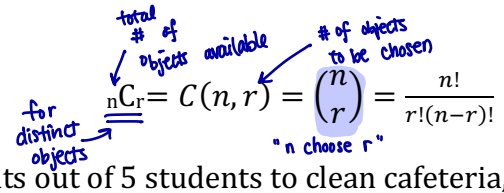
Lesson: Combination

Definition of combination

– a combination of "n" distinct objects taken r at a time is a selection of r of the n objects without regard to order. The total number of combinations of n objects taken r at a time is denoted by ${}_nC_r$, read as "n choose r".

- Order is not important



Example: If you had to choose 3 students out of 5 students to clean cafeteria, order of students would not be important. The students are indistinguishable because they are all doing the same job. The same is true for those that are not chosen.

Examples PART A:

Given 2 distinct objects, how many groups of:	0 objects can you make? $2C_0 = C(2,0)$ $= \binom{2}{0}$ $= \frac{2!}{0!(2-0)!}$ $= 1$	1 object can you make? $2C_1 = C(2,1)$ $= \binom{2}{1}$ $= 2$	2 objects can you make? $2C_2 = C(2,2)$ $= \binom{2}{2}$ $= 1$		
Given 3 distinct objects, how many groups of:	0 objects can you make? $3C_0 = \binom{3}{0}$ $= 1$	1 object can you make? $\binom{3}{1}$ $= 3$	2 objects can you make? $\binom{3}{2}$ $= 3$	3 object can you make? $\binom{3}{3}$ $= 1$	
Given 4 distinct objects, how many groups of:	0 objects can you make? $\binom{4}{0}$ $= 1$	1 object can you make? $\binom{4}{1}$ $= 4$	2 objects can you make? $\binom{4}{2}$ $= 6$	3 objects can you make? $\binom{4}{3}$ $= 4$	4 object can you make? $\binom{4}{4}$ $= 1$

Pascal's Triangle!!

COMBINATIONS

1. If there are 7 teachers, how many different groups of 3 teachers can be chosen to be on the social committee?

$_7C_3 = C(7,3) = \binom{7}{3} = 35$ ways

2. If you have 5 different candies, how many ways can you put 2 or 3 candies into a bag?

case ① $\binom{5}{2}$ + case ② $\binom{5}{3}$ = 20 ways

3. If there are 5 different fruits and 7 different vegetables, how many ways can you make a:
   a. Group of 4 fruits?

$\binom{5}{4} = 5$ ways ✓ ✓

   b. Group of 4 vegetables?

$\binom{7}{4} = 35$ ways ✓ ✓

   c. Group of 4 fruits or 4 vegetables?

$\binom{5}{4} + \binom{7}{4} = 40$ ways ✓ ✓ ✓

   d. Group of 4 fruits and 4 vegetables?

$\binom{5}{4} \times \binom{7}{4} = 175$ ways ✓ ✓ ✓

4. If there are 3 kinds of gum, 5 kinds of candies and 4 kinds of toys, how many different kinds of loot bags could be made if:
   a. They include 2 from each grouping (a total of 6 items)?

$\binom{3}{2} \times \binom{5}{2} \times \binom{4}{2} = 180$ ways

2 Gums
2 Candies
2 Toys

   b. They include 3 each from two groups (a total of 6 items)?

① Gum Candies $\binom{3}{3}\binom{5}{3}$ or ② Gum Toys $\binom{3}{3}\binom{4}{3}$ or ③ Candies Toys $\binom{5}{3}\binom{4}{3} = 54$ ways

   c. They must include all the kinds of gum and all the kinds of candies and at least 2 kinds of toys?

Gum Candies Toys

2 Toys case ① $\binom{3}{3} \times \binom{5}{5} \times \binom{4}{2} = 6$ ✓

3 Toys + case ② $\binom{3}{3} \times \binom{5}{5} \times \binom{4}{3} = 4$ ✓

4 Toys + case ③ $\binom{3}{3} \times \binom{5}{5} \times \binom{4}{4} = 1$ ✓

= 11 ways ✓