Lesson 2: Recursive Sequences & Binomial Theorem

Two famous recursive sequences are the Fibonacci sequence and the Lucas sequence. They are best defined recursively (a general term is possible, but messy!)

Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

Recursive definition: $t_1 = 1,\ t_2 = 1,\ t_n = t_{n-1} + t_{n-2};\ n > 2$

Lucas sequence: 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, …

Recursive definition: $t_1 = 1,\ t_2 = 3,\ t_n = t_{n-1} + t_{n-2},\ n > 2$

In what ways are these number found in nature?

patterns on leaves, petals, vegetables . . .

What number does the ratio of consecutive terms approach?

phi $\phi \approx 1.618033989$

$\frac{360^\circ}{\phi} = 137.5^\circ$

Doodling in Math Class: Fibonacci, Spirals and Being a Plant

Doodling in Math Class: Fibonacci, Spirals and Being a Plant (part 2)

Jan 31-9:35 PM

Pascal's Triangle

There is a two-dimensional pattern of numbers that is usually called Pascal's Triangle. It is named after the French mathematician Blaise Pascal, though he was not the first to discover it! Identify the number pattern in the first few rows and complete the next five rows.

1	n = 0
1 1	n = 1
1 2 1	n = 2
1 3 3 1	n = 3
1 4 6 4 1	n = 4
1 5 10 10 5 1	n = 5
1 6 15 20 15 6 1	n = 6
1 7 21 35 35 21 7 1	n = 7
1 8 28 56 70 56 28 8 1	n = 8
1 9 36 84 126 126 84 36 9 1	n = 9
1 10 45 120 210 252 210 120 45 10 1	n = 10

Jun 2-12:03 PM

A **binomial** is any expression that is written in the form of $a + b$. For example $x + 1$, $2x - 3y$, $5w^2 - 15$ are all binomial expressions.

Ex 1) For each of these binomial examples, identify a & b.

$x+1$	$2x-3y$	$5w^2-15$
a: X	a: 2x	a: $5w^2$
b: 1	b: $-3y$	b: -15

We often have to work with powers of binomials, whose exponents are natural numbers. For example $(a + b)^2$, $(a + b)^{15}$, $(a + b)^3$

Notice the patterns when we expand the following:

$(a+b)^0$ 1

$(a+b)^1$ $a+b$

$(a+b)^2$ $a^2+2ab+b^2$

$(a+b)^3$ $a^3+3a^2b+3ab^2+b^3$

$(a+b)^4$ $a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$(a+b)^5$ $a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

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How do the **coefficients** of the binomial expansions (expanded binomials) relate to the numbers in Pascal's Triangle?

Coefficients for $(a+b)^n$ correspond to the nth row of Pascal's triangle (with first row as n=0).

What is the pattern of the exponents of the bases a and b?

Exponents of each term **add** to the exponent of the original expression.

eg for $(a+b)^n$ a → starts at n, decrease to 0.

b → starts at 0, increase to n.

Jun 12-8:27 AM

Ex 2) Use Pascal's Triangle to expand $(x+2y)^5$

Pascal's Triangle for 5th row: 1, 5, 10, 10, 5, 1

a : x

b : 2y

$(a+b)^5 = 1a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(x+2y)^5 = 1x^5 + 5x^4(2y)^1 + 10x^3(2y)^2 + 10x^2(2y)^3 + 5x(2y)^4 + (2y)^5$

$(x+2y)^5 = x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

Jun 2-12:29 PM

HW U6L2:

1. p.443 #1

2. p.466 #1, 2c, 3c, 4f, 5cf

Dec 15-11:39 AM