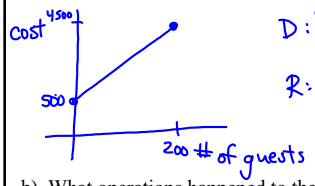
Lesson 6: Inverse Functions and Their Graphs

An **inverse operation** is an operation that reverses the effect of another operation.
An **inverse function** reverses the effect of a given function.

Ex 1) The function $C(n) = 20n + 500$, represents the cost, C, of holding a reception at a hall as a function of the number, n, of guests. The hall has a fire limit of 200 people.

$C(200) = 20(200) + 500$
$= 4500$

a) Determine the domain and range of $C(n)$.



$D: \{x \in \mathbb{R} \mid 0 \le x \le 200\}$

$R: \{y \in \mathbb{R} \mid 500 \le y \le 4500\}$

b) What operations happened to the input variable? What would be the reverse of those operations?

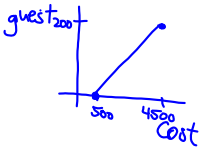
multiply by 20 → divide by 20
add 500 → subtract 500

c) Write the equation that would represent the number of guests as a function of the cost.

guests

$C(n) = \dfrac{C - 500}{20}$ ← an inverse function

d) Determine the domain and range of the inverse.



$D: \{x \in \mathbb{R} \mid 500 \le x \le 4500\}$

$R: \{y \in \mathbb{R} \mid 0 \le y \le 200\}$

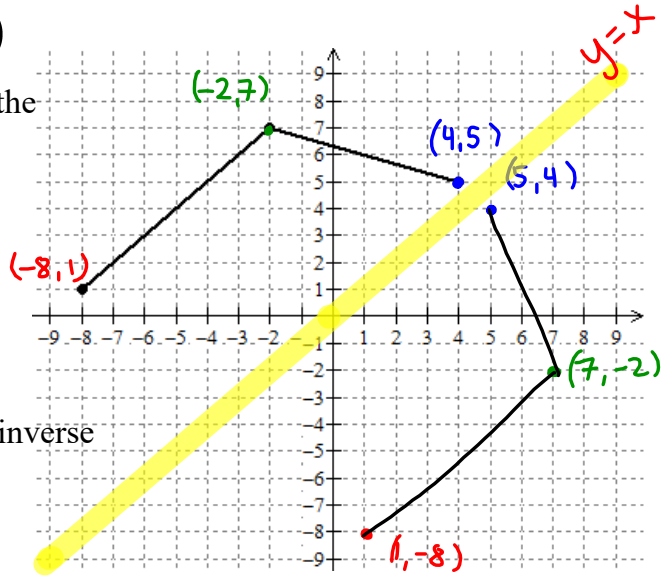
Jan 31-9:35 PM

Given a function $f(x)$, you can write its inverse as $f^{-1}(x)$ ***(this is not raising a value to the power –1)***

If the point (a, b) is on the graph of $y = f(x)$, then the point (b, a) is on the graph of $y = f^{-1}(x)$.

The graph of an inverse is the reflection of the graph $y = f(x)$ in the line $y = x$.

Ex 2) Given the graph of $y = f(x)$



a) Draw the graph of its inverse on the same grid

b) Is the inverse a function?

No, because it fails the VLT.

c) State the domain and range of the inverse

$D: \{x \in \mathbb{R} \mid 1 \le x \le 7\}$

$R: \{y \in \mathbb{R} \mid -8 \le y \le 4\}$

Apr 25-1:53 PM

Given the equation of a function, we can determine its inverse by **isolating the independent variable** and rewriting using inverse function notation.

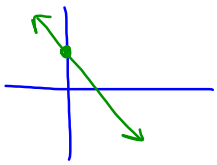
Ex 3) Given $f(x) = 3 - 5x$, determine $f^{-1}(x)$. Is the inverse a function?

① switch x and y
② isolate for y
③ change name to $f^{-1}(x)$

$y = 3 - 5x$
$x = 3 - 5y$
$x - 3 = -5y$
$\frac{x-3}{-5} = y$
$-\frac{1}{5}(x-3) = y$
$-\frac{1}{5}x + \frac{3}{5} = y$

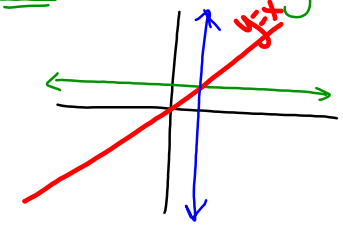
$f^{-1}(x) = -\frac{1}{5}x + \frac{3}{5}$

yes, it's a function



If the original function is linear, is the inverse always a function?

Yes, except when the original line is horizontal



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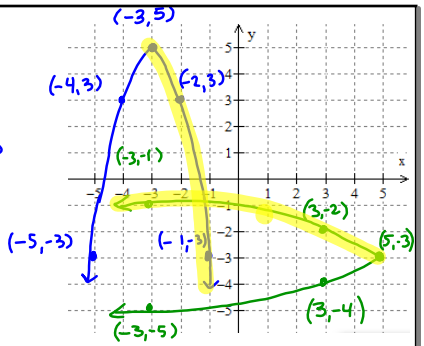
Ex 4) Graph $g(x) = -2(x+3)^2 + 5$ and its inverse on the grid.

vertex: (-3, 5)
1, 3, 5
x-2)
-2, -6, -10

a) Is the inverse a function?

No, fails the VLT

To make the inverse of a quadratic a function, we ***restrict the domain*** of the original function.



b) Given $g(x) = -2(x+3)^2 + 5, x \geq -3$, determine the equation for $f^{-1}(x)$

① switch x and y
② isolate for y
③ change name

$y = -2(x+3)^2 + 5$
$x = -2(y+3)^2 + 5$
$x - 5 = -2(y+3)^2$
$\frac{x-5}{-2} = (y+3)^2$

upper branch → $\pm\sqrt{\frac{x-5}{-2}} = y+3$

$\sqrt{\frac{x-5}{-2}} - 3 = g^{-1}(x)$

$\sqrt{\frac{-1}{2}x + \frac{5}{2}} - 3 = g^{-1}(x)$

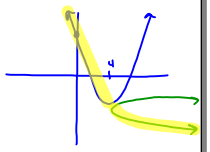
Apr 25-2:00 PM

To determine the inverse of a quadratic function algebraically, the equation must be in **vertex form**. **Complete the square** to put into vertex form, if needed.

Ex 5) Given $h(x) = 3x^2 - 24x + 30, x \leq 4$

a) Determine $h^{-1}(x)$ algebraically.

$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = 16$



CTS: $y = (3x^2 - 24x) + 30$

$= 3(x^2 - 8x) + 30$

$= 3(x^2 - 8x + 16 - 16) + 30$

$= 3(x^2 - 8x + 16) - 48 + 30$

$y = 3(x-4)^2 - 18$

① switch x and y

② isolate for y

③ change name $h^{-1}(x)$

$x = 3(y-4)^2 - 18$

$x + 18 = 3(y-4)^2$

$\frac{1}{3}x + 6 = (y-4)^2$

lower branch $\rightarrow \pm\sqrt{\frac{1}{3}x + 6} = y - 4$

$-\sqrt{\frac{1}{3}x + 6} + 4 = h^{-1}(x)$

$-\sqrt{\frac{1}{3}(x+18)} + 4 = h^{-1}(x)$

b) State the transformations on the parent function for both $h(x)$ and $h^{-1}(x)$. What do you notice?

$h(x) = 3(x-4)^2 - 18$

① v. stretch bafo 3

② h. translation 4 right

③ v. translation 18 down

$h^{-1}(x) = -\sqrt{\frac{1}{3}(x+18)} + 4$

① R ITXA

② horizontal stretch bafo 3

③ horizontal translation 18 left

④ vertical translation 4 up

We notice that the horizontal and verticals switch.

Apr 25-2:04 PM

HW U4L6:

1. sign and correct quizzes
2. p. 46 #1, 2ac, 5ade, 9a-e, 10de
3. p. 160 #2b, 3, 4bc, 5, 7, 8*(eq'n only)* , 10
4. study for unit test Friday