7.2 Geometric Sequences

May 28

– HAVE A COMMON RATIO, r.

ex. 1, 2, 4, 8, 16, 32, ..., 2^{n-1}

$\times 2 \quad \times 2 \quad \times 2$

$t_n \quad 2^x$ 1, 2, 4, 8

$a = 1, \; r = 2$

ex. 64, 32, 16, 8, 4, 2, 1, 1/2, 1/4 ,1/8..... $64\left(\frac{1}{2}\right)^{n-1}$

$a = 1, \; r = \frac{1}{2}$

GENERAL TERM $\boxed{t_n = a r^{n-1}}$ t_n

ex. Determine the 10th term of each sequence 3,-4.5,6.75,-10.125,15.1875......

$\times(-1.5)$

$a = 3, \; r = -1.5$

$t_n = a r^{n-1}$

$= 3(-1.5)^{n-1}$

$t_{10} = 3(-1.5)^9$

$= -115.3300781$

$$1, \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{8}}, \frac{1}{4}, \frac{1}{\sqrt{32}}, ...$$

$a = 1, \; r = \frac{1}{\sqrt{2}}$

$t_n = 1\left(\frac{1}{\sqrt{2}}\right)^{n-1}$

$t_{10} = 1\left(\frac{1}{\sqrt{2}}\right)^9 = \frac{1}{16\sqrt{2}}$

$\sqrt{2}^9 = \sqrt{2}^8 \cdot \sqrt{2}$

$= 2^4 \sqrt{2}$

$= 16\sqrt{2}$

A company has 3 kg of radioactive material that must be stored until it becomes safe to the environment. After one year, 95% of the radioactive material remains. How much radioactive material will be left after 100 years?

t_{101}

$3, \underset{}{\overset{t_2}{3(.95)}}, 3(.95)^2, 3(.95)^3 \dots$

$a = 3, \ r = .95$

$t_n = ar^{n-1}$

$t_n = 3(.95)^{n-1}$

$t_{101} = 3(.95)^{100} = \underline{0.01776 \text{ kg}}$

How many terms are in the geometric sequence 52 612 659, 17 537 553, ... , 11?

$a = 52612659, \ r = \frac{1}{3}$

$t_n = ar^{n-1}$

$11 = 52612659\left(\frac{1}{3}\right)^{n-1}$

$\frac{11}{52612659} = \left(\frac{1}{3}\right)^{n-1}$

$n-1 = \dfrac{\log\left(\frac{11}{52612659}\right)}{\log\left(\frac{1}{3}\right)}$

$a^x = b$

$x = \dfrac{\log b}{\log a}$

$n - 1 = 14$

$\therefore \underline{n = 15}$

Half-life is the time required for a radioactive material to decay to one-half of its original mass. Uranium-238, used in nuclear reactors, has a very long half-life, 4.5×10^9 years. Only a small fraction of uranium-238 has decayed since the Earth was formed. However, carbon-11, used in medical applications, has a half-life of only 20 min. $\frac{1}{3}$ h

Determine the mass of carbon-11 remaining after 24 h.

if the original mass is 100mg

t_{73}

$\overset{t_1}{100}, \overset{t_2}{50}, 25, 12.5, 6.25, \dots$

$\frac{1}{3}$ h

$\dfrac{24 \text{ h}}{\frac{1}{3} \text{ h}} = 72$

$a = 100, \ r = \frac{1}{2}$

$t_n = 100\left(\frac{1}{2}\right)^{n-1}$

$t_{73} = 100\left(\frac{1}{2}\right)^{72} = 2.12 \times 10^{-20}$

$\therefore 2.12 \times 10^{-20}$ mg remaining after 24h.

For each geometric sequence, find a, r, and t_n.

$t_n = ar^{n-1}$

$t_4 = 54,\ t_7 = 1458$

$54 = ar^3$ ①

$1458 = ar^6$ ②

②÷① $27 = r^3 \longrightarrow r = 3$

$r \to$ ① $54 = a(3)^3 \longrightarrow a = 2$

$$\boxed{\therefore a = 2,\ r = 3 \quad t_n = 2(3)^{n-1}}$$

The first three terms of the sequence 8, a, b, 36 form an arithmetic sequence, but the last three terms form a geometric sequence. Find all the possible values of a and b.

$8, a, b$ ARITHMETIC $\qquad a, b, 36$ GEOMETRIC

$b - a = a - 8$ ① $\qquad \frac{36}{b} = \frac{b}{a}$ ②

$b - 2a = -8$ ①

$36a = b^2$ ② $\longrightarrow a = \frac{b^2}{36}$

$a \to$ ① $b - 2\left(\frac{b^2}{36}\right) = -8$

$b - \frac{b^2}{18} = -8 \qquad \times 18$

$18b - b^2 = -144$

$0 = b^2 - 18b - 144$

$0 = (b + 6)(b - 24)$

$b = -6, 24$

$b \to$ ② $a = \frac{b^2}{36} = \frac{(-6)^2}{36}$ or $\frac{(24)^2}{36}$

$a = 1$ or 16

$$\boxed{\therefore (a, b) = \{(1, -6), (16, 24)\}}$$

Find the tenth term of the sequence $\frac{a^2}{b}, -a, b, \frac{-b^2}{a}, \ldots$.

$a = \frac{a^2}{b} \qquad r = -\frac{b}{a}$

$t_n = ar^{n-1}$

$= \frac{a^2}{b}\left(-\frac{b}{a}\right)^{n-1}$

$t_{10} = \frac{a^2}{b}\left(-\frac{b}{a}\right)^9 = -\frac{b^8}{a^7}$

p. 430#3,4,6i,iii,10,11,18,19