Unit 6: Discrete Functions & Applications

Lesson 1: Sequences: Arithmetic & Geometric

A sequence is an ordered list of numbers. Each number in the sequence is called a term. We can identify each term by its position in the list.

Generally, sequences take the form $t_1, t_2, t_3, \ldots t_n$, where t is the value of the sequence at position n.

There are two types of sequences: Arithmetic and Geometric.

An **arithmetic** sequence has the same difference between consecutive pairs of terms (called the **common difference**)

Ex 1) Consider the sequence 3, 5, 7, 9. This is an arithmetic sequence with 4 terms.

The first term, t_1, is 3. What is t_3?

$t_3 = 7$

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Sequences can be defined using a **general term**, a **recursive formula** (which relates the general term to the previous term(s)), or a **discrete linear function**.

The **general term** of an arithmetic sequence:	$\boldsymbol{t_n = a + (n-1)d}$
The **recursive formula** for the same arithmetic sequence:	$\boldsymbol{t_1 = a, t_n = t_{n-1} + d}$, **where** $\boldsymbol{n > 1}$

In all cases, $n \in \mathbf{N}$, a is the first term, and d is the common difference.

Ex 2) Given the sequence 6, 3, 0, -3…

a) Determine the **general term**, t_n.

$a_1 = 6$

$d = -3$

$a_n = 6 + (n-1)(-3)$

$= 6 - 3n + 3$

$\boxed{a_n = -3n + 9}$

b) Determine the **recursive formula** for t_n.

$t_1 = 6, \; t_n = t_{n-1} - 3$

c) Determine t_{10}.

$t_{10} = -3(10) + 9$

$\boxed{t_{10} = -21}$

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A **geometric** sequence has the same ratio between any pair of consecutive terms (called **common ratio**)

The **general term** of a geometric sequence: $t_n = a r^{n-1}$

The **recursive formula** for the same geometric sequence: $t_1 = a,\ t_n = r\,t_{n-1}$, where $n > 1$

In all cases, $n \in \mathbf{N}$, a is the first term, and r is the common ratio.

Ex 3) Given the sequence 18, 9, $4\frac{1}{2}$, …

a) Determine the next 3 terms of the sequence

$r = \frac{1}{2}$

$t_3 = \frac{9}{2}$ $t_4 = \frac{9}{4}$ $t_5 = \frac{9}{8}$ $t_6 = \frac{9}{16}$

b) Determine the **general term**, t_n

$t_n = 18\left(\frac{1}{2}\right)^{n-1}$

c) Determine the **recursive formula** for t_n.

$t_1 = 18,\ t_n = \left(\frac{1}{2}\right)t_{n-1}$

d) Determine t_{10}.

$t_{10} = 18\left(\frac{1}{2}\right)^{10-1}$

$t_{10} = \frac{9}{256}$

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n?

Ex 4) Given the sequence 9, 16, 23, 30, …, 100 determine the number of terms in the sequence.

arithmetic

$d = +7$

$a_1 = 9$

$a_n = 9 + (n-1)(7)$

$100 = 9 + (n-1)(7)$

$\boxed{n = 14}$

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HW U6L1:

1. p.424 #5ab, 6ab, 8be, 13ae, 15
2. p.430 #5acf, 6ae, 7acd, 8c, 11