3.8 Linear-Quadratic Systems

Each shoe represents a digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). The style of shoe always represents the same digit in all of the number sentences. Examine the number sentences.

5, 7

Pair each shoe with a digit. Explain your reasoning.

Systems of linear equations

How would I solve 3x +4y = 26 and -x +5y = 23?

Elimination:

$$3x+4y=26$$
$$3(-x+5y=23)$$

$$\begin{array}{r} 3x+4y=26 \\ + \; -3x+15y=69 \\ \hline 0x \quad 19y=95 \\ y=5 \end{array}$$

$$-x+5(5)=23$$
$$-x+25=23$$
$$x=2$$

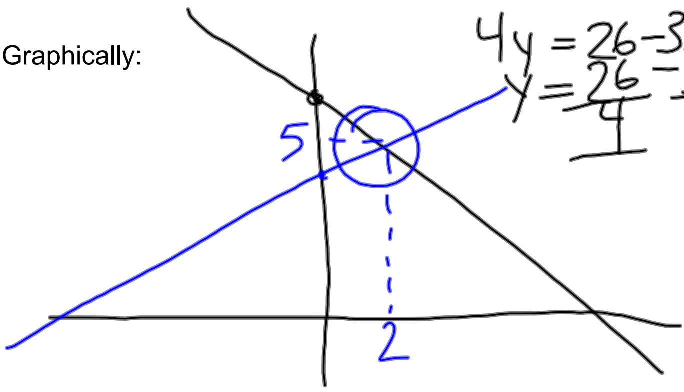
Substitution:

$$3x+4y=26$$
$$x=5y-23$$

$$3(5y-23)+4y=26$$
$$15y-69+4y=26$$
$$19y=95$$
$$y=5$$

$$x=2$$

Graphically:



$$4y=26-3x$$
$$y=\frac{26}{4}-\frac{3}{4}x$$

$$5y=x+23$$
$$y=\frac{x}{5}+\frac{23}{5}$$

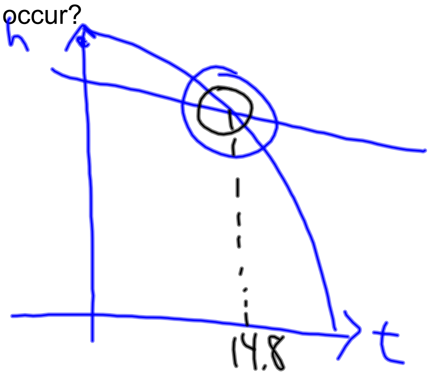
Linear - Quadratic Systems

Adam has decided to celebrate his birthday by going skydiving. He loves to freefall, so he will wait for some time before opening his parachute.

His height after jumping from the airplane before he opens his parachute can be modelled by the quadratic function $h_1(t) = -4.9t^2 + 5500$

After he releases his parachute, he begins falling at a constant rate. His height above ground can be modelled by the linear function $h_2(t) = -5t + 4500$

How long after jumping out of the airplane should Adam release his parachute? At what height will this occur?



$$-4.9t^2 + 5500 = -5t + 4500$$

$$-4.9t^2 + 5t + 1000 = 0$$

$$t = \frac{-5 \pm \sqrt{25 - 4(-4.9)(1000)}}{2(-4.9)}$$

$$t = \cancel{-13.76}, \ 14.8$$

∴ Adam should release parachute after 14.8 sec when he is at the height of 4426m.

$$h(t) = -5(14.8) + 4500$$
$$= 4426\text{m}$$

Determine the number of points of intersection of the following two functions:

$f(x) = 3x^2 + 12x + 14$

$g(x) = 2x-8$

if $D = b^2-4ac > 0 \Rightarrow 2$ pts. of intersection

$b^2-4ac = 0 \Rightarrow 1$ point of intersection

$b^2-4ac < 0 \Rightarrow 0$ points of intersection

$3x^2+12x+14 = 2x-8$

$3x^2+10x+22=0$

b^2-4ac

$10^2-4(3)(22)$

$100-264$

$=-164$

$\therefore$ 0 pts. of intersection

Given the functions: $p(x)= x^2 + 4x +4$ and $q(x)= -2x +k$, find the value(s) of k that the linear-quadratic system

has 2 points of intersection, 1 point of intersection, and 0 points of intersection.

$x^2+4x+4=-2x+K$

$x^2+6x+\underbrace{4-K}_{K}=0$

$x^2+6x\pm K=0$

$b^2-4ac>0$

$36-4(1)(K)>0$

$36-4K>0$

$36>4K$

$9>K$

$\therefore K<9$ we'll get 2 intersections

$b^2-4ac=0$

$36-4K=0$

$36=4K$

$K=9$

$\therefore$ when $K=9$ we get 1 point of intersection

$b^2-4ac<0$

$36-4K<0$

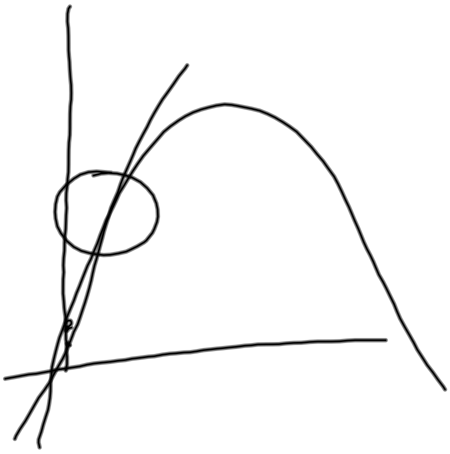
$36<4K$

$9<K$

$\therefore K>9$ we get 2 points of intersection.

Justin is skeet shooting. The height of the clay pigeon is modelled by the function h(t)= $-5t^2 + 32t + 2$, where h(t) is the height, in metres, t seconds after the clay pigeon was released. The path of Justin's bullet is modelled by the function g(x)= 31.5t +1.

After how many seconds will the bullet hit the skeet? How high off the ground will the skeet be when it is hit?



$$-5t^2 + 32t + 2 = 31.5t + 1$$

$$-5t^2 + 0.5t + 1 = 0$$

$$t = \frac{-0.5 \pm \sqrt{0.5^2 - 4(-5)(1)}}{-10}$$

$$t = \frac{-0.5 \pm \sqrt{20.25}}{-10}$$

$$t = \frac{-0.5 \pm 4.5}{-10}$$

$$t = \cancel{-0.4}$$
$$= 0.5\text{sec}$$

$$g(x) = 31.5(0.5) + 1$$
$$= 16.75\text{m}$$

∴ After 0.5 sec the clay pigeon will be hit at a height of 16.75m

HMWK: pg.198 #4-6,8,10-13