2.4 Problem Solving with Permutations

Example 1: How many arrangements are there of the letters in the word **WILL**?

WILL
WILL

1 arrangement $\quad \frac{4!}{2!} = 12$ arrangements

PERMUTATIONS WITH REPETITION: The number of arrangements of n letters with a identical items of one kind, b identical items of another kind, etc. is:

$$\frac{n!}{a!b!\ldots}$$

Example 2: How many **unique** arrangements are there of the letters in the word ***MISSISSIPPI*** if,

a) The permutations must be unique?

4 I's
4 S's
4 P's
2

$$\frac{11!}{4!4!2!} = 34650 \text{ ways}$$

b) The first letter must be an M?

M _____

$$1 \times \frac{10!}{4!4!2!} = 3150 \text{ ways}$$

c) The last letter must be a vowel?

_____ I

$$\frac{10!}{3!4!2!} = 12\,600 \text{ ways}$$

d) The last letter must be a consonant?

Indirect

Total − end in vowel

$= 34650 - 12\,600$

$= 22050$ ways

Example 3: How many 4-digit numbers are there that ...

a) Have no restrictions?

$9 \times 10 \times 10 \times 10 = 9000$ numbers

b) Have no repeated digits?

$9 \times 9 \times 8 \times 7 = 4536$ numbers

can't be 0 — 1st no repeat

c) Have some repeated digits?

Indirect: Total − no repeated digits

$= 9000 - 4536$

$= 4464$ numbers

d) Have no repeated digits and are even?

① Case 1: ends in 0

$9 \times 8 \times 7 \times 1 = 504$

Case 2: ends in 2, 4, 6, 8

$8 \times 8 \times 7 \times 4 = 1792$ (4 = 2468)

Total $= 504 + 1792$

$= 2296$

② Indirect:

Total − odd

(not repeated) (not repeated)

$= 4536 - 8875$

$= 4536 - 2240$

$= 2296$