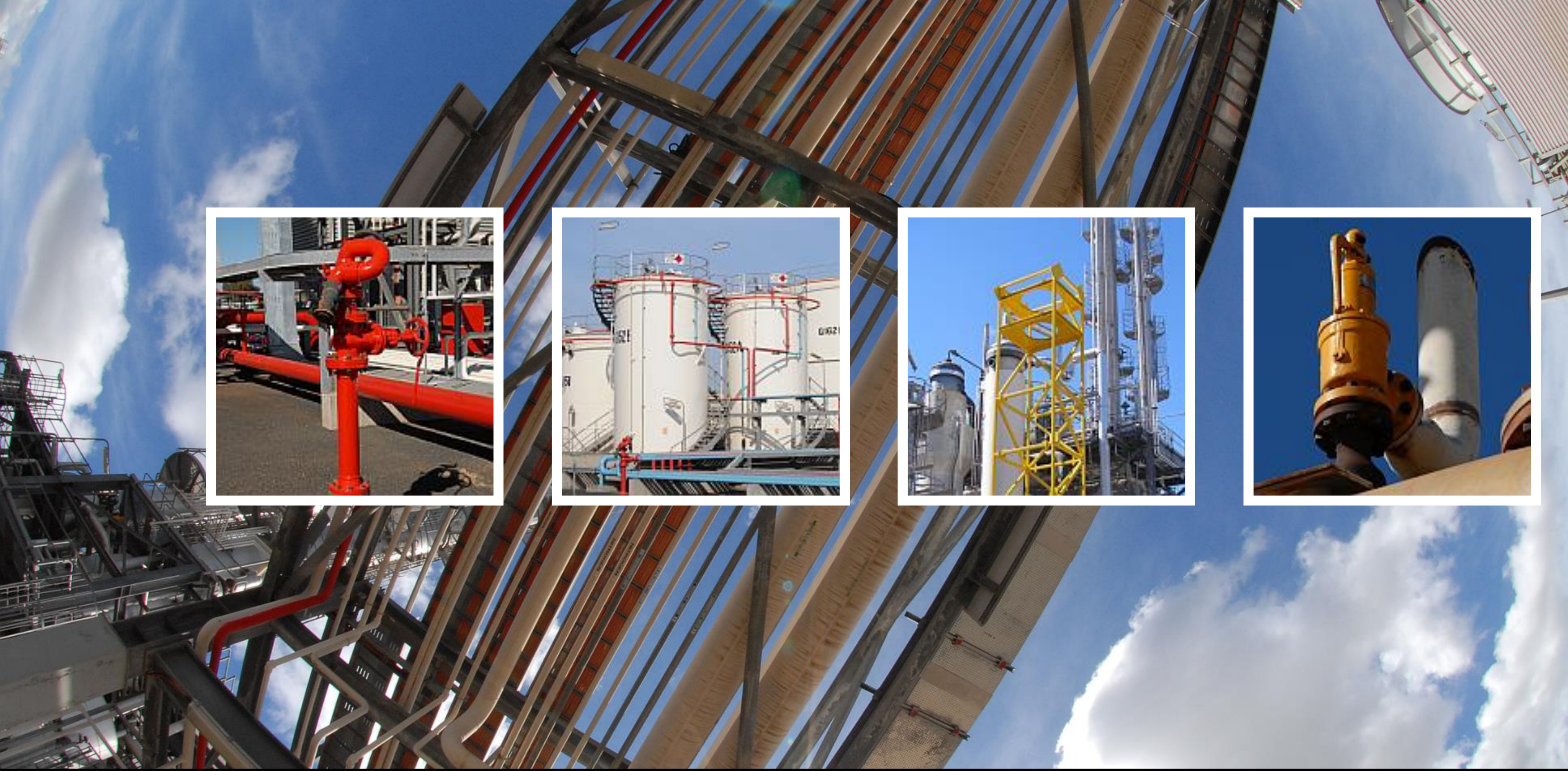


CHEN20010 Material and Energy Balances

Units and Dimensions

THE UNIVERSITY OF MELBOURNE

Units and Dimensions – Module Learning Outcomes

A student is expected to be able to:

- xx

The Importance of Units

Engineers need to be able to unambiguously express and interpret quantities in the designs and systems that they work with.

Engineers need to be able to work with different systems of units (e.g., SI and British) and they must be able to freely convert the units of these systems without mistake.

Much valuable numeric data is only available expressed in old units of measurement.

Every country in the world except one either uses the metric system or is (slowly) moving towards the metric system.

Which country is not metric or going metric?

Units and Dimensions

Base Quantities

Length	L
Time	T
Mass	M
Temperature	θ
Amount of Substance	N
Current	I

Derived Quantities

All other quantities may be derived from the base quantities

e.g. Force = Mass $\times$ Acceleration

$$[F] = M L T^{-2}$$

Work = Force $\times$ Distance

$$[W] = M L^{2} T^{-2}$$

Thus **Work** has dimensions of Mass x Length ^{2} per Time ^{2}

Unit Systems

e.g. Force = Mass × Acceleration

$$1\ \text{N} = 1\ \text{kg} \times 1\ \text{m/s}^2$$

One newton is that force required to move **one** kilogram at an acceleration of **one** metre per second per second.

Six newtons is that force required to move **two** kilogram at an acceleration of **three** metre per second per second.

Unit Systems

Coherent Systems — The equation between the numerical values is the same As the equation between the physical quantities.

FPS (foot – pound – second) **System of Units** — $\boxed{\text{Not coherent}}$

e.g. Force = Mass $\times$ Acceleration

$$1\ lb_f = 1\ lb_m \times 32.174\ ft/s^2$$

One pound-force is that force required to move one pound-mass at an acceleration of 32.174 feet per second per second.

i.e if	force	is expressed in	lb_f
	mass		lb_m
	acceleration		ft/sec/sec

then Force = (Mass $\times$ Acceleration) / g_c

where, $g_c = 32.174\ lb_m\ ft\ sec^2/\ lb_f$

Unit Systems

Coherent Systems	The equation between the numerical values is the same as the equation between the physical quantities.

SI System of Units

e.g. Force = Mass $\times$ Acceleration

$$1\ \mathrm{N} = 1\ \mathrm{kg} \times 1\ \mathrm{m/s^2}$$

SI (Systeme International d'Unites)

Base Quantities :

Length	m	metre
Mass	kg	kilogram
Time	s	second
Temperature	K	kelvin
Amount of Substance	mol	mole
Electric Current	A	ampere

Derived Quantities :

Force	N	newton	$kg\ m / s^2$
Energy	J	joule	N m
Power	W	watt	J / s
Pressure	Pa	pascal	$kg / m\ s^2$
Frequency	Hz	hertz	1 / s

SI (Systeme International d'Unites)

Prefixes :

10^{12}	tera	T	10^{-1}	deci	d
10^{9}	giga	G	10^{-2}	centi	c
10^{6}	mega	M	10^{-3}	milli	m
10^{3}	kilo	k	10^{-6}	micro	m
10^{2}	hecto	h	10^{-9}	nano	n
10	deca	da	10^{-12}	pico	p

The use of hecto, deca, deci and centi is NOT preferred.

Use prefixes in numerator only e.g. 123 GN/m^2, NOT 123 kN/mm^2

The prefix is part of the unit 1 mm^3 is 1 $(mm)^3$ NOT 1 $m(m)^3$

CGS System (centrimetre, gram, second)

Base unit of	length is the	centimetre	cm
	mass	gram	g
	time	second	s

partly coherent

mechanical energy IS coherent

force	1 dyne	=	$1\ \mathrm{g\ cm\ s^{-2}}$
work	1 erg	=	$1\ \mathrm{g\ cm^{2}\ s^{-2}}$

thermal energy IS NOT coherent calorie

You should avoid working in this system of units.

FPS (Foot – Pound – Second System)

Base unit of	length is the	foot	ft
	mass	pound	lb
	time	second	s

not coherent so AVOID !

unit of force is lb_f

unit of mass is lb_m

unit of energy is BTU (British Thermal Unit)

Note that in this system the prefix M denotes 10^3.

Do not confuse with the SI system where M denotes 10^6.

Thus, MBTU = 10^3 BTU

and MMBTU = 10^6 BTU

Amounts of Substances

Unit : mole

Symbol : mol

A mole is defined as the amount of substance which contains as many elementary entities (e.g. atoms, molecules, ionic groups) as there are atoms in exactly 12 g of Carbon 12.

Avogadro's Constant is 6.0220×10^{23} per mole

1 mol of He contains 6.0220×10^{23} atoms of He

1 mol of CO_2 contains 6.0220×10^{23} molecules of CO_2

1 mol of CH_4 contains 6.0220×10^{23} atoms of C and 24.088×10^{23} atoms of H

Amounts of Substances

Other units in common use are lb-mol and kg-mol

A lb-mole is defined as the amount of substance which contains as many elementary entities (e.g. atoms, molecules, ionic groups) as there are atoms in exactly 12 lb_m of Carbon 12.

A kg-mole is defined as the amount of substance which contains as many elementary entities (e.g. atoms, molecules, ionic groups) as there are atoms in exactly 12 kg of Carbon 12.

1 kg-mol = 1 kmol

Atomic Weights

So, the mass of 1 mole of a substance measured in grams is the equal to the atomic weight.

Atom	Atomic Weight	Mass of 1 mole
H	1.008	1.008 g
He	4.003	4.003 g
C	12.01	12.01 g
N	14.01	14.01 g
O	16.00	16.00 g
S	32.01	32.01 g
Cl	35.45	35.45 g
Cu	63.55	63.55 g
Au	196.97	196.97 g
U	238.03	238.03 g

Molecular Weights

So, the mass of 1 mole of a substance measured in grams is the equal to the molecular weight.

Molecule	Molecular Weight	Mass of 1 mole
H_2	2.016	2.016 g
CH_4	16.04	16.04 g
H_2O	18.02	18.02 g
CO	28.01	28.02 g
O_2	32.00	32.00 g
CO_2	44.01	44.01 g
CH_3OH	32.04	32.04 g
SO_2	64.07	64.07 g
C_2H_5OH	64.51	64.51 g
C_6H_6	78.11	78.11 g

What are the units of molecular weight ?

Length Units

metre

inch
foot
yard

12 inches = 1 foot 12” = 1’
3 feet = 1 yard 3’ = 1 yd

1 foot = 0.3048 m
1 mile = 5280 feet
1 mile ≈ 1.6 km

Other length units are chains, furlongs and fathoms.

The distance that a computer mouse moves across a mouse pad is measured in mickeys.

Area Units

m^2 ft^2

$$1 \text{ are} = 100 \text{ m}^2$$

$$1 \text{ hectare} = 100 \text{ are} = 10^4 \text{ m}^2$$

$$1 \text{ acre} = 4840 \text{ yd}^2 \approx 4067 \text{ m}^2$$

$$1 \text{ sq mi} \approx 2.59 \times 10^6 \text{ m}^2$$

Volume Units

m^3 litre

litre (l or L) = 1 dm^3

1000 L $\equiv$ 1.000 m^3

gallon : two definitions

Imperial (British) Gallon = 4.546 L
U.S. Gallon = 3.785 L

So, 1 Imp. Gallon = 1.20 U.S. Gallon

barrel : used in the oil industry

1 bbl = 42 U.S. gallons

Other volume units are pints, quarts and fluid ounces.

One noggin is usually taken to equal ¼ pint though it is sometimes taken to equal ½ pint.

Mass Units

kg tonne lb

1 tonne $\equiv$ 1000 kg

1 ton $\equiv$ 1016 kg Beware

1 lb_m (pound mass) $\equiv$ 0.4536 kg

Other mass units are grains, ounces (two definitions), stones and hundredweight.

Density

Mass per unit Volume

$$1\ \text{kg/l} \equiv 1000\ \text{kg/m}^3$$

$$\text{Specific Gravity} = \frac{(\text{density})_{\text{material}}}{(\text{density})_{\text{reference}}}$$

- reference material is usually water
- the density of water at 4 °C = 1000 kg/m^3
- since densities vary with temperatures, both temperatures must be specified

Specific Gravity

e.g. $SG = 0.856 \quad {}^{20^\circ}\!/_{4^\circ}$

This is the temperature of the material of interest.

This is the temperature of the reference material.

Specific Gravity

$$\text{e.g. SG} = 0.856 \qquad {}^{20^\circ}\!/_{4^\circ}$$

If water is the reference material, then because the density of water at 4° is 1000 kg/m^3 we can easily find the density of the material of interest.

$$\text{SG} = 0.856 = \frac{\text{density}}{1000\ \text{kg/m}^3} \qquad \therefore \quad \text{density} = 856\ \text{kg/m}^3$$

°API Gravity

The oil industry uses °API

$$°API = \frac{141.5}{SG^{60°F}/_{60°F}} - 131.5$$

heavy crude oils : 10 to 16 °API

light crude oil : > 35 °API

Specific Volume

Density - Mass per unit Volume

$$\text{Specific Volume} = \frac{1}{\text{Density}}$$

Temperature

Basic SI unit is kelvin, K

There are four temperature scales:

• kelvin	K (not °K)
• Celsius (once known as Centigrade)	°C
• Rankine	°R
• Fahrenheit	°F

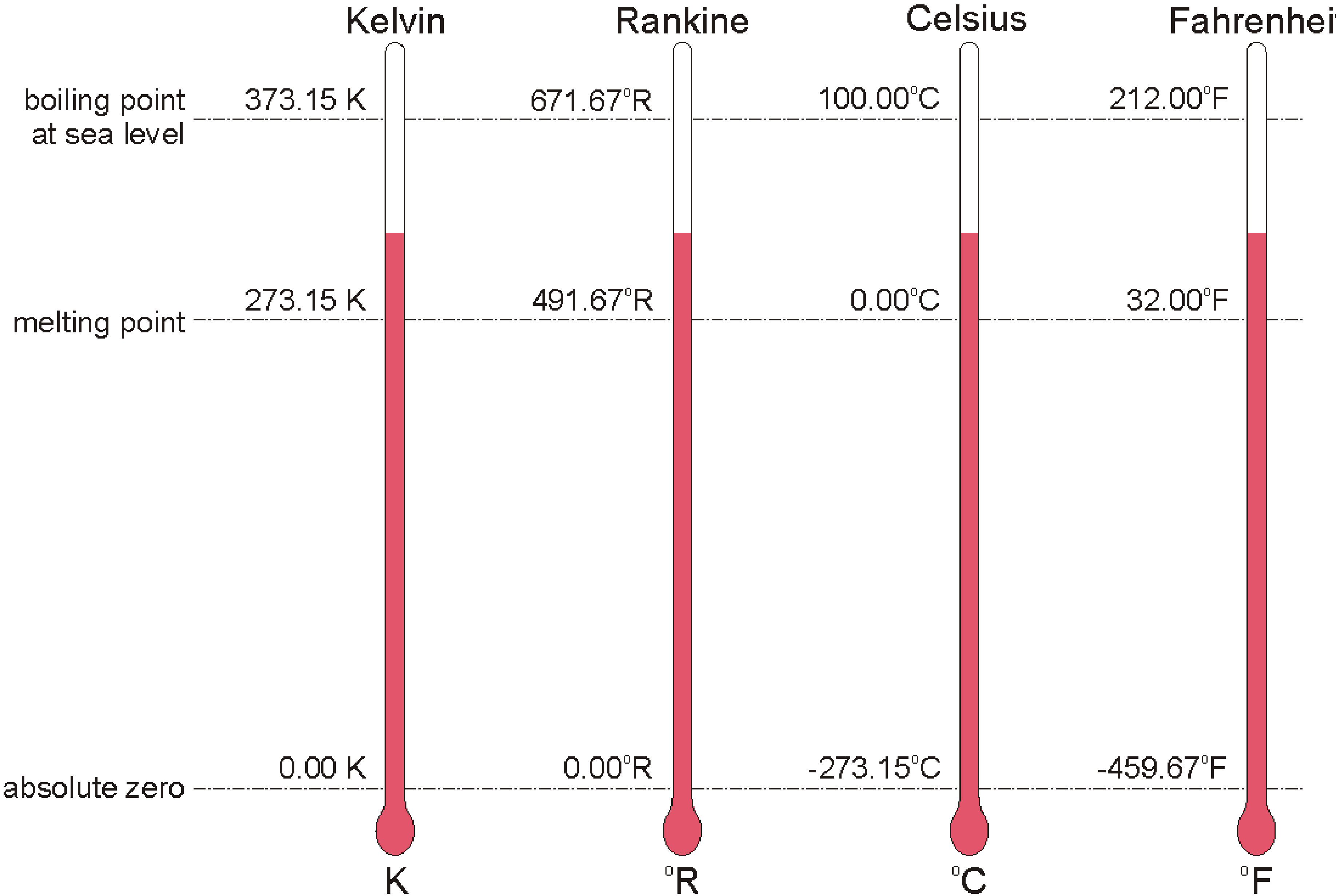
Temperature

Celsius and Fahrenheit are relative temperature scales as their zero points are arbitrarily set.

Celsius	0 °C	melting point of ice
	100 °C	boiling point of water at 1 atm pressure
Fahrenheit	0 °F	melting point of snow/chemical mixture
	600 °F	boiling point of mercury at 1 atm pressure
	32 °F	melting point of ice

Kelvin and Rankine are absolute temperature scales as their zero points are at absolute zero, the lowest conceivable temperature.

kelvin	273.15 K	melting point of ice
	373.15 K	boiling point of water at 1 atm pressure
Rankine	459.6°R	melting point of snow/chemical mixture
	1059.6°R	boiling point of mercury at 1 atm pressure


Kelvin
Rankine
Celsius
Fahrenheit
boiling point
at sea level
373.15 K
671.67°R
100.00°C
212.00°F
melting point
273.15 K
491.67°R
0.00°C
32.00°F
absolute zero
0.00 K
0.00°R
-273.15°C
-459.67°F
K
°R
°C
°F

Temperature

Remember this conversion:

$$°C = (°F - 32)\frac{5}{9}$$

Temperature Difference

$$\Delta ^\circ C \equiv \Delta K$$

$$\Delta ^\circ F \equiv \Delta ^\circ R$$

If the temperature is increased by 4 °C then the temperature is increased by 4 K.

If point A is 27 °F hotter than point B, then point B is 27 °R cooler than point A.

Caution : Do not confuse **TEMPERATURE** with **TEMPERATURE DIFFERENCE**

Pressure Units

Pa kPa bar atm psi (pounds per square inch)

$$1 \text{ Pa} \equiv 1 \text{ N/m}^2$$

gauge pressure - zero at the local barometric pressure

absolute pressure - zero at absolute vacuum (the lowest conceivable pressure)

Pressure Units

Bourdon Gauge

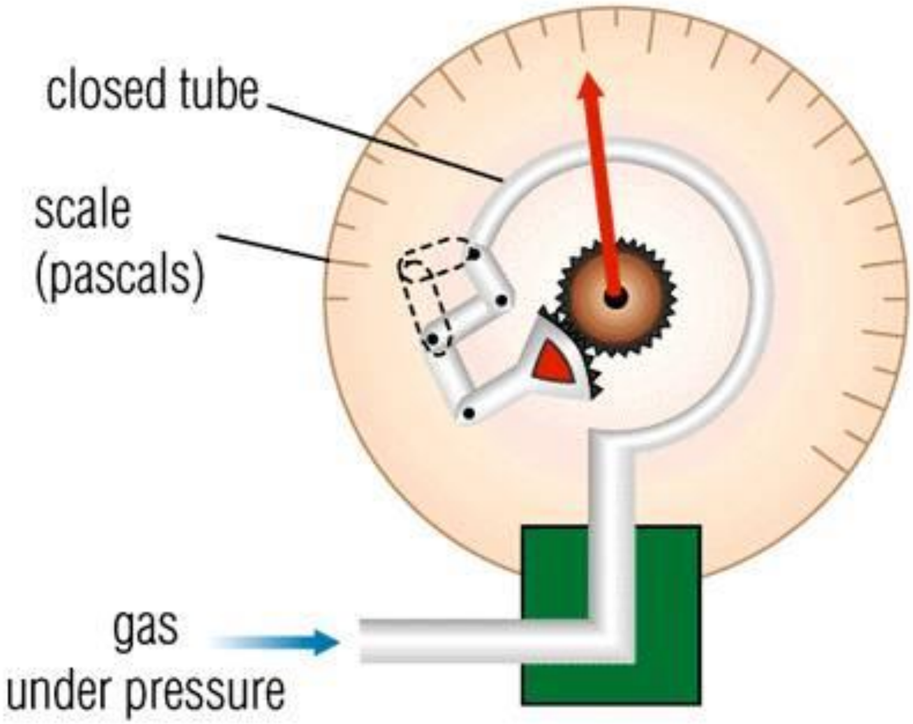
Why use gauge pressure ?

It all depends on the way in which pressure is measured.

Nearly all pressure gauges measure *pressure difference* and not *pressure*.

Pressure Units

Pressure Units



Bourdon Gauge

Pressure Units

Pa kPa bar atm psi (pounds per square inch)

$$1 \text{ Pa} \equiv 1 \text{ N/m}^2$$

gauge pressure - zero at the local barometric pressure

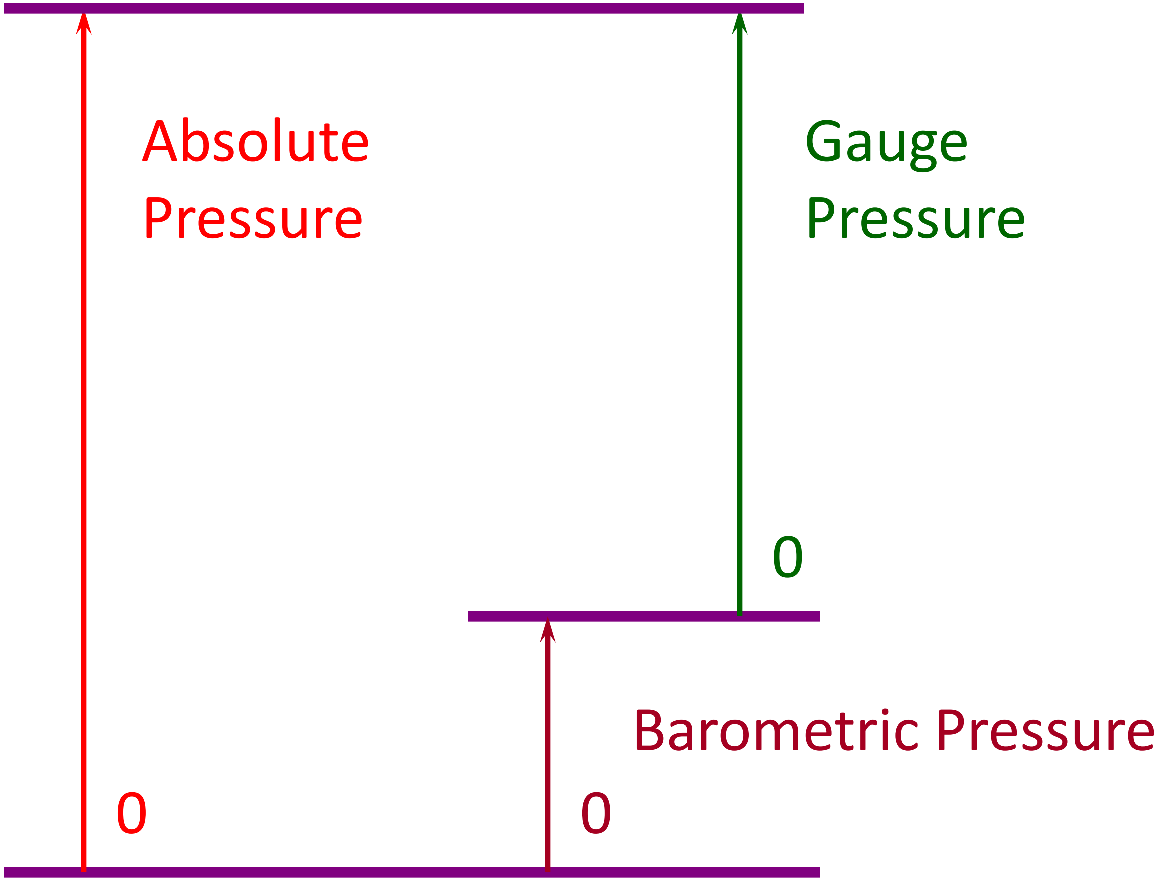
absolute pressure - zero at absolute vacuum (the lowest conceivable pressure)

$$\left(\begin{array}{c}\text{Absolute}\\ \text{Pressure}\end{array}\right) = \left(\begin{array}{c}\text{Gauge}\\ \text{Pressure}\end{array}\right) + \left(\begin{array}{c}\text{Barometric}\\ \text{Pressure}\end{array}\right)$$

Barometric pressure is the air or room pressure at the time the measurement is taken.

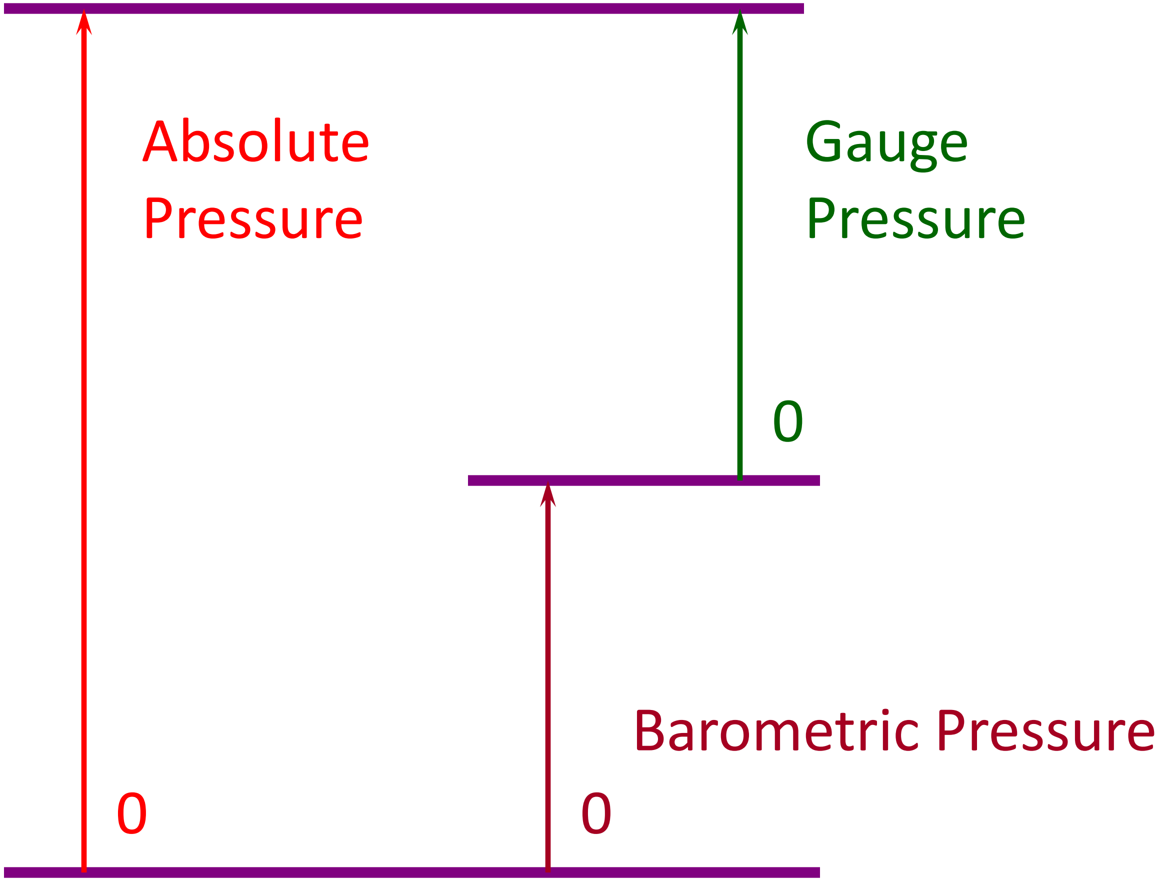
Barometric pressure varies with
weather
location
altitude.

Pressure



As barometric pressure changes, gauge pressure changes.

Pressure



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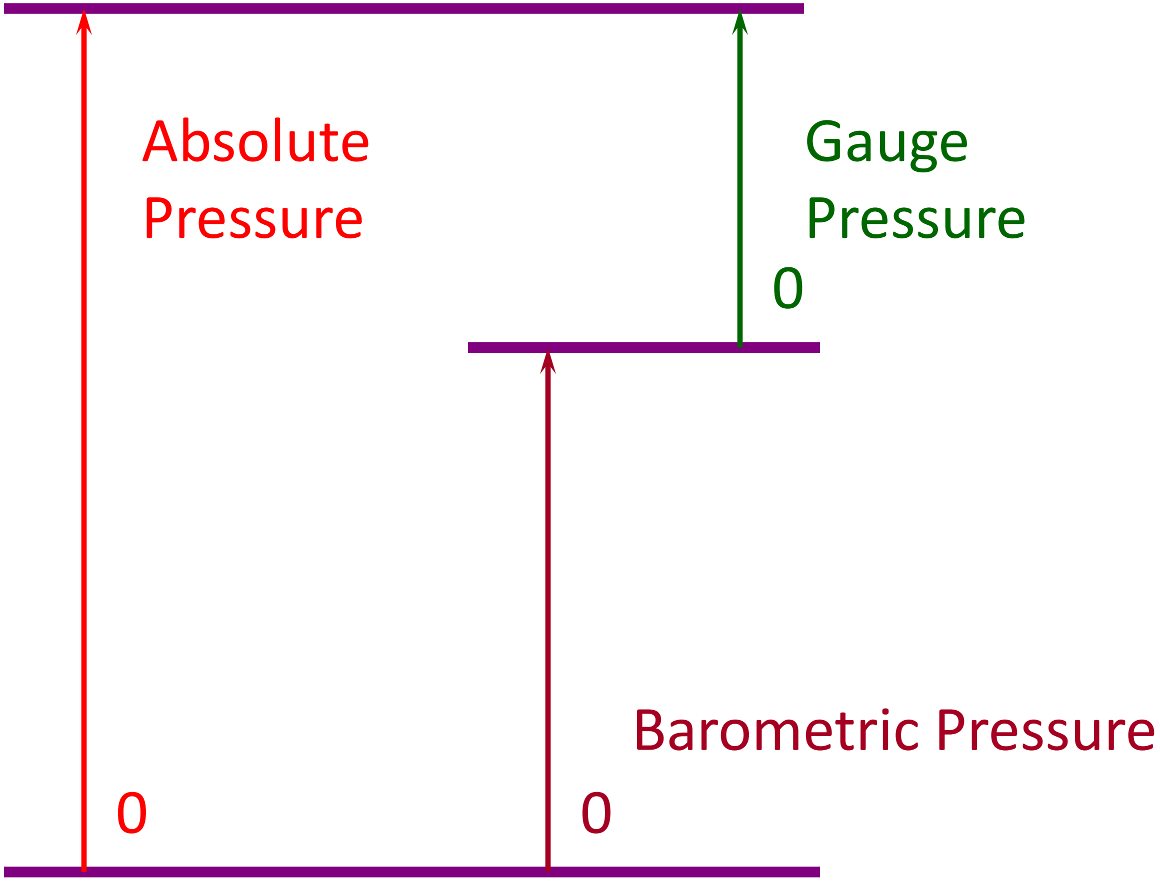

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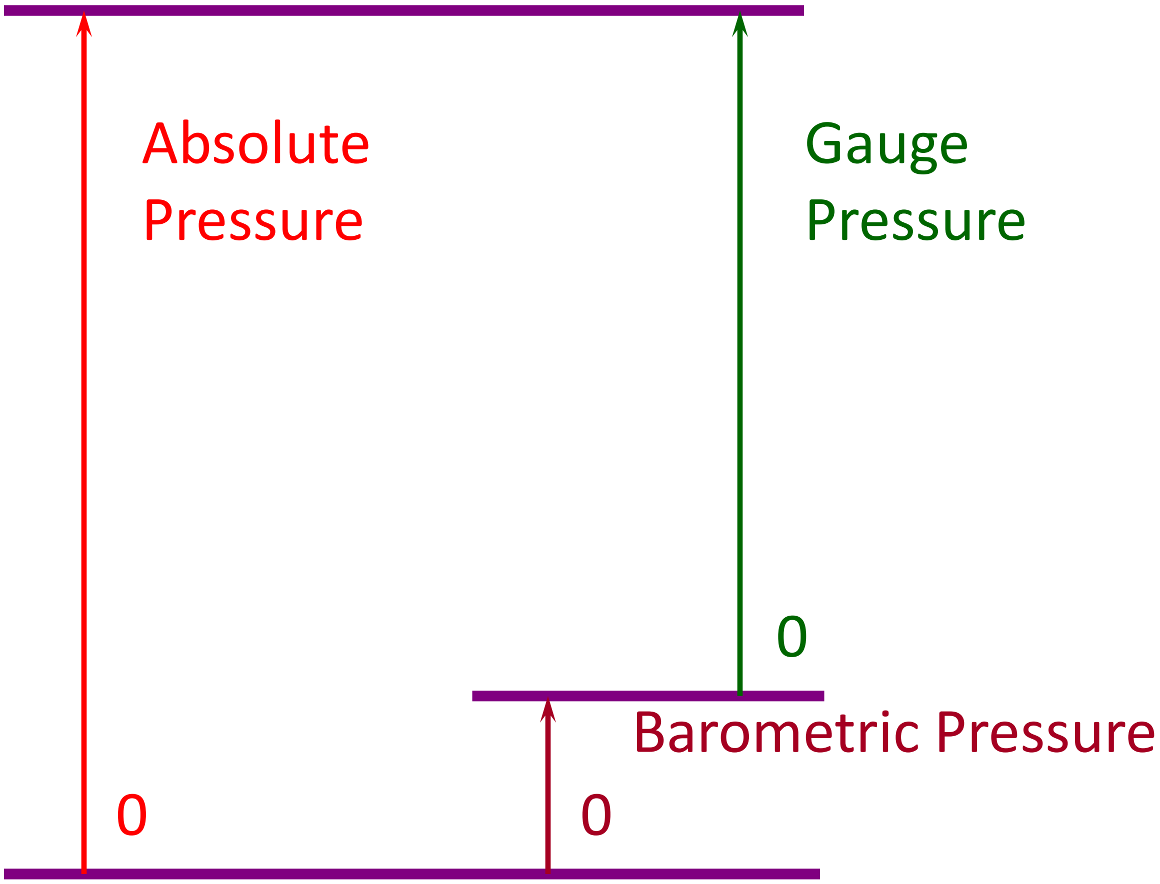
Pressure



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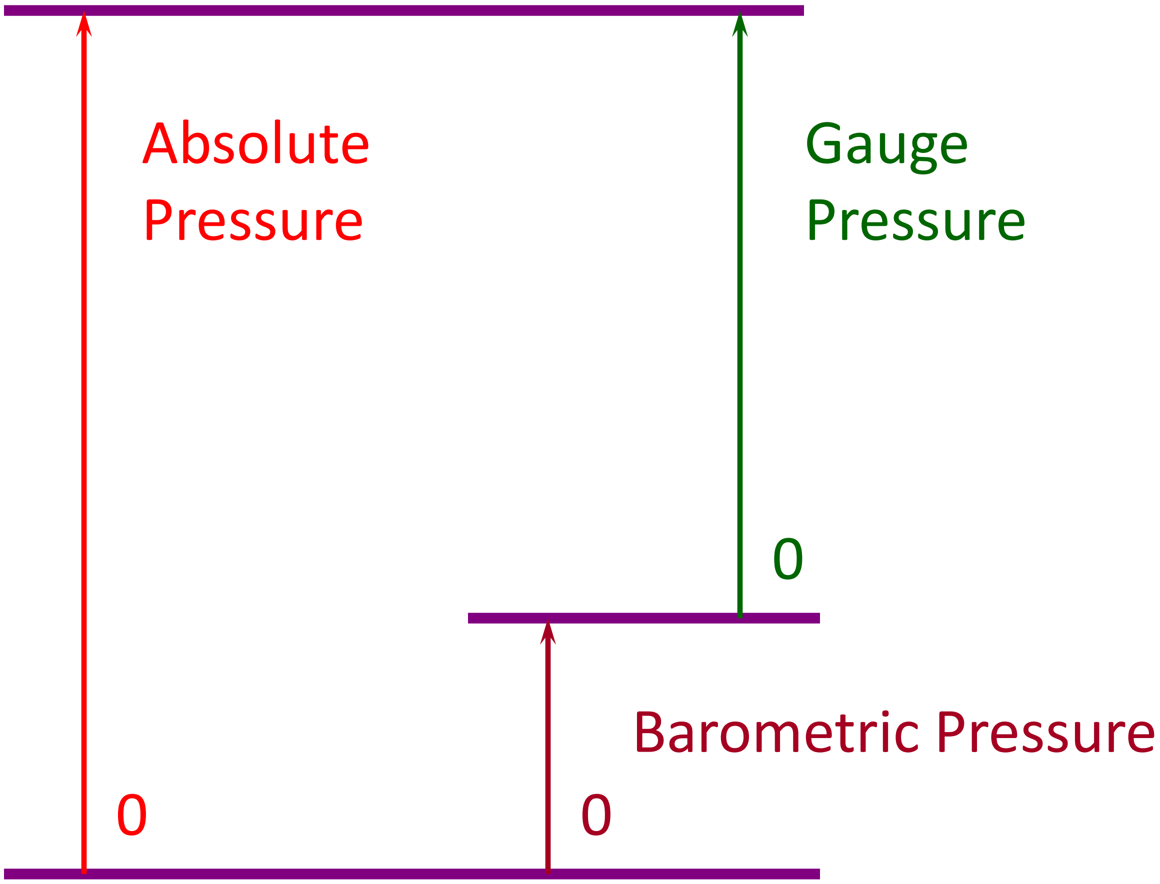
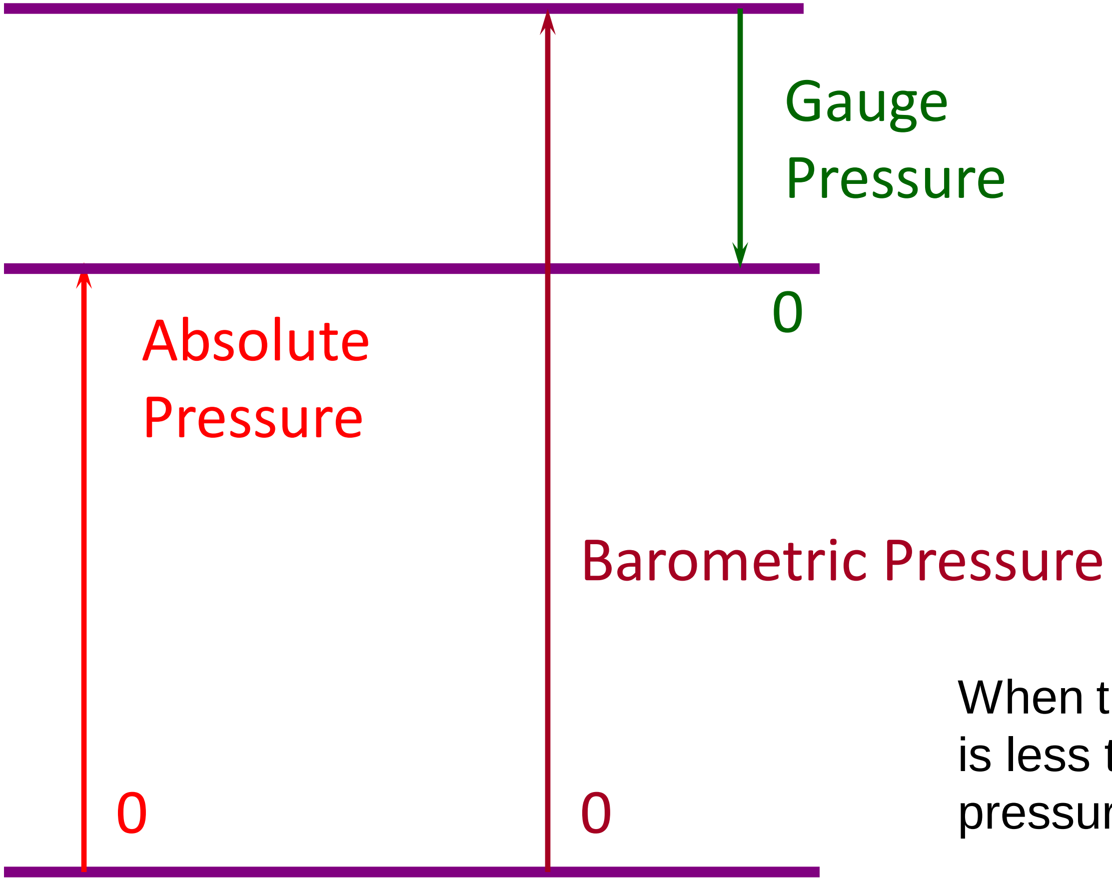
Pressure



As barometric pressure changes, gauge pressure changes.

Pressure



When the absolute pressure is less than the barometric pressure we have negative gauge, vacuum.

Pressure

force/area

SI System

$N/m^2 \equiv$ Pa (pascal) a small unit

More common to use bar or kPa.

1 bar $\equiv 10^5$ Pa $\equiv$ 100 kPa

Pressure

force/area

British System

lb_f /in^2	psi	pounds per square inch
	psia	absolute
	psig	gauge

Pressure

force/area

Atmosphere

standard	atm	1 atm $\equiv$ 1.01325 bar
technical	at	1 at $\equiv$ 1 kg_f /cm^2 not common unit

Pressure

force/area

Column of Liquid

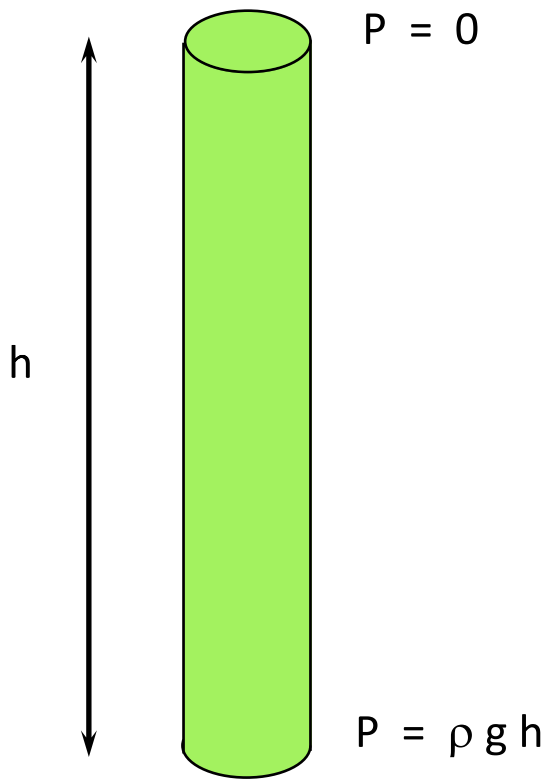
Pressure may also be expressed in terms of the pressure at the base of a column of liquid.

e.g. mm Hg, in Hg, ft H_2O

Pressure Units

Column of Liquid

1 atm	$\equiv$	101.325 kPa
	$\equiv$	14.696 psi
	$\equiv$	760 mm Hg
	$\equiv$	760 Torr
	$\equiv$	29.92 in Hg
	$\equiv$	33.93 ft H_2O



Energy

SI   Nm $\equiv$ J   (joule)

Other   BTU (British Thermal Unit)
calorie
CHU (Centigrade Heating Unit)

1 cal $\equiv$ 4.1868 J
1 BTU $\equiv$ 252 cal

Energy

1 BTU is the amount of energy required to increase the temperature of $1 lb_m$ of water by 1°F.

1 calorie is the amount of energy required to increase the temperature of 1 gram of water by 1°C.

1 CHU is the amount of energy required to increase the temperature of $1 lb_m$ of water by 1°C.

Power

SI $\quad$ J/s $\equiv$ W $\quad$ (watt)

1 W is the power required to apply an energy of 1 J for a period of 1 s.

Other $\quad$ hp $\quad$ (horsepower)

1 hp $\equiv$ 746 Watt

Simple maths error doomed Mars flight

Nasa scientists confused the imperial unit of poundals with the metric unit of newtons, sending the Mars Climate Orbiter off course by about 100 km

WASHINGTON — The loss of a US$125 million (S$212 million) satellite as it approached Mars last week has been blamed on a goof that should only have tripped up a novice science student — confusing imperial and metric units.

An internal review team at Nasa's Jet Propulsion Laboratory, in a preliminary conclusion, said that engineers at Lockheed Martin Corp, the spacecraft's builder, had specified certain measurements about the spacecraft's thrust in poundals, an imperial unit.

But Nasa scientists thought the information was a metric measurement known as newtons.

The resulting miscalculation, undetected for months, meant the Mars Climate Orbiter was off course by about 100 km as it approached Mars.

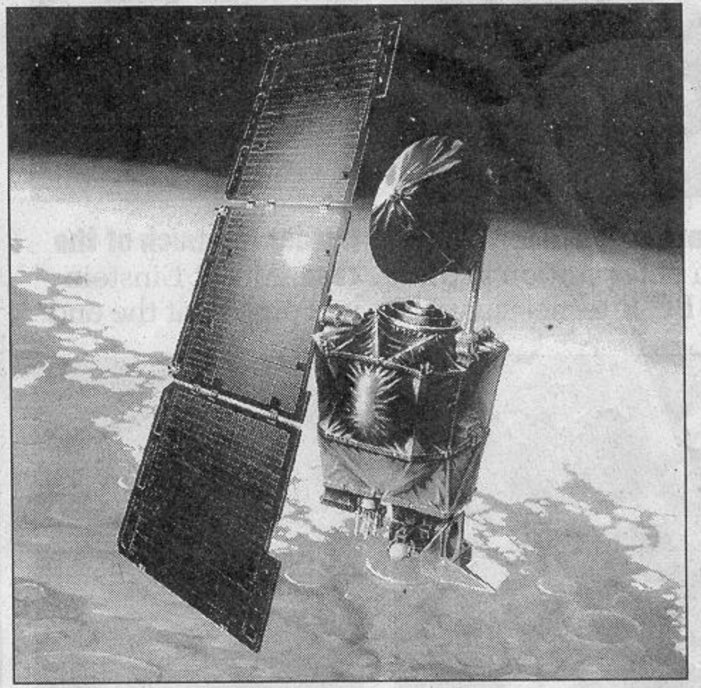
... ductions to the metric system in elementary school and high school and college physics till the end of time," said space-policy director John Pike at the Federation of American Scientists in Washington.

Lockheed's reaction was equally blunt.

'It can't be something that simple that could cause this to happen," said flight systems vice-president Noel Hinners at Lockheed Martin Astronautics in Denver.

The finding was a major embarrassment for Nasa (National Aeronautics and Space Administration), which said it was investigating how such a basic error could have gone through its checks and balances.

The fiasco could raise questions about whether Nasa and its contractors are skimping on safety to cut costs.



The error meant the Mars Climate Orbiter was off course by about 100 km as it neared Mars.

... planet's climate, was very inexpensive by Nasa's standards.

It is part of a new strategy to fly more but cheaper missions to Mars.

The failure is also anoth-

... special review board of experts from the lab and elsewhere.

An independent Nasa failure review board will be formed shortly.

It is uncertain what hap-

Straits Times, Singapore

October 2, 1999

Conversion of Units

Example 1 : Express length of 47 feet in metres.

length = 47 ft

Conversion of Units

Example 2 : Express speed of 80 feet/min in m/s.

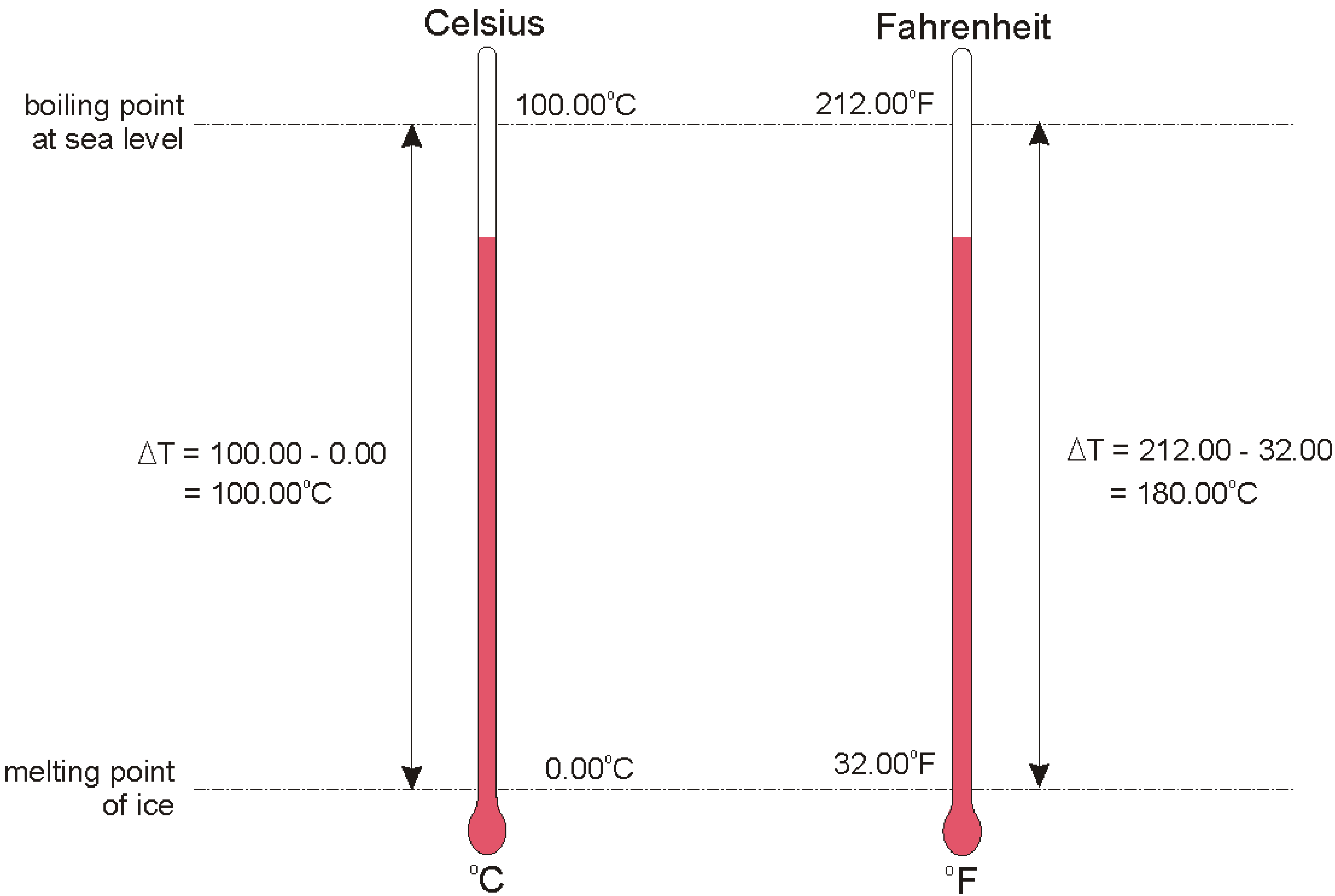
$$\text{speed} = 80 \; \frac{\text{ft}}{\text{min}}$$

Conversion of Units

Example 3 : Express density of $12\ lb_m/ft^3$ in kg/m^3.

$$\text{density} = \frac{12\ lb_m}{ft^3}$$

Example 4 : Express 100°C in °F.



Conversion of Units

Example 5 : Express 1.7 kJ/kg °C in Btu/g °F.

Conversion of Units

Example 6 : Density is given by $\rho = 965.4 - 0.408\, T$ where ρ is in kg/m^3 and T is in °C. Re-write so that ρ is in kg/m^3 and T is in °F.

We must first note that T is a unit of temperature **not** temperature difference.

Recall that $T = (t - 32)\,{}^5/_9$

Here T is in °C,
t is in °F.

Conversion of Units

Example 6 : Density is given by $\rho = 965.4 - 0.408\ T$
where ρ is in kg/m^3 and T is in °C. Re-write
so that ρ is in kg/m^3 and T is in °F.

Dimensional Homogeneity and Equations

Consider the equation:

$$C = 3\,(2 + D)$$

where, C has units of m^3/s

and D has units of m

What are the units of 3 and 2 ?

Dimensional Homogeneity and Equations

$C = 3 (2 + D)$

C has units of m^3/s
D has units of m

Only quantities of the same units (and thus dimensions) may be added to (or subtracted from) one another.

∴ 2 has same units and dimensions as D

D has dimensions of **LENGTH** and units of m

So, 2 has dimensions of **LENGTH** and units of m

Dimensional Homogeneity and Equations

$C = 3(2 + D)$

C has units of m^3/s
D has units of m

The units (and hence dimensions) on opposite sides of an equals sign must be equal.

C has dimensions of **$LENGTH^3/TIME$** and units of m^3/s

$\therefore$ $3(2 + D)$ has dimensions as **$LENGTH^3/TIME$** and units of m^3/s

but $(2 + D)$ has dimensions of **LENGTH** and units of m,

$\therefore$ 3 has dimensions of **$LENGTH^2/TIME$** and units of m^2/s

Dimensional Homogeneity and Equations

Consider the equation:

$$F = 7 D R^2 \qquad \text{where} \quad \begin{cases} F \text{ is in kg/s} \\ D \text{ is in kg/m}^3 \\ R \text{ is in m} \end{cases}$$

What are the dimensions and units of constants 7 and 2 ?

> Exponents (i.e., powers) must always be dimensionless and hence unitless. It is not possible to raise a quantity to the power 3 m^3 of 2.5 kg/s.

So, 2 must be dimensionless and therefore unitless.

Consider the equation:

$$F = 7\,D\,R^2$$

where

$$\begin{cases} F \text{ is in } kg/s \\ D \text{ is in } kg/m^3 \\ R \text{ is in } m \end{cases}$$

What are the dimensions and units of constants 7 and 2 ?

Re-arranging the equation we find : $7 = \dfrac{F}{D\,R^2}$

So the units of 7 are the units of $\dfrac{F}{D\,R^2}$

Units of $\dfrac{F}{D\,R^2} = \dfrac{kg/s}{(kg/m^3)(m)^2} = m/s$

So 7 has the units m/s and the dimensions of length/time.

Dimensional Homogeneity and Equations

Consider the equation:

$$SA = 0.657\, m_b^{0.425}\, h_b^{0.725} \quad \text{where} \quad \begin{cases} SA \text{ is in } ft^2 \\ m_b \text{ is in } lb_m \\ h_b \text{ is in } ft \end{cases}$$

Convert this equation to SI units

$$\text{i.e.} \quad SA = k\, m_b^{0.425}\, h_b^{0.725} \quad \text{where} \quad \begin{cases} SA \text{ is in } m^2 \\ m_b \text{ is in } kg \\ h_b \text{ is in } m \end{cases}$$

Find κ

What are the units of 0.657 ? $SA = 0.657\ m_b^{0.425}\ h_b^{0.725}$

$$0.657 = \frac{SA}{m_b^{0.425}\ h_b^{0.725}}$$

$$\therefore \text{ Units of } (0.657) = \text{Units of } \left(\frac{SA}{m_b^{0.425}\ h_b^{0.725}} \right)$$

Units of (0.657) =

Units of (k) =

What are the units of 0.657 ? $SA = 0.657\ m_b^{0.425}\ h_b^{0.725}$

Now convert units of 0.657 from $\dfrac{ft^{1.275}}{lb_m^{0.425}}$ to $\dfrac{m^{1.275}}{kg^{0.425}}$

$$SA = 0.657\, m_b^{0.425}\, h_b^{0.725}$$

SA is in ft^2
m_b is in lb_m
h_b is in ft

SA is in m^2
m_b is in kg
h_b is in m