Section 3.6 Worksheet – Confidence Intervals

MDM4U
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1) For each set of data, determine the margin of error and confidence interval for a 95% confidence level.

	n	$\bar{x}$	σ
a)	40	215	8
b)	130	35	3.4
c)	30	9.65	0.56

a)

Method 1: Formula

C.I. for $\mu = 215 \pm 1.96\left(\frac{8}{\sqrt{40}}\right)$

$C.I. = 215 \pm 2.479$

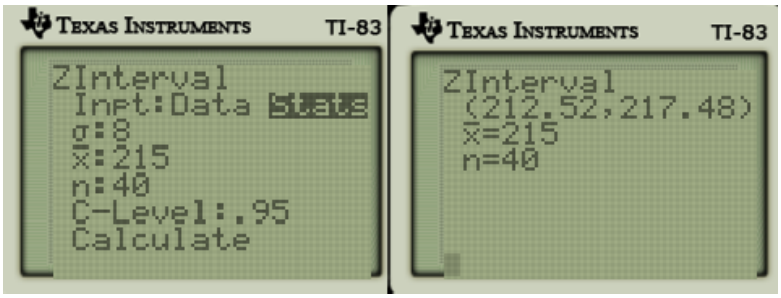
$C.I. = (212.5, 217.5)$

We can be 95% confident that the interval 212.5 to 217.5 captures the true population mean.

Method 2: Calculator

$\sigma = 8$
$\bar{x} = 215$
$n = 40$
C---Level: 0.95

C.I. = (212.52, 217.48)



We can be 95% confident that the interval 212.52 to 217.48 captures the true population mean.

b)

Method 1: Formula:

$C.I.$ for $\mu = 35 \pm 1.96\left(\frac{3.4}{\sqrt{130}}\right)$

$C.I. = 35 \pm 0.5845$

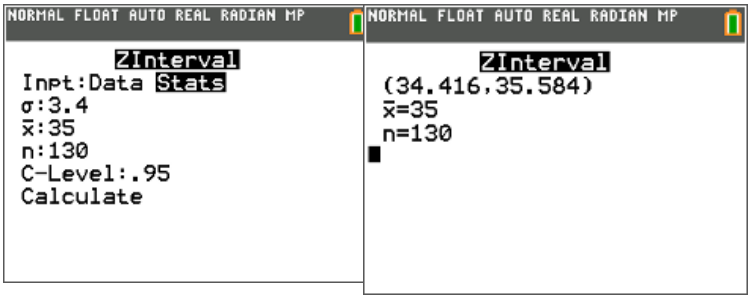
$C.I. = (34.4, 35.6)$

We can be 95% confident that the interval from 34.4 to 35.6 captures the true population mean.

Method 2: Calculator:

$\sigma = 3.4$
$\bar{x} = 35$
$n = 130$
C---Level: 0.95

$C.I. = (34.416, 35.584)$



We can be 95% confident that the interval from 34.416 to 35.584 captures the true population mean.

c)

Method 1: Formula:

$C.I.\ for\ \mu = 9.65 \pm 1.96\left(\frac{0.56}{\sqrt{30}}\right)$

$C.I. = 9.65 \pm 0.2$

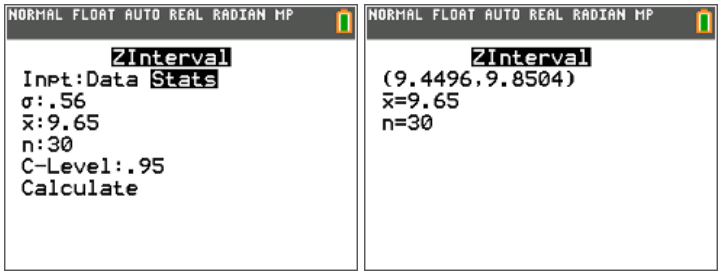
$C.I. = (9.45, 9.85)$

We can be 95% confident that the interval from 9.45 to 9.85 captures the true population mean.

Method 2: Calculator:

$\sigma = 0.56$
$\bar{x} = 9.65$
$n = 30$
C---Level: 0.95

$C.I. = (9.45, 9.85)$



We can be 95% confident that the interval from 9.45 to 9.85 captures the true population mean.

2) A machine that produces control arms for a vehicle gas pedal generates pedals that have a length with standard deviation of 0.08 cm. Thirty pedals are tested to see if their lengths are acceptable. The sample has a mean of 18.2 cm. What would be the acceptable range of lengths for a 95% confidence level for the mean length?

Method 1: Formula

$C.I. for\ \mu = 18.2 \pm 1.96\left(\frac{0.08}{\sqrt{30}}\right)$

$C.I. = 18.2 \pm 0.0286$

$C.I. = (18.17, 18.23)$

We can be 95% confident that the interval from 18.17 cm to 18.23 cm captures the true mean length of pedals.

Method 2: Calculator

$\sigma = 0.08$
$\bar{x} = 18.2$
$n = 30$
C---Level: 0.95

$C.I. = (18.17, 18.23)$

We can be 95% confident that the interval from 18.17 cm to 18.23 cm captures the true mean length of pedals.

3) You have a part---time job maintaining a water---jug---refilling machine. The machine rarely fills each jug to the same volume and sometimes needs recalibrating. The manufacturer states that the standard deviation of the machine is 0.3 L. You monitor the next 20 fillings and determine that their mean volume is 18.8 L. Assuming the data are normally distributed, determine the acceptable range of volumes for a confidence level of 95%.

Method 1: Formula

$C.I. for\ \mu = 18.8 \pm 1.96\left(\frac{0.3}{\sqrt{20}}\right)$

$C.I. = 18.8 \pm 0.1315$

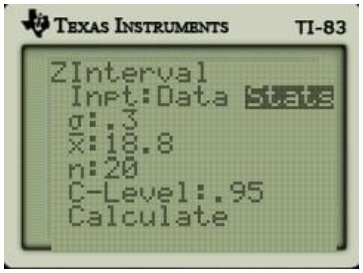
$C.I. = (18.67, 18.93)$

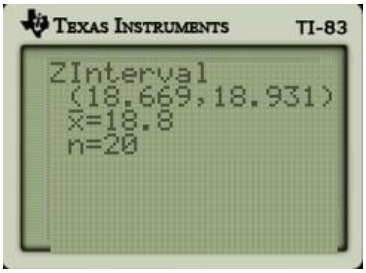
We can be 95% confident that the interval from 18.67 L to 18.93 L captures the true mean volume.

Method 2: Calculator

$\sigma = 0.3$
$\bar{x} = 18.8$
$n = 20$
C---Level: 0.95

$C.I. = (18.67, 18.93)$





We can be 95% confident that the interval from 18.67 L to 18.93 L captures the true mean volume.

4) An exit poll is done outside a voting location. People who have just voted are asked if they will state who they voted for. In a close election, an exit poll states that Larry Liberal has 48% of the vote, while Constance Conservative has 46% of the vote, with the rest split up among other candidates. The polling firm states that 500 people were polled. Find a 95% confidence interval for the proportion of people who support each candidate.

Larry

Method 1: Formula

$C.I.\ for\ p = 0.48 \pm 1.96\left(\frac{\sqrt{0.48 \cdot 0.52}}{\sqrt{500}}\right)$

$C.I. = 0.48 \pm 0.0438$

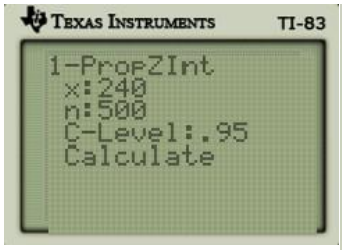
$C.I.\ (0.4362, 0.5238)$

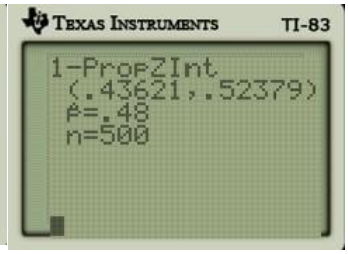
We can be 95% confident that the interval from 0.4362 to 0.5238 captures the true proportion of voters who support Larry.

Method 2: Calculator

$x = 0.48 \times 500 = 240$
$n = 500$
C-Level = 0.95

$C.I.\ (0.4362, 0.5238)$





We can be 95% confident that the interval from 0.4362 to 0.5238 captures the true proportion of voters who support Larry.

Constance

Method 1: Formula

$C.I. for\ p = 0.46 \pm 1.96\left(\frac{\sqrt{0.46 \cdot 0.54}}{\sqrt{500}}\right)$

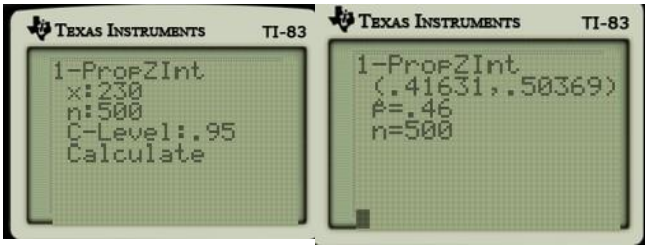
$C.I. = 0.46 \pm 0.0437$

$C.I. = (0.4163, 0.5037)$

We can be 95% confident that the interval from 0.4163 to 0.5037 captures the true proportion of voters who support Constance.

Method 2: Calculator

$x = 0.46 \times 500 = 230$
$n = 500$
C-Level = 0.95



$C.I. = (0.4163, 0.5037)$

We can be 95% confident that the interval from 0.4163 to 0.5037 captures the true proportion of voters who support Constance.

5) A market---research firm asked 300 people in Toronto who their favourite hockey team is. 55 said the Leafs are their favourite team. Determine a 99% confidence interval for the proportion of people in Toronto that have chosen the Leafs to be their favourite team.

Note: $p = \frac{55}{300} = 0.183$

Method 1: Formula

$C.I. for\ p = 0.183 \pm 2.576\left(\frac{\sqrt{0.183 \cdot 0.817}}{\sqrt{300}}\right)$

$C.I. = 0.183 \pm 0.0575$

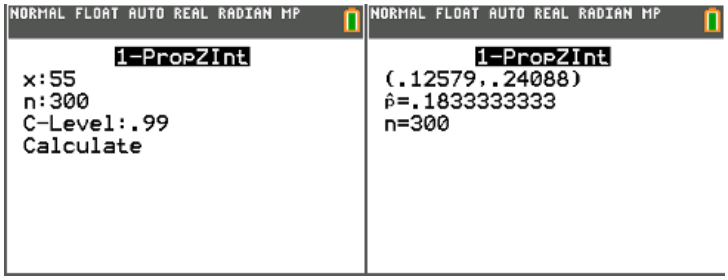
$C.I. = (0.1255, 0.2405)$

We can be 99% confident that the interval from 0.1255 to 0.2405 captures the true population proportion of people in Toronto who have chosen the Leafs to be their favourite team.

Method 2: Calculator

$x = 55$
$n = 300$
C-Level = 0.99

$C.I. = (0.1258, 0.2409)$



We can be 99% confident that the interval from 0.1258 to 0.2409 captures the true population proportion of people in Toronto who have chosen the Leafs to be their favourite team.