MCR 3U

U3L5 The Ambiguous Case of the Sine Law Handout MCR3U

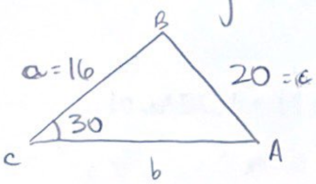
For each question, draw a labelled **sketch** and determine the **number of distinct triangles (justify your answer!)** Then, **solve the triangles**, if possible, rounding angles to the nearest degree and lengths to the nearest tenth of a unit.

1. In ΔABC, $a = 16$ cm, $c = 20$ cm, and $\angle C = 30º$.

$\measuredangle C$ is acute

$c > a$

$\therefore$ 1 triangle exists



$$\frac{\sin 30^\circ}{20} = \frac{\sin A}{16}$$

$$\boxed{\measuredangle A^\circ = 24^\circ}$$

$$\measuredangle B^\circ = 180^\circ - 24^\circ - 30^\circ$$

$$\boxed{\measuredangle B^\circ = 126^\circ}$$

$$\frac{\sin 126^\circ}{b} = \frac{\sin 30^\circ}{20}$$

$$\boxed{b = 32.4\text{ m}}$$

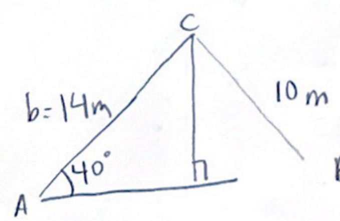
2. In ΔDEF, $d = 15$ cm, $f = 17$ cm, and $\angle D = 95º$.

$\measuredangle D$ is obtuse

d is $\leq f$

$\therefore$ No triangle exists

3. In ΔABC, $a = 10$ m, $b = 14$ m, and $\angle A = 40º$.



$$\sin 40^\circ = \frac{h}{14}$$

$$\boxed{h = 8.99\text{m}}$$

$\measuredangle A$ is acute

a is smaller than b, so check h.

h is 8.99, $\therefore$ 2 triangles!

#1 $\frac{\sin B}{14} = \frac{\sin 40^\circ}{10}$

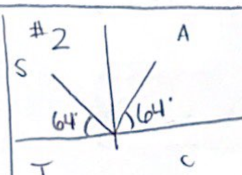
$\boxed{\measuredangle B_1 = 64^\circ}$

$\boxed{\measuredangle C_1 = 76^\circ}$ $(180^\circ - 64 - 40 = \measuredangle C^\circ)$

$\boxed{c_1 = 15.1\text{m}}$ $\frac{\sin 76^\circ}{c} = \frac{\sin 40^\circ}{10}$

$c_1 = 15.1$ m

#2



$\measuredangle B_2 = 180^\circ - 64$

$\boxed{\measuredangle B_2 = 116^\circ}$

$\boxed{\measuredangle C_2 = 24^\circ}$ $(\measuredangle C_2 = 180 - 116^\circ - 40)$

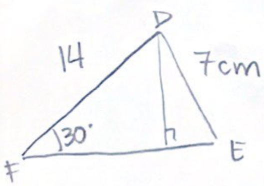
$\boxed{c_2 = 6.33\text{m}}$ $\frac{\sin 24^\circ}{c} = \frac{\sin 40^\circ}{10}$

$c = 6.33$m

MCR 3U

4. In ΔDEF, f = 7 cm, e = 14 cm, and $\angle$F = 30º.

acute, $f < e$, $\therefore$ find h



$\frac{h}{14} = \sin 30°$

$\boxed{h = 7\text{cm}}$

since $h = f$, there is 1 triangle
its a ⊾ Δ!

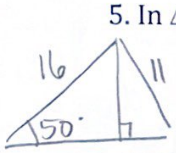
$\measuredangle D = 180° - 30° - 90°$

$\boxed{\begin{array}{l} \measuredangle D = 60° \\ \measuredangle E = 90° \\ d = 12.1 \text{ m} \end{array}}$

$\frac{\sin 60°}{d} = \frac{\sin 30°}{7}$

$d = 12.1$ m

5. In ΔABC, b = 11 mm, c = 16 mm, and $\angle$B = 50º. since $a < b$, find h.



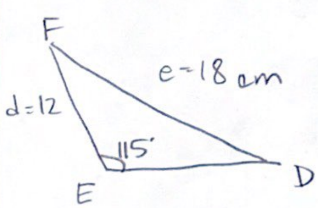
$\sin 50° = \frac{h}{16}$

$h = 12.2$ mm

since $h > b$, no triangle exists.

6. In ΔDEF, d = 12 cm, e = 18 cm, and $\angle$E = 115º.

obtuse
$a > b$, so 1 triangle exists



$\frac{\sin 115°}{18} = \frac{\sin D}{12}$

$\boxed{\measuredangle D = 37°}$

$\measuredangle F = 180 - 37° - 115°$

$\boxed{\measuredangle F = 28°}$

$\frac{\sin 28°}{f} = \frac{\sin 115°}{18}$

$\boxed{f = 9.3 \text{ cm}}$

1) 1 triangle; $c \geq a$ $\angle A = 24°$ $\angle B = 126°$ $b = 32.2$ cm 2) 0 triangles; $d \leq f$ 3) 2 triangles; $h = 9.0$ & $h < a < b$ $\angle B_1 = 64°$ $\angle C_1 = 76°$ $c_1 = 15.1$ m or $\angle B_2 = 116°$ $\angle C_2 = 24°$ $c_2 = 6.4$ m 4) 1 triangle; $h = 7.0$ cm & $f = h$ $\angle E = 90°$ $\angle D = 60°$ $d = 12.1$ cm 5) 0 triangles; $h = 12.3$ cm & $b < h$ 6) 1 triangle; $e > d$ $\angle D = 37°$ $\angle F = 28°$ $f = 9.3$ cm