Recursive Sequences

Topic Index | Algebra2/Trig Index | Regents Exam Prep Center

Recursion is the process of choosing a starting term and repeatedly applying the same process to each term to arrive at the following term. Recursion requires that you know the value of the term immediately before the term you are trying to find.

A recursive formula always has two parts:

1. the **starting value** for a_1.
2. the **recursion equation** for a_n as a function of a_{n-1} (the term before it.)

Recursive formula:	Same recursive formula:
$a_1 = 4$ $a_n = 2a_{n-1}$	$a_1 = 4$ $a_{n+1} = 2a_n$

Be sure you understand that the two formulas at the left say the same thing. Different textbooks write recursive formulas in different ways.

Always 2 parts!

A recursive formula may list the first two (or more) terms as starting values, depending upon the natrue of the sequence. In such caes, the a_n portion of the formula is dependent upon the previous two (or more) terms..

Recursion is described as an "iterative" procedure.

Examples:

1. Write the first four terms of the sequence: $a_1 = -4$, $a_n = a_{n-1} + 5$

$a_1 = -4$

$n = 2: \quad a_2 = a_{2-1} + 5 = 1$

$n = 3: \quad a_3 = a_{3-1} + 5 = 6$

$n = 4: \quad a_4 = a_{4-1} + 5 = 11$

In recursive formulas, each term is used to produce the next term. Follow the movement of the terms through the set up at the left.

Answer: -4, 1, 6, 11

2. Consider the sequence 2, 4, 6, 8, 10, ...

Explicit formula:

$a_n = 2n$

Recursive formula:

$a_1 = 2$

$a_n = a_{n-1} + 2$

Certain sequences, such as this arithmetic sequence, can be represented in more than one manner. This sequence can be represented as either an explicit (general) formula or a recursive formula.

3. Consider the sequence 3, 9, 27, 81, ...

Explicit formula:
$a_n = 3^n$

Recursive formula:
$a_1 = 3$
$a_n = 3a_{n-1}$

Certain sequences, such as this geometric sequence, can be represented in more than one manner. This sequence can be represented as either an explicit formula or a recursive formula.

4. Consider the sequence 2, 5, 26, 677, ...

Recursive formula:
$a_1 = 2$
$a_n = (a_{n-1})^2 + 1$

This sequence is neither arithmetic nor geometric. It does, however, have a pattern of development based upon each previous term.

5. Write the first 5 terms of the sequence $\begin{aligned} a_1 &= 3 \\ a_n &= (-1)^n \cdot 5a_{n-1} \end{aligned}$

$a_1 = 3$
$a_2 = (-1)^2 \cdot 5a_{2-1} = 5 \cdot 3 = 15$
$a_3 = (-1)^3 \cdot 5a_{3-1} = (-1) \cdot 5 \cdot 15 = -75$
$a_4 = (-1)^4 \cdot 5a_{4-1} = 5 \cdot (-75) = -375$
$a_5 = (-1)^5 \cdot 5a_{5-1} = (-1) \cdot 5 \cdot (-375) = 1875$

Notice how the value of n is used as the exponent for the value (-1). Also, remember that in recursive formulas, each term is used to produce the next term. Follow the movement of the terms through the set up at the left.

Answer: 3, 15, -75, -375, 1875

p. 424 #2,5ii,6,8ii

p. 430 #2,5ii,6ii,8,9i

p. 443 #3

p. 447 #4,5,10