Unit 5, Lesson 1: Solving Problems with Quadratic Functions

When solving problems involving quadratic functions, we are either interested in finding the **optimum value** (maximum or minimum) for a situation, or finding the value of the independent variable that produces a particular output (solving an equation). You must pay careful attention to which type of question is being asked.

- Is the question asking for a minimum, maximum, smallest or largest possible value?
  - **Find the vertex** by completing the square, or factoring to determine the axis of symmetry
- Does the question give you enough information to create one equation with a single variable, or two equations with 2 variables (*linear/quadratic systems*)?
  - **Solve the equation(s)** by factoring, quadratic formula, or inverse operations (if possible!)
  - Quadratic equations often have 2 roots, consider if you need both in your solution.

You will need to ***create equations*** to represent the situation.

- Pay careful attention to how you define your variables!
- Make sure you write a therefore statement to answer the question being asked.

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Ex 1) Phil and Shelly are playing a number game. Shelly says, "I am thinking of two numbers that add to 17 and the sum of their squares is 185." What are the two numbers?

Let x and y represent the 2 numbers

$x + y = 17 \longrightarrow y = 17 - x$

$x^2 + y^2 = 185$

$x^2 + (17-x)^2 = 185$

$x^2 + (17-x)(17-x) = 185$

$x^2 + 289 - 17x - 17x + x^2 = 185$

$2x^2 - 34x + 104 = 0$

Now factor: $2(x^2 - 17x + 52) = 0$

$2(x-13)(x-4) = 0$

$\boxed{x=13}$ $\boxed{x=4}$

M +52
A −17
x −13
x −4

check: $x+y=17$
$13+y=17$
$y=4$

$4+y=17$
$y=13$

∴ The 2 numbers must be 13 & 4.

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When defining your variable, always consider if you can successfully create an equation with that variable.

Research for a given orchard has shown that if 100 pear trees are planted, then the annual profit is $91.50 per tree. If more trees are planted they have less room to grow and generate fewer pears per tree. As a result, the annual profit per tree is reduced by $0.75 for each additional tree planted. How many pear trees should be planted to maximize the profit from the orchard for one year?

Let x rep. the additional trees

Let P rep. the profit

Think vertex (x of the vertex) (AOS)

$P = (91.50)(100)$

$P = (91.50 - 0.75x)(100 + x)$

$0 = 91.50 - 0.75x$

$-91.50 = -0.75x$

$\frac{-91.50}{-0.75} = x$

$122 = x$

$x = -100$

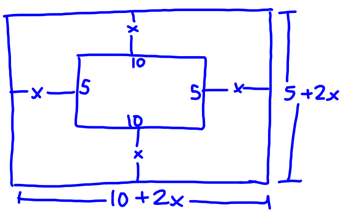
$AOS = \frac{122 + (-100)}{2}$

$\boxed{AOS = 11}$

∴ The orchard will make the max profit when they plant 11 extra trees.

For questions involving "Uniform width", always ***draw a picture*** and define your independent variable to be the uniform width.

A rectangular pool measures 10m by 5m. A deck, of uniform width, is to be built all the way around the pool such that the total area of the pool and deck will be 126 m^2. Algebraically determine the width of the deck.



$A = L \times W$

$126 = (10 + 2x)(5 + 2x)$

$126 = 50 + 20x + 10x + 4x^2$

$0 = 4x^2 + 30x + 50 - 126$

$0 = 4x^2 + 30x - 76$

Now factor it to solve for x:

$0 = 2(2x^2 + 15x - 38)$

$0 = 2(2x + 19)(x - 2)$

M −76
A +15
x +19 → $\frac{2x}{+19}$
x − 4 → $\frac{2x}{-4}$ → $\frac{x}{-2}$

$0 = 2x + 19$

$-19 = 2x$

$\frac{-19}{2} = x$

$\boxed{\cancel{-9.5 = x}}$

impossible because the width can't be −9.5 m

$\boxed{x = 2}$

∴ The width of the deck must be 2m.

Many economic problems are based on the **demand function**, and how it relates to **revenue** and **profits**.

Demand Function	$p(x)$	Price of a commodity as a function of the number of items sold (the # of items people are willing to purchase at the price)
Revenue Function	$R(x) = x \cdot p(x)$	Income as a function of the number of items sold
Cost Function	$C(x)$	Expenses incurred as a function of the number of items
Profit Function	$P(x) = R(x) - C(x)$	Difference between the revenue and costs

Ex 4) The demand function for a new product is $\boxed{p(x) = -5x + 39}$ where p represents the selling price of the product and x is the number sold in thousands. The cost function is $\boxed{C(x) = 4x + 30}$. How many items must be sold to maximize profit?

$R(x) = x(-5x+39)$

$R(x) = -5x^2+39x$

$P(x) = R(x) - C(x)$

$P(x) = (-5x^2+39x) - (4x+30)$

$P(x) = -5x^2+39x-4x-30$

$P(x) = -5x^2+35x-30$

$AOS = \frac{-b}{2a}$

$= \frac{-35}{2(-5)}$

$\boxed{x = 3.5}$ $\times 1000$

∴ They need to sell 3500 items to maximize their profit.

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HW U5L1:

1. p. 153 #5bd, 7ac, 12
2. p. 178 #9-12, 14

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