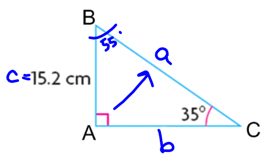
Review: Solving Triangles

"Solving a Triangle" means to determine the lengths of all the sides and the measure of all the angles. Unless otherwise stated, round all lengths to one decimal place and angles to the nearest degree.

Solving right triangles

If you are solving a right triangle, use the **primary trig ratios** and the **Pythagorean theorem**.
Using sine or cosine law will cost you technical marks!

Ex 1) Solve ΔABC



① $\angle B = 180° - 90° - 35°$

$\boxed{\angle B = 55°}$

② $\tan 35° = \dfrac{15.2}{b}$

$b = \dfrac{15.2}{\tan 35°}$

$\boxed{b = 21.7\text{ cm}}$

③ $\sin 35° = \dfrac{15.2}{a}$

$\boxed{a = 26.5\text{ cm}}$

$\angle A = 90°$	$a = 26.5$ cm
$\angle B = 55°$	$b = 21.7$ cm
$\angle C = 35°$	$c = 15.2$ cm

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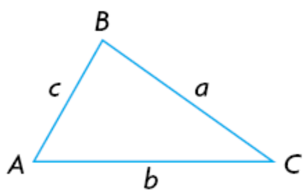
Solving oblique triangles

An oblique triangle is a non-right triangle that is either acute (all angles < 90°) or obtuse (1 angle > 90°). To solve an oblique triangle, you must use either the **sine law** or the **cosine law**. When labelling triangles, **angles** are **uppercase letters**, **sides** are **lowercase letters**. Sides and angles **opposite** each other **share the same letter**.



Sine law is used to determine:

- the **length** of the side of a triangle if you are given **any two angles** and **1 side (AAS or ASA)**
- an **angle** if you are given **two sides** and **an angle opposite one of these two sides (SSA)**

There are 2 forms of sine law: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ or $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

☆ $\boxed{\dfrac{\cdot}{\cdot} = \dfrac{\cdot}{?}}$

Multiply the diagonals, divide by the lonely one ☆

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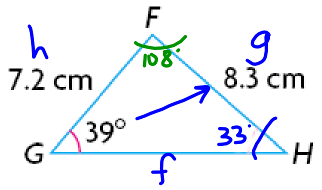
Ex 2) Solve ΔFGH



$\angle F = 108^\circ$	$f = 12.5\text{ cm}$
$\angle G = 39^\circ$	$g = 8.3\text{ cm}$
$\angle H = 33^\circ$	$h = 7.2\text{ cm}$

① $\dfrac{\sin 39^\circ}{8.3} = \dfrac{\sin H}{7.2}$

$\boxed{\angle H = 33^\circ}$

② $\angle F = 180^\circ - 33^\circ - 39^\circ$

$\boxed{\angle F = 108^\circ}$

③ $\dfrac{\sin 39^\circ}{8.3} = \dfrac{\sin 108^\circ}{f}$

$\boxed{f = 12.5\text{ cm}}$

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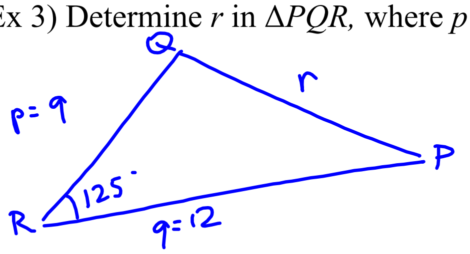
Cosine law is used to determine:

- the **length** of the side of a triangle if you are given **two sides** and **the contained angle (SAS)**
- any **angle** if you are given **all three sides (SSS)**

There are 3 forms of cosine law:

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ **or** $b^2 = a^2 + c^2 - 2ac\cos B$ **or** $c^2 = a^2 + b^2 - 2ab\cos C$

Ex 3) Determine r in ΔPQR, where p = 9cm, q = 12cm and R = 125º



$r^2 = p^2 + q^2 - 2pq\cos R$

$r^2 = 9^2 + 12^2 - 2(9)(12)(\cos 125^\circ)$

$r = \sqrt{348.8925}$

$\boxed{r \doteq 18.7\text{ cm}}$

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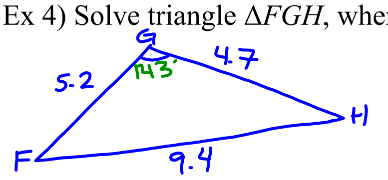
When solving a triangle given 3 sides, always start by using cosine law to find the angle opposite the LARGEST side.

- Use cosine law to determine the largest angle – which is always opposite the largest side.
- Then use sine law to determine either of the two remaining angles.
- Subtract the two known angles from 180° to determine the third.

Ex 4) Solve triangle ΔFGH, where $h = 5.2$m, $g = 9.4$m and $f = 4.7$m



① $9.4^2=4.7^2+5.2^2-2(4.7)(5.2)\cos G$

$9.4^2-4.7^2-5.2^2=[-2(4.7)(5.2)]\cos G$

$\frac{9.4^2-4.7^2-5.2^2}{(-2)(4.7)(5.2)}=\cos G$

$\boxed{143^\circ=\angle G}$

② $\frac{\sin 143^\circ}{9.4}=\frac{\sin F}{4.7}$

$\boxed{\angle F=18^\circ}$

③ $\angle H=180^\circ-18^\circ-143^\circ$

$\boxed{\angle H=19^\circ}$

$\angle F=18^\circ$	$f=4.7$
$\angle G=143^\circ$	$g=9.4$
$\angle H=19^\circ$	$h=5.2$

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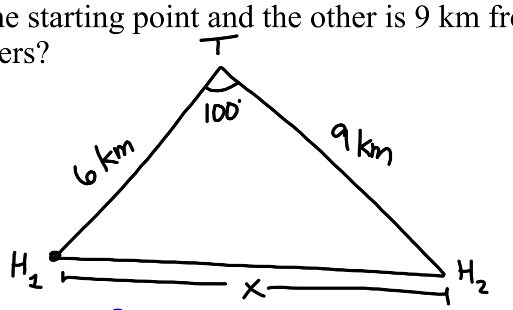
Ex 5) Two hikers set out in different directions from a marked tree on the Bruce Trail. The angle formed between their paths measures 100°. After 2 hours, one hiker is 6 km from the starting point and the other is 9 km from the starting point. How far apart are the hikers?



$x^2=6^2+9^2-2(6)(9)(\cos 100^\circ)$

$\boxed{x=11.7}$

∴ The hikers are 11.7 km apart.

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HW:

1. Sign and correct quizzes