The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7
Solutions

1. The value of $202-101+9$ is equal to
(A) 120 (B) 110 (C) 111 (D) 109 (E) 92

Source: 2012 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: B

Solution:

Evaluating, $202-101+9=101+9=110$.

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2. Which of the following numbers is equal to 33 million?
(A) 3 300 000 (B) 330 000 (C) 33 000 (D) 33 000 000 (E) 330 000 000

Source: 2012 Gauss Grade 7 #2

Primary Topics: Number Sense

Secondary Topics: Digits

Answer: D

Solution:

Written numerically, the number 33 million is 33 000 000.

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3. A six-sided die has the numbers one to six on its sides. What is the probability of rolling a five?
(A) $\frac{2}{6}$ (B) $\frac{1}{6}$ (C) $\frac{5}{6}$ (D) $\frac{3}{6}$ (E) $\frac{4}{6}$

Source: 2012 Gauss Grade 7 #3

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: B

Solution:

Each of the numbers $1, 2, 3, 4, 5$, and 6 is equally likely to appear when the die is rolled.
Since there are six numbers, then each has a one in six chance of being rolled.
The probability of rolling a 5 is $\frac{1}{6}$.

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4. The largest fraction in the set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\right\}$ is
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{1}{10}$

Source: 2012 Gauss Grade 7 #4

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: A

Solution:

A positive fraction increases in value as its numerator increases and also increases in value as its denominator decreases.
Since the numerators of all five fractions are equal, then the largest of these is the fraction with the smallest denominator.
The largest fraction in the set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\right\}$ is $\frac{1}{2}$.

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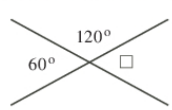
5. Two straight lines intersect as shown. The measure of the angle marked $\square$ is



(A) 60° (B) 120° (C) 30° (D) 300° (E) 180°

Source: 2012 Gauss Grade 7 #5

Primary Topics: Geometry and Measurement

Secondary Topics: Angles

Answer: A

Solution:

Solution 1
The angle marked $\square$ is vertically opposite the angle measuring 60°.
Since vertically opposite angles are equal, then the measure of the angle marked $\square$ is also 60°
.

Solution 2
The measure of a straight angle is 180°.
Together, the angle measuring 120° and the angle marked $\square$ make up a straight angle.
That is, $120^\circ + \square = 180^\circ$.
Therefore, the angle marked $\square$ is 60°.

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6. Fifteen times a number equals three hundred. The number is

(A) 20 (B) 10 (C) 60 (D) 30 (E) 25

Source: 2012 Gauss Grade 7 #6

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: A

Solution:

Since 15 times the number is 300, then the number equals 300 divided by 15, or 20.

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7. Which of the following statements is true?
(A) 0 is less than -5 (B) 7 is less than -1 (C) 10 is less than $\frac{1}{4}$
(D) -1 is less than -3 (E) -8 is less than -2

Source: 2012 Gauss Grade 7 #7

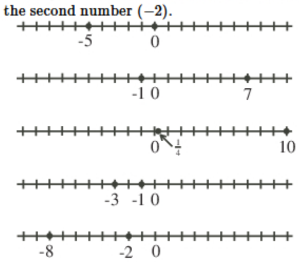
Primary Topics: Number Sense

Secondary Topics: Inequalities | Logic | Inequalities

Answer: E

Solution:

Consider placing each of the two numbers from each answer on a number line.
Numbers decrease in value as we move to the left along a number line.
Since the first number listed in the pair must be less than the second, we want the first number to be to the left of the second number on the number line.
The five possible answers are each given on the number lines shown.
The last number line is the only one for which the first number listed (-8) is positioned to the left of (is less than) the second number (-2).



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8. Bailey scores on six of her eight shots. The percentage of shots that she *does not* score on is
(A) 2 (B) 40 (C) 10 (D) 20 (E) 25

Source: 2012 Gauss Grade 7 #8

Primary Topics: Number Sense

Secondary Topics: Percentages | Fractions/Ratios

Answer: E

Solution:

Since Bailey scores on 6 of her 8 shots, then she misses on $8 - 6 = 2$ shots.
If she misses on $\frac{2}{8} = \frac{1}{4}$ of her shots, then the percentage of shots that she does not score is $\frac{1}{4} \times 100\% = 25\%$.

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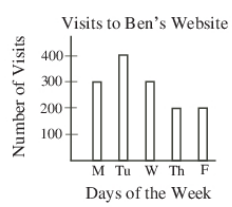
9. Ben recorded the number of visits to his website from Monday to Friday as shown in the bar graph. The mean (average) number of visits per day to his website over the 5 days is



(A) less than 100 (B) between 100 and 200 (C) between 200 and 300
(D) between 300 and 400 (E) more than 400

Source: 2012 Gauss Grade 7 #9

Primary Topics: Data Analysis

Secondary Topics: Averages | Graphs

Answer: C

Solution:

The number of visits to Ben's website from Monday to Friday can be read from the graph.
These are: $300, 400, 300, 200, 200$.
The mean number of visits per day is found by adding these five totals and then dividing the sum by 5.
Thus, the mean is $(300 + 400 + 300 + 200 + 200) \div 5 = 1400 \div 5$ or 280.
The mean number of visits per day to Ben's website over the 5 days is between 200 and 300.

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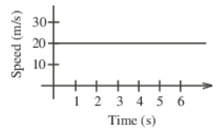
10. Using the graph, the number of seconds required for a vehicle to travel a total distance of 100 m is

Vehicle's Speed vs. Time Graph



(A) 2.5 (B) 20 (C) 8 (D) 10 (E) 5

Source: 2012 Gauss Grade 7 #10
Primary Topics: Data Analysis
Secondary Topics: Measurement | Rates | Graphs

Answer: E
Solution:

The graph shows that the vehicle travels at a constant speed of 20 m/s.
Travelling at 20 m/s, it will take the vehicle $100 \div 20 = 5$ seconds to travel 100 metres.

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11. The perimeter of a square is 36 cm. The area of the square, in cm^2, is
(A) 24 (B) 81 (C) 36 (D) 1296 (E) 324

Source: 2012 Gauss Grade 7 #11
Primary Topics: Geometry and Measurement
Secondary Topics: Perimeter | Area

Answer: B
Solution:

Since the four sides of a square are equal in length and the perimeter is 36 cm, then each side has length- $\frac{36}{4} = 9$ cm.
The area of the square is the product of the length and width, which are each equal to 9 cm.
Therefore, the area of the square, in cm^2, is $9 \times 9 = 81$.

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12. Which of the following is *not* equal to $\frac{15}{4}$?
(A) 3.75 (B) $\frac{14+1}{3+1}$ (C) $\frac{3}{4} + 3$ (D) $\frac{5}{4} \times \frac{3}{4}$ (E) $\frac{21}{4} - \frac{5}{4} - \frac{1}{4}$

Source: 2012 Gauss Grade 7 #12
Primary Topics: Number Sense
Secondary Topics: Fractions/Ratios | Operations

Answer: D
Solution:

Since $\frac{14+1}{3+1} = \frac{15}{4}$ and $\frac{21}{4} - \frac{5}{4} - \frac{1}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$, then answers (B) and (E) both simplify to $\frac{15}{4}$.
Written as a mixed fraction, $\frac{15}{4}$ is equal to $3\frac{3}{4}$.
Since $3.75 = 3\frac{3}{4} = 3 + \frac{3}{4}$, then answers (A) and (C) both simplify to $3\frac{3}{4}$ and thus are equivalent to $\frac{15}{4}$.
Simplifying answer (D), $\frac{5}{4} \times \frac{3}{4} = \frac{5\times3}{4\times4} = \frac{15}{16}$.
Thus, $\frac{5}{4} \times \frac{3}{4}$ is not equal to $\frac{15}{4}$.

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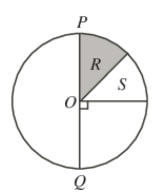
13. On the spinner shown, PQ passes through centre O. If areas labelled R and S are equal, then what percentage of the time will a spin stop on the shaded region?

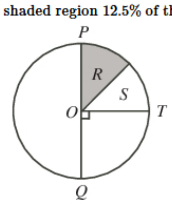


(A) 50% (B) 22.5% (C) 25% (D) 45% (E) 12.5%

Source: 2012 Gauss Grade 7 #13
Primary Topics: Geometry and Measurement
Secondary Topics: Probability | Percentages

Answer: E
Solution:

Since PQ passes through centre O, then it is a diameter of the circle.
Since $\angle QOT = 90^\circ$, then $\angle POT = 180^\circ - 90^\circ = 90^\circ$.
Thus, the area of sector POT is $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ or 25% of the area of the circle.
Since the areas labelled R and S are equal, then each is
$25\% \div 2 = 12.5\%$ of the area of the circle.
Therefore, a spin will stop on the shaded region 12.5% of the time.



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14. The digits 2, 4, 6 and 8 are each used once to create two 2-digit numbers. What is the largest possible difference between the two 2-digit numbers?
(A) 66 (B) 62 (C) 58 (D) 44 (E) 36

Source: 2012 Gauss Grade 7 #14

Primary Topics: Number Sense

Secondary Topics: Digits | Operations

Answer: B

Solution:

To make the difference as large as possible, we make one number as large as possible and the other number as small as possible.
The tens digit of a number contributes more to its value than its units digit.
Thus, we construct the largest possible number by choosing 8 (the largest digit) to be its tens digit, and by choosing 6 (the second largest digit) to be the ones digit.
Similarly, we construct the smallest possible number by choosing 2 (the smallest digit) to be its tens digit, and 4 (the second smallest digit) to be its ones digit.
The largest possible difference is $86 - 24 = 62$.

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15. If snow falls at a rate of 1 mm every 6 minutes, then how many *hours* will it take for 1 m of snow to fall?
(A) 33 (B) 60 (C) 26 (D) 10 (E) 100

Source: 2012 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: E

Solution:

Since 1 mm of snow falls every 6 minutes, then 10 mm will fall every $6 \times 10 = 60$ minutes.
Since 10 mm is 1 cm and 60 minutes is 1 hour, then 1 cm of snow will fall every 1 hour.
Since 1 cm of snow falls every 1 hour, then 100 cm will fall every $1 \times 100 = 100$ hours.

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16. The number 503 is a prime number. How many positive integers are factors of 2012?
(A) 2 (B) 3 (C) 7 (D) 6 (E) 8

Source: 2012 Gauss Grade 7 #16

Primary Topics: Number Sense

Secondary Topics: Prime Numbers | Factoring

Answer: D

Solution:

Both 1 and 2012 are obvious positive integer factors of 2012.
Since 2012 is an even number and $2012 \div 2 = 1006$, then both 2 and 1006 are factors of 2012.
Since 1006 is also even then 2012 is divisible by 4.
Since $2012 \div 4 = 503$, then both 4 and 503 are factors of 2012.
We are given that 503 is a prime number; thus there are no additional factors of 503 and hence there are no additional factors of 2012.
The factors of 2012 are 1 and 2012, 2 and 1006, 4 and 503.
Therefore, there are 6 positive integers that are factors of 2012.

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17. The ratio of boys to girls at Gauss Public School is 8 : 5. If there are 128 boys at the school, then how many students are there at the school?
(A) 218 (B) 253 (C) 208 (D) 133 (E) 198

Source: 2012 Gauss Grade 7 #17

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: C

Solution:

Solution 1
Since the ratio of boys to girls is 8 : 5, then for every 5 girls there are 8 boys.
That is, the number of girls at Gauss Public School is $\frac{5}{8}$ of the number of boys.
Since the number of boys at the school is 128, the number of girls is $\frac{5}{8} \times 128 = \frac{640}{8} = 80$.
The number of students at the school is the number of boys added to the number of girls or $128 + 80 = 208$.
Solution 2
Since the ratio of boys to girls is 8 : 5, then for every 8 boys there are $8 + 5 = 13$ students.
That is, the number of students at Gauss Public School is $\frac{13}{8}$ of the number of boys.
Since the number of boys at the school is 128, the number of students is $\frac{13}{8} \times 128 = \frac{1664}{8} = 208$.

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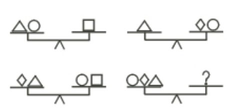
18. All four scales shown are balanced. One possible replacement for the ? is



(A) △□ (B) ◇△ (C) ○□ (D) □◇ (E) △○

Source: 2012 Gauss Grade 7 #18

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving | Operations

Answer: D

Solution:

In turn, we may use each of the three known scales to find a way to balance a circle, a diamond and a triangle.
Since many answers are possible, we must then check our solution to see if it exists among

the five answers given.
First consider the scale at the top right.
A diamond and a circle are balanced by a triangle.
If we were to add a triangle to both sides of this scale, then it would remain balanced and the right side of this scale would contain what we are trying to balance, a circle, a diamond and a triangle.
That is, a circle, a diamond and a triangle are balanced by two triangles.
However, two triangles is not one of the five answers given.
Next, consider the scale at the top left.
A triangle and a circle are balanced by a square.
If we were to add a diamond to both sides of this scale, then it would remain balanced and the left side of this scale would contain what we are trying to balance, a circle, a diamond and a triangle.
That is, a circle, a diamond and a triangle are balanced by a square and a diamond.
This answer is given as one of the five answers.
In the context of a multiple choice contest, we do not expect that students will verify that the other four answers do not balance a circle, a diamond and a triangle. However, it is worth noting that it can be shown that they do not.

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19. A set of five different positive integers has a mean (average) of 20 and a median of 18. What is the greatest possible integer in the set?
(A) 60 (B) 26 (C) 46 (D) 12 (E) 61

Source: 2012 Gauss Grade 7 #19

Primary Topics: Data Analysis

Secondary Topics: Averages

Answer: A

Solution:

In an ordered list of five integers, the median is the number in the middle or third position.
Thus if we let the set of integers be a, b, c, d, e, ordered from smallest to largest, then $c = 18$.
Since the average is fixed (at 20), e (the largest number in the set) is largest when a, b and d are as small as possible.
Since the numbers in the set are different positive integers, the smallest that a can be is 1 and the smallest that b can be is 2.
Our set of integers is now $1, 2, 18, d, e$.
Again, we want d to be as small as possible, but it must be larger than the median 18.
Therefore, $d = 19$.
Since the average of the 5 integers is 20, then the sum of the five integers is $5 \times 20 = 100$.
Thus, $1 + 2 + 18 + 19 + e = 100$ or $40 + e = 100$, and so $e = 60$.
The greatest possible integer in the set is 60.

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20. Chris lies on Fridays, Saturdays and Sundays, but he tells the truth on all other days. Mark lies on Tuesdays, Wednesdays and Thursdays, but he tells the truth on all other days. On what day of the week would they both say: "Tomorrow, I will lie."?
(A) Monday (B) Thursday (C) Friday (D) Sunday (E) Tuesday

Source: 2012 Gauss Grade 7 #20

Primary Topics: Other

Secondary Topics: Logic | Games

Answer: B

Solution:

If either Chris or Mark says, "Tomorrow, I will lie."-on a day that he tells a lie, then it actually means that tomorrow he will tell the truth (since he is lying).
This can only occur when he lies and then tells the truth on consecutive days.
For Chris, this only happens on Sunday, since he lies on Sunday but tells the truth on Monday.
For Mark, this only happens on Thursday, since he lies on Thursday but tells the truth on Friday.
Similarly, if either Chris or Mark says, "Tomorrow, I will lie."-on a day that they tell the truth, then it means that tomorrow they will lie (since they are telling the truth).
This can only occur when they tell the truth and then lie on consecutive days.
For Chris, this only happens on Thursday, since he tells the truth on Thursday but lies on Friday.
For Mark, this only happens on Monday, since he tells the truth on Monday but lies on Tuesday.
Therefore, the only day of the week that they would both say, "Tomorrow, I will lie.", is Thursday.

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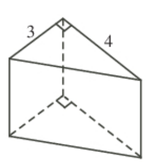
21. A triangular prism has a volume of 120 cm^3. Two edges of the triangular faces measure 3 cm and 4 cm, as shown. The height of the prism, in cm, is



(A) 12 (B) 20 (C) 10 (D) 16 (E) 8

Source: 2012 Gauss Grade 7 #21

Primary Topics: Geometry and Measurement

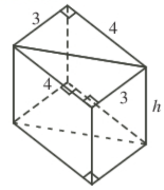
Secondary Topics: Volume | Prisms | Measurement

Answer: B

Solution:

The triangular prism given can be created by slicing the 3 cm by 4-cm base of a rectangular prism with equal height across its diagonal.
That is, the volume of the triangular prism in question is one half of the volume of the rectangular prism shown.

Since the volume of the triangular prism is 120 cm^3, then the volume of this rectangular prism is $2 \times 120 = 240$ cm^3.
The volume of the rectangular prism equals the area of its base 3×4 times its height, h.
Since the volume is 240, then $3 \times 4 \times h = 240$ or $12h = 240$, so $h = \frac{240}{12} = 20$.
Since the height of this rectangular prism is equal to the height of the triangular prism in question, then the required height is 20-cm.



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22. A quiz has three questions, with each question worth one mark. If 20% of the students got 0 questions correct, 5% got 1 question correct, 40% got 2 questions correct, and 35% got all 3 questions correct, then the overall class mean (average) mark was
(A) 1.8 (B) 1.9 (C) 2 (D) 2.1 (E) 2.35

Source: 2012 Gauss Grade 7 #22

Primary Topics: Data Analysis

Secondary Topics: Averages | Percentages

Answer: B

Solution:

Without changing the overall class mean, we may consider that the class has 100 students.
That is, 20 students got 0 questions correct, 5 students got 1 question correct, 40 students got 2 questions correct, and 35 students got 3 questions correct.
The combined number of marks achieved by all 100 students in the class is then,

$$(20 \times 0) + (5 \times 1) + (40 \times 2) + (35 \times 3) = 0 + 5 + 80 + 105 = 190.$$

Since the 100 students earned a total of 190 marks, then the overall class average was $\frac{190}{100} = 1.9$.

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23. The number N is the product of all positive odd integers from 1 to 99 that do not end in the digit 5. That is,
$N = 1 \times 3 \times 7 \times 9 \times 11 \times 13 \times 17 \times 19 \times \cdots \times 91 \times 93 \times 97 \times 99$. The units digit of N is
(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

Source: 2012 Gauss Grade 7 #23

Primary Topics: Number Sense

Secondary Topics: Digits | Divisibility

Answer: A

Solution:

The units digit of any product is given by the units digit of the product of the units digits of the numbers being multiplied.
For example, the units digit of the product 12×53 is given by the product 2×3, so it is 6.
Thus to determine the units digit of N, we need only consider the product of the units digits of the numbers being multiplied to give N.
The units digits of the numbers in the product N are $1, 3, 7, 9, 1, 3, 7, 9, \ldots$, and so on.
That is, the units digits $1, 3, 7, 9$ are repeated in each group of four numbers in the product.
There are ten groups of these four numbers, $1, 3, 7, 9$, in the product.
We first determine the units digit of the product $1 \times 3 \times 7 \times 9$.
The units digit of 1×3 is 3.
The units digit of the product 3×7 is 1 (since $3 \times 7 = 21$).
The units digit of 1×9 is 9.
Therefore, the units digit of the product $1 \times 3 \times 7 \times 9$ is 9.
(We could have calculated the product $1 \times 3 \times 7 \times 9 = 189$ to determine the units digit.)
This digit 9 is the units digits of the product of each group of four successive numbers in-N.
Thus, to determine the units digit of N we must determine the units digit of
$9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$.
This product is equal to $81 \times 81 \times 81 \times 81 \times 81$.
Since we are multiplying numbers with units digit 1, then the units digit of the product is 1.

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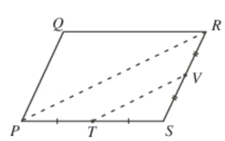
24. $PQRS$ is a parallelogram with area 40. If T and V are the midpoints of sides PS and RS respectively, then the area of $PRVT$ is



(A) 10 (B) 12 (C) 15 (D) 16 (E) 18

Source: 2012 Gauss Grade 7 #24

Primary Topics: Geometry and Measurement

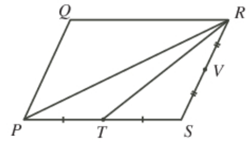
Secondary Topics: Area | Quadrilaterals

Answer: C

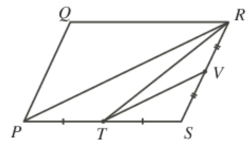
Solution:

Diagonal PR divides parallelogram $PQRS$ into two equal areas.
That is, the area of $\triangle PRS$ is one half of the area of parallelogram $PQRS$, or 20.
In $\triangle PRS$, we construct median RT.
(A median is a line segment that joins a vertex of a triangle to the midpoint of its opposite side.)
Median RT divides $\triangle PRS$ into two equal areas since its base, PS, is halved while the height

Median RT divides $\triangle PRS$ into two equal areas since its base, PS, is halved while the height remains the same.
That is, the area of $\triangle TRS$ is one half of the area of $\triangle PRS$, or 10.



Similarly, we construct median TV in $\triangle TRS$, as shown.
Median TV divides $\triangle TRS$ into two equal areas since its base, SR, is halved while the height remains the same.
That is, the area of $\triangle TVS$ is one half of the area of $\triangle TRS$, or 5.
The area of $PRVT$ is equal to the area of $\triangle TVS$ subtracted from the area of $\triangle PRS$, or $20 - 5 = 15$.



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25. The positive integers are arranged in rows and columns as shown below.

Row 1	1					
Row 2	2	3				
Row 3	4	5	6			
Row 4	7	8	9	10		
Row 5	11	12	13	14	15	
Row 6	16	17	18	19	20	21
			⋮			

More rows continue to list the positive integers in order, with each new row containing one more integer than the previous row. How many integers less than 2000 are in the *column* that contains the number 2000?
(A) 15 (B) 19 (C) 17 (D) 16 (E) 18

Source: 2012 Gauss Grade 7 #25

Primary Topics: Number Sense

Secondary Topics: Patterning/Sequences/Series

Answer: D

Solution:

A very useful and well-known formula allows us to determine the sum of the first n positive integers, $1 + 2 + 3 + 4 + \cdots + (n-1) + n$.
The formula says that this sum, $1 + 2 + 3 + 4 + \cdots + (n-1) + n$, is equal to $\frac{n(n+1)}{2}$ (justification of this formula is included at the end of the solution).
For example, if $n = 6$ then $1 + 2 + 3 + 4 + 5 + 6 = \frac{6(6+1)}{2} = \frac{6 \times 7}{2} = \frac{42}{2} = 21$.
You can check that this formula gives the correct sum, 21, by mentally adding the positive integers from 1 to 6.
In the table given, there is 1 number in Row 1, there are 2 numbers in Row 2, 3 numbers in Row 3, and so on, with n numbers in Row n.
The numbers in the rows list the positive integers in order beginning at 1 in Row 1, with each new row containing one more integer than the previous row.
Thus, the last number in each row is equal to the sum of the number of numbers in the table up to that row.
For example, the last number in Row 4 is 10, which is equal to the sum of the number of numbers in rows $1, 2, 3$, and 4.
But the number of numbers in each row is equal to the row number.
So 10 is equal to the sum $1 + 2 + 3 + 4$.
That is, the last number in Row n is equal to the sum $1 + 2 + 3 + 4 + \cdots + (n-1) + n$, which is equal to $\frac{n(n+1)}{2}$.
We may now use this formula to determine in what row the number 2000 appears.
Using trial and error, we find that since $\frac{62(63)}{2} = 1953$, then the last number in Row 62 is 1953.
Similarly, since $\frac{63(64)}{2} = 2016$, then the last number in Row 63 is 2016.
Since 2000 is between 1953 and 2016, then 2000 must appear somewhere in Row 63.
To find how many integers less than 2000 are in the column that contains the number 2000, we must determine in which column the number 2000 appears.
Further, we must determine how many numbers there are in that column above the 2000 (since all numbers in that column in rows below the 63^{rd} are larger than 2000).
We know that 2016 is the last number in Row 63 and since it is the last number, it will have no numbers in the column above it.
Moving backward (to the left) from 2016, the number 2015 will have 1 number in the column above it, 2014 will have 2 numbers in the column above it, and so on.
That is, if we move k numbers to the left of 2016, that table entry will have k numbers in the column above it.
In other words, if the number $2016 - k$ appears in Row 63, then there are k integers less than it in the column that contains it.
Since we know that 2000 appears in this 63^{rd} row, then $2016 - k = 2000$ means that $k = 16$.
Thus, there are 16 integers less than 2000 in the column that contains the number 2000.
Verification of the Formula: $1 + 2 + 3 + 4 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$
If we let the sum of the first n positive integers be S, then
$S = 1 + 2 + 3 + 4 + \cdots + (n-1) + n$.
If this same sum is written in the reverse order, then
$S = n + (n-1) + (n-2) + (n-3) + \cdots + 2 + 1$.
Adding the right sides of these two equations,

$$\begin{array}{cccccccccccccccc} & 1 & + & 2 & + & 3 & + & 4 & + & \ldots & + & (n-1) & + & n \\ + & n & + & (n-1) & + & (n-2) & + & (n-3) & + & \ldots & + & 2 & + & 1 \\ \hline = & (n+1) & + & (n+1) & + & (n+1) & + & (n+1) & + & \ldots & + & (n+1) & + & (n+1) \end{array}$$

In this sum there are n occurrences of $(n+1)$, hence the sum is $n(n+1)$.
However, this sum represents $S + S$ or $2S$, so if $2S = n(n+1)$ then $S = \frac{n(n+1)}{2}$.