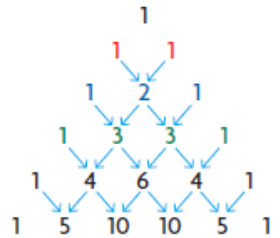
7.7 Pascal's Triangle and Binomial Expansions

June 3

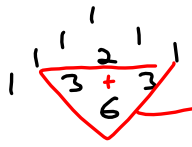


	Sums
0th row	1
1st row	2
2nd row	4
3rd row	8
4th row	16
5th row	32

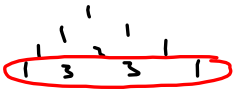
$a_{6,0}\ a_{6,1}\ a_{6,2} \ldots a_{6,6}$

ELEMENT $a_{i,j}$ i-ROW #, j-ENTRY #

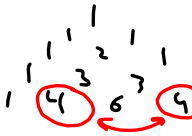
PATTERNS



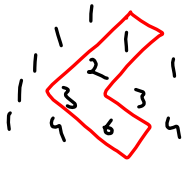
① RECURSIVE $t_{i,j} + t_{i,j+1} = t_{i+1,j+1}$



② ROW SUMMATION $\sum t_{i,j} = 2^i$



③ SYMMETRIC $t_{i,j} = t_{i,i-j}$



④ DIAGONAL (HOCKEY STICK)

ex. $t_{2,2} + t_{3,2} + t_{4,2} + t_{5,2} = t_{6,3}$

The Binomial Theorem

$(x+y)^0$ $\quad 1$

$(x+y)^1$ $\quad 1x + 1y$

$(x+y)^2$ $\quad 1x^2 + 2xy + 1y^2$

$(x+y)^3$ $\quad 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x+y)^4$ $\quad 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

ex. Expand $(x-2)^5$

$x^5 + 5x^4(-2)^1 + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + (-2)^5$

$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

ex. Expand $(5x+2y)^3$

$1(5x)^3 + 3(5x)^2(2y)^1 + 3(5x)^1(2y)^2 + 1(2y)^3$

$= 125x^3 + 150x^2y + 60xy^2 + 8y^3$

ex. Expand $\left(1+3x^2\right)^4$

$1(1)^4 + 4(1)^3(3x^2)^1 + 6(1)^2(3x^2)^2 + 4(1)^1(3x^2)^3 + 1(3x^2)^4$

$= 1 + 12x^2 + 54x^4 + 108x^6 + 81x^8$

ex. Expand $\left(x+\frac{1}{x}\right)^5$

$1(x)^5 + 5(x)^4\left(\frac{1}{x}\right)^1 + 10(x)^3\left(\frac{1}{x}\right)^2 + 10(x)^2\left(\frac{1}{x}\right)^3 + 5(x)^1\left(\frac{1}{x}\right)^4 + 1\left(\frac{1}{x}\right)^5$

$= x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$

p. 466#1-5,8