5.2 Worksheet – Hypergeometric Probability Distributions

MDM4U
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1) A customer randomly selects two RAM modules from a shipment of six known to contain two defective modules.

a) Create the probability distribution for x, the number of defective modules in the purchase.

# of defective modules purchased, x	$P(x)$
0	$\frac{\binom{2}{0}\binom{4}{2}}{\binom{6}{2}} = \frac{6}{15}$
1	$\frac{\binom{2}{1}\binom{4}{1}}{\binom{6}{2}} = \frac{8}{15}$
2	$\frac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}} = \frac{1}{15}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{2}{x}\binom{4}{2-x}}{\binom{6}{2}}$$

b) Compute the expected number of defective RAM modules the customer would purchase.

$E(X) = \sum x \cdot P(x) = 0\left(\frac{6}{15}\right) + 1\left(\frac{8}{15}\right) + 2\left(\frac{1}{15}\right) = \frac{10}{15} = \frac{2}{3}$ or 0.67

2) A drawer contains four red socks and two blue socks. Three socks are drawn from the drawer without replacement.

a) Create a probability distribution in which the random variable represents the number of red socks.

# of Red Socks (X)	$P(X)$
0	$\frac{\binom{4}{0}\binom{2}{3}}{\binom{6}{3}} = \frac{0}{20} = 0$
1	$\frac{\binom{4}{1}\binom{2}{2}}{\binom{6}{3}} = \frac{4}{20} = \frac{1}{5}$
2	$\frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} = \frac{12}{20} = \frac{3}{5}$
3	$\frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = \frac{4}{20} = \frac{1}{5}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{4}{x}\binom{2}{3-x}}{\binom{6}{3}}$$

b) Determine the expected number of red socks if three are drawn from the drawer without replacement.

$$E(X) = \sum x \cdot P(x) = 0\left(\frac{0}{20}\right) + 1\left(\frac{4}{20}\right) + 2\left(\frac{12}{20}\right) + 3\left(\frac{4}{20}\right) = \frac{40}{20} = 2$$

3) There are five cats and seven dogs in a pet shop. Four pets are chosen at random for a visit to a children's hospital.

a) Create a probability distribution for the number of dogs chosen for a random visit to the hospital.

# of Dogs (X)	$\boldsymbol{P(X)}$
0	$\frac{\binom{7}{0}\binom{5}{4}}{\binom{12}{4}} = \frac{5}{495} = \frac{1}{99}$
1	$\frac{\binom{7}{1}\binom{5}{3}}{\binom{12}{4}} = \frac{70}{495} = \frac{14}{99}$
2	$\frac{\binom{7}{2}\binom{5}{2}}{\binom{12}{4}} = \frac{210}{495} = \frac{14}{33}$
3	$\frac{\binom{7}{3}\binom{5}{1}}{\binom{12}{4}} = \frac{175}{495} = \frac{35}{99}$
4	$\frac{\binom{7}{4}\binom{5}{0}}{\binom{12}{4}} = \frac{35}{495} = \frac{7}{99}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{7}{x}\binom{5}{4-x}}{\binom{12}{4}}$$

b) What is the probability that at least one dog is chosen to go?

$$P(\geq 1\ dog) = 1 - P(0\ dogs) = 1 - \frac{1}{99} = \frac{98}{99}$$

c) What is the expected number of dogs chosen?

$$E(X) = \sum x \cdot P(x) = 0\left(\frac{5}{495}\right) + 1\left(\frac{70}{495}\right) + 2\left(\frac{210}{495}\right) + 3\left(\frac{175}{495}\right) + 4\left(\frac{35}{495}\right) = \frac{1155}{495} = 2.33$$

4) A 12---member jury for a criminal case will be selected from a pool of 14 men and 11 women.

a) What is the probability that the jury will have an equal number of men and women?

$$P(6\ men, 6\ women) = \frac{\binom{14}{6}\binom{11}{6}}{\binom{25}{12}} = \frac{1\ 387\ 386}{5\ 200\ 300} = 0.2668$$

b) What is the probability that at least 3 jurors will be women?

$$P(\geq 3\ women) = 1 - P(0\ women) - P(1\ woman) - P(2\ women)$$

$$P(\geq 3\ women) = 1 - \frac{\binom{14}{12}\binom{11}{0}}{\binom{25}{12}} - \frac{\binom{14}{11}\binom{11}{1}}{\binom{25}{12}} - \frac{\binom{14}{10}\binom{11}{2}}{\binom{25}{12}}$$

$$P(\geq 3\ women) = 1 - \frac{91}{5\ 200\ 300} - \frac{4\ 004}{5\ 200\ 300} - \frac{55\ 055}{5\ 200\ 300}$$

$$P(\geq 3\ women) = \frac{5\ 141\ 150}{5\ 200\ 300}$$

$$P(\geq 3\ women) = 0.9886$$

c) What is the expected number of women? *(Note: the formula $E(x) = r\left(\frac{a}{n}\right)$ can be used for hypergeometric distributions)*

$$E(x) = 12\left(\frac{11}{25}\right) = 5.28$$

5) The door prizes at a dance are four \$10 gift certificates, five \$20 gift certificates, and three \$50 gift certificates. The prize envelopes are mixed together in a bag, and five prizes are drawn at random.

a) Create a probability distribution for the number of \$10 gift certificates drawn.

# of \$10 Certificates Drawn (X)	$P(X)$
0	$\frac{\binom{4}{0}\binom{8}{5}}{\binom{12}{5}} = \frac{56}{792} = \frac{7}{99}$
1	$\frac{\binom{4}{1}\binom{8}{4}}{\binom{12}{5}} = \frac{280}{792} = \frac{35}{99}$
2	$\frac{\binom{4}{2}\binom{8}{3}}{\binom{12}{5}} = \frac{336}{792} = \frac{14}{33}$
3	$\frac{\binom{4}{3}\binom{8}{2}}{\binom{12}{5}} = \frac{112}{792} = \frac{14}{99}$
4	$\frac{\binom{4}{4}\binom{8}{1}}{\binom{12}{5}} = \frac{8}{792} = \frac{1}{99}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r!x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{4}{x}\binom{8}{5!x}}{\binom{12}{5}}$$

b) What is the expected number of \$10 gift certificates drawn?

$$E(X) = \sum x \cdot P(x) = 0\left(\frac{56}{792}\right) + 1\left(\frac{280}{792}\right) + 2\left(\frac{336}{792}\right) + 3\left(\frac{112}{792}\right) + 4\left(\frac{8}{792}\right) = \frac{1320}{792} = 1.67$$

OR

$$E(x) = r\left(\frac{a}{n}\right) = 5\left(\frac{4}{12}\right) = 1.67$$