Thermodynamics – MECH32102

Solution to Tutorial Questions

Gas Mixtures

Q1

Problem Statement

A gas mixture has the following composition on a mole basis: 60 percent N_2 and 40 percent CO_2. Determine the gravimetric analysis of the mixture, its molar mass, and the gas constant.

Solution

The molar fractions of the constituents of a gas mixture are given. The gravimetric analysis of the mixture, its molar mass, and gas constant are to be determined.

Properties The molar masses of N_2, and CO_2 are 28.0 and 44.0 kg/kmol, respectively

Analysis Consider 100 kmol of mixture. Then the mass of each component and the total mass are

$$N_{\mathrm{N_2}} = 60\ \mathrm{kmol} \longrightarrow m_{\mathrm{N_2}} = N_{\mathrm{N_2}} M_{\mathrm{N_2}} = (60\ \mathrm{kmol})(28\ \mathrm{kg/kmol}) = 1680\ \mathrm{kg}$$

$$N_{\mathrm{CO_2}} = 40\ \mathrm{kmol} \longrightarrow m_{\mathrm{CO_2}} = N_{\mathrm{CO_2}} M_{\mathrm{CO_2}} = (40\ \mathrm{kmol})(44\ \mathrm{kg/kmol}) = 1760\ \mathrm{kg}$$

$$m_m = m_{\mathrm{N_2}} + m_{\mathrm{CO_2}} = 1680\ \mathrm{kg} + 1760\ \mathrm{kg} = 3440\ \mathrm{kg}$$

Then the mass fraction of each component (gravimetric analysis) becomes

mole

60% N_2
40% CO_2

$$\mathrm{mf}_{\mathrm{N_2}} = \frac{m_{\mathrm{N_2}}}{m_m} = \frac{1680\ \mathrm{kg}}{3440\ \mathrm{kg}} = 0.488 \ \text{ or } \ \mathbf{48.8\%}$$

$$\mathrm{mf}_{\mathrm{CO_2}} = \frac{m_{\mathrm{CO_2}}}{m_m} = \frac{1760\ \mathrm{kg}}{3440\ \mathrm{kg}} = 0.512 \ \text{ or } \ \mathbf{51.2\%}$$

The molar mass and the gas constant of the mixture are determined from their definitions,

$$M_m = \frac{m_m}{N_m} = \frac{3{,}440\ \mathrm{kg}}{100\ \mathrm{kmol}} = \mathbf{34.40\ kg/kmol}$$

and

$$R_m = \frac{R_u}{M_m} = \frac{8.314\ \mathrm{kJ/kmol \cdot K}}{34.4\ \mathrm{kg/kmol}} = \mathbf{0.242\ kJ/kg \cdot K}$$

Q2

Problem Statement

A gas mixture at 300 K and 200 kPa consists of 1 kg of CO_2 and 3 kg of CH_4. Determine the partial pressure of each gas and the apparent molar mass of the gas mixture.

Solution

The masses of the constituents of a gas mixture at a specified pressure and temperature are given. The partial pressure of each gas and the apparent molar mass of the gas mixture are to be determined.

Assumptions Under specified conditions both CO_2 and CH_4 can be treated as ideal gases, and the mixture as an ideal gas mixture.

Properties The molar masses of CO_2 and CH_4 are 44.0 and 16.0 kg/kmol, respectively

Analysis The mole numbers of the constituents are:

$$m_{CO2} = 1 \text{ kg} \rightarrow N_{CO2} = \frac{m_{CO2}}{M_{CO2}} = \frac{1 \text{ kg}}{44 \text{ kg/kmol}} = 0.0227 \text{ kmol}$$

$$m_{CH4} = 3 \text{ kg} \rightarrow N_{CH4} = \frac{m_{CH4}}{M_{CH4}} = \frac{3 \text{ kg}}{16 \text{ kg/kmol}} = 0.1875 \text{ kmol}$$

$$N_m = N_{CO_2} + N_{CH_4} = 0.0227 \text{ kmol} + 0.1875 \text{ kmol} = 0.2102 \text{ kmol}$$

$$y_{CO_2} = \frac{N_{CO_2}}{N_m} = \frac{0.0227 \text{ kmol}}{0.2102 \text{ kmol}} = 0.108$$

$$y_{CH_4} = \frac{N_{CH_4}}{N_m} = \frac{0.1875 \text{ kmol}}{0.2102 \text{ kmol}} = 0.892$$

1 kg CO_2
3 kg CH_4

300 K
200 kPa

Then the partial pressures become:

$$P_{CO_2} = y_{CO_2} P_m = (0.108)(200 \text{ kPa}) = \mathbf{21.6 \text{ kPa}}$$

$$P_{CH_4} = y_{CH_4} P_m = (0.892)(200 \text{ kPa}) = \mathbf{178.4 \text{ kPa}}$$

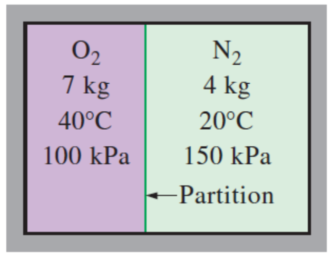
The apparent molar mass of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{4 \text{ kg}}{0.2102 \text{ kmol}} = \mathbf{19.03 \text{ kg / kmol}}$$

Q3

Problem Statement

An insulated rigid tank is divided into two compartments by a partition. One compartment contains 7 kg of oxygen gas at 40°C and 100 kPa, and the other compartment contains 4 kg of nitrogen gas at 20°C and 150 kPa. Now the partition is removed, and the two gases are allowed to mix. Determine (*a*) the mixture temperature and (*b*) the mixture pressure after equilibrium has been established.



Solution

A rigid tank contains two gases separated by a partition. The pressure and temperature of the mixture are to be determined after the partition is removed.

Assumptions

1. We assume both gases to be ideal gases, and their mixture to be an ideal-gas mixture. This assumption is reasonable since both the oxygen and nitrogen are well above their critical temperatures and well below their critical pressures. **2.** The tank is insulated and thus there is no heat transfer. **3.** There are no other forms of work involved.

Properties

The constant volume specific heats of O_2 and N_2 at room temperature are 0.658 kJ/kg.°C and 0.743 kJ/kg.°C, respectively.

Solution continued on next page...

Analysis

We take the entire contents of the tank (both compartments) as the system. This is a *closed system* since no mass crosses the boundary during the process. We note that the volume of a rigid tank is constant and thus there is no boundary work done.

(*a*) Noting that there is no energy transfer to or from the tank, the energy balance for the system can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$
$$0 = \Delta U = \Delta U_{O_2} + \Delta U_{N_2}$$
$$0 = \left[m c_v \left(T_m - T_1 \right) \right]_{O_2} + \left[m c_v \left(T_m - T_1 \right) \right]_{N_2}$$

Using c_v values at room temperature, the final temperature of the mixture is determined to be

$$(7 \text{ kg})(0.658 \text{ kJ/kg}\cdot{}^\circ\text{C})(T_m - 40)^\circ\text{C} + (4 \text{ kg})(0.743 \text{ kJ/kg}\cdot{}^\circ\text{C})(T_m - 20)^\circ\text{C} = 0$$
$$T_m = \mathbf{32.2^\circ C}$$

(*b*) The final pressure of the mixture is determined from the ideal-gas relation

$$P_m V_m = N_m R_u T_m$$

where

$$N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{7 \text{ kg}}{32 \text{ kg/kmol}} = 0.219 \text{ kmol}$$
$$N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{4 \text{ kg}}{28 \text{ kg/kmol}} = 0.143 \text{ kmol}$$
$$N_m = N_{N_2} + N_{O_2} = 0.219 \text{ kmol} + 0.143 \text{ kmol} = 0.362 \text{ kmol}$$

and

$$V_{O_2} = \left(\frac{N R_u T_1}{P_1} \right)_{O_2} = \frac{(0.219 \text{ kmol})(8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(313 \text{ K})}{100 \text{ kPa}} = 5.70 \text{ m}^3$$
$$V_{N_2} = \left(\frac{N R_u T_1}{P_1} \right)_{N_2} = \frac{(0.143 \text{ kmol})(8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(293 \text{ K})}{150 \text{ kPa}} = 2.32 \text{ m}^3$$
$$V_{\text{m}} = V_{O_2} + V_{N_2} = 5.70 \text{ m}^3 + 2.32 \text{ m}^3 = 8.02 \text{ m}^3$$

Thus,

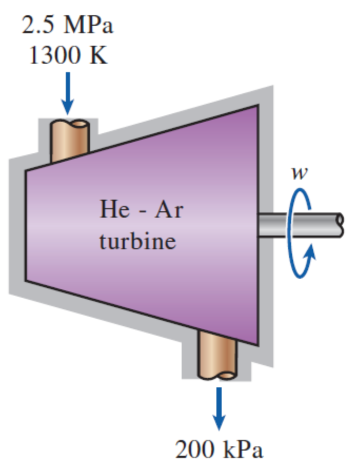
$$P_m = \frac{N_m R_u T_m}{V_m} = \frac{(0.362 \text{ kmol})(8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(305.2 \text{ K})}{8.02 \text{ m}^3} = \mathbf{114.5 \text{ kPa}}$$

Discussion We could also determine the mixture pressure by using $P_m V_m = N_m R_u T_m$ where R_m is the apparent gas constant of the mixture. This would require a knowledge of mixture composition in terms of mass or mole fractions.

Q4

Problem Statement

An equimolar mixture of helium and argon gases is to be used as the working fluid in a closed-loop gas-turbine cycle. The mixture enters the turbine at 2.5 MPa and 1300 K and expands isentropically to a pressure of 200 kPa. Determine the work output of the turbine per unit mass of the mixture.



Solution

An equimolar mixture of helium and argon gases expands in a turbine. The isentropic work output of the turbine is to be determined.

Assumptions

1. Under specified conditions both He and Ar can be treated as ideal gases, and the mixture as an ideal gas mixture. **2** The turbine is insulated and thus there is no heat transfer. **3** This is a steady-flow process. **4** The kinetic and potential energy changes are negligible.

Properties The molar masses and specific heats of He and Ar are 4.0 kg/kmol, 40.0 kg/kmol, 5.1926 kJ/kg·°C, and 0.5203 kJ/kg·°C, respectively.

Solution continued on next page...

Analysis The C_p and k values of this equimolar mixture are determined from

$$M_m = \sum y_i M_i = y_{\mathrm{He}} M_{\mathrm{He}} + y_{\mathrm{Ar}} M_{\mathrm{Ar}} = 0.5 \times 4 + 0.5 \times 40 = 22 \text{ kg/kmol}$$

$$\mathrm{mf}_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = \frac{y_i M_i}{M_m}$$

$$\begin{aligned} c_{p,m} &= \sum \mathrm{mf}_i\, c_{p,i} = \frac{y_{\mathrm{He}} M_{\mathrm{He}}}{M_m} c_{p,\mathrm{He}} + \frac{y_{\mathrm{Ar}} M_{\mathrm{Ar}}}{M_m} c_{p,\mathrm{Ar}} \\ &= \frac{0.5 \times 4 \text{ kg/kmol}}{22 \text{ kg/kmol}} (5.1926 \text{ kJ/kg} \cdot \text{K}) + \frac{0.5 \times 40 \text{ kg/kmol}}{22 \text{ kg/kmol}} (0.5203 \text{ kJ/kg} \cdot \text{K}) \\ &= 0.945 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

and

$$k_m = 1.667 \quad \text{since } k = 1.667 \text{ for both gases.}$$

Therefore, the He-Ar mixture can be treated as a single ideal gas with the properties above. For isentropic processes,

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = (1300 \text{ K}) \left(\frac{200 \text{ kPa}}{2500 \text{ kPa}} \right)^{0.667/1.667} = 473.2 \text{ K}$$

From an energy balance on the turbine,

$$\begin{aligned} \dot{E}_{\text{in}} - \dot{E}_{\text{out}} &= \Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0 \\ \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ h_1 &= h_2 + w_{\text{out}} \\ w_{\text{out}} &= h_1 - h_2 \\ w_{\text{out}} &= c_p (T_1 - T_2) = (0.945 \text{ kJ/kg} \cdot \text{K})(1300 - 473.2) \text{K} = \mathbf{781.3 \text{ kJ/kg}} \end{aligned}$$

Q5

Problem Statement

A piston–cylinder device contains a mixture of 0.5 kg of H_2 and 1.2 kg of N_2 at 100 kPa and 300 K. Heat is now transferred to the mixture at constant pressure until the volume is doubled. Assuming constant specific heats at the average temperature, determine (*a*) the heat transfer and (*b*) the entropy change of the mixture.

Solution

A piston-cylinder device contains a gas mixture at a given state. Heat is transferred to the mixture. The amount of heat transfer and the entropy change of the mixture are to be determined.

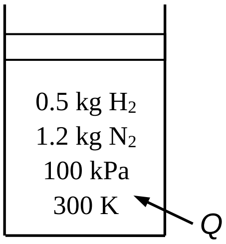
Assumptions

1. Under specified conditions both H_2 and N_2 can be treated as ideal gases, and the mixture as an ideal gas mixture. **2.** Kinetic and potential energy changes are negligible.

Properties The constant pressure specific heats of H_2 and N_2 at 450 K are 14.501 kJ/kg.K and 1.049 kJ/kg.K, respectively.

Analysis (*a*) Noting that $P_2 = P_1$ and $V_2 = 2V_1$

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \longrightarrow T_2 = \frac{2V_1}{V_1} T_1 = 2T_1 = (2)(300\text{ K}) = 600\text{ K}$$



From the closed system energy balance relation,

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$
$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U \rightarrow Q_{\text{in}} = \Delta H$$

since W_b and ΔU combine into ΔH for quasi-equilibrium constant pressure processes.

$$\begin{aligned} Q_{\text{in}} &= \Delta H = \Delta H_{H_2} + \Delta H_{N_2} = \left[mc_{p,\text{avg}}(T_2 - T_1)\right]_{H_2} + \left[mc_{p,\text{avg}}(T_2 - T_1)\right]_{N_2} \\ &= (0.5\text{ kg})(14.501\text{ kJ/kg}\cdot\text{K})(600 - 300)\text{K} + (1.2\text{ kg})(1.049\text{ kJ/kg}\cdot\text{K})(600 - 300)\text{K} \\ &= \mathbf{2553\ kJ} \end{aligned}$$

Solution continued on next page...

(*b*) Noting that the total mixture pressure, and thus the partial pressure of each gas, remains constant, the entropy change of the mixture during this process is

$$\Delta S_{H_2} = \left[m(s_2 - s_1)\right]_{H_2} = m_{H_2}\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}^{\nearrow 0}\right)_{H_2} = m_{H_2}\left(c_p \ln\frac{T_2}{T_1}\right)_{H_2}$$

$$= (0.5\text{ kg})(14.501\text{ kJ/kg}\cdot\text{K})\ln\frac{600\text{ K}}{300\text{ K}} = 5.026\text{ kJ/K}$$

$$\Delta S_{N_2} = \left[m(s_2 - s_1)\right]_{N_2} = m_{N_2}\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}^{\nearrow 0}\right)_{N_2} = m_{N_2}\left(c_p \ln\frac{T_2}{T_1}\right)_{N_2}$$

$$= (1.2\text{ kg})(1.049\text{ kJ/kg}\cdot\text{K})\ln\frac{600\text{ K}}{300\text{ K}} = 0.8725\text{ kJ/K}$$

$$\Delta S_{\text{total}} = \Delta S_{H_2} + \Delta S_{N_2} = 5.026\text{ kJ/K} + 0.8725\text{ kJ/K} = \mathbf{5.90\text{ kJ / K}}$$