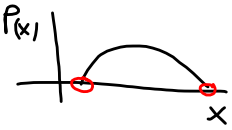
3.5 Quadratic Function Models

Mar 23

Anthony owns a business that sells parts for electronic game systems. The profit function for his business can be modelled by the equation $P(x) = -0.5x^2 + 8x - 24$, where x is the quantity sold, in thousands, and $P(x)$ is the profit in thousands of dollars.

How many parts must Anthony sell in order for his business to break even?



$P(x) = 0$

3 methods

① $-.5x^2 + 8x - 24 = 0$

BY FACTORING

$-0.5(x^2 - 16x + 48) = 0$

$-0.5(x - 4)(x - 12) = 0$

$x = 4, 12$

∴ BREAK EVEN WHEN 4000 AND 12 000 SYSTEMS ARE SOLD.

② $-.5x^2 + 8x - 24 = 0$

BY QUADRATIC FORMULA $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{8^2 - 4(-.5)(-24)}}{2(-.5)}$

$x = \frac{-8 \pm \sqrt{16}}{-1}$

$x = \frac{-8 \pm 4}{-1} = \frac{-12}{-1}, \frac{-4}{-1} = 4, 12$

③ $-.5x^2 + 8x - 24 = 0$

COMPLETE THE SQUARE.

$-.5(x^2 - 16x) - 24 = 0$

$-.5(x^2 - 16x + 64 - 64) - 24 = 0$

$-0.5(x - 8)^2 + 32 - 24 = 0$

$-0.5(x - 8)^2 + 8 = 0$

$-.5(x - 8)^2 = -8$

$(x - 8)^2 = 16$

$x - 8 = \pm 4$

$x = 8 \pm 4 = 4, 12$

A water balloon is catapulted into the air from the top of a building. The height, $h(t)$, in metres, of the balloon after t seconds is $h(t) = -5t^2 + 30t + 10$.

a) What are the domain and range of this function? REALISTIC

b) When will the balloon reach a height of 30 m?

a) ZEROS & VERTEX

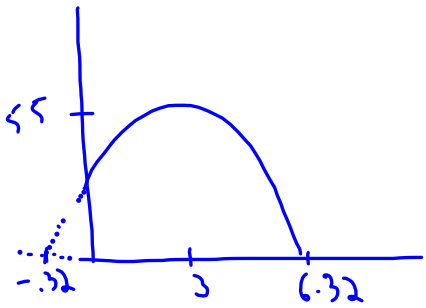
$h(t) = -5(t^2 - 6t - 2)$ DOESN'T FACTOR.

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

QF. $t = \frac{-30 \pm \sqrt{30^2-4(5)(10)}}{2(-5)}$

$t = -.32, 6.32$

VERTEX $t = \frac{-.32+6.32}{2} = 3$



$h(3) = -5(3)^2 + 30(3) + 10 = 55$

$\therefore D = \{t \in \mathbb{R} \mid 0 \le t \le 6.32\}$

$R = \{h \in \mathbb{R} \mid 0 \le h \le 55\}$

(b) $h(t) = 30$?

$30 = -5t^2 + 30t + 10$

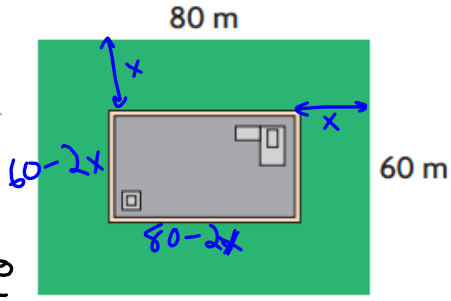
$5t^2 - 30t + 20 = 0$

$5(t^2 - 6t + 4) = 0 \rightarrow$ DOESN'T FACTOR.

QF $\rightarrow t = \frac{6 \pm \sqrt{6^2-4(4)}}{2} = 0.76, 5.24$

$\therefore$ HEIGHT IS 30 m AFTER .76 s AND 5.24 s

A factory is to be built on a lot that measures 80 m by 60 m. A lawn of uniform width, equal to the area of the factory, must surround it. How wide is the strip of lawn, and what are the dimensions of the factory?



$(80-2x)(60-2x) = \frac{60\times 80}{2}$

OR

$80\times 60 - (80-2x)(60-2x) = \frac{60\times 80}{2}$

$4800 - 160x - 120x + 4x^2 = 2400$

$4x^2 - 280x + 2400 = 0$

$4(x^2 - 70x + 600) = 0$

$4(x-60)(x-10) = 0$

OR QF

$x = \frac{70 \pm \sqrt{70^2 - 4(600)}}{2}$

$= \frac{70 \pm \sqrt{2500}}{2}$

$= 60, 10$

$\therefore x = 10, 60$, BUT $x < 30$

$\therefore x = 10$

$\therefore$ STRIP IS 10 m WIDE

FACTORY DIMENSIONS ARE 40 x 60 m.

Homework p.177 #4,5,6ac,7,8c,9-12,14