An Application: Desalination Processes

The potable water needs of the world are increasing steadily due to population growth, rising living standards, industrialization, and irrigation in agriculture. There are over 10,000 desalination plants in the world, with a total desalted water capacity of over 5 billion gallons a day. Saudi Arabia is the largest user of desalination with about 25 percent of the world capacity, and the United States is the second-largest user with 10 percent. The major desalination methods are distillation and reverse osmosis. The relations can be used directly for desalination processes by taking the water (the solvent) to be component A and the dissolved salts (the solute) to be component B. Then the minimum work needed to produce 1 kg of pure water from a large reservoir of brackish or seawater at temperature T_0 in an environment at T_0 is, from Eq. 13–59,

Desalination: $$w_{\text{min,in}} = -R_w T_0 \ln(1/y_w) \qquad \text{(kJ/kg pure water)} \tag{13–60}$$

where $R_w = 0.4615$ kJ/kg·K is the gas constant of water and y_w is the mole fraction of water in brackish or seawater. This relation also gives the maximum amount of work that can be produced as 1 kg of fresh water (from a river, for example) mixes with seawater whose water mole fraction is y_w.

The reversible work associated with liquid flow can also be expressed in terms of pressure difference ΔP and elevation difference Δz (potential energy) as $w_{\text{min,in}} = \Delta P/\rho = g\,\Delta z$ where ρ is the density of the liquid. Combining these relations with Eq. 13–60 gives

$$\Delta P_{\text{min}} = \rho w_{\text{min,in}} = \rho R_w T_0 \ln(1/y_w) \qquad \text{(kPa)} \tag{13–61}$$

and

$$\Delta z_{\text{min}} = w_{\text{min,in}}/g = R_w T_0 \ln(1/y_w)/g \qquad \text{(m)} \tag{13–62}$$

where ΔP_{min} is the **osmotic pressure**, which represents the pressure difference across a semipermeable membrane that separates fresh water from the saline water under equilibrium conditions, ρ is the density of saline water, and Δz_{min} is the **osmotic rise**, which represents the vertical distance the saline water would rise when separated from the fresh water by a membrane that is permeable to water molecules alone (again at equilibrium). For desalination processes, ΔP_{min} represents the minimum pressure that the saline water must be compressed in order to force the water molecules in saline water through the membrane to the fresh-water side during a reverse osmosis desalination process. Alternately, Δz_{min} represents the minimum height above the fresh-water level that the saline water must be raised to produce the required osmotic pressure difference across the membrane to produce fresh water. The Δz_{min} also represents the height that the water with dissolved organic matter inside the roots will rise through a tree when the roots are surrounded by fresh water with the roots acting as semipermeable membranes. The reverse osmosis process with semipermeable membranes is also used in dialysis machines to purify the blood of patients with failed kidneys.

EXAMPLE 13–6 Obtaining Fresh Water from Seawater

Fresh water is to be obtained from seawater at 15°C with a salinity of 3.48 percent on mass basis (or TDS = 34,800 ppm). Determine (*a*) the mole fractions of the water and the salts in the seawater, (*b*) the minimum work input required to separate 1 kg of seawater completely into pure water and pure salts, (*c*) the minimum work input required to obtain 1 kg of fresh water from the sea, and (*d*) the minimum gauge pressure that the seawater must be raised if fresh water is to be obtained by reverse osmosis using semipermeable membranes.

SOLUTION Fresh water is to be obtained from seawater. The mole fractions of seawater, the minimum works of separation needed for two limiting cases, and the required pressurization of seawater for reverse osmosis are to be determined.

Assumptions **1** The seawater is an ideal solution since it is dilute. **2** The total dissolved solids in water can be treated as table salt (NaCl). **3** The environment temperature is also 15°C.

Properties The molar masses of water and salt are M_w = 18.0 kg/kmol and M_s = 58.44 kg/kmol. The gas constant of pure water is R_w = 0.4615 kJ/kg·K (Table A–1). The density of seawater is 1028 kg/m^3.

Analysis (*a*) Noting that the mass fractions of salts and water in seawater are mf_s = 0.0348 and $\mathrm{mf}_w = 1 - \mathrm{mf}_s = 0.9652$, the mole fractions are determined from Eqs. 13–4 and 13–5 to be

$$M_m = \frac{1}{\sum \frac{\mathrm{mf}_i}{M_i}} = \frac{1}{\frac{\mathrm{mf}_s}{M_s} + \frac{\mathrm{mf}_w}{M_w}} = \frac{1}{\frac{0.0348}{58.44} + \frac{0.9652}{18.0}} = 18.44 \text{ kg/kmol}$$

$$y_w = \mathrm{mf}_w \frac{M_m}{M_w} = 0.9652 \frac{18.44 \text{ kg/kmol}}{18.0 \text{ kg/kmol}} = \mathbf{0.9888}$$

$$y_s = 1 - y_w = 1 - 0.9888 = \mathbf{0.0112} = 1.12\%$$

(*b*) The minimum work input required to separate 1 kg of seawater completely into pure water and pure salts is

$$\begin{aligned} \overline{w}_{\text{min,in}} &= -R_u T_0 (y_A \ln y_A + y_B \ln y_B) = -R_u T_0 (y_w \ln y_w + y_s \ln y_s) \\ &= -(8.314 \text{ kJ/kmol·K})(288.15 \text{ K})(0.9888 \ln 0.9888 + 0.0112 \ln 0.0112) \\ &= 147.2 \text{ kJ/kmol} \end{aligned}$$

$$w_{\text{min,in}} = \frac{\overline{w}_{\text{min,in}}}{M_m} = \frac{147.2 \text{ kJ/kmol}}{18.44 \text{ kg/kmol}} = \mathbf{7.98 \text{ kJ/kg seawater}}$$

Therefore, it takes a minimum of 7.98 kJ of work input to separate 1 kg of seawater into 0.0348 kg of salt and 0.9652 kg (nearly 1 kg) of fresh water.

(*c*) The minimum work input required to produce 1 kg of fresh water from seawater is

$$\begin{aligned} w_{\text{min,in}} &= R_w T_0 \ln (1/y_w) \\ &= (0.4615 \text{ kJ/kg·K})(288.15 \text{ K}) \ln(1/0.9888) \\ &= \mathbf{1.50 \text{ kJ/kg fresh water}} \end{aligned}$$

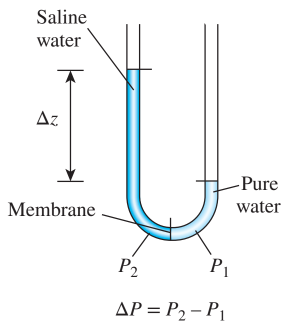

FIGURE 13–24
The osmotic pressure and the osmotic rise of saline water.

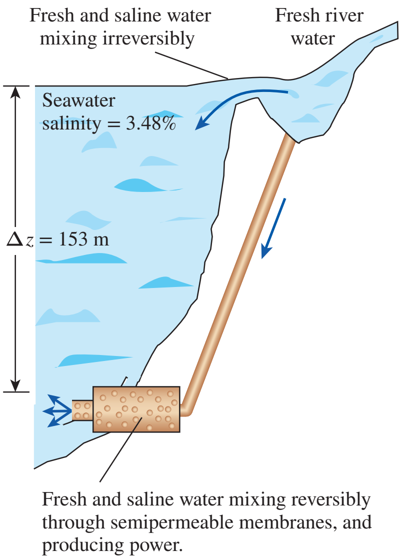

FIGURE 13–25
Power can be produced by mixing solutions of different concentrations reversibly.

Note that it takes about five times more work to separate 1 kg of seawater completely into fresh water and salt than it does to produce 1 kg of fresh water from a large amount of seawater.

(*d*) The osmotic pressure in this case is

$$\begin{aligned}\Delta P_{\text{min}} &= \rho_m R_w T_0 \ln(1/y_w)\\ &= (1028\ \text{kg/m}^3)(0.4615\ \text{kPa·m}^3/\text{kg·K})(288.15\ \text{K})\ln(1/0.9888)\\ &= \mathbf{1540\ kPa}\end{aligned}$$

which is equal to the minimum gauge pressure to which seawater must be compressed if the fresh water is to be discharged at the local atmospheric pressure. As an alternative to pressurizing, the minimum height above the fresh-water level that the seawater must be raised to produce fresh water is (Fig. 13–24)

$$\Delta z_{\text{min}} = \frac{w_{\text{min,in}}}{g} = \frac{1.50\ \text{kJ/kg}}{9.81\ \text{m/s}^2}\left(\frac{1\ \text{kg·m/s}^2}{1\ \text{N}}\right)\left(\frac{1000\ \text{N·m}}{1\ \text{kJ}}\right) = 153\ \text{m}$$

Discussion The minimum separation works determined above also represent the maximum works that can be produced during the reverse process of mixing. Therefore, 7.98 kJ of work can be produced when 0.0348 kg of salt is mixed with 0.9652 kg of water reversibly to produce 1 kg of saline water, and 1.50 kJ of work can be produced as 1 kg of fresh water is mixed with seawater reversibly. Therefore, the power that can be generated as a river with a flow rate of 10^5 m^3/s mixes reversibly with seawater through semipermeable membranes is (Fig. 13–25)

$$\begin{aligned}\dot{W}_{\text{max,out}} &= \rho \dot{V} w_{\text{max,out}} = (1000\ \text{kg/m}^3)(10^5\ \text{m}^3/\text{s})(1.50\ \text{kJ/kg})\left(\frac{1\ \text{MW}}{10^3\ \text{kJ/s}}\right)\\ &= 1.5 \times 10^5\ \text{MW}\end{aligned}$$

which shows the tremendous amount of power potential wasted as the rivers discharge into the seas.