Lesson 4: Solving Trig Equations

Recall: $\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

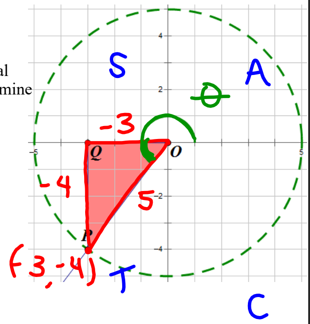
For point P(x, y) which lies on the terminal arm of an angle in standard position:

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

where $r^2 = x^2 + y^2$

Ex 1) Examine the graph on the right.

i) If the point P(-3, -4) lies on the terminal arm of an angle in standard position, determine the primary trig ratios.



$\sin\theta = -\frac{4}{5}$

$\cos\theta = -\frac{3}{5}$

$\tan\theta = \frac{4}{3}$

ii) Determine the principal angle to the nearest degree.

$\sin\theta = -\frac{4}{5}$

$\theta = \sin^{-1}\left(-\frac{4}{5}\right)$

$\theta = -53°$ ← This can't be right because the principal has to be between $180° \rightarrow 270°$ (Q3) ∴ It must be β!

$\beta = 53°$

$\theta = 180° + 53°$

$\boxed{\theta = 223°}$

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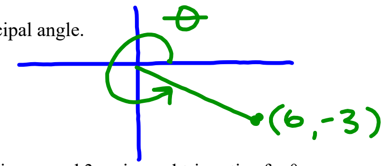
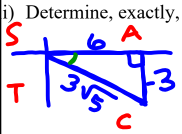
Ex 2) The point Q(6, -3) lies on the terminal arm of an angle in standard position.

i) Sketch and label the principal angle.



ii) Determine, exactly, the 3 primary and 3 reciprocal trig ratios for θ.



$6^2 + 3^2 = r^2$

$36 + 9 = r^2$

$\sqrt{45} = r$

$\sqrt{9}\sqrt{5} = r$

$\boxed{3\sqrt{5} = r}$

$\csc\theta = \frac{H}{O} = \frac{\cancel{3}\sqrt{5}}{-\cancel{3}}$

$\sec\theta = \frac{H}{A} = \frac{\cancel{3}\sqrt{5}}{\cancel{6}\,2}$

$\cot\theta = \frac{A}{O} = \frac{\cancel{6}}{-\cancel{3}} = -2$

$\sin\theta = -\frac{\cancel{3}}{\cancel{3}\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\cos\theta = \frac{\cancel{6}\,2}{\cancel{3}\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\tan\theta = \frac{-3}{6}$

$\boxed{\sin\theta = -\frac{\sqrt{5}}{5} \quad \cos\theta = \frac{2\sqrt{5}}{5} \quad \tan\theta = -\frac{1}{2}}$

$\boxed{\csc\theta = -\sqrt{5} \quad \sec\theta = \frac{\sqrt{5}}{2} \quad \cot\theta = -2}$

iii) Determine the principal angle to the nearest degree.

$\tan\theta = -\frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta = 27°$ ← This must be β!!!

$\therefore \theta = 360° - 27°$

$\boxed{\theta = 333°}$

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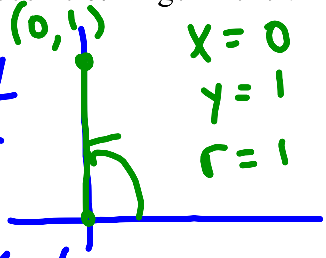
Using a point on the terminal arm to define trig ratios allows us to determine trig ratios for angles that cannot be found inside a right triangle.

Ex 3) Use the point P(0, 1) to determine the values of sine, cosine & tangent for 90°

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$



$x = 0$
$y = 1$
$r = 1$

$\sin\theta = 1$ $\cos\theta = 0$ $\tan\theta =$ undefined!

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When solving for an unknown angle, you must consider **all values** that would make the equation true!

1. Consider the ratio (sine, cosine, tangent, cosecant, secant, cotangent) and its sign (+/ –) to determine the quadrants where your angles will terminate.
2. Draw a sketch of the 2 angles
3. Solve for β, either with your calculator or using special triangles
4. Use β to determine θ_1 and θ_2

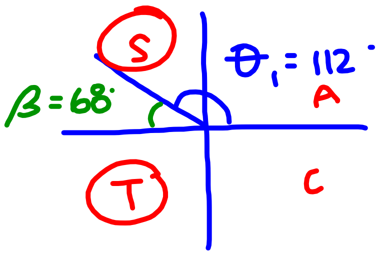
Ex 4) Solve for θ, if $0° \le \theta \le 360°$

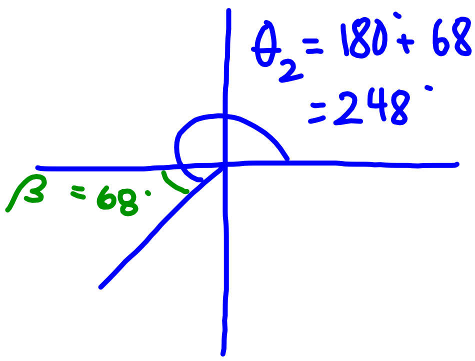
a) $4\cos\theta = -\frac{3}{2}$

$\cos\theta = -\frac{3}{2} \times \frac{1}{4}$

$\cos\theta = -\frac{3}{8}$

$$\boxed{\begin{aligned}\theta_1 &= 112° \\ \theta_2 &= 248°\end{aligned}}$$





$\theta_2 = 180° + 68°$
$= 248°$

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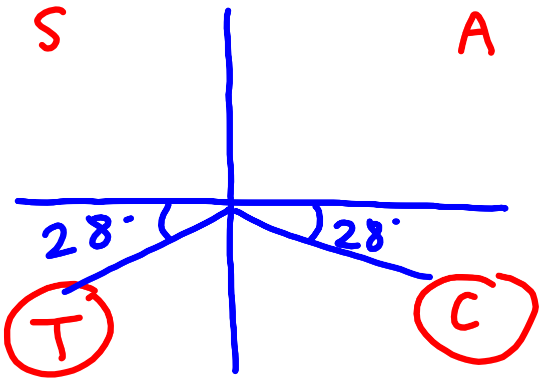
b) $\csc\theta = -2.1327$

$\sin\theta = \frac{-1}{2.1327}$

$\beta\ \cancel{\theta} = -28°$

$$\boxed{\begin{aligned}\theta_1 &= 332° \\ \theta_2 &= 208°\end{aligned}}$$



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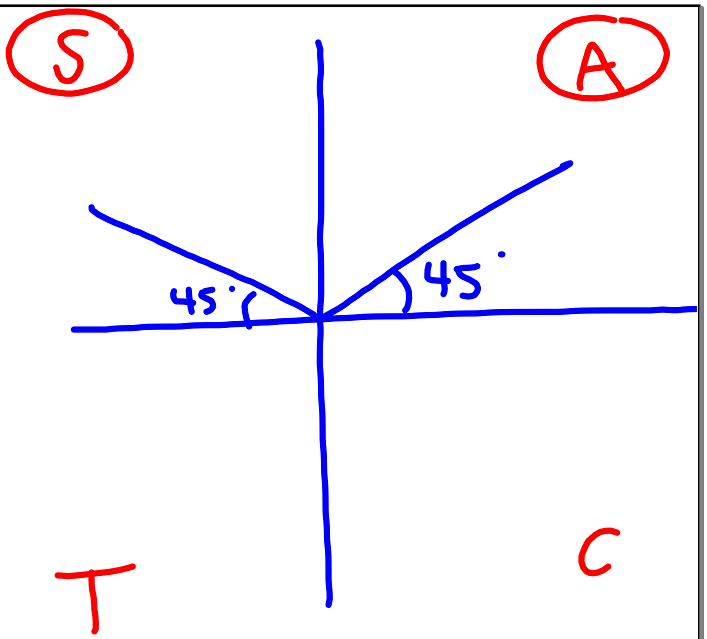
c) $3\sqrt{2}\sin\theta = 3$

$\sin\theta = \frac{\cancel{3}}{\cancel{3}\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$$\boxed{\begin{aligned}\theta_1 &= 45° \\ \theta_2 &= 135°\end{aligned}}$$



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d) $\cot\theta + 2\sqrt{3} = \sqrt{3}$

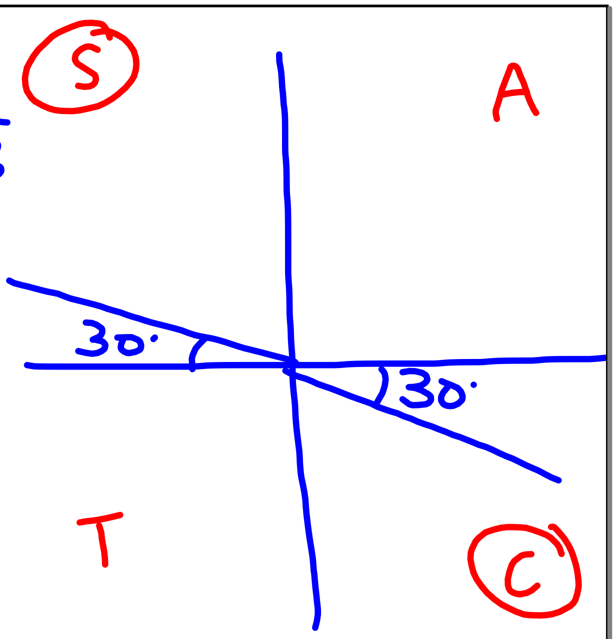
$\cot\theta = 1\sqrt{3} - 2\sqrt{3}$

$\cot\theta = -\sqrt{3}$

$\tan\theta = \frac{-1}{\sqrt{3}}$

$\beta \quad \cancel{\theta} = 30^\circ$

$$\boxed{\begin{aligned} \theta_1 &= 150^\circ \\ \theta_2 &= 330^\circ \end{aligned}}$$



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HW U3L4:

1. p.299 #2bcd (exact values for r), 3, 6ace, 8ace, 9ace,

2. Handout

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