The University of Manchester

MANCHESTER 1824

MECH32102

TWO HOURS

Department of Mechanical, Aerospace & Civil Engineering

UNIVERSITY OF MANCHESTER

ENGINEERING THERMODYNAMICS

SOLUTIONS

xxxxxx 2024

09:00 – 11:00

Special Instruction(s):

- Answer **all 4 questions**.
- Each question carries **25 marks**.
- A **formula sheet** is provided at the end of the paper.
- Several **property tables** are provided at the end of the paper, extracted from appendix a of ‘thermodynamics-an engineering approach (SI version) (9th ed.) (McGraw Hill) Cengel, Boles and Kanoglu.
- Electronic calculators **are permitted** as long as they cannot store text, perform algebra and have no graphing capability, in accordance with university regulations.

Q1. (a)

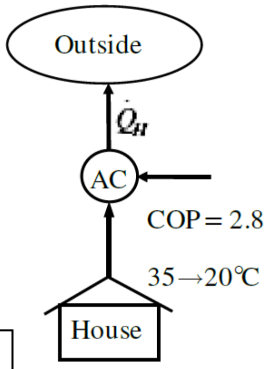
Assumptions **1** The air conditioner operates steadily. **2** The house is well-sealed so that no air leaks in or out during cooling. **3** Air is an ideal gas with constant specific heats at room temperature.

Properties The constant volume specific heat of air is given to be $c_v = 0.72$ kJ/kg.°C.

Analysis Since the house is well-sealed (constant volume), the total amount of heat that needs to be removed from the house is

[3 marks]

$$Q_L = \left(mc_v \Delta T\right)_{\text{House}} = (800\text{ kg})(0.72\text{ kJ/kg}\cdot{}^\circ\text{C})(35-20)^\circ\text{C} = 8640\text{ kJ}$$



This heat is removed in 30 minutes. Thus the average rate of heat removal from the house is

[3 marks]

$$\dot{Q}_L = \frac{Q_L}{\Delta t} = \frac{8640\text{ kJ}}{30 \times 60\text{ s}} = 4.8\text{ kW}$$

Using the definition of the coefficient of performance, the power input to the air-conditioner is determined to be

$$\dot{W}_{\text{net, in}} = \frac{\dot{Q}_L}{\text{COP}_\text{R}} = \frac{4.8\text{ kW}}{2.8} = \mathbf{1.71\text{ kW}}$$

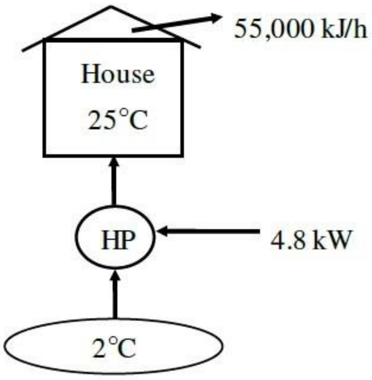
[3 marks]

Q1. (b)

Analysis (*a*) The coefficient of performance of this Carnot heat pump depends on the temperature limits in the cycle only, and is determined from

(b)(i) [3 marks]

$$\text{COP}_{\text{HP, rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\text{ K})/(25+273\text{ K})} = 12.96$$



The amount of heat the house lost that day is

$$Q_H = \dot{Q}_H\,(1\text{ day}) = (55{,}000\text{ kJ/h})(24\text{ h}) = 1{,}320{,}000\text{ kJ}$$

Then the required work input to this Carnot heat pump is determined from the definition of the coefficient of performance to be

$$W_{\text{net, in}} = \frac{Q_H}{\text{COP}_{\text{HP}}} = \frac{1{,}320{,}000\text{ kJ}}{12.96} = 101{,}880\text{ kJ}$$

Thus the length of time the heat pump ran that day is

$$\Delta t = \frac{W_{\text{net, in}}}{\dot{W}_{\text{net, in}}} = \frac{101{,}880\text{ kJ}}{4.8\text{ kJ/s}} = 21{,}225\text{ s} = \mathbf{5.90\text{ h}}$$

(b)(ii) [7 marks]

(*b*) The total heating cost that day is

$$\text{Cost} = W \times \text{price} = \left(\dot{W}_{\text{net, in}} \times \Delta t\right)(\text{price}) = (4.8\text{ kW})(5.90\text{ h})(0.11\text{ £/kWh}) = \mathbf{£3.11}$$

(b)(iii) [3 marks]

(*c*) If resistance heating were used, the entire heating load for that day would have to be met by electrical energy. Therefore, the heating system would consume 1,320,000 kJ of electricity that would cost

$$\text{New Cost} = Q_H \times \text{price} = (1{,}320{,}000\text{ kJ})\left(\frac{1\text{ kWh}}{3600\text{ kJ}}\right)(0.11\ \ /\text{kWh}) = \mathbf{£40.3}$$

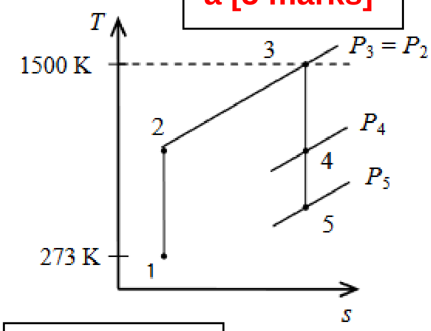
(b)(iv) [3 marks]

Q2.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air are given as $c_v = 0.718$ kJ/kg·K, $c_p = 1.005$ kJ/kg·K, $R = 0.287$ kJ/kg·K, $k = 1.4$.

a [5 marks]



Analysis (b) For the compression process,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (273\text{ K})(8)^{0.4/1.4} = 494.5\text{ K}$$

The power input to the compressor is equal to the power output from the high-pressure turbine. Then,

$$\dot{W}_{\text{Comp,in}} = \dot{W}_{\text{HP Turb,out}}$$
$$\dot{m}c_p(T_2 - T_1) = \dot{m}c_p(T_3 - T_4)$$
$$T_2 - T_1 = T_3 - T_4$$
$$T_4 = T_3 + T_1 - T_2 = 1500 + 273 - 494.5 = \mathbf{1278.5K}$$

b [5 marks]

The pressure at this state is

$$\frac{P_4}{P_3} = \left(\frac{T_4}{T_3}\right)^{k/(k-1)} \longrightarrow P_4 = rP_1\left(\frac{T_4}{T_3}\right)^{k/(k-1)} = 8(100\text{ kPa})\left(\frac{1278.5\text{ K}}{1500\text{ K}}\right)^{1.4/0.4} = \mathbf{457.3kPa}$$

b [5 marks]

(c) The temperature at state 5 is determined from

$$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{(k-1)/k} = (1278.5\text{ K})\left(\frac{100\text{ kPa}}{457.3\text{ kPa}}\right)^{0.4/1.4} = 828.1\text{ K}$$

The net power is that generated by the low-pressure turbine since the power output from the high-pressure turbine is equal to the power input to the compressor. Then,

$$\dot{W}_{\text{LP Turb}} = \dot{m}c_p(T_4 - T_5)$$
$$\dot{m} = \frac{\dot{W}_{\text{LP Turb}}}{c_p(T_4 - T_5)} = \frac{200{,}000\text{ kW}}{(1.005\text{ kJ/kg}\cdot\text{K})(1278.5 - 828.1)\text{K}}$$
$$= \mathbf{441.8kg/s}$$

c [5 marks]

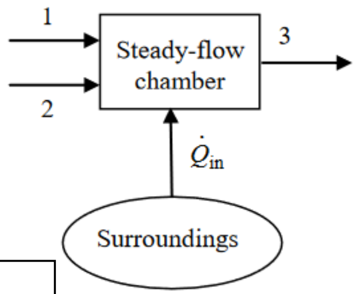
(d) Since density of inflowing air is inversely proportional to the local temperature, increasing the inlet temperature will result in decreased mass flow rate. Whereas, increasing the inlet temperature of the turbine T4 increases the net power output as well as efficiency of the power plant (see the formula above used in Q2(c)).

d [5 marks]

Q3.

Assumptions Kinetic and potential energy changes are negligible.

Analysis (a) Mass and energy balances for the mixing process:



$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$h = C_P T$$

$$\dot{m}_1 C_{P,1} T_1 + \dot{m}_2 C_{P,2} T_2 + \dot{Q}_{in} = \dot{m}_3 C_{P,m} T_3$$

$$C_{P,m} = \frac{\dot{m}_1}{\dot{m}_3} C_{P,1} + \frac{\dot{m}_2}{\dot{m}_3} C_{P,2}$$

$$T_3 = \frac{\dot{m}_1 C_{P,1}}{\dot{m}_3 C_{P,m}} T_1 + \frac{\dot{m}_2 C_{P,2}}{\dot{m}_3 C_{P,m}} T_2 + \frac{\dot{Q}_{in}}{\dot{m}_3 C_{P,m}}$$

[5 marks]

(b) The expression for the exit volume flow rate is obtained as follows:

$$\dot{V}_3 = \dot{m}_3 v_3 = \dot{m}_3 \frac{R_3 T_3}{P_3}$$

$$\dot{V}_3 = \frac{\dot{m}_3 R_3}{P_3} \left[\frac{\dot{m}_1 C_{P,1}}{\dot{m}_3 C_{P,m}} T_1 + \frac{\dot{m}_2 C_{P,2}}{\dot{m}_3 C_{P,m}} T_2 + \frac{\dot{Q}_{in}}{\dot{m}_3 C_{P,m}} \right]$$

$$\dot{V}_3 = \frac{C_{P,1} R_3}{C_{P,m} R_1} \frac{\dot{m}_1 R_1 T_1}{P_3} + \frac{C_{P,2} R_3}{C_{P,m} R_2} \frac{\dot{m}_2 R_2 T_2}{P_3} + \frac{R_3 \dot{Q}_{in}}{P_3 C_{P,m}}$$

$$P_3 = P_1 = P_2$$

$$\dot{V}_3 = \frac{C_{P,1} R_3}{C_{P,m} R_1} \dot{V}_1 + \frac{C_{P,2} R_3}{C_{P,m} R_2} \dot{V}_2 + \frac{R_3 \dot{Q}_{in}}{P_3 C_{P,m}}$$

$$R = \frac{R_u}{M}, \quad \frac{R_3}{R_1} = \frac{R_u}{M_3} \frac{M_1}{R_u} = \frac{M_1}{M_3}, \quad \frac{R_3}{R_2} = \frac{M_2}{M_3}$$

$$\dot{V}_3 = \frac{C_{P,1} M_1}{C_{P,m} M_3} \dot{V}_1 + \frac{C_{P,2} M_2}{C_{P,m} M_3} \dot{V}_2 + \frac{R_u \dot{Q}_{in}}{P_3 M_3 C_{P,m}}$$

The mixture molar mass M_3 is found as follows:

$$M_3 = \sum y_i M_i, \quad y_i = \frac{m_{fi} / M_i}{\sum m_{fi} / M_i}, \quad m_{fi} = \frac{\dot{m}_i}{\sum \dot{m}_i}$$

[10 marks]

(c) For adiabatic mixing $\dot{Q}_{in}$ is zero, and the mixture volume flow rate becomes

$$\dot{V}_3 = \frac{C_{P,1} M_1}{C_{P,m} M_3} \dot{V}_1 + \frac{C_{P,2} M_2}{C_{P,m} M_3} \dot{V}_2$$

[5 marks]

(d) When adiabatically mixing the same two ideal gases, the mixture volume flow rate becomes

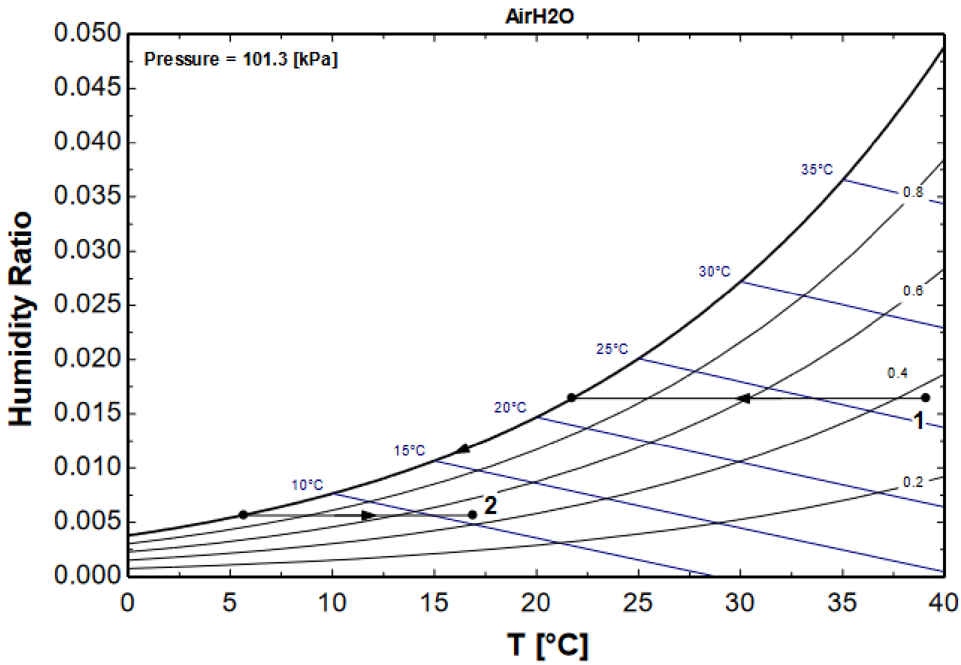
$$M_3 = M_1 = M_2$$
$$C_{P,3} = C_{P,1} = C_{P,2}$$
$$\dot{V}_3 = \dot{V}_1 + \dot{V}_2$$

[5 marks]

Q4 (a)

Assumptions **1** This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process $(\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a)$. **2** Dry air and water vapor are ideal gases. **3** The kinetic and potential energy changes are negligible.

Analysis (a) The schematic of the cooling and dehumidification process and the the process on the psychrometric chart are given below. The psychrometric chart is obtained from the Property Plot feature of EES.



[4 marks]

(b) The inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. The properties of the air at the inlet and exit states are determined from the psychrometric chart (Figure A-31 or EES) to be

$$T_{\text{dp}1} = \mathbf{26.7°C}$$
$$h_1 = 96.5 \text{ kJ/kg dry air}$$
$$\omega_1 = 0.0222 \text{ kg H}_2\text{O/kg dry air}$$

[4 marks]

and

$$T_{\text{dp}2} = \mathbf{5.3°C}$$
$$\phi_2 = 46\%$$
$$h_2 = 31.1 \text{ kJ/kg dry air}$$
$$\omega_2 = 0.00551 \text{ kg H}_2\text{O/kg dry air}$$

[4 marks]

Also,

$$h_w \cong h_{f\,@5.3°\text{C}} = 22.2 \text{ kJ/kg} \quad \text{(Table A-4)}$$

The enthalpy of water is evaluated at 5.3°C, which is the temperature of air at the end of the dehumidiffication process.

(c) Applying the water mass balance and energy balance equations to the combined cooling and dehumidification section,

Water Mass Balance:

$$\sum \dot{m}_{w,i} = \sum \dot{m}_{w,e} \longrightarrow \dot{m}_{a1}\omega_1 = \dot{m}_{a2}\omega_2 + \dot{m}_w$$

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$$

Energy Balance:

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta\dot{E}_{\text{system}}^{\nearrow 0\,(\text{steady})} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\sum \dot{m}_i h_i = \dot{Q}_{out} + \sum \dot{m}_e h_e$$

$$\begin{aligned} \dot{Q}_{\text{out}} &= \dot{m}_{a1}h_1 - (\dot{m}_{a2}h_2 + \dot{m}_w h_w) \\ &= \dot{m}_a(h_1 - h_2) - \dot{m}_w h_w \\ &= \dot{m}_a(h_1 - h_2) - \dot{m}_a(\omega_1 - \omega_2)h_w \end{aligned}$$

Substituting,

$$\begin{aligned} q_{\text{out}} &= (h_1 - h_2) - (\omega_1 - \omega_2)h_w \\ &= (96.5 - 31.1)\text{ kJ/kg} - (0.0222 - 0.00551)(22.2\text{ kJ/kg}) \\ &= \mathbf{65.1 kJ/kg} \end{aligned}$$

[8 marks]

Q4 (b)

Assumptions: Air leaves the head covering as saturated.

Analysis: Since the cloth behaves as the wick on a wet bulb thermometer, the temperature of the cloth will become the wet-bulb temperature. If we assume the liquid water is supplied at a temperature not much different from the exit temperature of the airstream, the evaporative cooling process follows a line of constant wet-bulb temperature on the psychrometric chart. That is,

$$\mathbf{T_{wb} = constant}$$

The wet-bulb temperature at 1 atm, 48°C, and 10 percent relative humidity is determined from the psychrometric chart to

$$\mathbf{T_2 = T_{wb} = 23.16\ ^oC}$$

[5 marks]

END OF THE EXAMINATION SOLUTIONS