Unit 5, Lesson 6: Solving Problems with Functions

So far we have seen 4 types of functions that model real-world problems

- Linear Functions $f(x) = mx + b$
- Quadratic Functions $f(x) = ax^2 + bx + c$
- Exponential Functions $f(x) = aB^{kx} + c$
- Sinusoidal Functions $f(x) = a\sin k(x-d) + c$ or $f(x) = a\cos k(x-d) + c$

Deciding which model to use for a given situation is an important skill that takes practice!

Some hints:

- Most Revenue & Profit functions are quadratic in nature. Demand functions are linear in this course, but can be non-linear as well.
- If a problem involves area, it will likely require a quadratic function.
- If a situation models growth or decay, with a rate given as a %, this will be an exponential function, and the base will depend on the growth or decay rate.
- Any situation involving half-life (medication in bloodstream, chemical isotopes) is exponential decay, with $B = \frac{1}{2}$.
- If a situation is periodic in some way (repeating motion, seasonal sales, etc), use a sinusoidal function.

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Ex 1) The half life of caffeine in the bloodstream for an adult is 5.5 hours. If Mrs. McKinnell drinks a Starbucks grande brewed coffee at 8:00 am, and then another one at 11:00 am, how much caffeine is still in her bloodstream at 9pm?

exponential decay: $\frac{t}{\text{period}}$

$f(x) = a\left(\frac{1}{2}\right)^{\frac{t}{\text{period}}}$

Starbucks Nutrition Information

Grande 330mg caffeine

Let t rep time in hours.

Let g(t) rep. the caffeine in bloodstream.

$g(t) = 330\left(\frac{1}{2}\right)^{\frac{t}{5.5}}$

8am → 11am 3h.

1 $g(3) = 330\left(\frac{1}{2}\right)^{\frac{3}{5.5}}$

$g(3) = 226.1$ mg (in bloodstream at 11am)

2 330mg + 226.1mg = 556.1mg

3 $g(10) = 556.1\left(\frac{1}{2}\right)^{\frac{10}{5.5}}$

11am → 9pm 10h

$\boxed{g(10) = 157.7\text{ mg}}$

∴ She will have 157.7mg of caffeine in her bloodstream at 9pm.

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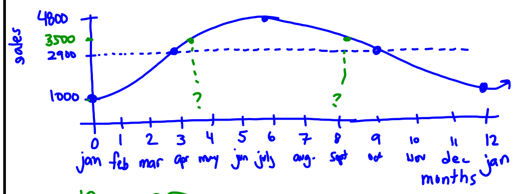
think sinusoidal

Ex 2) Sales of ice cream are seasonal. Daily sales at an ice cream parlour peak in July at \$4800 per day, while in January they are at their lowest at \$1000 per day. During which month(s) are sales **at least** \$3500 per day?

Let s rep. sales (\$)
Let t rep. months
(Jan is t = 0)



$a = -1900$
$k = 30$ ($\frac{360°}{12}$ ← period)
$d = 0$
$c = +2900$

$s(t) = -1900\cos 30t + 2900$

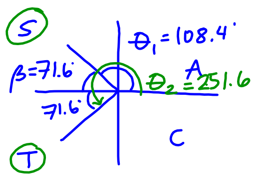
$3500 = -1900\cos 30t + 2900$

$3500 - 2900 = -1900\cos 30t$

$600 = -1900\cos 30t$

$\frac{600}{-1900} = \cos 30t$

$\cos^{-1}\left(-\frac{6}{19}\right) = 30t$



$108.4° = 30t$ $\quad$ $251.6 = 30t$

$\boxed{3.6 = t}$ $\quad$ $\boxed{8.4 = t}$

mid April $\quad$ mid September

∴ sales will be at least 3500\$ between mid April to mid September.

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Ex 3) You want to sell your handmade jewellery at the Carp Farmer's market. Some market research has shown that you will sell 300 necklaces per month when the price is \$10 per necklace, and will sell 250 necklaces per month when the price is \$15 per necklace. Each necklace costs you \$4.00 in materials to make, and the monthly rental on your market booth is \$800. Determine the price you should set for your necklaces to ensure the maximum profit.

(300 necklaces, 10 \$)
(250 necklaces, 15 \$)

$m = \frac{\Delta y}{\Delta x}$

$m = \frac{10-15}{300-250}$

$m = -0.1$

$y = mx + b$

$10 = -0.1(300) + b$

$10 = -30 + b$

$40 = b$

$\boxed{p(x) = -0.1x + 40}$ demand function

$\boxed{C(x) = 4x + 800}$ cost function

$R(x) = x \cdot p(x)$

$R(x) = x(-0.1x + 40)$

$\boxed{R(x) = -0.1x^2 + 40x}$ Revenue function

$P(x) = R(x) - C(x)$

$P(x) = -0.1x^2 + 40x - 4x - 800$

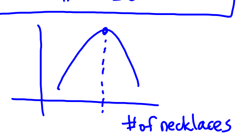
$\boxed{P(x) = -0.1x^2 + 36x - 800}$ profit function

$AOS = \frac{-b}{2a}$

$= \frac{-36}{2(-0.1)}$

$\boxed{x = 180}$

\# of necklaces you need to sell to maximize profit.



Now plug 180 into the demand function:

$p(x) = -0.1(180) + 40$

$\boxed{p(x) = 22}$

∴ you need to set the price at 22 \$ to maximize profit.

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HW U5L6:

1. handout (do 5-10 of the questions. We will do the rest in class Monday). I have posted the solutions so you can take a look if you are stumped.