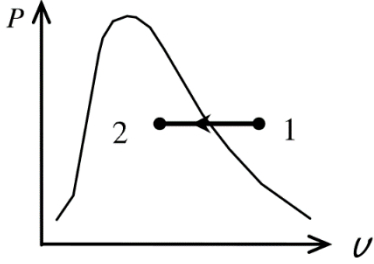
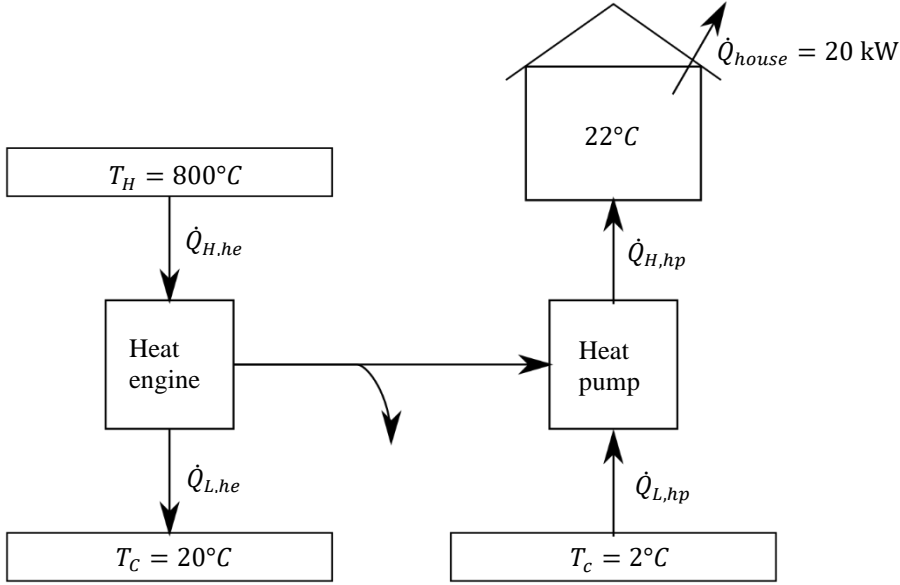
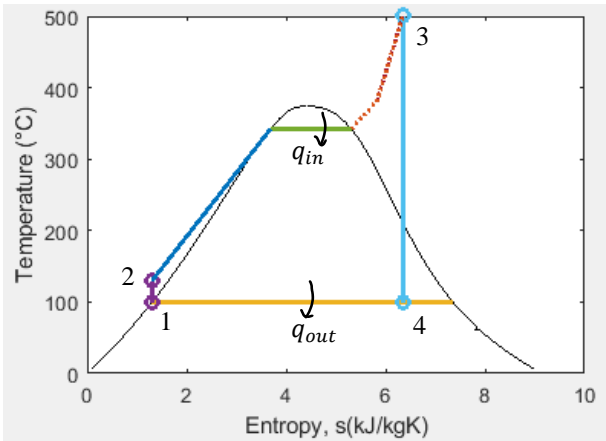


Question no & part	Answer	Mark (per part)
Q1(a)	<p>Diagram should be labelled as follows:</p>	
Q1(a)(i)	Labelling of "Critical point"	1
Q1(a)(ii)	Labelling of "Saturated liquid line" and "Saturated vapour line"	2
Q1(a)(iii)	Labelling of the "Compressed liquid region" and "Superheated Vapour region"	2
Q1(a)(iv)	Drawing and labelling of a single constant-temperature line, as per the blue $T = \text{const}$ line on the diagram above. The line should be of the correct shape in each of the three regions.	3
Q1(b)	The specific volume of superheated water vapour at 15 MPa and 400°C is to be determined using three different methods.	
Q1(b)(i)	From Table A-6, the value can be read directly as $v = 0.015671 \text{ m}^3/\text{kg}$	2
Q1(b)(ii)	<p>Using the ideal gas equation with <math>R = 0.4615 \text{ kJ/kgK}</math></p> $Pv = RT$ $v = \frac{RT}{P} = \frac{(0.4615 \text{ kJ/kgK})(400 + 273)\text{K}}{15,000 \text{ kPa}} = \mathbf{0.020706 \text{ m}^3/\text{kg}}$	2
Q1(b)(iii)	To use the generalized compressibility chart, we need to compute the reduced temperature and reduced temperature. Using $P_{cr} = 22.06 \text{ MPa}$ and $T_{cr} = 647.1 \text{ K}$	

	$P_r = \frac{P}{P_{cr}} = \frac{15}{22.06} = 0.68$ $T_r = \frac{T}{T_{cr}} = \frac{400 + 273}{647.1} = 1.04$ <p>The value of the compressibility factor <math>Z</math> is read from Figure A-15:  <math>Z = 0.76</math> (accept <math>0.74 &lt; Z &lt; 0.77</math>)</p> <p>Answers above 0.77 are incorrect, since this is where the <math>T_r = 1.05</math> line intercepts <math>P_r = 0.68</math></p> <p>Then the specific volume is calculated as:</p> $v = Zv_{ideal} = 0.76(0.020706) = \mathbf{0.015778 \text{ m}^3/\text{kg}}$ <p>Or</p> $v = \frac{ZRT}{P} = \frac{0.76(0.4615 \text{ kJ/kgK})(400 + 273)\text{K}}{15,000 \text{ kPa}} = \mathbf{0.015778 \text{ m}^3/\text{kg}}$ <p>where the value depends upon the value of <math>Z</math> calculated. For example:</p> $Z = 0.77 \Rightarrow v = Zv_{ideal} = 0.77(0.020706) = \mathbf{0.015944 \text{ m}^3/\text{kg}}$ $Z = 0.75 \Rightarrow v = Zv_{ideal} = 0.75(0.020706) = \mathbf{0.015530 \text{ m}^3/\text{kg}}$ $Z = 0.74 \Rightarrow v = Zv_{ideal} = 0.74(0.020706) = \mathbf{0.015322 \text{ m}^3/\text{kg}}$ <p>NOTE: Accept if they decide to compute <math>v_R</math>, and then use <math>v = v_R RT_{cr}/P_{cr}</math>, so long as the answer is similar to <math>v = 0.015778 \text{ m}^3/\text{kg}</math></p>	<p>1</p> <p>2</p> <p>1</p>
Q1(b)(iv)	<p>Comments similar to below.</p> <ul style="list-style-type: none"> <li>Value obtained with the ideal gas solution overestimates the specific volume compared to that from the steam tables.</li> <li>Value obtained with the compressibility chart is closer to the value from the steam tables than that from the ideal gas equation.</li> <li>The value obtained by treating superheated water vapour as an ideal gas is much less accurate than that obtained by using the compressibility chart.</li> <li>Using the ideal gas equation leads to an error of around 32%, whereas the error from using the compressibility chart is only 0.68%.</li> </ul> <p>NOTE: Percentage error not required to get full marks. Give 1 mark if previous errors mean that their conclusion is incorrect.</p>	2
Q1(c)(i)	<p>A piston-cylinder device is filled with 100kg of R-134a and has an initial volume of <math>12.322 \text{ m}^3</math></p> <p>The initial specific volume is:</p> $v_1 = \frac{V_1}{m} = \frac{12.322 \text{ m}^3}{100 \text{ kg}} = 0.12322 \text{ m}^3/\text{kg}$ <p>From Table A-12 at <math>P = 200 \text{ kPa}</math> we have <math>v_1 &gt; v_g</math> and thus the initial state is superheated.</p> <p>The final specific volume is:</p> $v_2 = \frac{v_1}{2} = \frac{0.12322 \text{ m}^3}{2} = 0.06161 \text{ m}^3/\text{kg}$ <p>This is a constant pressure process, so the final state is fixed by <math>v_2</math> and <math>P_1</math>.</p> <p>From Table A-12 at <math>P = 200 \text{ kPa}</math> we have <math>v_f &lt; v_2 &lt; v_g</math> and thus we have a saturated mixture. The temperature must be:</p> $T_{\text{sat@ } 200 \text{ kPa}} = \mathbf{-10.09^\circ \text{C}}$ <p>The internal energy in the initial state is read from Table A-13 at <math>P = 200 \text{ kPa}</math>:</p> $u_1 = 263.08 \text{ kJ/kg}$ <p>To determine the final internal energy, we need the dryness fraction:</p> $x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.06161 - 0.0007532}{0.099951 - 0.0007532} = 0.6135$ <p>The final internal energy is then:</p> $u_2 = u_f + x_2 u_{fg}$ $u_2 = 38.26 + 0.6135(186.25) = 152.52 \text{ kJ/kg}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p>

	$\Delta U = m(u_2 - u_1) = 100(152.52 - 263.08) = -110.6 \text{ kJ}$	
Q1(c)(ii)	 <p>NOTE: An unfortunate, but frequent, mistake is that students draw the line on the question paper. This attracts 0 marks.</p>	1
Q2(a)	<p>The diagram should be as below:</p>  <p>NOTE: Some students may draw the house below the heat pump. This is fine, so long as the directions are still correct.</p>	7
Q2(b)(i)	<p>COP of the heat pump:</p> $COP_{HP} = \frac{T_H}{T_H - T_L} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (2 + 273)/(22 + 273)} = 14.75$	2
Q2(b)(ii)	<p>If the temperature of the house is to be maintained, then heat must be supplied at the rate it is lost:</p> $\dot{Q}_{H,hp} = \dot{Q}_{house} = 20 \text{ kW}$ <p>Therefore, the power supplied to the heat pump</p> $COP_{HP} = \frac{\dot{Q}_{H,hp}}{\dot{W}_{net,hp}} \rightarrow \dot{W}_{net,hp} = \frac{\dot{Q}_{H,hp}}{COP_{HP}} = \frac{20 \text{ kW}}{14.75} = 1.356 \text{ kW}$	2 2
Q2(b)(iii)	<p>Half of the work output from the Carnot heat engine drives the heat pump. Therefore:</p> $\dot{W}_{net,hp} = 0.5\dot{W}_{net,he} \rightarrow \dot{W}_{net,he} = 2(\dot{W}_{net,hp}) = 2(1.356) = 2.712 \text{ kW}$ <p>Since this is a Carnot heat engine, the efficiency is:</p> $\eta_{he} = 1 - \frac{T_L}{T_H} = 1 - \frac{20 + 273 \text{ K}}{800 + 273 \text{ K}} = 0.727$ <p>The rate of heat supply can then be determined from the definition of thermal efficiency:</p> $\eta_{he} = \frac{\dot{W}_{net,he}}{\dot{Q}_{H,he}} \rightarrow \dot{Q}_{H,he} = \frac{\dot{W}_{net,he}}{\eta_{he}} = \frac{2.712 \text{ kW}}{0.727} = 3.73 \text{ kW}$	2 2 2

Q2(b)(iv)	<p>The heat rejected from the Carnot heat engine can be found by applying an energy balance:</p> $\dot{Q}_{H,he} = \dot{Q}_{L,he} + \dot{W}_{net,he} \rightarrow \dot{Q}_{L,he} = \dot{Q}_{H,he} - \dot{W}_{net,he} = 3.73 - 2.712 = \mathbf{1.01 \text{ kW}}$	2
Q2(c)	<p>Comments such as:</p> <ul style="list-style-type: none"> <li>No, because such an engine violates the Kelvin-Planck statement of the second law of thermodynamics.</li> <li>It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.</li> <li>A heat engine must reject heat in order to not violate the 2<sup>nd</sup> law of thermodynamics.</li> <li>Since we have a Carnot heat engine, we are already working at the maximum efficiency allowed by the 2<sup>nd</sup> law. Thus, we cannot improve the efficiency any further.</li> <li>All heat engines must reject heat, otherwise the system would be a perpetual motion machine.</li> </ul> <p>Deduct 1 mark if "Kelvin-Planck statement" is not mentioned, or if there is no reference to the 2<sup>nd</sup> law.</p>	4
Q3(a)	<p>The diagram should look similar to:</p>  <p>Does not need to be numerically precise, but:</p> <ul style="list-style-type: none"> <li>state points should be within the correct region</li> <li>numbers for state points can start at any point along the cycle but must be in the correct order</li> <li>isobars should be horizontal within the saturated mixture region.</li> </ul>	5
Q3(b)	<p>Steam is the working fluid for the simple ideal Rankine cycle. It enters the turbine at 15 MPa, 500°C and is cooled in the condenser at a pressure of 100 kPa until wet saturated.</p>	
Q3(b)(i)	<p>Fluid enters the pump as a wet saturated liquid at <math>P_1 = 100 \text{ kPa}</math>. From Table A-5:</p> $h_1 = h_{f@100\text{kPa}} = \mathbf{417.51 \text{ kJ/kg}}$ $v_1 = v_{f@100\text{kPa}} = 0.001043 \text{ m}^3/\text{kg}$ <p>The pump does work on the fluid at constant volume:</p> $w_{p,in} = v_1 \int_1^2 dP = v_1(P_2 - P_1) = 0.001043(15,000 - 100) = 15.54 \text{ kJ/kg}$ <p>Applying the steady flow energy equation across the pump:</p> $w_{p,in} = h_2 - h_1 \rightarrow h_2 = w_{p,in} + h_1 = 15.54 + 417.51 = \mathbf{433.05 \text{ kJ/kg}}$ <p>At state 3, <math>P_3 = 15 \text{ MPa}</math>, <math>T_3 = 500^\circ\text{C} \rightarrow</math> steam is superheated as per Table A-6:</p> $\left. \begin{array}{l} P_3 = 15 \text{ MPa} \\ T_3 = 500^\circ\text{C} \end{array} \right\} \begin{array}{l} h_3 = \mathbf{3310.8 \text{ kJ/kg}} \\ s_3 = 6.3480 \text{ kJ/kg} \cdot \text{K} \end{array}$	<p>2</p> <p>2</p> <p>2</p>

	At state 4, $P_4 = 100 \text{ kPa}$ and since the turbine is reversible adiabatic, this is modelled as isentropic in an ideal cycle, so $s_3 = s_4$ . From Table A-5: $s_{f@100\text{kPa}} = 1.3028 \text{ kJ/kg} \cdot \text{K}, \quad s_{g@100\text{kPa}} = 7.3589 \text{ kJ/kg} \cdot \text{K}$  Since $s_f < s_4 < s_g$ , we have a saturated mixture. $x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.3480 - 1.3028}{6.0562} = 0.833$ $h_4 = h_f + x_4 h_{fg} = 417.51 + 0.833(2257.5) = \mathbf{2298 \text{ kJ/kg}}$	2
Q3(b)(ii)	The heat transferred to working fluid in the boiler: $q_{in} = h_3 - h_2$ $q_{in} = 3310.8 - 433.05 = 2877.75 \text{ kJ/kg}$  The heat transferred from the working fluid in the condenser: $q_{out} = h_4 - h_1 = 2298 - 417.51 = 1880.49 \text{ kJ/kg}$ Or, we can compute the net work of the cycle: $w_{net} = w_T - w_P = (h_3 - h_4) - (h_2 - h_1)$ $w_{net} = (3310.8 - 2298.0) - (433.05 - 417.51) = 997.26 \text{ kJ/kg}$ The thermal efficiency of the cycle is then: $\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1880.49}{2877.75} = \mathbf{0.347 = 34.7\%}$ Or $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{997.26}{2877.75} = \mathbf{0.347 = 34.7\%}$	<div>1</div> <div> </div> <div>1</div> <div> </div> <div>2</div>
Q3(c)  Q3(c)(i)	The turbine is now irreversible, and the exit dryness fraction is given as 90%.  Our new state 4 is defined by $P_4 = 100 \text{ kPa}$ and $x_4 = 0.9$ . Therefore: $h_4 = h_f + x_4 h_{fg} = 417.51 + 0.9(2257.5) = 2449.26 \text{ kJ/kg}$  The isentropic efficiency of the turbine is defined as: $\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$ where the subscript $s$ refers to the isentropic case. Then: $\eta_T = \frac{3310.8 - 2449.26}{3310.8 - 2298} = 0.851 = \mathbf{85.1\%}$	<div> </div> <div>2</div> <div> </div> <div>2</div>
Q3(c)(ii)	$q_{out}$ becomes: $q_{out} = 2449.26 - 417.51 = 2031.76 \text{ kJ/kg}$ Or $w_{net}$ becomes: $w_{net} = (h_3 - h_4) - (h_2 - h_1) = (3310.8 - 2449.26) - (433.05 - 417.51)$ $w_{net} = 846.0 \text{ kJ/kg}$ The new thermal efficiency is: $\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2031.76}{2877.75} = \mathbf{0.294 = 29.4\%}$	<div> </div> <div>2</div>
Q3(c)(iii)	The figure drawn in Q3(a) should be updated to include the red dashed line below:	2

	State point 4s must lie to the right of state point 4 and along the same isobar.	
Q4(a)	<p>Using the molar masses provided in Table Q4:</p> $N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{0.5 \text{ kg}}{32 \text{ kg/kmol}} = \mathbf{0.01563 \text{ kmol}}$ $N_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{2 \text{ kg}}{44 \text{ kg/kmol}} = \mathbf{0.04545 \text{ kmol}}$ $N_{He} = \frac{m_{He}}{M_{He}} = \frac{0.25 \text{ kg}}{4 \text{ kg/kmol}} = \mathbf{0.0625 \text{ kmol}}$	<p>2</p> <p>2</p> <p>2</p>
Q4(b)	<p>The mole number of the mixture is:</p> $N_m = N_{O_2} + N_{CO_2} + N_{He} = 0.01563 + 0.04545 + 0.0625 = 0.12358 \text{ kmol}$ <p>The apparent molecular weight of the mixture is:</p> $M_m = \frac{m_m}{N_m} = \frac{2.75 \text{ kg}}{0.12358 \text{ kmol}} = 22.253 \text{ kg/kmol}$ <p>The gas constant of the mixture is then:</p> $R = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmolK}}{22.253 \text{ kg/kmol}} = \mathbf{0.374 \text{ kJ/kgK}}$ <p>where <math>R_u</math> is the universal gas constant, provided in the data sheet.</p>	<p>1</p> <p>2</p> <p>2</p>
Q4(c)	<p>The apparent constant-pressure specific heat of the mixture is given by:</p> $c_{p,m} = \sum_{i=1}^n mf_i c_{p,i}$ <p>The mass fractions <math>mf_i</math> are computed for each constituent as:</p> $mf_{O_2} = \frac{m_{O_2}}{m_m} = \frac{0.5 \text{ kg}}{2.75 \text{ kg}} = 0.182$ $mf_{CO_2} = \frac{m_{CO_2}}{m_m} = \frac{2 \text{ kg}}{2.75 \text{ kg}} = 0.727$ $mf_{He} = \frac{m_{He}}{m_m} = \frac{0.25 \text{ kg}}{2.75 \text{ kg}} = 0.091$ <p>Then:</p> $c_{p,m} = \sum_{i=1}^n mf_i c_{p,i} = mf_{O_2} c_{p,O_2} + mf_{CO_2} c_{p,CO_2} + mf_{He} c_{p,He}$ $c_{p,m} = 0.182(0.918) + 0.727(5.1926) + 0.091(0.846) = \mathbf{4.019 \text{ kJ/kgK}}$ <p>where the individual values of <math>c_{p,i}</math> are obtained from Table Q4.</p>	<p>1</p> <p>1</p> <p>1</p> <p>3</p>
Q4(d)	<p>We have a steady-flow, adiabatic, reversible expansion process. Therefore:</p> $\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$ <p>where <math>k</math> is the ratio of specific heats.</p> <p>To obtain the ratio of specific heats, we require <math>c_v</math>:</p> $c_v = c_p - R = 4.019 - 0.374 = 3.645 \text{ kJ/kgK}$ <p>Then:</p> $k = \frac{4.019}{3.645} = \mathbf{1.107}$ <p>Then:</p> $T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 600 \left( \frac{100}{1000} \right)^{\frac{1.107-1}{1.107}} = \mathbf{480.279 \text{ K}}$	<p>2</p> <p>3</p> <p>3</p>