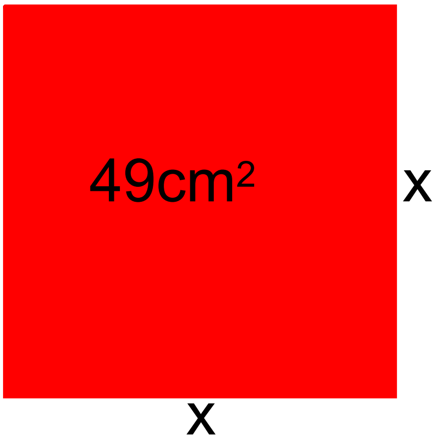
4.3 Rational Exponents



$A = 49$

$A = x \cdot x = x^2$

$x = A^n$

$A = x \cdot x = A^n \cdot A^n$

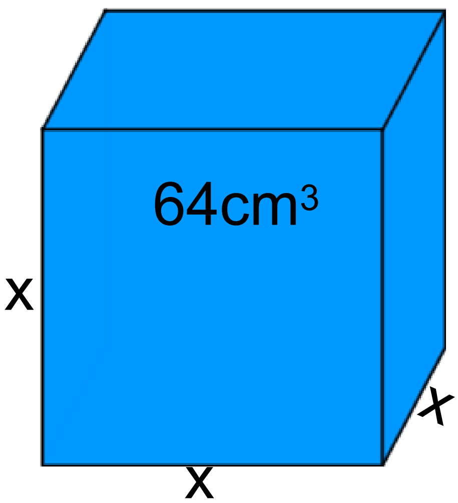
$A^1 = A^{2n}$

$1 = 2n$

$n = \frac{1}{2}$

$x = A^{\frac{1}{2}} = \sqrt{A}$

$x = \sqrt{49} = 7$



$V = 64$

$V = x \cdot x \cdot x = x^3$

$x = V^n$

$V = (V^n) \cdot (V^n)(V^n)$

$V^1 = V^{3n}$

$1 = 3n$

$n = \frac{1}{3}$

$x = V^{\frac{1}{3}}$

$x = \sqrt[3]{V}$

$x = \sqrt[3]{64} = 4$

Now we know:

$$X^{\frac{1}{2}} = \sqrt{X}$$

$$X^{\frac{1}{3}} = \sqrt[3]{X}$$

$$X^{\frac{1}{4}} = \sqrt[4]{X}$$

$$X^{\frac{1}{n}} = \sqrt[n]{X}$$

index

read as "n^{th} root of X"

radical

Do the rules of multiplying powers still apply if the exponents are rational?

Express the following in $\underbrace{\text{radical notation}}_{\text{A square root, cube root, higher root}}$. Then evaluate.

a) $49^{-\frac{1}{2}}$

$= \dfrac{1}{49^{\frac{1}{2}}}$ $\frac{1}{2} \to \sqrt[2]{\ }$

$= \dfrac{1}{\sqrt{49}}$ in radical notation

$= \dfrac{1}{7}$

b) $(-8)^{\frac{1}{3}}$ $\to \sqrt[3]{\ }$

$= \sqrt[3]{(-8)}$

$= -2$

because $(-2)(-2)(-2) = -\#$

Note: we can take $\sqrt[3]{-\#}$

we can't take $\sqrt{-\#}$

because $(-\#)(-\#) = +$

In general, if the index is even we can't have a negative # inside radical.

If index is odd, we can have a neg. inside the radical.

c) $10{,}000^{\frac{1}{4}}$ $\to \sqrt[4]{\ }$

$= \sqrt[4]{10{,}000}$

$= 10$

Evaluating a power with a rational exponent

a) $27^{\frac{2}{3}}$ exponent radical

$= 27^{2\cdot\frac{1}{3}}$
$= (27^2)^{1/3}$
$= \sqrt[3]{(27^2)}$
$= \sqrt[3]{729}$
$= 9$

OR $= 27^{1/3\cdot 2}$
$= (\sqrt[3]{27})^2$
$= (3)^2$
$= 9$

b) $(-27)^{\frac{4}{3}}$

$= (-27)^{4\cdot 1/3}$
$= (-27)^{1/3\cdot 4}$
$= ((-27)^{1/3})^4$
$= (\sqrt[3]{(-27)})^4$
$= (-3)^4$
$= 81$

c) $(16)^{-0.75}$

$= (16)^{-\frac{3}{4}}$
$\frac{1}{(16)^{3/4}}$
$= \frac{1}{16^{3\cdot 1/4}}$
$= \frac{1}{(16^{1/4})^3}$
$= \frac{1}{(\sqrt[4]{16})^3}$
$= \frac{1}{(2)^3} = \frac{1}{8}$

Simplifying rational exponents

a) $\dfrac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}}$

$= \dfrac{8^{\frac{5}{6}} \cdot 8^{\frac{1}{2}}}{8^{5/3}}$

$= \dfrac{8^{\frac{5}{6}+\frac{1}{2}}}{8^{5/3}}$

$= \dfrac{8^{\frac{5}{6}+\frac{3}{6}}}{8^{5/3}}$

$= \dfrac{8^{\frac{8}{6}}}{8^{5/3}}$

$= 8^{\frac{8}{6}-\frac{5}{3}} \cancel{\frac{3}{3}}$

$= 8^{\frac{4}{3}-\frac{5}{3}}$

$= 8^{-\frac{1}{3}}$

$= \dfrac{1}{8^{1/3}}$

$= \dfrac{1}{\sqrt[3]{8}}$

b) $\dfrac{\sqrt{75}\ 5^{-\frac{1}{2}}}{3^3}$

$= \dfrac{\sqrt{25 \cdot 3}\ 5^{-1/2}}{3^3}$

$= \dfrac{5\sqrt{3} \cdot 5^{-1/2}}{3^3}$

$= \dfrac{5^1 \cdot 3^{1/2} \cdot 5^{-1/2}}{3^3}$

$= \dfrac{5^1 \cdot 5^{-1/2} \cdot 3^{1/2}}{3^3}$

$= 5^{1/2} \cdot 3^{1/2-3}$

$= 5^{1/2} \cdot 3^{1/2-\frac{6}{2}}$

$= 5^{1/2}\ 3^{-\frac{5}{2}}$

$= \dfrac{5^{1/2}}{3^{5/2}} = \dfrac{\sqrt{5}}{(\sqrt{3})^5}$

c) $\dfrac{6^{2.5}\,6^{-\frac{1}{2}}}{6}$

$= \dfrac{6^{\frac{5}{2}}\,6^{-\frac{1}{2}}}{6}$

$= \dfrac{6^{4/2}}{6} = \dfrac{6^2}{6} = 6$

Key Ideas for Rational Exponents:

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

indicates our index

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m \quad \underline{\text{OR}} \quad \sqrt[n]{\left(b^m\right)}$$

because $b^{\frac{m}{n}} = b^{m \cdot \frac{1}{n}} = b^{\frac{1}{n} \cdot m}$

Spot It!

- $\sqrt{16}$
- 8^2
- 6^{-1}
- $\left(\frac{1}{6}\right)^{-2}$

$\left(\frac{1}{6}\right)^{-2}$
$= 6^2$
$= 36$

- 36
- $(8)^{\frac{1}{3}}$
- 2
- $\sqrt{64}$

- 4
- $4^{\left(\frac{3}{2}\right)}$
- $\left(-\frac{1}{2}\right)^3$
- $\left(\frac{1}{4}\right)$

$4^{\frac{1}{2}\cdot 3}$
$\left(\sqrt{4}\right)^3$
$= 2^3 = 8$

- $9^{\left(\frac{1}{2}\right)}$
- $-2^2 = -4$
- $-\frac{1}{2}$
- 8

- $\left(-\frac{1}{2}\right)^3$
- $-\frac{1}{2}$
- $9^{\left(-\frac{1}{2}\right)}$
- $4^{\left(\frac{3}{2}\right)}$

$\frac{1}{9^{\frac{1}{2}}}$
$= \frac{1}{\sqrt{9}} = \frac{1}{3}$

- $\frac{1}{81}$
- $(8)^{\frac{1}{3}}$
- $\frac{1}{3}$
- 4

HMWK: pg. 229 # 4,6,8,10,13,15