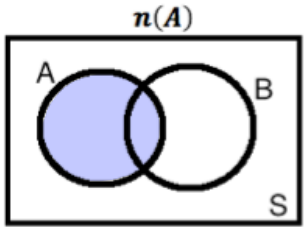
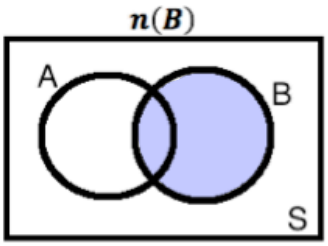
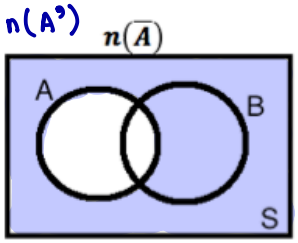
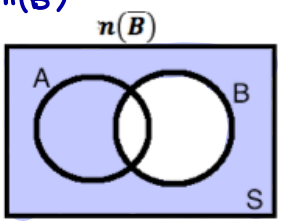
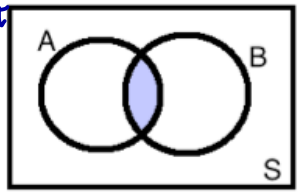
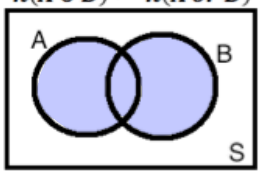
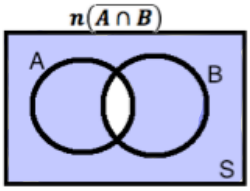
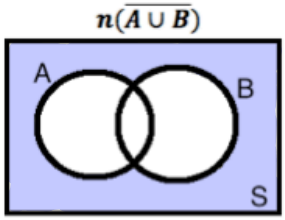
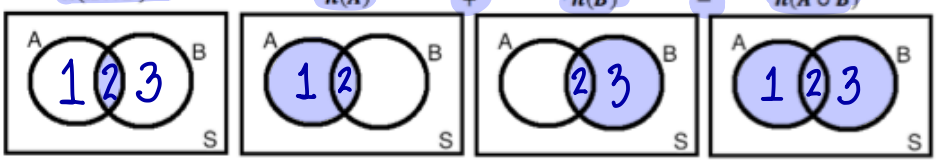
Lesson: Venn Diagrams

S = Universal Set $A, B, C, \ldots$ = subsets of the universal set $n(S)$ = number of elements in the universal set 4 suits: spade, club, heart, diamond A 2 3 4 5 6 7 8 9 10 J Q K	$n(A)$ = number of elements in subset "A" 	
$n(\overline{A})$ = complement of A OR (anything but A) $n(A')$ 	$n(\overline{B})$ = complement of B OR (anything but B) $n(B')$ 	$n(A \cap B) = n(A \text{ and } B)$ intersect $n(A \cap B) = n(A \text{ and } B)$ 
$n(A \cup B) = n(A \text{ or } B)$ union $n(A \cup B) = n(A \text{ or } B)$ 	$n(\overline{A \cap B})$ = complement of A and B 	$n(\overline{A \cup B})$ = complement of A or B 

Principle of Inclusion or Exclusion:

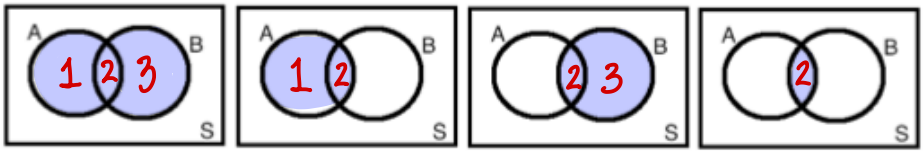
$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

2 = $n(A \cap B)$ = 1+2 ($n(A)$) + 2+3 ($n(B)$) − 1−2−3 ($n(A \cup B)$)

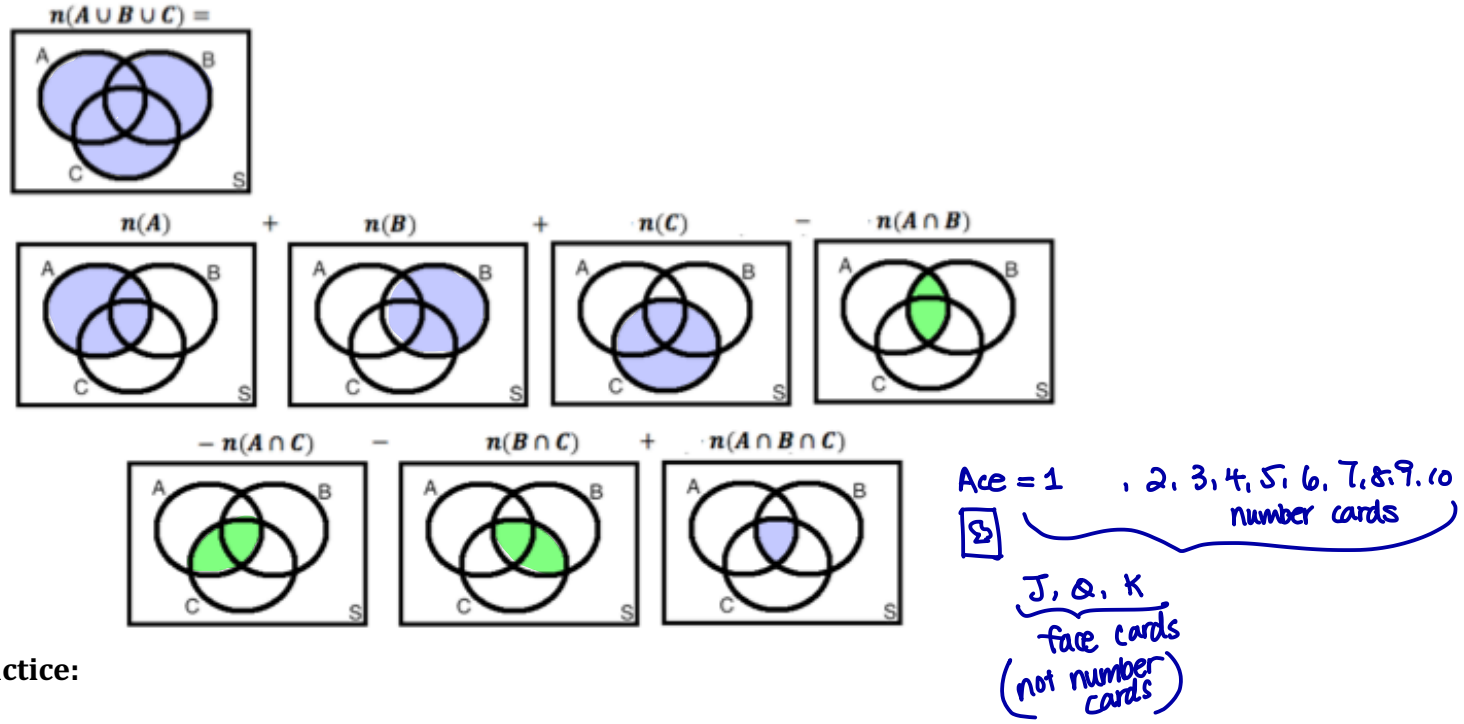


$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

1+2+3 = $n(A \cup B)$ = 1+2 ($n(A)$) + 2+3 ($n(B)$) − 2 ($n(A \cap B)$)

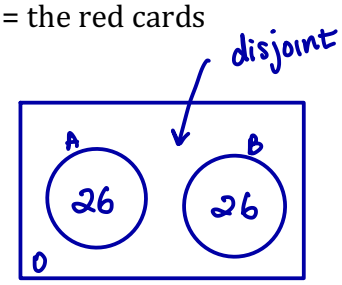
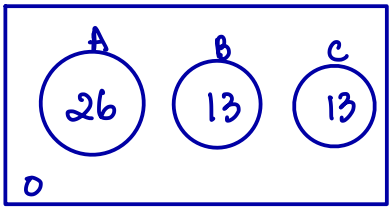
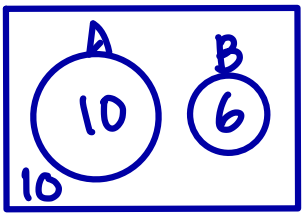
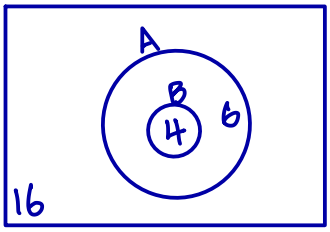
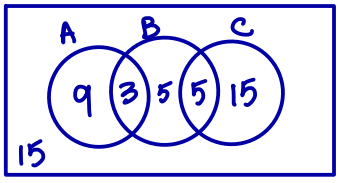
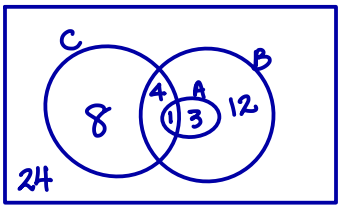


$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$



Practice:

You are given a deck of cards. Complete the following Venn diagrams by indicating how many elements are in each of the sets and subsets.

1. S = the whole deck of cards A = the black cards B = the red cards 	2. S = the whole deck of cards A = the black cards B = the hearts C = the diamonds 	3. S = the red cards A = the even cards B = the face cards 
4. S = the red cards = 26 A = the even cards B = the 2s and 4s 	5. S = the whole deck of cards A = the face cards B = the spades C = the even numbers 	6. S = the whole deck of cards A = Ace cards B = odd cards C = the hearts 

Principle of Inclusion or Exclusion:

For two sets A and B:

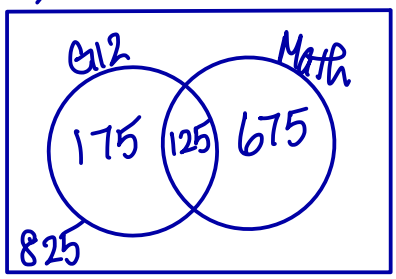
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

The notation $n(X)$ means the number of element in X.

Example 1:

Hollywood High School has about 1 800 students. There are about 300 Grade 12 students. 800 students are taking math. There are about 125 Grade 12 students taking math.

a) Make a Venn Diagram showing this relationship
b) How many students in Grade 12 are not taking math? 175
c) How many students in Grade 9, 10, and 11 are taking math? 675
d) How many students in Grade 9, 10, and 11 are not taking math? 825



If there are three overlapping sets, A, B, and C, the counting of elements in the union of the three sets becomes a little more difficult.

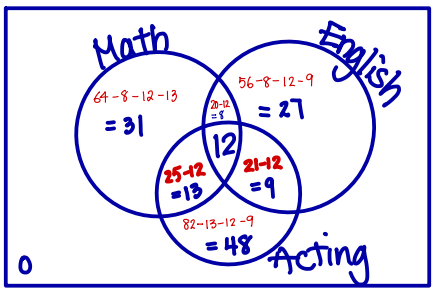
For two sets A and B:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Example 2:

The Hollywood School Service Department wants to count the number of students in Grade 12. They know that every student is taking Math, English, or Acting. They found that:

- 64 students are taking Math
- 56 students are taking English
- 82 students are taking Acting
- 20 students are taking Math and English
- 25 students are taking Math and Acting
- 21 students are taking English and Acting
- 12 students are taking all three courses



Use both methods, determine the number of students in Grade 12.

$n(M \cup E \cup A) = 64 + 56 + 82 - 20 - 25 - 21 + 12 = 148$ ∴ There are 148 students in Grade 12.

Venn Diagram Problems

1. The Swiss embassy in Ottawa has 65 employees. Of these workers, 47 speak German, 35 speak Italian, and 20 speak both German & Italian. How many embassy employees speak neither German nor Italian? Illustrate the situation with a Venn Diagram.

2. A survey of television viewers at "A Child's Place" preschool produces the following data:
   60% watch Sesame Street
   50% watch Captain Kangaroo
   50% watch Polka Dot Door
   30% watch Sesame Street and Captain Kangaroo
   20% watch Captain Kangaroo and Polka Dot Door
   30% watch Sesame Street and Polka Dot Door
   10% watch all three shows
   (a) What percentage view at least one of these programs?
   (b) What percentage view none of the shows?
   (c) What percentage view Sesame Street and Captain Kangaroo but not Polka Dot Door?
   (d) What percentage view exactly two of these programs?

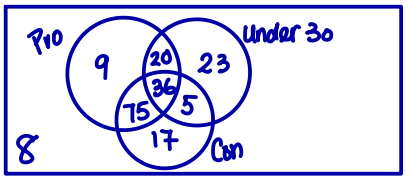
3. Of 1400 students at Tomlintown High, 800 attended the first school dance of the year. The music was not good so only 500 attended the next dance. If 300 attended both dances, how many did not go to either event?

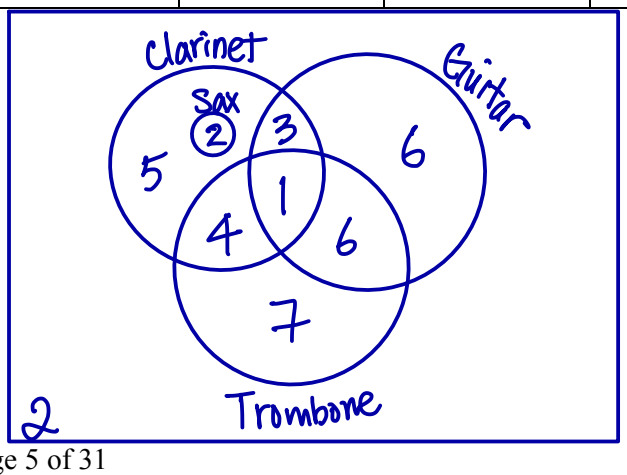
4. The 29 students of Mr. Vicker's class use a variety of forms of transportation to get to school. Twenty of them sometimes arrive at school in a car; 12 bicycle to school at least occasionally; 16 take the bus some days. If four students use all three of these options, six either bicycle or take the bus, ten come by car or bike, and nine arrive by car when they do not come on the bus, how many always use some other type of transportation?

5. In a recent election poll of 193 people, the following information was collected:
   140 of those polled were professionals; 84 were under 30 years of age; 133 voted Conservative in the last election; 56 were professionals under 30; 41 of those under 30 voted Conservative; 111 professionals voted Conservative; 36 of the professionals under 30 voted Conservative.
   Of those polled, how many non-professionals aged 30 or over did not vote Conservative?

6. In Mrs. Paul's Music Class, students learn to play only the clarinet, guitar, saxophone, and trombone. So far this term, no student can play all four; the one who plays three cannot play sax but all those who do play sax also play clarinet. If there are 36 students in the class and the following chart shows the skills of the students, how many students still cannot play any instrument?

Saxophone	Clarinet	Guitar	Trombone	Clarinet and Guitar	Clarinet and Trombone	Guitar and Trombone
2	15	16	18	4	5	7





Answers: 1)3 2a)90 b)10 c)20 d)50 3)400 4)2 5)8 6)2