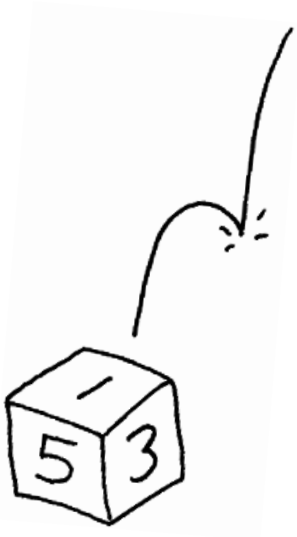
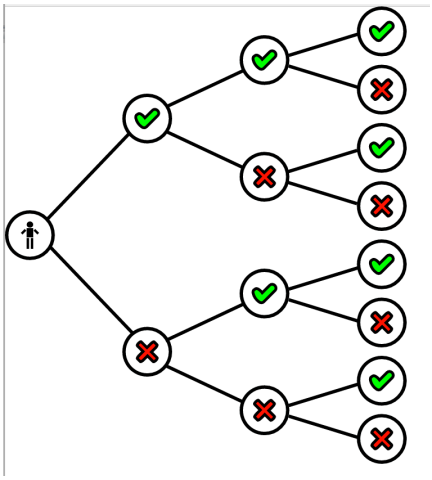
Unit 4
Probability Theory



Unit Overview

(Topics for this unit test include, but are not limited to the following)

1. Key terms and notations
   - Outcomes – favourable outcomes/ total possible outcomes
   - Events
   - Trials
   - Empirical Probability
   - Theoretical Probability
   - Subjective Probability



2. Counting Techniques
   - Combinations
   - Permutations
   - Tree diagrams
   - Dice chart
   - Venn diagrams

3. Mutually Exclusive Events vs. Non-Mutually Exclusive Events

4. Dependent Events vs. Independent Events
   - 'compound events' – two or more events

5. Odds
   - Odds in favor vs. odds against

Textbook Review Questions: pg. 357 #1-17, pg. 360 # 1-9

Learning Goal: Introduction to Probability

3 Basic Types of Probability:

1. _____
   - an estimate of likelihood based on intuition and experience
   - example: "I think…."
2. _____
   - an estimate of likelihood based on an experiment by the number of trials
   - example: " 3 out of 5 times I tried rolling a die is 2."
3. _____
   - an estimate of likelihood based on analysis
   - example: "There's 50% chance of getting a head when flipping a coin."

_____: possible results

_____: an occurrence

_____: an examination/experiment

_____: all possible outcomes of an event

Notation:

$0 = impossible; 1 = occurs\ 100\%\ of\ the\ time$

Example#1: Event: Flipping a coin

P(head) =

P(tail) =

Example#2: Event: Flipping a coin if **both** sides are head.

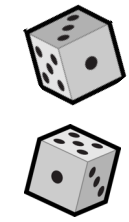
P(head) =

P(tail) =

Example#3:
Event: Rolling a die:

P(1) =	P(4) =
P(2) =	P(5) =
P(3) =	P(6) =

Example#4:
Event: Rolling **a pair** of standard dice to find **the sum**:



	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

P(1) =	P(evens) =
P(2) =	P(odds) =
P(3) =	P(doubles) =
P(4) =	P(prime) =
P(7) =	P(composite) =
P(not 7) =	

P(A')= Complement of even
A = event A doesn't happen

P(A)+P(A')=1

Example#5: Given the sample space {1,2,3,4,5,6,……50}

P(evens) =	P(odds) =
P(prime) =	P(composite) =
P(perfect squares) =	P(perfect cubes) =
P(not perfect squares) =	P(not perfect cubes) =

Tree Diagram:

Learning Goal: Probabilities Simulations

Textbook Page 322 Example#2:

A group of 3 members is to be randomly selected from 5 doctors and 7 technicians.

a) What is the probability that the group will be comprised of doctors only?

b) What is the probability that the group will not be comprised of doctors only?

Textbook Page 322 Example#1:

Two brothers enter a race with 5 other friends. The racers draw lots to determine their starting positions. What is the probability of having the elder brother in lane 1 and his brother is beside him in lane 2?

Textbook Page 323 Example#3:

What is the probability that two or more students out of a class of 19 will have the same birthday? (Assume no students were born in February 29)

Homework Question:

Page 324 #1: Four friends, two females and two males, are playing contract bridge. Partners are randomly assigned for each game. What is the probability that the two females will be partners for the first game?

Page 325 #11: Suki is enrolled in one data-management class at her school and Leo is in another. A school quiz team will have four volunteers, two randomly selected from each of the two classes. Suki is one of five volunteers from her class, and Leo is one of four volunteers from his. Calculate the probability of the two being on the team.

Learning Goal: Odds

Odds: A way to express a level of confidence about an outcome by the **ratio** of the probability that the event will occur to the probability that it will not occur. Odds is **always** presented in a form of a fraction or a ratio.

Odds in favour of A	Odds against A

Relationships between Odds and Probability

If $h = outcomes$

$k = complement\ of\ h$

Probability:	Odds in Favour:

Example#1: Given total of 10 baskets: 3 are red 5 are black and 2 are white:

a) What is the odds in favour of red basket?	b) What is the odds against white basket?

Example#2: Given the Odds in favour of passing the last unit test is 8:1, What is the probability of passing the last unit test?

Example#3:

If the chance of snowing in April is estimated at 0.4, what are the odds against having snows next April?

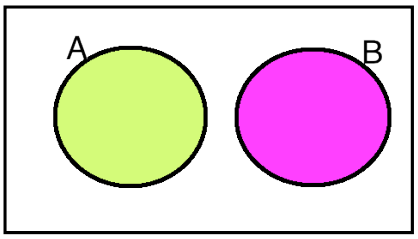
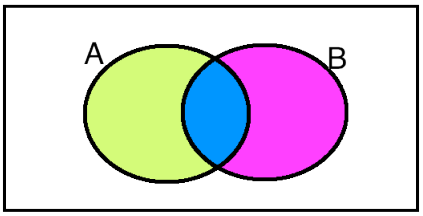
Example#4:

The odds of Kevin passing data management are 2:7. What is the probability of Kevin passing data management?

A) Round your answer to 3 decimal places.

B) Provide your answer in percent to 1 decimal place.

Learning Goal: Mutually Exclusive and Non-Mutually Exclusive Events

Mutually Exclusive Events (Disjoint)	**Non-Mutually Exclusive Events**
- events that cannot occur simultaneously Ex. A=alive B=dead Ex. A=standing up B=sitting down Ex. A=driving B=walking  $P(A \text{ or } B) =$	- events that can occur simultaneously Ex. A=eating B=watching TV Ex. A=texting B=talking Ex. A=listening to music B=doing homework  $P(A \text{ or } B) =$

Homework Question #1 from Textbook Page 340

Classify each pair of events as mutually exclusive or non-mutually exclusive:

	Event A	**Event B**	**Mutually Exclusive (ME) Or Non-Mutually Exclusive (NON)**
a)	Randomly drawing a grey sock from a drawer	Randomly drawing a wool sock from a drawer	
b)	Randomly selecting a student with brown eyes	Randomly selecting a student on the honour roll	
c)	Having an even number of students in your class	Having an odd number of students in your class	
d)	Rolling a six with a die	Rolling a prime number with a die	
e)	Your birthday falling on a Saturday next year	Your birthday falling on a weekend next year	
f)	Getting an A on the next test	Passing the next test	
g)	Calm weather at noon tomorrow	Stormy weather at noon tomorrow	
h)	Sunny weather next week	Rainy weather next week	

Textbook Page 337Example#1:
Teri attends a fundraiser at which 15 T-shirts are being given away as door prizes. Door prize winners are randomly given a shirt from a stock of 2 black shirts, 4 blue shirts, and 9 white shirts. Teri really likes the black and blue shirts, but is not too keen on the white ones. Assuming that Teri wins the first door prize, what is the probability that she will get a shirt that she likes?
Mutually Exclusive or Non-Mutually Exclusive?

Textbook Page 338 Example#2:
A card is randomly selected from a standard deck of cards. What is the probability that either a heart or a face card (Jack, Queen, King) is selected?
Mutually Exclusive or Non-Mutually Exclusive?

Example #3:
There are a red, white, blue, green and orange marbles. What is the probability that either white and blue be the first two or orange would be last?
Mutually Exclusive or Non-Mutually Exclusive?

Learning Goal: Independent and Dependent Events

Independent Events:	**Dependent Events:**
- the occurrence of one event has no effect on the occurrence of another Ex. P(A)=flipping a coin 1st time P(B)=flipping a coin 2nd time $P(A \text{ and } B) =$	- the probable outcome of an event, B, depends on the outcome of another event A. (B occurs, given that A has already occurred) $P(A \text{ and } B) =$ $P(B\|A) =$

Homework Question #1 from Textbook Page 334

Classify each pair of events as independent or dependent event:

	Event A	**Event B**	**Independent (I) OR Dependent (D) Event**
i)	Attending a rock concert on Tuesday night **(until 5 am on Wednesday morning)**	Passing a final examination the following Wednesday morning **(at 9 am for two hours)**	
j)	Eating chocolate	Winning at checkers	
k)	Having blue eyes	Having poor hearing	
l)	Attending an employee training session	Improving personal productivity	
m)	Graduating from university	Running a marathon	
n)	Going to a mall	~~Purchasing a new shirt~~ **(Trying on a shirt in a fitting room)**	

Textbook Page 328 Example#1:

a) A coin is flipped and turns up heads. What is the probability that the second flip will turn up head?
Independent or Dependent?

b) A coin is flipped four times and turns up heads each time. What is the probability that the fifth trial will be head?
Independent or Dependent?

c) Find the probability of tossing five heads in a row.
Independent or Dependent?

Textbook Page 329 Example#2:

A coin is flipped while a die is rolled. What is the probability of flipping heads and rolling 5 in a single trial?
Independent or Dependent?

Textbook Page 330 Example#3:

Soo-Ling travels the same route to work every day. She has determined that there is a 0.7 probability that she will wait for at least one red light and that there is a 0.4 probability that she will hear her favourite new song on her way to work.

a) What is the probability that Soo-Ling will not have to wait at a red light and will hear her favourite song?
Independent or Dependent?

b) What are the odds in favour of Soo-Ling having to wait at a red light and not hearing her favourite song?

Textbook Page 331 Example#4:
A professional hockey team has eight wingers. Three of these wingers are 30-goal scorers, or "snipers." Every fall the team plays an exhibition match with the club's farm team. In order to make the match more interesting for the fans, the coaches agree to select two wingers at random from the pro team to play for the farm team. What is the probability that two snipers will play for the farm team?
Independent or Dependent?

Modified from Textbook Page 332 Example#5:
Sina's computer sometimes crashes while he is trying to use his e-mail program, Outlook. When Outlook "hangs" (stop responding to commands), Sina is usually able to close Outlook without a system crash. In a computer magazine, he reads that the probability of Outlook hanging in any 15-min period is 2.5%, while the chance of Outlook and the operating system failing together in any 15-min period is 1%. If Outlook is hanging, what is the probability that the operating system will crash?
Independent or Dependent?

Practice

1. Determine the probability of:

a) Tossing exactly two heads if a coin is tossed three times	
b) Tossing at least two heads if a coin is tossed three times	
c) Rolling a total of 9 with a standard pair of dice	
d) Rolling a total that is less than 6 with a standard pair of dice	

2. Calculate the odds in favour:

If the probability of A is $\frac{10}{17}$	

3. The odds in favour of the Argos winning the Grey Cup are 10:7.

a) What would be the winnings if a $2.00 bet is placed and the Argos win?	
b) How much money must one wager in order to win $20.00 if the Argos wins?	

4. If a family is chosen at random from the set of all families with exactly two children, find the probability that

a) The family has two boys if it is known that one child is a boy	
b) The family has two boys if it is known that the first child is a boy	

5. If a committee of five is to be chosen randomly from 6 males and 8 females, what is the probability that the committee is either all male or all female?

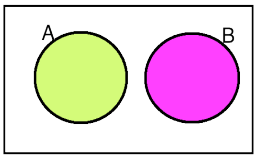
6. A card is drawn from a deck of cards, and then a second card is drawn without replacing the first card.

a) What is the probability of the first card is a heart?	
b) What is the probability of the second is also a heart?	

7. The probability of its raining this morning is 0.1. if it rains, the probability that Gen will be late for school is $\frac{3}{5}$; otherwise, the probability is $\frac{1}{4}$. What is the probability that Gen will be late for school?

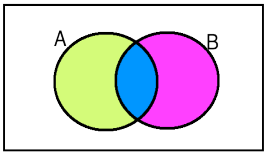
Learning Goal: Revisit Venn Diagrams

Mutually Exclusive and Non-mutually Exclusive Events:



Mutually Exclusive Events (Disjoint)

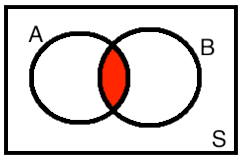
$$P(A \cup B) = P(A) + P(B)$$



Non-Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

Events:



$$P(A \cap B) = P(A) \times P(B)$$

Practice:

1. Everyone in a doctor's office is suffering from a cold or a hangmail. 13 have a cold, 8 have a hangnail, and 5 have both. What is the probability that a patient in the doctor's office is having cold only?

2. A survey of a machine shop reveals the following information about its employees, if a employee is selected, what is the probability that the employee is a cleaner?
   44 employees can run a lathe
   49 employees can run the milling machines
   56 employees can operate a punch press
   27 employees can run a lathe and a milling machine
   19 employees can run a milling machine and operate a punch press
   24 employees can run a lathe and operate a punch press
   10 employees can operate all three machines
   9 employees cannot operate any of the three (the cleaners)

3. On the first day of school, a teacher surveyed the students in her class to find out who owned drawing instrucments. There are 35 students in the class and all have at least one instrument. If a teacher selects a student, what is the probability that the student has square only?
   20 had compasses
   12 had compasses and protractors
   17 had protractors
   7 had squares and protractors
   10 had compasses and squares
   5 had all three instruments