

Section 4.3 Worksheet – Probability Using Sets

MDM4U

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1) Using the Venn diagram below, list the number of elements in each of the following sets.

a) $P \cap Q$

$$= 9 + 6$$

$$= 15$$

b) $P \cup Q$

$$= 5 + 2 + 9 + 6 + 10 + 12$$

$$= 44$$

c) $X \cap Q$

$$= 9 + 10$$

$$= 19$$

d) $X \cup Q$

$$= 4 + 2 + 9 + 10 + 6 + 12$$

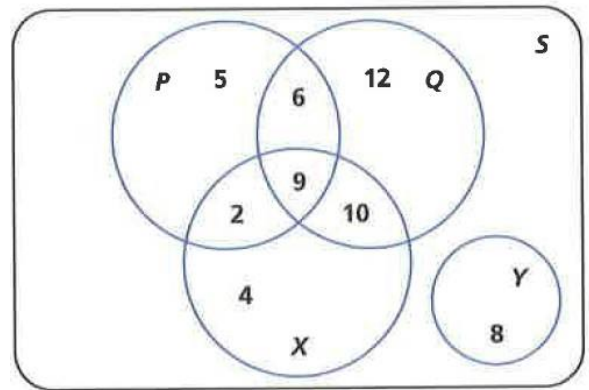
$$= 43$$

e) $Y \cap Q$

$$= 0$$

f) $P \cap Q \cap X$

$$= 9$$



2) For each of the following, find the indicated probability and state whether A and B are mutually exclusive.

a) $P(A) = 0.5$, $P(B) = 0.2$, $P(A \cup B) = 0.7$, $P(A \cap B) = ?$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.5 + 0.2 - P(A \cap B)$$

$$P(A \cap B) = 0$$

Since $P(A \cap B) = 0$, the events are mutually exclusive.

b) $P(A) = 0.7, P(B) = 0.2, P(A \cup B) = ?, P(A \cap B) = 0.15$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.7 + 0.2 - 0.15$$

$$P(A \cup B) = 0.75$$

Since $P(A \cap B) \neq 0$, the events are not mutually exclusive

c) $P(A) = 0.3, P(B) = ?, P(A \cup B) = 0.9, P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.9 = 0.3 + P(B) - 0$$

$$P(B) = 0.9 - 0.3$$

$$P(B) = 0.6$$

Since $P(A \cap B) = 0$, the events are mutually exclusive.

3) The probability that Kelly will make the volleyball team is $\frac{2}{3}$ and the probability that she will make the field hockey team is $\frac{3}{4}$. If the probability that she makes both teams is $\frac{1}{2}$, what is the probability that she makes at least one of the teams?

$$P(\text{volleyball} \cup \text{hockey}) = P(\text{volleyball}) + P(\text{hockey}) - P(\text{volleyball} \cap \text{hockey})$$

$$P(\text{volleyball} \cup \text{hockey}) = \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$$

$$P(\text{volleyball} \cup \text{hockey}) = \frac{8}{12} + \frac{9}{12} - \frac{6}{12}$$

$$P(\text{volleyball} \cup \text{hockey}) = \frac{11}{12}$$

4) An aquarium at a pet store contains 20 guppies (12 females and 8 males) and 36 tetras (14 females and 22 males). If the clerk randomly nets a fish, what is the probability that it is a female or a tetra?

$$P(\text{female} \cup \text{tetra}) = \frac{n(\text{female} \cup \text{tetra})}{n(\text{fish})}$$

$$P(\text{female} \cup \text{tetra}) = \frac{n(\text{female}) + n(\text{tetra}) - n(\text{female} \cap \text{tetra})}{n(\text{fish})}$$

$$P(\text{female} \cup \text{tetra}) = \frac{26 + 36 - 14}{56}$$

$$P(\text{female} \cup \text{tetra}) = \frac{48}{56}$$

$$P(\text{female} \cup \text{tetra}) = \frac{6}{7}$$

5) Teri attends a fundraiser at which 15 T-shirts are being given away as door prizes. Door prize winners are randomly given a shirt from a stock of 2 black shirts, 4 blue shirts, and 9 white shirts. Teri really likes the black and blue shirts, but is not too keen on the white ones. Assuming that Teri wins the first door prize, what is the probability that she will get a shirt that she likes?

$$P(\text{black} \cup \text{blue}) = P(\text{black}) + P(\text{blue})$$

$$P(\text{black} \cup \text{blue}) = \frac{2}{15} + \frac{4}{15}$$

$$P(\text{black} \cup \text{blue}) = \frac{6}{15}$$

6) A card is randomly selected from a standard deck of cards. What is the probability that either a heart or a face card (jack, queen, or king) is selected?

$$P(\text{heart} \cup \text{face card}) = P(\text{heart}) + P(\text{face card}) - P(\text{heart} \cap \text{face card})$$

$$P(\text{heart} \cup \text{face card}) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$P(\text{heart} \cup \text{face card}) = \frac{22}{52}$$

$$P(\text{heart} \cup \text{face card}) = \frac{11}{26}$$

7) An electronics manufacturer is testing a new product to see whether it requires a surge protector. The tests show that a voltage spike has a 0.2% probability of damaging the product's power supply, a 0.6% probability of damaging downstream components, and a 0.1% probability of damaging both. Determine the probability that a voltage spike will damage the product.

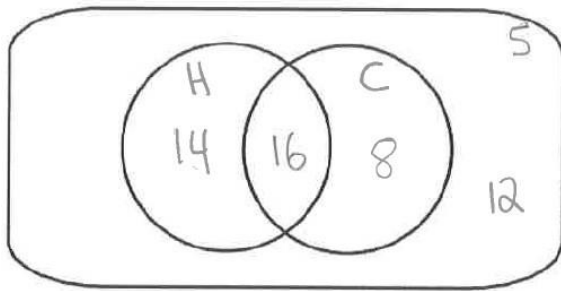
$$P(\text{power supply} \cup \text{components}) = P(\text{power supply}) + P(\text{components}) - P(\text{power supply} \cap \text{components})$$

$$P(\text{power supply} \cup \text{components}) = 0.2\% + 0.6\% - 0.1\%$$

$$P(\text{power supply} \cup \text{components}) = 0.7\%$$

8) At the start of flu season, Dr. Anna Ahmeed examines 50 patients over two days. Of those 50 patients, 30 have a headache, 24 have a cold, and 12 have neither symptom. Some patients have both symptoms.

a) Draw a Venn diagram and determine the number of patients that have both symptoms.



$$n(H \cup C) = n(H) + n(C) - n(H \cap C)$$

$$38 = 30 + 24 - n(H \cap C)$$

$$n(H \cap C) = 54 - 38$$

$$n(H \cap C) = 16$$

b) Find the probability that a patient selected at random...

i) has just a headache

$$P(\text{headache only}) = \frac{14}{50} = \frac{7}{25}$$

ii) has a headache or a cold

$$P(H \cup C) = \frac{n(H \cup C)}{n(S)} = \frac{38}{50} = \frac{19}{25}$$

iii) does not have cold symptoms

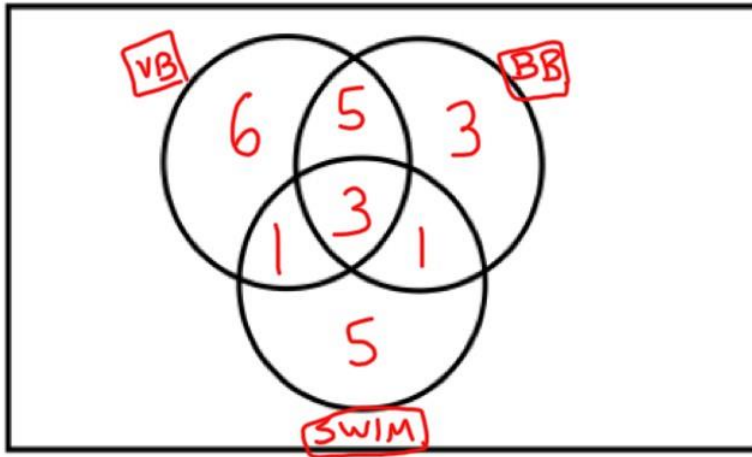
$$P(\text{cold}^c) = 1 - P(\text{cold})$$

$$P(\text{cold}^c) = 1 - \frac{24}{50}$$

$$P(\text{cold}^c) = \frac{26}{50}$$

$$P(\text{cold}^c) = \frac{13}{25}$$

9) a) Use the table to complete the following Venn Diagram:



Team	# of Students
Volleyball	15
Basketball	12
Swimming	10
VB and BB	8
VB and Swim	4
BB and Swim	4
All three	3

b) What is the probability that a student plays volleyball?

$$P(\text{volleyball}) = \frac{15}{24} = \frac{5}{8}$$

c) What is the probability that a student plays basketball?

$$P(\text{basketball}) = \frac{12}{24} = \frac{1}{2}$$

d) What is the probability that a student plays basketball and volleyball?

$$P(\text{basketball} \cap \text{volleyball}) = \frac{8}{24} = \frac{1}{3}$$

e) What is the probability that a student plays basketball or volleyball?

$$P(\text{basketball} \cup \text{volleyball}) = \frac{19}{24}$$