Thermodynamics – MECH32102

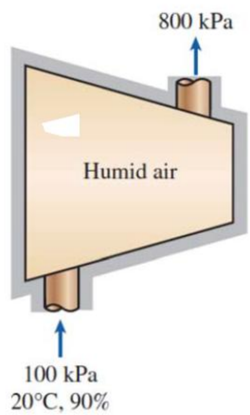
Solution to Tutorial Questions

Gas-Vapor Mixtures and Air-Conditioning

Q1

Problem Statement

Humid air at 100 kPa, 20°C, and 90 percent relative humidity is compressed in a steady-flow, isentropic compressor to 800 kPa. What is the relative humidity of the air at the compressor outlet?



Solution

Humid air is compressed in an isentropic compressor. The relative humidity of the air at the compressor outlet is to be determined.

Assumptions

The air and the water vapor are ideal gases.

Properties

The specific heat ratio of air at room temperature is k = 1.4. The saturation properties of water are to be obtained from water tables.

Analysis

At the inlet,

$$P_{v,1} = \phi_1 P_{g,1} = \phi_1 P_{\text{sat @ 20°C}} = (0.90)(2.3392 \text{ kPa}) = 2.105 \text{ kPa}$$

$$\omega_2 = \omega_1 = \frac{0.622 P_{v,1}}{P - P_{v,1}} = \frac{(0.622)(2.105 \text{ kPa})}{(100 - 2.105) \text{ kPa}} = 0.0134 \text{ kg H}_2\text{O/kg dry air}$$

Since the mole fraction of the water vapor in this mixture is very small,

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = (293 \text{ K}) \left(\frac{800 \text{ kPa}}{100 \text{ kPa}} \right)^{0.4/1.4} = 531 \text{ K}$$

The saturation pressure at this temperature is:

$$P_{g,2} = P_{\text{sat @ } 258°\text{C}} = 4542 \text{ kPa} \ \ \text{(from EES)}$$

The vapor pressure at the exit is:

$$P_{v,2} = \frac{\omega_2 P_2}{\omega_2 + 0.622} = \frac{(0.0134)(800)}{0.0134 + 0.622} = 16.87 \text{ kPa}$$

The relative humidity at the exit is then

$$\phi_2 = \frac{P_{v,2}}{P_{g,2}} = \frac{16.87}{4542} = \mathbf{0.0037 = 0.37\%}$$

Q2

Problem Statement

Atmospheric pressure is 98 kPa. Determine the relative humidity and specific humidity of the air.

Solution

Atmospheric air flows steadily into an adiabatic saturation device and leaves as a saturated vapor. The relative humidity and specific humidity of air are to be determined.

Assumptions

1. This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process $(\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a)$. **2.** Dry air and water vapor are ideal gases. **3.** The kinetic and potential energy changes are negligible.

Analysis

The exit state of the air is completely specified, and the total pressure is 98 kPa. The properties of the moist air at the exit state may be determined from EES to be

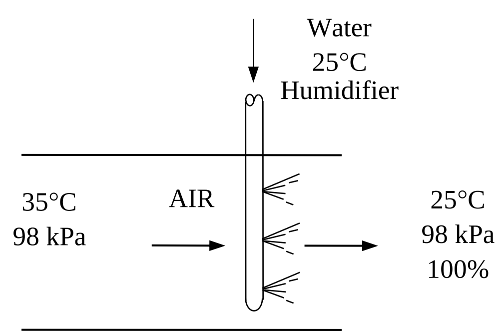
$$h_2 = 78.11 \text{ kJ/kg dry air}$$

$$w_2 = 0.02079 \text{ kg H}_2\text{O/kg dry air}$$

The enthalpy of makeup water is

$$h_{w2} = h_{f @ 25°\text{C}} = 104.83 \text{ kJ/kg} \quad \text{(Table A-4)}$$

An energy balance on the control volume gives



$$h_1 + (w_2 - w_1)h_w = h_2$$

$$h_1 + (0.02079 - w_1)(104.83 \text{ kJ/kg}) = 78.11 \text{ kJ/kg}$$

Pressure and temperature are known for inlet air. Other properties may be determined from this equation using EES. A hand solution would require a trial-error approach. The results are

$$h_1 = 77.66 \text{ kJ/kg dry air}$$

$$w_1 = \mathbf{0.01654 \text{ kg } H_2O \text{ /kg dry air}}$$

$$\phi_1 = \mathbf{0.4511}$$

Q3

Problem Statement

For an infiltration rate of 1.2 air changes per hour (ACH), determine sensible, latent, and total infiltration heat load of a building at sea level, in kW, that is 20 m long, 13 m wide, and 3 m high when the outdoor air is at 32°C and 35 percent relative humidity. The building is maintained at 24°C and 55 percent relative humidity at all times.

Solution

The infiltration rate of a building is estimated to be 1.2 ACH. The sensible, latent, and total infiltration heat loads of the building at sea level are to be determined.

Assumptions

1. Steady operating conditions exist. **2.** The air infiltrates at the outdoor conditions, and exfiltrates at the indoor conditions. **3.** Excess moisture condenses at room temperature of 24°C. **4** The effect of water vapor on air density is negligible.

Properties

The gas constant and the specific heat of air are R = 0.287 kPa.m^3/kg.K and c_p = 1.005 kJ/kg°C (Table A-2). The heat of vaporization of water at 24°C is $h_{fg} = h_{fg@\ 24°C} = 2444.1 \text{ kJ/kg}$ (Table A-4). The properties of the ambient and room air are determined from the psychrometric chart (Fig. A-31 or EES) to be:

Solution continued on next page...

$$\left.\begin{array}{l} T_{\text{ambient}} = 32°\text{C} \\ \phi_{\text{ambient}} = 35\% \end{array}\right\} w_{\text{ambient}} = 0.01039 \text{ kg/kg dry air}$$

$$\left.\begin{array}{l} T_{\text{room}} = 24°\text{C} \\ \phi_{\text{room}} = 55\% \end{array}\right\} w_{\text{room}} = 0.01024 \text{ kg/kg dry air}$$

Analysis

Noting that the infiltration of ambient air will cause the air in the cold storage room to be changed 0.9 times every hour, the air will enter the room at a mass flow rate of

$$\rho_{\text{ambient}} = \frac{P_0}{RT_0} = \frac{101.325 \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg.K})(32 + 273 \text{ K})} = 1.158 \text{ kg/m}^3$$

$$\dot{m}_{\text{air}} = \rho_{\text{ambient}} V_{\text{room}} \text{ACH} = (1.158 \text{ kg/m}^3)(20 \times 13 \times 3 \text{ m}^3)(1.2 \text{ h}^{-1}) = 1084 \text{ kg/h} = 0.3010 \text{ kg/s}$$

Then the sensible, latent, and total infiltration heat loads of the room are determined to be

$$\dot{Q}_{\text{infiltration, sensible}} = \dot{m}_{\text{air}} c_p (T_{\text{ambient}} - T_{\text{room}}) = (0.3010 \text{ kg/s})(1.005 \text{ kJ/kg.°C})(32 - 24)°\text{C} = \mathbf{2.42\ kW}$$

$$\dot{Q}_{\text{infiltration, latent}} = \dot{m}_{\text{air}} (w_{\text{ambient}} - w_{\text{room}}) h_{fg@24°\text{C}} = (0.3010 \text{ kg/s})(0.01039 - 0.01024)(2444.1 \text{ kJ/kg}) = \mathbf{0.110\ kW}$$

$$\dot{Q}_{\text{infiltration, total}} = \dot{Q}_{\text{infiltration, sensible}} + \dot{Q}_{\text{infiltration, latent}} = 2.42 + 0.110 = \mathbf{2.53\ kW}$$

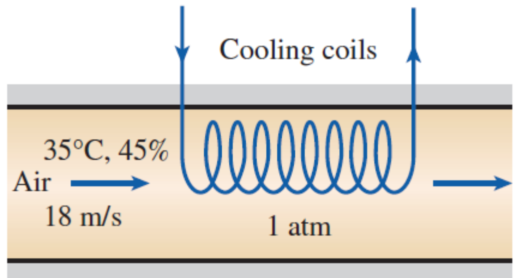
Discussion The specific volume of the dry air at the ambient conditions could also be determined from the psychrometric chart at ambient conditions.

Q4

Problem Statement

Air enters a 30-cm-diameter cooling section at 1 atm, 35°C, and 45 percent relative humidity at 18 m/s. Heat is removed from the air at a rate of 750 kJ/min. Determine (*a*) the exit temperature, (*b*) the exit relative humidity of the air, and (*c*) the exit velocity.

Answers: **(*a*) 26.5°C, (*b*) 73.1 percent, (*c*) 17.5 m/s**



Solution

Air enters a cooling section at a specified pressure, temperature, velocity, and relative humidity. The exit temperature, the exit relative humidity of the air, and the exit velocity are to be determined.

Assumptions

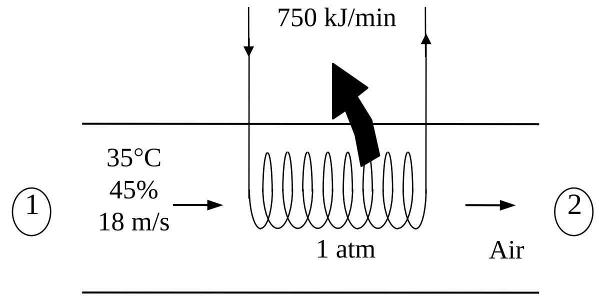
1. This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process $(\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a)$ **2.** Dry air and water vapor are ideal gases. **3.** The kinetic and potential energy changes are negligible.

Solution continued on next page...

Analysis

(*a*) The amount of moisture in the air remains constant ($\omega_1 = \omega_2$) as it flows through the cooling section since the process involves no humidification or dehumidification. The inlet state of the air is completely specified, and the total pressure is 1 atm. The properties of the air at the inlet state are determined from the psychrometric chart (Figure A-31 or EES) to be

$$h_1 = 76.14 \text{ kJ/kg dry air}$$
$$\omega_1 = 0.01594 \text{ kg H}_2\text{O/kg dry air } (= \omega_2)$$
$$v_1 = 0.8953 \text{ m}^3\text{/kg dry air}$$



The mass flow rate of dry air through the cooling section is

$$\begin{aligned}\dot{m}_a &= \frac{1}{v_1} V_1 A_1 \\ &= \frac{1}{(0.8953 \text{ m}^3\text{/kg})}(18 \text{ m/s})(\pi \times 0.3^2/4 \text{ m}^2) \\ &= 1.421 \text{ kg/s}\end{aligned}$$

From the energy balance on air in the cooling section,

$$\begin{aligned}-\dot{Q}_{\text{out}} &= \dot{m}_a (h_2 - h_1) \\ -(750/60) \text{ kJ/s} &= (1.421 \text{ kg/s})(h_2 - 76.14) \text{ kJ/kg} \\ h_2 &= 67.35 \text{ kJ/kg dry air}\end{aligned}$$

(*b*) The exit state of the air is fixed now since we know both h_2 and ω_2. From the psychrometric chart at this state we read

$$T_2 = \mathbf{26.5°C}$$
$$\phi_2 = \mathbf{73.1\%}$$
$$v_2 = 0.8706 \text{ m}^3\text{/kg dry air}$$

(*c*) The exit velocity is determined from the conservation of mass of dry air,

$$\dot{m}_{a1} = \dot{m}_{a2} \longrightarrow \frac{\dot{V}_1}{v_1} = \frac{\dot{V}_2}{v_2} \longrightarrow \frac{V_1 A}{v_1} = \frac{V_2 A}{v_2}$$

$$V_2 = \frac{v_2}{v_1} V_1 = \frac{0.8706}{0.8953}(18 \text{ m/s}) = \mathbf{17.5 \ m/s}$$

Q5

Problem Statement

Air at 1 atm, 15°C, and 60 percent relative humidity is first heated to 20°C in a heating section and then humidified by introducing water vapor. The air leaves the humidifying section at 25°C and 65 percent relative humidity. Determine (*a*) the amount of steam added to the air, and (*b*) the amount of heat transfer to the air in the heating section.

Answers: **(*a*) 0.0065 kg H_2O/kg dry air, (*b*) 5.1 kJ/kg dry air**

Solution

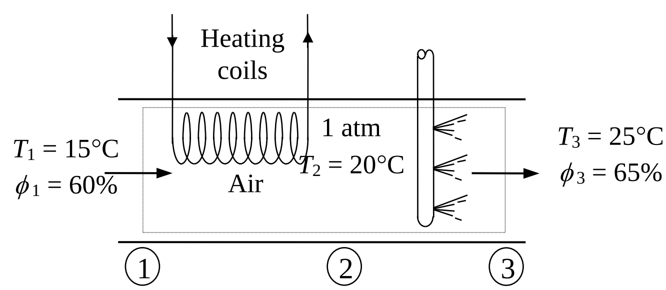
Air is first heated and then humidified by water vapor. The amount of steam added to the air and the amount of heat transfer to the air are to be determined.

Assumptions

1. This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process $(\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a)$. **2.** Dry air and water vapor are ideal gases. **3** The kinetic and potential energy changes are negligible.

Properties The inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. The properties of the air at various states are determined from the psychrometric chart (Figure A-31) to be

$$h_1 = 31.1 \text{ kJ/kg dry air}$$
$$\omega_1 = 0.0064 \text{ kg H}_2\text{O/kg dry air} \ (= \omega_2)$$
$$h_2 = 36.2 \text{ kJ/kg dry air}$$
$$h_3 = 58.1 \text{ kJ/kg dry air}$$
$$\omega_3 = 0.0129 \text{ kg H}_2\text{O/kg dry air}$$



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Analysis (*a*) The amount of moisture in the air remains constant it flows through the heating section ($\omega_1 = \omega_2$), but increases in the humidifying section ($\omega_3 > \omega_2$). The amount of steam added to the air in the heating section is

$$\Delta\omega = \omega_3 - \omega_2 = 0.0129 - 0.0064 = \mathbf{0.0065\ kg\ H_2O/kg\ dry\ air}$$

(*b*) The heat transfer to the air in the heating section per unit mass of air is

$$q_{in} = h_2 - h_1 = 36.2 - 31.1 = \mathbf{5.1\ kJ/kg\ dry\ air}$$

Q6

Problem Statement

On a summer day in New Orleans, Louisiana, the pressure is 1 atm: the temperature is 32°C; and the relative humidity is 95 percent. This air is to be conditioned to 24°C and 60 percent relative humidity. Determine the amount of cooling, in kJ, required and water removed, in kg, per 1000 m^3 of dry air processed at the entrance to the system.

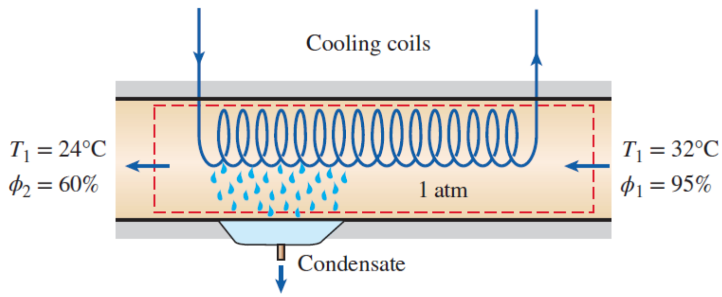


FIGURE P14–82

Solution

Air is cooled and dehumidified at constant pressure. The amount of water removed from the air and the rate of cooling are to be determined.

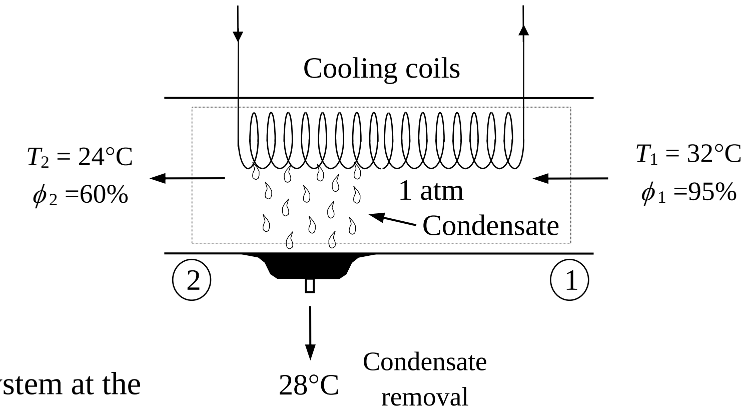
Assumptions

1. This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process $(\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a)$. **2.** Dry air and water vapor are ideal gases. **3.** The kinetic and potential energy changes are negligible.

Solution continued on next page...

Properties The inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. The properties of the air at various states are determined from the psychrometric chart to be

$$h_1 = 106.8 \text{ kJ/kg dry air}$$
$$\omega_1 = 0.0292 \text{ kg H}_2\text{O/kg dry air}$$
$$v_1 = 0.905 \text{ m}^3\text{/kg dry air}$$



and

$$h_2 = 52.7 \text{ kJ/kg dry air}$$
$$\omega_2 = 0.0112 \text{ kg H}_2\text{O/kg dry air}$$

We assume that the condensate leaves this system at the average temperature of the air inlet and exit. Then,

$$h_w \cong h_{f@28°\text{C}} = 117.4 \text{ kJ/kg} \qquad \text{(Table A-4)}$$

Analysis

The amount of moisture in the air decreases due to dehumidification ($\omega_2 < \omega_1$). The mass of air is

$$m_a = \frac{V_1}{v_1} = \frac{1000 \text{ m}^3}{0.905 \text{ m}^3 \text{ / kg dry air}} = 1105 \text{ kg}$$

Applying the water mass balance and energy balance equations to the combined cooling and dehumidification section,

Water Mass Balance:

$$\sum \dot{m}_{w,i} = \sum \dot{m}_{w,e} \quad \longrightarrow \quad \dot{m}_{a1}\omega_1 = \dot{m}_{a2}\omega_2 + \dot{m}_w$$

$$m_w = m_a(\omega_1 - \omega_2) = (1105 \text{ kg})(0.0292 - 0.0112) = \mathbf{19.89 \text{ kg}}$$

Energy Balance:

$$\begin{aligned}
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} &= \Delta\dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \\
\dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\
\sum \dot{m}_i h_i &= \dot{Q}_{out} + \sum \dot{m}_e h_e \\
\dot{Q}_{\text{out}} &= \dot{m}_{a1}h_1 - (\dot{m}_{a2}h_2 + \dot{m}_w h_w) = \dot{m}_a(h_1 - h_2) - \dot{m}_w h_w \\
Q_{\text{out}} &= m_a(h_1 - h_2) - m_w h_w \\
Q_{\text{out}} &= (1105 \text{ kg})(106.8 - 52.7)\text{kJ/kg} - (19.89 \text{ kg})(117.4 \text{ kJ/kg}) = \mathbf{57,450 \text{ kJ}}
\end{aligned}$$

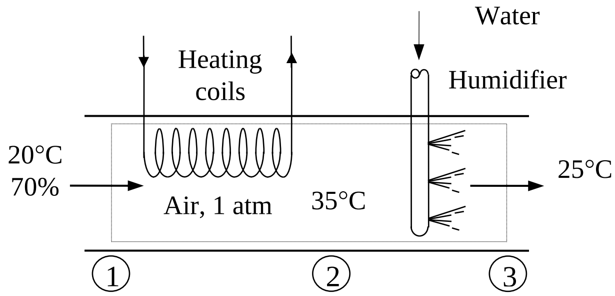
Q7

Problem Statement

Air at 1 atm, 20°C, and 70 percent relative humidity is first heated to 35°C in a heating section and then passed through an evaporative cooler, where its temperature drops to 25°C. Determine (*a*) the exit relative humidity and (*b*) the amount of water added to air, in kg H_2O/kg dry air.

Solution

Air is first heated in a heating section and then passed through an evaporative cooler. The exit relative humidity and the amount of water added are to be determined.



Analysis

(*a*) From the psychrometric chart (Fig. A-31 or EES) at 20°C and 70% relative humidity we read

$$\omega_1 = 0.01021 \text{ kg H}_2\text{O/kg dry air}$$

The specific humidity ω remains constant during the heating process. Therefore,

$$\omega_2 = \omega_1 = 0.01021 \text{ kg H}_2\text{O/kg dry air}$$

At this ω value and 35°C we read

$$T_{wb2} = 21.3°\text{C}$$

Assuming the liquid water is supplied at a temperature not much different than the exit temperature of the air stream, the evaporative cooling process follows a line of constant wet-bulb temperature. That is,

$$T_{wb3} \cong T_{wb2} = 21.3°\text{C}$$

At this T_{wb} value and 25°C we read

$$\phi_3 = 0.7225 = \mathbf{72.3\%}$$
$$\omega_3 = 0.01438 \text{ kg H}_2\text{O/kg dry air}$$

(*b*) The amount of water added to the air per unit mass of air is

$$\Delta\omega_{23} = \omega_3 - \omega_2 = 0.01438 - 0.01021 = \mathbf{0.00417 \text{ kg H}_2\text{O / kg dry air}}$$

Q8

Problem Statement

A stream of warm air with a dry-bulb temperature of 36°C and a wet-bulb temperature of 30°C is mixed adiabatically with a stream of saturated cool air at 12°C. The dry air mass flow rates of the warm and cool airstreams are 8 and 10 kg/s, respectively. Assuming a total pressure of 1 atm, determine (*a*) the temperature, (*b*) the specific humidity, and (*c*) the relative humidity of the mixture.

Solution

A stream of warm air is mixed with a stream of saturated cool air. The temperature, the specific humidity, and the relative humidity of the mixture are to be determined.

Assumptions

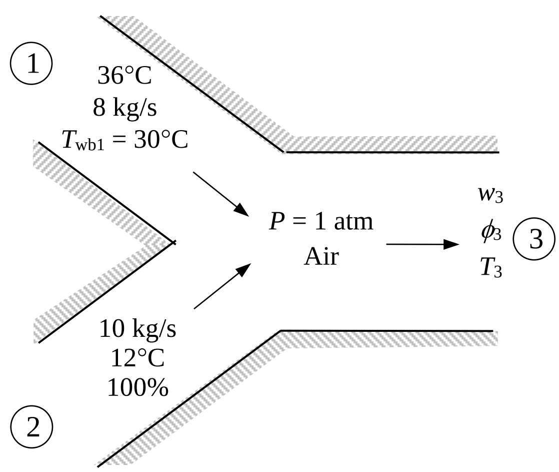
1. Steady operating conditions exist **2.** Dry air and water vapor are ideal gases. **3.** The kinetic and potential energy changes are negligible. **4.** The mixing section is adiabatic.

Properties The properties of each inlet stream are determined from the psychrometric chart (Fig. A-31 or EES) to be

$$h_1 = 99.4 \text{ kJ/kg dry air}$$
$$w_1 = 0.0246 \text{ kg } H_2O\text{/kg dry air}$$

and

$$h_2 = 34.1 \text{ kJ/kg dry air}$$
$$w_2 = 0.00873 \text{ kg } H_2O\text{/kg dry air}$$



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Analysis

The specific humidity and the enthalpy of the mixture can be determined from Eqs. 14-24, which are obtained by combining the conservation of mass and energy equations for the adiabatic mixing of two streams:

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$

$$\frac{8}{10} = \frac{0.00873 - \omega_3}{\omega_3 - 0.0246} = \frac{34.1 - h_3}{h_3 - 99.4}$$

which yields,

(*b*) $\omega_3 = \mathbf{0.0158\ kg\ H_2O/kg\ dry\ air}$

$h_3 = 63.1\ \text{kJ/kg dry air}$

These two properties fix the state of the mixture. Other properties of the mixture are determined from the psychrometric chart:

(*a*) $T_3 = \mathbf{22.8°C}$

(*c*) $\phi_3 = \mathbf{90.1\%}$