7.5 Arithmetic Series

June 1

Gauss' Method

(sum)

$S_n = (a) + (a+d) + (a+2d) + \ldots\ldots + (a+(n-1)d)$ ①

$S_n = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + \ldots + a$ ②

① + ②

$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + \ldots\ldots + (2a+(n-1)d)$

$2S_n = n(2a+(n-1)d)$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_n = \frac{n}{2}(a + a+(n-1)d)$

($a = t_1$, $a+(n-1)d = t_n$)

$S_n = \frac{n}{2}(t_1+t_n)$

TO FIND THE SUM OF AN ARITHMETIC SERIES,

USE $S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}(t_1+t_n)$

Marian goes to a party where there are 23 people present, including her. Each person shakes hands with every other person once and only once.

How can Marian determine the total number of handshakes that take place?

22, 21, 20, ..., 1

$S_n = 22+21+20+\ldots+1$

$S_n = \frac{n}{2}(t_1+t_n)$

$= \frac{22}{2}(22+1)$

$= 11(23)$

$= 253$

In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?

$23, 27, 31, \ldots$

$a = 23, \; d = 4$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{50} = \frac{50}{2}\left[2(23) + (50-1)(4)\right] = 6050$

Determine the sum of $-31 - 35 - 39 - \ldots - 403$.

$a = -31, \; d = -4$

$t_n = a + (n-1)d$

$-403 = -31 + (n-1)(-4)$

$-372 = -4(n-1)$

$93 = n - 1$

$n = 94$

$S_n = \frac{n}{2}(t_1 + t_n)$

$S_{94} = \frac{94}{2}(-31 - 403)$

$= -20398$

p. 452#5acef,6bf,7ac,10,15,16