Lesson 3: Series: Arithmetic & Geometric

A series is the *sum* of the terms of a sequence. The sum of the first n terms of a sequence is S_n , where

$$S_n = t_1 + t_2 + t_3 + t_4 + \dots + t_{n-1} + t_n$$

Recall: An <u>arithmetic sequence</u> has the general term $t_n = a + (n - 1)d$, where a is the first term and d is the common difference between terms. An *arithmetic series* is the sum of this sequence and is written

$$S_n = a + (a + d) + (a + 2d) + \dots + [a + (n-2)d] + [a + (n-1)d]$$

To determine a formula, we will write out the series twice, first forward and then backward. Then we will add them together (This is called Gauss's method)

$$S_n = a + (a+d) + \dots + [a+(n-2)d] + [a+(n-1)d] + S_n = [a+(n-1)d] + [a+(n-2)d] + \dots + (a+d) + a$$

$$2 S_n^- = [2a+(n-1)d] + [2a+(n-1)d] + \dots + [2a+(n-1)d] + [2a+(n-1)d]$$

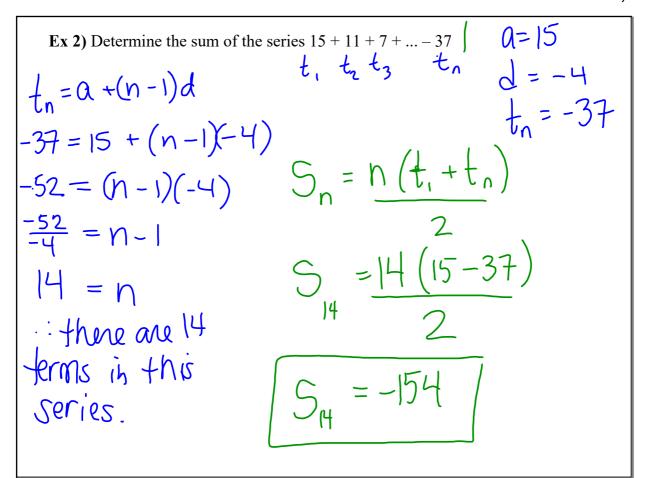
$$2 S_n = n' [2a + (n-1)d]$$

 $S_n = \frac{n}{2} [2a + (n-1)d]$, which is the *sum* of the first *n* terms of an arithmetic sequence.

Recognizing that $t_1 = a$ and $t_n = a + (n - 1)d$, we can also write: $S_n = \frac{n(t_1 + t_n)}{2}$

Jan 31-9:35 PM

Ex 1) Determine the sum of the arithmetic series given $t_1 = 88$ and $t_{15} = 4$ $S_n = \frac{n}{2} \left(\frac{t}{t} + \frac{t}{n} \right)$ $S_n = \frac{15}{88} \left(\frac{88}{44} + \frac{4}{15} \right)$ $S_n = \frac{15}{88} \left(\frac{88}{44} + \frac{4}{15} \right)$ $S_n = \frac{15}{88} \left(\frac{88}{44} + \frac{4}{15} \right)$



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Recall: A geometric sequence has the general term $t_n = ar^{n-1}$, where a is the first term and r is the common ratio. A geometric series is the sum of the terms of this sequence and is written

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1}$$

To determine a formula, multiply the series by the common ratio and subtract the original series.

$$r S_n = ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1} + ar^n$$

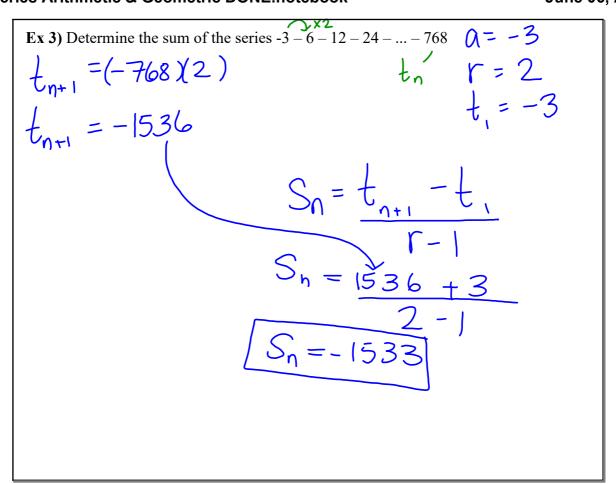
$$- S_n = -(a + ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1})$$

$$(r-1) S_n = -a + ar^n$$

 $S_n = \frac{a(r^n - 1)}{r - 1}$, which is the *sum* of the first *n* terms of a geometric sequence.

Recognizing that $t_1 = a$ and $t_{n+1} = ar^n$, we can also write: $S_n = \frac{t_{n+1} - t_1}{t_n}, r \neq 1$

$$S_n = \frac{t_{n+1} - t_1}{r - 1}, r \neq 1$$



Jun 2-1:17 PM

Ex 4) Determine
$$t_n$$
, S_n and S_0 for the series $81 + 27 + 9 + ...$

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