The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7 Solutions

1. $4.1 + 1.05 + 2.005$ equals
   (A) 7.155 (B) 7.2 (C) 8.1 (D) 7.605 (E) 8.63

Source: 2009 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations | Decimals

Answer: A

Solution:

Adding, $4.1 + 1.05 + 2.005 = 5.15 + 2.005 = 7.155$.

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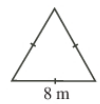
2. In the diagram, the equilateral triangle has a base of 8 m. The perimeter of the equilateral triangle is



   (A) 4 m (B) 16 m (C) 24 m (D) 32 m (E) 64 m

Source: 2009 Gauss Grade 7 #2

Primary Topics: Geometry and Measurement

Secondary Topics: Perimeter

Answer: C

Solution:

Since the triangle is equilateral, all sides are equal in length.
Therefore, the perimeter of the triangle is $8 + 8 + 8 = 8 \times 3 = 24$.

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3. How many numbers in the list 11, 12, 13, 14, 15, 16, 17 are prime numbers?
   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Source: 2009 Gauss Grade 7 #3

Primary Topics: Number Sense

Secondary Topics: Prime Numbers

Answer: D

Solution:

The numbers 12, 14 and 16 are even, and therefore divisible by 2 so not prime.
The number 15 is divisible by 5; therefore, it is also not prime.
Each of the remaining numbers, 11, 13 and 17, has no positive divisor other than 1 and itself.
Therefore, 3 numbers in the list are prime.

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4. The smallest number in the list $\{0.40, 0.25, 0.37, 0.05, 0.81\}$ is
   (A) 0.4 (B) 0.25 (C) 0.37 (D) 0.05 (E) 0.81

Source: 2009 Gauss Grade 7 #4

Primary Topics: Number Sense

Secondary Topics: Decimals

Answer: D

Solution:

Solution 1
Since each number in the set is between 0 and 1, they can be ordered from smallest to largest by comparing their tenths digits first. In order from smallest to largest the list is

$$\{0.05, 0.25, 0.37, 0.40, 0.81\}$$

The smallest number in the list is 0.05.

Solution 2
Consider the equivalent fraction for each decimal:
$0.40 = \frac{40}{100}$, $0.25 = \frac{25}{100}$, $0.37 = \frac{37}{100}$, $0.05 = \frac{5}{100}$, and $0.81 = \frac{81}{100}$.
Since the denominators all equal 100, we choose the fraction with the smallest numerator.
Therefore, $0.05 = \frac{5}{100}$ is the smallest number in the set.

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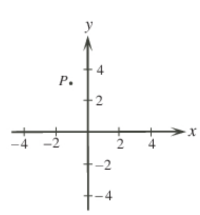
5. In the diagram, the coordinates of point P could be



(A) $(1,3)$ (B) $(1,-3)$ (C) $(-3,1)$ (D) $(3,-1)$ (E) $(-1,3)$

Source: 2009 Gauss Grade 7 #5

Primary Topics: Geometry and Measurement

Secondary Topics: Graphs | Estimation

Answer: E

Solution:

The x-coordinate of point P lies between -2 and 0. The y-coordinate lies between 2 and 4. Of the possible choices, $(-1,3)$ is the only point that satisfies both of these conditions.

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6. The temperature in Vancouver is 22°C. The temperature in Calgary is 19°C colder than the temperature in Vancouver. The temperature in Quebec City is 11°C colder than the temperature in Calgary. What is the temperature in Quebec City?
(A) 14°C (B) 3°C (C) −8°C (D) 8°C (E) −13°C

Source: 2009 Gauss Grade 7 #6

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: C

Solution:

The temperature in Vancouver is 22°C.
The temperature in Calgary is 22°C − 19°C = 3°C.
The temperature in Quebec City is 3°C − 11°C = −8°C.

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7. On a map of Nunavut, a length of 1 centimetre measured on the map represents a real distance of 60 kilometres. What length on the map represents a real distance of 540 kilometres?
(A) 9 cm (B) 90 cm (C) 0.09 cm (D) 0.11 cm (E) 5.4 cm

Source: 2009 Gauss Grade 7 #7

Primary Topics: Geometry and Measurement

Secondary Topics: Rates | Measurement

Answer: A

Solution:

Since a real distance of 60 km is represented by 1 cm on the map, then a real distance of 540-km is represented by $\frac{540}{60}$ cm or 9 cm on the map.

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8. In $\triangle PQR$, the sum of $\angle P$ and $\angle Q$ is 60°. The measure of $\angle R$ is
(A) 60° (B) 300° (C) 120° (D) 30° (E) 40°

Source: 2009 Gauss Grade 7 #8

Primary Topics: Geometry and Measurement

Secondary Topics: Angles | Triangles

Answer: C

Solution:

The sum of the three angles in any triangle is always 180°.
In $\triangle PQR$, the sum of $\angle P$ and $\angle Q$ is 60°, and thus $\angle R$ must measure $180^\circ - 60^\circ = 120^\circ$.

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9. In a class of 30 students, exactly 7 have been to Mexico and exactly 11 have been to England. Of these students, 4 have been to both Mexico and England. How many students in this class have not been to Mexico or England?
(A) 23 (B) 16 (C) 20 (D) 12 (E) 18

Source: 2009 Gauss Grade 7 #9

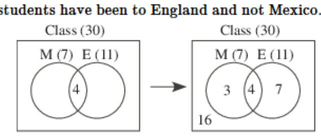
Primary Topics: Data Analysis

Secondary Topics: Counting

Answer: B

Solution:

The first Venn diagram below shows that there are 30 students in the class, 7 students have been to Mexico, 11 students have been to England and 4 students have been to both countries.
Of the 7 students that have been to Mexico, 4 have also been to England.
Therefore, $7 - 4 = 3$ students have been to Mexico and not England.
Of the 11 students that have been to England, 4 have also been to Mexico.
Therefore, $11 - 4 = 7$ students have been to England and not Mexico.



Therefore, 3 students have been to Mexico only, 7 students have been to England only, and 4 students have been to both.
In the class of 30 students, this leaves $30 - 3 - 7 - 4 = 16$ students who have not been to Mexico or England.

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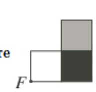
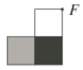
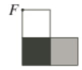
10. If the figure 

is rotated 180° about point F, the result could be

(A)  (B)  (C) 

(D) 

(E) 

Source: 2009 Gauss Grade 7 #10

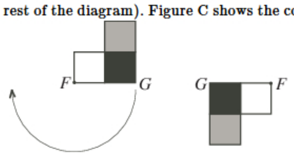
Primary Topics: Geometry and Measurement

Secondary Topics: Transformations

Answer: C

Solution:

Consider rotating the horizontal line segment FG (as shown below) 180° about point F. A 180° rotation is half of a full rotation. Point F stays fixed, while segment FG rotates to the left of F (as does the rest of the diagram). Figure C shows the correct result.



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11. Scott challenges Chris to a 100 m race. Scott runs 4 m for every 5 m that Chris runs. How far will Scott have run when Chris crosses the finish line?
(A) 75 m (B) 96 m (C) 20 m (D) 76 m (E) 80 m

Source: 2009 Gauss Grade 7 #11

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: E

Solution:

Since Scott runs 4 m for every 5 m Chris runs, Scott runs $\frac{4}{5}$ of the distance that Chris runs in the same time. When Chris crosses the finish line he will have run 100 m.
When Chris has run 100 m, Scott will have run $\frac{4}{5} \times 100 = 80$ m.

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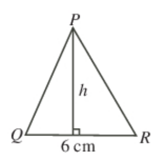
12. $\triangle PQR$ has an area of 27 cm^2 and a base measuring 6 cm. What is the height, h, of $\triangle PQR$?



(A) 9 cm (B) 18 cm (C) 4.5 cm (D) 2.25 cm (E) 7 cm

Source: 2009 Gauss Grade 7 #12

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Triangles

Answer: A

Solution:

The area of a triangle can be calculated using the formula Area $= \frac{1}{2} \times$ base $\times$ height.
The area is 27 cm^2 and the base measures 6 cm. Substituting these values into the formula, $A = \frac{1}{2} \times b \times h$ becomes $27 = \frac{1}{2} \times 6 \times h$ or $27 = 3h$. Therefore, $h = 9$ cm.

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13. The product $60 \times 60 \times 24 \times 7$ equals
(A) the number of minutes in seven weeks
(B) the number of hours in sixty days
(C) the number of seconds in seven hours
(D) the number of seconds in one week
(E) the number of minutes in twenty-four weeks

Source: 2009 Gauss Grade 7 #13

Primary Topics: Algebra and Equations

Secondary Topics: Operations | Measurement

Answer: D

Solution:

There are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day and 7 days in a week. Therefore, the number of seconds in one week is $60 \times 60 \times 24 \times 7$.

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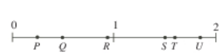
14. Which of the letters positioned on the number line best represents the value of $S \div T$?



(A) P (B) Q (C) R (D) T (E) U

Source: 2009 Gauss Grade 7 #14

Primary Topics: Number Sense

Secondary Topics: Estimation

Answer: C

Answer: C

Solution:

Solution 1
S represents a value of approximately 1.5 on the number line, while T is approximately 1.6. Then $S \div T$ is approximately equal to $1.5 \div 1.6 = 0.9375$. R is the only value on the number line that is slightly less than 1 and therefore best represents the value of $S \div T$.

Solution 2
S is slightly less than T, so $\frac{S}{T}$ is slightly less than 1. Thus, $\frac{S}{T}$ is best represented by R.

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15. The product of three *different* positive integers is 144. What is the maximum possible sum of these three integers?
(A) 20 (B) 75 (C) 146 (D) 52 (E) 29

Source: 2009 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Factoring

Answer: B

Solution:

For the sum to be a maximum, we try to use the largest divisor possible.
Although 144 is the largest divisor, using it would require that the remaining two divisors both equal 1 (since the divisors are integers).
Since the question requires the product of three different divisors, $144 = 144 \times 1 \times 1$ is not possible and the answer cannot be $144 + 1 + 1 = 146$ or (C).
The next largest divisor of 144 is 72 and $144 = 72 \times 2 \times 1$.
Now the three factors are different and their sum is $72 + 2 + 1 = 75$.
Since 75 is the largest possible answer remaining, we have found the maximum.

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16. A square has an area of 25. A rectangle has the same width as the square. The length of the rectangle is double its width. What is the area of the rectangle?
(A) 25 (B) 12.5 (C) 100 (D) 50 (E) 30

Source: 2009 Gauss Grade 7 #16

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Equations Solving

Answer: D

Solution:

Solution 1
For the square to have an area of 25, each side length must be $\sqrt{25} = 5$.
The rectangle's width is equal to that of the square and therefore must also be 5.
The length of the rectangle is double its width or $5 \times 2 = 10$.
The area of the rectangle is thus $5 \times 10 = 50$.

Solution 2
The rectangle has the same width as the square but twice the length.
Thus, the rectangle's area is twice that of the square or $2 \times 25 = 50$.

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17. Vanessa set a school record for most points in a single basketball game when her team scored 48 points. The six other players on her team averaged 3.5 points each. How many points did Vanessa score to set her school record?
(A) 21 (B) 25 (C) 32 (D) 17 (E) 27

Source: 2009 Gauss Grade 7 #17

Primary Topics: Data Analysis

Secondary Topics: Averages

Answer: E

Solution:

The six other players on the team averaged 3.5 points each.
The total of their points was $6 \times 3.5 = 21$.
Vanessa scored the remainder of the points, or $48 - 21 = 27$ points.

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18. If x, y and z are positive integers with $xy = 18$, $xz = 3$ and $yz = 6$, what is the value of $x + y + z$?
(A) 6 (B) 10 (C) 25 (D) 11 (E) 8

Source: 2009 Gauss Grade 7 #18

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving

Answer: B

Solution:

Since x and z are positive integers and $xz = 3$, the only possibilities are $x = 1$ and $z = 3$ or $x = 3$ and $z = 1$.
Assuming that $x = 1$ and $z = 3$, $yz = 6$ implies $3y = 6$ or $y = 2$.
Thus, $x = 1$ and $y = 2$ and $xy = 2$.
This contradicts the first equation $xy = 18$.
Therefore, our assumption was incorrect and it must be true that $x = 3$ and $z = 1$.
Then $yz = 6$ and $z = 1$ implies $y = 6$.
Checking, $x = 3$ and $y = 6$ also satisfies $xy = 18$, the first equation.
Therefore, the required sum is $x + y + z = 3 + 6 + 1 = 10$.

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19. A jar contains quarters (worth \$0.25 each), nickels (worth \$0.05 each) and pennies (worth \$0.01 each). The value of the quarters is \$10.00. The value of the nickels is \$10.00. The value of the pennies is \$10.00. If Judith randomly chooses one coin from the jar, what is the probability that it is a quarter?

(A) $\frac{25}{31}$ (B) $\frac{1}{31}$ (C) $\frac{1}{3}$ (D) $\frac{5}{248}$ (E) $\frac{1}{30}$

Source: 2009 Gauss Grade 7 #19

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: B

Solution:

The value of all quarters is \$10.00.
Each quarter has a value of \$0.25.
There are thus $10 \div 0.25 = 40$ quarters in the jar.
Similarly, there are $10 \div 0.05 = 200$ nickels, and $10 \div 0.01 = 1000$ pennies in the jar.
In total, there are $40 + 200 + 1000 = 1240$ coins in the jar.
The probability that the selected coin is a quarter is $\frac{\text{the number of quarters}}{\text{the total number of coins}} = \frac{40}{1240} = \frac{1}{31}$.

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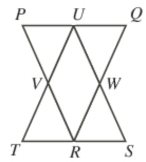
20. Each of $\triangle PQR$ and $\triangle STU$ has an area of 1. In $\triangle PQR$, U, W and V are the midpoints of the sides. In $\triangle STU$, R, V and W are the midpoints of the sides. What is the area of parallelogram $UVRW$?



(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) $\frac{2}{3}$

Source: 2009 Gauss Grade 7 #20

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Quadrilaterals

Answer: B

Solution:

Since V is the midpoint of PR, then $PV = VR$.
Since $UVRW$ is a parallelogram, then $VR = UW$.
Since W is the midpoint of US, then $UW = WS$.
Thus, $PV = VR = UW = WS$.
Similarly, $QW = WR = UV = VT$.
Also, R is the midpoint of TS and therefore, $TR = RS$.
Thus, $\triangle VTR$ is congruent to $\triangle WRS$, and so the two triangles have equal area.
Diagonal VW in parallelogram $UVRW$ divides the area of the parallelogram in half.
Therefore, $\triangle UVW$ and $\triangle RWV$ have equal areas.
In quadrilateral $VRSW$, $VR = WS$ and VR is parallel to WS.
Thus, $VRSW$ is a parallelogram and the area of $\triangle RWV$ is equal to the area of $\triangle WRS$.
Therefore, $\triangle VTR$, $\triangle WRS$, $\triangle RWV$, and $\triangle UVW$ have equal areas, and so these four triangles divide $\triangle STU$ into quarters.
Parallelogram $UVRW$ is made from two of these four quarters of $\triangle STU$, or one half of $\triangle STU$.
The area of parallelogram $UVRW$ is thus $\frac{1}{2}$ of 1, or $\frac{1}{2}$.

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21. Lara ate $\frac{1}{4}$ of a pie and Ryan ate $\frac{3}{10}$ of the same pie. The next day Cassie ate $\frac{2}{3}$ of the pie that was left. What fraction of the original pie was not eaten?
(A) $\frac{9}{10}$ (B) $\frac{3}{10}$ (C) $\frac{7}{60}$ (D) $\frac{3}{20}$ (E) $\frac{1}{20}$

Source: 2009 Gauss Grade 7 #21

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: D

Solution:

Together, Lara and Ryan ate $\frac{1}{4} + \frac{3}{10} = \frac{5}{20} + \frac{6}{20} = \frac{11}{20}$ of the pie.
Therefore, $1 - \frac{11}{20} = \frac{9}{20}$ of the pie remained.
The next day, Cassie ate $\frac{2}{3}$ of the pie that remained.
This implies that $1 - \frac{2}{3} = \frac{1}{3}$ of the pie that was remaining was left after Cassie finished eating.
Thus, $\frac{1}{3}$ of $\frac{9}{20}$, or $\frac{3}{20}$ of the original pie was not eaten.

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22. In the diagram, a 4×4 grid is to be filled so that each of the digits 1, 2, 3, and 4 appears in each row and each column. The 4×4 grid is divided into four smaller 2×2 squares. Each of these 2×2 squares is also to contain each of the digits 1, 2, 3 and 4. What digit replaces P?

1		3	
	2		
	P		
			4

(A) 1 (B) 2 (C) 3 (D) 4
(E) The digit cannot be determined

Source: 2009 Gauss Grade 7 #22

Primary Topics: Other

Secondary Topics: Games

Answer: A

Solution:

The first row is missing a 4 and a 2. Since there is already a 2 in the second column (in the second row), the first row, second column must contain a 4 and the first row, fourth column must contain a 2. To complete the upper left 2×2 square, the second row, first column must contain a 3. The second row is now missing both a 4 and a 1. But the fourth column already contains a 4 (in the fourth row), therefore the second row, fourth column must contain a 1. To complete the fourth column, we place a 3 in the third row. Now the P cannot be a 3, since there is already a 3 in the third row. Also, the P cannot be a 4 or a 2, since the second column already contains these numbers. By process of elimination, the digit 1 must replace the P.

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23. Each time Kim pours water from a jug into a glass, exactly 10% of the water remaining in the jug is used. What is the minimum number of times that she must pour water into a glass so that less than half the water remains in the jug?
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Source: 2009 Gauss Grade 7 #23

Primary Topics: Number Sense

Secondary Topics: Percentages | Fractions/Ratios

Answer: C

Solution:

Solution 1 We can suppose that the jug contains 1 litre of water at the start. The following table shows the quantity of water poured in each glass and the quantity of water remaining in each glass after each pouring, stopping when the quantity of water remaining is less than-0.5 L.

Number of glasses	Number of litres poured	Number of litres remaining
1	10% of $1 = 0.1$	$1 - 0.1 = 0.9$
2	10% of $0.9 = 0.09$	$0.9 - 0.09 = 0.81$
3	10% of $0.81 = 0.081$	$0.81 - 0.081 = 0.729$
4	10% of $0.729 = 0.0729$	$0.729 - 0.0729 = 0.6561$
5	10% of $0.6561 = 0.06561$	$0.6561 - 0.06561 = 0.59049$
6	10% of $0.59049 = 0.059049$	$0.59049 - 0.059049 = 0.531441$
7	10% of $0.531441 = 0.0531441$	$0.531441 - 0.0531441 = 0.4782969$

We can see from the table that the minimum number of glasses that Kim must pour so that less than half of the water remains in the jug is 7. Solution 2 Removing 10\% of the water from the jug is equivalent to leaving 90\% of the water in the jug. Thus, to find the total fraction remaining in the jug after a given pour, we multiply the previous total by 0.9. We make the following table, stopping when the fraction of water remaining in the glass is first less than 0.5 (one half).

Number of glasses poured	Fraction of water remaining
1	$0.9 \times 1 = 0.9$
2	$0.9 \times 0.9 = 0.81$
3	$0.9 \times 0.81 = 0.729$
4	$0.9 \times 0.729 = 0.6561$
5	$0.9 \times 0.6561 = 0.59049$
6	$0.9 \times 0.59049 = 0.531441$
7	$0.9 \times 0.531441 = 0.4782969$

We can see from the table that the minimum number of glasses that Kim must pour so that less than half of the water remains in the jug is 7.

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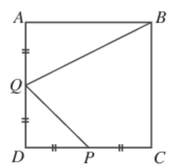
24. In square $ABCD$, P is the midpoint of DC and Q is the midpoint of AD. If the area of the quadrilateral $QBCP$ is 15, what is the area of square $ABCD$?



(A) 27.5 (B) 25 (C) 30 (D) 20 (E) 24

Source: 2009 Gauss Grade 7 #24

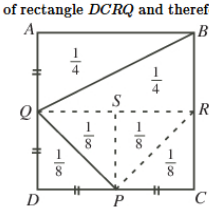
Primary Topics: Geometry and Measurement

Secondary Topics: Quadrilaterals | Area | Triangles

Answer: E

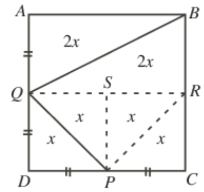
Solution:

Solution 1
Draw line segment QR parallel to DC, as in the following diagram. This segment divides square $ABCD$ into two halves. Since triangles ABQ and RQB are congruent, each is half of rectangle $ABRQ$ and therefore one quarter of square $ABCD$. Draw line segment PS parallel to DA, and draw line segment PR. Triangles PDQ, PSQ, PSR and PCR are congruent. Therefore each is one quarter of rectangle $DCRQ$ and therefore one eighth of square $ABCD$.



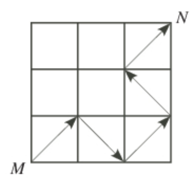
Quadrilateral $QBCP$ therefore represents $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$ of square $ABCD$. Its area is therefore $\frac{5}{8}$ of the area of the square.
Therefore, $\frac{5}{8}$ of the area of the square is equal to 15. Therefore, $\frac{1}{8}$ of the area of the square is equal to 3. Therefore the square has an area of 24.
Solution 2
Draw a line segment from Q to R, the midpoint of BC

Draw a line segment from Q to R, the midpoint of BC.
Draw a line segment from P to S, the midpoint of QR.
Let the area of $\triangle QSP$ equal x. Thus, the area of $\triangle QDP$ is also x and $QDPS$ has area $2x$.
Square $SPCR$ is congruent to square $QDPS$ and thus has area $2x$.
Rectangle $QDCR$ has area $4x$, as does the congruent rectangle $AQRB$.
Also, $\triangle AQB$ and $\triangle BRQ$ have equal areas and thus, each area is $2x$.



Quadrilateral $QBCP$ is made up of $\triangle BRQ$, $\triangle QSP$ and square $SPCR$, and thus has area $2x + x + 2x = 5x$.
Since quadrilateral $QBCP$ has area 15, then $5x = 15$ or $x = 3$.
Therefore, the area of square $ABCD$, which is made up of quadrilateral $QBCP$, $\triangle AQB$ and $\triangle QDP$, is $5x + 2x + x = 8x = 8(3) = 24$.

25. Kira can draw a connected path from M to N by drawing arrows along only the diagonals of the nine squares shown. One such possible path is shown. A path cannot pass through the interior of the same square twice. In total, how many different paths can she draw from M to N?



(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Source: 2009 Gauss Grade 7 #25

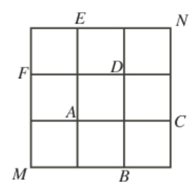
Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: E

Solution:

Labeling the diagram as shown below, we can describe paths using the points they pass through.



The path $MADN$ is the only path of length 3 (traveling along 3 diagonals).
Since the diagram is symmetrical about MN, all other paths will have a reflected path in the line MN and therefore occur in pairs. This observation alone allows us to eliminate (B) and (D) as possible answers since they are even.
The following table lists all possible paths from M to N traveling along diagonals only.

Path length	Path Name	Reflected Path
3	$MADN$	same
5	$MABCDN$	$MAFEDN$
9	$MABCDEFADN$	$MAFEDCBADN$
	$MABCDAFEDN$	$MAFEDABCDN$
	$MADCBAFEDN$	$MADEFABCDN$

At this point we have listed 9 valid paths. Since paths occur in pairs (with the exception of $MADN$), the next possible answer would be 11. Since 11 is not given as an answer (and 9 is the largest possible answer given), we can be certain that we have found them all.