Lesson: Introduction to Permutation

Factorial – $n!, n \epsilon \mathbb{N}$

$$n! = n(n-1)(n-2)(n-3) \ldots\ldots \times 3 \times 2 \times 1$$

$$0! = 1$$

i.e., $5! = 120$

Warm up: Give digits 5, 2, and 3,

(a) how many ways can you arrange 3-digit numbers?

523, 532, 325, 352, 235, 253 $\boxed{\therefore 6\text{ways}}$ $= \underset{2 \text{ or } 3 \text{ or } 5}{\underline{3}} \times \underline{2} \times \underline{1}$ ← line up 3 digits here

$= 3!$

(b) How many ways can you arrange 2-digit numbers?

$\underset{3 \text{ or } 2 \text{ or } 5}{\underline{3}} \times \underline{2} = 6$ ways

Using your calculator……

Permutation:

$P(n,r) = {}_n\mathrm{P}_r$, $n \geq r$, $n, r \in \mathbb{W}$

total number of objects available

# of objects need to be arranged.

permutation

$$= \frac{n!}{(n-r)!}$$

-Rearranging r objects from a total of n objects in a DEFINITE ORDER

i.e., $P(8,2) = \frac{8!}{(8-2)!} = \frac{8!}{6!}$ $\boxed{8}\,\boxed{nPr}\,\boxed{2}\,\boxed{=}$

$= 8 \times 7$

$= 56$

Using your calculator…..

Examples

(a) $P(8,3)$

$= 336$

(b) $P(6,6)$

$= 720$

$= \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$= 6!$

(c) $\frac{7!}{5!}$

$= \frac{n!}{(n-r)!}$ ($n = 7$, $r = 2$)

$= {}_nP_r$

$= {}_7P_2$ or $P(7,2)$

(d) $\frac{n!}{(n-1)!}$

$= P(n,1)$

$= {}_nP_1$

Recall:

Permutation: ${}_nP_r = \frac{n!}{(n-r)!}$

-Rearranging r objects from a total of n objects in a DEFINITE ORDER

Question	Answer
1. Given 4 distinct objects, in how many ways can you line up… (a) 1 of them?	$P(4,1) = \underline{4}$ ways $= \frac{4!}{(4-1)!}$
(b) 2 of them?	$P(4,2) = \underline{4} \times \underline{3} = 12$ ways
(c) 3 of them?	$P(4,3) = \underline{4} \times \underline{3} \times \underline{2} = 24$ ways
(d) 4 of them?	$P(4,4) = \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$ ways $= 4!$
2. Given 5 distinct objects, in how many ways can you line up … (a) 1 of them?	$P(5,1) = 5$ ways
(b) 2 of them?	$P(5,2) = 20$ ways

Rule of Sum: additive counting principle

- **if one mutually exclusive action can occur in m ways, a second in n ways, a third in p ways and so on, then there are m+n+p… ways in which one of these actions can occur**
- **"OR" – addition**

OR "+" (add)

Examples:

1. How many ways can you pick a 7 from a deck of cards?

   ${}_4P_1 = 4$ ways

2. How many ways can you pick a 7 OR a 3 from a deck of cards?

   $4 + 4 = 8$ ways

3. How many ways can you pick a jack OR a queen OR a king?

   $4 + 4 + 4 = 12$ ways

4. How many ways can you pick a black even number OR a red 6?

   $10 + 2 = 12$ ways

Fundamental Counting Principle:

- **If a task or process is made up of stages with separate choices, the total number of choices is $m \times n \times p \times \ldots$ ways**
- **"AND" – multiplication**

AND "x" (multiply)

Examples:

5. How many ways can you pick a 7 AND a 3?

   $4 \times 4 = 16$ ways

6. How many ways can you pick a jack AND a queen AND a king?

   $4 \times 4 \times 4 = 64$ ways.

7. How many ways can you pick a black even number and a red 6?

   $10 \times 2 = 20$ ways.

Complete the following questions, using the **Fundamental Principle of Counting, the Rule of Sum** and your generalization from Parts B and C.

1. Emily wants to get dressed in the morning. She has a choice of 6 tops, 4 bottoms and 3 pairs of socks. How many different outfits can she wear?

$$\underset{\text{tops}}{\underline{6}} \times \underset{\text{bottoms}}{\underline{4}} \times \underset{\text{socks}}{\underline{3}} = 72 \text{ ways}$$

2. Michelle is taking math, science, English and geography this semester. In how many ways could her timetable be made?

$$P(4,4) = 4! = \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24 \text{ ways}$$

3. Eric has signed up for 6 different courses this semester. He will only have 4 on his timetable. In how many ways could his timetable be made?

$$P(6,4) = \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} = 360 \text{ ways}$$

4. A baseball manager has 12 players. He has to set-up a batting order of 9 players. In how many ways can he set up his batting order?

$$_{12}P_9 = P(12,9)$$
$$= 79\ 833\ 600 \text{ ways}$$

5. Mr. Parent has 15 students in his most excellent Data Management class. He must pick 1 president, 1 vice-president, a secretary and a treasurer. In how many ways may he do this?

$$\underset{P}{\underline{15}} \times \underset{VP}{\underline{14}} \times \underset{S}{\underline{13}} \times \underset{T}{\underline{12}} = {_{15}P_4} = 32\ 760 \text{ ways}$$

6. A hockey coach has to set-up his starting line-up. He has to assign a center, left-wing and right-wing from 9 different forwards and he must assign a left defenseman and right defenseman from 6 different defensemen, and he must assign 1 goalie from 2 goalies. How many ways can he set up his starting line-up?

Forwards and Defenseman and Goalie

$$\underline{9} \times \underline{8} \times \underline{7} \quad \times \quad \underline{6} \times \underline{5} \quad \times \quad \underline{2}$$

$$= {_9P_3} \times {_6P_2} \times {_2P_1}$$

$$= 30\ 240 \text{ ways}$$