The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Gauss Contest Grade 7
Solutions

1. The value of $5+4-3+2-1$ is
(A) 0 (B) -5 (C) 3 (D) -3 (E) 7

Source: 2011 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: E

Solution:

Evaluating, $5+4-3+2-1 = 9-3+2-1 = 6+2-1 = 8-1 = 7$.

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2. The value of $\sqrt{9+16}$ is
(A) 5.2 (B) 7 (C) 5.7 (D) 25 (E) 5

Source: 2011 Gauss Grade 7 #2

Primary Topics: Number Sense

Secondary Topics: Operations

Answer: E

Solution:

We must first add 9 and 16. Thus, $\sqrt{9+16} = \sqrt{25} = 5$.

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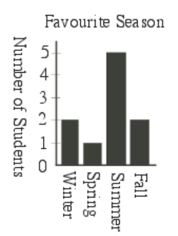
3. Students were surveyed about their favourite season. The results are shown in the bar graph. What percentage of the 10 students surveyed chose Spring?



(A) 50 (B) 10 (C) 25 (D) 250 (E) 5

Source: 2011 Gauss Grade 7 #3

Primary Topics: Data Analysis

Secondary Topics: Percentages | Graphs

Answer: B

Solution:

Reading from the bar graph, only 1 student chose spring.
Since 10 students were surveyed, then the percentage of students that chose spring was $\frac{1}{10} \times 100\%$ or 10%
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4. Ground beef sells for \$5.00 per kg. How much does 12 kg of ground beef cost?
(A) \$5.00 (B) \$12.00 (C) \$60.00 (D) \$17.00 (E) \$2.40

Source: 2011 Gauss Grade 7 #4

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: C

Solution:

Since ground beef sells for \$5.00 per kg, then the cost of 12 kg is $\$5.00 \times 12 = \$60.00$.

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5. The smallest number in the list $\{1.0101, 1.0011, 1.0110, 1.1001, 1.1100\}$ is
(A) 1.0101 (B) 1.0011 (C) 1.0110 (D) 1.1001 (E) 1.1100

Source: 2011 Gauss Grade 7 #5

Primary Topics: Number Sense

Secondary Topics: Decimals

Answer: B

Solution:

Since each of the numbers is between 1 and 2, we consider the tenths digits.
The numbers 1.0101, 1.0011 and 1.0110 are between 1 and 1.1, while 1.1001 and 1.1100 are both greater than 1.1.
We may eliminate answers (D) and (E).
Next, we consider the hundreths digits.
While 1.0101 and 1.0110 each have a 1 as their hundreths digit, 1.0011 has a 0 and is therefore the smallest number in the list.
The ordered list from smallest to largest is $\{1.0011, 1.0101, 1.0110, 1.1001, 1.1100\}$

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6. You are writing a multiple choice test and on one question you guess and pick an answer at random. If there are five possible choices (A,B,C,D,E), what is the probability that you guessed correctly?
(A) $\frac{1}{5}$ (B) $\frac{5}{5}$ (C) $\frac{4}{5}$ (D) $\frac{2}{5}$ (E) $\frac{3}{5}$

Source: 2011 Gauss Grade 7 #6

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: A

Solution:

Since you randomly choose one of the five answers, then each has an equally likely chance of being selected.
Thus, the probability that you select the one correct answer from the five is $\frac{1}{5}$.

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7. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ equals
(A) $3\frac{1}{3}$ (B) $7+\frac{1}{3}$ (C) $\frac{3}{7}$ (D) $7+3$ (E) $7\times\frac{1}{3}$

Source: 2011 Gauss Grade 7 #7

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: E

Solution:

Since we are adding $\frac{1}{3}$ seven times, then the result is equal to $7\times\frac{1}{3}$.

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8. Keegan paddled the first 12 km of his 36 km kayak trip before lunch. What fraction of his overall trip remains to be completed after lunch?
(A) $\frac{1}{2}$ (B) $\frac{5}{6}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$ (E) $\frac{3}{5}$

Source: 2011 Gauss Grade 7 #8

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: D

Solution:

Keegan paddled 12 km of his 36 km trip before lunch.
Therefore, Keegan has $36-12=24$ km left to be completed after lunch.
The fraction of his trip remaining to be completed is $\frac{24}{36}=\frac{2}{3}$.

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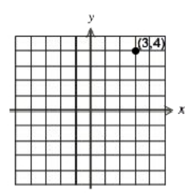
9. If the point $(3,4)$ is reflected in the x-axis, what are the coordinates of its image?



(A) $(-4,3)$ (B) $(-3,4)$ (C) $(4,3)$ (D) $(3,-4)$ (E) $(-3,-4)$

Source: 2011 Gauss Grade 7 #9

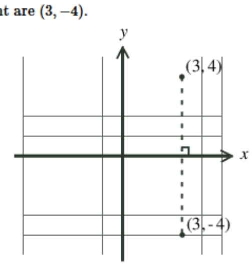
Primary Topics: Geometry and Measurement

Secondary Topics: Transformations

Answer: D

Solution:

After reflecting the point $(3,4)$ in the x-axis, the x-coordinate of the image will be the same as the x-coordinate of the original point, $x=3$.
The original point is a distance of 4 from the x-axis.
The image will be the same distance from the x-axis, but below the x-axis.
Thus, the image has y-coordinate -4.
The coordinates of the image point are $(3,-4)$.



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10. I bought a new plant for my garden. Anika said it was a red rose, Bill said it was a purple daisy, and Cathy said it was a red dahlia. Each person was correct in stating either the colour or the type of plant. What was the plant that I bought?
(A) purple dahlia (B) purple rose (C) red dahlia
(D) yellow rose (E) red daisy

Source: 2011 Gauss Grade 7 #10

Primary Topics: Other

Secondary Topics: Logic

Secondary Topics: Logic

Answer: E

Solution:

Anika said that the plant was a red rose.
Cathy said that the plant was a red dahlia.
If red was not the correct colour of the plant, then Anika was incorrect about the colour and therefore must be correct about the type; in other words, the plant is a rose.
Similarly, if red was not the correct colour of the plant, then Cathy was incorrect about the colour and therefore must be correct about the type; in other words, the plant is a dahlia.
But the plant cannot be both a rose and a dahlia.
Therefore, Cathy and Anika must have been correct about the colour being red.
Bill said that the plant was a purple daisy.
Since we know that the colour of the plant is red, then Bill was incorrect about it being purple.
Therefore, Bill must have been correct about it being a daisy.
Thus, the plant is a red daisy.

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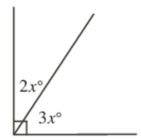
11. In the diagram, the value of x is



(A) 15 (B) 20 (C) 22 (D) 18 (E) 36

Source: 2011 Gauss Grade 7 #11

Primary Topics: Geometry and Measurement

Secondary Topics: Angles | Equations Solving

Answer: D

Solution:

The angles $2x^\circ$ and $3x^\circ$ shown are complementary and thus add to 90°.
Therefore, $2x + 3x = 90$ or $5x = 90$ and so $x = \frac{90}{5} = 18$.

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12. A square has a perimeter of 28 cm. The area of the square, in cm^2, is
(A) 196 (B) 784 (C) 64 (D) 49 (E) 56

Source: 2011 Gauss Grade 7 #12

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Perimeter

Answer: D

Solution:

Since the four sides of a square are equal in length and the perimeter is 28, then each side has length $\frac{28}{4} = 7$.
The area of the square is the product of the length and width, which are each equal to 7.
Therefore, the area of the square in cm^2 is $7 \times 7 = 49$.

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13. Five children had dinner. Chris ate more than Max. Brandon ate less than Kayla. Kayla ate less than Max but more than Tanya. Which child ate the second most?
(A) Brandon (B) Chris (C) Kayla (D) Max (E) Tanya

Source: 2011 Gauss Grade 7 #13

Primary Topics: Other

Secondary Topics: Logic | Inequalities

Answer: D

Solution:

Since Kayla ate less than Max and Chris ate more than Max, then Kayla ate less than Max who ate less than Chris.
Brandon and Tanya both ate less than Kayla.
Therefore, Max ate the second most.

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14. A *palindrome* is a positive integer that is the same when read forwards or backwards. For example, 545 and 1331 are both palindromes. The difference between the smallest three-digit palindrome and the largest three-digit palindrome is
(A) 909 (B) 898 (C) 888 (D) 979 (E) 878

Source: 2011 Gauss Grade 7 #14

Primary Topics: Number Sense

Secondary Topics: Digits

Answer: B

Solution:

The smallest three digit palindrome is 101.
The largest three digit palindrome is 999.
The difference between the smallest three digit palindrome and the largest three digit palindrome is $999 - 101 = 898$.

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15. A ski lift carries a skier at a rate of 12 km per hour. How many kilometres does the ski lift carry the skier in 10 minutes?
(A) 120 (B) 1.2 (C) 2 (D) 2.4 (E) 1.67

Source: 2011 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: C

Solution:

Since 10 minutes is equivalent to $\frac{10}{60} = \frac{1}{6}$ of an hour, the skier travels $12 \div 6 = 2$ km.

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16. A 51 cm rod is built from 5 cm rods and 2 cm rods. All of the 5 cm rods must come first, and are followed by the 2 cm rods. For example, the rod could be made from seven 5 cm rods followed by eight 2 cm rods. How many ways are there to build the 51 cm rod?
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Source: 2011 Gauss Grade 7 #16

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: A

Solution:

Any number of 2 cm rods add to give a rod having an even length.
Since we need an odd length, 51 cm, then we must combine an odd length from the 5-cm rods with the even length from the 2 cm rods to achieve this.
An odd length using 5 cm rods can only be obtained by taking an odd number of them.
All possible combinations are shown in the table below.

Number of 5 cm rods	Length in 5 cm rods	Length in 2 cm rods	Number of 2 cm rods
1	5	$51-5=46$	$46 \div 2 = 23$
3	15	$51-15=36$	$36 \div 2 = 18$
5	25	$51-25=26$	$26 \div 2 = 13$
7	35	$51-35=16$	$16 \div 2 = 8$
9	45	$51-45=6$	$6 \div 2 = 3$

Note that attempting to use 11 (or more) 5 cm rods gives more than the 51 cm length required.
Thus, there are exactly 5 possible combinations that add to 51 cm using 5 cm rods first followed by 2-cm rods.

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17. In Braydon's cafeteria, the meats available are beef and chicken. The fruits available are apple, pear and banana. Braydon is randomly given a lunch with one meat and one fruit. What is the probability that the lunch will include a banana?
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{5}$ (E) $\frac{3}{5}$

Source: 2011 Gauss Grade 7 #17

Primary Topics: Counting and Probability

Secondary Topics: Probability

Answer: A

Solution:

Solution 1
Choosing one meat and one fruit, the possible lunches are beef and apple, beef and pear, beef and banana, chicken and apple, chicken and pear, or chicken and banana.
Of these 6 lunches, 2 of them include a banana.
Thus when randomly given a lunch, the probability that it will include a banana is $\frac{2}{6}$ or $\frac{1}{3}$.
Solution 2
Each of the possible lunches that Braydon may receive contain exactly one fruit.
The meat chosen for each lunch does not affect what fruit is chosen.
Thus, the probability that the lunch includes a banana is independent of the meat that is served with it.
Since there are 3 fruits to choose from, then the probability that the lunch includes a banana is $\frac{1}{3}$.

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18. Three pumpkins are weighed two at a time in all possible ways. The weights of the pairs of pumpkins are 12 kg, 13 kg and 15 kg. How much does the lightest pumpkin weigh?
(A) 4 kg (B) 5 kg (C) 6 kg (D) 7 kg (E) 8 kg

Source: 2011 Gauss Grade 7 #18

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving | Logic

Answer: B

Solution:

Solution 1
In kilograms, let the weights of the 3 pumpkins in increasing order be A, B and C.
The lightest combined weight, 12 kg, must come from weighing the two lightest pumpkins.
That is, $A+B=12$.
The heaviest combined weight, 15 kg, must come from weighing the two heaviest pumpkins.
That is, $B+C=15$.
Then the third given weight, 13 kg, is the combined weight of the lightest and heaviest pumpkins.
That is, $A+C=13$.
Systematically, we may try each of the 5 possible answers.
If the lightest pumpkin weighs 4 kg (answer (A)), then $A+B=12$ gives $4+B=12$, or $B=8$.
If $B=8$, then $8+C=15$ or $C=7$.
Since C represents the weight of the heaviest pumpkin, C cannot be less than B and therefore the lightest pumpkin cannot weigh 4 kg.
If the lightest pumpkin weighs 5 kg (answer (B)), then $A+B=12$ gives $5+B=12$, or $B=7$.
If $B=7$, then $7+C=15$ or $C=8$.
If $A=5$ and $C=8$, then the third and final equation $A+C=13$ is also true.
Therefore, the weight of the smallest pumpkin must be 5 kg.
Solution 2
In kilograms, let the weights of the 3 pumpkins in increasing order be A, B and C.
The lightest combined weight, 12 kg, must come from weighing the two lightest pumpkins.
That is, $A+B=12$.
The heaviest combined weight, 15 kg, must come from weighing the two heaviest pumpkins.
That is, $B+C=15$.

Then the third given weight, 13 kg, is the combined weight of the lightest and heaviest pumpkins.
That is, $A + C = 13$.
Since $A + B = 12$ and $A + C = 13$, then C is one more than B.
Since $B + C = 15$ and C is one more than B, then $B = 7$ and $C = 8$.
Since $A + B = 12$, then $A + 7 = 12$ or $A = 5$.
When $A = 5$, $B = 7$ and $C = 8$, the weights of the pairs of pumpkins are 12 kg, 13 kg and 15 kg as was given.
Therefore, the weight of the lightest pumpkin is 5 kg.

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19. The sum of four numbers is T. Suppose that each of the four numbers is now increased by 1. These four new numbers are added together and then the sum is tripled. What is the value of this final result?
(A) $3T + 3$ (B) $3T + 4$ (C) $3T + 12$ (D) $T + 12$ (E) $12T$

Source: 2011 Gauss Grade 7 #19

Primary Topics: Algebra and Equations

Secondary Topics: Operations

Answer: C

Solution:

If each of the four numbers is increased by 1, then the increase in their sum is 4.
That is, these four new numbers when added together have a sum that is 4 more than their previous sum T, or $T + 4$.
This new sum $T + 4$ is now tripled.
The result is $3 \times (T + 4) = (T + 4) + (T + 4) + (T + 4)$ or $3T + 12$.

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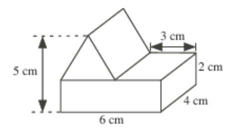
20. A triangular prism is placed on a rectangular prism, as shown. The volume of the combined structure, in cm^3, is



(A) 76 (B) 78 (C) 72 (D) 84 (E) 66

Source: 2011 Gauss Grade 7 #20

Primary Topics: Geometry and Measurement

Secondary Topics: Volume | Prisms

Answer: E

Solution:

The volume of the rectangular prism equals the area of its base 6×4 times its height-2.
That is, the rectangular prism has a volume of $6 \times 4 \times 2 = 48$ cm^3.
The volume of the triangular prism is found by multiplying the area of one of its triangular faces by its length.
The triangular face has a base of length $6 - 3 = 3$ cm.
This same triangular face has a perpendicular height of $5 - 2 = 3$ cm, since the height of the rectangular prism is 2 cm.
Thus, the triangular face has area $\frac{3 \times 3}{2} = \frac{9}{2}$ cm^2.
Since the length of the triangular prism is 4 cm, then its volume is $\frac{9}{2} \times 4 = \frac{36}{2} = 18$ cm^3.
The volume of the combined structure is equal to the sum of the volumes of the two prisms, or $48 + 18 = 66$ cm^3.

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21. Steve begins at 7 and counts forward by 3, obtaining the list 7, 10, 13, and so on. Dave begins at 2011 and counts backwards by 5, obtaining the list 2011, 2006, 2001, and so on. Which of the following numbers appear in each of their lists?
(A) 1009 (B) 1006 (C) 1003 (D) 1001 (E) 1011

Source: 2011 Gauss Grade 7 #21

Primary Topics: Algebra and Equations

Secondary Topics: Patterning/Sequences/Series

Answer: B

Solution:

Steve counts forward by 3 beginning at 7.
That is, the numbers that Steve counts are each 7 more than some multiple of 3.
We can check the given answers to see if they satisfy this requirement by subtracting 7 from each of them and then determining if the resulting number is divisible by 3.
We summarize the results in the table below.

Answers	Result after subtracting 7	Divisible by 3?
1009	1002	Yes
1006	999	Yes
1003	996	Yes
1001	994	No
1011	1004	No

Of the possible answers, Steve only counted 1009, 1006 and 1003.
Dave counts backward by 5 beginning at 2011.
That is, the numbers that Dave counts are each some multiple of 5 less than 2011.
We can check the given answers to see if they satisfy this requirement by subtracting each of them from 2011 and then determining if the resulting number is divisible by 5.
We summarize the results in the table below.

Answers	Result after being subtracted from 2011	Divisible by 5?
1009	1002	No
1006	1005	Yes
1003	1008	No
1001	1010	Yes

1011	1000	Yes

Of the possible answers, Dave only counted 1006, 1001 and 1011.
Thus while counting, the only answer that both Steve and Dave will list is 1006.

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22. A pool has a volume of 4000 L. Sheila starts filling the empty pool with water at a rate of 20 L/min. The pool springs a leak after 20 minutes and water leaks out at 2 L/min. Beginning from the time when Sheila starts filling the empty pool, how long does it take until the pool is completely full?
(A) 3 hours (B) 3 hours 40 minutes (C) 4 hours
(D) 4 hours 20 minutes (E) 3 hours 20 minutes

Source: 2011 Gauss Grade 7 #22

Primary Topics: Number Sense

Secondary Topics: Rates | Equations Solving

Answer: B

Solution:

In the first 20 minutes, Sheila fills the pool at a rate of 20 L/min and thus adds $20 \times 20 = 400$-L of water to the pool.
At this time, the pool needs $4000 - 400 = 3600$ L of water to be full.
After filling for 20 minutes, water begins to leak out of the pool at a rate of 2 L/min.
Since water is still entering the pool at a rate of 20 L/min, then the net result is that the pool is filling at a rate of $20 - 2 = 18$ L/min.
Since the pool needs 3600 L of water to be full and is filling at a rate of 18 L/min, then it will take an additional $3600 \div 18 = 200$ minutes before the pool is full of water.
Thus, the total time needed to fill the pool is $20 + 200 = 220$ minutes or 3 hours and 40 minutes.

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23. In the addition of the three-digit numbers shown, the letters A, B, C, D, and E each represent a single digit.

$$\begin{array}{cccc} & A & B & E \\ & A & C & E \\ + & A & D & E \\ \hline 2 & 0 & 1 & 1 \end{array}$$

The value of $A + B + C + D + E$ is
(A) 34 (B) 21 (C) 32 (D) 27 (E) 24

Source: 2011 Gauss Grade 7 #23

Primary Topics: Number Sense

Secondary Topics: Operations | Digits

Answer: C

Solution:

The sum of the units column is $E + E + E = 3E$.
Since E is a single digit, and $3E$ ends in a 1, then the only possibility is $E = 7$.
Then $3E = 3 \times 7 = 21$, and thus 2 is carried to the tens column.
The sum of the tens column becomes $2 + B + C + D$.
The sum of the hundreds column is $A + A + A = 3A$ plus any carry from the tens column. Thus, $3A$ plus the carry from the tens column is equal to 20.
If there is no carry from the tens column, then $3A = 20$.
This is not possible since A is a single digit positive integer.
If the carry from the tens column is 1, then $3A + 1 = 20$ or $3A = 19$.
Again, this is not possible since A is a single digit positive integer.
If the carry from the tens column is 2, then $3A + 2 = 20$ or $3A = 18$ and $A = 6$.
Since B, C, and D are single digits (ie.-they are each less than or equal to 9), then it is not possible for the carry from the tens column to be greater than 2.
Therefore, $A = 6$ is the only possibility.
Since the carry from the tens column is 2, then the sum of the tens column, $2 + B + C + D$, must equal 21.
Thus, $2 + B + C + D = 21$ or $B + C + D = 19$.
Since $A = 6$ and $E = 7$, then the sum $A + B + C + D + E = 6 + 19 + 7 = 32$.
Note that although we don't know B, C and D, it is only necessary that their sum be 19.
Although there are many possibilities, the example with $B = 2$, $C = 8$ and $D = 9$ is shown.

$$\begin{array}{cccc} & 6 & 2 & 7 \\ & 6 & 8 & 7 \\ + & 6 & 9 & 7 \\ \hline 2 & 0 & 1 & 1 \end{array}$$

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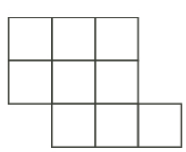
24. From the figure shown, three of the nine squares are to be selected. Each of the three selected squares must share a side with at least one of the other two selected squares. In how many ways can this be done?



(A) 19 (B) 22 (C) 15 (D) 16 (E) 20

Source: 2011 Gauss Grade 7 #24

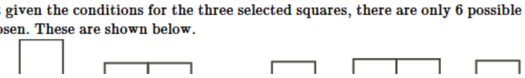
Primary Topics: Counting and Probability

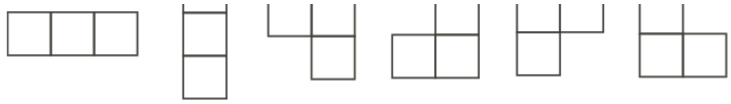
Secondary Topics: Counting

Answer: A

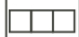
Solution:

First we recognize that given the conditions for the three selected squares, there are only 6 possible shapes that may be chosen. These are shown below.





To determine the number of ways that three squares can be selected, we count the number of ways in which each of these 6 shapes can be chosen from the given figure.
The results are summarized in the table below.

Shape						
Number of Each	3	2	4	3	3	4

Thus, three of the nine squares can be selected as described in $3 + 2 + 4 + 3 + 3 + 4 = 19$ ways.

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25. Ten circles are all the same size. Each pair of these circles overlap but no circle is exactly on top of another circle. What is the greatest possible total number of intersection points of these ten circles?
(A) 40 (B) 70 (C) 80 (D) 90 (E) 110

Source: 2011 Gauss Grade 7 #25

Primary Topics: Counting and Probability

Secondary Topics: Counting

Answer: D

Solution:

We first note that each circle can intersect any other circle a maximum of two times.
To begin, the first circle is drawn.
The second circle is then drawn overlapping the first, and two points of intersection are created.
Since each pair of circles overlap (but are not exactly on top of one another), then the third circle drawn can intersect the first circle twice and the second circle twice.
We continue in this manner with each new circle drawn intersecting each of the previously drawn circles exactly twice.
That is, the third circle drawn intersects each of the two previous circles twice, the fourth circle intersects each of the three previous circles twice, and so on.
Diagrams showing possible arrangements for 3, 4 and 5 circles, each giving the maximum number of intersections, are shown below.

The resulting numbers of intersections are summarized in the table below.

Circle number drawn	Number of new intersections	Total number of intersections
1	0	0
2	2	2
3	$2 \times 2 = 4$	$2 + 4$
4	$3 \times 2 = 6$	$2 + 4 + 6$
5	$4 \times 2 = 8$	$2 + 4 + 6 + 8$
6	$5 \times 2 = 10$	$2 + 4 + 6 + 8 + 10$
7	$6 \times 2 = 12$	$2 + 4 + 6 + 8 + 10 + 12$
8	$7 \times 2 = 14$	$2 + 4 + 6 + 8 + 10 + 12 + 14$
9	$8 \times 2 = 16$	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16$
10	$9 \times 2 = 18$	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18$

Thus, the greatest possible total number of intersection points using ten circles is

$$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 = 90.$$

To be complete, we technically need to show that this number is possible, though we don't expect students to do this to answer the question.
The diagram below demonstrates a possible positioning of the ten circles that achieves the maximum 90 points of intersection.
That is, every pair of circles intersects exactly twice and all points of intersection are distinct from one another.
It is interesting to note that this diagram is constructed by positioning each of the ten circles' centres at one of the ten vertices of a suitably sized regular decagon, as shown.