5.7 The Cosine Law

May 6

$$PT$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

A barn whose cross-section resembles half a regular octagon with a side length of 10 m needs some repairs to its roof. The roofers place a 22.9 m ramp against the side of the building, forming an angle of 26° with the ground. The ramp will be used to transport the materials needed for the repair. The base of the ramp is 15.6 m from the side of the building.
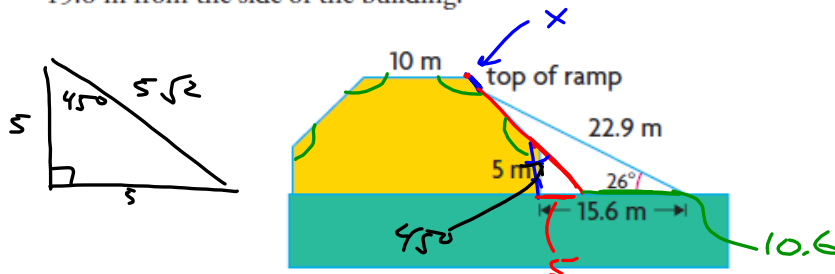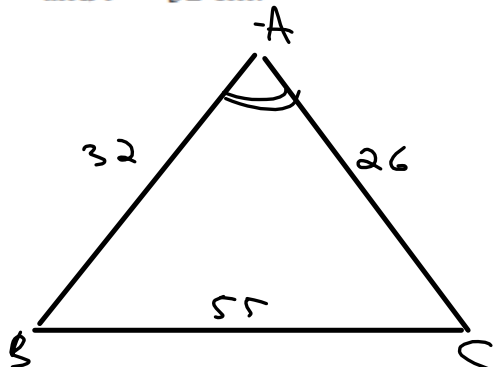
SUM OF ANGLES
OCTAGON

$= 6(180) = 1080°$

∴ EACH ANGLE $\frac{1080}{8} = 135°$

10 m   top of ramp

22.9 m

5 m   26°

15.6 m

10.6

$5\sqrt{2}$ 45°

5   45°

? How far, to the nearest tenth of a metre, is the top of the ramp from the flat roof of the building?

$10 + 5\sqrt{2}$ y 22.9   26°   10.6
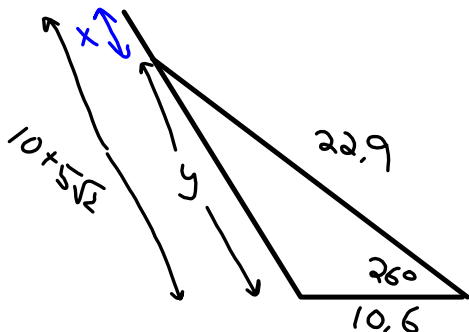
$$y^2 = 10.6^2 + 22.9^2 - 2(10.6)(22.9)\cos 26°$$
$$y^2 = 200.4$$
$$y = 14.2$$
$$x = 10 + 5\sqrt{2} - 14.2$$
$$\therefore x \doteq 2.9 \, m$$

In △ABC, determine ∠A to the nearest degree if $a = 55$ cm, $b = 26$ cm, and $c = 32$ cm.

-A   32   26   55   B   C

$$55^2 = 32^2 + 26^2 - 2(32)(26)\cos A$$
$$55^2 - 32^2 - 26^2 = -2(32)(26)\cos A$$
$$\frac{(55^2 - 32^2 - 26^2)}{(-2(32)(26))} = \cos A$$
$$A = 143°$$

Mitchell wants his 8.0 wide house to be heated with a solar hot-water system. The tubes form an array that is 5.1 m long. In order for the system to be effective, the array must be installed on the south side of the roof and the roof needs to be inclined by 60°. If the north side of the roof is inclined more than 40°, the roof will be too steep for Mitchell to install the system himself. Will Mitchell be able to install this system by himself?
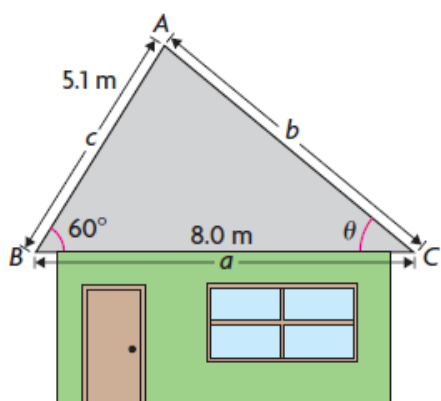
$$b^2 = 5.1^2 + 8^2 - 2(5.1)(8)\cos 60$$
$$b^2 = 49.21$$
$$b = 7.01498$$

$$\frac{\sin\theta}{5.1} = \frac{\sin 60}{7.01498}$$

$$\theta = \sin^{-1}\left(\frac{5.1\sin 60}{7.01498}\right)$$

$$\theta \doteq 39^\circ$$

∴ YES, HE CAN INSTALL HIMSELF

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