

Chapter 5 Exam Review – Probability Distributions

MDM4U

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Section 5.1 – Probability Distributions

1) What must be the value of $P(4)$ if this is a valid probability distribution? Why?

The probabilities of a valid probability distribution must add to 1. Therefore $P(4) = 0.35$

X	$P(X)$
0	0.1
1	0.2
2	0.05
3	0.2
4	0.35
5	0.1

2) Use the given frequency distribution to...

a) create a probability distribution for n , the number of dogs per household in a small town.

<i>Dogs</i>	<i>Households</i>
0	1500
1	430
2	175
3	52
4	16

n	$P(n)$
0	$\frac{1500}{2173}$ OR 0.6902899218
1	$\frac{430}{2173}$ OR 0.1978831109
2	$\frac{175}{2173}$ OR 0.0805338242
3	$\frac{52}{2173}$ OR 0.0239300506
4	$\frac{16}{2173}$ OR 0.0073630925

b) Determine the expected number of dogs in a home in the small town?

$$E(n) = 0 \left(\frac{1500}{2173} \right) + 1 \left(\frac{430}{2173} \right) + 2 \left(\frac{175}{2173} \right) + 3 \left(\frac{52}{2173} \right) + 4 \left(\frac{16}{2173} \right) = \frac{1000}{2173} \quad \text{OR} \quad 0.4601932812$$

Section 5.2 – Hypergeometric Probability Distributions

3) The door prizes at a dance are gift certificates from local merchants. There are four \$10 certificates, five \$20 certificates, and three \$50 certificates. The prize envelopes are mixed together in a bag and are drawn at random.

a) Create a probability distribution for the number of \$50 prizes drawn, n , on the first three draws.

# of \$50 prizes drawn (n)	$P(n)$
0	$\frac{\binom{3}{0}\binom{12}{3}}{\binom{15}{3}} = \frac{84}{220} = \frac{21}{55}$ OR 0.3818181818
1	$\frac{\binom{3}{1}\binom{12}{2}}{\binom{15}{3}} = \frac{108}{220} = \frac{27}{55}$ OR 0.4909090909
2	$\frac{\binom{3}{2}\binom{12}{1}}{\binom{15}{3}} = \frac{27}{220}$ OR 0.1227272727
3	$\frac{\binom{3}{3}\binom{12}{0}}{\binom{15}{3}} = \frac{1}{220}$ OR 0.0045454545

b) What is the expected number of \$50 certificates among the first three prizes drawn?

$$E(n) = r \left(\frac{a}{n} \right) = 3 \left(\frac{3}{12} \right) = \frac{9}{12} = 0.75$$

Note: you could also calculate the expected value using $\sum n \cdot P(n)$

c) What is the probability that at least 1 \$50 prize is drawn in the first three draws?

$$P(n \geq 1) = 1 - P(0) = 1 - \frac{21}{55} = \frac{34}{55} \text{ OR } 0.618181818$$

Section 5.3 – Binomial Distributions

4) A family plans on having four children. Assuming the probability of having a boy is equal to the probability of having a girl...

a) Create a probability distribution for the number of boys, X , the family will have

# of boys (X)	$P(X)$
0	$\text{binompdf}(4,0.5,0) = 0.0625$
1	$\text{binompdf}(4,0.5,1) = 0.25$
2	$\text{binompdf}(4,0.5,2) = 0.375$
3	$\text{binompdf}(4,0.5,3) = 0.25$
4	$\text{binompdf}(4,0.5,4) = 0.0625$

b) Find the expected number of boys in a family with four children

$$E(X) = np = 4(0.5) = 2$$

Note: you could also calculate the expected value using $\sum X \cdot P(X)$

5) A basketball player has a shooting percentage of 0.450

a) Create a probability distribution table for the number of baskets made in a quarter where he takes 4 shots.

Number of Baskets Made (X)	P(X)
0	$\text{binompdf}(4,0.45,0) = 0.09150625$
1	$\text{binompdf}(4,0.45,1) = 0.299475$
2	$\text{binompdf}(4,0.45,2) = 0.3675375$
3	$\text{binompdf}(4,0.45,3) = 0.200475$
4	$\text{binompdf}(4,0.45,4) = 0.04100625$

b) What is the expected number of baskets made in the quarter?

$$E(X) = np = 4(0.45) = 1.8$$

Note: you could also calculate the expected value using $\sum X \cdot P(X)$

6) The Choco---Latie Candies company makes candy---coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) Calculate the probability that exactly 5 of the candies in a box are red.

$$P(X = 5) = \text{binompdf}(10,0.4,5) = 0.2006581248$$

b) Calculate the probability that fewer than 5 in a box are red.

$$P(X < 5) = P(X \leq 4) = \text{binomcdf}(10,0.4,4) = 0.6331032576$$

c) Calculate the probability that at least 3 of the candies in a box are red.

$$P(X \geq 3) = 1 - P(X \leq 2) = 1 - \text{binomcdf}(10,0.4,2) = 0.8327102464$$

7) A certain type of rocket has a failure rate of 1.5%

a) Calculate the probability of there being exactly 1 failure in 100 launches. (answer to 6 decimal places)

$$P(X = 1) = \text{binompdf}(100, 0.015, 1) = 0.335953$$

b) Calculate the probability that there are more than 4 failures in 100 launches (answer to 6 decimal places)

$$P(X > 4) = 1 - P(X \leq 4) = 1 - \text{binomcdf}(100, 0.015, 4) = 0.017693$$

c) What is the expected number of failures in 100 launches of the rocket?

$$E(X) = np = 100(0.015) = 1.5$$

8) Suppose that 65% of the families in a town own computers. If eight families are surveyed at random...

a) What is the probability exactly 3 own a computer?

$$P(X = 3) = \text{binompdf}(8, 0.65, 3) = 0.0807733916$$

b) What is the probability that all 8 own a computer?

$$P(X = 8) = \text{binompdf}(8, 0.65, 8) = 0.0318644813$$

c) What is the probability that 6 or fewer families own a computer

$$P(X \leq 6) = \text{binomcdf}(8, 0.65, 6) = 0.8308731378$$

d) What is the expected number of families that will own a computer.

$$E(X) = np = 8(0.65) = 5.2$$

9) A recent survey of a gas---station's customers showed that 68% paid with credit cards, 29% used debit cards, and only 3% paid with cash. During her eight---hour shift as cashier at this gas station, Serena had a total of 223 customer. What is the probability that...

a) at least 142 customers used a credit card?

$$P(X \geq 142) = 1 - P(X \leq 141) = 1 - \text{binomcdf}(223, 0.68, 141) = 0.9260539149$$

b) fewer than 220 customers paid with credit or debit cards

$$P(X < 220) = P(X \leq 219) = \text{binomcdf}(223, 0.97, 219) = 0.9040253448$$

Section 5.4 – Geometric Distributions

10) From experience, you know that the probability that you will make a sale on any given telephone call is 0.23. Find the probability...

a) On any given day, your first sale won't be until your 5th call.

$$P(Y = 5) = \text{geometpdf}(0.23, 5) = 0.0808519943$$

b) It takes less than 4 calls to make a sale

$$P(Y < 4) = P(Y \leq 3) = \text{geometcdf}(0.23, 3) = 0.543467$$

c) It takes more than 10 calls to make a sale

$$P(Y > 10) = 1 - P(Y \leq 10) = 1 - \text{geometcdf}(0.23, 10) = 0.0732668047$$

11) Basketball player Shaquille O'Neal makes a free throw shot about 54% of the time. Find the probability...

a) The first free throw he makes is his 3rd free throw attempt

$$P(Y = 3) = \text{geometpdf}(0.54, 3) = 0.114264$$

b) It takes him more than 5 attempts to make his first free throw

$$P(Y > 5) = 1 - P(Y \leq 5) = 1 - \text{geometcdf}(0.54, 5) = 0.0205962976$$

12) A cereal maker places a game piece in its cereal boxes. The probability of winning a prize in the game is 1 in 4. Find the probability that...

a) You win your first prize with your fourth purchase

$$P(Y = 4) = \text{geompdf}(0.25, 4) = 0.10546875$$

b) It takes you fewer than 3 purchases to win a prize

$$P(Y < 3) = P(Y \leq 2) = \text{geomcdf}(0.25, 2) = 0.4375$$

Section 5.5 – Binomial Theorem

13) Find the binomial expansion of each expression in simplified form using the binomial theorem.

a) $(2x + 3)^4$

$$\begin{aligned} &= \binom{4}{0} (2x)^4 (3)^0 + \binom{4}{1} (2x)^3 (3)^1 + \binom{4}{2} (2x)^2 (3)^2 + \binom{4}{3} (2x)^1 (3)^3 + \binom{4}{4} (2x)^0 (3)^4 \\ &= (1)(16)(x^4)(1) + (4)(8)(x^3)(3) + (6)(4)(x^2)(9) + (4)(2)(x)(27) + (1)(1)(81) \\ &= 16x^4 + 96x^3 + 216x^2 + 216x + 81 \end{aligned}$$

b) $(2x - 1)^4$

$$\begin{aligned} &= \binom{4}{0} (2x)^4 (-1)^0 + \binom{4}{1} (2x)^3 (-1)^1 + \binom{4}{2} (2x)^2 (-1)^2 + \binom{4}{3} (2x)^1 (-1)^3 + \binom{4}{4} (2x)^0 (-1)^4 \\ &= (1)(16)(x^4)(1) + (4)(8)(x^3)(-1) + (6)(4)(x^2)(1) + (4)(2)(x)(-1) + (1)(1)(1) \\ &= 16x^4 - 32x^3 + 24x^2 - 8x + 1 \end{aligned}$$

c) $(3x - 2y)^5$

$$\begin{aligned} &= \binom{5}{0} (3x)^5 (-2y)^0 + \binom{5}{1} (3x)^4 (-2y)^1 + \binom{5}{2} (3x)^3 (-2y)^2 + \binom{5}{3} (3x)^2 (-2y)^3 + \binom{5}{4} (3x)^1 (-2y)^4 + \binom{5}{5} (3x)^0 (-2y)^5 \\ &= (1)(243)(x^5)(1) + (5)(81)(x^4)(-2)(y) + 10(27)(x^3)(4)(y^2) + 10(9)(x^2)(-8)(y^3) + 5(3)(x)(16)(y^4) + 1(1)(-32)(y^5) \\ &= 243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5 \end{aligned}$$

14) In the expansion of $\left(x^3 + \frac{2}{x}\right)^8$ find:

a) The number of terms

$$= 9$$

b) The general term

$$t^{r+1} = \binom{8}{r} (x^3)^{8-r} \left(\frac{2}{x}\right)^r$$

$$t_{r+1} = \binom{8}{r} (x^{24-3r})(2^r)(x^{-r})$$

$$t_{r+1} = \binom{8}{r} (2^r)(x^{24-4r})$$

c) The third term

$$t_{r+1} = \binom{8}{r} (2^r)(x^{24-4r})$$

$$t_{2+1} = \binom{8}{2} (2^2)(x^{24-4(2)})$$

$$t_3 = 28(4)(x^{24-8})$$

$$t_3 = 112x^{16}$$

d) The constant term

$$t_{r+1} = \binom{8}{r} (2^r)(x^{24-4r})$$

$$t_{6+1} = \binom{8}{6} (2^6)(x^{24-4(6)})$$

$$t_7 = 28(64)x^0$$

$$t_7 = 1792$$

15) Marshall is walking from his house to school. His route from home always takes him 4 blocks west and 9 blocks south to school but he likes to vary the path he takes.

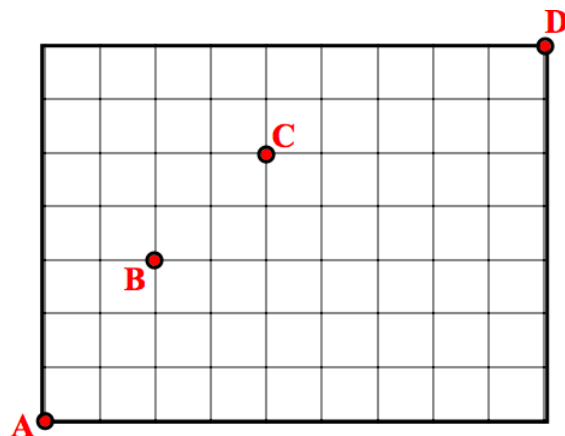
a) How many different routes can he take?

$$n(\text{routes}) = \binom{13}{4} = 715$$

b) If Marshall needs to stop for a coffee at the Tim Horton's that is 2 blocks west and three blocks south of his house, how many routes pass by this store?

$$n(\text{routes}) = \binom{5}{2} \binom{8}{2} = 10(28) = 280$$

16) The grid below shows the streets in your neighbourhood.



a) How many different routes are there to get from A to D?

$$n(\text{routes}) = \binom{16}{7} = 11440$$

b) How many different routes from A to D pass by the point B on the way?

$$n(\text{routes}) = \binom{5}{2} \binom{11}{4} = 10(330) = 3300$$

c) What is the probability that you pass C on your way from A to D?

$$P(\text{pass } C) = \frac{\binom{9}{5} \binom{7}{2}}{11440} = \frac{2646}{11440} = \frac{1323}{5720} \text{ OR } 0.2312937063$$