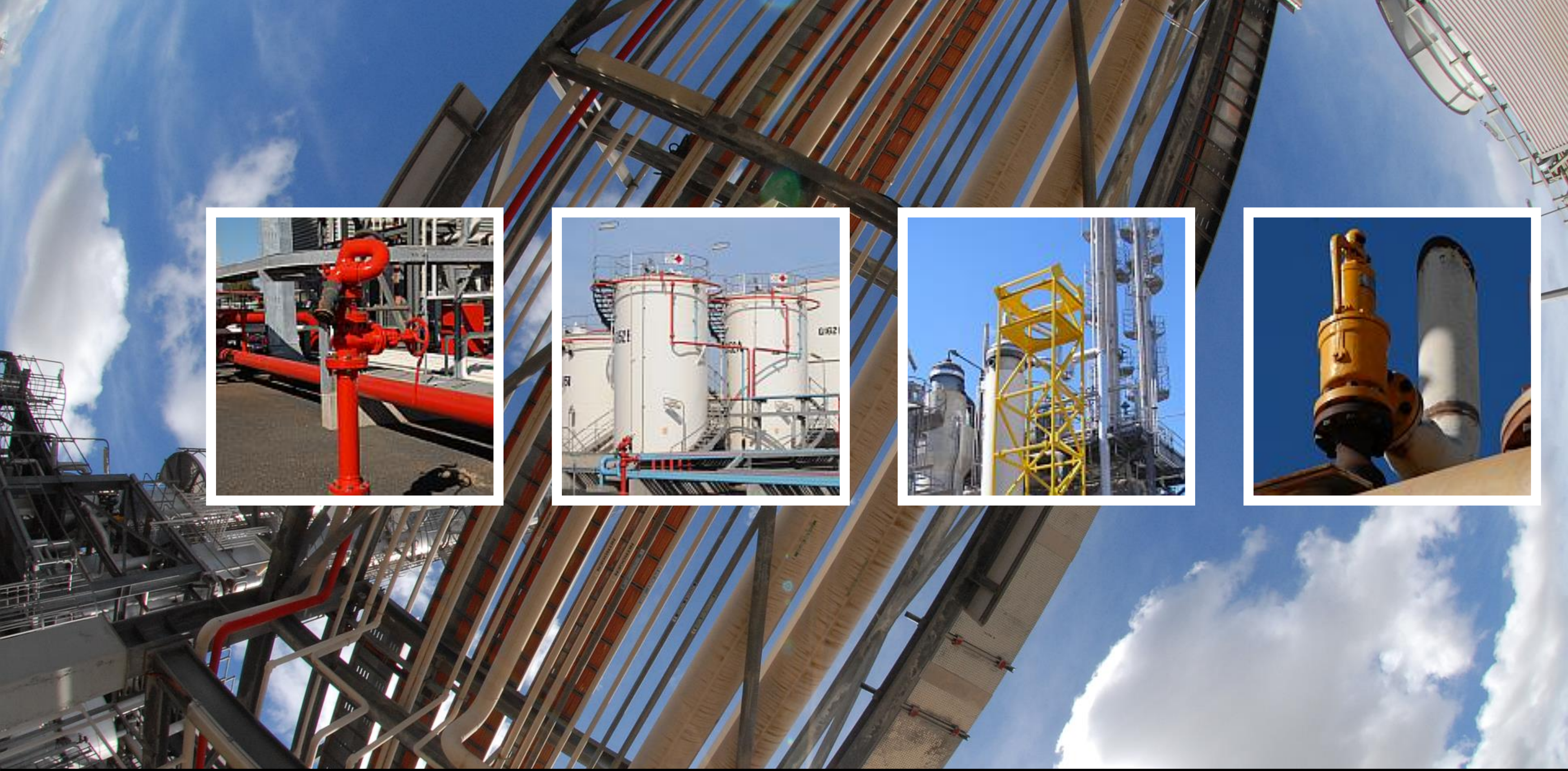


CHEN20010 Material and Energy Balances

Material Balances

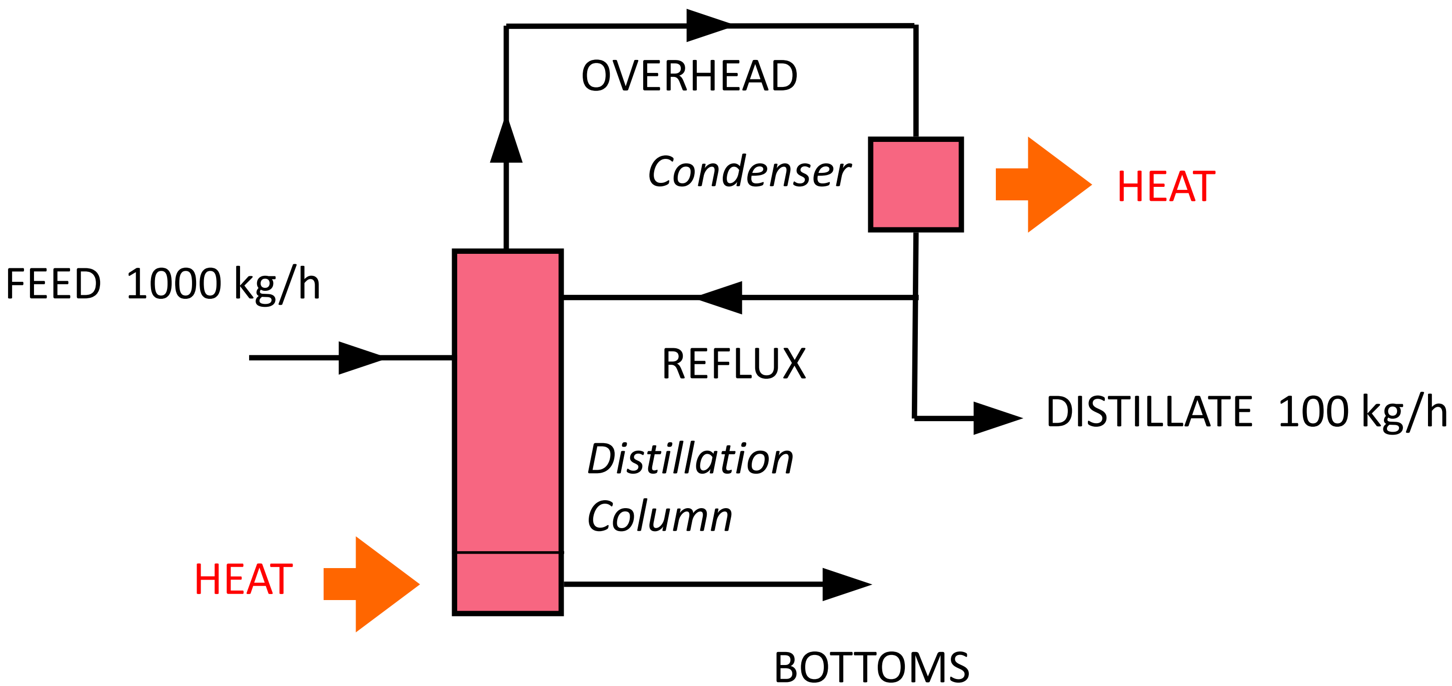
Material Balances – Module Learning Outcomes

A student is expected to be able to:

- Define the terms: system, surroundings, boundary, continuous, batch, steady-state
- Determine a suitable system boundary, and illustrate on a BFD
- Perform basic material balances (without chemical reactions):
  - Total, component, and elemental balances
  - Around single and multiple units
  - Calculate flow rates and composition, as required
- Define and treat appropriately in a balance: tie component, mixers, splitters, separators, feeds, product streams, recycles, bypasses, makeup.

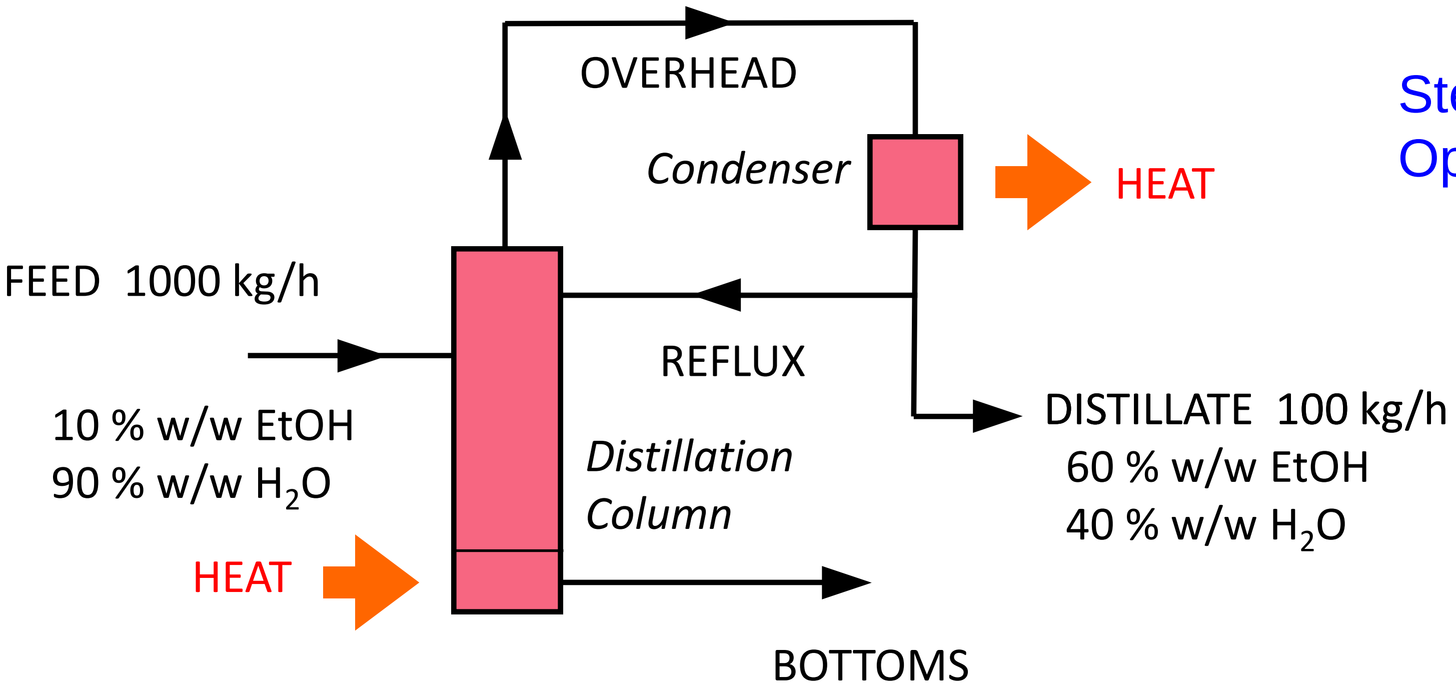
Material Balances

Distillation Problem - Direct Solution



Material Balances

Distillation Problem - Direct Solution



Steady-State Operation

What are the flow rate and composition of the bottoms stream ?

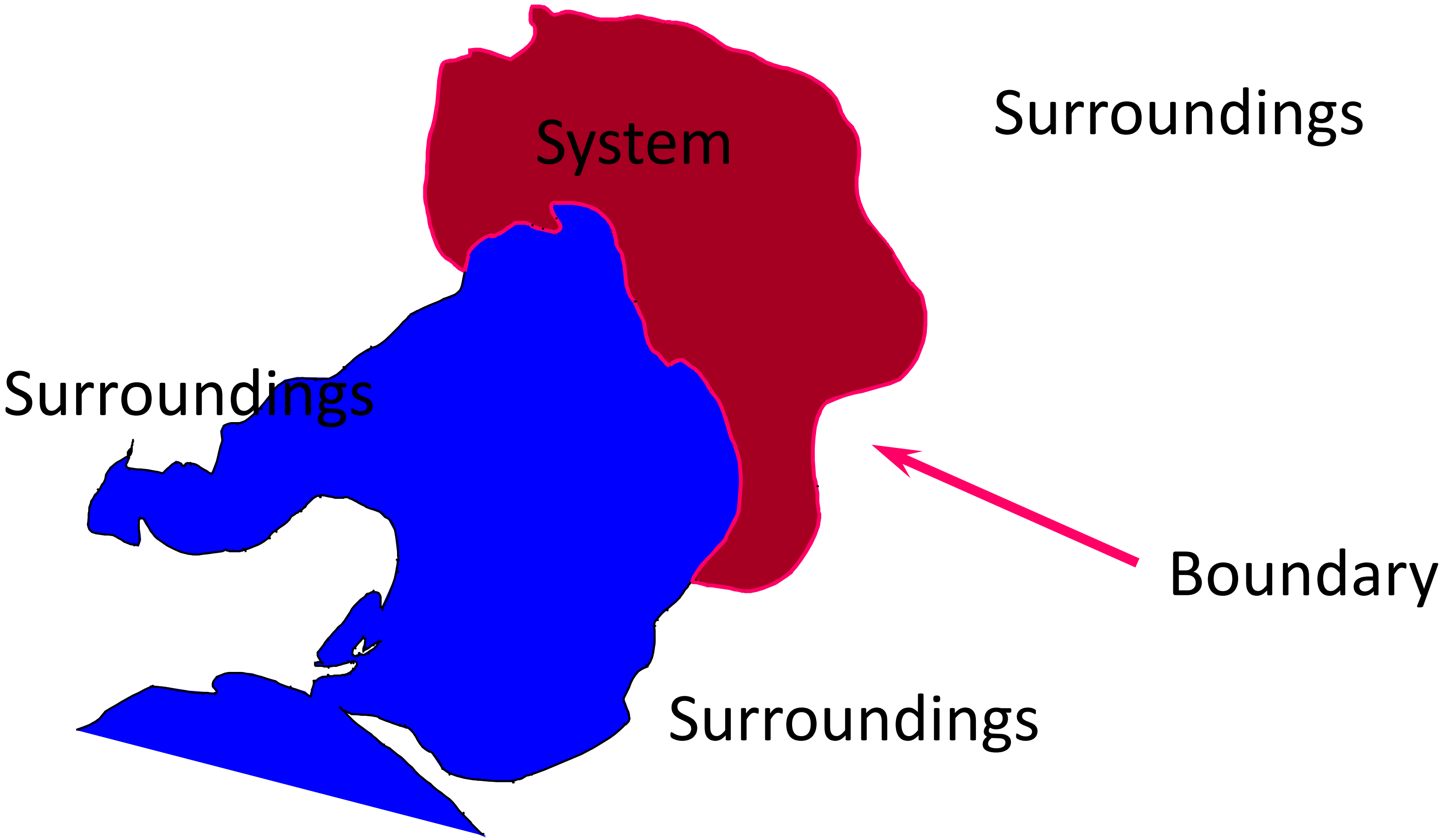
Consider the population of a city:

If we wish to determine the increase in population of Melbourne over a year what is the first thing which we must do ?


Essendon
City
Werribee
Oakleigh
Dandenong
Frankston
Mornington

Define what we mean by Melbourne

Consider the population of a city:



Define what we mean by Melbourne.

Consider the population of a city:

Over one year ....

$$\left(\text{Population Increase}\right) = \left(\text{People Entering}\right) - \left(\text{People Leavi}\right) + \left(\text{People Born}\right) - \left(\text{People Dyin}\right)$$

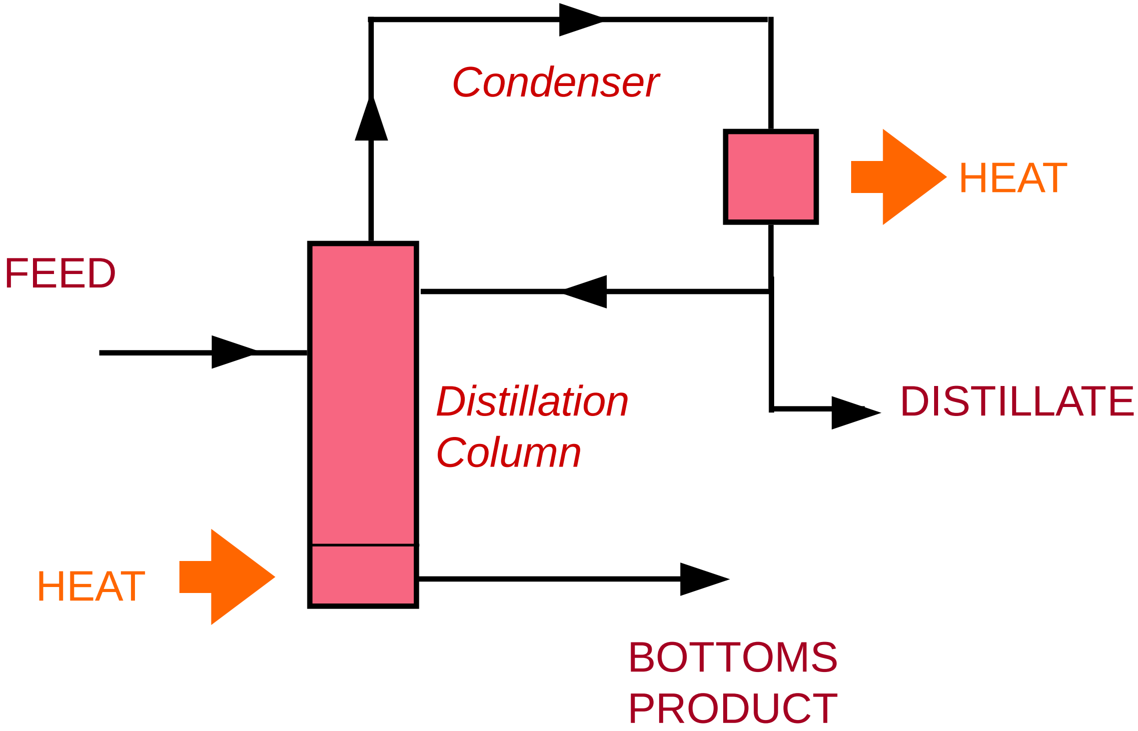
Consider the population of a city:

Over one year ....

$$\left(\text{Accumulation}\right) = \left(\text{Input}\right) - \left(\text{Output}\right) + \left(\text{Generation}\right) - \left(\text{Consumption}\right)$$

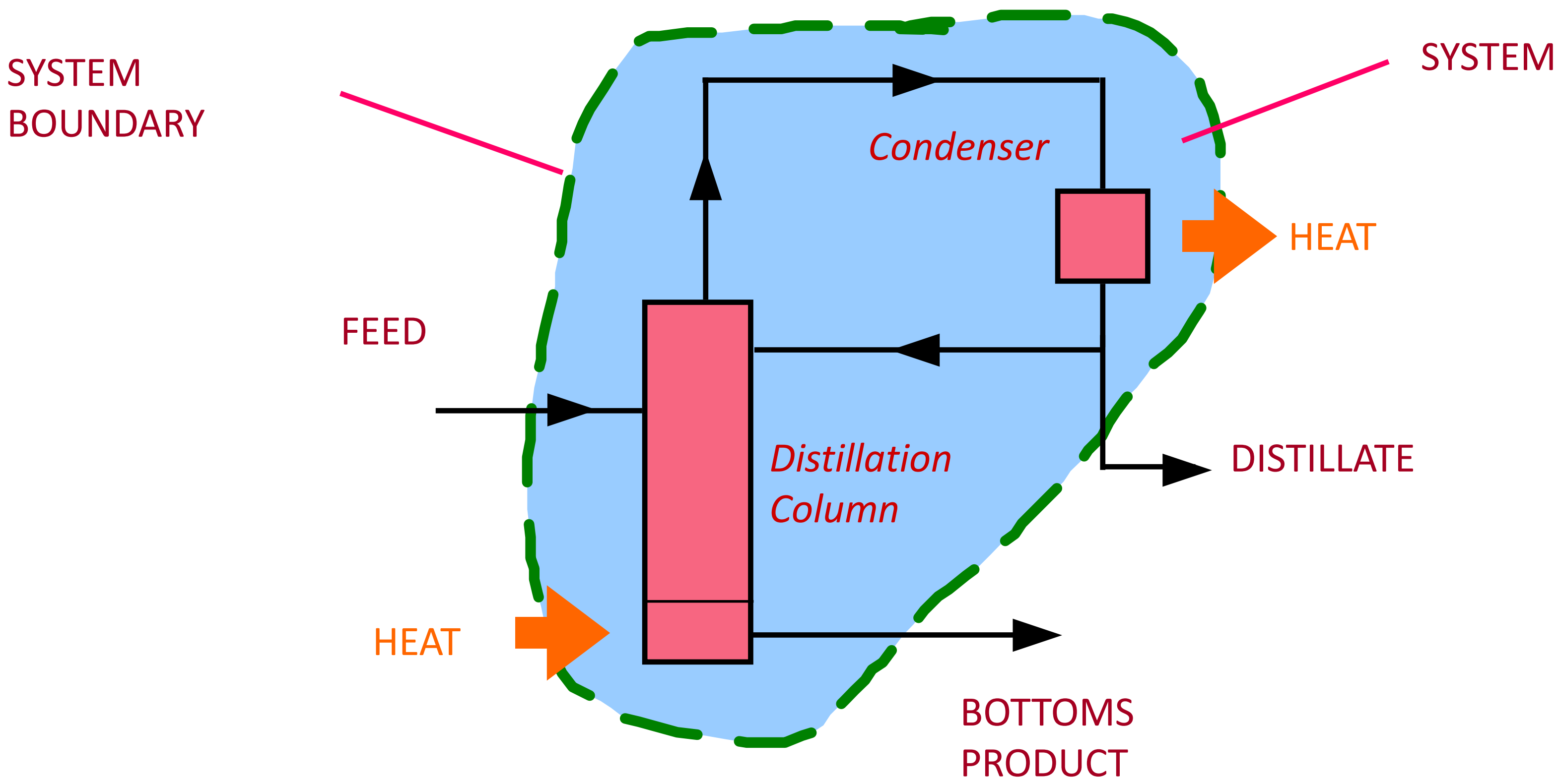
Material Balances

We perform MATERIAL BALANCES by first defining the SYSTEM.



Material Balances

SYSTEM : The volume of space under consideration, separated by **BOUNDARIES** from the **SURROUNDINGS**. Boundaries must be clearly defined.



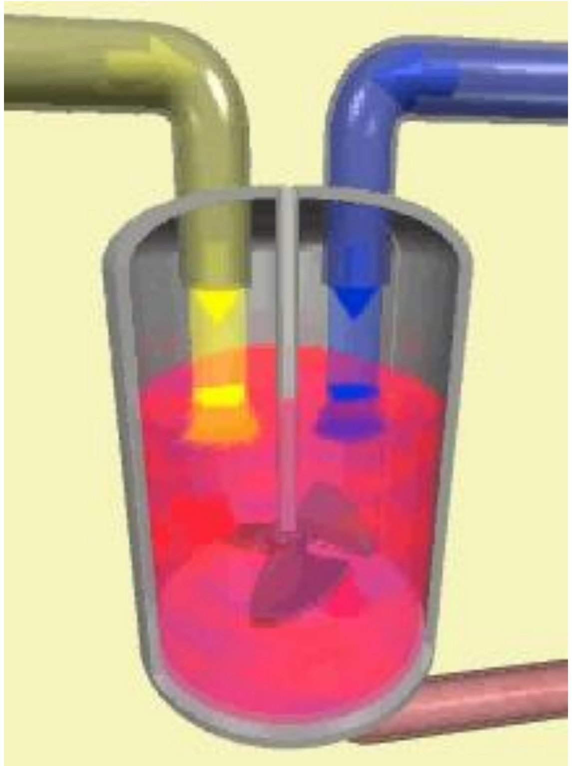
Material Balances

SYSTEM : The volume of space under consideration, separated by **BOUNDARIES** from the **SURROUNDINGS**. Boundaries must be clearly defined.

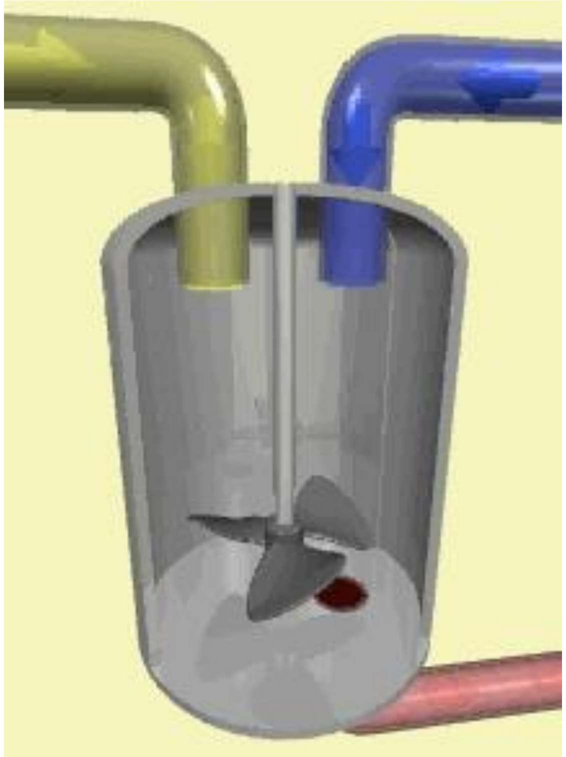
CONTINUOUS PROCESS (open system): Mass flows across the system boundary continuously. The distillation column is an example.

BATCH PROCESS (closed system): No mass flows across the system boundary. Baking a cake is an example.

STEADY-STATE OPERATION: Compositions, conditions and flow rates do not change with time.



Continuous Process



Batch Process

Material Balances

Basic Material Balance Equation:

This is a statement of the conservation of matter with regards to a system:

$$\begin{bmatrix} \text{ACCUMULATION} \\ \text{within system} \end{bmatrix} = \begin{bmatrix} \text{INPUT} \\ \text{across system} \\ \text{boundaries} \end{bmatrix} - \begin{bmatrix} \text{OUTPUT} \\ \text{across system} \\ \text{boundaries} \end{bmatrix}$$

$$+ \begin{bmatrix} \text{GENERATION} \\ \text{within system} \end{bmatrix} - \begin{bmatrix} \text{CONSUMPTION} \\ \text{within system} \end{bmatrix}$$

Material Balances

Balance Items - these must be consistent

MASS - Use consistent units (e.g. all items must be in the same units, i.e., all as kg, lb_m etc).

MOLES - Use with care when chemical reactions are involved. Use consistent units.

RATES - Items may be amounts (e.g. kg, moles) or rates of amounts (e.g. kg/hr, moles/min).

BASIS OF CALCULATION - Must be clearly stated

Material Balances

Total Balance

Balance items are total amounts or flow rates of streams, irrespective of the composition.

Mass units : generation, consumption terms must be zero (except for a nuclear reactor).

Mole units : generation, consumption terms may not be zero if a chemical reaction is involved.

Material Balances

Component Balance

Balance items are in terms of a particular, specified chemical compound e.g., H_2SO_4, ash, CH_4

No chemical reactions occurring :

generation and consumptions terms are both zero (for mass and moles).

Chemical reactions occurring :

generation and consumption terms must be evaluated based upon the reaction stoichiometry.

Material Balances

Component Balance

Balance items are in terms of a particular, specified chemical compound e.g., H_2SO_4, ash, CH_4

Consider H_2SO_4 :

$$\left[\begin{array}{c}\text{Accumulation}\\ \text{of } H_2SO_4\\ \text{within system}\end{array}\right] = \left[\begin{array}{c}\text{Input}\\ \text{of } H_2SO_4\\ \text{into system}\end{array}\right] - \left[\begin{array}{c}\text{Output}\\ \text{of } H_2SO_4\\ \text{out of system}\end{array}\right]$$

$$+ \left[\begin{array}{c}\text{Generation}\\ \text{of } H_2SO_4\\ \text{within system}\end{array}\right] - \left[\begin{array}{c}\text{Consumption}\\ \text{of } H_2SO_4\\ \text{within system}\end{array}\right]$$

Material Balances

Elemental Balance

Balance items are in terms of the elements (e.g. O, C, H, S and N) irrespective of the state they are in (e.g. whether the C is present as C, CO, CO_2 or CH_4).

Since we can neither create, destroy nor transmute elements outside a nuclear reactor then the generation and consumption terms are zero.

Consider C :

$$\begin{bmatrix} \text{Accumulation} \\ \text{of C} \\ \text{within system} \end{bmatrix} = \begin{bmatrix} \text{Input} \\ \text{of C} \\ \text{into system} \end{bmatrix} - \begin{bmatrix} \text{Output} \\ \text{of C} \\ \text{out of system} \end{bmatrix}$$

Material Balances

Accumulation Term

Accumulation accounts for the change in the amount of material with the system boundaries.

For steady state processes, accumulation term = 0

So material balances become :

TOTAL :

	INPUT = OUTPUT	mass
	INPUT = OUTPUT - GENERATION + CONSUMPTION	moles

COMPONENT :

INPUT = OUTPUT - GENERATION + CONSUMPTION

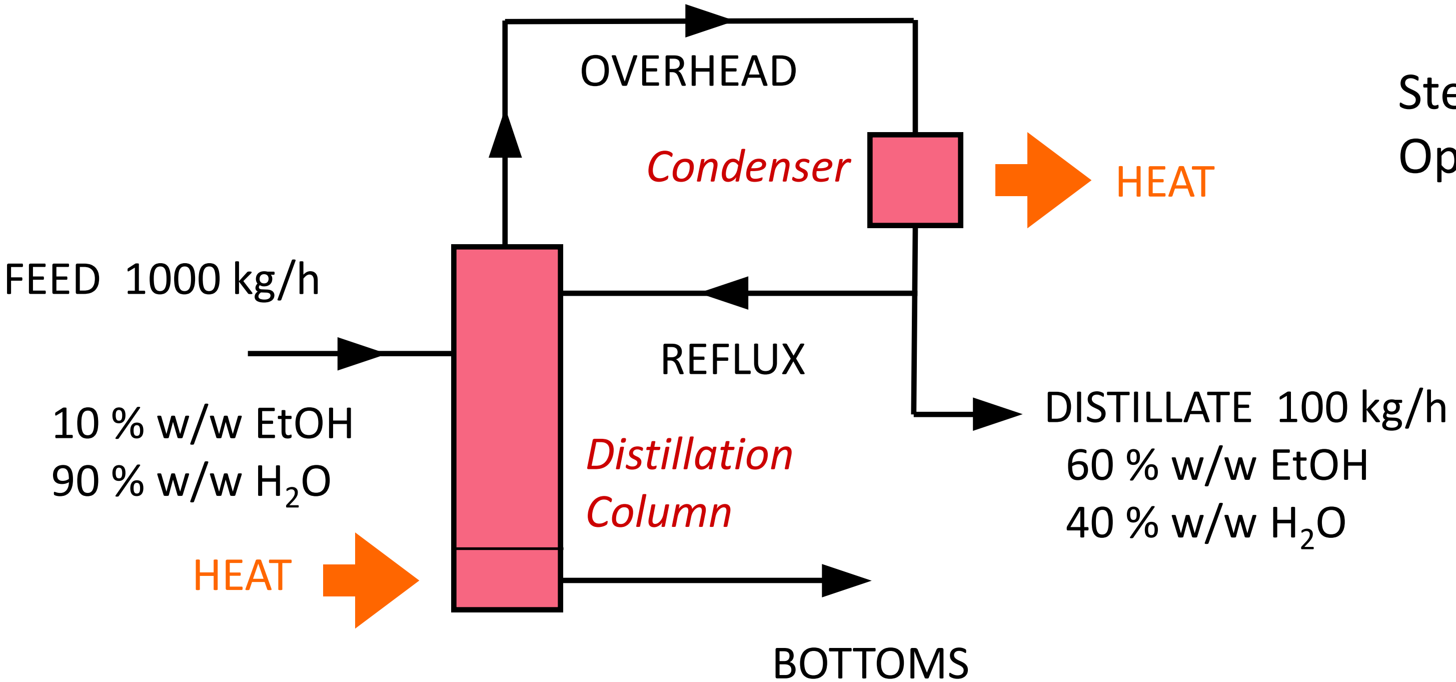
ELEMENTAL :

INPUT = OUTPUT

Accumulation must only be evaluated for unsteady state processes.

Material Balances

Distillation problem : Direction Solution

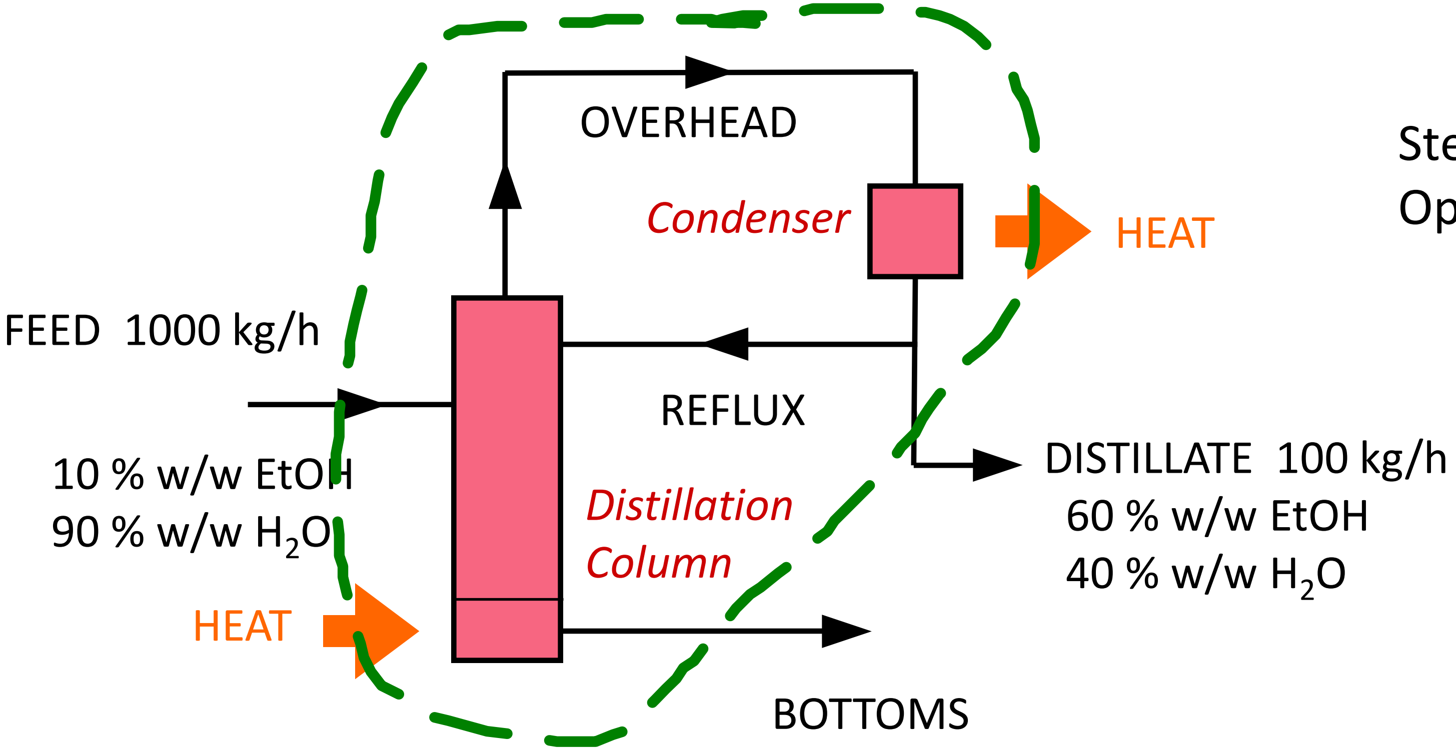


Steady-State Operation

What are the flow rate and composition of the bottoms stream ?

Material Balances

Distillation problem : Direction Solution

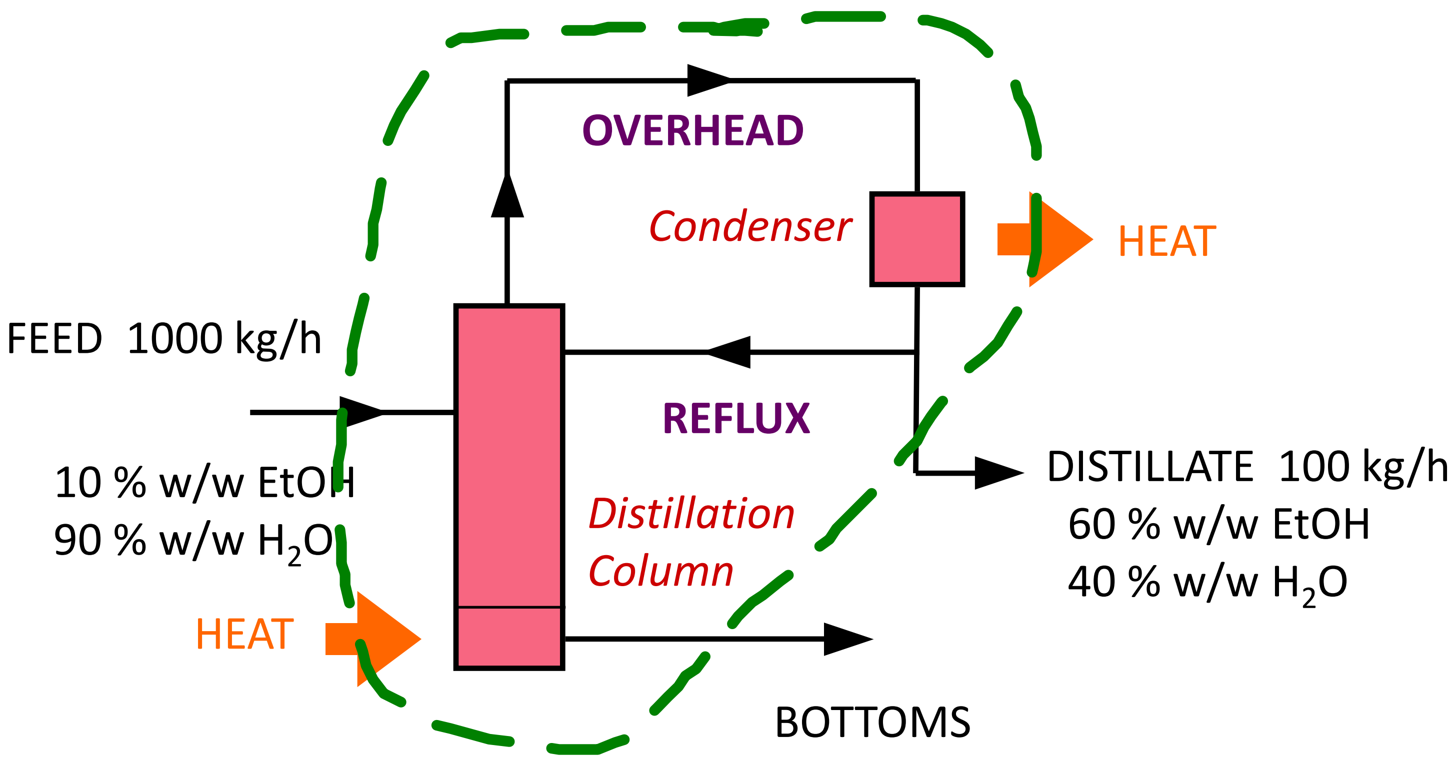


Steady-State Operation

The SYSTEM BOUNDARY is chosen so that it crosses the streams with the unknown flow rates and compositions, and the streams with the known parameters.

Material Balances

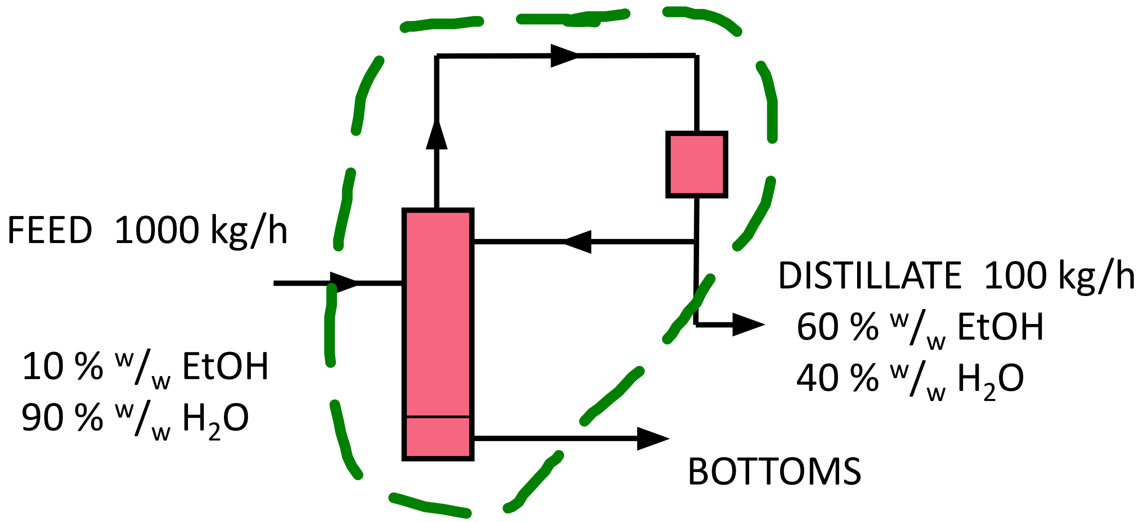
Distillation problem : Direction Solution



The OVERHEAD and REFLUX streams are inside the system and DO NOT CROSS the boundary, so do not appear in the balance.

Material Balances

Distillation problem : Direction Solution



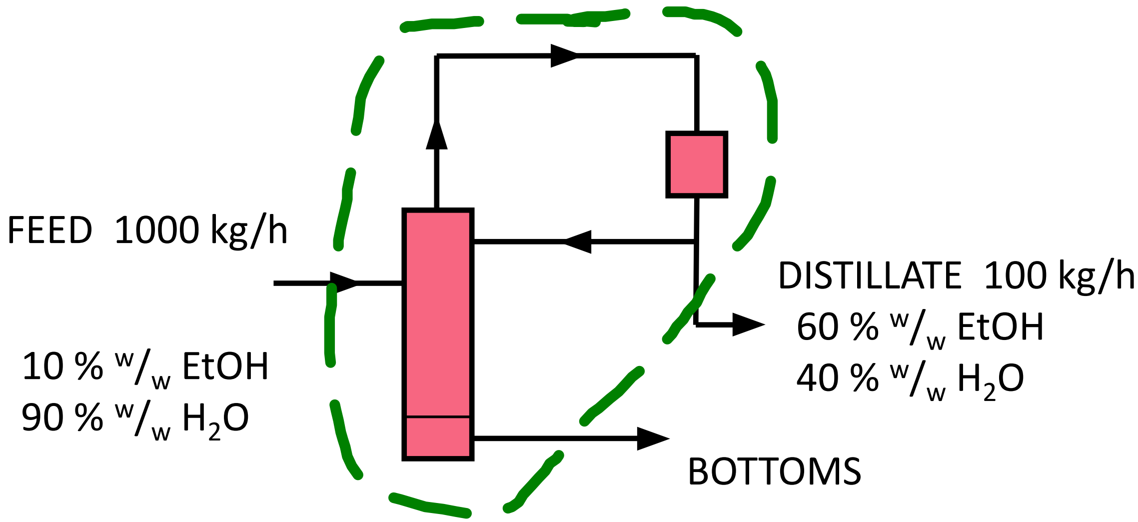
Basis of Calculation:
1000 kg of feed
i.e. 1 hour

Steady State Operation, so Accumulation = 0
No Chemical Reactions, so Generation = 0
and Consumption = 0

$\therefore$ Input = Output

Material Balances

Distillation problem : Direction Solution

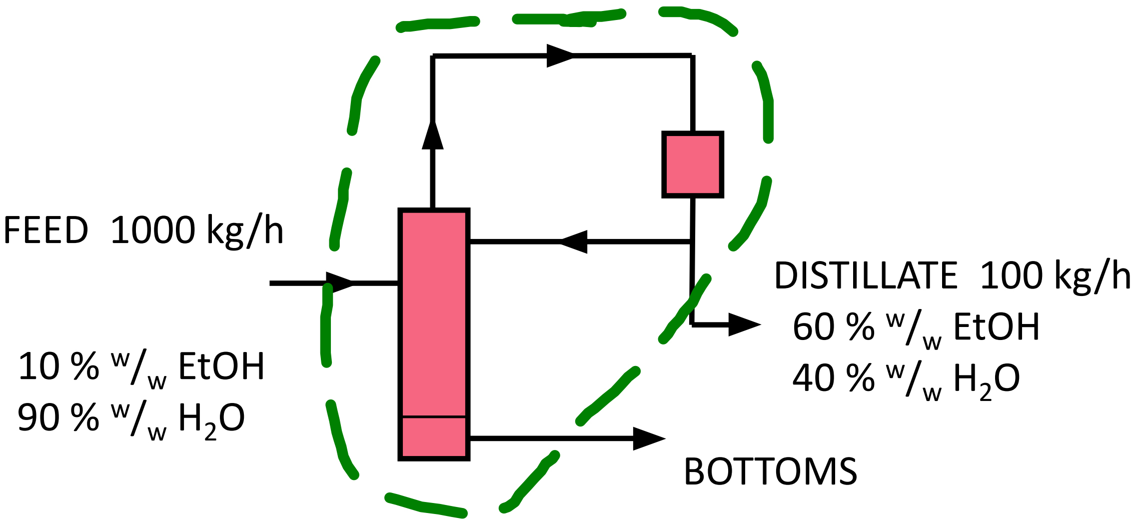


Basis of Calculation:
1000 kg of feed
i.e. 1 hour

	Feed (1000 kg)		Distillate (100 kg)		Bottoms
	% $^w/_w$	kg	% $^w/_w$	kg	kg
EtOH	10	100	60	60	
H_2O	90	900	40	40	
		1000		100	

Material Balances

Distillation problem : Direction Solution



Basis of Calculation:
1000 kg of feed
i.e. 1 hour

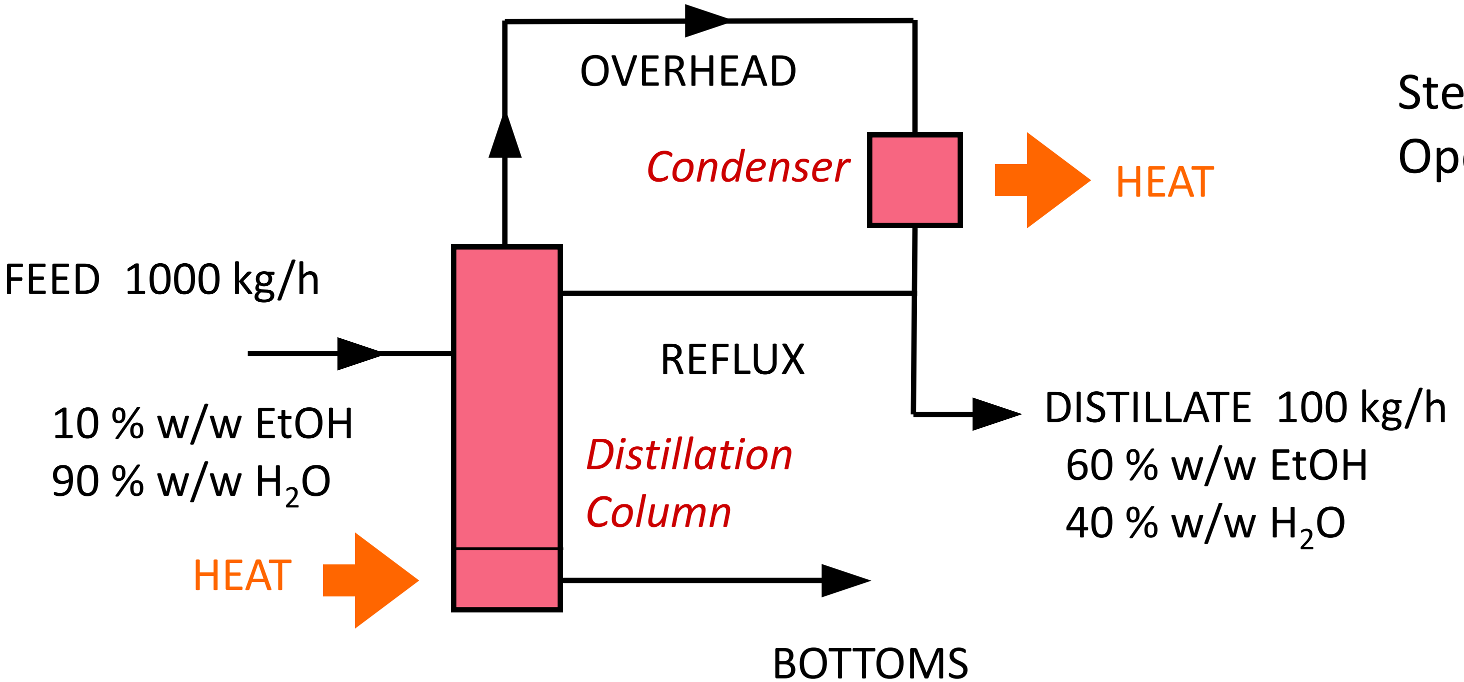
	Bottoms kg	
EtOH	40	(= 100 - 60)
H_2O	860	(= 900 - 40)
	900	

Composition of bottoms

=

Material Balances

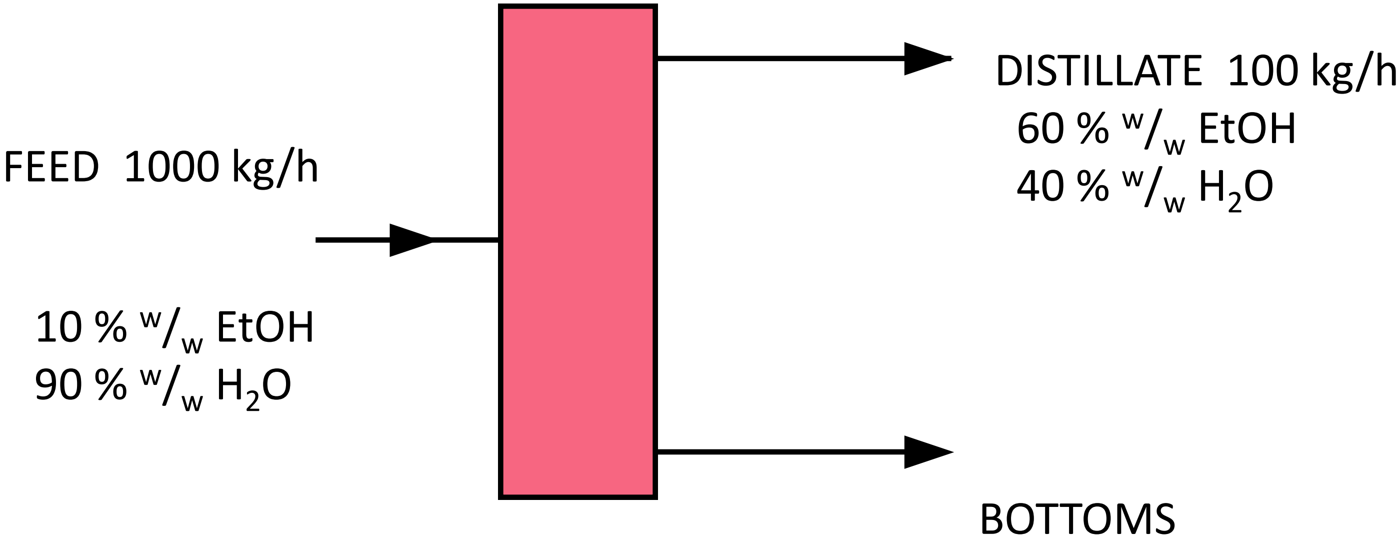
This diagram can be re-drawn as .....



Steady-State Operation

What are the flow rate and composition of the bottoms stream ?

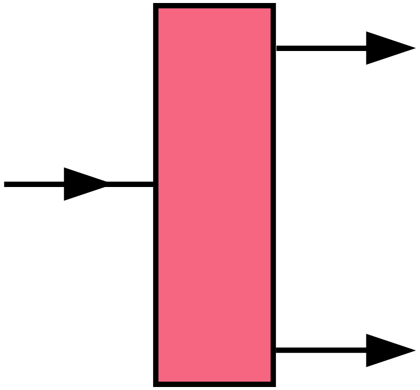
Material Balances



..... as this diagram.

Material Balances

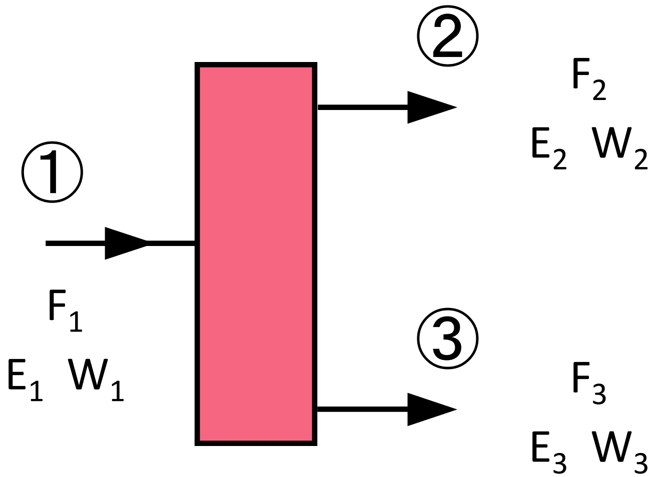
Distillation problem : Algebraic Solution



Note that the **block** at left is not the distillation column alone, but represents the complete system, including the column, reboiler and condenser, and the overhead and reflux streams.

Material Balances

Distillation problem : Algebraic Solution



Always use a simple and logical notation system.

Let F_i denote mass flow rate of stream i.

Let E_i denote mass fraction of EtOH in stream i.

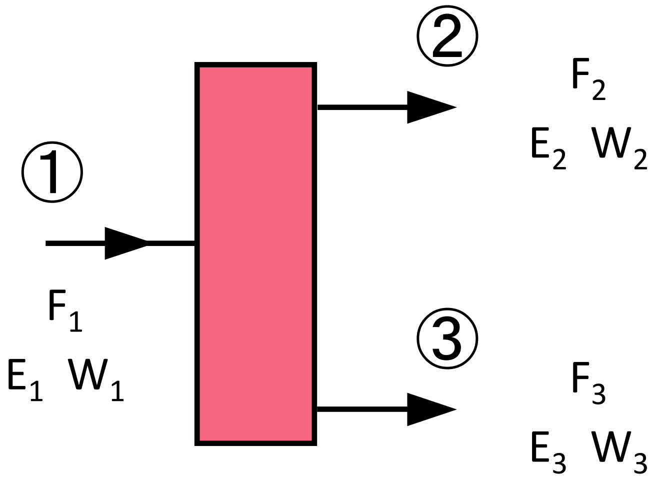
Let W_i denote mass fraction of H_2O in stream i.

$\therefore$ F_2 is total mass flow rate of stream 2

and, E_3 is mass fraction of EtOH in stream 3.

Material Balances

Distillation problem : Algebraic Solution



Basis of Calculation: 1 hour

Total balance :

$$F_1 = F_2 + F_3 \qquad (1)$$

Component balance for EtOH :

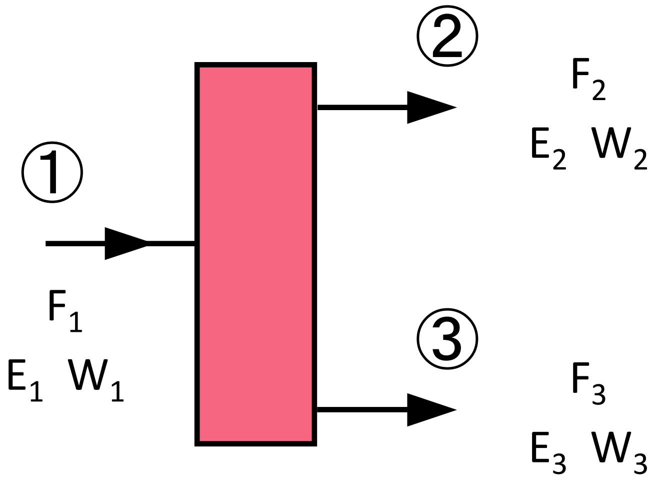
$$F_1 E_1 = F_2 E_2 + F_3 E_3 \qquad (2)$$

Component balance for H_2O :

$$F_1 W_1 = F_2 W_2 + F_3 W_3 \qquad (3)$$

Material Balances

Distillation problem : Algebraic Solution



$$F_1 = F_2 + F_3 \tag{1}$$

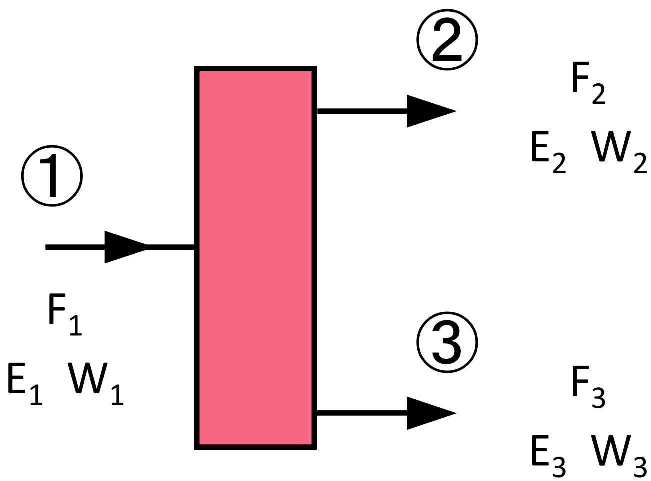
$$F_1 E_1 = F_2 E_2 + F_3 E_3 \tag{2}$$

$$F_1 W_1 = F_2 W_2 + F_3 W_3 \tag{3}$$

Note that since there are only 2 components, E and W, we may write $E_i + W_i = 1$ for all streams. Thus only 2 of the 3 equations are INDEPENDENT as any one equation may be derived directly from the other 2.

Material Balances

Distillation problem : Algebraic Solution



$$F_1 = F_2 + F_3 \quad (1)$$

$$F_1 E_1 = F_2 E_2 + F_3 E_3 \quad (2)$$

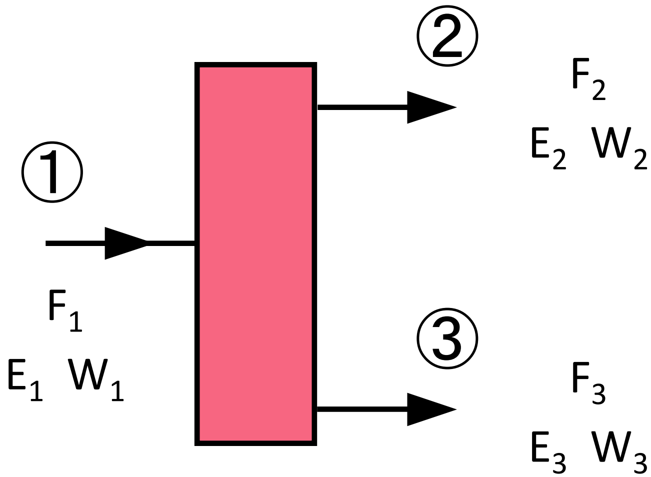
$$F_1 W_1 = F_2 W_2 + F_3 W_3 \quad (3)$$

Adding equations (2) and (3) yields:

But we know that $E_1 + W_1 = 1$, etc

Material Balances

Distillation problem : Algebraic Solution



$$F_1 = F_2 + F_3 \quad (1)$$

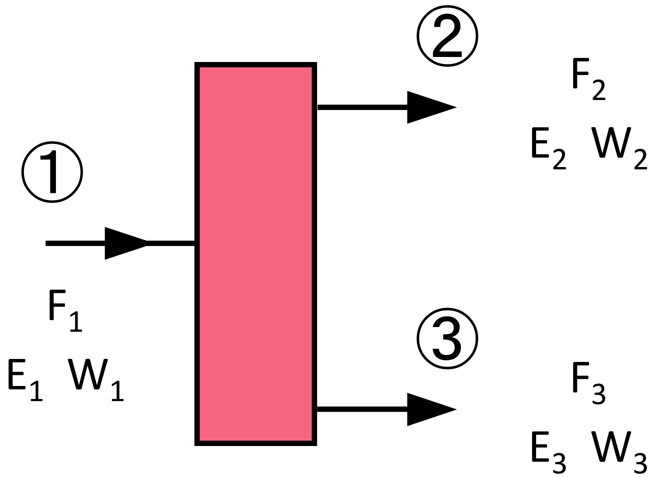
$$F_1 E_1 = F_2 E_2 + F_3 E_3 \quad (2)$$

$$F_1 W_1 = F_2 W_2 + F_3 W_3 \quad (3)$$

- Whenever we have 2 components and no chemical reactions we may write 2 independent material balances.
- Whenever we have N components and no chemical reactions we may write N independent material balances.

Material Balances

Distillation problem : Algebraic Solution



$$F_1 = F_2 + F_3 \qquad (1)$$

$$F_1 E_1 = F_2 E_2 + F_3 E_3 \qquad (2)$$

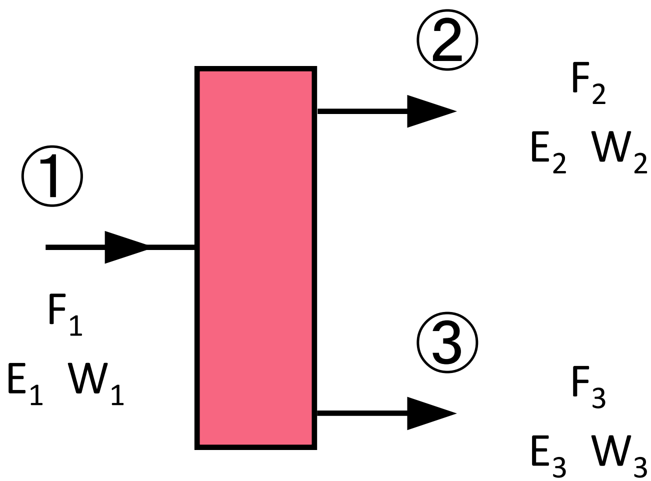
$$F_1 W_1 = F_2 W_2 + F_3 W_3 \qquad (3)$$

We know the values of these variables.

These variables are unknown.

Material Balances

Distillation problem : Algebraic Solution



$$F_1 = F_2 + F_3 \qquad (1)$$

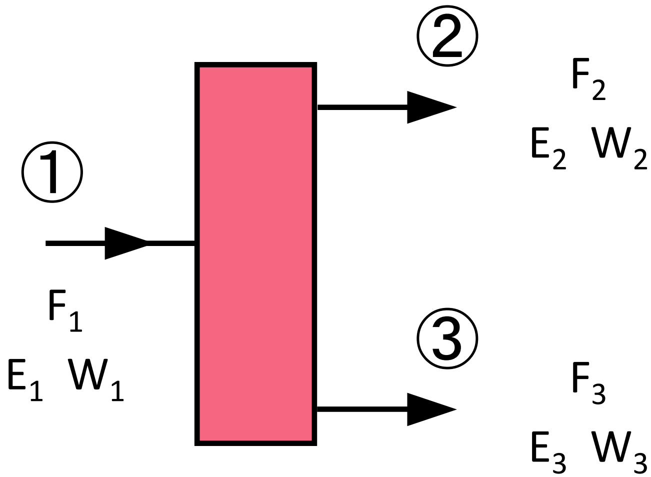
$$F_1 E_1 = F_2 E_2 + F_3 E_3 \qquad (2)$$

$$F_1 W_1 = F_2 W_2 + F_3 W_3 \qquad (3)$$

Since, no. of unknowns = no. of equations, we can solve these equations.

Material Balances

Distillation problem : Algebraic Solution

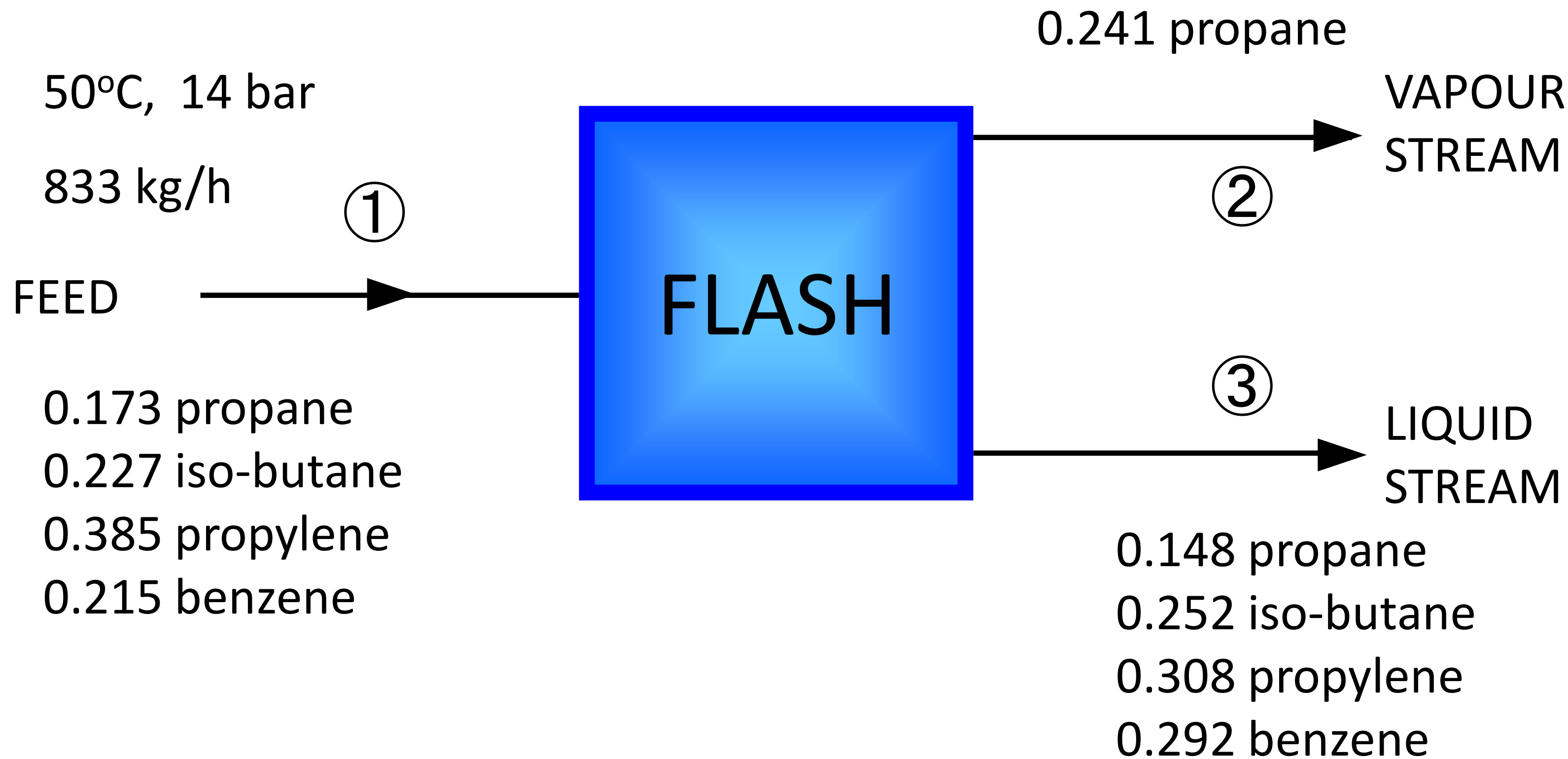


$$F_1 = F_2 + F_3 \quad (1)$$

$$F_1 E_1 = F_2 E_2 + F_3 E_3 \quad (2)$$

$$F_1 W_1 = F_2 W_2 + F_3 W_3 \quad (3)$$

Material Balances



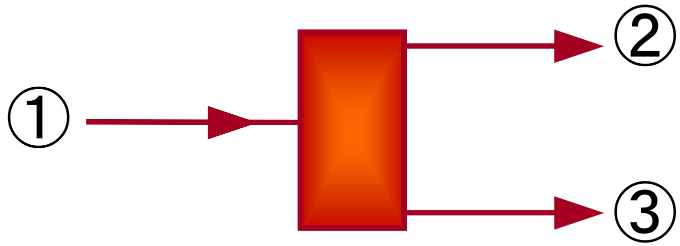
Calculate :

i) product stream flow rates

ii) vapour stream composition

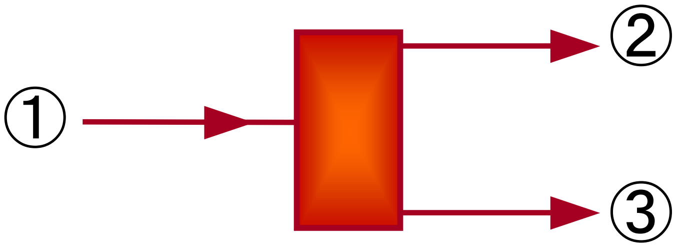
Material Balances

Denote propane as P
iso-butane as I
propylene as R
benzene as B



It is always very useful to choose a simple and obvious nomenclature system.

Material Balances



Denote propane as P
iso-butane as I
propylene as R
benzene as B

Let F_i be mass flow rate of stream i

Let P_i be mass fraction of P in stream i

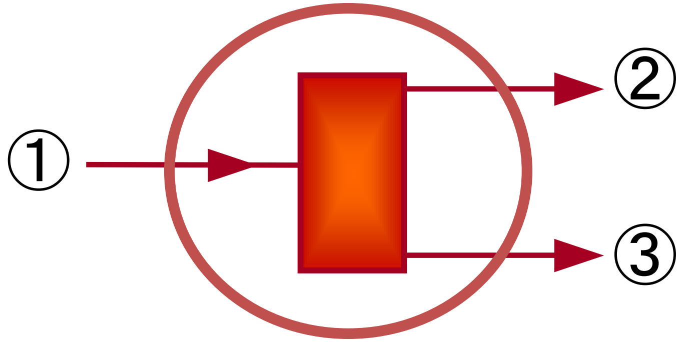
Basis of Calculation : 1 hour

Assume : steady state operation

no reactions are occurring

Material Balances

We begin by putting a system boundary around the entire process.



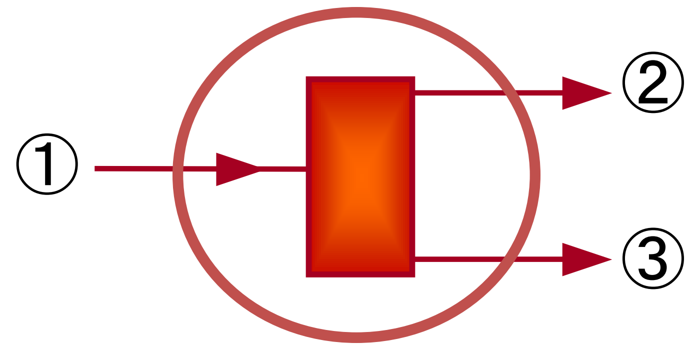
Total balance over the FLASH is :

$$F_1 = F_2 + F_3$$

Component balance for P over the FLASH is :

$$P_1F_1 = P_2F_2 + P_3F_3$$

Material Balances

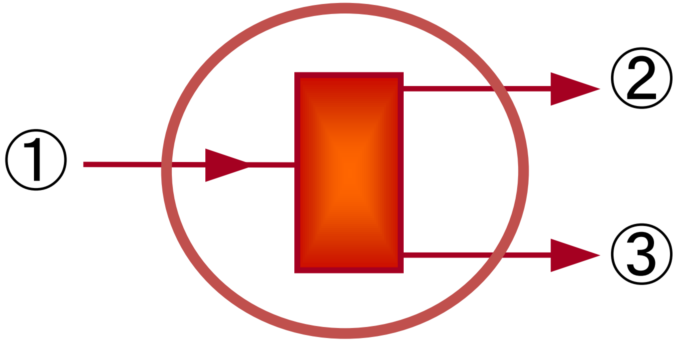


Substituting this expression for F_3 in equation (2) :

Simplifying :

So, we now know the flow rates of the two product streams. Now let's find the composition of the vapour stream.

Material Balances



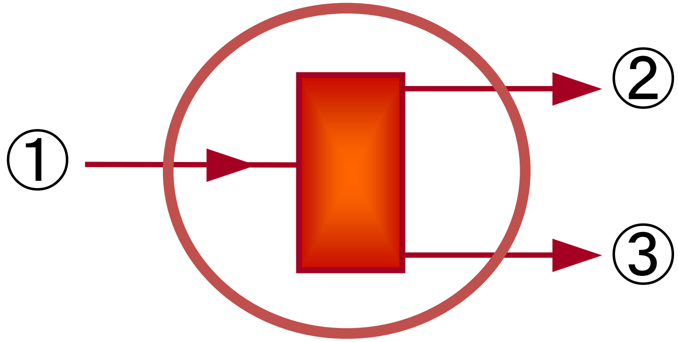
Component balance for I over the FLASH is :

$$I_1F_1 = I_2F_2 + I_3F_3$$

Component balance for R over the FLASH is :

$$R_1F_1 = R_2F_2 + R_3F_3$$

Material Balances



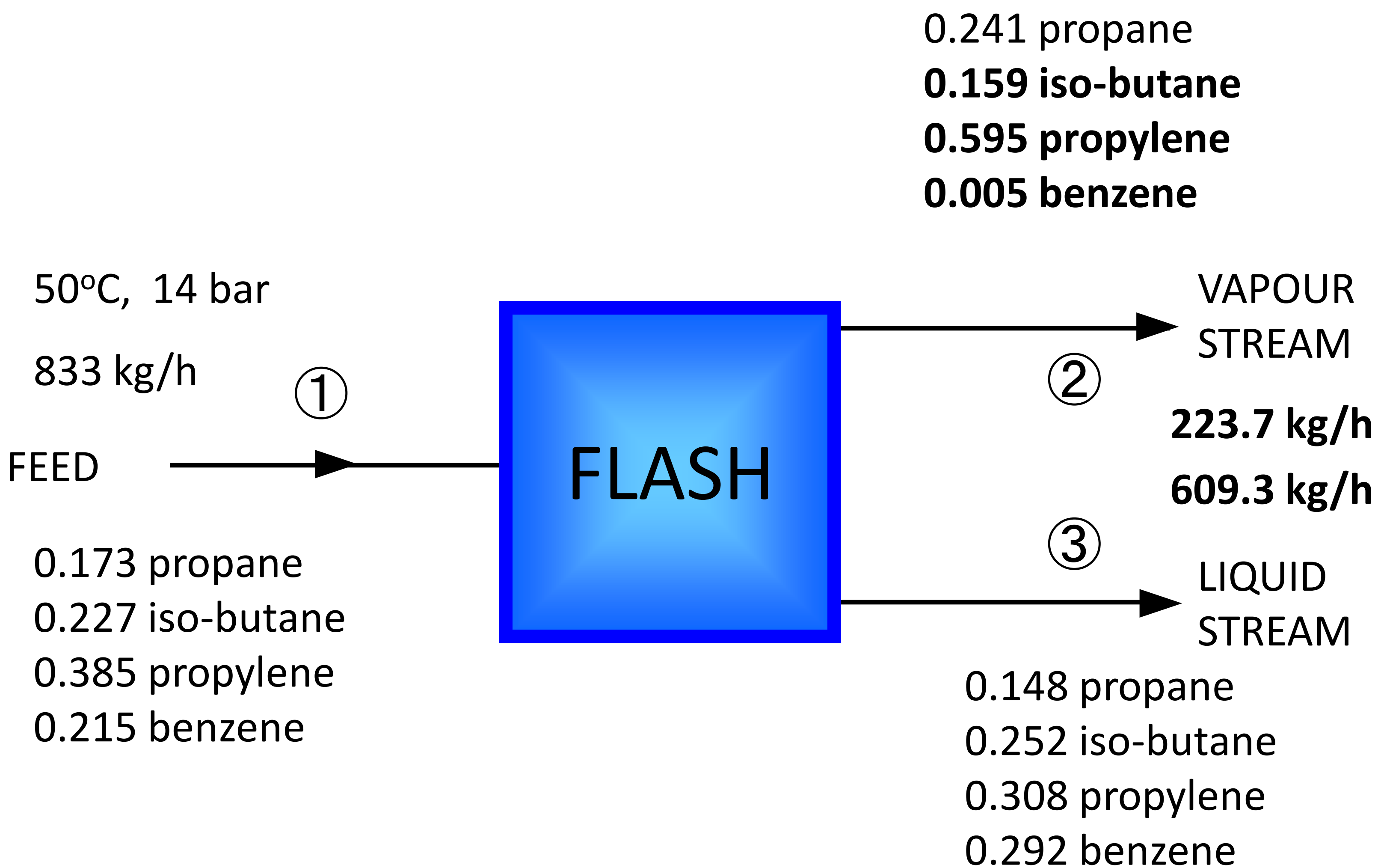
Component balance for B over the FLASH is :

$$B_1F_1 = \quad B_2F_2 \quad + \quad B_3F_3$$

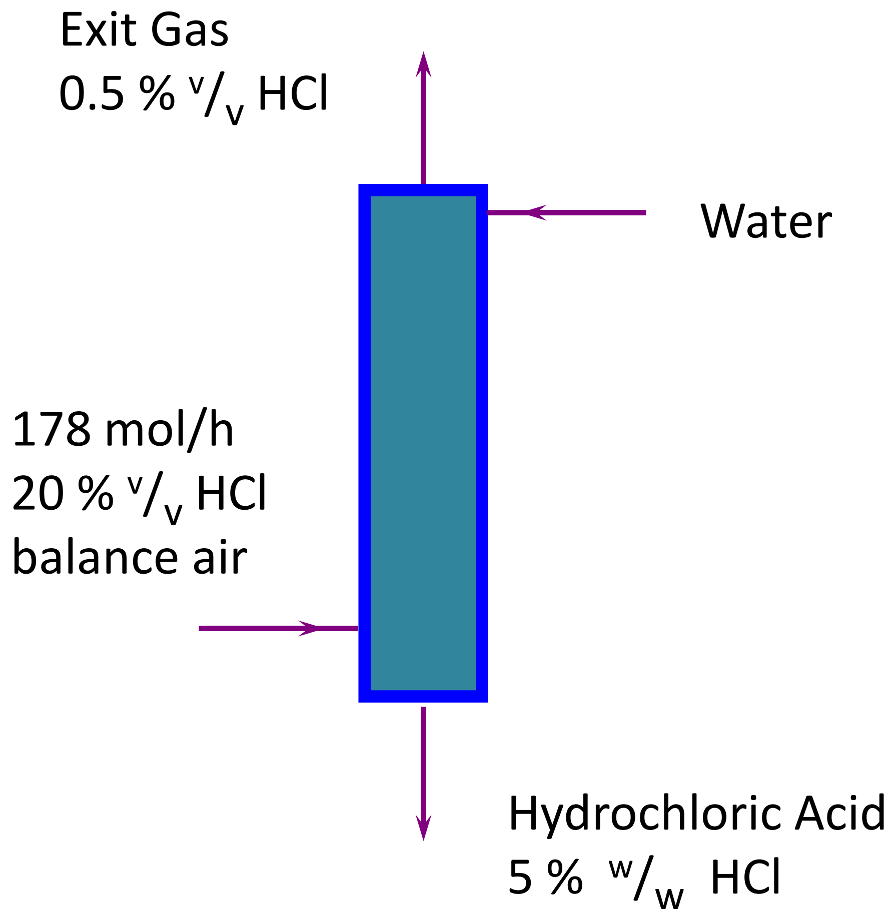
but, we know that $P_2 + I_2 + R_2 + B_2 = 1$

Material Balances

The problem has been solved.



Gas Absorption Problem – Direction Solution

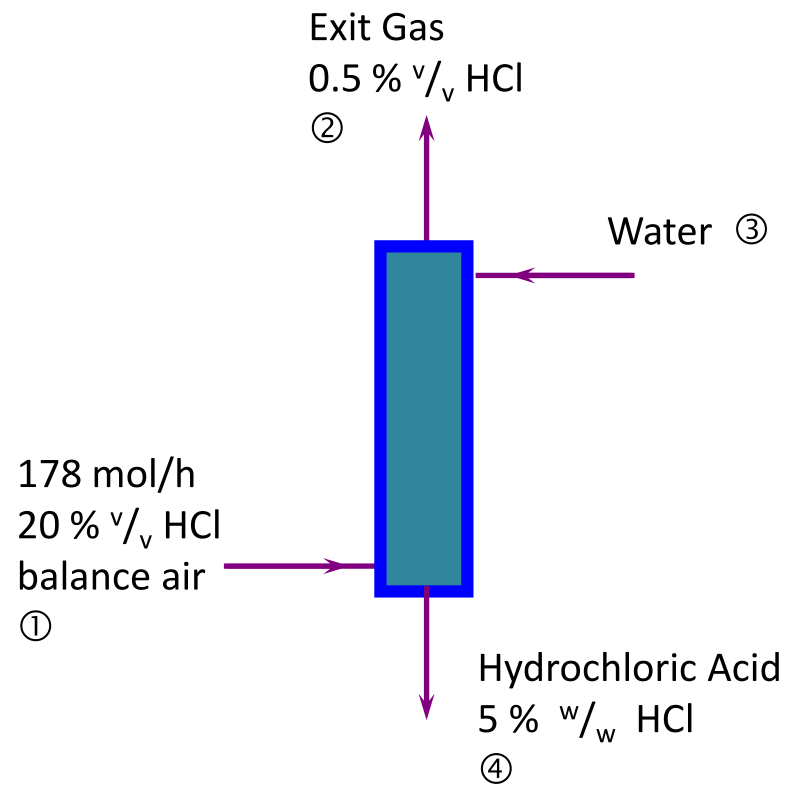


Assume :

- no evaporation of H_2O
- air is insoluble in aqueous phase
- no chemical reactions
- steady-state operation

What is the production rate of the acid ?

Gas Absorption Problem



Basis of Calculation : 1 hour

Input gas stream ①:

HCl =

Air =

Output gas stream ②:

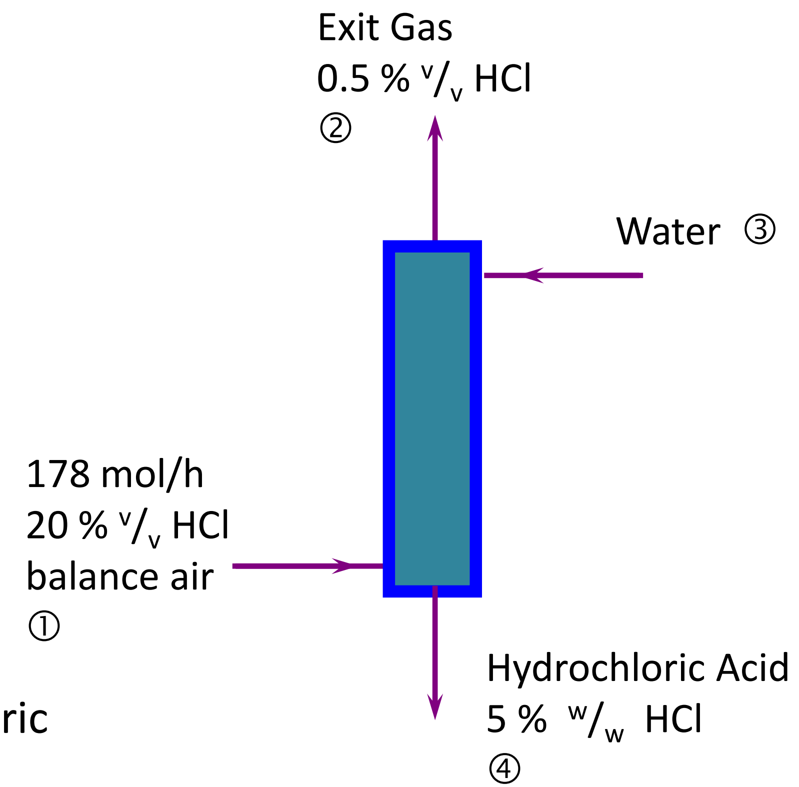
Air out =

This amount of air is 99.5 mol % of the total flow rate of the exit gas.

The remaining 0.5 mol % is the gaseous HCl.

∴ HCl out in the exit gas =

Gas Absorption Problem



HCl out in the exit gas = 0.715 mol

So how much of the HCl is absorbed in the water and leaves the tower in the hydrochloric acid stream ?

HCl absorbed by water

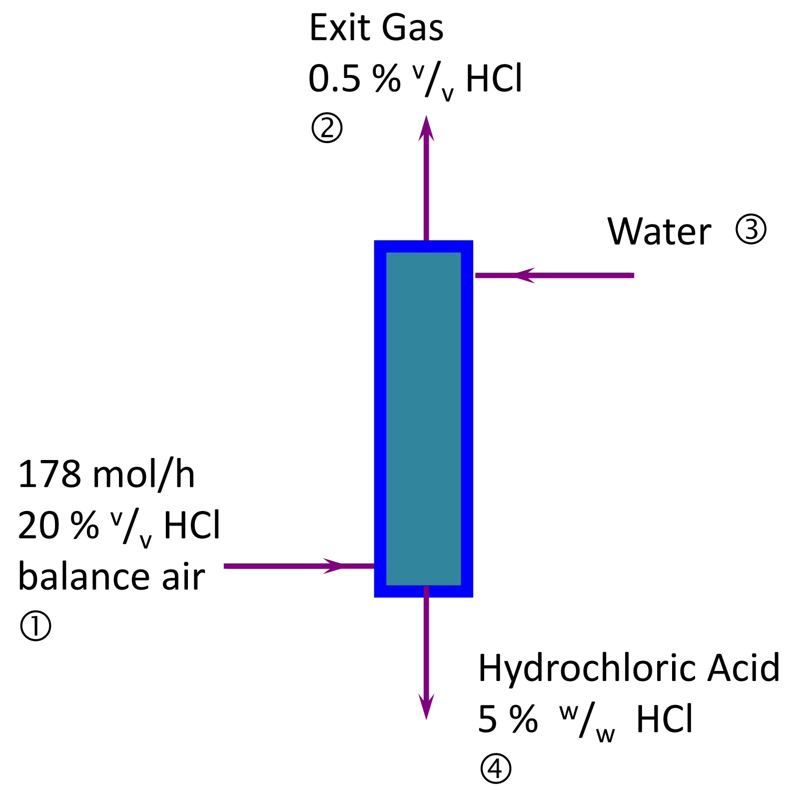
=

The HCl concentration in the acid stream is given on a weight basis, so we must convert to a mass.

MW (HCl) =

∴ HCl absorbed by the water =

Gas Absorption Problem



HCl absorbed in water =

This is 5 % of the total mass of the hydrochloric acid leaving the tower. Now we calculate the total mass of the acid stream produced.

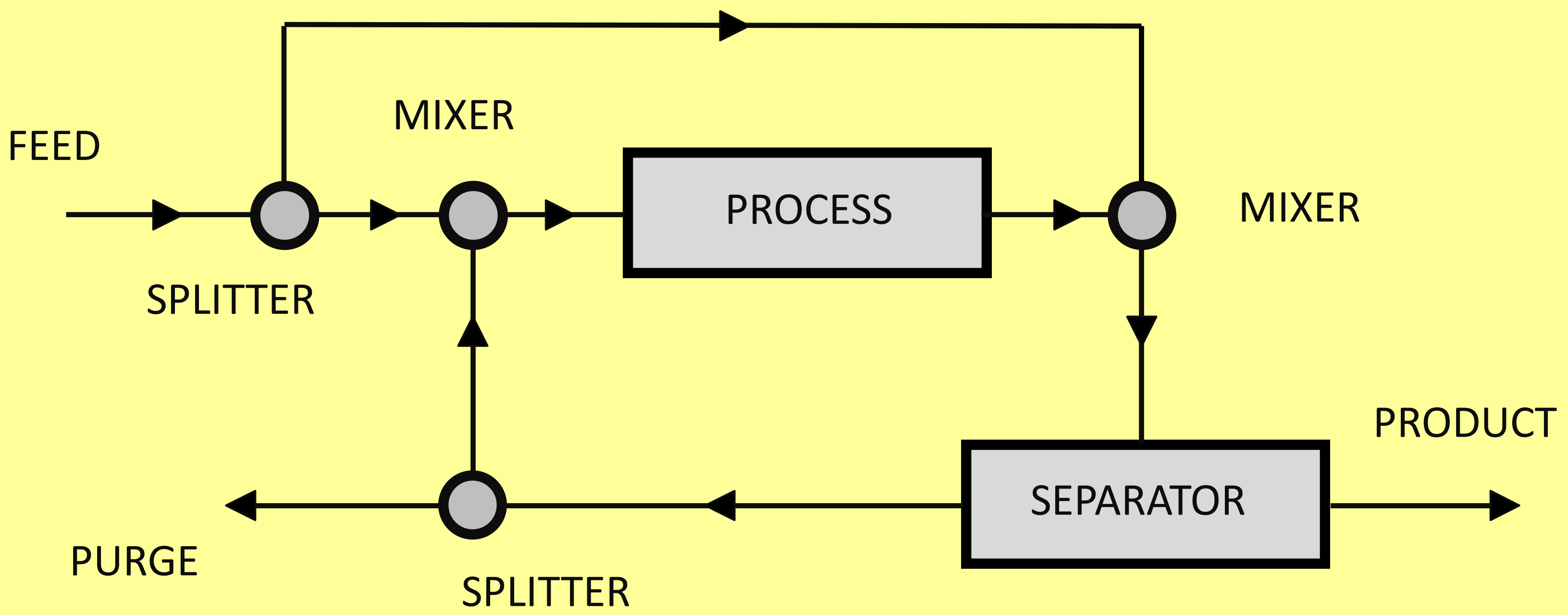
Acid production =

Tie Component or Tie Element

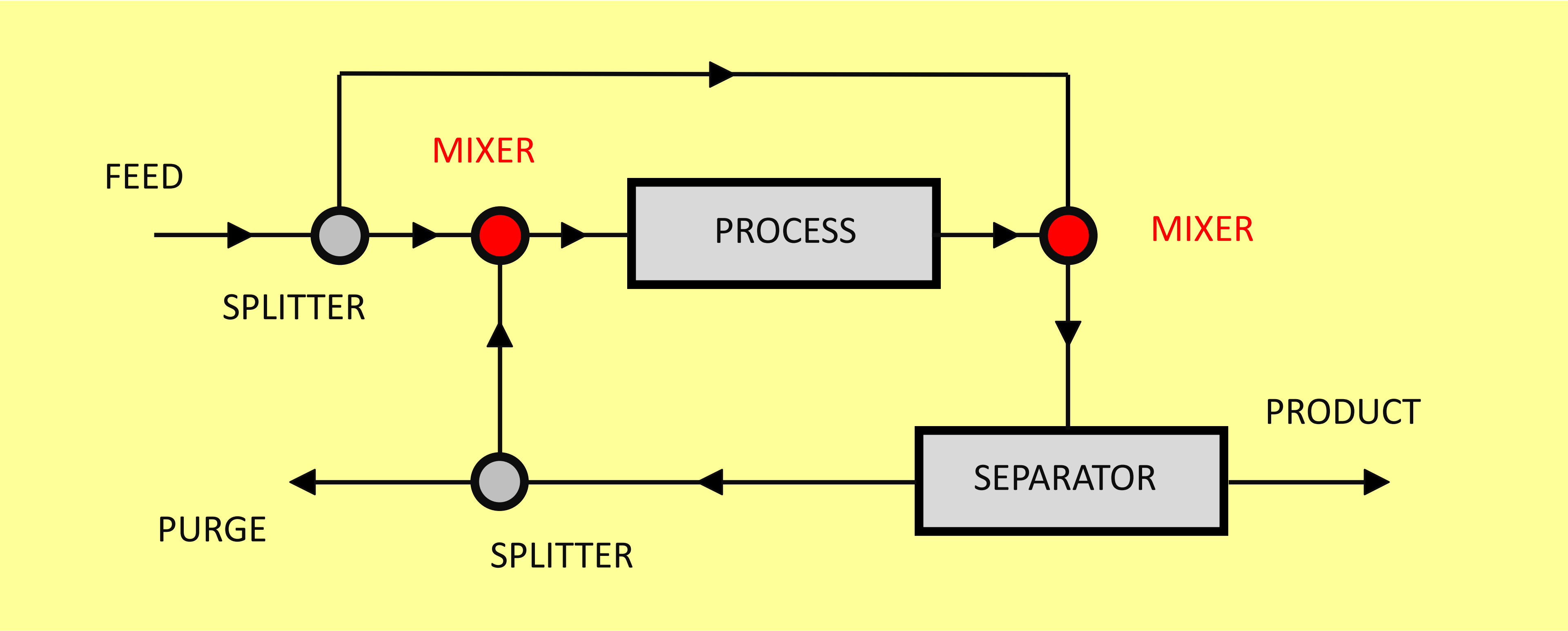
A tie component is a component that enters the system in one stream only, leaves in one stream only, and is not consumed or generated within the system.

Tie components allow simple material balances to be constructed quickly.

Multiple Unit Operations

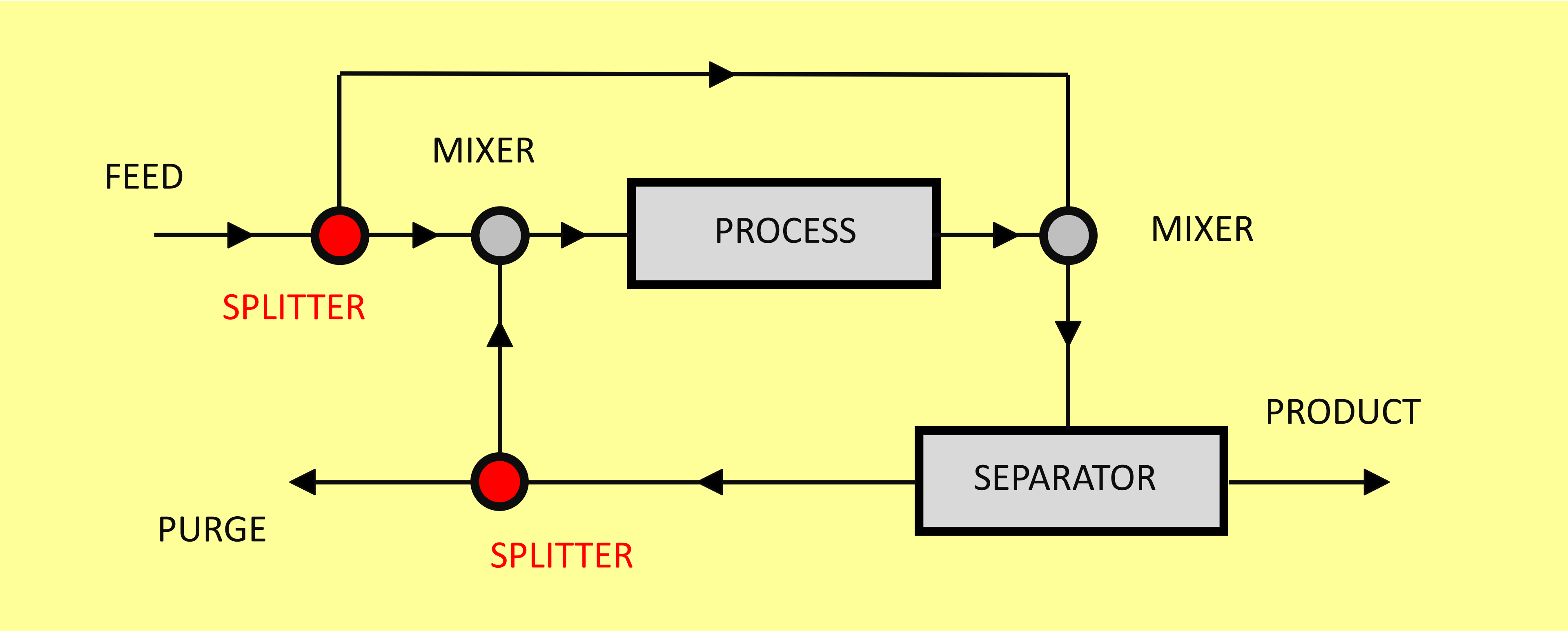


Multiple Unit Operations



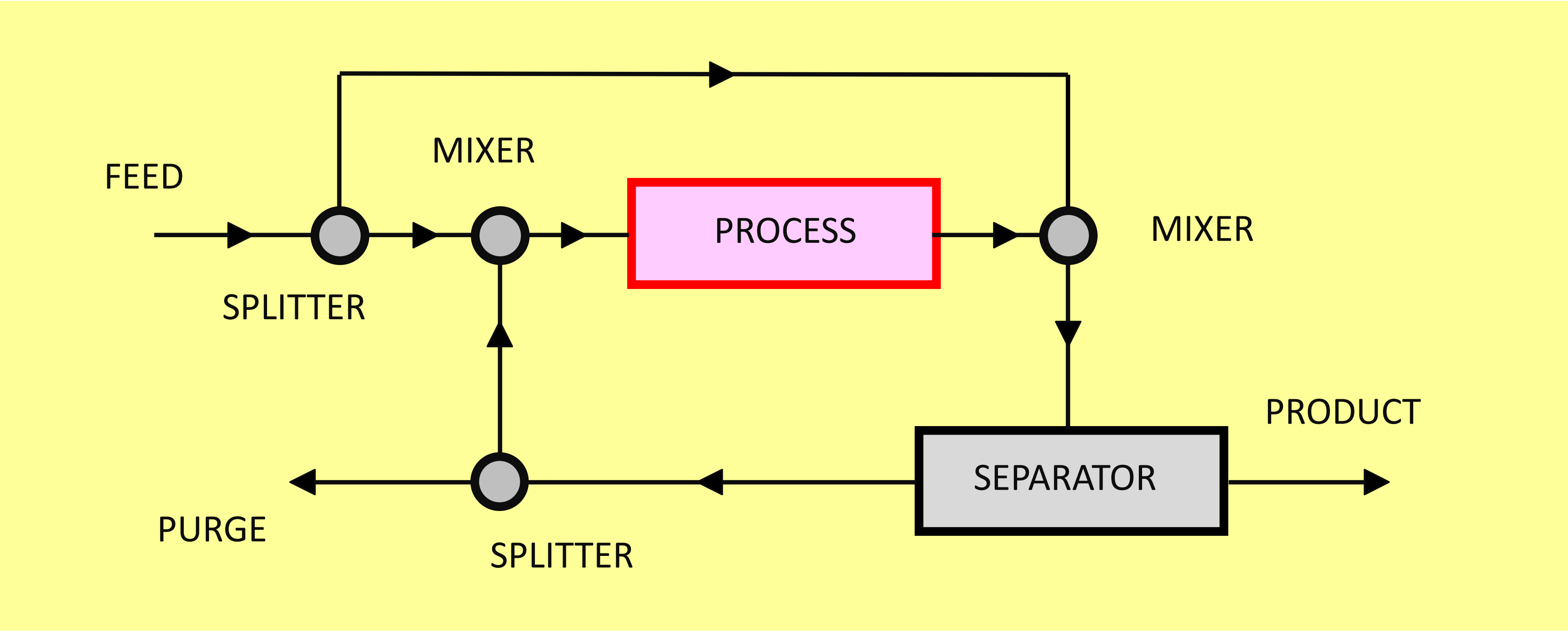
MIXER : Unit with one outlet stream but two or more inlet streams.

Multiple Unit Operations



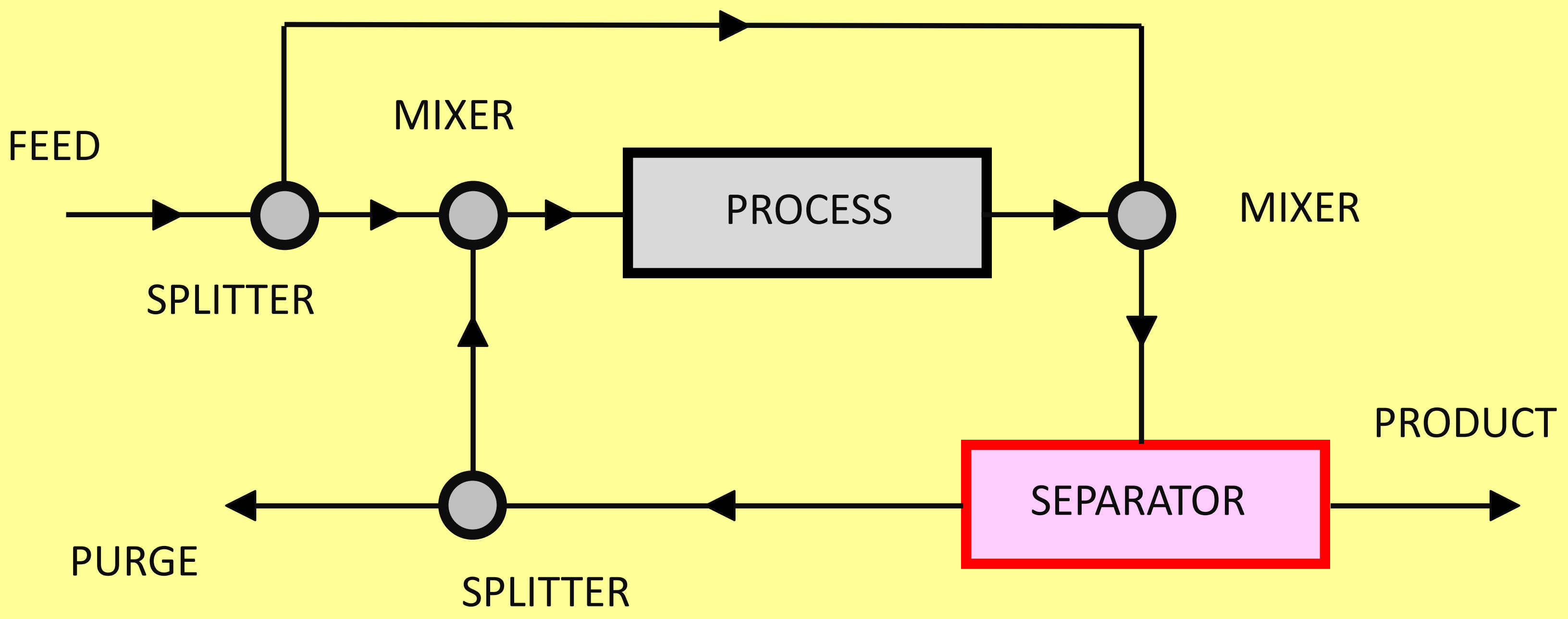
SPLITTER : Unit with one inlet stream but two or more outlet streams all having the same composition but possibly different flow rates.

Multiple Unit Operations



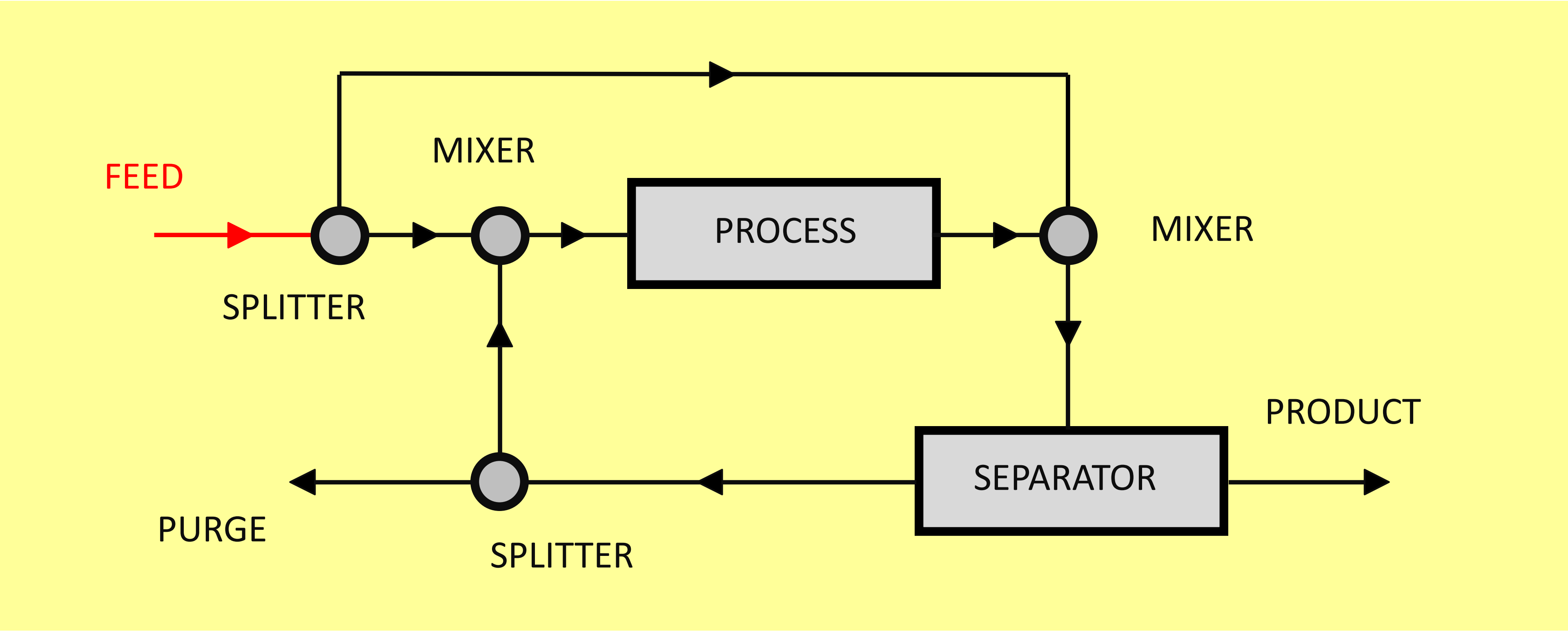
PROCESS : General process unit such as a reactor.

Multiple Unit Operations



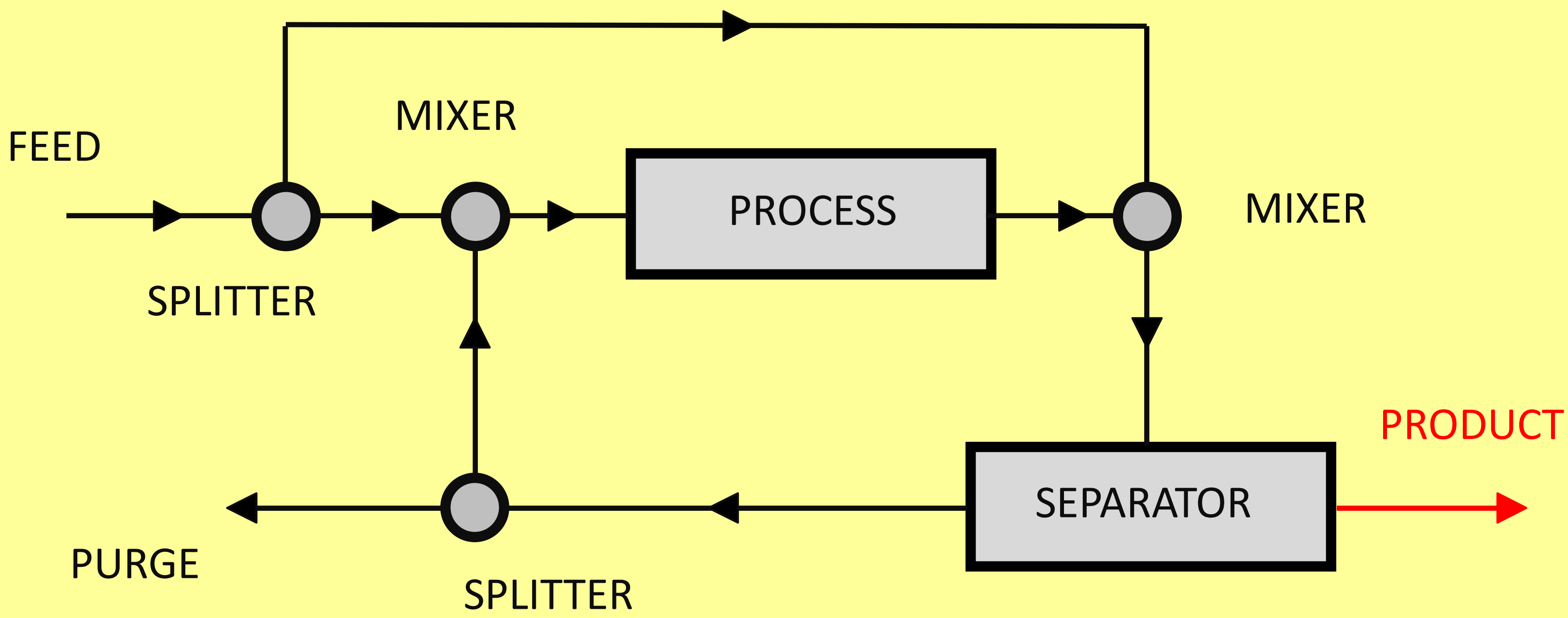
SEPARATOR : Unit with more than one outlet stream, all having different compositions. Note the difference between the **SEPARATOR** and the **SPLITTER**.

Multiple Unit Operations



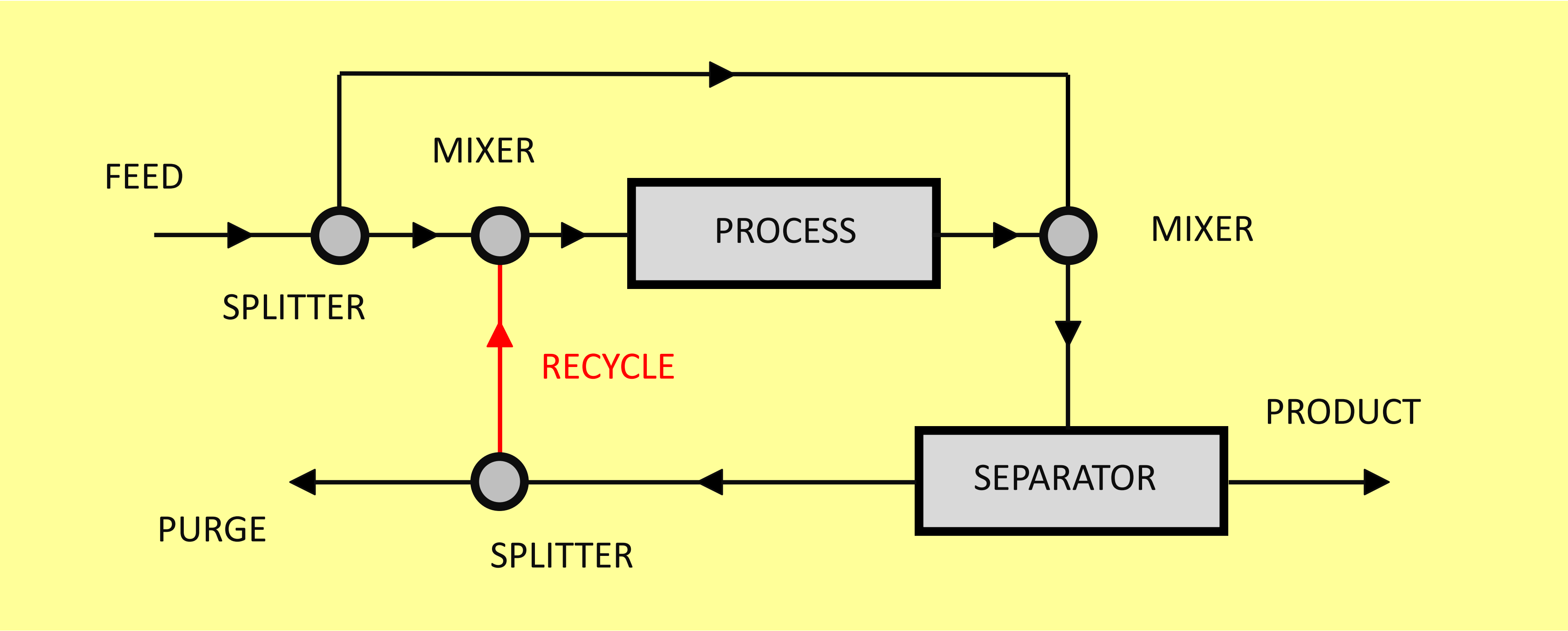
FEED : Stream that enters any process unit. Note that **FRESH FEED** is the name usually given to a stream that feeds an entire process or subprocess.

Multiple Unit Operations



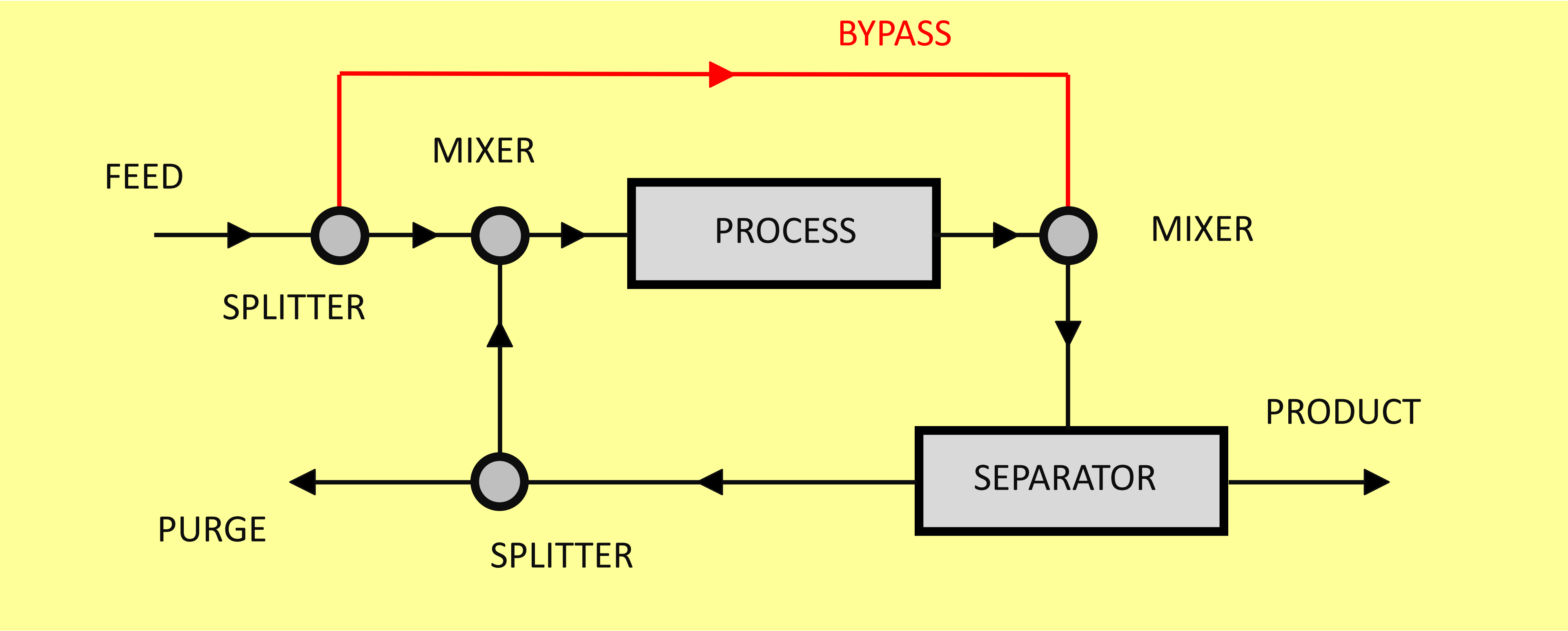
PRODUCT : Stream that leaves a process unit. It is the stream that usually contains the desired product.

Multiple Unit Operations



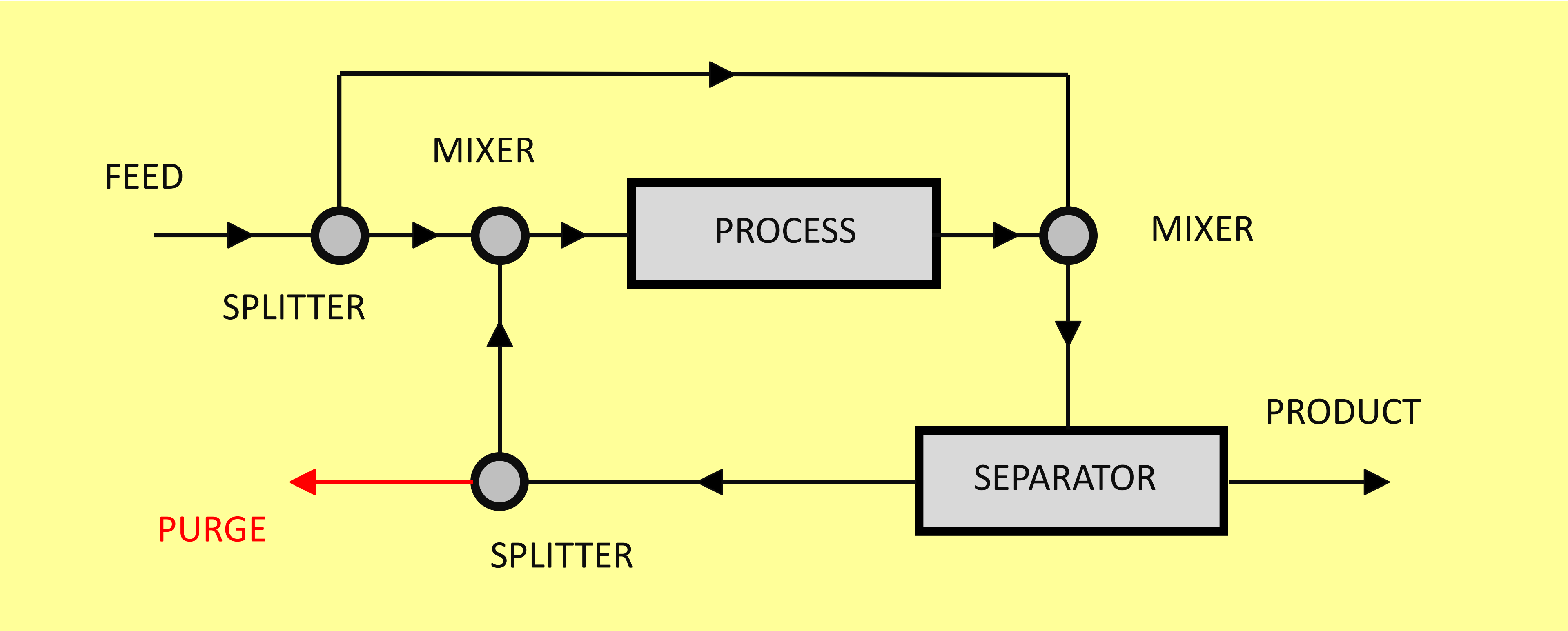
RECYCLE : Stream that passes back towards the beginning of the process.

Multiple Unit Operations



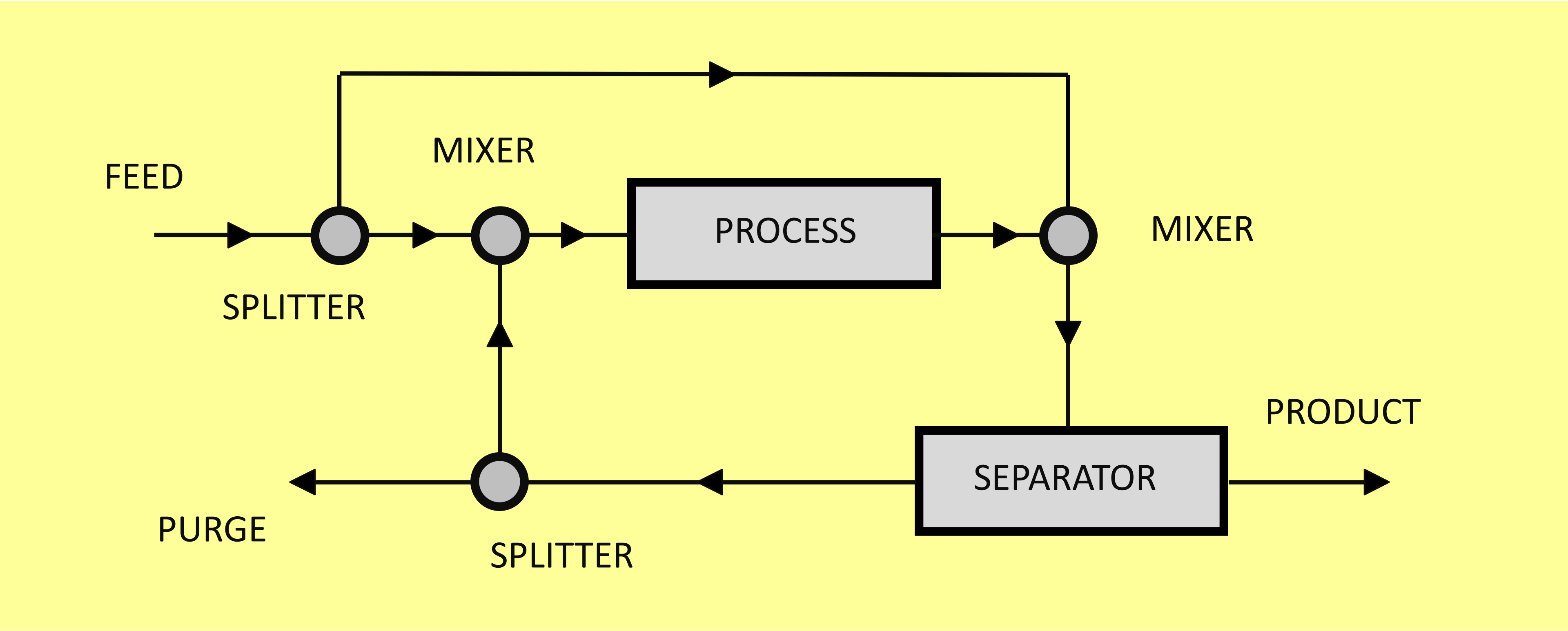
BYPASS : Stream that bypasses around a process unit or group of units.

Multiple Unit Operations



PURGE : Stream that is used to get rid of undesired material which would otherwise buildup within the process.

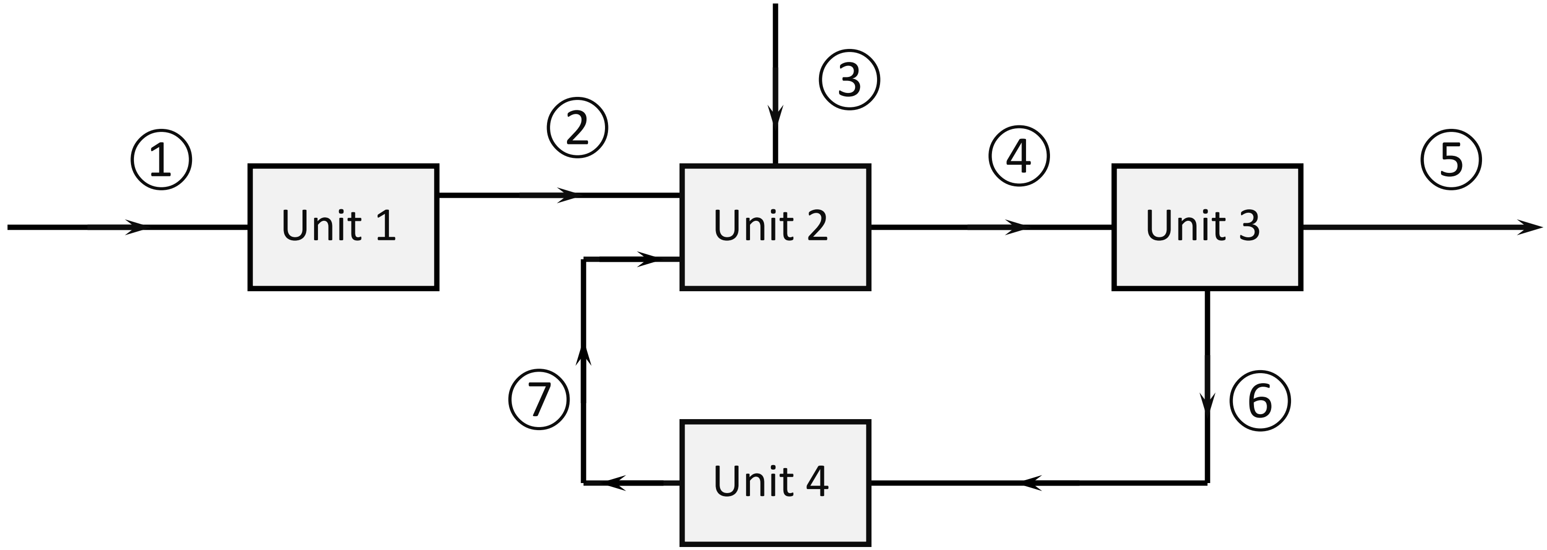
Multiple Unit Operations

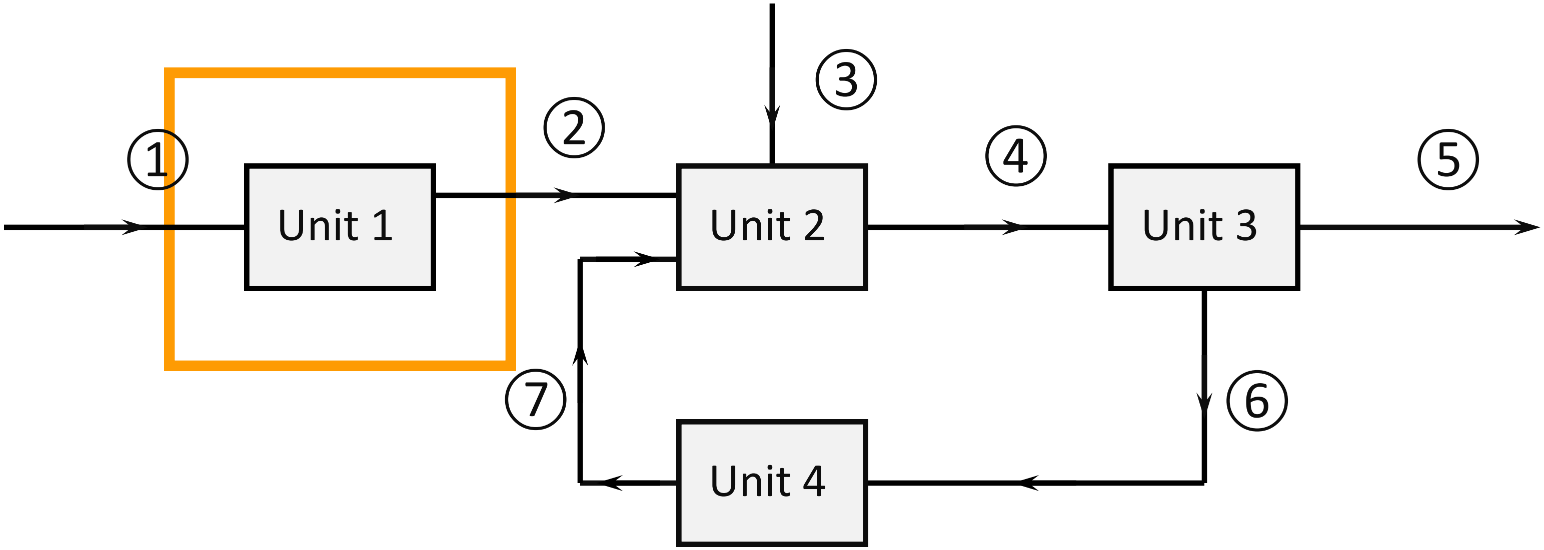


MAKE-UP STREAM : An inlet stream to the process as a whole, but not the primary feed stream. (Not shown above).

Multiple Unit Operations

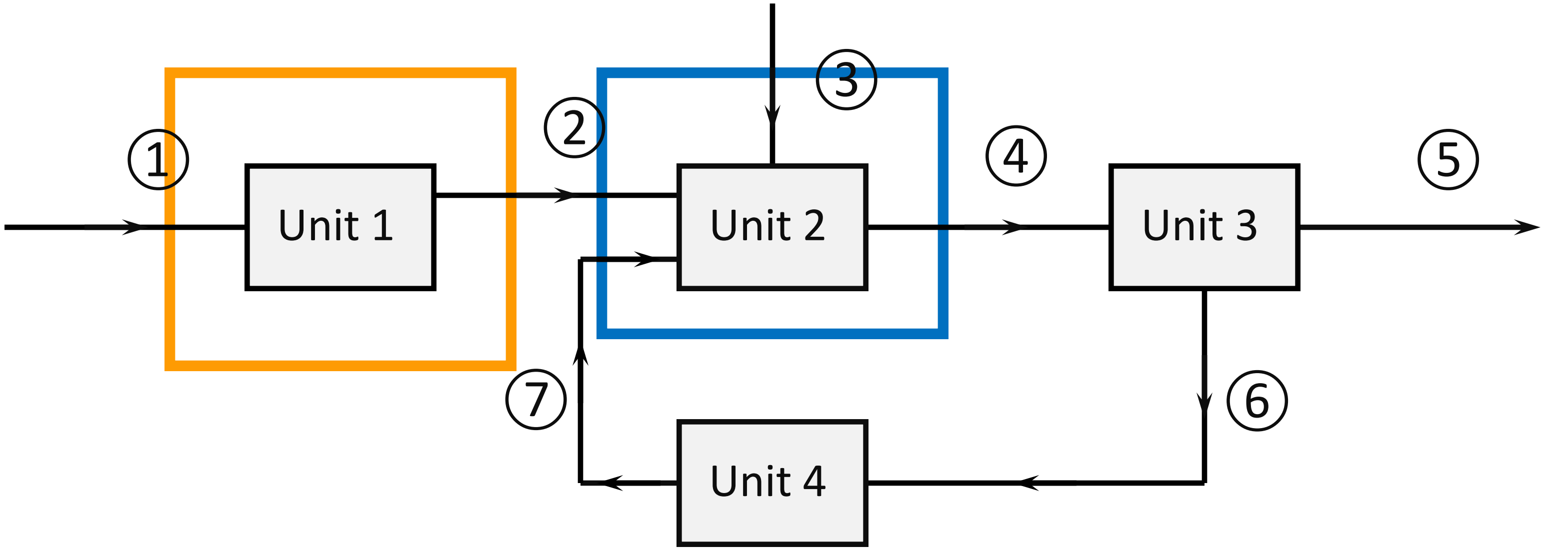
Now let's consider the following system ...


1
Unit 1
2
3
Unit 2
4
Unit 3
5
6
7
Unit 4



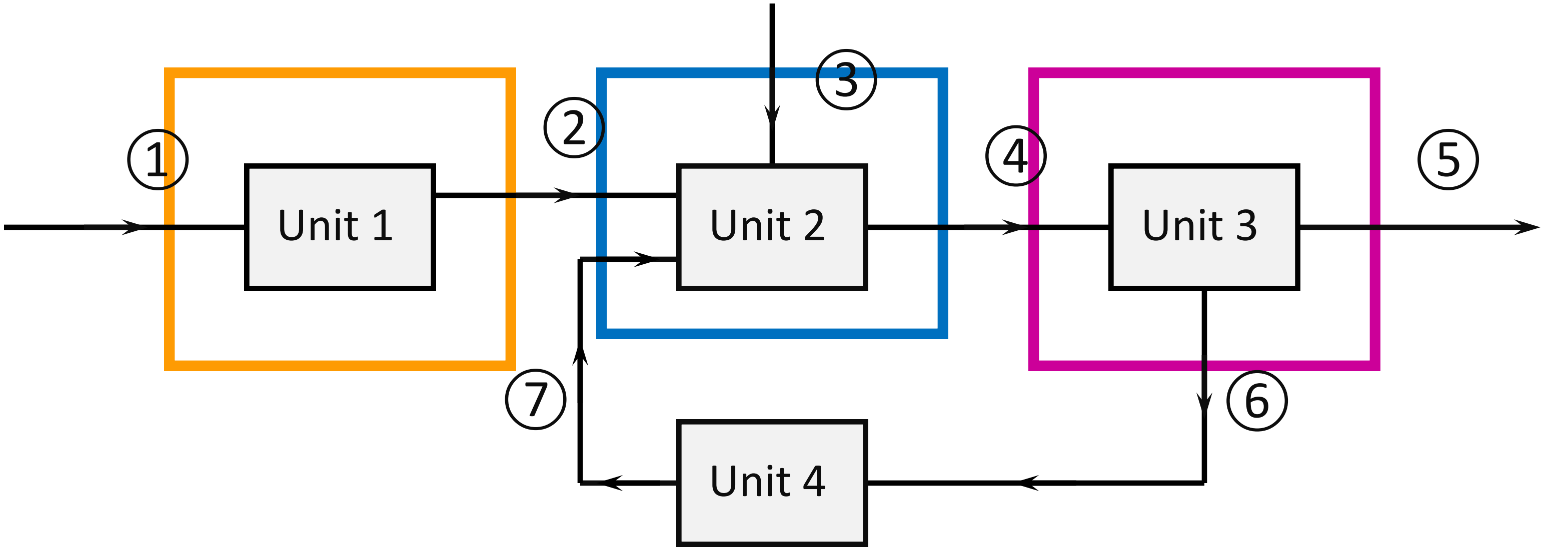
Unit 1

$$F_1 = F_2$$

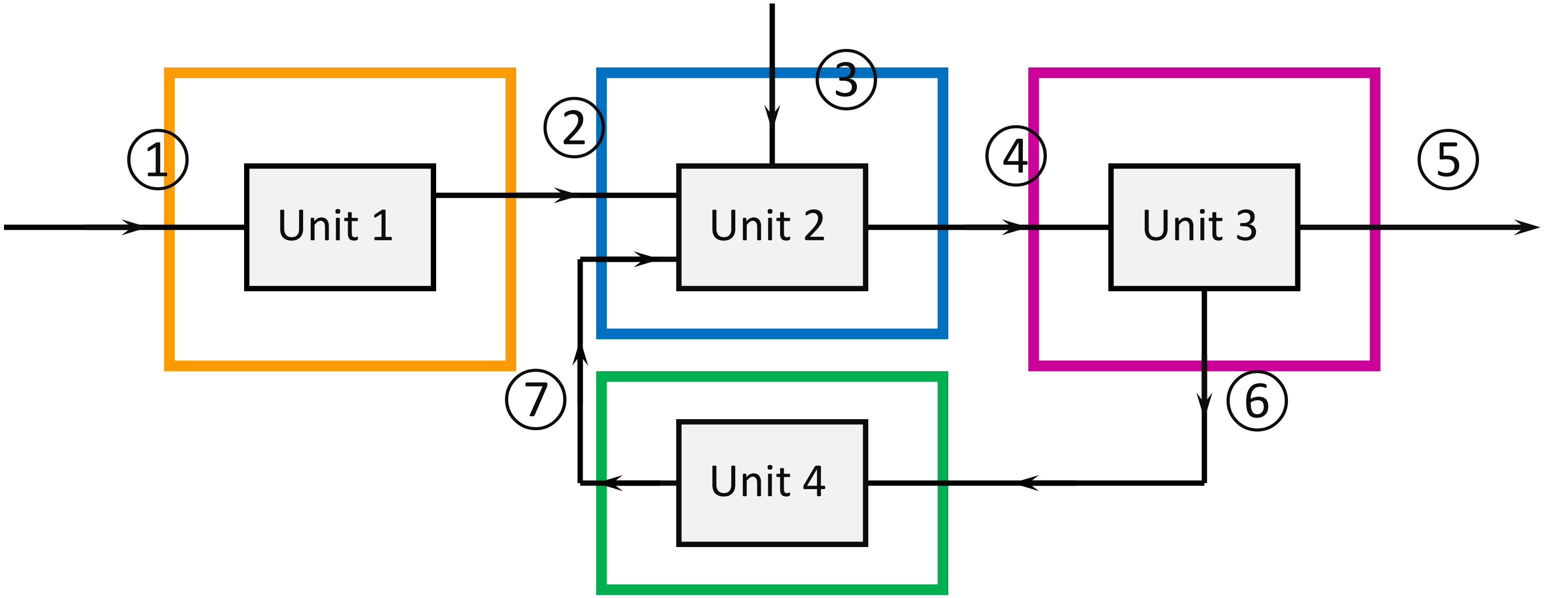


Unit 1 $F_1 = F_2$

Unit 2 $F_2 + F_3 + F_7 = F_4$



Unit 1	$F_1 = F_2$
Unit 2	$F_2 + F_3 + F_7 = F_4$
Unit 3	$F_4 = F_5 + F_6$

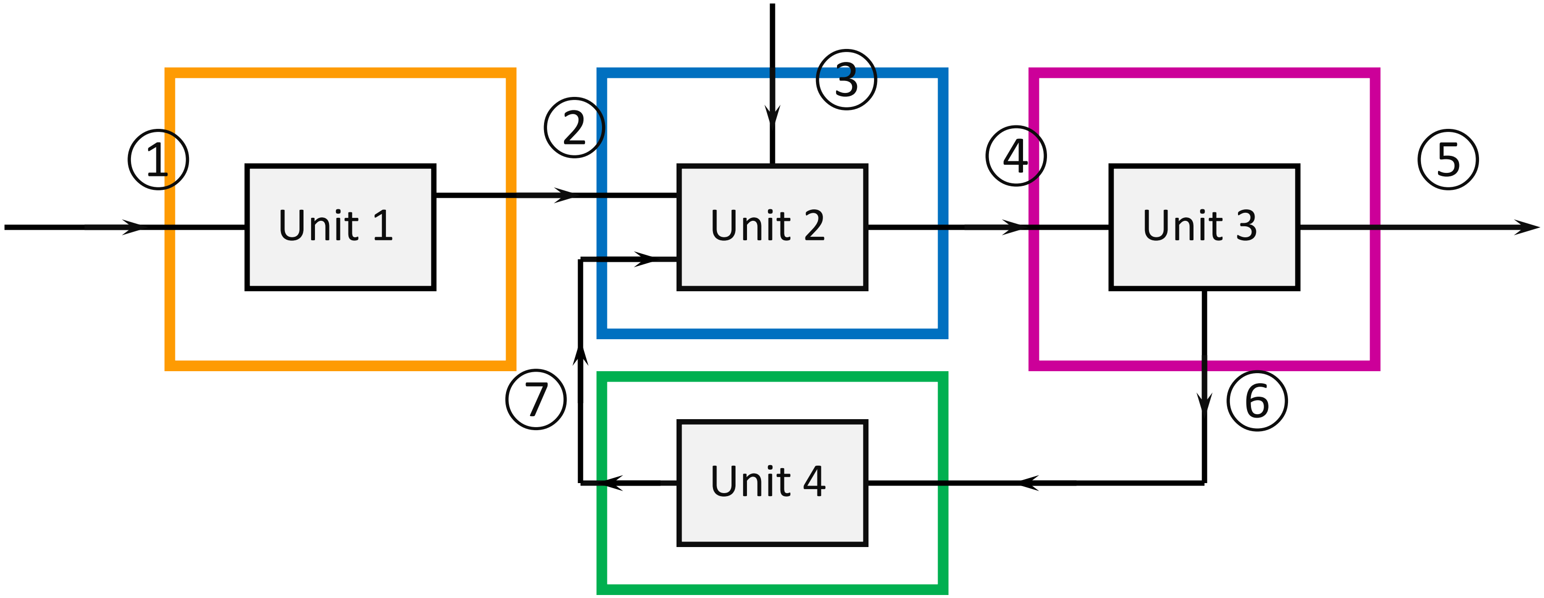


Unit 1 $F_1 = F_2$

Unit 2 $F_2 + F_3 + F_7 = F_4$

Unit 3 $F_4 = F_5 + F_6$

Unit 4 $F_6 = F_7$



Unit 1 $$F_1 = F_2$$

Unit 2 $$F_2 + F_3 + F_7 = F_4$$

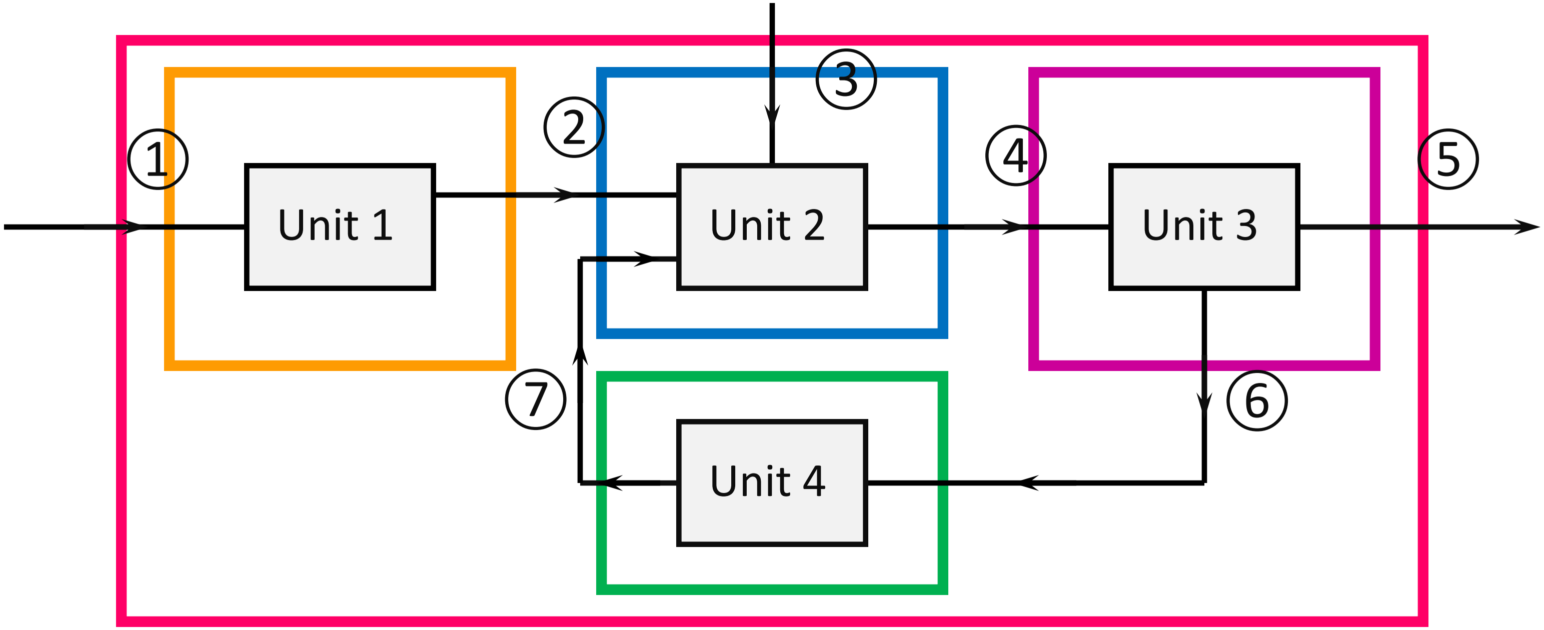
Unit 3 $$F_4 = F_5 + F_6$$

Unit 4 $$F_6 = F_7$$

Summing,

$$F_1 + \cancel{F_2} + F_3 + \cancel{F_7} + \cancel{F_4} + \cancel{F_6} = \cancel{F_2} + \cancel{F_4} + F_5 + \cancel{F_6} + \cancel{F_7}$$

$$\therefore \quad F_1 + F_3 = F_5$$



Unit 1 $$F_1 = F_2$$

Unit 2 $$F_2 + F_3 + F_7 = F_4$$

Unit 3 $$F_4 = F_5 + F_6$$

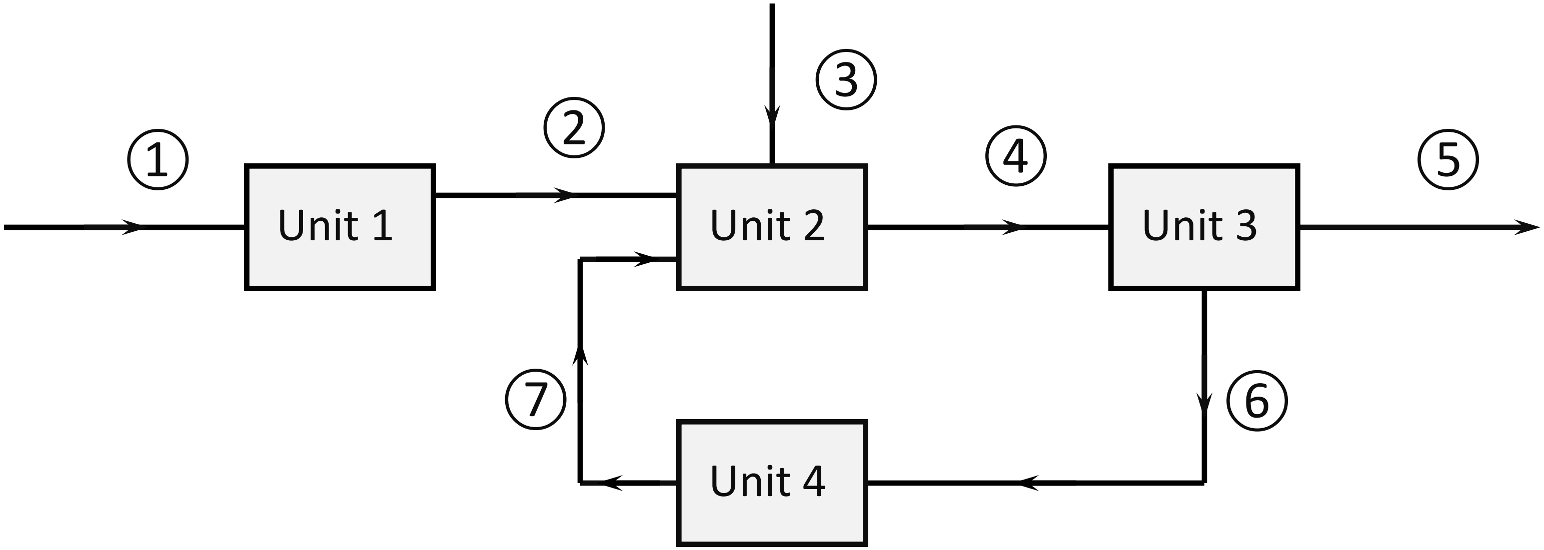
Unit 4 $$F_6 = F_7$$

Summing,

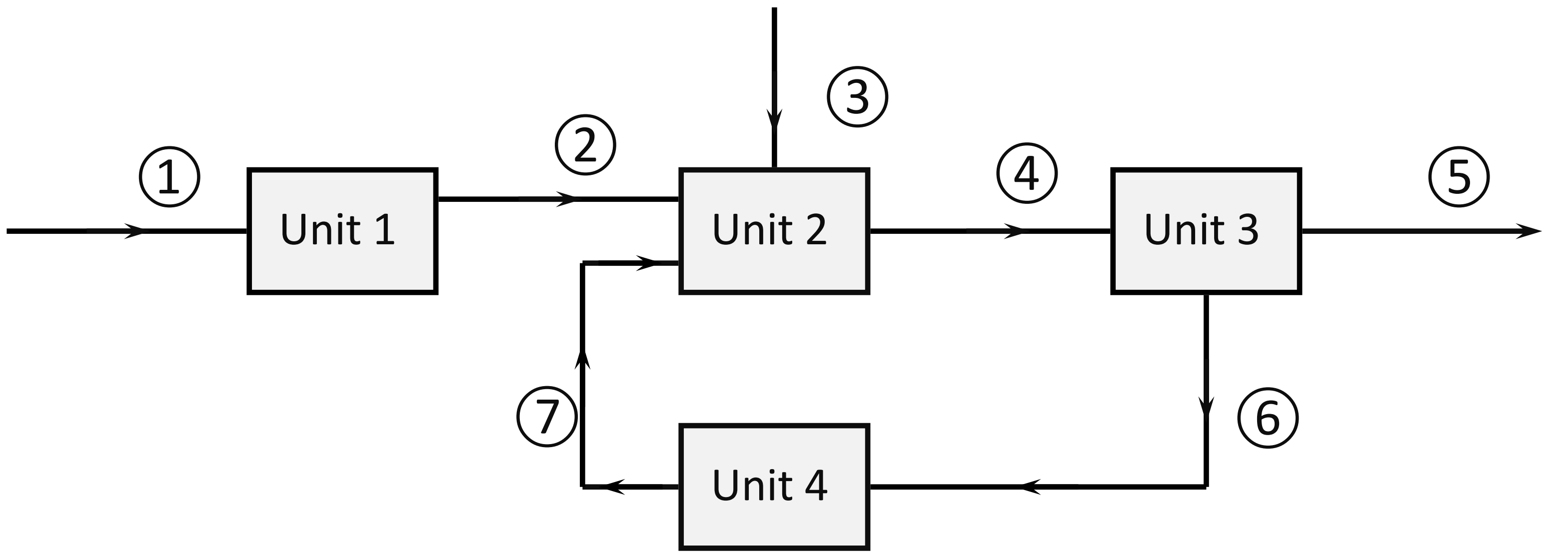
$$F_1 + F_2 + F_3 + F_7 + F_4 + F_6 = F_2 + F_4 + F_5 + F_6 + F_7$$

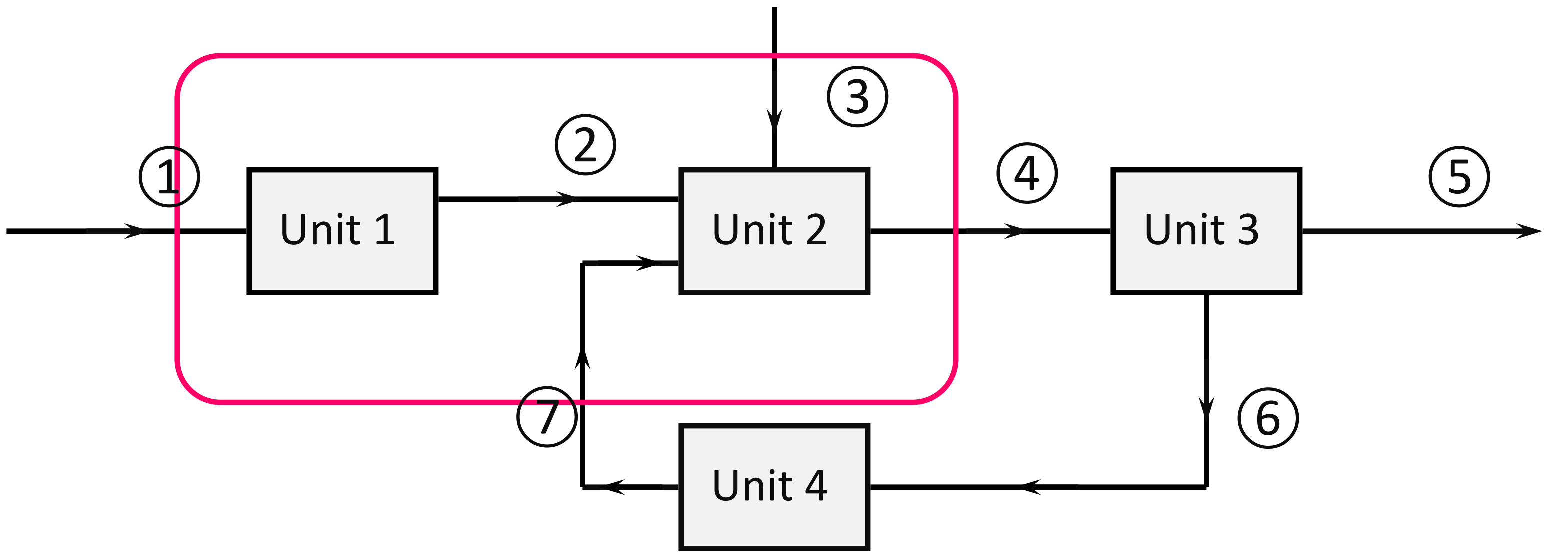
$$\therefore \quad F_1 + F_3 = F_5$$

Units 1,2,3,4 $$F_1 + F_3 = F_5$$



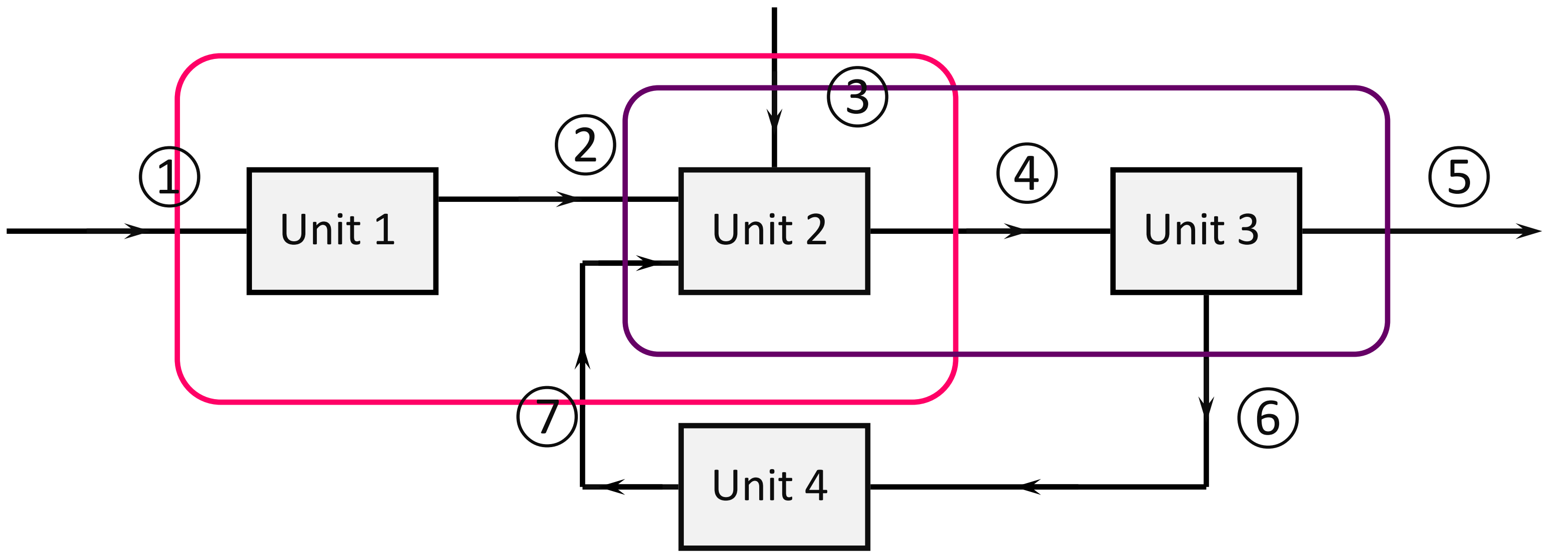
In general, if there are N process units, then we may write N sets of independent material balances.


1
2
3
4
5
6
7
Unit 1
Unit 2
Unit 3
Unit 4



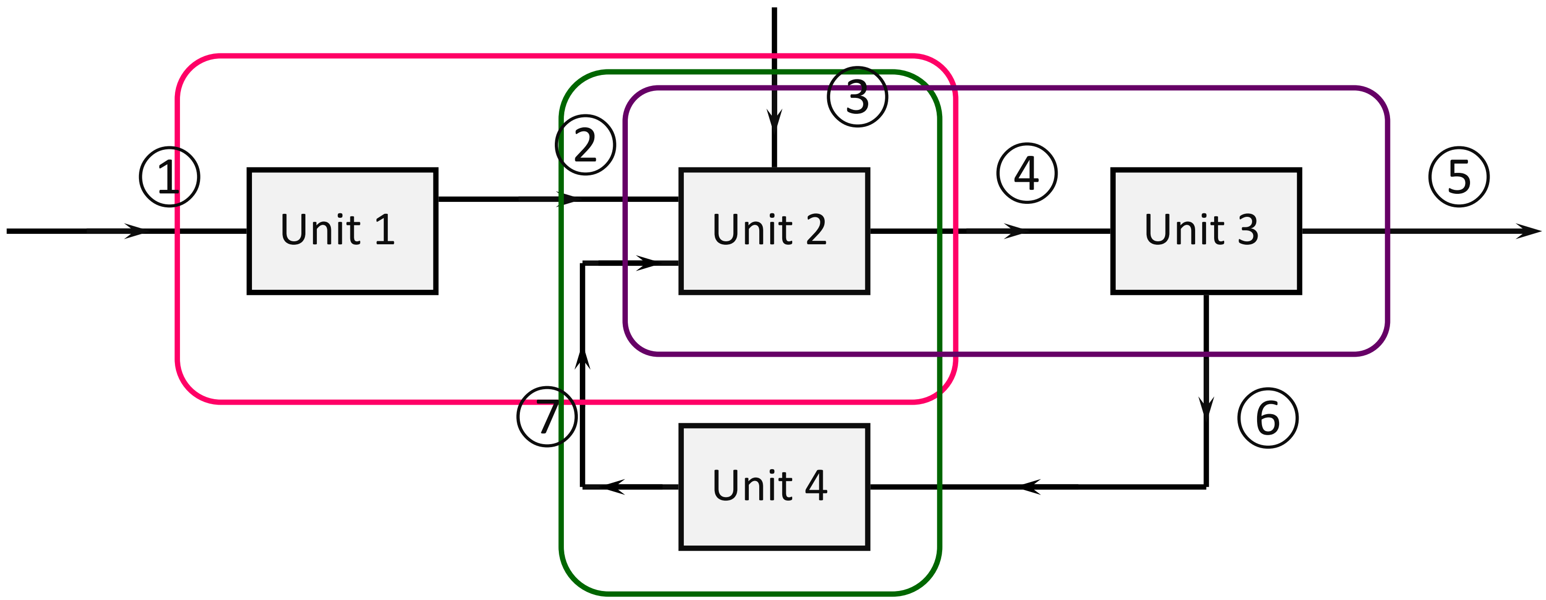
A system boundary may be placed around

Units 1 and 2



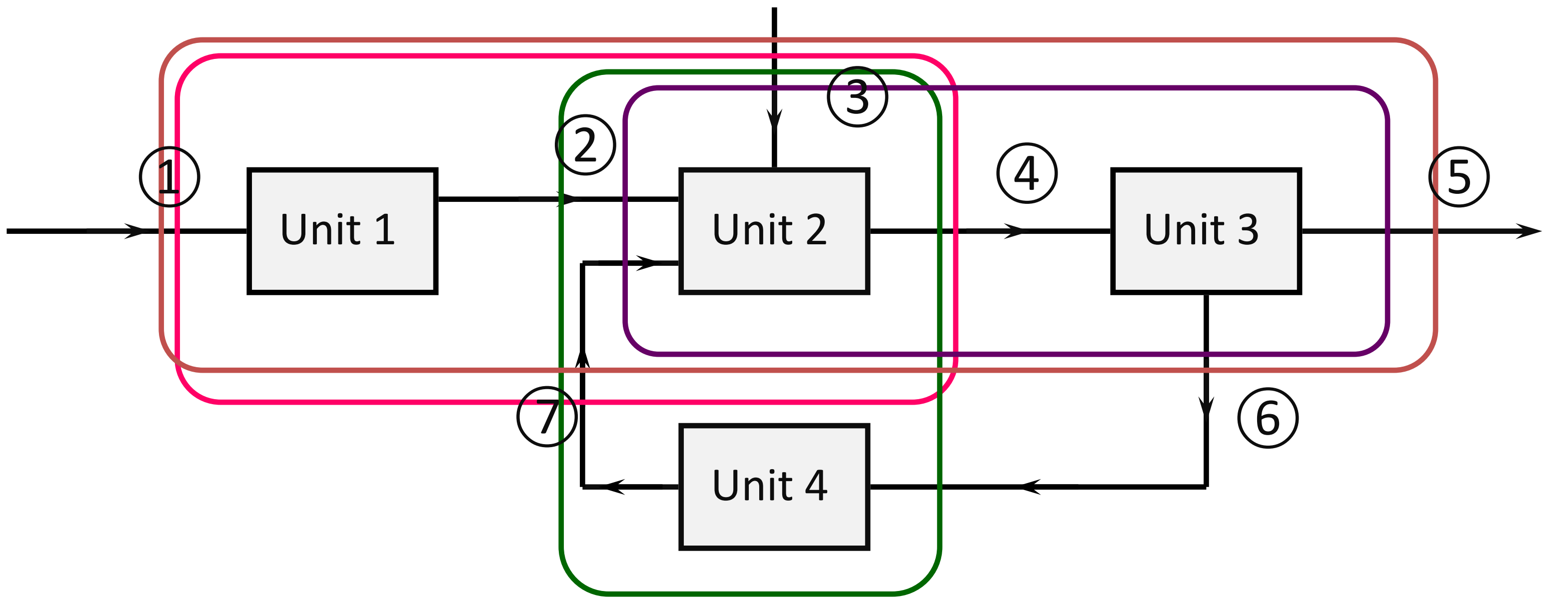
A system boundary may be placed around

Units 2 and 3



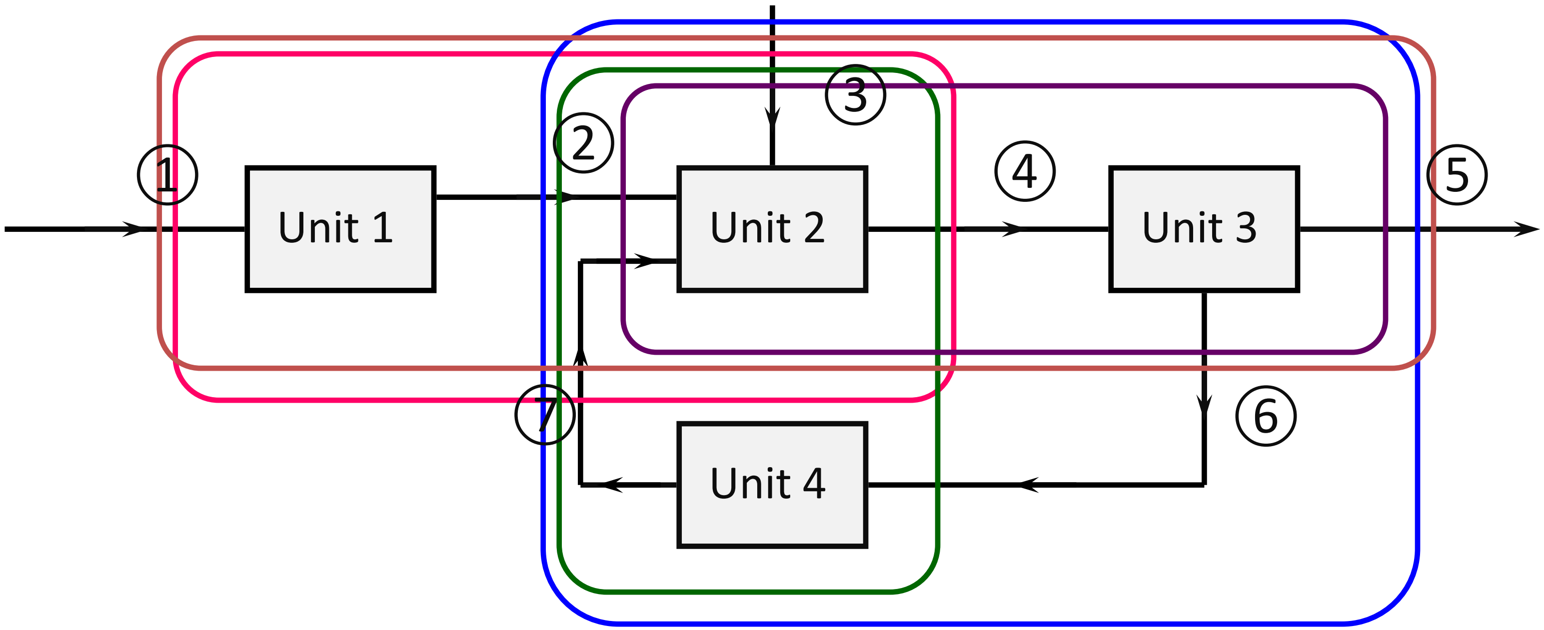
A system boundary may be placed around

Units 2 and 4



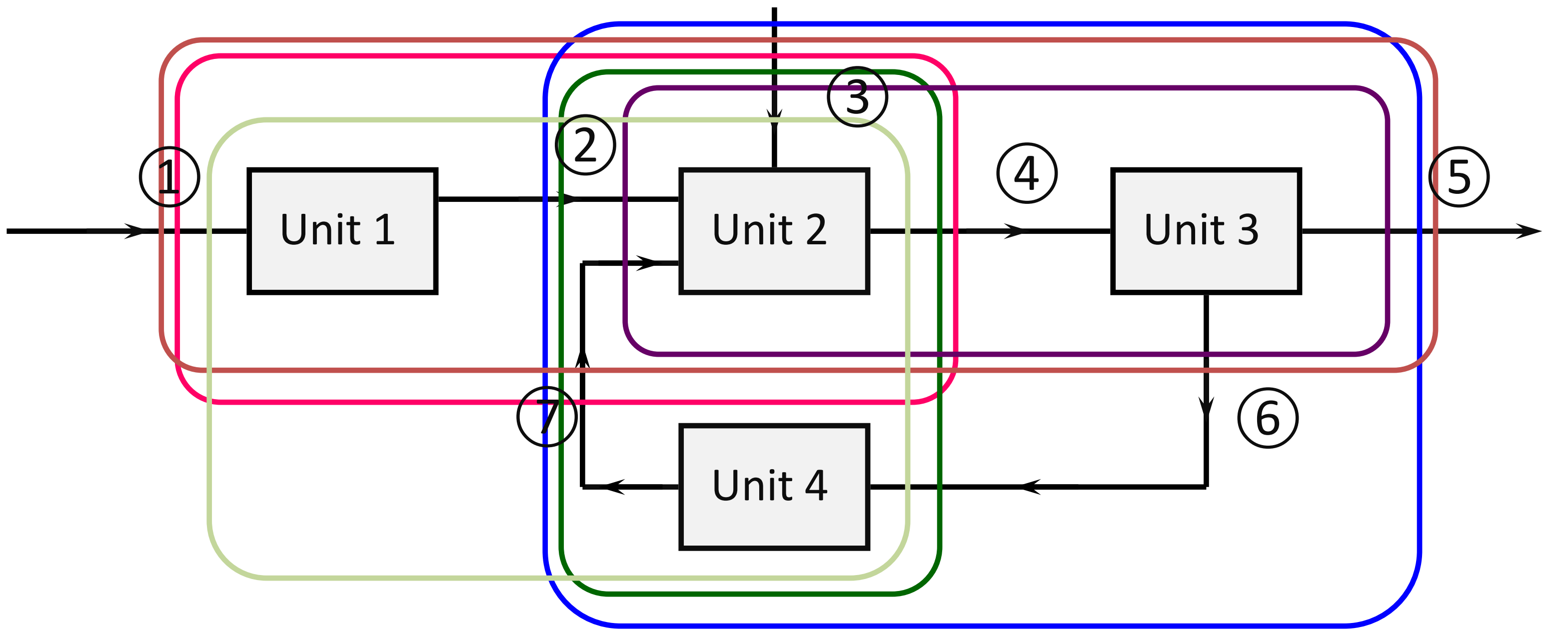
A system boundary may be placed around

Units 1, 2 and 3



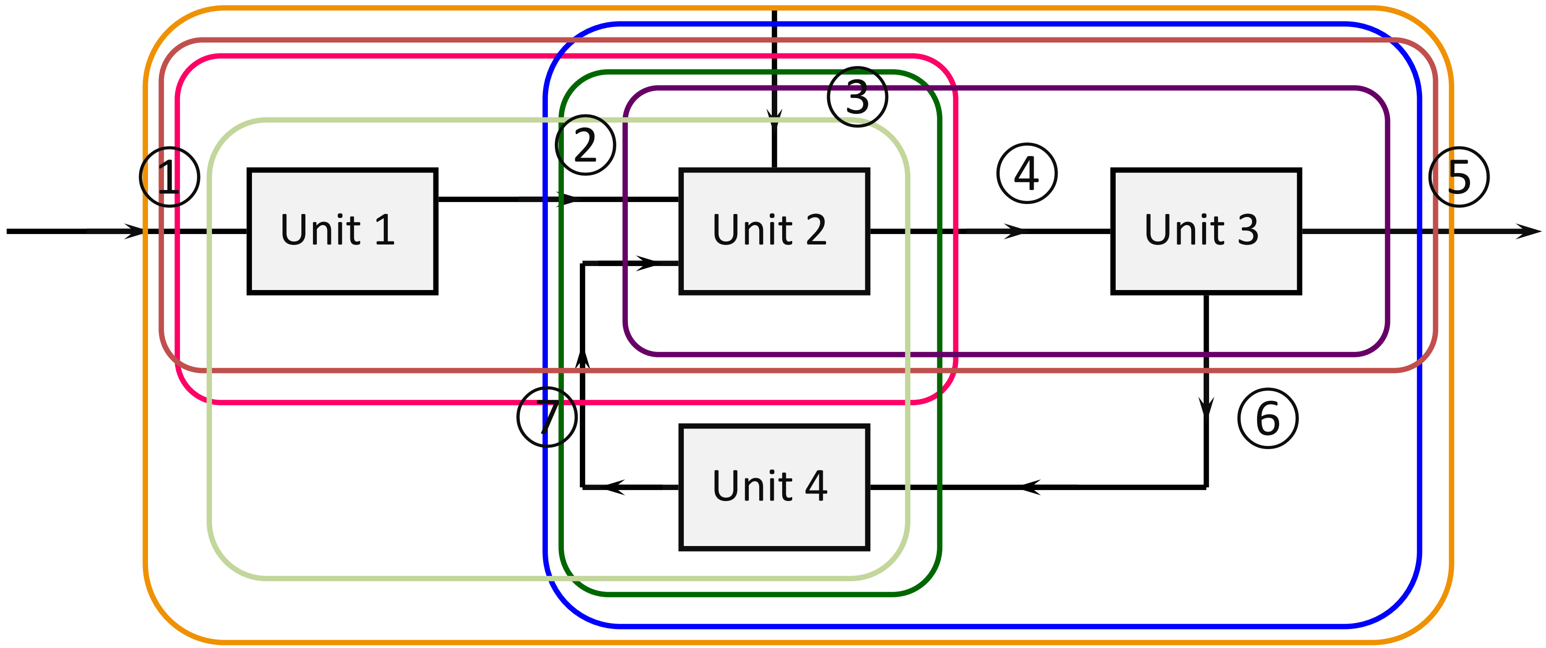
A system boundary may be placed around

Units 2, 3 and 4



A system boundary may be placed around

Units 1, 2 and 4



A system boundary may be placed around

Units 1, 2, 3 and 4

Always take care when setting up your material balances around multiple process units to ensure that all the sets of equations are independent.

Procedure for Solving Material Balance Problems

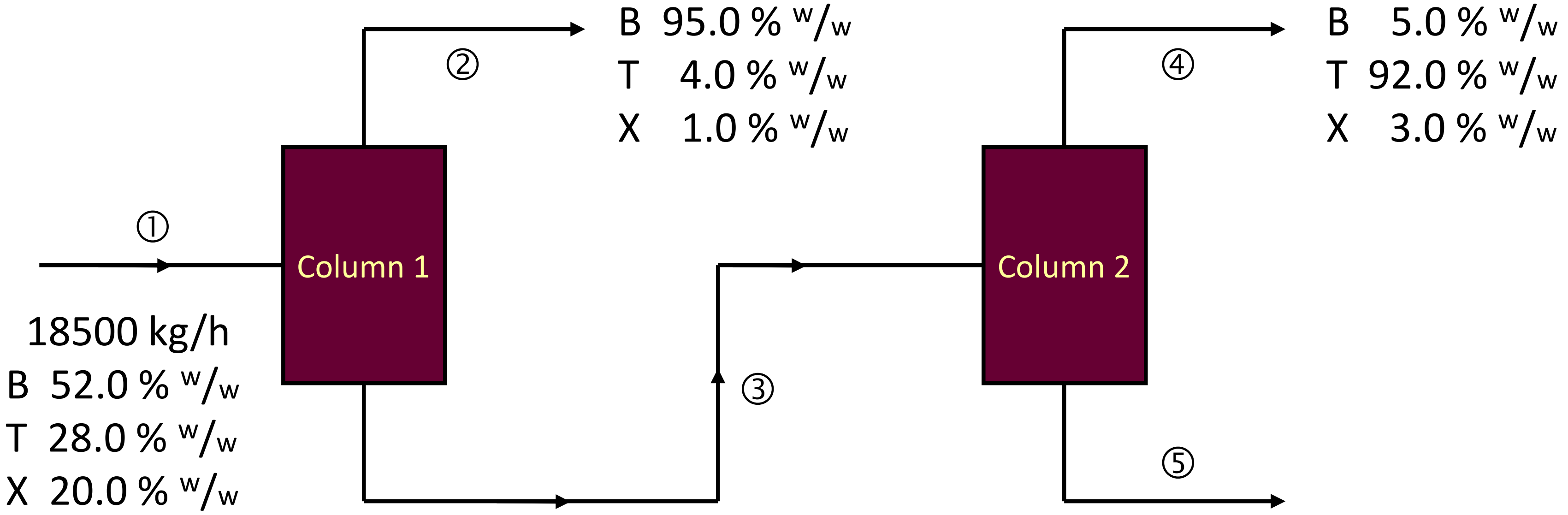
- 1. **Definition** : Draw a sketch or flow sheet showing streams as arrowed lines. Label each stream with flow rate, composition and any other data as appropriate, and identify with a name or number. List missing, unknown data using simple, practical notation.
- 2. **Select a Basis of Calculation** : Select the basis carefully as it is the starting point for all subsequent calculations. You can make an easy problem hard with the wrong choice of a basis of calculation.
- 3. **Select a Unit System** : Material balance must be in terms of mass or moles, but not volume. Be consistent with the units used.

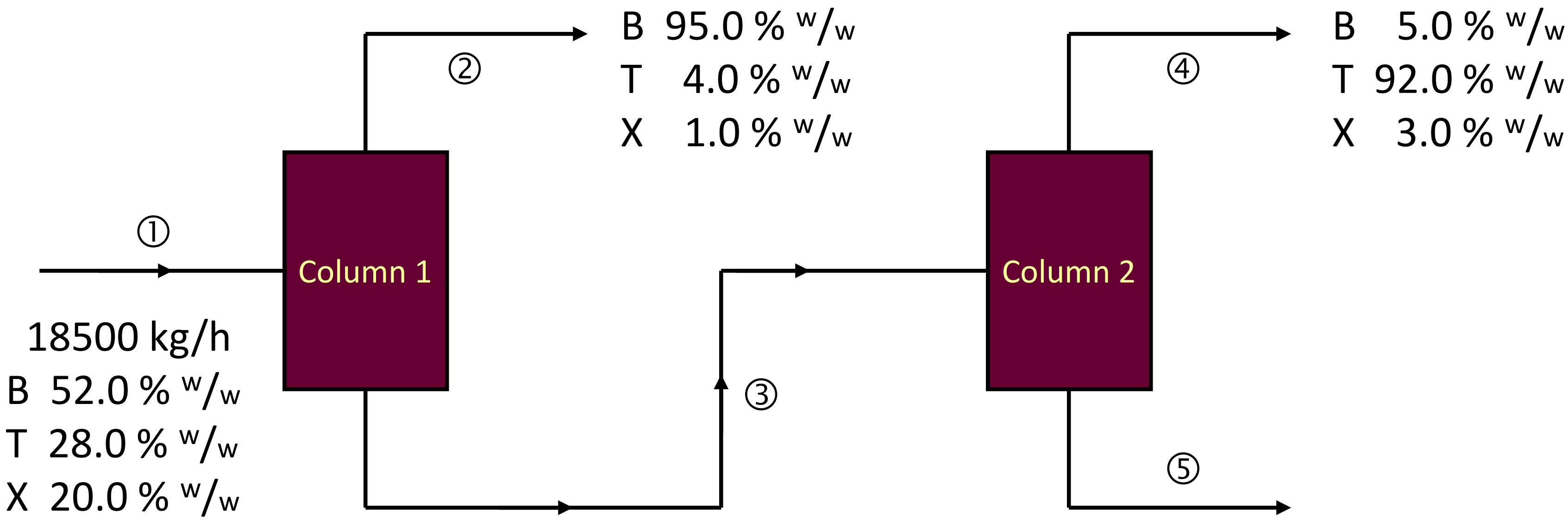
Procedure for Solving Material Balance Problems

- 4. **In a Unique Solution Possible ?** : Check the number of unknowns against the number of independent equations.
- 5. **List Assumptions** : List all assumptions made before and during calculations.
- 6. **Select System Boundaries** : Select the boundaries such that the streams with the derived, unknown data cross the boundaries.
- 7. **Solve** : Use a solution method appropriate to the problem.
- 8. **Check the Solution** : Is the answer semi-sensible (i.e., realistic) ?

Material Balance Example

The feed to a two-column fractionating system is 18,500 kg/h of a mixture containing 52.0 % benzene (B), 28.0 % toluene (T), and 20.0 % xylene (X) on a mass basis. The feed is introduced into column I and results in an overhead consisting of 95.0 % benzene, 4.0 % toluene, and 1.0 % xylene on a mass basis. The bottoms from column I are fed to the second column, resulting in an overhead from column II containing 5.0 % benzene, 92.0 % toluene, and 3.0 % xylene on a mass basis. Assume that 52.0 % of the feed appears as overhead in the first column and that 48.0 % of the benzene fed to the second column appears as overhead, calculate the composition and flow of the bottoms stream from the second column.





Assume steady-state operation.

Assume no chemical reactions occur.

Use a simple and unambiguous notation.

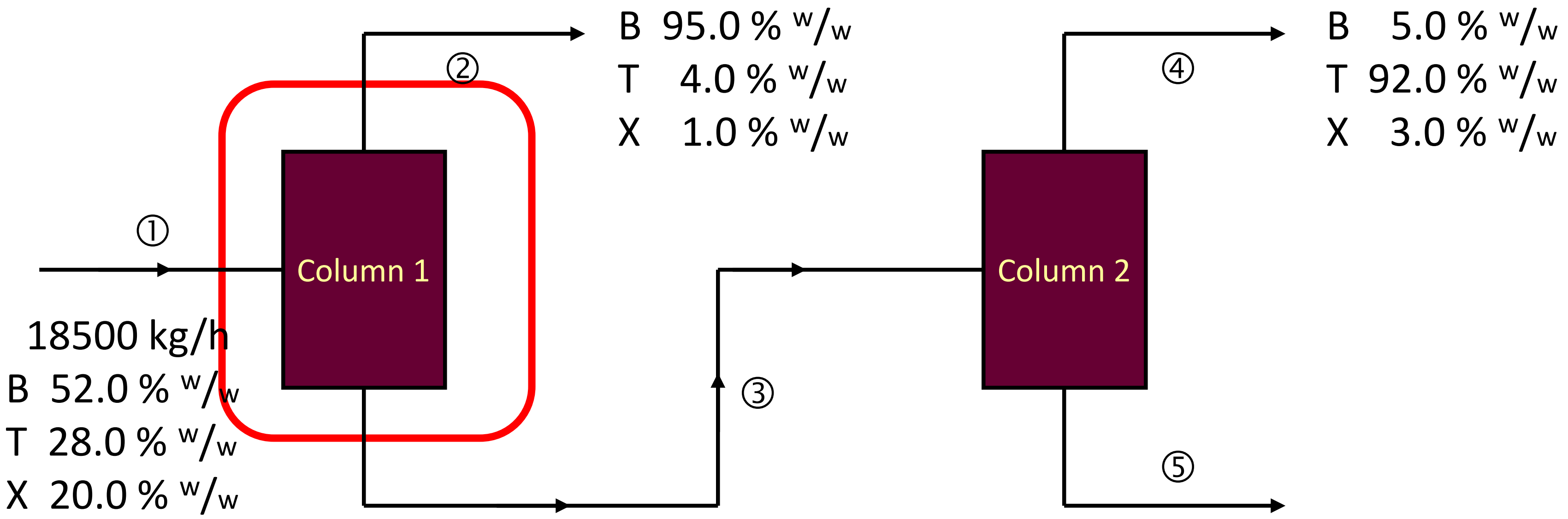
Let F_i denote the total mass flow rate of stream i.

Let B_i be the mass fraction of benzene in stream i.

Let T_i be the mass fraction of toluene in stream i.

Let X_i be the mass fraction of xylene in stream i.

So, $B_i F_i$ is the mass of benzene in stream i.



Basis of calculation : 1 hour

Total balance around Column 1 :

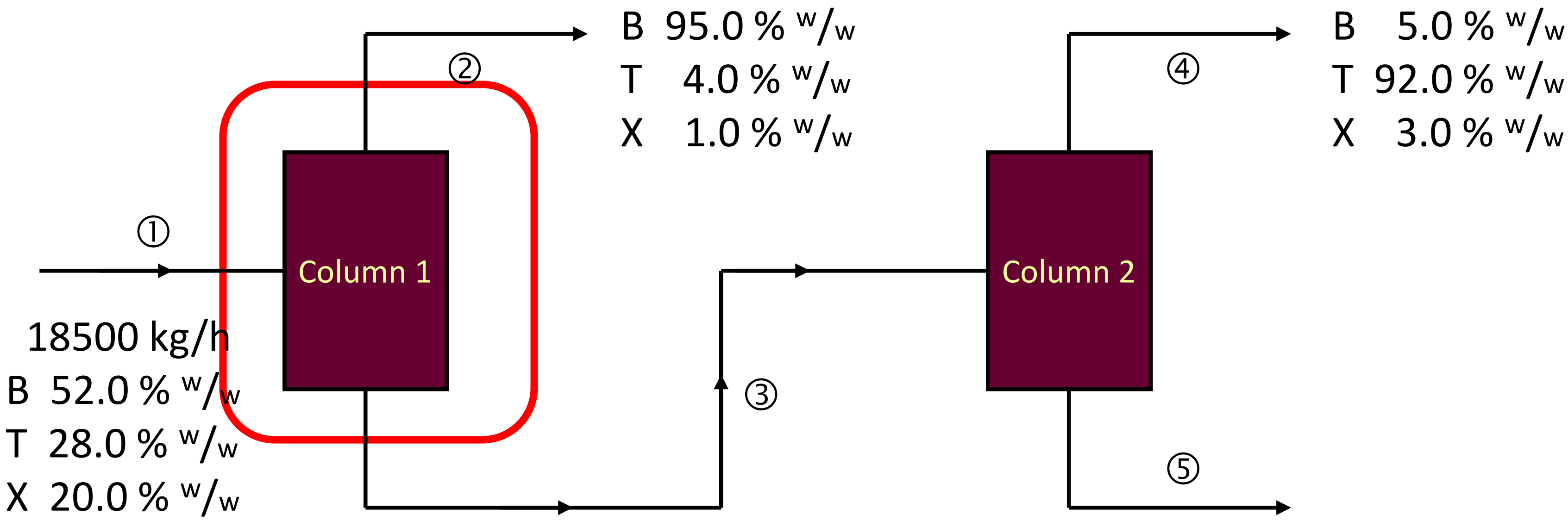
$$F_1 = F_2 + F_3$$

We know that $F_1 = 18500$ kg

So, $18500 = F_2 + F_3$ (1)

We are told that 52% of stream ① leaves Column 1 in stream ②.

So,



The benzene component balance around the Column 1 :

$$B_1 F_1 = B_2 F_2 + B_3 F_3$$

The only unknown in this equation is B_3.

$$0.52 \times 18500 = 0.95 \times 9620 + B_3 \times 8880$$

$$\Rightarrow \quad B_3 =$$

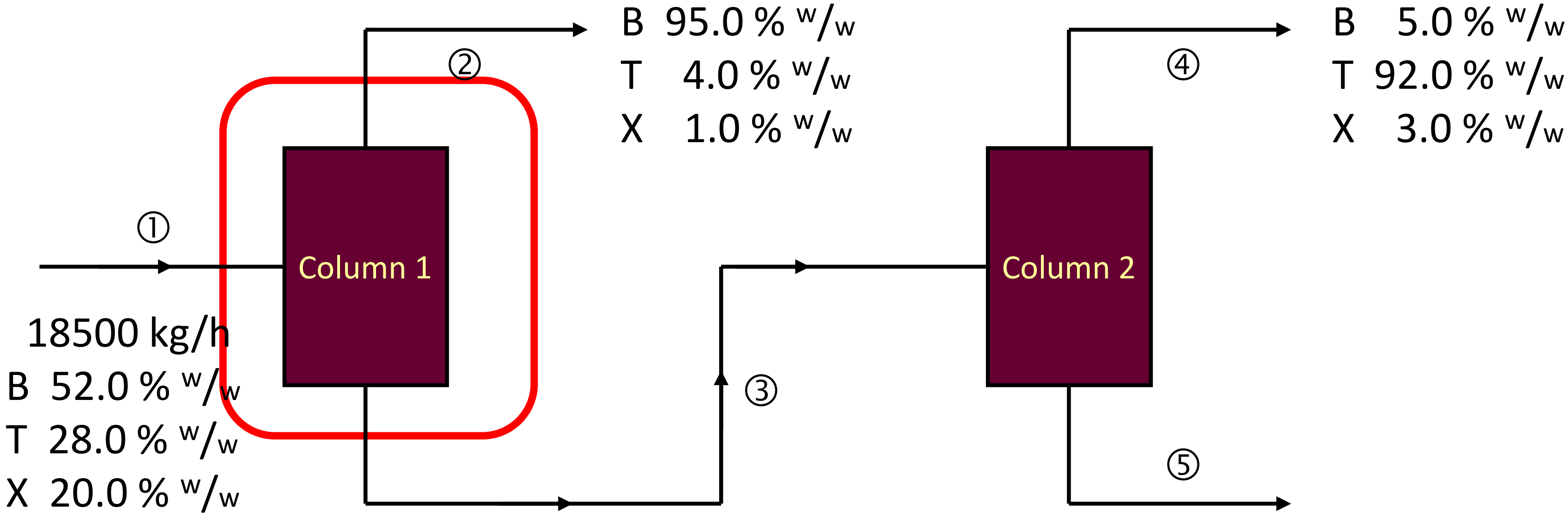
The toluene component balance around the Column 1 :

$$T_1 F_1 = T_2 F_2 + T_3 F_3$$

The only unknown in this equation is T_3.

$$0.28 \times 18500 = 0.04 \times 9620 + T_3 \times 8880$$

$$\Rightarrow \quad T_3 =$$



$B_3 = 0.0542$

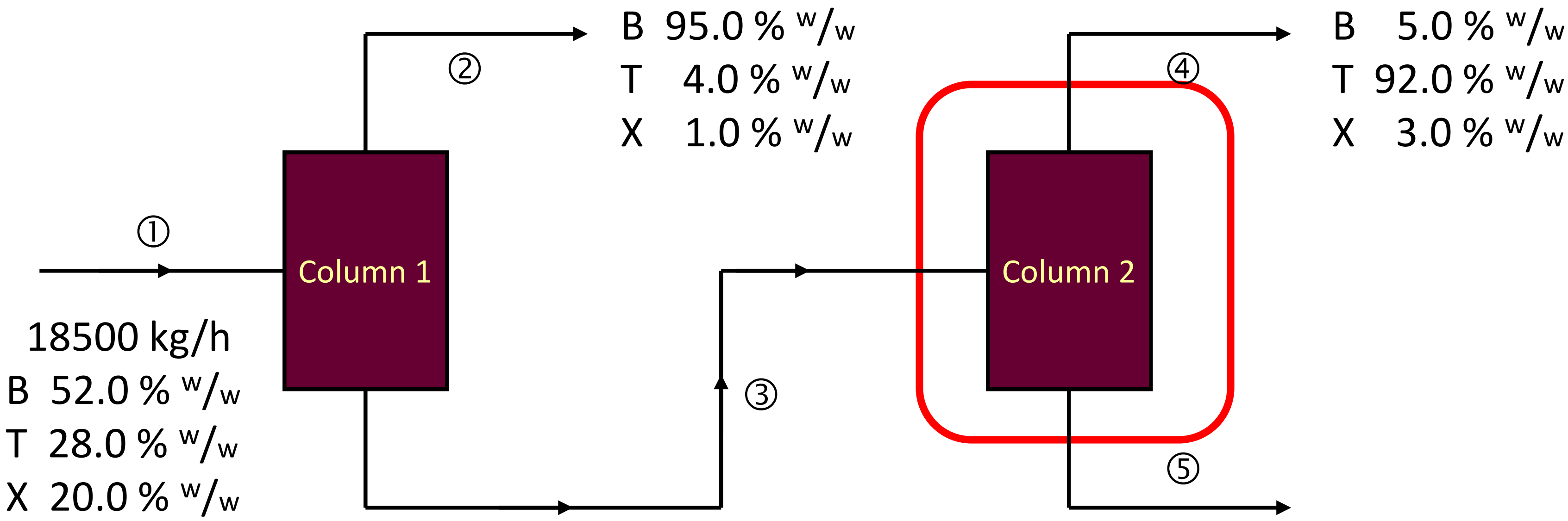
$T_3 = 0.5400$

The sum of the mass fractions must be one.

$$1 = B_3 + T_3 + X_3$$

$\therefore \quad X_3 =$

The composition of stream ③ is



Next consider Column 2. We are told that 48.0 % of the benzene entering Column 2 leaves in stream ④.

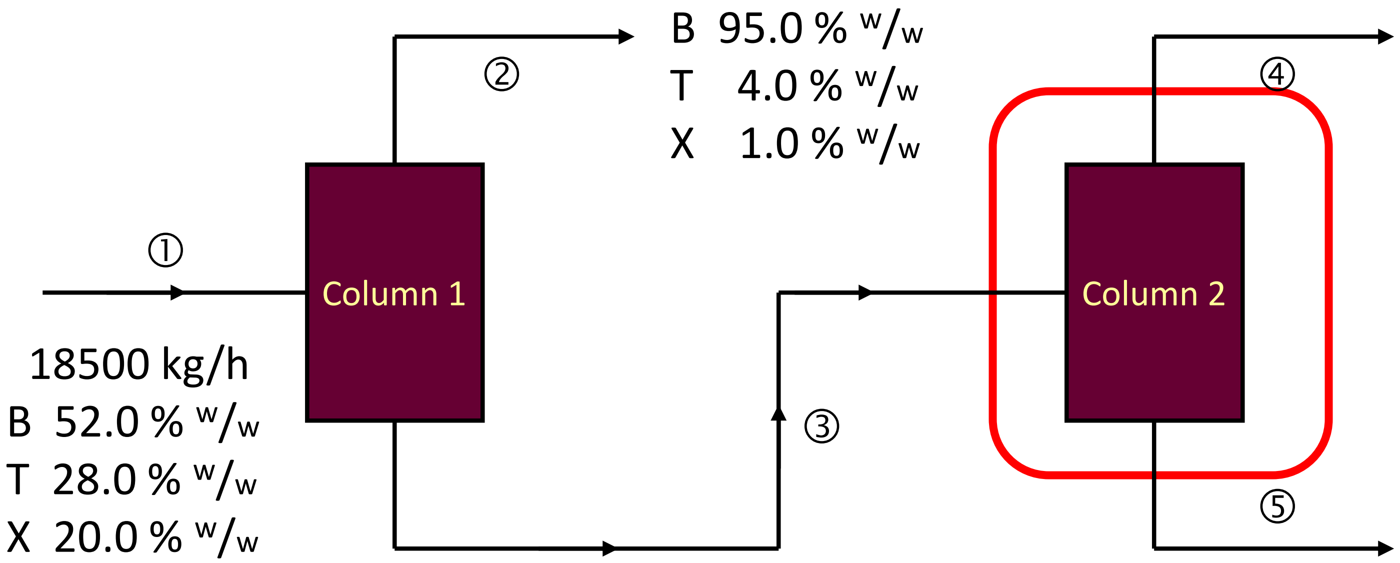
$$0.48 \text{ x} \begin{pmatrix} \text{Mass of benzene} \\ \text{in stream ③} \end{pmatrix} = \begin{pmatrix} \text{Mass of benzene} \\ \text{in stream ④} \end{pmatrix}$$

So, $0.48\, B_3\, F_3 = B_4\, F_4$

The only unknown in this equation is F_4.

$$0.48 \text{ x } 0.0542 \text{ x } 8880 = 0.05\, F_4$$

$\Rightarrow \quad F_4 =$



F_2 = 9620 kg/h

F_3 = 8880 kg/h

F_4 = 4620 kg/h

F_5 = 4260 kg/h

Total balance around Column 2 :

$$F_3 = F_4 + F_5$$

So, $F_5 = F_3 - F_4 =$

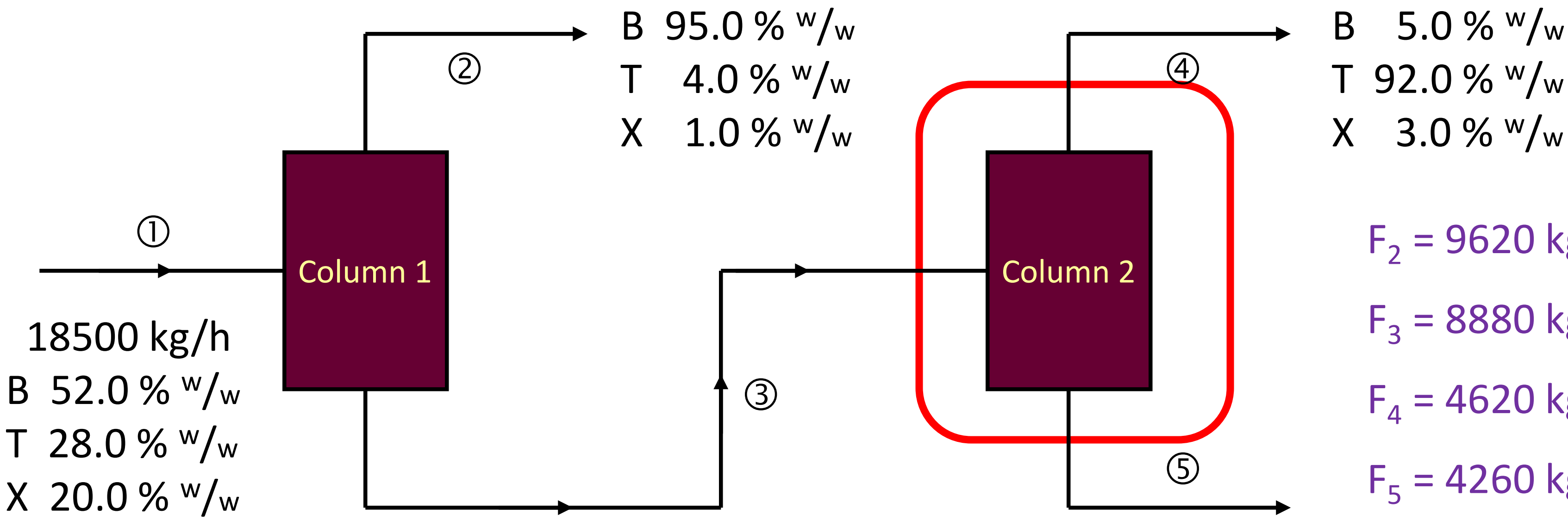
The benzene component balance around the Column 2 :

$$B_3\ F_3 = B_4\ F_4 + B_5\ F_5$$

The only unknown in this equation is B_5.

$$0.0542 \text{ x } 8880 = 0.05 \text{ x } 4620 + B_5 \text{ x } 4260$$

$\Rightarrow$ $B_5 =$



$F_2 = 9620$ kg/h

$F_3 = 8880$ kg/h

$F_4 = 4620$ kg/h

$F_5 = 4260$ kg/h

The toluene component balance around the Column 2 :

$$T_3\, F_3 = T_4\, F_4 + T_5\, F_5$$

The only unknown in this equation is T_5.

$$0.5400 \times 8880 = 0.92 \times 4620 + T_5 \times 4260$$

$\Rightarrow \quad T_5 =$

The sum of the mass fractions must be one.

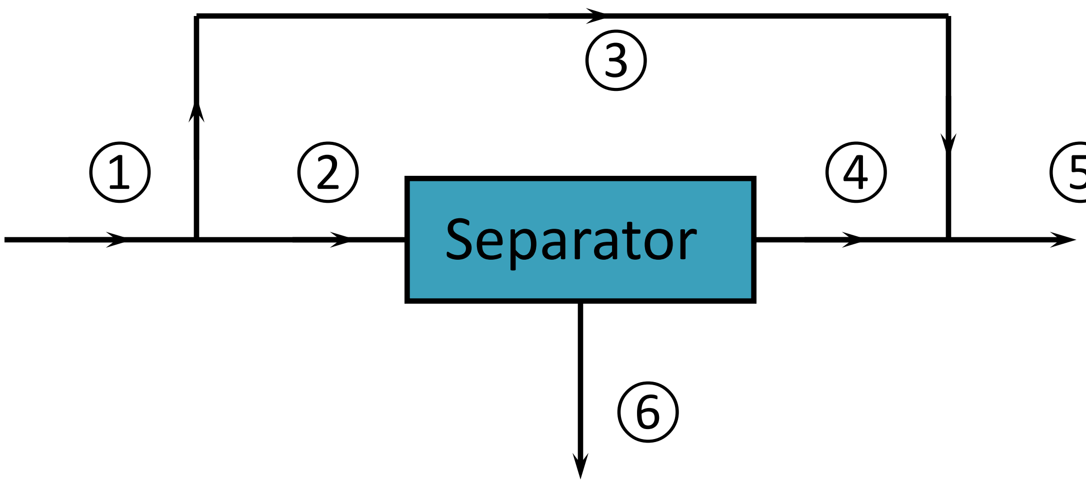
$$1 = B_5 + T_5 + X_5$$

$\therefore \quad X_5 =$

The composition of stream ⑤ is

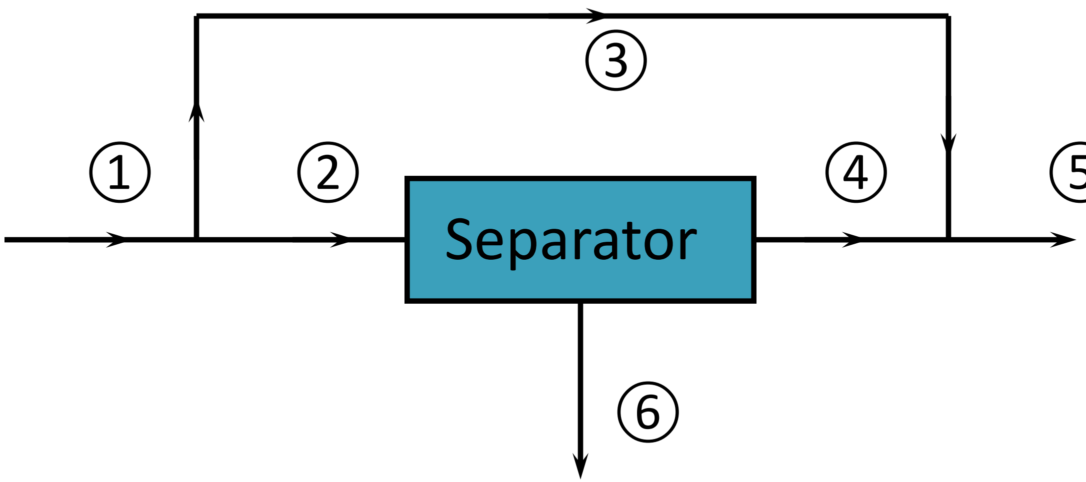
Material Balance Example

Consider the following problem which involves a bypass stream.

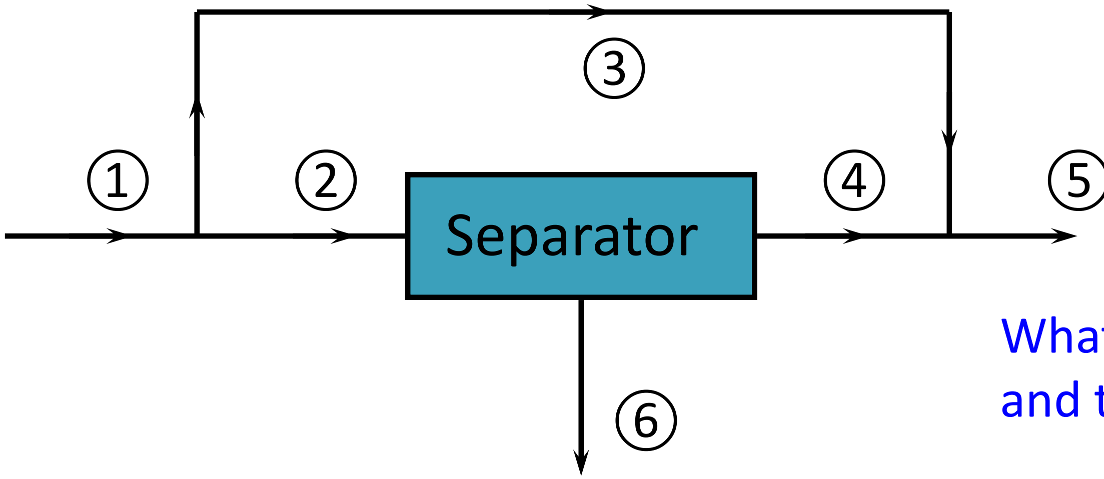


The feed to a separator sub-system has a composition of 42.0 % w/w A, 37.0 % w/w B with the balance C. The flow rate is 1273 kg/h. The only separator available has a capacity limited to 800.0 kg/h, so a portion of the feed bypasses the separator as shown below.

Of the feed entering in stream ②, 6.00 % of A leaves in ⑥, 5.00 % of B leaves in ⑥, 97.0 % of C leaves in ⑥.



What are the flow rate of stream ⑥, and the composition of stream ⑤ ?



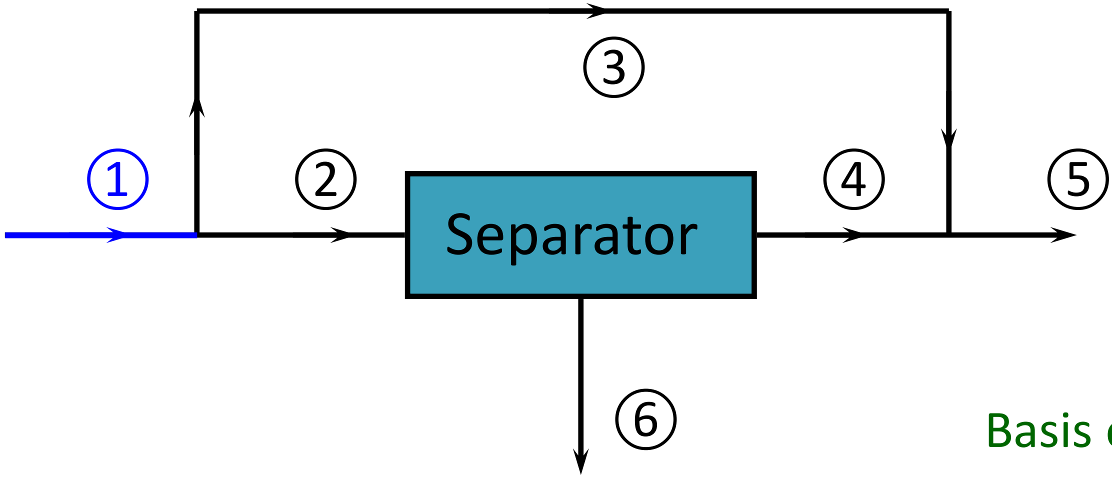
What are the flow rate of stream ⑥, and the composition of stream ⑤ ?

Assumptions Steady state operations, i.e., no accumulations.

No reactions occur.

Therefore:

$$\left[\begin{array}{c} \text{INPUT} \\ \text{across system} \\ \text{boundaries} \end{array} \right] = \left[\begin{array}{c} \text{OUTPUT} \\ \text{across system} \\ \text{boundaries} \end{array} \right]$$



Basis of Calculations : 1 hour

- Stream 1 is 1273 kg/h, 42.0 % $^w/_w$ A, 37.0 % $^w/_w$ B with the balance C

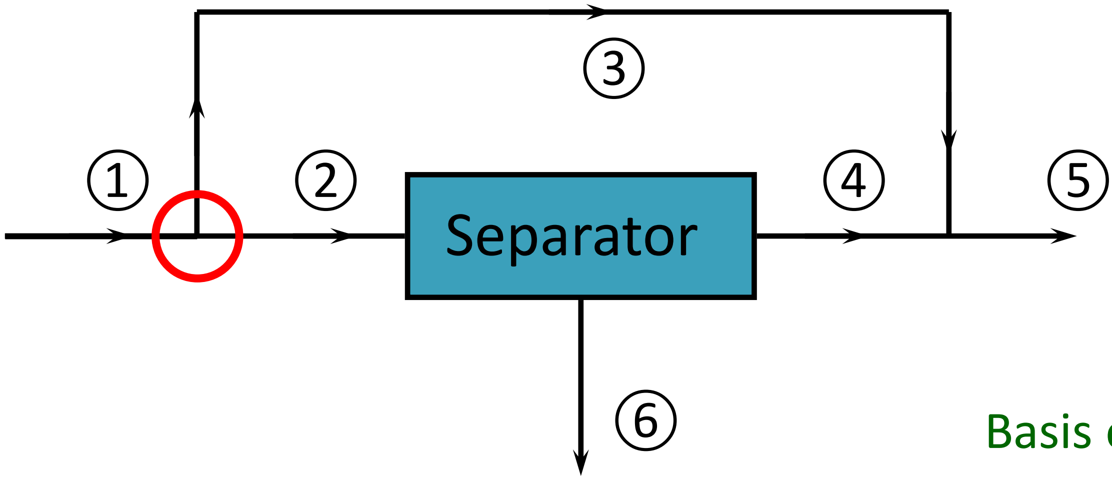
Stream 1 contains :

$A_1\,F_1 = 0.420 \textbf{ x } 1273$ = kg A

$B_1\,F_1 = 0.370 \textbf{ x } 1273$ = kg B

$C_1\,F_1 = 0.210 \textbf{ x } 1273$ = kg C

and, of course, $F_1 = 1273$ kg



Basis of Calculations : 1 hour

Total balance over the splitter is :

$$F_1 = F_2 + F_3$$

but $F_1 = 1273$ kg and $F_2 = 800$ kg

$\therefore$ $F_3 =$

$=$

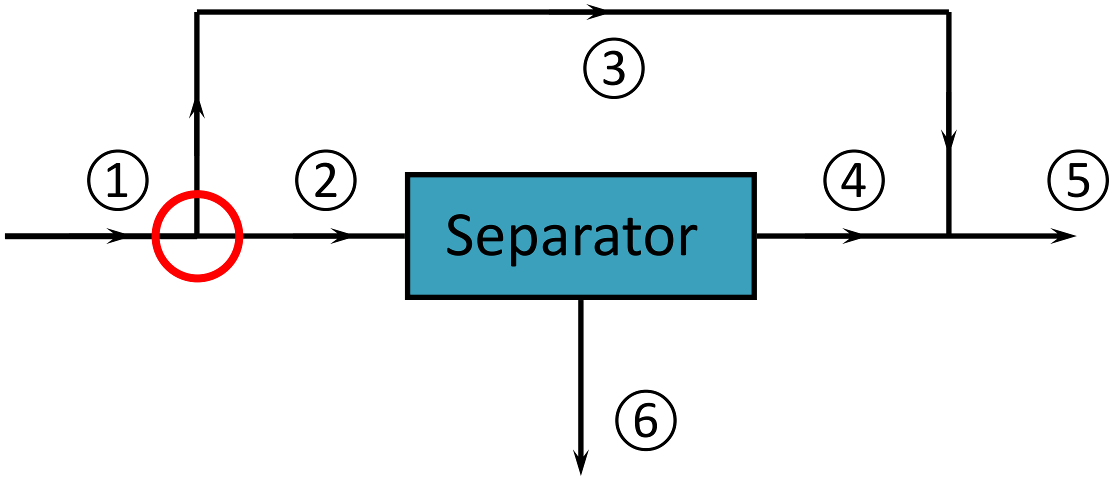
Compositions of stream 2 and 3 are the same as that of stream 1.

$\therefore$ $A_1 = A_2 = A_3 =$

$B_1 = B_2 = B_3 =$

$C_1 = C_2 = C_3 =$

where, A_i is the mass fraction of component A in stream i.



So, stream 2 contains :

$A_2 F_2 = 0.420 \times 800 =$ kg A

$B_2 F_2 = 0.370 \times 800 =$ kg B

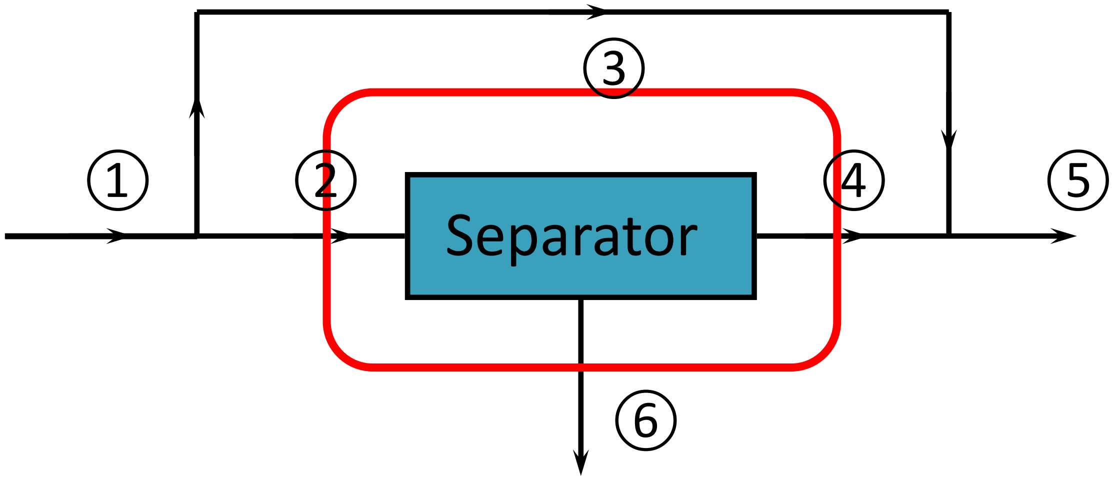
$C_2 F_2 = 0.210 \times 800 =$ kg C

and stream 3 contains :

$A_3 F_3 = 0.420 \times 473 =$ kg A

$B_3 F_3 = 0.370 \times 473 =$ kg B

$C_3 F_3 = 0.210 \times 473 =$ kg C



Now consider the separator. From the information given we may write:

$$\begin{pmatrix}\text{Mass of A}\\ \text{in ⑥}\end{pmatrix} = 0.06 \begin{pmatrix}\text{Mass of A}\\ \text{in ②}\end{pmatrix}$$

i.e. $A_6\,F_6 = 0.06 \times 336.0$

$=$

$$\begin{pmatrix}\text{Mass of B}\\ \text{in ⑥}\end{pmatrix} = 0.050 \begin{pmatrix}\text{Mass of B}\\ \text{in ②}\end{pmatrix}$$

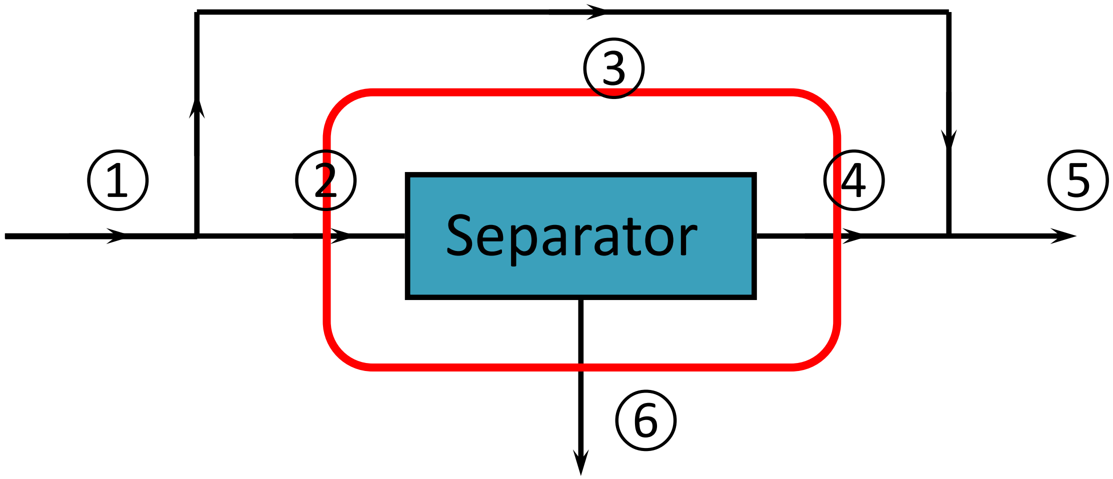
i.e. $B_6\,F_6 = 0.050 \times 296.0$

$=$

$$\begin{pmatrix}\text{Mass of C}\\ \text{in ⑥}\end{pmatrix} = 0.970 \begin{pmatrix}\text{Mass of C}\\ \text{in ②}\end{pmatrix}$$

i.e. $C_6\,F_6 = 0.970 \times 168.0$

$=$

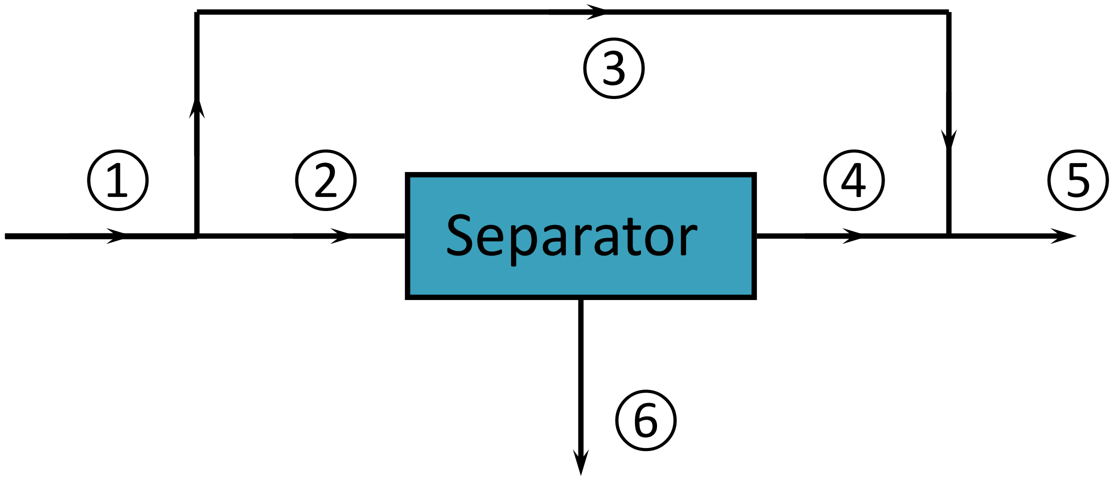


So,

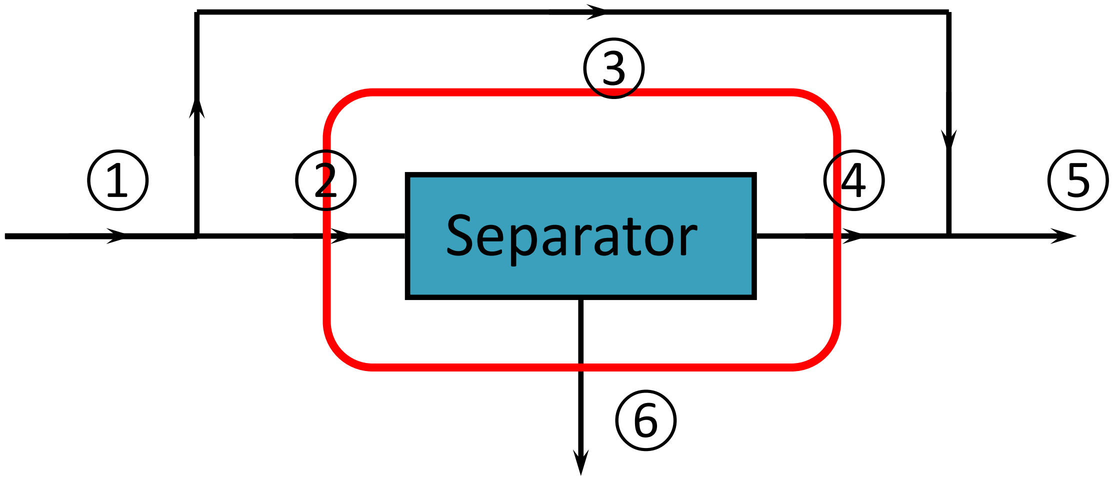
$$F_6 = \left(\begin{array}{c} \text{Mass of A} \\ \text{in ⑥} \end{array} \right) + \left(\begin{array}{c} \text{Mass of B} \\ \text{in ⑥} \end{array} \right) + \left(\begin{array}{c} \text{Mass of C} \\ \text{in ⑥} \end{array} \right)$$

$$=$$

So flow rate of stream ⑥ is



From this point there are two different strategies to solve the problem.



As a first approach we may write a component balance for A around the separator :

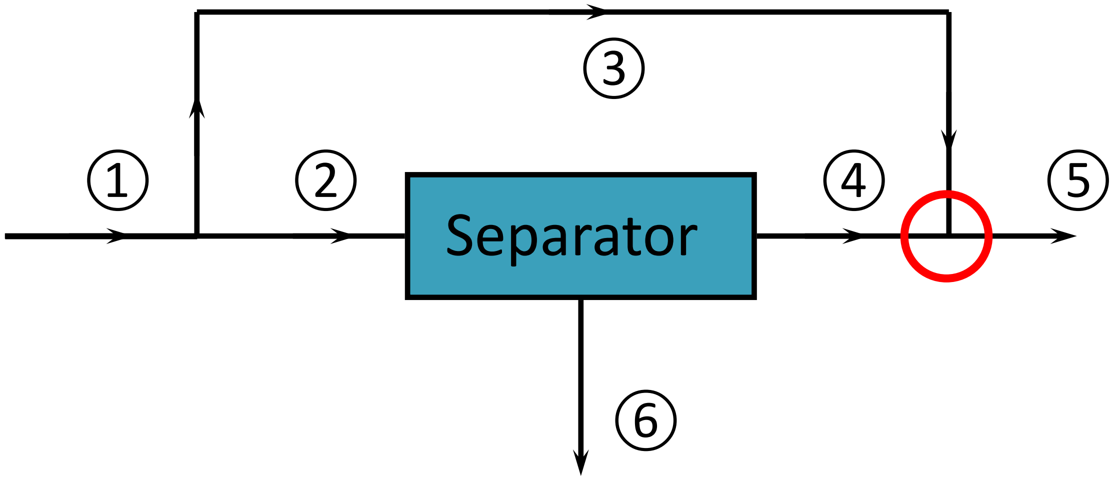
$$\left(\begin{array}{c} \text{Mass of A} \\ \text{in } ④ \end{array} \right) = \left(\begin{array}{c} \text{Mass of A} \\ \text{in } ② \end{array} \right) - \left(\begin{array}{c} \text{Mass of A} \\ \text{in } ⑥ \end{array} \right)$$

i.e. $A_4 F_4 = 336.0 - 20.16 =$

Also, we may write component balances for B and C :

$B_4 F_4 = 296.0 - 14.80 =$

and, $C_4 F_4 = 168.0 - 162.96 =$



Now consider the mixing point. A component balance around the mixing point for component A is :

$$\left(\begin{array}{c}\text{Mass of A}\\ \text{in } ⑤\end{array}\right) = \left(\begin{array}{c}\text{Mass of A}\\ \text{in } ③\end{array}\right) + \left(\begin{array}{c}\text{Mass of A}\\ \text{in } ④\end{array}\right)$$

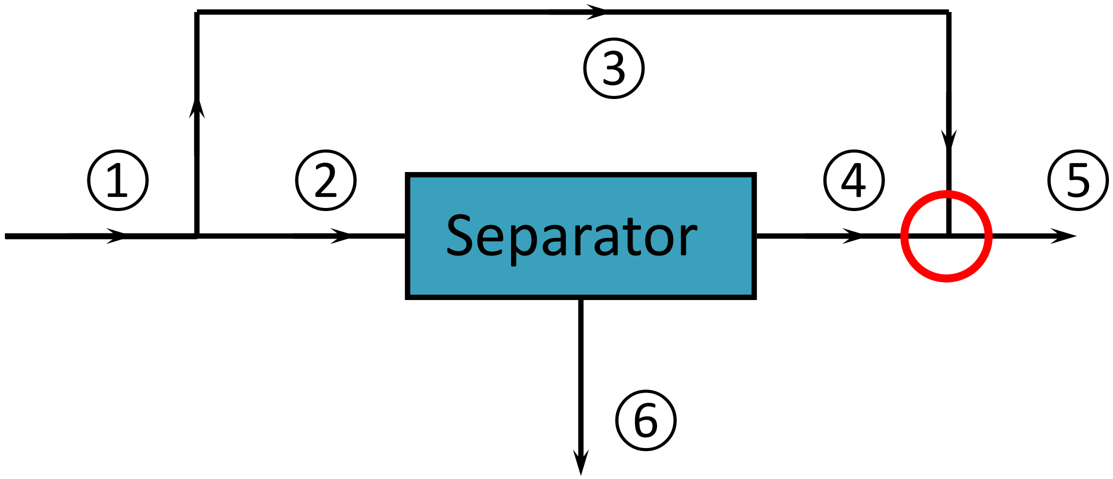
$A_5\,F_5 = 198.7 + 315.8 =$

Also, we may write component balances for B and C :

$B_5\,F_5 = 175.0 + 281.2 =$

and, $C_5\,F_5 = 99.3 + 5.0 =$

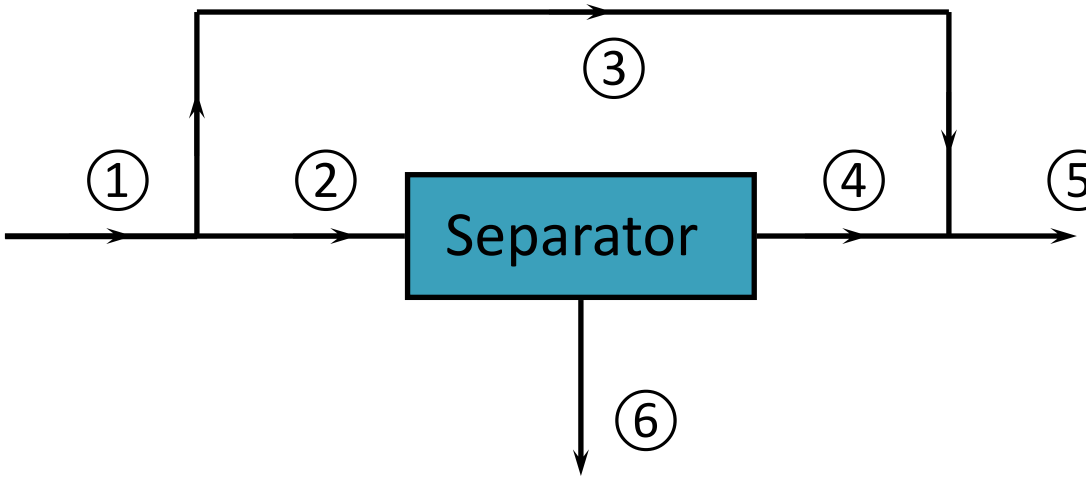
So, $F_5 = 514.5 + 456.2 + 104.3 =$



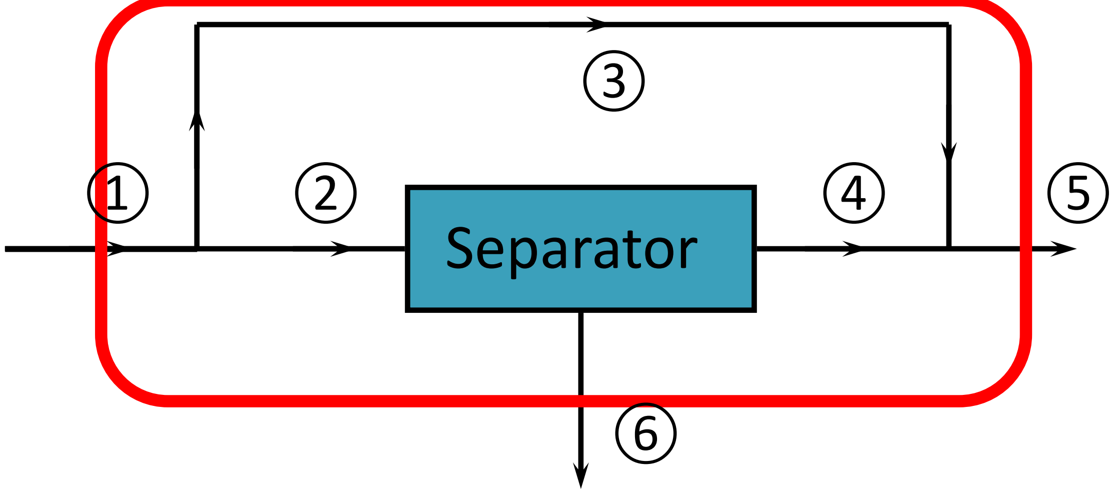
So composition of stream 5 is :

A	514.5 / 1075.0 **x** 100.0 =	% $^w/_w$
B	456.2 / 1075.0 **x** 100.0 =	% $^w/_w$
C	104.3 / 1075.0 **x** 100.0 =	% $^w/_w$
		100.0 % $^w/_w$

The composition of stream 5 is



From this point there are two different strategies to solve the problem.



As an alternative approach we could have written the overall mass balance :

$$F_1 = F_5 + F_6$$

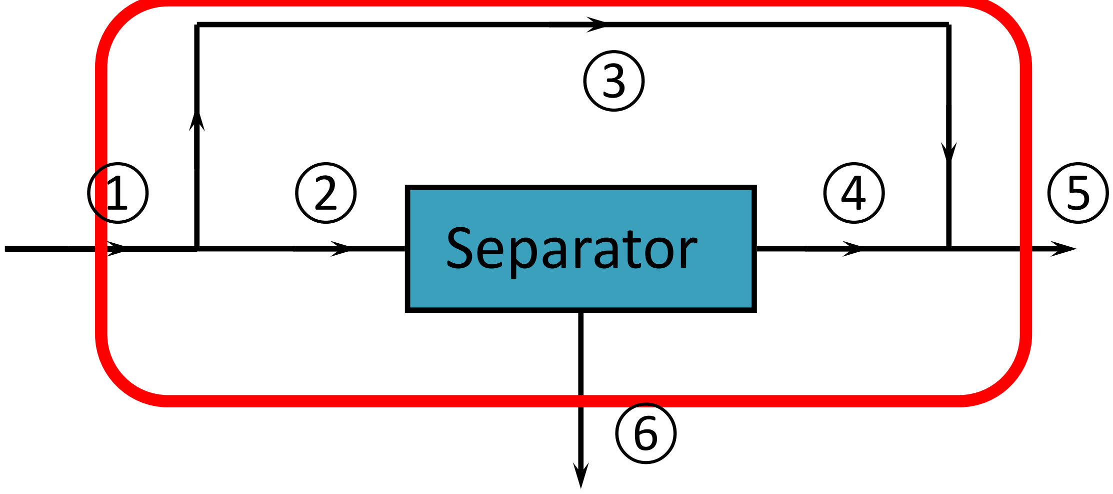
$$\therefore \quad F_5 = 1273 - 197.9$$

$$=$$

Then we may write the component balance for A:

$$F_1 A_1 = F_5 A_5 + F_6 A_6$$

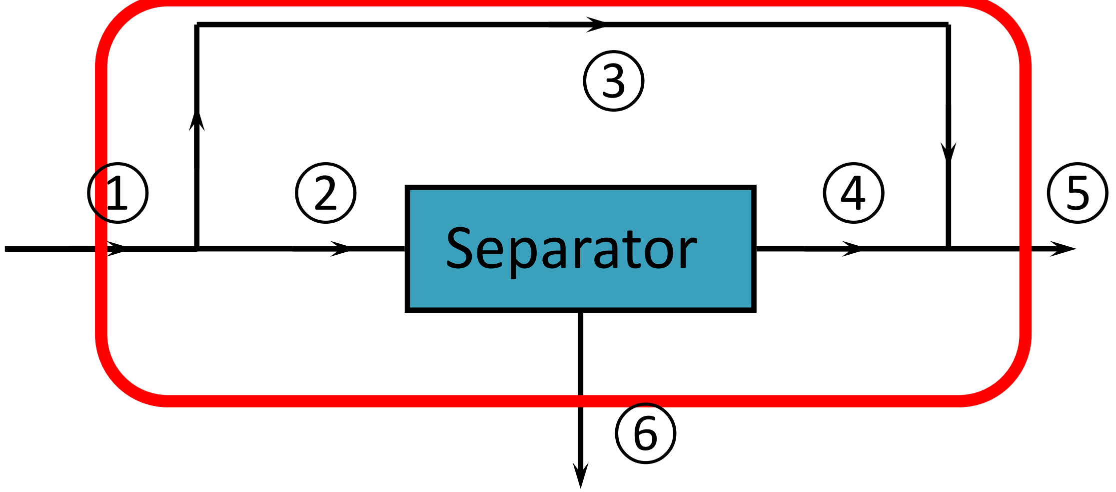
$$\therefore \quad A_5 = \frac{F_1 A_1 - F_6 A_6}{F_5} =$$



Similarly we may find B_5 and C_5 using the other component balances:

$$B_5 = \frac{F_1 B_1 - F_6 B_6}{F_5} =$$

and $$C_5 = \frac{F_1 C_1 - F_6 C_6}{F_5} =$$



The composition of stream ⑤ is

Material Balances – Module Learning Outcomes

A student is expected to be able to:

- Define the terms: system, surroundings, boundary, continuous, batch, steady-state
- Determine a suitable system boundary, and illustrate on a BFD
- Perform basic material balances (without chemical reactions):
  - Total, component, and elemental balances
  - Around single and multiple units
  - Calculate flow rates and composition, as required
- Define and treat appropriately in a balance: tie component, mixers, splitters, separators, feeds, product streams, recycles, bypasses, makeup