MCR 3U

Unit 5, Lesson 3: Solving Problems with Exponential Functions (Compound Interest)

When you **deposit** money in a bank account, you are actually lending money to the bank. In return, the bank pays out money to you in the form of interest. When you **borrow** money from the bank, you must pay them a fee, in the form of interest, for the convenience of borrowing.

Most banks use **compound interest,** which is interest that is added to the principal *before* new interest earned is calculated. This is done at regular intervals, called **compounding periods**.

The equation for compound interest is given by $A = P\left(1 + \frac{r}{n}\right)^{tn}$, where:

- A is the future value (the total value of an investment or loan)
- P is the principal (a sum of money that is borrowed or invested)
- i is the annual interest rate for the period (expressed as a decimal)
- n is the number of compounding periods (see chart)

t is total # of years.

To calculate the total interest, I, use the formula $I = A - P$

Compounding frequency	n
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Bi-Weekly	26
Weekly	52
Daily	365

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Ex 1) Complete the chart

Term	Annual Interest Rate, r (%)	Compounding Frequency	Principal, P	Future Value, A ($)	Compound Interest Earned, I ($)
$t=$ 3 years	4% $r=0.04$	Semi-annually $n=2$	$5100	$A=5100\left(1+\frac{0.04}{2}\right)^{3\times2}$ $A=5743.43\$$	$5743.43-5100.00$ $643.43\$$
$t=$ 2 years	$r=0.052$ 5.2%	$n=52$ weekly	$550	$610.25\$$	$60.25\$$
21 months $1\text{ year}=12\text{ month}$ 21 month $t=1.75\text{ years}$	$r=0.1055$ 10.55%	$n=4$ quarterly	$2000	$2399.79\$$	$399.79\$$

Using the compound interest formula, we can solve for the principal or annual interest of an investment or loan by using inverse operations.

"Per annum"

Ex 2) Jordan's brother borrows some money from him and he is charged 20%/a compounded semi-annually. He repays Jordan $512 after 2 years. How much did he borrow?

$A = 512\$$

$P = ?$

$r = 0.2$

$n = 2$

$t = 2$

$A = P\left(1 + \frac{r}{n}\right)^{tn}$

$512 = P\left(1 + \frac{0.2}{2}\right)^{2\times 2}$

$P = 349.70$

$\therefore$ he borrowed 349.70$.

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Ex 3) Chris has $2500 to invest right now. He hopes to have $3500 in 3 years. At what annual interest rate, compounded monthly, must he invest his principal?

$A = 3500$

$P = 2500$

$r = ?$

$n = 12$

$t = 3$

$A = P\left(1 + \frac{r}{n}\right)^{tn}$

$3500 = 2500\left(1 + \frac{r}{12}\right)^{3 \times 12}$

$\frac{3500}{2500} = \left(1 + \frac{r}{12}\right)^{36}$

$\sqrt[36]{\frac{3500}{2500}} = 1 + \frac{r}{12}$

$\left(\sqrt[36]{\frac{3500}{2500}} - 1\right) \times 12 = r$

$0.113 = r$

$\therefore$ the interest rate is 11.3%

Ex 4) Jackie is looking to borrow $4000. She is offered 2 options ✱ assume 1 year ✱

- An annual interest rate of 5.5%, compounded semi-annually, or
- An annual interest rate of 5.45%, compounded weekly.

Which has a lower cost of borrowing?

5.5% semi-annual

$A = 4000\left(1+\frac{0.055}{2}\right)^{1\times 2}$

$A = 4223.03$

$I = 223.03\$$

5.45% weekly

$A = 4000\left(1+\frac{0.0545}{52}\right)^{52}$

$A = 4223.93$

$I = 223.93\$$

∴ The 5.5% semi annual has a lower cost of borrowing.