Lesson 2: Properties of Parent Functions (Exponential and Sinusoidal)

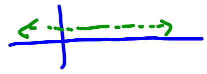
Today you will be investigating properties of more parent functions by graphing them. For all graphs, you will be identifying the **intervals of increase and decrease**: these are the ***x*-values** where the function's ***y*-values** are **increasing** or **decreasing,** respectively.

Some of the functions you are investigating are exponential. There are an infinite number of exponential parent functions (also called base function). The general form for an exponential function is $f(x) = B^x$, where $B > 0$ and $B \neq 1$. Why do we need these restrictions on B? *Hint*: domain is $\{x \in \Re\}$

Negative B : some values would make the function undefined ex: $f(x) = (-2)^x$

$f(x) = (-2)^{\frac{1}{2}}$

B can't be = 1 : $\boxed{\text{ex: } f(x) = 1^x}$ because this is just a horizontal line



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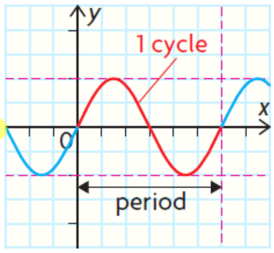
Ex 1) Identify the equation of the parent (base) function for each exponential function.

a) $g(x) = 3(4)^{2x} - 5$ b) $h(x) = 2\left(\frac{1}{3}\right)^{x-4} + 1$ c) $g(x) = (3)5^{2x-4}$

$\boxed{g(x) = 4^x}$ $\boxed{h(x) = \left(\frac{1}{3}\right)^x}$ $\boxed{g(x) = 5^x}$

Some of the functions you are investigating are periodic. A **periodic function** has a **self-repeating** graph. A periodic graph has the following properties:

- **Cycle**: the cycle of a graph is the smallest repeating pattern.
- **Period**: length of one cycle is called the period (read off the x axis)
- **Maximum (max)**: the *y*-value of the highest point of the graph. Also called the peak.
- **Minimum (min)**: the *y*-value of the lowest point of the graph. Also called the trough.
- **Amplitude**: the amplitude of the function is the vertical distance from the equation of the axis to either the maximum or minimum value. $a = \frac{\text{max} - \text{min}}{2}$
- **Equation of the axis**: the equation of the axis is the horizontal line halfway between the maximum and minimum values on the graph. $y = \frac{\text{max} + \text{min}}{2}$



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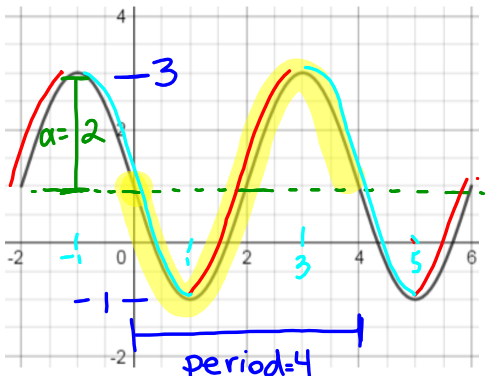
Ex 1) Determine the properties of the periodic graph to the right

cycle: highlighted
period: 4
max: 3 (peak)
min: −1 (trough)
amplitude: 2
Eq of A: y = 1
$y = \frac{max + min}{2}$
$y = \frac{3 + (-1)}{2}$
$y = 1$



increasing: $-2 \le x \le -1$, $1 \le x \le 3$, $5 \le x \le 6$

decreasing: $-1 \le x \le 1$, $3 \le x \le 5$.

For each parent function provided:

- **Name** the function and complete the **table of values**
- **Graph** the function. Verify with graphing technology. State the **domain** and **range**.
- Choose between 3 and 5 "**key points**" needed to properly sketch the function.
- Identify the **intervals** of **increase** and **decrease (inc/dec)**
- State any other **key features** of the graph/table (intercepts, asymptotes, 1st/2nd differences, period, max, min, amplitude, equation of the axis, etc.)

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1) $f(x) = \left(\frac{1}{2}\right)^x$

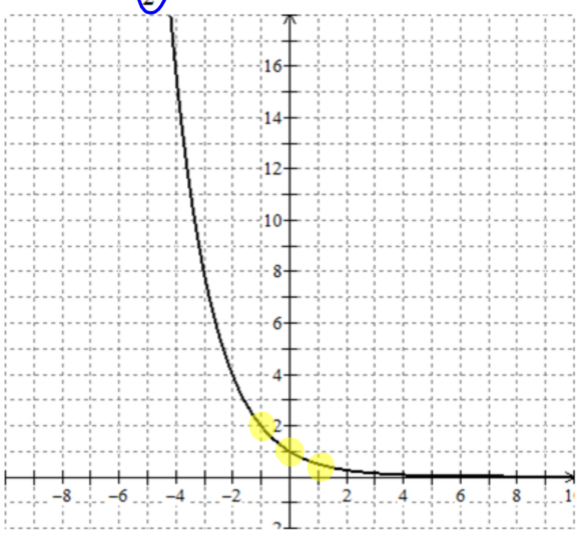
Name: Exponential Function, base $\frac{1}{2}$

First ratios

x	$f(x)$
-4	16
-3	8
-2	4
-1	2
0	1
1	0.5
2	0.25
3	0.125
4	0.0625

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$\div 2$



Domain: $\{x \in \Re\}$

Range: $\{y \in \Re \mid y > 0\}$

Key Points: (-1, 2), (0, 1), (1, 0.5)

Intervals of inc/dec: decreasing for all $x \in \Re$

Features: always decreasing curve; y-intercept (0, 1); no x-intercept; horizontal asymptote at $y = 0$; 1st and 2nd differences follow a similar pattern to y-values; y-values have a constant ratio (equal to the base)

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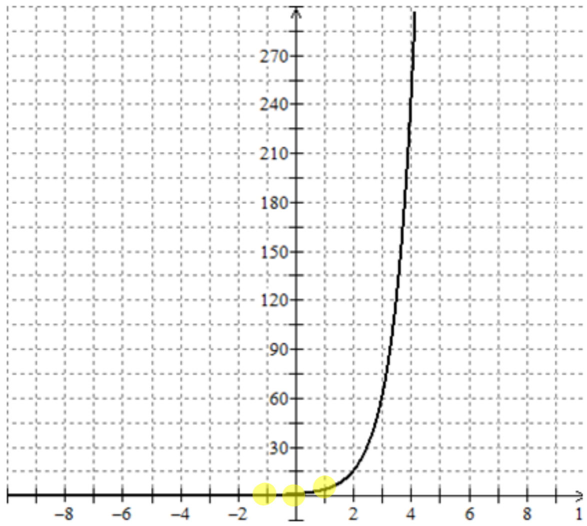
2) $f(x)=4^x$ Name: Exponential Function, base 4

1st ratios

x	$f(x)$
-4	0.00390625
-3	0.15625
-2	0.0625
-1	0.25
0	1
1	4
2	16
3	64
4	256

4
4
4
÷ 4



Domain: $\{x \in \Re\}$

Range: $\{y \in \Re \mid y > 0\}$

Key Points: (-1, 0.25), (0, 1), (1, 4)

Intervals of inc/dec: increasing for all $x \in \Re$

Features: always increasing curve; y-intercept (0, 1); no x-intercept; horizontal asymptote at $y = 0$; 1st and 2nd differences follow a similar pattern to y-values; y-values have a constant ratio (equal to the base)

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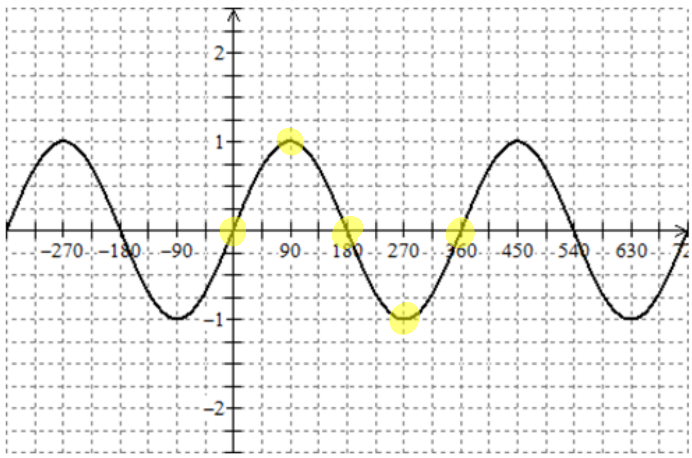
3) $f(x)=\sin(x)$ Name: Sine Function

x (°)	-360	-270	-180	-90	0	90	180	270	360	450	540	630	720
$f(x)$	0	1	0	-1	0	1	0	-1	0	1	0	-1	0



Domain: $\{x \in \Re\}$

Range: $\{y \in \Re \mid -1 \le y \le 1\}$

Key Points: (0°, 0), (90°, 1), (180°, 0), (270°, -1), (360°, 0)

Intervals of inc/dec: (for $0° \le x \le 360°$)
inc: $0° < x < 90°$ and $270° < x < 360°$
dec: $90° < x < 270°$

Features: y-intercept (0°, 0); x-intercepts at (0°, 0), (180°, 0), (360°, 0) over domain of $0° \le x \le 360°$; period of 360°; max value = 1; min value = -1; amplitude = 1; equation of the axis $y = 0$; table shows a repeating pattern of y-values.

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4) $f(x) = \cos(x)$ Name: Cosine Function

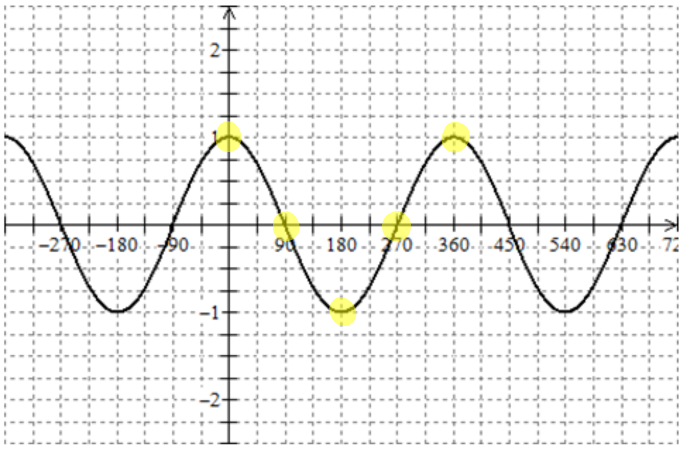
x (°)	-360	-270	-180	-90	0	90	180	270	360	450	540	630	720
$f(x)$	1	0	-1	0	1	0	-1	0	1	0	-1	0	1

Domain: $\{x \in \Re\}$

Range: $\{y \in \Re \mid -1 \le y \le 1\}$

Key Points: (0°, 1), (90°, 0), (180°, -1), (270°, 0), (360°, 1)

Intervals of inc/dec: (for $0° \le x \le 360°$)
inc: $180° < x < 360°$
dec: $0° < x < 180°$



Features: y-intercept (0°, 1); x-intercepts at (90°, 0), (270°, 0) over domain of $0° \le x \le 360°$; period of 360°; max value = 1; min value = -1; amplitude = 1; equation of the axis $y = 0$; table shows a repeating pattern of y-values.

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HW U4L2:

1. p. 243 #1, 2
2. p. 352 #2, 6, 8a-d, 10
3. p. 364 #12

*Bring laptops/ iPads tomorrow! *