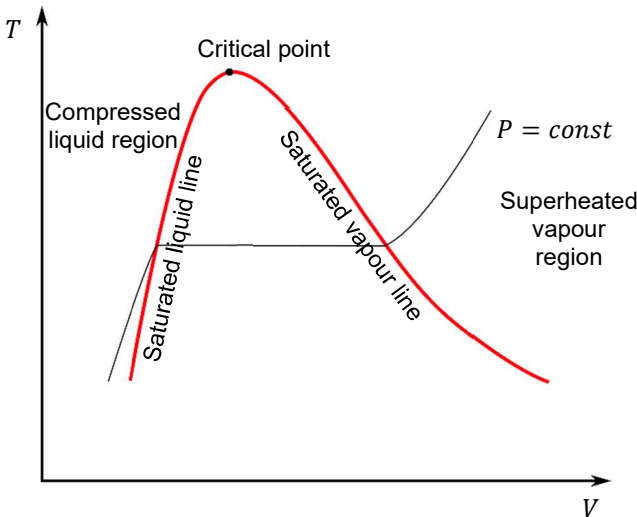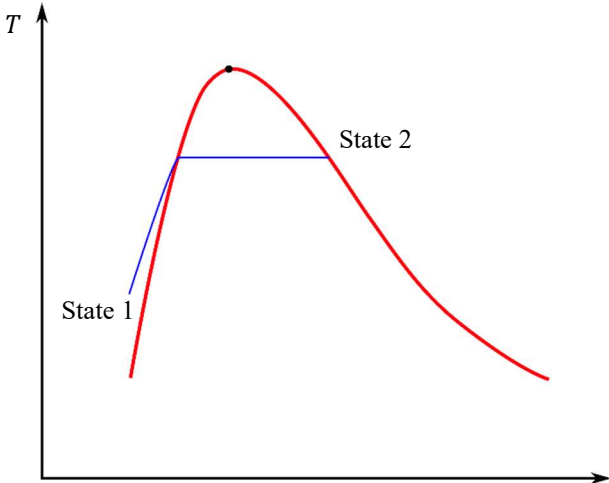Examination Question Solution Sheet

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Made by: Dean Wilson

Name of paper: **Engineering Thermodynamics**

Code number of paper: MACE32102

| Question no & part | Answer | Mark (per part) |
|---|---|---|
| Q1(a) | Diagram should be labelled as follows:  | |
| Q1(a)(i) | Labelling of "Critical point" | 1 |
| Q1(a)(ii) | Labelling of "Saturated liquid line" and "Saturated vapour line" | 2 |
| Q1(a)(iii) | Labelling of the "Compressed liquid region" and "Superheated Vapour region" | 2 |
| Q1(a)(iv) | Drawing of a single constant-pressure line, as $P = const$ on the diagram above. The line should be of the correct shape in each of the three regions | 3 |
| Q1(b) | A piston-cylinder device that is initially filled with water is heated at constant pressure until all the liquid has vaporized. Initial conditions are given as $T_1 = 40°C$, $V_1 = 0.050 \text{m}^3$ and the pressure remains constant at $P = 200$ kPa. | |
| Q1(b)(i) | From Table A-4, at $T_1 = 40°C$ the saturation pressure is $P_{sat} = 7.3851$ kPa. Since $P_1 > P_{sat}$, the cylinder initially contains compressed liquid that can be approximated as a saturated liquid at the specified temperature ($T_1$). | 2 |
| | The specific volume is then (Table A-4): $$v_1 \cong v_{f@40°C} = 0.001008 \text{ m}^3/\text{kg}$$ | 1 |
| | Then the mass can be calculated as: $$m = \frac{V_1}{v_1} = \frac{0.050 \text{ m}^3}{0.001008 \text{ m}^3/\text{kg}} = \mathbf{49.61 \text{ kg}}$$ | 1 |

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| | At the final state, the liquid has been completely vaporized. Thus, the cylinder contains saturated vapour, and the final temperature must be the saturation temperature at the final pressure ($P = 200$ kPa) (Table A-4): $$T_2 = T_{\text{sat @ 200 kPa}} = \mathbf{120.21\,°C}$$ | 2 |
| Q1(b)(ii) | The initial enthalpy is (Table A-4): $$h_1 \cong h_{f@40°C} = 167.53 \text{ kJ/kg}$$ | 1 |
| | The final enthalpy is (Table A-4): $$h_2 = h_{g\,@\,200\,kPa} = 2706.3 \text{ kJ/kg}$$ | 1 |
| | The total change in enthalpy is then: $$\Delta H = m(h_2 - h_1) = (49.61 \text{ kg})(2706.3 - 167.53) \text{ kJ/kg} = \mathbf{125,950 \text{ kJ}}$$ | 2 |
| Q1(b)(iii) | The process should start within the compressed liquid region, meet the saturated liquid line and proceed horizontally stopping at the saturated vapour line, as below. It should either be clearly indicated on the Figure drawn in Q1(a) or drawn on a separate figure.  | 2 |
| Q1(c) | The pressure and temperature of oxygen gas in a storage tank are given. The mass of oxygen is to be determined. Initial conditions are $V_1 = 2.5$ m$^3$, $P_g = 500$ kPa, $P_{\text{atm}} = 97$ kPa, $T_1 = 28°C$. At these conditions, we assume that oxygen behaves as an ideal gas. | |
| | The gas constant of oxygen is (Table A-1): $$R = 0.2598 \text{ kJ/kgK}$$ | 1 |
| | We are provided with the gauge pressure, $P_g$, thus the absolute pressure of the oxygen is: $$P = P_g + P_{\text{atm}} = 500 + 97 = 597 \text{ kPa}$$ | 2 |
| | Treating $O_2$ as an ideal gas, we can determine the mass from: $$PV = mRT$$ $$m = \frac{PV}{RT} = \frac{(597 \text{ kPa})(2.5 \text{ m}^3)}{(0.2598 \text{ kJ/kgK})(28 + 273)\text{K}} = \mathbf{19.08 \text{ kg}}$$ | 2 |
| Q2(a) | The geothermal water remains liquid as it passes through the evaporator. Thus, our inlet and outlet dryness fractions are zero. Then from Table A-4: | 2 |
| | $$\left.\begin{array}{l} T_{w,1} = 60°C \\ x_{w,1} = 0 \end{array}\right\} h_{w,1} = 251.18 \text{ kJ/kg}$$ | 2 |
| | $$\left.\begin{array}{l} T_{w,2} = 40°C \\ x_{w,2} = 0 \end{array}\right\} h_{w,2} = 167.53 \text{ kJ/kg}$$ | 2 |

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| | The rate of heat transferred from the water is equal to the energy change of the water from the geothermal inlet to the outlet.<br>$$\dot{Q}_L = \dot{m}_w\left(h_{w,1} - h_{w,2}\right)$$<br>$$\dot{Q}_L = (0.065 \text{ kg/s})(251.18 - 167.53) \text{ kJ/kg} = \mathbf{5.437\ kW}$$ | 2 |
| Q2(b) | We determine the enthalpy of the refrigerant at the inlet and outlet of the evaporator using the state information provided. Using Table A-11:<br><br>$\left.\begin{array}{l} T_1 = 12°C \\ x_1 = 0.15 \end{array}\right\}$ $h_1 = h_f + x_1 h_{fg} = 68.17 \text{ kJ/kg} + (0.15)(189.16 \text{ kJ/kg}) = 96.54 \text{ kJ/kg}$<br>$\qquad\qquad\qquad\qquad P_1 = P_{\text{sat}} = 443.3 \text{ kPa}$ | 2 |
| | $\left.\begin{array}{l} P_2 = P_1 = 443.3 \text{ kPa} \\ x_2 = 1 \end{array}\right\}$ $h_2 = h_{g@P_2} = 257.33 \text{ kJ/kg}$ | 2 |
| | The energy increase of the refrigerant is equal to the energy decrease of the water in the evaporator: | 2 |
| | $$\dot{Q}_L = \dot{m}_R(h_2 - h_1) \rightarrow \dot{m}_R = \frac{\dot{Q}_L}{(h_2 - h_1)} = \frac{5.437 \text{ kW}}{(257.33 - 96.54) \text{ kJ/kg}} = \mathbf{0.0338\ kg/s}$$ | 2 |
| Q2(c) | The rate of heat supply to the space, $\dot{Q}_H$ can be determined from an energy balance on the overall heat pump. The work done by the compressor is given as $\dot{W}_{\text{in}} = 1.6$ kW.<br>$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{in}} = 5.437 + 1.6 = \mathbf{7.04\ kW}$$ | 3 |
| Q2(d) | The coefficient of performance (COP) is determined from its definition:<br>$$\text{COP} = \frac{\dot{Q}_H}{\dot{W}_{\text{in}}} = \frac{7.04 \text{ kW}}{1.6 \text{ kW}} = \mathbf{4.40}$$ | 2 |
| Q2(e) | The COP of a reversible heat pump operating between the same temperature limits, $T_L = 25°C, T_H = 60°C$, is given by:<br>$$\text{COP}_{\text{max}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (25 + 273)/(60 + 273)} = 9.51$$<br>The minimum power input to the compressor required to drive the same heating load is then: | 2 |
| | $$\dot{W}_{\text{in,min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{max}}} = \frac{7.04 \text{ kW}}{9.51} = \mathbf{0.740\ kW}$$ | 2 |
| Q3(a) | The diagram should look like:<br><br>The labelling of TDC and BDC and/or the 0-1 stroke does not carry any marks but is not penalised. | 4 |
| Q3(b) | Air is the working fluid for the ideal Otto cycle considered, and it is assumed to be an ideal gas with constant specific heats taken at room temperature.<br>Properties can be taken from Table A-2: $c_p = 1.005 \text{ kJ/kgK}, \ c_v = 0.718 \text{ kJ/kgK}, \ R = 0.287 \text{ kJ/kgK}, \ k = 1.4$ | |

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| | The end of the heat addition phase is state point 3 in the diagram in Q3(a). Thus, we need to compute the temperature and pressure changes across processes 1-2 and 2-3.<br><br>Process 1-2: isentropic compression.<br>$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1}, \qquad r = 8 \rightarrow \frac{V_1}{V_2} = 8$$<br>$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1} = (300\ \text{K})(8)^{1.4-1} = 689\ \text{K}$$ | 2 |
| | $$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \rightarrow P_2 = \frac{V_1}{V_2}\frac{T_2}{T_1} P_1 = (8)\left(\frac{689\ \text{K}}{300\ \text{K}}\right)(95\ \text{kPa}) = 1745\ \text{kPa}$$<br>Process 2-3: constant volume heat addition | 2 |
| | $$q_{\text{in}} = u_3 - u_2 = c_v(T_3 - T_2)$$<br>$$\rightarrow T_3 = \frac{q_{\text{in}}}{c_v} + T_2 = \frac{750\ \text{kJ/kg}}{0.718\ \text{kJ/kgK}} + 689\ \text{K} = \mathbf{1734\ K}$$ | 2 |
| | $$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}, \qquad V_2 = V_3$$<br>$$\rightarrow P_3 = \frac{T_3}{T_2} P_2 = \left(\frac{1734\ \text{K}}{689\ \text{K}}\right)(1745\ \text{kPa}) = \mathbf{4392\ kPa}$$ | 2 |
| Q3(c) | To calculate the net work output, we need to consider the remaining processes.<br>Process 3-4: isentropic expansion<br>$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} \rightarrow T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{k-1} = T_3 \left(\frac{1}{r}\right)^{k-1} = (1734\ \text{K})\left(\frac{1}{8}\right)^{1.4-1} = 755\ \text{K}$$ | 2 |
| | Process 4-1: constant volume heat rejection<br>$$q_{\text{out}} = u_4 - u_1 = c_v(T_4 - T_1)$$<br>$$\rightarrow q_{\text{out}} = (0.718\ \text{kJ/kgK})(755 - 300)\text{K} = 327\ \text{kJ/kg}$$ | 2 |
| | $$w_{\text{net,out}} = q_{\text{in}} - q_{\text{out}} = 750 - 327 = 423\ \text{kJ/kg}$$ | 2 |
| Q3(d) | The mean effective pressure is determined from:<br>$$\text{MEP} = \frac{w_{\text{net,out}}}{v_{\text{max}} - v_{\text{min}}}$$<br>Noting that $v_{\text{max}} = v_1 = v_4$,<br>$$P_1 v_1 = RT_1 \rightarrow v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kJ/kgK})(300\ \text{K})}{95\ \text{kPa}} = 0.906\ \text{m}^3/\text{kg}$$ | 2 |
| | Then, since $v_{\text{min}} = v_2 = v_1/r$,<br>$$\text{MEP} = \frac{w_{\text{net,out}}}{v_1 - v_2} = \frac{w_{\text{net,out}}}{v_1\left(1 - \frac{1}{r}\right)} = \frac{423\ \text{kJ/kg}}{(0.906\ \text{m}^3/\text{kg})\left(1 - \frac{1}{8}\right)} = \mathbf{534\ kPa}$$ | 2 |
| Q3(e) | The thermal efficiency of an ideal Otto cycle can be computed either from:<br>$$\eta_{th} = \frac{w_{\text{net,out}}}{q_{\text{in}}} = \frac{423\ \text{kJ/kg}}{750\ \text{kJ/kg}} = 0.564 = \mathbf{56.4\%}$$<br>Or<br>$$\eta_{th} = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{8^{1.4-1}} = 0.5647 = \mathbf{56.5\%}$$ | 1 |
| | For a Carnot cycle, the efficiency is:<br>$$\eta_{th} = 1 - \frac{T_L}{T_H}$$<br>Noting that $T_L = T_1 = 300\ \text{K}$, and $T_H = T_3 = 1734\ \text{K}$:<br>$$\eta_{th} = 1 - \frac{300}{1734} = 0.827 = \mathbf{82.7\%}$$ | 2 |
| Q4(a) | To compute the mole numbers we require the molar masses of the constituent gases. From Table A-1:<br>$$M_{O2} = 32.0\ \text{kg/kmol}, \qquad M_{CO2} = 44.0\ \text{kg/kmol}, \qquad M_{He} = 4\ \text{kg/kmol}$$<br>Then: | 2 |

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| | $N_{O2} = \dfrac{m_{O2}}{M_{O2}} = \dfrac{0.1 \text{ kg}}{32 \text{ kg/kmol}} = \mathbf{0.003125 \text{ kmol}}$ | 2 |
| | $N_{CO2} = \dfrac{m_{CO2}}{M_{CO2}} = \dfrac{1 \text{ kg}}{44 \text{ kg/kmol}} = \mathbf{0.02273 \text{ kmol}}$ | 2 |
| | $N_{He} = \dfrac{m_{He}}{M_{He}} = \dfrac{0.5 \text{ kg}}{4 \text{ kg/kmol}} = \mathbf{0.125 \text{ kmol}}$ | 2 |
| Q4(b) | The mole number of the mixture is:<br>$$N_m = N_{O2} + N_{CO2} + N_{He} = 0.003125 + 0.02273 + 0.125 = 0.15086 \text{ kmol}$$ | 1 |
| | The apparent molecular weight of the mixture is:<br>$$M_m = \frac{m_m}{N_m} = \frac{1.6 \text{ kg}}{0.15086 \text{ kmol}} = 10.61 \text{ kg/kmol}$$ | 2 |
| | The gas constant of the mixture is then:<br>$$R = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmolK}}{10.61 \text{ kg/kmol}} = \mathbf{0.7836 \text{ kJ/kgK}}$$<br>where $R_u$ is the universal gas constant. | 2 |
| Q4(c) | The change in volume in the mixture is computed from the ideal gas law as:<br>$$\Delta V_m = \frac{m_m R \Delta T}{P}, \qquad \Delta T = (260 - 10) = 250 \text{ K}$$ | 2 |
| | $$\rightarrow \frac{(1.6 \text{ kg})(0.7836 \text{ kJ/kgK})(250 \text{ K})}{350 \text{ kPa}} = \mathbf{0.8955 \text{ m}^3}$$ | 2 |
| Q4(d) | To compute the heat transfer we require the constant-pressure specific heat of the mixture, given by:<br>$$c_{p,m} = \sum_{i=1}^{n} \text{mf}_i \, c_{p,i}$$<br>The mass fractions $\text{mf}_i$ are computed for each constituent as: | |
| | $$\text{mf}_{O2} = \frac{m_{O2}}{m_m} = \frac{0.1 \text{ kg}}{1.6 \text{ kg}} = 0.0625$$ | 1 |
| | $$\text{mf}_{CO2} = \frac{m_{CO2}}{m_m} = \frac{1 \text{ kg}}{1.6 \text{ kg}} = 0.625$$ | 1 |
| | $$\text{mf}_{He} = \frac{m_{He}}{m_m} = \frac{0.5 \text{ kg}}{1.6 \text{ kg}} = 0.3125$$ | 1 |
| | Then:<br>$$c_{p,m} = \sum_{i=1}^{n} \text{mf}_i \, c_{p,i} = \text{mf}_{O2} c_{p,O2} + \text{mf}_{CO2} c_{p,CO} + \text{mf}_{He} c_{p,He}$$<br>$$c_p, m = 0.0625(0.918) + 0.625(0.846) + 0.3125(5.1926) = 2.209 \text{ kJ/kgK}$$<br>where the individual values of $c_{p,i}$ are obtained from Table A-2. | 3 |
| | The heat transfer is then determined to be<br>$$q_{in} = c_p(T_2 - T_1) = (2.209 \text{ kJ/kgK})(260 - 10)\text{K} = \mathbf{552 \text{ kJ/kg}}$$ | 2 |