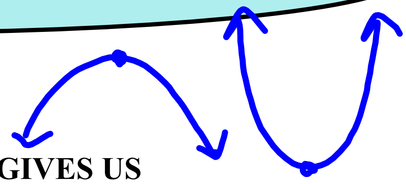
Lesson 3: Max/Min of a Quadratic



Different forms of a quadratic function

FORM	GIVES US
Standard form: $f(x) = ax^2 + bx + c$ (Pretty but almost useless)	*Direction of opening* $+a$ $-a$ *Vertical stretch* "a" *y-intercept* "c"
$3(x+2)^2-3$	
Vertex form: $f(x) = a(x-h)^2 + k$ Found by completing the square	*Direction of opening* a *Vertical stretch* a *Vertex* (h, k) $(-2,-3)$
Factored form: $f(x) = a(x-r)(x-s)$ Found by factoring	*Direction of opening* a *Vertical stretch* a *Zeros or x-intercepts* $(r, 0)$ $(s, 0)$

Jan 31-9:35 PM

The **maximum** or **minimum (optimal) value** of a quadratic function is the ***y*-coordinate of the vertex**. There are a variety of strategies to determine the vertex of a quadratic function.

Method 1: Factoring to determine the zeroes & use to determine vertex

Example: $f(x) = -3x^2 - 12x + 15$

$f(x) = -3(x^2+4x-5)$

$f(x) = -3(x-1)(x+5)$

zeros : -5 & $+1$

AOS : $-2 = \frac{(-5+1)}{2}$

vertex: $(-2, 27)$

$y=-3(-2)^2-12(-2)+15$

$y=-3(4)+24+15$

$y=-12+24+15$

$y=27$

Feb 1-7:55 PM

Method 2: Partial Factoring to determine the axis of symmetry (x-coordinate of the vertex), then substitute.

Example: $f(x) = 4x^2 + 10x + 3$

$$f(x) = 2x(2x+5) + 3$$

$\to 0?$

$x = 0$

$$0 = 2x + 5$$
$$-5 = 2x$$
$$-\frac{5}{2} = x$$

$$AOS = \frac{0 + \frac{-5}{2}}{2}$$

$$AOS = -\frac{5}{2} \times \frac{1}{2}$$

$$AOS = \frac{-5}{4}$$

$$f\left(\tfrac{-5}{4}\right) = 4\left(\tfrac{-5}{4}\right)^2 + 10\left(\tfrac{-5}{4}\right) + 3$$
$$= \cancel{4}^{1}\left(\frac{25}{\cancel{16}_{4}}\right) - \frac{50}{4} + 3$$
$$= \frac{25}{4} - \frac{50}{4} + \frac{12}{4}$$
$$f\left(\tfrac{-5}{4}\right) = \frac{-13}{4}$$

vertex: $\left(\frac{-5}{4}, \frac{-13}{4}\right)$

Feb 1-7:59 PM

Method 3: Completing the square & read vertex (h, k) from equation in vertex form.

Example: $f(x) = 7x^2 - 9x - 2$

$$\left(\tfrac{9}{7} \div 2\right)^2$$
$$\left(\tfrac{9}{7} \times \tfrac{1}{2}\right)^2 = \left(\tfrac{9}{14}\right)^2$$

$$= 7\left(x^2 - \tfrac{9}{7}x\right) - 2$$
$$= 7\left(x^2 - \tfrac{9}{7}x + \left(\tfrac{9}{14}\right)^2 - \left(\tfrac{9}{14}\right)^2\right) - 2$$
$$= 7\left(x^2 - \tfrac{9}{7}x + \left(\tfrac{9}{14}\right)^2\right) - 7\left(\tfrac{9}{14}\right)^2 - 2$$
$$= 7\left(x - \tfrac{9}{14}\right)^2 - \cancel{7}^{1}\left(\frac{81}{\cancel{196}_{28}}\right) - 2$$
$$= 7\left(x - \tfrac{9}{14}\right)^2 - \frac{81}{28} - \frac{56}{28}$$
$$= 7\left(x - \tfrac{9}{14}\right)^2 - \frac{137}{28}$$

vertex: $\left(\frac{9}{14}, -\frac{137}{28}\right)$

Feb 1-7:59 PM

When reading word problems, pay close attention to how variables are defined.

Example: The cost function in a computer manufacturing plant is $C(x) = 0.28x^2 - 0.7x + 1$, where $C(x)$ is the cost per hour in millions of dollars and x is the number of items produced per hour in thousands.

vertex!

Determine the number of items that will produce the minimum cost and give the minimum production cost.

$\left(\frac{2.5}{2}\right)^2 = 1.25^2$

$$C(x) = 0.28(x^2 - 2.5x) + 1$$

$$= 0.28(x^2 - 2.5x + 1.25^2 - 1.25^2) + 1$$

$$= 0.28(x - 2.5x + 1.25^2) - 0.28(1.25^2) + 1$$

$$= 0.28(x - 1.25)^2 - 0.4375 + 1$$

$$= 0.28(x - 1.25)^2 + 0.5625$$

vertex: $(1.25, 0.5625)$

$\therefore$ the number of items: 1250

the min cost: 562 500 \$

Feb 1-8:00 PM

HW: U1L3

p.153 # 1, 2, 4ace, 8, 11ab,

handout (in your package)

Make a note of anything you don't understand in this lesson and I will take it up next class :)

Mrs McKinnell

Feb 14-12:28 PM