

**TWO HOURS**

**Department of Mechanical, Aerospace & Civil Engineering**

**UNIVERSITY OF MANCHESTER**

**ENGINEERING THERMODYNAMICS**

**xxxxxx 2022**

**09:00 – 11:00**

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**Special Instruction(s):**

- ANSWER **ALL** QUESTIONS
  - A FORMULA SHEET IS PROVIDED AT THE END OF THE PAPER.
  - LIMITED THERMODYNAMIC PROPERTY TABLES ARE ALSO PROVIDED AT THE END OF THE PAPER (TAKEN FROM '**PROPERTY TABLES AND CHARTS (SI UNITS)**', 'THERMODYNAMICS-AN ENGINEERING APPROACH (SI VERSION) (9TH ED.) (MCGRAW HILL) CENGEL, BOLES AND KANOGLU.)
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**Q1.**

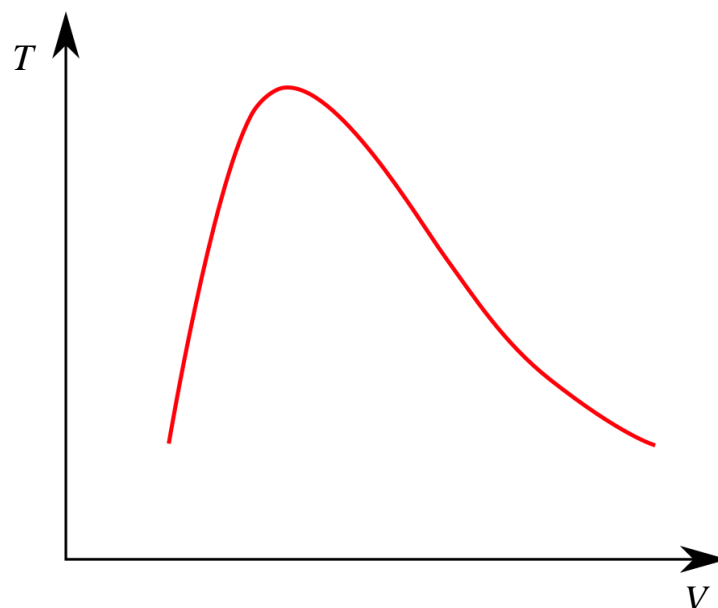
**(a)** A Temperature-Volume (T-v) diagram is typically used to aid analysis of phase change processes. One such diagram is shown in Figure Q1 for a generic pure substance. Use Figure Q1 to indicate:

- (i) the critical point, **[1 mark]**
- (ii) the saturated liquid and saturated vapour lines, **[2 marks]**
- (iii) the compressed liquid and superheated vapour regions, **[2 marks]**
- (iv) the path of a constant-pressure phase-change process that passes below the critical point. **[3 marks]**

**(b)** A piston-cylinder device initially contains 50 L of liquid water at 40°C and 200 kPa. The liquid is heated at constant pressure until it is saturated vapour.

- (i) Calculate the mass and final temperature of the water. **[6 marks]**
- (ii) Determine the total change in enthalpy. **[4 marks]**
- (iii) Indicate what the process may look like on the T-v diagram constructed in Figure Q1. **[2 marks]**

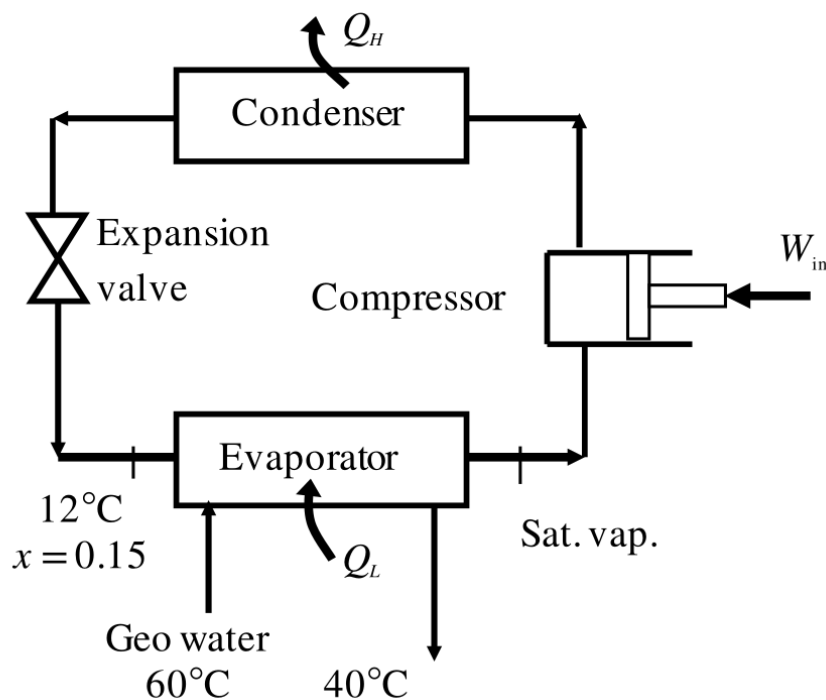
**(c)** The pressure gage on a 2.5 m<sup>3</sup> oxygen tank reads 500 kPa. Determine the mass of oxygen in the tank if the temperature is 28°C and the atmospheric pressure is 97 kPa. You may treat the Oxygen as an ideal gas. **[5 marks]**



**Figure Q1**

**Q2.** The heat pump shown in **Figure Q2** uses R-134a as the working fluid to maintain the temperature within a space at 25°C. Heat is absorbed from geothermal liquid water that enters the evaporator at 60 °C at a rate of 0.065 kg/s and leaves at 40°C. The refrigerant enters the evaporator at 12°C with a quality of 15% and leaves at the same pressure as saturated vapour. The compressor consumes 1.6 kW of power. You may also assume that the heat pump operates steadily and that changes in potential and kinetic energy are negligible. Determine:

- (a) the rate of heat transferred from the water,  $Q_L$  in kW [8 marks]
- (b) the mass flow rate of the refrigerant. [8 marks]
- (c) the rate of heat supply to the space,  $Q_H$  in kW [3 marks]
- (d) the Coefficient of Performance (COP) [2 marks]
- (e) the power input to the compressor for the same rate of heat supply if the heat pump can be considered reversible [4 marks]



**Figure Q2**

- Q3.** Consider an ideal Otto cycle that has a compression ratio of 8. At the beginning of the compression process, air is at 95 kPa and 27 °C. During the constant-volume heat-addition process, 750 kJ/kg of heat is transferred to the air. You may ignore any kinetic and potential energy changes and assume air is an ideal gas with constant specific heats at room temperature.
- (a) Sketch the P-v diagram for the cycle. Label the four state points on the diagram and any heat transfer in or out. **[4 marks]**
  - (b) Calculate the pressure and temperature at the end of the heat-addition phase. **[8 marks]**
  - (c) Calculate the net work output. **[6 marks]**
  - (d) Calculate the mean effective pressure. **[4 marks]**
  - (e) Calculate the thermal efficiency and compare it to the thermal efficiency of a Carnot cycle operating between the minimum and maximum temperatures found in this Otto cycle. **[3 marks]**
- Q4.** A mixture of gases consists of 0.1 kg of oxygen, 1 kg of carbon dioxide, and 0.5 kg of helium. This mixture is heated from 10°C to 260°C while its pressure is maintained constant at 350 kPa. Calculate:
- (a) the mole numbers of the components within the mixture **[8 marks]**
  - (b) the gas constant of the mixture **[5 marks]**
  - (c) the change in the volume of the mixture **[4 marks]**
  - (d) the heat transferred to the mixture **[8 marks]**

**END OF THE EXAMINATION PAPER**

## DATA SHEETS

### 1 Work

- **positive work:** is done by a system on the surroundings (a system does positive work if it can raise a weight)
- **negative work:** is done by the surroundings on a system.

Incremental piston, or displacement work, is  $\delta W = p dV$ ,  
and for a process in which the pressure varies with volume the work is  
 $W = \int p dV$

**Constant pressure (isobaric) process:  $pV^0 = c$**

$$W_{12} = \int_1^2 p dV = p(V_2 - V_1)$$

**Constant volume (isochoric) process:  $pV^\infty = c$**

$$W_{13} = \int_1^3 p dV = 0, \text{ because } dV = 0$$

**Process defined by  $pV = c$**

$$W_{14} = p_1 V_1 \ln \frac{V_4}{V_1} = p_1 V_1 \ln \frac{p_1}{p_4} = \text{etc}$$

**Process defined by  $pV^n = c$**

$$W_{15} = \frac{p_1 V_1 - p_5 V_5}{n-1} = \frac{p_5 V_5 - p_1 V_1}{1-n}$$

### 2 First Law of Thermodynamics - closed systems

$$Q - W_s = m \left( u_2 + \frac{V_2^2}{2} + g z_2 \right) - m \left( u_1 + \frac{V_1^2}{2} + g z_1 \right)$$

**First Law for a closed system in the absence of kinetic and potential energy**

$$\delta Q = dU + \delta W$$

**Specific heat at constant volume**

$$c_v = \left( \frac{\partial u}{\partial T} \right)_v = \left( \frac{\partial u}{\partial t} \right)_v = \left( \frac{\partial Q}{\partial T} \right)_v$$

**Enthalpy, H**

$$H = U + pV$$

**Specific enthalpy, h**

$$h = \frac{H}{m} = \frac{U + pV}{m} = \frac{U}{m} + \frac{pV}{m} = u + pv$$

**Specific heat at constant pressure**

$$c_p = \left( \frac{\partial h}{\partial T} \right)_p = \left( \frac{\partial h}{\partial t} \right)_p = \left( \frac{\partial q}{\partial T} \right)_p$$

### 3 Steady flow energy equation

$$\dot{Q} - \dot{W} = \dot{m} \left( h_e - h_i + \frac{V_e^2 - V_i^2}{2} + g(z_e - z_i) \right)$$

#### Stagnation enthalpy

$$h_0 = h + \frac{V^2}{2}$$

#### Velocity at exit to a nozzle

$$V_2 = \sqrt{2 \left\{ (h_1 - h_2) + \frac{V_1^2}{2} \right\}}$$

#### Work from an adiabatic machine

$$-\dot{W}_s = \dot{m}(h_e - h_i) = \dot{m}(h_2 - h_1).$$

### 4 Second Law of Thermodynamics

#### Efficiency

$$\text{Thermal efficiency, } \eta_{th} = \frac{\text{Useful work output}}{\text{Thermal energy input}},$$

for a heat engine operating in a cycle

#### Thermal efficiency of heat engine

$$\text{Thermal efficiency, } \eta_{th} = \frac{\text{Net work}}{\text{Heat supplied}} \quad \eta_{th} = \frac{W_s}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

#### Coefficient of performance of refrigerator

$$\text{Coefficient of performance, } \beta = \frac{\text{Heat transferred from cold reservoir}}{\text{Work done}}, = \frac{Q_2}{W_s} = \frac{Q_2}{Q_1 - Q_2}.$$

#### Coefficient of performance of heat pump

$$\text{Coefficient of performance, } \beta' = \frac{\text{Heat transferred to hot reservoir}}{\text{Work done}}, = \frac{Q_1}{W_s} = \frac{Q_1}{Q_1 - Q_2}.$$

#### Relationship between coefficients of performance

$$\beta' = 1 + \frac{Q_2}{W_s} = 1 + \beta$$

#### Entropy

Entropy is denoted by the symbol - S; specific entropy is denoted by - s.

$$\text{The change of entropy between states 1 and 2 is } S_2 - S_1 = \int_1^2 \frac{\delta Q_R}{T}$$

#### Central Equation of Thermodynamics

$$Tds = du + pdv = dh - vdp$$

#### Steady flow entropy equation for an adiabatic machine

$$\dot{S}_i = \dot{m}(s_e - s_i) \geq 0$$

## 5 Properties of pure substances

### Dryness fraction, or quality

$$\text{Dryness fraction, } x = \frac{\text{Mass of dry vapour}}{\text{Total mass of liquid + vapour}}$$

$$v = xv_g + (1-x)v_f = v_f + xv_{fg}$$

$$u = xu_g + (1-x)u_f = u_f + xu_{fg}$$

$$h = xh_g + (1-x)h_f = h_f + xh_{fg}$$

$$s = xs_g + (1-x)s_f = s_f + xs_{fg}$$

## 6 Perfect gases, and mixtures of perfect gases

**Ideal gas**  $\frac{pv}{T} = \text{const}, R$

**Universal Gas Constant**  $\Re = MR$

S.I. units  $\Re = 8.3145 \text{ kJ/kmol K}$

Imperial  $\Re = 1545 \text{ ft.lbf/lb mol } ^\circ\text{R}$   
 $= 1.986 \text{ Btu/lb mol } ^\circ\text{R}$

### Molar Masses for Common Gases/Elements

Gas/Element	M (kg/kmol)
H <sub>2</sub>	2
O <sub>2</sub>	32
N <sub>2</sub>	28
CO	28
CO <sub>2</sub>	44
H <sub>2</sub> O	18
C	12

**Internal energy**  $u = \int_{T_0}^T c_v dT + u_0$ , where  $u_0$  is the value of  $u$  at temperature  $T_0$

**Enthalpy**  $h = u + pv = u + RT$

$$h = \int_{T_0}^T c_p dT + h_0$$
, where  $h_0$  is the enthalpy at temperature  $T_0$ .

### Relationship between $c_p$ and $c_v$

$$c_p = c_v + R$$

$$\gamma = c_p / c_v$$

### Entropy change

$$ds = \frac{c_v dT + pdv}{T} = c_v \frac{dT}{T} + \frac{p}{T} dv$$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v}{T} dT + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p}{T} dT - R \ln \frac{p_2}{p_1}$$

## 7 Isentropic or process efficiencies

**For compressors:**  $\eta_c = \frac{h_{2i} - h_1}{h_2 - h_1}$

**For turbines:**  $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2i}}$

where the process is from state 1 to state 2 and subscript  $i$  denotes ideal (isentropic) values

## 8 Ideal cycle efficiencies

**Otto cycle:**  $\eta = 1 - r_v^{-(\gamma-1)}$

**Diesel cycle:**  $\eta = 1 - r_v^{-(\gamma-1)} \frac{(r_c^\gamma - 1)}{\gamma(r_c - 1)}$

**Dual cycle:**  $\eta = 1 - r_v^{-(\gamma-1)} \frac{(r_p r_c^\gamma - 1)}{(r_p - 1) + \gamma r_p (r_c - 1)}$

where  $r_v$  is the volumetric compression ratio  
 $r_c$  is the volumetric cut-off ratio  
 $r_p$  is the constant-volume heat input pressure ratio  
 $\gamma$  is the ratio of specific heats

## 9 Mean effective pressures of reciprocating engine cycles

The indicated mean effective pressure (imep)

$$p_m = \frac{W_{net}}{V_s}$$

where  $W_{net}$  is the net cycle work,  $V_s$  is the swept volume

The brake mean effective pressure (bmep)

$$p_b = \frac{W_{net} - W_f}{V_s}$$

where  $W_f$  is the work lost to friction.

The friction mean effective pressure (fmep)

$$p_f = p_m - p_b$$