Section 4.5 – Multiplication of Independent Events

MDM4U
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1) A truck driver has a choice of routes as he travels among four cities. He can choose from four routes between Toronto and Oakville, two between Oakville and Hamilton, and three between Hamilton and Guelph. Find the total number of routes possible for the complete Toronto-Oakville-Hamilton-Guelph trips.

$n(routes) = 4 \times 2 \times 3 = 24$

2) A test has four true/false questions. What is the probability that they will get all four correct by guessing?

$$P(all\ 4\ correct) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

3) A test has three multiple choice questions, each question has four possible answers. What is the probability that you get all three questions correct by guessing?

$$P(all\ 3\ correct) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

4) A standard deck of cards has had all the face cards (jacks, queens, and kings) removed so that only the ace through ten of each suit remain. A game is played in which a card is drawn from this deck and a six-sided die is rolled. For the purpose of this game, an ace is considered to have a value of 1.

a) Determine the total number of possible outcome for this game.

$n(outcomes) = n(card) \times n(die) = 40 \times 6 = 240$

b) Find the probability of each of these events:

i) an even card and an even roll of the die

$$P(even\ card, even\ die) = P(even\ card) \times P(even\ die) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

ii) an even card and a roll of 3.

$$P(even\ card, die\ 3) = P(even\ card) \times P(die\ 3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

iii) a card of 3 and a roll of the die of 3 or less

$$P(card\ 3, die\ \leq 3) = P(card\ 3) \times P(die \leq 3) = \frac{4}{40} \times \frac{3}{6} = \frac{12}{24\#} = \frac{1}{20}$$

5) Suppose the two joker cards are left in a standard deck of cards. One of the jokers is red and the other is black. A single card is drawn from the deck of 54 cards, returned, and then a second card is drawn. Determine the probability of drawing:

a) one of the jokers on the first draw and an ace on the second

$$P(joker, ace) = P(joker) \times P(ace) = \frac{2}{54} \times \frac{4}{54} = \frac{8}{2916} = \frac{2}{729}$$

b) the red joker on the second draw and a numbered card of any suit on the first

$$P(numbered\ card, red\ joker) = P(numbered\ card) \times P(red\ joker) = \frac{36}{54} \times \frac{1}{54} = \frac{36}{2916} = \frac{1}{81}$$

c) a queen on both draws

$$P(queen, queen) = P(queen) \times P(queen) = \frac{4}{54} \times \frac{4}{54} = \frac{16}{2916} = \frac{4}{729}$$

d) any black card on both draws

$$P(black, black) = P(black) \times P(black) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

e) any numbered card less than 10 on the first draw and a card with the same number on the second

$$P(< 10, same\ \#) = P(< 10) \times P(same\ \#) = \frac{32}{54} \times \frac{4}{54} = \frac{128}{2916} = \frac{32}{729}$$

6) A paper bag contains a mixture of 3 types of candy. There are ten chocolate bars, seven fruit bars, and three packages of toffee. Suppose a game is played in which a candy is randomly taken from the bag, replaced, and then a second candy is drawn from the bag. If you are allowed to keep the second candy only if it was the same type as the one that was drawn the first time, calculate the probability of each of the following:

a) you will be able to keep a chocolate bar

$$P(chocolate, chocolate) = P(chocolate) \times P(chocolate) = \frac{10}{20} \times \frac{10}{20} = \frac{100}{400} = \frac{1}{4}$$

b) you well be able to keep any candy

$$P(keep\ any) = P(chocolate, chocolate) + P(fruit, fruit) + P(toffee, toffee)$$

$$P(keep\ any) = \left(\frac{10}{20}\right)\left(\frac{10}{20}\right) + \left(\frac{7}{20}\right)\left(\frac{7}{20}\right) + \left(\frac{3}{20}\right)\left(\frac{3}{20}\right)$$

$$P(keep\ any) = \frac{100}{400} + \frac{49}{400} + \frac{9}{400}$$

$$P(keep\ any) = \frac{158}{400}$$

$$P(keep\ any) = \frac{79}{200}$$

c) you won't be able to keep any candy

$P(keep\ any') = 1 - P(keep\ any)$

$P(keep\ any') = 1 - \dfrac{79}{200}$

$P(keep\ any') = \dfrac{121}{200}$

7) A coin is tossed and a standard six-sided die is rolled.

a) How many different outcomes are possible?

$n(outcomes) = 2 \times 6 = 12$

b) What is the probability of flipping tails and rolling a number greater than 4?

$P(tails, > 4) = P(tails) \times P(> 4) = \frac{1}{2} \times \frac{2}{6} = \frac{2}{12} = \frac{1}{6}$

8) The probability that a salmon swims successfully through a dam is 0.85.

a) Find the probability that three salmon swim successfully through the dam.

$P(success, success, success) = 0.85 \times 0.85 \times 0.85 = 0.614$

b) Find the probability that none of the three salmon is successful.

$P(fail, fail, fail) = 0.15 \times 0.15 \times 0.15 = 0.003$

c) Find the probability that at least one of the three salmon is successful in swimming through the dam.

$P(atleast\ 1) = 1 - P(none) = 1 - 0.003 = 0.997$

9) There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that

a) both tests give the correct result

$P(A\ correct, B\ correct) = 0.95 \times 0.89 = 0.8455$

b) neither test gives the correct result

$P(A\ wrong, B\ wrong) = 0.05 \times 0.11 = 0.0055$

c) at least one of the tests gives the correct result

$P(at\ least\ one\ correct) = 1 - P(A\ wrong, B\ wrong) = 1 - 0.0055 = 0.9945$