Unit 2 – Rational Expressions & Exponents

Day	**Lesson**	**Practice Questions**	**Struggles?**
1	U2L1 – Exploring Graphs of Exponential & Reciprocal Functions	**Read:** Pg. 242 **Do:** **Handout**	
2	U2L2 – Integer Exponents	**Read:** Pg. 217 - 221 **Do:** Pg. 221 # 4 – 7ace, 8, 9ace, 11acd, 13 – 14ace, 15 *(Calculator permitted for #7, 11)*	
3	*Skill Builder: Simplifying Rational Functions*	**Read:** Pg. 108 – 112 **Do:** Pg. 113 # 4 – 7, 10, 14a **Try**: 14b, 17 Pg. 89 # 13 *#4d s/b* $\frac{1}{a(3a^2-2b)}$ *#5a s/b* $\frac{1}{a-1}$	
4	U2L3 – Rational Exponents	**Read:** Pg. 224 – 228 **Do:** Pg. 229 # 4 – 6ace, 8 – 11, 12ace, 14, 15a *(Calculator permitted for #8, 9, 12)*	
5	*Skill Builder: Multiplying/Dividing Rational Expressions*	**Read:** Pg. 117 – 121 **Do:** Pg. 122 # 4 – 8, 11, 12a **Try:** 13	
6	U2L4 – Simplifying Algebraic Expressions with Exponents	**Read:** Pg. 231 – 235 **Do:** Pg. 235 # 1 – 2ace, 3, 4 – 9ace, 11 *(Calculator permitted for #11b)*	
7	U2L5 – Solving Equations with Exponents	**Do:** Pg. 261 #1 Pg. 223 #16, 17 **Handout**	
8	Review	**Read:** Pg. 131, Pg. 238 **Do:** Pg. 132 # 2, 9 – 13 *(skip 10a)*, 17a *(n > 4)* Pg. 267 #1 → 8 *(5f: answer in back should be positive)*	
9	TEST		

Unit 2, Lesson 1: Exploring Exponential & Reciprocal Functions

For each function provided:

- Complete the table of values
- Plot the points and graph the function
- State the **domain** and **range** and the **equation(s) of the asymptote(s)**
- Verify your graphs using graphing technology

1. $f(x) = 2^x$

x	***f*** **(*x*)**
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

Asymptotes:

2. $g(x) = 3^x$

x	***g*** **(*x*)**
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

Asymptotes:

3. $h(x)=\left(\frac{1}{4}\right)^{x}$

x	***h* (*x*)**
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

Domain:

Range:

Asymptotes:

4. $k(x)=\frac{1}{x}$

x	***k* (*x*)**
-4	
-3	
-2	
-1	
-0.5	
-0.25	
-0.125	
0	
0.125	
0.25	
0.5	
1	
2	
3	
4	

Domain:

Range:

Asymptotes:

Unit 2, Lesson 2: Simplifying Expressions with Integer Exponents

Recall: Exponent Laws

Rule	**Numeric Example**	**Algebraic Example**
Product	$2^3 \times 2^4 = 2^7$	$a^m \times a^n = a^{m+n}$
Quotient	$5^6 \div 5^2 = 5^4$	$a^m \div a^n = a^{m-n}$
Power of a power	$(3^3)^2 = 3^6$	$(a^m)^n = a^{mn}$
Power of a product	$(2 \times 3)^4 = 2^4 \times 3^4$	$(xy)^m = x^m y^m$
Power of a quotient	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}, y \neq 0$

Test yourself: True or False? Circle the correct choice.

$x^9 \div x^{-9} = x^0$	true	false	$(3a^2)^2 = 9a^4$	true	false	$-6^2 = -36$	true	false
$(2y^3)^4 = 2y^{12}$	true	false	$(x^5)(x^4) = x^{20}$	true	false	$(x^7)(x^5) = x^{12}$	true	false
$(5^2)(5^2) = 25^4$	true	false	$(4a^2)^0 = 1$	true	false	$(a^4b^{-3})^{-3} = a^{-12}b^9$	true	false
$(-1)^6 = 1$	true	false	$(-6)^2 = -36$	true	false			

Zero Exponent Rule $a^0 = 1, a \neq 0$

Test yourself: Is the answer equal to one, or not equal to one? Circle the correct choice.

$-(-x)^0$	$=1$	$\neq 1$	-3^0	$=1$	$\neq 1$	$(-1)^{100}$	$=1$	$\neq 1$
$2000x^0$	$=1$	$\neq 1$	$(-1)^{101}$	$=1$	$\neq 1$	$\frac{(2^3)}{(2^{-3})}$	$=1$	$\neq 1$
$5x^0$	$=1$	$\neq 1$	$(-120x)^0$	$=1$	$\neq 1$			
-1^{50}	$=1$	$\neq 1$	$4^{-2} \div 4^{-2}$	$=1$	$\neq 1$			

Negative Exponents

Any base raised to a **negative exponent** equals the **reciprocal of the base** to the **positive exponent**!

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

Ex: $x^{-4} = \frac{1}{x^4}$

Ex: $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$

Ex 1) Rewrite as a positive power then evaluate and express in rational (fractional) form.

a) -2^3 b) 2^{-3} c) -2^{-3} d) $(-2)^{-3}$ e) $-(-2)^{-3}$

Ex 2) Simplify, then evaluate each expression. Express answers in rational form.

a) $\left(5^{-2}\right)^{3}\left(5^{3}\right)$

b) $\dfrac{\left(6^{-2}\right)^{5}\left(6^{7}\right)}{6^{5}}$

c) $3^{-5} \div \left(\dfrac{3}{3^{5}}\right)$

d) $\dfrac{\left(16^{-2}\right)^{3}(2)^{3}}{(-8)^{-6}}$

Ex 3) Simplify $\dfrac{(12)^{-5}\left(3^{-2}\right)^{-3}}{\left(2^{4}\right)^{-2}}$ using the power of a product rule. Then evaluate.

Ex 4) Evaluate $(x^{n} - y^{n})^{n}$ where $x = -1$, $y = -2$, and $n = -3$

Do ALL homework questions without a calculator, unless specified otherwise.

Skill Builder: Simplifying Rational Functions

A rational function is the ratio of two polynomial functions. A rational function can be expressed as $R(x)=\dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are each polynomial functions and $q \neq 0$.

To determine the **domain** of a rational function, consider the value(s) of x that make the polynomial in the denominator, $\boldsymbol{q(x)=0}$ (i.e. the **zeros of the denominator**). The domain will **exclude** these values. These values are also called the **restrictions** of the corresponding rational expression.

Ex 1) Determine the domain of the rational function $f(x)=\dfrac{5x^2-10x}{3x^2+9x}$

Recall: To simplify a fraction, we determine the GCF of the numerator & denominator, then divide both numerator & denominator by the GCF (i.e. "cancel out" the GCF). e.g. $\dfrac{12}{18}=\dfrac{(6)(2)}{(6)(3)}=\dfrac{2}{3}$

Think: Why are we allowed the "cancel out" the 6?

We can simplify rational functions and rational expressions in a similar manner.

- **Factor** both the numerator and the denominator (*using all of your factoring strategies*)
- **Divide** both numerator and denominator by the GCF ("**Cancel out**" all common factors)

When asked for the **restrictions**, you must determine the **zeros of the ORIGINAL denominator**

CAUTION: YOU CAN ONLY CANCEL FACTORS, NOT TERMS

Ex. $\dfrac{1+6}{6}=\dfrac{7}{6}$ $\quad \dfrac{1+6}{6}\neq 1$ **Ex.** $\dfrac{3}{3+7}=\dfrac{3}{10}$ $\quad \dfrac{3}{3+7}\neq\dfrac{1}{7}$

This applies to variables as well, so...

Ex. $\dfrac{x-8}{x+3}\neq-\dfrac{8}{3}$ **Ex.** $\dfrac{2x^2-8}{2x^2+3x}\neq-\dfrac{8}{3x}$

Ex 2) Simplify. State any restrictions on the variables

a) $\dfrac{15x^2y^3}{10x^4y^2}$ b) $\dfrac{2n^2+n-1}{n-1}$ c) $\dfrac{5(x+3)+2}{x+3}$

d) $\dfrac{x+2xy}{xy}$

e) $\dfrac{(a-9)(a^2-2)}{(9-a)}$

f) $\dfrac{6x^2-5xy-4y^2}{3x^2+8xy-16y^2}$

Ex 3) Simplify $f(x)$ and state the domain, where

a) $f(x)=\dfrac{3x+6}{x^2-4}$

b) $f(x)=\dfrac{x^2+5x-6}{2x-2}$

Equivalence

Two functions are considered **equivalent** if they have the **same domain** and yield the **same values** (output) for **all numbers in their domain** (input).

- To show **equivalence**, you must show that they both **simplify to the same expression**, with the **same domain**.
- To show **non-equivalence**, you can **choose an input** (i.e. substitute a number for "*x*") and show that each function yields a **different output**. This **does not work** to show **equivalence** since some functions intersect.

Ex 4) For each pair of functions, determine if they are equivalent.

a) $f(x)=\dfrac{8x^2+2x-21}{12x^2+29x+14}$

$g(x)=\dfrac{2x+3}{3x+2};x\neq -1\dfrac{3}{4}$

b) $f(x)=\dfrac{8x^2+10x-3}{6x^2+13x+6}$

$g(x)=\dfrac{4x-1}{3x+2};x\neq -1\dfrac{1}{2}$

Unit 2, Lesson 3: Rational Exponents Investigation

Consider the following pattern:

A. Fill in the blanks based off of the examples. Then answer the questions to the right.

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$2^3 =$ ______ $\cdot$ ______ $\cdot$ ______ $=$ ______

$2^2 =$ ______ $\cdot$ ______ $=$ ______

$2^1 =$ ______

$2^0 =$ ______

$2^{-1} =$ ______

$2^{-2} = \dfrac{1}{2^2} = \dfrac{1}{2} \cdot \dfrac{1}{2} =$ ______

$2^{-3} =$ ______ $=$ ______ $\cdot$ ______ $\cdot$ ______ $=$ ______

$2^{-4} =$ ______ $=$ ______ $\cdot$ ______ $\cdot$ ______ $\cdot$ ______ $=$ ______

1. With the number system in mind, what type of exponents is being used?

2. What is the most specific number classification for the results? (Final products)

B. Now consider $2^{\frac{1}{2}}$. Where would this fit in the pattern above? Draw an arrow where you think $2^{\frac{1}{2}}$ should be placed. What do you think the value will be?

My estimated value of $2^{\frac{1}{2}}$: [ ]

- Now enter $2^{\frac{1}{2}}$ in your calculator. Record the value below. What is the most specific number classification for the result?

- What is another key sequence on your calculator to find $2^{\frac{1}{2}}$?

C. Now consider $2^{\frac{1}{3}}$. Where would this fit in the pattern above? Draw an arrow where you think $2^{\frac{1}{3}}$ should be placed. What do you think the value will be?

My estimated value of $2^{\frac{1}{3}}$: [ ] Calculator value: [ ] Number classification:

- What is another key sequence on your calculator to find $2^{\frac{1}{3}}$?

D. Evaluate the following using your calculator:

$36^{\frac{1}{2}} =$

$81^{\frac{1}{2}} =$

$64^{\frac{1}{2}} =$

$144^{\frac{1}{2}} =$

$25^{\frac{1}{2}} =$

Write a statement about what the exponent $\frac{1}{2}$ represents.

Try to write this symbolically in *radical form*: $a^{\frac{1}{2}} =$

E. Based on your observations from part D, try to evaluate the following **without** your calculator.

$8^{\frac{1}{3}} =$

$27^{\frac{1}{3}} =$

$1000^{\frac{1}{3}} =$

$125^{\frac{1}{3}} =$

Write a statement about what the exponent $\frac{1}{3}$ represents?

Try to write this symbolically in *radical form*: $a^{\frac{1}{3}} =$

F. Look back at parts D and E to complete the following symbolic rule in *radical form*:

$$a^{\frac{1}{n}} =$$

G. Another way of understanding this rule:

Evaluate $\left(4^{\frac{1}{2}}\right)\left(4^{\frac{1}{2}}\right)$ using product rule

Evaluate $\left(\sqrt{4}\right)\left(\sqrt{4}\right)$

Evaluate $\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)$ using product rule

Evaluate $\left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right)$

What do you notice?

Unit 2, Lesson 3: Working with Rational Exponents

Rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$ **, means the n^th root of *x***

Ex 1) Evaluate

a) $49^{\frac{1}{2}}$ b) $(-64)^{\frac{1}{3}}$ c) $8^{-\frac{1}{3}}$ d) $\left(\frac{1}{36}\right)^{\frac{1}{2}}$

Rule: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$**, means the n*^th^* root of the *m^th^* power of *x***

Ex 2) Evaluate. Write in radical form first.

a) $8^{\frac{2}{3}}$ b) $-25^{\frac{5}{2}}$ c) $81^{-\frac{3}{4}}$

d) $16^{0.75}$ e) $\left(-\frac{1}{64}\right)^{\frac{2}{3}}$ f) $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Ex 3) Evaluate, no decimals:

$$128^{-\frac{5}{7}} - 16^{0.75}$$

Ex 4) **Simplify**, then evaluate to 4 decimal places.

$$3^{-\frac{4}{5}}\left(3^{\frac{1}{15}}\right) \div 3^{\frac{2}{3}}$$

Ex 5) Express as a single, positive power, then evaluate.

$$\left(\sqrt[3]{27}\right)\left(\sqrt[4]{81}\right)^3$$

Do ALL homework questions without a calculator, unless specified otherwise.

Skill Builder: Multiplying and Dividing Rational Expressions

Example:

$$\frac{10}{27}\times\frac{36}{35}=\frac{(2)(5)}{(3)(9)}\times\frac{(4)(9)}{(5)(7)}$$
$$=\frac{(2)(4)}{(3)(7)}$$
$$=\frac{8}{21}$$

Recall: The procedure for **multiplying numeric fractions**

- **Check** all the numerators and all the denominators for common factors
- **Divide** out ALL common factors (**"Cancel out"** common factors)
- Multiply **numerator by numerator** and **denominator by denominator**.

We can multiply rational expressions in a similar manner.

- **Factor** the numerator and the denominator of both rational expressions
- **Divide out** any factors common to the numerator and denominator **("Cancel out"** all common factors)
- **Multiply** numerator by numerator and denominator by denominator.
    - You do NOT need to expand your final expressions. Leave final answers in factored form.

When asked for the **restrictions**, you must determine the **zeros of ALL ORIGINAL denominators.**

Ex 1) Multiply. State any restrictions on the variables.

a) $\frac{9x^2}{4xy}\times\frac{12xy^2}{3x}$

b) $\frac{2x^2+5x+2}{4x^2-8x-5}\times\frac{2x^2-11x+15}{3x^2+7x+2}$

Recall: The procedure for **dividing numeric fractions**

- Take the **reciprocal of the divisor** (the 2nd fraction, the one you are "dividing by") and **change the $\div$ to a $\times$.**
- Proceed with the same steps as **multiplying**

$$\frac{8}{15} \div \frac{20}{9} = \frac{8}{15} \times \frac{9}{20} = \frac{(2)(4)}{(3)(5)} \times \frac{(3)(3)}{(4)(5)} = \frac{(2)(3)}{(5)(5)} = \frac{6}{25}$$

We can divide rational expressions in a similar manner.

- Take the **reciprocal of the divisor (**the 2nd rational expression) and change the $\div$ to a $\times$**.**
- **Factor** the numerator and the denominator of both rational expressions
- **Divide out** any factors common to the numerator and denominator **("Cancel out"** all common factors)
- **Multiply** numerator by numerator and denominator by denominator.
  - You do NOT need to expand your final expressions. Leave final answers in factored form.

When asked for the **restrictions**, you must determine the **zeros of ALL ORIGINAL denominators,** and the **ORIGINAL numerator of the divisor.**

The **order of operations** still applies for rational expressions: Multiplication and division are done from **LEFT to RIGHT**.

Ex 2) Divide. State any restrictions on the variables.

$$\frac{x^2+3x+2}{x^4-4x^2} \div \frac{x^2-x-2}{5x^3-9x^2-2x}$$

Ex 3) Simplify. State any restrictions on the variables.

$$\frac{3x^2+10x-8}{5x^2-18x-8} \div \frac{x^2-16}{2x^2+7x+3} \times \frac{5x^2+17x+6}{6x^2-x-2}$$

Unit 2, Lesson 4: Simplifying Algebraic Expressions with Exponents

Summary of Exponent Laws

Rule	**Numeric Example**	**Algebraic Example**
Product	$2^3 \times 2^4 = 2^7$	$a^m \times a^n = a^{m+n}$
Quotient	$5^6 \div 5^2 = 5^4$	$a^m \div a^n = a^{m-n}$
Power of a power	$(3^3)^2 = 3^6$	$(a^m)^n = a^{mn}$
Power of a product	$(2 \times 3)^4 = 2^4 \times 3^4$	$(xy)^m = x^m y^m$
Power of a quotient	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}, y \neq 0$
Zero exponent	$4^0 = 1$	$a^0 = 1, a \neq 0$
Negative exponents	$6^{-2} = \frac{1}{6^2}$	$a^{-n} = \frac{1}{a^n}, a \neq 0$
Rational exponents	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$	$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{8}\right)^m$

- When simplifying expressions involving exponents follow the **laws and rules for exponents** and the **order of operations**.
    - Power of a power rule must be done BEFORE product rule (exponents before multiplication).
    - Simplify all expressions in the numerator and denominator FIRST, before using quotient rule to divide (large fraction bar is a grouping symbol)
    - When you have nested grouping symbols, simplify the innermost first.
- Rewrite any decimal exponents as fractions.
- Rewrite numbers as powers with the same bases, if possible.
- Express all final answers using **positive exponents**.
- Express all answers in **rational form** (no decimals!)

Ex 1) Simplify, then evaluate. Express answers in rational form with positive exponents

a) $\dfrac{(2x^3)^4(-x^2)}{8x^{-4}}$ b) $(-2x^2y^6)(-3x^3y)^2$ c) $\dfrac{(2x^2y^8)(x^3y^2)}{(-2x^3y^2)^2}$

Ex 2) Simplify. Express answers with positive exponents

a) $\left(\dfrac{x^5\left(y^2\right)^3}{x^3y^8}\right)^{-2}$

b) $\left(\dfrac{\left(xy^{-2}\right)^3}{x^{-3}y^4}\right)^{-\frac{1}{2}}$

Ex 3) Simplify, then evaluate. Express answers in rational form with positive exponents

a) $\dfrac{\sqrt{16p^{-2}}}{\sqrt[3]{(125p^{-6})^{-2}}}$

b) $\left(\dfrac{\left(16x^3\right)^2\left(8y^2\right)}{32(xy)^4}\right)^{-1.5}$

Do ALL homework questions without a calculator, unless specified otherwise.

Unit 2, Lesson 5: Solving Equations with Exponents

Recall: Solving any equation means find the **value of the variable** that **makes the equation true**. When solving equations involving exponents, pay attention to the location of your variable in the equation.

Variable already isolated

- Apply correct order of operations (exponents before multiplication) and evaluate

Ex 1) Solve $A = 100(1.07)^5$

Variable is being multiplied by a power

- Solve using inverse operations

Ex 2) Solve $7500 = N(1.25)^{1.50}$

Variable is the base of a power

- Use inverse operations to isolate the power
- The exponent in the power becomes the type of root needed to solve for the base
    - exponent of 2 → square root
    - exponent of 3 → cube root
    - exponent of 4 → 4th root
    - etc …
- When taking an even root, ask yourself: should I consider the negative answer as well?

Ex 3) Solve $5000 = 2000(B)^{10}$

Variable is the exponent

Strategy #1 – Guess and Check

- Since we don't (yet!) know how to "undo" the raising of a base to an unknown variable, we can use a "guess and check" strategy

Ex 4) Solve $1000 = 500(1.10)^t$

Strategy #2 – Change of Base

Consider the equation $a^x = a^y$. Since the bases are equal, it follows that the exponents must be the same as well.

If $a^x = a^y$, Then $x = y$.

IMPORTANT: We are NOT "Cancelling the bases."
We ARE creating a NEW equation that has the same solution as our original equation.

Steps to follow:

- Rewrite all powers with a common base.
- Simplify to get a single power on each side of the equation.
- Create a new equation with the exponents
- Solve the new equation to get the solution(s) of the original equation

Ex 5) Solve

a) $3^{3x} = 81$

b) $5^{2x-1} = \frac{1}{125}$

c) $(2^x)(64) = (\sqrt{32})^x$

U2L5 Solving Equations with Exponents Handout — MCR3U

1. Solve each equation. Round final answer to 2 decimal places.

a) $A = 300\,(1.03)^8$ b) $2000 = P\,(1.05)^{15}$ c) $50 = N\,(0.5)^6$ d) $35 = T\,(0.5)^5 + 23$

2. Solve for the unknown base. Round final answer to 2 decimal places.

a) $650 = 300\,(B)^{12}$ b) $30 = 100\,(B)^9$ c) $5000 = 800\,(B)^{20}$ d) $1 = 10\,(B)^7$

3. Solve each equation using guess and check. Answer must be accurate to 2 decimal places

a) $20 = 50(0.5)^x$ b) $272 = 20(2.5)^x$ c) $2500 = 1500(1.08)^x$

4. Express each as a power of 3:

a) 27 b) 81 c) $\frac{1}{9}$ d) 9^{2x} e) $\left(\frac{1}{27}\right)^x$

5. Determine the exact solutions algebraically:

a) $2^x = 2^7$ b) $5^x = 5^3$ c) $3^{x+6} = 3^{12}$ d) $10^{2x-1} = 10^3$

e) $2^{2x-1} = 2^{x+9}$ f) $7^{3x+2} = 7^{2x+5}$ g) $4^{2x} = 4^{8}$ h) $5^{x} = 5^{3x-12}$

6. Find the exact roots of each equation:

a) $2^{x} = 32$ b) $3^{x} = 27$ c) $3^{x} = 9^{x-1}$ d) $5^{x} = 3125$

e) $4(2^{x}) = 32$ f) $5^{x} = \frac{1}{125}$ g) $6^{x} = \sqrt[3]{6}$ h) $3^{-x} = \frac{1}{81}$

7. Solve each equation:

a) $4^{x} = 8\sqrt{2}$ b) $3^{x} = \sqrt[5]{9}$ c) $125^{x} = 25\sqrt{5}$ d) $8^{x} = 16\sqrt[3]{2}$

e) $2^{7-x} = \frac{1}{2}$ f) $2^{x-2} = 4^{x+2}$ g) $9^{2x+1} = 81(27^{x})$

1) a) 380.03 b) 962.03 c) 3200 d) 384 2) a) ±1.07 b) 0.87 c) ±1.10 d) 0.72 3) a) 1.32 b) 2.85 c) 6.64 4) a) 3^{3} b) 3^{4} c) 3^{-2} d) 3^{4x} e) 3^{-3x}
5) a) 7 b) 3 c) 6 d) 2 e) 10 f) 3 g) 4 h) 6 6) a) 5 b) 3 c) 2 d) 5 e) 3 f) -3 g) $\frac{1}{3}$ h) 4 7) a) $\frac{7}{4}$ b) $\frac{2}{5}$ c) $\frac{5}{6}$ d) $\frac{13}{9}$ e) 8 f) -6 g) 2