Lesson: Permutation Involving Distinct Object

PERMUTATIONS

Permutations of n DISTINCT objects taken r at a time is an arrangement of n objects in a definite order. The total of such arrangements is denoted as

$P(n,r)$ or ${}_nP_r$ or "n arrange r" = $\frac{n!}{(n-r)!}$

note! These are examples of "Permutation" since the arrangement in which the song is played as well as definite order is important.

Example 1: A group of students learned 8 songs for a performance at an assembly. In how many different ways can the director arrange these songs if:

(a) all the songs are to be sung (no repetition)

$P(8,8) = 8! = 40\,320$ ways

(b) 5 songs are to be sung (no repetition)

$P(8,5) = {}_8P_5 = 6720$ ways

Example 2: A boy has 4 differently coloured blocks (R,Y,B,G). He selects one block at time without replacement and sets them out in an order drawn. How many arrangements can be made if:

(a) he selects 4 blocks?

$4! = P(4,4) = 24$ ways

(b) He selects 2 blocks?

$4P_2 = P(4,2) = 12$ ways

(c) He selects 3 out of 4 blocks and one must be yellow?

METHOD I: Direct Method (List all possible cases out)

Case 1: Yellow 1st block

$\underline{1}_{Y} \times \underline{3} \times \underline{2} = 6$

Case 2: Yellow 2nd block

$\underline{3} \times \underline{1}_{Y} \times \underline{2} = 6$

Case 3: Yellow 3rd block

$\underline{3} \times \underline{2} \times \underline{1}_{Y} = 6$

$6+6+6 = 18$ ways

METHOD II: Indirect Reasoning

indirect method =

All combination – "not possible case(s)"

All (without restrictions) – impossible case (no yellow)

$4P_3 - \underline{3} \times \underline{2} \times \underline{1}$

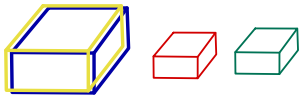
$= 24 - 6$

$= 18$ ways

(d) he selects 4, but green must be selected 2nd?

$\underline{3} \times \underline{1}_{G} \times \underline{2} \times \underline{1} = 6$ ways

(e) If he selects 4, but yellow and blue must be adjacent (side by side)?



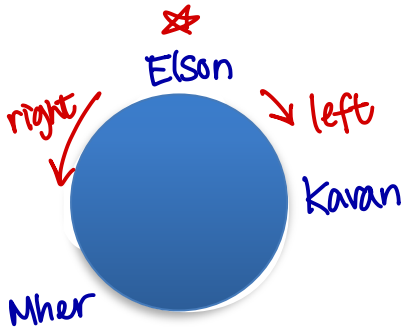
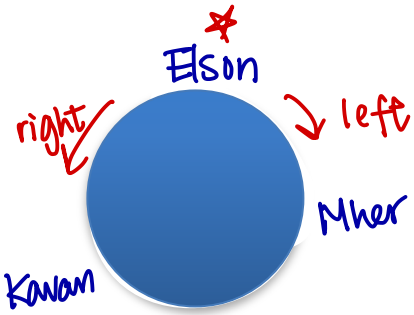
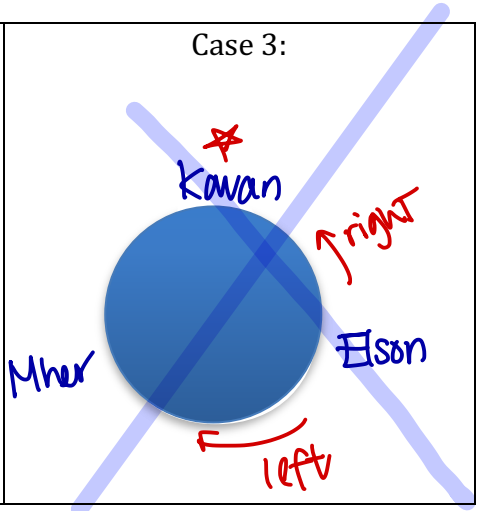
$\underline{2!} \times \underline{3} \times \underline{2} \times \underline{1} = 12$ ways

↑ YB AND Three blocks to arrange

Example 3: Circular permutation

3 people are to be seated at a round table for dinner. In how many ways can the people be seated for dinner?

Draw all possible solutions.

Case 1:	Case 2:	Case 3:
		

$(3-1)! = 2! = 2$ ways

Formula for circular permutation:

$$(n-1)!$$

Homework: Permutations of Distinct Elements Worksheet

1. Polygons are usually labelled by placing letters at the vertices.
   a) How many ways are there of labelling a triangle, using any three different letters of the alphabet? [26P3]
   b) How many ways are there of labelling a hexagon, using any six different letters of the alphabet? [26P6]

2. What is the number of four-letter words that can be formed using the letters found in each of the following words?

a) FOUR [24] b) PANEL [120] c) GROUND [360]
d) BROUGHT [840] e) STICKLER [1680] f) CANDY [120]

3. How many five-digit odd numbers can be formed from the digits of the number 5 390 462? [900]

4. In how many ways can all the letters of the word MASONITE be written so that the vowels occupy the even-numbered positions? [576]

5. If a penny, a nickel, and a dime are tossed simultaneously, in how many different ways can the three coins fall? [8]

P N D
2 × 2 × 2 = 8 ways

6. Given the digits 0, 1, 2, 3, 4 and 5, if no digit can be repeated,
   a) How many different three-digit numbers can be formed from? (Recall that 0 cannot be the first digit) [100]
   b) How many of the numbers formed in part (a) are even? [52]

7. Given the letters LINDSAY,
   a) How many distinct five-letter words can be made by arranging the letters. [2520]
   b) How many words can be made if D must be included? [1800]
   c) How many words can be made if D must NOT be included? [720]

a) 7 × 6 × 5 × 4 × 3

b) Direct: (1 × 6 × 5 × 4 × 3) × 5 (D)
Indirect: All − impossible
7P5 − no "D" 6P5

c) 6P5

8. Given the letter FERMAT,
   a) How many 4-permutations of the letters are there? [360]
   b) Find the number of distinct four-letter words containing at least one vowel that can be formed. [336]
   c) How many 6-permutations of the letters are there? [720]
   d) Find the number of distinct six-letter words in which the vowels appear adjacent to one another. [240]

9. There are six women and five men,
   a) How many ways can a photographer arrange them if the women must sit in the first row and the men stand in the second? [86400]
   b) How many different arrangements are there if the tallest man and the two tallest women must be at the centre of their respective rows? [1152]

10. In how many ways can four married couples be arranged around a circular table, (Thinking Question!!!)
   a) If there are no restrictions? [5040]
   b) If each man must sit beside his wife? [96]
   c) If each man sits beside his wife and men and women alternate? [12]
   d) If each man sits opposite his wife (that is, there are three people in between on either side)? [48]

8b) FRMT 4 consonants
EA 2 vowels

Direct methods: _ _ _ _

Case 1: 1 vowel + 3 consonants: (1 × 4 × 3 × 2) × 4

Case 2: 2 vowels: (2 × 1 × 4 × 3) × 6

Indirect method:
All without restrictions − impossible cases (no vowels)
6P4 − 4 × 3 × 2 × 1
= 6P4 − 4!
= 336

10a) (n−1)!
(8−1)! = 7!

b) Men: (4−1)! × 2 × 2 × 2 × 2

c) Men: (4−1)! × 2 × 1 × 1 × 1

d) m#2 m#3 m#4
1 × 6 × 1 × 4 × 1 × 2 × 1
wife from 1st man; W#2; W#3; W#4

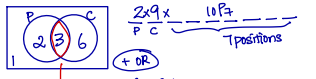
CP Pg 13-14 5) 7b) 7c) 10d) 13b) 15) 16) 17)

Thinking!!

#B 12 people
need only 9 people (different position)
↑ permutation

13a) $_{12}P_9$

b)



Case 1: $2 \times 9 \times \_\_\_\_\_$ ($_{10}P_7$ positions)
P C
+ OR
Case 2: $3 \times 6 \times \_\_\_\_\_$ ($_{10}P_7$ positions)
P C
+ OR
Case 3: $3 \times 2 \times \_\_\_\_\_$ ($_{10}P_7$ positions)
P C
$_3P_2$

① + ② + ③ = final answer

17) _ _ _ _

① $\frac{1}{5} \times 8 \times 7 \times 4$ (1, 3, 7, 9)

+ ② $\frac{7}{\times} \times \frac{1}{5} \times 7 \times 4$

+ ③ $\frac{7}{\times} \times 7 \times \frac{1}{5} \times 4$

+ ④ $8 \times 8 \times 7 \times \frac{1}{5}$

final answer

18) Indirect method:

_ _ _ _ _

All − impossible case (all separated by vowels)

$5! - \underset{C}{3} \times \underset{V}{2} \times \underset{C}{2} \times \underset{V}{1} \times \underset{C}{1}$

2 vowels
3 consonants.

$= 120 - 12$

$= 108$ ✓

15a) Direct

$(1 \times 4 \times 3 \times 2) \times 4$

① A

② A

③ A

④ A

Indirect

All − Imp (no "A")

$5P_4 - 4P_4$

15b) $\left(\frac{2!}{A\ B} \times 3 \times 2\right) \times 3$

case 1

case 2 _ $\overline{AB}$ _

case 3 _ _ $\overline{AB}$

16) $\frac{5}{①} \times \frac{5}{②} \times \frac{5}{③} \times \frac{5}{④} \times \frac{5}{⑤} \times \frac{5}{⑥} \times \frac{5}{⑦} \times \frac{5}{⑧}$

a) $= 5^8$

b) $\frac{5}{A} \times \frac{4}{\cancel{B}} \times \frac{4}{C} \times \frac{4}{\cancel{C}} \times \frac{4}{\cancel{C}} \times 4 \times 4 \times 4$

$= 5 \times 4^7$

11. In how many ways can a 5-member basketball team form a circle around its coach? [24]

12. Given the word STEVIN, how many distinct 5-letter words can be made
    a) if only one vowel is to be used? [240]
    b) if both vowels are to be used and E is to precede (come before) I? [240]
    c) if both vowels must be used, they must be adjacent and E is to precede I? [96]
    d) if either one vowel is to be used or E is to precede I? [480]

13. There are 12 candidates for a baseball team of 9 members. In how many ways can the 9 different positions on the field be allocated under the condition that: (Thinking Question!!!!)
    a) There is no restrictions? [79 833 600]
    b) Only 5 of the 12 candidates can be considered as a pitcher and 9 of the 12 as a catcher, and only 3 of the players are suitable for both a pitching and a catching position? All players can play any other position on the team. [25 401 600]

14. Suppose that you are at Bob's Restaurant and see the following items on the menu.

Dinners		Beverages		Deserts	
Big boy Hamburger	1.70	Coffee	0.45	Hot Fudge Sundae	1.20
Fried Chicken	3.95	Coca-Cola	0.45	Apple Pie	1.00
Shrimp	3.95	Root Beer	0.45	Banana Cream Pie	1.00
Fish and Fries	3.75	Milk	0.50	Strawberry Pie	1.00
Bob's Special Shark	5.50	Iced Tea	0.45	Cheese Cake	0.90
				Fudge Brownies	0.55

    a) If you ordered a dinner and a beverage, without dessert, how many different meals can you choose from? [25]
    b) If you also ordered a desert, how many meals are now available? [150]
    c) How many choices do you have if you order a dinner, beverage and desert, but decide that you do not want seafood? [60]

15. How many 4-letter words can be form from the letters A, B, C, D, E if,
    a) A is always included and no letter is to be used twice? [96]
    b) The A and B must be together and no letter is used twice? [36]

16. In how many different ways can an eight question multiple choice exam be answered
    a) if each question has 5 possible choices, A, B, C, D, and E? [390 625]
    b) if no two consecutive questions can have the same answer? [81 920]

17. how many odd four-digit numbers (all digit different) may be formed using the digits 0 to 9 if there must be a 5 in the number? [1064]

18. How many 5-letter words may be formed from BCDEA (with no repetition of letters) so that the consonants are not all separated from each other by vowels? [108]