Section 4.2 - - - Theoretical Probability

MDM4U
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The probability of any outcome of a chance process is a number between 0 and 1 that describes the proportion of times the outcome would occur in a very long series of repetitions.

Part 1: Video on Probability

http://www.learner.org/courses/againstallodds/unitpages/unit19.html

Answer the following questions while watching the video:

1. What is a probability model (distribution)?

A probability model is the set of all possible outcomes together with the probabilities associated with those outcomes.

2. Describe the sample space for the sum of two dice.

S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

3. What is the probability of rolling two dice and getting a sum of seven?

P(7) = 6/36 = 1/6

4. If you know the probability that event A occurs, how do you calculate the probability that event A does not occur?

P(not A) = 1 – P(A)

5. What probability can you find using the Addition Rule? 6. What probability can you find using the Multiplication Rule?

If events A and B are mutually exclusive, you can use the Addition Rule to calculate P(A or B), the probability that either A or B occurs. $P(A \cup B) = P(A) + P(B)$

6. What probability can you find using the Multiplication Rule?

If events A and B are independent, you can use the Multiplication Rule to calculate P(A and B), the probability that both A and B occur. $P(A \cap B) = P(A) \times P(B)$

Part 2: Theoretical Probability

Assuming that all outcomes are equally likely, the probability of an event in an experiment is the ratio of the number of outcomes that make up that event to the total number of possible outcomes.

The formula for the probability of an event *A* is:

$$P(A) = \frac{n(A)}{n(S)}$$

S is the collection of all possible outcomes of the experiment (the sample space)

A is the collection of outcomes that correspond to the event of interest (the event space)

n(S) and n(A) are the numbers of elements in the two sets.

Examples of Theoretical Probability

We know that there are 6 possible outcomes when a die is rolled. Only one of these outcomes is the event of rolling a 4. Since the outcomes are all equally likely to happen, it is reasonable to expect that the fraction of the time you roll a 4 is the ratio of the number of ways a 4 can occur to the number of possible outcomes.

$$\boldsymbol{P(4) = \frac{n(4)}{n(S)} = \frac{1}{6}}$$

What is the probability of rolling an even number?

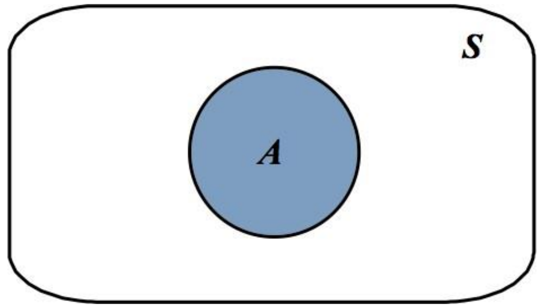
$$\boldsymbol{P(even) = \frac{n(even)}{n(S)} = \frac{3}{6} = \frac{1}{2}}$$

Given a large enough number of trials, the experimental probability should trend towards the theoretical probability

Venn Diagrams

A Venn diagram can be used to show the relationship between the event space, $\underline{A}$, and the sample space, $\underline{S}$. Venn diagrams will be used more in section 4.3.

The Venn diagram shows A as the shaded region within S.



Example 1

If a single die is rolled, determine the probability of rolling:

a) an odd number

A = odd = {1, 3, 5}

S = {1, 2, 3, 4, 5, 6}

$$P(odd) = \frac{n(A)}{n(S)} = \frac{n(odd)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

b) a 5 or a 2

A = 5 or 2 = {2, 5}

S = {1, 2, 3, 4, 5, 6}

$$P(5 \text{ or } 2) = \frac{n(A)}{n(S)} = \frac{n(5 \text{ or } 2)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

c) A number greater than 2

A = >2 = {3, 4, 5, 6}

S = {1, 2, 3, 4, 5, 6}

$$P(>2) = \frac{n(A)}{n(S)} = \frac{n(!2)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Example 2

A bag contains 20 marbles

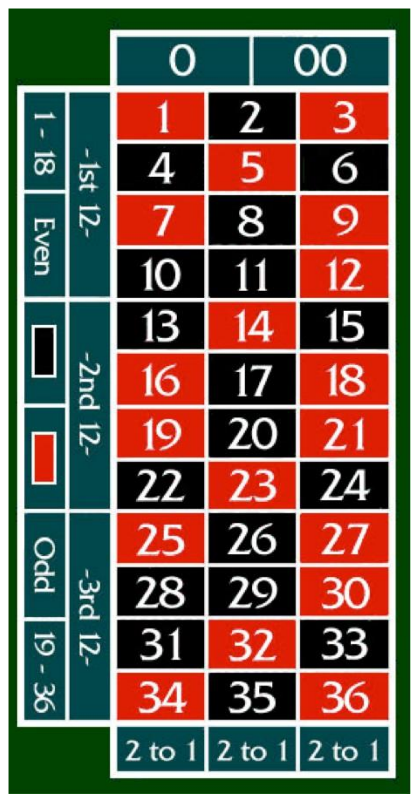
4 Blue, 6 Orange, 7 Green, 3 Purple

What is the probability of pulling a green marble?

$$P(green) = \frac{n(green)}{n(S)} = \frac{7}{20}$$

Example 3

Roulette was invented by Blaise Pascal in his search for a perpetual motion machine...



$$P(red) = \frac{n(red)}{n(S)} = \frac{18}{38} = \frac{9}{19}$$

$$P(even) = \frac{n(even)}{n(S)} = \frac{18}{38} = \frac{9}{19}$$

$$P(14) = \frac{n(14)}{n(S)} = \frac{1}{38}$$

$$P(1st\ 12) = \frac{n(1st\ 12)}{n(S)} = \frac{12}{38} = \frac{6}{19}$$

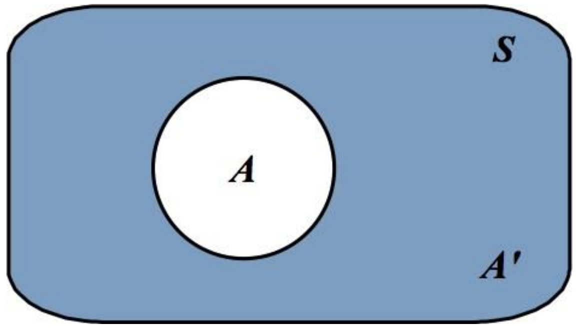
Part 3: Probability and Complimentary Events

The complement of a set (A), is written as $P(A')$ and consists of all the outcomes in the sample space that are NOT in A.

A' = {outcomes in S that are NOT in A}

$$\mathbf{P(A') = 1 - P(A)}$$

The Venn diagram shows A' as the shaded region within S that is entirely outside of A.



Example 4

In example 1b, we calculated the probability of rolling a 2 or a 5 to be $\frac{1}{3}$. Use this to calculate the probability of NOT rolling a 2 or a 5.

$$P(2\ or\ 5') = 1 - P(2\ or\ 5)$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Example 5

A standard deck of cards comprises 52 cards in four suits --- clubs, hearts, diamonds, and spades. Each suit consists of 13 cards --- ace through 10, jack, queen, and king.

a) What is the probability of drawing an ace from a well---shuffled deck?

$$P(ace) = \frac{n(ace)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

b) What is the probability of drawing anything but an ace?

$$P(ace') = 1 - P(ace)$$

$$= 1 - \frac{1}{13}$$

$$= \frac{12}{13}$$

c) What is the probability of selecting a face card (J, Q, K)?

$$P(face) = \frac{n(face)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

d) What is the probability of NOT selecting a face card (J, Q, K)?

$P(face') = 1 - P(face)$

$= 1 - \frac{3}{13}$

$= \frac{10}{13}$