The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7
Solutions

1. The value of $(5 \times 3) - 2$ is
(A) 5 (B) 9 (C) 6 (D) 8 (E) 13

Source: 2013 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: E

Solution:

Evaluating, $(5 \times 3) - 2 = 15 - 2 = 13$.

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2. Which of the following numbers is a multiple of 9?
(A) 50 (B) 40 (C) 35 (D) 45 (E) 55

Source: 2013 Gauss Grade 7 #2

Primary Topics: Number Sense

Secondary Topics: Divisibility

Answer: D

Solution:

Solution 1
A number is a multiple of 9 if it is the result of multiplying 9 by an integer.
Of the answers given, only 45 results from multiplying 9 by an integer, since $45 = 9 \times 5$.
Solution 2
A number is a multiple of 9 if the result after dividing it by 9 is an integer.
Of the answers given, only 45 results in an integer after dividing by 9, since $45 \div 9 = 5$.

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3. Thirty-six hundredths is equal to
(A) 0.36 (B) 360 (C) 3.6 (D) 0.036 (E) 0.0036

Source: 2013 Gauss Grade 7 #3

Primary Topics: Number Sense

Secondary Topics: Digits | Decimals

Answer: A

Solution:

Thirty-six hundredths equals $\frac{36}{100}$ or 0.36.

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4. The value of $1 + 1 - 2 + 3 + 5 - 8 + 13 + 21 - 34$ is
(A) -32 (B) 1 (C) 88 (D) 0 (E) -34

Source: 2013 Gauss Grade 7 #4

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: D

Solution:

By grouping terms using brackets as shown, $(1 + 1 - 2) + (3 + 5 - 8) + (13 + 21 - 34)$, we can see that the result inside each set of brackets is 0.
Thus the value of $1 + 1 - 2 + 3 + 5 - 8 + 13 + 21 - 34$ is 0.

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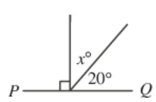
5. If PQ is a straight line segment, then the value of x is



(A) 160 (B) 70 (C) 110 (D) 20 (E) 80

Source: 2013 Gauss Grade 7 #5

Primary Topics: Geometry and Measurement

Secondary Topics: Angles

Answer: B

Solution:

Since PQ is a straight line segment, the three angles given sum to 180°.
That is, $90^\circ + x^\circ + 20^\circ = 180^\circ$ or $x^\circ = 180^\circ - 90^\circ - 20^\circ$ or $x = 70$.

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6. Nick has six nickels (5¢), two dimes (10¢) and one quarter (25¢). In cents (¢), how much money does Nick have?
(A) 65 (B) 75 (C) 35 (D) 15 (E) 55

Source: 2013 Gauss Grade 7 #6

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: B

Solution:

Nick has 6 nickels and each nickel is worth 5¢. So Nick has 6×5¢ or 30¢ in nickels.
Nick has 2 dimes and each dime is worth 10¢. So Nick has 2×10¢ or 20¢ in dimes.
Nick has 1 quarter and each quarter is worth 25¢. So Nick has 1×25¢ or 25¢ in quarters.
In total, Nick has 30¢ + 20¢ + 25¢ or 75¢.

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7. The smallest number in the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$ is
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{5}{6}$ (E) $\frac{7}{12}$

Source: 2013 Gauss Grade 7 #7

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Optimization

Answer: C

Solution:

Solution 1
To determine the smallest number in the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$, we express each number with a common denominator of 12. The set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$ is equivalent to the set $\left\{\frac{1\times 6}{2\times 6}, \frac{2\times 4}{3\times 4}, \frac{1\times 3}{4\times 3}, \frac{5\times 2}{6\times 2}, \frac{7}{12}\right\}$ or to the set $\left\{\frac{6}{12}, \frac{8}{12}, \frac{3}{12}, \frac{10}{12}, \frac{7}{12}\right\}$.
The smallest number in this set is $\frac{3}{12}$, so $\frac{1}{4}$ is the smallest number in the original set.
Solution 2
With the exception of $\frac{1}{4}$, each number in the set is greater than or equal to $\frac{1}{2}$.
We can see this by recognizing that the numerator of each fraction is greater than or equal to one half of its denominator.
Thus, $\frac{1}{4}$ is the only number in the list that is less than $\frac{1}{2}$ and so it must be the smallest number in the set.

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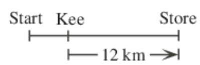
8. Ahmed is going to the store. One quarter of the way to the store, he stops to talk with Kee. He then continues for 12 km and reaches the store. How many kilometres does he travel altogether?



(A) 15 (B) 16 (C) 24 (D) 48 (E) 20

Source: 2013 Gauss Grade 7 #8

Primary Topics: Geometry and Measurement

Secondary Topics: Fractions/Ratios | Measurement

Answer: B

Solution:

Since Ahmed stops to talk with Kee one quarter of the way to the store, then the remaining distance to the store is $1 - \frac{1}{4} = \frac{3}{4}$ of the total distance.
Since $\frac{3}{4} = 3 \times \frac{1}{4}$, then the distance that Ahmed travelled from Kee to the store is 3 times the distance that Ahmed travelled from the start to reach Kee.
That is, 12 km is 3 times the distance between the start and Kee. So the distance between the start and Kee is $\frac{12}{3} = 4$ km. Therefore, the total distance travelled by Ahmed is $4 + 12$ or 16 km.

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9. An expression that produces the values in the second row of the table shown, given the values of n in the first row, is

n	1	2	3	4	5
value	1	3	5	7	9

(A) $3n - 2$ (B) $2(n - 1)$ (C) $n + 4$ (D) $2n$ (E) $2n - 1$

Source: 2013 Gauss Grade 7 #9

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving | Equations of Lines

Answer: E

Solution:

When $n = 4$, we are looking for an expression that produces a value of 7.
The results of substituting $n = 4$ into each expression and evaluating are shown below.

Expression	Value
(A) $3n - 2$	$3(4) - 2 = 12 - 2 = 10$
(B) $2(n - 1)$	$2(4 - 1) = 2(3) = 6$
(C) $n + 4$	$4 + 4 = 8$
(D) $2n$	$2(4) = 8$
(E) $2n - 1$	$2(4) - 1 = 8 - 1 = 7$

Since $2n - 1$ is the only expression which gives a value of 7 when $n = 4$, it is the only possible answer. We check that the expression $2n - 1$ does give the remaining values, $1, 3, 5, 9$, when $n = 1, 2, 3, 5$, respectively.
(Alternately, we may have began by substituting $n = 1$ and noticing that this eliminates answers (B), (C) and (D). Substituting $n = 2$ eliminates (A). Substituting $n = 3$, $n = 4$ and $n = 5$ confirms the answer (E).)

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10. UVW and XYZ are each 3-digit integers. U, V, W, X, Y, and Z are different digits chosen from the integers 1 to 9. What is the largest possible value for $UVW - XYZ$?
(A) 678 (B) 864 (C) 885 (D) 888 (E) 975

Source: 2013 Gauss Grade 7 #10

Primary Topics: Number Sense

Secondary Topics: Digits | Optimization

Answer: B

Solution:

To make the difference $UVW - XYZ$ as large as possible, we make UVW as large as possible and XYZ as small as possible.
The hundreds digit of a number contributes more to its value than its tens digit, and its tens digit contributes more to its value than its units digit.
Thus, we construct the largest possible number UVW by choosing 9 (the largest digit) to be its hundreds digit, U, and by choosing 8 (the second largest digit) to be its tens digit, V, and by choosing 7 (the third largest digit) to be the units digit, W.
Similarly, we construct the smallest possible number XYZ by choosing 1 (the smallest allowable digit) to be its hundreds digit, X, and 2 (the second smallest allowable digit) to be its tens digit, Y, and by choosing 3 (the third smallest allowable digit) to be its units digit, Z.
The largest possible difference is $UVW - XYZ$ or $987 - 123$ or 864.

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11. The length of each edge of a cube is 1 cm. The surface area of the cube, in cm^2, is
(A) 24 (B) 1 (C) 4 (D) 12 (E) 6

Source: 2013 Gauss Grade 7 #11

Primary Topics: Geometry and Measurement

Secondary Topics: Surface Area | Prisms

Answer: E

Solution:

Each face of a cube is a square. The dimensions of each face of the cube are 1 cm by 1 cm.
Thus, the area of each face of the cube is $1 \times 1 = 1$ cm^2.
Since a cube has 6 identical faces, the surface area of the cube is $6 \times 1 = 6$ cm^2.

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12. Which of the following pairs of numbers has a greatest common factor of 20?
(A) 200 and 2000 (B) 40 and 50 (C) 20 and 40
(D) 20 and 25 (E) 40 and 80

Source: 2013 Gauss Grade 7 #12

Primary Topics: Number Sense

Secondary Topics: Divisibility | Factoring

Answer: C

Solution:

The greatest common factor of two numbers is the largest positive integer which divides into both numbers with no remainder.
For answer (B), 50 divided by 20 leaves a remainder, so we may eliminate (B) as a possible answer.
Similarly for answer (D), 25 divided by 20 leaves a remainder, so we may eliminate (D) as a possible answer.
For answer (A), since 200 divides 200 and 200 divides 2000, the greatest common factor of 200 and 2000 cannot be 20.
For answer (E), since 40 divides 40 and 40 divides 80, the greatest common factor of 40 and 80 cannot be 20.
The largest positive integer which divides both 20 and 40 is 20, and so (C) is the correct answer.

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13. Jack, Kelly, Lan, Mihai, and Nate are sitting in the 5 chairs around a circular table. Lan and Mihai are sitting beside each other. Jack and Kelly are not sitting beside each other. The 2 people who are seated on either side of Nate are
(A) Jack and Lan (B) Jack and Kelly (C) Kelly and Mihai
(D) Lan and Mihai (E) Mihai and Jack

Source: 2013 Gauss Grade 7 #13

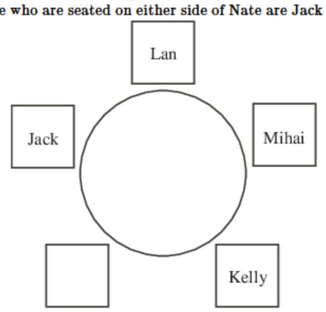
Primary Topics: Other

Secondary Topics: Logic

Answer: B

Solution:

Since Lan and Mihai are seated beside each other, while Jack and Kelly are not, the only possible location for the remaining chair (Nate's chair) is between Jack and Kelly, as shown.
Therefore, the 2 people who are seated on either side of Nate are Jack and Kelly.



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14. If $x = 4$ and $3x + 2y = 30$, what is the value of y?
(A) 18 (B) 6 (C) 3 (D) 4 (E) 9

Source: 2013 Gauss Grade 7 #14

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: E

Solution:

Substituting $x = 4$ into $3x + 2y$ we get, $3(4) + 2y = 12 + 2y$.
Since this expression $12 + 2y$ is equal to 30, then $2y$ must equal $30 - 12$ or 18.
If $2y = 18$, then y is $18 \div 2$ or 9.

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15. Daniel begins with 64 coins in his coin jar. Each time he reaches into the jar, he removes half of the coins that are in the jar. How many times must he reach in and remove coins from his jar so that exactly 1 coin remains in the jar?
(A) 5 (B) 32 (C) 6 (D) 7 (E) 63

Source: 2013 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Patterning/Sequences/Series

Answer: C

Solution:

Each time Daniel reaches into the jar, he removes half of the coins that are in the jar.
Since he removes half of the coins, then the other half of the coins remain in the jar.
We summarize Daniel's progress in the table below.

Number of Times Coins are Removed	0	1	2	3	4	5	6
Number of Coins Remaining in the Jar	64	32	16	8	4	2	1

For exactly 1 coin to remain in the jar, Daniel must reach in and remove coins from the jar 6-times.

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16. The mean (average) of five consecutive even numbers is 12. The mean of the smallest and largest of these numbers is
(A) 12 (B) 10 (C) 14 (D) 8 (E) 16

Source: 2013 Gauss Grade 7 #16

Primary Topics: Data Analysis

Secondary Topics: Averages

Answer: A

Solution:

Solution 1
Consider the set of five consecutive even numbers $8, 10, 12, 14, 16$.
The mean of these five numbers is $\frac{8+10+12+14+16}{5} = \frac{60}{5}$ or 12.
If the five consecutive even numbers were smaller, their mean would be less than 12.
If the five consecutive even numbers were larger, their mean would be greater than 12.
Therefore, this is the set of five consecutive even numbers that we seek.
The mean of the smallest and largest of these five numbers is $\frac{8+16}{2} = \frac{24}{2}$ or 12.
Solution 2
The mean of five consecutive even numbers is the middle (third largest) number.
To see this, consider that the smallest of the five numbers is 4 less than the middle number, while the largest of the five numbers is 4 more than the middle number.
Thus, the mean of the smallest and largest numbers is the middle number.
Similarly, the second smallest of the five numbers is 2 less than the middle number, while the fourth largest of the five numbers is 2 more than the middle number.
Thus, the mean of these two numbers is also the middle number.
Since the mean of the five numbers is 12, then the middle number is 12.
Therefore, the mean of the largest and smallest of the five numbers is also 12.

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17. For every 3 chocolates that Claire buys at the regular price, she buys a fourth chocolate for 25 cents. Claire buys 12 chocolates in total for \$6.15. What is the regular price of one chocolate, in cents?
(A) 180 (B) 45 (C) 60 (D) 54 (E) 57

Source: 2013 Gauss Grade 7 #17

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: C

Solution:

For every 3 chocolates that Claire buys at the regular price, she buys a fourth for 25 cents.
Consider dividing the 12 chocolates that Claire buys into 3 groups of 4 chocolates.
In each group of 4, Claire buys 3 chocolates at the regular price and the fourth chocolate is purchased for 25 cents.
That is, of the 12 chocolates that Claire buys, 3 are bought at 25 cents each while the remaining 9 are purchased at the regular price.
The total cost to purchase 3 chocolates at 25 cents each is $3 \times 25 = 75$ cents.
Since Claire spent \$6.15 and the total cost of the 25 cent chocolates was 75 cents, then the cost of the regular price chocolates was $\$6.15 - \$0.75 = \$5.40$.
Since 9 chocolates were purchased at the regular price for a total of \$5.40, then the regular price of one chocolate is $\frac{\$5.40}{9} = \$0.60$ or 60 cents.

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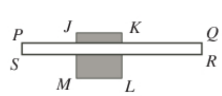
18. $JKLM$ is a square and $PQRS$ is a rectangle. If JK is parallel to PQ, $JK = 8$ and $PS = 2$, then the total area of the shaded regions is



(A) 32 (B) 16 (C) 56 (D) 48 (E) 62

Source: 2013 Gauss Grade 7 #18

Primary Topics: Geometry and Measurement

Secondary Topics: Area

Answer: D

Solution:

The total area of the shaded regions is the difference between the area of square $JKLM$ and the area of the portion of rectangle $PQRS$ that overlaps $JKLM$.
Since $JK = 8$, then the area of square $JKLM$ is 8×8 or 64.
Since JK is parallel to PQ, then the portion of $PQRS$ that overlaps $JKLM$ is a rectangle, and has length equal to JK or 8, and width equal to PS or 2.
So the area of the portion of $PQRS$ that overlaps $JKLM$ is $8 \times 2 = 16$.
Therefore the total area of the shaded regions is $64 - 16 = 48$.

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19. A special six-sided die is rolled. The probability of rolling a number that is a multiple of three is $\frac{1}{2}$. The probability of rolling an even number is $\frac{1}{3}$. A possibility for the numbers on the die is

(A) $1, 2, 3, 5, 5, 6$ (B) $1, 2, 3, 3, 5, 6$ (C) $1, 2, 3, 4, 6, 6$
(D) $1, 2, 3, 3, 4, 6$ (E) $2, 3, 3, 3, 5, 6$

Source: 2013 Gauss Grade 7 #19

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: B

Solution:

Using the special six-sided die, the probability of rolling a number that is a multiple of three is-$\frac{1}{2}$.
Since $\frac{1}{2}$ of 6 is 3, then exactly 3 numbers on the die must be multiples of 3.
Since the probability of rolling an even number is $\frac{1}{3}$ and $\frac{1}{3}$ of 6 is 2, then exactly 2 numbers on the die must be even.
The die in (A) has only 2 numbers that are multiples of 3 (3 and 6), and thus may be eliminated.
The die in (C) has 4 numbers that are even $(2, 4, 6, 6)$, and thus may be eliminated.
The die in (D) has 3 numbers that are even $(2, 4, 6)$, and thus may be eliminated.
The die in (E) has 4 numbers that are multiples of 3 $(3, 3, 3, 6)$, and thus may be eliminated.
The die in (B) has exactly 3 numbers that are multiples of 3 $(3, 3, 6)$, and exactly 2 even numbers (2 and 6), and is therefore the correct answer.

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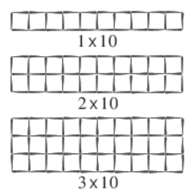
20. Toothpicks are used to make rectangular grids, as shown. Note that a total of 31 identical toothpicks are used in the 1×10 grid. How many toothpicks are used in a 43×10 grid?



(A) 913 (B) 860 (C) 871 (D) 903 (E) 946

Source: 2013 Gauss Grade 7 #20

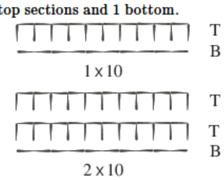
Primary Topics: Counting and Probability

Secondary Topics: Patterning/Sequences/Series | Counting

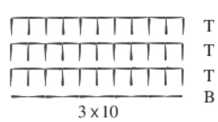
Answer: A

Solution:

In the diagram shown, the 31 identical toothpicks used in the 1×10 grid are separated into 2 sections.
The top section, T, is made from 11 vertical toothpicks and 10 horizontal toothpicks, or 21 toothpicks in total.
The bottom section, B, is made of 10 horizontal toothpicks.
The 2×10 and 3×10 grids are similarly separated into top and bottom sections, as shown.
We observe that the 1×10 grid consists of 1 top section and 1 bottom.
The 2×10 grid consists of 2 top sections and 1 bottom.



The 3×10 grid consists of 3 top sections and 1 bottom.
Continuing in this way, a grid of size $n \times 10$ will consist of n top sections and 1 bottom section, for any positive integer n.
So then a grid of size 43×10 consists of 43 top sections and 1 bottom section.
Each top section is made from 21 toothpicks and each bottom section is made from 10 toothpicks.
Thus, the total number of toothpicks in a 43×10 grid is $(43 \times 21) + (1 \times 10)$ or $903 + 10$ or 913.



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21. In the addition shown, P and Q each represent single digits, and the sum is $1PP7$. What is $P + Q$?

$$\begin{array}{r} 7\,7\,P \\ 6\,Q\,P \\ +\quad Q\,Q\,P \\ \hline 1\,P\,P\,7 \end{array}$$

(A) 9 (B) 12 (C) 14 (D) 15 (E) 13

Source: 2013 Gauss Grade 7 #21

Primary Topics: Number Sense

Secondary Topics: Digits

Answer: C

Solution:

The sum of the units column is $P+P+P=3P$.
Since P is a single digit, and $3P$ ends in a 7, then the only possibility is $P=9$.
This gives:

$$\begin{array}{cccc} & 7 & 7 & 9 \\ & 6 & Q & 9 \\ + & Q & Q & 9 \\ \hline 1 & 9 & 9 & 7 \end{array}$$

Then $3P=3\times 9=27$, and thus 2 is carried to the tens column.
The sum of the tens column becomes $2+7+Q+Q$ or $9+2Q$.
Since $9+2Q$ ends in a 9 (since $P=9$), then $2Q$ ends in $9-9=0$.
Since Q is a single digit, there are two possibilities for Q such that $2Q$ ends in 0.
These are $Q=0$ and $Q=5$.
If $Q=0$, then the sum of the tens column is 9 with no carry to the hundreds column.
In this case, the sum of the hundreds column is $7+6+Q$ or 13 (since $Q=0$); the units digit of this sum does not match the 9 in the total.
Thus, we conclude that Q cannot equal 0 and thus must equal 5.
Verifying that $Q=5$, we check the sum of the tens column again.
Since $2+7+5+5=19$, then 1 is carried to the hundreds column.
The sum of the hundreds column is $1+7+6+5=19$, as required.

Thus, $P+Q=9+5=14$ and the completed addition is shown below.

$$\begin{array}{cccc} & 7 & 7 & 9 \\ & 6 & 5 & 9 \\ + & 5 & 5 & 9 \\ \hline 1 & 9 & 9 & 7 \end{array}$$

22. An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, $2,4,6,8$ and $1,4,7,10$ are arithmetic sequences.
In the grid shown, the numbers in each row must form an arithmetic sequence and the numbers in each column must form an arithmetic sequence. The value of x is

1			
4			25
7			x
10		36	

(A) 37 (B) 28 (C) 36 (D) 43.75 (E) 46

Source: 2013 Gauss Grade 7 #22

Primary Topics: Number Sense

Secondary Topics: Patterning/Sequences/Series | Logic

Answer: A

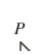
Solution:

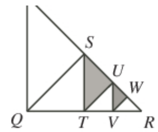
We use labels, m and n, in the fourth row of the grid, as shown.
Then, $10,m,36,n$ are four terms of an arithmetic sequence.
Since 10 and 36 are two terms apart in this sequence, and their difference is $36-10=26$, the constant added to one term to obtain the next term in the fourth row is $\frac{26}{2}$ or 13.
That is, $m=10+13=23$, and $n=36+13=49$.
(We confirm that the terms $10,23,36,49$ do form an arithmetic sequence.)
In the fourth column, 25 and n (which equals 49) are two terms apart in this sequence, and their difference is $49-25=24$. Thus, the constant added to one term to obtain the next term in the fourth column is $\frac{24}{2}$ or 12.
That is, $x=25+12=37$ (or $x=49-12=37$).
The completed grid is as shown.

1			
4			25
7			x
10	m	36	n

1	5	9	13
4	11	18	25
7	17	27	37
10	23	36	49

23. In the right-angled triangle PQR, $PQ=QR$. The segments QS,TU and VW are perpendicular to PR, and the segments ST and UV are perpendicular to QR, as shown. What fraction of $\triangle PQR$ is shaded?





(A) $\frac{3}{16}$ (B) $\frac{3}{8}$ (C) $\frac{5}{16}$ (D) $\frac{5}{32}$ (E) $\frac{7}{32}$

Source: 2013 Gauss Grade 7 #23

Primary Topics: Geometry and Measurement | Number Sense

Secondary Topics: Triangles | Fractions/Ratios | Area

Answer: D

Solution:

Since $\triangle PQR$ is isosceles with $PQ = QR$ and $\angle PQR = 90^\circ$, then $\angle QPR = \angle QRS = 45^\circ$.
Also in $\triangle PQR$, altitude QS bisects PR ($PS = SR$) forming two identical triangles, SQP and SQR.
Since these two triangles are identical, each has $\frac{1}{2}$ of the area of $\triangle PQR$.
In $\triangle SQR$, $\angle QSR = 90^\circ$, $\angle QRS = 45^\circ$, and so $\angle SQR = 45^\circ$.
Thus, $\triangle SQR$ is also isosceles with $SQ = SR$.
Then similarly, altitude ST bisects QR ($QT = TR$) forming two identical triangles, SQT and SRT.
Since these two triangles are identical, each has $\frac{1}{2}$ of the area of $\triangle SQR$ or $\frac{1}{4}$ of the area of $\triangle PQR$.
Continuing in this way, altitude TU divides $\triangle STR$ into two identical triangles, STU and RTU.
Each of these two triangles has $\frac{1}{2}$ of $\frac{1}{4}$ or $\frac{1}{8}$ of the area of $\triangle PQR$.
Continuing, altitude UV divides $\triangle RTU$ into two identical triangles, RUV and TUV.
Each of these two triangles has $\frac{1}{2}$ of $\frac{1}{8}$ or $\frac{1}{16}$ of the area of $\triangle PQR$.
Finally, altitude VW divides $\triangle RUV$ into two identical triangles, UVW and RVW.
Each of these two triangles has $\frac{1}{2}$ of $\frac{1}{16}$ or $\frac{1}{32}$ of the area of $\triangle PQR$.
Since the area of $\triangle STU$ is $\frac{1}{8}$ of the area of $\triangle PQR$, and the area of $\triangle UVW$ is $\frac{1}{32}$ of the area of $\triangle PQR$, then the total fraction of $\triangle PQR$ that is shaded is $\frac{1}{8} + \frac{1}{32} = \frac{4+1}{32}$ or $\frac{5}{32}$.

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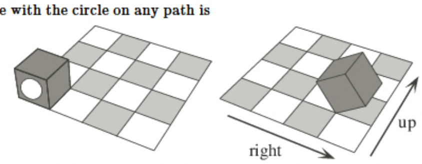
24. One face of a cube contains a circle, as shown. This cube rolls without sliding on a four by four checkerboard. The cube always begins a path on the bottom left square in the position shown and completes the path on the top right square. During each move, an edge of the cube remains in contact with the board. Each move of the cube is either to the right or up. For each path, a face of the cube contacts seven different squares on the checkerboard, including the bottom left and top right squares. The number of different squares that will not be contacted by the face with the circle on any path is



(A) 9 (B) 11 (C) 8 (D) 12 (E) 10

Source: 2013 Gauss Grade 7 #24

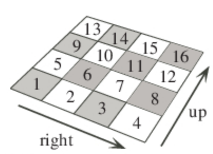
Primary Topics: Counting and Probability

Secondary Topics: Counting | Logic

Answer: C

Solution:

We begin by numbering the checkerboard squares from 1 to 16, as shown, so that we may refer to each of them specifically.
We denote a move "up" by the letter U and a move "right" by R.
We will begin by determining which of the 16 squares will not be touched by the face with the circle and then proceed to show that the remaining squares will be touched by the face with the circle.
Since the cube begins on square 1 and the circle is facing out, square 1 will not be touched by the face with the circle.



Similarly, each of the squares 2, 3, 4 can only be reached by moving the cube right (the sequence of moves to reach each of these three squares is R, RR and RRR, respectively), and in each case the circle remains facing out.
Squares 2, 3 and 4 will not be touched by the face with the circle.
Squares 5 and 9 can only be reached by moving the cube up (the sequence of moves to reach each of these two squares is U and UU, respectively).
In either case, the face with the circle will not touch squares 5 and 9.
Square 6 can be reached with two different sequences of moves, RU or UR.
In both cases, the face with the circle will not touch square 6.
Square 10 can be reached with three different sequences of moves, UUR, URU or RUU.
In all three cases, the face with the circle will not touch square 10.
In turns out that these eight squares $(1, 2, 3, 4, 5, 6, 9, 10)$ are the only squares that will not be touched by the face with the circle on any path.
The table below lists sequences of moves that demonstrate how each of the remaining eight squares will be touched by the face with the circle.
The second column lists the sequence of moves, while the third column lists the position of the face with the circle as the cube progresses through the sequence of moves.
We have used the letters F for front, B for back, T for top, O for bottom, L for left, and R for right to indicate the location of the face containing the circle.

Square	Sequence of Moves	Position of the Circle
7	URR	TRO
8	$RURR$	$FTRO$
11	$URUR$	$TRRO$
12	$RURUR$	$FTRRO$

13	UUU	TBO
14	$RUUU$	$FTBO$
15	$RRUUU$	$FFTBO$
16	$RRRUUU$	$FFFTBO$

Therefore, the number of different squares that will not be contacted by the face with the circle on any path is 8.

25. A box contains a total of 400 tickets that come in five colours: blue, green, red, yellow and orange. The ratio of blue to green to red tickets is $1:2:4$. The ratio of green to yellow to orange tickets is $1:3:6$. What is the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected?
(A) 50 (B) 246 (C) 148 (D) 196 (E) 115

Source: 2013 Gauss Grade 7 #25

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Optimization

Answer: D

Solution:

We denote the number of tickets of each of the five colours by the first letter of the colour.
We are given that $b:g:r=1:2:4$ and that $g:y:o=1:3:6$.
Through multiplication by 2, the ratio $1:3:6$ is equivalent to the ratio $2:6:12$.
Thus, $g:y:o=2:6:12$.
We chose to scale this ratio by a factor of 2 so that the only colour common to the two given ratios, green, now has the same number in both of these ratios.
That is, $b:\mathbf{g}:r=1:\mathbf{2}:4$ and $\mathbf{g}:y:o=\mathbf{2}:6:12$ and since the term g is 2 in each ratio, then we can combine these to form a single ratio, $b:g:r:y:o=1:2:4:6:12$.
This ratios tells us that for every blue ticket, there are 2 green, 4 red, 6 yellow, and 12 orange tickets.
Thus, if there was only 1 blue ticket, then there would be $1+2+4+6+12=25$ tickets in total.
However, we are given that the box contains 400 tickets in total.
Therefore, the number of blue tickets in the box is $\frac{400}{25}=16$.
Through multiplication by 16, the ratio $b:g:r:y:o=1:2:4:6:12$ becomes $b:g:r:y:o=16:32:64:96:192$.
(Note that there are $16+32+64+96+192=400$ tickets in total.)
Next, we must determine the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected.
It is important to consider that up to 49 tickets of any one colour could be selected without being able to ensure that 50 tickets of one colour have been selected.
That is, it is possible that the first 195 tickets selected could include exactly 49 orange, 49 yellow, 49 red, all 32 green, and all 16 blue tickets $(49+49+49+32+16=195)$.
Since all green and blue tickets would have been drawn from the box, the next ticket selected would have to be the 50^{th} orange, yellow or red ticket.
Thus, the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected is 196.