4.4 - Simplifying Algebraic Expressions Involving Exponents

The ratio of the surface area to the volume of Microorganisms affects their ability to survive. A higher ratio means the cell is more buoyant. And the closer they are to the surface of the liquid, the more availability of food.

Assume cells a spherical

$$\text{ratio} = \frac{SA}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3}$$

Let's calculate the ratio for different sized cells.

$\frac{16\pi}{\frac{32}{3}\pi} = \frac{16}{\left(\frac{32}{3}\right)} = 16 \times \frac{3}{32} = \frac{3}{2} = 1.5$

Radius	1	1.5	2	2.5	3
SA/V Ratio	$\frac{4\pi}{\frac{4}{3}\pi} = 3$	$\frac{4\pi(1.5)^2}{\frac{4}{3}\pi(1.5)^3} = 2$	$\frac{4\pi(2)^2}{\frac{4}{3}\pi(2)^3} = 1.5$	$\frac{3}{2.5}$	$\frac{3}{3} = 1$

Is there an easier way? YES! how? simplify

$$\text{ratio} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{\pi(4r^2)}{\pi\left(\frac{4}{3}r^3\right)} = \frac{r^2(4)}{r^2\left(\frac{4}{3}r\right)} = 4 \div \frac{4r}{3}$$

$$\frac{4}{1} \times \frac{3}{4r} = \frac{3}{r}$$

Using the simplified ratio, how can we explain why the ratios keep decreasing as the radius increases?

$\text{ratio} = \frac{3}{r}$ As denominator increases the fraction (or ratio) gets smaller

ex: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$

Using exponent rules to simplify

$$\frac{(2x^{-3}y^{2})^{3}}{(x^{3}y^{-4})^{2}} = \frac{2^{3}(x^{-3})^{3}(y^{2})^{3}}{(x^{3})^{2}(y^{-4})^{2}} = \frac{8x^{-9}y^{6}}{x^{6}y^{-8}} = 8x^{-15}y^{14} = \frac{8y^{14}}{x^{15}}$$

$$\frac{(x^{2n+1})(x^{3n-1})}{x^{2n-5}} = \frac{x^{(2n+1)+(3n-1)}}{x^{2n-5}} = \frac{x^{5n}}{x^{2n-5}} = x^{5n-(2n-5)} = x^{3n+5}$$

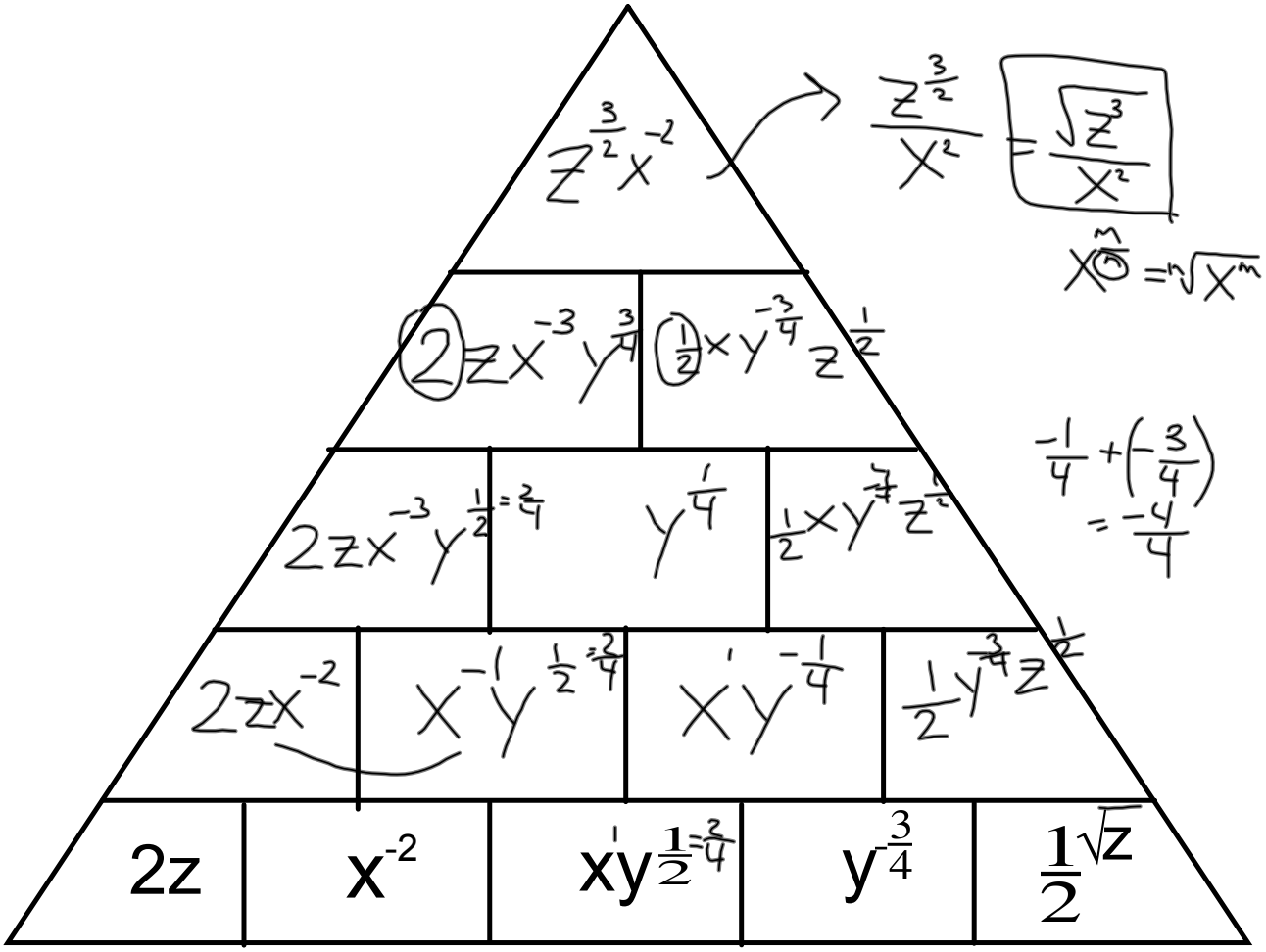
$-1-(-4) = -1+4$

$$\frac{(27a^{-3}b^{12})^{\frac{1}{3}}}{(16a^{-8}b^{12})^{\frac{1}{2}}} = \frac{27^{1/3}(a^{-3})^{1/3}(b^{12})^{1/3}}{16^{1/2}(a^{-8})^{1/2}(b^{12})^{1/2}} = \frac{3a^{-1}b^{4}}{4a^{-4}b^{6}} = \frac{3}{4}a^{3}b^{-2} = \frac{3a^{3}}{4b^{2}}$$

$16^{1/2} = \sqrt[2]{16}$

$$\left(\frac{\sqrt[5]{x^{8}}}{\sqrt{x^{3}}}\right)^{3} = \left(\frac{x^{\frac{8}{5}}}{x^{\frac{3}{2}}}\right)^{3} = \left(x^{\frac{8}{5}-\frac{3}{2}}\right)^{3} = \left(x^{\frac{16}{10}-\frac{15}{10}}\right)^{3} = \left(x^{\frac{1}{10}}\right)^{3} = x^{\frac{1}{10}\cdot 3} = x^{\frac{3}{10}} = \sqrt[10]{x^{3}}$$

$x^{\frac{m}{n}} = \sqrt[n]{x^{m}}$



HMWK: pg.236 #4,5,6,8,11