The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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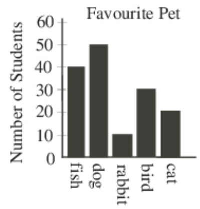
Gauss Contest Grade 7
Solutions

1. The grade 7 students at Gauss Public School were asked, "What is your favourite pet?" The number of students who chose fish is



(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

Source: 2010 Gauss Grade 7 #1
Primary Topics: Data Analysis
Secondary Topics: Graphs

Answer: D
Solution:
Reading the number on the vertical axis corresponding to the pet \emph{fish}, we find that 40 students chose fish as their favourite pet.

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2. Tanya scored 20 out of 25 on her math quiz. What percent did she score?
(A) 75 (B) 95 (C) 80 (D) 20 (E) 45

Source: 2010 Gauss Grade 7 #2
Primary Topics: Data Analysis
Secondary Topics: Percentages

Answer: C
Solution:
By dividing, we find the fraction $\frac{20}{25}$ is equivalent to the decimal 0.80.
We convert this to a percent by multiplying by 100%.
Thus, Tanya scored $0.80 \times 100\% = 80\%$ on her math quiz.

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3. The value of $4 \times 5 + 5 \times 4$ is
(A) 160 (B) 400 (C) 100 (D) 18 (E) 40

Source: 2010 Gauss Grade 7 #3
Primary Topics: Number Sense
Secondary Topics: Operations

Answer: E
Solution:
Using the correct order of operations, $4 \times 5 + 5 \times 4 = 20 + 20 = 40$.

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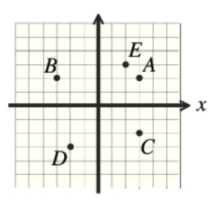
4. In the diagram, the point with coordinates $(-2, -3)$ is located at



(A) A (B) B (C) C (D) D (E) E

Source: 2010 Gauss Grade 7 #4
Primary Topics: Geometry and Measurement
Secondary Topics: Graphs

Answer: D
Solution:
To find the location of the point $(-2, -3)$, we begin at the origin, $(0, 0)$, and move left 2 units and down 3 units.
The point $(-2, -3)$ is located at D.

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5. Chaz gets on the elevator on the eleventh floor. The elevator goes down two floors, then stops. Then the elevator goes down four more floors and Chaz gets off the elevator. On what floor does Chaz get off the elevator?
(A) 7th floor (B) 9th floor (C) 4th floor (D) 5th floor (E) 6th floor

Source: 2010 Gauss Grade 7 #5
Primary Topics: Number Sense

Primary Topics: Number Sense
Secondary Topics: Operations

Answer: D

Solution:

Going down 2 floors from the 11th floor brings Chaz to the 9th floor.
Going down 4 floors from the 9th floor brings Chaz to the 5th floor.
Thus, Chaz gets off the elevator on the 5th floor.

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6. If $10.0003 \times \square = 10000.3$, the number that should replace the $\square$ is
(A) 100 (B) 1000 (C) 10000 (D) 0.001 (E) 0.0001

Source: 2010 Gauss Grade 7 #6

Primary Topics: Algebra and Equations

Secondary Topics: Decimals | Digits

Answer: B

Solution:

The answer, 10000.3, is 1000 times bigger than 10.0003. This can be determined either by dividing 10000.3 by 10.0003 or by recognizing that the decimal point in 10.0003 is moved three places to the right to obtain 10000.3. Thus, the number that should replace the $\square$ is 1000.

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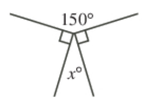
7. In the diagram, the value of x is



(A) 40 (B) 35 (C) 150 (D) 30 (E) 25

Source: 2010 Gauss Grade 7 #7

Primary Topics: Geometry and Measurement

Secondary Topics: Angles

Answer: D

Solution:

The four angles shown, 150°, 90°, x°, and 90°, form a complete rotation, a 360° angle.
Thus, $150^\circ + 90^\circ + x^\circ + 90^\circ = 360^\circ$, or $x^\circ = 360^\circ - 150^\circ - 90^\circ - 90^\circ = 30^\circ$.

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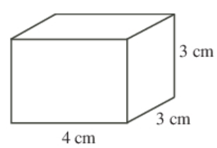
8. How many 1 cm $\times$ 1 cm $\times$ 1 cm blocks are needed to build the solid rectangular prism shown?



(A) 10 (B) 12 (C) 33 (D) 66 (E) 36

Source: 2010 Gauss Grade 7 #8

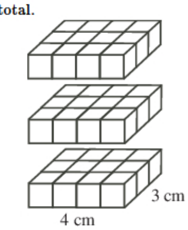
Primary Topics: Geometry and Measurement

Secondary Topics: Volume | Prisms

Answer: E

Solution:

Solution 1
To build the solid rectangular prism, we could first construct the 4-cm by 3 cm base using $4 \times 3 = 12$ of the 1 cm $\times$ 1 cm $\times$ 1 cm blocks.
Two more layers identical to the first layer, placed on top of the first layer, would give the prism its required 3 cm height.
This would require 12 more 1 cm $\times$ 1 cm $\times$ 1 cm blocks in layer two and 12 more in layer three, or $12 \times 3 = 36$ blocks in total.



Solution 2
Equivalently, this question is asking for the volume of the rectangular prism.
The volume of a prism is the area of the base times the height,
or $V = 4\text{ cm} \times 3\text{ cm} \times 3\text{ cm} = 36\text{ cm}^3$.
Since the volume of each of the 1 cm $\times$ 1 cm $\times$ 1 cm blocks is 1 cm^3,
then 36 blocks are needed to build the solid rectangular prism.
(The prism can actually be built with 36 blocks as seen in Solution 1.)

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9. The time on a digital clock reads 3:33. What is the shortest length of time, in minutes, until all of the digits are again equal to each other?
(A) 71 (B) 60 (C) 142 (D) 222 (E) 111

Source: 2010 Gauss Grade 7 #9

Source: 2010 Gauss Grade 7 #9

Primary Topics: Number Sense

Secondary Topics: Logic | Operations

Answer: A

Solution:

If the time reads 3:33, the next time that all of the digits on the clock are equal to one another is 4:44. Since the amount of time between 3:33 and 4:44 is 1 hour and 11 minutes, the shortest length of time in minutes is $60 + 11 = 71$.

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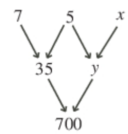
10. Each number below the top row is the product of the number to the right and the number to the left in the row immediately above it. What is the value of x?



(A) 8 (B) 4 (C) 7 (D) 5 (E) 6

Source: 2010 Gauss Grade 7 #10

Primary Topics: Algebra and Equations

Secondary Topics: Factoring

Answer: B

Solution:

Since 700 is the product of 35 and y, then $35 \times y = 700$ or $y = 700 \div 35 = 20$.
Since 20 is the product of 5 and x, then $5 \times x = 20$ or $x = 20 \div 5 = 4$.

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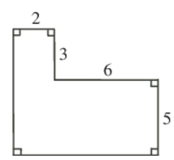
11. The area of the figure, in square units, is



(A) 36 (B) 64 (C) 46 (D) 58 (E) 32

Source: 2010 Gauss Grade 7 #11

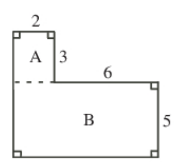
Primary Topics: Geometry and Measurement

Secondary Topics: Area

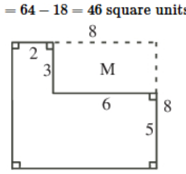
Answer: C

Solution:

Solution 1
We divide the shape into two rectangles, A and B, by constructing the dotted line segment of length 2 units shown.
The area of rectangle-A is $2 \times 3 = 6$ square units.
The length of rectangle B is 6 units plus the length of the dotted line segment, or $6 + 2 = 8$.
Thus, the area of rectangle B is $8 \times 5 = 40$ square units.
The area of the entire figure is the sum of the areas of rectangles A and B, or $6 + 40 = 46$ square units.



Solution 2
By constructing the dotted lines shown, we form a rectangle with length $2 + 6 = 8$ units and width $5 + 3 = 8$ units (in fact, this large rectangle is a square).
We find the required area by subtracting the area of rectangle-M from the area of the 8 by 8 square.
Thus, the area is $(8 \times 8) - (6 \times 3) = 64 - 18 = 46$ square units.



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12. Recycling 1 tonne of paper will save 24 trees. If 4 schools each recycle $\frac{3}{4}$ of a tonne of paper, then the total number of trees this will save is
(A) 24 (B) 72 (C) 18 (D) 126 (E) 80

Source: 2010 Gauss Grade 7 #12

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: B

Solution:

If 4 schools each recycle $\frac{3}{4}$ of a tonne of paper, then combined, they recycle $4 \times \frac{3}{4} = \frac{12}{4} = 3$ tonnes of paper.

Since recycling 1 tonne of paper will save 24 trees, recycling 3 tonnes of paper will save $3 \times 24 = 72$ trees.

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13. If the mean (average) of five consecutive integers is 21, the smallest of the five integers is
(A) 17 (B) 21 (C) 1 (D) 18 (E) 19

Source: 2010 Gauss Grade 7 #13

Primary Topics: Number Sense

Secondary Topics: Patterning/Sequences/Series | Averages

Answer: E

Solution:

Solution 1
The mean of 5 consecutive integers is equal to the number in the middle.
Since the numbers have a mean of 21, if we were to distribute the quantities equally, we would have 21, 21, 21, 21, and 21.
Since the numbers are consecutive, the second number is 1 less than the 21 in the middle, while the fourth number is 1 more than the 21 in the middle.
Similarly, the first number is 2 less than the 21 in the middle, while the fifth number is 2 more than the 21 in the middle.
Thus, the numbers are $21 - 2$, $21 - 1$, 21, $21 + 1$, $21 + 2$.
The smallest of 5 consecutive integers having a mean of 21, is 19.
Solution 2
Since 21 is the mean of five consecutive integers, the smallest of these five integers must be less than 21.
Suppose that the smallest number is 20.
The mean of $20, 21, 22, 23$, and 24 is $\frac{20 + 21 + 22 + 23 + 24}{5} = 22$.
This mean of 22 is greater than the required mean of 21; thus, the smallest of the 5 consecutive integers must be less than 20.
Suppose that the smallest number is 19.
The mean of $19, 20, 21, 22$, and 23, is $\frac{19 + 20 + 21 + 22 + 23}{5} = 21$, as required.
Thus, the smallest of the 5 consecutive integers is 19.

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14. A bag contains green mints and red mints only. If 75% of the mints are green, what is the ratio of the number of green mints to the number of red mints?
(A) 3 : 4 (B) 3 : 1 (C) 4 : 3 (D) 1 : 3 (E) 3 : 7

Source: 2010 Gauss Grade 7 #14

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Percentages

Answer: B

Solution:

Solution 1
Since the bag contains green mints and red mints only, the remaining $100\% - 75\% = 25\%$ of the mints must be red.
Thus, the ratio of the number of green mints to the number of red mints is $75 : 25 = 3 : 1$.

Solution 2
Since 75% of the mints are green, then $\frac{3}{4}$ of the mints are green.
Since the bag contains only green mints and red mints, then $1 - \frac{3}{4} = \frac{1}{4}$ of the mints in the bag are red.
Thus, there are 3 times as many green mints as red mints.
The ratio of the number of green mints to the number of red mints is $3 : 1$.

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15. Square M has an area of 100 cm^2. The area of square N is four times the area of square M. The perimeter of square N is
(A) 160 cm (B) 400 cm (C) 80 cm (D) 40 cm (E) 200 cm

Source: 2010 Gauss Grade 7 #15

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Perimeter

Answer: C

Solution:

The area of square N is four times the area of square M or $4 \times 100 \text{ cm}^2 = 400 \text{ cm}^2$.
Thus, each side of square N has length $\sqrt{400} = 20$ cm.
The perimeter of square N is $4 \times 20 \text{ cm} = 80$ cm.

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16. In a magic square, all rows, columns, and diagonals have the same sum. The magic square shown uses each of the integers from -6 to $+2$. What is the value of Y?

+1		Y
−4		
−3		−5

(A) −1 (B) 0 (C) −6 (D) +2 (E) −2

Source: 2010 Gauss Grade 7 #16

Primary Topics: Other

Secondary Topics: Games | Logic

Answer: A

Solution:

First we must find the magic constant, that is, the sum of each row, column and diagonal.
From column one, we find that the magic constant is $(+1)+(-4)+(-3) = -6$.
In the diagonal extending from the top left corner to the bottom right corner, the two existing numbers $+1$ and -5 have a sum of -4.
Thus, to obtain the magic constant of -6 in this diagonal, -2 must occupy the centre square. In the diagonal extending from the bottom left corner to the top right corner, the two numbers -3 and -2, have a sum of -5.
Thus, to obtain the magic constant of -6 in this diagonal, Y must equal -1.
The completed magic square is shown below.

+1	−6	−1
−4	−2	0
−3	+2	−5

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17. How many three-digit integers are exactly 17 more than a two-digit integer?
(A) 17 (B) 16 (C) 10 (D) 18 (E) 5

Source: 2010 Gauss Grade 7 #17

Primary Topics: Number Sense

Secondary Topics: Digits | Logic

Answer: A

Solution:

The smallest possible three-digit integer that is 17 more than a two-digit integer is 100 (100 is 17 more than 83 and 100 is in fact the smallest possible three-digit integer).
Notice that 101 is 17 more than 84, 102 is 17 more than 85, and so on. This continues until we reach 117 which is 17 more than 100, but 100 is not a two-digit integer. Thus, 116 is the largest possible three-digit integer that is 17 more than a two-digit integer (116 is 17 more than 99). Therefore, all of the integers from 100 to 116 inclusive, or 17 three-digit integers, are exactly 17 more than a two-digit integer.

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18. Distinct points are placed on a circle. Each pair of points is joined with a line segment. An example with 4 points and 6 line segments is shown. If 6 distinct points are placed on a circle, how many line segments would there be?

(A) 13 (B) 16 (C) 30 (D) 15 (E) 14

Source: 2010 Gauss Grade 7 #18

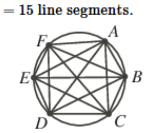
Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: D

Solution:

Solution 1
We label the 6 points A through F as shown and proceed to connect the points in all possible ways.
From point A, 5 line segments are drawn, 1 to each of the other points, B through-F.
From point B, 4 new line segments are drawn, 1 to each of the points C through F, since the segment AB has already been drawn.
This continues, with 3 line segments drawn from point C, 2 from point D, 1 from point E, and 0 from point F since it will have already been joined to each of the other points.
In total, there are $5+4+3+2+1 = 15$ line segments.



Solution 2
Label the 6 points A through F as shown above.
From each of the 6 points, 5 line segments can be drawn leaving the point, 1 to each of the other 5 points.
Thus, the total number of line segments leaving the 6 points is $6 \times 5 = 30$.
However, this counts each of the line segments twice, since each segment will be counted as leaving both of its ends.
For example, the segment leaving point A and ending at point D is also counted as a segment leaving point D and ending at point A.
Thus, the actual number of line segments is $30 \div 2 = 15$.

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19. If each of the four numbers 3, 4, 6, and 7 replaces a $\square$, what is the largest possible sum of the fractions shown? $\frac{\square}{\square} + \frac{\square}{\square}$
(A) $\frac{19}{12}$ (B) $\frac{13}{7}$ (C) $\frac{5}{2}$ (D) $\frac{15}{4}$ (E) $\frac{23}{6}$

Source: 2010 Gauss Grade 7 #19

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Optimization

Answer: E

Solution:

The value of any positive fraction is increased by increasing the numerator and/or decreasing the denominator.
Thus, to obtain the largest possible sum, we choose 6 and 7 as the numerators, and 3 and 4 as the denominators.
We then calculate:

$$\frac{7}{3}+\frac{6}{4}=\frac{28}{12}+\frac{18}{12}=\frac{46}{12}=\frac{23}{6}$$

We recognize that $\frac{23}{6}$ is the largest of the 5 possible answers, and thus is the correct response.
(This means that we do not need to try $\frac{7}{4}+\frac{6}{3}$.)

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20. Andy, Jen, Sally, Mike, and Tom are sitting in a row of five seats. Andy is not beside Jen. Sally is beside Mike. Who *cannot* be sitting in the middle seat?
(A) Andy (B) Jen (C) Sally (D) Mike (E) Tom

Source: 2010 Gauss Grade 7 #20

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: E

Solution:

Solution 1
To determine who cannot be sitting in the middle seat, we may eliminate the 4 people who can be sitting in the middle seat.
First, assume that Sally and Mike, who must be beside one another, are in seats 1 and 2, or in seats 2 and 1.
Since Andy and Jen are not beside each other, either Andy is in seat 3 (the middle seat) and Jen is in seat 5, or vice versa.
Thus, Andy and Jen can each be sitting in the middle seat and are eliminated as possible choices.
Next, assume that Sally and Mike are in seats 2 and 3, or in seats 3 and 2.
That is, either Sally is in the middle (seat 3), or Mike is.
In either case, seats 1, 4 and 5 are empty, allowing either Andy or Jen to choose seat 1 and hence, they are not next to one another.
This demonstrates that Sally and Mike can each be sitting in the middle seat.
Having eliminated Andy, Jen, Sally, and Mike, it must be Tom who cannot be sitting in the middle seat.

Solution 2
Assume that Tom is sitting in the middle (seat 3).
Since Sally and Mike are seated beside each other, they are either sitting in seats 4 and 5 or seats 1 and 2.
In either case, seats 1 and 2 remain empty or seats 4 and 5 remain empty.
However, Andy and Jen cannot sit beside each other.
Therefore, this arrangement is not possible.
Thus, Tom cannot be sitting in the middle seat.
Since the question implies that there is a unique answer, then Tom is the answer.

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21. A bicycle travels at a constant speed of 15 km/h. A bus starts 195 km behind the bicycle and catches up to the bicycle in 3 hours. What is the average speed of the bus in km/h?
(A) 65 (B) 80 (C) 70 (D) 60 (E) 50

Source: 2010 Gauss Grade 7 #21

Primary Topics: Number Sense

Secondary Topics: Rates | Averages

Answer: B

Solution:

Traveling at a constant speed of 15 km/h, in 3 hours the bicycle will travel $15\times 3=45$ km.
At the start, the bicycle was 195 km ahead of the bus.
Therefore, in order to catch up to the bicycle, the bus must travel 195 km plus the additional 45-km that the bicycle travels, or $195+45=240$ km.
To do this in 3 hours, the bus must travel at an average speed of $240\div 3=80$ km/h.

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22. In the *Coin Game*, you toss three coins at the same time. You win only if the 3 coins are all showing heads, or if the 3 coins are all showing tails. If you play the game once only, what is the probability of winning?
(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{2}{27}$ (D) $\frac{2}{3}$ (E) $\frac{1}{3}$

Source: 2010 Gauss Grade 7 #22

Primary Topics: Number Sense

Secondary Topics: Probability

Answer: B

Solution:

When tossing a single coin, there are two possible outcomes, a head (H) or a tail (T).
When tossing 2 coins, there are $2\times 2=4$ possible outcomes.
These are HH, HT, TH, and TT.
When tossing 3 coins, there are $2\times 2\times 2=8$ possible outcomes.
These are HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT.
Of these 8 possible outcomes, there are 2 winning outcomes, HHH and TTT.
Thus, the probability of winning the Coin Game is $\frac{2}{8}=\frac{1}{4}$.

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23. Molly assigns every letter of the alphabet a *different* whole number value. She finds the value of a word by *multiplying* the values of its letters together. For example, if D has a value of 10, and I has a value of 8, then the word DID has a value of $10\times 8\times 10=800$. The table shows the value of some words. What is the value of the word MATH?

Word	Value
$TOTE$	18
$TEAM$	168
MOM	49
$HOME$	70
$MATH$	?

(A) 19 (B) 840 (C) 420 (D) 190 (E) 84

Source: 2010 Gauss Grade 7 #23

Primary Topics: Data Analysis

Secondary Topics: Operations

Answer: C

Solution:

Since $\mathrm{M} \times \mathrm{O} \times \mathrm{M} = 49$, either $\mathrm{M} = 7$ and $\mathrm{O} = 1$ or $\mathrm{M} = 1$ and $\mathrm{O} = 49$.
However, since the value of the word TOTE is 18, O cannot have a value of 49 because 18 is not divisible by 49.
Thus, $\mathrm{M} = 7$ and $\mathrm{O} = 1$.
Since $\mathrm{T} \times \mathrm{O} \times \mathrm{T} \times \mathrm{E} = 18$ and $\mathrm{O} = 1$, we have $\mathrm{T} \times \mathrm{T} \times \mathrm{E} = 18$.
Therefore, either $\mathrm{T} = 3$ and $\mathrm{E} = 2$ or $\mathrm{T} = 1$ and $\mathrm{E} = 18$.
However, $\mathrm{O} = 1$, and since every letter has a different value, T cannot be equal to 1.
Thus, $\mathrm{T} = 3$ and $\mathrm{E} = 2$.
The value of the word TEAM is 168, so $\mathrm{T} \times \mathrm{E} \times \mathrm{A} \times \mathrm{M} = 168$, or $3 \times 2 \times \mathrm{A} \times 7 = 168$.
Thus, $42 \times \mathrm{A} = 168$ or $\mathrm{A} = 168 \div 42 = 4$.
The value of the word HOME is 70, so $\mathrm{H} \times \mathrm{O} \times \mathrm{M} \times \mathrm{E} = 70$, or $\mathrm{H} \times 1 \times 7 \times 2 = 70$.
Thus, $14 \times \mathrm{H} = 70$ or $\mathrm{H} = 70 \div 14 = 5$.
Finally, the value of the word MATH is $\mathrm{M} \times \mathrm{A} \times \mathrm{T} \times \mathrm{H} = 7 \times 4 \times 3 \times 5 = 420$.

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24. How many different pairs (m, n) can be formed using numbers from the list of integers $\{1, 2, 3, \ldots, 20\}$ such that $m
25. 55
26. 90
27. 140
28. 110
29. 50

Source: 2010 Gauss Grade 7 #24

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: B

Solution:

The sum of two even numbers is even. The sum of two odd numbers is even.
The sum of an odd number and an even number is odd.
Thus, for the sum $m + n$ to be even, both m and n must be even, or they must both be odd.
If $m = 2$, then n must be even and greater than 2.
Thus, if $m = 2$ then n can be $4, 6, 8, 10, 12, 14, 16, 18$, or 20.
This gives 9 different pairs (m, n) when $m = 2$.
If $m = 4$, then n must be even and greater than 4.
Thus, if $m = 4$ then n can be $6, 8, 10, 12, 14, 16, 18$, or 20.
This gives 8 different pairs (m, n) when $m = 4$.
Continuing in this manner, each time we increase m by 2, the number of choices for n, and thus for (m, n), decreases by 1.
This continues until $m = 18$, at which point there is only one choice for n, namely $n = 20$.
Therefore, the total number of different pairs (m, n) where both m and n are even is,
$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$.
Similarly, if $m = 1$, then n must be odd and greater than 1.
Thus, if $m = 1$, then n can be $3, 5, 7, 9, 11, 13, 15, 17$, or 19.
This gives 9 different pairs (m, n) when $m = 1$.
If $m = 3$, then n must be odd and greater than 3.
Thus, if $m = 3$ then n can be $5, 7, 9, 11, 13, 15, 17$, or 19.
This gives 8 different pairs (m, n) when $m = 3$.
Continuing in this manner, each time we increase m by 2, the number of choices for n, and thus for (m, n), decreases by 1.
This continues until $m = 17$, at which point there is only one choice for n, namely $n = 19$.
Therefore, the total number of different pairs (m, n) where both m and n are odd is,
$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$.
Thus, the total number of different pairs (m, n) using numbers from the list $\{1, 2, 3, \ldots, 20\}$ such that m<n and $m + n$ is even is $45 + 45 = 90$.

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25. Tanner wants to fill his swimming pool using two hoses, each of which sprays water at a constant rate. Hose A fills the pool in a hours when used by itself, where a is a positive integer. Hose B fills the pool in b hours when used by itself, where b is a positive integer. When used together, Hose A and Hose B fill the pool in 6 hours. How many different possible values are there for a?
(A) 5 (B) 6 (C) 9 (D) 10 (E) 12

Source: 2010 Gauss Grade 7 #25

Primary Topics: Counting and Probability | Number Sense

Secondary Topics: Rates | Divisibility

Answer: C

Solution:

Together, Hose A and Hose B fill the pool in 6 hours.
Thus, it must take Hose A more than 6 hours to fill the pool when used by itself.
Therefore, $a \geq 7$, since a is a positive integer.
Similarly, it must take Hose B more than 6 hours to fill the pool when used by itself.
Therefore, $b \geq 7$, since b is a positive integer.
When used by itself, the fraction of the pool that Hose A fills in 6 hours is $\frac{6}{a}$.
When used by itself, the fraction of the pool that Hose B fills in 6 hours is $\frac{6}{b}$.
When used together, Hose A and Hose B fill the pool \emph{once} in 6 hours. Thus,
$\frac{6}{a} + \frac{6}{b} = 1$.
Since $a \geq 7$, $b \geq 7$, and both a and b are integers, then we can find values for a and b that satisfy the equation $\frac{6}{a} + \frac{6}{b} = 1$ by using systematic trial and error.
For example, let $a = 7$. Then $\frac{6}{7} + \frac{6}{b} = 1$, or $\frac{6}{b} = 1 - \frac{6}{7}$, or $\frac{6}{b} = \frac{1}{7}$.
Since $\frac{6}{42} = \frac{1}{7}$, then $b = 42$ and $a = 7$ is one possible solution to the equation $\frac{6}{a} + \frac{6}{b} = 1$.
Compare this to what happens when we let $a = 11$.
We have $\frac{6}{11} + \frac{6}{b} = 1$, or $\frac{6}{b} = 1 - \frac{6}{11}$, or $\frac{6}{b} = \frac{5}{11}$, or $5b = 66$.
Since there is no integer value for b that makes $5b$ equivalent to 66, then $a = 11$ does not give a possible solution.
The possible solutions found by systematic trial and error are shown below

The possible solutions found by systematic trial and error are shown below.

a	7	8	9	10	12	15	18	24	42
b	42	24	18	15	12	10	9	8	7

Any value for a larger than 42 requires b to be smaller than 7, but we know that $b \geq 7$.
Thus, there are only 9 different possible values for a.
Note:
We can reduce the time it takes to complete this trial and error above by recognizing that in the equation $\frac{6}{a} + \frac{6}{b} = 1$, the a and b are interchangeable.
That is, interchanging a and b in the equation, gives $\frac{6}{b} + \frac{6}{a} = 1$, which is the same equation.
For example, this tells us that since $a = 7$, $b = 42$ satisfies the equation, then $a = 42$, $b = 7$ satisfies the equation as well.
Moreover, if the pair (a, b) satisfies the equation, then (b, a) satisfies the equation, and if (a, b) does not satisfy the equation, then (b, a) does not satisfy the equation.
This interchangeability of a and b is seen in the symmetry of the list of possible solutions above.
Recognizing that this symmetry must exist allows us to quickly determine the 4 remaining solutions that follow after $a = 12$, $b = 12$.