7.6 Geometric Series June 2

$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$ ①

$rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n$ ②

② - ①

$rS_n - S_n = ar^n - a$

$S_n(r-1) = a(r^n - 1)$

$$S_n = \frac{a(r^n-1)}{r-1}$$

ex. $2 + 6 + 18 + 54 + 162 + 486 + 1458 + 4374$

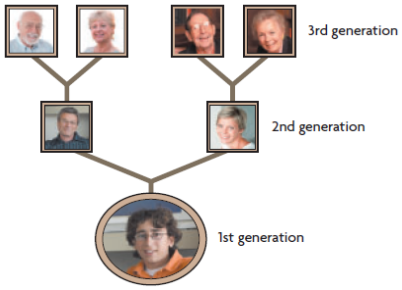
$n = 8,\ a = 2,\ r = 3$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$S_8 = \frac{2(3^8-1)}{3-1} = 6560$$

6560

An ancestor tree is a family tree that shows only the parents in each generation. John started to draw his ancestor tree, starting with his own parents. His complete ancestor tree includes 13 generations.



How many people are in John's ancestor tree?

$1 + 2 + 4 + 8 + \ldots$

$a = 1 \quad r = 2 \quad n = 13$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{13} = \frac{1(2^{13} - 1)}{2 - 1} = 8191$

ex. Calculate the sum, 7971615+5314410+3542940+.....+92160

$a = 7971615 \quad r = \frac{2}{3}$

$t_n = ar^{n-1}$

$92160 = 7971615\left(\frac{2}{3}\right)^{n-1}$

$\frac{92160}{7971615} = \left(\frac{2}{3}\right)^{n-1}$

$0.011561\ldots = \left(\frac{2}{3}\right)^{n-1}$

$n - 1 = \frac{\log 0.011561\ldots}{\log\left(\frac{2}{3}\right)} = 11$

$n = 12$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{12} = \frac{7971615\left(\left(\frac{2}{3}\right)^{12} - 1\right)}{\left(\frac{2}{3} - 1\right)}$

$= 23730525$

At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that hatched on each of the first four days after fertilization was 2, 10, 50, and 250, respectively. If the pattern continues, calculate the total number of fish hatched during the first 10 days.

$2, 10, 50, 250 \ldots$

$n = 10, \quad a = 2, \quad r = 5$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{2(5^{10} - 1)}{(5 - 1)} = 488281 2$$

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