5.1 Worksheet – Probability Distributions

MDM4U

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1) Which of the following are valid probability distributions? Explain

a)

x	$P(x)$
0	0.5
1	0.25
2	0.25

b)

x	$P(x)$
0.5	0.2
0.2	0.3
0.3	0.25

c)

x	$P(x)$
0	0.3
1	0.25
2	0.25
3	0.2

a & c are valid. b is not valid because the probabilities do not add to 1.

2) Given the following probability distributions, determine the expected values.

a)

x	$P(x)$
5	0.3
10	0.25
15	0.45

b)

x	$P(x)$
1 000	0.25
100 000	0.25
1 000 000	0.25
10 000 000	0.25

c)

x	$P(x)$
1	$\frac{1}{6}$
2	$\frac{1}{5}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{20}$

a) $E(x) = \sum x \cdot P(x) = 5(0.3) + 10(0.25) + 15(0.45) = 10.75$

b) $E(x) = \sum x \cdot P(x) = 1000(0.25) + 100000(0.25) + 1000000(0.25) + 10000000(0.25) = 2\,775\,250$

c) $E(x) = \sum x \cdot P(x) = 1\left(\frac{!}{6}\right) + 2\left(\frac{!}{5}\right) + 3\left(\frac{!}{4}\right) + 4\left(\frac{!}{3}\right) + 5\left(\frac{!}{20}\right) = 2.9$

3) A spinner has eight equally---sized sectors, numbered 1 through 8.

a) Create a probability distribution for the outcome of a spin

Spin, x	$P(x)$
1	$\frac{1}{8}$
2	$\frac{1}{8}$
3	$\frac{1}{8}$
4	$\frac{1}{8}$
5	$\frac{1}{8}$
6	$\frac{1}{8}$
7	$\frac{1}{8}$
8	$\frac{1}{8}$

b) What is the probability that the arrow on the spinner will stop on a prime number?

$$P(prime) = P(2) + P(3) + P(5) + P(7) = \frac{4}{8} = \frac{1}{2}$$

c) What is the expected outcome?

$$E(x) = \sum x \cdot P(x) = 1\left(\frac{1}{8}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 6\left(\frac{1}{8}\right) + 7\left(\frac{1}{8}\right) + 8\left(\frac{1}{8}\right) = 4.5$$

4) A lottery has a \$1 000 000 first prize, a \$25 000 second prize, and five \$1 000 third prizes. A total of 2 000 000 tickets are sold.

a) Create a probability distribution for the amount of money you could win

Winnings, x	Probability, $P(x)$
1 000 000	$\frac{1}{2\,000\,000}$
25 000	$\frac{1}{2\,000\,000}$
1 000	$\frac{5}{2\,000\,000}$
0	$\frac{1\,999\,993}{2\,000\,000}$

b) Calculate the expected winnings

$$E(x) = \sum x \cdot P(x) = 1\,000\,000\left(\frac{1}{2\,000\,000}\right) + 25\,000\left(\frac{1}{2\,000\,000}\right) + 1\,000\left(\frac{5}{2\,000\,000}\right) + 0\left(\frac{1\,999\,993}{2\,000\,000}\right)$$

$= \frac{1\,030\,000}{2\,000\,000}$

$= 0.515$

The expected winnings is about \$0.52

c) If you buy a ticket for \$2.00, what is your expected profit?

$Expected\ profit = expected\ winnings - cost\ of\ ticket = 0.52 - 2 = -1.48$

You should expect to lost \$1.48.

5) A game consists of rolling a die. If an even number shows, you receive double the value of the upper face in points. If an odd number shows, you lose points equivalent to triple the value of the upper face.

a) Create a probability distribution for the amount of points you receive on a single roll

Points, x	$P(x)$
-3	$\frac{1}{6}$
4	$\frac{1}{6}$
-9	$\frac{1}{6}$
8	$\frac{1}{6}$
-15	$\frac{1}{6}$
12	$\frac{1}{6}$

b) How many points would you expect to get on a single roll?

$$E(x) = \sum x \cdot P(x) = -3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) - 9\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) - 15\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) = -0.5$$

6) Make a probability distribution from the following frequency distribution to represent the number of fish caught in a 6---hour period. Then calculate the expected number of fish caught in 6 hours.

# of fish caught	0	1	2	3	4
frequency	88	72	30	8	2

# of fish caught, x	0	1	2	3	4
$P(x)$	$\frac{88}{200}$	$\frac{72}{200}$	$\frac{30}{200}$	$\frac{8}{200}$	$\frac{2}{200}$

$$E(x) = \sum x \cdot P(x) = 0\left(\frac{88}{200}\right) + 1\left(\frac{72}{200}\right) + 2\left(\frac{30}{200}\right) + 3\left(\frac{8}{200}\right) + 4\left(\frac{2}{200}\right) = \frac{164}{200} = 0.82$$