2.3 Factorials and Permutations

Factorials: the product of consecutive decreasing natural numbers

Example 1: Calculate each factorial.

a) $3!$

$= 3 \times 2 \times 1$

$= 6$

b) $69!$

$= 1.7 \times 10^{98}$

c) $70!$

= error (too large)

d) $\frac{8!}{6!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 8 \times 7$

$= 56$

e) $\frac{n+1}{(n+1)!}$

$= \frac{n+1}{(n+1)(n)(n-1)\cdots 1}$

$= \frac{1}{n!}$

f) $\frac{n!}{(n-1)!}$

$= \frac{n(n-1)(n-2)\cdots}{(n-1)(n-2)\cdots}$

$= n$

g) $(n+2)(n+1)!$

$= (n+2)!$

h) $\frac{(n+1)!}{(n-1)!}$

$= \frac{(n+1)(n)(n-1)!}{(n-1)!}$

$= n^2 + n$

Example 2: Solve for $n, n \in W$

a) $\frac{(n+3)!}{(n+2)!} = 5$

$\frac{(n+3)(n+2)!}{(n+2)!} = 5$

$n+3 = 5$

$n = 2$

b) $\frac{(n-4)!}{(n-6)!} = 6$

$\frac{(n-4)(n-5)(n-6)!}{(n-6)!} = 6$

$n^2 - 9n + 20 - 6 = 0$

$n^2 - 9n + 14 = 0$

$(n-7)(n-2) = 0$

$n = 7$

$n = 2 \rightarrow$ (would reject) like $2-4 = -2$.

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Example 3: The senior band has rehearsed five songs for an upcoming assembly. In how many orders can the band perform the songs?

$5 \times 4 \times 3 \times 2 \times 1 = 5!$

$= 120$ ways

Example 4: In a card game, a player is dealt a facedown reserve of 13 cards than can be turned up one at a time and used during the game. How many different sequences of the reserve cards could the player have?

$52 \times 51 \times 50 \times \cdots \times 40$

$$\frac{52!}{39!} = {}_{52}P_{13}$$

$$= 3.9 \times 10^{21} \text{ ways}$$

Example 5: A librarian wants to display 8 books on a bookshelf. There are 3 books by JK Rowling and the rest are by different authors. In how many ways could he arrange them if the Rowling books must **not** all be side by side?

Total − Books together

Indirect $= 8! - 6! \times 3!$

$= 36000$ ways

Permutations: An arrangement of some or all items from n distinct items.

1) To arrange **all** of n distinct items... $_nP_n = n! = P(n,n)$

$$_nP_n = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 = n!$$

2) To arrange **some** of n didistinct items... $_nP_r = \frac{n!}{(n-r)!}$

* Calculator button

Example 6: In how many ways can a president and a vice president be selected from a group of 10 candidates?

$_{10}P_2 = \frac{10!}{(10-2)!}$

$= \frac{10!}{8!}$

$= 10 \times 9$

$= 90$