# Exam Review Part 2b – Transformations of Functions

*MCR3U*

SOLUTIONS

## Section 1: Transformations of $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$

**1)** Below is the graph of $f(x) = x^2$. Describe the transformations to a) and b) and use transformations to graph them.

**a)** $f(x) = -(x+6)^2 + 4$

- vertical reflection $(-y)$
- shift left 6 $(x-6)$
- shift up 4 $(y+4)$

$f(x) = x^2$

| $x$ | $y$ |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

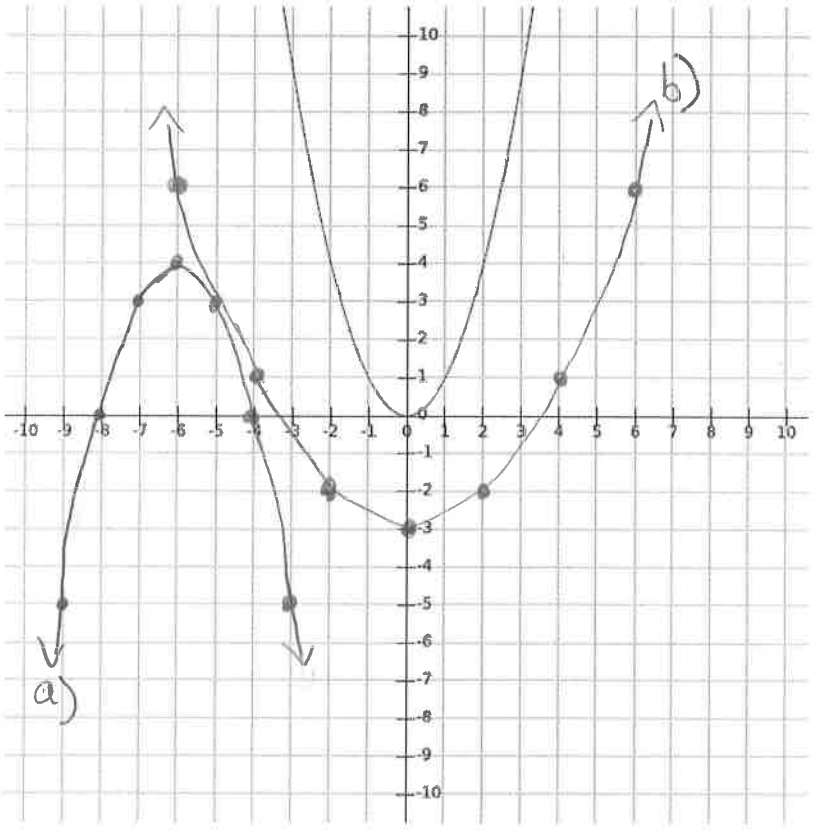
$f(x) = -(x+6)^2 + 4$

| $x-6$ | $-y+4$ |
|---|---|
| -9 | -5 |
| -8 | 0 |
| -7 | 3 |
| -6 | 4 |
| -5 | 3 |
| -4 | 0 |
| -3 | -5 |

**b)** $f(x) = \left(-\frac{1}{2}x\right)^2 - 3$

- horizontal reflection $(-x)$
- horizontal stretch b.af.o. 2 $(2x)$
- shift down 3 $(y-3)$

| $-2x$ | $y-3$ |
|---|---|
| 6 | 6 |
| 4 | 1 |
| 2 | -2 |
| 0 | -3 |
| -2 | -2 |
| -4 | 1 |
| -6 | 6 |



$g(x) = af[k(x-d)]+c$

**2)** For the function $f(x) = \sqrt{x}$, write the new function equation for each transformation.

**a)** translation up 4 and right 9.

$c=4$
$d=9$

$g(x) = \sqrt{x-9} + 4$

**b)** vertical stretch by 6 and translation left 5.

$a=6$
$d=-5$

$g(x) = 6\sqrt{x+5}$

**c)** horizontal reflection in the y-axis and horizontal compression by $\frac{1}{1}$.

$k=-4$

$g(x) = \sqrt{-4x}$

**3)** Write the new function for the following description, given that the transformations are applied to the parent function $f(x) = x^2$.

Vertical stretch by 2, horizontal stretch by 3, vertical reflection over the x-axis, a vertical translation 2 units up and a horizontal translation 6 units left.

$a=-2$
$k=\frac{1}{3}$
$d=-6$
$c=2$

$g(x) = -2\left[\frac{1}{3}(x+6)\right]^2 + 2$

**4)** List all the transformations, in words, of $f(x)$ for each of the following functions.

**a)** $g(x) = -f(x-3) - 4$

- vertical reflection
- shift right 3
- shift down 4

**b)** $h(x) = -\frac{1}{3}f(2x) + 10$

- vertical reflection
- vertical compression bafo $\frac{1}{3}$
- horizontal compression bafo $\frac{1}{2}$
- shift up 10 units.

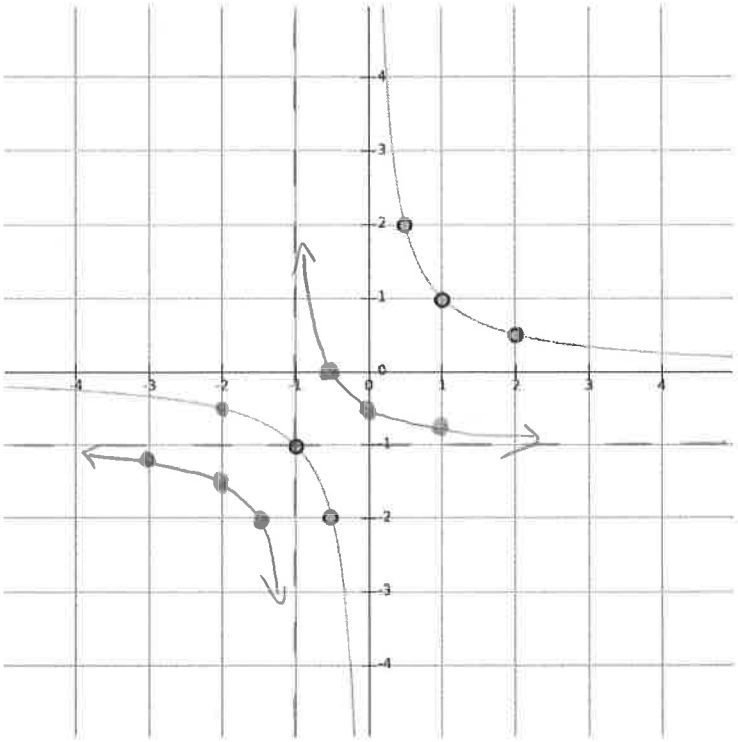
**c)** $j(x) = 5f(x+4) - 5$

- vertical stretch bafo 5
- shift left 4
- shift down 5

**d)** $k(x) = -2f\left(-\frac{1}{6}x\right) + 6$

- vertical reflection
- vertical stretch bafo 2
- horizontal reflection
- horizontal stretch bafo 6
- shift up 6.

**5)** Graph $g(x) = \frac{1}{2}f(x+1) - 1$ using transformations to the function $f(x) = \frac{1}{x}$ that is shown.



$f(x)$

| $x$ | $y$ |
|---|---|
| -2 | -0.5 |
| -1 | -1 |
| -0.5 | -2 |
| 0 | undefined |
| 0.5 | 2 |
| 1 | 1 |
| 2 | 0.5 |

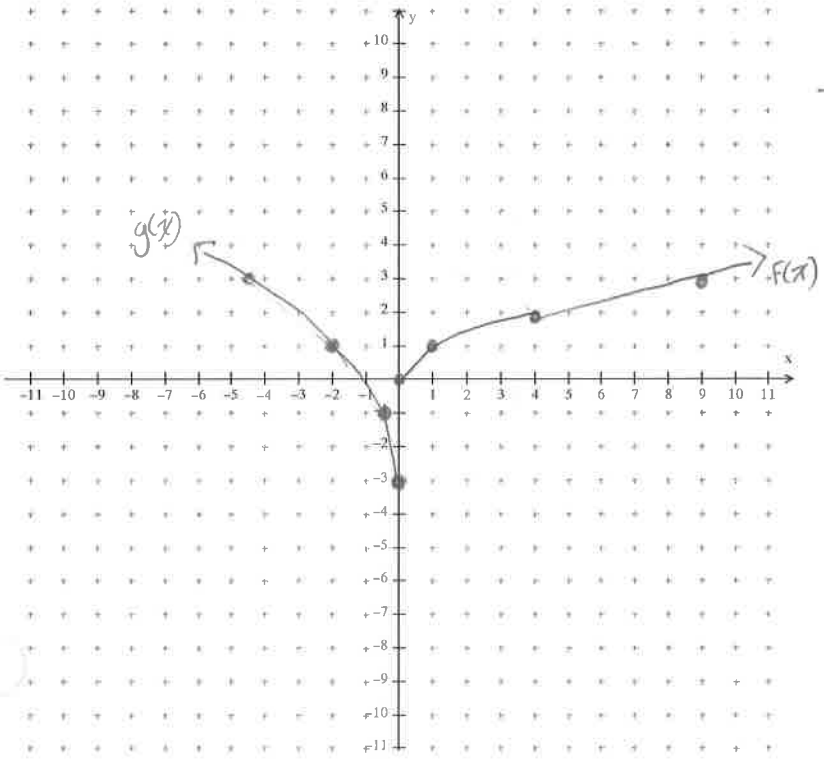
$g(x)$

| $x-1$ | $\frac{y}{2}-1$ |
|---|---|
| -3 | -1.25 |
| -2 | -1.5 |
| -1.5 | -2 |
| -1 | undefined |
| -0.5 | 0 |
| 0 | -0.5 |
| 1 | -0.75 |

Horizontal asymptote at $y=c \Rightarrow y=-1$

Vertical asymptote at $x=d \Rightarrow x=-1$

**6)** $f(x) = \sqrt{x}$. Graph $g(x) = 2f(-2x) - 3$ using transformations.



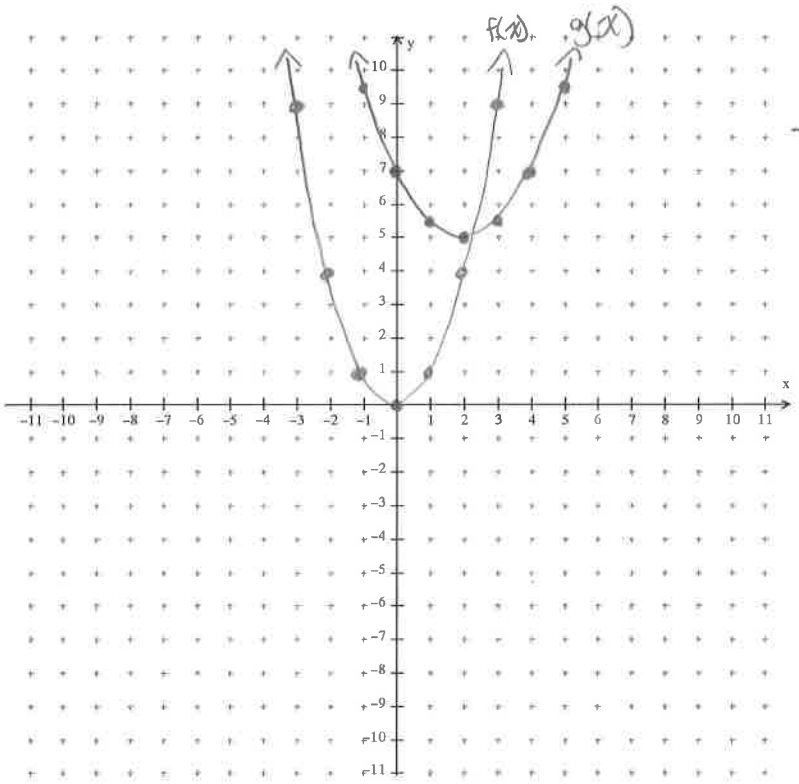

$f(x)$

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

$g(x)$

| $\frac{x}{-2}$ | $2y-3$ |
|---|---|
| 0 | -3 |
| -0.5 | -1 |
| -2 | 1 |
| -4.5 | 3 |

**7)** Graph the parent function of $g(x) = \frac{1}{2}(x-2)^2 + 5$ and $g(x)$ using transformations.



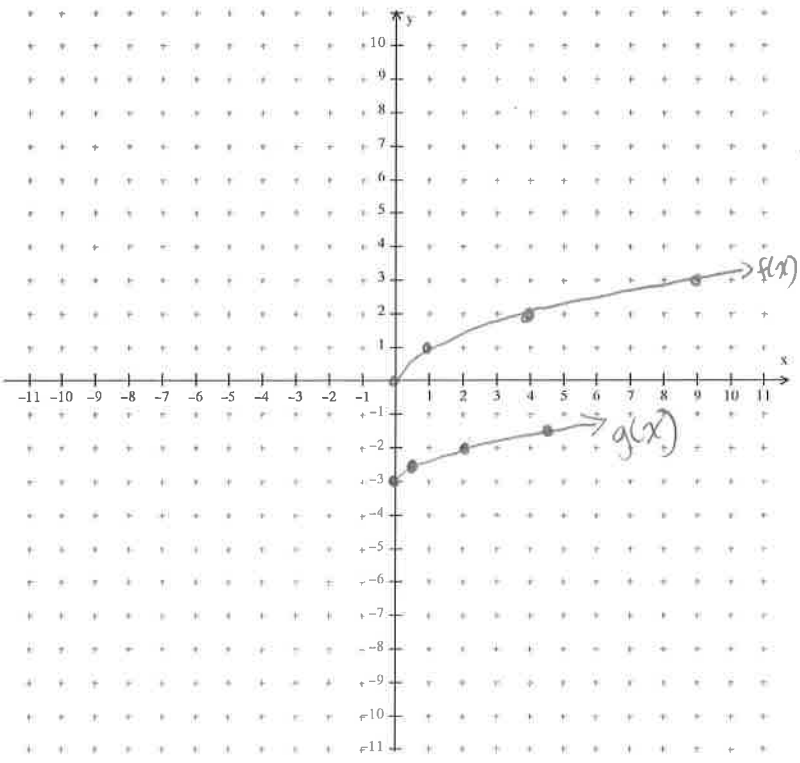

$f(x) = x^2$

| $x$ | $y$ |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$g(x) = \frac{1}{2}(x-2)^2 + 5$

| $x+2$ | $\frac{y}{2}+5$ |
|---|---|
| -1 | 9.5 |
| 0 | 7 |
| 1 | 5.5 |
| 2 | 5 |
| 3 | 5.5 |
| 4 | 7 |
| 5 | 9.5 |

**8)** Graph the parent function of $h(x) = \frac{1}{2}\sqrt{2x} - 3$ and $h(x)$ using transformations.




$f(x) = \sqrt{x}$

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

$g(x) = \frac{1}{2}\sqrt{2x} - 3$

| $\frac{x}{2}$ | $\frac{y}{2}-3$ |
|---|---|
| 0 | -3 |
| 0.5 | -2.5 |
| 2 | -2 |
| 4.5 | -1.5 |

## Section 2: Inverse of a Function

**9)** For each function listed below, determine the equation of the inverse, $f^{-1}(x)$.

**a)** $f(x) = 3x + 9$

$y = 3x + 9$

$x = 3y + 9$

$x - 9 = 3y$

$\frac{x-9}{3} = y$

$f^{-1}(x) = \frac{x-9}{3}$

**b)** $f(x) = \frac{1}{3}x^2 - 4$

$y = \frac{1}{3}x^2 - 4$

$x = \frac{1}{3}y^2 - 4$

$x + 4 = \frac{1}{3}y^2$

$3(x+4) = y^2$

$\pm\sqrt{3(x+4)} = y$

$f^{-1}(x) = \pm\sqrt{3(x+4)}$

**10)** Determine the equation of the inverse of $f(x) = 2x^2 + 16x + 30$ by first completing the square.

$f(x) = 2(x^2+8x) + 30$

$f(x) = 2(x^2+8x+16-16) + 30$

$f(x) = 2(x^2+8x+16) - 32 + 30$

$f(x) = 2(x+4)^2 - 2$

$f(x) = 2(x+4)^2 - 2$

$y = 2(x+4)^2 - 2$

$x = 2(y+4)^2 - 2$

$\frac{x+2}{2} = (y+4)^2$

$\pm\sqrt{\frac{x+2}{2}} = y+4$

$-4 \pm \sqrt{\frac{x+2}{2}} = y$

$f^{-1}(x) = -4 \pm \sqrt{\frac{x+2}{2}}$

**11)** Calculate the inverse of $f(x) = 2(x-1)^2 + 2$.

$y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$\frac{x-2}{2} = (y-1)^2$

$\pm\sqrt{\frac{x-2}{2}} = y-1$

$1 \pm \sqrt{\frac{x-2}{2}} = y$

$f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$

**12)** Graph $f(x)$ from the previous question and its inverse below.

$y = x^2$

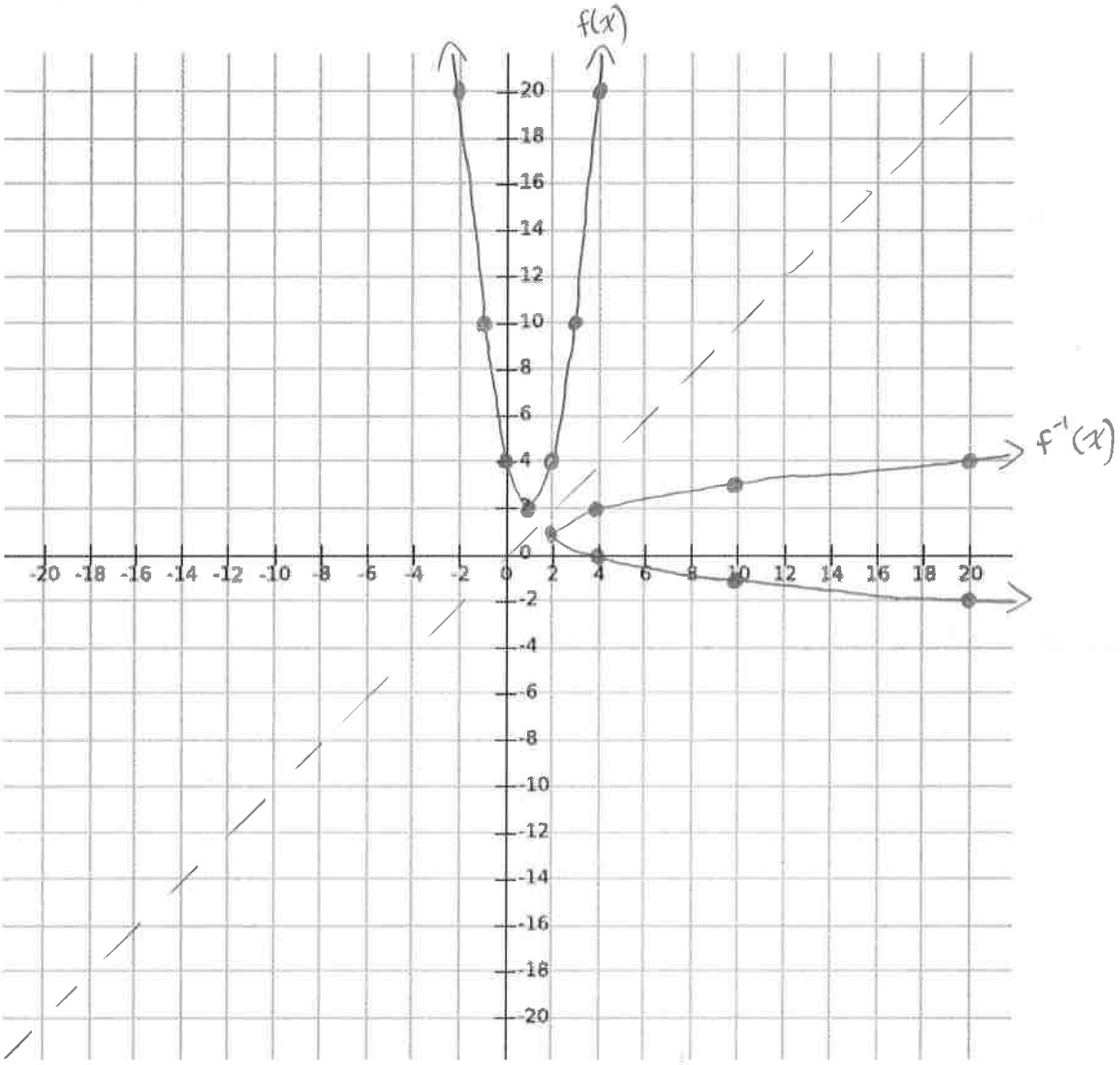
| $x$ | $y$ |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$f(x) = 2(x-1)^2 + 2$

| $x+1$ | $2y+2$ |
|---|---|
| -2 | 20 |
| -1 | 10 |
| 0 | 4 |
| 1 | 2 |
| 2 | 4 |
| 3 | 10 |
| 4 | 20 |

$f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$

| $x$ | $y$ |
|---|---|
| 20 | -2 |
| 10 | -1 |
| 4 | 0 |
| 2 | 1 |
| 4 | 2 |
| 10 | 3 |
| 20 | 4 |



## Answers

**1)** see posted solutions

**2) a)** $g(x) = \sqrt{x-9} + 4$ **b)** $g(x) = 6\sqrt{x+5}$ **c)** $g(x) = \sqrt{-4x}$

**3)** $g(x) = -2\left|\frac{1}{3}(x+6)\right|^2 + 2$

**4) a)** vertical reflection, right 3 units, down 4 units
**b)** vertical reflection, vertical compression bafo $\frac{1}{3}$, horizontal compression bafo $\frac{1}{2}$, up 10 units
**c)** vertical stretch bafo 5, left 4 units, down 5 units
**d)** vertical reflection, vertical stretch bafo 2, horizontal reflection, horizontal stretch bafo 6, up 6 units

**5)** through **8)** check posted solutions

**9) a)** $f^{-1}(x) = \frac{x-9}{3}$ **b)** $f^{-1}(x) = \pm\sqrt{3(x+4)}$

**10)** $f^{-1}(x) = -4 \pm \sqrt{\frac{x+2}{2}}$

**11)** $f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$

**12)** See posted solutions