Would you rather:

A) Work for 10 days and receive $100 per day

OR

B) Work for 10 days. Receive $2 on day 1, and every day after that you get double what you got the previous day.

	day 1	day 2	day 3	day 4	day 5	day 6	day 7	day 8	day 9	day 10
A)	100	100	100	→						
B)	2	2·2=4	8	16	32	64	128	256	512	1024

2   2·2   (2·2·2)   ... ||

2^x: 2^1 2^2 2^3 . . . 2^{10}

Total for plan A) : $100 · 10 = $1,000

Total for plan B) : $\boxed{}$ > $1,000

4.2 Working with integer exponents

Name	Symbol	Multiple of the Metre	Multiple as a Power of 10
terametre	Tm	1 000 000 000 000	10^{12}
gigametre	Gm	1 000 000 000	10^{9}
megametre	Mm	1 000 000	10^{6}
kilometre	km	×10 1 000	10^{3}
hectometre	hm	×10 100 $10^1 \times 10^1 = 10^2$	10^{2}
decametre	dam	×10 10 $= 10^1$	10^{1}
metre	m	1	

How do we represent multiples that are less than or equal to 1 as a power of 10?

Name	Symbol	Multiple of the Metre	Multiple as a Power of 10
terametre	Tm	1 000 000 000 000	10^{12}
gigametre	Gm	1 000 000 000	10^{9}
megametre	Mm	1 000 000	10^{6}
kilometre	km	1 000	10^{3}
hectometre	hm	100	10^{2}
decametre	dam $\times 10$	10	10^{1}
metre	m	1	10^{0}
decimetre	dm $\div 10$	0.1 $\frac{1}{10}$	10^{-1}
centimetre	cm $\div 10$	0.01 $\frac{10^{-1}}{10^{1}} = 10^{-1-1}$	10^{-2}
millimetre	mm $\div 10$	0.001 $10^{-2} \div 10$	10^{-3}
micrometre	μm	0.000 1	10^{-4}
nanometre	nm	0.000 01	10^{-5}
picometre	pm	0.000 001	10^{-6}
femtometre	fm	0.000 000 001	10^{-9}
attometre	am	0.000 000 000 001	10^{-12}

The rule for negative exponents is:

How is 10^2 related to 10^{-2}?

Use the quotient rule to show that $x^0 = 1$, when x is not zero.

$$\frac{10^{-1}}{10^{1}} = 10^{-1-1} = 10^{-2}$$

$$1 = \frac{x}{x} = \frac{x^n}{x^n} = x^{n-n} = \underbrace{x}_{\text{base}}{}^{0}\ \text{exponent}$$

$$(10{,}225)^0 = 1$$

Powers with integer bases in rational form

Evaluate:

a) 6^{-3} b) $(-9)^{-2}$ c) -2^{-5} d) $\left(\frac{2}{3}\right)^{-3}$

$\frac{6^{-3}}{1} = \frac{1}{6^3}$

0.006

0.06

6^(-3)

$= 0.0046$

$\frac{(-9)^{-2}}{1} = \frac{1}{(-9)^2}$

$= \frac{1}{81}$

$\frac{-2^{-5}}{1} = \frac{-1}{2^5}$

$= \frac{-1}{2^5} = \frac{-1}{32}$

$= \left(\frac{3}{2}\right)^3$

$= \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2}$

$= \frac{3^3}{2^3} = \frac{27}{8}$

Determine whether the following are true or false

a) $4^{-4} = \frac{1}{4^4}$ $4^{-4} = \frac{1}{4^4}$ T

b) $\left(\frac{1}{5}\right)^{-6} = \frac{1}{5^6}$ $\left(\frac{1}{5}\right)^{-6} = \left(\frac{5}{1}\right)^6 = \frac{5^6}{1^6} = 5^6$ F

c) $\left(-\frac{4}{5}\right)^{-3} = \frac{-4^3}{5^3}$ $\left(-\frac{4}{5}\right)^{-3} = \left(-\frac{5}{4}\right)^3 = \frac{(-5)^3}{4^3}$ F

d) $\left(-\frac{3}{8}\right)^{-2} = \frac{8^2}{3^2}$ $\left(-\frac{3}{8}\right)^{-2} = \left(-\frac{8}{3}\right)^2 = \frac{(-8)^2}{3^2} = \frac{8^2}{3^2}$ T

Simplify and Evaluate the following:

$$\frac{3^5 \times 3^{-2}}{\underbrace{(3^{-3})^2}_{\text{multiply exponents}}} = \frac{\overbrace{3^5 \cdot 3^{-2}}^{\text{add exponents}}}{3^{-6}} = \frac{3^{5-2}}{3^{-6}} = \frac{3^3}{3^{-6}} \Big\} \text{subtract exponents} = 3^{3-(-6)} = 3^9$$

$$\left[\underbrace{(-6^{-1})^{-2}}\right]^{-1} = \left[\underbrace{\left(\frac{-1}{6}\right)^{-2}}\right]^{-1} = \left[\left(\frac{-6}{1}\right)^2\right]^{-1} = \left[6^2\right]^{-1}$$

$$= 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

$$\frac{9^{-2}}{3^{-3}} = \frac{(3^2)^{-2}}{3^{-3}} = \frac{3^{-4}}{3^{-3}} = 3^{-4-(-3)} = 3^{-1} = \left(\frac{1}{3}\right)^1 = \frac{1}{3}$$

Key Ideas:

- Product rule: $x^m \cdot x^n = x^{m+n}$
- Quotient rule: $\frac{x^m}{x^n} = x^{m-n}$
- Negative exponent rule: $x^{-m} = \frac{1}{x^m}$

$$\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^{m} = \frac{y^m}{x^m}$$

- Zero exponent rule: $x^0 = 1$ as long as $x \neq 0$
- Power rule: $(x^m)^n = x^{m \cdot n}$ OR $\left(\frac{x^n}{y^n}\right)^m = \frac{x^{n \cdot m}}{y^{n \cdot m}}$

HMWK: pg. 222 #4,6,7,8,11,15