Section 3.5a Worksheet – Applying the Normal Distribution

MDM4U
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1) Calculate a z---score for each x---value given $\mu = 6$ and $\sigma = 2$.

a) $x = 5.3$

$$z_{5.3} = \frac{x-\mu}{\sigma} = \frac{5.3-6}{2} = -0.35$$

b) $x = 7.2$

$$z_{7.2} = \frac{x-\mu}{\sigma} = \frac{7.2-6}{2} = 0.6$$

c) $x = 9.9$

$$z_{9.9} = \frac{x-\mu}{\sigma} = \frac{9.9-6}{2} = 1.95$$

d) $x = 0.8$

$$z_{0.8} = \frac{x-\mu}{\sigma} = \frac{0.8-6}{2} = -2.6$$

2) Using the z---score table (or your calculator), find the percentile that corresponds to each of the following z---scores.

a) $z = 2.33$

$$area\ to\ the\ left\ of\ 2.33 = normalcdf(lower = -E99, upper = 2.33, \mu = 0, \sigma = 1) = 0.99$$

A z---score of 2.33 is in the 99th percentile.

b) $z = -0.83$

$$area\ to\ the\ left\ of\ -0.83 = normalcdf(lower = -E99, upper = -0.83, \mu = 0, \sigma = 1) = 0.203$$

A z---score of ---0.83 is in the 20th percentile.

3) Given a normally distributed data set whose mean is 50 and whose standard deviation is 10, what value of x would a z---score of 2.5 have?

$$z = \frac{x - \mu}{\sigma}$$

$$2.5 = \frac{x - 50}{10}$$

$$25 = x - 50$$

$$x = 75$$

4) Adrian's average bowling score is 174, and is normally distributed with a standard deviation of 35. In what percent of games does Adrian score more than 200 points?

	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
0.5	0.6915	0.6950	0.6985	0.7019	0.7054
0.6	0.7257	0.7291	0.7324	0.7357	0.7389
0.7	0.7580	0.7611	0.7642	0.7673	0.7704

Method 1: z---score

$$z_{200} = \frac{x!\mu}{\sigma} = \frac{200!174}{35} = 0.74$$

From table:

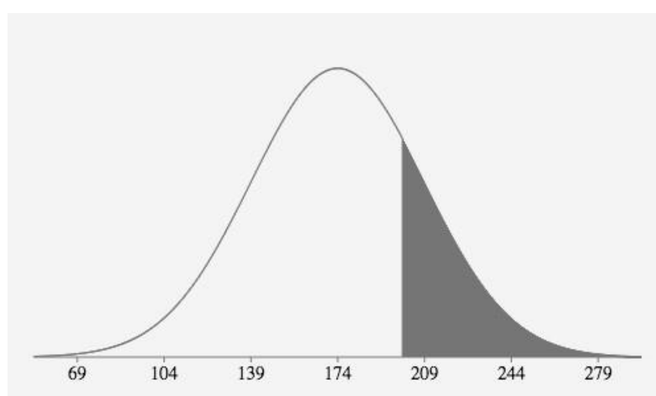
Area to right of 0.74 = 1 – area to left of 0.74 = 1 – 0.7704 = 0.2296

About 22.96% of her games she has a score more than 200 points.

Method 2: Calculator

Area to the right of 200 $= normalcdf(lower = 200, upper = E99, \mu = 174, \sigma = 35) = 0.2288$

About 22.88% of her games she has a score more than 200 points.



5) The top 10% of bowlers in Adrian's league get to play in an all---star game. If the league average is 170, with a standard deviation of 11 points, and is normally distributed what average score does Adrian need to have to obtain a spot in the all---star game?

Method 1: z---score

We are looking for the x value in the 90th percentile. From the table, the z---score with area to the left closest to 0.90 is 1.28.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997

$$z = \frac{x - \mu}{\sigma}$$

$$1.28 = \frac{x - 170}{11}$$

$$14.08 = x - 170$$

$x = 184.08$ *Adrian needs a score of about 184 to get a spot in the all---star game.*

Method 2: Calculator

x---value in 90th percentile = $invnorm(area = 0.9, \mu = 170, \sigma = 11) = 184.1$

Adrian needs a score of about 184 to get a spot in the all---star game.

6) IQ score of people around the world are normally distributed, with a mean of 100 and a standard deviation of 15. A genius is someone with an IQ greater than or equal to 140. What percent of the population is considered genius?

Method 1: z---score

$$z_{140} = \frac{140 - 100}{15} = 2.67$$

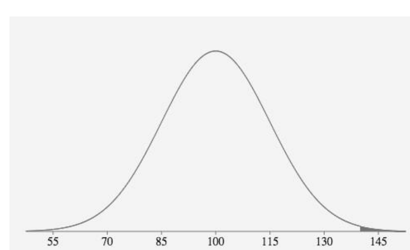
From the z---score table, the area to the right of 2.67 is = 1 – area to the left = 1 – 0.9962 = 0.0038

About 0.38% of the population is considered genius.

Method 2: calculator

% IQ > 140 = $normalcdf(lower = 140, upper = E99, \mu = 100, \sigma = 15) = 0.0038$

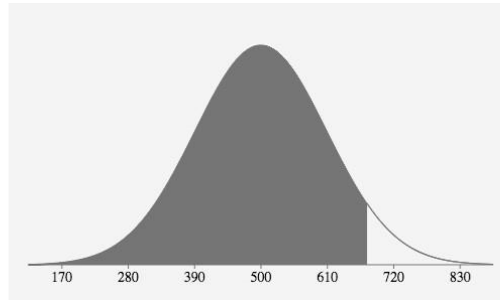
About 0.38% of the population is considered genius.



7) A standardized test is known to be normally distributed with a mean of 500 and a standard deviation of 110.

a) A student's score is 675, what percentile is she in?

Method 1: z-‐‑score



$$z_{675} = \frac{675 - 500}{110} = 1.59$$

From the z-‐‑score table, the area to the left of 1.59 is 0.9441

She is in about the 94th percentile.

Method 2: Calculator

% scores < 675 = $normalcdf(lower = -E99, upper = 675, \mu = 500, \sigma = 110) = 0.9442$

She is in about the 94th percentile.

b) Another student taking the same test wants to score in the 80th percentile. What score must he get?

Method 1: z-‐‑score

We are looking for the x value in the 80th percentile. From the table, the z-‐‑score with area to the left closest to 0.80 is 0.84.

	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
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0.6	0.7257	0.7291	0.7324	0.7357	0.7389
0.7	0.7580	0.7611	0.7642	0.7673	0.7704
0.8	0.7881	0.7910	0.7939	0.7967	0.7995

$$z = \frac{x - \mu}{\sigma}$$

$$0.84 = \frac{x - 500}{110}$$

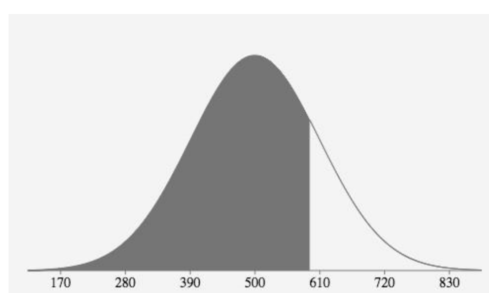
$$92.4 = x - 500$$

$x = 592.4$ *He must get a score of about 592 to be in the 80th percentile.*

Method 2: Calculator

x-‐‑value in 80th percentile = $invnorm(area = 0.8, \mu = 500, \sigma = 110) = 592.6$

He must get a score of about 593 to be in the 80th percentile.



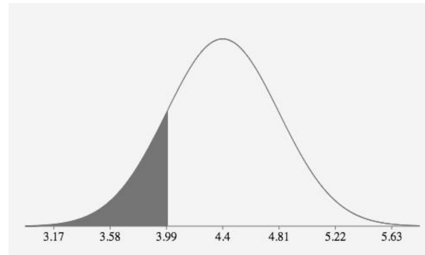
8) The weights of 75 model planes at a local convention are normally distributed. The average weight is 4.4 kg, with a standard deviation of 0.41 kg.

a) How many planes have a mass less than 4 kg?

% planes < 4 = $normalcdf(lower = -E99, upper = 4, \mu = 4.4, \sigma = 0.41) = 0.1646$

planes < 4 = $0.1646 \times 75 = 12.345$

About 12 planes have a mass less than 4 kg.

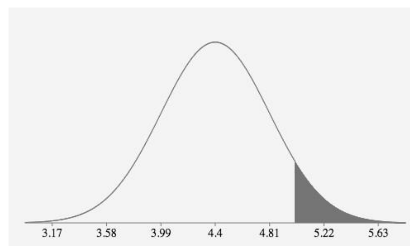


b) How many planes would be disqualified if it were against the rules to have a plane with a mass of more than 5 kg?

% planes > 5 = $normalcdf(lower = 5, upper = E99, \mu = 4.4, \sigma = 0.41) = 0.0717$

planes > 5 = $0.0717 \times 75 = 5.38$

Approximately 5 planes would be disqualified.

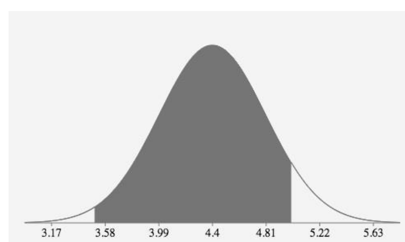


c) How many planes have a mass between 3.5 kg and 5 kg?

% 3.5 < planes < 5 = $normalcdf(lower = 3.5, upper = 5, \mu = 4.4, \sigma = 0.41) = 0.9142$

3.5 < planes < 5 = $0.9142 \times 75 = 68.57$

Approximately 69 planes are between 3.5 and 5 kg.

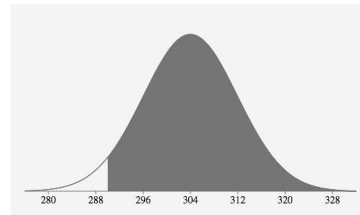


9) On the driving range, Tiger Woods practices his swing with a particular club by hitting many, many balls. Suppose that wen Tiger hits his driver, the distance the ball travels follows a normal distribution with mean 304 yards and standard deviation 8 yards.

a) What percent of Tiger's drives travel at least 290 yards?

% drives > 290 = $normalcdf(lower = 290, upper = E99, \mu = 304, \sigma = 8) = 0.9599$

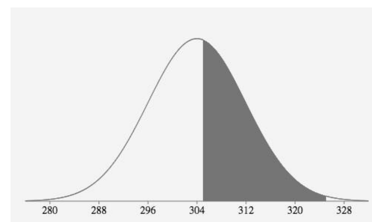
Approximately 95.99% of his drives travel at least 290 yards.



b) What percent of Tiger's drives travel between 305 and 325 yards?

% 305 < drives < 325 = $normalcdf(lower = 305, upper = 325, \mu = 304, \sigma = 8) = 0.4459$

Approximately 44.59% of his drives travel between 305 and 325 yards.



10) For the distribution $X \sim N(3, 0.74^2)$, determine the percent of the data that is within the given interval

a) $X > 2.44 = normalcdf(lower = 2.44, upper = E99, \mu = 3, \sigma = 0.74) = 0.7754 = 77.54\%$
b) $1.8 < X < 2.3 = normalcdf(lower = 1.8, upper = 2.3, \mu = 3, \sigma = 0.74) = 0.1196 = 11.96\%$
c) $X < 1.91 = normalcdf(lower = -E99, upper = 1.91, \mu = 3, \sigma = 0.74) = 0.0704 = 7.04\%$

11) Perch in a lake have a mean length of 20 cm and a standard deviation of 5 cm. What would be the length of a fish in the 95th percentile?

Length of fish in the 95th percentile $= invnorm(area = 0.95, \mu = 20, \sigma = 5) = 28.2$

The 95th percentile length of fish is about 28.2 cm.