y-coordinate 2.
The fourth line segment is drawn vertically upward, has length 4 and thus ends with a y-coordinate of $2 + 4 = 6$.
The sixth line segment is drawn vertically upward, has length 6 and thus ends with a y-coordinate of $2 + 4 + 6 = 12$.
The endpoint of the 45th line segment has y-coordinate equal to $2 + 4 + 6 + \cdots + 40 + 42 + 44$.
Rearranging the terms of this sum, we get

$$(2 + 44) + (4 + 42) + (6 + 40) + \cdots + (20 + 26) + (22 + 24) = 46 \times 11 = 506.$$

Thus, $(529, 506)$ is the endpoint of the 45th line segment.
The 46th line segment is vertical and has length 46.
Therefore the y-coordinate of the endpoint of the 46th line segment is

$$2 + 4 + 6 + \cdots + 40 + 42 + 44 + 46 = 506 + 46 = 552.$$

Since the x-coordinate of the endpoint of the 46th line segment is the same as that of the 45th, then the endpoint of the next line segment that Paul draws is $(529, 552)$.