

3.1 Properties of Quadratic Functions

Mar 10

Francisco owns a business that sells snowboards. His accountants have presented him with data on the business' profit in a table and a graph.

Snowboards Sold, x (1000s)	0	1	2	3	4	5	6	7	8	9
Profit, $f(x)$ (\$10 000s)	-32	-14	0	10	16	18	16	10	0	-14

What function models Francisco's profit?

Snowboards Sold (1000s)	Profit (\$10 000s)	D_1	D_2
0	-32	18	-4
1	-14	14	-4
2	0	10	-4
3	10	6	-4
4	16	2	-4
5	18	-2	-4
6	16	-6	-4
7	10	-10	-4
8	0	-14	-4
9	-14		

FINITE DIFFERENCES

SINCE SECOND DIFFERENCES ARE CONSTANT, FUNCTION IS QUADRATIC

METHOD 1

zeros (2, 0) (8, 0)

$$P(x) = a(x-2)(x-8)$$

ANY POINT $(7, 10) \rightarrow 10 = a(7-2)(7-8)$

$$10 = -5a$$

$$-2 = a$$

$$\therefore P(x) = -2(x-2)(x-8)$$

$(x-r)(x-s)$
FACTORED FORM

METHOD 2

VERTEX (5, 18)

$$P(x) = a(x-5)^2 + 18$$

$(7, 10) \rightarrow 10 = a(7-5)^2 + 18$

$$-8 = 4a$$

$$-2 = a$$

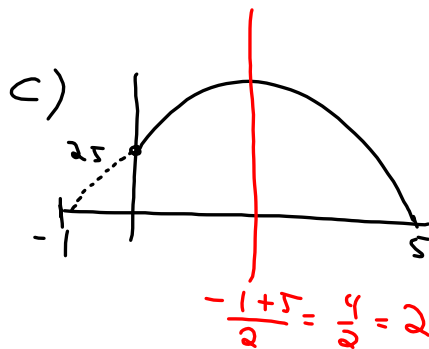
$$\therefore P(x) = -2(x-5)^2 + 18$$

VERTEX FORM
 $y = a(x-h)^2 + k$

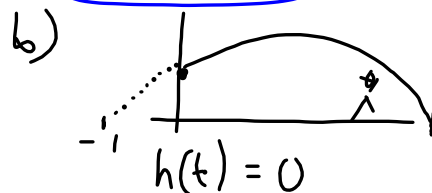
A construction worker repairing a window tosses a tool to his partner across the street. The height of the tool above the ground is modelled by the quadratic function $h(t) = -5t^2 + 20t + 25$, where $h(t)$ is height in metres and t is the time in seconds after the toss.

- How high above the ground is the window?
- If his partner misses the tool, when will it hit the ground?
- If the path of the tool's height were graphed, where would the axis of symmetry be?
- Determine the domain and range of the function in this situation.

a) $h(0) = 0 + 0 + 25$
 \therefore window is 25 m high.



\therefore Axis of symmetry is $x = 2$



$$-5t^2 + 20t + 25 = 0$$

$$-5(t^2 - 4t - 5) = 0$$

$$-5(t + 1)(t - 5) = 0$$

$$(t + 1) = 0 \quad \text{or} \quad (t - 5) = 0$$

$$t = -1 \quad \quad \quad t = 5$$

\therefore HITS GROUND AFTER 5 s.

d) $D = \{x \in \mathbb{R} / 0 \leq x \leq 5\}$
 $R = \{y \in \mathbb{R} / 0 \leq y \leq 45\}$

MAX HEIGHT

$$h(2) = -5(2)^2 + 20(2) + 25$$

$$= 45$$

vertex

Given $f(x) = 2(x - 1)^2 - 5$, state the vertex, axis of symmetry, direction of opening, y -intercept, domain, and range. Graph the function.

vertex $(1, -5)$

y -intercept,

$$y_{\text{INT}} = -3$$

$$\begin{aligned} f(0) &= 2(0-1)^2 - 5 \\ &= 2(1) - 5 \\ &= 2 - 5 \\ &= -3 \end{aligned}$$

axis of symmetry,

$$x = 1$$

domain

$$\{x \in \mathbb{R}\}$$

direction of opening
up

range

$$\{y \in \mathbb{R} / y \geq -5\}$$

Homework p. 145 #1-8,10,11,12,15