

MCR3U Trig Identities Assignment
McKinnell

Name: _____
Date: _____

Proving Identities Part 2

When proving Identities, here are some strategies to consider:

- All reciprocal trig ratios ($\csc\theta$, $\sec\theta$, $\cot\theta$) can be changed to one of the 3 primary trig ratios ($\sin\theta$, $\cos\theta$, $\tan\theta$) by flipping the fraction.
- It is usually recommended to change any instance of $\tan\theta$ to $\sin\theta$ and $\cos\theta$ using:
 $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- When sine or cosine is squared, they can easily be changed to one another using:
 $\sin^2\theta = 1 - \cos^2\theta$
 $\cos^2\theta = 1 - \sin^2\theta$
- The right side of the two identities above are differences of squares and can be factored accordingly: eg: $1 - \cos^2\theta = (1 - \cos\theta)(1 + \cos\theta)$
- Compare both sides of the equation; identify what the left side has in common with the right and what is different.
- When proving an identity, start working with the side that seems to have more details to simplify it down to the side that seems to have less detail.

Prove the following:

$\frac{\sin x}{\tan x} = \cos x$	$2\sin^2 x - 1 = \sin^2 x - \cos^2 x$
$\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$	$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{1}{\cos^2 x \sin^2 x}$

Prove the following trig identities:

$\tan x = \frac{\sin x + \sin^2 x}{\cos x(1 + \sin x)}$	$\frac{1}{\cos x} - \cos x = \sin x \tan x$
$\tan\theta + \cot\theta = \sec\theta \csc\theta$	$\sin\theta + \sin\theta \cot^2\theta = \csc\theta$

$\frac{2\sin\theta - \cos^2\theta - 2}{\sin\theta + 3} = \sin\theta - 1$	$\frac{\tan\theta}{1 - \cos\theta} = \frac{1 + \cos\theta}{\sin\theta \cos\theta}$
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$$\frac{\sin x}{\tan x} = \cos x$$

$$\sin x \div \tan x$$

$$\sin x \div \frac{\sin x}{\cos x}$$

$$\cancel{\sin x} \times \frac{\cos x}{\cancel{\sin x}}$$

$$\cos x = \cos x$$

$$\therefore LS = RS \quad \text{!!}$$

$$2\sin^2 x - 1 = \sin^2 x - \cos^2 x$$

$$2\sin^2 x - \sin^2 x - 1 = -\cos^2 x$$

$$\sin^2 x - 1 = -\cos^2 x$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 = 1$$

$$\therefore LS = RS \quad \text{!!}$$

$$\frac{\sin^2 x}{(1 - \cos x)} = 1 + \cos x$$

$$\sin^2 x \quad | \quad (1 + \cos x)(1 - \cos x)$$

$$1 - \cos^2 x$$

$$\sin^2 x$$

$$\therefore LS = RS \quad \text{!!}$$

$$\frac{1 \cdot \cos^2 x}{\sin^2 x \cdot \cos^2 x} + \frac{1 \cdot \sin^2 x}{\cos^2 x \cdot \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x} =$$

$$\frac{1}{\sin^2 x \cos^2 x} =$$

$$LS = RS \quad \text{!!}$$

$$\tan x = \frac{\sin x + \sin^2 x}{\cos x (1 + \sin x)}$$

$$= \frac{\sin x (1 + \cancel{\sin x})}{\cos x (1 + \cancel{\sin x})}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$\text{!! } LS = RS$$

$$\frac{1}{\cos x} - \cos x \cdot \frac{\cos x}{\cos x} = \sin x \tan x$$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \sin x \cdot \frac{\sin x}{\cos x}$$

$$\frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} =$$

$$\therefore LS = RS$$

$$\tan \theta + \cot \theta = \sec \theta \csc \theta$$

$$\frac{\sin \theta \cancel{\sin \theta}}{\cos \theta \cdot \cancel{\sin \theta}} + \frac{\cos \theta \cdot \cancel{\cos \theta}}{\sin \theta \cdot \cancel{\cos \theta}} = \frac{1}{\cos \theta} \times \frac{1}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta}$$

$$\frac{1}{\cos \theta \sin \theta} =$$

$$\therefore LS = RS \quad \text{!!}$$

$$\sin \theta + \sin \theta \cot^2 \theta = \csc \theta$$

$$\sin \theta (1 + \cot^2 \theta) = \frac{1}{\sin \theta}$$

$$\sin \theta (\csc^2 \theta) =$$

$$\cancel{\sin \theta} \left(\frac{1}{\cancel{\sin \theta}} \right) =$$

$$\frac{1}{\sin \theta} =$$

$$LS = RS \quad \text{!!}$$

$$\frac{2\sin\theta - \cos^2\theta - 2}{(\sin\theta + 3)} = \sin\theta - 1$$

$$2\sin\theta - \cos^2\theta - 2 = (\sin\theta - 1)(\sin\theta + 3)$$

$$2\sin\theta - (1 - \sin^2\theta) - 2 = \sin^2\theta + 2\sin\theta - 3$$

$$2\sin\theta - 1 + \sin^2\theta - 2 =$$

$$\sin^2\theta + 2\sin\theta - 3 =$$

$$\implies LS = RS$$

$$\frac{\tan\theta}{1 - \cos\theta} = \frac{1 + \cos\theta}{\sin\theta \cos\theta}$$

$$\tan\theta(\sin\theta \cos\theta) = (1 - \cos\theta)(1 + \cos\theta)$$

$$\frac{\sin\theta \times (\sin\theta \cancel{\cos\theta})}{\cancel{\cos\theta}} = 1 - \cos^2\theta$$

$$\sin^2\theta = \sin^2\theta$$

$$LS = RS \implies$$