5.3 Binomial Probability Distributions

MDM4U
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Part 1: Properties of a Binomial Experiment

In a Binomial Experiment, the number of successes in n trials is a discrete random variable --- X. X is termed a **binomial random variable** and its probability distribution is called a **binomial distribution.**

Properties of Binomial Experiments (Bernoulli Trials)

1. There are n identical trials

2. There are only two possible outcomes. Success or failure.

3. The probability of success is the same in every trial (trials are independent of one another)

Part 2: Investigating Binomial Experiments

Consider an experiment where you roll a single die 4 times.

a) What is the probability that your first two rolls are 6's and your next two rolls will be something other than a 6?

P[(roll 1 = 6) and (roll 2 = 6) and (roll 3 ≠ 6) and (roll 4 ≠ 6)]

$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{1296}$

> ***Note:*** *This is a Bernoulli Trial because there are only two possible outcomes - success is a roll of 6, failure is a roll that is not 6. The probability of success is the same for every roll. The trials are independent of each other.*

b) Find the probability of the event that the roll of 6 will appear exactly twice in any of the four available positions in the table.

Note: *The number of ways 6 can be placed in two of the four entries is the same as counting the number of ways two objects can be selected from four available objects. C(4, 2)*

$$P(\text{two } 6'\text{s}) = \frac{25}{1296} \times C(4,2) = \frac{25}{1296} \times 6 = \frac{150}{1296} = \frac{25}{216}$$

c) Complete a theoretical probability distribution for the number of 6's showing in four rolls

Probability of zero 6's

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

Probability of one 6

$$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \binom{4}{1} = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 \binom{4}{1} = \frac{500}{1296} = \frac{125}{324}$$

Probability of two 6's

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \binom{4}{2} = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \binom{4}{2} = \frac{150}{1296} = \frac{25}{216}$$

Probability of three 6's

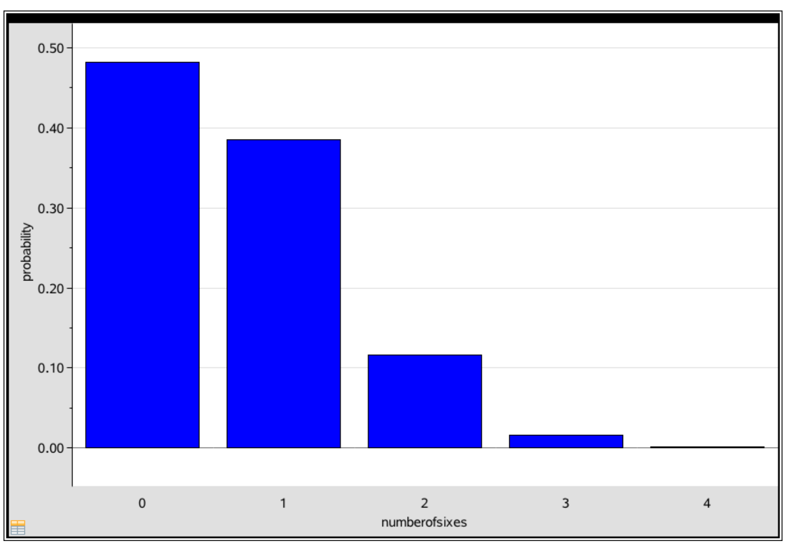
$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \binom{4}{3} = \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 \binom{4}{3} = \frac{20}{1296} = \frac{5}{324}$$

Probability of four 6's

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

Use a chart to display the probability distribution…

# of 6's, x	$P(x)$
0	$\binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4$ $\frac{\quad}{1296}$
1	$\binom{4}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3$ $\frac{\quad}{1296}$
2	$\binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$ $\frac{\quad}{1296}$
3	$\binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1$ $\frac{\quad}{1296}$
4	$\binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0$ $\frac{\quad}{1296}$



Part 2: Binomial Probability Formula

In a binomial experiment with *n* Bernoulli trials, each with a probability of success *p*, the probability of *k* successes in the *n* trials is given by:

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n!k}$$

Where *X* is the discrete random variable corresponding to the number of successes.

Example 1: Each child of a particular set of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

a) Find the probability that exactly 3 of the children have type O blood

$n = 5$

$p = 0.25$

$k = 3$

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n!k}$$

$$P(X = 3) = \binom{5}{3} (0.25)^3 (0.75)^2$$

$$P(X = 3) = 0.08789$$

There is an 8.79% chance that 3 of their children will have type O blood.

b) Should the parents be surprised if fewer than 2 of the children have type O blood?

$$P(X < 2) = P(X = 0) + P(X = 1)$$

$$P(X < 2) = \binom{5}{0} (0.25)^0 (0.75)^5 + \binom{5}{1} (0.25)^1 (0.75)^4$$

$$P(X < 2) = 0.6328125$$

There is a 63.28% chance that fewer than 2 of their children have type O blood so they should not be surprised.

Example 2: During the 2010 NHL season when Crosby led the NHL in Goals and Points, he had a 17% shooting percentage. Determine the probability that in a game where he takes four shots, he gets three goals.

$n = 4$

$p = 0.17$

$k = 3$

$$P(X = 3) = \binom{4}{3} (0.17)^3 (0.83)^1 = 0.0163$$

There is approximately a 1.63% chance of Crosby getting a hat---trick when he takes 4 shots.

Example 3: A basketball player has 3 shots in a free---throw competition. Historically, the player has a probability of 65% of scoring on a free throw. Assuming the probability is constant...

a) Determine the probability distribution for the number of free throws made

# of Free Throws Made, x	Probability, $P(x)$
0	$\binom{3}{0}(0.65)^0(0.35)^3 = 0.042875$
1	$\binom{3}{1}(0.65)^1(0.35)^2 = 0.238875$
2	$\binom{3}{2}(0.65)^2(0.35)^1 = 0.443625$
3	$\binom{3}{3}(0.65)^3(0.35)^0 = 0.274625$

b) Calculate the players expected number of free throws made in the competition.

Note: to calculate the expected value for a binomial probability distribution, you can use either of the following two formulas:

i) $E(X) = \sum x \cdot P(x)$ (this formula works for ALL types of probability distributions)

$= 0(0.042875) + 1(0.238875) + 2(0.443625) + 3(0.274625)$

$= 1.95$

ii) $E(X) = np$ (this formula only works for binomial probability distributions)

$= 3(0.65)$

$= 1.95$

Example 4: A candy company makes candy---coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that 3 of the candies in a given box are red?

$P(X = 3) = \binom{10}{3}(0.4)^3(0.6)^7 = 0.21499$

There is about a 21.50% chance that exactly three of the candies are red.

b) What is the probability that a given box has at least 2 red candies?

$P(X \geq 2) = 1 - [P(0) + P(1)]$

$P(X \geq 2) = 1 - \left[\binom{10}{0}(0.4)^0(0.6)^{10} + \binom{10}{1}(0.4)^1(0.6)^9\right]$

$P(X \geq 2) = 1 - 0.0463574016$

$P(X \geq 2) = 0.9536$

There is about a 95.36% chance that a given box has at least 2 red candies.

Part 3: Using the Ti---84

binompdf(n, p, k) computes $P(X = k)$

binomcdf(n, p, k) computes $P(X \leq k)$

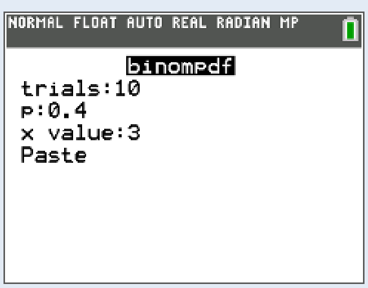
Note: the binomcdf command only computes the probability of getting k or FEWER successes

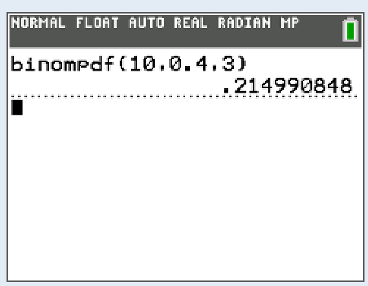
Example 4: A candy company makes candy---coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that 3 of the candies in a given box are red?

How to use the calculator:

--- 2nd → VARS (DISTR) → binompdf(→ trials: 10 → p: 0.4 → x value: 3 → PASTE





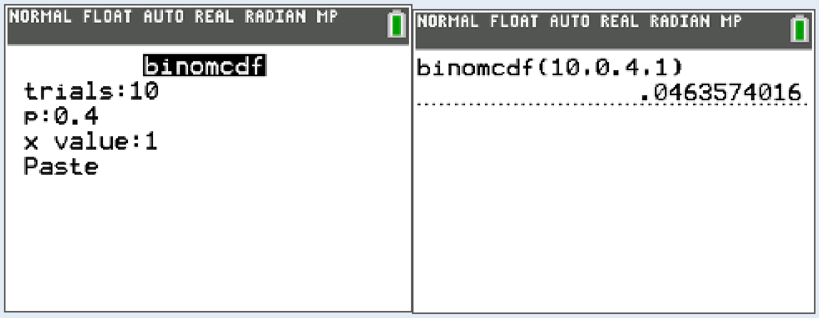
What you need to write:

$P(X = 3)$ = binompdf(n=10, p=0.4, k=3) = 0.21499

There is about a 21.50% chance that exactly three of the candies are red.

b) What is the probability that a given box has at least 2 red candies?

--- 2nd → VARS (DISTR) → binomcdf(→ trials: 10 → p: 0.4 → x value: 1 → PASTE



What you need to write:

$P(X \geq 2) = 1 - P(X \leq 1)$

$=$ 1 − binomcdf(n=10, p=0.4, k=1) = 0.0463574016

$= 0.9536$

There is about a 95.36% chance that a given box has at least 2 red candies.