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## Unit 4 – Graphing Functions

| Day | Lesson | Practice Questions | Struggles? |
|---|---|---|---|
| **1** | U4L1 – Properties of Parent Functions | **Read:** Pg. 25 → 28<br>**Do:** **Handout** | |
| **2** | U4L2 – Properties of More Parent Functions<br>*BRING HIGHLIGHTERS OR MARKERS* | **Read:** Pg. 240 – 242, Pg. 357 – 363<br>**Do:** **Handout** , Pg. 243 #1, 2<br>Pg. 352 #2, 6, 8a-d, 10 Pg. 364 #12 | |
| **3** | U4L3 – Investigating Transformations with Technology<br>*BRING YOUR LAPTOP OR TABLET!* | **In-class:** Desmos Investigation<br>**At home:** Point Mapping, Marbleslides (Desmos) | |
| **4-5** | U4L4 – Graphing Functions with Transformations | **Read:** Pg. 61 – 70 Pg. 247 – 251 Pg. 380 – 383<br>**Day 1:** Pg. 70 #1, 2, 6, 7c, 8c, 9c, 10abde, 12, 16, 17<br>**Day 2:** Pg. 251 #2, 3 *(for #2 only)*, 4c, 5, 11<br>Pg. 383 #1, 2, 4ace, 6abe, 7ace<br>*6e) domain* $17 \le t \le 23$ | |
| **6-7** | U4L5 – Representing Functions with Equations | **Read:** Pg. 67 – 70 Pg. 249 – 250 Pg. 386 – 391<br>**Day 1:** Pg. 73 #18, 22 Pg. 252 #9, 10, **Handout**<br>**Day 2:** Pg. 383 #5 Pg. 391 #1, 3, 4, 6, 7, **Handout**<br>*1b) cos(2(x-90))+2* | |
| **8** | U4L6 – Inverse Functions & Their Graphs | **Read:** Pg. 42 – 46 Pg. 155 – 160<br>**Do:** Pg. 46 #1, 2ac, 5ade, 9a-e, 10de<br>Pg. 160 #2b, 3, 4bc, 5, 7, 8 *(eq'n only)*, 10 | |
| **9** | Review | **Read:** Pg. 74 – 75, Pg. 265 – 266, Pg. 403<br>**Do:** Pg. 77 #10 – 13, 15, 16, 18, 19 Pg. 170 #9 – 11<br>*15.* $\left(-\frac{5}{4},10\right)$, *16a) left 12*<br>Pg. 268 #9 – 12 Pg. 404 #2, 3, 8 – 10, 12<br>*12a) y=sin(30(x-4))+2.5, b) y=2sin[120(x-1)]+4* | |
| **10** | TEST | | |
| **11-12** | TASK | | |

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# Unit 4, Lesson 1: Properties of Parent Functions

A family of functions is a set of functions whose equations have a similar form. The "**parent**" of the family is the equation in the family with the **simplest form**.

Ex 1) Identify the equation of the parent function for each function

a) $g(x)=\dfrac{3}{x-2}$ b) $h(x)=3\sqrt{5x-10}-2$ c) $k(x)=3(x+2)^2-5$

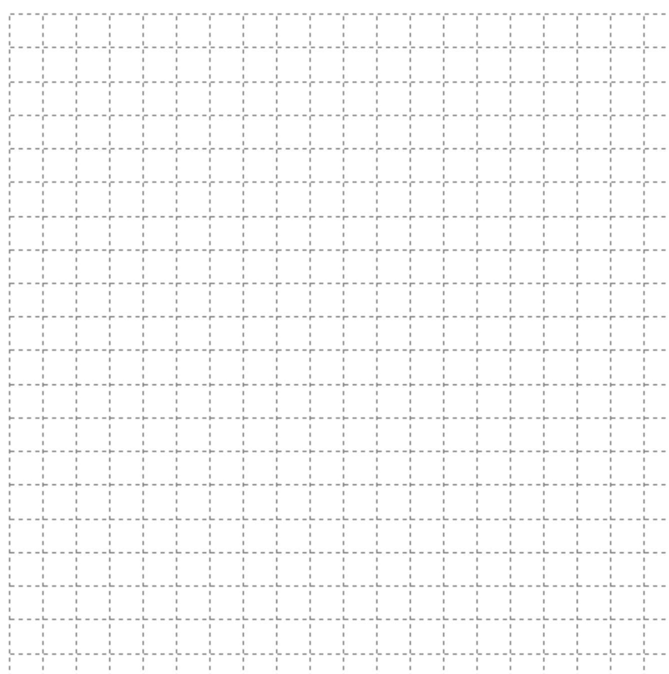
Today you will be investigating properties of some parent functions by graphing them. These graphs will be referred to throughout the unit, so make them precise and accurate!

For each parent function provided:

- **Name** the function (look it up if you don't know!)
- Complete the **table of values**
- **Graph** the function. Verify with graphing technology before continuing.
- State the **domain** and **range**
- Identify the **quadrant(s)** the function resides in
- Choose between 3 and 5 "**key points**" of the graph that would enable someone else to do a proper sketch of the graph
- State any other **key features** of the graph/table (slope, intercepts, vertex, asymptotes, 1st/2nd differences, etc. Anything that would help you identify the function!) ***Recall: An asymptote is a line (often horizontal or vertical) that a function approaches***.

1) $f(x)=x$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |



Domain:

Range:

Quadrant(s):

Key Points:

Features:

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2) $f(x) = x^2$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

Domain:

Range:

Quadrant(s):

Key Points:

Features:

3) $f(x) = |x|$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

Domain:

Range:

Quadrant(s):

Key Points:

Features:

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4) $f(x)=\sqrt{x}$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -1 | |
| 0 | |
| 1 | |
| 4 | |
| 9 | |
| 16 | |

Domain:

Range:

Quadrant(s):

Key Points:

Features:

5) $f(x)=\dfrac{1}{x}$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -2 | |
| -1 | |
| -½ | |
| -¼ | |
| 0 | |
| ¼ | |
| ½ | |
| 1 | |
| 2 | |
| 4 | |

Domain:

Range:

Quadrant(s):

Key Points:

Features:

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## U4L1 Handout – Identifying Parent Functions

For each function listed, identify the **parent function**. For the tables, you may find it helpful to graph the points to determine the parent function.

1) $g(x)=\frac{4}{5}x^2-\frac{3}{5}x+\frac{1}{10}$

2) $h(x)=3|x-2|+5$

3) $k(x)=\frac{3}{5}(x-2)+8$

4) $m(x)=3\sqrt{4-x}+2$

5)

| ***x*** | ***n* (*x*)** |
|---|---|
| -4 | 9 |
| -3 | 12 |
| -2 | 18 |
| 1 | -36 |
| 4 | -9 |
| 6 | -6 |

6)

| ***x*** | ***g* (*x*)** |
|---|---|
| -4 | 20 |
| -2 | 14 |
| 0 | 8 |
| 2 | -2 |
| 4 | 8 |
| 6 | 14 |

7)

| ***x*** | ***h* (*x*)** |
|---|---|
| 0 | 0 |
| 2 | 1.5 |
| 8 | 3.0 |
| 18 | 4.5 |
| 32 | 6.0 |
| 50 | 7.5 |

8)

| ***x*** | ***k* (*x*)** |
|---|---|
| -2 | 20 |
| 0 | 2 |
| 2 | -4 |
| 4 | 2 |
| 6 | 20 |
| 8 | 50 |

9)

| ***x*** | ***m* (*x*)** |
|---|---|
| -1 | -3 |
| 2 | 4.5 |
| 11 | 12 |
| 26 | 19.5 |
| 47 | 27 |
| 74 | 34.5 |

10)

| ***x*** | ***n* (*x*)** |
|---|---|
| -5 | 29 |
| -1 | 22 |
| 3 | 15 |
| 7 | 8 |
| 11 | 1 |
| 15 | -6 |

1) quadratic 2) absolute value 3) linear 4) square root 5) reciprocal
6) absolute value 7) square root 8) quadratic 9) square root 10) linear

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# Unit 4, Lesson 2: Properties of Parent Functions (Exponential & Sinusoidal)

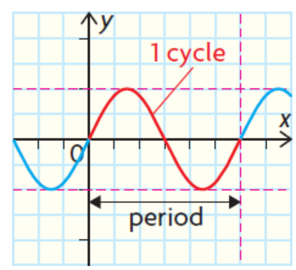
Today you will be investigating properties of more parent functions by graphing them. For all graphs, you will be identifying the **intervals of increase and decrease**: these are the ***x*-values** where the function's ***y*-values** are **increasing** or **decreasing,** respectively.

Some of the functions you are investigating are exponential. There are an infinite number of exponential parent functions (also called base function). The general form for an exponential function is $f(x) = B^x$, where $B > 0$ and $B \neq 1$. Why do we need these restrictions on *B*? *Hint*: domain is $\{x \in \Re\}$

**Example 1:** Identify the equation of the parent (base) function for each exponential function.

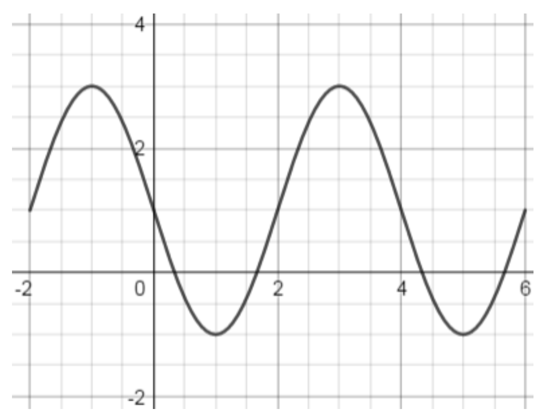
a) $g(x) = 3(4)^{2x} - 5$ b) $h(x) = 2\left(\frac{1}{3}\right)^{x-4} + 1$ c) $g(x) = (3)5^{2x-4}$

Some of the functions you are investigating are periodic. A **periodic function** has a **self-repeating** graph. A periodic graph has the following properties:




- **Cycle**: the cycle of a graph is the smallest repeating pattern.
- **Period**: length of one cycle is called the period
- **Maximum (max)**: the *y*-value of the highest point of the graph. Also called the peak.
- **Minimum (min)**: the *y*-value of the lowest point of the graph. Also called the trough.
- **Amplitude**: the amplitude of the function is the vertical distance from the equation of the axis to either the maximum or minimum value. $a = \dfrac{\max - \min}{2}$
- **Equation of the axis**: the equation of the axis is the horizontal line halfway between the maximum and minimum values on the graph. $y = \dfrac{\max + \min}{2}$

**Example 2:** Determine the properties of the periodic graph to the right




For each parent function provided:

- **Name** the function and complete the **table of values**
- **Graph** the function. Verify with graphing technology. State the **domain** and **range**.
- Choose between 3 and 5 "**key points**" needed to properly sketch the function.
- Identify the **intervals** of **increase** and **decrease (inc/dec)**
- State any other **key features** of the graph/table (intercepts, asymptotes, 1st/2nd differences, period, max, min, amplitude, equation of the axis, etc.)

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1) $f(x)=\left(\frac{1}{2}\right)^x$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

Domain:

Range:

Key Points:

Intervals of inc/dec:

Features:

2) $f(x)=4^x$ Name:

| $x$ | $f(x)$ |
|---|---|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

Domain:

Range:

Key Points:

Intervals of inc/dec:

Features:

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3) $f(x) = \sin(x)$ Name:

| **$x$ (°)** | -360 | -270 | -180 | -90 | 0 | 90 | 180 | 270 | 360 | 450 | 540 | 630 | 720 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **$f(x)$** | | | | | | | | | | | | | |

Domain:

Range:

Key Points:

Intervals of inc/dec:

Features:

4) $f(x) = \cos(x)$ Name:

| **$x$ (°)** | -360 | -270 | -180 | -90 | 0 | 90 | 180 | 270 | 360 | 450 | 540 | 630 | 720 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **$f(x)$** | | | | | | | | | | | | | |

Domain:

Range:

Key Points:

Intervals of inc/dec:

Features:

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## Unit 4, Lesson 3: Investigating Transformations with Technology

**Warm up:** go to www.student.desmos.com and enter code:__________ (10 minutes)

**Exploration:** go to www.student.desmos.com and enter code: ___________
As you go through this activity (approximately 30 minutes), take notes and create a "summary page" of what you are learning.

**Homework:** go to www.student.desmos.com and do the following 4 activities:

Marbleslides (Parabolics) code:____________
Marbleslides (Exponentials) code:____________
Marbleslides (Periodics) code:____________
Marbleslides (Rationals) code:____________

**Recall:**

$$g(x) = af[k(x-d)] + c$$

## Unit 4, Lesson 4: Graphing Functions with Transformations – Day 1

You can graph functions of the form $g(x) = af[k(x-d)] + c$ by applying the appropriate transformations to the key points of the parent function $f(x)$. Always apply *a* and *k* before *d* and *c*, since transformations should follow the same order of operations as numerical expressions.

To assist in transforming key points, we will use a "Mapping Rule." This rule tells us what operations to perform on the *x*- and *y*-coordinates of each key point.

| Parameter | Description of Transformation | Mapping Rule |
|---|---|---|
| $a$ | **Vertical Dilation** by a factor of ***a***<br>If ***a*** > **1** or ***a*** < **-1** the graph is stretched vertically<br>If **-1** < ***a*** < **1** the graph is compressed vertically<br>If ***a*** < **0** the graph is **Reflected in the *x*-axis** | $(x, y) \rightarrow (x, ay)$ |
| $k$ | **Horizontal Dilation** by a factor of $\mathbf{\frac{1}{k}}$<br>If ***k*** > **1** or ***k*** < **-1** the graph is compressed horizontally<br>If **-1** < ***k*** < **1** the graph is stretched horizontally<br>If ***k*** < **0** the graph is **Reflected in the *y*-axis** | $(x, y) \rightarrow (\frac{x}{k}, y)$ |
| $d$ | **Horizontal Translation (Shift)** left or right ***d*** units<br>If ***d*** > **0** the graph shifts right<br>If ***d*** < **0** the graph shifts left | $(x, y) \rightarrow (x + d, y)$ |
| $c$ | **Vertical Translation (Shift)** up or down ***c*** units<br>If ***c*** > **0** the graph shifts up<br>If ***c*** < **0** the graph shifts down | $(x, y) \rightarrow (x, y + c)$ |

Ex 1) Given the function $g(x) = -2f\left[\frac{1}{3}(x+3)\right] + 5$

a) State the transformations that have been applied to *f*(*x*)

b) State the mapping rule

b) The point (2, 5) is a point on the graph of *f*(*x*). Determine the coordinates of the transformed point.

**Equations** of transformed functions with **parameters *a, k, d* & *c***:

**Linear:** $g(x) = a[k(x-d)] + c$ **Quadratic:** $g(x) = a[k(x-d)]^2 + c$ **Reciprocal:** $g(x) = \frac{a}{k(x-d)} + c$

**Square Root:** $g(x) = a\sqrt{k(x-d)} + c$ **Absolute Value:** $g(x) = a|k(x-d)| + c$

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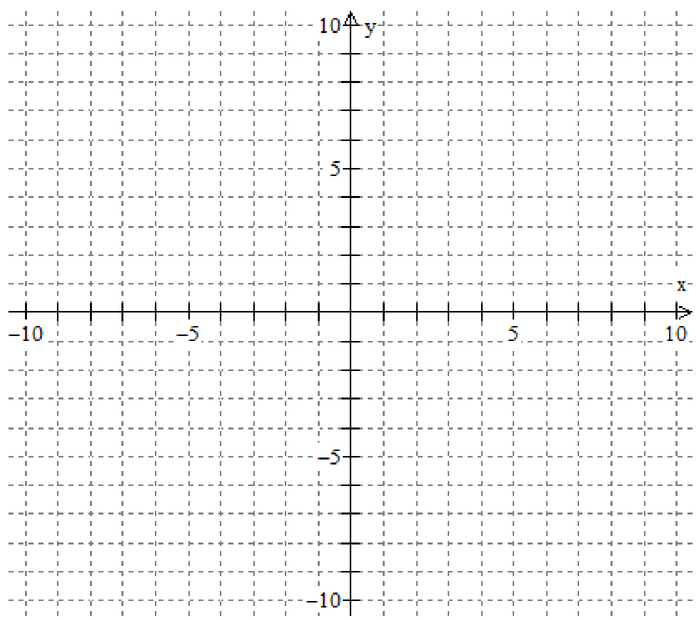
**If you have both a horizontal dilation/reflection AND a horizontal shift, you must FACTOR out *k*!**

Ex 2) Given $f(x)=\frac{1}{x}$ and $g(x)=2f\left(-\frac{1}{4}x-1\right)+3$

a) State the transformations that have been applied to $f(x)$

b) Using a mapping rule, graph both functions on the grid provided. Include the asymptotes.




c) Write the equation for $g(x)$ and state its domain and range.

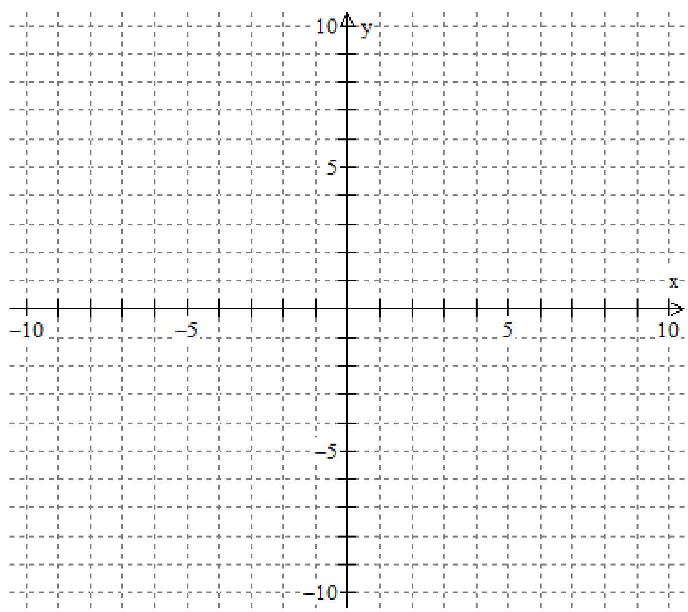
Ex 3) Given $g(x) = -2\sqrt{-x+3}+5$

a) Identify the parent function.

b) State the transformations that have been applied to the parent function

c) Using a mapping rule, graph both functions on the grid provided.

d) State the domain and range of $g(x)$



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## Unit 4, Lesson 4: Graphing Functions with Transformations – Day 2

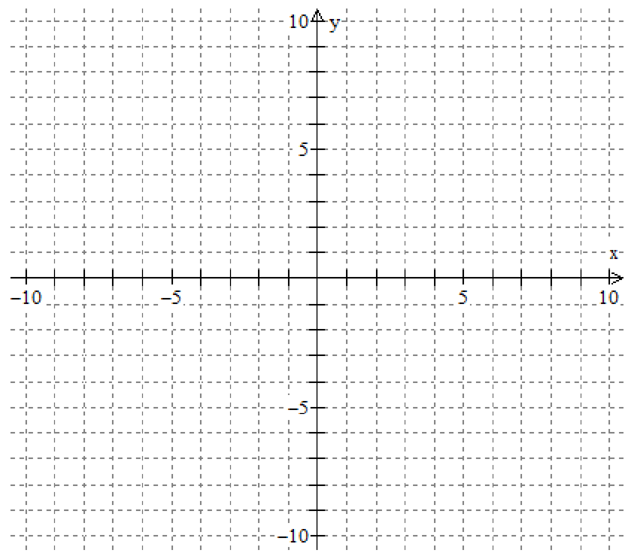
An exponential function with **base *B*** that has been transformed has the form: $g(x) = aB^{k(x-d)} + c$ with the following properties:

- The **horizontal asymptote** is only affected by the vertical translation.
- The ***y*-intercept** can be determined by finding $g(0)$ algebraically or looking at the graph of the transformed function.
- The **domain** is always the real numbers
- The **range** is affected by the **reflection in the *x*-axis** and the **vertical translation**.

We can graph these functions using a mapping rule on the key points of the parent function.

Ex 4) If $f(x) = 3^x$ and $g(x) = -4f\left(-\frac{1}{2}x - 3\right) + 5$

a) State the transformations that have been applied to $f(x)$




b) State the equation of $g(x)$

c) Using a mapping rule, graph both functions on the grid provided. Include the asymptotes.

d) Determine the *y*-intercept of $g(x)$

e) State the equation of the asymptote of $g(x)$

f) State the domain and range of $g(x)$.

Ex 5) If $g(x) = 9^x$ and $h(x) = 3^x$, describe the transformations you could apply to $h(x)$ to obtain $g(x)$.

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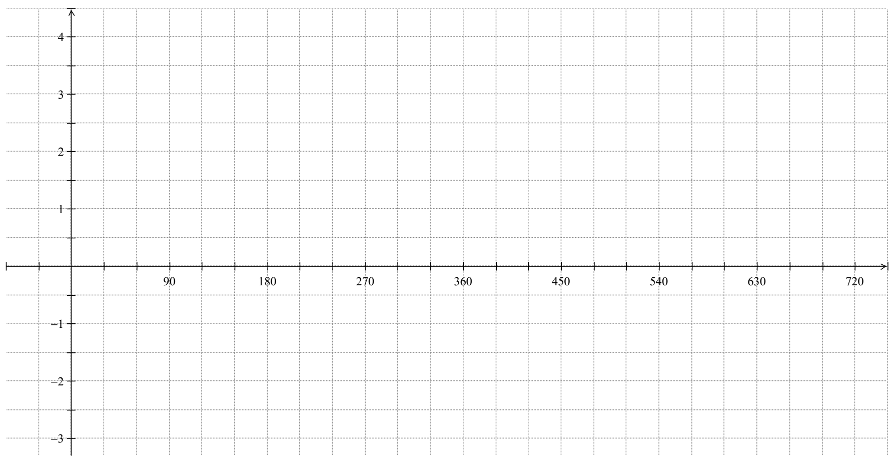
A sinusoidal function that has been transformed has the form: $g(\theta) = a\sin[k(\theta - d)] + c$ or $g(\theta) = a\cos[k(\theta - d)] + c$ with the following properties:

- The **amplitude** is **|*a*|**
- The **period** is $\dfrac{\mathbf{360°}}{\boldsymbol{k}}$
- The **# of cycles** (# of times a graph repeats within the domain of the base curve) is ***k***
- The phase shift is ***d*** *(remember to factor out k, if needed!)*
- The **equation of the axis** is ***y* = *c***
- The **vertical displacement** is ***c***
- The **range** is affected by the **amplitude** and the **vertical displacement**.

Ex 6) Given $g(\theta) = -3\sin(2\theta + 60°) + 1$




a) Identify the parent function (base curve)

b) State the transformations that have been applied to the base curve

c) Using a mapping rule, graph both functions over the base domain $\{\theta \in \Re \mid 0° \le \theta \le 720°\}$.

d) What is the amplitude?

e) What is the period of *g*?

f) What is the equation of the axis?

g) What is the domain and range of *g*?

Ex 7) Given $f(x) = \cos(x)$ and $g(x) = -2f\left(\frac{1}{3}x\right) + 2$, state the equation of *g*(*x*), its amplitude, period, equation of the axis, domain and range.

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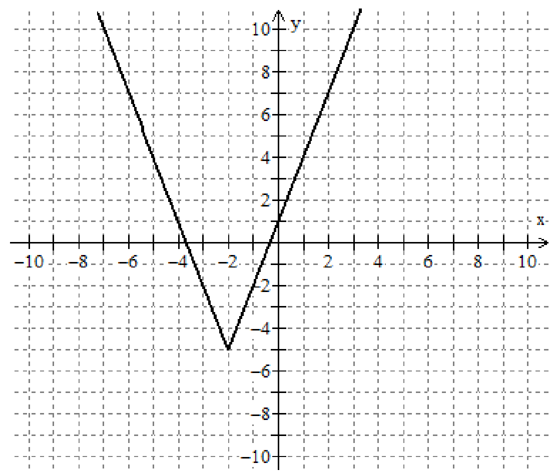
# Unit 4, Lesson 5: Representing Functions with Equations – Day 1

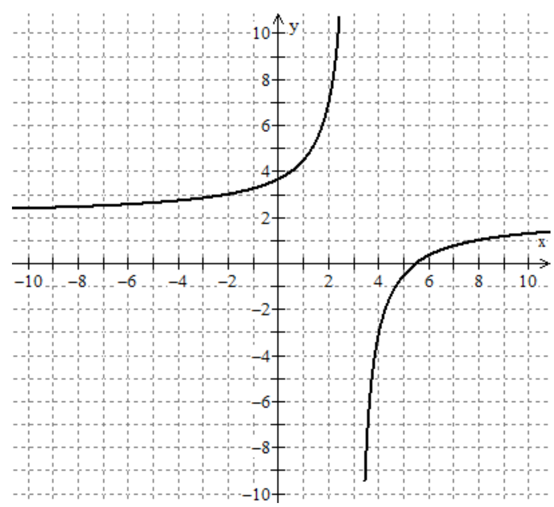
Given the graph of a function, you can determine the transformations that have happened to the parent function to determine its equation.

1. Identify the **parent function** by observing the shape of the graph
   - Each parent function has a distinct shape.
   - For exponential functions, this includes determining the base *B* if not given in the question.
2. Identify any **translations (shifts)** that have occurred.
   - It's often easiest to look at the vertex or the asymptotes, since those are not affected by dilations
   - Write the **equation** of the transformed function ***with the appropriate shifts***.
3. Identify any **reflections** that have occurred. Represent those in the **equation**.
4. Finally, **choose a point** on the graph (not the vertex) and **substitute** its coordinates ($x$, $y$) into your equation to **solve algebraically** for either $a$ or $k$. (usually identified in the question)

**Example 1:** The graph has undergone a transformation in the form $g(x) = f[k(x-d)]+c$. Determine the equation of the transformed function.



**Example 2:** The graph has undergone a transformation in the form $g(x) = af(x-d)+c$. Determine the equation of the transformed function.



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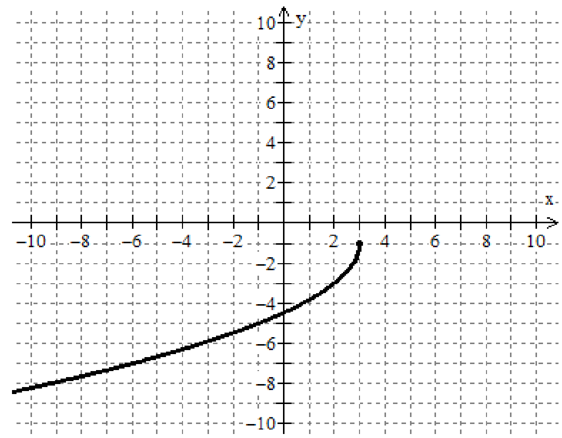
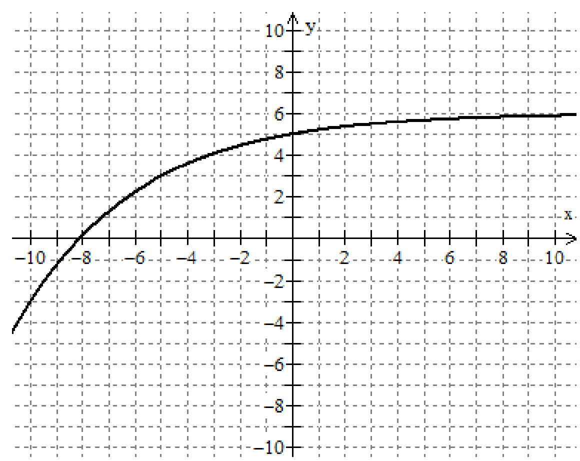
**Example 3:** Determine an equation for the graph shown.



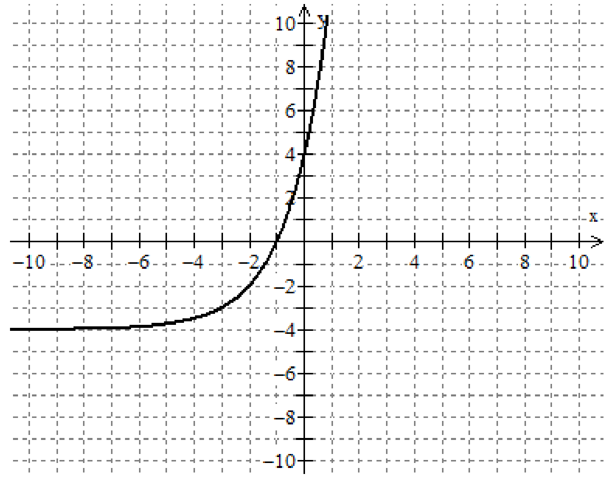

Determining the equation of an exponential function is especially challenging, since there are so many ways to write equivalent exponential functions using exponent laws.

**Example 4:** Using exponent rules, find 3 equivalent functions for $f(x)=4^x$

**Example 5:** The graph $f(x)=3^x$ has undergone transformations including a horizontal dilation. Determine an equation for the transformed function.



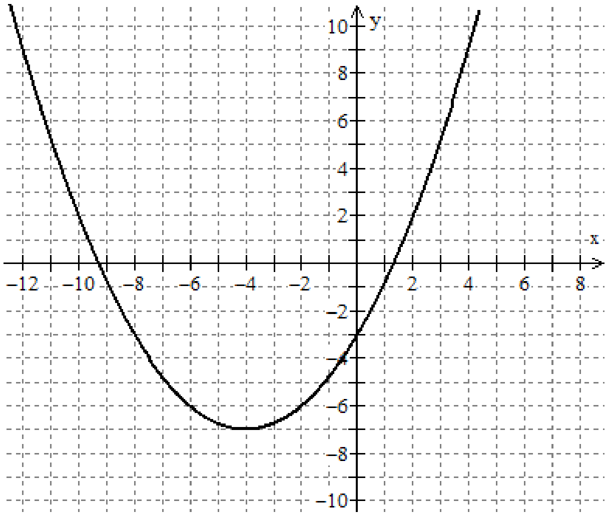
**Example 6:** The graph $f(x) = 2^x$ has undergone transformations. Determine an equation for the function using

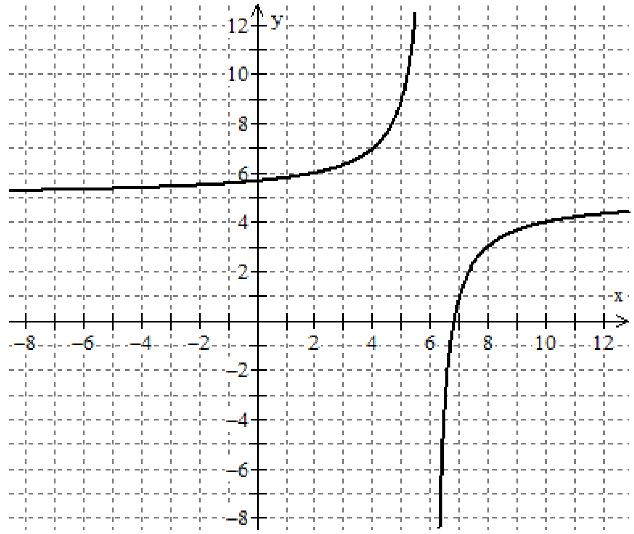


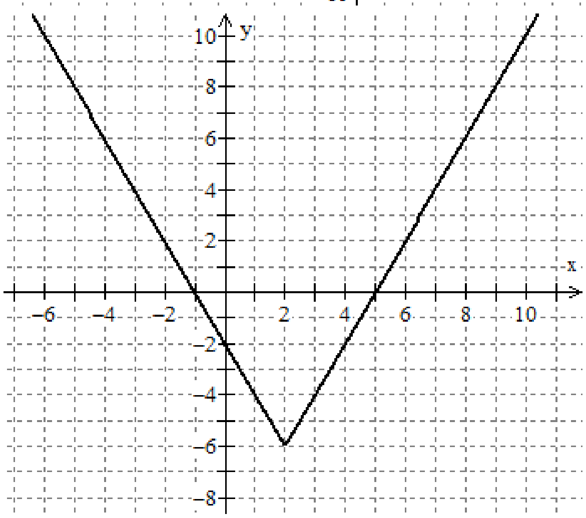

a) A vertical dilation

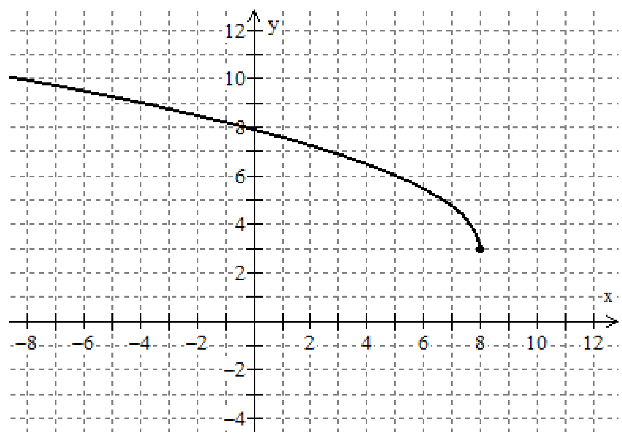
b) A horizontal shift

1. Each of the following graphs have undergone transformations in the form $g(x) = f(k(x-d)) + c$. Determine the equation of each transformed function.

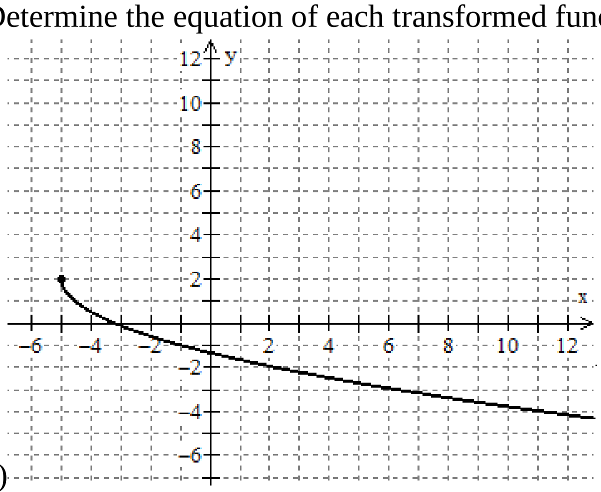
a) 

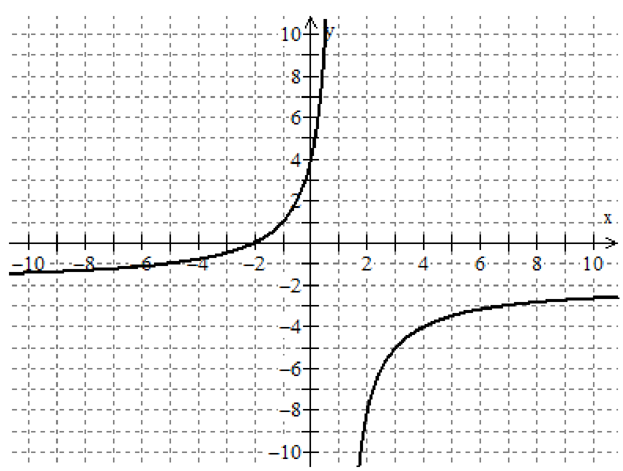

b) 


c) 


d) 


2. Each of the following graphs have undergone transformations in the form $g(x) = af(x-d) + c$. Determine the equation of each transformed function.

a) 


b) 

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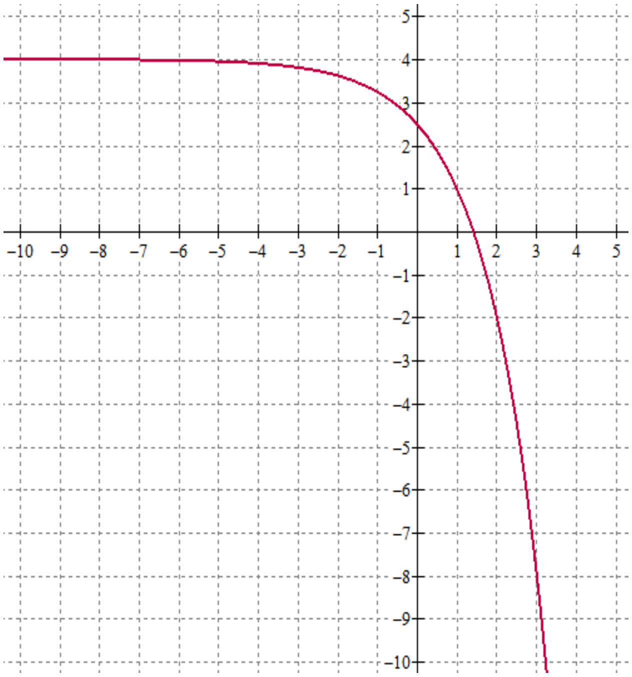
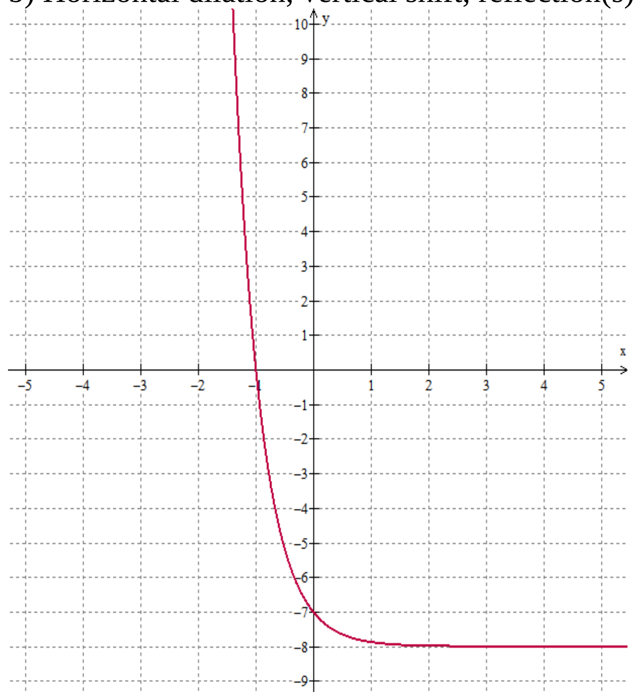
3. The following graphs have been obtained by applying the indicated transformations to the graph of $f(x)=2^x$. Determine an equation to represent each graph.

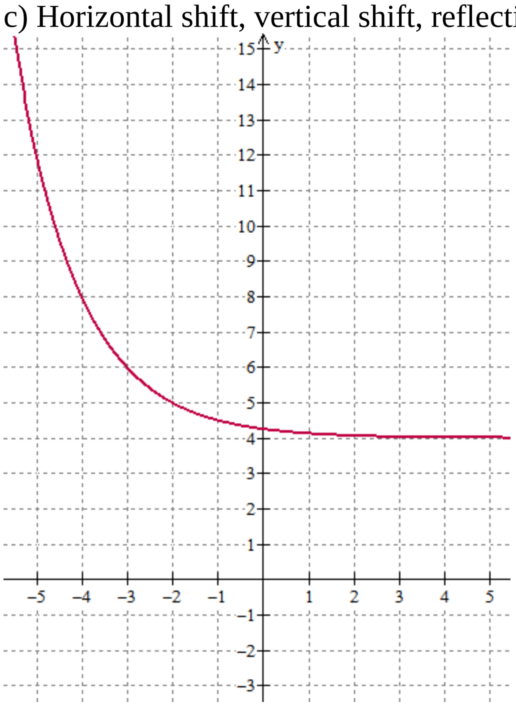
a) Vertical dilation, vertical shift, reflection(s)



b) Horizontal dilation, vertical shift, reflection(s)



c) Horizontal shift, vertical shift, reflection(s)



d) Horizontal shift, vertical shift, reflection(s)

1) a) $g(x)=\left(\frac{1}{2}(x+4)\right)^2-7$ b) $g(x)=\frac{1}{-\frac{1}{4}(x-6)}+5$ c) $g(x)=|2(x-2)|-6$ d) $g(x)=\sqrt{-3(x-8)}+3$ 2) a) $g(x)=-\frac{3}{2}\sqrt{x+5}+2$

b) $g(x)=\frac{-6}{x-1}-2$ 3) a) $g(x)=-\frac{3}{2}(2)^x+4$ b) $g(x)=2^{-3x}-8$ c) $g(x)=2^{-(x+2)}+4$ d) $g(x)=-(2)^{-(x-5)}-3$

**MCR 3U**

# Unit 4, Lesson 5: Representing Functions with Equations – Day 2

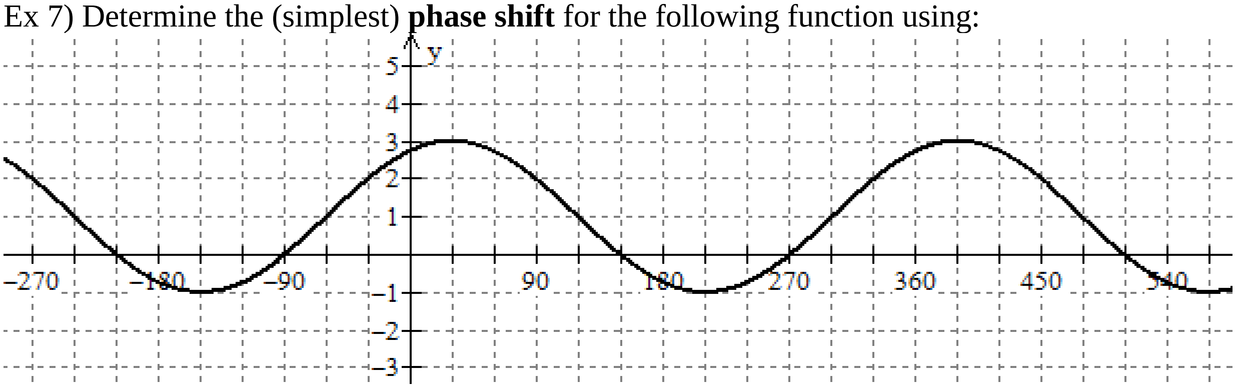
For graphs of sinusoidal functions, we are often interested in obtaining the ***simplest*** equation. This means:

- Avoid phase shifts (horizontal translations), if possible
  - Sometimes you can use a reflection in the *x*-axis instead of a phase shift
  - If possible, choose a base curve (sine or cosine) that avoids a phase shift
- Avoid using reflections in the *y*-axis

From the graph, it is easiest to identify its properties, and then use those properties to determine the transformations and the equation.

| **Parameter** | **Property** | **How to determine/calculate** |
|---|---|---|
| $a$ | Amplitude | $\lvert a \rvert = \dfrac{\max - \min}{2}$<br>$a < 0$ if a reflection is needed to avoid a phase shift |
| $k$ | # of cycles in the domain of the base curve | $k = \dfrac{360°}{period}$ |
| $d$ | Phase shift | Sine as base curve<br>• Locate the equation of the axis.<br>• Find the **POI** between the **curve** and the **equation of the axis** closest to the *y*-axis where the graph is ***increasing***.<br>• The $\theta$-value of this point is the phase shift.<br>Cosine as base curve<br>• Go to the ***maximum*** closest to the *y*-axis<br>• The $\theta$-value of this point is the phase shift. |
| $c$ | Vertical displacement, also the *y*-intercept of the equation of the axis | $c = \dfrac{\max + \min}{2}$ |

Ex 7) Determine the (simplest) **phase shift** for the following function using:




a) sine as the base curve

b) cosine as the base curve

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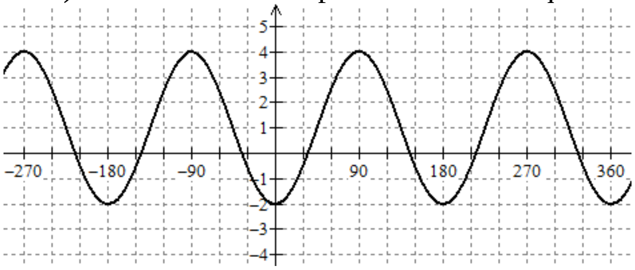
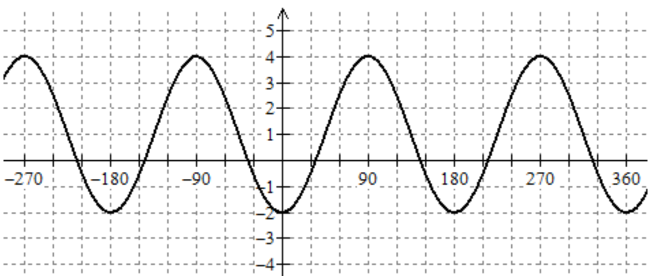
Ex 8) Determine the simplest sinusoidal equation for the following function, using:

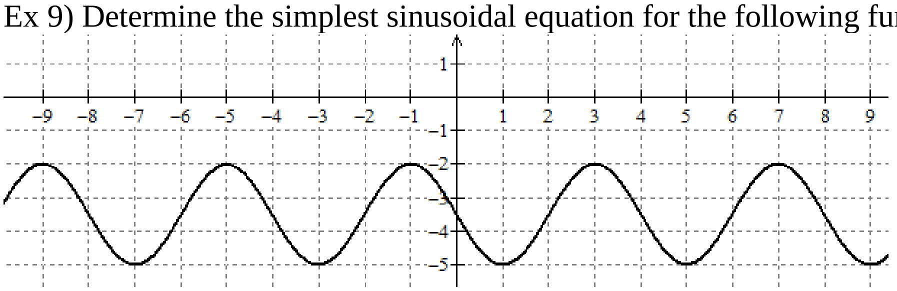



a) sine as the base curve

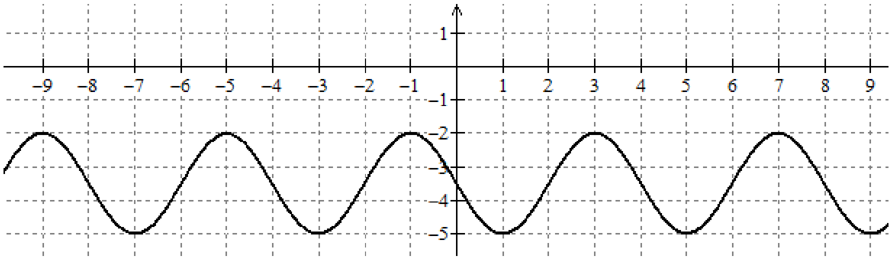



b) cosine as the base curve

Ex 9) Determine the simplest sinusoidal equation for the following function, using:




a) sine as the base curve



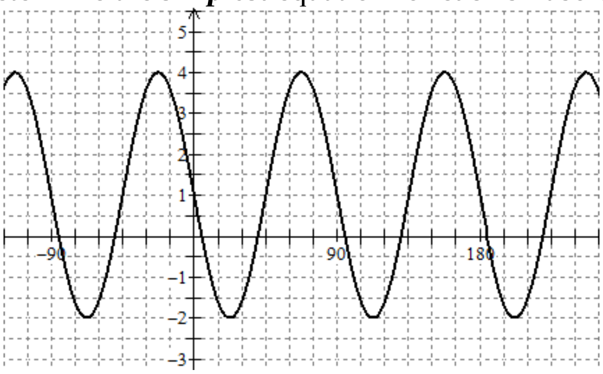

b) cosine as the base curve

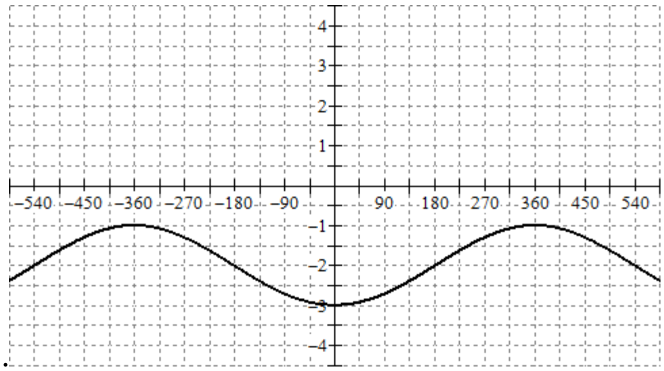
Ex 10) Determine a sinusoidal equation for a function with the following properties:
amplitude 3 units, minimum at (0, -4), period of 90°

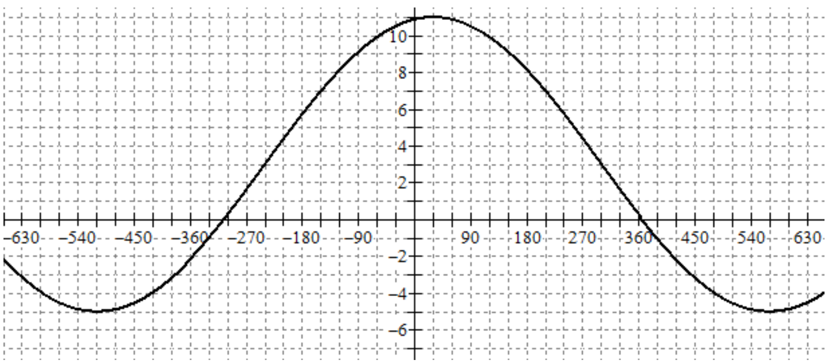
a) using sine as the base curve

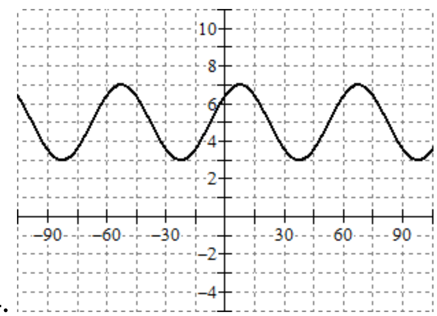
b) using cosine as the base curve

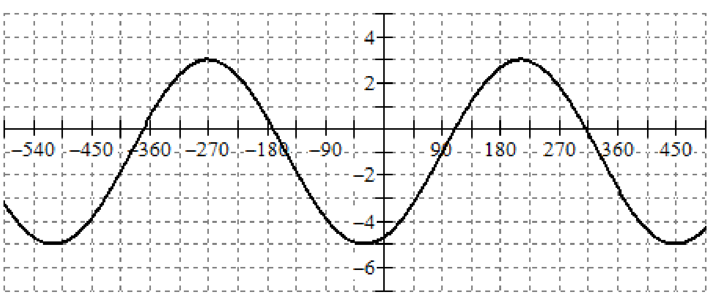
Determine the ***simplest*** equation for each sinusoidal function.

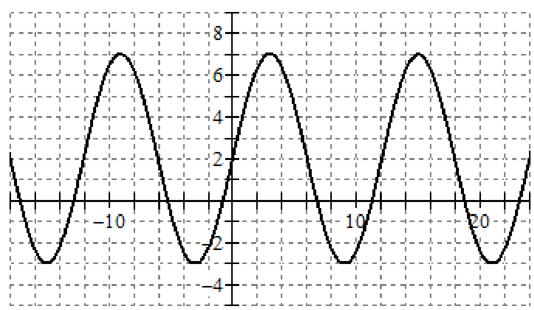
1. 

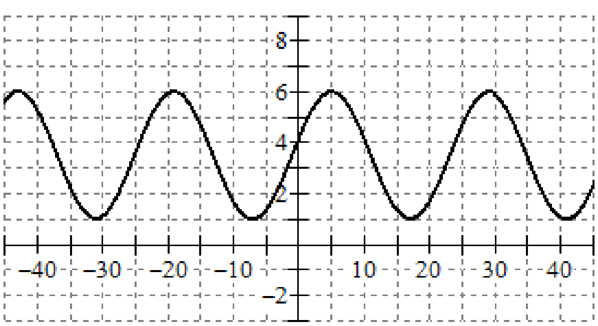

2 . 

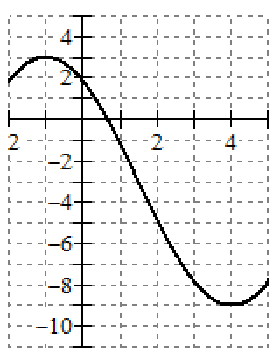

3. 


4. 


5. 


6. 


7. 


8. 

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Determine the equation of ***two sinusoidal functions*** (sine as base curve, cosine as base curve) given the following properties:

9. number of cycles ½, amplitude 4, minimum at -2

10. amplitude 6 units, maximum at 4, period of 45°

11. equation of the axis $y = 9$, maximum 12, phase shift 30°

12. amplitude 8 units, maximum at (90°, 13), number of cycles 1

13. vertical displacement 1, period 720°, maximum (0°, 3)

14. amplitude 4 units, minimum at (0°, 6), period 120°

15. vertical displacement -5, period 180°, minimum at (90°, -11)

1. $f(\theta) = -3\sin 4\theta + 1$ 2. $f(\theta) = -\cos\frac{1}{2}\theta - 2$ 3. $f(\theta) = 8\cos\frac{1}{3}(\theta - 30°) + 3$ 4. $f(\theta) = 2\sin 6(\theta + 7.5°) + 5$

5. $f(\theta) = 4\sin\frac{3}{4}(\theta - 90°) - 1$ 6. $f(\theta) = 5\sin 30\theta + 2$ 7. $f(\theta) = \frac{5}{2}\cos 15(\theta - 5°) + \frac{7}{2}$ 8. $f(\theta) = 6\cos 36(\theta + 1°) - 3$

9. $f(\theta) = 4\sin\frac{1}{2}\theta + 2$ or $f(\theta) = 4\cos\frac{1}{2}\theta + 2$ 10. $f(\theta) = 6\sin 8\theta - 2$ or $f(\theta) = 6\cos 8\theta - 2$

11. $f(\theta) = 3\sin(\theta - 30°) + 9$ or $f(\theta) = 3\cos(\theta - 30°) + 9$ 12. $f(\theta) = 8\sin\theta + 5$ or $f(\theta) = 8\cos(\theta - 90°) + 5$

13. $f(\theta) = 2\sin\frac{1}{2}(\theta + 180°) + 1$ or $f(\theta) = 2\cos\frac{1}{2}\theta + 1$ 14. $f(\theta) = 4\sin 3(\theta - 30°) + 10$ or $f(\theta) = 4\cos 3\theta + 10$

15. $f(\theta) = 6\sin 2(\theta + 45°) - 5$ or $f(\theta) = 6\cos 2\theta - 5$

**MCR 3U**

## Unit 4, Lesson 6: Inverse Functions & Their Graphs

An **inverse operation** is an operation that reverses the effect of another operation.
An **inverse function** reverses the effect of a given function.

**Example 1:** The function $C(n) = 20n + 500$, represents the cost, *C*, of holding a reception at a hall as a function of the number, *n*, of guests. The hall has a fire limit of 200 people.

a) Determine the domain and range of $C(n)$.

b) What operations happened to the input variable? What would be the reverse of those operations?

c) Write the equation that would represent the number of guests as a function of the cost.

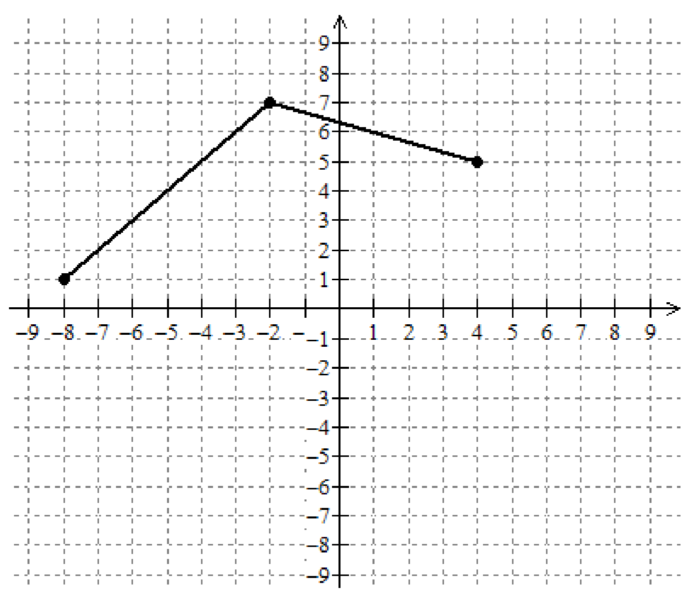
d) Determine the domain and range of the inverse.

Given a function $f(x)$, you can write its inverse as $f^{-1}(x)$ ***(this is not raising a value to the power –1)***
If the point ($a$, $b$) is on the graph of $y = f(x)$, then the point ($b$, $a$) is on the graph of $y = f^{-1}(x)$.
The graph of an inverse is the reflection of the graph $y = f(x)$ in the line $y = x$

**Example 2:** Given the graph of $y = f(x)$
a) Draw the graph of its inverse on the same grid

b) Is the inverse a function?

c) State the domain and range of the inverse



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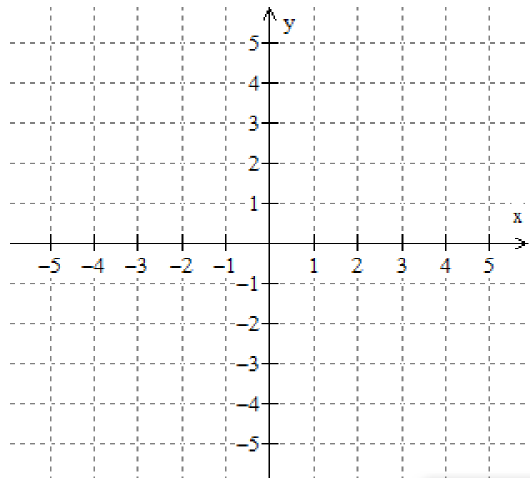
Given the equation of a function, we can determine its inverse by **isolating the independent variable** and rewriting using inverse function notation.

**Example 3:** Given $f(x)=3-5x$, determine $f^{-1}(x)$. Is the inverse a function?

If the original function is linear, is the inverse always a function?

**Example 4:** Graph $g(x)=-2(x+3)^2+5$ and its inverse on the grid. a) Is the inverse a function?




To make the inverse of a quadratic a function, we ***restrict the domain*** of the original function.

b) Given $g(x)=-2(x+3)^2+5, x\geq -3$, determine the equation for $f^{-1}(x)$

To determine the inverse of a quadratic function algebraically, the equation must be in **vertex form**. **Complete the square** to put into vertex form, if needed.

**Example 5:** Given $h(x)=3x^2-24x+30, x\leq 4$

a) Determine $h^{-1}(x)$ algebraically.

b) State the transformations on the parent function for both $h(x)$ and $h^{-1}(x)$. What do you notice?