

Lesson 4: Graphing Functions with Transformations

You can graph functions of the form $g(x) = af\left[k(x-d)\right] + c$ by applying the appropriate transformations to the key points of the parent function $f(x)$. Always apply a and k before d and c , since transformations should follow the same order of operations as numerical expressions.

To assist in transforming key points, we will use a "Mapping Rule." This rule tells us what operations to perform on the x - and y -coordinates of each key point.

Parameter	Description of Transformation	Mapping Rule
a	Vertical Dilation by a factor of a If $a > 1$ or $a < -1$ the graph is stretched vertically If $-1 < a < 1$ the graph is compressed vertically If $a < 0$ the graph is Reflected in the x-axis	$(x, y) \rightarrow (x, ay)$
k	Horizontal Dilation by a factor of $\frac{1}{k}$ If $k > 1$ or $k < -1$ the graph is compressed horizontally If $-1 < k < 1$ the graph is stretched horizontally If $k < 0$ the graph is Reflected in the y-axis	$(x, y) \rightarrow \left(\frac{x}{k}, y\right)$
d	Horizontal Translation (Shift) left or right d units If $d > 0$ the graph shifts right If $d < 0$ the graph shifts left	$(x, y) \rightarrow (x+d, y)$
c	Vertical Translation (Shift) up or down c units If $c > 0$ the graph shifts up If $c < 0$ the graph shifts down	$(x, y) \rightarrow (x, y+c)$

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Ex 1) Given the function $g(x) = -2f\left[\frac{1}{3}(x+3)\right] + 5$

a) State the transformations that have been applied to $f(x)$

- ① R I T X A
- ② vertically stretch bafo 2
- ③ horizontally stretched bafo 3.
- ④ horizontally translated 3 units left.
- ⑤ vertically translated up 5 units

b) State the mapping rule

recall: $(x, y) \rightarrow \left(\frac{1}{k}x + d, ay + c\right)$

$$(x, y) \rightarrow (3x - 3, -2y + 5)$$

c) The point $(2, 5)$ is a point on the graph of $f(x)$. Determine the coordinates of the transformed point.

$$(2, 5) \rightarrow (3(2) - 3, -2(5) + 5)$$

$$\rightarrow (3, -5)$$

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Equations of transformed functions with parameters a , k , d & c :

Linear: $g(x) = a[k(x-d)] + c$

Quadratic: $g(x) = a[k(x-d)]^2 + c$

Reciprocal: $g(x) = \frac{a}{k(x-d)} + c$

Square Root: $g(x) = a\sqrt{k(x-d)} + c$

Absolute Value: $g(x) = a|k(x-d)| + c$

k and d
If you have both a horizontal dilation/reflection AND a horizontal shift, you must FACTOR out k !

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Ex 2) Given $f(x) = \frac{1}{x}$ and $g(x) = 2f\left(-\frac{1}{4}x-1\right) + 3$

$g(x) = 2f\left[-\frac{1}{4}(x+4)\right] + 3$

$a=2$
 $k=-\frac{1}{4} \therefore \frac{1}{k} = -4$
 $d=-4$
 $c=+3$

a) State the transformations that have been applied to $f(x)$

- ① vertically stretch b.af. a 2
- ② R I T Y A
- ③ horizontally stretched b.af. 4
- ④ horizontally translated left 4 units.
- ⑤ vertically translated up 3 units.

b) Using a mapping rule, graph both functions on the grid provided. Include the asymptotes. $(x, y) \rightarrow (-4x-4, 2y+3)$

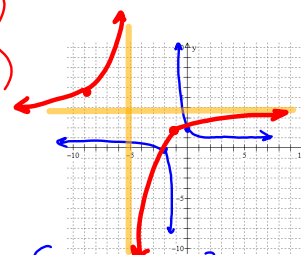
mainpoints:

$(-1, -1) \rightarrow (-4(-1)-4, 2(-1)+3) \rightarrow (0, 1)$

$(1, 1) \rightarrow (-4(1)-4, 2(1)+3) \rightarrow (-8, 5)$

$x=0 \rightarrow -x = -4(0)-4 \rightarrow x = -4$

$y=0 \rightarrow y = 2(0)+3 \rightarrow y = 3$



c) Write the equation for $g(x)$ and state its domain and range.

$g(x) = \frac{2}{-\frac{1}{4}(x+4)} + 3$

domain: $\{x \in \mathbb{R} \mid x \neq -4\}$
range: $\{y \in \mathbb{R} \mid y \neq 3\}$

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Ex 3) Given $g(x) = -2\sqrt{-x+3} + 5$

a) Identify the parent function.

square root function
 $f(x) = \sqrt{x}$

b) State the transformations that have been applied to the parent function

- ① R I T X A
- ② vertically stretch by 2
- ③ R I T Y A
- ④ horizontally translated 3 units right
- ⑤ vertically translated 5 units up.

c) Using a mapping rule, graph both functions on the grid provided.

$(x, y) \rightarrow (-x+3, -2y+5)$

Main Points:

$(0, 0) \rightarrow (-0+3, -2(0)+5) \rightarrow (3, 5)$

$(1, 1) \rightarrow (-1+3, -2(1)+5) \rightarrow (2, 3)$

$(4, 2) \rightarrow (-4+3, -2(2)+5) \rightarrow (-1, 1)$

$(9, 3) \rightarrow (-9+3, -2(3)+5) \rightarrow (-6, -1)$

d) State the domain and range of $g(x)$

$D: \{x \in \mathbb{R} \mid x \leq 3\}$

$R: \{y \in \mathbb{R} \mid y \leq 5\}$

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HW U4L4 Day 1:

1. p. 70 #1, 2, 6, 7c, 8c, 9c, 10abde, 12, 16, 17

2. study parent functions in preparation for the quiz Wednesday