Lesson 4: Graphing Functions with Transformations (Day 2)

An exponential function with **base *B*** that has been transformed has the form: $g(x) = aB^{k(x-d)} + c$ with the following properties:

- The **horizontal asymptote** is only affected by the vertical translation.
- The ***y*-intercept** can be determined by finding $g(0)$ algebraically or looking at the graph of the transformed function.
- The **domain** is always the real numbers
- The **range** is affected by the **reflection in the *x*-axis** and the **vertical translation**.

We can graph these functions using a mapping rule on the key points of the parent function.

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Ex 4) If $f(x) = 3^x$ and $g(x) = -4f\left(-\frac{1}{2}x - 3\right) + 5$ $g(x) = -4f\left[-\frac{1}{2}(x+6)\right] + 5$

a) State the transformations that have been applied to $f(x)$

1. R ITXA
2. vertical stretch bafo 4
3. R ITYA
4. horizontal stretch bafo 2
5. horizontal translation left 6 units
6. vertical translation up 5 units

$a = -4$
$k = -\frac{1}{2}$ $\therefore \frac{1}{k} = -2$
$d = -6$
$c = +5$

b) State the equation of $g(x)$

$g(x) = aB^{k(x-d)} + c$

$\boxed{g(x) = -4(3)^{-\frac{1}{2}(x+6)} + 5}$

c) Using a mapping rule, graph both functions on the grid provided. Include the asymptotes. $(x, y) \rightarrow \left(\frac{1}{k}x + d, ay + c\right)$ $\boxed{(-2x-6, -4y+5)}$

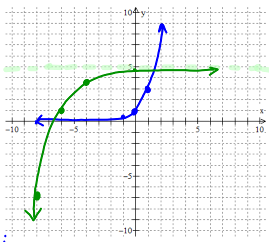
Main Points:

$\left(-1, \frac{1}{3}\right) \rightarrow \left(-2(-1)-6, -4\left(\frac{1}{3}\right)+5\right) \rightarrow \left(-4, 3\frac{2}{3}\right)$

$(0, 1) \rightarrow (-2(0)-6, -4(1)+5) \rightarrow (-6, 1)$

$(1, 3) \rightarrow (-2(1)-6, -4(3)+5) \rightarrow (-8, -7)$

$y = 0 \rightarrow y = -4(0)+5 \rightarrow y = 5$



d) Determine the *y*-intercept of $g(x)$

plug in $x=0$ into the transformed function:

$g(0) = -4(3)^{-\frac{1}{2}(0+6)} + 5$

$g(0) = -4(3)^{-3} + 5$

$g(0) = \frac{131}{27}$ or $\boxed{4\frac{23}{27}}$

e) State the equation of the asymptote of $g(x)$

$y = 5$

f) State the domain and range of $g(x)$.

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y < 5\}$

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Ex 5) If $g(x) = 9^x$ and $h(x) = 3^x$, describe the transformations you could apply to $h(x)$ to obtain $g(x)$.

$g(x) = (3^2)^x$

$g(x) = (3)^{2x}$

$k = 2$ $\quad \frac{1}{k} = \frac{1}{2}$

$\therefore$ a horizontal compression bafo $\frac{1}{2}$

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A sinusoidal function that has been transformed has the form:
$g(\theta) = a\sin\left[k(\theta - d)\right] + c$ or $g(\theta) = a\cos\left[k(\theta - d)\right] + c$ with the following properties:

- The **amplitude** is $|a|$
- The **period** is $\frac{360°}{k}$
- The # **of cycles** (# of times a graph repeats within the domain of the base curve) is $\boldsymbol{k}$
- The phase shift is $\boldsymbol{d}$ *(remember to factor out k, if needed!)*
- The **equation of the axis** is $y = c$
- The **vertical displacement** is c
- The **range** is affected by the **amplitude** and the **vertical displacement**.

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Ex 6) Given $g(\theta) = -3\sin(2\theta + 60°) + 1$ $g(\theta) = -3\sin[2(\theta + 30)] + 1$

a) Identify the parent function (base curve)

$f(\theta) = \sin\theta$

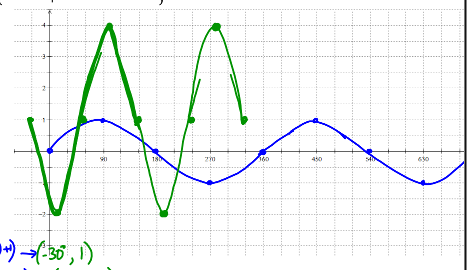
b) State the transformations that have been applied to the base curve

$a = -3$
$k = 2$ $\therefore \frac{1}{k} = \frac{1}{2}$
$d = -30°$
$c = +1$

1. R ITXA
2. stretched vertically bafo 3
3. compressed horizontally bafo $\frac{1}{2}$
4. horizontally translated left 30°
5. vertically translated up 1.

c) Using a mapping rule, graph both functions over the base domain $\{\theta \in \Re \mid 0° \le \theta \le 720°\}$. means draw 2 cycles of $y = \sin x$

$\boxed{\left(\frac{1}{2}x - 30°, -3y + 1\right)}$



Key Points:

$(0, 0) \rightarrow (\frac{1}{2}(0) - 30°, -3(0)+1) \rightarrow (-30°, 1)$
$(90°, 1) \rightarrow (\frac{1}{2}(90°) - 30°, -3(1)+1) \rightarrow (15°, -2)$
$(180°, 0) \rightarrow (\frac{1}{2}(180) - 30°, -3(0)+1) \rightarrow (60°, 1)$
$(270°, -1) \rightarrow (\frac{1}{2}(270) - 30°, -3(-1)+1) \rightarrow (105°, 4)$
$(360°, 0) \rightarrow (\frac{1}{2}(360) - 30°, -3(0)+1) \rightarrow (150°, 1)$

d) What is the amplitude?

$|a| = 3$

e) What is the period of g? recall: period $= \frac{360°}{k}$

$180°$

f) What is the equation of the axis?

$y = 1$

g) What is the domain and range of g?

$D: \{x \in \mathbb{R} \mid -30° \le x \le 330°\}$
$R: \{y \in \mathbb{R} \mid -2 \le y \le 4\}$

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Ex 7) Given $\boxed{f(x) = \cos(x)}$ and $g(x) = -2f\left(\frac{1}{3}x\right) + 2$, state the equation of $g(x)$, its amplitude, period, equation of the axis, domain and range.

$a = -2$
$k = \frac{1}{3}$

$g(x) = -2\cos\left(\frac{1}{3}x\right) + 2$

amplitude = 2

period = 1080°   recall: $\frac{360°}{k}$ $360° \div \frac{1}{3} = 1080°$

equation of the axis: $y = 2$

$D: \{x \in \mathbb{R}\}$

$R: \{y \in \mathbb{R} \mid 0 \le y \le 4\}$

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HW U4L4 Day 2:

1. p. 251 #2,3(for #2 only), 4c, 5, 11

2. p. 383 #1, 2, 4ace, 6abe, 7ace *6e) domain 17<t<23

3. study for quiz.

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