# Exam Review Part 4 – Discrete Functions

MCR3U

SOLUTIONS

**1)** Find the formula for the general term $t_n$ and then use it to calculate $t_{12}$ for each of the following sequences:

**a)** $9, \overset{+6}{15}, 21, \ldots$

$a=9$
$d=6$

$t_n = 9+(n-1)(6)$

$t_{12} = 9+(12-1)(6)$

$t_{12} = 75$

**b)** $-1, \overset{\times(-2)}{2}, -4, 8, \ldots$

$a=-1$
$r=-2$

$t_n = -1(-2)^{n-1}$

$t_{12} = -1(-2)^{12-1}$

$t_{12} = 2048$

**2)** Determine the general term for each of these sequences. Are they arithmetic, geometric or neither?

**a)** $1, \overset{+3}{4}, 7, 10, 13$ arithmetic

$a=1$
$d=3$

$t_n = 1+(n-1)(3)$

**b)** $2187, \overset{\times 1/3}{729}, 243, 81, 27$ geometric.

$a=2187$
$r=\frac{1}{3}$

$t_n = 2187\left(\frac{1}{3}\right)^{n-1}$

**3)** For those sequences which are arithmetic or geometric in question ~~3~~ 2:

**i)** determine the value of the 10th term, $t_{10}$

a) $t_{10} = 1+(10-1)(3)$

$t_{10} = 28$

b) $t_{10} = 2187\left(\frac{1}{3}\right)^{10-1}$

$t_{10} = \frac{1}{9}$

**ii)** determine the sum of the series up to the 12th term, $S_{12}$.

a) $S_{12} = \frac{12}{2}\left[2(1)+(12-1)(3)\right]$

$= 6(35)$

$= 210$

b) $S_{12} = \dfrac{2187\left[\left(\frac{1}{3}\right)^{12}-1\right]}{\frac{1}{3}-1}$

$= \dfrac{\left(\frac{-531440}{243}\right)}{\left(\frac{-2}{3}\right)}$

$= \dfrac{265720}{81}$

**4)** In an arithmetic series of 50 terms, the 17th term is 53 and the 28th term is 86. Determine, a, d and $S_{50}$.

$t_{17} = 53$

$53 = a + 16d$

$t_{28} = 86$

$86 = a + 27d$

$$\begin{array}{r} 86 = a + 27d \\ 53 = a + 16d \quad - \\ \hline 33 = 11d \end{array}$$

$d = 3$

$86 = a + 27(3)$

$86 = a + 81$

$a = 5$

$S_{50} = \frac{50}{2}\left[2(5) + (50-1)(3)\right]$

$= 25(157)$

$= 3925$

**5)** In an arithmetic series, the 12th term is 15 and the sum of the first 15 terms is 105. Determine the sum of the first three terms in the series.

$t_{12} = 15$

$15 = a + 11d$

$S_{15} = 105$

$105 = \frac{15}{2}\left[2a + 14d\right]$

$105 = 15(a + 7d)$

$7 = a + 7d$

$$\begin{array}{r} 15 = a + 11d \\ 7 = a + 7d \quad - \\ \hline 8 = 4d \end{array}$$

$d = 2$

$7 = a + 7(2)$

$7 = a + 14$

$a = -7$

$S_3 = \frac{3}{2}\left[2(-7) + (3-1)(2)\right]$

$S_3 = -15$

**6)** The fifth term of a geometric series is 405 and the sixth term is 1215. Find the sum of the first nine terms.

$t_5 = 405$

① $405 = a(r)^4$

$a = \frac{405}{r^4}$

$t_6 = 1215$

② $1215 = a(r)^5$

sub 'a' from ① into ②

$1215 = \frac{405}{r^4}(r)^5$

$1215 = 405r$

$r = 3$

$a = \frac{405}{r^4}$

$a = \frac{405}{3^4}$

$a = 5$

$S_9 = \frac{5\left[(3)^9 - 1\right]}{3-1}$

$S_9 = 49205$

**7)** Find the sum of each of the following series:

**a)** $251 \overset{-8}{+} 243 + 235 + \ldots -205$

$a = 251$

$d = -8$

$t_n = 251 + (n-1)(-8)$

$-205 = 251 + (n-1)(-8)$

$-456 = (n-1)(-8)$

$57 = n-1$

$n = 58$

$S_{58} = \frac{58}{2}(251 - 205)$

$= 1334$

**b)** $-4 \overset{\times 3}{-} 12 - 36 - \ldots -8748$

$a = -4$

$r = 3$

$t_n = -4(3)^{n-1}$

$-8748 = -4(3)^{n-1}$

$2187 = 3^{n-1}$

$\boxed{\frac{\log 2187}{\log 3} = 7}$

$3^7 = 3^{n-1}$

$7 = n-1$

$n = 8$

$S_8 = \frac{-4(3^8 - 1)}{3-1}$

$S_8 = -13\,120$

**c)** $21 \overset{+2}{+} 23 + 25 + \ldots + 43$

$a = 21$

$d = 2$

$t_n = 21 + (n-1)(2)$

$43 = 21 + (n-1)(2)$

$22 = (n-1)(2)$

$11 = n-1$

$n = 12$

$S_{12} = \frac{12}{2}(21 + 43)$

$S_{12} = 384$

**d)** $1280 \overset{\times(-\frac{1}{2})}{-} 640 + 320 - \ldots + 5$

$a = 1280$

$r = -\frac{1}{2}$

$t_n = 1280\left(-\frac{1}{2}\right)^{n-1}$

$5 = 1280\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{256} = \left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^8 = \left(-\frac{1}{2}\right)^{n-1}$

$8 = n-1$

$n = 9$

$S_9 = \frac{1280\left[\left(-\frac{1}{2}\right)^9 - 1\right]}{-\frac{1}{2} - 1}$

$S_9 = 855$

**8)** Write the first 4 term of each of the following sequences:

a) $t_1 = -6$; $t_n = t_{n-1} + 5$

$t_1 = -6$

$t_2 = -6+5 = -1$

$t_3 = -1+5 = 4$

$t_4 = 4+5 = 9$

$\boxed{-6, -1, 4, 9}$

b) $t_1 = -2$; $t_2 = -1$; $t_n = t_{n-1} \times t_{n-2}$

$t_1 = -2$

$t_2 = -1$

$t_3 = (-1)(-2) = 2$

$t_4 = 2(-1) = -2$

$\boxed{-2, -1, 2, -2}$

**9)** Determine the recursive formula of each of these sequences. ~~Are they arithmetic, geometric or neither?~~

**a)** 1, 1, 2, 3, 5, 8,...

$t_n = t_{n-1} + t_{n-2}$

**b)** 3, 8, 13, 18, 23, 28, 33, 38

$t_n = t_{n-1} + 5$

**10)** In an arithmetic sequence, the 3rd term is 25 and the 9th term is 43. How many terms are less than 100?

$t_3 = 25$ $t_9 = 43$

$25 = a + 2d$ $43 = a + 8d$

$43 = a + 8d$
$25 = a + 2d$ $-$
$18 = 6d$
$d = 3$

$43 = a + 8(3)$
$a = 19$

$t_n = 19 + (n-1)(3)$

$100 = 19 + (n-1)(3)$

$81 = (n-1)(3)$

$27 = n-1$

$n = 28$

$\boxed{\therefore 27 \text{ terms are } < 100.}$

**11)** The sum of the first 6 terms is 297 and the sum of the first 8 terms is 500. Determine the 5th term if the sequence is arithmetic.

$S_6 = 297$

$297 = \frac{6}{2}[2a + (6-1)d]$

$297 = 3(2a+5d)$

$99 = 2a+5d$

$S_8 = 500$

$500 = \frac{8}{2}[2a+(8-1)d]$

$500 = 4(2a+7d)$

$125 = 2a+7d$

$t_n = 17 + (n-1)(13)$

$t_5 = 17 + (5-1)(13)$

$t_5 = 69$

$$\begin{array}{r} 99 = 2a+5d \\ 125 = 2a+7d \\ \hline -26 = -2d \end{array}$$

$d = 13$

$125 = 2a + 7(13)$

$34 = 2a$

$a = 17$

**12)** For $(1-x)^{11}$: use row 11 of Pascal's triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
⋮
1 11 55 165 330 462 462 330 165 55 11 1

**a)** find $t_3$

$= 55(1)^9(-x)^2$

$= 55x^2$

**b)** how many terms are in the expansion?

12

**c)** explain where the numerical coefficients of the expansion are coming from

Row 11 of Pascal's Triangle.

**13)** Expand $(x^2 - 2y)^4$ using binomial theorem; take the coefficients from Pascal triangle n = 4

$= 1(x^2)^4(-2y)^0 + 4(x^2)^3(-2y)^1 + 6(x^2)^2(-2y)^2 + 4(x^2)^1(-2y)^3 + 1(x^2)^0(-2y)^4$

$= x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$

**14)** Expand $(4x + 2x^3)^3$ using binomial theorem.

1
1 1
1 2 1
1 3 3 1

$= 1(4x)^3(2x^3)^0 + 3(4x)^2(2x^3)^1 + 3(4x)^1(2x^3)^2 + 1(4x)^0(2x^3)^3$

$= 64x^3 + 96x^5 + 48x^7 + 8x^9$

**Answers**

**1) a)** $t_n = 9 + (n-1)6\,;\, t_{12} = 75$ **b)** $t_n = -1(-2)^{n-1}\,;\, t_{12} = 2048$

**2) a)** arithmetic; $t_n = 1 + (n-1)3$ **b)** geometric; $t_n = 2187\left(\frac{1}{3}\right)^{n-1}$

**3) i) a)** $t_{10} = 28$ **b)** $t_{10} = \frac{1}{9}$ **ii) a)** $S_{12} = 210$ **b)** $S_{12} = \frac{265720}{81}$

**4)** $S_{50} = 3925$

**5)** $S_3 = -15$

**6)** $S_9 = 49205$

**7) a)** $S_{58} = 1334$ **b)** $S_8 = -13120$ **c)** $S_{12} = 384$ **d)** $S_9 = \cancel{2555}\ 855$

**8) a)** -6, -1, 4, 9 **b)** -2, -1, 2, -2

**9) a)** $t_n = t_{n-1} + t_{n-2}$ **b)** $t_n = t_{n-1} + 5$

**10)** 27

**11)** $t_5 = 69$

**12) a)** $55x^2$ **b)** 12 **c)** 11th row of Pascal's triangle

**13)** $x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$

**14)** $64x^3 + 96x^5 + 48x^7 + 8x^9$