Skill Builder: Adding & Subtracting Rational Expressions

Recall: The procedure for **adding or subtracting numeric fractions**

- Determine the **lowest common denominator**
- Create **equivalent fractions** all with the **same denominator**
- **Add or subtract** the **numerators** as needed, keeping the **denominator the same**
- **Simplify** your final answer, if possible.

Example:

$$\frac{7}{12}+\frac{5}{8}=\frac{7(2)}{12(2)}+\frac{5(3)}{8(3)}$$
$$=\frac{14}{24}+\frac{15}{24}$$
$$=\frac{29}{24}$$

However, for rational expressions, we must use the **prime factor method** to determine the LCD.

- Determine the **prime factorization** of each denominator (use powers if necessary)
- The LCD will be the **product of all prime factors**, with each factor given the **highest power** of its occurrence in any denominator
- To check if your LCD is correct, it should "contain" all the prime factors needed for each original denominator, but no extras!

Example:

$12 = 2^2 \times 3^1$

$8 = 2^3$

$LCD = 2^3 \times 3^1$

$LCD = 24$

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Ex 1) Determine the LCD for each set of denominators

a) $\frac{}{ab^2}$ & $\frac{}{a^2b}$

LCD: a^2b^2

c) $\frac{}{15(3x-2)}$ & $\frac{}{10(3x-2)^2}$

LCD: $30(3x-2)^2$

$\frac{x}{15} + \frac{x}{10}$

b) $\frac{}{(2x-3)}$ & $\frac{}{(2x+3)}$

LCD: $(2x-3)(2x+3)$

d) $\frac{}{20x}$ & $\frac{}{35y^2}$ & $\frac{}{14xy}$

LCD: $140xy^2$

20: 20, 40, 60, 80, 100, 120, 140 ...

35: 35, 70, 105, 140

14: 14, 28 ...

140

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To add or subtract rational expressions:
- **Factor** all numerators and denominators.
- **Check** to see if each rational expression is simplified; if not, cancel common factors **within** the rational expression.
- **Determine the LCD** and create equivalent rational expressions
  - each rational expression needs to be multiplied by the factor(s) it is "missing"
- **Add or subtract** the numerators as indicated (expand & gather like terms)
  - when **subtracting**, use **brackets** to ensure your signs are correct!
- **Simplify** the final rational expression, if possible.

Recall: For restrictions, determine the **zeros of ALL ORIGINAL denominators**

Ex 2) Simplify. State any restrictions on the variables.

a) $\frac{7}{12}+\frac{1}{8y}$ LCD: $24y$

$\frac{7\cdot 2y}{12\cdot 24}\cdot\frac{1\cdot 3}{8y\cdot 3}$

$\frac{14y}{24y}+\frac{3}{24y}$

$\frac{(14y+3)}{24y}$, $y\neq 0$

b) $\frac{2}{x}+\frac{3}{x^2}+\frac{1}{2x}$ LCD: $2x^2$

$\frac{2\cdot 2x}{x\cdot 2x}+\frac{3\cdot 2}{x^2\cdot 2}+\frac{1\cdot x}{2x\cdot x}$

$\frac{4x}{2x^2}+\frac{6}{2x^2}+\frac{x}{2x^2}$

$\frac{5x+6}{2x^2}$, $x\neq 0$

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c) $\frac{3}{xy}+\frac{2}{x^2}+\frac{y}{x^2y^2}$ LCD: x^2y^2

$\frac{3\cdot xy}{xy\cdot xy}+\frac{2\cdot y^2}{x^2\cdot y^2}+\frac{y}{x^2y^2}$

$\frac{3xy}{x^2y^2}+\frac{2y^2}{x^2y^2}+\frac{y}{x^2y^2}$

$\frac{(3xy+2y^2+y)}{x^2y^2}$

$\frac{y(3x+2y+1)}{x^2y^2}$

$\frac{3x+2y+1}{x^2y}$, $x\neq 0$, $y\neq 0$

d) $\frac{(5x-7y)}{12x}\cdot 2+\frac{(2x-9y)}{8y}\cdot 3x$ LCD: $24xy$

$\frac{10xy-14y^2}{24xy}+\frac{6x^2-27xy}{24xy}$

$\frac{6x^2-17xy-14y^2}{24xy}$

$\frac{(2x-7y)(3x+2y)}{24xy}$, $x\neq 0$, $y\neq 0$

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e) $\frac{6}{y+1} - \frac{3}{y-1}$

LCD: $(y+1)(y-1)$

$\frac{6(y-1)}{(y+1)(y-1)} - \frac{3(y+1)}{(y-1)(y+1)}$

$\frac{(6y-6)}{(y+1)(y-1)} - \frac{(3y+3)}{(y-1)(y+1)}$

$\frac{6y-6-3y-3}{(y+1)(y-1)}$

$\frac{3y-9}{(y+1)(y-1)}$

$\frac{3(y-3)}{(y+1)(y-1)}$, $y \neq \pm 1$

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HW Skill Builder Day 1:

p. 128 # 1cd, 2cd, 3a, 5bc, 6ace, 10bc

★ Correct Trig Unit tests and sign if under 70%.

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Ex 3) Determine the LCD for each set of denominators

a) $\frac{}{9x^2-12x+4}$ & $\frac{}{15x^2-25x+10}$

$(3x-2)^2$ $\quad 5(x-1)(3x-2)$

LCD: $5(3x-2)^2(x-1)$

b) $\frac{}{6x^2+x-2}$ & $\frac{}{10x^2+9x-7}$ & $\frac{}{9x^2+12x+4}$ $\rightarrow (3x+2)^2$

$(3x+2)(2x-1)$ $\quad (5x+7)(2x-1)$

LCD: $(3x+2)^2(5x+7)(2x-1)$

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Ex 4) Simplify. State any restrictions on the variables.

a) $\frac{3}{(x-1)(x-2)}+\frac{x-2}{(x+2)(x-1)}$ LCD: $(x-1)(x-2)(x+2)$

$= \frac{3(x+2)}{(x-1)(x-2)(x+2)} + \frac{(x-2)(x-2)}{(x+2)(x-1)(x-2)}$

$= \frac{3x+6}{(x-1)(x-2)(x+2)} + \frac{x^2-4x+4}{(x-1)(x-2)(x+2)}$

$= \frac{x^2-x+10}{(x-1)(x-2)(x+2)}, \quad x \neq 1, \pm 2$

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b) $\frac{x-3}{x^2+x-12} - \frac{x-2}{x^2+3x-4}$ LCD: $(x+4)(x-3)(x-1)$

$(x+4)(x-3)$ $(x+4)(x-1)$

$= \frac{(x-3)(x-1)}{(x+4)(x-3)(x-1)} - \frac{(x-2)(x-3)}{(x+4)(x-1)(x-3)}$

$= \frac{x^2-4x+3 - (x^2-5x+6)}{(x-4)(x-3)(x-1)}$

$= \frac{x^2-4x+3-x^2+5x-6}{(x+4)(x-3)(x-1)}$

$= \frac{(x-3)}{(x+4)(x-3)(x-1)}$

$= \frac{1}{(x+4)(x-1)}$; $x \neq -4, 3, 1$

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The **order of operations** still applies for rational expressions: Multiplication and division are done BEFORE addition and subtraction. LCD: $2(2a+1)(a+5)$

Ex 5) Simplify. State any restrictions on the variables.

$\left(\frac{3a+2}{2a^2+11a+5}\right) - \left(\frac{a-2}{6a^2-7a-5} \div \frac{2a}{3a^2-5a}\right)$

$= \frac{3a+2}{(2a+1)(a+5)} - \left(\frac{(a-2)}{(2a+1)(3a-5)} \times \frac{a(3a-5)}{2a}\right)$

$= \frac{(3a+2)\cdot 2}{(2a+1)(a+5)\cdot 2} - \frac{(a-2)(a+5)}{2(2a+1)(a+5)}$

$= \frac{6a+4 - (a^2+3a-10)}{2(2a+1)(a+5)}$

$= \frac{-a^2+3a+14}{2(2a+1)(a+5)}$; $a \neq -\frac{1}{2}, -5, \frac{5}{3}, 0$

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Try on your own:

$$\frac{3x^2-7}{3x^3+6x^2-7x-14}-\frac{16x^2-56x+49}{9x^2-25}\div\frac{4x^2+x-14}{6x^2+10x}$$

$$=\frac{3x^2-7}{3x^2(x+2)-7(x+2)}-\left[\frac{(4x-7)(4x-7)}{(3x-5)(3x+5)}\times\frac{2x(3x+5)}{(4x-7)(x+2)}\right]$$

$$=\frac{(3x^2-7)}{(3x^2-7)(x+2)}-\left[\frac{2x(4x-7)}{(3x-5)(x+2)}\right]$$

$$=\frac{1(3x-5)}{(x+2)(3x-5)}-\frac{8x^2-14x}{(3x-5)(x+2)}$$

$$=\frac{3x-5-8x^2+14x}{(x+2)(3x-5)}$$

$$=\frac{-8x^2+17x-5}{(x+2)(3x-5)}\,;\quad x\neq 0,-2,\pm\sqrt{\frac{7}{3}}$$

$$\pm\frac{5}{3},\frac{7}{4}$$

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