TWO HOURS

Department of Mechanical, Aerospace & Civil Engineering

UNIVERSITY OF MANCHESTER

ENGINEERING THERMODYNAMICS

xxxxxx 2024

09:00 – 11:00

Special Instruction(s):

- Answer **all 4 questions**.
- Each question carries **25 marks**.
- A **formula sheet** is provided at the end of the paper.
- Several **property tables** are provided at the end of the paper, extracted from appendix a of 'thermodynamics-an engineering approach (SI version) (9th ed.) (McGraw Hill) Cengel, Boles and Kanoglu.
- Electronic calculators **are permitted** as long as they cannot store text, perform algebra and have no graphing capability, in accordance with university regulations.

Q1. (a) When a man returns to his well-sealed house (no air leaks in or out of the house) on a summer day, he finds that the house is at 35°C. He turns on the air conditioner, which cools the entire house to 20°C in 30 min. If the Coefficient of Performance (COP) of the air-conditioning system is 2.8, determine the power drawn by the air conditioner. Assume the entire mass within the house is equivalent to 800 kg of air for which c_v = 0.72 kJ/kg·K and c_p = 1.0 kJ/kg·K. Also assume air as an ideal gas and the air conditioner operating steadily. **[9 marks]**

(b) A Carnot heat pump is to be used to heat a house and maintain it at 25°C in winter. On a day when the average outdoor temperature remains at about 2°C, the house is estimated to lose heat at a rate of 55,000 kJ/h. If the heat pump consumes 4.8 kW of power while operating, determine:

(i) the COP of the heat pump. **[3 marks]**

(ii) how long the heat pump ran on that day. **[7 marks]**

(iii) the total heating costs, assuming an average price of £0.11/kWh for electricity. **[3 marks]**

(iv) the heating cost for the same day if resistance heating is used instead of a heat pump. **[3 marks]**

Q2. A gas-turbine power plant operates on a modified Brayton cycle shown in **Figure Q2** with an overall pressure ratio of 8. Air enters the compressor at 0°C and 100 kPa. The maximum cycle temperature is 1500 K. The compressor and the turbines are isentropic. The high-pressure turbine develops just enough power to run the compressor. Assume constant properties for air at 300 K with c_v = 0.718 kJ/kg·K, c_p = 1.005 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.4.

(a) Sketch the T-s diagram for the cycle. Label the data states. **[5 marks]**

(b) Determine the temperature and pressure at state 4, the exit of the high-pressure turbine. **[10 marks]**

(c) If the net power output is 200 MW, determine the mass flow rate of the air into the compressor in kg/s. **[5 marks]**

(d) Assuming all parameters of the problem remain the same, explain the impact of raising the compressor inlet temperature on the inlet mass flow rate and net power output (for a fixed compressor inlet velocity and flow area). **[5 marks]**

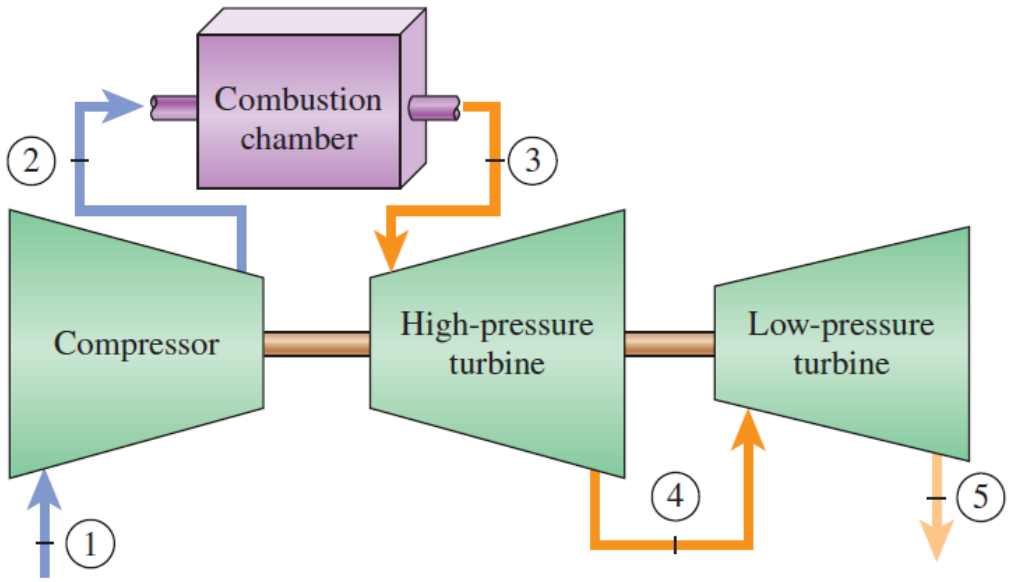

Figure Q2. Schematic diagram of gas-turbine power plant.

Q3. Two mass streams of two different ideal gases are mixed in a steady-flow chamber while receiving energy by heat transfer from the surroundings, as shown in **Figure Q3**. The mixing process takes place at constant pressure with no work and negligible changes in kinetic and potential energies. Assume the gases have constant specific heats.

(a) Determine the expression for the final temperature of the mixture in terms of the rate of heat transfer to the mixing chamber and the mass flow rates, specific heats, and temperatures of the three mass streams. **[5 marks]**

(b) Obtain an expression for the exit volume flow rate in terms of the rate of heat transfer to the mixing chamber, mixture pressure, universal gas constant, and the specific heats and molar masses of the inlet gases and exit mixture. **[10 marks]**

(c) For the special case of adiabatic mixing, show that the exit volume flow rate is a function of the two inlet volume flow rates and the specific heats and molar masses of the inlets and exit. **[5 marks]**

(d) For the special case of adiabatic mixing of the same ideal gases, show that the exit volume flow rate is a function of the two inlet volume flow rates. **[5 marks]**

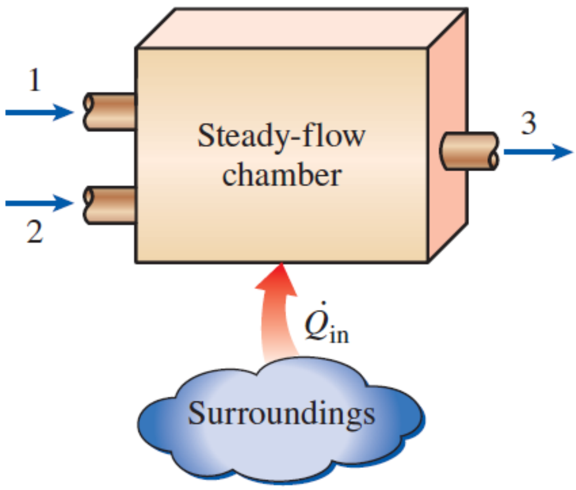

Figure Q3. Schematic diagram of the problem.

Q4. (a) As shown in **Figure Q4**, humid air is to be conditioned in a constant-pressure process at 1 atm from 39°C dry bulb and 50 percent relative humidity to 17°C dry bulb and 10.8°C wet bulb. The air is first passed over cooling coils to remove all of the moisture necessary to achieve the final moisture content and then is passed over heating coils to achieve the final state.

(i) Sketch the psychometric diagram for the process. **[4 marks]**

(ii) Determine the dew-point temperature of the mixture at the inlet of the cooling coils and at the inlet of the heating coils. **[8 marks]**

(iii) Determine the heat removal by the cooling coils, the heat addition by the heating coils, and the net heat transfer for the entire process, all in kJ/kg dry air. **[8 marks]**

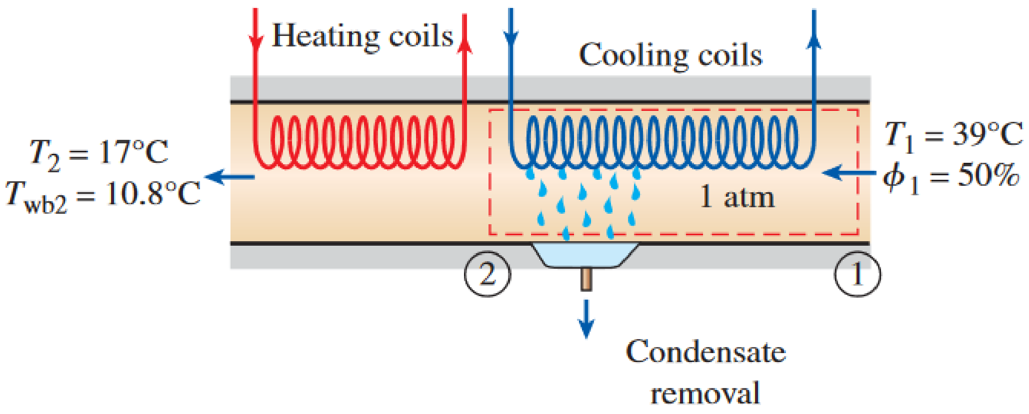

Figure Q4. Schematics of air-cooling arrangement.

(b) Desert dwellers often wrap their heads with a water soaked porous cloth. On a desert where the pressure is 1 atm, temperature is 48°C, and relative humidity is 10 percent, what is the temperature of this cloth? **[5 marks]**

END OF THE EXAMINATION PAPER

DATA SHEETS

1 Work

- **positive work:** is done by a system on the surroundings (a system does positive work if it can raise a weight)
- **negative work:** is done by the surroundings on a system.

Incremental piston, or displacement work , is $\delta W = p\mathrm{d}V$, and for a process in which the pressure varies with volume the work is $W = \int p\mathrm{d}V$

Constant pressure (isobaric) process: $pV^0 = c$

$$W_{12} = \int_1^2 p dV = p(V_2 - V_1)$$

Constant volume (isochoric) process: $pV^\infty = c$

$$W_{13} = \int_1^3 p dV = 0, \text{because } dV = 0$$

Process defined by $pV = c$

$$W_{14} = p_1 V_1 \ln \frac{V_4}{V_1} = p_1 V_1 \ln \frac{p_1}{p_4} = etc$$

Process defined by $pV^n = c$

$$W_{15} = \frac{p_1 V_1 - p_5 V_5}{n-1} = \frac{p_5 V_5 - p_1 V_1}{1-n}$$

2 First Law of Thermodynamics - closed systems

$$Q - W_s = m\left(u_2 + \frac{V_2^2}{2} + gz_2\right) - m\left(u_1 + \frac{V_1^2}{2} + gz_1\right)$$

First Law for a closed system in the absence of kinetic and potential energy

$$\delta Q = dU + \delta W$$

Specific heat at constant volume

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u}{\partial t}\right)_v = \left(\frac{\partial Q}{\partial T}\right)_v$$

Enthalpy, H

$$H = U + pV$$

Specific enthalpy, h

$$h = \frac{H}{m} = \frac{U + pV}{m} = \frac{U}{m} + \frac{pV}{m} = u + pv$$

Specific heat at constant pressure

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{\partial h}{\partial t}\right)_p = \left(\frac{\partial q}{\partial T}\right)_p$$

3 Steady flow energy equation

$$\dot{Q} - \dot{W} = \dot{m}\left(h_e - h_i + \frac{V_e^2 - V_i^2}{2} + g(z_e - z_i) \right)$$

Stagnation enthalpy

$$h_0 = h + \frac{V^2}{2}$$

Velocity at exit to a nozzle

$$V_2 = \sqrt{2\left\{ (h_1 - h_2) + \frac{V_1^2}{2} \right\}}$$

Work from an adiabatic machine

$$-\dot{W}_s = \dot{m}(h_e - h_i) = \dot{m}(h_2 - h_1).$$

4 Second Law of Thermodynamics

Efficiency

$$\textit{Thermal efficiency}, \ \eta_{th} = \frac{\text{Useful work output}}{\text{Thermal energy input}},$$
$$\text{for a heat engine operating in a cycle}.$$

Thermal efficiency of heat engine

$$\text{Thermal efficiency}, \ \eta_{th} = \frac{\text{Net work}}{\text{Heat supplied}} \qquad \eta_{th} = \frac{W_s}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Coefficient of performance of refrigerator

$$\text{Coefficient of performance}, \ \beta = \frac{\text{Heat transferred from cold reservoir}}{\text{Work done}}, = \frac{Q_2}{W_s} = \frac{Q_2}{Q_1 - Q_2}.$$

Coefficient of performance of heat pump

$$\text{Coefficient of performance}, \ \beta' = \frac{\text{Heat transferred to hot reservoir}}{\text{Work done}}, = \frac{Q_1}{W_s} = \frac{Q_1}{Q_1 - Q_2}.$$

Relationship between coefficients of performance

$$\beta' = 1 + \frac{Q_2}{W_s} = 1 + \beta$$

Entropy

Entropy is denoted by the symbol - S; specific entropy is denoted by - s.

The change of entropy between states 1 and 2 is $S_2 - S_1 = \int_1^2 \frac{\delta Q_R}{T}$

Central Equation of Thermodynamics

$$Tds = du + pdv = dh - vdp$$

Steady flow entropy equation for an adiabatic machine

$$\dot{S}_i = \dot{m}(s_e - s_i) \geq 0$$

5 Properties of pure substances

Dryness fraction, or quality

$$\text{Dryness fraction, } x = \frac{\text{Mass of dry vapour}}{\text{Total mass of liquid + vapour}}$$

$$v = xv_g + (1-x)v_f = v_f + xv_{fg}$$
$$u = xu_g + (1-x)u_f = u_f + xu_{fg}$$
$$h = xh_g + (1-x)h_f = h_f + xh_{fg}$$
$$s = xs_g + (1-x)s_f = s_f + xs_{fg}$$

6 Perfect gases, and mixtures of perfect gases

Ideal gas $\frac{pv}{T} = const, R$

Universal Gas Constant $\Re = MR$

S.I. units $\Re$ = 8.3145 kJ/kmol K

Imperial $\Re$ = 1545 ft.lbf/lb mol °R

= 1.986 Btu/lb mol °R

Molar Masses for Common Gases/Elements

Gas/Element	**M** (kg/kmol)
H_2	2
O_2	32
N_2	28
CO	28
CO_2	44
H_2O	18
C	12

Internal energy $u = \int_{T_o}^{T} c_v dT + u_o$, where u_0 is the value of u at temperature T_0

Enthalpy $h = u + pv = u + RT$

$$h = \int_{T_0}^{T} c_p dT + h_0, \text{ where } h_0 \text{ is the enthalpy at temperature } T_0.$$

Relationship between c_p and c_v

$$c_p = c_v + R \qquad \gamma = c_p/c_v$$

Entropy change

$$ds = \frac{c_v dT + pdv}{T} = c_v \frac{dT}{T} + \frac{p}{T}dv$$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v}{T} dT + R\lambda n \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p}{T} dT - R\lambda n \frac{p_2}{p_1}$$

7 Isentropic or process efficiencies

For compressors: $\eta_C = \frac{h_{2i} - h_1}{h_2 - h_1}$

For turbines: $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2i}}$

where the process is from state 1 to state 2 and subscript *i* denotes ideal (isentropic) values

8 Ideal cycle efficiencies

Otto cycle: $\eta = 1 - r_V^{-(\gamma-1)}$

Diesel cycle: $\eta = 1 - r_V^{-(\gamma-1)} \frac{(r_C^{\gamma} - 1)}{\gamma(r_C - 1)}$

Dual cycle: $\eta = 1 - r_V^{-(\gamma-1)} \frac{(r_p r_C^{\gamma} - 1)}{(r_p - 1) + \gamma r_p (r_C - 1)}$

where r_v is the volumetric compression ratio
r_c is the volumetric cut-off ratio
r_p is the constant-volume heat input pressure ratio
γ is the ratio of specific heats

9 Mean effective pressures of reciprocating engine cycles

The indicated mean effective pressure (imep)

$$p_m = \frac{W_{net}}{V_s}$$

where W_{net} is the net cycle work, V_s is the swept volume

The brake mean effective pressure (bmep)

$$p_b = \frac{W_{net} - W_f}{V_s}$$

where W_f is the work lost to friction.

The friction mean effective pressure (fmep)

$$p_f = p_m - p_b$$

10 Specific and Relative Humidity

Absolute Humidity

$$\omega = \frac{m_v}{m_a} \quad (\text{kg water vapor / kg dry air})$$

$$\omega = \frac{m_v}{m_a} = \frac{P_v V / R_v T}{P_a V / R_a T} = \frac{P_v / R_v}{P_a / R_a} = 0.622 \frac{P_v}{P_a}$$

$$\omega = \frac{0.622 P_v}{P - P_v} \quad (\text{kg water vapor / kg dry air})$$

Relative Humidity

$$\phi = \frac{m_v}{m_g} = \frac{P_v V / R_v T}{P_g V / R_v T} = \frac{P_v}{P_g}$$

$$\phi = \frac{\omega P}{(0.622 + \omega) P_g} \quad \text{and} \quad \omega = \frac{0.622 \phi P_g}{P - \phi P_g}$$

Enthalpy

$$h = h_a + \omega h_g \quad (\text{kJ / kg dry air})$$

Adiabatic Saturation and Wet-bulb Temperatures

$$\omega_1 = \frac{c_p (T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$

$$\omega_2 = \frac{0.622 P_{g_2}}{P_2 - P_{g_2}}$$

EXTRACTED TABLES FROM APPENDIX A OF 'THERMODYNAMICS-AN ENGINEERING APPROACH (SI VERSION) (9TH ED.) (MCGRAW HILL) CENGEL, BOLES AND KANOGLU

- **TABLE A1** Molar mass, gas constant, and critical-point properties
- **TABLE A2** Ideal-gas specific heats of various common gases
- **TABLE A4** Saturated water—Temperature table
- **TABLE A5** Saturated water—Pressure table
- **TABLE A6** Superheated water
- **TABLE A11** Saturated refrigerant-134a—Temperature table
- **TABLE A12** Saturated refrigerant-134a—Pressure table
- **TABLE A13** Superheated refrigerant-134a
- **TABLE A17** Ideal-gas properties of air
- **TABLE A23** Ideal-gas properties of water vapor, H_2O
- **TABLE A26** Enthalpy of formation, Gibbs function of formation, and absolute entropy at 25°C, 1 atm
- **TABLE A27** Properties of some common fuels and hydrocarbons
- **TABLE A31** Psychrometric chart at 1 atm total pressure