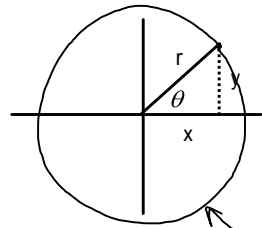


5.5 Trigonometric Identities



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$x^2 + y^2 = r^2$$

Prove each of the following trigonometric identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

LS	RS
$\sin^2 \theta + \cos^2 \theta$ $= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$ $= \frac{x^2 + y^2}{r^2}$ $= \frac{r^2}{r^2}$ $= 1$	1

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\begin{aligned} \text{RS} &= \frac{\frac{y}{r}}{\frac{x}{r}} \\ &= \frac{y}{r} \cdot \frac{r}{x} \\ &= \frac{y}{x} \\ &= \tan \theta \\ &= \text{LS} \end{aligned}$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\begin{aligned} \text{LS} &= 1 + \frac{1}{\tan^2 \theta} \\ &= 1 + \frac{1}{\frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= 1 + \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \\ &= \frac{1}{\sin^2 \theta} \\ &= \csc^2 \theta \\ &= \text{RS} \end{aligned}$$

$$\tan \theta = \frac{\sin \theta + \sin^2 \theta}{\cos \theta (1 + \sin \theta)}$$

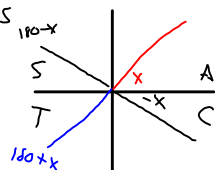
$$\begin{aligned} \text{RS} &= \frac{\sin \theta (1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{LS} \end{aligned}$$

$$\sin x \tan x = \sec x - \cos x$$

$$\begin{aligned} \text{RS} &= \frac{1}{\cos x} - \cos x \\ &= \frac{1 - \cos^2 x}{\cos x} \\ &= \frac{\sin^2 x}{\cos x} \\ &= \sin x \cdot \frac{\sin x}{\cos x} \\ &= \sin x \tan x \\ &= \text{LS} \end{aligned}$$

$$\cos x = \sin x \tan^2 x \cot^3 x$$

$$\begin{aligned} \text{RS} &= \cancel{\sin x} \cdot \frac{\cancel{\sin^2 x}}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^3 x}}{\cancel{\sin^3 x}} \\ &= \cos x \\ &= \text{LS} \end{aligned}$$



$$\cos(-x) + \cos(180^\circ - x) = \cos(180^\circ + x) + \cos x$$

LS	RS
$\cos x - \cos x$ $= 0$	$-\cos x + \cos x$ $= 0$