Course Review

Chapter 1 Functions

1. Determine the domain and the range for each relation. Sketch a graph of each.

a) $y = \frac{3}{x - 9}$ **b)** $y = \sqrt{2 - x} - 4$

2. Which of the following is NOT true?

A All functions are also relations.

B The vertical line test is used to determine if the graph of a relation is a function.

C All relations are also functions.

D Some relations are also functions.

3. The approximate time for an investment to double can be found using the function $n(r) = \frac{72}{r}$, where n represents the number of years and r represents the annual interest rate, as a percent.

a) How long will it take an investment to double at each rate?

i) 3% **ii)** 6% **iii)** 9%

b) Graph the data to illustrate the function.

c) Determine the domain and range in this context.

4. Determine the vertex of each quadratic function by completing the square. Verify your answer by using partial factoring. State if the vertex is a minimum or a maximum.

a) $f(x) = 3x^2 + 9x + 1$

b) $f(x) = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$

5. A small company manufactures a total of x items per week. The production cost is modelled by the function $C(x) = 50 + 3x$. The revenue is given by the function $R(x) = 6x - \frac{x^2}{100}$. How many items per week should be manufactured to maximize the profit for the company?

Hint: Profit = Revenue − Cost

6. Simplify.

a) $2\sqrt{243} - 5\sqrt{48} + \sqrt{108} - \sqrt{192}$

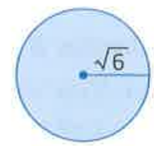
b) $\frac{2}{3}\sqrt{125} - \frac{1}{3}\sqrt{27} + 2\sqrt{48} - 3\sqrt{80}$

7. Expand. Simplify where possible.

a) $(\sqrt{5} + 2\sqrt{3})(3\sqrt{5} + 4\sqrt{3})$

b) $(4 - \sqrt{6})(1 + \sqrt{6})$

8. Find a simplified expression for the area of the circle.



9. Solve $3x^2 + 9x - 30 = 0$ by

a) completing the square

b) using a graphing calculator

c) factoring

d) using the quadratic formula

10. The length of a rectangle is 5 m more than its width. If the area of the rectangle is 15 m², what are the dimensions of the rectangle, to the nearest tenth of a metre?

11. Find an equation for the quadratic function with the given zeros and containing the given point. Express each function in standard form. Graph each function to check.

a) $2 \pm \sqrt{3}$, point $(4, -6)$

b) 4 and −1, point $(1, -4)$

12. An arch of a highway overpass is in the shape of a parabola. The arch spans a distance of 16 m from one side of the road to the other. At a horizontal distance of 1 m from each side of the arch, its height above the road is 6 m.

a) Sketch the quadratic function if the vertex of the parabola is on the y-axis and the road is along the x-axis.

b) Use this information to determine the equation of the function that models the arch.

c) Find the maximum height of the arch.

13. At a fireworks display, the path of the biggest firework can be modelled using the function $f(x) = -0.015x^2 + 2.24x + 1.75$, where x is the horizontal distance from the launching platform. The profile of a hill, some distance away from the platform, can be modelled with the equation $h(x) = 0.7x - 83$, with all distances in metres. Will the firework reach the hill? Justify your answer.

Chapter 2 Transformations of Functions

14. Test whether the functions in each pair are equivalent by
    - **i)** testing three different values of x
    - **ii)** simplifying the expressions on the right sides
    - **iii)** graphing using graphing technology

    a) $f(x) = -2(x + 3)^2 + (5x + 1)$, $g(x) = -2x^2 - 7x - 17$

    b) $f(x) = \dfrac{x^2 - 2x - 15}{x^2 - 9x + 20}$, $g(x) = \dfrac{x + 3}{x - 4}$

15. Simplify and state the restrictions.

    a) $\dfrac{-x + 1}{8x} \div \dfrac{2x - 2}{14x^2}$

    b) $\dfrac{x^2 + 5x - 36}{x^2 - 2x} \div \dfrac{x^2 + 11x + 18}{8x^2 - 4x^3}$

    c) $\dfrac{x^2 - 25}{x - 4} \times \dfrac{x^2 - 6x + 8}{3x + 15}$

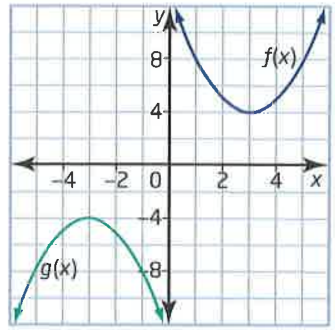
16. For each function $g(x)$, state the corresponding base function $f(x)$. Describe the transformations that must be applied to the base function using function notation and words. Then, transform the graph of $f(x)$ to sketch the graph of $g(x)$ and state the domain and range of each function.

    a) $g(x) = \dfrac{1}{x + 5} - 1$

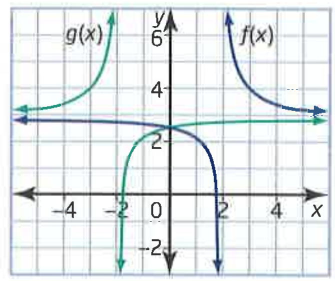
    b) $g(x) = \sqrt{x + 7} - 9$

17. For each graph, describe the reflection that transforms $f(x)$ into $g(x)$.

    a)



    b)



18. For each of the functions $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \dfrac{1}{x}$, write an equation to represent $g(x)$ and $h(x)$ and describe the transformations. Then, transform the graph of $f(x)$ to sketch graphs of $g(x)$ and $h(x)$ and state the domain and range of the functions.

    a) $g(x) = 4f(-x)$ and $h(x) = \dfrac{1}{4}f(x)$

    b) $g(x) = f(4x)$ and $h(x) = -f\left(\dfrac{1}{4}x\right)$

19. A ball is dropped from a height of 32 m. Acceleration due to gravity is -9.8 m/s². The height of the ball is given by $h(t) = -4.9t^2 + 32$.

    a) State the domain and range of the function.

    b) Write the equation for the height of the object if it is dropped on a planet with acceleration due to gravity of -11.2 m/s².

    c) Compare the domain and range of the function in part b) to those of the given function.

20. Describe the combination of transformations that must be applied to the base function $f(x)$ to obtain the transformed function $g(x)$. Then, write the corresponding equation and sketch its graph.

a) $f(x) = x$, $g(x) = -2f[3(x - 4)] - 1$

b) $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{3}f\left[\frac{1}{4}(x - 2)\right] + 3$

21. For each function $f(x)$,

i) determine $f^{-1}(x)$

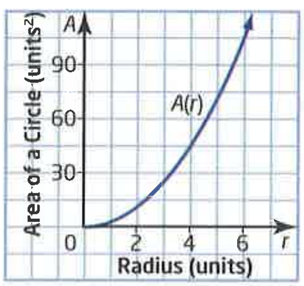
ii) graph $f(x)$ and its inverse

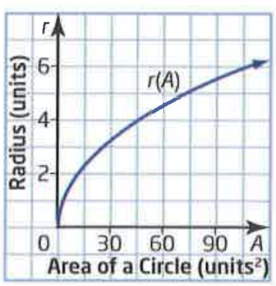
iii) determine whether the inverse of $f(x)$ is a function

a) $f(x) = 4x - 5$

b) $f(x) = 3x^2 - 12x + 3$

22. The relationship between the area of a circle and its radius can be modelled by the function $A(r) = \pi r^2$, where A is the area and r is the radius. The graphs of this function and its inverse are shown.





a) State the domain and range of the function $A(r)$.

b) Determine the equation of the inverse of the function. State its domain and range.

Chapter 3 Exponential Functions

23. A petri dish contains an initial sample of 20 bacteria. After 1 day, the number of bacteria has tripled.

a) Determine the population after each day for 1 week.

b) Write an equation to model this growth.

c) Graph the relation. Is it a function? Explain why or why not.

d) Assuming this trend continues, predict the population after

i) 2 weeks

ii) 3 weeks

e) Describe the pattern of finite differences for this relationship.

24. Tritium is a substance that is present in radioactive waste. It has a half-life of approximately 12 years. How long will it take for a 50-mg sample of tritium to decay to 10% of its original mass?

25. Apply the exponent rules first, if possible, and then evaluate.

a) $(-8)^{-2} + 2^{-6}$

b) $(3^{-3})^{-2} \div 3^{-5}$

c) $\left(\frac{2^3}{3^2}\right)^{-2}$

d) $\frac{(6^6)(6^{-3})}{6^2}$

26. Simplify.

a) $(4n^{-2})(-3n^5)$

b) $\frac{12c^{-3}}{15c^{-5}}$

c) $(3a^2b^{-2})^{-3}$

d) $\left(\frac{-2p^3}{3q^4}\right)^{-5}$

27. Evaluate.

a) $16^{-\frac{3}{4}}$

b) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$

c) $\left(-\frac{8}{125}\right)^{-\frac{2}{3}}$

28. Simplify. Express your answers using only positive exponents.

a) $\frac{a^{-2}b^3}{a^{\frac{1}{4}}b^{\frac{2}{3}}}$

b) $(u^{-\frac{2}{3}}v^{\frac{1}{4}})^{\frac{3}{5}}$

c) $w^{\frac{7}{8}} \div w^{-\frac{3}{4}}$

29. Graph each exponential function. Identify the

- domain
- range
- x- and y-intercepts, if they exist
- intervals of increase/decrease
- asymptote

a) $y = 5\left(\frac{1}{3}\right)^x$ **b)** $y = -4^{-x}$

30. A radioactive sample has a half-life of 1 month. The initial sample has a mass of 300 mg.

a) Write a function to relate the amount remaining, in milligrams, to the time, in months.

b) Restrict the domain of the function so the mathematical model fits the situation it is describing.

31. Sketch the graph of each function, using the graph of $y = 8^x$ as the base. Describe the effects, if any, on the

- asymptote
- domain
- range

a) $y = 8^{x-4}$ **b)** $y = 8^{x+2} + 1$

32. Write the equation for the function that results from each transformation applied to the base function $y = 11^x$.

a) reflect in the x-axis and stretch vertically by a factor of 4

b) reflect in the y-axis and stretch horizontally by a factor of $\frac{4}{3}$

33. At midnight, one hospital patient contracts an unknown virus. By 1 a.m., three other hospital patients are diagnosed with the same virus. One hour later, nine more patients are found to have the virus, and by 3 a.m., 27 more patients have the virus. The virus continues to spread this way through the hospital.

a) Make a table of values to relate the number of new patients who are diagnosed with the virus to time, in 1-h intervals.

b) Make a scatter plot. Describe the trend.

c) What type of function represents the spread of this virus? Justify your answer.

d) Determine an equation to model this relation. Explain the method you chose to determine the equation.

Chapter 4 Trigonometry

34. a) To find trigonometric ratios for 240° using a unit circle, a reference angle of 60° is used. What reference angle should you use to find the trigonometric ratios for 210°?

b) Use the unit circle to find exact values of the three primary trigonometric ratios for 210° and 240°.

35. A fishing boat is 15 km south of a lighthouse. A yacht is 15 km west of the same lighthouse.

a) Use trigonometry to find an exact expression for the distance between the two boats.

b) Check your answer using another method.

36. Without using a calculator, determine two angles between 0° and 360° that have a sine of $\frac{\sqrt{3}}{2}$.

37. The point P(−2, 7) is on the terminal arm of ∠A.

a) Determine the primary trigonometric ratios for ∠A and ∠B, such that ∠B has the same sine as ∠A.

b) Use a calculator and a diagram to determine the measures of ∠A and ∠B, to the nearest degree.

38. Consider right $\triangle PQR$ with side lengths $PQ = 5$ cm and $QR = 12$ cm, and $\angle Q = 90°$.

a) Determine the length of side PR.

b) Determine the six trigonometric ratios for $\angle P$.

c) Determine the six trigonometric ratios for $\angle R$.

39. Determine two possible measures between 0° and 360° for each angle, to the nearest degree.

a) $\csc A = \frac{7}{3}$

b) $\sec B = -6$

c) $\cot C = -\frac{9}{4}$

40. An oak tree, a chestnut tree, and a maple tree form the corners of a triangular play area in a neighbourhood park. The oak tree is 35 m from the chestnut tree. The angle between the maple tree and the chestnut tree from the oak tree is 58°. The angle between the oak tree and the chestnut tree from the maple tree is 49°.

a) Sketch a diagram of this situation. Why is the triangle formed by the trees an oblique triangle?

b) Is it necessary to consider the ambiguous case? Justify your answer.

c) Determine the unknown distances, to the nearest tenth of a metre. If there is more than one possible answer, determine both.

41. At noon, two cars travel away from the intersection of two country roads that meet at a 34° angle. Car A travels along one of the roads at 80 km/h and car B travels along the other road at 100 km/h. Two hours later, both cars spot a jet in the air between them. The angle of depression from the jet to car A is 20° and the distance between the jet and the car is 100 km. Determine the distance between the jet and car B.

42. Isra parks her motorcycle in a lot on the corner of Canal and Main streets. She walks 60 m west to Maple Avenue, turns 40° to the left, and follows Maple Avenue for 90 m to the office building where she works. From her office window on the 18th floor, she can see her motorcycle in the lot. Each floor in the building is 5 m in height.

a) Sketch a diagram to represent this problem, labelling all given measurements.

b) How far is Isra from her motorcycle, in a direct line?

43. Prove each identity.

a) $\dfrac{1}{\sin^2 \theta} + \dfrac{1}{\cos^2 \theta} = \left(\tan \theta + \dfrac{1}{\tan \theta}\right)^2$

b) $\csc \theta\left(\dfrac{1}{\cot \theta} + \dfrac{1}{\sec \theta}\right) = \sec \theta + \cot \theta$

Chapter 5 Trigonometric Functions

44. a) Sketch a periodic function, $f(x)$, with a maximum value of 5, a minimum value of -3, and a period of 4.

b) Select a value a for x, and determine $f(a)$.

c) Determine two other values, b and c, such that $f(a) = f(b) = f(c)$.

45. While visiting a town along the ocean, Bashira notices that the water level at the town dock changes during the day as the tides come in and go out. Markings on one of the piles supporting the dock show a high tide of 4.8 m at 6:30 a.m., a low tide of 0.9 m at 12:40 p.m., and a high tide again at 6:50 p.m.

a) Estimate the period of the fluctuation of the water level at the town dock.

b) Estimate the amplitude of the pattern.

c) Predict when the next low tide will occur.

46. Consider the following functions.

i) $y = 4 \sin \left[\frac{1}{3}(x + 30°)\right] - 1$

ii) $y = -\frac{1}{2} \cos [4(x + 135°)] + 2$

a) What is the amplitude of each function?

b) What is the period of each function?

c) Describe the phase shift of each function.

d) Describe the vertical shift of each function.

e) **Use Technology** Graph each function. Compare the graph to the characteristics expected.

47. A sinusoidal function has an amplitude of 6 units, a period of 150°, and a maximum at (0, 4).

a) Represent the function with an equation using a sine function.

b) Represent the function with an equation using a cosine function.

48. The height, h, in metres, above the ground of a rider on a Ferris wheel after t seconds can be modelled by the sine function $h(t) = 9 \sin [2(t - 30)] + 10$.

a) **Use Technology** Graph the function.

b) Determine

i) the maximum and minimum heights of the rider above the ground

ii) the height of the rider above the ground after 30 s

iii) the time required for the Ferris wheel to complete one revolution

49. Marcia constructs a model alternating current (AC) generator in physics class and cranks it by hand at 4 revolutions per second. She is able to light up a flashlight bulb that is rated for 6 V. The voltage can be modelled by a sine function of the form $y = a \sin [k(x - d)] + c$.

a) What is the period of the AC produced by the generator?

b) Determine the value of k.

c) What is the amplitude of the voltage function?

d) Model the voltage with a suitably transformed sine function.

e) **Use Technology** Graph the voltage function over two cycles. Explain what the scales on the axes represent.

Chapter 6 Discrete Functions

50. Write the ninth term, given the explicit formula for the nth term of the sequence.

a) $t_n = \frac{n^2 - 1}{2n}$

b) $f(n) = (-3)^{n-2}$

51. Write the first five terms of each sequence.

a) $t_1 = 3$, $t_n = \frac{t_{n-1}}{0.2}$

b) $f(1) = \frac{2}{5}$, $f(n) = f(n - 1) - 1$

52. A hospital patient, recovering from surgery, receives 400 mg of pain medication every 5 h for 3 days. The half-life of the pain medication is approximately 5 h. This means that after 5 h, about half of the medicine is still in the patient's body.

a) Create a table of values showing the amount of medication remaining in the body after each 5-h period.

b) Write the amount of medication remaining after each 5-h period as a sequence. Write a recursion formula for the sequence.

c) Graph the sequence.

d) Describe what happens to the medicine in the patient's body over time.

Use Pascal's triangle for questions 53 and 54.

53. Expand each power of a binomial.

a) $(x - y)^6$

b) $\left(\frac{x}{3} - 2x\right)^4$

54. Write each as the sum of two terms, each in the form $t_{n,r}$.

a) $t_{5,2}$

b) $t_{10,7}$

55. **a)** State whether or not each sequence is arithmetic. Justify your answer.

i) $9, 5, 1, -3, \ldots$

ii) $\frac{1}{5}, \frac{3}{5}, 1, \frac{7}{5}, \frac{9}{5}, \ldots$

iii) $-4.2, -3.8, -3.5, -3.3, -3.2, \ldots$

b) For those sequences that are arithmetic, write the formula for the general term.

56. For each geometric sequence, determine the formula for the general term and then write t_{10}.

a) $90, 30, 10, \ldots$

b) $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \ldots$

c) $-0.0035, 0.035, -0.35, \ldots$

57. Determine the sum of the first 10 terms of each arithmetic series.

a) $a = 2$, $d = -3$, $t_{10} = 56$

b) $a = -5$, $d = 1.5$

58. Determine the sum of each geometric series.

a) $45 + 15 + 5 + \cdots + \frac{5}{729}$

b) $1 - x + x^2 - x^3 + \cdots - x^{15}$

59. A bouncy ball bounces to $\frac{3}{5}$ of its height when dropped on a hard surface. Suppose the ball is dropped from 45 m.

a) What height will the ball bounce back up to after the seventh bounce?

b) What is the total distance travelled by the ball after 12 bounces?

Chapter 7 Financial Applications

60. Richard deposits $1000 into a guaranteed investment certificate (GIC) that earns 4.5% per year, simple interest.

a) Develop a linear model to relate the amount in the GIC to time. Identify the fixed part and the variable part. Graph the function.

b) How long will it take, to the nearest month, for the investment to double?

c) What annual rate of interest must be earned so that the investment doubles in 6 years?

61. Suppose you have $2500 to invest for 6 years. Two options are available:

- Top Bank: earns 6% per year, compounded quarterly
- Best Credit Union: earns 5.8% per year, compounded weekly

Which investment would you choose and why?

62. Five years ago, money was invested at 6.75% per year, compounded annually. Today the investment is worth $925.

a) How much money was originally invested?

b) How much interest was earned?

63. At the end of every month, Cassie deposits $120 into an account that pays 5.25% per year, compounded monthly. She does this for 5 years.

a) Draw a time line to represent this annuity.

b) Determine the amount in the account after 5 years.

c) How much interest will have been earned?

64. Murray plans to withdraw $700 at the end of every 3 months, for 5 years, from an account that earns 7% interest, compounded quarterly.

a) Draw a time line to represent this annuity.

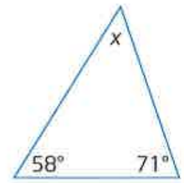
b) Determine the present value of the annuity.

c) How much interest will have been earned?

Prerequisite Skills Appendix

Angle Sum of a Triangle

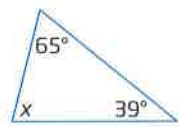
Use the fact that the sum of the interior angles of a triangle is 180° to find the measure of x.



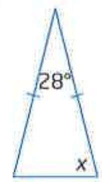
$$x + 58° + 71° = 180°$$
$$x = 180° - 129°$$
$$x = 51°$$

1. Find the measure of each angle x.

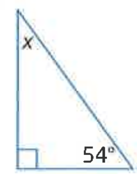
a)



b)



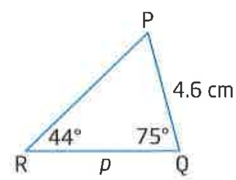
c)



Apply the Sine Law and the Cosine Law

Use the sine law to solve any acute triangle given
- the measures of two angles and any side
- the measures of two sides and the angle opposite one of the given sides

To find the length of side p, first determine the measure of ∠P.



Apply the angle sum of a triangle.

$$\angle P = 180° - 44° - 75°$$
$$= 61°$$

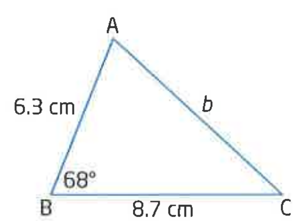
Then, use the sine law.

$$\frac{p}{\sin 61°} = \frac{4.6}{\sin 44°}$$
$$p = \frac{4.6 \sin 61°}{\sin 44°}$$
$$\doteq 5.8$$

The length of side p is approximately 5.8 cm.

Use cosine law to solve any triangle given
- the measures of two sides and the contained angle
- the measures of three sides

Use the cosine law to find b.

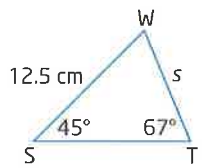


$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$b^2 = 8.7^2 + 6.3^2 - 2(8.7)(6.3) \cos 68°$$
$$b^2 \doteq 74.316$$
$$b \doteq 8.6$$

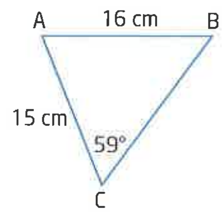
The length of side b is approximately 8.6 cm.

1. Determine the measure of the angle or side indicated.

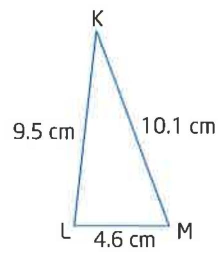
a) length of s



b) measure of $\angle$B



c) measure of $\angle$M

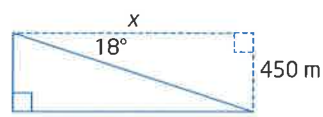


Apply Trigonometric Ratios to Problems

Various problems involving right triangles can be solved using trigonometric ratios.

An airplane that is 450 m above the ground is coming down for a landing at an angle of depression of 18°. How far, horizontally, is the plane from its landing point?

Draw and label a diagram to represent the given information. Use the tangent ratio and solve for x.



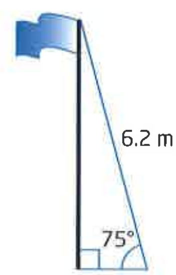
$$\tan 18° = \frac{450}{x}$$

$$x = \frac{450}{\tan 18°}$$

$$\doteq 1385$$

The plane is about 1385 m horizontally from its landing point.

1. A guy wire attached to the ground and the top of a flagpole is 6.2 m long. The wire makes an angle of 75° with the ground. How tall is the flagpole?



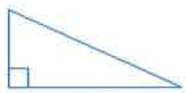
2. The ramp from the back of a truck is 4.0 m long. If the back of the truck is 0.6 m above the ground, at what angle is the ramp inclined?

Classify Triangles

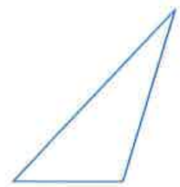
Triangles can be classified according to their side lengths or the measures of their angles.

Classification by Side Length
- Equilateral: all three sides equal
- Isosceles: two sides equal
- Scalene: no sides equal

Classification by Angle Measure
- Right triangle: contains a 90° angle
- Acute triangle: has all angles less than 90°
- Obtuse triangle: contains an angle greater than 90°

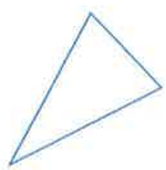

This is a scalene right triangle.


This is an isosceles acute triangle.

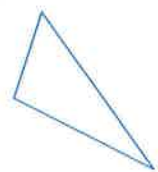

This is a scalene obtuse triangle.

1. Classify each triangle by its sides and angles.

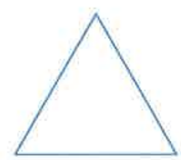
a)



b)



c)



Common Factors

To factor an expression, determine the greatest common factor (GCF) of each term.

To factor $6m^2 - 15m$, write each term in expanded form.
$6m^2 = 2 \times 3 \times m \times m$
$15m = 3 \times 5 \times m$

$3m$ is the GCF.
Find the second factor by dividing the GCF into each term in the original expression.

$$\frac{6m^2}{3m} - \frac{15m}{3m} = 2m - 5$$

Therefore, $6m^2 - 15m = 3m(2m - 5)$.
A common factor can have more than one term. For example,
$3(x - 2) + x^2(x - 2) = (x - 2)(3 + x^2)$.

1. Find the GCF of each set of terms.

a) $8x$, $12y$

b) $-12a^3$, $6a^2b$, $9ab$

c) $10xy^3$, $35x^3y^2$

d) $24m^3n^2$, $-72m^2n^4$, $96m^2n^3$

e) $6(a^2 + 3)$, $-5(a^2 + 3)$

f) $3x^2y + 12xy$, $15xy^3 - 6x^2y$

2. Factor fully.

a) $16x^2 + 20x$

b) $5x^2y^2 + 10xy^3$

c) $3a^3 - 9a^2$

d) $4r^5s^2 + 16r^2s$

e) $8a^3b - 10ab + 4a^2b^2$

f) $-6x^3y - 18x^4y^3 - 36x^2y^4$

g) $12p(p + 3q) - q(p + 3q)$

h) $8x(y - 2x^2) + 20xy(y - 2x^2)$

Determine an Angle Given a Trigonometric Ratio

To determine the measure of an acute angle, use the corresponding inverse function on a scientific or graphing calculator.

If $\sin A = 0.3897$, then
$\angle A = \sin^{-1} 0.3897$
$\angle A \doteq 22.9°$

1. Determine the measure of each acute angle, to the nearest degree.

a) $\cos A = 0.2598$

b) $\sin Q = 0.8339$

c) $\tan T = 2.4591$

d) $\cos P = 0.7662$

e) $\sin X = 0.3478$

f) $\tan C = 0.6264$

Direct Variation and Partial Variation

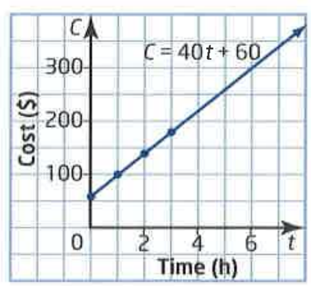
In a direct variation, one variable is a constant multiple of another variable. For example, $y = 4x$ is a direct variation. It is a linear relation that passes through the origin.

A partial variation is a linear relation between two variables that involves a fixed amount plus a variable amount.

A plumber charges \$60, plus \$40/h for labour.
The equation of the relationship is $C = 40t + 60$, where C represents the total cost, in dollars, and t represents the time, in hours.
The fixed part is 60 and the variable part is 40.

Use a table of values to graph the relationship.

Time (h)	Cost (\$)
0	60
1	100
2	140
3	180



The slope of the line is $\dfrac{100 - 60}{1 - 0} = 40$.

The vertical intercept is 60.

The slope is the coefficient of the variable part and represents the hourly cost of labour. The vertical intercept is the fixed part and represents the fixed or initial cost of labour.

1. For an advertisement in the classified section, a newspaper charges \$25 plus \$12 per day.
   a) Write an equation to relate the total cost, C, in dollars, of the advertisement to the number, d, of days.
   b) Identify the fixed part and the variable part of this relation.
   c) Graph the relation.
   d) Determine the slope and the vertical intercept of the graph.
   e) Explain how your answers in parts b) and d) are related.

2. Irene works part-time at a clothing store. She earns \$200 per week, plus a commission of 5% of her sales.
   a) Write an equation to relate her weekly salary to her sales.
   b) Identify the fixed part and the variable part of this relation.
   c) Graph the relation.
   d) Determine the slope and the vertical intercept of the graph.
   e) Explain how your answers in parts b) and d) are related.

Distributive Property

According to the distributive property, $a(x + y) = ax + ay$.
An algebraic expression in factored form can be expanded by multiplying each term in the brackets by the term outside.

For example, $3(x + 7) = 3x + 21$.

1. Expand.

a) $2(a + b)$	**b)** $6(x - 4)$	**c)** $4(k^2 + 5)$	**d)** $-3(x - 2)$
e) $5(x^2 - 2x + 1)$	**f)** $2x(3x - 4)$	**g)** $8a(3 + a)$	**h)** $-2x(x + y - 3)$

Evaluate Expressions

To find the percent of a number, change the percent to a decimal number. Then, multiply by the number.

$$\begin{aligned} 16\% \text{ of } 50 &= 0.16 \times 50 \\ &= 8 \end{aligned}$$

1. Evaluate.

a) 45% of 120	**b)** 3% of 64	**c)** 20% of 95
d) 5.5% of 2036	**e)** 4.25% of 600	**f)** 140% of 230

To add or subtract rational numbers in fraction form, find the least common denominator (LCD), multiply accordingly to get equivalent fractions, and add or subtract the numerators.

To evaluate $\frac{3}{8} - \frac{7}{12}$, use the LCD of 24.

$$\begin{aligned} \frac{3}{8} - \frac{7}{12} &= \frac{9}{24} - \frac{14}{24} \\ &= -\frac{5}{24} \end{aligned}$$

Refer to **Work With Fractions** on page 494.

To multiply rational numbers in fraction form, multiply numerators and denominators. To divide, multiply by the reciprocal of the second fraction. For all operations, convert any mixed numbers to improper fractions first.

$$
\begin{aligned}
-\frac{5}{6} \div 2\frac{1}{2} &= -\frac{5}{6} \div \frac{5}{2} \\
&= -\frac{5}{6} \times \frac{2}{5} \\
&= -\frac{10}{30} \\
&= -\frac{1}{3}
\end{aligned}
$$

2. Evaluate.

a) $\frac{3}{4} + \left(-\frac{1}{2}\right)$ **b)** $1\frac{2}{3} - \frac{5}{12}$ **c)** $-\frac{5}{8} + \left(-1\frac{1}{6}\right)$

d) $\frac{7}{9} \times \left(-\frac{3}{4}\right)$ **e)** $3\frac{1}{8} \div \left(-1\frac{1}{4}\right)$ **f)** $-1\frac{1}{5} \div 6$

Exponent Rules

To multiply powers with the same base, add the exponents.

$$
\begin{aligned}
x^3 \times x^2 &= x^{3+2} \\
&= x^5
\end{aligned}
$$

To divide powers with the same base, subtract the exponents.

$$
\begin{aligned}
x^6 \div x^2 &= x^{6-2} \\
&= x^4
\end{aligned}
$$

To raise a power to a power, multiply the exponents.

$$
\begin{aligned}
(x^2)^3 &= x^{2 \times 3} \\
&= x^6
\end{aligned}
$$

1. Simplify, using the exponent rules. Leave answers in exponential form.

a) $2^3 \times 2^4$ **b)** $5^2 \times 5^4$ **c)** $3^5 \div 3^2$

d) $4^8 \div 4^3$ **e)** $(6^4)^2$ **f)** $(9^3)^7$

g) $a^5 \times a^5$ **h)** $z^4 \times z^4$ **i)** $3x^2 \times 2x^3$

j) $y^8 \div y^5$ **k)** $(p^3)^6$ **l)** $n^6 \div n$

m) $(12x^7) \div (-3x^4)$ **n)** $(2t^4)^3$ **o)** $(-4x^2)^4$

2. Evaluate.

a) $8^6 \div 8^4$ **b)** $2^2 \times 2^3 \div 2^4$ **c)** $3^9 \div 3^3$

d) $(2^4)^2$ **e)** $(4^2)^3$ **f)** $\left(\frac{1}{4}\right)^3$

Factor Quadratic Expressions

To factor a quadratic expression:

- check for common factors
- for expressions in the form $x^2 + bx + c$, find two integers, m and n, that have a sum of b and a product of c, and factor as $(x + m)(x + n)$
- for expressions in the form $ax^2 + bx + c$, find two integers, m and n, that have a sum of b and a product of ac, and then rewrite as $ax^2 + mx + nx + c$ and factor by grouping
- look for special products such as differences of squares or perfect squares

To factor $x^2 + 2x - 15$, note that $5 + (-3) = +2$ and $(5)(-3) = -15$.
Therefore, $x^2 + 2x - 15 = (x + 5)(x - 3)$.

In the expression $12x^2 - 60x + 75$, the number 3 is a common factor.
So, $12x^2 - 60x + 75 = 3(4x^2 - 20x + 25)$.
The coefficients of the first term and the last term are perfect squares, so that the trinomial inside the brackets may be a perfect square.
Since the coefficient of the middle term, -20, is twice the product of the square roots of 4 and 25, it is a perfect square trinomial.
Therefore, $12x^2 - 60x + 75 = 3(2x - 5)^2$.

1. Factor fully.

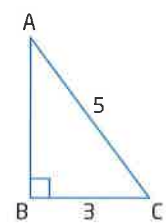
a) $x^2 + 6x + 8$ **b)** $x^2 - 7x + 12$ **c)** $2x^2 + 6x - 36$

d) $3x^2 - 48$ **e)** $3x^2 - 11x + 10$ **f)** $x^2 - 6x + 9$

g) $4x^2 - 100$ **h)** $2x^2 + 3x - 20$ **i)** $4x^2 - 15x + 14$

Find Primary Trigonometric Ratios

To determine the primary trigonometric ratios for $\angle$A, determine the length of the third side using the Pythagorean theorem.

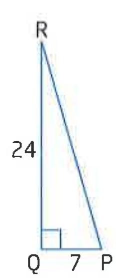


$$c^2 = 5^2 - 3^2$$
$$c = \sqrt{16}$$
$$= 4$$

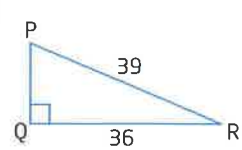
$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

1. Find the primary trigonometric ratios for $\angle$P in each triangle.

a)



b)



Use a calculator to find the primary trigonometric ratios for any angle. For the angle 25°, sin 25° = 0.4226, cos 25° = 0.0963, and tan 25° = 0.4663, rounded to four decimal places.

2. Use a calculator to find the primary trigonometric ratios for each angle. Round each answer to four decimal places. Be sure your calculator is in degree mode.

a) 58° **b)** 79° **c)** 15°

Finite Differences

Given a table of values where the x-values change in constant steps, it is possible to determine the type of relationship that exists between the variables by calculating finite differences. The first differences are found by subtracting successive y-values. If the first differences are constant, the relationship is linear. If not, calculate the second differences by subtracting successive first differences. If the second differences are constant, the relationship is quadratic. If not, then the relationship is neither linear nor quadratic.

x	y	First Differences
1	-3	
2	3	3 – (–3) = 6
3	13	13 – 3 = 10
4	27	27 – 13 = 14
5	45	45 – 27 = 18

The first differences are not constant. So, the relationship is not linear.

x	y	First Differences	Second Differences
1	-3		
2	3	3 – (–3) = 6	
3	13	13 – 3 = 10	10 – 6 = 4
4	27	27 – 13 = 14	14 – 10 = 4
5	45	45 – 27 = 18	18 – 14 = 4

The second differences are constant. So, the relationship is quadratic.

1. Use finite differences to determine whether each relationship is linear, quadratic, or neither.

a)

x	y
1	5
2	1
3	-3
4	-7
5	-11

b)

x	y
1	-2
2	2
3	18
4	52
5	110

c)

x	y
1	15
2	28
3	39
4	48
5	55

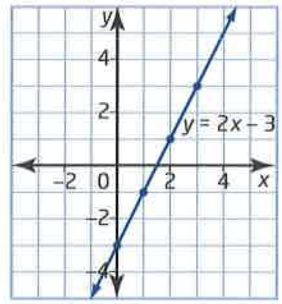
Graphs and Lines

Graph a linear relation by using

- a table of values
- the slope and the y-intercept
- the intercepts

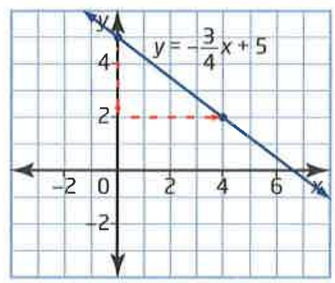
To graph $y = 2x - 3$, make a table of values by choosing simple values for x and substituting to calculate y.

x	y
0	−3
1	−1
2	1
3	3



Plot the four points and draw a line through them.

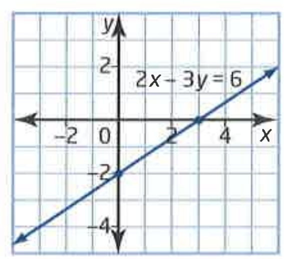
To graph the line $y = -\frac{3}{4}x + 5$, use the y-intercept, 5, and the slope, or $\frac{\text{rise}}{\text{run}}$, $\frac{-3}{4}$. Start on the y-axis at (0, 5). Then, use the slope to get to another point on the line. Here, counting down 3 and right 4 leads to (4, 2). Then, draw a line through the two points.



To graph $2x - 3y = 6$, use intercepts.
At the x-intercept, $y = 0$.

$$\begin{aligned} 2x - 3(0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

The x-intercept is 3. A point on the line is (3, 0).



At the y-intercept, $x = 0$.

$$\begin{aligned} 2(0) - 3y &= 6 \\ -3y &= 6 \\ y &= -2 \end{aligned}$$

The y-intercept is −2. A point on the line is (0, −2). Draw a line through the two points.

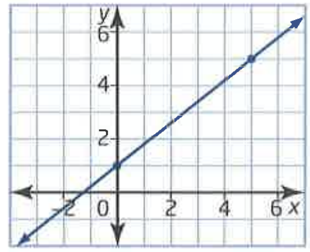
1. Graph each line using a convenient method.

a) $y = \frac{2}{3}x - 4$ **b)** $y = -3x + 6$ **c)** $x + 2y = 8$

d) $5x - 3y = 15$ **e)** $y = -\frac{1}{2}x + 5$ **f)** $y = 4x - 7$

To determine the equation of a line when given a graph, identify the y-intercept and the slope. Then, write the equation in slope y-intercept form, $y = mx + b$.

The y-intercept of this line is 1.
The line passes through the point (5, 5).
Starting from the y-intercept, the rise is 4 and the run is 5.
The slope is $\frac{4}{5}$.
The equation of the line is $y = \frac{4}{5}x + 1$.



If you are given two points that are on the line, determine an equation for the line by finding the slope and the y-intercept.
Find the slope of the line passing through (3, −6) and (15, 2).

$$m = \frac{2 - (-6)}{15 - 3}$$
$$= \frac{8}{12} \text{ or } \frac{2}{3}$$

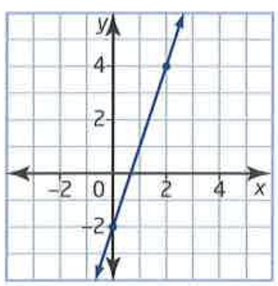
Therefore, $y = \frac{2}{3}x + b$.
Substitute the point (3, −6) to solve for b.

$$-6 = \frac{2}{3}(3) + b$$
$$-6 = 2 + b$$
$$b = -8$$

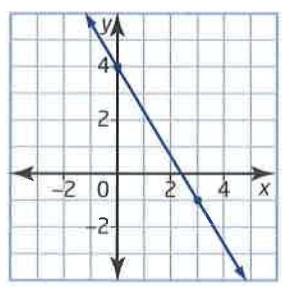
The equation of the line is $y = \frac{2}{3}x - 8$.

2. Determine the equation in the form $y = mx + b$ for each line.

a)



b)



3. Determine the equation in the form $y = mx + b$ for the line that passes through each pair of points.

a) (1, 9) and (3, 13) **b)** (3, 1) and (−12, 8) **c)** (−5, −14) and (10, −5)

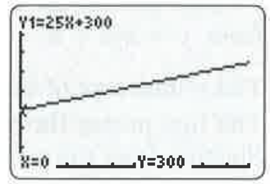
Identify Patterns

1. Identify the next three terms in each pattern.

a) Z, ZY, ZYX, ...
b) −3, 2, 7, 12, ...
c) 3, 9, 27, ...
d) 19, 11, 3, −5, ...
e) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$
f) $54x, 18x^2, 6x^3, \ldots$

Linear and Exponential Growth

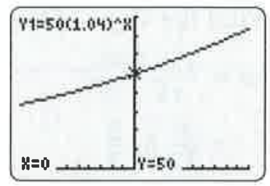
A graph of $y = 25x + 300$ is shown.
The relation is linear.
The y-intercept is 300 and the slope is 25.



A table of values with first differences shows that the first differences are constant.

x	y	First Differences
0	300	
		325 – 300 = 25
1	325	
		350 – 325 = 25
2	350	
		375 – 350 = 25
3	375	

A graph of $y = 50(1.04)^x$ is shown.
The relationship is exponential.
The y-intercept is 50.



A table of values with first differences, second differences, and common ratios is shown.

x	y	First Differences	Second Differences	Common Ratios
0	50			
		52 – 50 = 2		$\frac{52}{50} = 1.04$
1	52		2.08 – 2 = 0.08	
		54.08 – 52 = 2.08		$\frac{54.08}{52} = 1.04$
2	54.08		2.163 – 2.08 = 0.083	
		56.243 – 54.08 = 2.163		$\frac{56.243}{54.08} = 1.04$
3	56.243			

Successive first and second differences can be obtained by multiplying by 1.04. The common ratios are all equal to 1.04.

1. Graph the relation $y = 30x + 200$.

a) What kind of relation is this?

b) Identify the slope and the y-intercept.

c) Construct a table of values for $x = 0, 1, 2, 3$.

d) Calculate the first differences and describe their pattern.

2. Graph the relation $y = 25(1.1)^x$.

a) What type of relation is this?

b) Identify the y-intercept.

c) Construct a table of values for $x = 0, 1, 2, 3$.

d) Calculate the first and second differences and describe any patterns.

e) Calculate the common ratios of consecutive terms and describe their pattern.

Quadratic Relations

An equation in vertex form $y = a(x - h)^2 + k$ or standard form $y = ax^2 + bx + c$, $a \neq 0$, represents a quadratic relation. The graph of such a relation has a characteristic shape called a parabola.

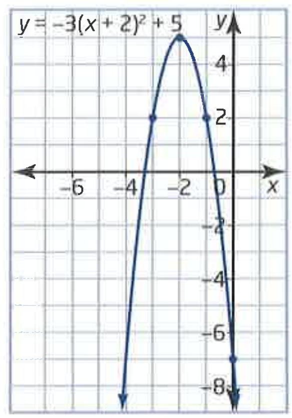
Given an equation in vertex form, (h, k) represents the coordinates of the parabola's vertex, $x = h$ is the equation of the axis of symmetry, and a represents the vertical stretch factor. If a is positive, the parabola opens upward; if a is negative, the parabola opens downward.

For the quadratic relation $y = -3(x + 2)^2 + 5$, the coordinates of the vertex are $(-2, 5)$ and the axis of symmetry is $x = -2$. The vertical stretch factor is 3 and the parabola opens downward because a is negative.

To determine the y-intercept, substitute $x = 0$.

$$\begin{aligned} y &= -3(0 + 2)^2 + 5 \\ &= -3(4) + 5 \\ &= -7 \end{aligned}$$

The y-intercept is -7.



To sketch the graph, plot the vertex $(-2, 5)$. Substitute to find that when $x = -1$, $y = 2$. By symmetry, another point is $(-3, 2)$. Plot the y-intercept. Then, draw a smooth U-shaped curve passing through all four points.

1. For each quadratic relation, state

i) the coordinates of the vertex

ii) the equation of the axis of symmetry

iii) the direction of opening

iv) the y-intercept

Then, sketch the graph of the relation.

a) $y = 2(x - 3)^2 - 8$ **b)** $y = -4(x + 1)^2 + 3$ **c)** $y = 3(x - 5)^2 + 1$

2. Compare the graph of each quadratic function to the graph of $y = x^2$. Identify the direction of opening and state whether the parabola has been vertically stretched or compressed. Justify your answer.

a) $y = 5x^2$ **b)** $y = -\frac{1}{4}(x - 1)^2$ **c)** $y = -3(x + 5)^2 + 2$

To identify the coordinates of the vertex of a quadratic relation using an equation in standard form, change to vertex form by completing the square.

If $a = 1$, then complete the square as described in the second step below.

If a is any number other than 1, then the first step is to factor this from the x-terms.

Write $y = 2x^2 - 12x + 19$ in vertex form as follows.

$y = 2(x^2 - 6x) + 19$	Factor a (i.e., 2) from the first two terms.
$= 2(x^2 - 6x + 9 - 9) + 19$	Divide the coefficient of the x-term by 2 and square the result. This yields 9. Add this number to and subtract it from the expression in brackets.
$= 2(x - 3)^2 + 2(-9) + 19$	Express the perfect square, $x^2 - 6x + 9$, in factored form. Multiply the subtracted value (i.e., 9) by a.
$= 2(x - 3)^2 + 1$	Simplify.

The vertex is (3, 1).

3. Complete the square to express each quadratic relation in the form $y = a(x - h)^2 + k$. Then, state the coordinates of the vertex.

a) $y = x^2 + 4x + 7$
b) $y = x^2 - 12x + 3$
c) $y = 3x^2 + 18x - 2$
d) $y = -2x^2 + 16x + 9$

Rearrange Formulas

To rearrange a formula to isolate a variable, apply the same steps as for solving an equation.

Solve for a in the formula $I = \dfrac{50d}{a + b}$.

$I = \dfrac{50d}{a+b}$	
$I(a + b) = 50d$	Multiply both sides by $a + b$.
$a + b = \dfrac{50d}{I}$	Divide both sides by I.
$a = \dfrac{50d}{I} - b$	Subtract b from both sides.

1. Solve each formula for the variable indicated.

a) $y = -4x + 5$ for x
b) $P = 2\ell + 2w$ for w
c) $x^2 + y^2 = r^2$ for y
d) $V = \frac{1}{3}\pi r^2 h$ for h
e) $s = \dfrac{2 - 10e}{t}$ for e
f) $A = P(1 + rt)$ for r

Solve Equations

To solve $4(x + 2) = x + 5$, expand to remove brackets and then isolate the variable.

$4(x + 2) = x + 5$	
$4x + 8 = x + 5$	Expand.
$3x + 8 = 5$	Subtract x from both sides.
$3x = -3$	Subtract 8 from both sides.
$x = -1$	Divide both sides by 3.

To check, substitute -1 for x in the original equation.

L.S. $= 4(x + 2)$	R.S. $= x + 5$
$= 4(-1 + 2)$	$= -1 + 5$
$= 4(1)$	$= 4$
$= 4$	

Since L.S. = R.S., the solution is $x = -1$.

To solve $\frac{x+5}{2} - \frac{x}{3} = 1$, multiply both sides by the least common denominator, 6, to eliminate fractions.

$$\begin{aligned}
\frac{x+5}{2} - \frac{x}{3} &= 1 \\
6 \times \left(\frac{x+5}{2} - \frac{x}{3}\right) &= 6 \\
3(x+5) - 2x &= 6 \\
3x + 15 - 2x &= 6 \\
x + 15 &= 6 \\
x &= -9
\end{aligned}$$

To solve $600(1+i)^5 = 747.72$, first isolate the variable expression in brackets.

$$600(1+i)^5 = 747.72$$

$$(1+i)^5 = \frac{747.72}{600} \qquad \text{Divide both sides by 600.}$$

$$(1+i)^5 = 1.2462$$

$$\sqrt[5]{(1+i)^5} = \sqrt[5]{1.2462} \qquad \text{Take the fifth root of both sides.}$$

$$1 + i \doteq 1.0450 \qquad \text{Simplify.}$$

$$i \doteq 0.0450$$

1. Solve and check.

a) $7x - 5 = 3x - 17$

b) $3x - 7 = 5(x - 3)$

c) $\frac{x+1}{3} + \frac{x+5}{5} = 4$

d) $\frac{x+1}{2} - \frac{x-7}{6} = 3$

2. Solve. Round your answers to four decimal places, if necessary.

a) $\frac{850}{w-5} = 200$

b) $-6(p-3)^2 = -31.74$

c) $275.38 = 200(1+i)^{10}$

d) $\frac{2026.12}{(k+4)^3} = 5$

Solve Equations Involving Rational Expressions

To solve $\frac{200}{x} = 10$, isolate x.

$$\frac{200}{x} = 10$$

$$200 = 10x \qquad \text{Multiply both sides by } x.$$

$$20 = x \qquad \text{Divide both sides by 10.}$$

1. Solve.

a) $\frac{16}{x} = 8$

b) $\frac{a}{5} = 7$

c) $45 = \frac{135}{c}$

d) $\frac{12}{r} = \frac{4}{9}$

e) $\frac{k}{20} = \frac{3}{8}$

f) $\frac{36}{t} = \frac{2}{15}$

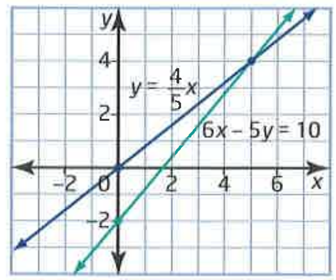
Solve Linear Systems of Equations

To determine the point of intersection of a linear system, graph the linear relations or solve the system algebraically.

To solve the linear system $y = \frac{4}{5}x$ and $6x - 5y = 10$ by graphing, find the point of intersection of the two lines. First, express the second equation in the form $y = mx + b$.

$$
\begin{aligned}
6x - 5y &= 10 \\
-5y &= -6x + 10 \\
y &= \frac{6}{5}x - 2
\end{aligned}
$$

Graph each line using the y-intercept and the slope. Then, identify the point of intersection. The point of intersection is (5, 4).



To solve a linear system algebraically, use either the substitution or the elimination method.
Substitution is suitable when one of the variables is easily isolated. The following system is best solved using the elimination method.

$5x + 2y = 5$
$2x + 3y = 13$

To make the coefficients of the y-terms the same, multiply the first equation by 3 and the second equation by 2. Then, subtract to eliminate y.

$$
\begin{array}{lll}
5x + 2y = 5 & ① \times 3 \rightarrow & 15x + 6y = 15 \quad ③ \\
2x + 3y = 13 & ② \times 2 \rightarrow & 4x + 6y = 26 \quad ④ \\
\hline
 & & 11x = -11 \quad ③ - ④ \\
 & & x = -1
\end{array}
$$

Substitute $x = -1$ in ①.

$$
\begin{aligned}
5(-1) + 2y &= 5 \\
2y &= 10 \\
y &= 5
\end{aligned}
$$

The point of intersection is $(-1, 5)$.

1. Graph to find the point of intersection of each pair of lines.

a) $y = 3x - 5$, $y = 2x - 4$ **b)** $x + y = 1$, $y = \frac{2}{5}x - 6$ **c)** $x - 3y - 2 = 0$, $2x + y = 4$

2. Solve algebraically to find the point of intersection.

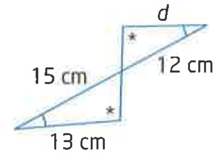
a) $x - 2y = 7$, $2x - 3y = 13$ **b)** $y = 2x - 7$, $3x + y = -17$ **c)** $4x - 7y = 20$, $x - 3y = 10$

d) $4x - 3y = 8$, $6x - 3y = 18$ **e)** $4x + 3y = -2$, $4x + y = -6$ **f)** $5x - 2y = 20$, $2x + 5y = 8$

Use Similar Triangles

In similar triangles, the corresponding angles are equal and the lengths of corresponding sides are proportional.

The triangles shown are similar because all corresponding pairs of angles are equal. Write a proportion involving corresponding sides to solve for the length of side d.

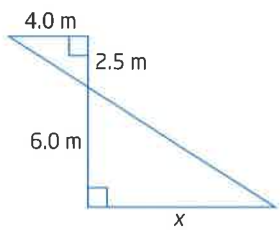


$$\frac{d}{13} = \frac{12}{15}$$
$$d = 13 \times \frac{12}{15}$$
$$d = 10.4$$

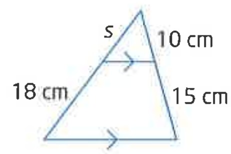
The length of side d is 10.4 cm.

1. Use similar triangles to determine the unknown length in each. If necessary, round answers to the nearest tenth.

a)



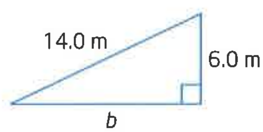
b)



Use the Pythagorean Theorem

The Pythagorean theorem states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

To find the length, b, to the nearest tenth, write an equation using the Pythagorean theorem.

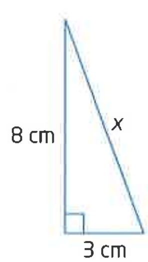


$$b^2 + 6^2 = 14^2$$
$$b^2 + 36 = 196$$
$$b^2 = 160$$
$$b = \sqrt{160}$$
$$b \doteq 12.6$$

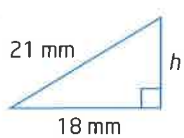
The length of side b is 12.6 m, to the nearest tenth of a metre.

1. Determine the measure of the unknown side in each triangle. Round to the nearest tenth, if necessary.

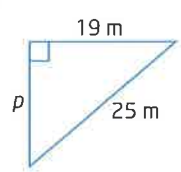
a)



b)



c)



Work With Fractions

To evaluate or simplify expressions involving adding or subtracting fractions, it is best to determine the least common denominator (LCD), which is the same as determining the least common multiple (LCM) of the denominators.

To find the LCM of 24 and 30, write each number in factored form and determine the greatest common factor (GCF).
$24 = 2 \times 2 \times 2 \times 3$
$30 = 2 \times 3 \times 5$

The GCF is 2×3, or 6. Find the LCM by multiplying the GCF by the other factors of the original numbers.
$2 \times 3 \times 2 \times 2 \times 5 = 120$
The LCM is 120.

To find the LCM of $x^2 + x - 12$ and $x^2 - 8x + 15$, factor each expression and determine the GCF.
$x^2 + x - 12 = (x + 4)(x - 3)$
$x^2 - 8x + 15 = (x - 3)(x - 5)$
The GCF is $(x - 3)$.
The LCM is $(x - 3)(x + 4)(x - 5)$.

To add or subtract rational numbers in fraction form, find the LCM of the denominators, multiply the numerators accordingly to get equivalent fractions, and add or subtract the numerators.

To simplify $\frac{3x}{4} + \frac{2y}{10}$, first find the LCM of 4 and 10.

$$\frac{3x}{4} + \frac{2y}{10} = \frac{3x \times 5}{4 \times 5} + \frac{2y \times 2}{10 \times 2}$$
The LCM is $2 \times 2 \times 5 = 20$. Multiply to write equivalent fractions with denominator 20.

$$= \frac{15x}{20} + \frac{4y}{20}$$
Simplify.

$$= \frac{15x + 4y}{20}$$

1. Determine the LCM of each set.

a) 24, 40 **b)** $-10x^2, 35x, -55x^3$ **c)** $x^2 + 7x + 12, x^2 - 9$

2. Add or subtract.

a) $\frac{5}{9} + \frac{5}{6}$ **b)** $\frac{5}{24} - \frac{7}{60}$ **c)** $\frac{3a}{16} + \frac{5b}{36}$ **d)** $\frac{4x}{45} - \frac{k}{18}$

To simplify $\left(-\frac{14}{9}\right)\left(\frac{6}{7}\right)$, look for common factors to reduce the fractions.

Then, multiply numerators and multiply denominators.

$$\left(-\frac{14}{9}\right)\left(\frac{6}{7}\right) = \left(-\frac{\overset{2}{\cancel{14}}}{\underset{3}{\cancel{9}}}\right)\left(\frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}}\right)$$
Divide by common factors.

$$= -\frac{4}{3}$$
Simplify.

To divide, multiply by the reciprocal of the second fraction.

3. Simplify.

a) $\left(\frac{8}{15}\right)\left(\frac{3}{4}\right)$ **b)** $\left(\frac{5}{6}\right)\left(-\frac{3}{10}\right)$ **c)** $\frac{3}{8} \div \left(-\frac{9}{20}\right)$ **d)** $-\frac{11}{30} \div \frac{33}{9}$

Work With Polynomials

To expand $2(x - 5)(x + 4)$, expand the brackets. Then, multiply by the outside term.

$$\begin{aligned} 2(x-5)(x+4) &= 2(x^2 + 4x - 5x - 20) \\ &= 2(x^2 - x - 20) && \text{Collect like terms.} \\ &= 2x^2 - 2x - 40 && \text{Multiply.} \end{aligned}$$

A perfect square trinomial has the general form $a^2 + 2ab + b^2$ and can be expressed in factored form as $(a + b)^2$.

To factor $4x^2 + 20x + 25$, check for the general form.

$4x^2 = (2x)^2$ $20x = 2(2x)(5)$ $25 = 5^2$

Therefore, $4x^2 + 20x + 25 = (2x + 5)^2$.

1. Expand and simplify.

a) $3(x + 2)(x + 5)$ **b)** $-2(x - 4)(x + 7)$ **c)** $(x - 6)^2$

2. Factor fully.

a) $x^2 + 3x - 10$ **b)** $x^2 + 14x + 49$ **c)** $9y^2 - 25$

d) $3a^2 + 48a + 192$ **e)** $25x^2 - 60x + 36$ **f)** $-p^2 - 20p - 100$

3. What value of k makes each quadratic expression a perfect square trinomial?

a) $x^2 + 16x + k$ **b)** $x^2 - 30x + k$ **c)** $x^2 + 40x + k$

d) $4x^2 + 12x + k$ **e)** $x^2 - 7x + k$ **f)** $9x^2 - 4x + k$

Zero and Negative Exponents

Any base, other than zero, raised to the exponent zero is equal to one.

$3^0 = 1$

A base raised to a negative exponent is equal to the reciprocal of the base raised to the positive value of the exponent.

1. Evaluate.

a) $10^3 \times 10^0$ **b)** 4^{-3} **c)** -7^0

d) 5^{-2} **e)** $(3^{-3})^2$ **f)** $6^4 \times 6^{-3} \times 6^2$

g) $2^3 \div 2^{-2}$ **h)** $2(3^4)^{-1}$ **i)** $\left(\frac{2}{3}\right)^{-2}$

2. Simplify. Write your answers using only positive exponents.

a) $3x^{-4}$ **b)** $(5y^{-2})^2$ **c)** $2(3x)^{-3}$

d) $\frac{4x^5y^6}{8x^2y^8}$ **e)** $\frac{(3a^{-4})2b^3}{6ab^{-3}}$ **f)** $\left(\frac{4m^3n^{-2}}{3m^{-4}n}\right)^{-3}$

Technology Appendix

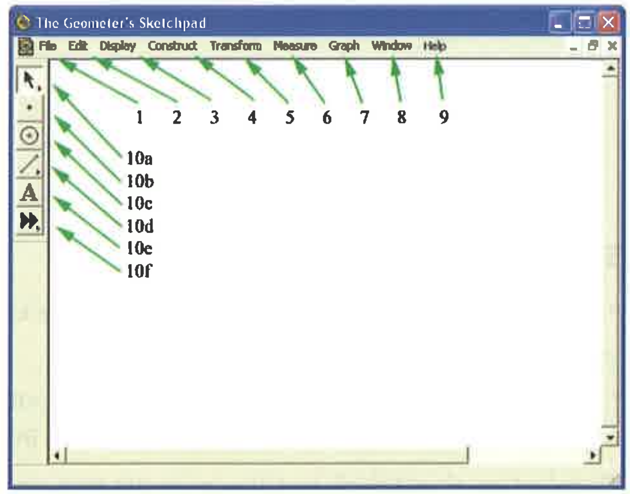
CONTENTS

The Geometer's Sketchpad®, Geometry Software Package

TI-83 Plus and TI-84 Plus Graphing Calculators

The Geometer's Sketchpad®, Geometry Software Package

Menu Bar

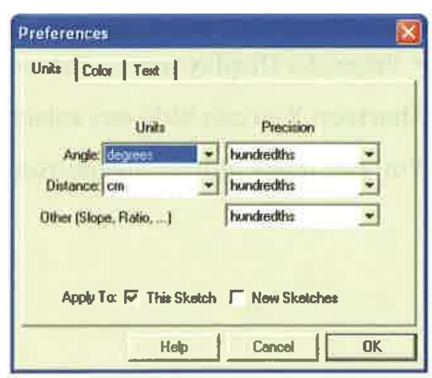


- **1** **File** menu—open/save/print sketches
- **2** **Edit** menu—undo/redo actions/ set preferences
- **3** **Display** menu—control appearance of objects in sketch
- **4** **Construct** menu—construct new geometric objects based on objects in sketch
- **5** **Transform** menu—apply geometric transformations to selected objects
- **6** **Measure** menu—make various measurements on objects in sketch
- **7** **Graph** menu—create axes and plot measurements and points
- **8** **Window** menu—manipulate windows
- **9** **Help** menu—access the help system, an excellent reference guide
- **10** **Toolbox**—access tools for creating, marking, and transforming points, circles, and straight objects (segments, lines, and rays); also includes text and information tools
- **10a** **Selection Arrow Tool** (Arrow)—select and transform objects
- **10b** **Point Tool** (Dot)—draw points
- **10c** **Compass Tool** (Circle)—draw circles
- **10d** **Straightedge Tool**—draw line segments, rays, and lines
- **10e** **Text Tool** (Letter A)—label points and write text
- **10f** **Custom Tool** (Double Arrow)—create or use special "custom" tools

Creating a Sketch

- From the **File** menu, choose **New Sketch** to start with a new work area.

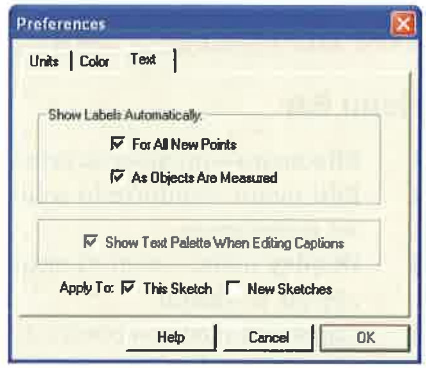
Setting Preferences



- From the **Edit** menu, choose **Preferences....**
- Click the **Units** tab.
- Set the units and precision for angles, distances, and calculated values such as slopes and ratios.
- Click the **Text** tab.
- If you check the auto-label box **For All New Points**, then *The Geometer's Sketchpad®* will label points as you create them.
- If you check the auto-label box **As Objects Are Measured**, then *The Geometer's Sketchpad®* will label any measurements that you define.

You can also choose whether the auto-labelling functions will apply only to the current sketch, or also to any new sketches that you create.

Be sure to click **OK** to apply your preferences.



Selecting Points and Objects

- Choose the **Selection Arrow Tool**. The mouse cursor appears as an arrow.

To select a single point:
- Select the point by moving the cursor to the point and clicking it.

The selected point will now appear as a darker point, similar to a bull's-eye ⊙.

To select an object such as a line segment or a circle:
- Move the cursor to a point on the object until it becomes a horizontal arrow.
- Click the object. The object will change appearance to show it is selected.

To select a number of points or objects:
- Select each object in turn by moving the cursor to the object and clicking it.

To deselect a point or an object:
- Move the cursor over it, and then click the left mouse button.
- To deselect all selected objects, click in an open area of the workspace.

Hiding Points and Objects

Open a new sketch. Draw several objects, such as points and line segments.

To hide a point:
- Select the point.
- From the **Display** menu, choose **Hide Point**.

To hide an object:
- Select another point and a line segment.
- From the **Display** menu, choose **Hide Objects**.

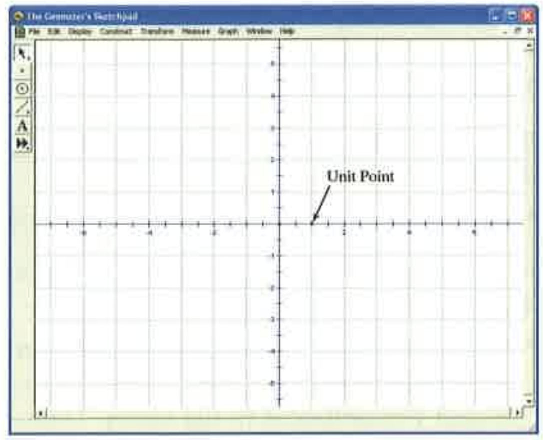
Shortcut: You can hide any selected objects by holding down the CTRL key and typing H.

You can make hidden objects reappear by choosing **Show All Hidden** from the **Display** menu.

Using a Coordinate System and Axes

- From the **Graph** menu, choose **Show Grid**.

The default coordinate system has an origin point in the centre of your screen and a unit point at (1, 0). Drag the origin to relocate the coordinate system and drag the unit point to change the scale.



Graphing Relations

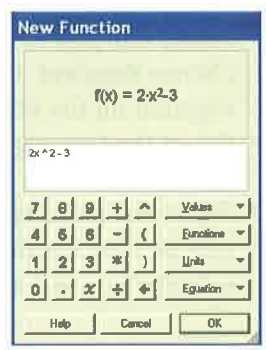
Consider the equations $y = 2x^2 - 3$ and $y = 2^{-x}$ as examples.

- From the **Graph** menu, select **Show Grid**.
- From the **Graph** menu, select **Plot New Function....**

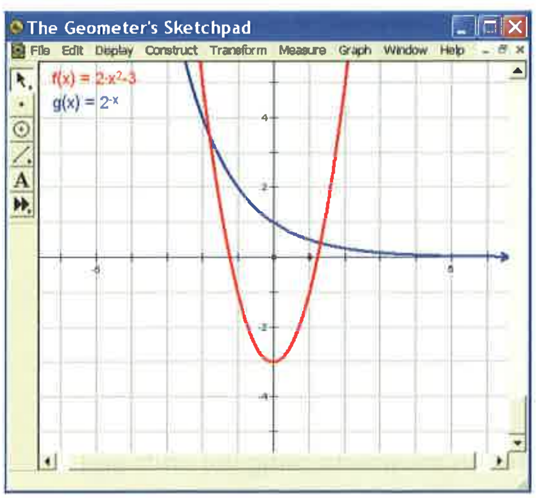
The calculator interface will appear.

Enter the first equation: 2 * x ^ 2 − 3.

- Press **OK**. The graph of the first equation appears, along with the equation in function notation. You can move the equation next to the line.



Use the same procedure to graph the second equation.

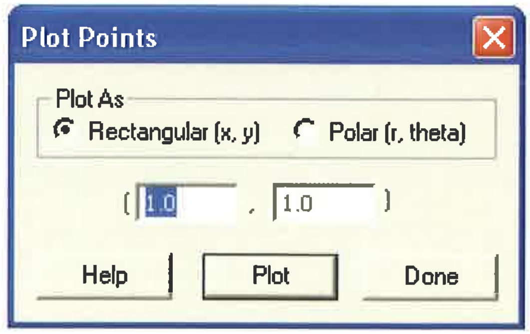


Plotting Points

- From the **Graph** menu, choose **Show Grid**.
- If you want points plotted exactly at grid intersections, also choose **Snap Points**.
- Choose the **Point Tool**.

If you have enabled **Snap Points**, a point will "snap" to the nearest grid intersection as you move the cursor over the grid.

- Click the left mouse button to plot the point.

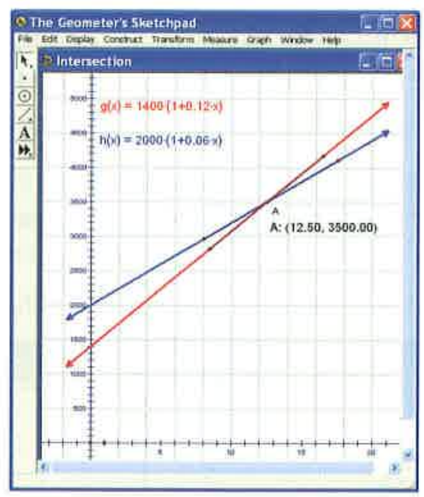


Alternatively, you can plot points by typing in the desired coordinates.

- From the **Graph** menu, choose **Plot Points....** A dialogue box will appear. Type the desired *x*- and *y*-coordinates in the boxes. Then, press **Plot**.
- When you are finished plotting points, click **Done**.

Finding a Point of Intersection

Consider the equations $y = 1400(1 + 0.12x)$ and $y = 2000(1 + 0.06x)$ as examples.



- Turn on the grid, and then plot the graphs of the two equations.
- Use the **Point Tool** to plot two points on each line, such that the intersection lies between the points.
- Select one pair of points on a line. From the **Construct** menu, choose **Segment**. Use the same procedure to construct a segment on the other line.
- Select the two segments. From the **Construct** menu, choose **Intersection**. The point of intersection will appear.
- Select the point of intersection. From the **Measure** menu, choose **Coordinates**. The coordinates of the point of intersection will appear.

Using the Measure Menu

To measure the distance between two points:

- Ensure that nothing is selected.
- Select the two points.
- From the **Measure** menu, choose **Distance**.

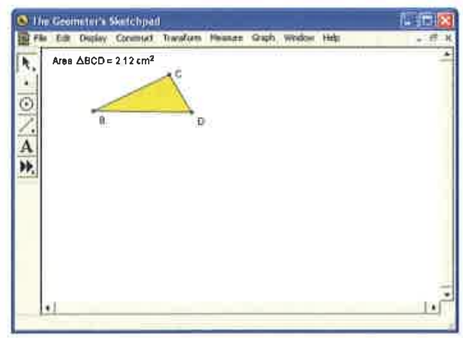
The Geometer's Sketchpad® will display the distance between the points using the units and accuracy selected in **Preferences...** under the **Edit** menu.

To measure the length of a line segment:
- Ensure that nothing is selected.
- Select the line segment (but not the endpoints).
- From the **Measure** menu, choose **Length**.

To measure an angle:
- Ensure that nothing is selected.
- Select the three points that define the angle so that the second point selected is the vertex of the angle.
- From the **Measure** menu, choose **Angle**.

To calculate the ratio of two lengths:
- Select the two lengths to be compared.
- From the **Measure** menu, choose **Ratio**.

To measure the area of a triangle or other closed figure:
- Use the **Straightedge Tool** to draw a triangle.
- Select the points on the vertices of the triangle.
- From the **Construct** menu, choose **Triangle Interior**.
- Select the triangle interior.
- From the **Measure** menu, choose **Area**.

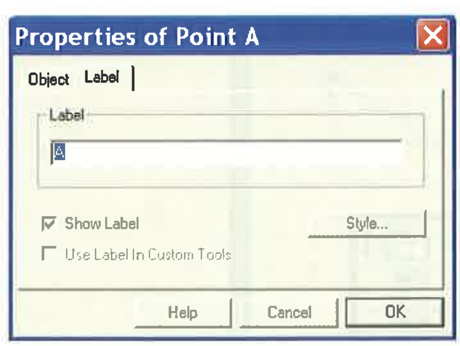


You can construct and measure the area of other closed figures in a similar manner.

To measure the slope of a line, ray, or line segment:
- Select the line, ray, or line segment.
- From the **Measure** menu, choose **Slope**.

Labelling a Vertex

To label a point on a line segment:
- Use the cursor to select the **Text Tool**.
- Move the cursor over the point on the line segment and click once.
- A letter label will appear.
- Move the cursor to the other point on the segment and click once.
- The next letter label in the alphabet will appear.
- If you double-click the point you can input the letter desired.
- Click **OK**.



Changing Labels of Measures

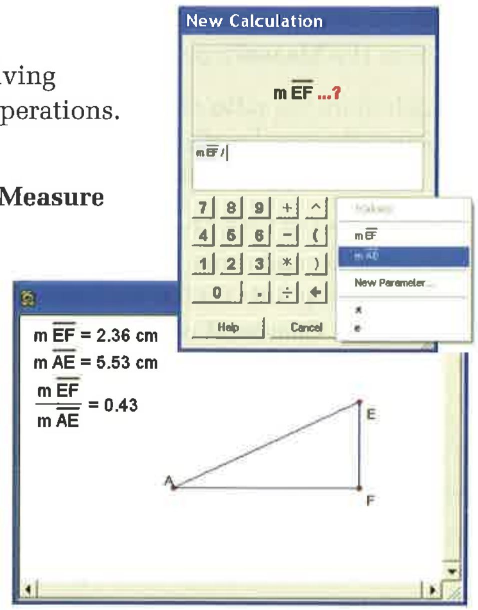
- Right-click the measure and choose **Label Measurement** (or **Label Distance Measurement** depending on the type of measure) from the drop-down menu.
- Type in the new label.
- Click **OK**.

Using the On-Screen Calculator

You can use the on-screen calculator to do calculations involving measurements, constants, functions, or other mathematical operations.

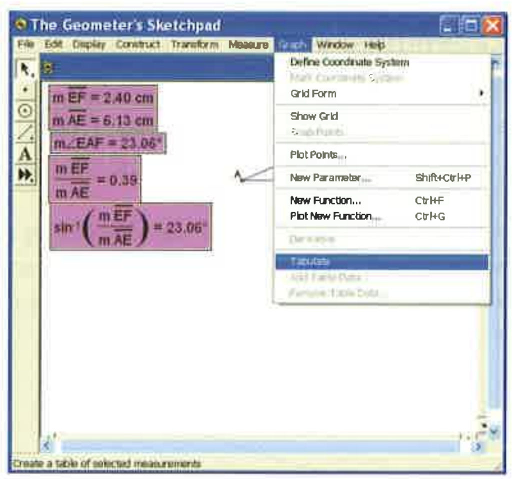
To calculate the sine ratio of two lengths:

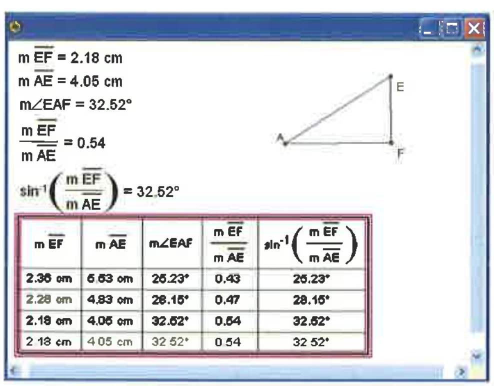
- Select the two measures. Then, choose **Calculate** from the **Measure** menu.
- Click the **Values** button and select the first measure.
- Press the (÷) key.
- Click the **Values** button and select the second measure.
- Click **OK** and the ratio will be calculated.



Using Tabulate to Construct a Data Table

- Click the measures in the order that they are to appear in the table. This will highlight them.
- Choose **Graph** and then **Tabulate**.
- The first row of the data table will be completed.
- Select a vertex and manipulate the object.
- Double-click the table to add a row of data.
  Note: The bottom row is always the active row.
  It changes as you manipulate the object. If you highlight only the table, you can choose the **Graph** menu and select **Add Table Data** to add a row to the table.

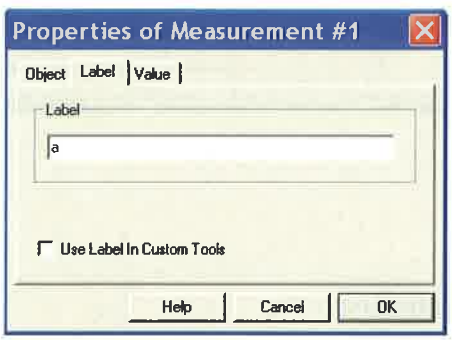


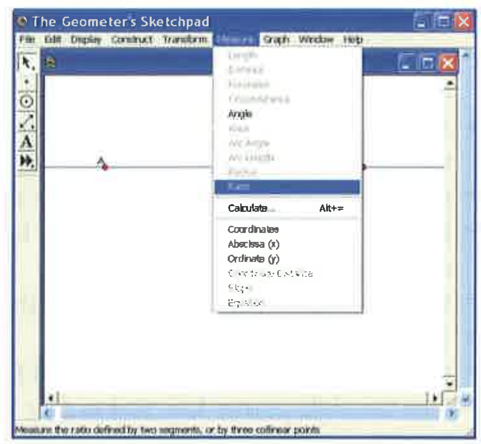


Constructing a Slider for a Function

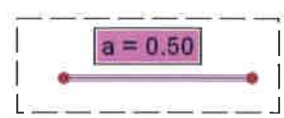
To construct a slider for a function of the form $y = a(x + h)^2 + k$:

- Construct a line through two points.
- Use the **Text Tool** to label the points A and B.
- Use the **Point Tool** to create another point on the line between A and B.
- Label the point C.
- Click A, B, and then C, in that order.
- From the **Measure** menu, choose **Ratio**.
- Click the white background to turn off the highlight on the ratio. Click the line and the point B to highlight them.
- Hide the line and point B by holding down the Ctrl key and pressing H. The points A and C and the ratio measure will remain.
- Construct a line segment from point A to point C.
- Select the **Text Tool** and double-click the ratio measure.
- Change its label to the letter a.

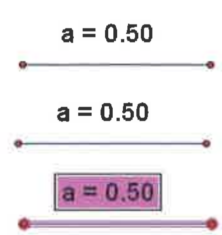




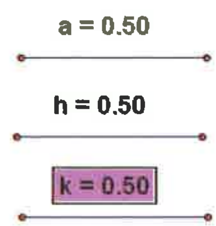
- Click the endpoints of the line segment to hide the letters A and C.
- Click the **Selection Arrow Tool** and click on the white background to turn off the highlighted ratio measure.
- Move the measure of a to the line segment.
- Select the right point and move it. **Note:** The value of a is positive when this point is on the right but negative when it is to the left of the other point.
- Manipulate the slider for the parameter a so that its value is 0.5.
- Select the measure and the slider for parameter a by holding the left mouse button until a rectangle is drawn around the objects as shown on the right.



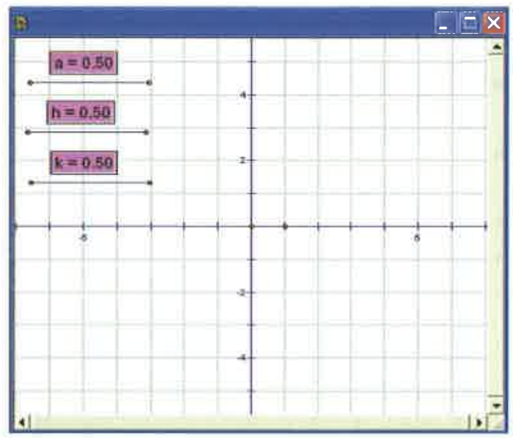
- From the **Edit** menu, choose **Copy**.
- From the **Edit** menu, choose **Paste** to insert a copy of the slider in the document.
- Move the copy below the original.
- From the **Edit** menu, choose **Paste** to insert another copy of the slider in the document.
- Move this copy below the last copy.



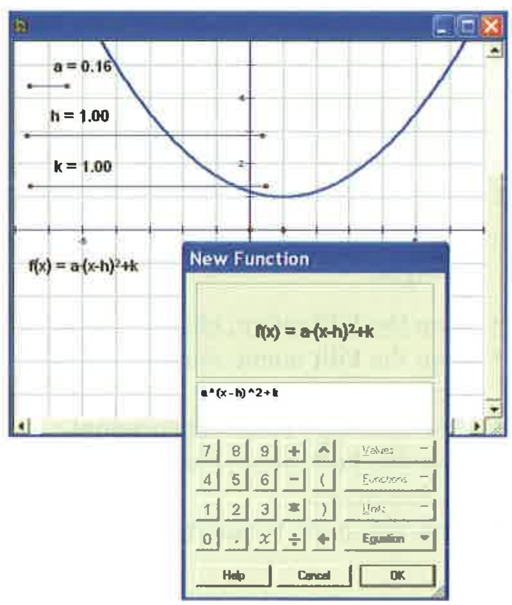
- Click the white background to turn off the highlight.
- Select the **Text Tool** and double-click the middle measure of a.
- Change its label to h.
- Click **OK**.
- Double-click the last measure.
- Change its label to k.
- Click **OK**.



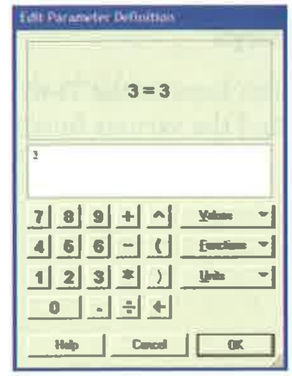
- Click the **Selection Arrow Tool**.
- Move the sliders and their values to the left side of the window.
- From the **Graph** menu, choose **Show Grid**.
- Click each of the measures a, h, and k (not the line segments).
- From the **Graph** menu, choose **Plot New Function**.



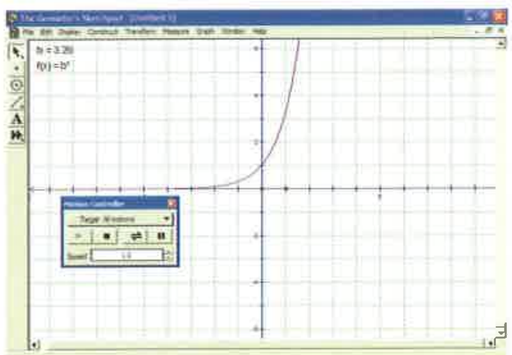
- Click (x) ^ 2 to begin the quadratic equation.
- Use the left arrow key and cursor to the beginning of the expression.
- Click the **Values** button and select the parameter a.
- Insert a multiplication symbol (*) between a and the opening parenthesis.
- Use the right arrow and cursor to the right of x.
- Insert a negative sign (–).
- Click the **Values** button and select the parameter h.
- Use the right arrow key and cursor to the far right of the expression.
- Insert an addition symbol (+).
- Click the **Values** button and select the parameter k.
- Click **OK**. The function will appear highlighted on your sketch.
- From the **Graph** menu, choose **Plot Function**. The graph of the quadratic function will appear.
- Move the right endpoints of the sliders and the values and the graph will change.



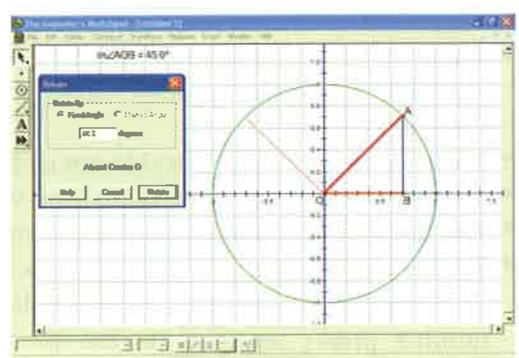
Changing the Value of a Parameter



- From the **Graph** menu, choose **New Parameter**. Set the name as *b* and its initial value to 3. Click **OK**.
- From the **Graph** menu, choose **Plot New Function**. Click on the parameter *b*, and then click on ^*x* and **OK**.
- Click on parameter *b* and press the [+] and [−] keys on the keyboard to increase or decrease the value of *b* in unitary increments.
- Right-click on parameter *b* and choose **Edit Parameter** to enter a specific value.
- Right-click on parameter *b* and choose **Animate Parameter**. Use the various buttons on the **Motion Controller** to see the effects of changing *b* continuously.



Rotating a Terminal Arm About the Origin



- From the **Graph** menu, choose **Show Grid**.
- Move the cursor to one of the increment numbers on either axis. Then, you can change the scale of both axes by dragging, so that you can create a large unit circle.
- Use the **Compass Tool** to draw a unit circle with centre (0, 0).
- Draw a line segment from the origin to the unit circle to create a terminal arm and then another one from the origin on the *x*-axis to create an initial arm that will form a right triangle when joined.
- To create the initial arm, select the x-axis and the point on the unit circle from the terminal arm. From the **Construct** menu, choose **Perpendicular Line**.
- Select the perpendicular line and the *x*-axis. From the **Construct** menu, choose **Intersection**. Select the origin and the point of intersection. From the **Construct** menu, choose **Segment**. You should now have an initial arm.
- Select the three points, making sure that the origin is your second choice. From the **Measure** menu, choose **Angle**. (Ensure that this is 45° by moving the point on the unit circle).
- Select the terminal arm. From the **Transform** menu, choose **Rotate**. Rotating the terminal arm by 90° is shown.

TI-83 Plus and TI-84 Plus Graphing Calculators

Keys

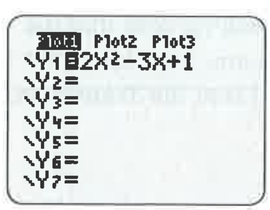
The keys on the TI-83 Plus and TI-84 Plus are colour-coded to help you find the various functions.

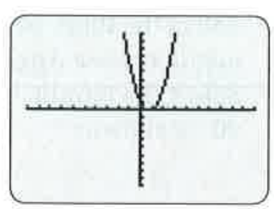
- The white keys include the number keys, decimal point, and negative sign. When entering negative values, use the white (−) key and not the grey − key.
- The grey keys on the right side are math operations.
- The grey keys across the top are used when graphing.
- The primary function of each key is printed on the key, in white.
- The secondary function of each key is printed in blue and is activated by pressing the 2nd key. For example, to find the square root of a number, press 2nd x^2 for [$\sqrt{\ }$].
- The alpha function of each key is printed in green and is activated by pressing the green ALPHA key.

Graphing Relations and Equations

Press Y=. Enter the equation.
To display the graph, press GRAPH.

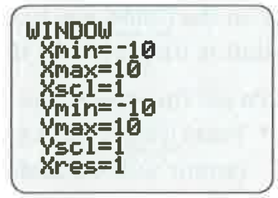
For example, enter $y = 2x^2 - 3x + 1$ by pressing Y= 2 X,T,θ,n x^2 − 3 X,T,θ,n + 1.
Press GRAPH.





Setting Window Variables

The WINDOW key defines the appearance of the graph.
The standard (default) window settings are shown.

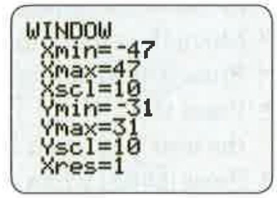


To change the window settings:

- Press WINDOW. Enter the desired window settings.

In the example shown,

- the minimum x-value is -47
- the maximum x-value is 47
- the scale of the x-axis is 10
- the minimum y-value is -31
- the maximum y-value is 31
- the scale of the y-axis is 10
- the resolution is 1, so equations are graphed at each horizontal pixel



Note: The greater the resolution, the faster the graph plots because the horizontal pixels are omitted.

Setting Up a Table of Values

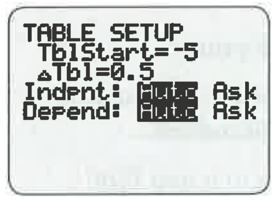
The standard (default) table settings are shown.
This feature allows you to specify the x-values of the table.

To change the **Table Set up** settings:

- Press 2nd WINDOW. Enter the desired values.

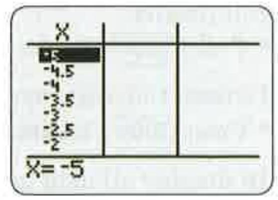
In the example shown,

- The starting x-value of the table is -5.
- The change in x-values is 0.5.



- Press 2nd GRAPH.

The table of values will appear as shown.



Tracing a Graph

- Enter a function using Y=.
- Press TRACE.
- Press ◄ and ► to move along the graph.

The x- and y-values are displayed at the bottom of the screen.

If you have more than one graph plotted, use the ▲ and ▼ keys to move the cursor to the graph you wish to trace.

You may want to turn off all Stat Plots before you trace a function:

- Press 2nd Y= for [STAT PLOT]. Select **4:PlotsOff**.
- Press ENTER.

Using Zoom

Use the (ZOOM) key to change the area of the graph that is displayed in the graphing window.

To set the size of the area you want to zoom in on:

- Press (ZOOM). Select **1:ZBox**. The graph screen will be displayed, and the cursor will be flashing.
- If you cannot see the cursor, use the (▶), (◀), (▲), and (▼) keys to move the cursor until you see it.
- Move the cursor to an area on the edge of where you would like a closer view.
- Press (ENTER) to mark that point as a starting point.
- Press the (▶), (◀), (▲), and (▼) keys, as needed, to move the sides of the box to enclose the area you want to look at.
- Press (ENTER) when you are finished. The area will now appear larger.

To zoom in on an area without identifying a boxed-in-area:

- Press (ZOOM). Select **2:Zoom In**.

To zoom out of an area:

- Press (ZOOM). Select **3:Zoom Out**.

To display the viewing area where the origin appears in the centre and the x- and y-axes intervals are equally spaced:

- Press (ZOOM). Select **4:ZDecimal**.

To display an equation with the minimum and maximum y-values constructed to match the scale of the x-axis and without changing the current minimum and maximum x-values, so that a square grid results:

- Press (ZOOM). Select **5:ZSquare**.

To reset the axes range on your calculator:

- Press (ZOOM). Select **6:ZStandard**.

To display all data points in a Stat Plot:

- Press (ZOOM). Select **9:ZoomStat**.

To display an equation with the minimum and maximum y-values constructed to best show the equation without changing the current minimum and maximum x-values:

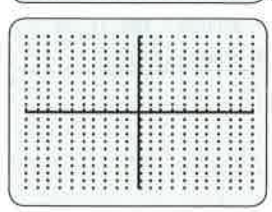
- Press (ZOOM). Select **0:ZFit**.

Setting the Format

To define a graph's appearance:

- Press (2nd) (ZOOM) for [FORMAT] to view the choices available.

The default settings, shown here, have some of the features on the left turned on.



To use **GridOn**:

- Pressing (2nd) (ZOOM) for [FORMAT].

Cursor down and right to **GridOn**.

- Press (ENTER).
- Press (GRAPH) to see the grid turned on.
- Press (2nd) (MODE) for [QUIT].

Entering Data Into Lists

To enter data:
- Press STAT. The cursor will highlight the **Edit** menu.
- Press 1 or ENTER to select **1:Edit...**.

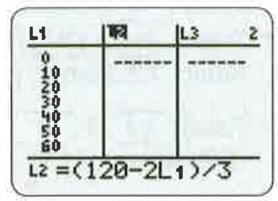
This allows you to enter new data, or edit existing data, in lists **L1** to **L6**.

For example, press STAT, select **1:Edit...**, and then enter values in **L1**.
- Use the cursor keys to move around the editor screen.
- Complete each data entry by pressing ENTER.
- Press 2nd MODE for [QUIT] to exit the list editor when the data are entered.

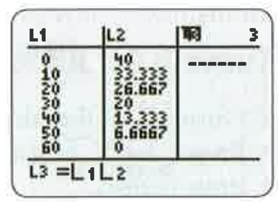
Using List Operations

You can use a list in the list editor and apply the order of operations to produce another list. This is useful when tabulating data in a list affected by the same order of operations.

For example, for the area of a fenced-in yard:
Width = (120 − 2 × length) ÷ 3
Area = length × width



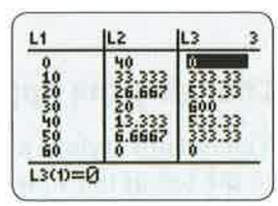
Use the list operations to compute the width values.
- Press STAT 1 to enter the list editor.
- Enter the given lengths into **L1**. For the example given, use 0, 10, 20, 30, 40, 50, 60.
- Press ► for **L2**. Cursor up to the title for **L2**.
- Press (120 − 2 2nd 1) ÷ 3 ENTER.



The width data are pasted into **L2**.

Use the list operations to compute the area values.
- Press ► for **L3**. Cursor up to the title for **L3**.
- Press 2nd 1 × 2nd 2 ENTER.

The area data are pasted into **L3**.

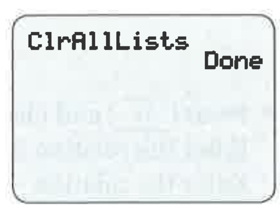


This process is useful when constructing the data for simple and compound interest problems.

Clearing Lists

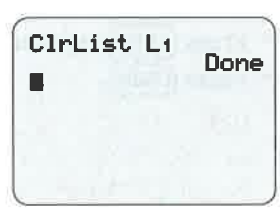
To clear all lists from the calculator without resetting the RAM:
- Press 2nd + for [MEM]. Select **4:ClrAllLists**.

This will paste the **ClrAllLists** command to the home screen of the calculator.



To clear all the lists:
- Press ENTER.

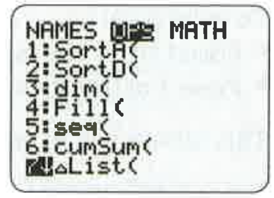
You may need to clear only a specific list before you enter data into it. For example, to clear list **L1**:
- Press STAT and select **4:ClrList**.
- Press 2nd 1 for [L1] and press ENTER.



Calculating First and Second Differences

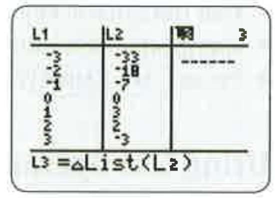
The calculator can compute the first and second differences of a list.

- Press STAT then select **1:Edit…**.
- Enter data into **L1**.
- Enter data into **L2**.



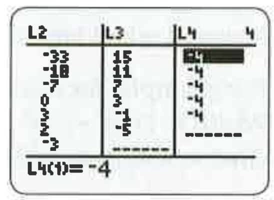
To find the first differences:

- Press ► and cursor over to **L3**.
- Press ▲ and cursor up to the title for **L3**.
- Press 2nd STAT for [LIST]. Cursor over to the **OPS** menu. Select **7:Δ List(** to paste the command in the title for **L3**.
- Press 2nd 2) ENTER to compute the first differences of **L2**.



To find the second differences:

- Press ► and cursor over to **L4**.
- Press ▲ and cursor up to the title for **L4**.
- Press 2nd STAT for [LIST]. Cursor over to the **OPS** menu. Select **7:Δ List(** to paste the command in the title for **L4**.
- Press 2nd 3) ENTER to compute the second differences of **L2** (first differences of **L3**).



Turning Off All Plots

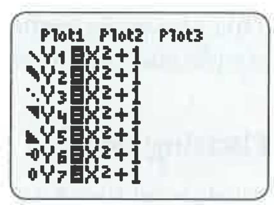
To turn off all the plots without resetting the RAM:

- Press 2nd Y= and select **4:PlotsOff**.
- Press ENTER.

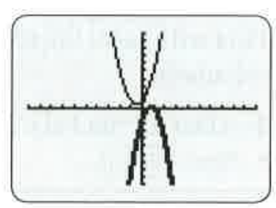
Changing the Appearance of a Line

The default style is a thin solid line. The line style is displayed to the left of the equation.
There are seven options for the appearance of a line.

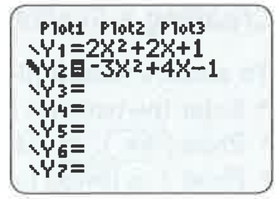
Thin line
Thick line
Dotted line
Shade upper
Shade lower
Animate with trace
Animate without trace



- Press Y= and clear any previously entered equations. Enter the relation $2x^2 + 2x + 1$ for **Y1**. Enter the relation $-3x^2 + 4x - 1$ for **Y2**.
- Press ◄ until the cursor is to the left of **Y2 =**.
- Press ENTER repeatedly until the thick solid line shows.
- Press GRAPH.

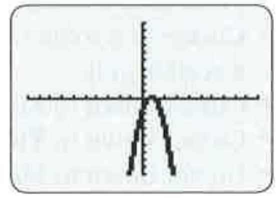


Turning Off an Equation



In the equation editor, move the cursor over the equal sign of the equation of the graph you do not want to display.

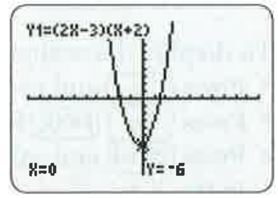
- Press ENTER to turn off the graph for that equation.



- Press GRAPH.

Note: Any equation without a highlighted equal sign is turned off and will not be plotted.

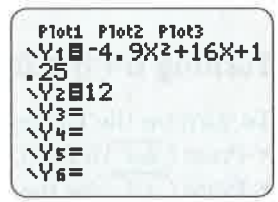
Finding a *y*-Intercept



To find the *y*-intercept of a function:
Enter $(2x - 3)(x + 2)$ for **Y1**.

- Press GRAPH.
- Press TRACE 0 ENTER.

Finding an Intersection Point



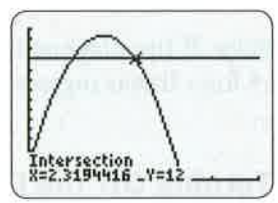
There must be at least two equations in the calculator's Equation Editor.

- Press Y= and enter the relations for **Y1** and **Y2**.

Be sure to use the appropriate window settings.

- Press GRAPH.

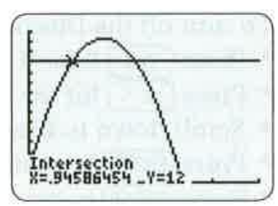
Note: An intersection point *must* be visible in order to find its coordinates. If an intersection point is not visible, adjust the window settings accordingly.



To find an intersection point:

- Press 2nd TRACE and select **5:Intersect.**
- Press ENTER ENTER ENTER.

The coordinates of the intersection point will appear at the bottom of the screen.



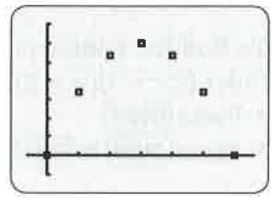
To find the other intersection point:

- Press 2nd TRACE and select **5:Intersect.**
- Press and hold the ◄ or ► key to move closer to the other intersection point.
- Press ENTER ENTER ENTER.

Creating a Scatter Plot

To create a scatter plot:

- Enter the two sets of data in lists **L1** and **L2**.
- Press 2nd Y= for [STATPLOT].
- Press 1 or ENTER to select **1:Plot1…**.
- Press ENTER to select **On**.
- Cursor down then press ENTER to select the top left graphing option, a scatter plot.
- Cursor down to **Xlist** and press 2nd 1 for [L1].
- Cursor down to **Ylist** and press 2nd 2 for [L2].
- Cursor down to **Mark** and select a mark style by pressing ENTER.
- Press 2nd MODE for [QUIT] to exit the Stat Plots editor after you have entered the data.



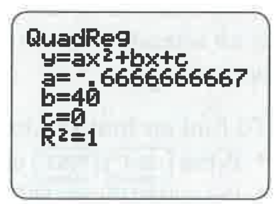
To display the scatter plot:

- Press Y= and use the CLEAR key to remove any graphed equations.
- Press 2nd MODE for [QUIT].
- Press ZOOM and select **9:ZoomStat** to display the scatter plot for the data in the lists.
- Press WINDOW to change **Xscl** and **Yscl** appropriately to place tick marks on both axes if they are not visible.

Turning On the Diagnostic Mode

To turn on the Diagnostic mode:

- Press 2nd 0 for [CATALOG].
- Press x^{-1} for the Ds in the alphabetic list of commands.
- Scroll down to **DiagnosticOn**.
- Press ENTER to paste the command to the home screen of the calculator.
- Press ENTER to turn on the Diagnostic mode.

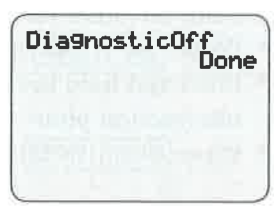


Note: If the diagnostic mode is turned on, you will see the values for r and r^2 for a linear regression and the value of R^2 for a quadratic regression.

Turning Off the Diagnostic Mode

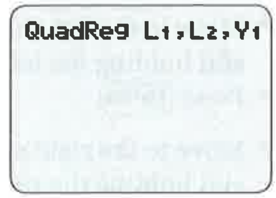
To turn off the Diagnostic mode:

- Press 2nd 0 for [CATALOG].
- Press x^{-1} for the Ds in the alphabetic list of commands.
- Scroll down to **DiagnosticOff**.
- Press ENTER to paste the command to the home screen of the calculator.
- Press ENTER to turn off the Diagnostic mode.

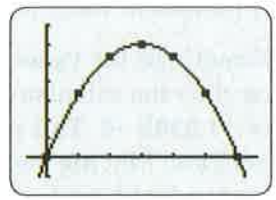


Finding a Curve or a Line of Best Fit

You can add a curve of best fit to data in the lists by using an appropriate model.



- With the scatter plot displayed, press STAT ▶ for the **CALC** menu.
- Select the model that would best represent the data displayed in the scatter plot:
  4:LinReg(ax+b) for a linear regression
  5:QuadReg for a quadratic regression
  0:ExpReg for an exponential regression
  C:SinReg for a sinusoidal regression
- Press 2nd 1 for **L1** and press , .
- Press 2nd 2 for **L2** and press , .
- Press VARS ▶ for the **Y-VARS** menu. Select **1:Function...**, and then select **1:Y1**.
- Press ENTER to perform the regression analysis.
- Press GRAPH to view the model.

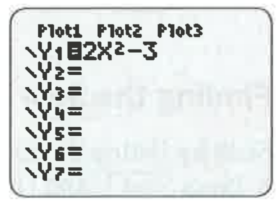


Finding a Zero or a Maximum/Minimum Value

There must be at least one equation in the Equation Editor.

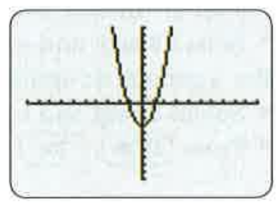
- Press Y=.

The example shows a parabola.



To calculate a zero (*x*-intercept), maximum, or minimum there *must* be one visible.

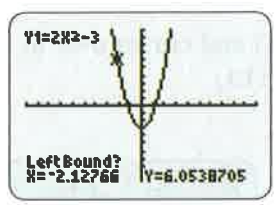
- Press GRAPH.



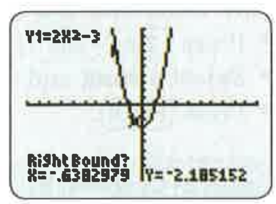
Note: If a zero (*x*-intercept), maximum, or minimum is not visible, adjust the window settings accordingly.

To find a zero:

- Press 2nd TRACE 2.
- Move to the left side of a zero (*x*-intercept) by pressing and holding the left cursor key.
- Press ENTER.



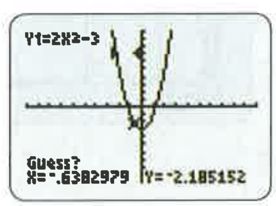
- Move to the right side of a zero (*x*-intercept) by pressing and holding the right cursor key.
- Press ENTER.

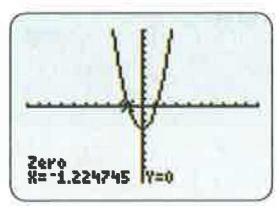


To find the value of a zero (*x*-intercept) using the calculator's guess feature:

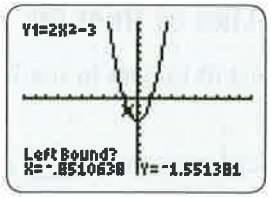
- Press ENTER.

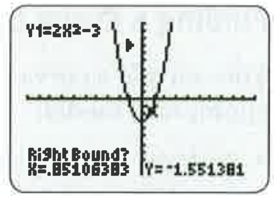
A zero (*x*-intercept) is shown at the bottom of the screen.



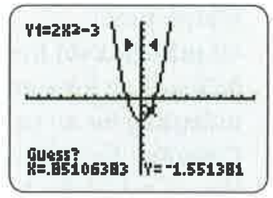


To find a minimum:





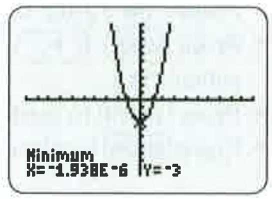
- Press 2nd TRACE 3.
- Move to the left side of a minimum by pressing and holding the left cursor key.
- Press ENTER.
- Move to the right side of a minimum by pressing and holding the right cursor key.
- Press ENTER.



To find the value of a minimum using the calculator's guess feature:

- Press ENTER.

A minimum value is shown at the bottom of the screen.



Sometimes the values of x and/or y are not exact because of the method used by the calculator to determine the values. The value for x, shown, is $-1.938\text{E}-6$. This means the number is -1.938×10^{-6} in scientific notation. Moving the decimal six places to the left will give the number in standard form ($-0.000\,001\,938$). In this case, assume that the x-value of the minimum is 0 rather than $-0.000\,001\,938$.

To find a maximum:

- Press 2nd TRACE 4. Follow a similar procedure as above.

Finding the Sum of an Arithmetic or Geometric Series

Start by listing the first ten terms and storing them **L1**.

- Press 2nd STAT for [LIST] and cursor over to the **OPS** menu.

For an arithmetic series, such as one with $a = 2$, $d = 4$, to find S_{10}:

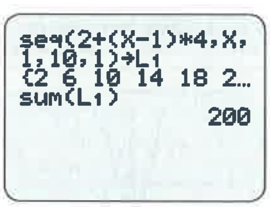
- Select **5:seq(** and enter $2 + (x - 1) \times 4$, x, 1, 10, 1).

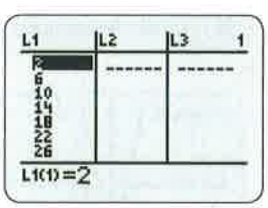
For a geometric series, such as one with $a = 2$, $r = -3$, to find S_{10}:

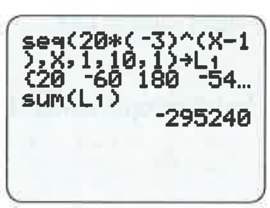
- Select **5:seq(** and enter $20 \times (-3)^{x-1}$, x, 1, 10, 1).
- Press STO▶, 2nd 1 for [L1], ENTER.

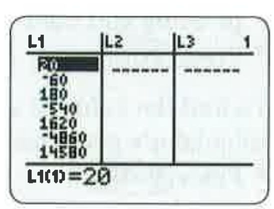
Now determine the sum.

- Press 2nd STAT [LIST] and cursor over to the **MATH** menu.
- Select **5:sum(** and enter **L1**).
- Press ENTER.









Repeating Calculations for Other Scenarios

- Press (2nd) (ENTER) for [ENTRY] to recall the previous calculation.
  Use the cursor keys, the (DEL) key, and the [INS] command to modify the equation to fit each scenario.
- Press (ENTER) to perform the new calculation.
  In the example shown, press (2nd) (ENTER) for [ENTRY] twice to see the first calculation again, and then alter the amount, 2600.

```
2600((1+0.06)^15
-1)/0.06
          60517.5217
650((1+0.015)^60
-1)/0.015
         62539.52361
650((1+0.015)^60
-1)/0.015
```

```
2600((1+0.06)^15
-1)/0.06
          60517.5217
650((1+0.015)^60
-1)/0.015
         62539.52361
2600((1+0.06)^15
-1)/0.06
```

About the Finance Applications: The TVM Solver

The **TVM Solver** is used to work with annuities (for example, loans and investments with regular payments, and mortgages) and can also be used for non-annuities (for example, loans or investments with no regular payments). **TVM** stands for **T**ime **V**alue of **M**oney.

To open the TVM Solver:

- On the TI-83 Plus/TI-84 Plus, press (APPS) 1 and 1.

What the TVM Solver Variables Represent

When There Are Regular Payments (Ordinary Annuities and Mortgages)

N	Number of Payments
I%	Annual Interest Rate
PV	Present Value
PMT	Payment
FV	Future Value
P/Y	Number of Payments/Year
C/Y	Number of Compounding Periods/Year
PMT: END BEGIN	Payments at End of Payment Interval

```
 N=12.00
 I%=7.00
 PV=0.00
 PMT=-200.00
■FV=2645.02
 P/Y=4.00
 C/Y=4.00
 PMT:END BEGIN
```

A savings annuity invested at 7%, compounded quarterly, with quarterly deposits of $200, for 3 years has a future value of $2645.02.

When There Are No Regular Payments

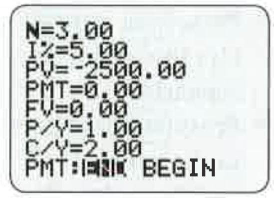
N	Number of Years
I%	Annual Interest Rate
PV	Present Value, or Principal
PMT	Always set **PMT=0.00**.
FV	Future Value, or Final Amount
P/Y	Always set **P/Y=1.00**.
C/Y	Number of Compounding Periods/Year
PMT: END BEGIN	END or BEGIN

```
 N=7.00
 I%=5.00
 PV=-1000.00
 PMT=0.00
■FV=1418.04
 P/Y=1.00
 C/Y=12.00
 PMT:END BEGIN
```

$1000 invested at 5%, compounded monthly, for 7 years has a future value of $1418.04.

Investments and Loans (No Regular Payments)

Final Amount If you know the principal, or present value, interest rate, compounding frequency, and term of an investment or loan, you can determine its final amount.

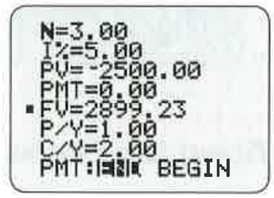


Term is 3 years.
Annual interest rate is 5%.
Principal is $2500.
Final amount is unknown.
2 compounding periods/year

For example, to determine the final amount of a **$2500** investment earning 5% interest, **compounded semi-annually**, for **3 years**, follow these steps:

Open the **TVM Solver** and enter the values as shown:

To solve for **FV**, cursor to **FV=0.00** and press ALPHA ENTER.



The final amount is $2899.23.

Present Value, or Principal If you know the final amount, interest rate, compounding frequency, and term of an investment or loan, you can determine its present value, or principal.

Open the **TVM Solver** and enter the known values.
To solve for **PV**, cursor to **PV=0.00** and press ALPHA ENTER.

Interest Rate To find the annual interest rate, enter the known values for **N**, **PV**, **FV**, and **C/Y**. Set **I%=0.00**, **PMT=0.00**, and **P/Y=1.00**. Then, cursor to **I%=0.00** and press ALPHA ENTER.

Term To find the term, in years, enter the known values for **I%**, **PV**, **FV**, and **C/Y**. Set **N=0.00**, **PMT=0.00**, and **P/Y=1.00**. Then, cursor to **N=0.00** and press ALPHA ENTER.

Important Points About the TVM Solver

- Set the number of decimal places to 2.
- A value must be entered for each variable.
- Money paid out (cash outflow), such as a loan payment, is negative.
- Money received (cash inflow), such as the final amount of an investment, is positive.
- To quit the **TVM Solver** and return to the Home Screen, press 2nd MODE.

Ordinary Annuities (Regular Payments)

Future Value Enter the known values for **N**, **I%**, **PMT**, **P/Y**, and **C/Y** and set **PV = 0.00**, **FV = 0.00**, and **PMT:END**. Then, cursor to **FV = 0.00** and press ALPHA ENTER.

Present Value Enter the know values for **N**, **I%**, **PMT**, **P/Y**, and **C/Y** and set **PV = 0.00**, **FV = 0.00**, and **PMT:END**. Then, cursor to **PV = 0.00** and press ALPHA ENTER.

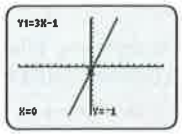
Payment To find the payment given the present value or the future value, enter the known values for **N**, **I%**, **PV** or **FV**, **P/Y**, and **C/Y** and set **PV = 0.00** or **FV = 0.00**, and **PMT:END**. Then, cursor to **PMT = 0.00** and press ALPHA ENTER.

Answers

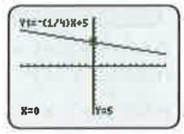
Chapter 1

Prerequisite Skills, pages 2–3

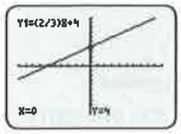
1. a)



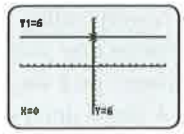
b)



c)



d)



2. a) $y = 2x - 3$ **b)** $y = -\frac{1}{3}x + 2$

3. a) $y = -\frac{5}{4}x + 8$ **b)** $y = -3x + 4$

c) $y = \frac{5}{4}x - 6$

4. a) $(-1, 2)$ **b)** $(4, -3)$ **c)** $(2, 2)$

5. a) $(1, 8)$ **b)** $(5, 9)$ **c)** $(5, -1)$

6. a) $x^2 + 4x + 4$ **b)** $n^2 - 9$

c) $\frac{1}{2}t^2 - 4t + 8$ **d)** $3x^2 + 3x - 18$

e) $4k^2 - 4$ **f)** $4x^2 + \frac{14}{3}x - 2$

7. a) $(x + 5)(x - 3)$ **b)** $(x + 3)^2$

c) $(3n + 5)(3n - 5)$ **d)** $-(x + 4)(x - 3)$

e) $3(t + 1)^2$ **f)** $-5(x - 4)^2$

8. a) no **b)** yes; $(x - 6)^2$

c) no **d)** no

e) yes; $(x + 2)^2$ **f)** yes; $(2n + 3)^2$

9. a) 16 **b)** 25 **c)** 1 **d)** 49

e) $\frac{25}{4}$ **f)** $\frac{121}{4}$ **g)** $\frac{1}{4}$ **h)** $\frac{9}{4}$

10. a) $\frac{1}{2}(x^2 - 3x)$ **b)** $\frac{2}{3}\left(x^2 + \frac{15}{2}x\right)$

c) $-\frac{1}{5}(x^2 + 10x)$ **d)** $-\frac{3}{4}(x^2 - 12x)$

11. a) **i)** $(-1, -3)$ **ii)** $x = -1$

iii) upward **iv)** -1

b) **i)** $(3, 1)$ **ii)** $x = 3$

iii) downward **iv)** -14

12. a) $y = -(x - 2)^2 + 3$ **b)** $y = (x + 5)^2 - 1$

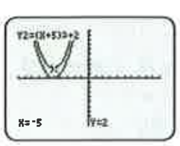
13. a) $y = (x + 2)^2 - 3$; $(-2, -3)$

b) $y = (x - 5)^2 - 30$; $(5, -30)$

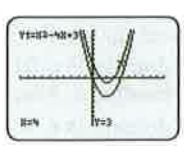
14. a) The graph of $y = (x + 5)^2 + 2$ is the graph of $y = (x + 5)^2$ translated vertically up 2 units.

b) The graph of $y = x^2 - 4x$ is the graph of $y = x^2 - 4x + 3$ translated vertically down 3 units.

15. a)



b)



1.1 Functions, Domain, and Range, pages 12–15

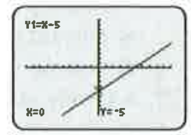
1. a) This relation is a function. No vertical line can be drawn that will pass through more than one point on the line.

b) This relation is a function. No vertical line can be drawn that will pass through more than one point on the line.

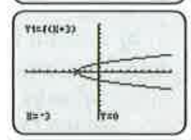
c) This relation is a function. No vertical line can be drawn that will pass through more than one point on the line.

d) This relation is not a function. An infinite number of vertical lines can be drawn that will pass through more than one point on the curve.

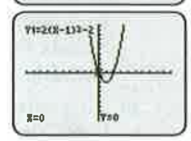
2. a) This relation is a function. No vertical line can be drawn that will pass through more than one point on the line.



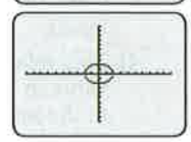
b) This relation is not a function. An infinite number of vertical lines can be drawn that will pass through more than one point on the curve.



c) This relation is a function. No vertical line can be drawn that will pass through more than one point on the line.



d) This relation is not a function. An infinite number of vertical lines can be drawn that will pass through more than one point on the circle.



3. a) domain {5, 6, 7, 8, 9}, range {5, 6, 7, 8, 9}; this relation is a function because for each value in the domain there is exactly one value in the range.

b) domain {3, 4, 5, 6}, range {−1}; this relation is a function because for each value in the domain there is exactly one value in the range.

c) domain {1}, range {−14, −8, 0, 6, 11}; this relation is not a function. The x-value 1 has five corresponding y-values.

d) domain {1, 3, 4, 5, 11}, range {1, 4, 5, 9,11}; this relation is a function because for each value in the domain there is exactly one value in the range.

e) domain {1, 2, 3}, range {−2, −1, 0, 1, 2}; this relation is not a function. The x-values 2 and 3 have two corresponding y-values.

4. a) This relation is a function because for each value in the domain there is exactly one value in the range.

b) This relation is a function because for each value in the domain there is exactly one value in the range.

c) This relation is not a function. The domain has three elements but the range has five elements. So one or more values in the domain must be associated with two values in the range.

d) This relation is not a function. The domain has one element but the range has five elements. So one value in the domain must be associated with every value in the range.

5. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

c) domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}\}$

d) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq -1\}$

e) domain $\{x \in \mathbb{R}, x \neq 3\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

6. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -4\}$

c) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 1\}$

d) domain $\{x \in \mathbb{R}, -3 \leq x \leq 3\}$; range $\{y \in \mathbb{R}, -3 \leq y \leq 3\}$

e) domain $\{x \in \mathbb{R}, x \neq -3\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

f) domain $\{x \in \mathbb{R}, x \geq -0.5\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

7. Answers may vary.

8. a) $A = -2x^2 + 45x$

b) domain $\{0 < x < 22.5\}$, range $\{0 < A < 253.125\}$

9. a) This relation is a function because for each value in the domain there is exactly one value in the range.
Independent variable: the number of raffle tickets that a hockey team sells; dependent variable: the amount of money taken in.

b) This relation is not a function. For any grade level, there may be students of different age levels.

c) This relation is a function because for each value in the domain there is exactly one value in the range.
Independent variable: the speed at which Jung Yoo walks; dependent variable: the time it takes Jung Yoo to walk to school

10. Answers may vary. Sample answers:

a) The fourteen 3-m sections of preassembled fencing can be assembled so that the length is 12 m and the width is 9 m. This will give an area of 108 m² for the parking lot.

b) Since the sections are 3 m in length, the sections can be assembled to give an area of 12 m by 9 m. The greatest area that can be enclosed with 42 m of fencing will occur when the width of the area is 10.5 m and the length of the area of is 10.5 m. The greatest area is 110.25 m².

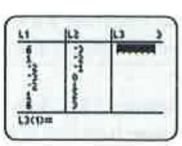
11. a) range $\{0, 6, 12, 18, 24\}$ **b)** range $\{-3, 0, 5, 12, 21\}$

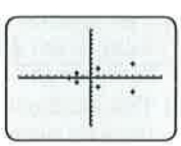
c) range $\{3\}$ **d)** range $\{-1, 1, 7, 17, 31\}$

e) range $\left\{\frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}\right\}$

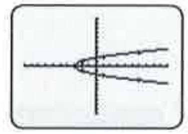
f) range $\{-2\sqrt{6}, -\sqrt{21}, -4, -3, 0, 3, 4, \sqrt{21}, 2\sqrt{6}\}$

12. a)





b)



c) Answers may vary. Sample answers: The relation $x = y^2 - 3$ can be defined using the relations $y = \sqrt{x + 3}$ and $y = -\sqrt{x + 3}$.

d) The x-values 6, 1, and -2 each have two corresponding y-values.

13. Answers may vary.

14. Answers may vary. Sample answers:

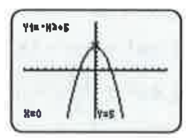
a) A single point on a graph has one entry in the domain and one entry in the range.

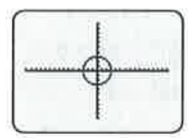
b) A vertical line on a graph has one entry in the domain and many entries in the range.

c) A horizontal line on a graph has many entries in the domain and one entry in the range.

15. Answers may vary. Sample answers:

a) $y = -x^2 + 5$ **b)** $x^2 + y^2 = 9$

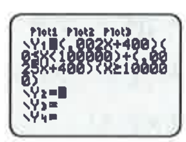


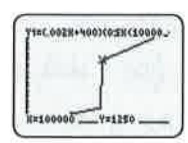


16. a) $P = 0.002s + 400$:
domain $\{x \in \mathbb{R}, 0 \leq x \leq 100\,000\}$,
range $\{y \in \mathbb{R}, 400 \leq y < 600\}$.
$P = 0.0025s + 400$;
domain $\{x \in \mathbb{R}, x \geq 100\,000\}$,
range $\{y \in \mathbb{R}, y \geq 650\}$

b) Both relations are functions because for each value in the domain there is exactly one value in the range.

c)





d) Answers may vary. Sample answer: At $s = 100\,000$, there is a jump discontinuity on the graph. At this number of sales, the weekly earnings for the salesperson increase from less than \$600 per week to \$1250 per week, and for sales greater than \$100 000, the salesperson's earnings will increase more rapidly than for sales less than \$100 000.

17. Answers may vary. Sample answer: Yes. The function $y = (x + 2)^2 + 5$ has domain $\{x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}, y \geq 5\}$. The function $y = (x - 2)^2 + 5$ has domain $\{x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}, y \geq 5\}$. The domain and range of both functions are the same.

18. a) domain $\{\theta \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, 1 \leq y \leq 5\}$; the relation is a function.

b) domain $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$, range $\{y \in \mathbb{R}, -4 \leq y \leq 4\}$; the relation is not a function.

19. $\{x \in \mathbb{R}, 3 \leq x < 5\}$

20. \$160

21. 48 factors

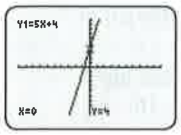
22. $-2 < x < 2$

1.2 Functions and Function Notation, pages 21–24

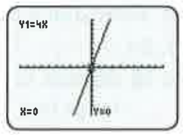
1. a) $\frac{63}{5}, 9, \frac{161}{15}$ **b)** $57, 66, 1$ **c)** $128, 2, \frac{200}{9}$

d) $-6, -6, -6$ **e)** $\frac{1}{4}, -\frac{1}{5}, -\frac{3}{2}$ **f)** $3, 0, \sqrt{\frac{13}{3}}$

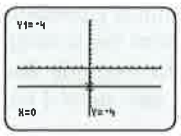
2. a) 4



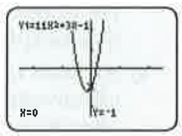
b) 0



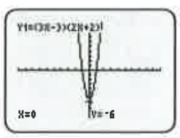
c) -4



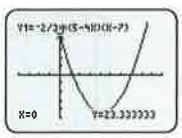
d) -1



e) -6



f) $\frac{70}{3}$

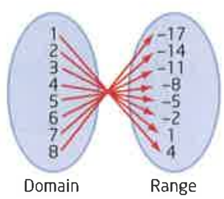


3. a) $-4; y = -4x$ **b)** $3; y = 3x$

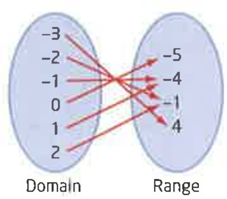
c) $\frac{2}{3}; y = \frac{2}{3}x$ **d)** $-1; y = -x$

4. Answers will vary. For example: An example of a linear function is $y = 2x + 3$ and an example of a constant function is $y = 2$. The functions have the same domains $\{x \in \mathbb{R}\}$, but different ranges. For the function $y = 2x + 3$ the range is $\{y \in \mathbb{R}\}$and for the function $y = 2$ the range is $\{y \in \mathbb{R}, y = 2\}$.

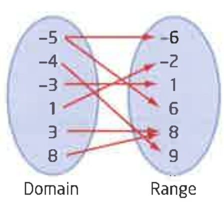
5. a)



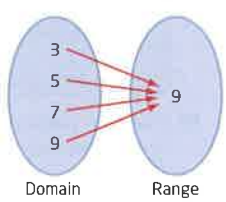
b)



c)



d)



6. a) This relation is a function because for each value in the domain there is exactly one value in the range.

b) This relation is a function because for each value in the domain there is exactly one value in the range.

c) This relation is not a function. The x-value -5 has two corresponding y-values.

d) This relation is a function because for each value in the domain there is exactly one value in the range.

7. a) $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$

b) $\{(-5, 11), (-4, 6), (-2, -4), (0, -14), (2, -24)\}$

c) $\{(-4, 6), (3, 6), (1, 6), (5, 6)\}$

8. a) This relation is a function because for each value in the domain there is exactly one value in the range.

b) This relation is a function because for each value in the domain there is exactly one value in the range.

c) This relation is a function because for each value in the domain there is exactly one value in the range.

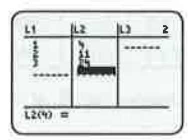
9. Answers may vary. Sample answer: Mapping diagrams are visual representations of relations that are sets of ordered pairs. For visual learners, it may be easier to determine if a relation that is represented by a mapping diagram is a function by determining if there is exactly one arrow leading from each value in the domain.

10. a) $f : x \to -x + 4$ **b)** $g : x \to x^2 + 5x - 3$

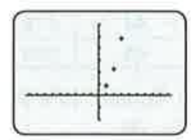
c) $s : x \to \sqrt{4x - 4}$ **d)** $r : k \to -\frac{1}{2k - 1}$

11. Answers may vary. Sample answer: A relation is a function if for each value in the domain there is exactly one value in the range. In a mapping diagram a relation is a function if there is exactly one arrow leading from each value in the domain.

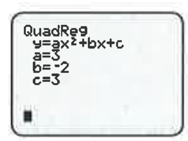
12. a)



b)

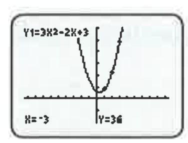


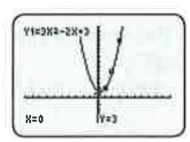
c)

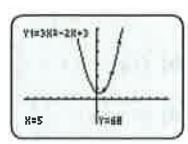


$y = 3x^2 - 2x + 3$

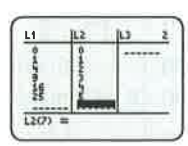
d)

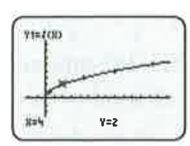






13. a)



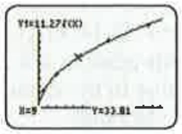


b) Answers may vary. Sample answer: This relation is a function because for each value in the domain there is exactly one value in the range.

c) Answers may vary. Sample answer: Yes. For each value in the domain there is exactly one value in the range.

14. a) domain $\{d \in \mathbb{R}, d \geq 0\}$, range $\{v \in \mathbb{R}, v \geq 0\}$

b)



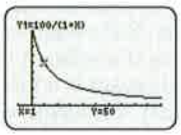
15. a) \$24 000 **b) i)** \$3090.90 **ii)** \$2769.12

c) 22 years

d) Answers may vary. Sample answer: The relation is a function because for each value in the domain there is exactly one value in the range.

16. a) domain $\{i \in \mathbb{R}, i \neq -1\}$, range $\{A \in \mathbb{R}, 0 \leq A \leq 100\}$

b)



c) \$95.24 **d)** 11.1%

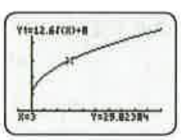
17.–18. Answers may vary.

19.

Age, n \ Rating, r	1	2	3	4	5
40	570	560	550	540	530
41	572	562	552	542	532
42	574	564	554	544	534
43	576	566	556	546	536
44	578	568	558	548	538
45	580	570	560	550	540

21. a) domain $\{d \in \mathbb{R}, d \geq 0\}$, range $\{v \in \mathbb{R}, v \geq 8\}$

b)



c) Answers may vary. Sample answer: This relation is a function because for each value in the domain there is exactly one value in the range.

22. D **23.** 65 **24.** C

1.3 Maximum or Minimum of a Quadratic Function, pages 31–32

1. a) $y = (x + 2)^2 - 4$ **b)** $f(x) = \left(x + \frac{7}{2}\right)^2 - \frac{5}{4}$

c) $g(x) = \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$ **d)** $y = \left(x - \frac{11}{2}\right)^2 - \frac{137}{4}$

e) $f(x) = \left(x + \frac{13}{2}\right)^2 - \frac{161}{4}$ **f)** $y = \left(x - \frac{9}{2}\right)^2 - \frac{117}{4}$

2. a) $(-5, -19)$; minimum **b)** $(-3, -2)$; minimum

c) $(1, 4)$; maximum **d)** $(6, 31)$; maximum

e) $(-1, 2)$; maximum **f)** $\left(-4, -\frac{7}{3}\right)$; minimum

3. a) $(1, 8)$; minimum **b)** $(2, 5)$; maximum

c) $\left(3, \frac{7}{2}\right)$; minimum **d)** $\left(\frac{3}{2}, -\frac{25}{4}\right)$; maximum

e) $\left(5, -\frac{3}{2}\right)$; minimum **f)** $\left(-7, \frac{22}{5}\right)$; maximum

4. Answers may vary.

5. \$1000

6. 130

7. 2.4 m

8. 83 km/h

9. 12 m by 6 m

10. short side 5 m, longer side 10 m

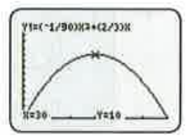
11. 25

12. a) domain $\{n \in \mathbb{R}, 0 \leq n \leq 60\}$; range $\{e \in \mathbb{R}, 0 \leq e \leq 10\}$

b) vertex: (30, 10); maximum; Answers may vary. For example: The graph is a parabola opening downward, so the vertex is a maximum.

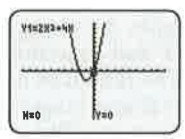
c) Answers may vary. For example: the maximum effectiveness of a TV commercial will occur after 30 viewings.

d)

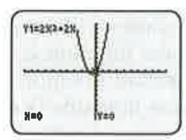


13. Answers may vary. Sample answers:

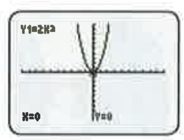
a) $y = 2x^2 + 4x$



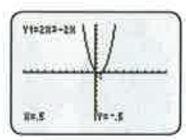
$y = 2x^2 + 2x$



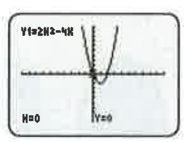
$y = 2x^2$



$y = 2x^2 - 2x$



$y = 2x^2 - 4x$



b) The five parabolas are congruent and they all open upward.

c) $(-1, -2)$; $(-0.5, -0.5)$; $(0, 0)$; $(0.5, -0.5)$; $(1, -2)$

d) The x-coordinate of each vertex is the value of b multiplied by $-\frac{1}{4}$.

14. 7.5 cm

15. Answers may vary.

16. C **17.** C **18.** B

1.4 Skills You Need: Working With Radicals, pages 39–40

1. a) $12\sqrt{5}$ **b)** $5\sqrt{6}$ **c)** $-2\sqrt{35}$
d) $-20\sqrt{15}$ **e)** $6\sqrt{6}$ **f)** $6\sqrt{22}$

2. a) $2\sqrt{3}$ **b)** $11\sqrt{2}$ **c)** $7\sqrt{3}$
d) $2\sqrt{5}$ **e)** $6\sqrt{7}$ **f)** $14\sqrt{2}$

3. a) $\sqrt{3}$ **b)** $-4\sqrt{5}$ **c)** 0
d) $5\sqrt{2} - 4\sqrt{5}$ **e)** $4\sqrt{6} - 5\sqrt{2}$ **f)** $-3\sqrt{10} + \sqrt{5}$

4. a) $4\sqrt{2}$ **b)** $37\sqrt{2}$ **c)** $-3\sqrt{5} - 6\sqrt{3}$
d) $9\sqrt{7} + 8\sqrt{6}$ **e)** $14\sqrt{3} - 4\sqrt{2}$ **f)** $5\sqrt{11} + 5\sqrt{22}$

5. a) $30\sqrt{2}$ **b)** $-16\sqrt{7}$ **c)** $40\sqrt{2}$
d) $-18\sqrt{5}$ **e)** $55\sqrt{6}$ **f)** -24

6. a) $24 - 3\sqrt{5}$ **b)** $5\sqrt{6} + 12$ **c)** $3\sqrt{2} - 3$
d) $8\sqrt{5} - 20$ **e)** $64 + 48\sqrt{6}$ **f)** $6\sqrt{21} - 15\sqrt{6}$

7. a) $27 + 10\sqrt{2}$ **b)** $-12 - 4\sqrt{2}$
c) $5\sqrt{3} + 5\sqrt{6} + 10\sqrt{2} + 20$
d) $-5 - 7\sqrt{5}$ **e)** -4 **f)** $-17 + \sqrt{7}$

8. a) $-\frac{1}{2}\sqrt{6}$ **b)** $2\sqrt{5}$ **c)** $2\sqrt{2}$
d) $\sqrt{5} - 4\sqrt{3}$

9. a) $3\sqrt{2}$ **b)** $60\sqrt{2}$ **c)** 45 **d)** 2π

10. Answers may vary. Sample answer:
$\sqrt{2880} = \sqrt{576 \times 5} = \sqrt{576} \times \sqrt{5} = 24\sqrt{5}$.

11. $15\sqrt{3}$ cm

12. 100

13. area: $12\sqrt{7}$ cm²; perimeter: $12 + 4\sqrt{7}$ cm

14. Answers may vary.

15. a) Answers may vary. Sample answer: Yes.
$1 + \sqrt{3}$ is a solution to the equation $x^2 - 2x - 2 = 0$. Substitute $1 + \sqrt{3}$ for x in the left side of the equation. Then,
$$(1 + \sqrt{3})^2 - 2(1 + \sqrt{3}) - 2$$
$$= 1 + 2\sqrt{3} + 3 - 2 - 2\sqrt{3} - 2$$
$$= 0$$

16. a) $2 + 3\sqrt{5}$ **b)** $3 - \sqrt{6}$ **c)** $\sqrt{7}$
d) $3 - \sqrt{3}$ **e)** $-2 + \sqrt{2}$

17. a) $3\sqrt[3]{2}$ **b)** $10\sqrt[3]{3}$ **c)** $5\sqrt[3]{9}$

18. Explanations may vary.
a) $\sqrt{a} < a$ for $\{a \in \mathbb{R}, a > 1\}$.
b) $\sqrt{a} > a$ for $\{a \in \mathbb{R}, 0 < a < 1\}$.

19. D **20.** C **21.** C **22.** 1

1.5 Solve Quadratic Equations, pages 49–51

1. a) $-3, 1$ **b)** $-5, 2$ **c)** $-3, 3$
d) $\frac{4}{3}, 1$ **e)** $\frac{1}{5}, \frac{1}{3}$ **f)** $-\frac{5}{2}, -\frac{2}{3}$

2. Answers may vary.

3. a) $\frac{17 \pm \sqrt{73}}{4}$ **b)** $\frac{3 \pm \sqrt{137}}{8}$
c) $\frac{-1 \pm \sqrt{29}}{2}$ **d)** $-3 \pm \sqrt{13}$
e) $\frac{-1 \pm \sqrt{133}}{6}$ **f)** $4 \pm \sqrt{14}$

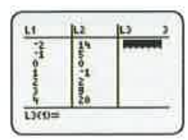
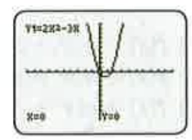
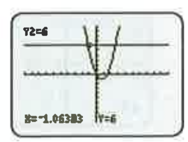
4. a) no real roots **b)** one root
c) two roots **d)** two roots

5. a) $\frac{-3 - \sqrt{57}}{12}, \frac{-3 + \sqrt{57}}{12}$
b) $6 - 2\sqrt{3}, 6 + 2\sqrt{3}$
c) $-2, \frac{14}{3}$ **d)** 4

6. a) two distinct real roots
b) two equal real roots
c) no real roots
d) two distinct real roots

7. Answers may vary. Sample answers:
a) factor; $-\frac{3}{2}, 4$ **b)** factor; ± 5
c) quadratic formula, complete the square, graphing calculator; $\frac{-3 \pm \sqrt{17}}{4}$
d) factor; $-8, 0$
e) quadratic formula, complete the square, graphing calculator; no solution
f) factor; 2
g) quadratic formula, graphing calculator; $\frac{3.7 \pm \sqrt{19.39}}{1.14}$
h) factor; $\frac{4}{3}$

8. a) $k = -6, k = 6$ **b)** $k < -6, k > 6$

9. a)

b)

c)

d) $-1.1, 2.6$ **e)** $\frac{3 - \sqrt{57}}{4}, \frac{3 + \sqrt{57}}{4}$

10. Answers may vary.

11. a) $-8, 8, -7, 7, -13, 13$ **b)** $-7, 7, -2, 2$
c) Answers may vary. Sample answer: if k can be factored as $k = ab$, where $a + b = -3$, then the equation can be solved
$k = -2, 0, 4, 10, 18, 24, 28, 40, 54, 70, 88, 108, \ldots$

12. 4.1 s

13. a) 64.0 km/h **b)** 95.2 km/h **c)** 115.5 km/h

14. a) 10 m **b)** 8.7 m

15. width: 3.6 m; length: 5.6 m

16. 23.0 m

17. 6 m

18. 1413 km

19. a) Yes. The stock will reach this volume at 16.3 weeks and 33.7 weeks.

b) No. The stock will not reach a volume of 400 000 shares.

20. a) $x = -26$, $x = -24$; two distinct real roots

b) $x = -25$, $x = -25$; one double real root

c) no real roots

22. Answers may vary.

23. edge length: 10.1 cm; volume: 1030.3 cm^3

24. a) 3.7 m **b)** 20.6 m^2

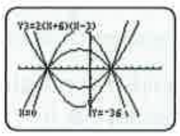
25. B

26. D

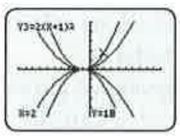
27. $\text{Area}_{\text{KROQ}} = \frac{1}{4}\,\text{Area}_{\text{MNOP}}$

1.6 Determine a Quadratic Equation Given Its Roots, pages 57–59

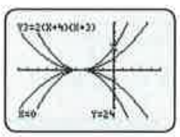
1. a) $f(x) = a(x + 6)(x - 3)$



b) $f(x) = a(x + 1)(x + 1)$



c) $f(x) = a(x + 4)(x + 3)$

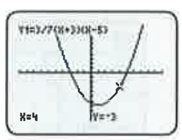


2. a) $f(x) = a(x^2 + 3x - 18)$

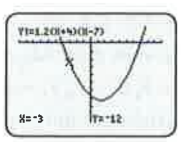
b) $f(x) = a(x^2 + 2x + 1)$

c) $f(x) = a(x^2 + 7x + 12)$

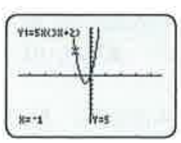
3. a) $f(x) = \frac{3}{7}(x + 3)(x - 5)$



b) $f(x) = 1.2(x + 4)(x - 7)$



c) $f(x) = 5x(3x + 2)$



4. a) $f(x) = \frac{3}{7}x^2 - \frac{6}{7}x - \frac{45}{7}$

b) $f(x) = \frac{6}{5}x^2 - \frac{18}{5}x - \frac{168}{5}$

c) $f(x) = 15x^2 + 10x$

5. a) $f(x) = 3x^2 - 6x - 30$ **b)** $f(x) = x^2 + 4x - 3$

c) $f(x) = -2x^2 - 20x - 46$

6. a) $f(x) = \frac{1}{2}(x - 1)^2 - 2$ **b)** $f(x) = -2\left(x + \frac{1}{2}\right)^2 + \frac{9}{2}$

c) $f(x) = \frac{1}{3}(x - 1)^2 - \frac{16}{3}$

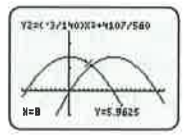
7. a) $f(x) = -\frac{3}{140}x^2 + \frac{4107}{560}$

b) 7.3 m **c)** 18.5 m

d) $f(x) = -\frac{3}{140}x^2 + \frac{111}{140}x$

e) The graphs in parts a) and d) are congruent. The graph of $f(x) = -\frac{3}{140}x^2 + \frac{111}{140}x$ is the graph of $f(x) = -\frac{3}{140}x^2 + \frac{4107}{560}$ translated 18.5 units to the right.

f)



8. a) $f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$

b) $f(x) = 3x^2 + 15x - 18$

c) $f(x) = -4x^2 - 8x + 60$

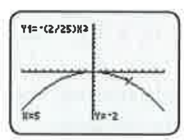
9. Graphs may vary.

10. Answers may vary.

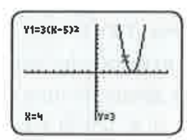
11. a) $f(x) = -\frac{2}{25}x^2$ **b)** $f(x) = 3(x - 5)^2$

c) $f(x) = \frac{2}{3}(x + 1)^2$

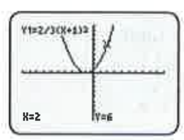
12. a)



b)



c)



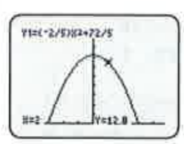
13. $c = \frac{25}{4a}$

14. a) $f(x) = -2x^2 + 28x - 90$

b) $f(x) = -2x^2 + 28x - 68$

c) for $x = 6$, $y = 28$; for $x = 7$, $y = 30$

15. a)



b) $f(x) = -\frac{2}{5x^2} + \frac{72}{5}$

c) 14.4 m

16. a) $f(x) = -\frac{2}{5}x^2 + \frac{24}{5}x$

b) 14.4 m; the maximum heights are the same.

17. Answers may vary. Sample answer: The graph of equation developed for the arch in question 16 is congruent to the one for the arch in question 15. The graph of the arch in question 16 is the graph of the arch in question 15 translated horizontally 6 units to the right.

19. a) Yes. Answers may vary. Sample answer: The equation of a quadratic function with an x-intercept of 5 and vertex (3, 2) can be found by substituting $x = 5$, $y = 0$, $p = 3$, and $q = 2$ in $y = a(x - p)^2 + q$ and solving the resulting equation to find the value of $a = -\frac{1}{2}$. The defining equation is $f(x) = -\frac{1}{2}(x - 3)^2 + 2$.

b) Yes. Answers may vary. Sample answer: The equation of a quadratic function with a vertex (2, 4) that passes through the point (5, 7) can be found by substituting $x = 5$, $y = 7$, $p = 2$, and $q = 4$ in $y = a(x - p)^2 + q$ and solving the resulting equation to find the value of $a = \frac{1}{3}$. The defining equation is $f(x) = \frac{1}{3}(x - 2)^2 + 4$.

c) Yes. Substitute and solve three equations of the form $y = ax^2 + bx + c$ for a, b, and c.

20. Answers may vary. Sample answer: $f(x) = x^2 + \frac{2}{3}x - \frac{2}{3}$.

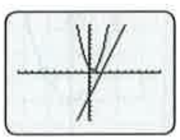
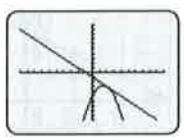
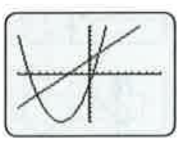
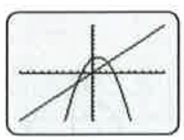
22. Answers may vary.

1.7 Solve Linear-Quadratic Systems, pages 67–69

1. a) (4, 3), (5, 5) **b)** (2, 17), (6, 49)
c) (−4, 13), (2, −5) **d)** (−4, 6), (1, 1)

2. Answers may vary.

3. a) The functions do not intersect.
b) The functions intersect once.
c) The functions intersect twice.
d) The functions intersect twice.

4. a)

b)

c)

d)


5. a) 6 **b)** −4 **c)** $-\frac{3}{2}$ **d)** $\frac{2}{3}$

6. Answers may vary.

7. (−11, 265), (3, 55)

8. Yes, at the points (117, −24 524) and (−262, −214 024).

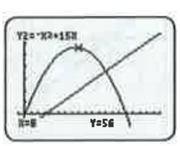
9. Answers may vary.

10. a) $k > -6$ **b)** $k = -6$ **c)** $k < -6$

11. a) $k > -\frac{1}{2}$ **b)** $k = -\frac{1}{2}$ **c)** $k < -\frac{1}{2}$

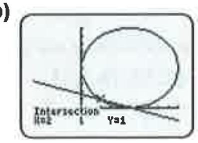
12. 49

13. Answers may vary. Sample answer: The vertical line $x = 2$ intersects the graph of $y = x^2 - 9$ so that part of the line is above the graph of $y = x^2 - 9$ and part of the line is below it.

14. a)

b) (12, 36)
c) The fence should be located 37.1 m up the hill.

15. 7.5 s

16. $11{:}00 < t < 15{:}00$, or between 11 a.m. and 3 p.m.

18. a) (2, 1), (5, 0) **b)**


19. (5, −7), (6, 0)

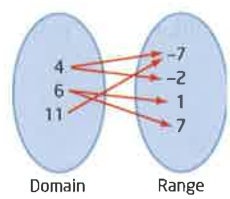
20. B

Chapter 1 Review, pages 70–71

1. a) domain $\{x \in \mathbb{R}, -2 \le x \le 2\}$, range $\{y \in \mathbb{R}, -3 \le y \le 3\}$
b) domain {−2, 3, 5, 11}, range {1, 2, 3, 7}
c) domain {1, 2, 3, 4, 5}, range {4, 6, 10, 18, 29}
d) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \ge 11\}$

2. a) not a function **b)** not a function
c) Answers may vary. Sample anwer: This relation is a function because for each value in the domain there is exactly one value in the range.
d) Answers may vary. Sample anwer: This relation is a function because for each value in the domain there is exactly one value in the range.

3. a) $y = 4x - 3$
b) Answers may vary. Sample anwer: It is possible for a second function to exist that will generate these values, but the function will not be linear.

4. a)


b) Answers may vary. Sample anwer: This relation is not a function. The x-values 4 and 6 have two corresponding y-values.

5. 160

6. a) 10 A **b)** 500 W

7. a) $-70 + 8\sqrt{3}$ **b)** $-9 - 15\sqrt{6}$
c) 22 **d)** $-12 + 43\sqrt{2}$

8. a) $5\sqrt{6} - 2$ square units **b)** $\frac{27\pi}{2}$ square units

9. a) $\frac{1 - \sqrt{7}}{3}, \frac{1 + \sqrt{7}}{3}$ **b)** $\frac{4}{3}, \frac{5}{2}$

10. a) two distinct real roots
b) two distinct real roots
c) one real root

11. Answers may vary. Sample answer: Jessica is not correct. Since $\sqrt{2} + \sqrt{2} = 2\sqrt{2} \doteq 2.8$ and $\sqrt{2} \times \sqrt{2} = 2$, then $\sqrt{2} + \sqrt{2} \ne \sqrt{2} \times \sqrt{2}$.

12. a) $f(x) = -\frac{1}{2}x^2 + \frac{3}{2}x + 5$
b) $f(x) = -5x^2 - 20x + 5$

13. a) $f(x) = -\frac{1}{40}x^2 + \frac{53}{40}x$ **b)** 17.6 m

c) Answers may vary. Sample answer: Yes. The second quadratic function is $f(x) = -\frac{1}{40}x^2 + \frac{2809}{40}$.

14. Answers may vary.

15. a) (2, 6), (3, 11) **b)** (−1, −2), (7, −26)

16. $b = -4$

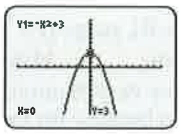
17. Answers may vary. Sample answer: No. The line $y = 2x + 1$ and the parabola $y = x^2 - 3$ do not intersect.

Chapter 1 Practice Test, pages 72–73

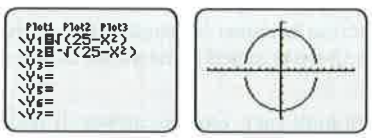
1. a) false **b)** true **c)** true
d) false **e)** false

2. A **3.** B **4.** C **5.** B **6.** B

7. a) $y = -x^2 + 3$

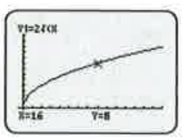


b) $x^2 + y^2 = 25$



8. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$
b) domain {0, 1, 2, 3}, range {−21, −12, −8, −5}

9. a) domain $\{\ell \in \mathbb{R}, \ell \geq 0\}$, range $\{T \in \mathbb{R}, T \geq 0\}$;
b)



c) Answers may vary. Sample answer: Yes. For each value of ℓ there is exactly one value for T.

10. {(1, 3), (3, 2), (4, 2), (4, 3), (7, 2), (7, 3)}. Answers may vary. Sample answer: No. This relation is not a function. The x-value and 7 have two corresponding y-values.

11. a) (4, 11)
b) maximum; the parabola opens downward
c) two x-intercepts

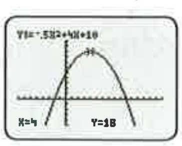
12. a) 3.75 m by 5 m **b)** 18.75 m²

13. a) $R(x) = -10(x - 10)^2 + 16\,000$
b) \$40 **c)** \$16 000

14. a) $-18 + 6\sqrt{6}$ **b)** $2 - x^2$

15. $\sqrt{x} + \sqrt{x} = \sqrt{x} \times \sqrt{x}$ is true for $x = 4$.
$\sqrt{4} + \sqrt{4} = 2 + 2 = 4$ and
$\sqrt{4} \times \sqrt{4} = 2 \times 2 = 4$.

16. a) −2, 10
b) Answers may vary. Sample answer: (4, 18)
c)



17. 5 m by 13 m

18. $f(x) = 8x^2 + 80x + 176$

19. a) $f(x) = -\frac{2}{5}x^2 + \frac{8}{5}x + \frac{24}{5}$ **b)** 6.4

20. (−3, −16), (6, 2)

21. a) Answers may vary.
b) Answers may vary. Sample answer: The equation needs to be changed from $f(x) = 3x^2 - 4$ to $f(x) = 3x^2 - 12$.
c) $f(x) = a\left(x^2 - \frac{7}{5}\right)$

22. Answers may vary. Sample answer: No. The quadratic function that models the path of the baseball does not intersect the linear function that models the profile of the bleachers in the first quadrant. The ball lands on the ground before it reaches the bleachers.

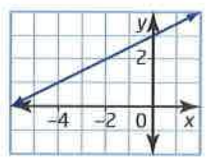
Chapter 2

Prerequisite Skills, pages 76–77

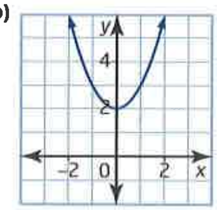
1. a) up; vertically stretched by a factor of 3 because $a = 3$
b) down; vertically compressed by a factor of 0.5 because $a = -0.5$
c) up; vertically compressed by a factor of 0.1 because $a = 0.1$
d) up; vertically compressed by a factor of $\frac{3}{5}$ because $a = \frac{3}{5}$

2. a) (5, 10) **b)** (−6, 20)
c) (1, −5) **d)** (−3, −4)

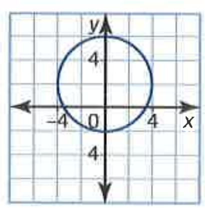
3. a)



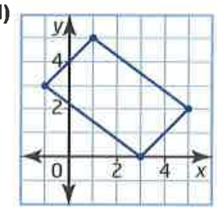
b)



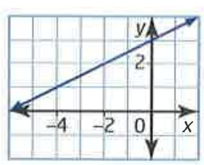
c)



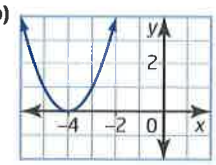
d)



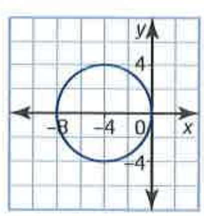
4. a)



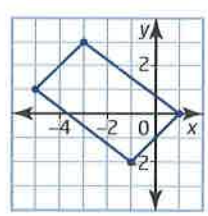
b)

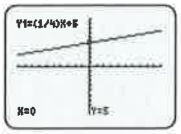
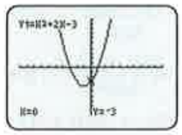


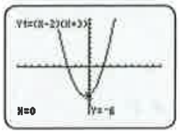
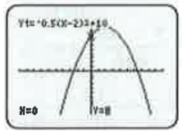
c)



d)



5. a)  **b)** 

c)  **d)** 

6. a) $15x^2 - 24x$ **b)** $60x^3y - 120x^2y^2$
c) $-12x^3 - 39x^2 + 21x$ **d)** $6x^2 - 7x - 20$
e) $16x^2 - 25$ **f)** $52x^2 - 39x$

7. a) 8 **b)** 3 **c)** 12
d) $4xy^2$ **e)** $x^2 + 2x$ **f)** $x + 4$

8. a) $5x(3x + 2)$ **b)** $-5x^2(7x + 9)$
c) $6y^3(3x^2 - 6x + 1)$ **d)** $-5x^2(x^3 + 20x^2 + 6)$
e) $2(2x + 5)(2x - 5)$ **f)** $(6 - 11x)(x - 1)$

9. a) $(x + 2)(x + 3)$ **b)** $(x + 2)(x - 6)$
c) $(x + 9)(x - 3)$ **d)** $(x - 7)(x - 7)$
e) $(x + 8)(x - 8)$ **f)** $3(x + 5)(x - 8)$
g) $2(x + 5)(x + 5)$ **h)** $4(x + 8)(x - 8)$

10. a) $(2x + 3)(x - 5)$ **b)** $(3x + 4)(3x + 4)$
c) $2(3x + 1)(2x - 1)$ **d)** $2(3x + 1)(3x - 10)$
e) $2(2x - 3)(x + 4)$ **f)** $(5x + 8)(4x + 3)$

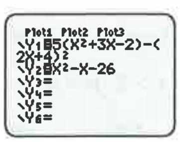
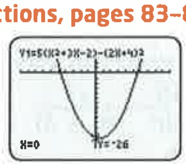
11. a) 630 **b)** $12x^2y^2$
c) $(x + 8)(x - 5)(x - 6)$

12. a) $\frac{17}{15}$ **b)** $\frac{23}{36}$
c) $\frac{2x + 3y}{12}$ **d)** $\frac{16x - 9y}{24}$

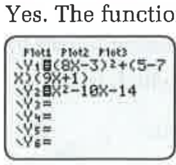
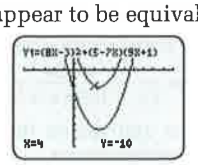
13. a) $-\frac{1}{8}$ **b)** $\frac{16}{5}$ **c)** $\frac{128}{27}$ **d)** 10

14. a) $r = \pm\sqrt{\frac{A}{\pi}}$ **b)** $w = \frac{P - 2\ell}{2}$
c) $y = \pm\sqrt{16 - x^2}$ **d)** $y = \pm\sqrt{y + 20}$
e) $x = \pm\sqrt{y^2 + 5}$ **f)** $h = \frac{A - 2\pi r^2}{2\pi r}$

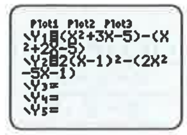
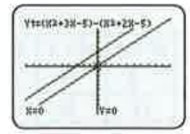
2.1 Transformations of Functions, pages 83–85

1. a)  

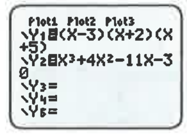
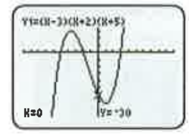
Yes. The functions appear to be equivalent.

b)  

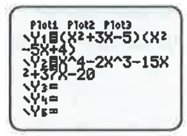
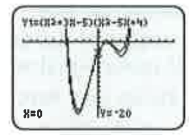
No. The functions do not appear to be equivalent.

c)  

No. The functions do not appear to be equivalent.

d)  

Yes. The functions appear to be equivalent.

e)  

No. The functions do not appear to be equivalent.

2. a) Answers may vary.
b) $f(0) = 14$; $g(0) = -14$ **c)** $f(0) = 0$; $g(0) = 3$
d) Answers may vary. **e)** $f(3) = -26$; $g(3) = -17$

3. a) $x \neq 3$ **b)** $x \neq -2$

4. a) $x \geq 1, x \neq 5$ **b)** $x \neq -3, x \neq -2$

5. a) Yes; $x \neq -6$ **b)** No; $f(x) = \frac{x + 4}{x - 4}, x \neq 4$
c) No; $f(x) = x + 1, x \neq -5$
d) Yes; $x \neq -8, x \neq 6$ **e)** Yes; $x \neq 0, x \neq \frac{2}{3}$
f) No; $f(x) = x - 5, x \neq -\frac{2}{5}$

6. a) $\frac{1}{x - 5}, x \neq 5, x \neq 8$ **b)** $3(x - 7), x \neq 7, x \neq 10$
c) $\frac{x + 3}{x + 7}, x \neq -7, x \neq 6$ **d)** $\frac{x + 9}{x + 5}, x \neq -5, x \neq 2$
e) $\frac{1}{x - 2}, x \neq -8, x \neq 2$
f) $\frac{5x + 4}{2(x + 3)}, x \neq -3, x \neq \frac{2}{5}$

7. a) 32, 33, 34, 37, 44
b) -4, undefined, 0, 6, 20; the expression cannot be evaluated for $x = -1$ since it simplifies to $2x, x \neq -5, x \neq -1$.

8. a) $A = \pi r^2 - 9\pi$
b) domain $\{r \in \mathbb{R}, r > 3\}$, range $\{A \in \mathbb{R}, A > 0\}$

9. a) $V(x) = (2x + 0.5)(x - 0.5)(x + 0.5)$
b) $SA(x) = 2(2x + 0.5)(x - 0.5)$
$+ 2(2x + 0.5)(x + 0.5) + 2(x - 0.5)(x + 0.5)$
c) 0.625 m³, 6.625 m²; 1.875 m³, 11.5 m²; 7 m³, 25 m²
d) $V(x)$: domain $\{x \in \mathbb{R}, x > 0.5\}$, range $\{V \in \mathbb{R}, V > 0\}$;
$SA(x)$: domain $\{x \in \mathbb{R}, x > 0.5\}$, range $\{SA \in \mathbb{R}, SA > 3\}$

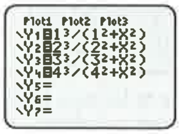
10. a) $f(x) = (18 + x)(12 - x)$
b) $(18 + x)(12 - x) = 216 - 6x - x^2$
c) 15 green lights per hour

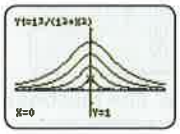
11. a)

x	x^2+1	x^2-1	$2x$	$(x^2+1)^2 = (x^2-1)^2 + (2x)^2$
2	5	3	4	$5^2 = 3^2 + 4^2$
3	10	8	6	$10^2 = 8^2 + 6^2$
4	17	15	8	$17^2 = 15^2 + 8^2$

b) hypotenuse is $x^2 + 1$; in all three cases, this side is the longest side

c) Answers may vary.

12. a) 



b) The value of the expression in the denominator will never equal zero.

c) Answers may vary.

13. The simplified form of the function is $f(x) = 2x + 3$, $x \neq -6$, $x \neq 2$. The graph is the line $y = 2x + 3$ with holes at $(-6, -9)$ and $(2, 7)$.

14. $A(x) = 100 - x^2$; domain $\{x \in \mathbb{R}, \sqrt{50} < x < 10\}$, range $\{A \in \mathbb{R}, 0 < A < 50\}$

15. division by zero in the last line of the proof: $a^2 - ab = a(a - b)$ which is zero because $a = b$

16. $(2, 0)$

2.2 Skills You Need: Operations With Rational Expressions, pages 94–96

1. a) $\frac{22y^2}{x^2}$, $x \neq 0$ **b)** $25x^6$, $x \neq 0$

c) $\frac{5b}{2}$, $b \neq 0$ **d)** $\frac{1}{b}$, $a \neq 0, b \neq 0$

2. a) $2y$, $x \neq 0, y \neq 0$ **b)** $330x^3$, $x \neq 0, y \neq 0$

c) $\frac{2b^2}{a^4}$, $a \neq 0, b \neq 0$ **d)** $8ac^2$, $a \neq 0, b \neq 0, c \neq 0$

3. a) 5, $x \neq -10$ **b)** 2, $x \neq 0, x \neq 1$

c) $\frac{x+5}{x+7}$, $x \neq -7, x \neq 3$ **d)** 1, $x \neq -8, x \neq -\frac{3}{2}$

4. a) $\frac{x}{3(x+10)}$, $x \neq -10, x \neq -\frac{3}{2}, x \neq 0$

b) $\frac{16x}{x+8}$, $x \neq -8, x \neq -6, x \neq 0$

c) 1, $x \neq -7, x \neq -3, x \neq -2$

d) 1, $x \neq -5, x \neq 3, x \neq 6$

5. a) 2, $x \neq -1, x \neq 0$ **b)** x, $x \neq 3$

c) $\frac{x-5}{x+10}$, $x \neq -12, x \neq -10, x \neq 5$

d) 1, $x \neq -3, x \neq 7$

6. a) $\frac{x+15}{4}$, $x \neq -6, x \neq 0$ **b)** $\frac{1}{6}$, $x \neq 0, x \neq 9$

c) $\frac{5x(x+13)}{x-5}$, $x \neq -2, x \neq 0, x \neq 5$

d) $\frac{x+8}{x-8}$, $x \neq -3, x \neq 1, x \neq 8$

7. a) $\frac{7x+3}{90}$, no restrictions

b) $\frac{-x+18}{20}$, no restrictions

c) $\frac{5}{12x}$, $x \neq 0$ **d)** $\frac{37}{24x}$, $x \neq 0$

e) $\frac{12+5a}{4ab}$, $a \neq 0, b \neq 0$

f) $\frac{26b+55a^2}{20a^2b^2}$, $a \neq 0, b \neq 0$

g) $\frac{6b+3ab+4a-a^2}{3a^2b^2}$, $a \neq 0, b \neq 0$

h) $\frac{7-ab}{9ab}$, $a \neq 0, b \neq 0$

8. a) $\frac{12}{(x-6)(x+6)}$, $x \neq -6, x \neq 6$

b) $\frac{15x-84}{(x+8)(x-9)}$, $x \neq -8, x \neq 9$

c) $\frac{23x+22}{(x-6)(x+4)}$, $x \neq -4, x \neq 6$

d) $\frac{2(x+4)(x-1)}{(x+1)(x-2)}$, $x \neq -1, x \neq 2$

9. a) $\frac{3x-2}{(x-1)(x-8)}$, $x \neq 1, x \neq 8$

b) $\frac{x^2+2x-4}{(x+5)(x-2)}$, $x \neq -5, x \neq 2$

c) $\frac{-2x^2+3x+4}{(x+1)(x+2)(x+7)}$, $x \neq -7, x \neq -2, x \neq -1$

d) $\frac{-(x-23)(x-1)}{(x+11)(x-11)(x-3)}$, $x \neq -11, x \neq 3, x \neq 11$

10. a) Total time $= \frac{20x-20}{x(x-2)}$, $x > 2$

b) 2.25 h

11. a) $\frac{2}{x-2}$, $x \neq 2$ **b)** $\frac{x+2}{x-3}$, $x \neq 3$

c) $\frac{-2a+3}{2a-5}$, $a \neq \frac{5}{2}$ **d)** $\frac{b-3}{4b-1}$, $b \neq \frac{1}{4}$

12. a) $V(x) = x(80-2x)(100-2x)$

b) $SA(x) = 2x(80-2x) + (80-2x)(100-2x) + 2x(100-2x)$

c) $\frac{V(x)}{SA(x)} = \frac{x(40-x)(50-x)}{2000-x^2}$

d) $x \neq 20\sqrt{5}$, but in context $x < 40$

13. a) $R_T = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$

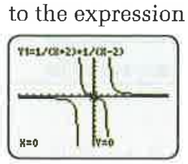
b) Answers may vary. Sample answer: $R_T = \frac{R_1}{3}$

c) Answers may vary. Sample answer: $R_T = \frac{R_1}{9}$

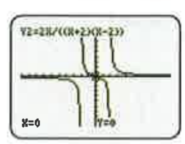
14. a) $\frac{V(r)}{SA(r)} = \frac{rh}{2(r+h)}$ **b)** $r > 0, h > 0$

15. a) $\frac{800}{v}$ **b)** $\frac{400}{v-0.5} + \frac{400}{v+0.5}$

c) No. Simplifying the expression in part b) gives $\frac{800v}{(v-0.5)(v+0.5)}$, which is not equivalent to the expression in part a).

16. a) 

b) $f(x) = \frac{2x}{(x+2)(x-2)}$



c) Answers may vary. Sample answer: The two graphs are the same. The restrictions for both graphs are $x \neq -2$ and $x \neq 2$. Both graphs are discontinuous at $x = -2$ and $x = 2$.

18. a) $\dfrac{V_{\text{sphere}}(r)}{V_{\text{cylinder}}(r)} = \dfrac{2}{3}$ **b)** $\dfrac{SA_{\text{sphere}}(r)}{SA_{\text{cylinder}}(r)} = \dfrac{2}{3}$

c) Answers may vary. Sample answer: The ratios are the same for part a) and part b).

19. $\dfrac{-x^2 - 7x - 7}{(2x + 5)(x + 2)}$, $x \neq -8$, $x \neq -3$, $x \neq -\dfrac{5}{2}$, $x \neq -2$, $x \neq 3$

20. a) 2.718 279 57

b) 2.718 281 828... The answer for part a) is equal to e^1 up to four decimal places.

c) The result gets closer and closer to the value of e.

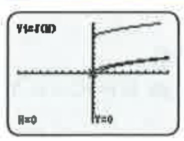
$$\ddots$$
$$1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{8}}}}$$

21. B **22.** A **23.** −34 **24.** A

2.3 Horizontal and Vertical Translations of Functions, pages 102–104

1. a)

x	$f(x) = \sqrt{x}$	$r(x) = f(x) + 7$	$s(x) = f(x - 1)$
0	0	7	no value
1	1	8	0
4	2	9	$\sqrt{3}$
9	3	10	$\sqrt{8}$

b) 

c) Answers may vary. Sample answer: Since $r(x)$ is a vertical translation of 7 units up of $f(x)$, the y-coordinate of each point of $r(x)$ is 7 more than the corresponding y-coordinate of $f(x)$. Since $s(x)$ is a horizontal translation of 1 unit to the right of $f(x)$, the x-coordinate of each point of $s(x)$ is 1 more than the corresponding x-coordinate of $f(x)$.

2. a) A'(−4, 7), B'(−2, 7), C'(−1, 3), D'(1, 3), E'(2, 4), F'(4, 4)

b) A'(−4, −5), B'(−2, −5), C'(−1, −9), D'(1, −9), E'(2, −8), F'(4, −8)

c) A'(4, 2), B'(6, 2), C'(7, −2), D'(9, −2), E'(10, −1), F'(12, −1)

d) A'(−10, 2), B'(−8, 2), C'(−7, −2), D'(−5, −2), E'(−4, −1), F'(−2, −1)

3. a) A'(−1, 8), B'(1, 8), C'(2, 4), D'(4, 4), E'(5, 5), F'(7, 5)

b) A'(−2, −8), B'(0, −8), C'(1, −12), D'(3, −12), E'(4, −11), F'(6, −11)

c) A'(−9, 6), B'(−7, 6), C'(−6, 2), D'(−4, 2), E'(−3, 3), F'(−1, 3)

d) A'(−16, −1), B'(−14, −1), C'(−13, −5), D'(−11, −5), E'(−10, −4), F'(−8, −4)

4. a) A'(−4, 1), B'(−3, 3), C'(−1, 1), D'(0, 3)

b) A'(−4, −8), B'(−3, −6), C'(−1, −8), D'(0, −6)

c) A'(0, −2), B'(1, 0), C'(3, −2), D'(4, 0)

d) A'(−11, −2), B'(−10, 0), C'(−8, −2), D'(−7, 0)

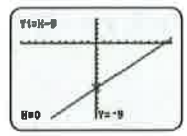
5. a) A'(−2, 8), B'(−1, 10), C'(1, 8), D'(2, 10)

b) A'(1, −11), B'(2, −9), C'(4, −11), D'(5, −9)

c) A'(−12, 7), B'(−11, 9), C'(−9, 7), D'(−8, 9)

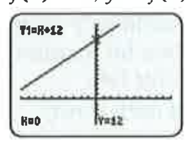
d) A'(−5, −13), B'(−4, −11), C'(−2, −13), D'(−1, −11)

6. a) $f(x) = x$; $y = f(x) - 9$; translate 9 units down:



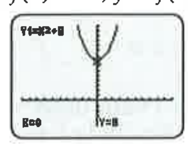
domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) $f(x) = x$; $y = f(x) + 12$; translate 12 units up:



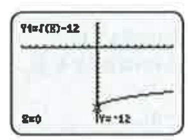
domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

c) $f(x) = x^2$; $y = f(x) + 8$; translate 8 units up;



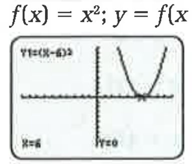
domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 8\}$

d) $f(x) = \sqrt{x}$; $y = f(x) - 12$; translate 12 units down;



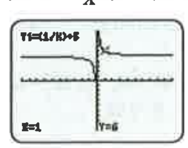
domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq -12\}$

e) $f(x) = x^2$; $y = f(x - 6)$; translate 6 units right;



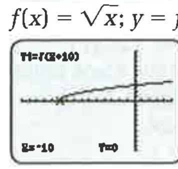
domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

f) $f(x) = \dfrac{1}{x}$; $y = f(x) + 5$; translate 5 units up;



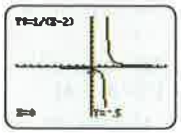
domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 5\}$

g) $f(x) = \sqrt{x}$; $y = f(x + 10)$; translate 10 units left;



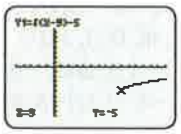
domain $\{x \in \mathbb{R}, x \geq -10\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

h) $f(x) = \frac{1}{x}$; $y = f(x - 2)$; translate 2 units right;



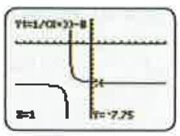
domain $\{x \in \mathbb{R}, x \neq 2\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$

i) $f(x) = \sqrt{x}$; $y = f(x - 9) - 5$; translate 9 units right and 5 units down; domain $\{x \in \mathbb{R}, x \geq 9\}$,



range $\{y \in \mathbb{R}, y \geq -5\}$

j) $f(x) = \frac{1}{x}$; $y = f(x + 3) - 8$; translate 3 units left and 8 units down; domain $\{x \in \mathbb{R}, x \neq -3\}$,



range $\{y \in \mathbb{R}, y \neq -8\}$

7. Answers may vary. Sample answer: The translation for question 6a) could be 9 units down or 9 units right. The translation for question 6b) could be 12 units up or 12 units left.

8. a) translate 9 units left and 3 units down;
$g(x) = f(x + 9) - 3$;
$f(x)$: domain $\{x \in \mathbb{R}, 0 \leq x \leq 8\}$,
range $\{y \in \mathbb{R}, 0 \leq y \leq 4\}$;
$g(x)$: domain $\{x \in \mathbb{R}, -9 \leq x \leq -1\}$,
range $\{y \in \mathbb{R}, -3 \leq y \leq 1\}$

b) translate 4 units right and 3 units down;
$g(x) = f(x - 4) - 3$; $f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq -3\}$

c) translate 4 units left and 2 units up;
$g(x) = f(x + 4) + 2$; $f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \geq -4\}$,
range $\{y \in \mathbb{R}, y \geq 2\}$

d) translate 6 units right and 3 units down;
$g(x) = f(x - 6) - 3$;
$f(x)$: domain $\{x \in \mathbb{R}, 0 \leq x \leq 12\}$,
range $\{y \in \mathbb{R}, -3 \leq y \leq 3\}$;
$g(x)$: domain $\{x \in \mathbb{R}, 6 \leq x \leq 18\}$,
range $\{y \in \mathbb{R}, -6 \leq y \leq 0\}$

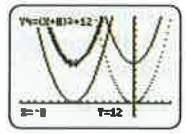
9. a) translate 8 units left and 12 units up

Answers may vary in parts b) and c). Sample answers:

b) A(−1, 1), B(0, 0), C(1, 1); A'(−9, 1), B'(−8, 0), C'(−7, 1); A"(−9, 13), B"(−8, 12), C"(−7, 13)

c) A(−1, 1), B(0, 0), C(1, 1); A'(−1, 13), B'(0, 12), C'(1, 13); A"(−9, 13), B"(−8, 12), C"(−7, 13). The final image points are the same regardless of the order of the translations. So, the order of translations is not important.

d)



e) Answers may vary.

10. Horizontal translations have the same effect as vertical translations when $c = -d$. Answers may vary.

11. a) $n(x) = x - 10$ **b)** $r(x) = x + 11$
c) $s(x) = x - 1$ **d)** $t(x) = x - 7$

12. a) $n(x) = (x - 4)^2 - 6$ **b)** $r(x) = (x + 2)^2 + 9$
c) $s(x) = (x + 6)^2 - 7$ **d)** $t(x) = (x - 11)^2 + 4$

13. a) $g(x) = \sqrt{x - 4} - 6$ **b)** $g(x) = \sqrt{x + 2} + 9$
c) $g(x) = \sqrt{x + 6} - 7$ **d)** $g(x) = \sqrt{x - 11} + 4$

14. a) $n(x) = \frac{1}{x - 4} - 6, x \neq 4$

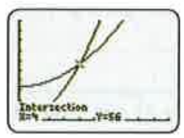
b) $r(x) = \frac{1}{x + 2} + 9, x \neq -2$

c) $s(x) = \frac{1}{x + 6} - 7, x \neq -6$

d) $t(x) = \frac{1}{x - 11} + 4, x \neq 11$

15. a) The hybrid electric car would have to be given a head start of 33.6 m.

b)



16. a) domain $\{x \in \mathbb{N}, x \geq 0\}$; the number of units of the product produced and sold is greater than or equal to 0 units; range $\{c \in \mathbb{R}, c \geq 500\}$; the cost to produce the units of the product is greater than or equal to \$500.

b) $c(x) = \sqrt{x + 10} + 500$

c) translate 10 units to the left

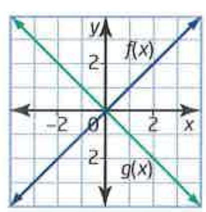
d) domain $\{x \in \mathbb{N}, x \geq 0\}$, range $\{c \in \mathbb{R}, c \geq 503.16\}$

17. Answers may vary.

18. D **19.** D **20.** B

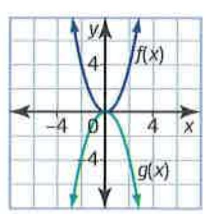
2.4 Reflections of Functions, pages 110–112

1. a)

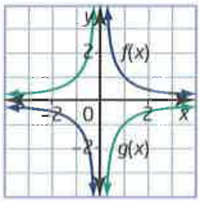


$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}\}$

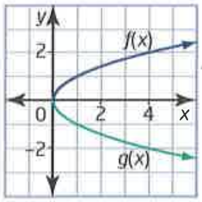
b)



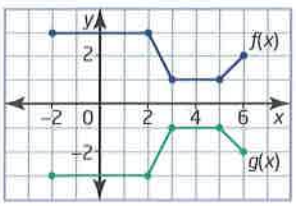
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \leq 0\}$

c) 

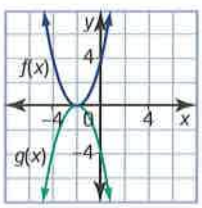
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

d) 

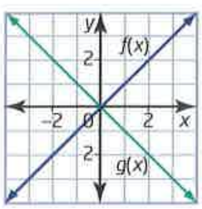
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \leq 0\}$

e) 

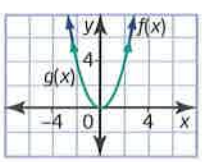
$f(x)$: domain $\{x \in \mathbb{R}, -2 \leq x \leq 6\}$, range $\{y \in \mathbb{R}, 1 \leq y \leq 3\}$; $g(x)$: domain $\{x \in \mathbb{R}, -2 \leq x \leq 6\}$, range $\{y \in \mathbb{R}, -3 \leq y \leq -1\}$

f) 

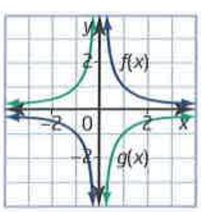
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 0\}$

2. a) 

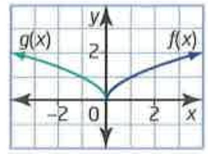
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) 

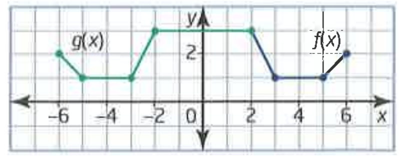
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

c) 

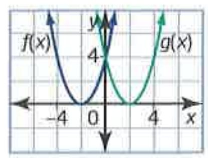
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

d) 

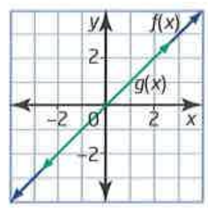
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \leq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

e) 

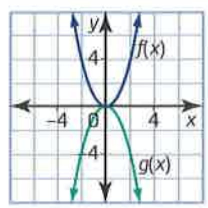
$f(x)$: domain $\{x \in \mathbb{R}, -2 \leq x \leq 6\}$, range $\{y \in \mathbb{R}, 1 \leq y \leq 3\}$; $g(x)$: domain $\{x \in \mathbb{R}, -6 \leq x \leq 2\}$, range $\{y \in \mathbb{R}, 1 \leq y \leq 3\}$

f) 

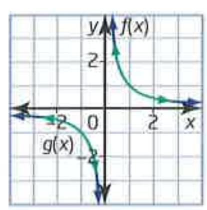
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

3. a) 

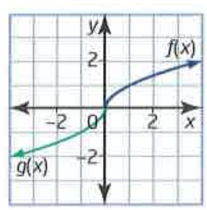
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) 

$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 0\}$

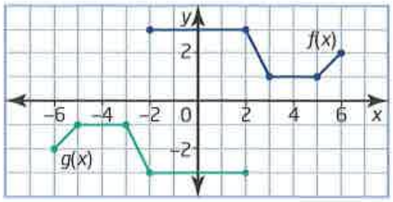
c) 

$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

d) 

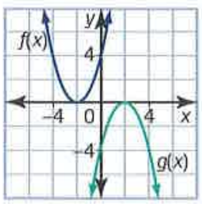
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \leq 0\}$, range $\{y \in \mathbb{R}, y \leq 0\}$

e)



$f(x)$: domain $\{x \in \mathbb{R}, -2 \le x \le 6\}$,
range $\{y \in \mathbb{R}, 1 \le y \le 3\}$;
$g(x)$: domain $\{x \in \mathbb{R}, -6 \le x \le 2\}$,
range $\{y \in \mathbb{R}, -3 \le y \le -1\}$

f)



$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \ge 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \le 0\}$

4. **a)** $g(x) = -\sqrt{x+4} + 4$ **b)** $g(x) = (-x+1)^2 - 4$
c) $g(x) = -(-x-5)^2 - 9$ **d)** $g(x) = -\dfrac{1}{(-x-3)} + 6$
e) $g(x) = -\sqrt{-x-2} + 5$ **f)** $g(x) = -\sqrt{-x+9} + 1$

5. **a)** $g(x) = f(-x)$; reflection in the y-axis
b) $g(x) = -f(x)$; reflection in the x-axis
c) $g(x) = -f(-x)$; reflection in the x-axis followed by a reflection in the y-axis

6. If a set of axes is superimposed with the origin at the horizontal and vertical centre of the logo, "az" is a reflection in the x-axis followed by a reflection in the y-axis of "Te."

7. Answers may vary. Sample answer: A reflection in the x-axis leaves the domain unchanged, but the range may change. A reflection in the y-axis leaves the range unchanged, but the domain may change.

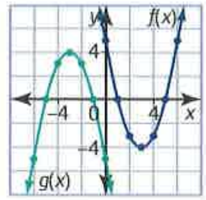
8. **a)** **i)** $(-1, 0)$ and $(5, 0)$ **ii)** $(0, -5)$
b) Answers may vary. Sample answer: $f(x) = x^3$

9. **a)** Since $g(x) = f(-x)$, $g(x)$ is a reflection of $f(x)$ in the y-axis.
b) Since $g(x) = f(-x)$, $g(x)$ is a reflection of $f(x)$ in the y-axis.
c) Since $g(x) = -f(x)$, $g(x)$ is a reflection of $f(x)$ in the x-axis.
d) Since $g(x) = -f(x)$, $g(x)$ is a reflection of $f(x)$ in the x-axis.
e) Since $g(x) = -f(x)$, $g(x)$ is a reflection of $f(x)$ in the x-axis. Since $g(x) \ne f(-x)$, $g(x)$ is not a reflection of $f(x)$ in the y-axis. Since $g(x) \ne -f(-x)$, $g(x)$ is not a reflection of $f(x)$ in both axes.
f) Since $g(x) = f(-x)$, $g(x)$ is a reflection of $f(x)$ in the y-axis.

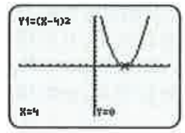
10. Answers may vary.

11. **a)** Answers may vary. Sample answer: The x-value and the y-value in each ordered pair of the function is multiplied by -1 to give the coordinates of each ordered pair of the function that is reflected in the origin.

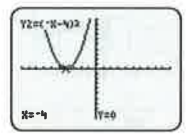
b)



12. **a)**

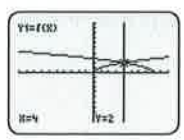


b) $g(x) = (-x-4)^2$

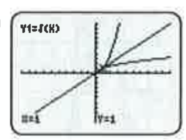


c) Translate $f(x)$ to the left 8 units.
d) The result of a translation of 8 units to the left of $f(x) = (x-4)^2$ is $h(x) = (x+4)^2$.
$g(x) = (-x-4)^2$
$= [-1(x+4)]^2$
$= (-1)^2(x+4)^2$
$= (x+4)^2$
e) Answers may vary. Sample answer: No, it changes the orientation.
f) Answers may vary. Sample answer: Yes, it works for any function that is symmetrical about a vertical line.

14. **a)**



b)



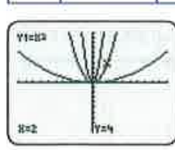
15. 0
16. D
17. Answers may vary.
18. domain $\{x \in \mathbb{R}, x \ne -2, x \ne 3\}$, range $\{y \in \mathbb{R}\}$
19. C

2.5 Stretches of Functions, pages 119–122

1. **a)**

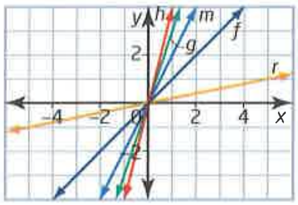
x	$f(x) = x^2$	$g(x) = 5f(x)$	$h(x) = f\left(\frac{1}{4}x\right)$
0	0	0	0
2	4	20	$\frac{1}{4}$
4	16	80	1
6	36	180	$\frac{9}{4}$

b)

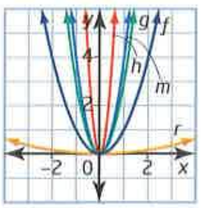


c) Answers may vary. Sample answer: The graph of $g(x)$ is the graph of $f(x)$ stretched vertically by a factor of 5. Each y-value of $g(x)$ is five times as far from the x-axis as the corresponding y-value of $f(x)$. The graph of $h(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 4. Each x-value of $h(x)$ is four times as far from the y-axis as the corresponding x-value of $f(x)$.

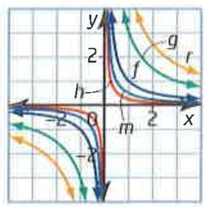
2. a)



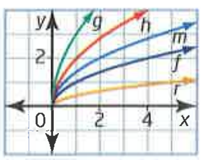
b)



c)



d)



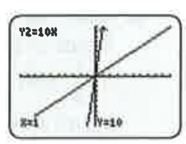
3. a) $a = 10$; the graph of $g(x)$ is a vertical stretch by a factor of 10 of the graph of $f(x)$.

b) $k = 9$; the graph of $g(x)$ is a horizontal compression by a factor of $\frac{1}{9}$ of the graph of $f(x)$.

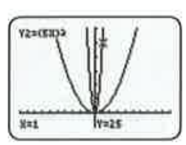
c) $a = \frac{1}{5}$; the graph of $g(x)$ is a vertical compression by a factor of $\frac{1}{5}$ of the graph of $f(x)$.

d) $k = \frac{1}{20}$; the graph of $g(x)$ is a horizontal stretch by a factor of 20 of the graph of $f(x)$.

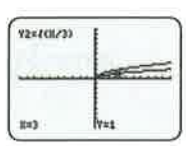
4. a) $f(x) = x$; $a = 10$, vertical stretch by a factor of 10



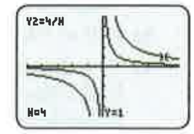
b) $f(x) = x^2$; $k = 5$, horizontal compression by a factor of $\frac{1}{5}$



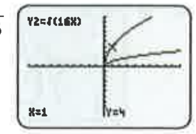
c) $f(x) = \sqrt{x}$; $k = \frac{1}{3}$, horizontal stretch by a factor of 3



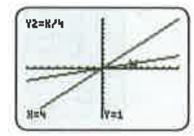
d) $f(x) = \frac{1}{x}$; $a = 4$, vertical stretch by a factor of 4



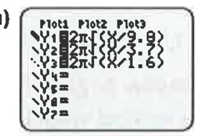
e) $f(x) = \sqrt{x}$; $k = 16$, horizontal compression by a factor of $\frac{1}{16}$

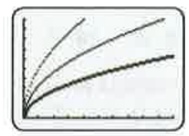


f) $f(x) = x$; $a = \frac{1}{4}$, vertical compression by a factor of $\frac{1}{4}$

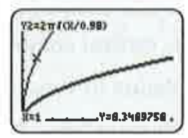


5. a)





b)



6. a) vertical stretch by a factor of 3

b) vertical stretch by a factor of 5

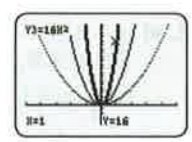
7. Answers may vary. Sample answers:

a) A(−2, 4), B(0, 0), C(2, 4)

b) A'(−2, 16), B'(0, 0), C'(2, 16); A"(−1, 16), B"(0, 0), C"(1, 16)

c) A'(−1, 4), B'(0, 0), C'(1, 4); A"(−1, 16), B"(0, 0), C"(1, 16)

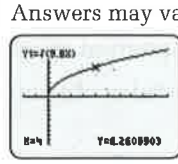
d) The graphs from parts b) and c) are the same.



e) vertical stretch by a factor of 16

f) Answers may vary.

8. a)



b) approximately 4.4 m/s

c) approximately 198.0 m/s

9. a)

b) 13.8 km/h

10. Answers may vary. Sample answer: Each x-value of $g(x)$ is $\frac{1}{k}$ times as far from the y-axis as the corresponding x-value of $f(x)$.

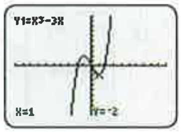
11. a) domain $\{t \in \mathbb{R}, 0 \leq t \leq 2.02\}$, range $\{h \in \mathbb{R}, 0 \leq h \leq 20\}$

b) $h(t) = -6.2t^2 + 20$

c) The domain and range of the function in part b) are the same as the domain and range of the given function.

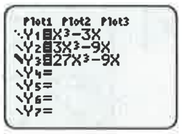
12. Answers may vary.

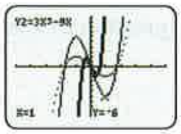
13. a)



b) $g(x) = 3x^3 - 9x$; $h(x) = 27x^3 - 9x$

c)





d) No.

14. B **15.** A **16.** A **17.** (2, 1) **18.** $y = \frac{7}{5}x + 6$

2.6 Combinations of Transformations, pages 129–131

1. a) $a = 4$, $k = 1$, $d = 3$, $c = 0$; vertical stretch by a factor of 4 then translation of 3 units right

b) $a = \frac{1}{3}$, $k = 1$, $d = 0$, $c = 1$; vertical compression by a factor of $\frac{1}{3}$ then translation of 1 unit up

c) $a = 1$, $k = 1$, $d = -5$, $c = 9$; translation 5 units left and 9 units up

d) $a = 1$, $k = \frac{1}{4}$, $d = 0$, $c = 2$; horizontal stretch by a factor of 4 then translation of 2 units up

e) $a = 1$, $k = 5$, $d = 0$, $c = -2$; horizontal compression by a factor of $\frac{1}{5}$ then translation of 2 units down

f) $a = 2$, $k = 1$, $d = 0$, $c = -7$; vertical stretch by a factor of 2 then translation of 7 units down

2. a) $a = 3$, $k = 2$, $d = 0$, $c = -1$; vertical stretch by a factor of 3, horizontal compression by a factor of $\frac{1}{2}$, and then translation of 1 unit down

b) $a = -2$, $k = 1$, $d = 0$, $c = 1$; reflection in the x-axis, vertical stretch by a factor of 2, and then translation of 1 unit up

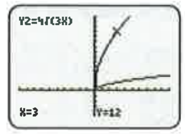
c) $a = \frac{1}{2}$, $k = 1$, $d = 4$, $c = 5$; vertical compression by a factor of $\frac{1}{2}$, then translation of 4 units right and 5 units up

d) $a = 1$, $k = -3$, $d = 0$, $c = 4$; reflection in the y-axis, horizontal compression by a factor of $\frac{1}{3}$, and then translation of 4 units up

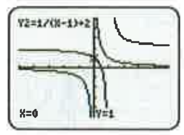
e) $a = -1$, $k = \frac{1}{2}$, $d = 0$, $c = -3$; reflection in the x-axis, horizontal stretch by a factor of 2, and then translation of 3 units down

f) $a = \frac{1}{4}$, $k = 3$, $d = 0$, $c = -6$; vertical compression by a factor of $\frac{1}{4}$, horizontal compression by a factor of $\frac{1}{3}$, and then translation of 6 units down

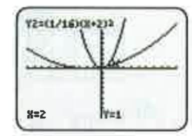
3. a) vertical stretch by a factor of 4 and horizontal compression by a factor of $\frac{1}{3}$; $g(x) = 4\sqrt{3x}$



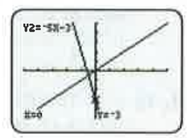
b) translation 1 unit right and 2 units up; $g(x) = \frac{1}{x - 1} + 2$



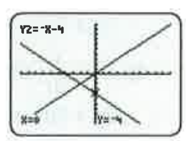
c) horizontal stretch by a factor of 4 then translation of 2 units left; $g(x) = \frac{1}{16}(x + 2)^2$



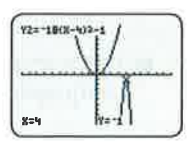
d) reflection in the x-axis, vertical stretch by a factor of 5, and then translation of 3 units down; $g(x) = -5x - 3$



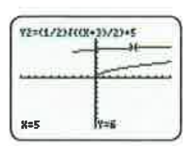
4. a) reflection in the x-axis, vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{2}$, and then translation of 1 unit left and 3 units down; $g(x) = -x - 4$



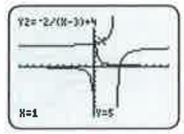
b) reflection in the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{3}$, and then translation of 4 units right and 1 unit down; $g(x) = -18(x - 4)^2 - 1$



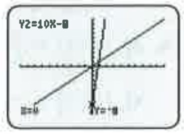
c) vertical compression by a factor of $\frac{1}{2}$, horizontal stretch by a factor of 2, and then translation of 3 units left and 5 units up; $g(x) = \frac{1}{2}\sqrt{\frac{1}{2}(x + 3)} + 5$



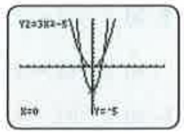
d) vertical stretch by a factor of 2, reflection in the y-axis, and then translation 3 units right and 4 units up;
$g(x) = -\frac{2}{x - 3} + 4$



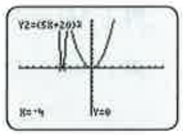
5. a) $f(x) = x$
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$;
$b(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$



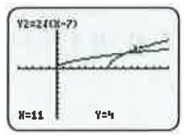
b) $f(x) = x^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$;
$e(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -5\}$



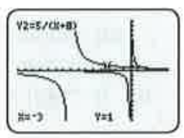
c) $f(x) = x^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$;
$h(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



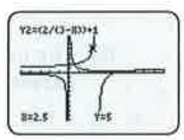
d) $f(x) = \sqrt{x}$;
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$;
$j(x)$: domain $\{x \in \mathbb{R}, x \geq 7\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



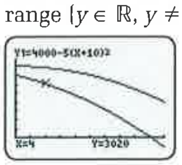
e) $f(x) = \frac{1}{x}$;
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$;
$m(x)$:
domain $\{x \in \mathbb{R}, x \neq -8\}$, range $\{y \in \mathbb{R}, y \neq 0\}$



f) $f(x) = \frac{1}{x}$
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$;
$r(x)$: domain $\{x \in \mathbb{R}, x \neq 3\}$, range $\{y \in \mathbb{R}, y \neq 1\}$



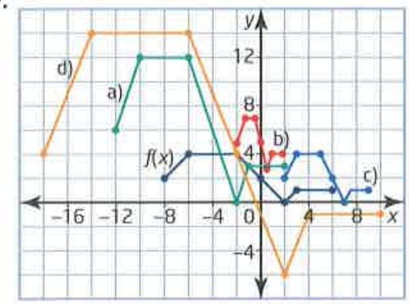
6. a)



b) No. The second skydiver will not catch up to the first skydiver before they have to open their parachutes at 800 m.

c) $g(t)$: domain $\{t \in \mathbb{R}, t \geq -10\}$, range $\{g \in \mathbb{R}, 0 \leq g \leq 4000\}$;
$h(t)$: domain $\{t \in \mathbb{R}, t \geq 0\}$, range $\{h \in \mathbb{R}, 0 \leq h \leq 4000\}$

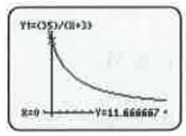
7.



8. a) approximately 121 Hz
b) domain $\{v \in \mathbb{R}, 0 \leq v \leq 40\}$, range $\left\{f \in \mathbb{R}, \frac{83\,000}{93} \leq f \leq \frac{83\,000}{73}\right\}$

9. Answers may vary.

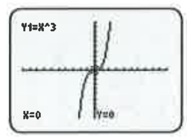
10. a)



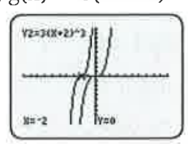
b) \$11 666.67

c) i) \$8750 **ii)** \$7000 **iii)** \$2692.31

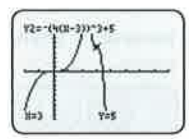
12. a)



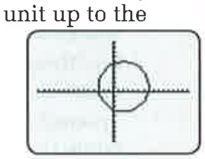
b) i) $g(x) = 3(x + 2)^3$



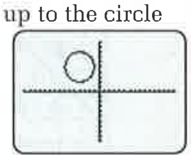
ii) $h(x) = -64(x - 3)^3 + 5$



13. a) Apply a horizontal translation of 2 units right and a vertical translation of 1 unit up to the circle $x^2 + y^2 = 25$.



b) Apply a horizontal translation of 4 units left and a vertical translation of 5 units up to the circle $x^2 + y^2 = 9$.



14. Answers may vary.

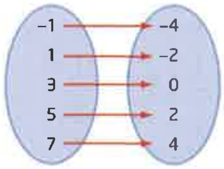
15. C

2.7 Inverse of a Function, pages 138–141

1. a) $\{(5, 1), (2, 4), (-3, 5), (0, 7)\}$;
function: domain $\{1, 4, 5, 7\}$, range $\{-3, 0, 2, 5\}$
inverse: domain $\{-3, 0, 2, 5\}$, range $\{1, 4, 5, 7\}$

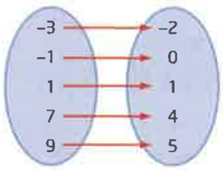
b) $\{(5, 3), (0, 4), (-5, 5), (-10, 6)\}$;
function: domain $\{3, 4, 5, 6\}$, range $\{-10, -5, 0, 5\}$
inverse: domain $\{-10, -5, 0, 5\}$, range $\{3, 4, 5, 6\}$

c)



function: domain {−4, −2, 0, 2, 4},
range {−1, 1, 3, 5, 7}
inverse: domain {−1, 1, 3, 5, 7},
range {−4, −2, 0, 2, 4}

d)

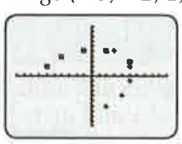


function: domain {−2, 0, 1, 4, 5},
range {−3, −1, 1, 7, 9}
inverse: domain {−3, −1, 1, 7, 9},
range {−2, 0, 1, 4, 5}

2. a)

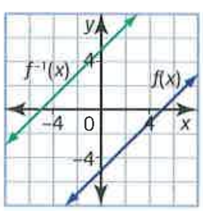
function: domain {−5, −2, 1, 4, 7},
range {−2, 0, 2, 4, 6}
inverse: domain {−2, 0, 2, 4, 6},
range {−5, −2, 1, 4, 7}

b)



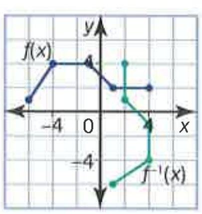
function: domain {−6, −4, −1, 2, 5},
range {2, 3, 4, 5}
inverse: domain {2, 3, 4, 5},
range {−6, −4, −1, 2, 5}

3. a)



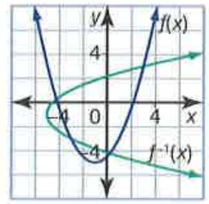
The inverse of $f(x)$ is a function because it passes the vertical line test.

b)



The inverse of $f(x)$ is not a function. It does not pass the vertical line test.

c)



The inverse of $f(x)$ is not a function. It does not pass the vertical line test.

4. a) $f^{-1}(x) = \frac{x}{2}$ **b)** $f^{-1}(x) = \frac{x + 5}{6}$

c) $f^{-1}(x) = -x + 10$ **d)** $f^{-1}(x) = \frac{5x - 4}{2}$

5. a) $f^{-1}(x) = \pm\sqrt{x - 6}$ **b)** $f^{-1}(x) = \pm\sqrt{\frac{x}{4}}$

c) $f^{-1}(x) = \pm\sqrt{x} - 8$ **d)** $f^{-1}(x) = \pm\sqrt{2x - 20}$

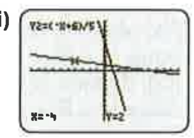
6. a) $f^{-1}(x) = \pm\sqrt{x - 6} - 3$

b) $f^{-1}(x) = \pm\sqrt{-x + 1} + 10$

c) $f^{-1}(x) = \pm\sqrt{\frac{x + 75}{2}} - 6$

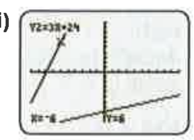
d) $f^{-1}(x) = \pm\sqrt{\frac{x - 8}{-3}} - 6$

7. a) i) $f^{-1}(x) = \frac{-x + 6}{5}$ **ii)**



iii) The inverse of $f(x)$ is a function because it passes the vertical line test.

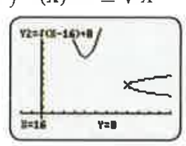
b) i) $f^{-1}(x) = 3x + 24$ **ii)**



iii) The inverse of $f(x)$ is a function because it passes the vertical line test.

c) i) $f^{-1}(x) = \pm\sqrt{x - 16} + 8$

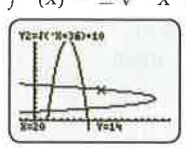
ii)



iii) The inverse of $f(x)$ is not a function because it does not pass the vertical line test. The x-value $x = 20$ has two corresponding y-values.

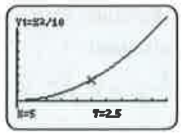
d) i) $f^{-1}(x) = \pm\sqrt{-x + 36} + 10$

ii)

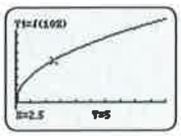


iii) The inverse of $f(x)$ is not a function because it does not pass the vertical line test.

8. a) domain $\{v \in \mathbb{R}, v \geq 0\}$, range $\{d \in \mathbb{R}, d \geq 0\}$

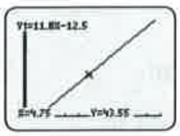


b) $v = \sqrt{10d}$ domain $\{d \in \mathbb{R}, d \geq 0\}$, range $\{v \in \mathbb{R}, v \geq 0\}$; The inverse function represents the muzzle speed, in metres per second, for distances, in metres, travelled by the projectile.



c) Answers may vary.

9. a) approximate domain $\{t \in \mathbb{R}, t \geq 1\}$, range $\{d \in \mathbb{R}, d \geq 0\}$



b) Answers may vary. Sample answer: The first second is pure acceleration.

c) $t = \dfrac{d + 12.5}{11.8}$; domain $= \{d \in \mathbb{R}, d \geq 0\}$; approximate range $= \{t \in \mathbb{R}, t \geq 1\}$

d) Yes. Bolt's coach was close to being correct. The inverse function can be used to determine that Bolt could have run the 100-m race in 9.53 s.

10. Answers may vary.

11. a) $g(x)$ is not the inverse of $f(x)$ since $g(x)$ is not a reflection of $f(x)$ in the line $y = x$.

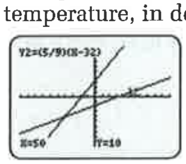
b) $g(x)$ is the inverse of $f(x)$ since $g(x)$ is a reflection of $f(x)$ in the line $y = x$.

c) $g(x)$ is not the inverse of $f(x)$ since $g(x)$ is not a reflection of $f(x)$ in the line $y = x$.

d) $g(x)$ is the inverse of $f(x)$ since $g(x)$ is a reflection of $f(x)$ in the line $y = x$.

12. a) $y = \frac{5}{9}(x - 32)$; x represents the temperature, in degrees Fahrenheit; y represents the temperature, in degrees Celsius

b)



c) $-40°$; Answers may vary. Sample answer: The original and the inverse function intersect at the point $(-40, -40)$.

13. a) $t = 5 - \sqrt{\dfrac{-d + 70}{2.8}}$; the inverse represents the time, t, in seconds, before the brakes are applied, for a given stopping distance.

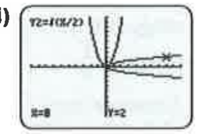
b) function: domain $\{t \in \mathbb{R}, 5 \leq t \leq 10\}$, range $\{d \in \mathbb{R}, 0 \leq d \leq 70\}$
inverse: domain $\{d \in \mathbb{R}, 0 \leq d \leq 70\}$, range $\{t \in \mathbb{R}, 5 \leq t \leq 10\}$

c) Answers may vary. Sample answer: The distance travelled will be less in successive time intervals.

14. Eve's car: $\sqrt{40}$ s;
Byron's car with 1-s head start: $\sqrt{80} - 1$ s;
Byron's car with 2-s head start: $\sqrt{80} - 2$ s;
Byron's car with 3-s head start: $\sqrt{80} - 3$ s;
Byron's car with 4-s head start: $\sqrt{80} - 4$ s

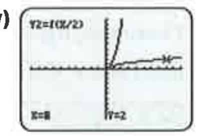
15. a) i) $f^{-1}(x) = \pm\sqrt{\dfrac{x}{2}}$

ii)



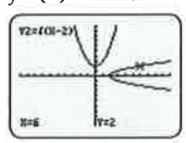
iii) $\{x \in \mathbb{R}, x \geq 0\}$

iv)



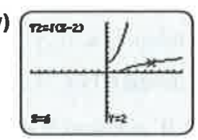
b) i) $f^{-1}(x) = \pm\sqrt{x - 2}$

ii)



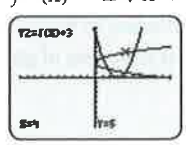
iii) $\{x \in \mathbb{R}, x \geq 0\}$

iv)



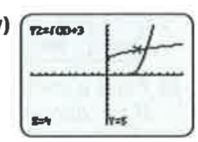
c) i) $f^{-1}(x) = \pm\sqrt{x} + 3$

ii)



iii) $\{x \in \mathbb{R}, x \geq 3\}$

iv)



16. a) $f^{-1}(x) = \dfrac{x - 7}{3}$

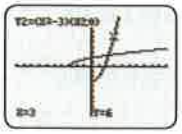
b) $f(f^{-1}) = x$ and $f^{-1}(f) = x$

c) $f^{-1}(x) = \pm\sqrt{x + 6}$; $f(f^{-1}) = x$ and $f^{1}(f) = x$

d) Answers may vary. Sample answer: For a linear or quadratic function, f, and its inverse, f^{-1}, $f(f^{-1}) = x$ and $f^{-1}(f) = x$.

18. a) $f^{-1}(x) = x^2 - 3, x \geq 0$

b) function: domain $\{x \in \mathbb{R}, x \geq -3\}$, range $\{y \in \mathbb{R}, y \geq 0\}$
inverse: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq -3\}$

c) 

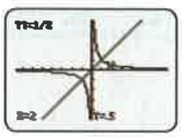
19. a) $r = \sqrt[3]{\frac{3V}{4\pi}}$ the inverse of the equation represents the radius, r, of a sphere in terms of the volume, V, of the sphere.

b) $V = \frac{4}{3}\pi r^3$: domain $\{r \in \mathbb{R}, r \geq 0\}$, range $\{V \in \mathbb{R}, V \geq 0\}$

$r = \sqrt[3]{\frac{3V}{4\pi}}$; domain $\{V \in \mathbb{R}, V \geq 0\}$, range $\{r \in \mathbb{R}, r \geq 0\}$

20. a) $f^{-1}(x) = \frac{1}{x}$

b) Answers may vary. Sample answer: The graph of $f(x)$ is a reflection of itself in the line $y = x$. Therefore, the inverse of $f(x) = \frac{1}{x}$ is the same function: $f^{-1}(x) = \frac{1}{x}$.



21. a) $f^{-1}(x) = \frac{1}{x} + \frac{16}{5}$

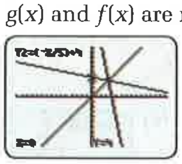
b) $f(x)$: domain $\left\{x \in \mathbb{R}, x \neq \frac{16}{5}\right\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $f^{-1}(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\left\{y \in \mathbb{R}, y \neq \frac{16}{5}\right\}$

22. a) i) $g(x)$ and $f(x)$ are inverses of each other

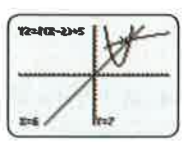
ii) $g(x)$ and $f(x)$ are inverses of each other

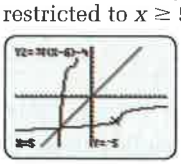
iii) $g(x)$ and $f(x)$ are inverses of each other

iv) $g(x)$ and $f(x)$ are not inverses of each other

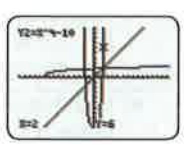
b) i) 

ii) f and g are inverses if the domain of f is restricted to $x \geq 5$.



iii) 

iv) f and g are inverses if the domain of g is restricted to $x \geq 0$.



23. A 24. A 25. C

26. (3, 4) and (4, 3)

Chapter 2 Review, pages 142–143

1. a) equivalent b) not equivalent

2. a) $\frac{1}{x+3}$, $x \neq -7$, $x \neq -3$

b) $x + 8$, $x \neq 8$

3. a) $1600 - 4x^2$ b) $0 < x < 20$

4. a) xy, $x \neq 0$, $y \neq 0$ b) $10a^2b^4$, $a \neq 0$, $b \neq 0$

c) $\frac{2x + 15}{6x^2}$, $x \neq 0$

d) $\frac{x + 2}{(x - 4)(x - 6)}$, $x \neq 4$, $x \neq 6$

5. a) $\frac{x(x + 1)}{3}$, $x \neq -7$, $x \neq -2$

b) 2, $x \neq -10$, $x \neq 2$, $x \neq 6$

c) $\frac{-5x^2 - 22x - 6}{(x + 5)(x + 2)(x - 2)}$, $x \neq -5$, $x \neq -2$, $x \neq 2$

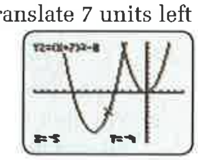
d) $\frac{2x(x + 62)}{(x + 16)(x + 2)(x - 10)}$, $x \neq -16$, $x \neq -2$, $x \neq 10$

6. $\frac{V}{SA} = \frac{x(40 - 2x)}{(40 + 2x)}$, $0 < x < 20$

7. a) A′(0, 6), B′(1, 7), C′(4, 8), D′(9, 9)

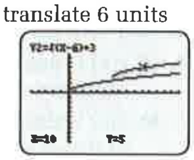
b) A′(3, 0), B′(4, 1), C′(7, 2), D′(12, 3)

8. a) $f(x) = x^2$; $y = f(x + 7) - 8$; translate 7 units left and 8 units down;



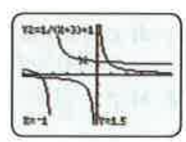
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -8\}$

b) $f(x) = \sqrt{x}$; $y = f(x - 6) + 3$; translate 6 units right and 3 units up;

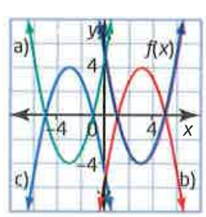


$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \geq 6\}$, range $\{y \in \mathbb{R}, y \geq 3\}$

c) $f(x) = \frac{1}{x}$; $y = f(x + 3) + 1$; translate 3 units left and 1 unit up;



$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq -3\}$, range $\{y \in \mathbb{R}, y \neq 1\}$

9. 

$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -4\}$

a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -4\}$

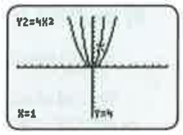
b) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 4\}$

c) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 4\}$

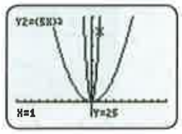
10. a) i) $g(x) = -\sqrt{x} - 5$ ii) $g(x) = -\frac{1}{x} + 7$

b) i) $h(x) = \sqrt{-x} + 5$ ii) $h(x) = -\frac{1}{x} - 7$

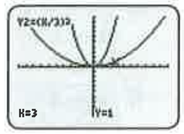
11. a) $a = 4$; $g(x) = 4x^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



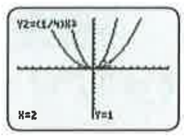
b) $k = 5$; $g(x) = (5x)^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $= \{y \in \mathbb{R}, y \geq 0\}$



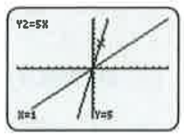
c) $k = \frac{1}{3}$; $g(x) = \left(\frac{1}{3}x\right)^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



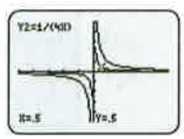
d) $a = \frac{1}{4}$; $g(x) = \frac{1}{4}x^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



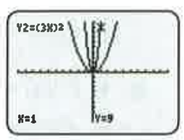
12. a) $f(x) = x$; vertical stretch by a factor of 5



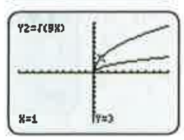
b) $f(x) = \frac{1}{x}$; horizontal compression by a factor of $\frac{1}{4}$



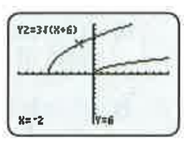
c) $f(x) = x^2$; horizontal compression by a factor of $\frac{1}{3}$



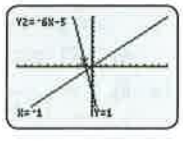
d) $f(x) = \sqrt{x}$; horizontal compression by a factor of $\frac{1}{9}$



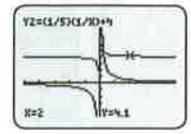
13. a) vertical stretch by a factor of 3, then translation of 6 units left; $g(x) = 3\sqrt{x + 6}$



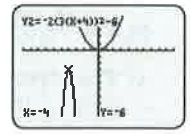
b) reflection in the x-axis, horizontal compression by a factor of $\frac{1}{6}$, and then translation of 5 units down; $g(x) = -6x - 5$



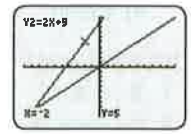
c) vertical compression by a factor of $\frac{1}{5}$, then translation of 4 units up;
$g(x) = \frac{1}{5}\left(\frac{1}{x}\right) + 4$ or
$g(x) = \frac{1}{5x} + 4$



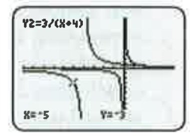
d) reflection in the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{3}$, and then translation of 4 units left and 6 units down;
$g(x) = -2[3(x + 4)]^2 - 6$



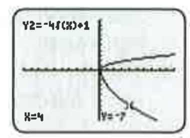
14. a) $f(x) = x$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}\}$



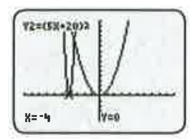
b) $f(x) = \frac{1}{x}$;
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \neq -4\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$



c) $f(x) = \sqrt{x}$;
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \leq 1\}$

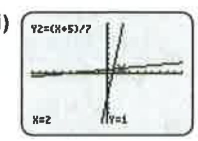


d) $f(x) = x^2$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



15. a) i) $f^{-1}(x) = \frac{x + 5}{7}$

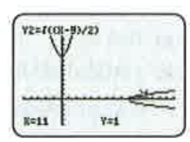
ii)



iii) Yes. $f^{-1}(x)$ is a function.

b) i) $f^{-1}(x) = \pm\sqrt{\frac{x - 9}{2}}$

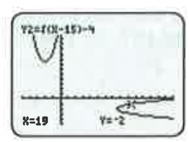
ii)



iii) No. $f^{-1}(x)$ is not a function.

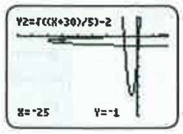
c) i) $f^{-1}(x) = \pm\sqrt{x - 15} - 4$

ii)



iii) No. $f^{-1}(x)$ is not a function.

d) **i)** $f^{-1}(x) = \pm\sqrt{\frac{x + 30}{5}} - 2$

ii) 

iii) No. $f^{-1}(x)$ is not a function.

16. a) $E = 600 + 0.05s$, where E represents Jai's weekly earnings, in dollars, and s represents Jai's weekly sales, in dollars.

b) $s = \frac{E - 600}{0.05}$

c) The inverse function represents Jai's weekly sales, s, in dollars, as a function of her weekly earnings, E, in dollars.

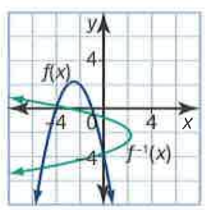
d) \$3500

Chapter 2 Practice Test, pages 144–145

1. B **2.** C **3.** D **4.** C **5.** B

6. Answers may vary. Sample answer: No. The two expressions are not equivalent. The first expression is not defined for $x = 7$. The second expression is defined for $x = 7$. The first expression can be simplified to $6x + 15$. The second expression can be simplified to $5x + 15$. Since the expressions are not equivalent when simplified, the given expressions are not equivalent.

7. The inverse of $f(x)$ is not a function.



8. a) $\frac{x - 8}{(x + 7)(x - 3)}$, $x \neq -15$, $x \neq -7$, $x \neq 3$

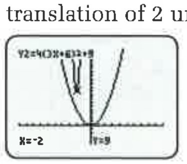
b) $\frac{(x + 10)(x + 2)}{(x + 5)(x - 3)}$, $x \neq -10$, $x \neq -5$, $x \neq 3$

c) $\frac{-x + 57}{(x - 7)(x - 2)}$, $x \neq 2$, $x \neq 7$

d) $\frac{x^2 + 15x + 42}{(x + 6)(x + 3)}$, $x \neq -6$, $x \neq -3$

9. a) $f(x) = x^2$

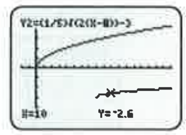
b) vertical stretch by a factor of 4, horizontal compression by a factor of $\frac{1}{3}$, and then translation of 2 units left and 9 units up

c) 

$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 9\}$

10. a) $f(x) = \sqrt{x}$

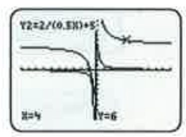
b) vertical compression by a factor of $\frac{1}{5}$, horizontal compression by a factor of $\frac{1}{2}$, and then translation of 8 units right and 3 units down

c) 

$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \geq 8\}$, range $\{y \in \mathbb{R}, y \geq -3\}$

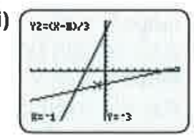
11. a) $f(x) = \frac{1}{x}$

b) vertical stretch by a factor of 2, horizontal stretch by a factor of 2, and then translation of 5 units up

c) 

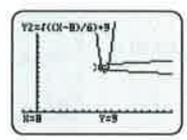
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 5\}$

12. a) i) $f^{-1}(x) = \frac{x - 8}{3}$

ii) 

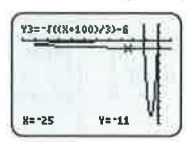
iii) Yes. $f^{-1}(x)$ is a function.

b) **i)** $f^{-1}(x) = \pm\sqrt{\frac{x - 8}{6}} + 9$

ii) 

iii) No. $f^{-1}(x)$ is not a function.

c) **i)** $f^{-1}(x) = \pm\sqrt{\frac{x + 100}{3}} - 6$

ii) 

iii) No. $f^{-1}(x)$ is not a function.

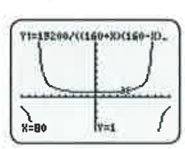
13. a) $R = (80 + 5x)(120 - 15x)$

b) R: domain $\{x \in \mathbb{Z}, 0 \leq x \leq 8\}$, range $\{R \in \mathbb{R}, 0 \leq R \leq 10\,800\}$

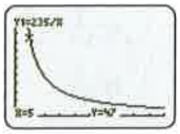
c) $x = \pm\sqrt{\frac{R - 10\,800}{-75}} - 4$; the number of \$5 increases as a function of the revenue, R, in dollars; domain $\{R \in \mathbb{R}, 0 \leq R \leq 10\,800\}$, range $\{x \in \mathbb{Z}, 0 \leq x \leq 8\}$

d) 2 \$5 increases

14. a) $T = \frac{19\,200}{(160 + w)(160 - w)}$; domain $\{w \in \mathbb{R}, -160 < x < 160\}$, range $\{T \in \mathbb{R}, T \geq 0.75\}$

b) 

c) Answers may vary. Sample answer: No. The minimum time to complete the round trip occurs when there is no prevailing wind. While some time will be made up, it will always take longer to complete the trip when wind is involved.

15. a) 

b) For lesser values of ℓ, in litres, the graph has greater values of m, in litres per 100 km.

c) $m_{\text{imperial}} = \dfrac{235}{1.2\,\ell}$

d) Answers may vary. Sample answer: horizontal compression by a factor of $\dfrac{1}{1.2}$

Chapter 3

Prerequisite Skills, pages 148–149

1. a) C **b)** A **c)** B

2. Answers may vary.

3. a) x^5 **b)** y^9 **c)** m^2 **d)** h
e) a^7b^6 **f)** x^2y^2 **g)** $a^4b^8c^{12}$ **h)** $9u^2v^6$
i) $\dfrac{a^2b^4}{16}$ **j)** $-\dfrac{27w^6}{64r^9}$

4. a) 128 **b)** 729 **c)** 32 000 **d)** 1
e) 64 **f)** 5 **g)** 6561 **h)** 64

5. a) missing values in the y-column: 4, 2, 1

b) Answers may vary. Sample answer: Divide each previous term by 2.

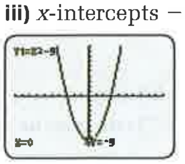
c) missing values in the y-column: $4, 2, 1, \dfrac{1}{2^2}, \dfrac{1}{2^3}, \dfrac{1}{2^n}$

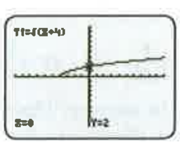
6. a) 1 **b)** $\dfrac{1}{16}$ **c)** $-\dfrac{1}{216}$ **d)** $\dfrac{1}{9}$
e) 1 **f)** -1 **g)** $\dfrac{25}{16}$ **h)** $\dfrac{1}{9}$

7. a) $\dfrac{1}{x}$ **b)** $\dfrac{1}{y^6}$ **c)** $\dfrac{u}{v}$ **d)** $\dfrac{1}{16a^4b^2}$

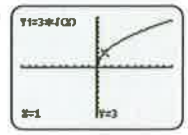
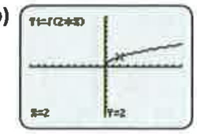
8. a) **i)** $\{x \in \mathbb{R}\}$ **ii)** $\{y \in \mathbb{R}\}$
iii) x-intercept 8; y-intercept 4

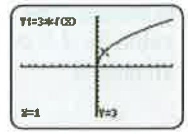
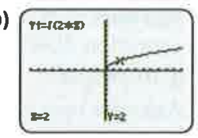
b) **i)** $\{x \in \mathbb{R}\}$ **ii)** $\{y \in \mathbb{R}, y \geq -5\}$
iii) x-intercepts -8, 2; y-intercept -2

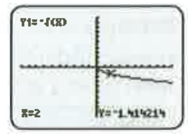
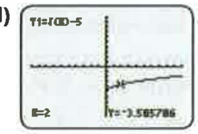
9. a) 
i) $\{x \in \mathbb{R}\}$
ii) $\{y \in \mathbb{R}, y \geq -9\}$
iii) x-intercepts -3, 3; y-intercept -9

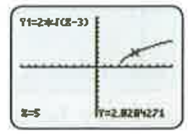
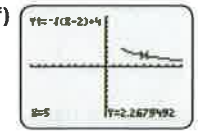
b) 
i) $\{x \in \mathbb{R}, x \geq -4\}$
ii) $\{y \in \mathbb{R}, y \geq 0\}$
iii) x-intercept -4; y-intercept 2

10. Translate 2 units right and 1 unit down.

11. a)  **b)** 

11. a)  **b)** 

c)  **d)** 

e)  **f)** 

3.1 The Nature of Exponential Growth, pages 155–157

1. a)

Day	Population	First Differences	Second Differences
0	20	60	180
1	80	240	720
2	320	960	2 880
3	1 280	3 840	11 520
4	5 120	15 360	
5	20 480		

Answers for parts b) to e) may vary. Sample answers:

b) Yes; the values in each difference column increase by a factor of 4.

c) Differences in each column are 4 times the previous difference in that column.

d) Yes.

e)

Third Differences	Fourth Differences
540	1620
2160	6480
8640	

Yes.

2. Answers may vary. Sample answer: Since $10^3 = 1000$, $10^2 = 100$, and $10^1 = 10$, following the pattern of dividing by 10 suggests $10^0 = 1$.

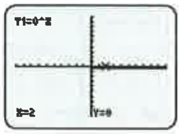
3. a) $\dfrac{a \times a \times a}{a \times a \times a}$ **b)** 1 **c)** a^0

d) Answers may vary. Sample answer: $a^0 = 1$

4. a) 1 **b)** 1 **c)** 1 **d)** 1

5. a)–b) Answers may vary.
c) **i)** 81 **ii)** 19 683 **d)** 6 **e)** 6

6. a) **i)** $\{x \in \mathbb{R}, x > 0\}$ **ii)** $\{y \in \mathbb{R}, y = 0\}$
iii) linear and horizontal along the x-axis

b) 

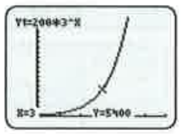
c) Answers may vary. Sample answer: The **Trace** operation does not give a value for $x = 0$.

7. a) i) 10 people **ii)** 20 people

b) Answers may vary.

c) Answers may vary. Sample answer: Yes; the ratio of successive first differences is 2.

8. Answers may vary. Sample answer: Take the Double Deal; it is worth the most after 2 weeks.

9. a) $p = 200 \times 3^t$ **b)** 

Answers for parts c) and d) may vary. Sample answers:

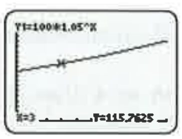
c) 961; the graph is easier to use.

d) 106 288 200; the equation is easier to use.

10. a) 0.05; $A = P(1.05)^t$

b)–c)

Number of Compounding Periods (years)	Amount ($)	First Differences	Second Differences
0	100		
1	105	5	
2	110.25	5.25	0.25
3	115.76	5.51	0.26
4	121.55	5.79	0.28

d) 

Answers for parts e) and f) may vary. Sample answers:

e) The points between values have no meaning because the payment of interest is not continuous.

f) The function is exponential because each value in the chart is greater than the previous by a factor of 1.05.

11. a) $2249.73 **b)** $2737.14

12. a) approximately 10.24 years

b) approximately 4 more years

13. Answers may vary.

14. a) approximately 5.7 days, approximate population 25 600 bacteria cells

b) approximately 2.6 days faster

15. Answers may vary. Sample answers:

a) The Double Deal is still the best choice, because it still pays more.

b) $0.03

16. C **17.** D **18.** B

19. $\frac{1}{9}$

20. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}; \frac{1}{4} + \frac{1}{4} + \frac{1}{2}$

3.2 Exponential Decay: Connecting to Negative Exponents, pages 166–169

1. a) $\frac{1}{3}$ **b)** $\frac{1}{x}$ **c)** $\frac{1}{y^2}$

d) $\frac{1}{ab}$ **e)** $-\frac{1}{x^2}$ **f)** $\frac{1}{x^2}$

2. a) 5^{-2} **b)** k^{-3}

3. a) $\frac{1}{36}$ **b)** $\frac{1}{32}$ **c)** $\frac{1}{10\,000}$

d) $\frac{1}{729}$ **e)** $\frac{1}{2}$ **f)** $\frac{5}{8}$

4. a) 16 **b)** $\frac{1}{256}$ **c)** 100 000

d) $\frac{1}{5}$ **e)** 49 **f)** $\frac{1}{16}$

g) 9 **h)** 4

5. a) m **b)** $-\frac{6}{v^9}$ **c)** p^7

d) $\frac{3}{w^2}$ **e)** k^{12} **f)** $\frac{b^6}{4a^2}$

6. a) 64 **b)** $\frac{1\,000\,000}{729}$ **c)** $\frac{16}{81}$

d) $-\frac{8}{125}$ **e)** 8 **f)** 1600

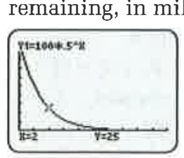
7. a) a^2b^2 **b)** $512u^3$ **c)** $\frac{w^4}{g^8}$

d) $\frac{27b^6}{64a^9}$ **e)** $\frac{b^9}{27a^6}$ **f)** $\frac{x^4}{y^2}$

8. a)

Time (days)	Amount of W-187 remaining (mg)
0	100
1	50
2	25
3	12.5
4	6.25

b) $f(x) = 100\left(\frac{1}{2}\right)^x$, where f is the amount of W-187 remaining, in milligrams, on day x.

c) 

d) 0.781 25 mg **e)** 4.3 days

f) The function $f(x) = 100(2^{-x})$ can also be used, as $\left(\frac{1}{2}\right)^x = (2^{-1})^x$, which is equal to 2^{-x}.

9. a) i) $\frac{1}{9}$ **ii)** $\frac{1}{27}$ **b)** $\frac{1}{243}$ **c)** $\frac{1}{243}$

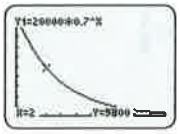
d) Answers may vary. Sample answer: The two answers are the same. This illustrates that the product rule applies to negative exponents.

e) Answers may vary.

10.–11. Answers may vary.

12. a) 20 000 is the initial value of the car; 0.7 represents the remaining value of the car (as a percent) after each year; and t represents the time since purchase, in years.

b) i) \$14 000 **ii)** \$9800

c) 

d) approximately 6.5 years

13. a) Answers may vary. Sample answer: Substituting negative values for n will give amounts of Pu-239 before the beginning of the study.

b) i) 100 mg **ii)** 400 mg

14.–15. Answers may vary.

17. \$767.90

18. 2.1×10^{20} N

19. a) $F = \dfrac{GMm}{r^2}$

b) Answers may vary. Sample answer: This is an inverse square relationship because the force between two objects is proportional to the inverse of the square of the distance, r, separating those objects.

c) Answers may vary.

20. Answers may vary.

21. A **22.** D **23.** B

3.3 Rational Exponents, pages 175–177

1. a) 4 **b)** -10 **c)** $\frac{1}{2}$ **d)** $\frac{2}{3}$

2. a) 3 **b)** $\frac{2}{5}$ **c)** 2 **d)** -10

3. a) 4 **b)** 16 **c)** -1024 **d)** $\frac{1}{1000}$

4. a) $\frac{1}{2}$ **b)** $\frac{1}{125}$ **c)** 128

d) 4 **e)** $\frac{27}{1000}$ **f)** $\frac{9}{4}$

5. a) $x^{\frac{1}{2}}$ **b)** $m^{\frac{13}{12}}$ **c)** $w^{\frac{1}{6}}$

d) $a^{\frac{1}{2}}b^{\frac{5}{3}}$ **e)** $y^{\frac{1}{3}}$ **f)** $u^{\frac{1}{6}}v^{\frac{1}{9}}$

6. a) k **b)** $\dfrac{1}{p^{\frac{3}{2}}}$ **c)** $\dfrac{1}{y^2}$

d) $w^{\frac{2}{3}}$ **e)** $\frac{4}{3}x^{\frac{1}{3}}$ **f)** $\frac{5}{49}y^{\frac{4}{3}}$

7. 4000 cm^2

8. a) $A = \ell^2$ **b)** $\ell = A^{\frac{1}{2}}$

c) i) 6 m **ii)** 13 cm **iii)** $4\sqrt{5}$ m

d) $S = 6\ell^2$, $\ell = \left(\dfrac{S}{6}\right)^{\frac{1}{2}}$

e) i) 5 m **ii)** 10 cm **iii)** $5\left(\dfrac{5}{3}\right)^{\frac{1}{2}}$ m

9. a) $V = \ell^3$ **b)** $\ell = V^{\frac{1}{3}}$

c) i) 4 m **ii)** 7 cm **iii)** approximately 2.49 m

10. a) B and C are correct

Answers for parts b) and c) may vary. Sample answers:

b) Since B and C are equivalent, both are correct.

c) It is called the square-cube law because the square of the volume is related to the cube of the surface area.

11. a) i) 600 cm^2 **ii)** approximately 205.2 m^2

Answers for parts b) to d) may vary. Sample answers:

b) I used $S = 6V^{\frac{2}{3}}$, since it gives the surface area in terms of the volume.

c) i) 343 m^3 **ii)** approximately 15.2 cm^3

d) I used $V = \left(\dfrac{S}{6}\right)^{\frac{3}{2}}$, since it gives the volume in terms of the surface area.

12. a) 1.72×10^{-17} **b)** 1.9 times as long

c) 1.1×10^{11} m

13. a) $r = \left(\dfrac{T}{k}\right)^{\frac{2}{3}}$ **b)** 7.87×10^{11} m

15. a) $V = \pi r^2 h$ **b)** $r = \sqrt{\dfrac{V}{\pi h}}$

16. a) $SA = 2\pi r^2 + 2\pi rh$

b) $SA = \dfrac{2V}{h} + 2\sqrt{\pi hV}$

c) $SA = \dfrac{1}{5}V + 2\sqrt{10\pi V}$

d) 554.5 m^2

17. a) $V = \sqrt[3]{\dfrac{850}{P^2}}$ **b)** 2.04 m^3

c) 

d) Answers may vary. Sample answer: This relation is a function since there is exactly one volume for each given pressure value.

18. A **19.** 5

3.4 Properties of Exponential Functions, pages 185–187

1. a) B **b)** D **c)** C **d)** A

2. a) Answers may vary.

b) Answers may vary. Sample answer: No; there are many exponential functions with these properties.

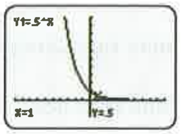
3. a) Answers may vary.

b) Answers may vary. Sample answer: No; there are many exponential functions with these properties.

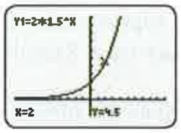
4. $y = 4(2^x)$

5. $y = 24\left(\dfrac{1}{2}\right)^x$

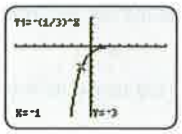
6. a) C **b)** approximately 2.2 mg

7. a) 

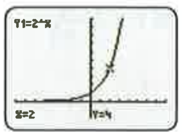
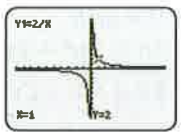
i) $\{x \in \mathbb{R}\}$
ii) $\{y \in \mathbb{R}, y > 0\}$
iii) no x-intercept; y-intercept 1
iv) always decreasing **v)** $y = 0$

b) 

i) $\{x \in \mathbb{R}\}$
ii) $\{y \in \mathbb{R}, y > 0\}$
iii) no x-intercept; y-intercept 2
iv) always increasing **v)** $y = 0$

c) 

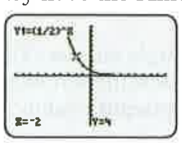
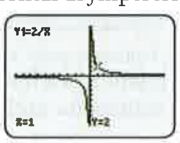
i) $\{x \in \mathbb{R}\}$
ii) $\{y \in \mathbb{R}, y < 0\}$
iii) no x-intercept; y-intercept -1
iv) always increasing **v)** $y = 0$

8. a) i)  **ii)** 

Answers for parts b) and c) may vary. Sample answers:

b) They are alike in that they both have a horizontal asymptote at $y = 0$. They differ in that $f(x)$ is increasing for all $x \in \mathbb{R}$ and $r(x)$ is decreasing on the intervals $x < 0$ and $x > 0$ ($r(x)$ is not defined at $x = 0$). $r(x)$ also has a vertical asymptote at $x = 0$.

c) They have the same horizontal asymptotes.

9. a) i)  **ii)** 

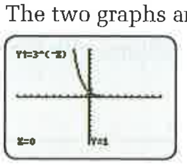
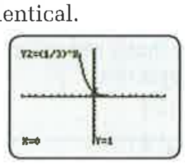
Answers for parts b) and c) may vary. Sample answers:

b) They both have a horizontal asymptote at $y = 0$. They differ in that $g(x)$ is decreasing for all $x \in \mathbb{R}$ and $r(x)$ is decreasing on the intervals $x < 0$ and $x > 0$ (it is not defined at $x = 0$). $r(x)$ also has a vertical asymptote at $x = 0$.

c) They have the same horizontal asymptotes.

10. Answers may vary. Sample answers:

a) The two graphs are identical.

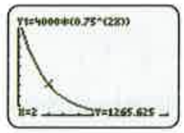
b)  

c) Since $\frac{1}{3} = 3^{-1}$, $\left(\frac{1}{3}\right)^x = (3^{-1})^x$, which can be simplified to 3^{-x}.

11. a) $\{x \in \mathbb{R}, x \geq 0\}$ **b)** $\{y \in \mathbb{R}, 0 < y \leq 2\}$
c) 2 V **d)** 0 V
e) approximately 1 ms

12. Answers may vary. Sample answers:

a) The value of 0.75 is the base of the exponential function and the 2 represents a horizontal compression by a factor of $\frac{1}{2}$.

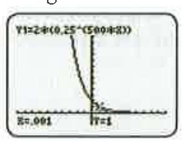
b) 

c) The base of the exponential function is between 0 and 1, indicating that the number of revolutions per minute is decreasing.

d) i) 2250 **ii)** 712

13. a) $\frac{1}{4}$

b) Answers may vary. Sample answer: Substitute $t = 0$ and $V = 2$ to find V_0. Then, substitute $t = 0.001$ and $V = 1$ from the graph, as well as the given values for R and C to find b.

c) 

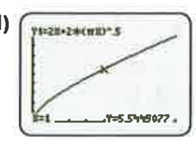
d) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y > 0\}$

e) domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, 0 < y \leq 2\}$. Answers may vary. Sample answer: The restriction is made because the circuit only begins to discharge at $t = 0$.

14. a) $S = \frac{(3V)^{\frac{1}{2}}}{5}$

b) Answers may vary. Sample answer: domain $\{V \in \mathbb{R}, V > 0\}$

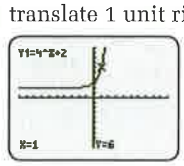
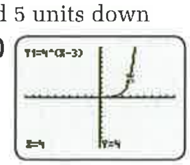
c) The side length of the base will increase by a factor of $\sqrt{2}$.

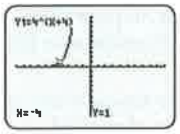
15. a) $SA = 2V + 2(V\pi)^{\frac{1}{2}}$
b) $SA \doteq 4.77$ m^2; $d \doteq 1.00$ m
c) $\{V \in \mathbb{R}, V > 0\}$ **d)** 

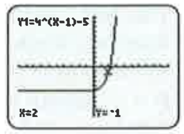
16. 1, 3 **17.** A **18.** A

3.5 Transformations of Exponential Functions, pages 196–198

1. a) translate 2 units up
b) translate 3 units right
c) translate 4 units left
d) translate 1 unit right and 5 units down

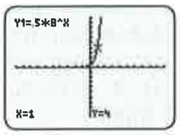
2. a)  **b)** 

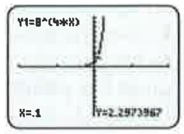
c) 

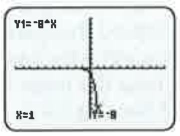
d) 

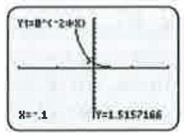
3. a) $y = 5^x - 3$ **b)** $y = 5^{x-2}$
c) $y = 5^{x+\frac{1}{2}}$ **d)** $y = 5^{x+2.5} + 1$

4. a) vertical compression by a factor of $\frac{1}{2}$
b) horizontal compression by a factor of $\frac{1}{4}$
c) reflection in the x-axis
d) reflection in the y-axis and horizontal compression by a factor of $\frac{1}{2}$

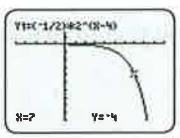
5. a) 

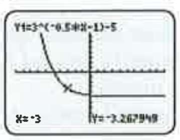
b) 

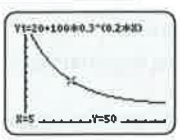
c) 

d) 

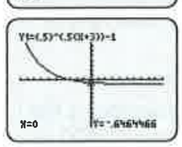
6. a) $y = -7^x$ **b)** $y = 3(7^x)$
c) $y = 7^{\frac{x}{2.4}}$ **d)** $y = 7(7^{-x})$

7. 

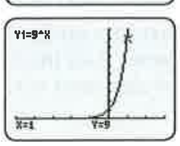
8. 

9. a) 

b) $T = 20$; room temperature
c) approximately 28.7 min

10. a) 

b) i) $\{x \in \mathbb{R}\}$
ii) $\{x \in \mathbb{R}, y > 21\}$
iii) $y = -1$

11. a) 

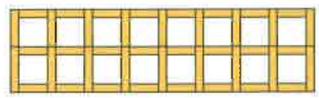
b) $y = 3^{2x}$; horizontal compression of the graph of $y = 3^x$ by a factor of $\frac{1}{2}$
c) $y = 81^{\frac{1}{2}x}$; horizontal stretch of the graph of $y = 81^x$ by a factor of 2
d) Answers may vary. Sample answer: Since 3^{2x} and $81^{\frac{1}{2}x}$ both equal 9^x, all three functions are equivalent.

12. Answers may vary. Sample answers:
a) $y = 2^{3x}$, $y = 64^{\frac{1}{2}x}$
b) Since 2^{3x} and $64^{\frac{1}{2}x}$ both equal 8^x, all three functions are equivalent.

13. Answers may vary. Sample answers:
a) $y = 2^{x+1} + 4$
b) There are many possible functions. Many transformations of different functions can produce the function in part a).

14. Answers may vary. Sample answers:
a) $y = 2(2^x)$ **b)** $y = 2^x + 1$, $y = 2^{x+1}$
c) All functions have base 2, and when $x = 0$ is substituted into each equation, the result is $y = 2$.

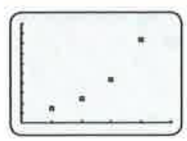
15. Answers may vary.

16. a) 

b) Answers may vary. Sample answer: The number of squares is growing exponentially as a power of 2.
c) $s = 2^n$ **d) i)** 32 **ii)** 1024

17. a)–b)

Term Number (n)	Number of Toothpicks (t)	First Differences	Second Differences
1	7		
2	12	5	
3	22	10	5
4	42	20	10

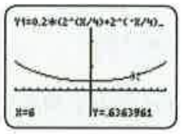
c) 

Answers for parts d) to f) may vary. Sample answers:
d) The function that expresses t in terms of n is exponential, since the ratio of successive finite differences is always 2.
e) $t = 5(2^{n-1}) + 2$
f) This function results from a vertical stretch by a factor of 5 and a translation of 1 unit right and 2 units up.

18.–19. Answers may vary.

21. Answers may vary. Sample answers:
a) 2 white squares, 3 short horizontal rectangles, 2 short vertical rectangles, 2 slightly longer vertical rectangles
b) The growing patterns of the different types of pieces all seem to be exponential.
c) $t = 7(2^{n-1}) + 2$

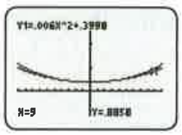
d) They are all translations of the function $y = 2^n$. The function in part c) results from a vertical stretch of the function in question 16 by a factor of 5 and a shift of 1 unit right and 2 units up. The function in part c) results from a vertical stretch of the function in question 17 by a factor of $\frac{7}{5}$ and a shift of $\frac{4}{5}$ units down.

22. a) 

Answers for parts b) to f) may vary. Sample answers:

b) This graph resembles a quadratic function.

c) $y = 0.006x^2 + 0.3998$

d) 

e) The graphs are similar, but the quadratic equation is slightly lower than the catenary function for the graphed domain.

f) This difference is magnified as the values of x get larger in the positive and negative directions.

23. 117 **24.** B **25.** $\{y \in \mathbb{R}, y > -3\}$ **26.** A

3.6 Making Connections: Tools and Strategies for Applying Exponential Models, pages 207–209

1. a) D **b)** C **c)** A

2. Answers may vary.

3. Answers may vary. Sample answers:

a) Yes; the data appear to be exponential.

b) I estimate that $a = 100$ and $b = 1.1$. I arrived at these values by calculating the ratio of successive first differences.

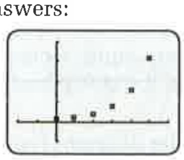
d) $259.37

e) approximately 7 years

4. a)

Time (half-hour intervals)	Number of People Who Just Heard the News
0	1
1	2
2	4
3	8
4	16
5	32

Answers for parts b) to d) may vary. Sample answers:

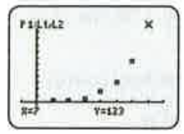
b)  The trend is increasing.

c) The data seem to follow an exponential function, since the ratio of successive first differences is constant.

d) $P = 2^n$, where P represents the number of people that know and n represents the number of half-hour increments.

5. Answers may vary. Sample answers:

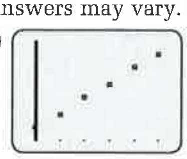
a) No, as eventually everyone in the company will know the news.

b) 

At the beginning of the 24-h period, only Gina knows the news. After each half-hour interval, twice as many people hear the news, which gives the points (0, 1), (1, 2), (2, 4), (3, 8), (4, 16), (5, 32), and (6, 64). Since $1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$, after six half-hour intervals have elapsed there are only $250 - 127 = 123$ people are left who know the news. All of these people hear the news in the next interval, at (7, 123). After this time, no one will learn of the news, and so all remaining coordinates have a y-value of 0.

6.–7. Answers may vary.

8. Answers may vary. Sample answers:

a)  The data appear to be linear.

b) The data seem to follow a linear relation, as the increase in y-values is consistently around 30 to 40 pandas per year. A linear model is $P = 35x + 800$ and an exponential model is $P = 800(1.04^x)$, where x is the number of years and P is the number of pandas.

c) linear model: 1220 pandas; exponential model: 1281 pandas

d) Answers may vary. Sample answer: linear model: 34.3 years, exponential model: 23.4 years

9. Answers may vary. Sample answers:

a) $y = 100(1.023)^x$, where y represents the CPI and x represents the time, in years (2002 is $x = 0$).

b) The average Canadian's earnings are growing at a rate that is slightly greater than the CPI, as the value of the base of the exponent is slightly greater.

10. Answers may vary.

11. Answers may vary. Sample answers:

a) Earnings for Ontarians are greater than earnings for all Canadians.

b) The only province that has a greater average weekly salary than Ontario is Alberta. The three territories have consistently had greater average weekly earnings than Ontario.

12.–15. Answers may vary.

Chapter 3 Review, pages 210–211

1. D

2. Answers may vary.

3. a) 1 **b)** Answers may vary.

4. a) $A = 250\left(\frac{1}{2}\right)^n$, where n is the number of years and A is the amount radioactive material remaining, in milligrams.

b) 0.244 mg **c)** approximately 2.3 years

5. Answers may vary. Sample answers:

a) $A = 250(2^{-n})$

b) Since $b^{-x} = \frac{1}{b^x}$, which can be written as $\left(\frac{1}{b}\right)^x$, then $\left(\frac{1}{2}\right)^n = 2^{-n}$, so the equations are equivalent.

6. a) $\frac{1}{10}$ **b)** $\frac{1}{16}$ **c)** $\frac{2}{9}$

d) $\frac{126}{125}$ **e)** 5 **f)** $\frac{64}{27}$

7. a) $\frac{1}{x^3}$ **b)** $\frac{6}{k}$ **c)** $\frac{1}{w}$

d) uv^5 **e)** z^6 **f)** $\frac{b^2}{4a^2}$

8. a) 4 **b)** 5 **c)** -5

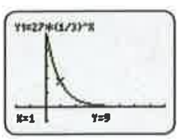
d) $\frac{1}{2}$ **e)** 9 **f)** 10 000

g) $-\frac{1}{64}$ **h)** $\frac{16}{9}$ **i)** $\frac{25}{9}$

9. a) $x = 2^{\frac{1}{2}}U^{\frac{1}{2}}k^{-\frac{1}{2}}$

b) $x = \sqrt{\frac{2U}{k}}$ **c)** 8 cm

10. a)



b) i) $\{x \in \mathbb{R}\}$ **ii)** $\{y \in \mathbb{R}, y > 0\}$

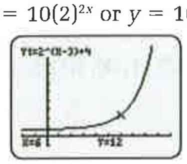
iii) no x-intercepts; y-intercept 27

iv) The function is decreasing over its domain.

v) $y = 0$

11. $y = 10(2)^{2x}$ or $y = 10(4^x)$

12. a)



b) i) $\{x \in \mathbb{R}\}$

ii) $\{y \in \mathbb{R}, y > 4\}$

iii) $y = 4$

13. a) vertical stretch by a factor of 2

b) horizontal compression by a factor of $\frac{1}{2}$

c) reflection in the x-axis and the y-axis

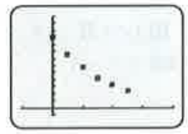
d) reflection in the y-axis, horizontal compression by a factor of $\frac{1}{5}$, translation of 2 units left

14. a)

Number of Bounces, n	Height, h (cm)	First Differences	Second Differences
0	100		
		−24	
1	76		5
		−19	
2	57		5
		−14	
3	43		3
		−11	
4	32		3
		−8	
5	24		

Answers for parts b) to f) may vary. Sample answers:

b)



The data seem to follow an exponential curve.

c) $y = 100(0.75)^x$

d) i) According to the mathematical model, the ball should never stop bouncing, as it will always bounce to a height that is 75% of the previous bounce, which will never equal 0.

ii) In the real situation, the ball will eventually stop bouncing.

e) There is also a slight loss of energy due to air resistance and friction. Eventually, these factors will cause the ball to stop bouncing.

Chapter 3 Practice Test, pages 212–213

1. B **2.** C **3.** A **4.** A

5. a) 7 **b)** $\frac{1}{125}$ **c)** 1 **d)** 2

e) -32 **f)** $\frac{256}{81}$ **g)** $\frac{4}{3}$ **h)** $\frac{625}{16}$

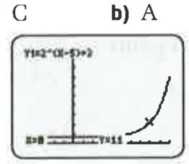
6. a) $\frac{1}{x^3}$ **b)** $\frac{1}{p^5}$ **c)** $\frac{1}{2k^4}$

d) $a^{\frac{7}{6}}$ **e)** $\frac{1}{y^4}$ **f)** $\frac{v^6}{u}$

7. Answers may vary, but should be of the form $y = a(2^x)$ for some value of a.

8. a) C **b)** A **c)** D **d)** B

9. a)



b) i) $\{x \in \mathbb{R}\}$

ii) $\{y \in \mathbb{R}, y > 3\}$

iii) $y = 3$

10. a) vertical compression by a factor of $\frac{1}{3}$

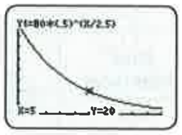
b) horizontal compression by a factor of $\frac{1}{4}$

c) reflection in the x-axis, reflection in the y-axis

d) reflection in the y-axis, horizontal compression by a factor of $\frac{1}{3}$, translation of 2 units left

11. Answers may vary. Sample answers:

a) $M = 80\left(\frac{1}{2}\right)^{\frac{t}{2.5}}$, M is the mass remaining, in milligrams, after t days.

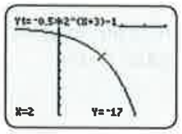
b) 

The function is a decreasing exponential function: as the values of t increase, the values of M decrease.

c) $\{t \in \mathbb{R}, t \geq 0\}$ **d) i)** approximately 5 mg

ii) approximately 1.25 mg

e) approximately 10.8 days

12. a) 

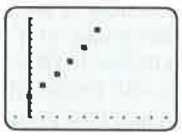
b) i) $\{x \in \mathbb{R}\}$

ii) $\{y \in \mathbb{R}, y < -1\}$

iii) $y = -1$

13. 30 m

14. Answers may vary. Sample answers:

a) 

Yes; the data appear to be exponential in nature as are not increasing the y-values linearly.

b) $y = 1987.6(1.0805)^x$ **c)** $\{x \in \mathbb{R}, x \geq 0\}$

d) approximately 3417

e) approximately 9 years, assuming the rate of population growth increases at the same rate

Chapters 1 to 3 Review, pages 214–217

1. a) domain $\{x \in \mathbb{R}, -4 \leq x \leq 4\}$, range $\{y \in \mathbb{R}, -3 \leq y \leq 3\}$; this is not a function because a vertical line can be drawn that will pass through more than one point on the relation.

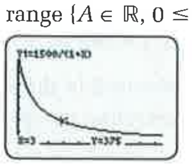
b) domain $\{-2, -1, 0, 1, 2\}$, range $\{1, 4, 9, 25\}$; this is a function because for each value in the domain there is exactly one value in the range.

c) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -4\}$; this is a function because every vertical line intersects the relation at exactly one point.

2. a) $f\colon x \to \sqrt{1 - 3x}$; $f(-1) = 2$

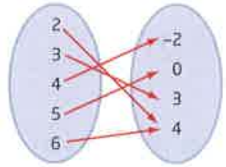
b) $f\colon x \to \dfrac{2x + 1}{x^2 - 4}$; $f(-1) = \dfrac{1}{3}$

3. a) domain $\{i \in \mathbb{R}, i \geq 0\}$, range $\{A \in \mathbb{R}, 0 \leq A \leq 1500\}$

b) 

c) approximately \$1456.31

d) 50%

4. a) 

b) Since every value in the domain maps to exactly one value in the range, this is a function.

5. 333.3 m by 500 m

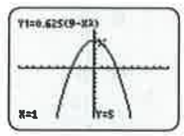
6. a) \$17.50 **b)** \$9187.50

7. a) $x = \dfrac{2 \pm \sqrt{10}}{2}$ **b)** $x = \dfrac{6 \pm 2\sqrt{6}}{3}$

8. a) 2 **b)** 2 **c)** 2

9. The length is approximately 8.81 m and the width is approximately 2.27 m.

10. a) $f(x) = \frac{1}{3}x^2 - \frac{1}{3}x - 4$ **b)** $f(x) = 2x^2 - 8x + 2$

11. a) 

b) $f(x) = 9 - x^2$

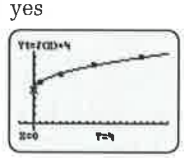
c) 9 m

12. a) $\left(\dfrac{5 + \sqrt{33}}{4}, \dfrac{15 + \sqrt{33}}{2}\right), \left(\dfrac{5 - \sqrt{33}}{4}, \dfrac{15 - \sqrt{33}}{2}\right)$

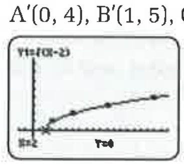
b) $\left(\dfrac{17 + \sqrt{321}}{4}, \dfrac{-9 - \sqrt{321}}{8}\right)$, $\left(\dfrac{17 - \sqrt{321}}{4}, \dfrac{-9 + \sqrt{321}}{8}\right)$

13. $b = 5$

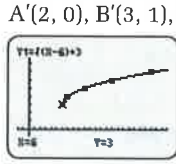
14. a) yes **b)** no

15. a) 

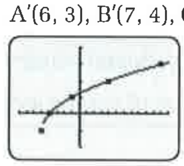
A′(0, 4), B′(1, 5), C′(4, 6), D′(9, 7), E′(16, 8)

b) 

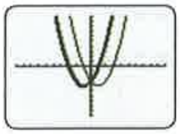
A′(2, 0), B′(3, 1), C′(6, 2), D′(11, 3), E′(18, 4)

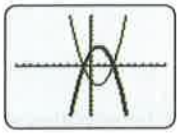
c) 

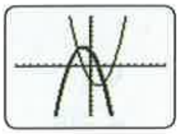
A′(6, 3), B′(7, 4), C′(10, 5), D′(15, 6), E′(22, 7)

d) 

A′(−5, −1), B′(−4, 0), C′(−1, 1), D′(4, 2), E′(11, 3)

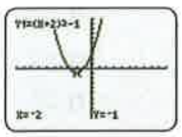
16. a)  domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -4\}$

b)  domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 4\}$

c)  domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 4\}$

17. a) i) $f(x) = x^2$

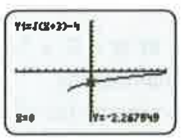
ii) $g(x) = f(x + 2) - 1$; translation of 2 units to the left and 1 unit down

iii) 

iv) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -1\}$

b) i) $f(x) = \sqrt{x}$

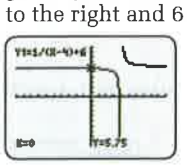
ii) $g(x) = f(x + 3) - 4$; translation of 3 units to the left and 4 units down

iii) 

iv) $f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \geq -3\}$, range $\{y \in \mathbb{R}, y \geq -4\}$

c) i) $f(x) = \frac{1}{x}$

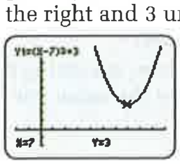
ii) $g(x) = f(x - 4) + 6$; a translation of 4 units to the right and 6 units up

iii) 

iv) $f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 4\}$, range $\{y \in \mathbb{R}, y \neq 6\}$

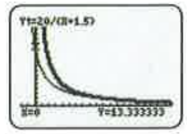
d) i) $f(x) = x^2$

ii) $g(x) = f(x - 7) + 3$; translation of 7 units to the right and 3 units up

iii) 

iv) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 3\}$

18. a) $t = \frac{20}{v + 1.5}$ **b)** $t = \frac{20}{v}$

c) 

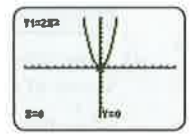
d) Shane's time would be approximately 48 min.

19. a) i) $g(x) = -2x^2 + 7x - 3$ **ii)** $h(x) = 2x^2 + 7x + 3$

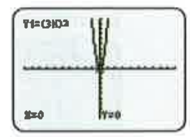
b) i) $g(x) = -\sqrt{x} + 3$ **ii)** $h(x) = \sqrt{-x} - 3$

c) i) $g(x) = -\frac{1}{x + 2}$ **ii)** $h(x) = -\frac{1}{x - 2}$

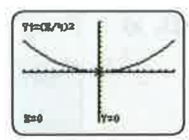
20. a) $a = 2$; $g(x) = 2x^2$; domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



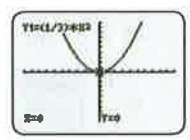
b) $a = 9$; $g(x) = 9x^2$ or $g(x) = (3x)^2$; domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



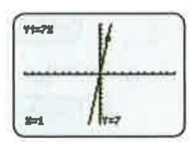
c) $a = \frac{1}{16}$; $g(x) = \frac{1}{16}x^2$ or $g(x) = \left(\frac{x}{4}\right)^2$; domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



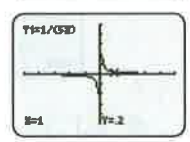
d) $a = \frac{1}{3}$; $g(x) = \frac{1}{3}x^2$; domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



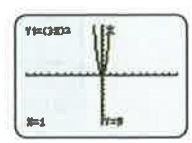
21. a) base function $f(x) = x$; vertical stretch by a factor of 7



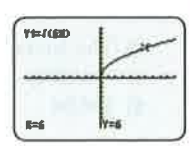
b) base function $f(x) = \frac{1}{x}$; horizontal compression by a factor of $\frac{1}{5}$



c) base function $f(x) = x^2$; horizontal compression by a factor of $\frac{1}{3}$

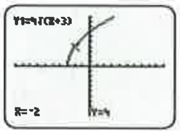


d) base function $f(x) = \sqrt{x}$; horizontal compression by a factor of $\frac{1}{6}$

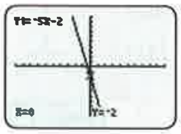


22. Answers may vary. Sample answers:

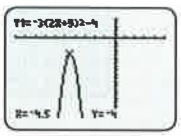
a) vertical stretch by a factor of 4, horizontal translation of 3 units to the left



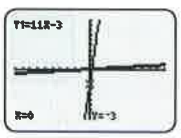
b) reflection in the x-axis, horizontal compression by a factor of $\frac{1}{5}$, vertical translation of 2 units down



c) reflection in the x-axis, vertical stretch by a factor of 3, horizontal compression by a factor of $\frac{1}{2}$, translation of $\frac{9}{2}$ units to the left and 4 units down

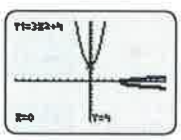


23. a) i) $f^{-1}(x) = \frac{x + 3}{11}$

ii) 

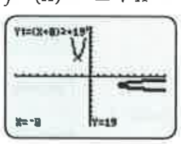
iii) function

b) i) $f^{-1}(x) = \pm\sqrt{\frac{x - 4}{3}}$

ii) 

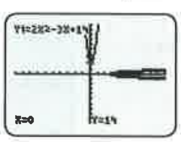
iii) not a function

c) i) $f^{-1}(x) = \pm\sqrt{x - 19} - 8$

ii) 

iii) not a function

d) i) $f^{-1}(x) = \pm\sqrt{\frac{1}{2}\left(x - \frac{103}{8}\right)} + \frac{3}{4}$

ii) 

iii) not a function

24. a) $f(x) = 450 + 0.06x$ b) $f^{-1}(x) = \frac{x - 450}{0.06}$

c) The inverse represents Issa's total sales as a function of her total weekly earnings.

d) $9500

25. a) $y = 3^x$ b) $y = x^2$

c) $y = \left(\frac{1}{3}\right)^x$ d) $y = 3x$

26. C

27. a) $M(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{3}}$ b) approximately 19.8 g

c) approximately 10 years

d) $M(t) = 200(2^{-\frac{t}{3}})$

e) Since $b^{-x} = \frac{1}{b^x}$, which can be rewritten as $\left(\frac{1}{b}\right)^x$, then $\left(\frac{1}{2}\right)^{\frac{t}{3}} = 2^{-\frac{t}{3}}$.

28. a) $\frac{1}{9}$ b) $\frac{1}{25}$ c) $\frac{1}{8}$

d) $\frac{28}{27}$ e) 25 f) $\frac{243}{32}$

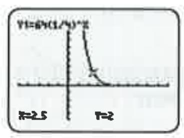
29. a) $\frac{1}{x^5}$ b) $\frac{1}{2n^5m^8}$ c) a

d) $\frac{b}{mn^4}$ e) s^{20} f) $\frac{b^6}{9a^2}$

30. a) 3 b) -10 c) -2

d) 7 e) $\frac{5}{6}$ f) 27

g) 16 h) $-\frac{1}{625}$ i) $\frac{243}{32}$

31. a) 

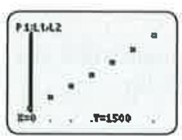
b) i) $\{x \in \mathbb{R}\}$ ii) $\{y \in \mathbb{R}, y > 0\}$

iii) no x-intercept; y-intercept 64

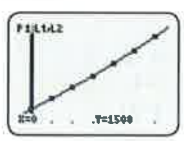
iv) The function is decreasing for all values of x.

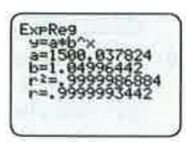
v) $y = 0$

c) $y = 64(4^{-x})$; since $b^{-x} = \frac{1}{b^x}$, which can be rewritten as $\left(\frac{1}{b}\right)^x$, then $\left(\frac{1}{4}\right)^x = 4^{-x}$.

32. a) 

Yes; an exponential curve best describes the data.





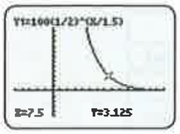
b) $y = 1500(1.05^x)$

c) $\{x \in \mathbb{R}, x \geq 0\}$

d) approximately 2327, assuming the population increases at the same rate

e) approximately 14.2 years, assuming the population increases at the same rate

33. a) $M(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{1.5}}$

b) 

The graph follows an exponential decay curve.

c) $\{t \in \mathbb{R}, t \geq 0\}$

d) i) approximately 2.48 mg

ii) approximately 0.16 mg

e) approximately 7.6 days

Chapter 4

Prerequisite Skills, pages 220–221

1. a) scalene, right **b)** isosceles
c) equilateral **d)** isosceles, right

2. a) 60° **b)** 70°, 70°
c) 60°, 60°, 60° **d)** 45°, 45°

3. a) 17 m **b)** 8.5 cm

4. a) 8 cm **b)** 3.6 m

5. a) $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$, $\sin C = \frac{4}{5}$, $\cos C = \frac{3}{5}$, $\tan C = \frac{4}{3}$

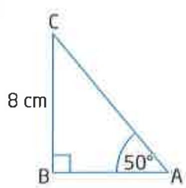
b) $\sin A = \frac{12}{13}$, $\cos A = \frac{5}{13}$, $\tan A = \frac{12}{5}$, $\sin C = \frac{5}{13}$, $\cos C = \frac{12}{13}$, $\tan C = \frac{5}{12}$

6. a) $\sin 30° = 0.5$, $\cos 30° = 0.8660$, $\tan 30° = 0.5774$

b) $\sin 45° = 0.7071$, $\cos 45° = 0.7071$, $\tan 45° = 1$

c) $\sin 60° = 0.8660$, $\cos 60° = 0.5$, $\tan 60° = 1.7321$

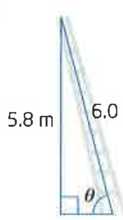
7. a) 32° **b)** 41° **c)** 30°

8. a) 

Answers may vary. Sample answer: Use the tangent ratio because the opposite side is given and the adjacent side is asked for.

b) 6.7 cm **c)** 40°

9. Answers may vary. Sample answers:

a) 

b) Yes.

c) Using the sine ratio, the angle is 75°, which is within the safe angle range.

10. 16.4 m **11.** 10.4 cm

12. a) Answers may vary. Sample answer: Use the cosine law because two sides and their contained angle are given.

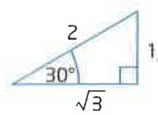
b) 14 m **c)** 82°

13. Answers may vary. Sample answer: The two given angles add to 90°, so the third angle is 90°, and the primary trigonometric ratios can be applied to find the measure of *c*, which is approximately 21.4 cm.

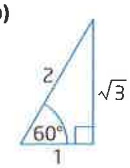
4.1 Special Angles, pages 228–231

1. Answers may vary. Sample answer: All exact values for the trigonometric ratios are either identical to the value displayed on the calculator or are found to be the same when the exact value is calculated with a CAS.

2. In all cases, exact values are identical to those found by a calculator.

3. a) 

$\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$

b) 

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

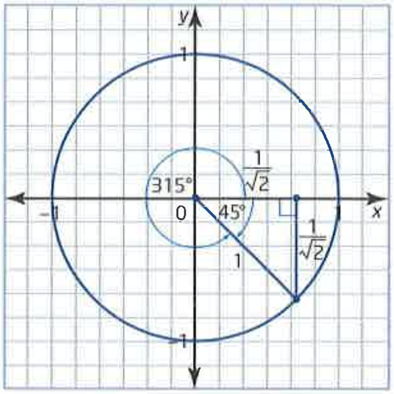
4.

	sin θ		cos θ		tan θ	
θ	**Exact**	**Calculator**	**Exact**	**Calculator**	**Exact**	**Calculator**
0°	0	0	1	1	0	0
30°	$\frac{1}{2}$	0.5	$\frac{\sqrt{3}}{2}$	0.8660	$\frac{1}{\sqrt{3}}$	0.5774
45°	$\frac{1}{\sqrt{2}}$	0.7071	$\frac{1}{\sqrt{2}}$	0.7071	1	1
60°	$\frac{\sqrt{3}}{2}$	0.8660	$\frac{1}{2}$	0.5	$\sqrt{3}$	1.7321
90°	1	1	0	0	undefined	error

5. a) 60°

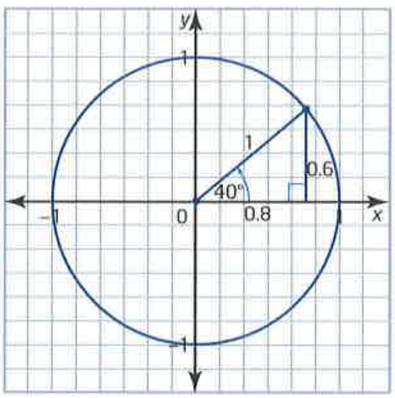
b) $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$, $\tan 120° = -\sqrt{3}$

6.



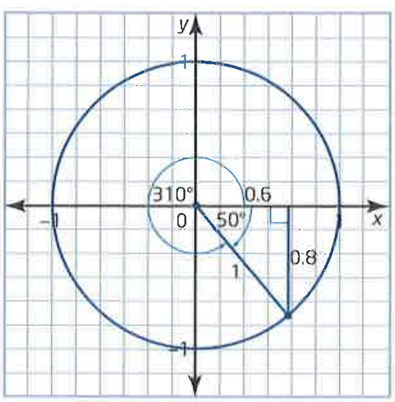
$\sin 315° = -\frac{1}{\sqrt{2}}$, $\cos 315° = \frac{1}{\sqrt{2}}$,
$\tan 315° = -1$

7.



sin 40°		cos 40°		tan 40°	
Diagram	Calculator	Diagram	Calculator	Diagram	Calculator
0.6	0.6428	0.8	0.7660	0.8	0.8391

8.



sin 310°		cos 310°		tan 310°	
Diagram	Calculator	Diagram	Calculator	Diagram	Calculator
−0.8	−0.7660	0.6	0.6428	−1.3	−1.1918

9.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
90°	1	0	undefined
180°	0	−1	0
270°	−1	0	undefined
360°	0	1	0

10. a) First quadrant: $\sin\theta$, $\cos\theta$, $\tan\theta$; second quadrant: $\sin\theta$; third quadrant: $\tan\theta$; fourth quadrant: $\cos\theta$

b) C: only cosine is positive; A: all are positive; S: only sine is positive; T: only tangent is positive

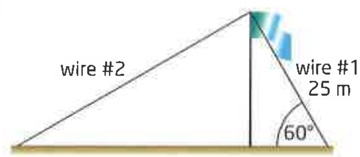
11. a)

b) 5 m

12. a) $12\sqrt{2}$ km

b) Answers may vary. Sample answer: Using the Pythagorean theorem, the answer is $12\sqrt{2}$ km.

13. a)



b) 43.3 m

c) Answers may vary. Sample answer: The angle that the second wire makes with the ground is not needed because you can use similar triangles to find the wire's length or you can calculate the length using the Pythagorean theorem.

d) 30°

14. a)

θ	$\sin\theta$	Quadrant	Sign
30°	0.5	first	+
150°	0.5	second	+
210°	−0.5	third	−
330°	−0.5	fourth	−

b) Yes, because the signs all follow the CAST rule.

c) i) 30° ii) −30°

d)

θ	$\cos\theta$	Quadrant	Sign
60°	0.5	first	+
120°	−0.5	second	−
240°	−0.5	third	−
300°	0.5	fourth	+

e) Answers may vary. Sample answer:

θ	$\tan\theta$	Quadrant	Sign
45°	1	first	+
135°	−1	second	−
225°	1	third	+
315°	−1	fourth	−

15. Answers may vary.

16. 30°, 30 m

17. 40 m

19.–21. Answers may vary.

22. B **23.** A **24.** C

4.2 Co-Terminal and Related Angles, pages 237–240

1. a) $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

b) $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$

c) $\sin\theta = -\frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = \frac{4}{3}$

d) $\sin\theta = \frac{5}{\sqrt{29}}$, $\cos\theta = \frac{2}{\sqrt{29}}$, $\tan\theta = \frac{5}{2}$

e) $\sin\theta = -\frac{3}{\sqrt{10}}$, $\cos\theta = -\frac{1}{\sqrt{10}}$, $\tan\theta = 3$

2. a) $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

b) $\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$

c) $\sin\theta = -\frac{8}{17}$, $\cos\theta = -\frac{15}{17}$, $\tan\theta = \frac{8}{15}$

d) $\sin\theta = -\frac{5}{\sqrt{34}}$, $\cos\theta = \frac{3}{\sqrt{34}}$, $\tan\theta = -\frac{5}{3}$

e) $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$, $\tan\theta = 2$

f) $\sin\theta = -\frac{1}{\sqrt{10}}$, $\cos\theta = \frac{3}{\sqrt{10}}$, $\tan\theta = -\frac{1}{3}$

3. a) $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$

b) $\sin B = -\frac{4}{5}$, $\tan B = -\frac{4}{3}$

c) $\sin C = \frac{5}{13}$, $\cos C = -\frac{12}{13}$

d) $\cos D = -\frac{\sqrt{5}}{3}$, $\tan D = \frac{2}{\sqrt{5}}$

e) $\sin E = \frac{\sqrt{11}}{6}$, $\tan E = -\frac{\sqrt{11}}{5}$

f) $\sin F = \frac{12}{\sqrt{193}}$, $\cos F = \frac{7}{\sqrt{193}}$

4. Answers may vary. Sample answers:
a) 315° **b)** 30° **c)** 660° **d)** 80° **e)** 590° **f)** 170°

5. Answers may vary. Sample answers:
a) 480°, 840°, 1200° **b)** −240°, −600°, −960°

6. a) $\sin A = -\frac{1}{\sqrt{2}}$, $\cos A = \frac{1}{\sqrt{2}}$, $\tan A = -1$

b) $\sin B = -\frac{\sqrt{3}}{2}$, $\cos B = -\frac{1}{2}$, $\tan B = \sqrt{3}$

c) $\sin C = 0$, $\cos C = -1$, $\tan C = 0$

d) $\sin D = \frac{1}{\sqrt{2}}$, $\cos D = \frac{1}{\sqrt{2}}$, $\tan D = 1$

e) $\sin E = \frac{\sqrt{3}}{2}$, $\cos E = \frac{1}{2}$, $\tan E = \sqrt{3}$

f) $\sin F = 1$, $\cos F = 0$, $\tan F$ is undefined

7. 150°, 210°

8. 135°, 315°

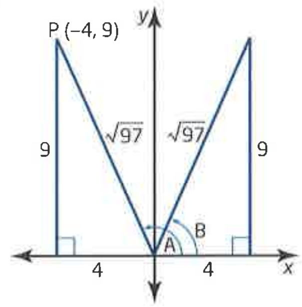
9. 45°, 315°

10. 90°, 270°

11. a) $\sin A = \frac{9}{\sqrt{97}}$, $\cos A = -\frac{4}{\sqrt{97}}$, $\tan A = -\frac{9}{4}$;

$\sin B = \frac{9}{\sqrt{97}}$, $\cos B = \frac{4}{\sqrt{97}}$, $\tan B = \frac{9}{4}$

b)

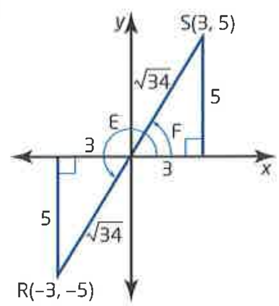


$\angle A = 114°$, $\angle B = 66°$

12. a) $\sin E = -\frac{5}{\sqrt{34}}$, $\cos E = -\frac{3}{\sqrt{34}}$, $\tan E = \frac{5}{3}$;

$\sin F = \frac{5}{\sqrt{34}}$, $\cos F = \frac{3}{\sqrt{34}}$, $\tan F = \frac{5}{3}$

b)



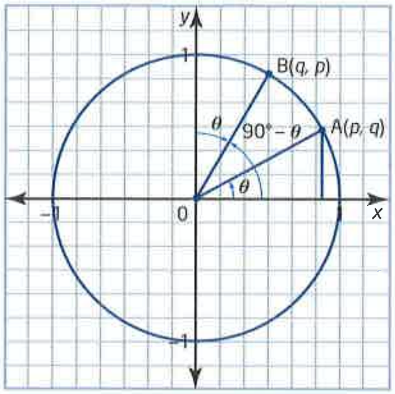
$\angle E = 239°$, $\angle F = 59°$

13. Answers may vary.

14. a) $\sqrt{p^2 + q^2}$

b) $\sin\theta = \frac{q}{\sqrt{p^2 + q^2}}$, $\cos\theta = \frac{p}{\sqrt{p^2 + q^2}}$, $\tan\theta = \frac{q}{p}$

c)



d) $\sin (90° - \theta) = \dfrac{p}{\sqrt{p^2 + q^2}}$,

$\cos (90° - \theta) = \dfrac{q}{\sqrt{p^2 + q^2}}$,

$\tan (90° - \theta) = \dfrac{p}{q}$

e) The sine and cosine ratios are reversed, while the two tangent ratios are reciprocals of each other.

15. 90°, 40 m

17. $\sqrt{3}s$

18. $\sqrt{2 + \sqrt{2}}\ell$

19. Answers may vary.

20. A **21.** C **22.** C

4.3 Reciprocal Trigonometric Ratios, pages 246–248

1. a) $\sin 20° = 0.342$, $\cos 20° = 0.940$, $\tan 20° = 0.364$, $\csc 20° = 2.924$, $\sec 20° = 1.064$, $\cot 20° = 2.747$

b) $\sin 42° = 0.669$, $\cos 42° = 0.743$, $\tan 42° = 0.900$, $\csc 42° = 1.494$, $\sec 42° = 1.346$, $\cot 42° = 1.111$

c) $\sin 75° = 0.966$, $\cos 75° = 0.259$, $\tan 75° = 3.732$, $\csc 75° = 1.035$, $\sec 75° = 3.864$, $\cot 75° = 0.268$

d) $\sin 88° = 0.999$, $\cos 88° = 0.035$, $\tan 88° = 28.636$, $\csc 88° = 1.001$, $\sec 88° = 28.654$, $\cot 88° = 0.035$

e) $\sin 153° = 0.454$, $\cos 153° = -0.891$, $\tan 153° = -0.510$, $\csc 153° = 2.203$, $\sec 153° = -1.122$, $\cot 153° = -1.963$

f) $\sin 289° = -0.946$, $\cos 289° = 0.326$, $\tan 289° = -2.904$, $\csc 289° = -1.058$, $\sec 289° = 3.072$, $\cot 289° = -0.344$

2. $\sin 315° = -\dfrac{1}{\sqrt{2}}$, $\cos 315° = \dfrac{1}{\sqrt{2}}$, $\tan 315° = -1$,

$\csc 315° = -\sqrt{2}$, $\sec 315° = \sqrt{2}$, $\cot 315° = -1$

3. $\sin 120° = \dfrac{\sqrt{3}}{2}$, $\cos 120° = -\dfrac{1}{2}$, $\tan 120° = -\sqrt{3}$,

$\csc 120° = \dfrac{2}{\sqrt{3}}$, $\sec 120° = -2$, $\cot 120° = -\dfrac{1}{\sqrt{3}}$

4. $\sin 270° = -1$, $\cos 270° = 0$, $\tan 270°$ is undefined, $\csc 270° = -1$, $\sec 270°$ is undefined, $\cot 270° = 0$

5. a) 42° **b)** 53° **c)** 67°

d) 63° **e)** 41° **f)** 53°

g) no such angle, because the cosecant ratio is positive in the first quadrant

h) no such angle, because the reciprocal of a secant ratio with value $\frac{2}{5}$ is a cosine ratio with value $\frac{5}{2}$, which exceeds 1, the cosine ratio's maximum value

6. 135°, 225°

7. 135°, 315°

8. a) $\sin \theta = \dfrac{12}{13}$, $\cos \theta = -\dfrac{5}{13}$, $\tan \theta = -\dfrac{12}{5}$,

$\csc \theta = \dfrac{13}{12}$, $\sec \theta = -\dfrac{13}{5}$, $\cot \theta = -\dfrac{5}{12}$

b) $\sin \theta = -\dfrac{3}{5}$, $\cos \theta = -\dfrac{4}{5}$, $\tan \theta = \dfrac{3}{4}$,

$\csc \theta = -\dfrac{5}{3}$, $\sec \theta = -\dfrac{5}{4}$, $\cot \theta = \dfrac{4}{3}$

c) $\sin \theta = \dfrac{15}{17}$, $\cos \theta = -\dfrac{8}{17}$, $\tan \theta = -\dfrac{15}{8}$,

$\csc \theta = \dfrac{17}{15}$, $\sec \theta = -\dfrac{17}{8}$, $\cot \theta = -\dfrac{15}{8}$

d) $\sin \theta = -\dfrac{7}{25}$, $\cos \theta = \dfrac{24}{25}$, $\tan \theta = -\dfrac{7}{24}$,

$\csc \theta = -\dfrac{25}{7}$, $\sec \theta = \dfrac{25}{24}$, $\cot \theta = -\dfrac{24}{7}$

e) $\sin \theta = \dfrac{40}{41}$, $\cos \theta = \dfrac{9}{41}$, $\tan \theta = \dfrac{40}{9}$,

$\csc \theta = \dfrac{41}{40}$, $\sec \theta = \dfrac{41}{9}$, $\cot \theta = \dfrac{9}{40}$

f) $\sin \theta = -\dfrac{3}{\sqrt{13}}$, $\cos \theta = -\dfrac{2}{\sqrt{13}}$, $\tan \theta = \dfrac{3}{2}$,

$\csc \theta = -\dfrac{\sqrt{13}}{3}$, $\sec \theta = -\dfrac{\sqrt{13}}{2}$, $\cot \theta = \dfrac{2}{3}$

g) $\sin \theta = -\dfrac{3}{\sqrt{34}}$, $\cos \theta = \dfrac{5}{\sqrt{34}}$, $\tan \theta = -\dfrac{3}{5}$,

$\csc \theta = -\dfrac{\sqrt{34}}{3}$, $\sec \theta = \dfrac{\sqrt{34}}{5}$, $\cot \theta = -\dfrac{5}{3}$

h) $\sin \theta = \dfrac{7}{\sqrt{53}}$, $\cos \theta = -\dfrac{2}{\sqrt{53}}$, $\tan \theta = -\dfrac{7}{2}$,

$\csc \theta = \dfrac{\sqrt{53}}{7}$, $\sec \theta = -\dfrac{\sqrt{53}}{2}$, $\cot \theta = -\dfrac{2}{7}$

9. $\sin P = \dfrac{15}{17}$, $\cos P = \dfrac{8}{17}$, $\tan P = \dfrac{15}{8}$;

$\csc P = \dfrac{17}{15}$, $\sec P = \dfrac{17}{8}$, $\cot P = \dfrac{15}{8}$

10. 12°, 168°

11. 102°, 258°

12. 162°, 342°

13. 124°

14. Answers may vary.

15. 90°, 60 m

17. Answers may vary.

18. $\sin B = \dfrac{d}{\sqrt{c^2 + d^2}}$, $\cos B = -\dfrac{c}{\sqrt{c^2 + d^2}}$,

$\tan B = -\dfrac{d}{c}$, $c \neq 0$, $\csc B = \dfrac{\sqrt{c^2 + d^2}}{d}$, $d \neq 0$,

$\sec B = -\dfrac{\sqrt{c^2 + d^2}}{c}$, $c \neq 0$

19. $\sin A = -\dfrac{2\sqrt{t}}{t + 1}$, $t \geq 0$

20. a) 14 **b)** 23 **c)** 300 m²

d) Answers may vary. Sample answer: Because angle parking spaces extend into the street more than 3 m, more of the street would be taken up for parking.

e) 707 m², which is consistent with the prediction in part d) for the reason stated

21. a)

Answers may vary. Sample answer: Assume that the sun's rays are parallel when they reach Earth. This is valid, as the distance from Earth to the sun is so large. Also assume that the 800-km separation between the two cities is a linear measure, instead of an arc measure, even though the surface of Earth curves. This is also valid, because the measure of 800 km is small relative to Earth's dimensions.

b) 6370 km **c)** 6371.0 km **d)** 24° north

22. $\sqrt{22}$ **23.** 58 **24.** 65 km

4.4 Problems in Two Dimensions, pages 254–258

1. a) Since the triangle is a right triangle, primary trigonometric ratios are the most appropriate tool.

b) Since two angles and a side are given, the most appropriate tool is the sine law.

c) Since two angles and a side are given, the most appropriate tool is the sine law.

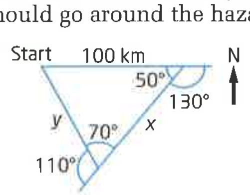
d) Since two sides and their contained angle are given, the most appropriate tool is the cosine law.

2. a) 6.3 cm **b)** 5.2 m **c)** 10.6 cm **d)** 16.8 km

3. 53°

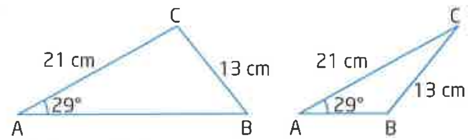
4. Answers may vary. Sample answer: The golfer needs to hit the ball at least 67 m to clear the water. Since he can only hit the ball 60 m, he should go around the hazard.

5. a)



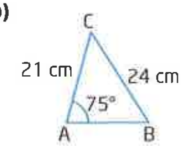
b) 274 km

6. a)



26.5 cm, 10.3 cm

b)



18.3 m. Explanations may vary. Sample explanation: The second diagram cannot be solved because the sum of the given angle and the calculated value of the second angle (an obtuse angle) is greater than 180°.

7. $20\sqrt{181 + 90\sqrt{2}}$ m

8. $50(\sqrt{3} - 1)$ m

9. 32.2°

10. 38 m

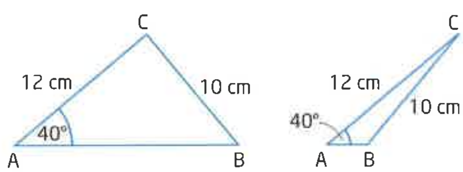
11. a) 42 km **b)** 22.7° east of south

12. $25(\sqrt{2} - 1)$ m

13. Answers may vary.

14. a) Answers may vary. Sample answer: Since the given angle is not contained within the two given sides, two triangles are possible.

b)



c) 15.6 cm, 2.8 cm; both are valid solutions

d) Answers may vary. Sample answer: There is only one valid solution; it is not possible to draw the second triangle.

e) 18.4 cm

15. a) Answers may vary. Sample answer: There is no solution; the 7-cm-long side is not long enough to enclose a triangle.

b) 7.7135 cm

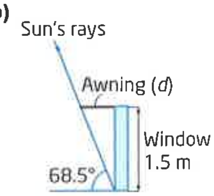
16. Answers may vary.

17. 55°, 50 m

18. 74 min

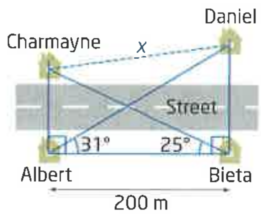
19. a) 23.5° is the tilt angle of Earth as it spins on its axis.

b)



c) 0.59 m **d)** 1.27 m

20. a)



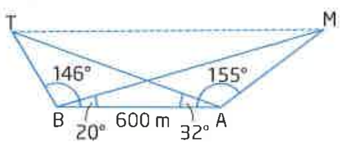
b) Answers may vary. Sample answer: You can use primary trigonometric ratios to determine the distance from Albert's house to Charmayne's house, and do the same for the distance from Bieta's house to Daniel's house. The difference between these two measures represents the length of one of the legs of a right triangle, with the other leg being 200 m in length. Use the Pythagorean theorem to find the hypotenuse, which represents the required distance.

c) 202 m

22. Answers may vary.

23. 3.8°

24. a)



b) Answers may vary. Sample answer: Use the known angles to determine the measures of the missing angles in the triangle. Use the sine law in each triangle to find the missing side lengths, and then use the cosine law to find the distance from T to M.

c) 11 073 m

25. a) 10:14 a.m., 1:46 p.m.

b) Answers may vary. Sample answer: The shadow can be on either side of the flagpole, which means that an equal number of hours and minutes on each side of noon will create the required shadow.

26. Answers may vary.

27. B

28. 7 min 45 s

29. B

4.5 Problems in Three Dimensions, pages 265–269

1. 38°

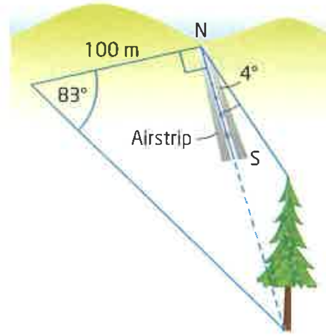
2. a) $\sqrt{3} + 1$ m

b) $2 + 2\sqrt{3}$ m

c) $4\sqrt{2 + \sqrt{3}}$ m

3. 47.8°

4. a)



b) Answers may vary. Sample answer: Use the tangent ratio to calculate the distance from the departure point of the runway to the base of the tree: 814.4 m. **c)** 57 m

d) The height is sufficient to clear the trees but not by enough to be safe.

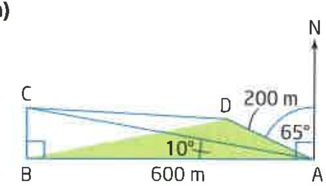
5. 35.3°

6. a) 3.3 km **b)** 3.3 km **c)** 173 m

d) Answers may vary. Sample answer: Assume that the minimum height above point C where the circle exists does not affect the radius of the circle. This is a reasonable assumption based on the calculated values.

7. Jodi, in approximately 3 min 10 s (versus approximately 3 min 33 s for Leanna)

8. a)



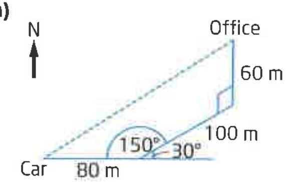
b) 9 min 29 s

9. a)–b) Answers may vary.

c) 109.5°

d) Answers may vary.

10. a)



b) 184 m

11. $\theta = 45°$, 50 m

12. 56.8 m

13.–14. Answers may vary.

15. $\sqrt{3}R - 2r$

16. $\sqrt{\frac{2}{3}}s$

17. 49.1°

18. a) Answers may vary. Sample answer: The vertices of the triangle are not on a flat surface, so the sum of the angles does not equal 180°. In other words, the shape is not really a triangle.

b) The sum of the angles will be greater than 180°, as Earth's surface is convex.

19. Answers may vary. Sample answers:

a) The intersection of the two spheres is a circle, as the two surfaces contact each other around a common radius.

b) The third satellite creates two points of intersection with the circle created by the intersection of the first two spheres.

c) The final satellite eliminates one of the two points of intersection of the three spheres, narrowing the location to a single point.

20. 20 min 47 s, by the cutter

21. B **22.** C

4.6 Trigonometric Identities, pages 273–275

1. Answers may vary.

2. The graph is a horizontal line through $y = 1$.

3.–10. Answers may vary.

11. Answers may vary. Sample answer: No, as the graph only shows that the expression is true for the range of values chosen on the display of the calculator. You would need to show that it is true for all values in the domain.

12.–13 Answers may vary.

14. 30°, 20 m

15.–16. Answers may vary.

16. Answers may vary.

18. Answers may vary. Sample answers:

a) Since the second equation traces over the first when graphed, it appears that the equation is an identity.

b) $\sin 60° = 2 \sin 30° \cos 30°$,
$\sin 90° = 2 \sin 45° \cos 45°$,
$\sin 180° = 2 \sin 90° \cos 90°$. Therefore, the equation is true for the given values of the angle.

19. Answers may vary. Sample answers:

a) Since the second equation traces over the first when graphed, it appears that the equation is an identity.

b) $\cos 30° = \sin 60°$, $\cos 45° = \sin 45°$, $\cos 60° = \sin 30°$. Therefore, the equation is true for the given values of the angle.

c) From the unit circle, it can be seen that the side opposite θ is the adjacent side for $90° - \theta$.

d) $\sin \theta = \cos (90° - \theta)$, $\tan \theta = \dfrac{1}{\tan(90° - \theta)}$

20. Answers may vary. Sample answers:

a) Since the second equation traces over the first when graphed, it appears that the equation is an identity.

b) $\sin 30° = \sin 150°$, $\sin 45° = \sin 135°$, $\sin 90° = \sin 90°$. Therefore, the equation is true for the given values of the angle.

c) From the unit circle and using the CAST rule, it can be seen that the relation is true.

d) $\cos \theta = -\cos (180° - \theta)$

21. C **22.** C **23.** D

Chapter 4 Review, pages 276–277

1. $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\tan 210° = \frac{1}{\sqrt{3}}$

2. $5(\sqrt{3} - \sqrt{2})$ m, 1.6 m

3. a) $\sin \theta = \frac{12}{13}$, $\cos \theta = -\frac{5}{13}$, $\tan \theta = -\frac{12}{5}$

b) $\sin \theta = -\frac{4}{5}$, $\cos \theta = \frac{3}{5}$, $\tan \theta = -\frac{4}{3}$

c) $\sin \theta = -\frac{4}{5}$, $\cos \theta = \frac{3}{5}$, $\tan \theta = -\frac{4}{3}$

d) $\sin \theta = -\frac{3}{\sqrt{13}}$, $\cos \theta = -\frac{2}{\sqrt{13}}$, $\tan \theta = \frac{3}{2}$

e) $\sin \theta = -\frac{5}{\sqrt{26}}$, $\cos \theta = \frac{1}{\sqrt{26}}$, $\tan \theta = -5$

f) $\sin \theta = \frac{4}{\sqrt{65}}$, $\cos \theta = -\frac{7}{\sqrt{65}}$, $\tan \theta = -\frac{4}{7}$

4. a) $\cos A = \frac{3}{5}$, $\tan A = \frac{4}{3}$

b) $\sin B = -\frac{15}{17}$, $\tan B = -\frac{15}{8}$

c) $\sin C = \frac{12}{13}$, $\cos C = -\frac{5}{13}$

d) $\cos D = -\frac{\sqrt{33}}{7}$, $\tan D = \frac{4}{\sqrt{33}}$

5. a) 194°, 346° **b)** 37°, 323° **c)** 212°, 32°

6. a) $\sin \theta = -0.9231$, $\cos \theta = -0.3846$, $\tan \theta = 2.4$, $\csc \theta = -1.0833$, $\sec \theta = -2.6$, $\cot \theta = 0.4167$

b) $\sin \theta = 0.6$, $\cos \theta = -0.8$, $\tan \theta = -0.75$, $\csc \theta = 1.6667$, $\sec \theta = -1.25$, $\cot \theta = -1.3333$

c) $\sin \theta = 0.8824$, $\cos \theta = 0.4706$, $\tan \theta = 1.875$, $\csc \theta = 1.1333$, $\sec \theta = 2.125$, $\cot \theta = 0.5333$

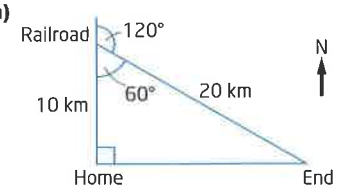
d) $\sin \theta = -0.96$, $\cos \theta = 0.28$, $\tan \theta = -3.4286$, $\csc \theta = -1.0417$, $\sec \theta = 3.5714$, $\cot \theta = -0.2917$

7. 104°, 256°

8. a) Yes; as there is a unique value for the angle.

b) 270°

9. a)

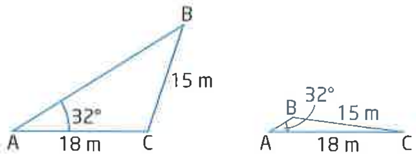


b) cosine law, since two sides and their contained angle are given

c) $10\sqrt{3}$ km, 17.3 km

10. a) Answers may vary.

b)



c) 27 m, 4 m

11. 321 m

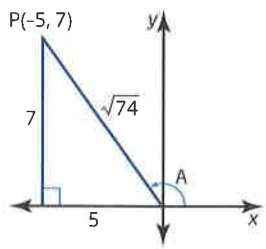
12. a) 11.7 km **b)** 8.2°

13.–15. Answers may vary.

Chapter 4 Practice Test, pages 278–279

1. B **2.** D **3.** D **4.** D **5.** C

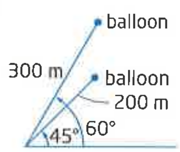
6. a)



b) $\sin A = \frac{7}{\sqrt{74}}$, $\cos A = -\frac{5}{\sqrt{74}}$, $\tan A = -\frac{7}{5}$,

$\csc A = \frac{\sqrt{74}}{7}$, $\sec A = -\frac{\sqrt{74}}{5}$

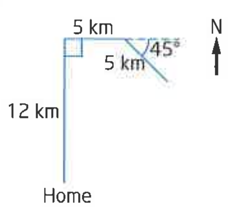
7. a)



b) $50(3 - 2\sqrt{2})$ m

c) The horizontal distance at the initial point (300 m of cable at an angle of 60°) is 150 m. At the second point (200 m of cable at an angle of 45°), the horizontal distance is 141.4 m. Therefore, the balloon moves horizontally toward the tether point.

8. a)



b) Answers may vary. Sample answer: Once the diagram is split into its component triangles, use the primary trigonometric ratios and the cosine law to determine the distance from home.

c) 12.0 km **d)** 45.2° west of south

9. 3.6 km

10. 2.3 km

11.–13 Answers may vary.

Chapter 5

Prerequisite Skills, pages 282–283

1. a) 2.6 cm **b)** 5 cm **c)** 10 cm

2. a) $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$,

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$

b) $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$, $\sin 150° = \frac{1}{2}$,

$\cos 150° = -\frac{\sqrt{3}}{2}$, $\sin 210° = -\frac{1}{2}$,

$\cos 210° = -\frac{\sqrt{3}}{2}$, $\sin 240° = -\frac{\sqrt{3}}{2}$,

$\cos 240° = -\frac{1}{2}$, $\sin 300° = -\frac{\sqrt{3}}{2}$, $\cos 300° = \frac{1}{2}$,

$\sin 330° = -\frac{1}{2}$, $\cos 330° = \frac{\sqrt{3}}{2}$

c) $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\sin 135° = \frac{1}{\sqrt{2}}$,

$\cos 135° = -\frac{1}{\sqrt{2}}$, $\sin 225° = -\frac{1}{\sqrt{2}}$,

$\cos 225° = -\frac{1}{\sqrt{2}}$, $\sin 315° = -\frac{1}{\sqrt{2}}$,

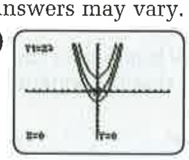
$\cos 315° = \frac{1}{\sqrt{2}}$

d) $\sin 0° = 0$, $\cos 0° = 1$, $\sin 90° = 1$, $\cos 90° = 0$, $\sin 180° = 0$, $\cos 180° = -1$, $\sin 270° = -1$, $\cos 270° = 0$

3. domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

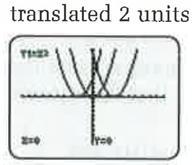
4. Answers may vary. Sample answer: $y = \sqrt{25 - x^2}$

5. a)



b) The second function is translated 3 units up from the first function and the third function is translated 2 units down from the first function.

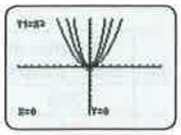
6. a)



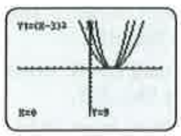
b) The second function is translated 3 units to the right of the first function and the third function is translated 2 units to the left of the first function.

7. a) $y = (x + 5)^2 - 3$ **b)** $y = (x - 4)^2 + 7$

8. The parabola is translated 2 units to the right and 2 units up. The equation of the transformed function is $y = (x - 2)^2 + 2$. The vertex of the transformed parabola is (2, 2).

9. a) 

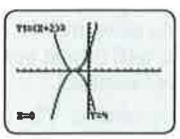
b) The second function is stretched vertically by a factor of 2 compared to the first function and the third function is compressed vertically by a factor of $\frac{1}{2}$ compared to the first function.

10. a) 

b) The second function is stretched vertically by a factor of 2 compared to the first function and the third function is compressed vertically by a factor of $\frac{1}{2}$ compared to the first function.

11. $y = 4x^2$

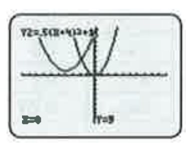
12. $y = 9(x - 4)^2$

13. a) 

b) Answers may vary. Sample answer: The second function is a reflection in the x-axis of the first function.

14. a) i) $y = (x + 2)^2$ **ii)** $y = (x - 2)^2$

b) Answers may vary. Sample answer: The two functions are reflections of each other in the y-axis.

15. a) $y = \frac{1}{2}(x + 4)^2 + 1$ **b)** 

16. The function has been stretched vertically by a factor of 3, reflected in the x-axis, and translated 4 units to the right and 8 units up.

17. $k = 12$

18. $k = 10\ 800$

5.1 Modelling Periodic Behaviour, pages 290–293

1. a) periodic; the pattern of y-values repeats on a regular basis

b) not periodic; the pattern of y-values does not repeat on a regular basis

c) periodic; the pattern of y-values repeats on a regular basis

d) periodic; the pattern of y-values repeats on a regular basis

2. a) amplitude 1.5, period 5

c) amplitude 1, period 6

d) amplitude 0.75, period 3.5

3.–4. Answers may vary.

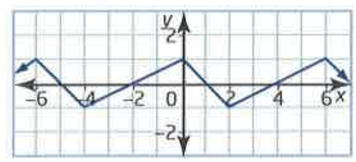
5. Answers may vary. Sample answer: My graphs have common characteristics with those of my classmates, but the actual graphs are different, because an infinite number of periodic functions can be drawn with each set of characteristics.

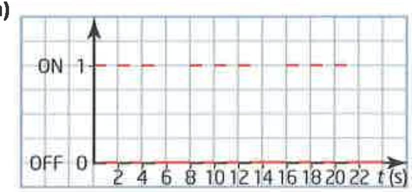
6. a) -3 **b)** 2 **c)** 8

d) Answers may vary. Sample answer: It is not possible to determine because the x-value of 40 is not a multiple of the period added to or subtracted from one of the given x-values of the given points on the function.

7. Answers may vary.

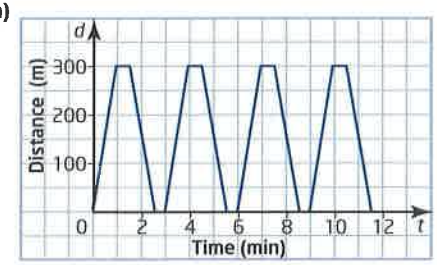
8. Answers may vary. Sample answer: No, because the two points may not be in the same relative position on the periodic function. In the graph shown, $f(-2) = f(1)$. However, the period is 6, not $1 - (-2) = 3$.



9. a) 

b) It is periodic because the pattern of y-values repeats on a regular basis.

c) 8 s **d)** 0.5

10. a) 

b) 3 min **c)** 150 m

11. Answers may vary. Sample answers:

a) This could possibly be periodic because the seasons will influence the prices of produce. However, factors such as drought could cause the prices to not follow a periodic function.

b) Interest rates tend to be governed by varying economic conditions that are not periodically predictable.

c) The phases of the moon are such that the illuminated portion of the moon is periodic.

d) As long as the breathing is normal, the function that describes the volume of air in the lungs as a function of time is periodic.

12. Answers may vary. Sample answer: Solar activity is periodic. This rise and fall in sunspot counts varies in a cyclical way. The length of the cycle is about 11 years on average.

13. Answers may vary. Sample answer: No, based on the strict definition of a periodic function. However, science describes some motions, such as damped harmonic motion, as "periodic" even if amplitude is either continuously increasing or decreasing. See the example in question 23 on page 293.

14. **a)** approximately 12 h 20 min
b) 1.3 m **c)** 1:00 a.m.

15. Answers may vary. Sample answers:
a) The distance will start at zero, increase to a maximum value of 12 800 km, and then decrease to zero as Quito returns to its starting position. As the rotation continues, the same values of d will exist as existed during the first rotation, thus forming a repeating pattern.
b) 24 h **c)** 6400 km
d) The cosine law is an appropriate tool, since y, representing the lengths of the two sides of the triangle, is constant, and it is the angle that is changing. Select a suitable step, such as 1 h, and determine the angle between the two sides of the triangle. Use the cosine law to calculate the value of d. Continue for 24 h.

16. Answers may vary. Sample answers:
a) In Ottawa, Ontario, the average maximum monthly temperature is about 21 °C (in July) and the average minimum monthly temperature is about −11 °C (in January). The amplitude of this pattern is 16 °C, being half the difference between the maximum value and the minimum value.
b) The period of the function will be 12 months, because the weather patterns in an area usually change in the same pattern on a yearly basis.

17. Answers may vary.

18. **a)** The wave form of the note played has a regular repeating pattern.
b) 0.002 s
c) As the frequency of the note increases, the period decreases. The frequency is the reciprocal of the period.

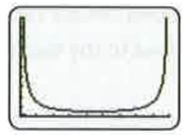
19. Answers may vary.

20. Answers may vary. Sample answers:
a) Because of the tilt of Earth on its axis, hours and minutes of daylight anywhere on any given day depend on the position of Earth as it orbits the sun. Because a complete orbit takes 1 year (about 365 days), the data will show a periodic pattern over several years through successive Earth orbits.
b) just over 14 h
c) approximately 12 h 30 min

22. **a)** 1 s, 200 m
b)

Angle of Beam (degrees)	Time (seconds)	Distance of Beam (metres)
30	1	200
60	2	115.5
90	3	100
120	4	115.5
150	5	200

c) As the beam approaches 180°, the distance that it travels becomes very large. When it reaches 180°, it no longer reaches the cliff.
d) After 6 s, the beam will reach the 0° position. After this, it will again hit the cliff.
e) The graph shows the function from 0° to 180°. From 180° to 360°, the light does not hit the cliff.

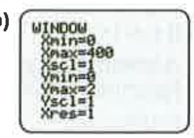


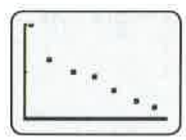
f) Answers may vary. Sample answer: It is periodic, since the pattern will repeat every 12 s (which is the period of the function).
g) Answers may vary. Sample answer: The pattern has no amplitude because there is no maximum value in the function.

23. Answers may vary. Sample answers:
a)

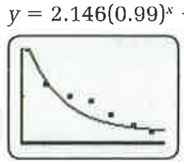
x	y
17	2
70	1.25
135	1
195	0.9
250	0.6
315	0.4
365	0.25

b)





c) The data seem to follow an exponential model.
d) $y = 2.146(0.99)^x + 0.25$
e)

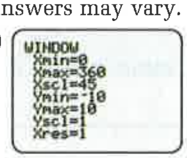


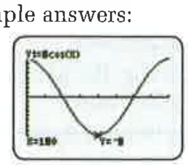
The fit is not perfect, but it seems to generally show the behaviour of the data.

24. C **25.** B **26.** C **27.** A

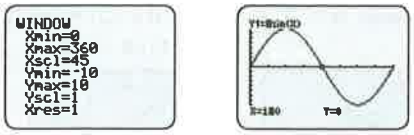
5.2 The Sine Function and the Cosine Function, pages 299–301

1. Answers may vary. Sample answers:
a)

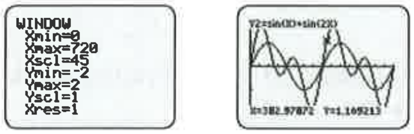




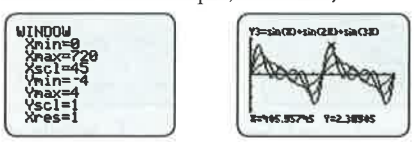
A cosine function models the horizontal displacement, because the horizontal displacement starts at 8 m and decreases to 0 at 90°, a characteristic of the cosine function.

b) 

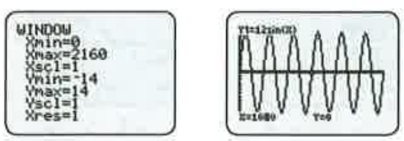
A sine function models the vertical displacement, because the vertical displacement starts at 0 and moves through to a maximum at 90°, a characteristic of the sine function.

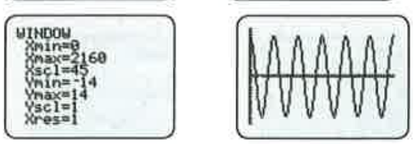
2. a) 

b) Answers may vary. Sample answer: The more complex function has more detail in its wave form, creating a more complex sound. They both have a period of 360° and all x-intercepts of $y = \sin x$ are also x-intercepts of the more complex function (the more complex function has more x-intercepts, however).

c) 

Answers may vary. Sample answer: Again, the second harmonic is far more complex than the $y = \sin x$ function, with more x-intercepts and a more complex wave form, even more complex than the $y = \sin x + \sin 2x$ wave form. The second harmonic and $y = \sin x$ have the same period and all x-intercepts for $y = \sin x$ are shared with the second harmonic.

3. a) 

b) 

c) 6 cycles **d)** 72 cycles

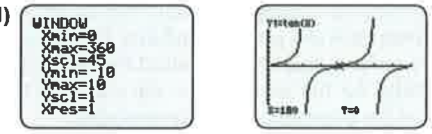
4. a)

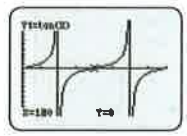
x	tan x
0	0
10	0.176
20	0.364
30	0.577
40	0.839
50	1.192
60	1.732
70	2.747

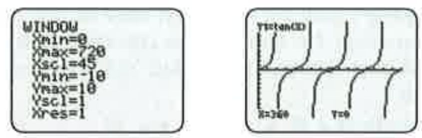
x	tan x
75	3.732
80	5.671
85	11.430
86	14.301
87	19.081
88	28.636
89	57.290

b) Answers may vary. Sample answer: As the angle gets closer to 90°, the tangent function value gets larger and larger. This means that as x approaches 90°, the tangent function value approaches infinity. At a value of 90°, the tangent function is undefined.

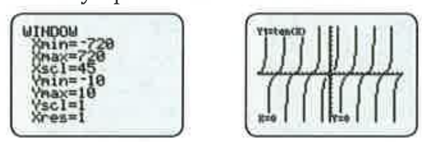
c) Answers may vary, as students will extend their tables of values differently, but should all lead to the graph in question 4d).

d) 

e) 

f) 

The asymptotes will be located at 450° and 630°.

g) 

The asymptotes will be located at −90°, −270°, −450°, and −630°.

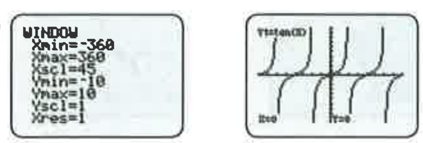
h) Answers may vary. Sample answer: Yes, the tangent function is periodic, as the y-values repeat in a periodic pattern. The period of the tangent function is 180°.

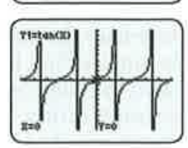
5. Answers may vary. Sample answers:

a) All vertical lines that can be drawn intersect $y = \tan x$ at exactly one point. Therefore, it is a function.

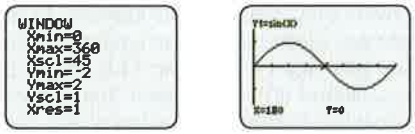
b) Since there is no maximum or minimum of a tangent function, there is no amplitude.

c) The tangent function is increasing on the intervals (0°, 90°), (90°, 270°), and (270°, 360°). The tangent function does not exist at 90° and 270°.

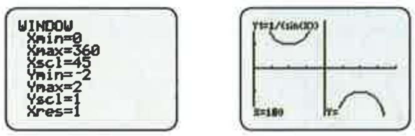
d) 

e) 

f) domain $\{x \in \mathbb{R}, x \neq 90 + 180n, n \in \mathbb{Z}\}$, range $\{y \in \mathbb{R}\}$

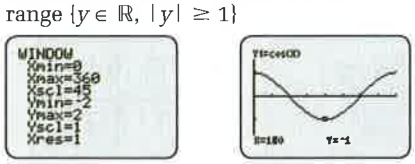
6. a) 

b) Answers may vary. Sample answer: The graph of $y = \csc x$ has asymptotes where the sine function is zero, has minimum values of 1 and maximum values of -1 in the appropriate areas between the asymptotes, and moves either upward toward positive infinity for the areas above the x-axis or downward toward negative infinity for the areas below the x-axis as the function approaches the asymptotes.

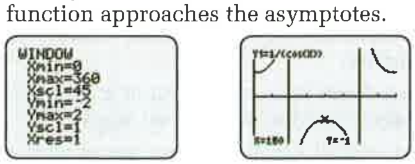
c) 

d) Answers may vary. Sample answer: $y = \csc x$ is a function. No vertical line can be drawn that will pass through more than one point on the line.

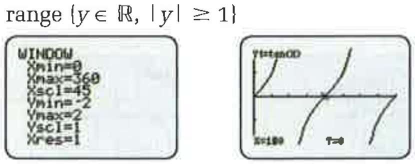
e) domain $\{x \in \mathbb{R}, x \neq 180n, n \in \mathbb{Z}\}$, range $\{y \in \mathbb{R}, |y| \geq 1\}$

7. a) 

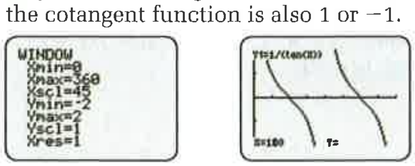
b) Answers may vary. Sample answer: The graph of $y = \sec x$ has asymptotes where the cosine function is zero, has minimum values of 1 and maximum values of -1 in the appropriate areas between the asymptotes, and moves either upward toward positive infinity for the areas above the x-axis or downward toward negative infinity for the areas below the x-axis as the function approaches the asymptotes.

c) 

e) domain $\{x \in \mathbb{R}, x \neq 90 + 180n, n \in \mathbb{Z}\}$, range $\{y \in \mathbb{R}, |y| \geq 1\}$

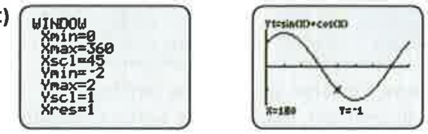
8. a) 

b) Answers may vary. Sample answer: The graph of $y = \cot x$ has asymptotes where the tangent function is zero. Where the tangent function has asymptotes, the cotangent function is zero. Anywhere the tangent function is either 1 or -1, the cotangent function is also 1 or -1.

c) 

e) domain $\{x \in \mathbb{R}, x \neq 180n, n \in \mathbb{Z}\}$, range $\{y \in \mathbb{R}\}$

9. a) 1 **b)** $(135 + 180n)°, n \in \mathbb{Z}$

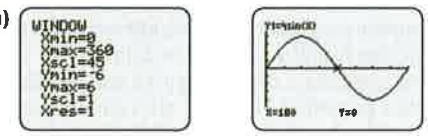
c) 

10. $0 \leq x < 45°$ or $225° < x \leq 360°$

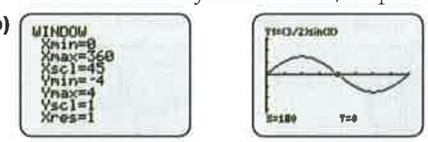
11. $-45°, 45°$

12. C

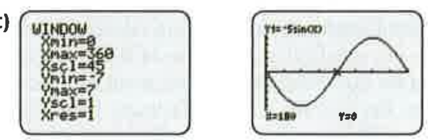
5.3 Transformations of Sine and Cosine Functions, pages 309–312

1. a) 

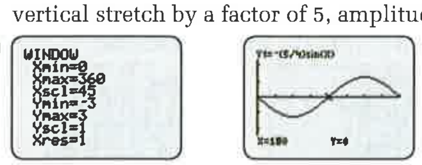
vertical stretch by a factor of 4, amplitude 4

b) 

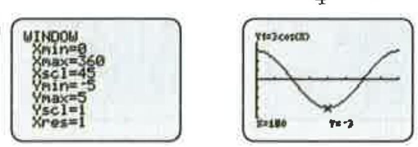
vertical stretch by a factor of $\frac{3}{2}$, amplitude $\frac{3}{2}$

c) 

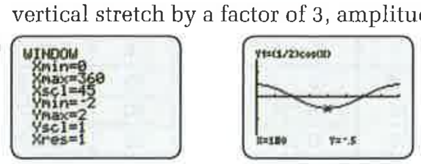
vertical stretch by a factor of 5, amplitude 5

d) 

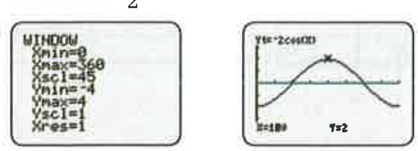
vertical stretch by a factor of $\frac{5}{4}$, amplitude $\frac{5}{4}$

2. a) 

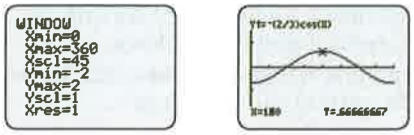
vertical stretch by a factor of 3, amplitude 3

b) 

vertical compression by a factor of $\frac{1}{2}$, amplitude $\frac{1}{2}$

c) 

vertical stretch by a factor of 2, amplitude 2

d) 

vertical compression by a factor of $\frac{2}{3}$, amplitude $\frac{2}{3}$

3. a) horizontal compression by a factor of $\frac{1}{5}$, period 72°

b) horizontal stretch by a factor of $\frac{3}{2}$, period 540°

c) horizontal stretch by a factor of 6, period 2160°

d) horizontal stretch by a factor of 2, period 720°

e) no horizontal stretch, period 360°

f) horizontal compression by a factor of $\frac{1}{8}$, period 45°

g) horizontal compression by a factor of $\frac{1}{12}$, period 30°

h) horizontal stretch by a factor of $\frac{4}{3}$, period 480°

4. Answers may vary. Sample answers:

a) $y = 5 \sin 3x$; $y = 5 \cos [3(x + 90°)]$

b) $y = -3 \sin 4x$; $y = 3 \cos [4(x + 67.5°)]$

5. Answers may vary. Sample answers:

a) $y = 4 \cos 2x$; $y = 4 \sin [2(x + 45°)]$

b) $y = -2 \cos 3x$; $y = 2 \sin [3(x - 30°)]$

6. a) phase shift right 50°, vertical shift up 3

b) phase shift left 45°, vertical shift down 1

c) phase shift right 25°, vertical shift up 4

d) phase shift left 60°, vertical shift down 2

7. a) phase shift left 30°, no vertical shift

b) phase shift right 32°, vertical shift up 6

c) phase shift left 120°, vertical shift down 5

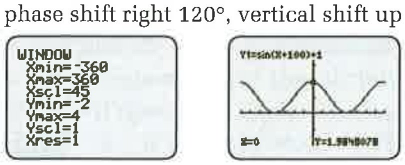
d) phase shift right 150°, vertical shift up 7

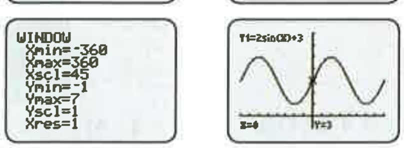
8. a) **i)** phase shift left 100°, vertical shift up 1

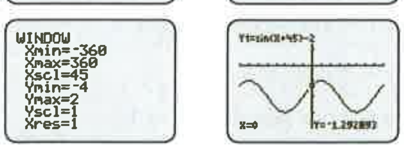
ii) no phase shift, vertical shift up 3

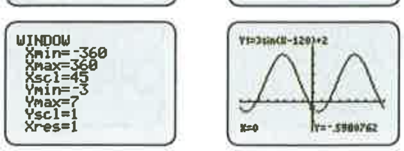
iii) phase shift left 45°, vertical shift down 2

iv) phase shift right 120°, vertical shift up 2

b) **i)** 

ii) 

iii) 

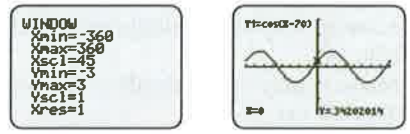
iv) 

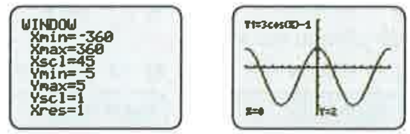
9. a) **i)** no vertical shift, amplitude 1

ii) vertical shift down 1, amplitude 3

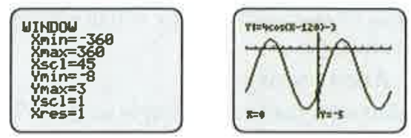
iii) vertical shift up 2, amplitude 1

iv) vertical shift down 3, amplitude 4

b) **i)** 

ii) 

iii) 

iv) 

10. a) 10 cm **b)** 90 cm **c)** 0.5 s

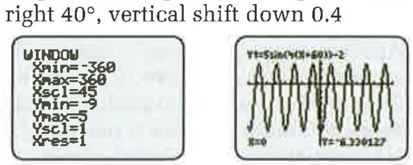
d) The value of k will change from 720 to 240, so the equation becomes $y = 40 \sin 240t + 50$.

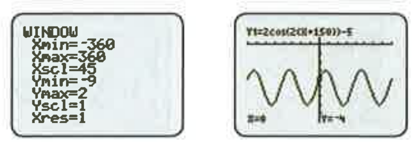
11. a) **i)** amplitude 5, period 90°, phase shift left 60°, vertical shift down 2

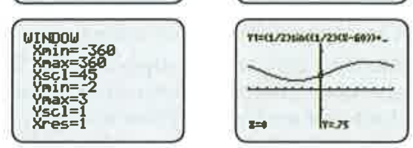
ii) amplitude 2, period 180°, phase shift left 150°, vertical shift down 5

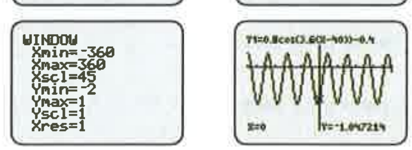
iii) amplitude 0.5, period 720°, phase shift right 60°, vertical shift up 1

iv) amplitude 0.8, period 100°, phase shift right 40°, vertical shift down 0.4

b) **i)** 

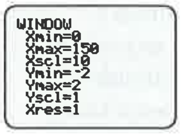
ii) 

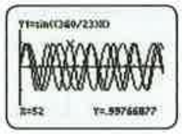
iii) 

iv) 

12. a) physical: $y = \sin\left(\frac{360°}{23}\right)t$; emotional: $y = \sin\left(\frac{360°}{28}\right)t$; intellectual: $y = \sin\left(\frac{360°}{33}\right)t$

b)





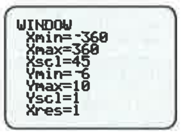
c) Answers may vary. Sample answer: days 6, 75, 120, 141

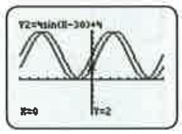
d) Answers may vary. Sample answer: days 20, 61, 89, 108, 132

13. a) $y = 4 \sin kt$ **b)** $y = \sin kt$

c) 16 m from the source

14. a) $y = 4 \sin x + 4$ **b)** 30° to the right



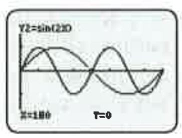


c) Answers may vary. Sample answer: No, because no matter what the period of the function is, the y-intercept of $\sin kx = 0$ is always 0 for any value of k.

15.–16. Answers may vary.

18. a) Answers may vary. Sample answer: There are five points of intersection, as there are three common zeros, and in the two intervals between the zeros, there is one more point of intersection each, for a total of five.

b)

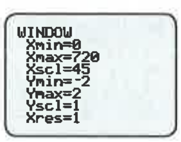


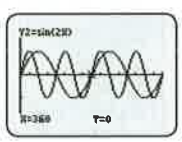
Yes. My prediction was correct.

c) $\left(60°, \frac{\sqrt{3}}{2}\right)$

d) Answers may vary. Sample answer: There are nine points of intersection, as there are five common zeros, and in the four intervals between the zeros, there is one more point of intersection each, for a total of nine.

e)





Yes. My prediction was correct.

f) Answers may vary. Sample answer: There are seven points of intersection. There are three common zeros. There are two points of intersection between each adjacent pair of zeros.

19. Answers may vary.

20. C

21. Answers may vary.

5.4 Graphing and Modelling With $y = a \sin[k(x - d)] + c$ and $y = a \cos[k(x - d)] + c$, pages 318–321

1. a) amplitude 5, period 90°, phase shift 25° to the right, vertical shift 3 units up

b) amplitude 2, period 20°, phase shift 40° to the left, vertical shift 5 units down

c) amplitude 3, period 3°, phase shift 30° to the right, vertical shift 2 units up

d) amplitude $\frac{3}{4}$, period 540°, phase shift 60° to the right, vertical shift $\frac{1}{2}$ unit up

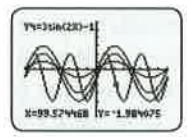
2. a) amplitude 3, period 72°, phase shift 45° to the right, vertical shift 4 units up

b) amplitude 2, period 15°, phase shift 80° to the left, vertical shift 1 unit down

c) amplitude 3, period 5°, phase shift 10° to the right, vertical shift 3 units up

d) amplitude $\frac{5}{2}$, period 480°, phase shift 40° to the right, vertical shift $\frac{1}{2}$ unit up

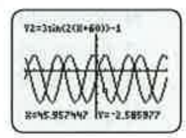
3. a) Answers may vary. Sample answer: Apply the amplitude of 3, apply the vertical shift of 1 unit down, and apply the horizontal compression by a factor of $\frac{1}{2}$.



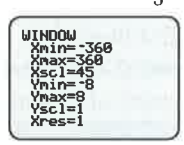
b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le x \le 1\}$

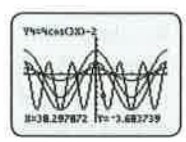
$g(x)$: domain $= \{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -4 \le x \le 2\}$

c) $h(x) = 3 \sin [2(x + 60°)] - 1$



4. a) Apply the amplitude 4, the vertical shift of 2 units down, and the horizontal compression by a factor of $\frac{1}{3}$.

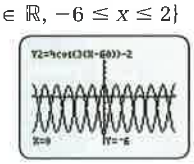




b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le x \le 1\}$

$g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -6 \le x \le 2\}$

c) $h(x) = 4 \cos [3(x - 60°)] - 2$



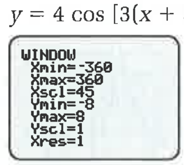
5. a) $y = 5 \sin [3(x + 30°)] - 2$ **b)** $y = 5 \cos 3x - 2$

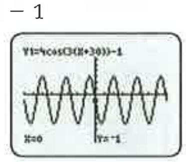
6. a) $y = \frac{1}{2} \sin\left[\frac{1}{2}(x + 180°)\right] + 1$

b) $y = \frac{1}{2} \cos \frac{1}{2}x + 1$

7. a) $y = 4 \cos [3(x + 30°)] - 1$

b)





8. a) amplitude 10, period 360°, phase shift right 45°, vertical shift 10 units up

b) maximum 20, minimum 0

c) 315°, 675°, 1035° **d)** 2.93

9. a) amplitude 5, period 180°, phase shift 30° to the right, no vertical shift

b) maximum 5, minimum −5

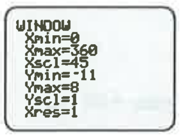
c) 75°, 165°, 255° **d)** 2.5

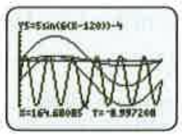
10. Answers may vary.

11. a) Apply the amplitude of 5, $y = 5 \sin x$; apply the vertical shift of 4 units down, $y = 5 \sin x - 4$; apply the horizontal compression by a factor of $\frac{1}{6}$, $y = 5 \sin 6x - 4$; and translate the function 120° to the right, $y = 5 \sin [6(x - 120°)] - 4$.

b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le x \le 1\}$
$g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -9 \le x \le 1\}$

c)

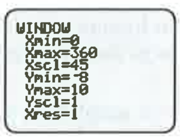


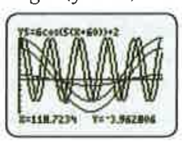


12. a) Apply the amplitude of 6, $y = 6 \cos x$; apply the vertical shift of 2 units up, $y = 5 \cos x + 2$; apply the horizontal compression by a factor of $\frac{1}{5}$, $y = 6 \cos 5x + 2$; and translate the function 60° to the left, $y = 6 \cos [5(x + 60°)] + 2$.

b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le x \le 1\}$
$g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -4 \le x \le 8\}$

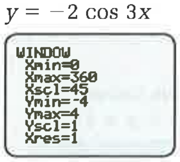
c)

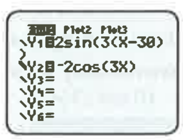


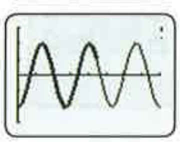


13. a) Answers may vary. Sample answer:
$y = -2 \cos 3x$

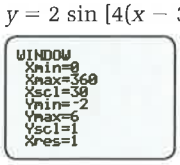
b)

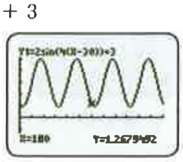




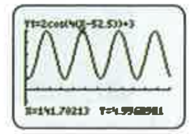


14. a) $y = 2 \sin [4(x - 30°)] + 3$



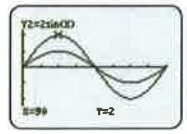


b) Answers may vary. Sample answer:
$y = 2 \cos [4(x - 52.5°)] + 3$

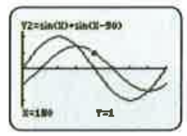


15. Answers may vary. Sample answers:

a) The graphs of $y = \sin x$ and $y = 2 \sin x$ have the same period, but different amplitudes, with $y = 2 \sin x$ having an amplitude that is twice the amplitude of $y = \sin x$.



b) The amplitude of the second curve is less than the amplitude of the first. The two curves look slightly phase-shifted from each other.



c) Answers may vary.

16. Answers may vary. Sample answers:

a) $y = 4 \cos 30x + 10$

b) $y = 4 \sin [30(x + 3)] + 10$

c) $y = 4 \sin [30(x - 3)] + 10$

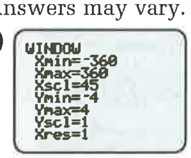
d) $y = 4 \cos [30(x - 6)] + 10$

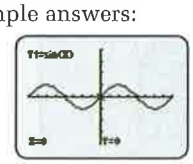
e) Answers may vary.

18. Answers may vary. Sample answer: No. This does not work if p is a zero of the function (that is, $q = 0$). It does work for all other points. If $q \neq 0$, then $a = \frac{q}{\sin p}$.

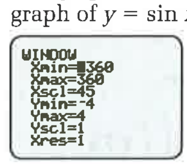
19. Answers may vary. Sample answers:

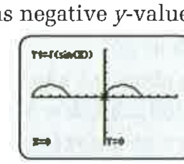
a)



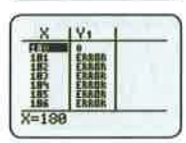


b) The graph will have missing sections where the graph of $y = \sin x$ has negative y-values.

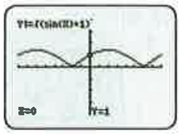




c)

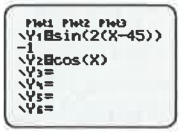


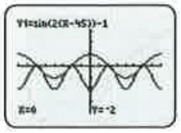
d) Since the entire function will now be above the x-axis, there will be no missing section of the curve.

e) 

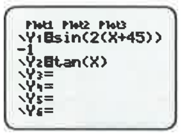
20. Answers may vary. Sample answers:

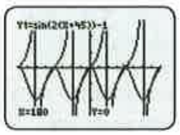
a) Three transformations are needed: a horizontal compression by a factor of $\frac{1}{2}$, a phase shift of 45° to the right, and a vertical shift of 1 unit down, giving the equation $y = \sin[2(x - 45°)] - 1$.





b) Three transformations are needed: a horizontal compression by a factor of $\frac{1}{2}$, a phase shift of 45° to the left, and a vertical shift of 1 unit down, giving the equation $y = \sin[2(x + 45°)] - 1$.

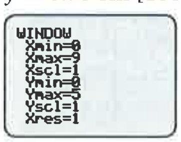


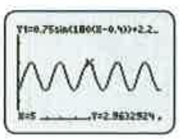


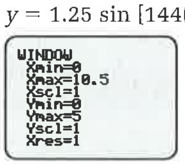
21. 75° **22.** C **23.** B

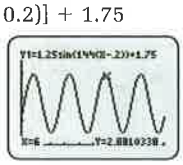
5.5 Data Collecting and Modelling, pages 328–332

In these answers, the data are modelled using sine functions. Answers using cosine functions are possible.

1. Answers may vary. Sample answers:
   a) maximum 3 m, minimum 1.5 m, amplitude 0.75
   b) 2.25 m up
   c) d is about 0.4 s to the right.
   d) period 2 s, $k = 180$
   e) $y = 0.75 \sin[180(x - 0.4)] + 2.25$
   f) 



2. Answers may vary. Sample answers:
   a) maximum 3 m, minimum 0.5 m, amplitude 1.25
   b) 1.75 m up
   c) d is about 0.2 s to the right.
   d) period 2.5 s, $k = 144$
   e) $y = 1.25 \sin[144(x - 0.2)] + 1.75$
   f) 



3. a) maximum 12 m, minimum 2 m
   b) high tide at 8:00 a.m.; low tide at 2:00 p.m.
   c) 11.3 m
   d) 12:46 a.m., 3:14 a.m., 12:46 p.m., 3:14 p.m.

4. a) maximum 13 000, minimum 3000
   b) maximum in October, at $t = 10$; minimum in April, at $t = 4$
   c) 12 330 people
   d) January 6, $t = 6$ days, and July 24, $t = 7$ months 24 days

5. Answers may vary. Sample answers:
   a) $y = 180.7 \sin[30(x - 2.6)] + 199.3$
   b) domain $\{x \in \mathbb{R}, 1 \le x \le 12\}$, range $\{y \in \mathbb{R}, 18 \le y \le 380\}$

6. a) $y = 10 \sin(x + 240°) + 14$
   b) $y = 10 \cos(x + 150°) + 14$
   c) The phase shift of the curves will be altered by an additional 30°. $y = 10 \sin(x + 210°) + 14$; $y = 10 \cos(x + 120°) + 14$

7. The new equations are as follows:
   a) $y = 10 \sin(x + 240°) + 16$
   b) $y = 10 \cos(x + 150°) + 16$

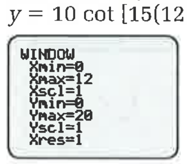
8. a) $\frac{1}{30}$ s
   b) maximum 70 mm, minimum −30 mm, amplitude 50 mm
   c) maximum at $\frac{1}{120}$ s, minimum at $\frac{1}{40}$ s
   d) 70 cm

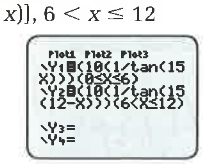
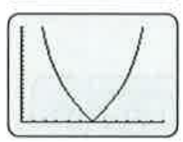
9. a) by a factor of $\sqrt{2}$
   b) You must decrease the length by a factor of 4.
   c) Answers may vary.
   d) The period would increase by a factor of $\sqrt{6}$.

10.–11. Answers may vary.

12. a) period 24 h. Answers may vary. Sample answer: Smog is often created from human activity that generally repeats from day to day, so a period of 24 h is appropriate.
    b) maximum 55, minimum −5, amplitude 30
    c) maximum at 10 a.m., minimum at 10 p.m.
    d) Answers may vary. Sample answer: The interval from 7:45 a.m. to 12:15 a.m.

13.–16. Answers may vary.

18. a) Answers may vary. Sample answer: $y = 10 \cot[15(12 - x)], 6 < x \le 12$
    b) 

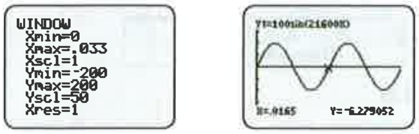



19. B **20.** D **21.** D

5.6 Use Sinusoidal Functions to Model Periodic Phenomena Not Involving Angles, pages 337–342

1. a) $y = 8 \sin[30(t - 2.5)]$
   b) high tide at 5:30 p.m., low tides at 11:30 a.m. and 11:30 p.m.

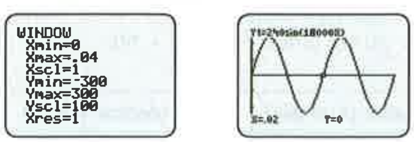
2. a) $y = 5 \cos 30t$
   b) Answers may vary. Sample answer: The graphs of $y = 5 \sin [30(t + 3)]$ and $y = 5 \cos 30t$ are the same.
3. Answers may vary. Sample answer: During a brownout, the voltage drops. This means that the amplitude of the function decreases. All other properties of the function remain the same. An example is $V = 100 \sin 21\,600t$, where the peak voltage drops from 170 V to 100 V.



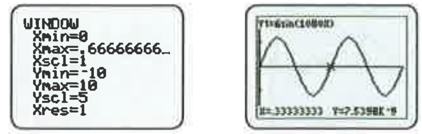
4. a) maximum 850, minimum 250, amplitude 300
   b) vertical shift up 550 c) phase shift 0
   d) 6 years, $k = 60$ e) $y = 300 \sin 60x + 550$
   f) Answers may vary, but all graphs should match the shape of the graph given.
5. a) maximum 26 m, minimum 2 m b) 12 h
   c)

   d) from 1:13 a.m. to 2:47 a.m., and from 1:13 p.m. to 2:47 p.m.
   e) Answers may vary. Sample answer: Factors such as wind conditions and boat traffic in the harbour need to be considered.
6. a) $y = 8 \sin [30(x - 2)] + 14$
   b)

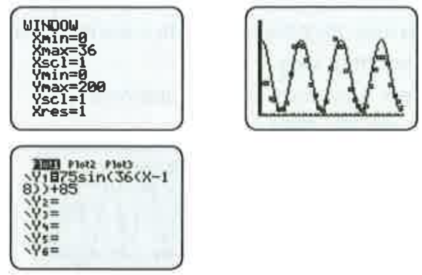
   c) Answers may vary. Sample answer: Since the water level has a minimum value of 6 m, and only 3 m of water are needed for a safe landing, it is never considered unsafe to land at this time of year.
7. a) $\frac{1}{50}$ s b) $k = 18\,000$ c) 240 V
   d) $V = 240 \sin 18\,000t$
   e)



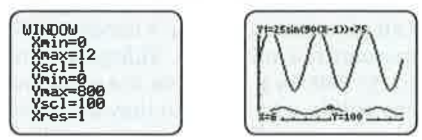
   Answers may vary. Sample answer: The domain is set from 0 s to $\frac{1}{25}$ s with each tick representing $\frac{1}{100}$ s. Each tick on the y-axis represents 100 V.
8. a) period $\frac{1}{3}$ s b) $k = 1080$
   c) 6 d) $V = 6 \sin 1080t$
   e)



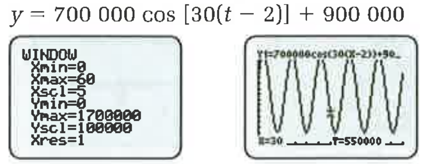
   Answers may vary. Sample answer: The domain is set from 0 to $\frac{2}{3}$ s with each tick representing $\frac{1}{6}$ s. Each tick on the y-axis represents 5 V.
9. Answers may vary. Sample answers:
   a) $y = 75 \sin [36(x - 18)] + 85$
   b)



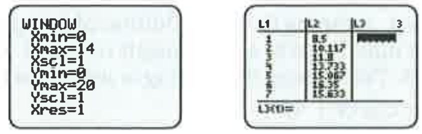
   c) Answers may vary.
10. Answers may vary. Sample answers:
   a) 2010, 2020, and 2030
   b) 2015, 2025, and 2035
   c) The variation in the maximum values is from approximately 123 to 163 and the variation in the minimum values is from approximately 8 to 12.
11. a) $N(t) = 25 \sin [90(t - 1)] + 75$
   b)

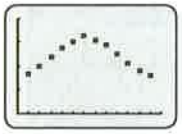


   c) Answers may vary. Sample answer: The phase shift to the right is because, as the prey population (the food supply) increases, the number of predators grows. This increase continues, until the predators begin to deplete the prey population (the food supply), which results in a decrease in the number of predators a short time later.
12. Answers may vary.
13. Answers may vary. Sample answers:
   a) $y = 700\,000 \cos [30(t - 2)] + 900\,000$
   b)



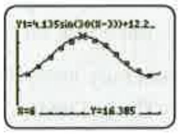
   c) 1 593 187 000 m^3
14. a)





b) Answers may vary. Sample answer: $y = 4.19 \sin [30(x + 8.98)] + 12.35$

c)



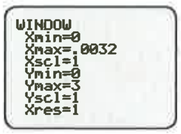
d) Answers may vary. Sample answer: 9 h 21 min

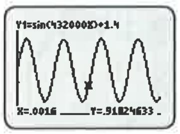
15. Answers may vary.

16. Answers may vary. Sample answers:

a) $y = \sin 432\,000t + 1.4$

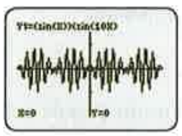
b)





c) 0.000 47 min, 0.000 78 min, or 0.028 s, 0.047 s

17. a)



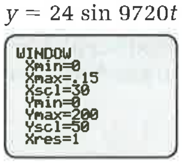
b) Answers may vary. Sample answer: $\sin x$ controls the amplitude of $\sin 10x$ such that the pattern is a grouped series of maximum and minimum values where, within the grouping, the amplitudes increase to a maximum before decreasing to a minimum. This pattern repeats twice in 360° as $y = \sin 10x$ has more wave forms within 0 to 360°, but they are controlled by the one wave form of $y = \sin x$ within this same domain.

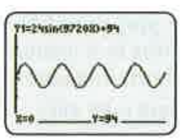
c) Answers may vary.

18. Answers may vary.

19. a) $y = 24 \sin 9720t + 94$

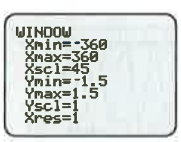
b)

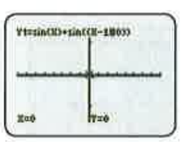




c) 0.0015 s and 0.0170 s

20. a)





b) Answers may vary. Sample answer: The two functions are completely out of phase with each other, meaning that the addition of two parts of the function will always result in a total y-value of 0. This is why the coating is non-reflective.

c) $y = \cos (x + 90°)$

21. Answers may vary.

22. D **23.** 121

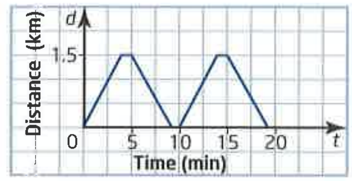
Chapter 5 Review, pages 344–345

1. a) The function is periodic because the y-values repeat in a regular pattern.

b) 3 **c)** maximum 5, minimum 1

d) 2 **e)** 360°

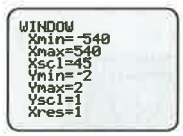
2. a)

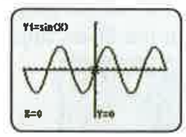


b) 0.75 km

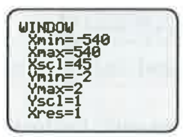
c) Answers may vary. Sample answer: It would take less time to get to and from the airport and parking lot, so more cycles would exist in a 20-min period. This would not affect the portions of the graph associated with the stops at the terminal and parking lot.

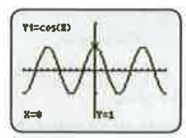
3.





4.

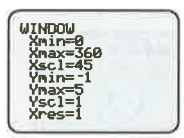


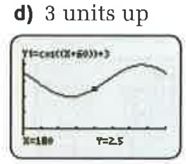


5. a) 1 **b)** 360°

c) 60° to the left **d)** 3 units up

e)





f) The new equation is $y = \cos (x - 60°) - 3$. The graph shifts to the right 120° and down 6 units compared to the graph of $y = \cos (x + 60) + 3$.

6. a) amplitude 30, period 1 s, phase shift 0.25 cm to the right, vertical shift 45 cm up

b) 15 cm

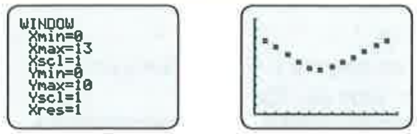
c) domain $\{t \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, 15 \le y \le 75\}$

d) The vertical shift must change from 45 cm to 50 cm. The new equation is $y = 30 \sin [360(t - 0.25)] + 50$.

7. a)

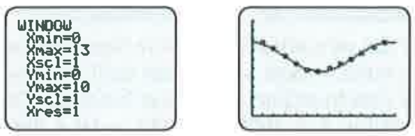
Month (21st day)	Time (decimal format)
1	7.67
2	7.07
3	6.28
4	5.42
5	4.78
6	4.62
7	4.92
8	5.47

Month (21st day)	Time (decimal format)
9	6.02
10	6.60
11	7.25
12	7.72



b) $y = 1.55 \sin [0.5(x + 3.3)] + 6.24$; amplitude 1.55, period 12 months, phase shift 3.3 left, vertical shift 6.24 up

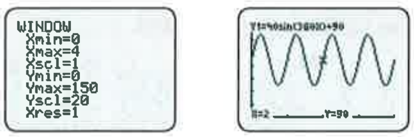
c)



8. January 7 at 7:39 a.m.; July 7 at 4:49 a.m.

9. a) $y = 40 \sin 360t + 90$

b)



c) 4.8 L

d) $y = 40 \cos [360(t - 0.25)] + 90$

e) $y = 40 \sin (720t) + 90$

f) Answers may vary. Sample answer: The amplitude does not change because the maximum and minimum volume of blood in the left ventricle does not change regardless of any increase in the heart rate.

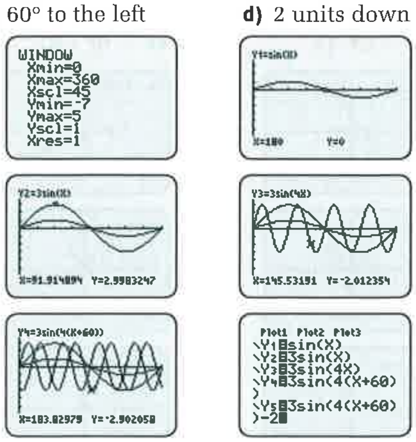
Chapter 5 Practice Test, pages 346–347

1. D **2.** B **3.** C **4.** A **5.** C **6.** B **7.** C **8.** A

9. a) 3 **b)** 90°

c) 60° to the left **d)** 2 units down

e)



f) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -5 \leq y \leq 1\}$

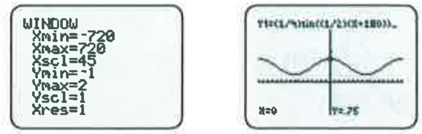
10. Answers may vary. Sample answers:

a) $y = 4 \cos 4x - 2$

b) $y = 4 \sin (4x + 22.5°) - 2$

11. a) $y = \frac{1}{4}\sin\left[\frac{1}{2}(x + 180°)\right] + \frac{1}{2}$

b)



12. a) amplitude 2, period 120°, phase shift 120° to the right, no vertical shift

b) maximum 2, minimum −2

c) 30°, 90°, 150° **d)** 2

13. a) $y = 2 \sin [6(x - 15°)] + 1$

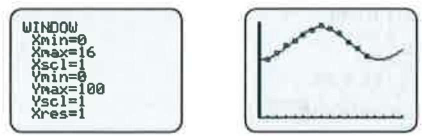
b) Answers may vary. Sample answer: The period of the function changes from 60° to 30°; so the equation is $y = 2 \sin [12(x - 15°)] + 1$.

14. Answers may vary. Sample answers:

a) $y = 17 \sin [30(x - 4)] + 79$

amplitude 17, period 12, phase shift 4, vertical shift 79

b)



The fit of the curve is good with respect to the data.

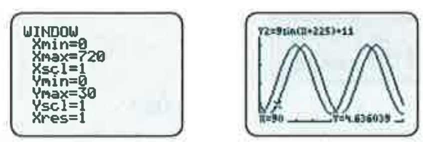
c) Approximately 95% will be employed on June 15.

d) This will result in a damped periodic function, with the amplitude decreasing to 90% of the value in the previous month, due to the 10% decrease.

15. a) $y = 9 \sin (x + 270°) + 11$

b) $y = 9 \sin (x + 225°) + 11$

c)



Answers may vary. Sample answer: Similarities: same amplitude, period, and vertical shift, because the wheel turns with these characteristics no matter where it is loaded. Differences: phase shift, because the loading dock of the ride changes position.

Chapters 4 and 5 Review, pages 348–349

1. a) $\sin 315° = -\frac{1}{\sqrt{2}}$, $\cos 315° = \frac{1}{\sqrt{2}}$, $\tan 315° = -1$

b) $\sin 315° = -0.7071$, $\cos 315° = 0.7071$, $\tan 315° = -1$

2. a) $\sin 255° = -1.0$, $\cos 255° = -0.3$, $\tan 255° = 3.7$

b) $\sin 255° = -0.9659$, $\cos 255° = -0.2588$, $\tan 255° = 3.7321$

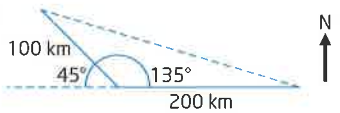
3. $\sin\theta = -\dfrac{1}{\sqrt{10}}$, $\cos\theta = \dfrac{3}{\sqrt{10}}$, $\tan\theta = -\dfrac{1}{3}$, $\csc\theta = -\sqrt{10}$, $\sec\theta = \dfrac{\sqrt{10}}{3}$, $\cot\theta = -3$

4. $\cos Q = -\dfrac{8}{17}$, $\tan Q = -\dfrac{15}{8}$

5. $\theta = 159°, 339°$

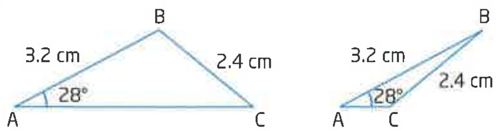
6. $187°, 353°$

7. a)



b) 280 km

8. a)



b) First triangle: $\angle C = 39°$, $\angle B = 113°$, $b = 4.7$ cm; second triangle: $\angle C = 141°$, $\angle B = 11°$, $b = 1.0$ cm

9. 3.0 km

10. 32.3 m, 11.4 m

11.–12. Answers may vary.

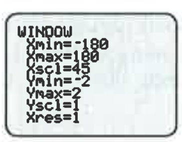
13. a) The y-values repeat in a regular pattern.

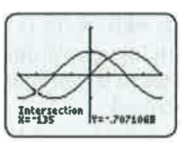
b) 7 **c)** maximum 3, minimum -4

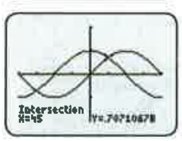
d) amplitude 3.5 **e)** period 2

14. a) $45°$ and $-135°$

b)



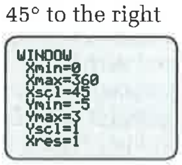


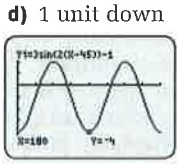


15. a) 3 **b)** $180°$

c) $45°$ to the right **d)** 1 unit down

e)

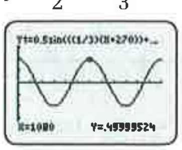




f) The equation becomes $y = 3\sin[4(x - 45°)] - 1$.

16. a) $y = \dfrac{1}{2}\sin\left[\dfrac{1}{3}(x + 270°)\right] + \dfrac{1}{4}$

b)



17. a) amplitude 2, period $120°$, phase shift $120°$, no vertical shift

b) maximum $\dfrac{1}{4}$, minimum $-\dfrac{1}{4}$

c) $45°, 135°, 325°$ **d)** $-\dfrac{1}{4}$

18. Answers may vary. Sample answers:

a) $d = 1.5\sin 24t + 0.2$

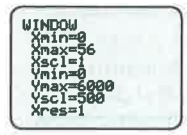
b) amplitude 1.5, period 15 s, no phase shift, vertical shift up 0.2

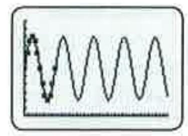
c) 1.5 m

19. Answers may vary. Sample answers:

a) $A = 2000\sin 30x + 3000$

b)



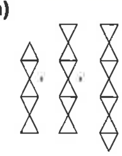


c) All the properties of the sine function remain the same, except for a phase shift to translate the sine function to a cosine function. Sample function: $A = 2000\cos[30(x - 3)] + 3000$

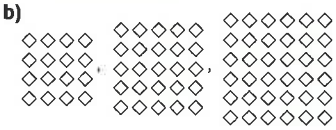
Chapter 6

Prerequisite Skills, pages 352–353

1. a)



b)



c) EEEEE, FFFFFF, GGGGGGG

d) PQRST, PQRSTU, PQRSTUV

e) 15, 18, 21 **f)** $-25, 30, -35$

g) $-9, -13, -17$ **h)** $\dfrac{1}{6}, \dfrac{1}{7}, \dfrac{1}{8}$

i) $5x, 6x, 7x$

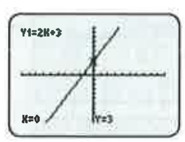
2. a) 2 **b)** -10 **c)** $\dfrac{1}{2}$ **d)** $\dfrac{-m + 7}{m + 2}$

3. a) 2 **b)** $\dfrac{1}{4}$ **c)** $2^{\frac{1}{3}}$ **d)** $2^{\frac{t-1}{3}}$

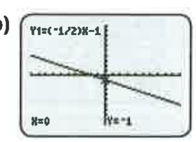
4. a) 1 **b)** 5

c) $t^2 - 7t + 11$ **d)** $4t^2 - 6t + 1$

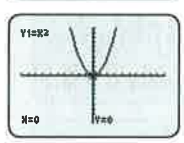
5. a)



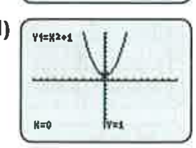
b)



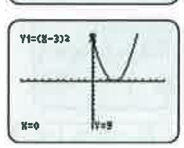
c)



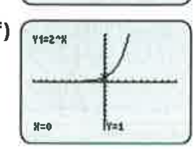
d)



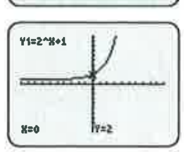
e)



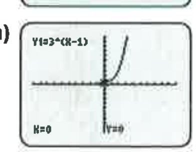
f)



g)



h)



6. a) $y = -\frac{3}{7}$ b) $t = 0$ c) $a = \frac{3}{2}$ d) $x = 44$ e) $x = 6$
7. a) 4.8 b) 105 c) 0.48
d) 20 e) 0.051 f) 980
8. a) $\frac{4}{5}$ b) $-\frac{3}{14}$ c) $\frac{24}{5}$
d) $-\frac{39}{2}$ e) $\frac{1}{6}$ f) $\frac{11}{5}$
9. a) quadratic b) linear c) neither
10. Answers may vary. Sample answer: For a linear function, the first differences are the same. The value of the first differences for the linear function $y = 3x - 1$ is 3. The slope of the linear function $y = 3x - 1$ is also 3. The value of the first differences for a linear function is equal to the value of m in a linear function of the form $y = mx + b$.
11. Answers may vary. Sample answer: For a quadratic function, the second differences are the same. The value the second differences for the quadratic function $y = 2x^2 - 5x - 3$ is 4. The value of the second differences of a quadratic function is equal to the value of $2a$ in a quadratic function of the form $y = ax^2 + bx + c$.
12. a) (3, 1) b) $\left(-\frac{8}{3}, \frac{49}{3}\right)$
c) $\left(\frac{228}{13}, -\frac{112}{3}\right)$ d) (5, 4)

6.1 Sequences as Discrete Functions, pages 360–363

1. a) 2, 5, 8 b) −3, −8, −13 c) 1, 3, 9
d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ e) $1, \frac{1}{2}, \frac{1}{3}$ f) 24, 48, 96
2. a) −35 b) 29 c) 142
d) $\frac{13}{12}$ e) 168 f) −2048
3. Answers may vary. Sample answers:
a) The first term is 4. Multiply each term by 4 to get the next term.
next three terms: 1024, 4096, 16 384
b) The first term is 7. Subtract 1 from each term to get the next term.
next three terms: 3, 2, 1
c) The first term is −3. Subtract three from each term to get the next term.
next three terms: −15, −18, −21
d) The first term is 100. Divide each term by 10 to get the next term.
next three terms: 0.01, 0.001, 0.0001
e) The first term is 5. To get each subsequent term in the sequence, increase the absolute value of the previous term by 5 and then multiply the result by $(-1)^{n+1}$.
next three terms: 25, −30, 35
f) The first term is $\frac{1}{3}$. Multiply each term by $\frac{1}{3}$ to get the next term.
next three terms: $\frac{1}{243}, \frac{1}{729}, \frac{1}{2187}$
g) The first term is x. Add $2x$ to each term to get the next term.
next three terms: $9x$, $11x$, $13x$
h) The first term is 4. Add 4 to each term to get the next term.
next three terms: 20, 24, 28
i) The first term is a. Multiply each term by r to get the next term.
next three terms: ar^4, ar^5, ar^6
j) The first term is 0.2 To get each subsequent term in the sequence, increase the absolute value of the previous term by 0.2 and then multiply the result by $(-1)^{n+1}$.
next three terms: 1, −1.2, 1.4

4. a)

Term Number, n	Term, t_n	First Differences
1	2	
2	4	2
3	6	2
4	8	2

$f(n) = 2n$; domain $\{n \in \mathbb{N}\}$

b)

Term Number, n	Term, t_n	First Differences
1	2	
2	1	−1
3	0	−1
4	−1	−1

$f(n) = -n + 3$; domain $\{n \in \mathbb{N}\}$

c)

Term Number, n	Term, t_n	First Differences
1	3	
2	6	3
3	9	3
4	12	3

$f(n) = 3n$; domain $\{n \in \mathbb{N}\}$

d)

Term Number, n	Term, t_n	First Differences	Second Differences
1	0		
2	3	3	
3	8	5	2
4	15	7	2

$f(n) = n^2 - 1$; domain $\{n \in \mathbb{N}\}$

e)

Term Number, n	Term, t_n	First Differences	Second Differences
1	3		
2	6	3	
3	11	5	2
4	18	7	2

$f(n) = n^2 + 2$; domain $\{n \in \mathbb{N}\}$

f)

Term Number, n	Term, t_n	First Differences	Second Differences
1	−10		
2	−9	1	
3	0	9	8
4	17	17	8

$f(n) = 4n^2 - 11n - 3$; domain $\{n \in \mathbb{N}\}$

5. a) $f(n) = n^2$, domain {1, 2, 3, 4}

b) $f(n) = 2n$, domain {1, 2, 3, 4}

c) $f(n) = \frac{1}{2}n - \frac{3}{2}$, domain {1, 2, 3, 4}

d) $f(n) = 4n - 11$, domain {1, 2, 3, 4}

6. a) discrete; Answers may vary. Sample answer: The function is a distinct set of points.

b) continuous; Answers may vary. Sample answer: The function is a continuous line.

c) discrete; Answers may vary. Sample answer: The function is a distinct set of points.

7. Answers may vary. Sample answers:

a) The first term and each alternate term after this term in the sequence is 1. The second term and each alternate term after this term in the sequence is the set of natural numbers.

Next three terms: 5, 1, 6

b) The first term and each alternate term after this term in the sequence is the set of natural numbers. The second term in the sequence is 5 and each alternate term after this term is 5 more than the preceding alternate term.

Next three terms: 4, 20, 5

c) The first term in the sequence is 3. Multiply each term by $\sqrt{5}$ to get the next term.

Next three terms: $75\sqrt{5}$, 375, $375\sqrt{5}$

d) The first term in the sequence is $\frac{1}{2}$. Multiply each term by $\frac{1}{2}$ to get the next term.

Next three terms: $\frac{1}{32}, \frac{1}{64}, \frac{1}{128}$

8. Answers may vary. Sample answers: Each term in the sequence is a multiple of 7.

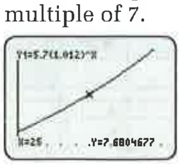
a) 98 is a part of this sequence. 98 is a multiple of 7.

b) 110 is not a part of this sequence. 110 is not a multiple of 7.

c) 378 is a part of this sequence. 378 is a multiple of 7.

d) 575 is not a part of this sequence. 575 is not a multiple of 7.

9. a)



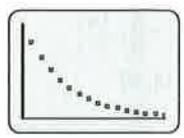
b) Answers may vary. Sample answer: The curve is an exponential curve. If the growth rate were greater the curve would be steeper.

c) 6.5772, 6.6561, 6.736, 6.8168, 6.8986, 6.9814, 7.0652, 7.15, 7.2358, where each number is in billions.

10. a)–b)

Year	Value
0	35 000.00
1	28 000.00
2	22 400.00
3	17 920.00
4	14 336.00
5	11 468.80
6	9 175.04
7	7 340.03
8	5 872.03
9	4 697.62
10	3 758.10
11	3 006.48
12	2 405.18
13	1 924.15
14	1 539.32
15	1 231.45

c)



d) $f(n) = 35\ 000(0.80)^n$

e) This is a discrete function. The depreciated value of the car is calculated on a yearly basis.

11. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

12. a) Answers may vary.

b)

Stage Number	Line Segment Length	Number of Line Segments	Perimeter of the Snowflake
1	1	3	3
2	$\frac{1}{3}$	12	4
3	$\frac{1}{9}$	48	$\frac{16}{3}$
4	$\frac{1}{27}$	192	$\frac{64}{9}$
5	$\frac{1}{81}$	768	$\frac{256}{27}$
6	$\frac{1}{243}$	3072	$\frac{1024}{81}$

c) Line Segment Length: $f(n) = \left(\frac{1}{3}\right)^{n-1}$;

Number of Line Segments: $f(n) = 3(4)^{n-1}$;

Perimeter of the Snowflake: $f(n) = 3\left(\frac{4}{3}\right)^{n-1}$

d) Line Segment Length: 1.0622×10^{-11};

Number of Line Segments: 2.1111×10^{14};

Perimeter of the Snowflake: 2242.3954

13. Answers may vary. Sample answers:

a) $f(n) = (-4)(-2)^{n-1}$; −65 536

b) $f(n) = \frac{n}{2n - 1}$; $\frac{15}{29}$

c) $f(n) = \sqrt{n}$; $\sqrt{15}$

d) $f(n) = 2^{n-1}$; 16 384

e) $f(n) = \frac{1}{n}$; $\frac{1}{15}$

f) $f(n) = (-1)^{n-1}$; 1

14. Answers may vary.

15. a) 50, 100, 200, 400, 800, 1600

b) $f(n) = 50(2)^{n-1}$

c) \$409 600.00; Answers may vary. Sample answer: No. This amount seems to be too great for the sales on the 14th day of a new small business.

16. $f(n) = 2100 - 110n$; 12 years

17. a) 1.7321, 1.3161, 1.1472

b) Answers may vary. Sample answer: The numbers are getting smaller and approaching 1.

c) 1

18. $f(n) = \frac{n(n + 1)(2n + 1)}{6}$

19. B **20.** B **21.** B

6.2 Recursive Procedures, pages 370–372

1. a) 4, 7, 10, 13

b) 7, 13, 25, 49

c) −3, −1.8, −1.56, −1.512

d) 50, 25, 12.5, 6.25

e) 8, −20, 66, −190

f) 100, 5000, 250 000, 12 500 000

2. a) 9, 7, 5, 3 **b)** −1, 3, −9, 27

c) 3, $\frac{3}{2}$, $\frac{1}{2}$, $\frac{1}{8}$ **d)** 18, 20, 22, 24

e) 0.5, −0.5, 0.5, −0.5 **f)** 25, −12.5, 6.25, −3.125

3. a) $t_1 = 5$, $t_n = t_{n-1} + 6$ **b)** $t_1 = 4$, $t_n = t_{n-1} - 3$

c) $t_1 = 4$, $t_n = 2t_{n-1}$ **d)** $t_1 = -4$, $t_n = \frac{1}{2}t_{n-1}$

e) $t_1 = -5$, $t_n = -3t_{n-1}$

4. a) 40, 70, 100, 130, ...; $t_1 = 40$, $t_n = t_{n-1} + 30$

b) 1, −2, 4, −8; $t_1 = 1$, $t_n = t_{n-1}(-2)$

5. Answers may vary. Sample answer:

$t_1 = 206$, $t_n = t_{n-1}$

6. a) 50, 54, 62, 74

b) Answers may vary. Sample answer: There are 50 seats in the first row. The number of seats in the second row is the number of seats in the first row increased by 4. The number of seats in the third row is equal to the number of seats in the second row increased by 8. The number of seats in the fourth row is equal to the number of seats in the third row increased by 12. The number of seats in subsequent rows is equal to the number of seats in the previous row increased by 4 multiplied by the result that you get if you subtract one from the number that the row is designated as.

c) $t_1 = 50$, $t_n = t_{n-1} + 4(n - 1)$

7. a)

Year	House Value (\$)
0	250 000
1	250 000 + 0.03 × 250 000 = 257 500
2	257 500 + 0.03 × 257 500 = 265 225
3	265 225 + 0.03 × 265 225 = 273 181.75
4	273 181.75 + 0.03 × 273 181.75 = 281 377.20
5	281 377.20 + 0.03 × 281 377.20 = 289 818.52
6	289 818.52 + 0.03 × 289 818.52 = 298 513.08
7	298 513.08 + 0.03 × 298 513.08 = 307 468.47
8	307 468.47 + 0.03 × 307 468.47 = 316 692.52
9	316 692.52 + 0.03 × 316 692.52 = 326 193.30
10	326 193.30 + 0.03 × 326 193.30 = 335 979.10

b) 257 500, 265 225, 273 181.75, 281 377.20, 289 818.52, 298 513.08, 307 468.47, 316 692.52, 326 193.30, 335 979.10

c) $t_n = 1.03t_{n-1}$; \$389 491.86

8. a) 1, 7, 58, 3376 **b)** 8, 4, 2, 1

c) 3, 6, 12, 24 **d)** −5, 14, −24, 52

e) $\frac{1}{2}$, 4, 18, 74

f) $a + 3b$, $a + 7b$, $a + 11b$, $a + 15b$

9. a) 2, 2, 6, 10 **b)** 1, 2, 2, 4

c) 5, 7, −2, 9 **d)** −2, 3, −3, 6

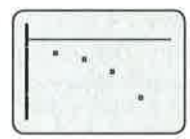
e) 1, −4, −4, 16 **f)** 3, 1, 7, −3

10. 0, 2, 5, 9, 14, 20; $t_1 = 0$, $t_n = t_{n-1} + n$

11. $t_1 = 1$, $t_n = t_{n-1} + 2n - 1$

12. a) −8, −12, −20, −36

b) $t_1 = -8$, $t_n = 2t_{n-1} + 4$



13. Answers may vary. Sample answers:

a) 1, 9, 25, 49; $t_1 = 1$, $t_n = t_{n-1} + 8(n - 1)$

b) 2, $\frac{5}{2}$, $\frac{10}{3}$, $\frac{17}{4}$; $t_1 = 2$, $t_n = t_{n-1} + \frac{n(n - 1) - 1}{n(n - 1)}$

c) $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$; $f(1) = \frac{1}{3}$, $f(n) = \frac{f(n - 1)}{3}$

d) 4, 7, 10, 13; $t_1 = 4$, $t_n = t_{n-1} + 3$

e) −3, 0, 5, 12; $f(1) = -3$, $f(n) = f(n - 1)$

f) 2, 8, 32, 128; $f(1) = 2$, $f(n) = 4f(n - 1)$

14. a) 3, 7, 15, 31; $t_n = 4(2^{n-1}) - 1$

b) 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$; $t_n = \frac{1}{2^{n-1}}$

c) 10, 0, −10, −20; $t_n = 20 - 10n$

d) −2, $-\frac{5}{2}$, $-\frac{8}{3}$, $-\frac{11}{4}$; $t_n = \frac{1 - 3n}{n}$

15. a) Answers will vary.

b) area = $\frac{3}{4}$ square units

c) Answers will vary.

d) 1, $\frac{3}{4}$, $\frac{9}{16}$, $\frac{27}{64}$, $\frac{81}{256}$, ...; recursive: $t_1 = 1$, $t_n = \frac{3}{4}t_{n-1}$; explicit $f(n) = \left(\frac{3}{4}\right)^{n-1}$

16. Answers may vary. Sample answers:
   a) $t_1 = 2, t_n = t_{n-1} + 2n$
   b) $t_1 = 3, t_n = t_{n-1} + n^2$
   c) $t_1 = 2, t_n = (t_{n-1})^2 + 1$
   d) $t_1 = -1, t_n = t_{n-1} + 3^{n-2}$
18. a) 1.5, −3, 6, −12, 24 b) −4, 0, 9, 25
19. Answers may vary.
20. a) sequence as exact numbers: $1, 1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \ldots;$
   sequence as approximate numbers: 1, 1, 2, 1.5, 1.667, 1.6, 1.625, 1,615, ...;
   Answers may vary. Sample answer: The sequence is converging and approaching 1.618.
   b) Answers will vary.
21. a) Answers may vary.
   b) $\frac{\pi}{4}, \frac{\pi}{4}, \pi, \frac{9\pi}{4}, \frac{25\pi}{4}, 16\pi$; 26π square units
22. Answers may vary.

6.3 Pascal's Triangle and Expanding Binomial Powers, pages 378–379

1. 6; Answers may vary. Sample answer: The number not on the diagonal is equal to the sum of the three numbers that are on the diagonal. Examples may vary.
2. a) 256 b) 4096
   c) 1 048 576 d) 2^n
3. a) $t_{5,4}$ b) $t_{9,6}$
   c) $t_{26,18}$ d) $t_{a+1,b+1}$
4. a) $t_{3,1} + t_{3,2}$ b) $t_{11,8} + t_{11,9}$
   c) $t_{27,13} + t_{27,14}$ d) $t_{16,x-1} + t_{16,x}$
5. a) $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
   b) $y^4 - 12y^3 + 54y^2 - 108y + 81$
   c) $4096 + 6144t + 3840t^2 + 1280t^3 + 240t^4 + 24t^5 + t^6$
   d) $1 - 5m + 10m^2 - 10m^3 + 5m^4 - m^5$
   e) $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$
   f) $a^{10} + 20a^8 + 160a^6 + 640a^4 + 1280a^2 + 1024$
6. a) 1 b) 26 c) 16 d) $n + 1$
7. a) 1 b) 7 c) 4 d) 495
8. a) 8 b) 11 c) 14 d) 16
9. Answers may vary. Sample answers:
   a) $t_{5,2} - t_{4,1}$ b) $t_{7,3} - t_{6,2}$
   c) $t_{13,9} - t_{12,8}$ d) $t_{29,14} - t_{28,15}$
10. a) top row: 8, 1; bottom row: 45, 10
   b) middle left: 35; bottom right: 28 ; middle cell 21
   c) middle left: 11; middle right: 165; bottom row: 66, 220
   d) top left: 21; middle right: 70; bottom row: 84, 126
11. Answers may vary.
12. Answers may vary. Sample answer: If you write Pascal's triangle as a right triangle, then the sum of the diagonals form the Fibonacci sequence, 1, 1, 2, 3, 5, 8, 13, ...
13. Row 0: 1; Row 1; 2: Row 2: 6; Row 3: 20; Row 4: 70; 1, 2, 6, 20, 70; Answers may vary. Sample answer: The numbers are located at the centre of every other row in Pascal's Triangle. Answers may vary. Sample answer: $t_n = t_{2(n-1), n-1}$
14. $x = -2$; $n = 9$
15. Answers may vary. Sample answer: The top term is 1. The first and last terms of each row n are generated by increasing the denominator of first and last terms of row $n - 1$ by 1. Every other term in a row is obtained by subtracting the term to its immediate right from the term immediately above andto the right. Using this pattern the next three rows in the triangular array are as follows:

$$\frac{1}{6} \quad \frac{1}{30} \quad \frac{1}{60} \quad \frac{1}{60} \quad \frac{1}{30} \quad \frac{1}{6}$$
$$\frac{1}{7} \quad \frac{1}{42} \quad \frac{1}{105} \quad \frac{1}{140} \quad \frac{1}{105} \quad \frac{1}{42} \quad \frac{1}{7}$$
$$\frac{1}{8} \quad \frac{1}{56} \quad \frac{1}{168} \quad \frac{1}{280} \quad \frac{1}{280} \quad \frac{1}{168} \quad \frac{1}{56} \quad \frac{1}{8}$$

16. Answers may vary. Sample answer: The numbers in the rows where the first term after the 1 is a prime number are multiples of the prime number.
17. 89 ways
18. C 19. B 20. A

6.4 Arithmetic Sequences, pages 385–387

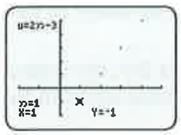
1. a) $a = 12, d = 3$; 21, 24, 27, 30
   b) $a = 6, d = -2$; 0, −2, −4, −6
   c) $a = 0.2, d = 0.15$; 0.65, 0.8, 0.95, 1.1
   d) $a = -30, d = 6$; −12, −6, 0, 6
   e) $a = 5, d = -6$; −13, −19, −25, −31
   f) $a = \frac{1}{2}, d = \frac{1}{2}$; $2, \frac{5}{2}, 3, \frac{7}{2}$
2. a) arithmetic; the first term is $a = 3$ and the common difference between the consecutive terms is $d = 2$
   b) not arithmetic; the first term is $a = 2$, the difference between consecutive terms is not equal
   c) not arithmetic; the first term is $a = 4$, the difference between consecutive terms is not equal
   d) arithmetic; the first term is $a = 13$ and the common difference between the consecutive terms is $d = -6$
   e) arithmetic; the first term is $a = -12$ and the common difference between the consecutive terms is $d = 7$
   f) arithmetic; the first term is $a = 0$ and the common difference between the consecutive terms is $d = 1.5$
3. a) 5, 7, 9; $t_n = 2n + 3$
   b) −2, −6, −10; $t_n = -4n + 2$
   c) 9, 5.5, 2; $t_n = -3.5n + 12.5$
   d) $0, -\frac{1}{2}, -1$; $t_n = -\frac{1}{2}n + \frac{1}{2}$
   e) 100, 110, 120; $t_n = 10n + 90$
   f) $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}$; $t_n = \frac{1}{2}n + \frac{1}{4}$

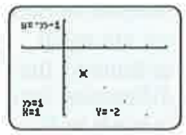
g) $10, 10 + t, 10 + 2t; t_n = tn + 10 - t$

h) $x, 3x, 5x; t_n = 2nx - x$

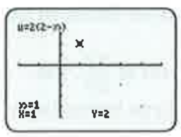
4. a) 40 **b)** -47 **c)** $\frac{15}{2}$ **d)** 2

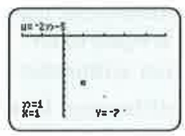
5. a) $-1, 1, 3$ **b)** $-2, -3, -4$



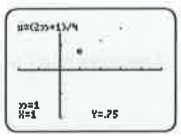


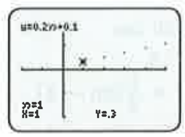
c) $2, 0, -2$ **d)** $-7, -9, -11$





e) $0.75, 1.25, 1.75$ **f)** $0.3, 0.5, 0.7$





6. t_{32}

7. a) 40 **b)** 30 **c)** 89 **d)** 35

8. Answers may vary.

9. a) $a = \frac{5}{2}, d = \frac{1}{2}; 1, \frac{1}{2}, 0$

b) $a = -6, d = \frac{5}{2}; \frac{3}{2}, 4, \frac{13}{2}$

c) $a = 2a, d = -b; 2a - 3b, 2a - 4b, 2a - 5b$

10. a) $a = 5, d = 4; t_n = 4n + 1$

b) $a = -4, d = 6; t_n = 6n - 10$

c) $a = -8, d = -3; t_n = -3n - 5$

d) $a = 3 - 22x, d = 4.5x; t_n = 4.5xn + 3 - 26.5x$

11. a) $t_1 = 5, t_n = t_{n-1} + 4$

b) $t_1 = -4, t_n = t_{n-1} + 6$

c) $t_1 = -8, t_n = t_{n-1} - 3$

d) $t_1 = 3 - 22x, t_n = t_{n-1} + 4.5x$

12. a) $t_n = 2n$ **b)** $t_n = 10n$

c) $t_n = -10n + 5$ **d)** $t_n = -3n + 2$

13. a) \$5500

b) 20 winners; Answers may vary. Sample answer: The 20th winner will receive \$500. The amount left after this prize is paid out will be \$0.

14. \$130 500, if $n = 0$ represents his current salary and $n = 1$ represents his salary at the end of the first year.

15. 822 members

16. $-9, -27$

17. 83

18 Answers may vary. Sample answers:

a) arithmetic; Each term after the first term is found by adding a common difference of 3 to the previous term.

b) not arithmetic; Each term after the first term and the second term is found by multiplying the previous term by 4 and then adding the term that is 2 before the current term. The terms in the sequence do not have a common difference.

c) not arithmetic; Each consecutive term is found by squaring the previous term. The terms in the sequence do not have a common difference.

d) not arithmetic; Each term after the first term is found by multiplying the previous term by -2 and then subtracting 5. The terms in the sequence do not have a common difference.

Answers may vary.

19. $\frac{7x + 10y}{4}, \frac{5x + 6y}{2}, \frac{13x + 14y}{4}$

20. $x = 5$; the first three terms are 5, 9.5, 14

21. 4, 11, 18, 25, ...

22. a) $(-1, 2)$ **b)** $(-1, 2)$

c) Answers may vary.

23. C

24. Answers may vary.

25. B **26.** C **27.** D

6.5 Geometric Sequences, pages 392–394

1. a) arithmetic; The first term is $a = 5$ and the common difference is $d = -2$.

b) geometric; The first term is $a = 5$ and the common ratio is $r = -2$.

c) geometric: The first term is $a = 4$ and the common ratio is $r = \frac{1}{10}$.

d) geometric: The first term is $a = \frac{1}{2}$ and the common ratio is $r = \frac{1}{3}$.

e) neither: The first term is $a = 1$, but there is no common difference or common ratio between the consecutive terms.

f) neither: The first term is $a = 1$, but there is no common difference or common ratio between the consecutive terms.

2. a) 2; 16, 32, 64

b) -3; $-243, 729, -2187$

c) -1; $\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}$

d) -0.5; $37.5, -18.75, 9.375$

e) 1; $-15, -15, -15$

f) 10; 3000, 30 000, 300 000

g) 0.5, 4.5, 2.25, 1.125

h) $x^2; x^9, x^{11}, x^{13}$

3. a) $t_n = 54\left(\frac{1}{3}\right)^{n-1}; \frac{2}{243}$

b) $t_n = 4(5)^{n-1}$; 1 562 500

c) $t_n = \left(\frac{1}{6}\right)\left(\frac{6}{5}\right)^{n-1}$; 0.716 636 16

d) $t_n = 0.0025(10)^{n-1}$; 250 000

4. a) 5, 10, 20, 40

b) $500, -2500, 12\,500, -62\,500$

c) $0.25, -0.75, 2.25, -6.75$

d) $2, 2\sqrt{2}, 4, 4\sqrt{2}$

e) $-1, -0.2, -0.04, -0.008$

f) $-100, 20, -4, 0.8$

5. a) 7 b) 6 c) 12
d) 21 e) 8 f) 7

6. a) arithmetic; $a = x$, $d = 2x$
b) geometric $a = 1$, $r = \frac{x}{2}$
c) neither
d) geometric; $a = \frac{5x}{10}$, $r = \frac{1}{100}$

7. 10

8. 7

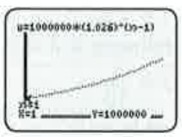
9. after 14 7-h periods, the bacteria count will be 1.64×10^6

10. a) $\frac{100}{9}$ Ci/km²
b) $1 = \frac{100}{9}\left(\frac{1}{2}\right)^{\frac{t}{30}}$; 104.2 years
c) Answers may vary.

11. 46 656

12. a) Answers may vary.
b) $A = 1\left(\frac{8}{9}\right)^{n-1}$
c) 0.106 684 square units
d) Answers may vary.

13. a) Number of Voters, V: $V = (1\,000\,000)(1.026)^{n-1}$, where n is the number of 4-year periods since 1850



b) Answers may vary. Sample answer: discrete; the elections are held once every 4 years
c) 2 791 865

14. a) 25 b) 12, 36, and 108

16. $y = \frac{1}{5}$, $y = 6$

17. a) $x = 6$, $y = 24$; $x = -6$, $y = -24$
b) $x = 16$, $y = -128$

18. $x^2 + x$, $x^3 + x^2$, $x^4 + x^3$

19. 22.47% per year

20. Case 1: $x = 2$, $y = 12$; Case 2: $x = 8$, $y = 24$

21. C 22. B 23. $50\sqrt[3]{2}$

6.6 Arithmetic Series, pages 400–401

1. a) 39 b) -12 c) -90
d) 130 e) $-\frac{49}{6}$ f) $180x$

2. a) $a = 5$, $d = 4$, $S_{20} = 860$
b) $a = 20$, $d = 5$, $S_{20} = 1350$
c) $a = 45$, $d = -6$, $S_{20} = -240$
d) $a = 2$, $d = 0.2$, $S_{20} = 78$
e) $a = \frac{1}{2}$, $d = \frac{1}{4}$, $S_{20} = 57.5$
f) $a = -5$, $d = -1$, $S_{20} = -290$

3. a) 18 b) 1020 c) 270 d) 1120

4. a) 375 b) 2170 c) -1480 d) 0

5. a) 4564 b) -3630 c) $-35\,409.3$ d) 87

6. $-27, -22, -17$

7. 3925

8. a) $1190\sqrt{7}$ b) $-550x$ c) $22b$ d) $\frac{90}{x}$

9. Answers may vary. Sample answers:
a) not arithmetic; the first term is -2, the differences between the four terms in the series are not equal
b) arithmetic; the first term is $2x^2$, the common differences between the three terms in the series is equal to x^2
c) arithmetic; the first term is a, the common differences between the three terms in the series is equal to $2b$
d) not arithmetic; the first term is $\frac{17}{20}$, the differences between the three terms in the series are not equal

10. 68 cans

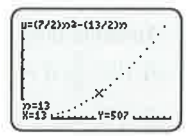
11. 1860 cm

12. a) 15 b) Answers may vary.

13. $S_n = \frac{n}{2}(3n - 1)$

14. a) $x = 0$, $x = 4$
b) if $x = 0$, sum = 45; if $x = 4$, sum = 305

16. $S_n = \frac{7}{2}n^2 - \frac{13}{2}n$



17. 124; 15

18. $2 + 6 + 10 + 14 + \dots$

19. A

20. C

21. 14 706

22. $x = 7$

23. Answers may vary.

24. $\frac{20}{21}$

6.7 Geometric Series, pages 408–409

1. a) geometric; The first term is $a = 4$ and the common ratio is $r = 5$.
b) geometric; The first term is $a = -150$ and the common ratio is $r = -\frac{1}{10}$.
c) not geometric: The first term is $a = 3$, but there is no common ratio between the consecutive terms.
d) geometric: The first term is $a = 256$ and the common ratio is $r = -\frac{1}{4}$.

2. a) $a = 2$, $r = 3$, $S_8 = 6560$
b) $a = 24$, $r = -\frac{1}{2}$, $S_{10} = \frac{1023}{64}$
c) $a = 0.3$, $r = 0.01$, $S_{15} \doteq \frac{10}{33}$
d) $a = 1$, $r = -\frac{1}{3}$, $S_{12} = \frac{132\,860}{177\,147}$
e) $a = 2.1$, $r = -2$, $S_9 = 359.1$
f) $a = 8$, $r = -1$, $S_{40} = 0$

3. **a)** 3066 **b)** -2730

c) 2 615 088 483 **d)** $2.999\ 999\ 97 \times 10^{10}$

e) 10 922.5 **f)** $\dfrac{29\ 524}{81}$

4. **a)** $\dfrac{9841}{243}$ **b)** $\dfrac{889}{64}$ **c)** ≈ 1333.3 **d)** $\dfrac{6305}{6561}$

5. **a)** 2735 **b)** -510 **c)** 64 125 **d)** $\dfrac{463}{729}$

6. **a)** $\dfrac{-242\sqrt{3}}{\sqrt{3}+1}$ **b)** $\dfrac{63\sqrt{2x}}{\sqrt{2}-1}$ **c)** $\dfrac{3(x^{15}-1)}{x-1}$

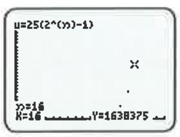
7. **a)** $\dfrac{1275}{64}$ **b)** $\dfrac{31\ 248\sqrt{5}}{\sqrt{5}-1}$ **c)** $\dfrac{1(x^k-1)}{x-1}$

8. 7 terms

9. $\dfrac{58\ 025}{48}$

10. $r = 2$; $S_k = 3(2^k - 1)$ or $r = -3$, $S_k = \dfrac{-3}{4}[(-3)^k - 1]$

11. **a)** $S = 25(2^x - 1)$, where S is the total amount of prize money and x is the total number of tickets drawn.

b) 

16 prizes can be given out if the total amount of prize money is \$2 million

12. **a)** $\dfrac{1280}{729}$ m **b)** approximately 98.6 m

13. **a)**

Stage	Line Segment Length	Total Length
1	1	1
2	$\frac{1}{3}$	3
3	$\frac{1}{9}$	9
4	$\frac{1}{27}$	27
5	$\frac{1}{81}$	81
6	$\frac{1}{243}$	243

b) $LSL = \left(\dfrac{1}{3}\right)^{n-1}$; $\dfrac{1}{243}$ units

c) $TL = (3)^{n-1}$; 243 units

d) Answers may vary.

15. $a = 5$, $b = 10$, $c = 20$ or $a = 20$, $b = 10$, $c = 5$

16. $3 + 6 + 12$ or $3 - 6 + 12$

17. $a = 6$, $r = 2$, $t_{10} = 3072$, $S_{10} = 6138$

18. $n = 8$

19. Sequence 1: $b = -\dfrac{1}{3}$, $S_5 = -\dfrac{11}{6}$;

Sequence 2: $b = 3$, $S_5 = 124$

Chapter 6 Review, pages 410–411

1. **a)** 6, 12, 22, 36 **b)** $1, \dfrac{3}{2}, \dfrac{5}{3}, \dfrac{7}{4}$

2. **a)**

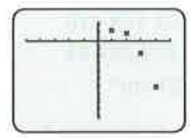
Term Number, n	Term, t_n	First Differences
1	−8	
2	−11	−3
3	−14	−3
4	−17	−3

$f(n) = -3n - 5$;
domain $= \{n \in \mathbb{N}\}$

b)

Term Number, n	Term, t_n	First Differences	Second Differences
1	3		
2	2	−1	
3	−3	−5	−4
4	−12	−9	−4

$f(n) = -2n^2 + 5n$;
domain $= \{n \in \mathbb{N}\}$



3. **a)** $5, 1, -3, -7$ **b)** 3, 4, 5, 6

4. **a)** $t_1 = -2$, $t_n = t_{n-1} + 9$

b) $t_1 = 1$, $t_n = -3t_{n-1}$

5. **a)** $x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$

b) $y^4 - 24y^3 + 216y^2 - 864y + 1296$

c) $m^4 + 8m^3n + 24m^2n^2 + 32mn^3 + 16n^4$

d) $729p^6 - 1458p^5q + 1215p^4q^2 - 540p^3q^3 + 135p^2q^4 - 18pq^5 + q^6$

6. **a)** 35, 56

b)

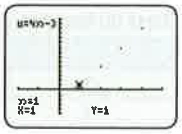
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

The tetrahedral numbers are found in the fourth diagonals of Pascal's triangle starting with the 1's on the ends of row 3. The tetrahedral numbers are 1, 4, 10, 20, 35, 56, 84, … . The two diagonals have the value 20 in common.

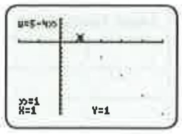
7. **a)** $a = 3$, $d = -2$; $t_n = 5 - 2n$, $-5, -7, -9, -11$

b) $a = \dfrac{2}{3}$, $d = \dfrac{1}{4}$; $t_n = \dfrac{n}{4} + \dfrac{5}{12}$, $\dfrac{5}{3}, \dfrac{23}{12}, \dfrac{13}{6}, \dfrac{29}{12}$

8. a) 1, 5, 9



b) 1, −3, −7



9. a) 145 seats **b)** 245 seats

10. a) arithmetic; The first term is $a = -1$ and the common difference is $d = 10$.

b) neither: The first term is $a = 1$, but there is no common difference or common ratio between the consecutive terms.

c) geometric; The first term is $a = -2$ and the common ratio is $r = -3$.

11. a) −2, 2, −2 **b)** −12, −24, −48

12. 1177.3 m

13. a) $a = 50$, $d = -5$; 50 **b)** $a = -27$, $d = 6$; 600

14. $48 985

15. a) −1450 **b)** 120

16. a) 94 158 416 **b)** −2728

17. a) 272.221 95 **b)** 9841.499 924

18. 1023 cm²

Chapter 6 Practice Test, pages 412–413

1. D **2.** B **3.** B **4.** C **5.** B

6. a) 4, −1, −6, −11, −16 **b)** 1, 10, 23, 40, 61

c) $\frac{1}{8}, \frac{1}{2}$, 2, 8, 32 **d)** 1, 1.2, 1.4, 1.6, 1.8

e) 2.5, 3, 3.5, 4, 4.5

f) −6, −12, −24, −48, −96

7. a) $f(n) = 64\left(\frac{1}{2}\right)^{n-1}$; $t_1 = 64$, $t_n = t_{n-1} \times \frac{1}{2}$

b) $f(n) = -23 + 3n$; $t_1 = -20$, $t_n = t_{n-1} + 3$

c) $f(n) = 84 - 4n$; $t_1 = 80$, $t_n = t_{n-1} - 4$

d) $f(n) = -4000\left(-\frac{1}{4}\right)^{n-1}$; $t_1 = -4000$, $t_n = t_{n-1} \times \left(-\frac{1}{4}\right)$

e) $f(n) = -3(2)^{n-1}$; $t_1 = -3$, $t_n = t_{n-1} \times 2$

f) $f(n) = -14\sqrt{2} + 2\sqrt{2}n$; $t_1 = -12\sqrt{2}$, $t_n = t_{n-1} + 2\sqrt{2}$

8. a) 46 **b)** −3072 **c)** 5120 **d)** −55

9. a) 86 093 442 **b)** 425

c) 1.0×10^{-29} **d)** −11.25

10. a) 20 **b)** 15

11. a) 1.6, 1.68, 1.764

b) $t_1 = 1.6$, $t_n = 1.05t_{n-1}$; 1.944 81 t

12. a) $\frac{25\,575}{64}$ **b)** −855

13. a) −2470 **b)** 2592

14. a) $\frac{671\,846}{81}$ **b)** 5115

15. a) $b^5 - 15b^4 + 90b^3 - 270b^2 + 405b - 243$

b) $64x^6 - 960x^5y + 6000x^4y^2 - 20\,000x^3y^3 + 37\,500x^2y^4 - 37\,500xy^5 + 15\,625y^6$

16. a) 8 **b)** −7

17. 1380

18. Answers may vary. Sample answer: A is equal to B. Each pair of numbers in A could be factored as a difference of squares as follows:

$$\begin{aligned} A &= (50 + 49)(50 - 49) + (48 + 47)(48 - 47) + \ldots + (4 + 3)(4 - 3) + (2 + 1)(2 - 1) \\ &= (99)(1) + (95)(1) + \ldots + (7)(1) + (3)(1) \\ &= 99 + 95 + \ldots + 7 + 3. \end{aligned}$$

The sum of this series is 1275. The sum of series B is 50 + 49 + 48 + 47 + ... + 2 + 1, or 1275.

19. 32 paths

20. 66.8%

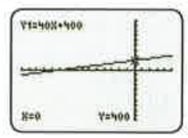
21. 3930

22. $54 908.48

23. Answers may vary.

Chapter 7

Prerequisite Skills, pages 416–417

1. a) 

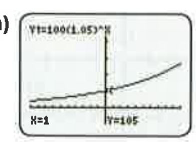
linear

b) $m = 40$, $b = 400$

c) and **d)**

x	$y = 40x + 400$	First Differences
0	400	
1	440	40
2	480	40
3	520	40
4	560	40

The first differences are constant.

2. a) 

exponential

b) (0, 100)

c) and **d)**

x	$y = 100(1.05)^x$	First Differences	Second Differences
0	100		
1	105	5.00	
2	110.25	5.25	0.25
3	115.76	5.51	0.26
4	121.55	5.79	0.28

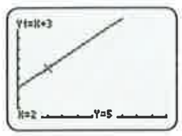
Neither the first differences nor the second differences are constant.

e) $\frac{105}{100} = 1.05$, $\frac{110.25}{105} = 1.05$, $\frac{115.75}{110.25} = 1.05$, $\frac{121.55}{115.76} = 1.05$. All common ratios are the same value, 1.05.

3. a) linear function; first differences are constant

b) exponential function; the ratio of successive y-values is the same, 0.9

4. a) direct b) partial
c) partial d) direct

5. a) $C = d + 3$
b) Fixed: \$3, Variable: \$1 per kilometre
c) 

d) slope = 1, vertical intercept = 3
e) Answers may vary. Sample answer: slope is the coefficient of the variable part and vertical intercept is the fixed part of the relation

6. a) All terms differ by a constant difference.
b) $a = 7, d = 3$ c) $t_n = 3n + 4$

7. a) $-2, 3, 8, 13$ b) $a = -2, d = 5$

8. 25 050

9. a) It is geometric because consecutive terms have a common ratio.
b) $a = 3, r = 2$ c) $t_n = 3(2)^{n-1}$

10. a) $-2, -6, -18, -54$ b) $a = -2, r = 3$

11. 21.578 563 59

12. $a = 2$

13. a) 80 b) 780 c) $\frac{250}{1.32}$ d) 0.065

14. a) 153.5791 b) -0.92
c) 0.0328 or -2.0328 d) ± 1.0309

15. a) 36 months b) $\frac{15}{52}$ years
c) $\frac{26}{73}$ years d) 39 weeks
e) $\frac{240}{73}$ months f) $\frac{75}{13}$ months

16. a) 104 b) 4 c) 42
d) 12 e) 4 f) 7

7.1 Simple Interest, pages 424–425

1. a) \$117 b) \$21.88 c) \$15.99 d) \$14.10

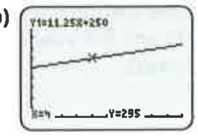
2. a) \$212, \$224, \$236, \$248, \$260
b) $a = 212, d = 12$
c) $t_n = 200 + 12n$; this is a linear model that represents the amount of the \$200 investment at the end of the nth year.

3. a) All first differences are \$39, representing the amount of simple interest earned each year.
b) \$650; $t = 0$ is the start of the investment
c) 6%

4. a) $A = 650 + 39t$
b) Answers may vary. Sample answer: This is a partial variation since the initial amount is not \$0. The linear model contains a fixed part, and a 650, variable part $39t$.
c) 16 years 8 months

5. a) \$1500 b) 8%
c) $A = 120t + 1500$ d) 12 years 6 months

6. a) $I = 120t$
b) 12 years 6 months; the same as 5d)

7. a) $A = 11.25t + 250$
b) 

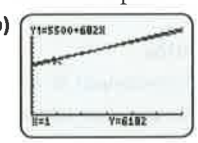
c) approximately 4 years 5 months
d) approximately 8.2%

8. a) \$561.88 b) \$61.88
c) repay in approximately 14 months

9. approximately 10.7%

10. a) \$280 b) \$2280

11. 17.8 months

12. a) Bank: $A = 5500 + 682t$,
Dealership: $A = 5700 + 605t$
b) 

c) Answers may vary. Sample answer: The bank loan is a better deal if the loan is repaid in less than 2.6 years. The dealership loan is a better deal if the loan is repaid in more than 2.6 years.

13. a) $r = \frac{A - P}{Pt}$ b) approximately 5.4%
c) Answers may vary.

14. a) $t = \frac{A - P}{Pr}$ b) Answer may vary.

7.2 Compound Interest, pages 433–435

1. a) \$632.66 b) \$132.66

2. a) \$1222.01 b) \$372.01

3. a) 0.0075 b) 0.02 c) 0.03 d) 0.005

4. a) 12 b) 8 c) 9
d) 14 e) 6

5. a) $n = 6, i = 0.0875$ b) $n = 12, i = 0.015$
c) $n = 24, i = 0.002$ d) $n = 15, i = 0.0225$

6. a) \$1661.54 b) \$261.54 c) \$241.50

7. \$2.06

8. a) i) \$360 ii) \$457.41
iii) \$483.67 iv) \$490.02
The order from best to worst for Karin is the order given in the problem i), ii), iii), iv).
b) Answers may vary. Sample answer: The greater the number of times interest is paid, the more interest accumulates.

9. approximately 11%

10. approximately 8%

11. Answers may vary. Sample answer: First Provincial Bank; it pays more interest.

12. a) \$2149.19 b) \$149.19
c) Answers may vary. Sample answer: She earns \$130.81 less, but Chloe can access the money in the chequing account with no penalty.

13. a) i) 9 years ii) 8 years iii) 6 years
b) Answers may vary. Sample answer: The Rule of 72 is close but not exact. The results using the compound interest formula are: i) 9 years, ii) 8.04 years, and iii) 6.12 years.

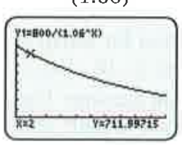
14. approximately 4.36 years

16.–17. Answers may vary.

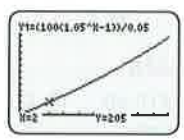
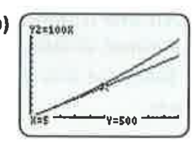
18. A 19. C

20. a) \$1610.51 b) \$1645.31
c) \$1648.61 d) \$1648.72

7.3 Present Value, pages 441–443

1. a) \$548.47 b) \$885.57
2. \$400.00
3. a) \$516.85 b) \$290.36
4. a) \$2499.98 b) \$921.42
5. annually
6. approximately 5 years 4 months
7. a) Investment A: \$7712.54, Investment B: \$7808.59
b) Answers may vary. Sample answer: Investment A; more interest is earned.
8. Bank A is better; he will pay slightly less in interest charges. Bank A will lend him \$1533.65, bank B will lend \$1531.10.
9. a) \$314.63 b) \$126.65
10. \$712.02
11. approximately 13.3%
12. a) Answers may vary. Sample answer: As the number of compounding periods increases, the present value decreases.
b) Answers may vary. Sample answer: The more frequent the interest payments, the smaller the amount you need to deposit to have a fixed amount in 5 years.
13. a) \$2.00 b) \$0.14
14. \$27 683.79
15. a) \$4429.19
b) \$7570.81; represents the future value of the last investment in her portfolio
c) 6%
16. a) $PV = \dfrac{800}{(1.06)^t}$
b) 
The graph is exponentially decaying.
c) Answers may vary. Sample answer: The horizontal scale of the graph is time in years.
d) Answers may vary. Sample answer: The graph provides information on the present value of the investment.
e) Answers may vary. Sample answer: No, in the context any negative value of t corresponds to a date after the time that the account's future value is \$800.
18. Answers may vary. Sample answer: The new graph is a mirror image of the original. It is exponentially growing and represents the future value of an \$800 investment as a function of time. The interest is 6% per year, compounded annually.
19. $PV = \dfrac{FV}{1 + rt}$
20. \$3402.67
21. D

7.4 Annuities, pages 453–455

1. \$2653.19
2. a) A time line for the future value of an annuity with $R = 250$, $n = 6$, and $i = 0.045$.
b) \$1679.22 c) \$179.22
3. a) A time line for the future value of an annuity with $R = 35$, $n = 104$, and $i = 0.001$.
b) \$3834.00 c) \$194.00
4. \$2277.45
5. \$449.44
6. a) \$3289.83 b) 6%
7. 1.6 percentage points
8. a) \$42 059.39 b) \$13 259.39
9. Answers may vary. Sample answer: I agree with his advisor that increasing the frequency of the deposit and the frequency of the compounding period will increase the amount of the annuity. He also invests more per year by depositing \$40 per week as
$\$40 \times 52 = \$2080$ and $\$160 \times 12 = \$1920$.
10. Answers may vary. Sample answer: He should switch to the weekly deposits since it is worth more than the other two possibilities at the end of the 7 years.
11. Option A pays \$533 704.47 more.
12. a) \$87.33 b) Answers may vary.
13. a)  The graph shows exponential growth.
b) Answers may vary.
c) regular payment = \$100 and interest rate = 5% per year
d) Answers may vary.
14. a) $P = 100n$ b) 
c) Answers will vary. Sample answer: The interest earned will be the difference between the two functions. $I(n) = \dfrac{100(1.05^n - 1)}{0.05} - 100n$
15. B
16. A
17. (0, 10), (6, 2), (10, 0)
18. B

7.5 Present Value of an Annuity, pages 461–463

1. \$106 593.02

2. a) A time line for the present value of an annuity with $R = 1000$, $n = 4$, and $i = 0.08$.

b) \$3312.13

3. a) A time line for present value of an annuity with $R = 650$, $n = 20$, and $i = 0.016$.

b) \$11 050.38 **c)** \$1949.62

4. \$1614.14

5. \$2830.60

6. \$152 917.37

7. a) Answers may vary. Sample answer: No, she will not have enough to retire and live off her lottery winnings.

b) \$1 242 519.27

8. approximately 8%

9. approximately 20.1%

10. a) Option A: \$567.36 Option B: \$187.33

b) Option A: \$808.32 Option B: \$743.88

c) Answers may vary. Sample answer: Option A earns more interest, but option B allows Jordan to withdraw more.

11. \$8217.75

12. \$1585.60

13. a) Answers may vary. **b)** Answers may vary.

c) \$632 490.38 **d)** \$54 274.33

e) Answers may vary.

15. Answers may vary.

16. a) 0.412 392% per month

b) \$1163.21

17. a) \$1022.99 **b)** \$235.58

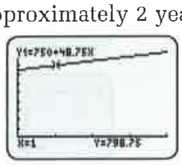
c) \$156 254

Chapter 7 Review, pages 464–465

1. a) \$102.60 **b)** \$822.60

2. approximately 2 years 10 months

3. a)



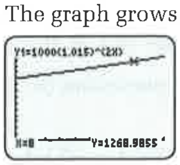
b) Answers may vary. Sample answer: The relation is linear, as the graph is a straight sloping line.

c) The vertical intercept of \$750 represents the amount borrowed.

d) The slope of \$48.75 represents the interest per year.

4. a) \$1241.15 **b)** \$406.15

5. a) The graph grows exponentially over time.



b) The vertical intercept of \$1000 represents the initial amount in the account.

c) The slope will be increasing for increasing values of time.

6. a) 10 years 9 months

b) Answers may vary. Sample answer: No. The interest rate and period is not changing and the ratio of the amount to the principal is still 2:1.

7. \$816.30

8. a) \$33 758.66 **b)** \$11 241.34

9. approximately 6.15%

10. quarterly

11. a) A time line for the future value of an annuity with $R = 2400$, $n = 4$, and $i = 0.043$.

b) Answers may vary. Sample answer: The series is geometric as consecutive terms have a common ratio.

c) \$10 237.14 **d)** \$637.14

12. a) \$48 100.11 **b)** \$9 700.11

13. \$341.94

14. a) 6 years

b) 6%; 4 compounding periods per year

c) \$5007.60 **d)** \$992.40

15. \$109 449.87

16. \$1798.97

Chapter 7 Practice Test, pages 466–467

1. D **2.** B **3.** B **4.** C **5.** A

6. a) \$33.60 **b)** \$223.20

c) 15.05%

7. \$516.99

8. a) 3 years, as the annuity lasts for 36 months

b) 6%, with 12 payments per year

c) \$15 734.44 **d)** \$1 334.44

9. Answers will vary. Sample answer: Option A is better because it pays more in interest.

10. approximately 6.9%

11. approximately 5.97%

12. a) \$808.35 **b)** \$151.81

13. \$3115.83

14. 817 days

15. a) A time line for the future value of an annuity with $R = 200$, $n = 1040$, and $i = 0.0005$.

b) \$272 723.63 **c)** \$64 723.63

16. a) A time line for the present value of an annuity with $R = 336$, $n = 1040$, and $i = 0.0005$.

b) \$336.36

c) \$141 814.40

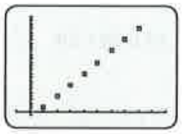
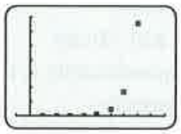
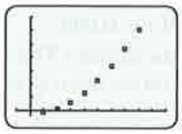
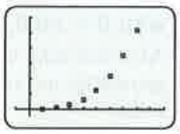
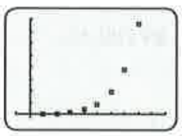
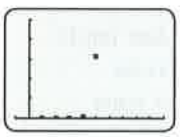
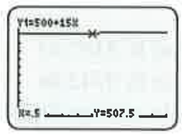
17. a) A time line for the future value of an annuity with R, $n = 52$, and $i = 0.0025$.

b) \$93.77

18. 33.6%

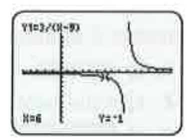
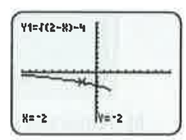
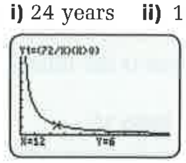
19. \$52.04

Chapters 6 and 7 Review, pages 468–469

1. a) 1, 4, 7; Answers may vary.
   b) 5, 17, 65; Answers may vary.
   c) -11, 4, 29; Answers may vary.
   d) $\frac{5}{2}$, 6, $\frac{31}{2}$; Answers may vary.
   e) 1, 7, 19; Answers may vary.
   f) -2, -4, 8; Answers may vary.
2. a)  b) 
   c)  d) 
   e)  f) 
3. a) \$38 250, \$36 377.50, \$34 520.63
   b) $V(n) = 38\,250(0.95)^{n-1}$
   c) \$14 433.78. Answers may vary.
4. 1, -1, 3, -5; $f(1) = 1$, $f(n) = f(n-1) + (-2)^{n-1}$
5. a) $t_n = 2n + 3$; $t_1 = 5$, $t_n = t_{n-1} + 2$
   b) $t_n = 2^{2^{n-1}}$; $t_1 = 2$, $t_n = (t_{n-1})^2$
6. Answers may vary.
7. a) $128x^7 + 2240x^6 + 16\,800x^5 + 70\,000x^4 + 175\,000x^3 + 262\,500x^2 + 218\,750x + 78\,125$
   b) $a^{10} - 15a^8b + 90a^6b^2 - 270a^4b^3 + 405a^2b^4 - 243b^5$
   c) $\frac{64}{x^6} + \frac{192}{x^3} + 240 + 160x^3 + 60x^6 + 12x^9 + x^{12}$
   d) $625 - \frac{1500}{\sqrt{n}} + \frac{1350}{n} - \frac{540}{n\sqrt{n}} + \frac{81}{n^2}$
8. a) neither
   b) arithmetic, $t_n = 4n - 7$, $t_{12} = 41$
   c) geometric, $t_n = 3(4)^{n-1}$, $t_{12} = 12\,582\,912$
   d) geometric, $t_n = 2\,657\,205\left(-\frac{1}{3}\right)^{n-1}$, $t_{12} = -15$
9. $t_1 = -15$, $t_2 = -8$
10. 13 terms
11. a) $t_n = 175n - 45$ b) 830 members
    c) after 12 weeks
12. a) 335 b) $-8\,138\,020$
    c) 511.5 d) $-\frac{155}{6}$
13. a) 26.84 cm b) 1383.71 cm
14. a) The principal is the amount of money initially invested or borrowed.
    b) The amount is the value of an investment or loan at the end of a time period.
    c) Simple interest is the interest calculated only on the original principal using the formula $I = Prt$, where I is the interest, in dollars; P is the principal, in dollars; r is the annual rate of interest, as a decimal; and t is the time, in years.
    d) Compound interest is the interest that is calculated at regular compounding periods.
    e) An annuity is a sum of money paid as a series of regular payments.
    f) The present value is the principal invested or borrowed today to result in a given future amount, given specified interest and time conditions.
    g) The compounding period is the time interval after which compound interest is calculated.
15. a) \$33.33 b) \$15.38 c) \$118.36
16. a) $A = 500 + 15t$
    b) 

    c) 8 months
17. \$186.90
18. a) Account A: \$8407.29 Account B: \$8523.25
    b) Answers may vary. Sample answer: Account A is the better choice because it requires a smaller initial amount.
19. \$1839.94
20. \$7031.73
21. 4.2%
22. \$17 808.46
23. \$2378.99

Course Review, pages 471–477

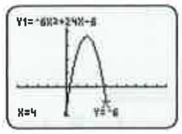
1. a) domain $\{x \in \mathbb{R}, x \neq 9\}$, range $\{y \in \mathbb{R}, y \neq 0\}$
   
   b) domain $\{x \in \mathbb{R}, x \leq 2\}$, range $\{y \in \mathbb{R}, y \geq -4\}$
   
2. C
3. a) i) 24 years ii) 12 years iii) 8 years
   b) 
   c) domain $\{r \in \mathbb{R}, r > 0\}$, range $\{n \in \mathbb{R}, n > 0\}$
4. a) minimum $\left(-\frac{3}{2}, -\frac{23}{4}\right)$ b) maximum (3, 2)
5. 150 items
6. a) $-4\sqrt{3}$ b) $7\sqrt{3} - \frac{26}{3}\sqrt{5}$
7. a) $39 + 10\sqrt{15}$ b) $3\sqrt{6} - 2$

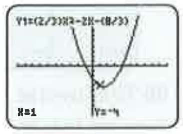
8. 6π

9. **a)–d)** all solutions are $x = 2$, $x = 25$

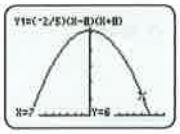
10. 2.1 m by 7.1 m

11. **a)** $y = -6x^2 + 24x - 6$ **b)** $y = \frac{2}{3}x^2 - 2x - \frac{8}{3}$





12. **a)**



b) $y = -\frac{2}{5}(x - 8)(x + 8)$

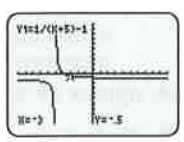
c) 25.6 m

13. Answers may vary. Sample answer: Yes, they intersect at a horizontal distance of approximately 142 m.

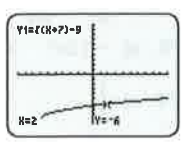
14. **a)** **i)** The functions appear to be equivalent.
ii) Algebraically the functions are equivalent.
iii) The functions seem to yield the same graph.
b) **i)** The functions appear to be equivalent.
ii) Algebraically the functions are not equivalent.
iii) The functions yield the same graph except at $x = 5$.

15. **a)** $-\frac{7x}{8}$, $x \neq 0$, $x \neq 1$
b) $\frac{-4x(x - 4)}{x + 2}$, $x \neq -9$, $x \neq -2$, $x \neq 0$, $x \neq 2$
c) $\frac{(x - 5)(x - 2)}{3}$, $x \neq -5$, $x \neq 4$

16. **a)** $f(x) = \frac{1}{x}$; $y = f(x + 5) - 1$; translate left 5 units and down 1 unit;
domain $\{x \in \mathbb{R}, x \neq -5\}$,
range $\{y \in \mathbb{R}, y \neq -1\}$

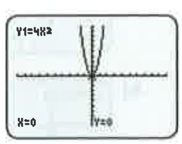


b) $f(x) = \sqrt{x}$, $y = f(x + 7) - 9$; translate left 7 units and down 9 units;
domain $\{x \in \mathbb{R}, x \geq -7\}$,
range $\{y \in \mathbb{R}, y \geq -9\}$

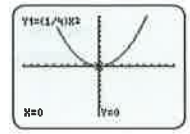


17. **a)** reflection in the x-axis and then the y-axis
b) reflection in the y-axis

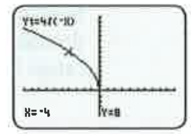
18. **a)** **i)** $g(x) = 4x^2$; reflection in the y-axis and a vertical stretch by a factor of 4;
domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



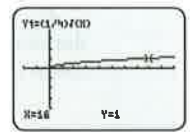
ii) $h(x) = \frac{1}{4}x^2$; vertical compression by a factor of $\frac{1}{4}$; domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



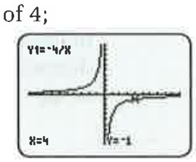
iii) $g(x) = 4\sqrt{-x}$; reflection in the y-axis and a vertical stretch by a factor of 4;
domain $\{x \in \mathbb{R}, x \leq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



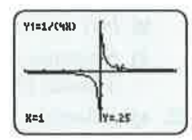
iv) $h(x) = \frac{1}{4}\sqrt{x}$; vertical compression by a factor of $\frac{1}{4}$;
domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



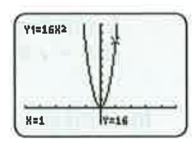
v) $g(x) = -\frac{4}{x}$; reflection in the y-axis and a vertical stretch by a factor of 4;
domain $\{x \in \mathbb{R}, x \neq 0\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$



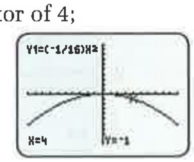
vi) $h(x) = \frac{1}{4x}$; vertical compression by a factor of $\frac{1}{4}$;
domain $\{x \in \mathbb{R}, x \neq 0\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$



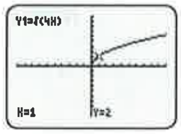
b) **i)** $g(x) = 16x^2$; horizontal compression by a factor of $\frac{1}{4}$;
domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



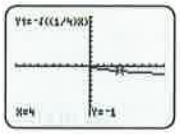
ii) $h(x) = -\frac{1}{16}x^2$; reflection in the x-axis and a horizontal stretch by a factor of 4;
domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \leq 0\}$



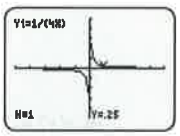
iii) $g(x) = \sqrt{4x}$; horizontal compression by a factor of $\frac{1}{4}$;
domain $\{x \in \mathbb{R}, x \geq 0\}$, range$\{y \in \mathbb{R}, y \geq 0\}$



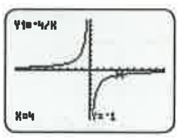
iv) $h(x) = -\sqrt{\frac{1}{4}x}$; reflection in the x-axis and a horizontal stretch by a factor of 4;
domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \leq 0\}$



v) $g(x) = \frac{1}{4x}$; horizontal compression by a factor of $\frac{1}{4}$;
domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

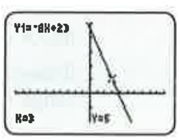


vi) $h(x) = -\frac{4}{x}$; reflection in the x-axis and a horizontal stretch by a factor of 4;
domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

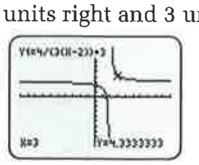


19. a) domain $\{t \in \mathbb{R}, 0 \leq t \leq 2.56\}$, range $\{h \in \mathbb{R}, 0 \leq h \leq 32\}$
b) $h(t) = -5.6t^2 + 32$
c) Answers may vary. Sample answer: only the domain changes, domain $\{t \in \mathbb{R}, 0 \leq t \leq 2.39\}$

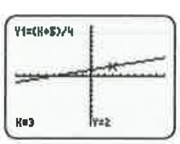
20. a) reflection in the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{3}$, and then translation of 4 units right and 1 unit down; $g(x) = -6x + 23$



b) vertical compression by a factor of $\frac{1}{3}$, horizontal stretch by a factor of 4, and then translation of 2 units right and 3 units up; $g(x) = \frac{4}{3(x-2)} + 3$



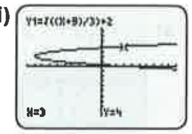
21. a) i) $f^{-1}(x) = \frac{x+5}{4}$ ii)



iii) The inverse is a function.

b) i) $f^{-1}(x) = \pm\sqrt{\frac{x+9}{3}} + 2$

ii)



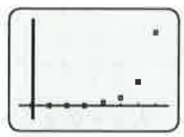
iii) The inverse is not a function.

22. a) domain $\{r \in \mathbb{R}, r \geq 0\}$, range $\{A \in \mathbb{R}, A \geq 0\}$
b) $r = \sqrt{\frac{A}{\pi}}$; domain $\{A \in \mathbb{R}, A \geq 0\}$, range $\{r \in \mathbb{R}, r \geq 0\}$

23. a)

Day	Number of Bacteria
0	20
1	60
2	180
3	540
4	1 620
5	4 860
6	14 580
7	43 740

b) $y = 20(3)^t$
c)



This is a function because each element in the domain corresponds to exactly one element in the range.
d) i) 95 659 380 ii) 209 207 064 100
e) Answers may vary. Sample answer: The consecutive values in each difference column increases by a factor of 3.

24. approx 40 years

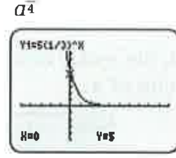
25. a) $\frac{1}{2^5} = \frac{1}{32}$ **b)** $3^{11} = 177\ 147$
c) $\frac{3^4}{2^6} = \frac{81}{64}$ **d)** 6

26. a) $-12n^3$ **b)** $\frac{4c^2}{5}$ **c)** $\frac{b^6}{27a^6}$ **d)** $-\frac{243q^{20}}{32p^{15}}$

27. a) $\frac{1}{8}$ **b)** $\frac{3}{2}$ **c)** $\frac{25}{4}$

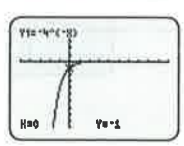
28. a) $\frac{b^{\frac{7}{3}}}{a^{\frac{9}{4}}}$ **b)** $\frac{v^{\frac{3}{20}}}{u^{\frac{2}{5}}}$ **c)** $w^{\frac{13}{8}}$

29. a)



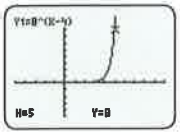
domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y > 0\}$, no x-intercept, y-intercept is 5, always decreasing, and asymptote $y = 0$

b)

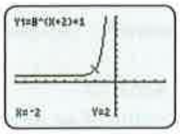


domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y < 0\}$, no x-intercept, y-intercept is -1, always increasing, and asymptote $y = 0$

30. a) $A = 300\left(\frac{1}{2}\right)^t$ **b)** domain $\{t \in \mathbb{R}, t \geq 0\}$

31. a) 

No effect on domain, range, or asymptote.

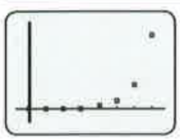
b) 

No effect on the domain, but the asymptote changes to $y = 1$ and the range changes to $\{y \in \mathbb{R}, y > 1\}$.

32. a) $y = -4(11)^x$ **b)** $y = 11^{-\frac{3}{4}x}$

33. a)

Time (1-h intervals)	Number of New Patients Diagnosed
0	1
1	3
2	9
3	27
4	81
5	243
6	729
7	2187

b) 

c) exponential

d) $y = 3^t$

34. a) 30°

b) $\sin 240° = -\frac{\sqrt{3}}{2}$, $\cos 240° = -\frac{1}{2}$, $\tan 240° = \sqrt{3}$; $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\tan 210° = \frac{1}{\sqrt{3}}$

35. a) $15\sqrt{2}$ km **b)** Answers may vary.

36. 60°, 120°

37. a) $\sin A = \frac{7}{\sqrt{53}}$, $\cos A = -\frac{2}{\sqrt{53}}$, $\tan A = -\frac{7}{2}$

$\sin B = \frac{7}{\sqrt{53}}$, $\cos B = \frac{2}{\sqrt{53}}$, $\tan B = \frac{7}{2}$

b) $\angle A = 106°$, $\angle B = 74°$

38. a) 13 cm

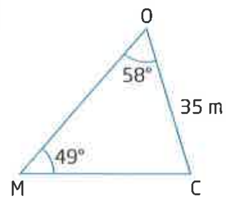
b) $\sin P = \frac{12}{13}$, $\cos P = \frac{5}{13}$, $\tan P = \frac{12}{5}$

$\csc P = \frac{13}{12}$, $\sec P = \frac{13}{5}$, $\cot P = \frac{5}{12}$

c) $\sin R = \frac{5}{13}$, $\cos R = \frac{12}{13}$, $\tan R = \frac{5}{12}$

$\csc R = \frac{13}{5}$, $\sec R = \frac{13}{12}$, $\cot R = \frac{12}{5}$

39. a) 25°, 155° **b)** 100°, 260° **c)** 156°, 336°

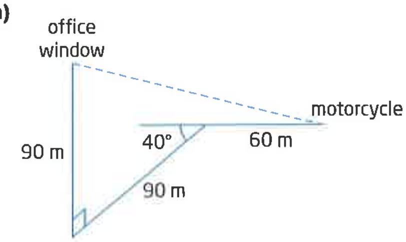
40. a) 

Since the triangle has no right angle, it is an oblique triangle.

b) It is not necessary to consider the ambiguous case because two angles and a side are given.

c) 39.3 m, 44.3 m

41. 38.7 km

42. a) 

b) 167.6 m

43.–44. Answers may vary.

45. a) 12 h and 20 min

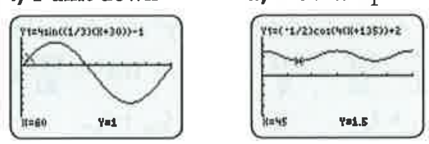
b) 1.95 m **c)** 1:00 a.m.

46. a) i) amplitude 4 **ii)** amplitude $\frac{1}{2}$

b) i) 1080° **ii)** 90°

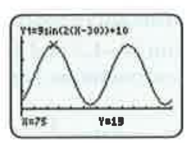
c) i) 30° to the left **ii)** 135° to the left

d) i) 1 unit down **ii)** 2 units up

e) 

47. a) $y = 6 \sin\left[\frac{12}{5}(x + 37.5°)\right] - 2$

b) $y = 6 \cos\left(\frac{12}{5}x\right) - 2$

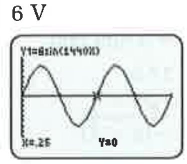
48. a) 

b) i) maximum 19 m, minimum 1 m

ii) 10 m **iii)** 3 min

49. a) 0.25 s **b)** 1440

c) 6 V **d)** $V = 6 \sin 1440t$

e) 

Each tick mark on the x-axis represents 0.125 s. Each tick mark on the y-axis represents 1V.

50. a) $\frac{40}{9}$ **b)** -2187

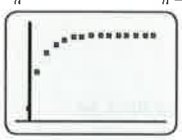
51. a) 3, 15, 75, 375, 1875

b) $\frac{2}{5}, -\frac{3}{5}, -\frac{8}{5}, -\frac{13}{5}, -\frac{18}{5}$

52. a)

5-h Interval	Amount of Medication (mg)
0	400
1	600
2	700
3	750
4	775
5	787.5
6	793.75
7	796.875
8	798.437 5
9	799.218 75
10	799.609 375
11	799.804 687 5
12	799.902 343 8
13	799.951 171 9
14	799.975 585 9

b) 400, 600, 700, 750, 775, …, 799.975 585 9; $t_n = 400 + 0.5t_{n-1}$

c) 

d) Answers may vary. Sample answer: Over time the amount of drug remaining in the body will vary between 800 mg right after a dose and 400 mg just before the next dose is taken.

53. a) $x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$

b) $\frac{x^4}{81} - \frac{8x^4}{27} + \frac{24x^4}{9} - \frac{32x^4}{3} + 16x^4 = \frac{625}{81}x^4$

54. a) $t_{4,1} + t_{4,2}$ **b)** $t_{9,6} + t_{9,7}$

55. a) i) arithmetic; the first term is 9 and the common difference between the consecutive terms is -4

ii) arithmetic; the first term is $\frac{1}{5}$ and the common difference between the consecutive terms is $\frac{2}{5}$

iii) not arithmetic; the first term is -4.2 and the difference between consecutive terms is not equal

b) i) $t_n = -4n + 13$ **ii)** $t_n = \frac{2}{5}n - \frac{1}{5}$

56. a) $t_n = 90\left(\frac{1}{3}\right)^{n-1}$, $t_{10} = \frac{10}{2187}$

b) $t_n = \frac{1}{4}\left(\frac{2}{3}\right)^{n-1}$, $t_{10} = \frac{128}{19\,683}$

c) $t_n = -0.0035(-10)^{n-1}$, $t_{10} = 3\,500\,000$

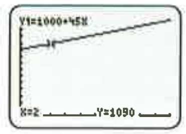
57. a) -115 **b)** 17.5

58. a) $67\frac{362}{729}$ **b)** $\frac{x^{16} - 1}{-(x + 1)}$

59. a) approximately 1.26 m

b) approximately 179.7 m

60. a) $A = 1000 + 45t$, with 1000 representing the fixed part and $45t$ representing the variable part



b) 22 years 3 months **c)** 16.7%

61. Answers will vary. Sample answer: Top Bank; it pays more in interest.

62. a) \$667.27 **b)** \$257.73

63. a) A time line for the future value of an annuity with $R = 120$, $n = 60$, and $i = 0.004\,375$.

b) \$8213.00 **c)** \$1013.00

64. a) A time line for the present value of an annuity with $R = 700$, $n = 20$, and $i = 0.0175$.

b) \$11 727.02 **c)** \$2272.98

Prerequisite Skills Appendix, pages 478–495

Angle Sum of a Triangle, page 478

1. a) $x = 76°$ **b)** $x = 76°$ **c)** $x = 36°$

Apply the Sine Law and the Cosine Law, page 478

1. a) 9.6 cm **b)** 53.5° **c)** 69.3°

Apply Trigonometric Ratios to Problems, page 479

1. 6.0 m **2.** 8.6°

Classify Triangles, page 480

1. a) isosceles acute triangle

b) scalene obtuse triangle

c) equilateral triangle

Common Factors, page 480

1. a) 4 **b)** 3a **c)** $5xy^2$

d) $24m^2n^2$ **e)** $a^2 + 3$ **f)** $3xy$

2. a) $4x(4x + 5)$ **b)** $5xy^2(x + 2y)$

c) $3a^2(a - 3)$ **d)** $4r^2s(r^3s + 4)$

e) $2ab(4a^2 - 5 + 2ab)$ **f)** $-6x^2y(x + 3x^2y^2 + 6y^3)$

g) $(p + 3q)(12p - q)$ **h)** $4x(2 + 5y)(y - 2x^2)$

Determine an Angle Given a Trigonometric Ratio, page 481

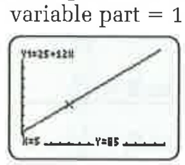
1. a) 75° **b)** 57° **c)** 70°

d) 40° **e)** 20° **f)** 32°

Direct Variation and Partial Variation, page 481

1. a) $C(d) = 25 + 12d$

b) fixed part = 25 (\$25); variable part = 12 (\$12 per day)

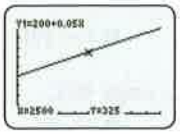
c) 

d) slope = 12, vertical intercept 25

e) Answers may vary. Sample Answer: The slope of the graph is equal to the variable part of the equation. The vertical intercept of the graph is equal to the fixed part of the equation.

2. a) $E = 200 + 0.05s$, where E represents Irene's total earnings and s represents her total sales.

b) fixed part = 200 (\$200); variable part = 0.05 (5% commission of her sales)

c) 

d) slope = 0.05, vertical intercept 200

e) Answers may vary. Sample Answer: The slope of the graph is equal to the variable part of the equation. The vertical intercept of the graph is equal to the fixed part of the equation.

Distributive Property, page 482

1. a) $2a + 2b$ b) $6x - 24$ c) $4k^2 + 20$
d) $-3x + 6$ e) $5x^2 - 10x + 5$ f) $6x^2 - 8x$
g) $24a + 8a^2$ h) $-2x^2 - 2xy + 6x$

Evaluate Expressions, page 482

1. a) 54 b) 1.92 c) 19
d) 111.98 e) 25.5 f) 322

2. a) $\frac{1}{4}$ b) $\frac{5}{4}$ c) $-\frac{43}{24}$
d) $-\frac{7}{12}$ e) $-\frac{5}{2}$ f) $-\frac{1}{5}$

Exponent Rules, page 483

1. a) 2^7 b) 5^6 c) 3^3
d) 4^5 e) 6^8 f) 9^{21}
g) a^{10} h) z^8 i) $6x^5$
j) y^3 k) p^{18} l) n^5
m) $-4x^3$ n) $8t^{12}$ o) $256x^8$

2. a) 64 b) 2 c) 729
d) 256 e) 4096 f) $\frac{1}{64}$

Factor Quadratic Expressions, page 484

1. a) $(x + 4)(x + 2)$ b) $(x - 3)(x - 4)$
c) $2(x + 6)(x - 3)$ d) $3(x + 4)(x - 4)$
e) $(3x - 5)(x - 2)$ f) $(x - 3)(x - 3)$
g) $4(x + 5)(x - 5)$ h) $92x - 5)(x + 4)$
i) $(x - 2)(4x - 7)$

Find Primary Trigonometric Ratios, page 484

1. a) $\sin P = \frac{24}{25}$; $\cos P = \frac{7}{25}$; $\tan P = \frac{24}{7}$

b) $\sin P = \frac{12}{13}$; $\cos P = \frac{5}{13}$; $\tan P = \frac{12}{5}$

2. a) $\sin 58° = 0.8480$; $\cos 58° = 0.5299$; $\tan 58° = 1.6003$

b) $\sin 79° = 0.9816$; $\cos 79° = 0.1908$; $\tan 79° = 5.1446$

c) $\sin 15° = 0.2588$; $\cos 15° = 0.9659$; $\tan 15° = 0.2679$

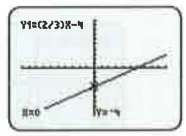
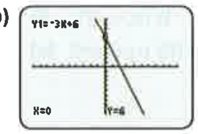
Finite Differences, page 485

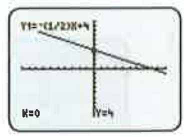
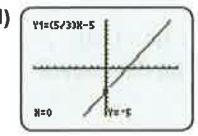
1. a) The first differences are constant, 4. So, the relationship is linear.

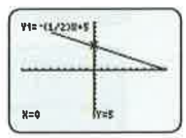
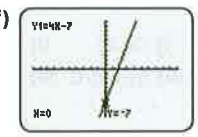
b) The first differences are not constant. So, the relationship is not linear. The second differences are not constant. So, the relationship is not quadratic.

c) The first differences are not constant. So, the relationship is not linear. The second differences are constant, −2. So, the relationship is quadratic.

Graphs and Lines, page 486

1. a)  b) 

c)  d) 

e)  f) 

2. a) $y = 3x - 2$ b) $y = -\frac{5}{3}x + 4$

3. a) $y = 2x + 7$ b) $y = -\frac{7}{15}x + \frac{12}{5}$

c) $y = \frac{3}{5}x - 11$

Identify Patterns, page 487

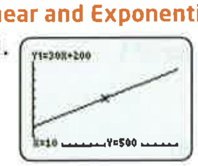
1. a) ZYXW, ZYXWV, ZYXWVU

b) 17, 22, 27 c) 81, 243, 729

d) −13, −21, −29 e) $\frac{5}{6}, -\frac{6}{7}, \frac{7}{8}$

f) $2x^4, \frac{2}{3}x^5, \frac{2}{9}x^6$

Linear and Exponential Growth, page 488

1. 

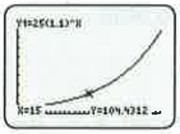
a) linear

b) slope = 30; y-intercept 200

c)

x	y	First Differences
0	200	
1	230	30
2	260	30
3	290	30

d) The first differences are all 30.

2. 

a) exponential
b) y-intercept 25

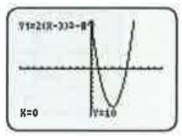
c)

x	y	First Differences	Second Differences	Common Ratios
0	25			
1	27.5	2.5		1.1
2	30.25	2.75	0.25	1.1
3	33.275	3.025	0.275	1.1

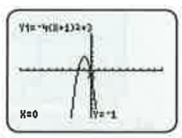
d) The common ratios are all 1.1.

Quadratic Relations, page 489

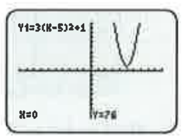
1. a) i) $(3, -8)$ ii) $x = 3$
iii) upward iv) 10



b) i) $(-1, 3)$ ii) $x = -1$
iii) downward iv) -1



c) i) $(5, 1)$ ii) $x = 5$
iii) upward iv) 76



2. a) upward, stretched vertically by a factor of 5; Since $a > 0$ $(a = 5)$, the parabola opens upward. Since $a > 1$, the parabola is stretched vertically by a factor of 5.
b) downward, compressed vertically by a factor of $\frac{1}{4}$; Since $a < 0$ $\left(a = -\frac{1}{4}\right)$ the parabola opens downward. Since $0 < a < 1$, the parabola is compressed vertically by a factor of $\frac{1}{4}$.
c) downward, stretched vertically by a factor of 3; Since $a < 0$ $(a = -3)$ the parabola opens downward. Since $a < -1$, the parabola is stretched vertically by a factor of 3.

3. a) $y = (x + 2)^2 + 3$; $(-2, 3)$
b) $y = (x - 6)^2 - 33$; $(6, -33)$
c) $y = 3(x + 3)^2 - 29$; $(-3, -29)$
d) $y = -2(x - 4)^2 + 41$; $(4, 41)$

Rearrange Formulas, page 490

1. a) $x = \frac{-y + 5}{4}$ b) $w = \frac{P - 2\ell}{2}$
c) $y = \pm\sqrt{r^2 - x^2}$ d) $h = \frac{3v}{\pi r^2}$
e) $e = \frac{2 - st}{10}$ f) $r = \frac{A - P}{Pt}$

Solve Equations, page 490

1. a) $x = -3$ b) $x = 4$ c) $x = 5$ d) $x = 4$
2. a) $w = 9.25$ b) $p = 0.7$ or $p = 5.3$
c) $i = -2.0325$ or $i = 0.0325$ d) $k = 3.4$

Solve Equations Involving Rational Expressions, page 491

1. a) $x = 2$ b) $a = 35$ c) $c = 3$
d) $r = 27$ e) $k = \frac{15}{2}$ f) $t = 270$

Solve Linear Systems Equations, page 492

1. a) $(1, -2)$ b) $(5, -4)$ c) $(2, 0)$
2. a) $(5, -1)$ b) $(-2, -11)$ c) $(-2, -4)$
d) $(5, 4)$ e) $(-2, 2)$ f) $(4, 0)$

Use Similar Triangles, page 493

1. a) $x = 9.6$ m b) $s = 12$ cm

Use the Pythagorean Theorem, page 493

1. a) 8.5 cm b) 10.8 mm c) 16.2 m

Work With Fractions, page 494

1. a) 120 b) 1008 c) $770x^4$
d) $(x + 3)(x + 4)(x - 3)$
2. a) $\frac{25}{18}$ b) $\frac{11}{120}$
c) $\frac{27a + 20b}{144}$ d) $\frac{8x - 5y}{90}$
3. a) $\frac{2}{5}$ b) $-\frac{1}{4}$ c) $-\frac{5}{6}$ d) $-\frac{1}{10}$

Work With Polynomials, page 495

1. a) $3x^2 + 21x + 30$ b) $-2x^2 - 6x + 56$
c) $x^2 - 12x + 36$
2. a) $(x + 5)(x - 2)$ b) $(x + 7)(x + 7)$
c) $(3y + 5)(3y - 5)$ d) $3(a + 8)(a + 8)$
e) $(5x - 6)(5x - 6)$ f) $-(p + 10)(p + 10)$
3. a) 64 b) 225 c) 400
d) 9 e) $\frac{49}{4}$ f) $\frac{4}{9}$

Zero and Negative Exponents, page 495

1. a) 1000 b) $\frac{1}{64}$ c) -1
d) $\frac{1}{25}$ e) $\frac{1}{729}$ f) 216
g) 32 h) $\frac{2}{81}$ i) $\frac{9}{4}$
2. a) $\frac{3}{x^4}$ b) $\frac{25}{y^4}$ c) $\frac{2}{27x^3}$
d) $\frac{x^3}{2y^2}$ e) $\frac{b^6}{a^5}$ f) $\frac{27n^9}{64m^{21}}$

Glossary

A

absolute value The distance of a number from zero on a real number line.

$|3| = 3$ and $|-3| = 3$

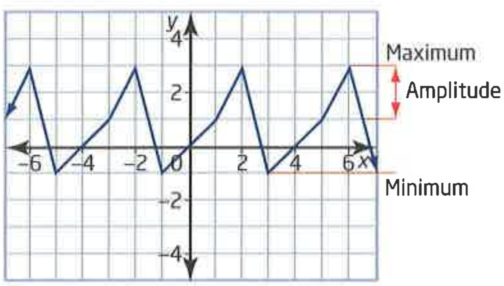
acute angle An angle whose measure is less than 90°.

acute triangle A triangle in which each of the three interior angles is acute.

algebraic modelling The process of representing a relationship by an equation or a formula, or representing a pattern of numbers by an algebraic expression.

amount The value of an investment or loan at the end of a time period, calculated by adding the principal and interest.

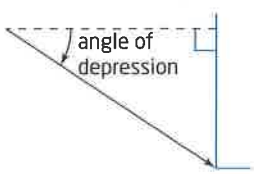
amplitude Half the distance between the maximum and minimum values of a periodic function.



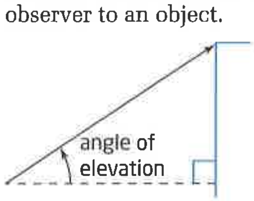
angle bisector A line that divides an angle into two equal parts.

angle in standard position An angle with vertex at the origin and initial arm on the positive x-axis.

angle of depression The angle, measured downward, between the horizontal and the line of sight from an observer to an object.



angle of elevation The angle, measured upward, between the horizontal and the line of sight from an observer to an object.



annual rate of interest The rate at which interest is charged, as a percent, per year.

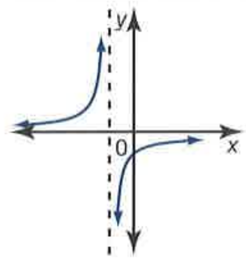
annuity A sum of money paid as a series of equal payments at regular intervals of time.

arithmetic sequence A sequence where the difference between consecutive terms is a constant.

arithmetic series The indicated sum of the terms of an arithmetic sequence. $S_n = \frac{n}{2}[2a + (n - 1)d]$ gives the sum of the first n terms of an arithmetic series with first term a and common difference d.

associative property $a + (b + c) = (a + b) + c$ for addition and $a \times (b \times c) = (a \times b) \times c$ for multiplication.

asymptote A line that a curve approaches more and more closely, but never touches.



B

base function The simplest form of various functions: linear $f(x) = x$, quadratic $f(x) = x^2$, radical $f(x) = \sqrt{x}$, rational $f(x) = \frac{1}{x}$.

base (of a power) The number used as a factor for repeated multiplication.

In 6^3, the base is 6.

binomial A polynomial with two terms.

$3x + 4$ is a binomial.

C

CAST rule An acronym that tells which primary trigonometric ratios are positive in which quadrant.

Sin	All
Tan	Cos

circle The set of all points in the plane that are equidistant from a fixed point called the centre.

coefficient The factor by which a variable is multiplied.
In the term $8y$, the coefficient is 8;
in the term ax, the coefficient is a.

common difference The difference between any two consecutive terms in an arithmetic sequence.
For the sequence 1, 4, 7, 10, ..., the common difference is 3.

common ratio The ratio of any two consecutive terms in a geometric sequence.
For the sequence 1, 2, 4, 8, 16, ..., the common ratio is 2.

commutative property $a + b = b + a$ for addition and $ab = ba$ for multiplication.

completing the square Part of a process by which a quadratic function in standard form can be written in vertex form.

compound interest Interest that is calculated at regular compounding periods and is added to the principal to earn interest for the next compounding period.

compounding period The time interval after which compound interest is calculated.

compression A transformation that is a stretch by a factor less than 1.

conjecture A generalization, or educated guess, made using inductive reasoning.

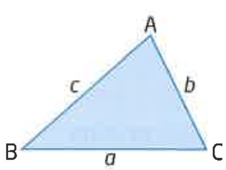
constant term A term that does not include a variable.

continuous function A function that maps real numbers to real numbers and has a graph that is a curve with no holes or jumps.

continuous graph A graph that consists of an unbroken line or curve.

cosecant ratio The reciprocal of the sine ratio.
$\csc A = \dfrac{1}{\sin A}$

cosine law The relationship between the lengths of the three sides and the cosine of an angle in any triangle.
$a^2 = b^2 + c^2 - 2bc \cos A$



cosine ratio In a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse.
$\text{cosine} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

cotangent ratio In trigonometry, the reciprocal of the tangent ratio.
$\cot A = \dfrac{1}{\tan A}$

coterminal angles Angles in standard position that have the same terminal arm.

counterexample An example that demonstrates that a conjecture is false.

cube root Given a number z, the number a such that $a^3 = z$.
$\sqrt[3]{8} = 2$, because $2 \times 2 \times 2 = 8$, or $2^3 = 8$.

cycle One complete pattern of a periodic function.

D

dependent variable In a relation, the variable whose value depends on the value of the independent variable. On a coordinate grid, the values of the dependent variable are on the vertical axis.
In $d = 4.9t^2$, d is the dependent variable.

depreciation The amount by which an item decreases in value.

discontinuity A point at which a function is not defined. The graph has a break at this point.
$f(x) = \frac{1}{x}$ has a discontinuity at $x = 0$.

discrete function A function that is made up of separate points that are not connected.

discriminant In the quadratic formula, the quantity under the radical sign, $b^2 - 4ac$.

distance between two points The length of the line segment joining the points.
For points (x_1, y_1) and (x_2, y_2),
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

distributive property $a(b + c) = ab + ac$.

domain The set of all values of the first coordinates of the ordered pairs, or the independent variable, in a relation.

double root The solution of an equation where two roots are the same.
The equation $(x - a)^2 = 0$ has a double root at $x = a$.

E

elements The individual members of a set.

entire radical A radical in the form $\sqrt{n}$, where $n > 0$.
$\sqrt{29}$ and $\sqrt{\frac{5}{3}}$ are entire roots.

equilateral triangle A triangle with all three sides equal.

equivalent expressions Expressions that have the same value for all values of the variable(s).

explicit formula A formula for the nth term of a sequence, from which the terms of the sequence may be obtained by substituting 1, 2, 3, and so on, for n.

$t_n = 2n + 1$ is an explicit formula for the arithmetic sequence 3, 5, 7, 9, ...

exponent The raised number that denotes repeated multiplication of a base.

In $3x^4$, the exponent is 4.

exponential decay A pattern of decay in which each term is multiplied by a constant amount, between 0 and 1, to produce the next term.

exponential equation An equation that has a variable in an exponent.

$3^x = 81$ is an exponential equation.

exponential function A function in which a variable is an exponent. It can be defined by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

exponential growth A pattern of growth in which each term is multiplied by a constant amount, greater than 1, to produce the next term.

extrapolate To estimate values lying outside the range of given data. To extrapolate from a graph means to estimate coordinates of points beyond those that are plotted.

F

factor To express a number as the product of two or more numbers, or an algebraic expression as the product of two or more other algebraic expressions. Also, the individual numbers or algebraic expressions in such a product.

Fibonacci sequence The sequence of numbers 1, 1, 2, 3, 5, 8, ... Each term, after the first two terms, is the sum of the preceding two terms.

finite differences Differences found from the y-values in a table of values with evenly spaced x-values. See first differences and second differences.

first differences Differences between consecutive y-values in a table of values with evenly spaced x-values.

x	y	First Differences
1	3	
		5 − 3 = 2
2	5	
		7 − 5 = 2
3	7	
		9 − 7 = 2
4	9	
		11 − 9 = 2
5	11	

fractal A curve that generates itself by replacing each side of its original shape with a generator and iterating the process.

function A relation in which each value of the independent variable (the first coordinate) corresponds to exactly one value of the dependent variable (the second coordinate).

future value The amount that a principal invested or borrowed will grow to, given specified interest and time conditions.

G

generalize To determine a general rule or conclusion from examples. Specifically, to determine a general rule to represent a pattern or relationship between variables.

geometric mean If a, x, and b are consecutive terms of a geometric sequence, then x is the geometric mean of a and b.

geometric sequence A sequence where the ratio of consecutive terms is a constant.

1, 3, 9, 27, ... is a geometric sequence with common ratio 3.

geometric series The indicated sum of the terms of a geometric sequence. $S^n = \dfrac{a(r^n - 1)}{r - 1}$ gives the sum of the first n terms of a geometric series with first term a and common ratio r.

graphing calculator A hand-held device capable of a wide range of mathematical operations, including graphing from an equation, constructing a scatter plot, determining the equation of a line of best fit for a scatter plot, making statistical calculations, and performing elementary symbolic manipulation. Many graphing calculators will attach to scientific probes that can be used to gather data involving physical measurements, such as position, temperature, and force.

graphing software Computer software that provides features similar to those of a graphing calculator.

greatest common factor (GCF) The expression with the greatest numerical coefficient and greatest degree that is a factor of two or more terms.

The GCF of $12ab$ and $8bc$ is $4b$.

H

half-life The length of time for an unstable element to spontaneously decay to one half of its original amount.

hypotenuse The longest side of a right triangle.

I

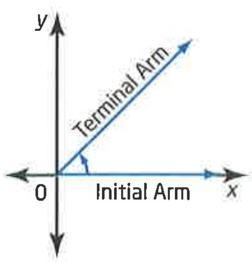
identity An equation that is true for all values of the variable for which the expressions on each side of the equation are defined.

image point Any point that has been transformed from a point on the original figure or graph.

independent variable In a relation, the variable whose value determines that of the dependent variable. On a coordinate grid, the values of the independent variable are on the horizontal axis.
In $d = 4.9t^2$, t is the independent variable.

inequality A mathematical statement that contains one of the symbols $<$, $\leq$, $>$, $\geq$, or $\neq$.

initial arm The arm, or ray, of an angle in standard position that is on the positive x-axis.



integer The set of whole numbers and their opposites, represented by $\mathbb{Z}$.

intercept The distance from the origin of the xy-plane to the point at which a line or curve crosses a given axis.

interest rate The percent of the principal that is earned, or paid, as interest.

interpolate To estimate values lying between elements of given data. To interpolate from a graph means to estimate coordinates of points between those that are plotted.

interval An unbroken part of the real number line, such as $-3 < x < 5$.

invariant points Points that are unaltered by a transformation.

inverse of a function A function and its inverse undo each other. A function f and its inverse f^{-1} have the property that if $f(a) = b$, then $f^{-1}(b) = a$.

irrational number A number that cannot be written in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$ and $b \neq 0$.
$\sqrt{2}$, $\sqrt{3}$, and π are irrational numbers.

isosceles triangle A triangle with exactly two equal sides.

L

least common denominator (LCD) The least common multiple of the denominators of two or more rational expressions.

least common multiple (LCM) The least multiple that two or more expressions share.
The LCM of $2x$ and $3x^2$ is $6x^2$.

like radicals Numbers that have the same radicand.
$2\sqrt{3}$ and $8\sqrt{3}$ are like radicals.

like terms Terms that have exactly the same variable(s) raised to exactly the same exponent(s).
$3x^2$, $-x^2$, and $2.5x^2$ are like terms.

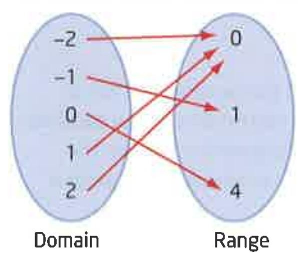
line of symmetry A line such that a figure coincides with its reflection image over the line.

line segment The part of a line that joins two points.

linear relation A relation between two variables that appears as a straight line when graphed on a coordinate system. May also be referred to as a linear function.

M

mapping diagram A graphical representation that relates the values in one set, the domain, to a second set, the range, using directed arrows from domain to range.



mapping notation A style of writing a function using an arrow.
$f(x) \rightarrow ax^2 + bx + c$.

mathematical model A mathematical description of a real situation. The description may include a diagram, a graph, a table of values, an equation, a formula, a physical model, or a computer model.

mathematical modelling The process of describing a real situation in mathematical form.

mean The sum of a set of values divided by the number of values.
The mean of 2, 4, 6, and 8 is 5.

midpoint The point that divides a line segment into two equal points. For the line segment joining points (x_1, y_1) and (x_2, y_2), the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

mixed radical A radical in the form $a\sqrt{b}$, where $a \neq \pm 1$ and $b > 0$.

$3\sqrt{10}$ and $\frac{1}{2}\sqrt{7}$ are mixed radicals.

monomial An algebraic expression with one term.

$7x$ is a monomial.

N

natural number A number in the sequence 1, 2, 3, 4, ..., represented by $\mathbb{N}$.

non-linear relation A relationship between two variables that does not fit a straight line when graphed.

O

oblique triangle A triangle that is not right-angled.

obtuse angle An angle that measures more than 90° but less than 180°.

obtuse triangle A triangle containing one obtuse angle.

ordinary annuity A series of equal payments made at the end of each payment period.

origin The point of intersection of the x-axis and the y-axis on a coordinate grid.

P

parabola A symmetrical U-shaped curve that is the graph of a quadratic function. The domain is any real number.

Pascal's triangle A triangular arrangement of numbers with 1 in the first row, 1 and 1 in the second row, and where each number in the succeeding rows is the sum of the two numbers above it in the preceding row.

payment interval The time between successive payments of an annuity.

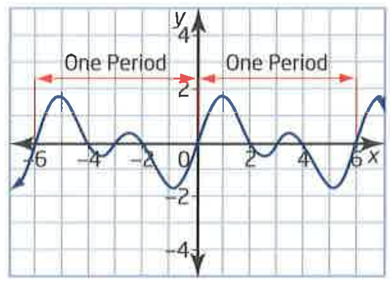
payment period Another name for the payment interval.

perfect square A number that is the square of a whole number.

perfect square trinomial The trinomial that results from squaring a binomial.

perimeter The distance around a polygon.

period The horizontal length of one cycle of a periodic function.



period (of a pendulum) The time it takes to complete one back-and-forth swing.

periodic function A function that has a pattern of y-values that repeats at regular intervals.

phase shift The horizontal translation of a trigonometric function.

point of intersection The point that is common to two non-parallel lines.

polynomial An algebraic expression formed by adding or subtracting monomials.

power A product obtained by using a base as a factor one or more times.

5^3, x^6, and a^m are powers.

present value The principal invested or borrowed today to result in a given final amount, given specified interest and time conditions.

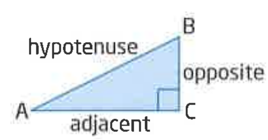
present value of an annuity The principal that must be invested today to finance a series of regular withdrawals.

primary trigonometric ratios The three ratios, sine cosine, and tangent, defined in right triangles.

$$\sin A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



prime number A number with exactly two factors—itself and 1.

2, 5, and 13 are prime numbers.

principal An amount of money invested or borrowed.

principal square root The positive square root of a number.

prism A three-dimensional figure with two parallel, congruent polygonal bases. A prism is named by the shape of its bases, for example, rectangular prism, triangular prism.

probability The ratio of the number of favourable outcomes to the number of possible outcomes.

product rule To multiply powers of the same base, add the exponents.

$x^a \times x^b = x^{a+b}$

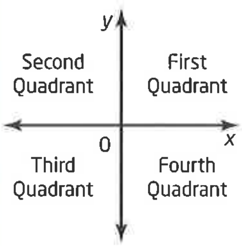
proportion An equation that states that two ratios are equal.

Pythagorean identity In trigonometry, $\sin^2\theta + \cos^2\theta = 1$ for all values of θ.

Pythagorean theorem In a right triangle, the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides.

Q

quadrant One of the four regions formed by the intersection of the x-axis and the y-axis.



quadratic equation An equation of the form $ax^2 + bx + c = 0$, where a, b, and $c \in \mathbb{R}$ and $a \neq 0$.

quadratic expression An expression of the form $ax^2 + bx + c$, where a, b, and $c \in \mathbb{R}$ and $a \neq 0$.

quadratic formula The formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the roots of a quadratic function of the form $y = ax^2 + bx + c$, where $a \neq 0$.

quadratic function A function defined by a quadratic equation.

In standard form: $y = ax^2 + bx + c$, where a, b, and $c \in \mathbb{R}$ and $a \neq 0$.

In factored form: $y = a(x - r)(x - s)$, where $a \neq 0$ and r and $s \in \mathbb{R}$ and are the x-intercepts.

In standard form: $y = a(x - h)^2 + k$, where $a \neq 0$ and the vertex is at (h, k).

quotient identity In trigonometry, $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ for all values of θ.

quotient rule To divide powers of the same base, subtract exponents.

$x^a \div x^b = x^{a-b}$

R

radical expression An expression involving the square root of an unknown.

radical sign The symbol $\sqrt{\ }$.

radicand A number or expression under a radical sign.

In $\sqrt{ab}$, the radicand is ab.

range of a relation The set of the second coordinates of the ordered pairs in a relation.

ratio A comparison of two quantities with the same units.

rational expression The quotient of two polynomials.

$\dfrac{3}{k-1}$ and $\dfrac{a^2+b^2}{a+b}$ are rational expressions.

rational number A number that can be expressed as the ratio of two integers, where the divisor is not zero.

0.75, $\dfrac{3}{8}$, and -2 are rational numbers.

real number A member of the set of all rational and irrational numbers, represented by $\mathbb{R}$.

reciprocal identities In trigonometry, the reciprocals of the primary trigonometric ratios.

$\csc\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$

reciprocals Two numbers that have a product of 1.

x and $\dfrac{1}{x}$ are reciprocals.

recursion formula A formula by which each term in the sequence is generated from the preceding term or terms.

$t_1 = 1$, $t_n = t_{n-1} + 3$ is a recursion formula for the arithmetic sequence 1, 4, 7, 10, ...

reflection A transformation in which a figure is reflected over a reflection line.

reflex angle An angle that measures more than 180° but less than 360°.

regular payments Payments of equal value made at equal time periods.

regular withdrawals Withdrawals of equal value made at equal time periods.

relation An identified pattern between two variables that may be expressed as ordered pairs, a table of values, a graph, or an equation.

restriction Any value that must be excluded for a variable.

$\frac{8y}{y+1}$ has restriction $y \neq -1$.

rhombus A parallelogram in which the lengths of all four sides are equal.

right angle An angle that measures 90°.

right bisector of a line segment A line that is perpendicular to a line segment and divides the line segment into two equal parts.

right prism A three-dimensional figure with two parallel, congruent polygonal bases and lateral faces that are perpendicular to the bases.

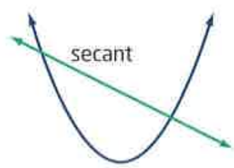
right triangle A triangle containing a 90° angle.

roots The solutions of an equation.

S

scalene triangle A triangle with no sides equal.

secant A line that intersects a curve at two distinct points.



secant ratio In trigonometry, the reciprocal of the cosine ratio.

$\sec A = \frac{1}{\cos A}$

second differences Differences between consecutive first differences in a table of values with evenly spaced x-values.

x	y	First Differences	Second Differences
1	2	5 – 2 = 3	5 – 3 = 2
2	5	10 – 5 = 5	7 – 5 = 2
3	10	17 – 10 = 7	9 – 7 = 2
4	17	26 – 17 = 9	
5	26		

sequence A set of numbers, terms, or items, usually separated by commas, arranged in an order that can be identified by a pattern or rule.

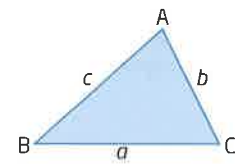
series The sum of the terms of a sequence.

similar triangles Triangles that have the same shape but different size. Corresponding sides are in proportion and corresponding angles are equal.

simple interest Interest calculated only on the original principal using the formula $I = Prt$, where I is the interest, in dollars; P is the principal, in dollars; r is the annual rate of interest, as a decimal; and t is the time, in years.

sine law The relationship between the lengths of the sides and their opposite angles in any triangle.

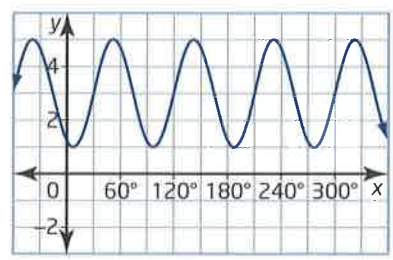
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



sine ratio In a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse.

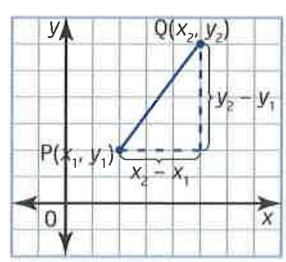
$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}$

sinusoidal function A function with the curved form of a sine wave that is used to model periodic data.



slope A measure of the steepness of a line. The slope of a line, m, containing the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$m = \frac{\text{vertical change}}{\text{horizontal change}} \text{ or } \frac{\text{rise}}{\text{run}}$$
$$= \frac{\Delta y}{\Delta x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$



slope *y*-intercept form of a linear equation The equation of the line with slope m and y-intercept b is given by $y = mx + b$.

square root A number that is multiplied by itself to give another number.

straight angle An angle that measures 180°.

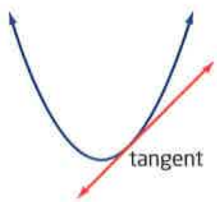
stretch A transformation that results in the distance from the x-axis of every point growing by a scale factor greater than 1 (vertical stretch) or the distance from the y-axis of every point growing by a scale factor greater than 1 (horizontal stretch).

symmetry A quality of a plane figure so that it can be folded along a fold line so that the halves of the figure match exactly.

system of equations Two or more equations that are considered together.

T

tangent line A line that touches a curve at one point and equals the slope of the curve at that point.



tangent ratio In a right triangle, the ratio of the length of the opposite side to the length of the adjacent side.

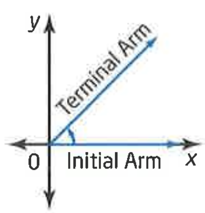
$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$$

term A number or a variable, or the product or quotient of numbers and variables.

The expression $x^2 + 5x$ has two terms: x^2 and $5x$.

term (of a sequence) A single value or object in a sequence.

terminal arm The arm, or ray, of an angle in standard position that is not on the positive x-axis.



time line A diagram used to illustrate the cash flow of an annuity.

transformation A change made to a figure or a relation such that the figure or the graph of the relation is shifted or changed in shape.

translation A slide transformation that results in a shift of the original figure without changing its shape.

translation image The image of a plane figure after a translation.

trigonometric equation An equation that contains one or more trigonometric functions.

trinomial A polynomial with three terms.

$x^2 + 3x - 1$ is a trinomial.

TVM (Time-Value-Money) Solver A feature of graphing calculators that is used for financial calculations.

U

unit circle A circle of radius 1 unit that is centred at the origin.

V

variable A letter or symbol, such as x, used to represent an unspecified number.

x and y are variables in the expression $2x + 3y$.

vertex A point at which two sides of a polygon meet.

vertex of a parabola The point of the parabola at which the graph intersects the axis of symmetry. It is the minimum point on a parabola that opens upward, or the maximum point on a parabola that opens downward.

vertical line test A method of determining whether a relation is a function. If every vertical line passes through exactly one point on the graph of a relation, then the relation is a function.

W

whole number A number in the sequence 0, 1, 2, 3, 4, 5, ...

X

x-intercept The x-coordinate of the point where a line or curve crosses the x-axis.

Y

y-intercept The y-coordinate of the point where a line or curve crosses the y-axis.

Z

zero of a function Any value of x for which the value of the function is 0.

zero product property The property that, if the product of two real numbers is zero, then one or both of the numbers must be zero.