

1.5 The Inverse Function

Feb 6

The inverse of a relation is found by interchanging the independent and dependent variables.

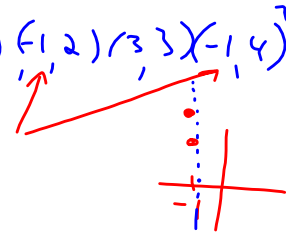
 x y

eg. $f = \{(1,2), (2,-1), (3,3), (4,-1)\}$

Is the relation a function? **Yes**

Find its inverse relation. $f^{-1} = \{(2,1), (-1,2), (3,3), (-1,4)\}$

Is the inverse relation a function? **NO**



To find the inverse of an equation, switch x and y and solve for y .

ex. $y = 3x - 2$

$x \leftrightarrow y$

$$x = 3y - 2$$

new y is y^{-1}
ACTUALLY y^{-1}

$$x + 2 = 3y$$

$$\frac{x+2}{3} = y$$

$$y^{-1} = \frac{x+2}{3}$$

$$\therefore f^{-1}(x) = \frac{x+2}{3}$$

ex. $f(x) = \frac{3-2x}{5}$

$$y = \frac{3-2x}{5}$$

$x \leftrightarrow y$

$$x = \frac{3-2y}{5}$$

$$5x = 3 - 2y$$

$$5x - 3 = -2y$$

$$\frac{5x-3}{-2} = y$$

$$\therefore f^{-1}(x) = \frac{5x-3}{-2}$$

$$\begin{array}{r} x \\ \downarrow \\ x(-2) \\ \downarrow \\ +3 \\ \downarrow \\ \div 5 \\ \downarrow \\ y \end{array}$$

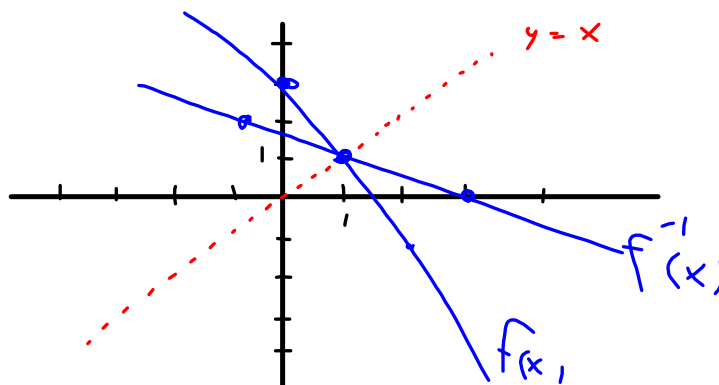
$$\begin{array}{r} y^{-1} \\ \uparrow \\ \div (-2) \\ \uparrow \\ -3 \\ \uparrow \\ +3 \\ \uparrow \\ x \end{array}$$

BEFORE \rightarrow SAME AS

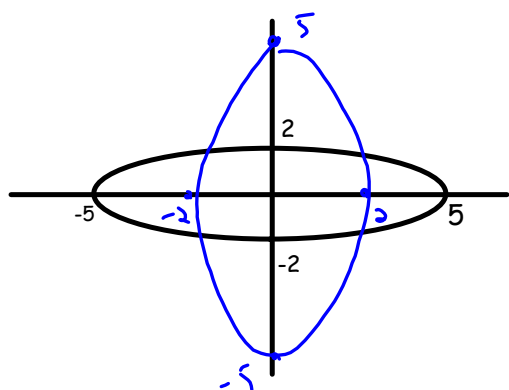
$$\frac{5x-3}{-2}$$

To graph an inverse, reflect the graph in the line $y=x$.

ex. $f(x) = -2x + 3$



ex. Sketch the inverse of the following ellipse.



inverse

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

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ex. Determine the domain and range of the inverse of the function

$$f(x) = -3(x-1)^2 - 4$$

$$D_f = \{x \in \mathbb{R}\} \longrightarrow R_{f^{-1}} = \{y \in \mathbb{R}\}$$

$$R_f = \{y \in \mathbb{R} / y \leq -4\} \longrightarrow D_{f^{-1}} = \{x \in \mathbb{R} / x \leq -4\}$$

Homework p. 46#1,2,4,6,8,10de,12,17,20