Skill Builder: Adding & Subtracting Rational Expressions

Recall: The procedure for adding or subtracting numeric fractions

- Determine the lowest common denominator
- Create equivalent fractions all with the same denominator
- Add or subtract the numerators as needed, keeping the denominator the same
- Simplify your final answer, if possible.

Example:

$$\frac{7}{12} + \frac{5}{8} = \frac{7(2)}{12(2)} + \frac{5(3)}{8(3)}$$

$$= \frac{14}{24} + \frac{15}{24}$$
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However, for rational expressions, we must use the **prime factor method** to determine the LCD.

- Determine the **prime factorization** of each denominator (use powers if necessary)
- The LCD will be the **product of all prime factors**, with each factor given the **highest power** of its occurrence in any denominator
- To check if your LCD is correct, it should "contain" all the prime factors needed for each original denominator, but no extras!

Example:

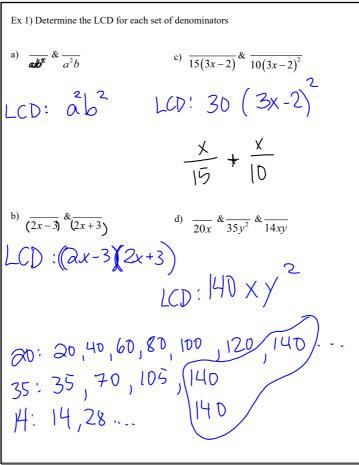
$$12 = 2^{2} \times 3^{1}$$

$$8 = 2^{3}$$

$$LCD = 2^{3} \times 3^{1}$$

$$LCD = 24$$

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To add or subtract rational expressions:

Factor all numerators and denominators.

Check to see if each rational expression is simplified; if not, cancel common factors within the rational expression.

Determine the LCD and create equivalent rational expressions

each rational expression needs to be multiplied by the factor(s) it is "missing"

Add or subtract the numerators as indicated (expand & gather like terms)

owhen subtracting, use brackets to ensure your signs are correct!

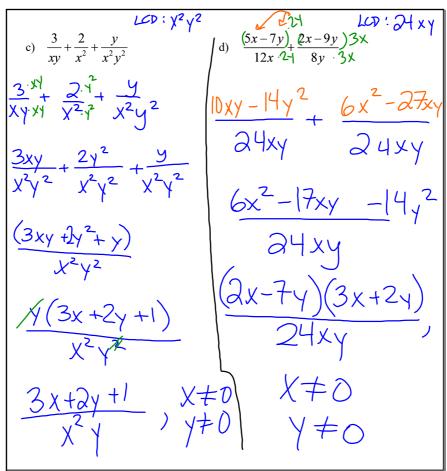
Simplify the final rational expression, if possible.

Recall: For restrictions, determine the zeros of ALL ORIGINAL denominators

Ex 2) Simplify. State any restrictions on the variables.

LCD: $\frac{34}{3}$ $\frac{7}{12} + \frac{1}{8y}$ $\frac{3}{12 \cdot 24} + \frac{3}{8y \cdot 3}$ LCD: $\frac{34}{3}$ $\frac{3}{12 \cdot 24} + \frac{3}{8y \cdot 3}$ $\frac{4x}{3} + \frac{1}{2x} + \frac{1}{2x}$ $\frac{3}{2x^2} + \frac{1}{2x} + \frac{x}{3} + \frac{1}{3} + \frac{x}{3} + \frac{x}{$

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e)
$$\frac{6}{y+1} - \frac{3}{y-1}$$

$$\frac{6(y-1)}{(y+1)(y-1)} - \frac{3(y+1)}{(y-1)(y+1)}$$

$$\frac{6y-6}{(y+1)(y-1)} - \frac{3(y+1)}{(y-1)(y+1)}$$

$$\frac{6y-6}{(y+1)(y-1)} - \frac{3(y+1)}{(y-1)(y+1)}$$

$$\frac{6y-6-3y-3}{(y+1)(y-1)}$$

$$\frac{3y-9}{(y+1)(y-1)}$$

$$\frac{3(y-3)}{(y+1)(y-1)}$$

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Ex 4) Simplify. State any restrictions on the variables.

a)
$$\frac{3}{(x-1)(x-2)} + \frac{x-2}{(x+2)(x-1)}$$

$$= \frac{3}{(x-1)(x-2)} (x+2) + \frac{(x-2)(x-2)}{(x+2)(x-2)} (x+2)$$

$$= \frac{3}{(x-1)(x-2)} (x+2) + \frac{(x-2)(x-2)}{(x+2)(x-2)} (x+2)$$

$$= \frac{3x+6}{(x-1)(x-2)(x+2)} + \frac{x^2-4x+4}{(x-1)(x-2)(x+2)}$$

$$= \frac{x^2-x+10}{(x-1)(x-2)(x+2)}$$

$$= \frac{x^2-x+10}{(x-1)(x-2)(x+2)}$$

b)
$$\frac{x-3}{x^2+x-12} - \frac{x-2}{x^2+3x-4}$$
 LCD: $(x+4)(x-3)(x-1)$
 $(x+4)(x-3)$ $(x+4)(x-1)$ $(x+4)(x-3)(x-1)$
 $=\frac{(x-3)(x-1)}{(x+4)(x-3)(x-1)} - \frac{(y-2)(x-3)}{(x+4)(x-1)(x-3)}$
 $=\frac{x^2-4x+3}{(x-4)(x-3)(x-1)} - \frac{x^2-4x+3}{(x-4)(x-3)(x-1)}$
 $=\frac{x^2-4x+3}{(x-4)(x-3)(x-1)} - \frac{x^2-4x+3}{(x-4)(x-3)(x-1)}$
 $=\frac{x^2-4x+3}{(x-4)(x-3)(x-1)}$
 $=\frac{x^2-4x+3}{(x-4)(x-3)(x-1)}$
 $=\frac{x^2-4x+3}{(x-4)(x-3)(x-1)}$

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The order of operations still applies for rational expressions: Multiplication and division are done BEFORE addition and subtraction
$$Q_{0}$$
. $Q(A+1)(A+5)$

Ex 5) Simplify. State any restrictions on the variables.

$$\begin{pmatrix}
3a+2 \\
2a^{2}+11a+3
\end{pmatrix}
\begin{pmatrix}
a-2 \\
6a^{2}-7a-5
\end{pmatrix}
\begin{pmatrix}
2a \\
3a^{2}-5a
\end{pmatrix}$$

$$= \frac{3a+2}{6a+1}(a+5) - \frac{(a-2)}{(a+1)(3a+5)} - \frac{(a-2)(a+5)}{(a+1)(a+5)}$$

$$= \frac{(3a+2)\cdot Q}{(2a+1)(a+5)} - \frac{(a-2)(a+5)}{(a+3a-10)}$$

$$= \frac{(3a+2)\cdot Q}{(2a+1)(a+5)} - \frac{(a-2)(a+5)}{(a+3a-10)}$$

$$= \frac{(3a+2)\cdot Q}{(2a+1)(a+5)} - \frac{(a-2)(a+5)}{(a+3a-10)}$$

$$= \frac{(3a+2)\cdot Q}{(2a+1)(a+5)} - \frac{(a-2)(a+5)}{(a+5)}$$

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$$= \frac{(3a+2)\cdot Q}{(2a+1)(a+5)} - \frac{(a-2)\cdot Q}{(a+5)}$$

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$$= \frac{(3a+2)\cdot Q}{(2a+1)(a+5)} - \frac{(a-2)\cdot$$

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Try on your own:
$$\frac{3x^{2}-7}{(3x^{3}+6x^{2}-7x-14)} - \frac{16x^{2}-56x+49}{9x^{2}-25} + \frac{4x^{2}+x-14}{6x^{2}+10x}$$

$$= \frac{3x^{2}-7}{3x^{2}(x+2)-7(x+2)} - \left(\frac{(4x-7)(4x-7)}{(3x-5)(3x+5)} \times \frac{2x(3x-5)}{(4x-7)(x+2)}\right)$$

$$= \frac{(3x^{2}-7)}{(3x^{2}-7)(x+2)} - \left(\frac{2x(4x-7)}{(3x-5)(x+2)} \times \frac{2x(3x-5)}{(4x-7)(x+2)}\right)$$

$$= \frac{1(3x-5)}{(x+2)(3x-5)} - \frac{8x^{2}+|4x|}{(3x-5)(x+2)}$$

$$= \frac{3x-5-8x^{2}+|4x|}{(x+2)(3x-5)}$$

$$= -\frac{8x^{2}+|7x-5|}{(x+2)(3x-5)} \times \frac{7}{3}$$

$$= \frac{7}{3}$$

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