Skill Builder: Simplifying Rational Expressions

A rational function is the ratio of two polynomial functions.

A rational function can be expressed as $R(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are each polynomial functions and $q \neq 0$.

To determine the **domain** of a rational function, consider the value(s) of x that make the polynomial in the denominator, $\boldsymbol{q(x)=0}$ (i.e. the **zeros of the denominator**). The domain will **exclude** these values. These values are also called the **restrictions** of the corresponding rational expression.

Ex 1) Determine the domain of the rational function $f(x)=\frac{5x^2-10x}{3x^2+9x}$

$D: \{x \in \mathbb{R} \mid x \neq -3, 0\}$

Factor the dominator:

$3x(x+3)$

$\downarrow \qquad \downarrow$

$x=0 \qquad x=-3$

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Recall: To simplify a fraction, we determine the GCF of the numerator & denominator, then divide both numerator & denominator by the GCF (i.e. "cancel out" the GCF).

e.g. $\frac{12}{18}=\frac{(\cancel{6})(2)}{(\cancel{6})(3)}=\frac{2}{3}$

Think: Why are we allowed the "cancel out" the 6?

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We can simplify rational functions and rational expressions in a similar manner.

- **Factor** both the numerator and the denominator (*using all of your factoring strategies*)
- **Divide** both numerator and denominator by the GCF ("**Cancel out**" all common factors)

When asked for the **restrictions**, you must determine the **zeros of the ORIGINAL denominator**

CAUTION: YOU CAN ONLY CANCEL FACTORS, NOT TERMS

Ex. $\frac{1+6}{6} = \frac{7}{6}$ $\quad \frac{1+6}{6} \neq 1$ **Ex.** $\frac{3}{3+7} = \frac{3}{10}$ $\quad \frac{3}{3+7} \neq \frac{1}{7}$

This applies to variables as well, so...

Ex. $\frac{x-8}{x+3} \neq -\frac{8}{3}$ **Ex.** $\frac{2x^2-8}{2x^2+3x} \neq -\frac{8}{3x}$

Ex 2) Simplify. State any restrictions on the variables

↳ list the values that make the denominator zero

a) $\frac{15x^2y^3}{10x^4y^2}$

$= \frac{(5\cancel{x^2}y^2)(3y)}{(5\cancel{x^2}y^2)(2x^2)}$

$= \frac{3y}{2x^2}$; $x \neq 0, y \neq 0$

b) $\frac{2n^2+n-1}{n-1}$

$= \frac{(n+1)(2n-1)}{(n-1)}$

This can't be Simplified

$n \neq 1$

c) $\frac{5(x+3)+2}{x+3}$

$= \frac{5x+15+2}{x+3}$

$= \frac{5x+17}{x+3}$

This can't be simplified.

$x \neq -3$

d) $\frac{(x+2xy)}{xy}$

$= \frac{\cancel{x}(1+2y)}{\cancel{x}y}$

$= \frac{1+2y}{y}$

$x \neq 0$

$y \neq 0$

e) $\frac{(a-9)(a^2-2)}{(9-a)}$

$= \frac{\cancel{(a-9)}(a^2-2)}{-1\cancel{(a-9)}}$

$= -(a^2-2)$

$a \neq 9$

f) $\frac{6x^2-5xy-4y^2}{3x^2+8xy-16y^2}$

$= \frac{\cancel{(3x-4y)}(2x+y)}{\cancel{(3x-4y)}(x+4y)}$

$= \frac{2x+y}{x+4y}$

$x \neq -4y, \frac{4}{3}y$

$3x-4y=0$

$3x=4y$

$x \neq \frac{4}{3}y$

$x+4y=0$

$x \neq -4y$

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Ex 3) Simplify $f(x)$ and state the domain, where

a) $f(x)=\frac{3x+6}{x^2-4}$ ← DOS

$= \frac{3\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$

$= \frac{3}{(x-2)}$

$\{x \in \mathbb{R} \mid x \neq \pm 2\}$

b) $f(x)=\frac{x^2+5x-6}{2x-2}$

$= \frac{(x+6)\cancel{(x-1)}}{2\cancel{(x-1)}}$

$= \frac{x+6}{2}$ or $= \frac{1}{2}x+3$

$D: \{x \in \mathbb{R} \mid x \neq 1\}$

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Equivalence

Two functions are considered **equivalent** if they have the **same domain** and yield the **same values** (output) for **all numbers in their domain** (input).

- To show **equivalence**, you must show that they both **simplify to the same expression**, with the **same domain**.
- To show **non-equivalence**, you can **choose an input** (choose zero) (i.e. substitute a number for "x") and show that each function yields a **different output**. This **does not work** to show **equivalence** since some functions intersect.

Ex 4) For each pair of functions, determine if they are equivalent.

a) $f(x)=\dfrac{8x^2+2x-21}{12x^2+29x+14}$ $\qquad g(x)=\dfrac{2x+3}{3x+2}; x\neq -1\dfrac{3}{4}$

$f(0)=\dfrac{8(0)^2+2(0)-21}{12(0)^2+29(0)+14}$

$f(0)=\dfrac{-21}{14}$

$\boxed{f(0)=-\dfrac{3}{2}}$

$g(0)=\dfrac{2(0)+3}{3(0)+2}$

$\boxed{g(0)=\dfrac{3}{2}}$

$\therefore$ because they have different outputs, they are NON equivalent.

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$f(x)=\dfrac{8x^2+10x-3}{6x^2+13x+6}$ $\qquad g(x)=\dfrac{4x-1}{3x+2}; x\neq -1\dfrac{1}{2}$

$f(0)=\dfrac{-3}{6}$

$f(0)=-\dfrac{1}{2}$

$g(0)=\dfrac{-1}{2}$

Since the outputs, we have to simplify to prove equivalence:

$f(x)=\dfrac{(4x-1)(\cancel{2x+3})}{(3x+2)(\cancel{2x+3})}$

$\boxed{f(x)=\dfrac{4x-1}{3x+2}}$

$x\neq -\dfrac{2}{3}, -\dfrac{3}{2}$

$\therefore$ the 2 functions are equivalent

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HW:

p. 113 #4-7, 10, 14a. TRY: 14b, 17

p. 89 #13 (check your handout for corrected answers for #4&5)