Unit 5, Lesson 2: Solving Problems with Exponential Functions (Growth/Decay)

Exponential functions can be used to model situations involving growth or decay.

Consider the function $f(x) = aB^x$

If $B > 1$, the function models growth | **If $0 < B < 1$, the function models decay**

$f(x)$ is the final amount
a is the starting or initial amount
B is the change factor
for exponential **growth**, $B = 1 +$ **growth rate**
for exponential **decay**, $B = 1 -$ **decay rate**
x is the number of growth or decay periods

May 12-12:27 PM

Ex 1) Determine the change factor (B) for each situation

% ÷ 100 = decimal

B: increasing = 1 + decimal
decreasing = 1 - decimal

a) money is growing at 5%/year?

0.05

B = 1 + 0.05
B = 1.05

b) a car is depreciating in value by 20%/year?

B = 1 - 0.2
B = 0.8

0.2

c) bacteria is doubling every 8 minutes?

B = 2

d) a chemical is decaying by half every 10 hours

B = 1 - 0.5
B = 0.5

The growth or decay rate can be found by evaluating $\boxed{B - 1 \text{ (then multiply by 100)}}$

Ex 2) Determine the growth or decay rate for each exponential function

a) $f(x) = 3(0.75)^x$

decay by 25%

$0.75 - 1 = -0.25 \times 100$

b) $f(x) = \frac{1}{2}(1.75)^x$

growth by 75%

$1.75 - 1 = +0.75 \times 100$

c) $f(x) = 3^{0.5x}$

growth by 200%

$3 - 1 = 2 \times 100$

May 12-12:32 PM

Ex 3) The following is an excerpt from the Stittsville Village Association website

Stittsville remained a small farming community serving the local area for many years. By 1899, the population was only 205 and by the mid-1950s, had only reached about 500 people. ... By 1993, the population had increased to nearly 10,000. ... The pace and distribution of current development indicates our population figures will continue to rise.

a) Determine the average annual growth rate, as a percent, from $\underbrace{\text{1899-1993}}_{\text{94 years}}$.

$a = 205$

$f(x) = 10\,000$

$x = 94$

$f(x) = aB^x$

$10\,000 = 205(B)^{94}$

$\frac{10\,000}{205} = B^{94}$

$\sqrt[94]{\frac{10000}{205}} = B$

$1.042 = B$

$B = (1 + \%)$

$B = (1.042)$

$B = (1 + 0.042)$

$\times 100$

$= 4.2\%$

$\therefore$ it is growing at a rate of 4.2%

May 12-12:35 PM

b) Create an equation to model the population of Stittsville as a function of years since 1899. Use your function to estimate Stittsville's current population.

Let x represent the numbers of years since 1899.

Let $P(x)$ represent the population.

$$\begin{array}{r} 2019 \\ -\ 1899 \\ \hline 120 \text{ years} \end{array}$$

$P(x) = 205(1.042)^x$

$P(120) = 205(1.042)^{120}$

$\boxed{P(120) \doteq 28\,568}$

$\therefore$ The population in 2019 is approximately 28 568 people.

Nov 30-11:04 AM

Ex 4) The half-life of a radioactive element is the time taken for the element to decay to one half of its initial amount. The half-life of Iodine 131 is 8 days. The function that models the mass, in g, of a 320g sample of Iodine, $m(t)$ after t days is given by $m(t) = 320\left(\frac{1}{2}\right)^{\frac{t}{8}}$

a) Determine the mass of Iodine after 10 days

$m(10) = 320(0.5)^{10/8}$ ← 1.25

$\boxed{m(10) = 134.5g}$

b) How long does it take for the mass of iodine to decay to 10g?

$10 = 320(0.5)^{t/8}$

$\frac{10}{320} = (0.5)^{\frac{t}{8}}$

$\frac{1}{32} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\left(\frac{1}{2}\right)^{5} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\therefore \; 5 = \frac{t}{8}$

$\boxed{40 = t}$

$2 \times 2 \times 2 \times 2 \times 2 = 32$

$2^5 = 32$

$\frac{1}{2^5} = \frac{1}{32}$

$\therefore$ it will 40 days to decay to 10g.

May 12-12:37 PM

Ex 5) Mrs. McKinnell's coffee cools at a rate defined by the function $T(t) = 65\left(\frac{1}{2}\right)^{\frac{t}{75}} + 23$ where T is temperature in degrees Celsius and t is time in minutes.

a) What is the initial temperature of the coffee?

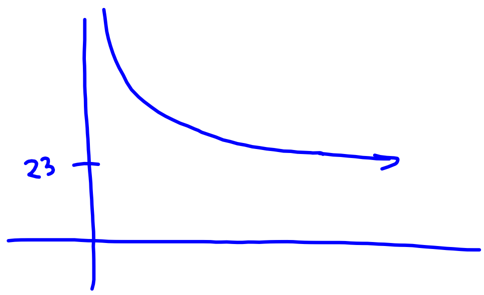
$T(0) = 65(0.5)^{0/75} + 23$

$T(0) = 65(1) + 23$

$\boxed{T(0) = 88^\circ C}$

b) What is the ambient temperature in the room? How do you know?

$23^\circ C$



Nov 30-11:02 AM

c) Mrs. McKinnell pours her coffee every morning at 7:30 am. All first and second period her students are in need of her attention so she doesn't have a chance to sip her coffee. When she finally drinks it at the start of lunch, what temperature will her coffee be?

$$T(t) = 65\left(\frac{1}{2}\right)^{\frac{t}{75}} + 23$$

Lunch : 10:50

from 7:30 → 10:50 3h 20min

$\boxed{200 \text{ min}}$

$T(200) = 65(0.5)^{200/75} + 23$

$\boxed{T(200) = 33.24° \text{ c}}$

Nov 30-11:01 AM

d) Sketch a graph of the function and use the graph to estimate when Mrs. McKinnell should drink her coffee, if her preferred temperature is 63°.

Coffee Temp.

$T(50) = 65(0.5)^{50/75} + 23$

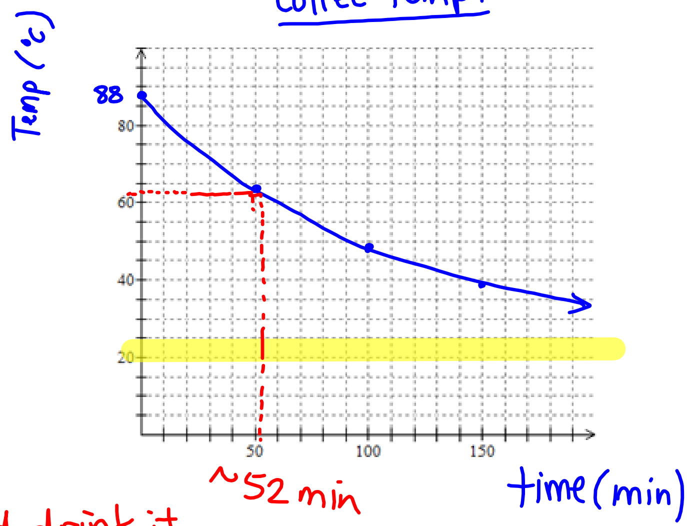
$T(50) \doteq 64°$

$T(100) = 65(0.5)^{100/75} + 23$

$T(100) \doteq 49°$

$T(150) = 65(0.5)^{150/75} + 23$

$T(150) = 39.25$



∴ She should drink it approximately 52 min.

Nov 30-11:00 AM

HW U5L2:

1. p. 261 #2,3/4a-c, 6, 9(use a graph for c),
   10, 11a-d, 12, 15

May 12-9:47 PM