MDM4U REVIEW QUESTIONS #1: EXAM PREPARATION

WARNING: ***The following questions are meant to <u>supplement</u> your own review of the course for the exam. Some of the concepts that we covered in the course may not come up in the following review; however, you are still responsible for knowing everything we did. Redo all of the questions we have done in the past for more practice. Also, try to create your own variation on questions that we have already done.***

Chapter 1: Permutations

1. In how many ways can three different awards be distributed among 20 students in the following situations?
   a. No student may receive more than one award.
   b. There is no limit on the number of awards won by one student.

   Answer: a) 6840 b)8000

2. Consider the word BASKETBALL:
   a. How many permutations are there?
   b. How many permutations begin with the letter K?
   c. How many permutations have the two L's together?

   Answer: a) 453 600 b) 45 360 c) 90 720

3. How many 3-digit numbers can be formed using only the digits 1 to 7, if the number 2 must be included? (Repetitions are allowed.) Answer: 127

4. How many arrangements of the word ALGORITHM bein with a vowel and end with a consonant? Answer: 90 720

5. Prove: $P(n,r) = P(n,2)P(n-2,r-2)$

Chapter 2: Combinations

6. How many bridge hands (13 cards) contain five clubs, two hearts, three diamonds, and three spades? Leave answer in factorial form.

   Answer: $\left(\frac{13!}{5!8!}\right)\left(\frac{13!}{2!11!}\right)\left(\frac{13!}{3!10!}\right)\left(\frac{13!}{3!10!}\right)$

7. In how many ways can 12 prizes be awarded evenly among four people?

   Answer: 369 600

8. From a deck of 52 cards, how many different four-card hands could be dealt which include one card from each suit? Answer: 28 561

9. Find the number of divisors of 540 other than 1. Answer: 23

10. The swise embassy in Ottawa has 65 employees. Of these workers, 47 speak German, 35 speak Italian, and 20 speak both German and Italian. How many embassy employees speak neither German nor Italian? Illustrate the situation with a Venn diagram. Answer: 3

11. Solve for $n \in N$: $5P(n,3) = 24\binom{n}{4}$ Answer: 8

Chapter 3: The Binomial Theorem

12. If $\sum_{r=0}^{n}\binom{n}{r} = 512$, find n. Answer: 9

13. In the expansion of $\left(x - \frac{3}{x^2}\right)^9$, find the following:
    a. The general term
    b. The term containing x^6
    c. The constant term. Answer: a) $t_{r+1} = \binom{9}{r}(-3)^r x^{9-3r}$ b) $-27x^6$ c) –2268

14. In the expansion of $(x^2-1)^5(1+x)^8$, find the term containing x. Answer: $-8x$

Chapter 6 (New Text): Introduction to Probability

15. An integer from 1 to 50 inclusive is chosen at random. What is the probability that the integer is:
    a. Odd?
    b. Not a perfect square? Answer: a) 0.5 b) 43/50

16. In a track meet, five entrants of equal ability are competing. What is the probability that:
    a. The finish wil be in the descending order of the entrants' ages?
    b. Sandy will be first? Answer: a) 1/120 b) 1/5

17. Hans has 12 good friends, five of them male and seven of them female. He decides to have a dinner party but can invite only seven because his dining room table will seat only 8 people. He decides to invite his guests by lot (picking names out of a hat.) What is the probability that
    a. There will be four males and four females at the party?
    b. Rivka will be among those invited? Answer: a) 0.44 b) 0.58

18. A is the event of rolling a prime number with a die.
    B is the event of rolling a perfect square with a die.
    C is the event of rolling an even number with a die.
Find :
    a. $P(A \cup B)$
    b. $P(C \cup B)$ Answer: a) 5/6 b) 2/3

19. A die is rolled twice. What is the probability that the sum of the rolls is less than 4 given that one of the rolls is a 1? Answer: 3/11

20. What is the probability that there are at least two people with the same birthday in a class of 40 students? Answer: 0.891

21. A confident and boastful coach claims that on the next league game the odds of his team winning are 3:1; the odds against losing are 5:1; and the odds against a tie are 7:1. Can these odds be right? Explain.

22. Andrea and Ling are evenly matched tennis players. However, each time Ling loses a game his probability of winning the next game is decreased by 1/5. But when he wins, his probability of winning the next game increases by 1/10.
    a. Make a tree diagram for a three-game sequence and label the diagram with the probabilities associated with each branch of the diagram.
    b. Find the probability that Andrea wins exactly two games. Answer: b) 0.245

Chapter 7 (New Text): Probability Distributions

23. A game consists of cutting a shuffled deck of cards. If you cut a face card you win 10 cents. If you cut an ace you win 25 cents. If you cut anything else you lose a nickel. What is the expectation for this game? Answer: approx 0.8 cents

24. A kennel is to be enclosed with 20m of fencing. The length of the kennel is to be an integer length and is to be chosen randomly. What is the expected area of the kennel?
Answer: 18.3 m^2

25. In a manufacturing process, it is estimated that only 2% of the bolts that are machined are declared defective, that is, they are either too large or too small. In a package of 50 bolts, what is the probability that there is at least one devedtive bolt? Answer: 0.636

26. If the probability of Michiko getting an A on her mathematics test is 0.6, what is the probability that Answer: a) 0.24 b) 0.936
    a. She will get her first A on the second test?
    b. She will get an A on one or more of her first three tests?

27. At an office party names were drawn out of a hat to pick teams for charades. There were ten females and six males at the party. What is the probability that the first team drawn (four members to a team) was all female? Answer: 0.115

Chapter 2 (New Text): Statistics of One Variable

See textbook for examples.

Chapter 3 (New Text): Statistics of Two Variables

See textbook for examples.

Chapter 8 (New Text): The Normal Distribution

28. If test marks are assumed to be normally distributed and 70% of the students scored greater than a mark of 60 (out of 100), and 10% scored over 90, determine the mean and standard deviation of the test marks. Answer: mean: 68.3 s.d.: 16.67

29. If X is $N(6,4)$, then find $P(x < 4.5)$ Answer: 0.2266

30. It is estimated that an average of 10% of the vehicles crossing an international border between two countries are smuggling undeclared goods. If the customs officers search 400 vehicles at random, what is the probability that more than 100 of the motorists are smuggling?
Answer: 0