Lesson 3: Series: Arithmetic & Geometric

A **series** is the *sum* of the terms of a sequence. The sum of the first n terms of a sequence is S_n, where

$$S_n = t_1 + t_2 + t_3 + t_4 + \dots + t_{n-1} + t_n$$

Recall: An **arithmetic sequence** has the general term $t_n = a + (n - 1)d$, where a is the first term and d is the common difference between terms. An *arithmetic series* is the sum of this sequence and is written

$$S_n = a + (a + d) + (a + 2d) + \dots + [a + (n - 2)d] + [a + (n - 1)d]$$

To determine a formula, we will write out the series twice, first forward and then backward. Then we will add them together (This is called Gauss's method)

$$\begin{array}{rlllll} & S_n = a & + (a + d) & + \dots + [a + (n - 2)d] & + [a + (n - 1)d] \\ + & S_n = [a + (n - 1)d] & + [a + (n - 2)d] & + \dots + (a + d) & + a \\ \hline & 2S_n = [2a + (n - 1)d] & + [2a + (n - 1)d] & + \dots + [2a + (n - 1)d] & + [2a + (n - 1)d] \end{array}$$

$$2S_n = n \times [2a + (n - 1)d]$$

$S_n = \frac{n}{2}[2a + (n-1)d]$, which is the *sum* of the first n terms of an arithmetic sequence.

Recognizing that $t_1 = a$ and $t_n = a + (n - 1)d$, we can also write: $S_n = \frac{n(t_1 + t_n)}{2}$

Jan 31-9:35 PM

Ex 1) Determine the sum of the arithmetic series given $\boxed{t_1 = 88}$ and $\boxed{t_{15} = 4}$

t_n $\therefore n = 15$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_n = \frac{15(88 + 4)}{2}$$

$$\boxed{S_n = 690}$$

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Ex 2) Determine the sum of the series 15 + 11 + 7 + ... – 37

$t_1 \quad t_2 \quad t_3 \qquad t_n$

$a = 15$
$d = -4$
$t_n = -37$

$t_n = a + (n-1)d$
$-37 = 15 + (n-1)(-4)$
$-52 = (n-1)(-4)$
$\frac{-52}{-4} = n - 1$
$14 = n$

$\therefore$ there are 14 terms in this series.

$S_n = \frac{n(t_1 + t_n)}{2}$

$S_{14} = \frac{14(15 - 37)}{2}$

$\boxed{S_{14} = -154}$

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Recall: A **geometric sequence** has the general term $t_n = ar^{n-1}$, where $\boldsymbol{a}$ is the first term and $\boldsymbol{r}$ is the common ratio. A *geometric series* is the sum of the terms of this sequence and is written

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1}$$

To determine a formula, multiply the series by the common ratio and subtract the original series.

$$\begin{aligned} rS_n &= \quad ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1} + ar^n \\ -\ S_n &= -(a + ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1}) \end{aligned}$$

$(r-1)\,S_n = -a + ar^n$

$S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$, which is the *sum* of the first n terms of a geometric sequence.

Recognizing that $t_1 = a$ and $t_{n+1} = ar^n$, we can also write: $S_n = \frac{t_{n+1} - t_1}{r - 1}, r \neq 1$

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Ex 3) Determine the sum of the series -3 – 6 – 12 – 24 – ... – 768

$\times 2$

t_n

$a = -3$

$r = 2$

$t_1 = -3$

$t_{n+1} = (-768)(2)$

$t_{n+1} = -1536$

$S_n = \frac{t_{n+1} - t_1}{r - 1}$

$S_n = \frac{1536 + 3}{2 - 1}$

$\boxed{S_n = -1533}$

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Ex 4) Determine t_n, S_n, and S_6 for the series 81 + 27 + 9 + ...

$r = \frac{1}{3}$

$a = t_1 = 81$

$t_n = ar^{n-1}$

$\boxed{t_n = 81\left(\frac{1}{3}\right)^{n-1}}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_n = \frac{81\left[\left(\frac{1}{3}\right)^n - 1\right]}{\frac{1}{3} - 1}$

$S_n = \frac{81\left[\left(\frac{1}{3}\right)^n - 1\right]}{-\frac{2}{3}}$

$= \left(-\frac{3}{2}\right)(81)\left[\left(\frac{1}{3}\right)^n - 1\right]$

$\boxed{S_n = \frac{-243}{2}\left[\left(\frac{1}{3}\right)^n - 1\right]}$

$\boxed{S_n = \frac{-3^5}{2}\left[\left(\frac{1}{3}\right)^n - 1\right]}$

$S_6 = \frac{-3^5}{2}\left[\left(\frac{1}{3}\right)^6 - 1\right]$

$= \frac{-3^5}{2}\left(\frac{1}{3^6} - \frac{3^6}{3^6}\right)$

$= \frac{-\cancel{3^5}}{2}\left(\frac{1 - 729}{3\cancel{^6}}\right)$

$= \frac{-1}{\cancel{2}}\left(\frac{-\cancel{728}}{3}\right)$ 364

$= \frac{364}{3}$

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L3 HW:

p.452 # 4abdf, 5acde, 6acd, 7aef, 11, 13

p.459 # 3ade, 4ade, 5ade, 6acef, 7, 11

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