

$$y = a(b)^x \quad y = a(1+r)^x \quad y = a(1-r)^x \quad A = P(1+i)^n$$

Exam Review Part 3 – Exponential Functions

MCR3U

SOLUTIONS

Section 1: Exponential Growth

1) An insect colony has an initial population of 15. The number of insects quadruples every day.

a) Determine a function that models this exponential growth.

$$a = 15$$

$$b = 4$$

$$P(n) = 15(4)^n$$

b) How many insects will be present in 1 week?

$$\begin{aligned} P(7) &= 15(4)^7 \\ &= 245\,760 \end{aligned}$$

$$245\,760$$

2) If the population of an ant colony is 213 and it doubles every week,

a) What will the population be in 4 weeks?

$$y = 213(2)^x$$

$$y = 213(2)^4$$

$$y = 3408$$

$$3\,408$$

b) How long will it take the population to reach 109 056 ants?

$$109\,056 = 213(2)^x$$

$$512 = 2^x$$

$$2^9 = 2^x$$

$$9 = x$$

$$9 \text{ weeks}$$

3) The population of a town in the Northwest Territories starts off at 20,000 and grows by 13% each year. Find the populations after 10 years.

$$y = 20\,000(1.13)^x$$

$$y = 20\,000(1.13)^{10}$$

$$y = 67\,891.35$$

$$67\,891$$

$$\frac{\log 512}{\log 2} = 9$$

- 4) There are 50 bacteria present initially in a culture. In 3min., the count is 204800.
What is the doubling period?

$$y = 50(2)^x$$

$$204800 = 50(2)^{3/t}$$

$$4096 = 2^{3/t}$$

$$2^{12} = 2^{3/t}$$

$$12 = \frac{3}{t}$$

$$t = \frac{3}{12}$$

$$t = \frac{1}{4}$$

The doubling period is 0.25 minutes OR 15 seconds.

- 5) A bacteria culture starts with a population of 12 000 and doubles every four hours.

- a) How many bacteria are present after 12 hours?

$$y = 12000(2)^{t/4}$$

$$y = 12000(2)^{12/4}$$

$$y = 96000$$

$$96000$$

- b) How many bacteria are present after 1 day?

$$y = 12000(2)^{24/4}$$

$$y = 12000(2)^6$$

$$y = 768000$$

$$768000$$

- c) How long will it take for the population of the bacteria to reach 49 152 000?

$$49152000 = 12000(2)^{t/4}$$

$$4096 = 2^{t/4}$$

$$2^{12} = 2^{t/4}$$

$$12 = \frac{t}{4}$$

$$48 = t$$

$$\frac{\log 4096}{\log 2} = 12$$

48 hours OR 2 days

$$b=2 \quad x = t/15 \quad a$$

6) A bacteria culture doubles every 15 minutes. There were 20 individuals initially.

a) How many bacteria will be present after 3 hours

= 180 minutes

$$y = 20(2)^{t/15}$$

$$y = 20(2)^{180/15}$$

$$y = 81920$$

b) How long will it take to grow a population of 163 840?

$$163\,840 = 20(2)^{t/15}$$

$$8192 = 2^{t/15}$$

$$2^{13} = 2^{t/15}$$

$$13 = \frac{t}{15}$$

$$t = 195$$

$$\frac{\log 8192}{\log 2} = 13$$

195 minutes

Section 2: Exponential Decay

7) In 1976, a research hospital bought half of a gram of radium for cancer research. Assuming the hospital still exists, how much of this radium will the hospital have in the year 6836, if the half-life of the radium is 1620 years?

$$a = 0.5$$

$$b = \frac{1}{2}$$

$$x = t/1620$$

$$y = 0.5\left(\frac{1}{2}\right)^{t/1620}$$

$$y = 0.5\left(\frac{1}{2}\right)^{4860/1620}$$

$$y = 0.0625$$

0.0625 grams

$$b = \frac{1}{2} \quad x = t/20 \quad a$$

8) Polonium-210 is a radioactive isotope that has a half-life of 20 days. Suppose you start with a 40-mg sample.

a) Write an equation that relates the amount of polonium-210 remaining and time.

$$f(t) = 40\left(\frac{1}{2}\right)^{t/20}$$

b) How much polonium-210 will remain after 10 weeks?

= 70 days

$$f(70) = 40 \left(\frac{1}{2}\right)^{70/20}$$

$$= 3.54$$

$$3.54 \text{ mg}$$

c) How long will it take for the amount of polonium-210 to decay to 8% of its initial mass?

$$0.08(40) = 40 \left(\frac{1}{2}\right)^{t/20}$$

$$0.08 = \left(\frac{1}{2}\right)^{t/20}$$

$$\log 0.08 = \left(\frac{t}{20}\right) \log \left(\frac{1}{2}\right)$$

$$\frac{\log 0.08}{\log 0.5} = \frac{t}{20}$$

$$3.643856 = \frac{t}{20}$$

$$t = 72.88$$

About 73 days

9) A cup of coffee contains approximately 96 mg of caffeine. When you drink the coffee, the caffeine is absorbed into the bloodstream and is eventually metabolized by the body. Every 5 hours the amount of caffeine present in the body is reduced by one-half. How many hours does it take for the amount of caffeine to be reduced to 12 mg?

$$a = 96$$

$$b = \frac{1}{2}$$

$$x = \frac{t}{5}$$

$$y = 96 \left(\frac{1}{2}\right)^{t/5}$$

$$12 = 96 \left(\frac{1}{2}\right)^{t/5}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/5}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/5}$$

$$3 = \frac{t}{5}$$

$$t = 15$$

15 hours

10) Daniel is very excited about his new motorcycle. Although the motorcycle costs \$13 500, its resale value will depreciate by 20% of its current value every year.

a) How much will the motorcycle be worth in 6 years?

$$y = 13500 (1 - 0.2)^x$$

$$y = 13500 (0.8)^6$$

$$y = 3538.94$$

$$\$ 3538.94$$

b) How long will it take for Daniel's motorcycle to depreciate to 50% of its original cost?

$$0.5 (13500) = 13500 (0.8)^x$$

$$0.5 = 0.8^x$$

$$\frac{\log 0.5}{\log 0.8} = x$$

$$x = 3.1$$

3.1 years

Section 3: Interest

1) An investment opportunity is found that makes 7% per year compounded annually. How much should you invest now if you need \$13,450 at the end of 9 years?

$$13450 = P(1.07)^9$$

$$P = \frac{13450}{(1.07)^9}$$

$$P = 7315.91$$

$$\boxed{\$7315.91}$$

12) Jacqueline deposits an inheritance of \$1500 into an account that earns interest of 3.5% per year, compounded annually.

a) How much is in the account after 8 years?

$$A = 1500(1.035)^8$$

$$A = 1975.21$$

$$\boxed{\$1975.21}$$

b) How long will it take for the money to double (round to the nearest year)?

$$2(1500) = 1500(1.035)^n$$

$$2 = (1.035)^n$$

$$\frac{\log 2}{\log 1.035} = n$$

$$n = 20.1488$$

$$\boxed{\text{About 20 years}}$$

13) Five years ago, Denise deposited an amount into an account that pays 7.5% per year, compounded annually. Today the account balance is \$4200.

a) What was the amount of Denise's initial deposit?

$$4200 = P(1.075)^5$$

$$P = \frac{4200}{(1.075)^5}$$

$$P = 2925.55$$

$$\boxed{\$2925.55}$$

b) How much was in the account 2 years ago?

$$4200 = P(1.075)^2$$

$$P = \frac{4200}{(1.075)^2}$$

$$P = 3634.40$$

$$\boxed{\$ 3634.40}$$

c) How much will be in the account 2 years from now?

$$A = 4200(1.075)^2$$

$$A = 4853.63$$

$$\boxed{\$ 4853.63}$$

Section 4: Properties of Exponential Functions and Transformations

14) Match each graph with its corresponding equation

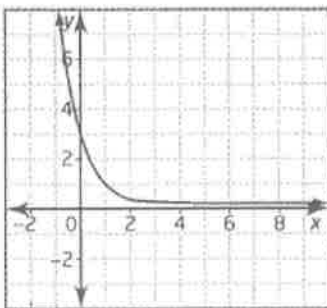
A $y = 3(3^x)$

B $y = 3\left(\frac{1}{3}\right)^x$

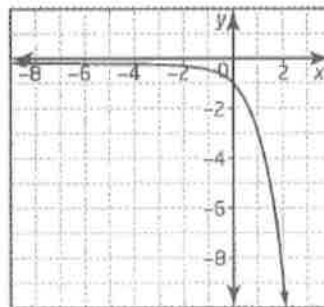
C $\frac{1}{3}(3^x)$

D $y = -3^x$

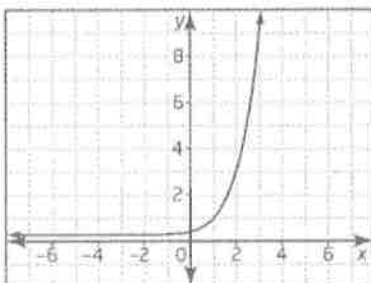
a) B



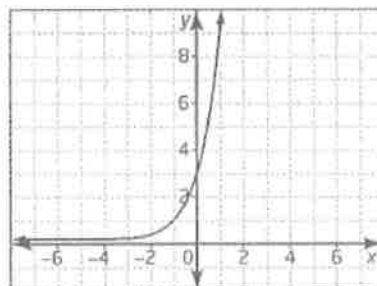
b) D



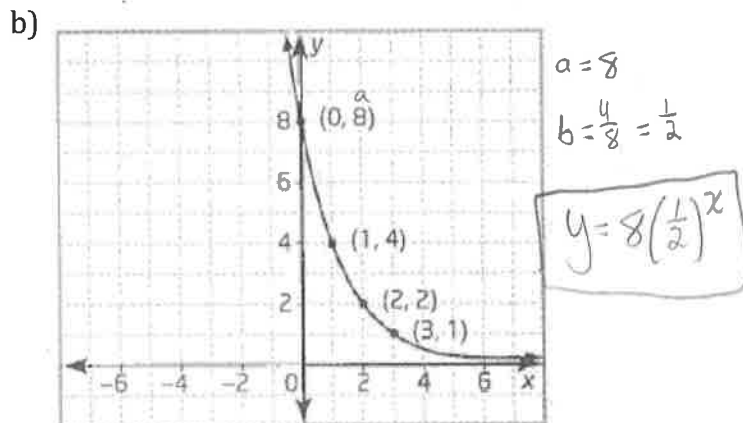
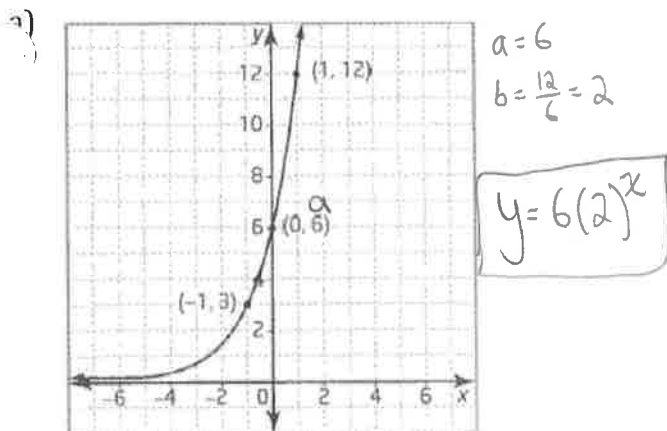
c) C



d) A



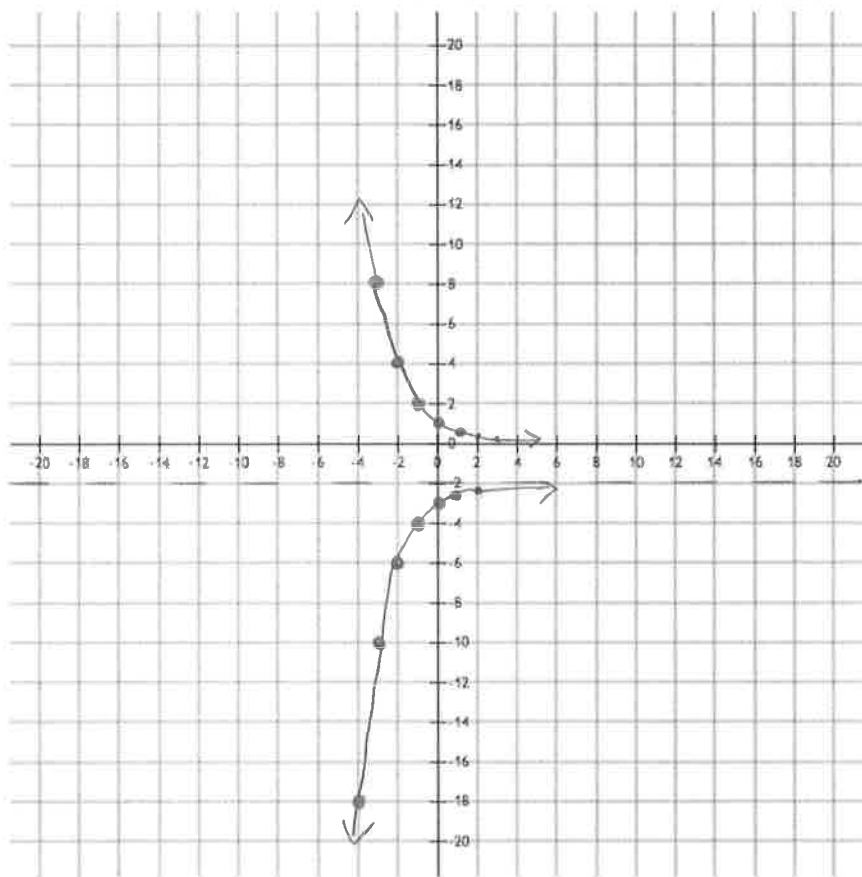
15) Write an exponential equation to match each graph.



16) Sketch the graph base graph of $f(x) = -2\left(\frac{1}{2}\right)^{x+1} - 2$ and use transformations to graph $f(x)$. Is this an increasing or decreasing function?

Handwritten notes:
 $y = \left(\frac{1}{2}\right)^x$
 $f(x) = -2\left(\frac{1}{2}\right)^{x+1} - 2$

x	y	$x-1$	$-2y-2$
-3	8	-4	-18
-2	4	-3	-10
-1	2	-2	-6
0	1	-1	-4
1	0.5	0	-3
2	0.25	1	-2.5
3	0.125	2	-2.25



Answers

1) a) $P(n) = 15 \times 4^n$ b) 245 760

2) a) 3408 b) 9 weeks

3) 67 891

4) 15 seconds

5) a) 96 000 b) 768 000 c) 2 days

6) a) 81 920 b) 195 minutes

7) 0.0625 g

8) a) $f(t) = 40 \left(\frac{1}{2}\right)^{\frac{t}{20}}$ b) 3.54 mg c) approximately 73 days

9) 15 hours

10) a) \$3538.94 b) 3.1 years

11) \$7315.91

12) a) \$1975.21 b) approximately 20 years

13) a) \$2925.55 b) \$3634.40 c) \$4853.63

14) a) B b) D c) C d) A

15) a) $y = 6(2^x)$ b) $y = 8 \left(\frac{1}{2}\right)^x$

16) See posted solutions