Section 4.4 – Conditional Probability

MDM4U

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Refer to part 1 of 4.4 lesson for help with the following questions

1) Joel surveyed his class and summarized responses to the question, "Do you like school?"

	Liked	Disliked	No Opinion	Total
Males	12	5	2	19
Females	10	3	1	14
Total	22	8	3	33

Find each of the following probabilities:

a) $P(likes\ school\ |student\ is\ male)$

$$P(likes\ school\ |\ student\ is\ male) = \frac{n(likes\ school \cap male)}{n(male)} = \frac{12}{19}$$

b) $P(student\ is\ female\ |student\ dislikes\ school)$

$$P(student\ is\ female\ |\ student\ dislikes\ school) = \frac{n(female \cap dislikes\ school)}{n(dislikes\ school)} = \frac{3}{8}$$

2) A person is chosen at random from shoppers at a department store. If the person's probability of having blonde hair and glasses is $\frac{2}{25}$ and the probability of wearing glasses is $\frac{9}{25}$, determine the probability that a person has blonde hair given that they wear glasses.

$$P(blonde\ hair\ |\ wears\ glasses) = \frac{P(blonde \cap glasses)}{P(glasses)} = \frac{\left(\frac{2}{25}\right)}{\left(\frac{9}{25}\right)} = \frac{2}{9}$$

3) From a medical study of 10 000 male patients, it was found that 2500 were smokers; 720 died from lung cancer and of these, 610 were smokers. Determine:

a) $P(dying\ from\ lung\ cancer\ |smoker)$

$$P(dying\ from\ lung\ cancer\ |smoker) = \frac{n(dying\ from\ lung\ cancer \cap smoker)}{n(smoker)} = \frac{610}{2500} = \frac{61}{250}$$

b) $P(dying\ from\ lung\ cancer\ |non-smoker)$

$$P(dying\ from\ lung\ cancer\ |non-smoker) = \frac{n(dying\ from\ lung\ cancer \cap non-smoker)}{n(non-smoker)} = \frac{110}{7500} = \frac{11}{750}$$

4) The table shows the results of a survey in which 146 families were asked if they own a computer and if they will be taking a summer vacation this year.

	Takes a Vacation	Does not Take a Vacation	Total
Owns a Computer	46	11	57
Does Not Own a Computer	55	34	89
Total	101	45	146

a) Find the probability a randomly selected family is taking a summer vacation this year given that they own a computer.

$$P(vacation \mid owns\ a\ computer) = \frac{n(vacation \cap computer)}{n(computer)} = \frac{46}{57}$$

b) Find the probability a randomly selected family is taking a summer vacation this year and owns a computer.

$$P(vacation \cap computer) = P(vacation) \times P(computer \mid vacation)$$

$$P(vacation \cap computer) = \frac{101}{146} \times \frac{46}{101} = \frac{46}{146} = \frac{23}{73}$$

Refer to part 2 of 4.4 lesson for help with the following questions

4) What is the probability of being dealt two clubs in a row from a well-shuffled deck of 52 playing cards without replacing the first card drawn?

$$P(1st\ club \cap 2nd\ club) = P(1st\ club) \times P(2nd\ club \mid 1st\ club) = \frac{13}{52} \times \frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$$

5) A bag contains three red marbles and five white marbles. What is the probability of drawing two red marbles at random if the first marble drawn is not replaced?

$$P(1st\ red \cap 2nd\ red) = P(1st\ red) \times P(2nd\ red \mid 1st\ red) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

6) A road has two stop lights at two consecutive intersections. The probability of getting a green light at the first intersection is 0.6, and the probability of getting a green light at the second intersection, given that you got a green light at the first intersection, is 0.8. What is the probability of getting a green light at both intersections?

$$P(1st\ green \cap 2nd\ green) = P(1st\ green) \times P(2nd\ green \mid 1st\ green) = 0.6 \times 0.8 = 0.48$$

You have a 48% chance of getting a green light at both intersections.

7) Suppose the two joker cards are left in a standard deck of cards. One of the jokers is red and the other is black. A single card is drawn from the deck of 54 cards but not returned to the deck, and then a second card is drawn. Determine the probability of drawing:

a) one of the jokers on the first draw and an ace on the second draw

$$P(1st\ joker \cap 2nd\ ace) = P(1st\ joker) \times P(2nd\ ace \mid 1st\ joker) = \frac{2}{54} \times \frac{4}{53} = \frac{8}{2862} = \frac{4}{1431}$$

b) a numbered card of any suit on the first draw and the red joker on the second draw

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = P(1st\ numbered\ card) \times P(2nd\ red\ joker \mid 1st\ numbered\ card)$$

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = \frac{36}{54} \times \frac{1}{53}$$

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = \frac{36}{2862}$$

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = \frac{2}{159}$$

c) a queen on both draws

$$P(1st\ queen \cap 2nd\ queen) = P(1st\ queen) \times P(2nd\ queen \mid 1st\ queen) = \frac{4}{54} \times \frac{3}{53} = \frac{12}{2862} = \frac{2}{477}$$

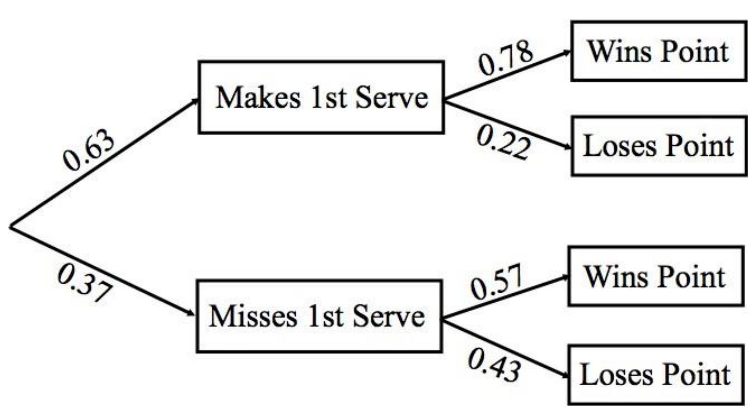
d) any black card on both draws

$$P(1st\ black \cap 2nd\ black) = P(1st\ black) \times P(2nd\ black \mid 1st\ black) = \frac{27}{54} \times \frac{26}{53} = \frac{702}{2862} = \frac{13}{53}$$

Refer to part 3 of 4.4 lesson for help with the following questions

8) Tennis great Roger Federer made 63% of his first serves in 2011 season. When Federer made his first serve, he won 78% of the points. When Federer missed his first serve and had to serve again, he won only 57% of the points. Suppose we randomly choose a point on which Federer served.

a) Start by creating a tree diagram to model the situation.



b) What is the probability that Federer makes the first serve and wins the point?

$P(makes\ 1st\ serve \cap wins\ point) = P(makes\ 1st\ serve) \times P(wins\ point | makes\ 1st\ serve)$

$P(makes\ 1st\ serve \cap wins\ point) = 0.63 \times 0.78$

$P(makes\ 1st\ serve \cap wins\ point) = 0.4914$

There is a 49.14% chance Federer makes the first serve and wins the point.

c) What his the probability the he loses the point?

There are two ways he can lost he point; he can make his first serve and lose OR he can miss his first serve and lose.

$P(loses) = P(wins\ 1st\ serve\ \cap loses) + P(misses\ 1st\ serve \cap loses)$

$P(loses) = 0.63 \times 0.22 + 0.37 \times 0.43$

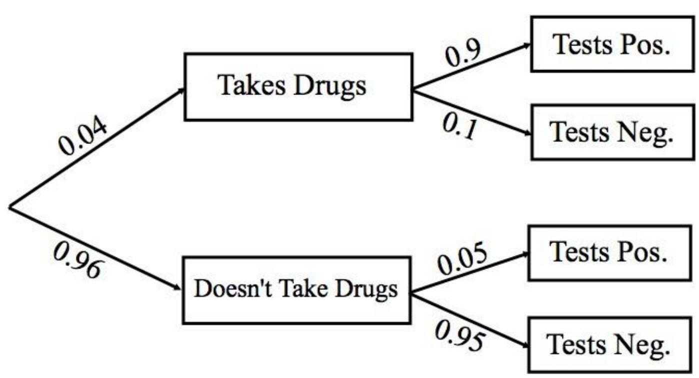
$P(loses) = 0.1386 + 0.1591$

$P(loses) = 0.2977$

There is a 29.77% chance Federer loses a point when he is serving.

9) Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs. Of the employees who use drugs, 90% would test positive. Of the employees who don't use drugs, 5% would test positive.

a) Start by creating a tree diagram to model the situation.



b) A randomly selected prospective employee tests positive for drugs. What is the probability that he actually took drugs?

$$P(took\ drugs | positive\ test) = \frac{P(took\ drugs \cap positive\ test)}{P(positive\ test)}$$

$$P(took\ drugs | positive\ test) = \frac{0.04 \times 0.9}{0.04 \times 0.9 + 0.96 \times 0.05} = \frac{0.036}{0.084} = 0.4286$$

There is about a 42.86% chance that he actually took drugs if he tested positive.