Lesson 5: Zeros of Quadratic Functions

<u>How to Determine the Number of Zeros (*x*-intercepts)</u>

Factored form: $f(x) = a(x-r)(x-s)$

- The number of zeros will be equivalent to the number of **unique factors**.
- If there are no zeros, the equation cannot be written in factored form.

Example 1: Without drawing the graph, find the number of zeros of the following functions.

a) $f(x) = 0.4(x-1)(x+2)$

2 zeros (1 & -2)

b) $g(x) = 3(x+5)^2$

1 zero (at -5)

c) $h(x) = 4x(x-2)$

2 zeros (2 & 0)

$= 4(x-0)(x-2)$

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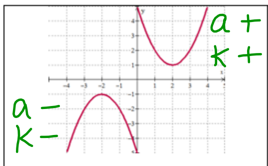
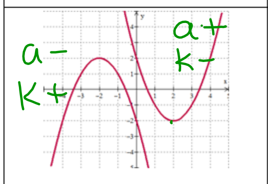
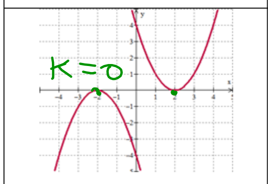
Vertex form: $f(x) = a(x-h)^2 + k$

	if a & k have the **same** sign $a > 0$ & $k > 0$ or $a < 0$ & $k < 0$ $\boxed{\text{no real zeros}}$
	if a & k have **opposite** signs $a > 0$ & $k < 0$ or $a < 0$ & $k > 0$ $\boxed{\text{two real zeros}}$
	if $k = 0$ $\boxed{\text{one real zero}}$

Example 2: Without drawing the graph, find the number of zeros of the following functions.

a) $f(x) = 1.3(x-4)^2 + 2.2$

No zeros (because a&k are the same sign)

b) $g(x) = 1.7(x+2)^2 - 4.5$

2 zeros because a & k are opposite signs

c) $h(x) = 3(x-2.4)^2$

1 zero because k = 0

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Standard form: $f(x) = ax^2 + bx + c$ quad. Form: $x = \frac{-b \mp \sqrt{b^2-4ac}}{2a}$

- Calculate the **Discriminant** → $\mathbf{b^2 - 4ac}$ (from the quadratic formula)
  - if $b^2 - 4ac > 0 \rightarrow$ 2 real zeros
  - if $b^2 - 4ac = 0 \rightarrow$ 1 real zero
  - if $b^2 - 4ac < 0 \rightarrow$ no real zeros (2 complex zeros)

Example 3: Determine the **number** of zeros.

a) $f(x) = x^2 - 8x + 16$

$= b^2 - 4ac$

$= (-8)^2 - 4(1)(16)$

$= 64 - 64$

$= 0$

∴ There is <u>ONE</u> root.

b) $g(x) = 3x^2 + 2x + 4$

$= b^2 - 4ac$

$= 2^2 - 4(3)(4)$

$= 4 - 48$

$= -44$

∴ There are <u>NO</u> roots.

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Example 4: For what values of k does the equation $2x^2 + kx + 8 = 0$ have

a b c

a) two distinct, real roots?

b) one real root?

c) no real roots?

a) 2 ROOTS

$0 < b^2 - 4ac$

$0 < k^2 - 4(2)(8)$

$0 < k^2 - 64$

$64 < k^2$

$\mp\sqrt{64} < k$

$\boxed{\pm 8 < k}$

b) ONE ROOT

$0 = b^2 - 4ac$

$0 = k^2 - 64$

$64 = k^2$

$\mp\sqrt{64} = k$

$\boxed{\mp 8 = k}$

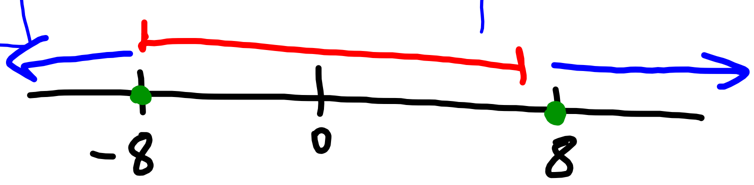
c) NO ROOTS

$0 > b^2 - 4ac$

$0 > k^2 - 64$

$64 > k^2$

$\boxed{\mp 8 > k}$



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Try: Determine the value(s) of k such that the function $f(x) = \underbrace{3}_{a}x^2 + \underbrace{k}_{b}x \underbrace{-3+k}_{c}$ has exactly one zero.

↳ discriminant = 0

$0 = b^2 - 4ac$

$0 = k^2 - 4(3)(-3+k)$

$0 = k^2 - 12(-3+k)$

$0 = k^2 + 36 - 12k$

$0 = k^2 - 12k + 36$

$0 = (k-6)(k-6)$

$\boxed{k = 6}$

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HW U1L5:

1. p. 185 #4ad, 5ab, 6-10, 14

2. study for quiz (DAY 4/5/6/7)

Thursday Feb 21