

3.7 Families of Quadratics

Take a look at these equations:

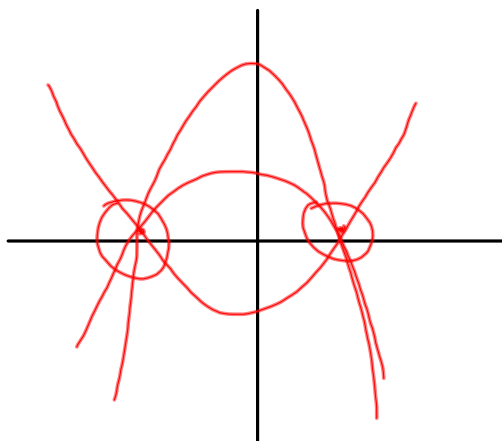
$$\begin{cases} f(x) = x^2 - 3x - 10 \\ g(x) = -2x^2 + 6x + 20 \\ h(x) = 4x^2 - 12x - 40 \\ k(x) = -0.5x^2 + 1.5x + 5 \end{cases}$$

A group of quadratics that
Share a common characteristic

What are some similarities? Differences?

All have same zeros

Now, let's graph them



Factoring:

$$f(x) = x^2 - 3x - 10 \\ = (x - 5)(x + 2)$$

$$g(x) = -2x^2 + 6x + 20 \\ = -2(x^2 - 3x - 10) \\ = -2(x - 5)(x + 2)$$

Concluding remarks:

A family of quadratics is created when the
value of 'a' is varied in factored form.

$$y = a(x - r)(x - s)$$

zeros

Now try these:

$$\begin{cases} m(x) = -2x^2 + 4x + 1 \\ n(x) = 0.5x^2 - 1x + 3.5 \\ p(x) = -6x^2 + 12x - 3 \\ q(x) = 10x^2 - 20x + 13 \end{cases}$$

What do you see?

All have the same vertex

$$y = a(x-h)^2 + k$$

$$\begin{aligned} m(x) &= -2x^2 + 4x + 1 \\ &= -2(x^2 - 2x + \underbrace{1}_{(-1)}) + 1 + 2 \\ &= \boxed{-2}(x-1)^2 + 3 \Rightarrow \text{vertex } (1, 3) \end{aligned}$$

$$\begin{aligned} n(x) &= 0.5x^2 - 1x + 3.5 \\ &= 0.5(x^2 - 2x + \underbrace{1}_{-1}) + 3.5 \\ &= \boxed{0.5}(x-1)^2 + 3 \Rightarrow \text{vertex } (1, 3) \end{aligned}$$

Concluding remarks:

A family of quadratics is created in vertex form when 'a' is varied. $y = a(x-h)^2 + k$

Last group:

$$r(x) = -3x^2 + 5x - 2$$

$$s(x) = 2x^2 + x - 2$$

$$t(x) = 7x^2 - 2x - 2$$

$$u(x) = -4x^2 - 4x - 2$$

What do you see?

All have the same y-int.

Concluding remarks:

A set of functions will share a y-int.
if 'a' and 'b' vary but 'c' is fixed

$$y = ax^2 + bx + c$$

Given the function $f(x) = -3(x+2)^2 - 1$, determine another quadratic function within the same family.

vertex of $f(x)$ is $(-2, -1)$

$$g(x) = \underline{5}(x+2)^2 - 1$$

$$h(x) = -7.23(x+2)^2 - 1$$

what about $g(x) = 2x^2 + 8x - 7$?

$$x = \frac{-b}{2a} = \frac{-8}{4} = -2$$

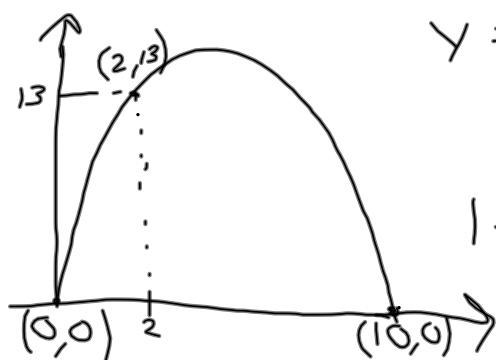
$$g(-2) = 2(4) + 8(-2) - 7 = 8 - 16 - 7 = -15$$

vertex $(-2, -15)$ $\therefore g(x)$ is not in the same family as $f(x)$.

Determine the equation of the quadratic function that passes through $(-3, 20)$ if its zeros are 2 and -1.

$$\begin{array}{l} \text{given} \\ (-3, 20) \\ (2, 0) \\ (-1, 0) \end{array} \quad \begin{array}{l} y = a(x-r)(x-s) \\ y = a(x-2)(x+1) \\ 20 = a(-3-2)(-3+1) \\ 20 = a(-5)(-2) \\ 20 = 10a \\ a = 2 \\ \therefore y = 2(x-2)(x+1) \end{array}$$

A roller coaster has a shape that can be modelled by the equation of a parabola. If the start of the roller coaster is the origin, and the roller coaster is 10m wide, what is the equation of the parabola if the height of the roller coaster 2m from the start is 13m.



$$y = a(x-0)(x-10) \\ = a(x)(x-10)$$

$$13 = a(2)(-8)$$

$$13 = -16a$$

$$a = \frac{-13}{16}$$

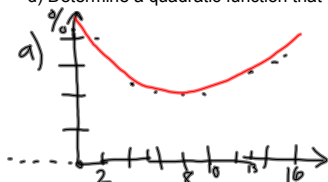
$$\therefore y = \frac{-13}{16}x(x-10)$$



The percent of 15- to 19-year-old males who smoke has been tracked by Health Canada. The data from 1981 to 1996 are given in the table.

Year	1981	1983	1985	1986	1989	1991	1994	1995	1996
Smokers (%)	43.4	39.6	26.7	25.2	22.6	22.6	27.3	28.5	29.1

- a) Draw a scatter plot
 b) Draw a curve of best fit
 c) Estimate the coordinates of the vertex
 d) Determine a quadratic function that will model the data



c) Vertex (9, 22)

$$d) y = a(x-h)^2 + k$$

$$y = a(x-9)^2 + 22$$

$$43.4 = a(81) + 22$$

$$21.4 = 81a$$

$$a = 0.264$$

$$\therefore y = 0.264(x-9)^2 + 22$$

Let's use the calculator to find the quadratic regression

- enter data: **stat** **enter**
- create scatter plot: **2nd** **y=** turn **Plot1 On** **graph**
- Regression analysis: **stat** scroll over to **Calc** then press **QuadReg** then **enter**
- Graph curve: **stat** scroll over to **Calc** then press **QuadReg** then **vars** scroll over to **Y-Vars** and press **enter** **enter**
Now hit **graph**

HMWK: pg.192 # 4-10, 12-14