5.3 Worksheet – Binomial Probability Distributions

MDM4U
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1) For each term, identify i) the number of trials ii) the probability p of a success iii) the number of successes

a) $\binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4$

i) $n = 10$ ii) $p = \frac{1}{2}$ iii) $k = 6$

b) $\binom{7}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^4$

i) $n = 7$ ii) $p = \frac{1}{3}$ iii) $k = 3$

2) Mail---order marketing companies have a response rate of 15% to their advertising flyers.

a) Compute the probability that exactly 3 people out of a sample of 20 respond to the flyers they receive.

$P(X = 3) = \binom{20}{3}(0.15)^3(0.85)^{17} = 0.243$

$n = 20$

$p = 0.15$

$k = 3$

OR

$P(X = 3) = binompdf(n = 20, p = 0.15, k = 3) = 0.243$

b) Find the expected number of people in a sample of 20 who will respond to the flyers.

$E(x) = np = 20(0.15) = 3$

c) Compute the probability that at least 3 people out of a sample of 20 respond to the flyers they receive.

$P(X \geq 3) = 1 - P(X \leq 2)$
$= 1 - binomcdf(n = 20, p = 0.15, k = 2)$
$= 1 - 0.4049$
$= 0.5951$

3) A study published in a consumer magazine indicated that when a husband and a wife shop for a car, the husband exerts the primary influence in the decision 70% of the time. Five couples who will be purchasing a car are selected at random. Determine the probability of each of the following.

a) In exactly two of the couples, the husband will exert the primary influence on the decision.

$$P(X = 2) = \binom{5}{2}(0.7)^2(0.3)^3 = 0.1323$$

$n = 5$

$p = 0.7$

$k = 2$

OR

$$P(X = 2) = binompdf(n = 5, p = 0.7, k = 2) = 0.1323$$

b) In all five couples, the husband will exert the primary influence on the decision.

$$P(X = 5) = \binom{5}{5}(0.7)^5(0.3)^0 = 0.16807$$

OR

$$P(X = 2) = binompdf(n = 5, p = 0.7, k = 5) = 0.16807$$

c) Find the expected number of couples in which the husband will exert primary influence.

$$E(x) = np = 5(0.7) = 3.5$$

4) A baseball player has a batting average of 0.280. Find each of the following probabilities

a) Exactly 4 hits in her next 10 times at bat

$$P(X = 4) = \binom{10}{4}(0.28)^4(0.72)^6 = 0.1798$$

OR

$$P(X = 4) = binompdf(n = 10, p = 0.28, k = 4) = 0.1798$$

b) More than 4 hits in her next 10 times at bat

$$P(X > 4) = 1 - P(X \leq 4)$$
$$= 1 - binomcdf(n = 10, p = 0.28, k = 4)$$
$$= 1 - 0.8819$$
$$= 0.1181$$

c) Less than 6 hits in her next 10 times at bat

$$P(X < 6) = P(X \leq 5)$$
$$= binomcdf(n = 10, p = 0.28, k = 5)$$
$$= 0.9658$$

d) What is the player's expected number of hits in her next 10 times at bat?

$E(x) = np = 10(0.28) = 2.8$

5) Ten percent of the keyboards a computer company manufactures are defective. Determine the probability that one or more of the next three keyboards to come off the assembly line will be defective.

$P(X \geq 1) = 1 - P(0)$
$= 1 - binompdf(n = 3, p = 0.1, k = 0)$
$= 1 - 0.729$
$= 0.271$

6) Determine the probability, correct to four decimal places, that a die rolled six times in a row will produce the following:

a) one 3

$$P(X = 1) = \binom{6}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5 = 0.4019$$

OR

$$P(X = 1) = binompdf\left(n = 6, p = \frac{1}{6}, k = 1\right) = 0.4019$$

b) at least five 3's

$P(X \geq 5) = 1 - P(X \leq 4)$
$= 1 - binompdf\left(n = 6, p = \frac{1}{6}, k = 4\right)$
$= 1 - 0.9993$
$= 0.0007$

c) two 3's or less

$$P(X \leq 2) = binomcdf\left(n = 6, p = \frac{1}{6}, k = 2\right) = 0.9377$$

7) A multiple---choice quiz has 10 questions. Each question has four possible answers. Sam is certain that he knows the correct answer for Questions, 3, 5, and 8. If he guesses on the other questions, determine the probability that he gets at least 50% on the quiz.

To get at least 50%, Sam will need to guess at least 2 of the remaining seven questions correctly.

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - binomcdf(n = 7, p = 0.25, k = 1) = 1 - 0.4449 = 0.5551$

8) A survey indicates that 41% of women in the United States consider reading as their favorite leisure---time activity. You randomly select four U.S. women and ask them if reading is their favorite leisure---time activity.

a) Create a binomial probability distribution for the number of women who respond yes.

# of women whose favorite leisure activity is reading (*X*)	**P(*X*)**
0	$\binom{4}{0}(0.41)^0(0.59)^4 = 0.121$ $binompdf(n = 4, p = 0.41, k = 0) = 0.121$
1	$\binom{4}{1}(0.41)^1(0.59)^3 = 0.337$ $binompdf(n = 4, p = 0.41, k = 1) = 0.337$
2	$\binom{4}{2}(0.41)^2(0.59)^2 = 0.351$ $binompdf(n = 4, p = 0.41, k = 2) = 0.351$
3	$\binom{4}{3}(0.41)^3(0.59)^1 = 0.163$ $binompdf(n = 4, p = 0.41, k = 3) = 0.163$
4	$\binom{4}{4}(0.41)^4(0.59)^0 = 0.028$ $binompdf(n = 4, p = 0.41, k = 4) = 0.028$

b) What is the probability that at least two of them respond yes?

$P(\geq 2) = P(2) + P(3) + P(4) = 0.351 + 0.163 + 0.028 = 0.542$

c) What is the expected number of women in the group of 4 that would choose reading as their favourite leisure time activity?

$E(x) = np = 4(0.41) = 1.64$