Factorial Notation (!)

In Combinatory, we often multiply consecutive natural numbers such as the following:

$$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 9!$$

We define for any natural factorial number n, $n \in \mathbb{N}$, natural numbers

$$n! = n\,(n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$$

and

$$0! = 1$$

Example 1: Evaluate and simplify.

(a) $4! = 4 \times 3 \times 2 \times 1 = 24$

(b) $1! = 1$

(c) $2! = 2$

(d) $17! = 3.556874\ldots \times 10^{14}$

(e) $7 \cdot 5! = 7 \times 5 \times 4 \times 3 \times 2 \times 1 = 840$

(f) $\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1}} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 7 \times 6 \times 5$

(g) $\frac{17!}{15!2!} = 210$

$$= \frac{17 \times \cancel{16}^{\,8} \times \cancel{15!}}{\cancel{15!}\,\cancel{2!}}$$

$$= 17 \times 8$$

$$= 136$$

Expand and simplify each of factorial with variables

(h) $(n+3)!$

$$= (n+3)(n+2)(n+1)(n)(n-1)(n-2)\cdots \times 3 \times 2 \times 1$$

(i) $\frac{(n+2)(n+1)!}{(n-1)!}$

$$= \frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$$= n(n+1)(n+2)\ \checkmark$$

Example 2: Solve for n, $n \in \mathbb{N}$

(a) $\frac{(n-1)!}{n!} = \frac{1}{2}$

$$\frac{\cancel{(n-1)!}}{n\cancel{(n-1)!}} = \frac{1}{2}$$

$$\frac{1}{n} = \frac{1}{2}$$

$$n = 2$$

(b) $\frac{(n+1)!}{n!} = 9$

$$\frac{(n+1)\cancel{(n!)}}{\cancel{n!}} = 9$$

$$n+1 = 9$$

$$n = 8$$

(c) $\frac{3(n+1)!}{(n-1)!} = 126$

$$\frac{3(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}} = 126$$

$$n^2 + n = \frac{126}{3}$$

$$n^2 + n - 42 = 0$$

$$(n+7)(n-6) = 0$$

$\cancel{n = -7}$ or $n = 6$

inadmissible because $n \in \mathbb{N}$

$\therefore n = 6$