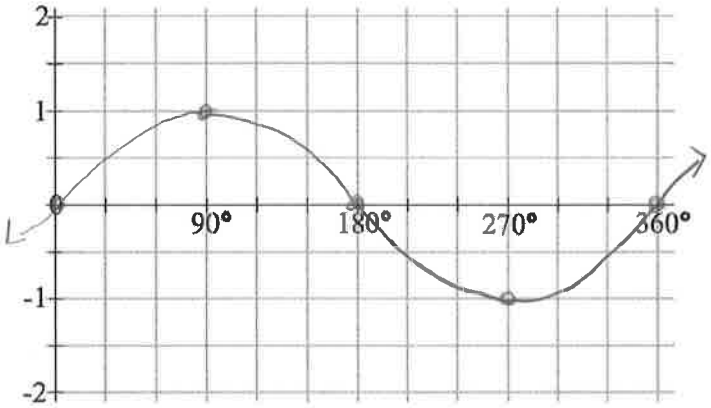
Exam Review Part 6 – Trig Functions

MCR3U

SOLUTIONS

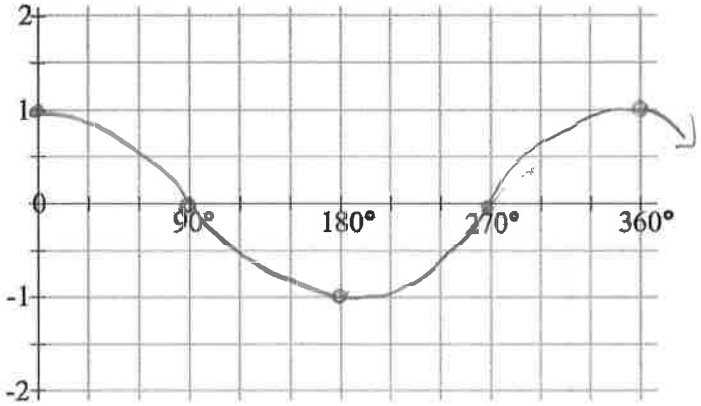
1) Graph the function $y = sinx$ using key points between 0° and 360° and then continuing the pattern.

x	y
0	0
90	1
180	0
270	-1
360	0



2) Graph the function $y = cosx$ using key points between 0° and 360°.

x	y
0	1
90	0
180	-1
270	0
360	1



3) Graph the three curves given on the axes below. Clearly label each of your graphs and the x-axis values.

a) $y = \sin x$

b) $y = \sin(x + 60) + 1$

c) $y = 2\sin\left[\frac{2}{3}(x - 60)\right] - 1$

a)

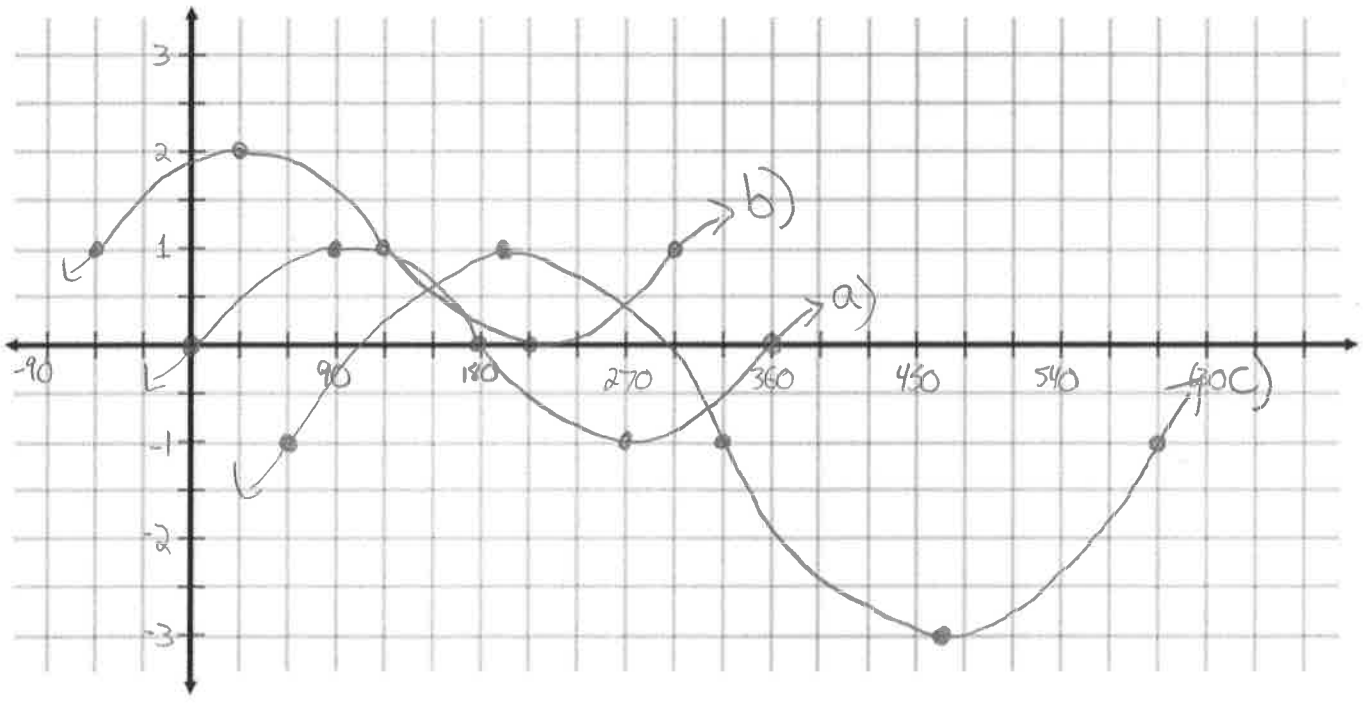
x	y
0	0
90	1
180	0
270	-1
360	0

b)

$x-60$	$y+1$
-60	1
30	2
120	1
210	0
300	1

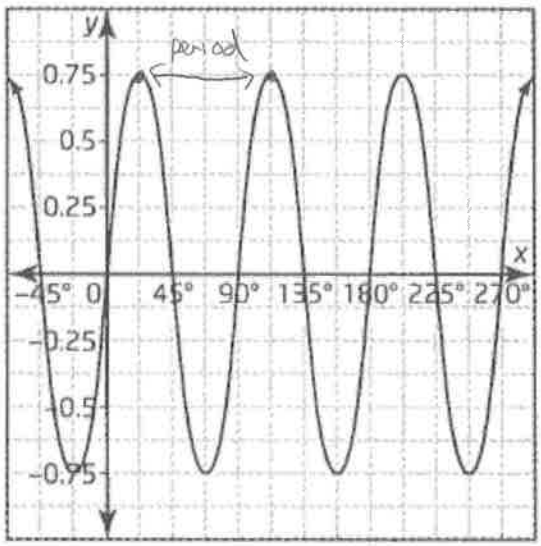
c)

$\frac{3}{2}x + 60$	$2y - 1$
60	-1
195	1
330	-1
465	-3
600	-1



4) Write the equations of sine function and a cosine function to match each graph.

a)



$a = \frac{max - min}{2} = \frac{0.75-(-0.75)}{2} = 0.75$

$k = \frac{360}{period} = \frac{360}{90} = 4$

$C = max - amp = 0.75 - 0.75 = 0$

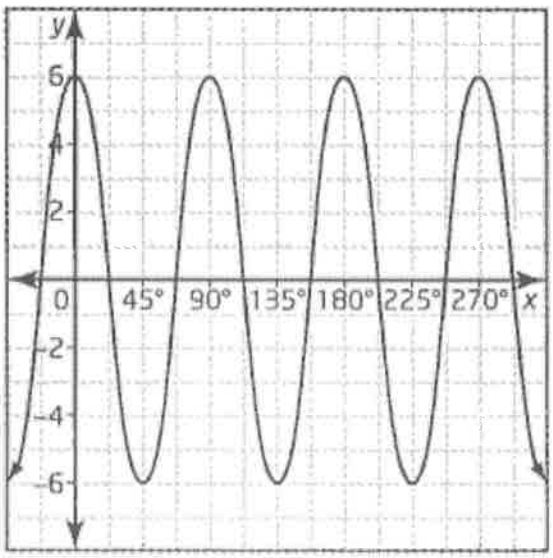
$d = 22.5$ (cos)

$d = 0$ (sin)

$y = 0.75 \sin(4x)$

$y = 0.75 \cos[4(x-22.5)]$

b)



$a = \frac{max - min}{2} = \frac{6-(-6)}{2} = 6$

$k = \frac{360}{period} = \frac{360}{90} = 4$

$C = max - amp = 6 - 6 = 0$

$d = 0$ (cos)

$d = -22.5$ (sin)

Note: rising midline is $\frac{90}{k} = \frac{90}{4} = 22.5$ to the left of the max.

$y = 6 \sin[4(x+22.5)]$

$y = 6 \cos(4x)$

5) Write sentences to explain the steps you would take to transform $y = \sin x$ into the graph of $y = -3\sin 4(x+30)+1$.

- vertical reflection
- vertical stretch bafo 3
- horizontal compression bafo $\frac{1}{4}$
- ~~shift left~~ 30
- ~~shift~~ up 1

6) For the transformed function $y = 4\cos[3(x - 20°)] + 5$ state the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function, also state the maximum and minimum values of the function.

Note: be sure to indicate the direction of the phase shift and vertical shift!

Amplitude = 4 Period = $\frac{360}{3} = 120°$ Phase shift = ~~Right~~ 20

Vertical Shift = up 5 Maximum Value = $5+4 = 9$ Minimum Value = $5-4 = 1$

7) For the transformed function $y = \frac{1}{4}\sin\left[\frac{1}{2}(x + 90°)\right] - 2$ state the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function, also state the maximum and minimum values of the function.

Note: be sure to indicate the direction of the phase shift and vertical shift!

Amplitude = 0.25 Period = $\frac{360}{0.5} = 720°$ Phase shift = ~~Left~~ 90

Vertical Shift = Down 2 Maximum Value = $-2 + 0.25 = -1.75$ Minimum Value = $-2 - 0.25 = -2.25$

8) Determine two equations for a sinusoidal wave that has a maximum at (0, 2/3), vertical shift of 1/3 up, and a period of 120.

$a = \text{max} - c = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

$k = \frac{360}{\text{period}} = \frac{360}{120} = 3$

$c = \frac{1}{3}$

$d = 0$ (cos)

$d = -30$ (sin) Note: rising midline is $\frac{90}{k} = \frac{90}{3} = 30$ to the ~~left~~ of the max.

$y = \frac{1}{3}\cos(3x) + \frac{1}{3}$

$y = \frac{1}{3}\sin[3(x+30)] + \frac{1}{3}$

9) Pitt Lake is a freshwater lake in southern British Columbia with the highest tidal change of any freshwater lake in the world. In a daily period, the highest tide is traditionally at 8:00 am, reaching 5.2 m, and the lowest tide is traditionally at 8:00 pm, reaching only 0.6 m. Consider the cosine function that gives the tidal height of the lake, y, in terms of the hours after midnight, x.

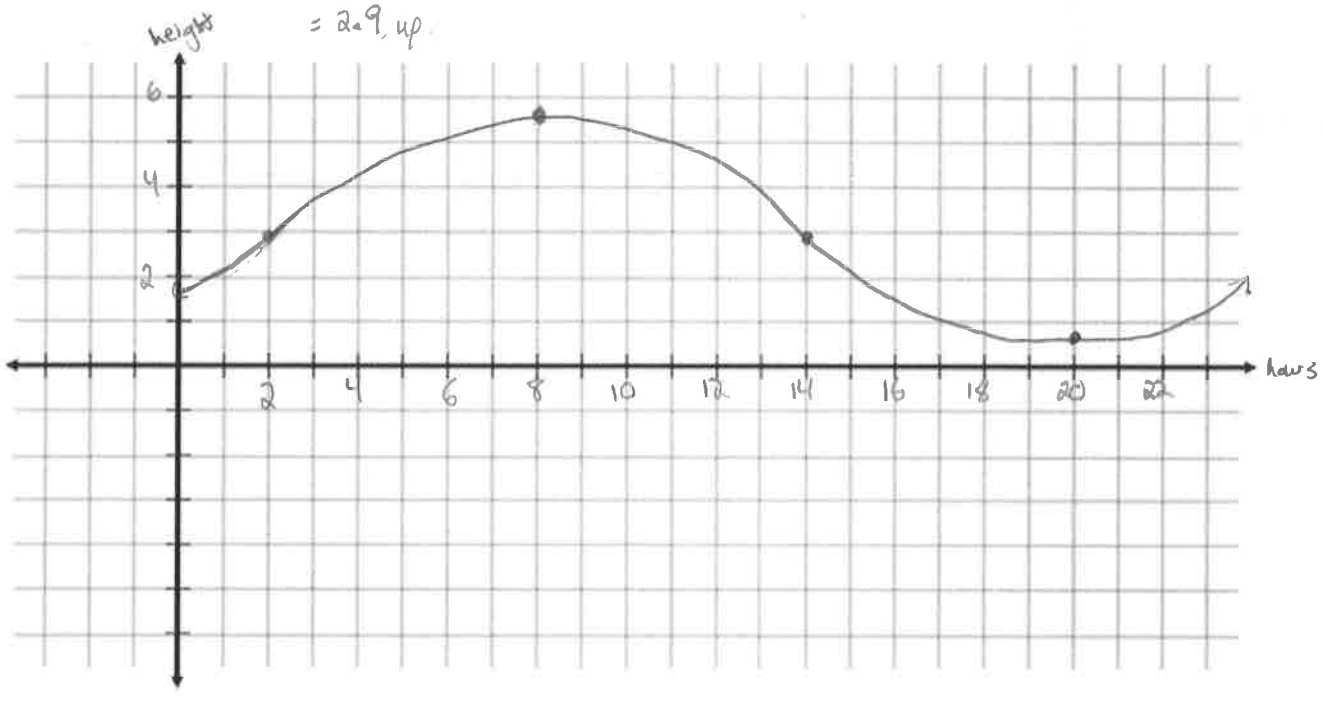
a) Draw a sketch of the function. What are the period, amplitude, phase shift and vertical shift of the function?

Period = 24 amplitude = $\frac{5.2 - 0.6}{2} = 2.3$ phase shift = 8 Right (max is 8 hours after midnight)

vertical shift = max - amp

= 5.2 - 2.3

= 2.9 up



b) What is the function equation in the form $y = a\cos k(x-d)+c$?

$a = 2.3$

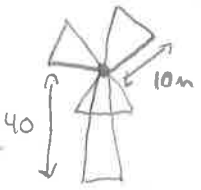
$k = \frac{360}{24} = 15$

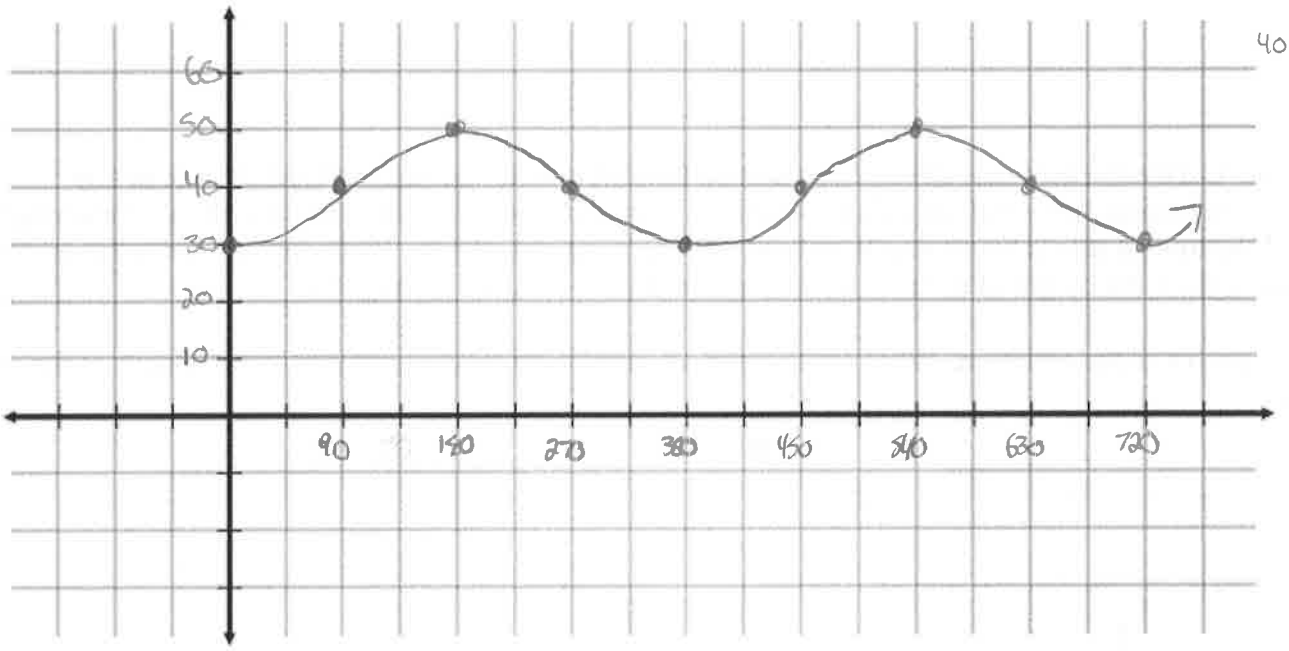
$c = 2.9$

$d = 8$

$y = 2.3\cos[15(x-8)]+2.9$

10) A windmill is 40 meters tall and has three blades each measuring 10m.
a) Graph the height of the tip of a blade that starts at the bottom of the windmill and rotates around counter clockwise. Graph two rotations.





b) Determine a sine and cosine function to represent the motion of the blade.

$a = \frac{50-30}{2} = 10$

$k = \frac{360}{360} = 1$

$c = 50-10 = 40$

$d = 90$ (sin)

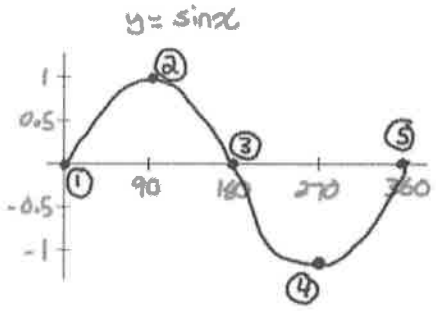
$d = 180$ (cos)

$y = 10\sin(x-90)+40$

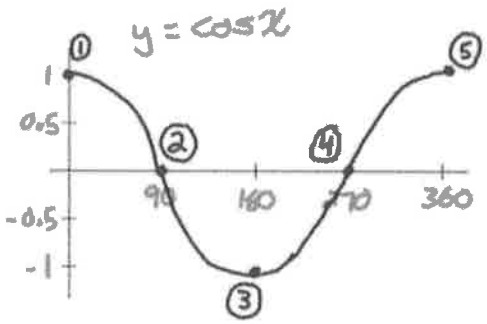
$y = 10\cos(x-180)+40$

Answers

1)



2)



3) Check posted solutions

4) a) $y = \frac{3}{4}\sin(4x)\,;\, y = \frac{3}{4}\cos[4(x - 22.5)]$ **b)** $y = 6\cos(4x)\,;\, y = 6\sin[4(x + 22.5)]$

5) reflection in the x-axis, vertical stretch bafo 3, horizontal compression bafo $\frac{1}{4}$, shift left 30°, up 1 unit

6) Amplitude = 4 Period = 120 Phase shift = 20 right Vertical Shift = up 5 Maximum = 9 Minimum = 1

7) Amplitude = $\frac{1}{4}$ Period = 720 Phase shift = 90 left Vertical Shift = down 2 Maximum = -1.75 Minimum = -2.25

8) $y = \frac{1}{3}\cos(3x) + \frac{1}{3}\,;\, y = \frac{1}{3}\sin 3(x + 30) + \frac{1}{3}$

9) a) amplitude = 2.3 period = 24 phase shift = 8 right vertical shift = 2.9 up

b) $y = 2.3\cos[15(x - 8)] + 2.9$

10) a) See posted solutions **b)** $y = 10\sin(x - 90) + 40\,;\; y = 10\cos(x - 180) + 40$