The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7
Solutions

1. The value of $(8 \times 4) + 3$ is
   (A) 96 (B) 15 (C) 56 (D) 35 (E) 28

Source: 2006 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: D

Solution:

Calculating, $(8 \times 4) + 3 = 32 + 3 = 35$.

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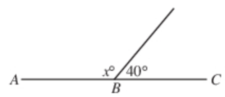
2. In the diagram, ABC is a straight line. The value of x is



   (A) 100 (B) 140 (C) 50 (D) 120 (E) 320

Source: 2006 Gauss Grade 7 #2

Primary Topics: Geometry and Measurement

Secondary Topics: Angles

Answer: B

Solution:

Since the angles along a straight line add to 180°, then $x^\circ + 40^\circ = 180^\circ$ or $x + 40 = 180$ or $x = 140$.

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3. Mikhail has \$10 000 in \$50 bills. How many \$50 bills does he have?
   (A) 1000 (B) 200 (C) 1250 (D) 500 (E) 2000

Source: 2006 Gauss Grade 7 #3

Primary Topics: Algebra and Equations

Secondary Topics: Operations

Answer: B

Solution:

To determine the number of \$50 bills, we divide the total amount of money by 50, to get $10\,000 \div 50 = 200$ bills.
Therefore, Mikhail has 200 \$50 bills.

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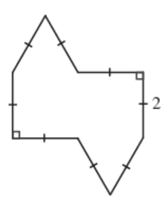
4. What is the perimeter of the figure shown?



   (A) 16 (B) 10 (C) 8 (D) 14 (E) 18

Source: 2006 Gauss Grade 7 #4

Primary Topics: Geometry and Measurement

Secondary Topics: Perimeter | Polygons

Answer: A

Solution:

The figure has 8 sides, each of equal length.
Since the length of each side is 2, then the perimeter of the figure is $8 \times 2 = 16$.

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5. The value of $\frac{2}{5} + \frac{1}{3}$ is
   (A) $\frac{3}{8}$ (B) $\frac{2}{15}$ (C) $\frac{11}{15}$ (D) $\frac{13}{15}$ (E) $\frac{3}{15}$

Source: 2006 Gauss Grade 7 #5

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Operations

Answer: C

Solution:

Using a common denominator, $\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$.

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6. The value of $6 \times 100,000 + 8 \times 1000 + 6 \times 100 + 7 \times 1$ is
   (A) 6867 (B) 608 067 (C) 608 607 (D) 6 008 607 (E) 600 000 867

Source: 2006 Gauss Grade 7 #6
Primary Topics: Number Sense
Secondary Topics: Operations | Digits

Answer: C
Solution:
Calculating by determining each product first,

$$6 \times 100\,000 + 8 \times 1000 + 6 \times 100 + 7 \times 1 = 600\,000 + 8000 + 600 + 7 = 608\,607$$

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7. If $3 + 5x = 28$, the value of x is
   (A) 20 (B) 3.5 (C) 5 (D) 6.2 (E) 125

Source: 2006 Gauss Grade 7 #7
Primary Topics: Algebra and Equations
Secondary Topics: Equations Solving

Answer: C
Solution:
Since $3 + 5x = 28$, then $5x = 28 - 3 = 25$ so $x = \frac{25}{5} = 5$.

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8. The value of $9^2 - \sqrt{9}$ is
   (A) 0 (B) 6 (C) 15 (D) 72 (E) 78

Source: 2006 Gauss Grade 7 #8
Primary Topics: Number Sense
Secondary Topics: Operations

Answer: E
Solution:
Calculating, $9^2 - \sqrt{9} = 9 \times 9 - \sqrt{9} = 81 - 3 = 78$.

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9. There are 2 red, 5 yellow and 4 blue balls in a bag. If a ball is chosen at random from the bag, without looking, the probability of choosing a yellow ball is
   (A) $\frac{2}{11}$ (B) $\frac{5}{11}$ (C) $\frac{4}{11}$ (D) $\frac{6}{11}$ (E) $\frac{7}{11}$

Source: 2006 Gauss Grade 7 #9
Primary Topics: Counting and Probability
Secondary Topics: Probability

Answer: B
Solution:
In total, there are $2 + 5 + 4 = 11$ balls in the bag.
Since there are 5 yellow balls, then the probability of choosing a yellow ball is $\frac{5}{11}$.

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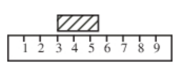
10. A small block is placed along a 10 cm ruler. Which of the following is closest to the length of the block?



   (A) 0.24 cm (B) 4.4 cm (C) 2.4 cm (D) 3 cm (E) 24 cm

Source: 2006 Gauss Grade 7 #10
Primary Topics: Geometry and Measurement
Secondary Topics: Decimals | Measurement | Estimation

Answer: C
Solution:
Since the left edge of the block is at the "3" on the ruler and the right edge of the block is between the "5" and "6", then the length of the block is between 2 and 3.
Looking at the possible choices, the only choice between 2 and 3 is (C) or 2.4 cm.
(Looking again at the figure, the block appears to end roughly halfway between the "5" and the "6", so 2.4 cm is reasonable.)

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11. The cost, before taxes, of the latest CD released by The Magic Squares is \$14.99. If the sales tax is 15%, how much does it cost to buy this CD, including tax?
   (A) \$17.24 (B) \$15.14 (C) \$2.25 (D) \$16.49 (E) \$16.50

Source: 2006 Gauss Grade 7 #11
Primary Topics: Number Sense
Secondary Topics: Percentages | Decimals

Answer: A
Solution:
Solution 1
Since the sales tax is 15%, then the total price for the CD including tax is
$1.15 \times \$14.99 = \$17.2385$ which rounds to \$17.24.

Solution 2
Since the sales tax is 15%, then the amount of tax on the CD which costs \$14.99 is
$0.15 \times \$14.99 = \$2.2485$, which rounds to \$2.25.
Therefore, the total price of the CD including tax is $\$14.99 + \$2.25 = \$17.24$.

12. A rectangular pool is 6 m wide, 12 m long and 4 m deep. If the pool is half full of water, what is the volume of water in the pool?
(A) 100 m^3 (B) 288 m^3 (C) 36 m^3 (D) 22 m^3 (E) 144 m^3

Source: 2006 Gauss Grade 7 #12

Primary Topics: Geometry and Measurement

Secondary Topics: Volume

Answer: E

Solution:

Solution 1
Since the pool has dimensions 6 m by 12 m by 4 m, then its total volume is
$6 \times 12 \times 4 = 288$ m^3.
Since the pool is only half full of water, then the volume of water in the pool is $\frac{1}{2} \times 288$ m^3 or 144 m^3.

Solution 2
Since the pool is half full of water, then the depth of water in the pool is $\frac{1}{2} \times 4 = 2$ m.
Therefore, the portion of the pool which is filled with water has dimensions 6 m by 12 m by 2-m, and so has volume $6 \times 12 \times 2 = 144$ m^3.

13. What number must be added to 8 to give the result -5?
(A) 3 (B) -3 (C) 13 (D) -13 (E) -10

Source: 2006 Gauss Grade 7 #13

Primary Topics: Algebra and Equations

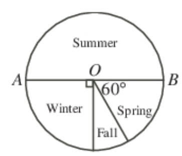
Secondary Topics: Equations Solving

Answer: D

Solution:

To determine the number that must be added 8 to give the result -5, we subtract 8 from -5 to get $(-5) - 8 = -13$. Checking, $8 + (-13) = -5$.

14. In the diagram, O is the centre of the circle, AOB is a diameter, and the circle graph illustrates the favourite season of 600 students. How many of the students surveyed chose Fall as their favourite season?



(A) 100 (B) 50 (C) 360 (D) 150 (E) 75

Source: 2006 Gauss Grade 7 #14

Primary Topics: Data Analysis

Secondary Topics: Graphs | Circles

Answer: B

Solution:

Solution 1
Since AOB is a diameter of the circle, then $\angle AOB = 180^\circ$.
We are told that the angle in the "Winter" sector is a right angle (or 90°). Also, we are told that the angle in the "Spring" sector is 60°.
Therefore, the angle in the "Fall" sector is $180^\circ - 90^\circ - 60^\circ = 30^\circ$.
What fraction of the complete circle is 30°?
Since the whole circle has 360°, then the fraction is- $\frac{30^\circ}{360^\circ} = \frac{1}{12}$.
Therefore, $\frac{1}{12}$ of the students chose fall as their favourite season, or $\frac{1}{12} \times 600 = 50$ students in total.

Solution 2
Since AOB is a diameter of the circle, then $\frac{1}{2}$ of the students chose summer as their favourite season, or $\frac{1}{2} \times 600 = 300$ students in total.
Since the angle in the "Winter" sector is a right angle (or 90°), then $\frac{1}{4}$ of the students (since 4-right angles make up a complete circle) chose Winter as their favourite season, or
$\frac{1}{4} \times 600 = 150$ students in total.
Since the angle in the "Spring" sector is 60°, then $\frac{60^\circ}{360^\circ} = \frac{1}{6}$ of the students chose Spring as their favourite season, or $\frac{1}{6} \times 600 = 100$ students in total.
Since there were 600 students in total, then the number who chose Fall as their favourite season was $600 - 300 - 150 - 100 = 50$.

15. Harry charges \$4 to babysit for the first hour. For each additional hour, he charges 50% more than he did for the previous hour. How much money in total would Harry earn for 4 hours of babysitting?
(A) \$16.00 (B) \$19.00 (C) \$32.50 (D) \$13.50 (E) \$28.00

Source: 2006 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Percentages | Rates

Answer: C

Solution:

Since Harry charges 50% more for each additional hour as he did for the previous hour, then he charges 1.5 or $\frac{3}{2}$ times as much as he did for the previous hour.
Harry charges \$4 for the first hour.
Harry then charges $\frac{3}{2} \times \$4 = \$6$ for the second hour.
Harry then charges $\frac{3}{2} \times \$6 = \$9$ for the third hour.

Harry then charges $\frac{3}{2} \times \$9 = \$\frac{27}{2} = \$13.50$ for the fourth hour.
Therefore, for 4 hours of babysitting, Harry would earn $\$4 + \$6 + \$9 + \$13.50 = \$32.50$.

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16. A fraction is equivalent to $\frac{5}{8}$. Its denominator and numerator add up to 91. What is the difference between the denominator and numerator of this fraction?
(A) 21 (B) 3 (C) 33 (D) 13 (E) 19

Source: 2006 Gauss Grade 7 #16

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: A

Solution:

Solution 1
We obtain fractions equivalent to $\frac{5}{8}$ by multiplying the numerator and denominator by the same number.
The sum of the numerator and denominator of $\frac{5}{8}$ is 13, so when we multiply the numerator and denominator by the same number, the sum of the numerator and denominator is also multiplied by this same number.
Since $91 = 13 \times 7$, then we should multiply the numerator and denominator both by 7 to get a fraction $\frac{5\times7}{8\times7} = \frac{35}{56}$ equivalent to $\frac{5}{8}$ whose numerator and denominator add up to 91.
The difference between the denominator and numerator in this fraction is $56 - 35 = 21$.

Solution 2
We make a list of the fractions equivalent to $\frac{5}{8}$ by multiplying the numerator and denominator by the same number, namely 2, 3, 4, and so on:

$$\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \frac{30}{48}, \frac{35}{56}, \ldots$$

Since the numerator and denominator of $\frac{35}{56}$ add to 91 (since $35 + 56 = 91$), then this is the fraction for which we are looking.
The difference between the denominator and numerator is $56 - 35 = 21$.

Solution 3
We obtain fractions equivalent to $\frac{5}{8}$ by multiplying the numerator and denominator by the same number.
If this number is n, then a fraction equivalent to $\frac{5}{8}$ is $\frac{5n}{8n}$.
For the numerator and denominator to add up to 91, we must have $5n + 8n = 91$ or $13n = 91$ or $n = 7$.
Therefore, the fraction for which we are looking is $\frac{5\times7}{8\times7} = \frac{35}{56}$.
The difference between the denominator and numerator is $56 - 35 = 21$.

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17. Bogdan needs to measure the area of a rectangular carpet. However, he does not have a ruler, so he uses a shoe instead. He finds that the shoe fits exactly 15 times along one edge of the carpet and 10 times along another. He later measures the shoe and finds that it is 28 cm long. What is the area of the carpet?
(A) 150 cm^2 (B) 4200 cm^2 (C) $22,500\text{ cm}^2$
(D) $630,000\text{ cm}^2$ (E) $117,600\text{ cm}^2$

Source: 2006 Gauss Grade 7 #17

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Measurement

Answer: E

Solution:

Solution 1
Since the shoe is 28 cm long and fits 15 times along one edge of the carpet, then one dimension of the carpet is $15 \times 28 = 420$ cm.
Since the shoe fits 10 times along another edge of the carpet, then one dimension of the carpet is $10 \times 28 = 280$ cm.
Therefore, then area of the carpet is $420 \times 280 = 117\,600\text{ cm}^2$.

Solution 2
Since the shoe fits along one edge of the carpet 15 times and along another edge 10 times, then the area of the carpet is $15 \times 10 = 150$ square shoes.
Since the length of the shoe is 28 cm, then

$$1\text{ square shoe} = 1\text{ shoe} \times 1\text{ shoe} = 28\text{ cm} \times 28\text{ cm} = 784\text{ cm}^2$$

Therefore, the area of the carpet in cm^2 is $150 \times 784 = 117\,600\text{ cm}^2$.

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18. Keiko and Leah run on a track that is 150 m around. It takes Keiko 120 seconds to run 3 times around the track, and it takes Leah 160 seconds to run 5 times around the track. Who is the faster runner and at approximately what speed does she run?
(A) Keiko, 3.75 m/s (B) Keiko, 2.4 m/s (C) Leah, 3.3 m/s
(D) Leah, 4.69 m/s (E) Leah, 3.75 m/s

Source: 2006 Gauss Grade 7 #18

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: D

Solution:

Solution 1
Since Keiko takes 120 seconds to run 3 times around the track, then it takes her $\frac{1}{3} \times 120 = 40$ seconds to run 1 time around the track.
Since Leah takes 160 seconds to run 5 times around the track, then it takes her $\frac{1}{5} \times 160 = 32$ seconds to run 1 time around the track.
Since Leah takes less time to run around the track than Keiko, then she is the faster runner.
Since Leah takes 32 seconds to run the 150 m around the track, then her speed is $\frac{150\text{ m}}{32\text{ s}} = 4.6875\text{ m/s} \approx 4.69\text{ m/s}$.
Therefore, Leah is the faster runner and her speed is approximately 4.69 m/s.

Solution 2
In 120 seconds, Keiko runs 3 times around the track, or $3 \times 150 = 450$ m in total. Therefore,

her speed is $\frac{450 \text{ m}}{120 \text{ S}} = 3.75$ m/s.
In 160 seconds, Leah runs 5 times around the track, or $5 \times 150 = 750$ m in total. Therefore, her speed is $\frac{750 \text{ m}}{160 \text{ S}} = 4.6875$ m/s ≈ 4.69 m/s.
Since Leah's speed is larger, she is the faster runner and her speed is approximately 4.69 m/s.

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19. Which of the following is closest to one million (10^6) seconds?
(A) 1 day (B) 10 days (C) 100 days (D) 1 year (E) 10 years

Source: 2006 Gauss Grade 7 #19

Primary Topics: Number Sense

Secondary Topics: Measurement | Estimation

Answer: B

Solution:

Solution 1
In one minute, there are 60 seconds.
In one hour, there are 60 minutes, so there are $60 \times 60 = 3600$ seconds.
In one day, there are 24 hours, so there are $24 \times 3600 = 86\,400$ seconds.
Therefore, 10^6 seconds is equal to $\frac{10^6}{86\,400} \approx 11.574$ days, which of the given choices is closest to 10 days.

Solution 2
Since there are 60 seconds in one minute, then 10^6 seconds is $\frac{10^6}{60} \approx 16\,666.67$ minutes.
Since there are 60 minutes in one hour, then $16\,666.67$ minutes is $\frac{16\,666.67}{60} \approx 277.78$ hours.
Since there are 24 hours in one day, then 277.78 hours is $\frac{277.78}{24} \approx 11.574$ days, which of the given choices is closest to 10 days.

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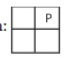
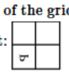
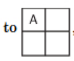
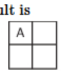
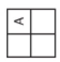
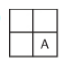
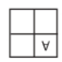
20. The letter P is written in a 2×2 grid of squares as shown:  A combination of rotations about the centre of the grid and reflections in the two lines through the centre achieves the result: 

When the same combination of rotations and reflect is applied to , the result is

(A)  (B)  (C)  (D)  (E) 

Source: 2006 Gauss Grade 7 #20

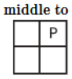
Primary Topics: Geometry and Measurement

Secondary Topics: Transformations

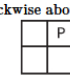
Answer: B

Solution:

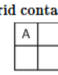
One possible way to transform the initial position of the P to the final position of the P is to reflect the grid in the vertical line in the middle to obtain



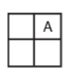
and then rotate the grid 90° counterclockwise about the centre to obtain



Applying these transformations to the grid containing the A, we obtain



and then



(There are many other possible combinations of transformations which will produce the same resulting image with the P; each of these combinations will produce the same result with the A.)

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21. Gail is a server at a restaurant. On Saturday, Gail gets up at 6:30 a.m., starts work at x a.m. and finishes at x p.m. How long does Gail work on Saturday?
(A) $24 - 2x$ hours (B) $12 - x$ hours (C) $2x$ hours
(D) 0 hours (E) 12 hours

Source: 2006 Gauss Grade 7 #21

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving | Measurement

Answer: E

Solution:

Solution 1
Between x a.m.-and x p.m. there are 12 hours. (For example, between 10 a.m.-and 10 p.m.-there are 12 hours.)
Therefore, Gail works for 12 hours on Saturday.

Solution 2
From x a.m.-until 12 noon, the number of hours which Gail works is $12 - x$.
From 12 noon until x p.m., she works x hours.
Thus, the total number of hours that Gail works is $(12 - x) + x = 12$.

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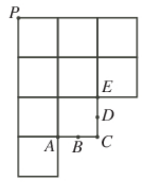
22. In the diagram, a shape is formed using unit squares, with B the midpoint of AC and D the midpoint of CE. The line which passes through P and cuts the area of the shape into two pieces of equal area also passes through the point



(A) A (B) B (C) C (D) D (E) E

Source: 2006 Gauss Grade 7 #22

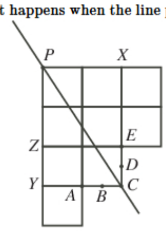
Primary Topics: Geometry and Measurement

Secondary Topics: Logic | Area

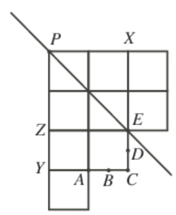
Answer: D

Solution:

As an initial guess, let us see what happens when the line passes through C.



Since each square is a unit square, then the area of rectangle $PXCY$ is $2 \times 3 = 6$, and so the line through P and C cuts this area in half, leaving 3 square units in the bottom piece and 3 square units in the top piece.
(A fact has been used here that will be used several times in this solution: If a line passes through two diagonally opposite vertices of a rectangle, then it cuts the rectangle into two pieces of equal area (since it cuts the rectangle into two congruent triangles).)
In the bottom piece, we have not accounted for the very bottom unit square, so the total area of the bottom piece is 4 square units.
In the top piece, we have not accounted for the two rightmost unit squares, so the total area of the top piece is 5 square units.
So putting the line through C does not produce two pieces of equal area, so C is not the correct answer.
Also, since the area of the bottom piece is larger than the area of the top piece when the line passes through C, we must move the line up to make the areas equal (so neither A nor B can be the answer).
Should the line pass through E?



If so, then the line splits the square $PXEZ$ (of area 4) into two pieces of area 2.
Then accounting for the remaining squares, the area of the bottom piece is $2 + 3 = 5$ and the area of the top piece is $2 + 2 = 4$.
So putting the line through E does not produce two pieces of equal area, so E is not the correct answer.
By elimination, the correct answer should be D.

(We should verify that putting the line through D does indeed split the area in half.
The total area of the shape is 9, since it is made up of 9 unit squares.
If the line goes through D, the top piece consists of $\triangle PXD$ and 2 unit squares.
The area of $\triangle PXD$ is $\frac{1}{2} \times 2 \times \frac{5}{2} = \frac{5}{2}$, since $PX = 2$ and $XD = \frac{5}{2}$.
Thus, the area of the top piece is $\frac{5}{2} + 2 = \frac{9}{2}$, which is exactly half of the total area, as required.)

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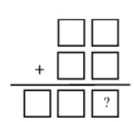
23. In the addition of two 2-digit numbers, each blank space, including those in the answer, is to be filled with one of the digits 0, 1, 2, 3, 4, 5, 6, each used exactly once. The units digit of the sum is



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Source: 2006 Gauss Grade 7 #23

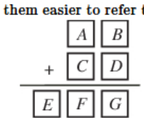
Primary Topics: Number Sense

Secondary Topics: Logic | Equations Solving | Digits

Answer: D

Solution:

We label the blank spaces to make them easier to refer to.



Since we are adding two 2-digit numbers, then their sum cannot be 200 or greater, so E

Since we are adding two 2-digit numbers, then their sum cannot be 200 or greater, so E must be a 1 if the sum is to have 3 digits.

Where can the digit 0 go?
Since no number can begin with a 0, then neither A nor C can be 0.
Since each digit is different, then neither B and D can be 0, otherwise both D and G or B and G would be the same.
Therefore, only F or G could be 0.

Since we are adding two 2-digit numbers and getting a number which is at least 100, then $A+C$ must be at least 9. (It could be 9 if there was a "carry" from the sum of the units digits.)
This tells us that A and C must be 3 and 6, 4 and 5, 4 and 6, or 5 and 6.

If G was 0, then B and D would have to 4 and 6 in some order. But then the largest that A and C could be would be 3 and 5, which are not among the possibilities above.
Therefore, G is not 0, so $F=0$.

$$\begin{array}{cccc} & & \boxed{A} & \boxed{B} \\ + & & \boxed{C} & \boxed{D} \\ \hline & \boxed{1} & \boxed{0} & \boxed{G} \end{array}$$

So the sum of A and C is either 9 or 10, so A and C are 3 and 6, 4 and 5, or 4 and 6.
In any of these cases, the remaining possibilities for B and D are too small to give a carry from the units column to the tens column.
So in fact, A and C must add to 10, so A and C are 4 and 6 in some order.
Let's try $A=4$ and $C=6$.

$$\begin{array}{cccc} & & \boxed{4} & \boxed{B} \\ + & & \boxed{6} & \boxed{D} \\ \hline & \boxed{1} & \boxed{0} & \boxed{G} \end{array}$$

The remaining digits are 2, 3 and 5. To make the addition work, B and D must be 2 and 3 and G must be 5. (We can check that either order for B and D works, and that switching the 4 and 6 will also work.)

So the units digit of the sum must be 5, as in the example

$$\begin{array}{cccc} & & \boxed{4} & \boxed{2} \\ + & & \boxed{6} & \boxed{3} \\ \hline & \boxed{1} & \boxed{0} & \boxed{5} \end{array}$$

(Note that we could have come up with this answer by trial and error instead of this logical procedure.)

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24. A triangle can be formed having side lengths 4, 5 and 8. It is impossible, however, to construct a triangle with side lengths 4, 5 and 10. Using the side lengths 2, 3, 5, 7 and 11, how many different triangles *with exactly two equal sides* can be formed?
(A) 8 (B) 5 (C) 20 (D) 10 (E) 14

Source: 2006 Gauss Grade 7 #24

Primary Topics: Geometry and Measurement

Secondary Topics: Triangles

Answer: E

Solution:

The sum of any two sides of a triangle must be bigger than the third side.
(When two sides are known to be equal, we only need to check if the sum of the two equal sides is longer than the third side, since the sum of one of the equal sides and the third side will always be longer than the other equal side.)
If the equal sides were both equal to 2, the third side must be shorter than $2+2=4$. The 1-possibility from the list not equal to 2 (since we cannot have three equal sides) is 3. So here there is 1 possibility.
If the equal sides were both equal to 3, the third side must be shorter than $3+3=6$. The 2-possibilities from the list not equal to 3 (since we cannot have three equal sides) are 2 and 5. So here there are 2 possibilities.
If the equal sides were both equal to 5, the third side must be shorter than $5+5=10$. The 3-possibilities from the list not equal to 5 (since we cannot have three equal sides) are 2, 3 and-7. So here there are 3 possibilities.
If the equal sides were both equal to 7, the third side must be shorter than $7+7=14$. The 4 possibilities from the list not equal to 7 (since we cannot have three equal sides) are 2, 3, 5 and 11. So here there are 4 possibilities.
If the equal sides were both equal to 11, the third side must be shorter than $11+11=22$. The 4 possibilities from the list not equal to 11 (since we cannot have three equal sides) are 2, 3, 5 and 7. So here there are 4 possibilities.
Thus, in total there are $1+2+3+4+4=14$ possibilities.

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25. Five students wrote a quiz with a maximum score of 50. The scores of four of the students were 42, 43, 46, and 49. The score of the fifth student was N. The average (mean) of the five students' scores was the same as the median of the five students' scores. The number of values of N which are possible is
(A) 3 (B) 4 (C) 1 (D) 0 (E) 2

Source: 2006 Gauss Grade 7 #25

Primary Topics: Number Sense

Secondary Topics: Averages | Counting

Answer: A

Solution:

The five scores are N, 42, 43, 46, and 49.
If $N<43$, the median score is 43.
If $N>46$, the median score is 46.
If $N\geq 43$ and $N\leq 46$, then N is the median.
We try each case.

If $N<43$, then the median is 43, so the mean should be 43.
Since the mean is 43, then the sum of the 5 scores must be $5\times 43=215$.

Therefore, $N + 42 + 43 + 46 + 49 = 215$ or $N + 180 = 215$ or $N = 35$, which is indeed less than-43.
We can check that the median and mean of 35, 42, 43, 46 and 49 are both 43.

If $N > 46$, then the median is 46, so the mean should be 46.
Since the mean is 46, then the sum of the 5 scores must be $5 \times 46 = 230$.
Therefore, $N + 42 + 43 + 46 + 49 = 230$ or $N + 180 = 230$ or $N = 50$, which is indeed greater than 46.
We can check that the median and mean of 42, 43, 46, 49, and 50 are both 46.

If $N \geq 43$ and $N \leq 46$, then the median is N, so the mean should be N.
Since the mean is N, then the sum of the 5 scores must be $5N$.
Therefore, $N + 42 + 43 + 46 + 49 = 5N$ or $N + 180 = 5N$ or $4N = 180$, or $N = 45$, which is indeed between 43 and 46.
We can check that the median and mean of 42, 43, 45, 46 and 49 are both 45.

Therefore, there are 3 possible values for N.