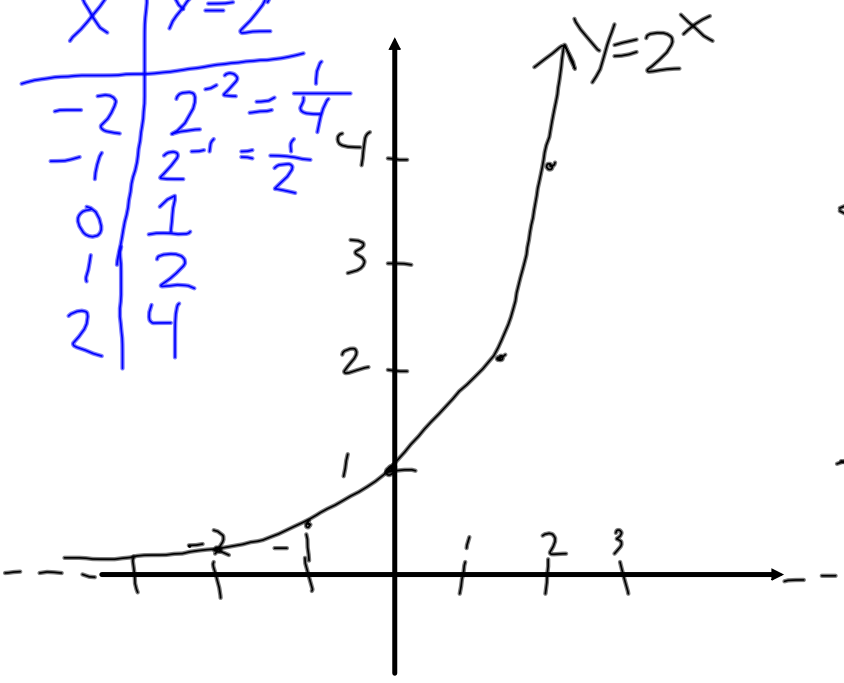
4.6 Transformations of Exponential Functions

Fold a sheet of paper into 2 equal parts. Count the number of rectangles formed. Record it in a table of values. Continue to fold the paper in half as many times as you can and record the number of rectangles along the way.

Fold stage	0	1	2	3	4	5	6	7
# of rectangles formed	1	2	4	8	16	32	64	128

2^0 2^1 2^2 2^3 2^4 ..... $y=2^x$

x	$y=2^x$
-2	$2^{-2}=\frac{1}{4}$
-1	$2^{-1}=\frac{1}{2}$
0	1
1	2
2	4



$y=b^x$ = Parent function

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y>0\}$

We just looked at a situation modelled by the function $Y = 2^x$. Now we are going to explore exponential functions in their general form.

$y = b^x$ parent function

$y = af[k(x-d)] + c$

ex: transform $y = 5^x$

$y = a5^{k(x-d)} + c$

$y = ab^{k(x-d)} + c$

b: base of my parent function

1. The effect of ***d***.

Graph the following functions on the same plane

$d = 0$

$f(x) = 2^x$

-2	$2^{-2} = \frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

$d = 1$

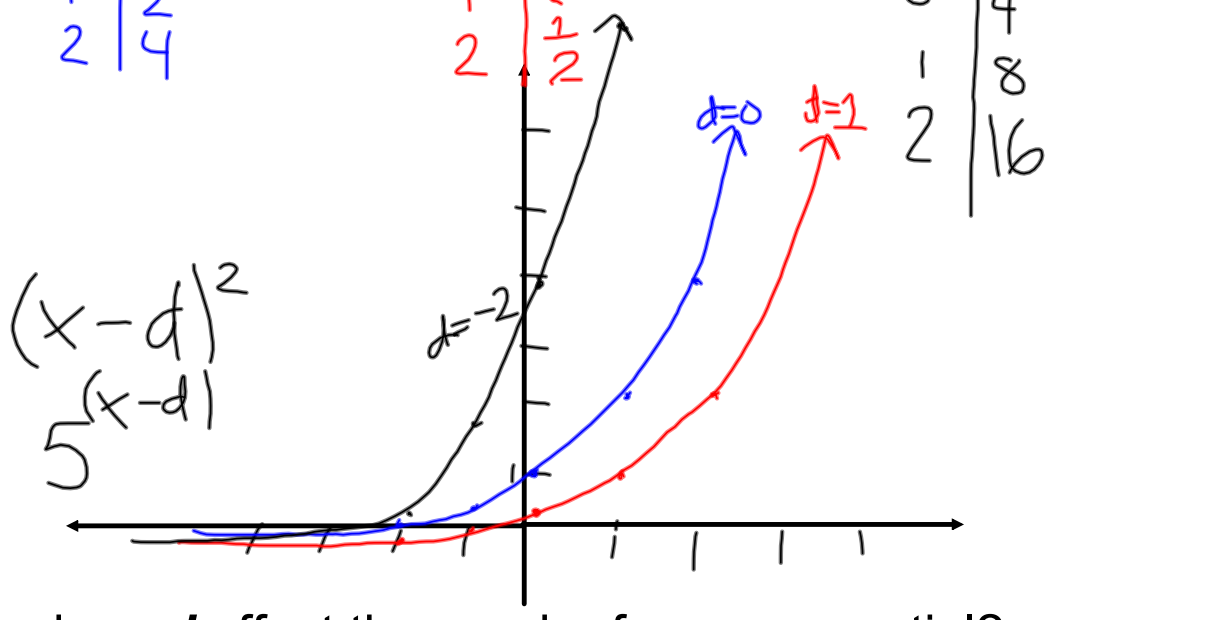
$f(x) = 2^{x-1}$

-2	$2^{-2-1} = 2^{-3} = \frac{1}{8}$
-1	$2^{-2} = \frac{1}{4}$
0	$\frac{1}{2}$
1	1
2	2

$d = -2$

$f(x) = 2^{x+2}$

-2	$2^{-2+2} = 2^0 = 1$
-1	$2^{1+2} = 2^1 = 2$
0	4
1	8
2	16



$(x-d)^2$

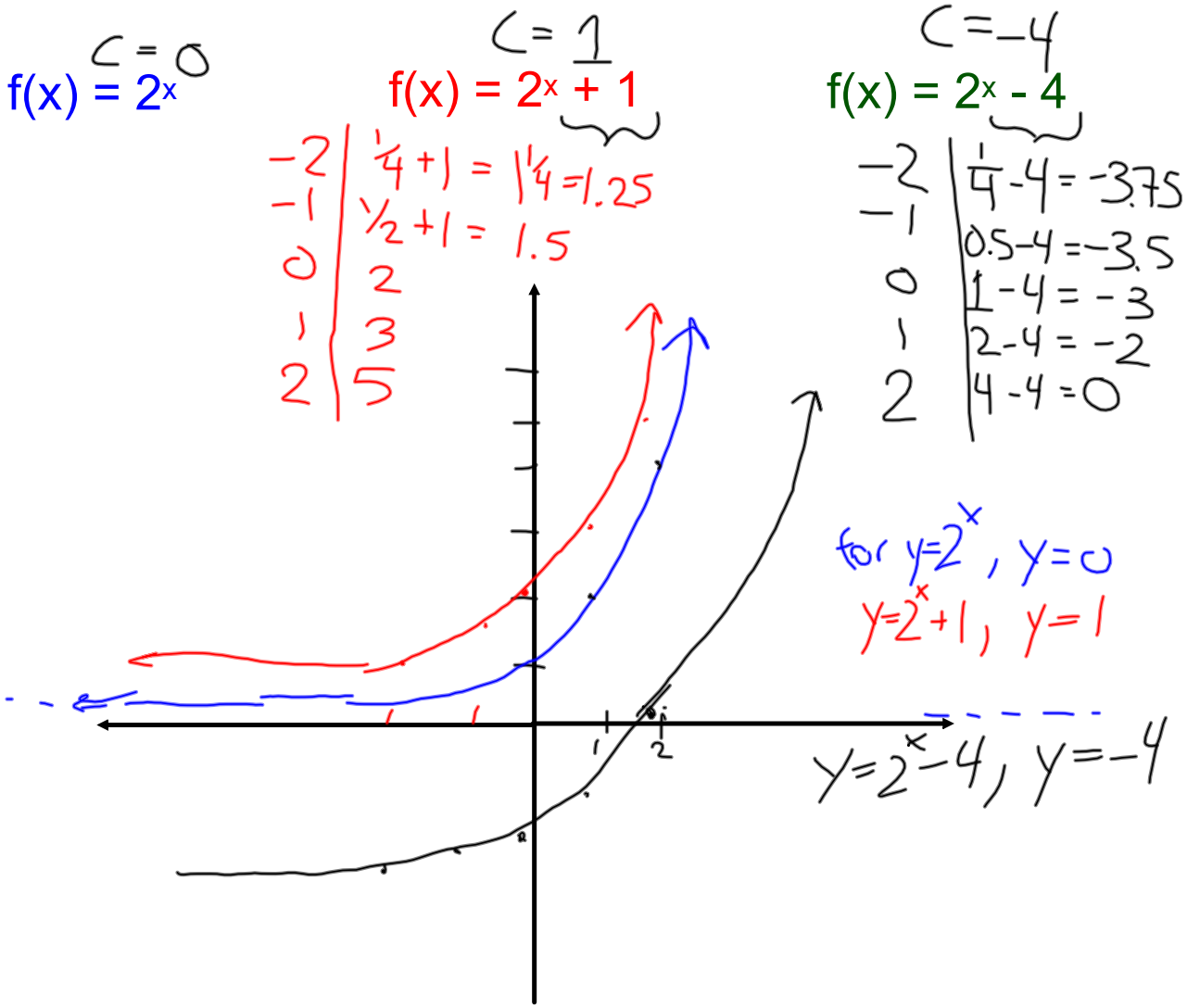
$5^{(x-d)}$

How does ***d*** effect the graph of an exponential?

If $d > 0$, a horizontal translation (or shift) of d units to the right.

If $d < 0$, shift to the left.

2. The effect of ***c***



How does ***c*** effect the graph of an exponential?

If $c > 0$, shift up c units (vertical translation)

If $c < 0$, shift down c units

c also tells us the equation of the asymptote. $y = c$

2. The effect of *a*

$a = 1$

$f(x) = 2^x$

$a = 3$

$f(x) = 3 \cdot 2^x$

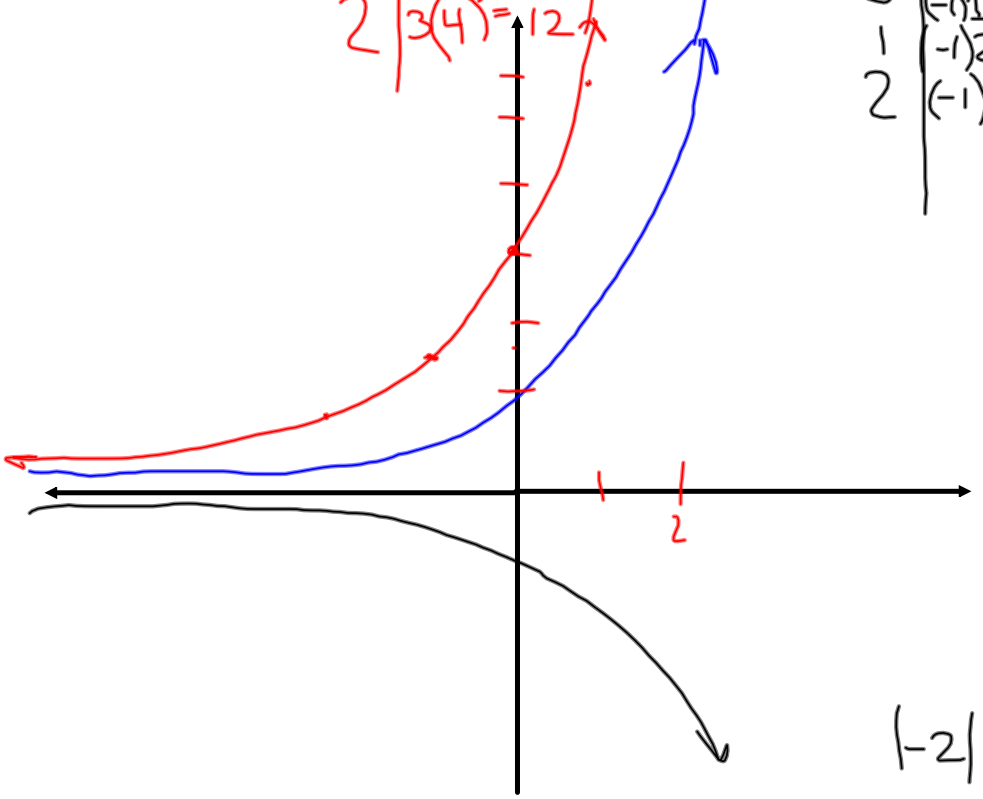
-2	$3\frac{1}{4} = \frac{3}{4} = 0.75$
-1	$3(\frac{1}{2}) = \frac{3}{2} = 1.5$
0	$3(1) = 3$
1	$3(2) = 6$
2	$3(4) = 12$

$a = -1$

$f(x) = -2^x$

$= (-1)2^x \neq (-2)^x$

-2	$(-1)(\frac{1}{4}) = -\frac{1}{4}$
-1	$(-1)(\frac{1}{2}) = -\frac{1}{2}$
0	$(-1)1 = -1$
1	$-1)2 = -2$
2	$(-1)4 = -4$



$|-2| = 2$

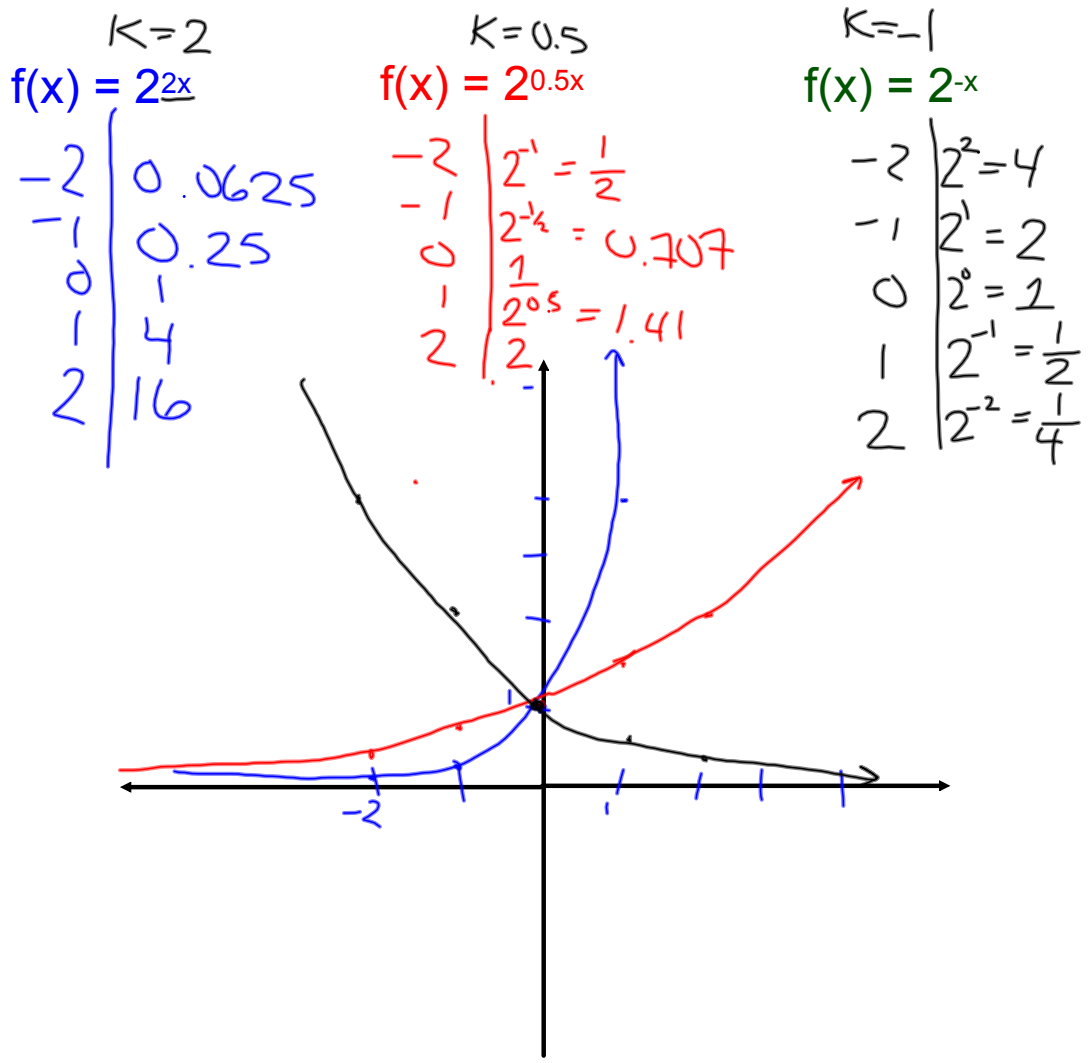
How does ***a*** effect the graph of an exponential?

If $|a| > 1$, a vertical stretch of factor $|a|$

If $0 < |a| < 1$, a vertical compression of $|a|$

If a is also neg. we get a reflection in the x-axis

2. The effect of *k*



How does ***k*** effect the graph of an exponential?

If $|k| > 1$ a horizontal compression by a factor of $\frac{1}{|k|}$.

If $0 < |k| < 1$ a horizontal stretch of factor $\frac{1}{|k|}$

If k is also neg. we get a reflection in the y-axis.

$2^{0.5x}$

$k = 0.5$

$\frac{1}{0.5} = 2$

$$y = ab^{k(x-d)} + c$$

b → base of my parent function

EX1: Describe the transformations used to obtain the graph of *g* from the graph of *f*

$f(x) = 2^x \longrightarrow g(x) = -3 \cdot 2^{(x+2)} - 2$

$a = -3$ → vertically stretch by 3, reflection in the x-axis

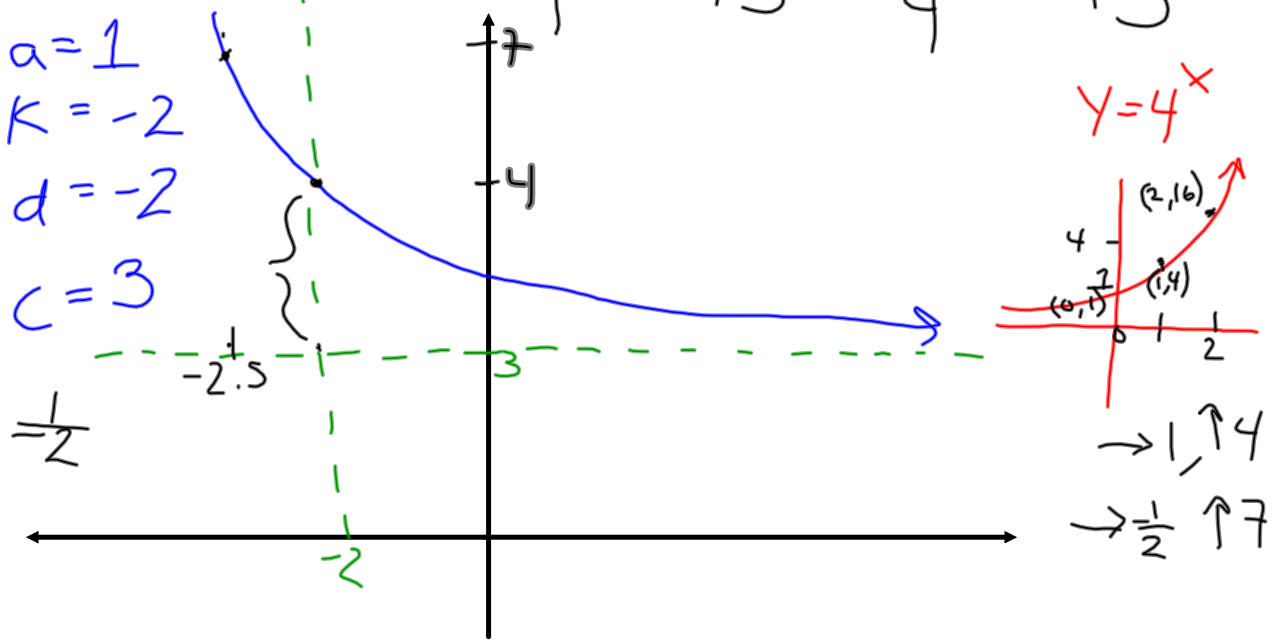
$k = 1$

$d = -2$ → shift left by 2.

$c = -2$ → shift down by 2.

EX2: Use transformations to sketch the function

$y = 4^{-2x-4} + 3 = 4^{(-2x-4)} + 3 = 4^{-2(x+2)} + 3$

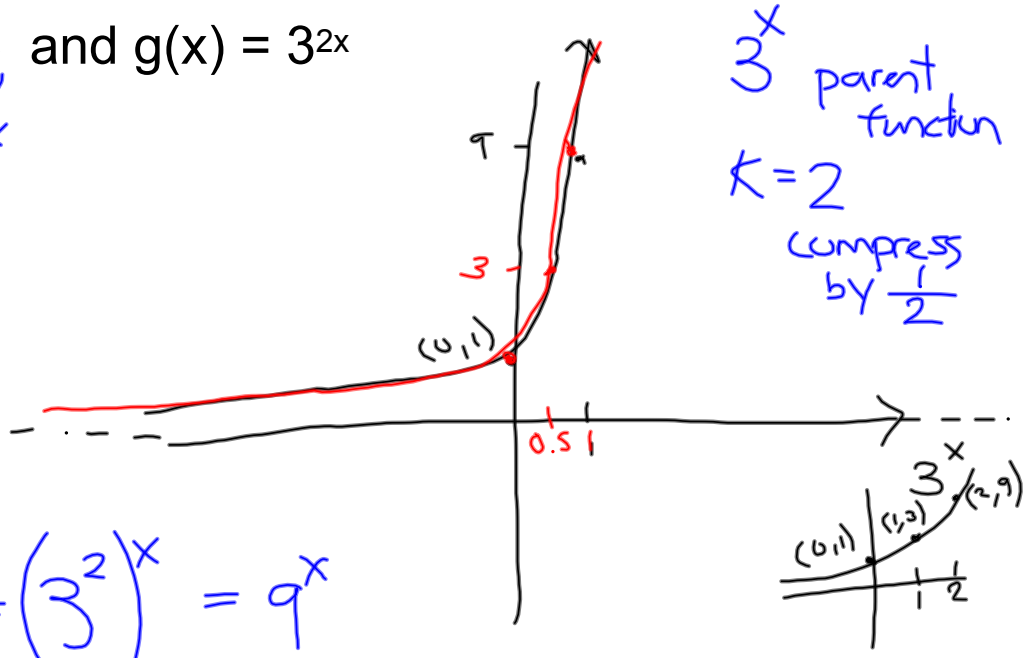


EX3:Compare and contrast the functions

$f(x) = 9^x$ and $g(x) = 3^{2x}$

b^x

x	$f(x) = 9^x$
0	$9^0 = 1$
1	$9^1 = 9$



3^x parent function

$k = 2$

compress by $\frac{1}{2}$

$3^{2x} = (3^2)^x = 9^x$

EX4: An exponential function with a base of 2 has been stretched vertically by a factor of 1.5 and reflected in the y-axis. Its asymptote is the line y=2. Write an equation for the new function and its domain and range.

parent function $y=2^x$

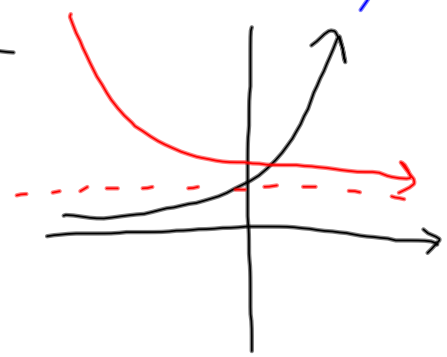
$y=ab^{k(x-d)}+c$

$a=1.5$

$k=-1$

$c=2$

$y=(1.5)2^{-(x)}+2$



domain: $\{x \in \mathbb{R}\}$

range: $\{y \in \mathbb{R} \mid y>2\}$

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