5.3 Binomial Probability Distributions

MDM4U
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Part 1: Properties of a Binomial Experiment

In a Binomial Experiment, the number of _________________ in n trials is a discrete random variable --- X. X is termed a **binomial random variable** and its probability distribution is called a **binomial distribution.**

Properties of Binomial Experiments (Bernoulli Trials)

1. There are n identical trials

2. There are only two possible outcomes. Success or failure.

3. The probability of success is the same in every trial (trials are independent of one another)

Part 2: Investigating Binomial Experiments

Consider an experiment where you roll a single die 4 times.

a) What is the probability that your first two rolls are 6's and your next two rolls will be something other than a 6?

Note: *This is a Bernoulli Trial because there are only two possible outcomes - success is a roll of 6, failure is a roll that is not 6. The probability of success is the same for every roll. The trials are independent of each other.*

b) Find the probability of the event that the roll of 6 will appear exactly twice in any of the four available positions in the table.

Note: *The number of ways 6 can be placed in two of the four entries is the same as counting the number of ways two objects can be selected from four available objects. C(4, 2)*

c) Complete a theoretical probability distribution for the number of 6's showing in four rolls

Probability of zero 6's

Probability of one 6

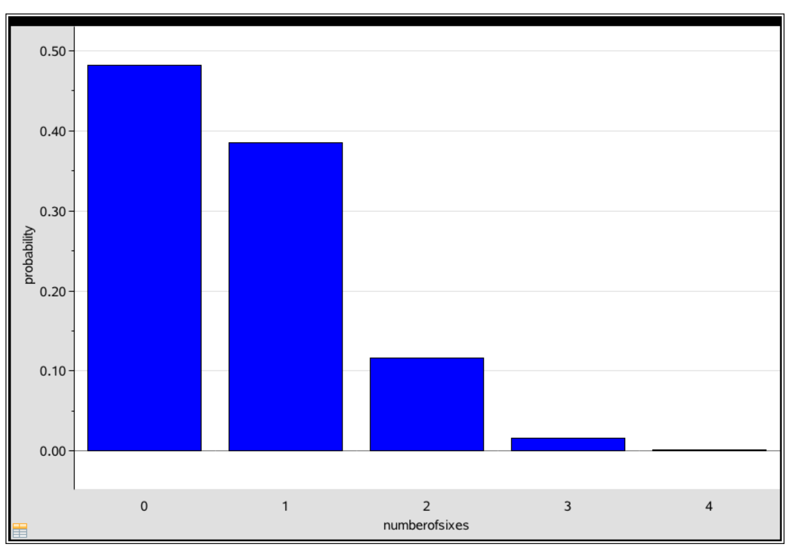
Probability of two 6's

Probability of three 6's

Probability of four 6's

Use a chart to display the probability distribution…

# of 6's, x	$P(x)$
0	
1	
2	
3	
4	



Part 2: Binomial Probability Formula

In a binomial experiment with n Bernoulli trials, each with a probability of success p, the probability of k successes in the n trials is given by:

Where X is the discrete random variable corresponding to the number of successes.

Example 1: Each child of a particular set of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

a) Find the probability that exactly 3 of the children have type O blood

b) Should the parents be surprised if fewer than 2 of the children have type O blood?

Example 2: During the 2010 NHL season when Crosby led the NHL in Goals and Points, he had a 17% shooting percentage. Determine the probability that in a game where he takes four shots, he gets three goals.

Example 3: A basketball player has 3 shots in a free---throw competition. Historically, the player has a probability of 65% of scoring on a free throw. Assuming the probability is constant...

a) Determine the probability distribution for the number of free throws made

# of Free Throws Made, x	**Probability, $P(x)$**

b) Calculate the players expected number of free throws made in the competition.

Note: to calculate the expected value for a binomial probability distribution, you can use either of the following two formulas:

Example 4: A candy company makes candy---coated chocolates, 49% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that 3 of the candies in a given box are red?

b) What is the probability that a given box has at least 2 red candies?

Part 3: Using the Ti---84

binompdf(n, p, k) computes $P(X = k)$

binomcdf(n, p, k) computes $P(X \leq k)$

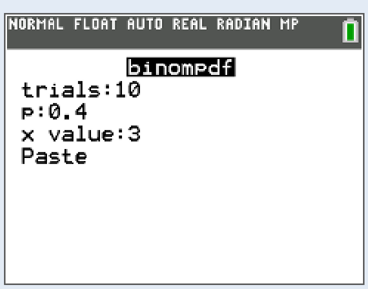
Note: the binomcdf command only computes the probability of getting k or FEWER successes

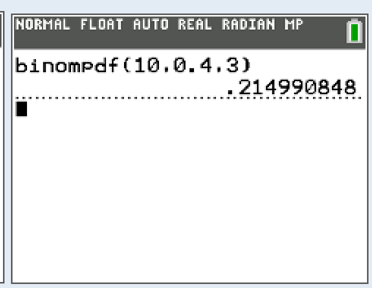
Example 4: A candy company makes candy---coated chocolates, 49% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that 3 of the candies in a given box are red?

How to use the calculator:

--- 2nd → VARS (DISTR) → binompdf(→ trials: 10 → p: 0.4 → x value: 3 → PASTE





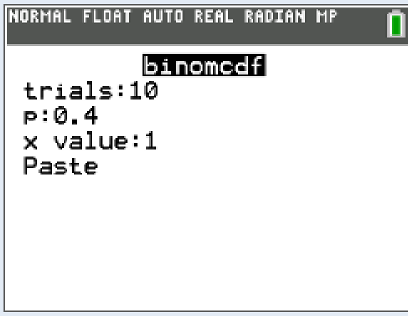
What you need to write:

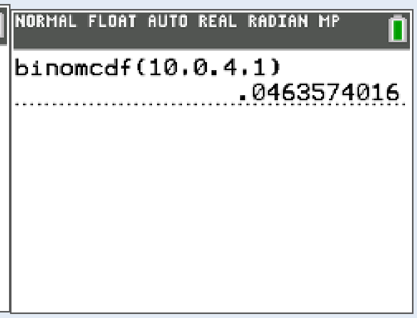
$P(X = 3)$ = binompdf(n=10, p=0.4, k=3) = 0.21499

There is about a 21.50% chance that exactly three of the candies are red.

b) What is the probability that a given box has at least 2 red candies?

--- 2[nd] → VARS (DISTR) → binomcdf(→ trials: 10 → p: 0.4 → x value: 1 → PASTE





What you need to write:

$P(X \geq 2) = 1 - P(X \leq 1)$

$=1-$ binomcdf(n=10, p=0.4, k=1) = 0.0463574016

$= 0.9536$

There is about a 95.36% chance that a given box has at least 2 red candies.