

Sequences

May 27

The Sierpinski gasket is a fractal created from an equilateral triangle. At each stage, the "middle" is cut out of each remaining equilateral triangle. The first three stages are shown.



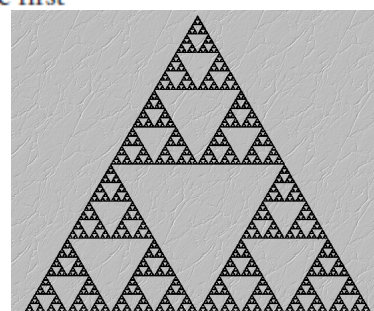
stage 1



stage 2



stage 3



- a) If the process continues indefinitely, the stages get closer to the Sierpinski gasket. How many shaded triangles would be present in the sixth stage?
- b) If the triangle in the first stage has an area of 80 cm^2 , what is the area of the shaded portion of the 20th stage?

$$\# \Delta = 3^{n-1}$$

STAGE n	1	2	3	4	5	6	20
# SHADED Δ	1	3	9	27	81	243	1162261467
	3^0	3^1	3^2	3^3	3^4	3^5	3^{19}
STAGE n	1	2	3	4	5	6	20
# NON SHADED Δ	0	1	4	13	40	121	581130733

Handwritten notes: Red arrows show the difference between shaded triangles in consecutive stages: $1 \rightarrow 3$ (diff 2), $3 \rightarrow 9$ (diff 6), $9 \rightarrow 27$ (diff 18), $27 \rightarrow 81$ (diff 54), $81 \rightarrow 243$ (diff 162). A blue arrow points from the 6th stage shaded count (243) to the 20th stage shaded count (1162261467), with the formula $(t-1) \div 2$ written next to it.

$$A_{\text{SHADED IN STAGE 20}} = \left(\frac{1162261467}{1162261467 + 581130733} \right) 80 = 53.\bar{3} \text{ cm}^2$$

7.1 Arithmetic Sequences

-sequences with a common difference

a - FIRST TERM

d - COMMON DIFFERENCE

t_n - GENERAL TERM.

$$\underset{t_1}{a}, \underset{t_2}{a+d}, \underset{t_3}{a+2d}, \underset{t_4}{a+3d}, \dots, \underset{t_n}{a+(n-1)d}$$

$$t_n = a + (n-1)d$$

Determine the 100th term of the sequence
3, 12, 21, 30,

$$a = 3, d = 9$$

$$t_n = a + (n-1)d$$

$$t_{100} = 3 + (100-1)9 = \underline{\underline{894}}$$

The 7th term of an arithmetic sequence is 53.
The 11th term is 97. Determine the 100th term.

$$t_7 = 53 \longrightarrow 53 = a + 6d \quad \textcircled{1}$$

$$t_{11} = 97 \longrightarrow 97 = a + 10d \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} \quad 44 = 4d \longrightarrow d = 11$$

$$d \rightarrow 0 \quad 53 = a + 6(11) \longrightarrow a = -13$$

$$t_n = a + (n-1)d$$

$$t_{100} = -13 + (100-1)(11) = \underline{\underline{1076}}$$

How many terms are in the sequence,
6, 17, 28, 39,, 435

$$a = 6$$

$$d = 11$$

$$t_n = a + (n-1)d$$

$$435 = 6 + (n-1)11$$

$$429 = 11(n-1)$$

$$\frac{429}{11} = n-1$$

$$39 = n-1$$

$$\boxed{40 = n}$$

p. 424#3,4,8i,iii,9,10,13abc,15,,17