Section 4.7 Worksheet – Combinations

MDM4U
David Chen

1) Evaluate each of the following

a) C(8, 3)

$$= \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = 56$$

b) ${}_7C_4$

$$= \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!} = 35$$

c) $\binom{!''}{11}$

$$= \frac{12!}{(12-11)!\,11!} = \frac{12!}{1!\,11!} = 12$$

d) C(10, 3)

$$= \frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!} = 120$$

2) In how many ways can a team of six female volleyball players be chosen to start the game from a roster of 12 players?

$$n_{(starting\ lineups)} = \binom{12}{6} = 924$$

3) In the card game Crazy Eights, how many different eight---card hands can be dealt from a standard 52---card deck?

$$n_{(hands)} = \binom{52}{8} = 752\,538\,150$$

4) From a group of 40 people, a jury of 12 people is selected. In how many different ways can a jury of 12 people be selected?

$$n_{(juries)} = \binom{40}{12} = 5\,586\,853\,480$$

5) There are 15 qualified applicants for 5 trainee positions in a fast---food management program. How many different groups of trainees can be selected?

$$n_{(groups)} = \binom{15}{5} = 3\,003$$

6) A pizza shop offers nine toppings. No topping is used more than once. In how many different ways can a three---topping pizza be formed?

$$n_{(3\ topping\ pizzas)} = \binom{9}{3} = 84$$

7) Ursula runs a small landscaping business. She has on hand 8 kinds of rose bushes, 10 kinds of small shrubs, 5 kinds of evergreen seedlings, and 7 kinds of flower lilies. In how many ways can Ursula fill an order if a customer wants 8 different varieties consisting of 3 roses, 3 shrubs, and 2 lilies?

$$n_{(3\ roses, 3\ shrubs, 2\ lilies)} = \binom{8}{3} \times \binom{10}{3} \times \binom{7}{2} = 56 \times 120 \times 21 = 141\ 120$$

8) From a group of five men and four women, determine how many committees of five people can be formed with

a) no restrictions

$$n_{(committees)} = \binom{9}{5} = 126$$

b) exactly three women

$$n_{(3\ women, 2\ men)} = \binom{4}{3} \times \binom{5}{2} = 4 \times 10 = 40$$

c) exactly four men

$$n_{(1\ woman, 4\ men)} = \binom{4}{1} \times \binom{5}{4} = 4 \times 5 = 20$$

d) no women

$$n_{(5\ men)} = \binom{5}{5} = 1$$

e) at least two men

$$n(\geq 2\ men) = 126 - n(1\ man, 4\ women)$$
$$= 126 - \binom{5}{1}\binom{4}{4}$$
$$= 126 - 5$$
$$= 121$$

f) at least three women

$$n(\geq 3\ women) = n(3\ women, 2\ men) + n(4\ women, 1\ man)$$
$$= \binom{4}{3}\binom{5}{2} + \binom{4}{4}\binom{5}{1}$$
$$= 40 + 5$$
$$= 45$$

9) One professor grades homework by randomly choosing 5 out of 12 homework problems to grade.

a) How many different groups of 5 problems can be chosen from the 12 problems?

$$n_{(groups\ of\ problems)} = \binom{12}{5} = 792$$

b) Jerry did only 5 problems of one assignment. What is the probability that the problems he did comprised the group that was selected to be graded?

$$P_{(right\ group)} = \frac{1}{792}$$

c) Silvia did 7 problems. How many different groups of 5 did she complete? What is the probability that one of the groups of 5 she completed comprised the group selected to be graded?

$$P(right\ group) = \frac{\binom{7}{5}}{\binom{12}{5}} = \frac{21}{792} = \frac{7}{264}$$

10) The qualified applicant pool for six management trainee positions consists of seven women and five men.

a) How many different groups of applicants can be selected for the positions?

$$n_{(groups)} = \binom{12}{6} = 924$$

b) How many different groups of trainees would consist entirely of women?

$$n_{(groups\ with\ only\ women)} = \binom{7}{6} = 7$$

c) If the positions are selected at random, what is the probability that the trainee class will consist entirely of women?

$$P_{(only\ women)} = \frac{7}{924} = \frac{1}{132}$$

11) Find the probability of being dealt five diamonds from a standard deck of playing cards.

$$P(5\ diamonds) = \frac{n(5\ diamonds)}{n(5\ card\ hands)} = \frac{\binom{13}{5}}{\binom{52}{5}} = \frac{1\ 287}{2\ 598\ 960}$$

12) Three cards are selected at random from a standard deck of 52 playing cards. Determine the probability that all three cards are

a) hearts

$$n(3\ hearts) = \frac{\binom{13}{3}}{\binom{52}{3}} = \frac{286}{22\ 100} = \frac{11}{850}$$

b) black

$$n(3\ black) = \frac{\binom{26}{3}}{\binom{52}{3}} = \frac{2\ 600}{22\ 100} = \frac{2}{17}$$

c) aces

$$n(3\ aces) = \frac{\binom{4}{3}}{\binom{52}{3}} = \frac{4}{22\ 100} = \frac{1}{5\ 525}$$

d) face cards

$$n(3\ face\ cards) = \frac{\binom{12}{3}}{\binom{52}{3}} = \frac{220}{22\ 100} = \frac{11}{1\ 105}$$

13) A paper bag contains a mixture of three types of candy. There are ten gum balls, seven candy bars, and three packages of toffee. Suppose a game is played in which a candy is randomly taken from the bag and then a second candy is drawn from the bag, without replacement. You are allowed to keep both candies, if, and only if, the second is the same type as the first.

a) Calculate the probability that you will be able to keep a gum ball on the first try.

$$P(win\ gum) = \frac{\binom{10}{2}}{\binom{20}{2}} = \frac{45}{190} = \frac{9}{38} \qquad \text{OR} \qquad P(win\ gum) = \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = \frac{9}{38}$$

b) Calculate the probability that you will be able to keep any candy on the first try.

$$P_{(win\ any\ candy)} = \frac{\binom{10}{2}}{\binom{20}{2}} + \frac{\binom{7}{2}}{\binom{20}{2}} + \frac{\binom{3}{2}}{\binom{20}{2}} = \frac{45}{190} + \frac{21}{190} + \frac{3}{190} = \frac{69}{190}$$

c) Calculate the probability that you will not be able to keep any candy on the first try.

$$P_{(lose)} = 1 - P_{(win\ any)} = 1 - \frac{69}{190} = \frac{121}{190}$$

14) Melik has five quarters and six dimes in his pocket. He pulls out one coin.

a) What are the odds of the coin being a quarter?

5:6

b) What are the odds of the coin being a dime?

6:5

15) Suppose the probability of rain tomorrow is 80%. What are the odds of rain tomorrow?

80:20 = 4:1

16) The coach says that the probability of winning the next game is 40%. What are the odds the team will win?

40:60 = 2:3