Thermodynamics – MECH32102

Solution to Tutorial Questions

Renewable Energy

Q1

The transmissivity of the glazing and the absorptivity of the absorber plate for a flat-plate solar collector are given. The maximum efficiency of the collector is to be determined.

Properties The transmissivity of glazing and the absorptivity of absorber plate are given in the problem statement.

Analysis The efficiency of a solar collector is defined as

$$\eta_{c} = \frac{\dot{Q}_{\text{useful}}}{\dot{Q}_{\text{incident}}} = \frac{\tau \alpha A \, G - U A \, (T_{c} - T_{a})}{A \, G} = \tau \alpha - U \, \frac{T_{c} - T_{a}}{G}$$

The collector efficiency is maximum when the collector temperature is equal to the air temperature $T_c = T_a$ and thus $T_c - T_a = 0$. Therefore,

$$\eta_{c,\text{max}} = \tau \alpha = (0.82)(0.94) = 0.771 = 77.1 \text{ percent}$$

That is, the maximum efficiency of this collector is 77.1 percent.

$\mathbf{Q2}$

A flat-plate solar collector is used to produce hot water. The characteristics of a flat-plate solar collector and the rate of radiation incident are given. The temperature of hot water provided by the collector is to be determined.

Properties The product of transmissivity of glazing and the absorptivity of absorber plate is given in the problem statement. The density of water is 1000 kg/m³ and its specific heat is 4.18 kJ/kg·°C.

Analysis The total rate of solar radiation incident on the collector is

$$\dot{Q}_{\text{incident}} = AG = (33 \text{ m}^2)(880 \text{ W/m}^2) = 29,040 \text{ W}$$

The mass flow rate of water is

$$\dot{m} = \rho \dot{V} = (1 \text{ L/kg})(6.3/60 \text{ L/s}) = 0.105 \text{ kg/s}$$

The rate of useful heat transferred to the water is determined from the definition of collector efficiency to be

$$\eta_c = \frac{\dot{Q}_{\text{useful}}}{\dot{Q}_{\text{incident}}} \longrightarrow \dot{Q}_{\text{useful}} = \eta_c \dot{Q}_{\text{incident}} = (0.70)(29,040 \text{ W}) = 20,328 \text{ W}$$

Then the temperature of hot water provided by the collector is determined from

$$\begin{split} \dot{Q}_{\rm useful} &= \dot{m}c_p (T_{w, \, \rm out} - T_{w, \, \rm in}) \\ 20,328 \; \mathrm{W} &= (0.105 \; \mathrm{kg/s}) (4.28 \; \mathrm{kJ/kg} \cdot ^{\circ}\mathrm{C}) (T_{w, \, \rm out} - 18)^{\circ}\mathrm{C} \\ T_{w, \, \rm out} &= \mathbf{64.3}^{\circ}\mathrm{C} \end{split}$$

That is, this collector is supplying water at 64.3°C.

Q3

A solar cell with a specified value of open circuit voltage is considered. The current output density, the load voltage for maximum power output, and the maximum power output of the cell for a unit cell area are to be determined.

Analysis (a) The current output density is determined from Eq. 18-18 to be

$$V_{oc} = \frac{kT}{e_o} \ln \left(\frac{J_s}{J_o} + 1 \right)$$

$$0.55 \text{ V} = \frac{(1.381 \times 10^{-23} \text{ J/K})(298 \text{ K})}{1.6 \times 10^{-19} \text{ J/V}} \ln \left(\frac{J_s}{1.9 \times 10^{-9} \text{ A/m}^2} + 1 \right)$$

$$J_s = \mathbf{3.676 \text{ A/m}^2}$$

(b) The load voltage at which the power output is maximum is determined from Eq. 18-22 to be

$$\exp\left(\frac{e_o V_{\text{max}}}{kT}\right) = \frac{1 + J_s / J_o}{1 + \frac{e_o V_{\text{max}}}{kT}}$$

$$\exp\left(\frac{(1.6 \times 10^{-19} \text{ J/V}) V_{\text{max}}}{(1.381 \times 10^{-23} \text{ J/K})(298 \text{ K})}\right) = \frac{1 + (3.676 \text{ A/m}^2 / 1.9 \times 10^{-9} \text{ A/m}^2)}{1 + \frac{(1.6 \times 10^{-19} \text{ J/V}) V_{\text{max}}}{(1.381 \times 10^{-23} \text{ J/K})(298 \text{ K})}}$$

$$V_{\text{max}} = \mathbf{0.4737 V}$$

(c) The maximum power output of the cell for a unit cell area is determined from

$$\dot{W}_{\rm max} = \frac{{\rm V}_{\rm max} \left(J_s + J_o\right)}{1 + \frac{kT}{e_o {\rm V}_{\rm max}}} = \frac{(0.4737 \ {\rm V})(3.676 \ {\rm A/m^2})(1.9 \times 10^{-9} \ {\rm A/m^2})}{1 + \frac{(1.381 \times 10^{-23} \ {\rm J/K})(298 \ {\rm K})}{(1.6 \times 10^{-19} \ {\rm J/V})(0.4737 \ {\rm V})} = {\rm \textbf{1.652}} \, {\rm \textbf{W/m^2}}$$

04

A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through an east window in a typical day in January.

Assumptions 1 The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 18-6). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is $4.55 \text{ W/m}^2 \cdot ^{\circ}\text{C}$. The total solar radiation incident at an East-facing surface in January during a typical day is 1863 Wh/m^2 (Table 18-3).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from

$$SHGC = 0.87 \times SC = 0.87 \times 0.88 = 0.7656$$

Then the solar heat gain through the window per unit area becomes

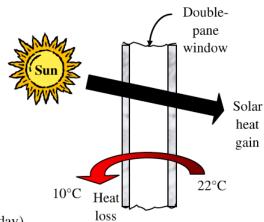
$$Q_{\text{solar gain}} = SHGC \times A_{\text{glazing}} \times q_{\text{solar, daily total}}$$
$$= 0.7656(1 \text{ m}^2)(1863 \text{ Wh/m}^2)$$
$$= 1426 \text{ Wh} = 1.426 \text{ kWh}$$

The heat loss through a unit area of the window during a 24-h period is

$$Q_{\text{loss, window}} = \dot{Q}_{\text{loss, window}} \Delta t = U_{\text{window}} A_{\text{window}} (T_i - T_{0, \text{ ave}}) (1 \text{ day})$$

$$= (4.55 \text{ W/m}^2 \cdot {}^{\circ}\text{C}) (1 \text{ m}^2) (22 - 10) {}^{\circ}\text{C} (24 \text{ h})$$

$$= 1310 \text{ Wh} = 1.31 \text{ kWh}$$



Therefore, the house is loosing less heat than it is gaining through the East windows during a typical day in January.

Q5

The wind power potential of a wind turbine at a specified wind speed is to be determined.

Assumptions Wind flows steadily at the specified speed.

Properties The gas constant of air is $R = 0.287 \text{ kJ/kg} \cdot \text{K}$.

Analysis The density of air is determined from the ideal gas relation to be

$$\rho = \frac{P}{RT} = \frac{(96 \text{ kPa})}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(296 \text{ K})} = 1.130 \text{ kg/m}^3$$

The blade span area is

$$A = \pi D^2 / 4 = \pi (50 \text{ m})^2 / 4 = 1963 \text{ m}^2$$

Then the wind power potential is

$$\dot{W}_{\text{available}} = \frac{1}{2} \rho A V_1^3 = \frac{1}{2} (1.130 \text{ kg/m}^3) (1963 \text{ m}^2) (7.5 \text{ m/s})^3 \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) =$$
468 kW

Q6

A wind turbine generates a certain amount of electricity during a week period. The average wind speed during this period is to be determined.

Properties The density of air is given to be $\rho = 1.16 \text{ kg/m}^3$.

Analysis The total operating hours during a week period is

Operating hours =
$$7 \times 24 \text{ h} = 168 \text{ h}$$

The average rate of electricity production is

$$\dot{W}_{\text{electric}} = \frac{W_{\text{electric}}}{\text{Operating hours}} = \frac{11,000 \text{ kWh}}{168 \text{ h}} = 65.48 \text{ kW}$$

The available wind power is

$$\dot{W}_{\text{available}} = \frac{\dot{W}_{\text{electric}}}{\eta_{\text{wt. overall}}} = \frac{65.48 \text{ kW}}{0.28} = 233.8 \text{ kW}$$

The blade span area is

$$A = \pi D^2 / 4 = \pi (40 \text{ m})^2 / 4 = 1257 \text{ m}^2$$

Finally, the average wind speed is determined from the definition of available wind power to be

$$\dot{W}_{\text{available}} = \frac{1}{2} \rho A V^3$$

$$233.8 \text{ kW} = \frac{1}{2} (1.16 \text{ kg/m}^3) (1257 \text{ m}^2) V^3 \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)$$

$$V = \textbf{6.85 m/s}$$

Q7

The power that can be produced by an ideal turbine from a large dam is to be determined.

Assumptions 1 The flow is steady. 2 Water levels at the reservoir and the discharge site remain constant.

Properties The density of water is taken to be $\rho = 1000 \text{ kg/m}^3 = 1 \text{ kg/L}$.

Analysis The total mechanical energy the water in a dam possesses is equivalent to the potential energy of water at the free surface of the dam (relative to free surface of discharge water), and it can be converted to work entirely for an ideal operation. Therefore, the maximum power that can be generated is equal to potential energy of the water. Noting that the mass flow rate is $\dot{m} = \rho \dot{V}$, the maximum power is determined from

$$\dot{W}_{\text{max}} = \dot{m}gH_{\text{gross}} = \rho \dot{V}gH_{\text{gross}}$$

$$= (1 \text{ kg/L})(1500 \text{ L/s})(9.81 \text{ m/s}^2)(65 \text{ m}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$

$$= 15.94 \text{ kW}$$

Q8

The irreversible head losses in the penstock and its inlet and those after the exit of the draft tube are given. The power loss due to irreversible head loss, the efficiency of the piping, and the electric power output are to be determined.

Assumptions 1 The flow is steady. 2 Water levels at the reservoir and the discharge site remain constant.

Properties The density of water is taken to be $\rho = 1000 \text{ kg/m}^3 = 1 \text{ kg/L}$.

Analysis (a) The power loss due to irreversible head loss is determined from

$$\begin{split} \dot{E}_{\text{loss, piping}} &= \rho \dot{V} g h_L \\ &= (1 \text{ kg/L})(4000/60 \text{ L/s})(9.81 \text{ m/s}^2)(7 \text{ m}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= \textbf{4.578 kW} \end{split}$$

(b) The efficiency of the piping is determined from the head loss and gross head to be

$$\eta_{\text{piping}} = 1 - \frac{h_L}{H_{\text{eross}}} = 1 - \frac{7 \text{ m}}{140 \text{ m}} = 0.950 =$$
95.0 percent

(c) The maximum power is determined from

$$\dot{W}_{\text{max}} = \dot{m}gH_{\text{gross}} = \rho \dot{V}gH_{\text{gross}}$$

$$= (1 \text{ kg/L})(4000/60 \text{ L/s})(9.81 \text{ m/s}^2)(140 \text{ m}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$

$$= 91.56 \text{ kW}$$

The overall efficiency of the hydroelectric plant is

$$\eta_{\text{plant}} = \eta_{\text{turbine-generator}} \eta_{\text{piping}} = (0.84)(0.950) = 0.798$$

The electric power output is

$$\dot{W}_{\text{electric}} = \eta_{\text{plant}} \dot{W}_{\text{max}} = (0.798)(91.56 \text{ kW}) = 73.1 \text{ kW}$$

Q9

A hydroelectric power plant operating 80 percent of the time is considered. The revenue that can be generated in a year is to be determined.

Assumptions 1 The flow is steady. 2 Water levels at the reservoir and the discharge site remain constant.

Properties The density of water is taken to be $\rho = 1000 \text{ kg/m}^3 = 1 \text{ kg/L}$.

Analysis The maximum power for one turbine is determined from

$$W_{\text{max}} = \dot{m}gH_{\text{gross}} = \rho V gH_{\text{gross}}$$

$$= (1 \text{ kg/L})(3300/60 \text{ L/s})(9.81 \text{ m/s}^2)(150 \text{ m}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$

$$= 80.93 \text{ kW}$$

The electric power output is

$$\dot{W}_{\text{electric}} = \eta_{\text{plant}} \dot{W}_{\text{max}} = (0.90)(80.93 \text{ kW}) = 72.84 \text{ kW}$$

Noting that the plant operates 80 percent of the time, the operating hours in one year is

Operating hours =
$$0.80 \times 365 \times 24 = 7008 \text{ h}$$

The amount of electricity produced per year by one turbine is

$$W_{\text{electric}} = \dot{W}_{\text{electric}} \times \text{Operating hours} = (72.84 \text{ kW})(7008 \text{ h}) = 510,500 \text{ kWh}$$

Finally, the revenue generated by 18 such turbines is

Revenue =
$$n_{\text{turbine}} \times W_{\text{electric}} \times \text{Unit price of electricity} = (18)(510,500 \text{ kWh})(\$0.095/\text{kWh}) = \$873,000$$