

5 cards in hand.

① 4 of a kind and a random card.

$\boxed{A} \boxed{A} \boxed{A} \boxed{A} \boxed{B}$

$$\binom{13}{1} \binom{4}{4} \binom{12}{1} \binom{4}{1} = 624 \text{ ways}$$

② 3 of a kind and a pair = 3744 ways

AAABB $\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2}$

③ 2 pairs and a random card.

AABBC = 123552 ways

$$\frac{\binom{13}{1} \binom{4}{2} \binom{12}{1} \binom{4}{2} \binom{11}{1} \binom{4}{1}}{2!} \quad \frac{n!}{(n-r)!} \quad \frac{n!}{r! (n-r)!}$$

OR - $\binom{13}{2} \binom{4}{2} \binom{4}{2} \binom{11}{1} \binom{4}{1}$ OR $\binom{13}{1} \binom{4}{1} \binom{12}{2} \binom{4}{2} \binom{4}{2}$

④ one pair and 3 random cards.

= 1098240 ways

$$\binom{13}{1} \binom{4}{2} \binom{12}{3} \binom{4}{1} \binom{4}{1} \binom{4}{1}$$

⑤ 5 card straight.
(consecutive)

high card &
low card are
Ace

= 10240 ways

$$10 \times 4^5 = \binom{10}{1} \binom{4}{1} \binom{4}{1} \binom{4}{1} \binom{4}{1} \binom{4}{1}$$

A2345

23456








34567

45678

⋮

10JQKA

10 1 1 1 1

Hand	Distinct hands	Frequency	Probability	Cumulative probability	Odds	Mathematical expression of absolute frequency
Royal flush 	1	4	0.000154%	0.000154%	649,739 : 1	$\binom{4}{1}$
Straight flush (excluding royal flush) 	9	36	0.00139%	0.0015%	72,192 : 1	$\binom{10}{1}\binom{4}{1} - \binom{4}{1}$
Four of a kind 	156	624	0.0240%	0.0256%	4,164 : 1	$\binom{13}{1}\binom{12}{1}\binom{4}{1}$
Full house 	156	3,744	0.1441%	0.17%	693 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$
Flush (excluding royal flush and straight flush) 	1,277	5,108	0.1965%	0.367%	508 : 1	$\binom{13}{5}\binom{4}{1} - \binom{10}{1}\binom{4}{1}$
Straight (excluding royal flush and straight flush) 	10	10,200	0.3925%	0.76%	254 : 1	$\binom{10}{1}\binom{4}{1}^5 - \binom{10}{1}\binom{4}{1}$
Three of a kind 	858	54,912	2.1128%	2.87%	46.3 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2$
Two pair 	858	123,552	4.7539%	7.62%	20.0 : 1	$\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}$
One pair 	2,860	1,098,240	42.2569%	49.9%	1.37 : 1	$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$
No pair / High card 	1,277	1,302,540	50.1177%	100%	0.995 : 1	$\left[\binom{13}{5} - 10\right] \left[\binom{4}{1}^5 - 4\right]$
Total	7,462	2,598,960	100%	---	0 : 1	$\binom{52}{5}$