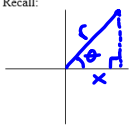
Unit 3, Lesson 7: Proving Simple Trigonometric Identities

Recall:



$x^2 + y^2 = r^2$

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$ $\quad \sin^2\theta = \frac{y^2}{r^2}$

$\cos\theta = \frac{x}{r}$ $\Rightarrow$ $\sec\theta = \frac{r}{x}$ $\quad \cos^2\theta = \frac{x^2}{r^2}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$ $\quad \tan^2\theta = \frac{y^2}{x^2}$

A trigonometric identity is a trigonometric equation that is true for **ALL** values of the variable.

To prove that an equation is an identity, show that both sides of the equation represent the same expression.

- Manipulate the more complex side of the equation to arrive at the expression on the other side or manipulate both sides to get the same expression.
- When simplifying, remember to use all of your algebraic skills including factoring or finding a common denominator.

Consider: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

LS $\frac{y}{x}$ | RS $\frac{y}{r} \div \frac{x}{r}$

$\frac{y}{r} \times \frac{r}{x}$

$\frac{y}{x}$

LS = RS ☺

Consider: $\cot\theta = \frac{\cos\theta}{\sin\theta}$

LS $\frac{x}{y}$ | RS $\frac{x}{r} \div \frac{y}{r}$

$\frac{x}{r} \times \frac{r}{y}$

$\frac{x}{y}$

LS = RS ☺

Consider: $\sin^2\theta + \cos^2\theta = 1$

Pythagorean: $x^2 + y^2 = r^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2 + x^2}{r^2}$

$= \frac{r^2}{r^2}$

$= 1$

LS = RS ☺

Consider: $\tan^2\theta + 1 = \sec^2\theta$

recall: $\cos^2\theta = \frac{x^2}{r^2}$

$\therefore \sec^2\theta = \frac{r^2}{x^2}$

$\frac{y^2}{x^2} + 1$ | $\frac{r^2}{x^2}$

$\frac{y^2}{x^2} + \frac{x^2}{x^2}$

$\frac{y^2 + x^2}{x^2}$

$\frac{r^2}{x^2}$

LS = RS ☺

Consider: $\cot^2\theta + 1 = \csc^2\theta$

$\frac{x^2}{y^2} + 1$ | $\frac{r^2}{y^2}$

$\frac{x^2}{y^2} + \frac{y^2}{y^2}$

$\frac{x^2 + y^2}{y^2}$

$\frac{r^2}{y^2}$

LS = RS ☺

SUMMARY:

Quotient Identity $\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Trigonometric equations can be factored and expanded in the same manner as regular equations are:

Ex 1) Factor $\sin^2\theta - 1$

$(\sin\theta + 1)(\sin\theta - 1)$

Think: $x^2 - 1$

$(x+1)(x-1)$

Difference of squares

Ex 2) Expand $(\cos\theta + 2)(\cos\theta - 1)$

$\cos^2\theta - \cos\theta + 2\cos\theta - 2$

$\cos^2\theta + \cos\theta - 2$

*When solving Trigonometric identies, use all the algebraic tools you know (FOIL, factoring, GCF, common denomitors etc)

Ex 3) Prove the following identities, using a formal L.S = RS proof

a) $\frac{\sin\theta}{\tan\theta} = \cos\theta$

$\sin\theta \div \tan\theta$ | $\cos\theta$

$\sin\theta \div \frac{\sin\theta}{\cos\theta}$

$\sin\theta \times \frac{\cos\theta}{\sin\theta}$

$\cos\theta$

$\therefore$ LS = RS ☺

b) $\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$

$\frac{\sin^2\theta}{\sin\theta}$ | $\sin\theta$

$\sin\theta$

LS = RS ☺

Since $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

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c) $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$ | $\frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$\frac{1}{\cos\theta\sin\theta}$

$\frac{1}{\cos\theta\sin\theta}$

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d) $\sin^2\theta\sec^2\theta = \sec^2\theta - 1$

$\sin^2\theta \times \frac{1}{\cos^2\theta}$ | $\tan^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta}$

$\tan^2\theta$

LS = RS ☺

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HW:

1. p. 310 # 2-3ac, 4, 5ac, 6, 7 bd, 8bdf, 12a

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