Unit 3 – Trigonometric Ratios

Day	Lesson	Practice Questions	Struggles?
1	U3L1 – Exploring Graphs of Sinusoidal Functions	**Read:** Pg. 362 - 363 **Do:** **Handout**	
2	U3L2 – Trig Ratios & Special Triangles	**Read:** Pg. 276 – 280, Pg. 283 – 286 **Do:** Pg. 280 # 1 – 3, 5a) i iv, 9 Pg. 286 # 3, 4, 5a, 6b, 8, 11	
3	U3L3 – Trig Ratios for Angles Between 0° & 360°	**Read:** Pg. 291 – 292 **Do:** Pg. 292 # 4 Pg 299 #1bc, 12ab, **Handout**	
4	U3L4 – Solving Trigonometric Equations	**Read:** Pg. 293 – 299 **Do:** Pg. 299 # 2bcd *(exact values for r)*, 3, 6ace, 8ace, 9ace, **Handout**	
5	U3L5 –The Ambiguous Case of the Sine Law	**Read:** Pg. 312 – 317 **Do:** **Handout** , Pg. 318 # 1b, 2, 4, 5 *(# of ∆, do not solve)* 13, 14 *#4b s/b 68° or 112°*	
6-7	U3L6 – 2D & 3D Trigonometric Problems	**Read:** Pg. 321 – 325, 328 – 331 **Do:** Pg. 319 # 8, 12 Pg. 327 # 10, **Handout** *#8 s/b 4139 m*	
8	Review	**Read:** Pg. 336 – 337 **Do:** Pg. 338 # 1 – 5, 8 – 13 *#9 s/b 5.7 km or 30.5 km #11 s/b 9.4 m #13 s/b 46°*	
9	TEST		
10-11	*Skill Builder: Adding & Subtracting Rational Expressions*	**Read:** Pg. 124 – 127 **Day 1:** Pg. 128 # 1cd, 2cd, 3a, 5bd, 6ace, 10bc **Day 2:** Pg. 128 # 3bc, 7 – 9 Pg. 133 # 15f *typo on #9b – all terms in numerator*	
12-13	U3L7 – Proving Trigonometric Identities	**Read:** Pg 305 – 309 **Do:** Pg. 310 # 2 – 3ac, 4, 5ac, 6, 7bd, 8bdf, 12a **Handout**	
14	Task: Trig Identities		

Unit 3, Lesson 1: Exploring Graphs of Sinusoidal Functions

For each function provided:

- Complete the table of values (round to 2 decimal places) *(**Make sure your calculator is in DEGREE mode**)*
- Plot the points and graph the function, choosing a scale that will utilize the entire grid.
- State the **domain** and **range**
- Verify your graphs using graphing technology

1. $f(x) = \sin(x)$

x (°)	$f(x)$
0	
30	
45	
60	
90	
120	
135	
150	
180	
210	
225	
240	
270	
300	
315	
330	
360	

Domain:

Range:

$g(x) = \cos(x)$

x (°)	**g (x)**
0	
30	
45	
60	
90	
120	
135	
150	
180	
210	
225	
240	
270	
300	
315	
330	
360	

Domain:

Range:

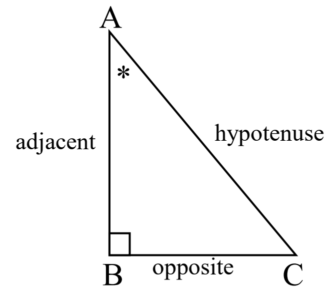
Unit 3, Lesson 2: Trig Ratios & Special Triangles

Recall: Primary Trigonometric Ratios

sine of ∠A ⟶ $\sin A = \dfrac{opposite}{hypotenuse}$ ⟶ $\sin A = \dfrac{O}{H}$

cosine of ∠A ⟶ $\cos A = \dfrac{adjacent}{hypotenuse}$ ⟶ $\cos A = \dfrac{A}{H}$

tangent of ∠A ⟶ $\tan A = \dfrac{opposite}{adjacent}$ ⟶ $\tan A = \dfrac{O}{A}$



** The easiest way to **MEMORIZE** these ratios is to use:

SOH, CAH, TOA

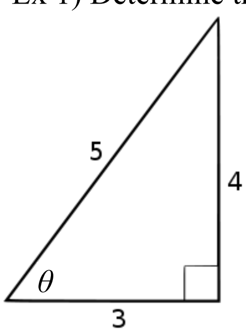
NEW: Reciprocal Trigonometric Ratios

cosecant of $\angle A$ ⟶ $\csc A = \dfrac{1}{\sin A}$ ⟶ $\csc A = \dfrac{H}{O}$

secant of $\angle A$ ⟶ $\sec A = \dfrac{1}{\cos A}$ ⟶ $\sec A = \dfrac{H}{A}$

cotangent of $\angle A$ ⟶ $\cot A = \dfrac{1}{\tan A}$ ⟶ $\cot A = \dfrac{A}{O}$

Ex 1) Determine the 6 trigonometric ratios for θ.

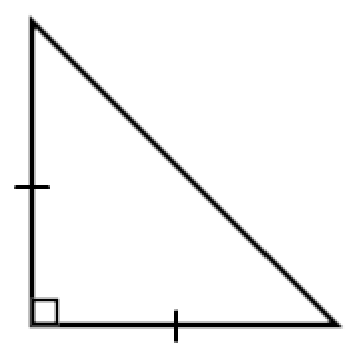


Special Triangles

Special triangles are used to determine **exact ratios** for certain special angles. **(no calculators!)**

45-45-90 triangle

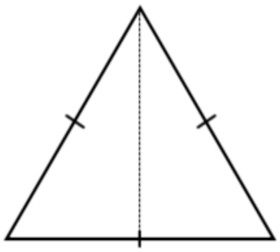
Consider a **right isosceles** triangle, with legs measuring 1 unit. Label the angles. Determine the exact measure of the hypotenuse.



From this we can determine the **exact values** of
$\sin 45° =$ $\cos 45° =$ $\tan 45° =$

30-60-90 triangle

Consider an **equilateral** triangle, with sides measuring 2 units. Divide into 2 congruent right triangles (along the altitude). Determine the interior angles & the exact measure of each side.



From this we can determine the **exact values** of

sin 30° = 				cos 30° = 				tan 30° =

sin 60° = 				cos 60° = 				tan 60° =

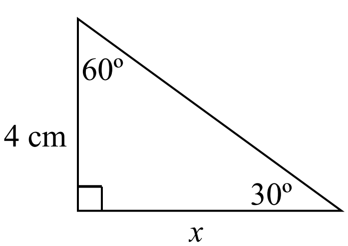
For all triangles, the ***smallest side*** is across from the ***smallest angle*** and the ***largest side*** is across from the ***largest angle***.

"Rationalizing the denominator" is a process that is used to change a rational expression so it does *not* have a radical in the denominator. It uses the ***identity property of 1*** (any number multiplied by 1 retains its value.)

NOTE: $\sin^2\theta = (\sin\theta)^2$ *in both cases we are squaring the ratio, not the angle* $(\sin^2\theta \neq \sin\theta^2)$

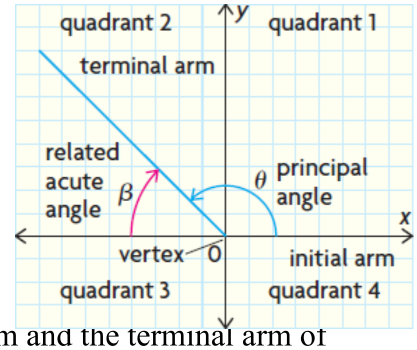
Ex 2) Determine the exact value of $\sin^2 60° \times \sin 45°$. Rationalize the denominator, if necessary.

Ex 4) Use special triangles to determine the exact length of side *x*.



Unit 3, Lesson 3: Trig Ratios for Angles Between 0° and 360°

Important Terminology



An angle has 3 parts: **Initial arm**, **vertex**, **terminal arm**

For an angle to be in **standard position**, it must meet the following criteria:

- Vertex must be at the origin
- Initial arm must be on the positive *x*-axis
- Angle is measured from initial arm to terminal arm

The **principal angle (θ)** is the counter clockwise angle between the initial arm and the terminal arm of an angle in standard position. Its value is between 0º and 360º.

The r**elated acute angle (β)** is the acute angle between the terminal arm of an angle in standard position and the *x*-axis when the terminal arm lies in quadrants 2, 3 or 4.

Exploration using GSP

A. P(4, 3) lies on the terminal arm of an angle in standard position. Determine the primary trigonometric ratios for BOTH θ and ß. Record these ratios in the first 2 lines of the chart below.

B. Reflect point P in the *y*-axis; it should now be in the ***second quadrant***. Record the values of the primary trig ratios for the principal angle θ and the related acute angle ß in the 3rd and 4th row of the table.

C. Reflect point P in the *x*-axis; it should now be in the ***third quadrant***. Record the values of the primary trig ratios for the principal angle θ and the related acute angle ß in the 5th and 6th row of the table.

D. Reflect point P in the *y*-axis; it should now be in the ***fourth quadrant***. Record the values of the primary trig ratios for the principal angle θ and the related acute angle ß in the 7th and 8th row of the table.

Angles	Quadrant	Sine Ratio	Cosine Ratio	Tangent Ratio
Principal angle θ =	1			
Related acute angle ß =				
Principal angle θ =	2			
Related acute angle ß =				
Principal angle θ =	3			
Related acute angle ß =				
Principal angle θ =	4			
Related acute angle ß =				

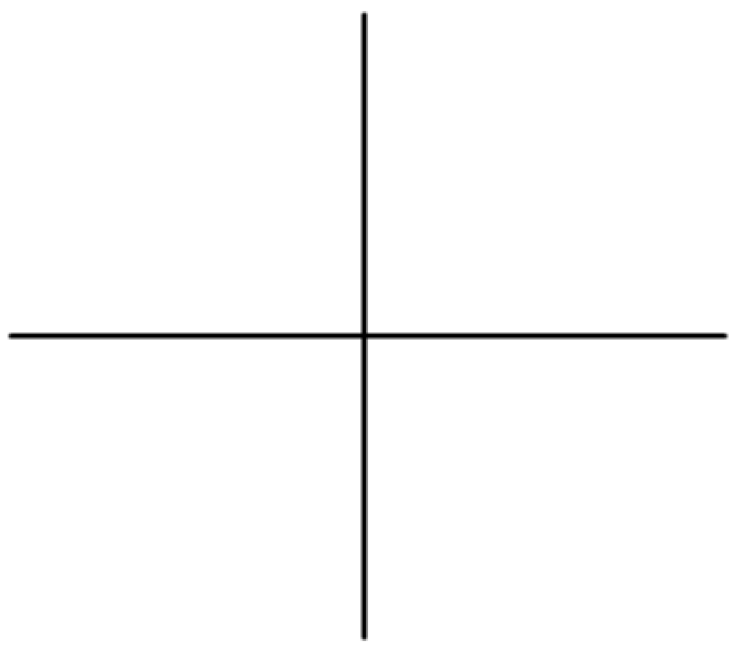
Summary

1. In which quadrants were each of the primary trig ratios for θ positive? (record on grid)

2. What do you notice about the sign of the primary trig ratios for ß for all quadrants?

3. What do you notice about the *absolute value* of the ratios for the principal angle compared to the ratios for the related acute angle?

4. For each quadrant, determine an expression for θ in terms of β. (record on grid)



Ex 1) Determine ß, if $\theta = 220^o$

Positive angles are formed by a **counter clockwise** rotation of the terminal arm.
Negative angles are formed by a **clockwise** rotation of the terminal arm.

Ex 2) Evaluate without the use of a calculator (special triangles!)

a) $\cos 135^{\circ}$

b) $\tan 210^{\circ}$

c) $\csc 300^{\circ}$

d) $\sec(-120^{\circ})$

Co-terminal angles have the same initial arm, the same terminal arm but have different angle measurements.

Ex 3) Determine 3 angles that are co-terminal to 45°

U3L3 Trig Ratios Handout — MCR3U

1. Use a sketch to determine in which **quadrant** the terminal arm of the principal angle lies, the value of the **related acute angle** and the **sign** of the ratio. Determine the **value of the ratio**, exactly (no calculators & rationalize the denominators!)

a) $\sin 225^\circ$

b) $\sec 135^\circ$

c) $\sin(-60^\circ)$

d) $\cot(300^\circ)$

e) $\cos 480^\circ$

f) $\csc(-330^\circ)$

2. Give another angle between 0° and 360° that makes the equation true.

a) $\sin 45^\circ = \sin \blacksquare$

b) $\tan 30^\circ = \tan \blacksquare$

c) $\cos 120^\circ = \cos \blacksquare$

d) $\tan 135^\circ = \tan \blacksquare$

3. Evaluate. Use exact values only (no calculators!) Reduce all answers to lowest terms, and rationalize all denominators.

a) $\tan 135° \times \sin 315°$

b) $2\tan 210° - 3\cos 330°$

c) $\cos 330° + \tan 315° - \sin 240°$

d) $\csc^2 135° - \sec 240°$

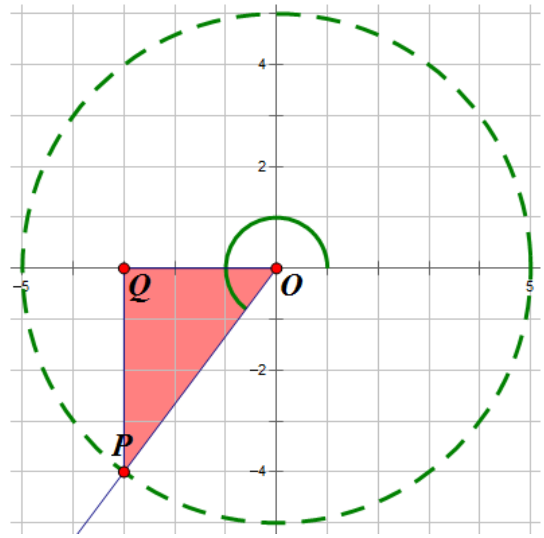
e) $\sin^2 390° + \cos^2 390°$

f) $\csc^2 120° - \cot^2 120°$

1) a) Q3, $\beta = 45°$, $-$, $-\frac{\sqrt{2}}{2}$ b) Q2, $\beta = 45°$, $-$, $-\sqrt{2}$ c) Q4, $\beta = 60°$, $-$, $-\frac{\sqrt{3}}{2}$ d) Q4, $\beta = 60°$, $-$, $-\frac{\sqrt{3}}{3}$ e) Q2, $\beta = 60°$, $-$, $-\frac{1}{2}$
f) Q1, $\beta = 30°$, $+$, 2) a) 135° b) 210° c) 240° d) 315° 3) a) $\frac{\sqrt{2}}{2}$ b) $\frac{-5\sqrt{3}}{6}$ c) $\sqrt{3}-1$ d) 4 e) 1 f) 1

Unit 3, Lesson 4: Solving Trig Equations

Recall: $\sin\theta = \dfrac{opp}{hyp}$ $\quad \cos\theta = \dfrac{adj}{hyp}$ $\quad \tan\theta = \dfrac{opp}{adj}$



For point P(x, y) which lies on the terminal arm of an angle in standard position:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

where $r^2 = x^2 + y^2$

Ex 1) Examine the graph on the right.

i) If the point P(-3, -4) lies on the terminal arm of an angle in standard position, determine the primary trig ratios.

ii) Determine the principal angle to the nearest degree.

Ex 2) The point Q(6, -3) lies on the terminal arm of an angle in standard position.

i) Sketch and label the principal angle.

ii) Determine, exactly, the 3 primary and 3 reciprocal trig ratios for θ.

iii) Determine the principal angle to the nearest degree.

Using a point on the terminal arm to define trig ratios allows us to determine trig ratios for angles that cannot be found inside a right triangle.

Ex 3) Use the point $P(0, 1)$ to determine the values of sine, cosine & tangent for $90°$

When solving for an unknown angle, you must consider **all values** that would make the equation true!

1. Consider the ratio (sine, cosine, tangent, cosecant, secant, cotangent) and its sign (+/ –) to determine the quadrants where your angles will terminate
2. Draw a sketch of the 2 angles
3. Solve for β, either with your calculator or using special triangles
4. Use β to determine θ_1 and θ_2

Ex 4) Solve for θ, if $0° \leq \theta \leq 360°$

a) $4\cos\theta = -\frac{3}{2}$

b) $\csc\theta = -2.1327$

c) $3\sqrt{2}\sin\theta = 3$

d) $\cot\theta + 2\sqrt{3} = \sqrt{3}$

1. Solve for θ, if $0º \leq \theta \leq 360º$. **(NO CALCULATORS!)**

a) $\cos\theta = -\frac{1}{2}$

b) $2\tan\theta = 2$

c) $\sqrt{2}\sin\theta - 1 = 0$

d) $2\sin\theta = -\sqrt{3}$

e) $2\cos\theta = \sqrt{3}$

f) $\cos\theta - 1 = -\cos\theta$

g) $3\sin\theta = \sin\theta + 1$

h) $5\cos\theta + \sqrt{3} = 3\cos\theta$

i) $4\sin\theta + 1 = 2\sin\theta$

j) $3\sec\theta = -6$

EXTRA CHALLENGE

k) $\csc^2\theta + \csc\theta = 2$

l) $\sec\theta\tan\theta - 2\tan\theta + \sec\theta - 2 = 0$

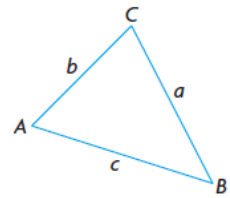
1) a) 120°, 240° b) 45°, 225° c) 45°, 135° d) 240°, 300° e) 30°, 330° f) 60°, 300° g) 30°, 150° h) 150°, 210° i) 210°, 330° j) 120°, 240° k) 90°, 210°, 330° l) 60°, 135°, 300°, 315°

Pg. 299 2) b) $r = \sqrt{73}$ $\sin\theta = \frac{3\sqrt{73}}{73}$ $\cos\theta = -\frac{8\sqrt{73}}{73}$ $\tan\theta = -\frac{3}{8}$ c) $r = \sqrt{89}$ $\sin\theta = -\frac{8\sqrt{89}}{89}$ $\cos\theta = -\frac{5\sqrt{89}}{89}$ $\tan\theta = \frac{8}{5}$

Unit 3, Lesson 5: The Ambiguous Case of the Sine Law

To solve a triangle means to determine the length of all sides and the measure of all angles.

The primary trig ratios are useful for solving right triangles, but NOT oblique triangles (triangles that are either acute or obtuse)



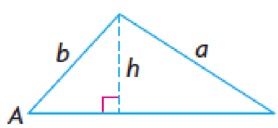
<u>Sine Law</u> $\quad \dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\quad$ or $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

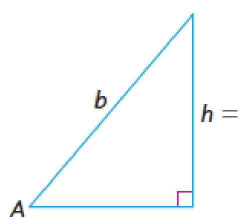
Sine law can be used if you know two angles and any side (AAS or ASA), or two sides and one angle opposite a given side (SSA – a.k.a "The Donkey Case")

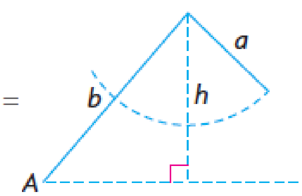
SSA: Given angle A is acute. There are 4 different possible outcomes as depicted in the chart. In one case, two triangles are actually possible based on the information provided. In this case the information is "ambiguous" and thus, you must solve for **both possibilities**.

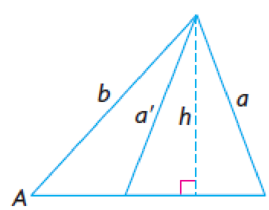
- Determine the vertical height of the triangle (using primary trig ratios)
    - ***Always draw the height so it doesn't cut into any given angles or sides.***
- Compare the height to the given sides. **Given ∠A & sides *a* & *b*:**

If ∠A is ACUTE			
a ≥ b	**a < b**		
	Check the height		
	h = a	**h > a**	**h < a**
1 triangle	1 triangle (right angle)	No triangles	2 triangles









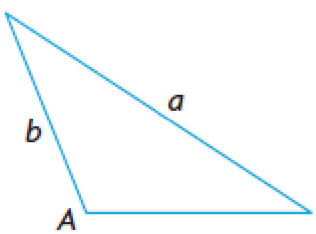
** ***CAREFUL! These outcomes are based on given angle A and given sides a & b. You will need to adapt if different letters are used!*****

Ex 1) In ΔABC, ∠A = 32°, *a* = 6 cm, *c* = 5 cm. Draw and label a diagram then determine the number of solutions. Do not solve.

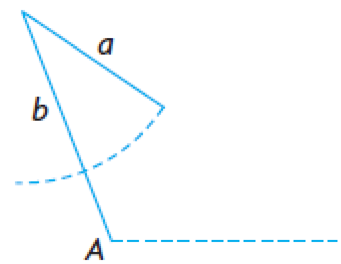
Ex 2) In ΔABC, ∠C is 16°, *c* = 3 m, *a* = 15 m. Draw and label a diagram then determine the number of solutions. Do not solve.

Ex 3) Solve ΔABC given $\angle B = 39°$, $a = 8$ cm, $b = 6$ cm.

SSA: Given angle A is obtuse. There are 2 different possible outcomes as depicted in the chart. You do NOT need to determine the height. **Given $\angle A$ & sides *a* & *b*:**



If $a > b$	One triangle exists
If $a \leq b$	No triangle exists



Ex 4) In ΔDEF, $\angle E = 120°$, $e = 6$ cm and $f = 7$ cm. Draw and label a diagram then determine the number of solutions. Do not solve.

Ex 5) Solve ΔRST given $\angle S = 130°$, $s = 11$ m and $t = 4$ m

For each question, draw a labelled **sketch** and determine the **number of distinct triangles** (**justify your answer!)** Then, **solve the triangles**, if possible, rounding angles to the nearest degree and lengths to the nearest tenth of a unit.

1. In ΔABC, a = 16 cm, c = 20 cm, and $\angle C = 30º$.

2. In ΔDEF, d = 15 cm, f = 17 cm, and $\angle D = 95º$.

3. In ΔABC, a = 10 m, b = 14 m, and $\angle A = 40º$.

4. In ΔDEF, f = 7 cm, e = 14 cm, and $\angle$F = 30º.

5. In ΔABC, b = 11 mm, c = 16 mm, and $\angle$B = 50º.

6. In ΔDEF, d = 12 cm, e = 18 cm, and $\angle$E = 115º.

1) 1 triangle; $c \geq a$ $\angle A = 24°$ $\angle B = 126°$ $b = 32.4$ cm 2) 0 triangles; $d \leq f$ 3) 2 triangles; $h = 9.0$ & $h < a < b$ $\angle B_1 = 64°$ $\angle C_1 = 76°$ $c_1 = 15.1$ m or $\angle B_2 = 116°$ $\angle C_2 = 24°$ $c_2 = 6.4$ m 4) 1 triangle; $h = 7.0$ cm & $f = h$ $\angle E = 90°$ $\angle D = 60°$ $d = 12.1$ cm 5) 0 triangles; $h = 12.3$ cm & $b < h$ 6) 1 triangle; $e > d$ $\angle D = 37°$ $\angle F = 28°$ $f = 9.3$ cm

Unit 3, Lesson 6: Solving Two & Three Dimensional Trig Problems

Two and three dimensional problems involving triangles can be solved using a combination of tools:

- Right Triangles
  - Primary trig ratios SOH CAH TOA
  - Pythagorean theorem $c^2 = a^2 + b^2$
- Oblique Triangles
  - Sine Law (ASA, AAS, SSA) $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
  - Cosine Law (SSS, SAS) $c^2 = a^2 + b^2 - 2ab\cos C$

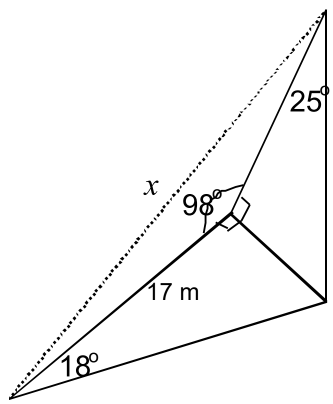
You will need to have these formulas memorized – they will NOT be provided on a test/exam.

Some useful tips:

- Always create a diagram if one is not provided in the problem.
  - Label with all given information.
  - Where possible, draw diagrams roughly to scale.
  - Make it large so you can fit all the information!
- Create a plan to solve for the angle or side indicated.
  - Start where you have the most information (at least 3 pieces of info)
  - Think: is this the most efficient way to solve the problem?
- Execute the plan
  - Make sure your calculator is in degree mode
  - Refer back to you diagram to help stay on track
  - Check to see if your answers make sense
- Don't forget about those presentation marks! ($\therefore$, units, $\doteq$, etc)

Ex 1) Jim uses a clinometer to measure the height of a building. He determines the angle of elevation to the top of the building is 63°. Then he steps back 10m and repeats the measurement and finds that the angle of elevation is now 57°. Determine the height of the building.

Ex 2) Determine the length of the dashed line, x, to the nearest tenth of a meter.



Ex 3) Two roads intersect at an angle of 60º. Two bicycles leave the intersection, one on each road. One bike travels at 20km/h and the other bike at 30km/h. After 6 min, a police helicopter 1200m directly above and between (not necessarily halfway between) the two bikes, notes the angle of depression to the slower bike is 30º. What is the horizontal distance from the helicopter to the faster bike to one decimal place?

Ex 4) Keith is looking out the window of his condo that is 166m above ground. He sees a robbery take to his left at an angle of depression of 40º and he sees a police officer to his right at an angle of depression of 34º. If the angle between the two people seen is 116º, find how far apart the police officer is from the robbery.

Complete all work on a separate sheet of paper. If a diagram is not provided, draw one.

1. Given Figure A, determine the value of x to the nearest cm.

2. Given Figure B, determine the value of x to the nearest tenth of a centimetre.

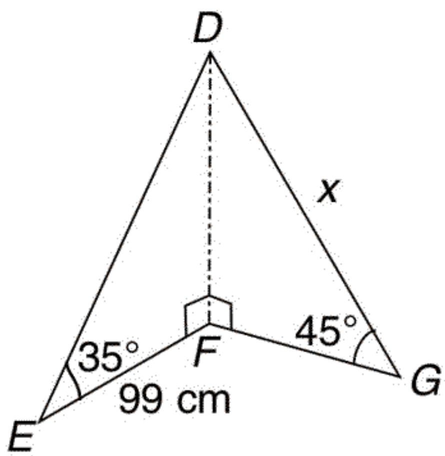


Figure A

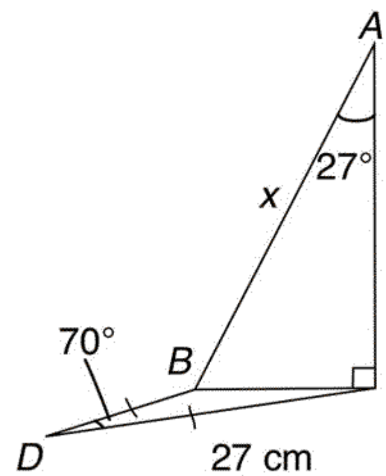


Figure B

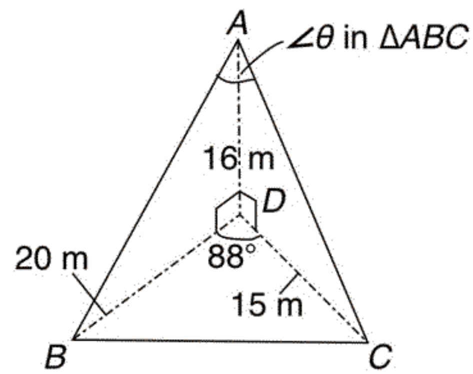
3. Joel is standing on the 10 m diving platform. His coach is on the ground to the left of him, while his parents are on the ground to the right of him. His coach is looking at him at an angle of 37° with the ground, while his parents are looking at him at an angle of 32° with the ground. If the angle at the base of the platform between Joel's coach and Joel's parents is 100°, how far apart are his coach and his parents to the nearest tenth of a metre?

4. Keith is looking at a cliff. He determines that the angle of elevation to the top is 70° from where he is standing. 70 m away from Keith, Alan estimates the angle between the base of the cliff, himself, and Keith to be 29° while Keith estimates the angle between the base of the cliff, himself, and Alan to be 48°. What is the height, h, of the cliff to the nearest tenth of a metre?

5. Kim stands on a balcony. Brian is on the left of the balcony looking up at her at an angle of 62° with the ground. Dan is on the right of the balcony looking up at her at an angle of 44° with the ground. If the height, *h*, is 5 m, how far apart are Brian and Dan standing to the nearest tenth of a metre? Assume the angle the base of the balcony makes between Brian and Dan is 90°.

6. Gary wants to know the height of a sign across a road. He stands directly across from the sign and notices the angle of elevation to the top of the sign is 19°. Gary then walks 20 m parallel to the road and observes the angle between the base of the sign and Gary's previous spot is 52°. What is the height, *h*, of the sign to the nearest tenth of a metre?

7. Coleen is standing at the top of a water slide. One base is 15 m, one base is 20 m, and the height is 16 m (see the figure below). The angle from the base to each of the posts (B and is 88°. What is the angle, θ, in which Coleen must slide to the nearest degree?



C)

8. Sarah and Megan are going to swim around a very small island. They part with an angle θ between them. Sarah swims 1800 m/h while Megan swims 2100 m/h. After 20 minutes, they reach opposite sides of the island. Sarah notices an eagle in a tree on the island. With her keen observation skills, she notices that the angle of elevation to the eagle is 38° and that the tree is 60 m tall. Megan realizes the eagle is at an angle of elevation of 32° for her. What angle did they part at when they started their swim?

9. Two roads intersect at 34°. Two cars leave the intersection on different roads at speeds of 80 km/h and 100 km/h. After 2 hours, a traffic helicopter that is above and between the two cars takes readings on them. The angle of depression to the slower car is 20°, and the distance from the helicopter to that car is 100 km. Calculate the distance from the helicopter to the faster car.

1) 98 cm   2) 68.2 cm   3) 22.5 m   4) 95.7 m   5) 5.8 m   6) 8.8 m   7) 62°   8) 12°   9) 39km

Skill Builder: Adding and Subtracting Rational Expressions – Day 1

Recall: The procedure for **adding or subtracting numeric fractions**

- Determine the **lowest common denominator**
- Create **equivalent fractions** all with the **same denominator**
- **Add or subtract** the **numerators** as needed, keeping the **denominator the same**
- **Simplify** your final answer, if possible.

Example:

$$\frac{7}{12}+\frac{5}{8}=\frac{7(2)}{12(2)}+\frac{5(3)}{8(3)}$$
$$=\frac{14}{24}+\frac{15}{24}$$
$$=\frac{29}{24}$$

However, for rational expressions, we must use the **prime factor method** to determine the LCD.

- Determine the **prime factorization** of each denominator (use powers if necessary)
- The LCD will be the **product of all prime factors**, with each factor given the **highest power** of its occurrence in any denominator
- To check if your LCD is correct, it should "contain" all the prime factors needed for each original denominator, but no extras!

Example:

$12=2^2\times 3^1$

$8=2^3$

$LCD=2^3\times 3^1$

$LCD=24$

Ex 1) Determine the LCD for each set of denominators

a) $\frac{\quad}{ab^2}$ & $\frac{\quad}{a^2b}$

b) $\frac{\quad}{2x-3}$ & $\frac{\quad}{2x+3}$

c) $\frac{\quad}{15(3x-2)}$ & $\frac{\quad}{10(3x-2)^2}$

d) $\frac{\quad}{20x}$ & $\frac{\quad}{35y^2}$ & $\frac{\quad}{14xy}$

To add or subtract rational expressions:

- **Factor** all numerators and denominators.
- **Check** to see if each rational expression is simplified; if not, cancel common factors **within** the rational expression.
- **Determine the LCD** and create equivalent rational expressions
  - each rational expression needs to be multiplied by the factor(s) it is "missing"
- **Add or subtract** the numerators as indicated (expand & gather like terms)
  - when **subtracting**, use **brackets** to ensure your signs are correct!
- **Simplify** the final rational expression, if possible.

Recall: For restrictions, determine the **zeros of ALL ORIGINAL denominators**.

Ex 2) Simplify. State any restrictions on the variables.

a) $\dfrac{7}{12}+\dfrac{1}{8y}$

b) $\dfrac{2}{x}+\dfrac{3}{x^2}+\dfrac{1}{2x}$

c) $\dfrac{3}{xy}+\dfrac{2}{x^2}+\dfrac{y}{x^2y^2}$

d) $\dfrac{5x-7y}{12x}+\dfrac{2x-9y}{8y}$

e) $\dfrac{6}{y+1}-\dfrac{3}{y-1}$

Skill Builder: Adding and Subtracting Rational Expressions – Day 2

Ex 3) Determine the LCD for each set of denominators

a) $\dfrac{}{9x^2-12x+4}$ & $\dfrac{}{15x^2-25x+10}$

b) $\dfrac{}{6x^2+x-2}$ & $\dfrac{}{10x^2+9x-7}$ & $\dfrac{}{9x^2+12x+4}$

Ex 4) Simplify. State any restrictions on the variables.

a) $\dfrac{3}{(x-1)(x-2)}+\dfrac{x-2}{(x+2)(x-1)}$

b) $\dfrac{x-3}{x^2+x-12}-\dfrac{x-2}{x^2+3x-4}$

The **order of operations** still applies for rational expressions: Multiplication and division are done BEFORE addition and subtraction.

Ex 5) Simplify. State any restrictions on the variables.

$$\frac{3a+2}{2a^2+11a+5}-\frac{a-2}{6a^2-7a-5}\div\frac{2a}{3a^2-5a}$$

Try on your own:

$$\frac{3x^2-7}{3x^3+6x^2-7x-14}-\frac{16x^2-56x+49}{9x^2-25}\div\frac{4x^2+x-14}{6x^2+10x}$$

Unit 3, Lesson 7: Proving Simple Trigonometric Identities

A trigonometric identity is a trigonometric equation that is true for **ALL** values of the variable.

To prove that an equation is an identity, show that both sides of the equation represent the same expression.

- Manipulate the more complex side of the equation to arrive at the expression on the other side or manipulate both sides to get the same expression.
- When simplifying, remember to use all of your algebraic skills including factoring or finding a common denominator.

Ex 1) Factor $\sin^2\theta - 1$

Ex 2) Expand $(\cos\theta + 2)(\cos\theta - 1)$

- Rewrite all expressions involving tangent and reciprocal trig ratios in terms of sine and cosine.

  Recall: $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Consider the expression $\dfrac{\sin\theta}{\cos\theta}$. Which trigonometric ratio is equivalent to the simplified expression?

This is called the **Quotient Identity**. It has 2 forms:

Consider the expression $\sin^2\theta + \cos^2\theta$. What does it simplify to?

This is called the **Pythagorean Identity**. It has 3 forms:

- Use the Quotient & Pythagorean Identities (or manipulated versions of them) to help you.

Quotient Identity	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, where $\cos\theta \neq 0$
Pythagorean Identity	$\sin^2\theta + \cos^2\theta = 1$

Ex 3) Prove the following identities, using a formal LS = RS proof

a) $\dfrac{\sin\theta}{\tan\theta} = \cos\theta$

b) $\dfrac{1-\cos^2\theta}{\sin\theta} = \sin\theta$

c) $\tan\theta + \cot\theta = \sec\theta\csc\theta$

d) $\sin^2\theta\sec^2\theta = \sec^2\theta - 1$

Prove each identity using a formal LS = RS proof.

1. $\sin\theta\cot\theta = \cos\theta$

2. $\cot\theta\sec\theta = \csc\theta$

3. $\dfrac{\sec x}{\tan x} = \csc x$

4. $\dfrac{\sin\theta}{1+\sin\theta} - \dfrac{\sin\theta}{1-\sin\theta} = -2\tan^2\theta$

5. $\dfrac{\cos x}{1-\sin x} - \sec x = \tan x$

6. $\dfrac{\sin\theta + \tan\theta}{1+\cos\theta} = \tan\theta$

7. $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

8. $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

9. $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{\csc\theta + 1}{\csc\theta - 1}$

10. $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$

11. $\cot x = \dfrac{1+\cos x}{\sin x + \tan x}$

12. $\sin^2 x + \tan^2 x = \sec^2 x - \cos^2 x$

13. $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x\sec x$

14. $(\sin\theta - \tan\theta)(\cos\theta - \cot\theta) = (\sin\theta - 1)(\cos\theta - 1)$