7. Math Skills

Chemistry is a *quantitative* subject. This is its strength. Only through careful *measurement* and *calculation* are chemists able to reveal the world of atoms and molecules.

7.1 Scientific Notation

You may be asked to do chemical calculations. These calculations frequently involve very large or very small numbers. It is inconvenient (sometimes ridiculous) to write these numbers in the usual way – e.g. 0.00071, 0.012, 15, 3700, 450000. Scientific notation was introduced to let us leave out all the leading or trailing zeros otherwise needed to write numbers. The set of numbers listed above can be written as $7.1{\times}10^{-4}$, $1.2{\times}10^{-2}$, $1.5{\times}10^{1}$, $3.7{\times}10^{3}$ and $4.5{\times}10^{5}$. The advantage of scientific notation is not necessarily evident here – e.g. we would rarely write 15 as $1.5{\times}10^{1}$. However, try to write the values $6.022{\times}10^{23}$ or $6.626{\times}10^{-34}$ without scientific notation!

Writing a number in scientific notation simply amounts to factoring the number into a power of 10 and a number in the range 1.0 to 9.999.... The exponent of 10 is the number of places the decimal moves to the left or minus the number of places the decimal moves to the right in order to get the number into the convenient range.

Example 7.1: Write the following numbers in scientific notation: (a) 53400 (b) 0.00618

> **Approach:** How many places do we need to move the decimal to get a number less than 10 or bigger than (or equal to) 1.0? The number of places moved to the left is the exponent of 10. Moving the decimal to the right means the exponent is negative.
>
> (a) If the decimal is moved 4 places to the left, we get 5.34. Therefore,
> $53400 = 5.34 \times 10^{4}$.
> (b) If the decimal is moved 3 places to the right, we get 6.18. Therefore,
> $0.00618 = 6.18 \times 10^{-3}$.

Scientific notation uses powers of 10 because our number system is the decimal system. Another way to get to work with more manageable numbers is to change the units. Again, powers of 10 are convenient for unit conversion. For example, we prefer to measure the distance between places in km rather than m. 1 km = 10^{3} m. Also, precision machine components must be accurate to within a micrometer (micron), 1 μm = 10^{-6} m. Powers of 10^{3} are common unit conversions in the metric system. Tiny masses might be measured in picograms (1 pg = 10^{-12} g), while very large masses measured in teragrams (1 Tg = 10^{12} g). The range of units spanned here is shown in the table.

Table 7.1. Units of Distance

Distance measure	unit	relation to m
picometer	pm	10^{-12} m
nanometer	nm	10^{-9} m
micrometer	μm	10^{-6} m
millimeter	mm	10^{-3} m
centimeter	cm	10^{-2} m
decimeter	dm	10^{-1} m
Meter	m	1 m
kilometer	km	10^{3} m
megameter	Mm	10^{6} m
gigameter	Gm	10^{9} m
terameter	Tm	10^{12} m

7.2 Dimensional Analysis

It is important to pay close attention to units in order to do correct calculations. For example, if you are given density in g mL^{-1} and a volume of the material in L, then you must convert one of the given units so that only one volume unit, mL or L, is used. The mass is determined by multiplying density and volume. The volume units must be the same so that they cancel. for example,

$$\text{mass} = \text{density} \times \text{volume}$$

units:

$$\text{g} = \text{g mL}^{-1} \times \text{mL}$$

$$= \frac{\text{g}}{\text{mL}} \times \text{mL}$$

Attention to units can prevent use of incorrect formulas. The units must be the same on both sides of a formula. If they are not, the formula is wrong. Here are some examples with correct units:

$$\text{amount} = \frac{\text{mass}}{\text{molar mass}}$$

units:

$$\text{mol} = \frac{\text{g}}{\text{g mol}^{-1}}$$

$$= \frac{\text{g mol}}{\text{g}}$$

$$p = \frac{nRT}{V}$$

units:

$$\text{atm} = \frac{\text{mol}\ \left(\text{L atm K}^{-1}\text{mol}^{-1}\right)\ \text{K}}{\text{L}}$$

Prove to yourself that the mol, L and K units all cancel appropriately in the equation above!

The value of the gas constant used in the ideal gas law, above, has the units, L atm K^{-1} mol^{-1}. The other form of the gas constant with units of J K^{-1} mol^{-1} is useful in work and other thermodynamic calculations.

Another common unit used in calculations is Avogadro's number, N_A, which has the value and unit of 6.0223×10^{23} mol^{-1}. We use this unit to scale quantities up to the molar scale, or down from the molar scale to the scale of individual particles. Carefully paying attention to this unit helps us use it correctly. We can have Avogadro's number of anything – atoms, molecules, ions, etc., so we can make the unit atoms mol^{-1}, molecules mol^{-1}, etc., as needed.

Example 7.2: How many Cu atoms are there in a penny? Let the mass of the penny be 3.35 g and assume it is pure copper.

Approach: Determine number of moles of copper. Convert to number of atoms.

Amount of Cu (mol) = mass / molar mass = 3.35 g / 63.546 g mol^{-1} = $5.27_2 \times 10^{-2}$ mol [Note: the final digit is shown as a subscript to indicate that it is not a significant digit. Rather, it is an extra digit which we will carry through the next stage of the calculation, in order to avoid rounding errors.]

To convert to number of atoms,

Number of atoms = $5.27_2 \times 10^{-2}$ mol $\times$ 6.0223×10^{23} atoms mol^{-1} = 3.17×10^{22} atoms

In this case we needed to multiply by Avogadro's number, in order to cancel the mol units.

7.3 Solving for a variable

Often in a calculation we are solving for at least one unknown quantity. Basic algebra skills will help us manipulate equations to isolate the variable we seek.

Example 7.3: A solution with volume 0.500 L and concentration 0.101 mol L^{-1} is diluted to 1.50 L. What is the new concentration?

Approach: The number of moles of solute is unchanged.
Reality check: The new concentration should be less than the initial.

Since mol (initial) = mol (final), and $n = cV$, then $c_i V_i = c_f V_f$.

$$0.101 \text{ mol L}^{-1} \times 0.500 \text{ L} = c_f \left(1.50 \text{ L}\right)$$

So

$$c_f = \frac{0.101 \text{ mol L}^{-1} \times 0.500 \text{ L}}{1.50 \text{ L}} = 0.0337 \text{ mol L}^{-1}$$

This matches our expectation (lower concentration). Notice how the units provide an excellent way to check that the calculation is set up correctly.

On occasion we may have more than one variable that is unknown. In such cases we will need more than one equation (or statement of variables) in order to solve for the unknowns. For example, if we have 2 unknowns, then we will need a minimum of 2 equations involving the variables, in order to solve for each one.

Example 7.4: We have information that tells us the following relationships:
$x + y = 0.295$ mol and $0.20 = \log [\, y / (0.075 + x) \,]$

What are x and y?

We can use the first equation to write one variable in terms of the second one, then make a substitution in the second equation, in order to solve.

$$x = 0.295 - y$$

Substitute this into the second equation to get

$$0.20 = \log [\, y / (0.075 + 0.295 - y) \,]$$

Raise both sides to the power of 10 to remove the log statement on the right side

$$10^{0.20} = y / (0.370 - y)$$
$$1.58 = y / (0.370 - y)$$
$$1.58\,(0.370 - y) = y$$
$$0.584_6 - 1.58\,y = y$$
$$0.584_6 = y + 1.58\,y$$
$$0.584_6 = 2.58\,y$$
$$0.584_6/2.58 = y$$
$$0.227 = y$$

Now we can return and solve for x:

$$x + y = 0.295 \text{ mol}$$
$$x + 0.227 = 0.295$$
$$x = 0.295 - 0.227 = 0.068 \text{ mol}$$

Note that this last problem required taking the inverse of a log – i.e., we raised both sides of the equation in the third line of the solution to the power of 10. This is the inverse of a log (antilog).

7.4 Logarithms

Chemistry deals with quantities that can vary over vast ranges, as we have seen. This arises because many quantities vary exponentially. It is often convenient – and sometimes essential – to introduce the logarithm function, For example, acidity (or basicity) is generally expressed via the pH (the "potential of hydrogen").

$$\text{pH} = -\log_{10}(\,[H^+]\,)$$

A base 10 logarithm is used here. When we use the natural logarithm, we use the notation, ln.

pH can vary from below 0 to above 14. This corresponds to $[H^+]$ varying over more than 14 orders of magnitude.

The key properties of logarithms are as follows:

1. $y = 10^x$ is the inverse of the logarithm:

$$\text{if } \; y = \log x, \; \text{ then } \; 10^y = x$$

2. The sign of a log determines the size of the argument relative to 1:

$$\text{If } \log x > 0, \text{ then } x > 1$$

If $\log x < 0$, then $0 < x < 1$.

Note that you cannot take the log of a negative number. Also, the argument of the logarithm (here, x) must not be negative.

3. The log of a product is the sum of the logarithms:

$$\log(x\,y) = \log x + \log y$$

4. The argument of a log must not have units. The $[H^+]$ that appears above in the definition of pH is the concentration of H^+ with the units of mol L^{-1} divided out. Note that we can take the log of an equilibrium constant because it has no units – any units of concentration (mol L^{-1}) and pressure (atm) are divided out.