The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7 Solutions

1. The value of $(4-3)\times 2$ is
(A) -2 (B) 2 (C) 1 (D) 3 (E) 5

Source: 2007 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: B

Solution:

Calculating, $(4-3)\times 2 = 1\times 2 = 2$.

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2. Which number represents ten thousand?
(A) 10 (B) 10 000 000 (C) 10 000 (D) 100 (E) 1 000

Source: 2007 Gauss Grade 7 #2

Primary Topics: Number Sense

Secondary Topics: Digits

Answer: C

Solution:

Since one thousand is 1000, then ten thousand is 10 000.

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3. What integer should be placed in the $\boxed{}$ to make the statement $\boxed{}$ - 5 = 2 true?
(A) 7 (B) 4 (C) 3 (D) 1 (E) 8

Source: 2007 Gauss Grade 7 #3

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: A

Solution:

When we subtract 5 from the missing number, the answer is 2, so to find the missing number, we add 5 to 2 and obtain 7. (Check: $7-5=2$.)

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4. If Mukesh got 80% on a test which has a total of 50 marks, how many marks did he get?
(A) 40 (B) 62.5 (C) 10 (D) 45 (E) 35

Source: 2007 Gauss Grade 7 #4

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: A

Solution:

Solution 1
As a fraction, 80% is $\frac{80}{100}$ or $\frac{4}{5}$.
Therefore, Mukesh got $\frac{4}{5}$ of the possible 50 marks, or $\frac{4}{5}\times 50 = 40$ marks.

Solution 2
Since Mukesh got 80% of the 50 marks, he got $\frac{80}{100}\times 50 = \frac{80}{2} = 40$ marks.

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5. The sum $\frac{7}{10}+\frac{3}{100}+\frac{9}{1000}$ is equal to
(A) 0.937 (B) 0.9037 (C) 0.7309 (D) 0.739 (E) 0.0739

Source: 2007 Gauss Grade 7 #5

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Decimals

Answer: D

Solution:

Solution 1

$$\begin{aligned}\frac{7}{10}+\frac{3}{100}+\frac{9}{1000} &= \frac{700}{1000}+\frac{30}{1000}+\frac{9}{1000} \quad \text{(using a common denominator)}\\ &= \frac{739}{1000}\\ &= 0.739\end{aligned}$$

Solution 2

$$\begin{aligned}\frac{7}{10}+\frac{3}{100}+\frac{9}{1000} &= 0.7+0.03+0.009 \quad \text{(converting each fraction to a decimal)}\\ &= 0.739\end{aligned}$$

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6. Mark has $\frac{3}{4}$ of a dollar and Carolyn has $\frac{3}{10}$ of a dollar. Together they have
(A) \$0.90 (B) \$0.95 (C) \$1.00 (D) \$1.10 (E) \$1.05

Source: 2007 Gauss Grade 7 #6

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios | Decimals

Answer: E

Solution:

Solution 1
Mark has $\frac{3}{4}$ of a dollar, or 75 cents.
Carolyn has $\frac{3}{10}$ of a dollar, or 30 cents.
Together, they have $75 + 30 = 105$ cents, or \$1.05.

Solution 2
Since Mark has $\frac{3}{4}$ of a dollar and Carolyn has $\frac{3}{10}$ of a dollar, then together they have $\frac{3}{4} + \frac{3}{10} = \frac{15}{20} + \frac{6}{20} = \frac{21}{20}$ of a dollar.
Since $\frac{21}{20}$ is equivalent to $\frac{105}{100}$, they have \$1.05.

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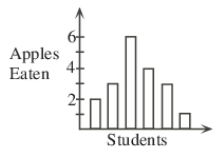
7. Six students have an apple eating contest. The graph shows the number of apples eaten by each student. Lorenzo ate the most apples and Jo ate the fewest. How many more apples did Lorenzo eat than Jo?



(A) 2 (B) 5 (C) 4 (D) 3 (E) 6

Source: 2007 Gauss Grade 7 #7

Primary Topics: Data Analysis

Secondary Topics: Measurement | Graphs

Answer: B

Solution:

From the graph, the student who ate the most apples ate 6 apples, so Lorenzo ate 6 apples.
Also from the graph, the student who ate the fewest apples ate 1 apple, so Jo ate 1 apple.
Therefore, Lorenzo ate $6 - 1 = 5$ more apples than Jo.

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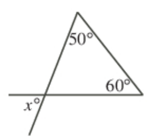
8. In the diagram, what is the value of x?



(A) 110 (B) 50 (C) 10 (D) 60 (E) 70

Source: 2007 Gauss Grade 7 #8

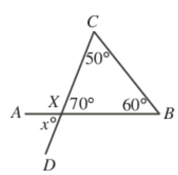
Primary Topics: Geometry and Measurement

Secondary Topics: Angles | Triangles

Answer: E

Solution:

Since the angles in a triangle add to 180°, then the missing angle in the triangle is $180^\circ - 50^\circ - 60^\circ = 70^\circ$.
We then have:



Since $\angle BXC = 70^\circ$, then $\angle AXC = 180^\circ - \angle BXC = 110^\circ$.
Since $\angle AXC = 110^\circ$, then $\angle DXA = 180^\circ - \angle AXC = 70^\circ$.
Therefore, $x = 70$.
(Alternatively, we could note that when two lines intersect, the vertically opposite angles are equal so $\angle DXA = \angle BXC = 70^\circ$.)

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9. The word BANK is painted exactly as shown on the outside of a clear glass window. Looking out through the window from the inside of the building, the word appears as

(A) BAИꓘ (B) KNAᗺ (C) ᗺAИꓘ (D) ꓘИAᗺ (E) KNAB

Source: 2007 Gauss Grade 7 #9

Primary Topics: Geometry and Measurement

Secondary Topics: Transformations

Answer: D

Solution:

When the word BANK is viewed from the inside of the window, the letters appear in the reverse order and the letters themselves are all backwards, so the word appears as BANK.

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10. A large box of chocolates and a small box of chocolates together cost \$15. If the large box costs \$3 more than the small box, what is the price of the small box of

chocolates?

(A) \$3 (B) \$4 (C) \$5 (D) \$6 (E) \$9

Source: 2007 Gauss Grade 7 #10

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: D

Solution:

Since a large box costs \$3 more than a small box and a large box and a small box together cost \$15, then replacing the large box with a small box would save \$3.
This tells us that two small boxes together cost \$12.
Therefore, one small box costs \$6.

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11. In the Fibonacci sequence 1, 1, 2, 3, 5, . . . ,, each number beginning with the 2 is the sum of the two numbers before it. For example, the next number in the sequence is $3+5=8$. Which of the following numbers is in the sequence?

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

Source: 2007 Gauss Grade 7 #11

Primary Topics: Number Sense

Secondary Topics: Patterning/Sequences/Series

Answer: B

Solution:

Since each number in the Fibonacci sequence, beginning with the 2, is the sum of the two previous numbers, then the sequence continues as $1, 1, 2, 3, 5, 8, 13, 21$.
Thus, 21 appears in the sequence.

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12. The Grade 7 class at Gauss Public School has sold 120 tickets for a lottery. One winning ticket will be drawn. If the probability of one of Mary's tickets being drawn is $\frac{1}{15}$, how many tickets did she buy?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Source: 2007 Gauss Grade 7 #12

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: D

Solution:

The probability that Mary wins the lottery is equal to the number of tickets that Mary bought divided by the total number of tickets in the lottery.
We are told that the probability that Mary wins is $\frac{1}{15}$.
Since there were 120 tickets in total sold, we would like to write $\frac{1}{15}$ as a fraction with 120 in the denominator.
Since $120 \div 15 = 8$, then we need to multiply the numerator and denominator of $\frac{1}{15}$ each by 8 to obtain a denominator of 120.
Therefore, the probability that Mary wins is $\frac{1\times8}{15\times8} = \frac{8}{120}$. Since there were 120 tickets sold, then Mary must have bought 8 tickets.

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13. What is the largest amount of postage in cents that *cannot* be made using only 3 cent and 5 cent stamps?

(A) 7 (B) 13 (C) 4 (D) 8 (E) 9

Source: 2007 Gauss Grade 7 #13

Primary Topics: Number Sense

Secondary Topics: Divisibility

Answer: A

Solution:

Solution 1
We look at each of the choices and try to make them using only 3 cent and 5 cent stamps:
(A): 7 cannot be made, since no more than one 5 cent and two 3 cent stamps could be used (try playing with the possibilities!)
(B): $13 = 5 + 5 + 3$
(C): 4 cannot be the answer since a larger number (7) already cannot be made
(D): $8 = 5 + 3$
(E): $9 = 3 + 3 + 3$
Therefore, the answer must be 7.
(We have not really justified that 7 is the largest number that cannot be made using only 3s and 5s; we have, though, determined that 7 must be the answer to this question, since it is the only possible answer from the given possibilities!)

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14. Harry, Ron and Neville are having a race on their broomsticks. If there are no ties, in how many different possible orders can they finish?

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

Source: 2007 Gauss Grade 7 #14

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: B

Solution:

We list all of the possible orders of finish, using H, R and N to stand for Harry, Ron and Neville.
The possible orders are HNR, HRN, NHR, NRH, RHN, RNH.
(It is easiest to list the orders in alphabetical order to better keep track of them.)
There are 6 possible orders.

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15. How many positive whole numbers, including 1, divide exactly into both 40 and 72?
(A) 9 (B) 12 (C) 4 (D) 2 (E) 5

Source: 2007 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Divisibility | Counting

Answer: C

Solution:

Solution 1
The positive whole numbers that divide exactly into 40 are 1, 2, 4, 5, 8, 10, 20, 40.
The positive whole numbers that divide exactly into 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72.
The numbers that occur in both lists are 1, 2, 4, 8, or four numbers in total.

Solution 2
The greatest common divisor of 40 and 72 is 8.
Any common divisor of 40 and 72 is a divisor of the greatest common divisor (namely 8) and vice-versa.
Since the positive divisors of 8 are 1, 2, 4, and 8, there are four such common positive divisors.

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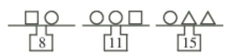
16. In the diagram, each scale shows the total mass (weight) of the shapes on that scale. What is the mass (weight) of a △



(A) 3 (B) 5 (C) 12 (D) 6 (E) 5.5

Source: 2007 Gauss Grade 7 #16

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: D

Solution:

The first scale tells us that a square and a circle together have a mass of 8.
The second scale tells us that a square and two circles together have a mass of 11.
We can replace the square and one circle on the second scale with an "8", so 8 plus the mass of a circle gives a mass of 11. This tells us that the mass of a circle is 3.
From the third scale, since the mass of a circle and two triangles is 15, then the mass of the two triangles only is $15 - 3 = 12$.
Therefore, the mass of one triangle is $12 \div 2 = 6$.

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17. To rent a kayak and a paddle, there is a fixed fee to use the paddle, plus a charge of \$5 per hour to use the kayak. For a three hour rental, the total cost is \$30. What is the total cost for a six hour rental?
(A) \$50 (B) \$15 (C) \$45 (D) \$60 (E) \$90

Source: 2007 Gauss Grade 7 #17

Primary Topics: Number Sense

Secondary Topics: Rates | Equations Solving

Answer: C

Solution:

The total cost to use the kayak for 3 hours is $3 \times \$5 = \$15$. Since the total rental cost for 3-hours is \$30, then the fixed fee to use the paddle is $\$30 - \$15 = \$15$.
For a six hour rental, the total cost is thus $\$15 + (6 \times \$5) = \$15 + \$30 = \$45$.

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18. Fred's birthday was on a Monday and was exactly 37 days after Pat's birthday. Julie's birthday was 67 days before Pat's birthday. On what day of the week was Julie's birthday?
(A) Saturday (B) Sunday (C) Monday (D) Tuesday (E) Wednesday

Source: 2007 Gauss Grade 7 #18

Primary Topics: Counting and Probability

Secondary Topics: Counting | Patterning/Sequences/Series

Answer: D

Solution:

Solution 1
Julie's birthday was $37 + 67 = 104$ days before Fred's birthday.
When we divide 104 by 7 (the number of days in one week), we obtain a quotient of 14 and a remainder of 6.
In 14 weeks, there are $14 \times 7 = 98$ days, so 98 days before Fred's birthday was also a Monday.
Since Julie's birthday was 104 days before Fred's, this was 6 days still before the Monday 98-days before Fred's birthday. The 6th day before a Monday is a Tuesday.
Therefore, Julie's birthday was a Tuesday.

Solution 2
37 days is 5 weeks plus 2 days. Since Fred's birthday was on a Monday and Pat's birthday was 37 days before Fred's, then Pat's birthday was on a Saturday.
67 days is 9 weeks plus 4 days. Since Pat's birthday was on a Saturday and Julie's birthday was 67 days before Pat's, then Julie's birthday was on a Tuesday.

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19. The whole numbers from 1 to 1000 are written. How many of these numbers have at least two 7's appearing side-by-side?

(A) 10 (B) 11 (C) 21 (D) 30 (E) 19

Source: 2007 Gauss Grade 7 #19

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: E

Solution:

The positive whole numbers less than 1000 that end with 77 are 77, 177, 277, 377, 477, 577, 677, 777, 877, 977.
The positive whole numbers less than 1000 which begin with 77 are 77, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779.
There is no other way for a positive whole number less than 1000 to contain at least two 7's side-by-side.
There are 10 numbers in the first list and 11 numbers in the second list. Since 2 numbers appear in both lists, the total number of whole numbers in the two lists is $10 + 11 - 2 = 19$.

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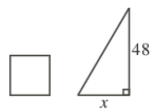
20. In the diagram, the square has a perimeter of 48 and the triangle has a height of 48. If the square and the triangle have the same area, what is the value of x?



(A) 1.5 (B) 12 (C) 6 (D) 3 (E) 24

Source: 2007 Gauss Grade 7 #20

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Perimeter

Answer: C

Solution:

Since the perimeter of the square is 48, its side length is $48 \div 4 = 12$.
Since the side length of the square is 12, its area is $12 \times 12 = 144$.
The area of the triangle is $\frac{1}{2} \times 48 \times x = 24x$.
Since the area of the triangle equals the area of the square, then $24x = 144$ or $x = 6$.

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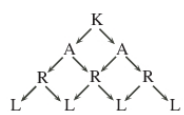
21. In the diagram, how many paths can be taken to spell "KARL"?



(A) 4 (B) 16 (C) 6 (D) 8 (E) 14

Source: 2007 Gauss Grade 7 #21

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: D

Solution:

Solution 1
Starting at the "K" there are two possible paths that can be taken. At each "A", there are again two possible paths that can be taken. Similarly, at each "R" there are two possible paths that can be taken.
Therefore, the total number of paths is $2 \times 2 \times 2 = 8$.
(We can check this by actually tracing out the paths.)

Solution 2
Each path from the K at the top to one of the L's at the bottom has to spell KARL.
There is 1 path that ends at the first L from the left. This path passes through the first A and the first R.
There are 3 paths that end at the second L. The first of these passes through the first A and the first R. The second of these passes through the first A and the second R. The third of these passes through the second A and the second R.
There are 3 paths that end at the third L. The first of these passes through the first A and the second R. The second of these passes through the second A and the second R. The third of these passes through the second A and the third R.
There is 1 path that ends at the last L. This path passes through the last A and the last R.
So the total number of paths to get to the bottom row is $1 + 3 + 3 + 1 = 8$, which is the number of paths that can spell KARL.

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22. The average of four different positive whole numbers is 4. If the difference between the largest and smallest of these numbers is as large as possible, what is the average of the other two numbers?
(A) $1\frac{1}{2}$ (B) $2\frac{1}{2}$ (C) 4 (D) 5 (E) 2

Source: 2007 Gauss Grade 7 #22

Primary Topics: Number Sense

Secondary Topics: Averages

Answer: B

Solution:

Since the average of four numbers is 4, their sum is $4 \times 4 = 16$.
For the difference between the largest and smallest of these numbers to be as large as possible, we would like one of the numbers to be as small as possible (so equal to 1) and the other (call it B for big) to be as large as possible.
Since one of the numbers is 1, the sum of the other three numbers is $16 - 1 = 15$.

For the B to be as large as possible, we must make the remaining two numbers (which must be different and not equal to 1) as small as possible. So these other two numbers must be equal to 2 and 3, which would make B equal to $15-2-3=10$.
So the average of these other two numbers is $\frac{2+3}{2}=\frac{5}{2}$ or $2\frac{1}{2}$.

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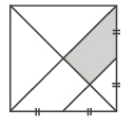
23. A square is divided, as shown. What fraction of the area of the square is shaded?



(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{6}$ (E) $\frac{3}{32}$

Source: 2007 Gauss Grade 7 #23

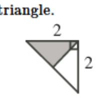
Primary Topics: Geometry and Measurement

Secondary Topics: Area | Fractions/Ratios

Answer: C

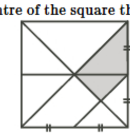
Solution:

Solution 1
Since we are dealing with fractions of the whole area, we may make the side of the square any convenient value.
Let us assume that the side length of the square is 4.
Therefore, the area of the whole square is $4\times 4=16$.
The two diagonals of the square divide it into four pieces of equal area (so each piece has area $16\div 4=4$).
The shaded area is made up from the ``right" quarter of the square with a small triangle removed, and so has area equal to 4 minus the area of this small triangle.
This small triangle is half of a larger triangle.



This larger triangle has its base and height each equal to half of the side length of the square (so equal to 2) and has a right angle. So the area of this larger triangle is $\frac{1}{2}\times 2\times 2=2$.
So the area of the small triangle is $\frac{1}{2}\times 2=1$, and so the area of the shaded region is $4-1=3$.
Therefore, the shaded area is $\frac{3}{16}$ of the area of the whole square.

Solution 2
Draw a horizontal line from the centre of the square through the shaded region.



The two diagonals divide the square into four pieces of equal area. The new horizontal line divides one of these pieces into two parts of equal area. Therefore, the shaded region above the new horizontal line is $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ of the total area of the square.
The shaded piece below this new horizontal line is half of the bottom right part of this right-hand piece of the square. (It is half of this part because the shaded triangle and unshaded triangle making up this part have the same shape.) So this remaining shaded piece is $\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}$ of the total area of the square.
In total, the shaded region is $\frac{1}{8}+\frac{1}{16}=\frac{3}{16}$ of the total area of the square.

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24. In the multiplication shown, P, Q and R are all different digits so that

$$\begin{array}{cccc} & P & P & Q \\ \times & & & Q \\ \hline R & Q & 5 & Q \end{array}$$

What is the value of $P+Q+R$?
(A) 20 (B) 13 (C) 15 (D) 16 (E) 17

Source: 2007 Gauss Grade 7 #24

Primary Topics: Number Sense

Secondary Topics: Digits | Operations

Answer: E

Solution:

First, we try to figure out what digit Q is.
Since the product is not equal to 0, Q cannot be 0. Since the product has four digits and the top number has three digits, then Q (which is multiplying the top number) must be bigger than-1.
Looking at the units digits in the product, we see that $Q\times Q$ has a units digit of Q.
Since $Q>1$, then Q must equal 5 or 6 (no other digit gives itself as a units digit when multiplied by itself).
But Q cannot be equal to 5, since if it was, the product $RQ5Q$ would end "55" and each of the two parts (PPQ and Q) of the product would end with a 5. This would mean that each of the parts of the product was divisible by 5, so the product should be divisible by $5\times 5=25$. But a number ending in 55 is not divisible by 25.
Therefore, $Q=6$.
So the product now looks like

$$\begin{array}{cccc} & P & P & 6 \\ \times & & & 6 \\ \hline R & 6 & 5 & 6 \end{array}$$

Now when we start the long multiplication, 6×6 gives 36, so we write down 6 and carry a 3.
When we multiply $P\times 6$ and add the carry of 3, we get a units digit of 5, so the units digit of $P\times 6$ should be 2.
For this to be the case, $P=2$ or $P=7$.
We can now try these possibilities: $226\times 6=1356$ and $776\times 6=4656$. Only the second ends "656" like the product should.
So $P=7$ and $R=4$, and so $P+Q+R=7+6+4=17$.

So $P = 7$ and $R = 4$, and so $P + Q + R = 7 + 6 + 4 = 17$.

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25. The CMC reception desk has a tray in which to stack letters as they arrive. Starting at 12:00, the following process repeats every five minutes:
    - Step 1 - Three letters arrive at the reception desk and are stacked on top of the letters already in the stack. The first of the three is placed on the stack first, the second letter next, and the third letter on top.
    - Step 2 - The top two letters in the stack are removed.

    This process repeats until 36 letters have arrived (and the top two letters have been immediately removed). Once all 36 letters have arrived (and the top two letters have been immediately removed), no more letters arrive and the top two letters in the stack continue to be removed every five minutes until all 36 letters have been removed. At what time was the 13th letter to arrive removed?
    (A) 1:15 (B) 1:20 (C) 1:10 (D) 1:05 (E) 1:25

Source: 2007 Gauss Grade 7 #25

Primary Topics: Counting and Probability

Secondary Topics: Counting | Rates

Answer: A

Solution:

The easiest way to keep track of the letters here is to make a table of what letters arrive at each time, what letters are removed, and what letters stay in the pile.

Time	Letters Arrived	Letters Removed	Remaining Pile (bottom to top)
$12:00$	$\{1,2,3\}$	$\{3,2\}$	$\{1\}$
$12:05$	$\{4,5,6\}$	$\{6,5\}$	$\{1,4\}$
$12:10$	$\{7,8,9\}$	$\{9,8\}$	$\{1,4,7\}$
$12:15$	$\{10,11,12\}$	$\{12,11\}$	$\{1,4,7,10\}$
$12:20$	$\{13,14,15\}$	$\{15,14\}$	$\{1,4,7,10,13\}$
$12:25$	$\{16,17,18\}$	$\{18,17\}$	$\{1,4,7,10,13,16\}$
$12:30$	$\{19,20,21\}$	$\{21,20\}$	$\{1,4,7,10,13,16,19\}$
$12:35$	$\{22,23,24\}$	$\{24,23\}$	$\{1,4,7,10,13,16,19,22\}$
$12:40$	$\{25,26,27\}$	$\{27,26\}$	$\{1,4,7,10,13,16,19,22,25\}$
$12:45$	$\{28,29,30\}$	$\{30,29\}$	$\{1,4,7,10,13,16,19,22,25,28\}$
$12:50$	$\{31,32,33\}$	$\{33,32\}$	$\{1,4,7,10,13,16,19,22,25,28,31\}$
$12:55$	$\{34,35,36\}$	$\{36,35\}$	$\{1,4,7,10,13,16,19,22,25,28,31,34\}$
$1:00$	$\emptyset$	$\{34,31\}$	$\{1,4,7,10,13,16,19,22,25,28\}$
$1:05$	$\emptyset$	$\{28,25\}$	$\{1,4,7,10,13,16,19,22\}$
$1:10$	$\emptyset$	$\{22,19\}$	$\{1,4,7,10,13,16\}$
$1:15$	$\emptyset$	$\{16,13\}$	$\{1,4,7,10\}$

(At 12:55, all 36 letters have been delivered, so starting at 1:00 letters are only removed and no longer added.)
Letter #13 is removed at 1:15.
