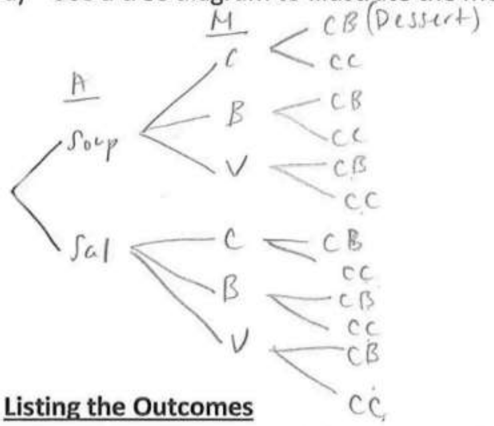
2.1 Organized Counting and Fundamental Counting Principle

Combinatorics: a branch of mathematics that deals with counting, particularly in complex situations.

There are many methods of counting:

1) **Tree Diagrams**
   Example 1: Mark is ordering a three course meal from a restaurant. There are two appetizers (soup or salad), three main courses (chicken, beef, or vegetarian), and two desserts (crème brule and chocolate cake) to choose from. If he can only choose one item from each of the three categories, how many different three-course meals can be formed?
   a) Use a tree diagram to illustrate the meals that Mark can order.



$\therefore$ 12 meals

or $2 \times 3 \times 2$.

2) **Listing the Outcomes**
   Example 2: How many 2-digit prime numbers are there?
   11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

   21 2-digit prime #'s

3) **Fundamental Counting Principle (Rule of multiplication)**

If a first action can be performed 'm' ways, AND a second action can be performed 'n' ways, then these two actions can be performed in $m \times n$ ways.

Example 3: A company makes gloves that are black, brown, grey, or burgundy. Each is available in a small, medium and large. How many different types of gloves does the company make?

$m = 4$ $\qquad m \times n = 4 \times 3$

$n = 3$ $\qquad = 12$

Example 4: How many 2-digit positive integers are there? $9 \times 10 = 90$

Example 5: How many 2 digit positive integers are there in which the digits are not repeated?

$9 \times 9 = 81$

4) **Additive Counting Principle (Rule of Sum):** will be covered in the following lesson.

5) **Indirect Method:**

Leora has four pairs of running shoes loose in her gym bag. In how many ways can she pull out two unmatched shoes one after the other?

# of ways to pull out any 2 shoes	–	# of ways to pull out 2 matching shoes	=	# ways to pull out 2 unmatched shoes

Case 1: How many ways can Leora pull out 2 shoes?

$\underset{\text{shoe 1}}{\underline{8}} \times \underset{\text{shoe 2}}{\underline{7}} = 56$

Case 2: How many ways can Leora pull out 2 matching shoes?

$\underset{\text{any of the 8}}{\underline{8}} \times \underset{\text{only 1 matching shoe}}{\underline{1}} = 8$

$58 - 8$

$= 48$

Additional Examples:

Ex 5) The rugby team is considering buying new uniforms. They are considering four different jerseys, shorts in white, blue or black and socks that are either striped or not. How many different uniform options are there?

$j = 4$

$sh = 3$

$soc = 2$

$4 \times 3 \times 2 = 24$ uniform options

Ex 6) Determine how many possible outcomes there are for flipping a coin:

a) once
$= 2$
$= 2^1$

b) twice
$= 2 \times 2$
$= 4$
$= 2^2$

c) 3 times
$= 2 \times 2 \times 2$
$= 8$
$= 2^3$

d) 4 times
$= 2 \times 2 \times 2 \times 2$
$= 16$
$= 2^4$

e) n times
$= 2^n$.

Ex 7) You roll a standard six-sided die. How many outcomes are possible with ...

a) one roll
$= 6$
$= 6$

b) two rolls
$= 6 \times 6$
$= 36$
$= 6^2$

c) 10 rolls
$= 6^{10}$
$= 60,466,176$

d) 'n' rolls
$= 6^n$

Ex 8) A jar contains a red, a blue and a yellow marble. A student removes three marbles, one after the other. How many possible outcomes are there if:

a) the marbles are replaced after each selection?

$3 \times 3 \times 3 = 27$ outcomes

b) the marbles are not replaced after each selection?

$3 \times 2 \times 1 = 6$ outcomes

Ex 9) Consider the letters in the word **M O N E Y**.

a) How many different ways can you arrange the letters?

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$ ways

b) How many different ways can you arrange the letters if the arrangement must begin in a vowel?

$\underset{O \text{ or } E}{\underline{2}} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 48$ ways

c) How many different ways can you arrange the letters if the arrangement **must not** begin in a vowel?

Method: ① Total − begin w/ vowel

$= 120 - 48$

$= 72$ ways

* using INDIRECT

Method ②: $\underline{3} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$= 72$ ways

Ex 10) Consider the letters in the word **H O C K E Y**. How many ways are there to arrange the letters if the 'c' and 'k' must be kept separate?

INDIRECT: Total − ck together

C, K or K, C.

$= (6 \times 5 \times 4 \times 3 \times 2 \times 1) - (5 \times 4 \times 3 \times 2 \times 1) \times \underline{2}$

$= 720 - 240$

$= 480$ arrangements