Rule of Sum

2.2 ~~Organized Counting and Fundamental Counting Principle~~

Rule of Sum (Additive Counting Principle):

> If a first action can be performed 'm' ways, AND a second action can be performed 'n' ways, and the actions are MUTUALLY EXCLUSIVE, then there are $m + n$ ways in which either the first **OR** second action can be performed.
>
> Note: mutually exclusive actions cannot be performed at the same time!

Example 1: how many 1 digit or 2-digit positive integers end in the digit 7?

$1 + 9$
$= 10$

$\underset{\text{just } 7}{\underline{1}} + \underset{1 \ldots 9}{\underline{9}} \times \underset{\text{just } 7}{\underline{1}}$

Example 2: How many 2-digit positive integers are divisible by either 2 or 5?

$\left(\frac{98-10}{2} + 1\right) + \left(\frac{95-10}{5} + 1\right) - 9 = x$

$45 + 18 - 9 = 54$

double counted
"end in 0": 20 – 90.

divisible by 2
1st digit: 1 ... 9
2nd digit: 2, 4, 6, 8, 0.
$9 \times 5 = 45$

divisible by 5
1st digit: 1 ... 9
2nd digit: 5 or 0
$9 \times 2 = 18$

Example 3: Sums of Dice Questions:
In how many ways can you roll either a sum of 4 or a sum of 11 with a pair of dice?

Method 1: Rule of Sum

Case 1: Sum of 4   1, 2, 3

$\underset{\text{\# choices from 1st dice}}{\underline{\quad 3 \quad}} \times \underset{\text{\# of choices from 2nd dice}}{\underline{\quad 1 \quad}}$ 3

Case 1: Sum of 11   5 + 6

+

Method 2: Array

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example 4: At an international conference, in how many different arrangement could the countries' flags be flown if ...

a) Eight **or** nine countries attend?

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880$

$+$

403200 ways

b) Seven, eight or nine countries attend?

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 + 40320 + 362880$

$= 408240$ ways

c) Seven, eight or nine countries attend **but** the host country's flag must be on the far left?

$1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$1 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

scoops, of 2.2 ✓

Example 5: I can make an ice cream cone by using **at most** 3 different flavors of ice cream. I have chocolate, vanilla, strawberry, mango and butterscotch flavors. How many different cones can I make? What assumption did you make?

@ most 3 → 1 or 2 or 3

$$\begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 5 & 5\times4 & 5\times4\times3 \\ & =20 & =60 \end{array}$$

* assuming it we repeat flavours are not ok. max of 3 scoops.

Total : $5 + 20 + 60$
$= 185$ ways

Example 6: In how many ways can you deal three cards that are ...

* review cards & totals
52 total, 13 / suit
26 red / 26 black
"face" cards

a) All black or all red?

$26 \times 25 \times 24 = 15600 \quad \underset{\text{black or red}}{\times 2} = 31\,200$ ways

b) All hearts or all aces?

(−1)

$\underbrace{13 \times 12 \times 11}_{\text{2 suits ♡ or aces}} + \underbrace{4 \times 3 \times 2}_{\text{4 aces possible}} = 1740$ ways

or

← (ace of ♡'s, not overlapped by itself)

c) All face cards or all red?

$\underset{\text{(12 face card}}{12 \times 11 \times 10} + \underset{\text{all red}}{26 \times 25 \times 24} - \underset{\text{overlap}}{6 \times 5 \times 4} = 16,800$ ways

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Example 7: Ten members of a basketball team are lining up for their medals after a tournament. In how many ways can they line up if ...

a) There are no restrictions?

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3{,}628{,}800$ ways

b) The captain and assistant captain must be together?

$(9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) \times 2 = 725{,}760$ ways

CA
AC

c) The captain and assistant captain must not be together?

Indirect : Total − CA together

$= 3{,}628{,}800 - 725{,}760$

$= 2\,903\,040$ ways