The University of Manchester

TWO HOURS

Department of Mechanical, Aerospace & Civil Engineering

UNIVERSITY OF MANCHESTER

ENGINEERING THERMODYNAMICS

SOLUTIONS

xxxxxx 2021

09:00 – 11:00

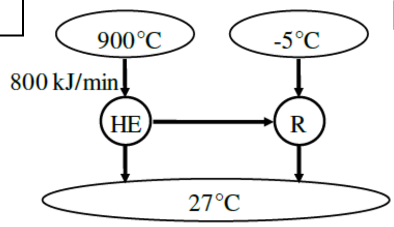
Special Instruction(s):

- ANSWER **ANY 5 QUESTIONS** OUT OF THE **TOTAL 6 QUESTIONS**
- A FORMULA SHEET IS PROVIDED AT THE END OF THE PAEPR.
- MAKE USE OF THE ‘**PROPERTY TABLES AND CHARTS (SI UNITS)**’, GIVEN AS APPENDIX OF ‘THERMODYNAMICS-AN ENGINEERING APPROACH (SI VERSION) (9TH ED.) (MCGRAW HILL) CENGEL, BOLES AND KANOGLU.

Q1.

The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

(a) **[5 marks]**



(b)

$$\eta_{\text{th, max}} = \eta_{\text{th, C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{1173\text{ K}} = 0.744$$

Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{\text{net, out}} = \eta_{\text{th}}\dot{Q}_H = (0.744)(800\text{ kJ/min}) = 595.2\text{ kJ/min}$$

which is also the power input to the refrigerator, $\dot{W}_{\text{net, in}}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is **[6 marks]**

(c)

$$\text{COP}_{\text{R, rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(27+273\text{ K})/(-5+273\text{ K})-1} = 8.37$$

Then the rate of heat removal from the refrigerated space becomes

$$\dot{Q}_{L,\text{R}} = \left(\text{COP}_{\text{R, rev}}\right)\left(\dot{W}_{\text{net, in}}\right) = (8.37)(595.2\text{ kJ/min}) = \mathbf{4982\text{ kJ/min}}$$ **[6 marks]**

(d) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,\text{HE}}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,\text{R}}$),

$$\dot{Q}_{L,\text{HE}} = \dot{Q}_{H,\text{HE}} - \dot{W}_{\text{net, out}} = 800 - 595.2 = 204.8\text{ kJ/min}$$

$$\dot{Q}_{H,\text{R}} = \dot{Q}_{L,\text{R}} + \dot{W}_{\text{net, in}} = 4982 + 595.2 = 5577.2\text{ kJ/min}$$

and

$$\dot{Q}_{\text{ambient}} = \dot{Q}_{L,\text{HE}} + \dot{Q}_{H,\text{R}} = 204.8 + 5577.2 = \mathbf{5782\text{ kJ/min}}$$ **[8 marks]**

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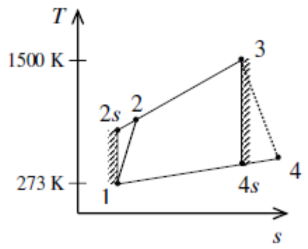
Q2.

A simple Brayton cycle with air as the working fluid operates at a specified pressure ratio and between the specified temperature and pressure limits. The cycle is to be sketched on the *T*-*s* cycle and the volume flow rate of the air into the compressor is to be determined. Also, the effect of compressor inlet temperature on the mass flow rate and the net power output are to be investigated.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air are given as $c_v = 0.718$ kJ/kg·K, $c_p = 1.005$ kJ/kg·K, $R = 0.287$ kJ/kg·K, $k = 1.4$.

(a) **[4 marks]**



(b) For the compression process,

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (273\text{ K})(7)^{0.4/1.4} = 476.0\text{ K}$$

$$\eta_{\text{Comp}} = \frac{\dot{W}_{\text{Comp},s}}{\dot{W}_{\text{Comp}}} = \frac{\dot{m}c_p(T_{2s}-T_1)}{\dot{m}c_p(T_2-T_1)} = \frac{T_{2s}-T_1}{T_2-T_1}$$

$$0.80 = \frac{476.0-273}{T_2-273} \longrightarrow T_2 = 526.8\text{ K}$$

[4 marks]

(c) For the expansion process,

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1500\text{ K})\left(\frac{1}{7}\right)^{0.4/1.4} = 860.3\text{ K}$$

$$\eta_{\text{Turb}} = \frac{\dot{W}_{\text{Turb}}}{\dot{W}_{\text{Turb},s}} = \frac{\dot{m}c_p(T_3-T_4)}{\dot{m}c_p(T_3-T_{4s})} = \frac{T_3-T_4}{T_3-T_{4s}}$$

$$0.90 = \frac{1500-T_4}{1500-860.3} \longrightarrow T_4 = 924.3\text{ K}$$

[4 marks]

(d) Given the net power, the mass flow rate is determined from

$$\dot{W}_{\text{net}} = \dot{W}_{\text{Turb}} - \dot{W}_{\text{Comp}} = \dot{m}c_p(T_3-T_4) - \dot{m}c_p(T_2-T_1)$$

$$\dot{W}_{\text{net}} = \dot{m}c_p\left[(T_3-T_4)-(T_2-T_1)\right]$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{c_p\left[(T_3-T_4)-(T_2-T_1)\right]}$$

$$= \frac{150{,}000\text{ kW}}{(1.005\text{ kJ/kg}\cdot\text{K})\left[(1500-924.3)-(526.8-273)\right]}$$

$$= 463.7\text{ kg/s}$$

The specific volume and the volume flow rate at the inlet of the compressor are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{ kJ/kg}\cdot\text{K})(273\text{ K})}{100\text{ kPa}} = 0.7835\text{ m}^3/\text{kg}$$

$$\dot{V}_1 = \dot{m}v_1 = (463.7\text{ kg/s})(0.7835\text{ m}^3/\text{kg}) = \mathbf{363.2\,m^3/s}$$

[8 marks]

(e) For a fixed compressor inlet velocity and flow area, when the compressor inlet temperature increases, the specific volume increases since $v = \frac{RT}{P}$. When specific volume increases, the mass flow rate decreases since $\dot{m} = \frac{\dot{V}}{v}$. Note that volume flow rate is the same since inlet velocity and flow area are fixed ($\dot{V} = AV$). When mass flow rate decreases, the net power decreases since $\dot{W}_{\text{net}} = \dot{m}(w_{\text{Turb}} - w_{\text{Comp}})$. Therefore, when the inlet temperature increases, both mass flow rate and the net power decrease. **[5 marks]**

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Q3.

Assumptions All gases will be modeled as ideal gases with constant specific heats.

Properties The molar masses of CH_4, C_3H_8, and C_4H_{10} are 16.0, 44.0, and 58.0 kg/kmol, respectively (Table A-1).

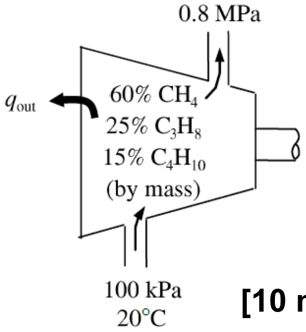
Analysis The mole numbers of each component are

(a)

$$N_{CH4} = \frac{m_{CH4}}{M_{CH4}} = \frac{60 \text{ kg}}{16 \text{ kg/kmol}} = 3.75 \text{ kmol}$$

$$N_{C3H8} = \frac{m_{C3H8}}{M_{C3H8}} = \frac{25 \text{ kg}}{44 \text{ kg/kmol}} = 0.5682 \text{ kmol}$$

$$N_{C4H10} = \frac{m_{C4H10}}{M_{C4H10}} = \frac{15 \text{ kg}}{58 \text{ kg/kmol}} = 0.2586 \text{ kmol}$$



The mole number of the mixture is

$$N_m = N_{CH4} + N_{C3H8} + N_{C4H10}$$
$$= 3.75 + 0.5682 + 0.2586 = 4.5768 \text{ kmol}$$

[10 marks]

(b) The apparent molecular weight of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{100 \text{ kg}}{4.5768 \text{ kmol}} = 21.85 \text{ kg/kmol}$$

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol}\cdot\text{K}}{21.85 \text{ kg/kmol}} = 0.3805 \text{ kJ/kg}\cdot\text{K}$$

[5 marks]

(c) For a reversible, isothermal process, the work input is

$$w_{in} = RT\ln\left(\frac{P_2}{P_1}\right) = (0.3805 \text{ kJ/kg}\cdot\text{K})(293 \text{ K})\ln\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = \mathbf{232\ kJ/kg}$$

An energy balance on the control volume gives

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 + \dot{W}_{in} = \dot{m}h_2 + \dot{Q}_{out}$$

$$\dot{W}_{in} - \dot{Q}_{out} = \dot{m}(h_2 - h_1)$$

$$w_{in} - q_{out} = c_p(T_2 - T_1) = 0 \quad \text{since} \quad T_2 = T_1$$

$$w_{in} = q_{out}$$

That is,

$$q_{out} = w_{in} = \mathbf{232\ kJ/kg}$$

[10 marks]

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Q4.

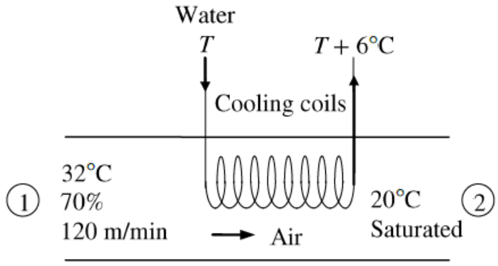
(a) ***Assumptions*** **1** This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process. **2** Dry air and water vapor are ideal gases. **3** The kinetic and potential energy changes are negligible.

Analysis (*a*) The saturation pressure of water at 32°C is 4.76 kPa (Table A-4). Then the dew point temperature of the incoming air stream at 32°C becomes

$$T_{dp} = T_{sat\,@\,P_v} = T_{sat\,@\,0.7\times4.76\text{ kPa}} = 25.8°\text{C} \quad \text{(Table A-5)}$$

since air is cooled to 20°C, which is below its dew point temperature, some of the moisture in the air will condense. The amount of moisture in the air decreases due to dehumidification $(\omega_2 < \omega_1)$. The inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. Then the properties of the air at both states are determined from the psychrometric chart (Fig. A-31 or EES) to be

$$h_1 = 86.35 \text{ kJ/kg dry air}$$
$$\omega_1 = 0.02114 \text{ kg } H_2O\text{/kg dry air}$$
$$\upsilon_1 = 0.8939 \text{ m}^3\text{/kg dry air}$$



and

$$h_2 = 57.43 \text{ kJ/kg dry air}$$
$$\omega_2 = 0.0147 \text{ kg } H_2O\text{/kg dry air}$$
$$\upsilon_2 = 0.8501 \text{ m}^3\text{/kg dry air}$$

Also, $h_w \cong h_{f\,@\,20°C} = 83.91 \text{ kJ/kg} \quad \text{(Table A-4)}$

Then,

$$\dot{V}_1 = V_1 A_1 = V_1 \frac{\pi D^2}{4} = (120 \text{ m/min})\left(\frac{\pi(0.4 \text{ m})^2}{4}\right) = 15.08 \text{ m}^3\text{/min}$$

$$\dot{m}_{a1} = \frac{\dot{V}_1}{\upsilon_1} = \frac{15.08 \text{ m}^3\text{/min}}{0.8939 \text{ m}^3\text{/kg dry air}} = 16.87 \text{ kg/min}$$

[7 marks]

(b) Applying the water mass balance and the energy balance equations to the combined cooling and dehumidification section (excluding the water),

Water Mass Balance:

$$\sum \dot{m}_{w,i} = \sum \dot{m}_{w,e} \longrightarrow \dot{m}_{a1}\omega_1 = \dot{m}_{a2}\omega_2 + \dot{m}_w$$

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_2) = (16.87 \text{ kg/min})(0.02114 - 0.0147) = 0.1086 \text{ kg/min}$$

Energy Balance:

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}} = 0 \longrightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e + \dot{Q}_{out} \longrightarrow Q_{out} = \dot{m}_{a1}h_1 - (\dot{m}_{a2}h_2 + \dot{m}_w h_w) = \dot{m}_a(h_1 - h_2) - \dot{m}_w h_w$$

$$\dot{Q}_{out} = (16.87 \text{ kg/min})(86.35 - 57.43) \text{ kJ/kg} - (0.1086 \text{ kg/min})(83.91 \text{ kJ/kg}) = \mathbf{478.7 \text{ kJ/min}}$$

[6 marks]

(c) Noting that the heat lost by the air is gained by the cooling water, the mass flow rate of the cooling water is determined from

$$\dot{Q}_{\text{cooling water}} = \dot{m}_{\text{cooling water}} \Delta h = \dot{m}_{\text{cooling water}} c_p \Delta T$$

$$\dot{m}_{\text{cooling water}} = \frac{\dot{Q}_w}{c_p \Delta T} = \frac{478.7 \text{ kJ/min}}{(4.18 \text{ kJ/kg}\cdot°\text{C})(6°\text{C})} = \mathbf{19.09 \text{ kg/min}}$$

[6 marks]

(d) The exit velocity is determined from the conservation of mass of dry air,

$$\dot{m}_{a1} = \dot{m}_{a2} \longrightarrow \frac{\dot{V}_1}{\upsilon_1} = \frac{\dot{V}_2}{\upsilon_2} \longrightarrow \frac{V_1 A}{\upsilon_1} = \frac{V_2 A}{\upsilon_2}$$

$$V_2 = \frac{\upsilon_2}{\upsilon_1} V_1 = \frac{0.8501}{0.8939}(120 \text{ m/min}) = \mathbf{114 \text{ m/min}}$$

[6 marks]

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