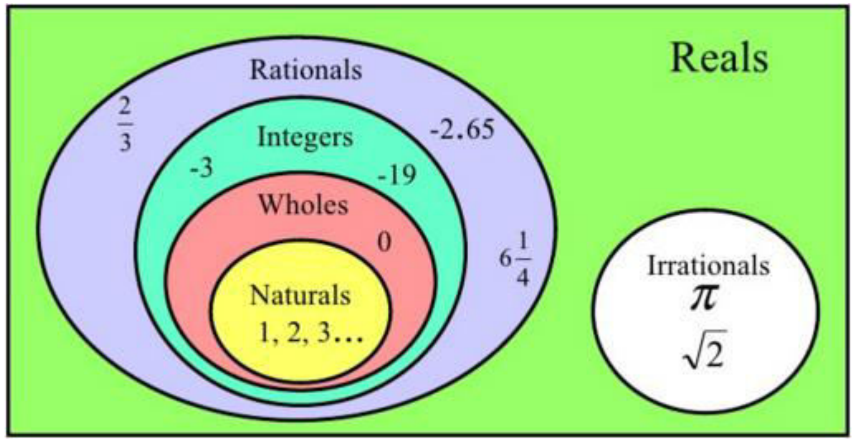
Unit 1 – Linear & Quadratic Functions

Day	Lesson	Practice Questions	Struggles?
1	U1L1: Domain and Range	**Do: Handout**	
2	U1L2: Functions & Function Notation	**Read:** Pg. 14 – 22 **Do: Handout** Pg. 22 # 1adf, 2, 5ac, 7ac, 10, 11bd, 12, 15c, 16b, 17	
3	*Skill Builder: Factoring*	**Read:** Pg. 98 – 101 **Do:** Pg. 102 # 2 *(just factor, don't describe)*, 4 – 7, 9 **Challenge:** #14	
4	U1L3: Max/Min of a Quadratic	**Read:** Pg. 148 – 153 **Do:** Pg. 153 # 1, 2, 4ace, 8, 11ab, **Handout** Pg. 147 # 12	
5	*Skill Builder: Radicals*	**Read:** Pg. 163 – 167 **Do:** Pg. 167 # 1 – 7ace, 15b, 17	
6	U1L4: Solve Quadratic Equations	**Read:** Pg. 172 – 177 **Do:** Pg. 177 # 1bd, 2ad, 4, 5, 6bd, 7, 8c	
7	U1L5: Zeros of a Quadratic Function	**Read:** Pg. 179 – 184 **Do:** Pg. 185 # 4ad, 5ab, 6 – 10, 14	
8	*Skill Builder: Polynomials*	**Read:** Pg. 84 – 87, 91 – 94 **Do:** Pg. 88 # 5 – 6ace Pg. 95 # 4 – 5ace, 10, 11ac	
9	U1L6: Equation of a Quadratic Function	**Read:** Pg. 187 – 191 **Do:** Pg. 192 # 1 – 3, 4cd, 6, 8, 9, 16	
10	U1L7: Linear/Quadratic Systems	**Read:** Pg. 194 – 198 **Do:** Pg. 198 # 1b, 3, 4ac, 6, 8, 10 – 12	
11	Review	**Read:** Pg. 38 – 39, 105 – 106, 169, 200 – 201 **Do:** Pg. 76 # 1, 2abcd, 4 – 7 Pg. 132 # 4c, 6g, 7 – 8 Pg. 202 # 4, 5, 9, 12, 13, 15–18, 20–22, 23a *#13b: change 300,000 to 30,000* *#16: one zero #18: standard form*	
12	TEST		

Unit 1, Lesson 1: Domain and Range

Domain: The domain of a relation is the complete set of possible values of the independent (x) variable.
Range: The range of a relation is the complete set of possible values of the dependent (y) variable.

Number Systems



Example 1: A diver jumps from the top of a 10 m cliff. He jumps 1 m into the air, does a front flip and then falls and hits the water 2 seconds after starting his jump.

a) Sketch a height vs. time graph for the function that models the diver's jump.

b) What is the domain and range for the function representing the diver's jump?

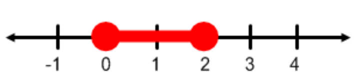
Different ways to describe domain and range

WORDS: We use words all the time, so this is a very natural way to describe domain and range.
For example: The domain is the time from 0 seconds to 2 seconds.

INEQUALITY STATEMENT: This is a more formal way of showing what we put in words.
For example: $0 \leq t \leq 2$

SET BUILDER NOTATION: The most formal mathematics way of showing domain and range.
For example: $\{ t \mid t \varepsilon \mathrm{R}, 0 \leq t \leq 2\}$

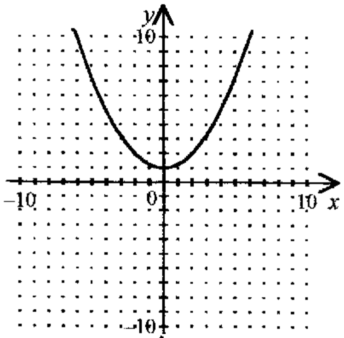
NUMBER LINE: People like seeing pictures, so we sometimes show a line.
For example:



LIST OF NUMBERS: Only use this method when we have a finite set of points so we can actually list all numbers.

Example 2:

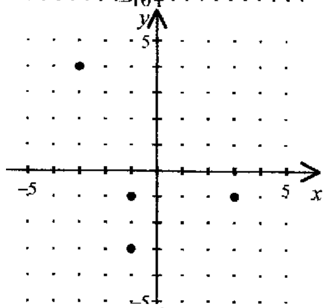
Ex.



D = ________________________________

R = ________________________________

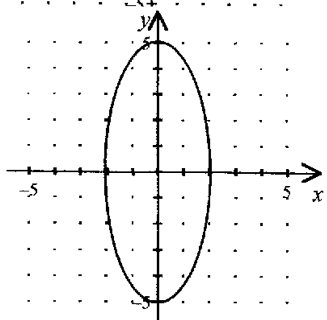
Ex.



D = ________________________________

R = ________________________________

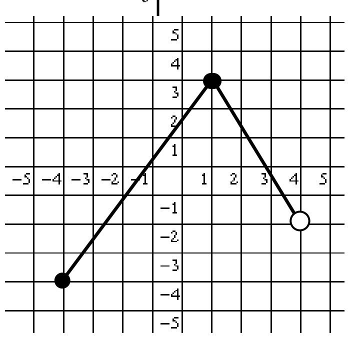
Ex.



D = ________________________________

R = ________________________________

Ex.



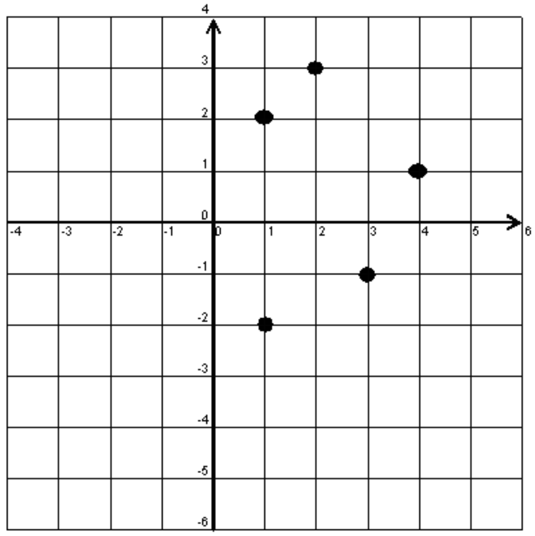
D = ________________________________

R = ________________________________

Example 3: A pool at a fitness centre is being drained. The number of kilolitres of water, N, in the pool after an elapsed time t, in minutes, is given by the formula
$N = 100 - 0.25t$. State the domain and range for this function.

State the domain and the range for each of the following graphs, using set notation.

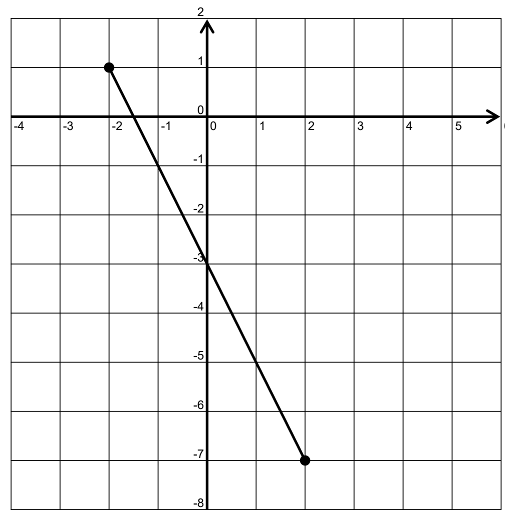
1\.



D = ___________________________

R = ___________________________

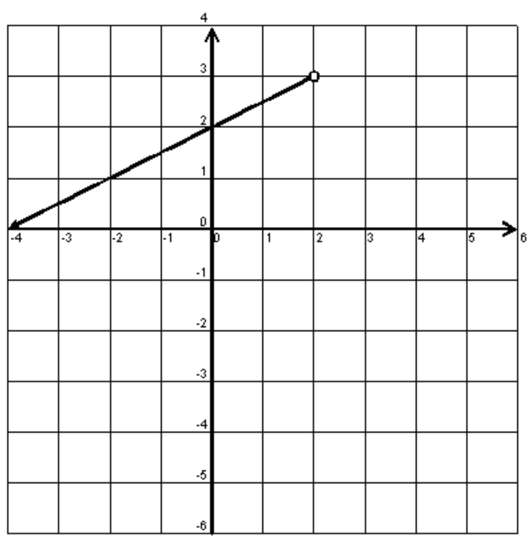
2\.



D = ___________________________

R = ___________________________

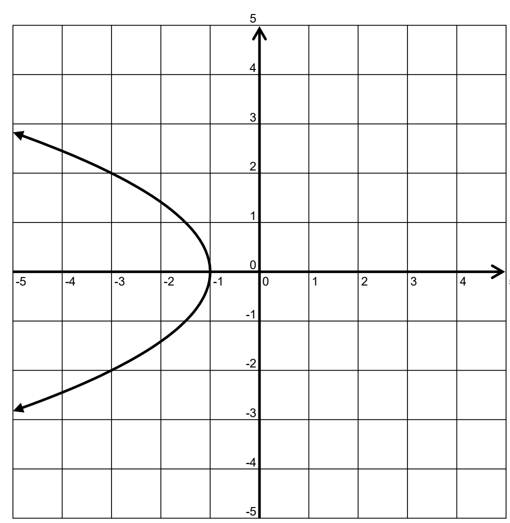
3\.



D = ___________________________

R = ___________________________

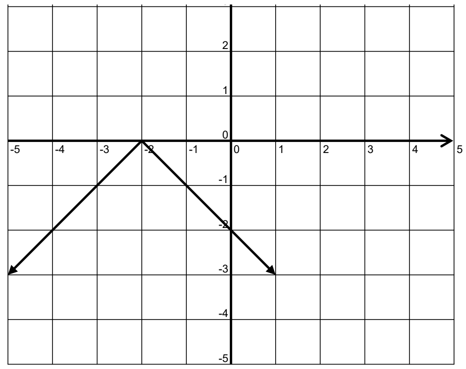
4\.



D = ___________________________

R = ___________________________

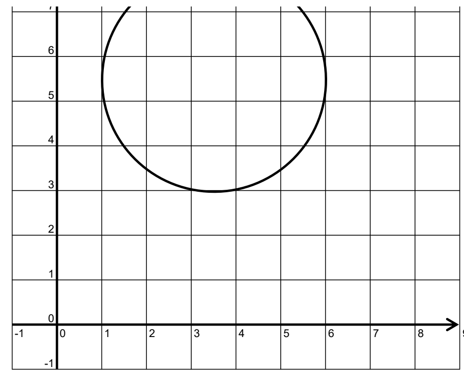
5.



D = ______________________

R = ______________________

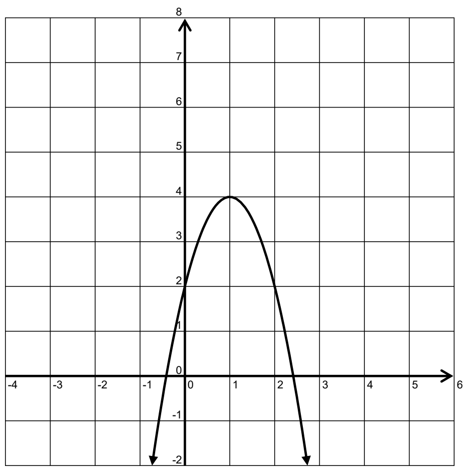
6.



D = ______________________

R = ______________________

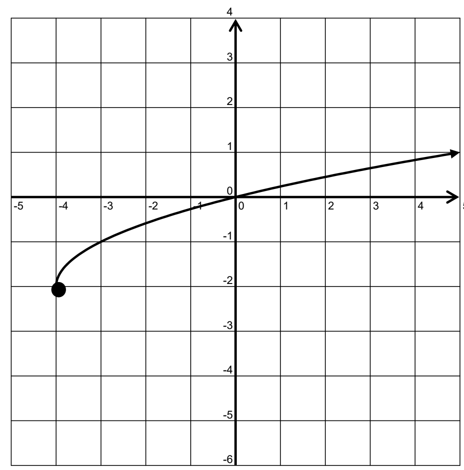
7.



D = ______________________

R = ______________________

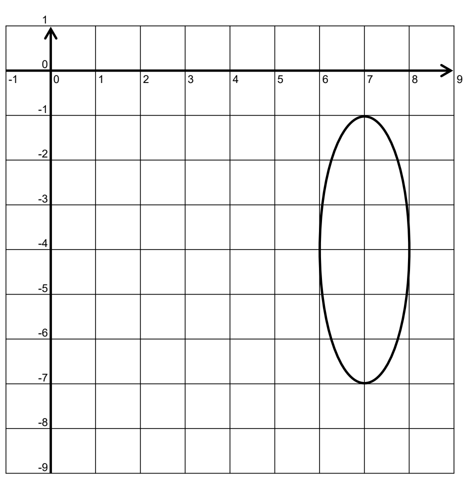
8.



D = ______________________

R = ______________________

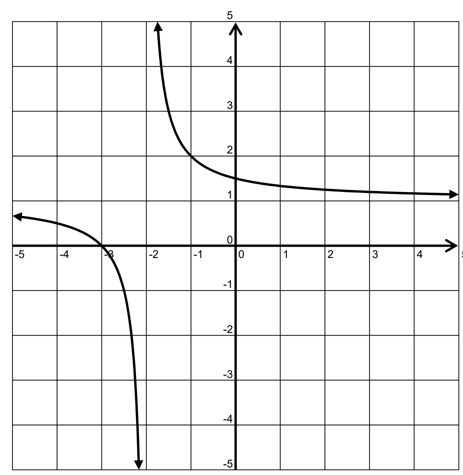
9.



D = ______________________

R = ______________________

10.



D = ______________________

R = ______________________

Unit 1, Lesson 2: Functions & Function Notation

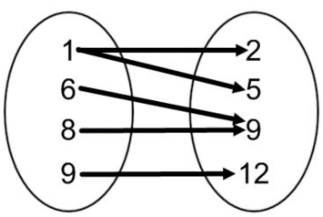
- **Relation**
  - Mapping between a domain and a range
  - Can be represented as: list of ordered pairs, mapping, table, graph, equation.
- **Function**
  - A relation where each element in the domain maps to a single element in the range
  - Given any *x* value there is only one *y* value associated with it
- **Vertical Line Test (VLT)**
  - Used to test if a graph represents a function
  - If a vertical line through any portion of the graph touches the graph more than once, the graph does not represent a function

Example: Classify each relation as a function or a non-function.

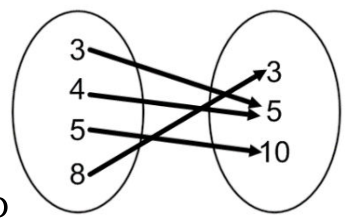
A {(1,2), (3,4), (4,5)}

B {(1,2), (1,4), (4,5)}

C



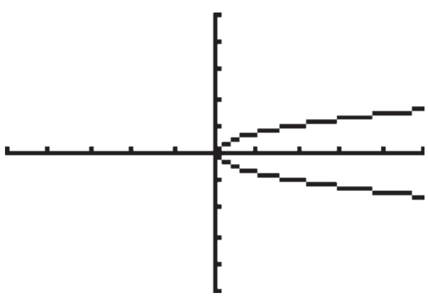
D



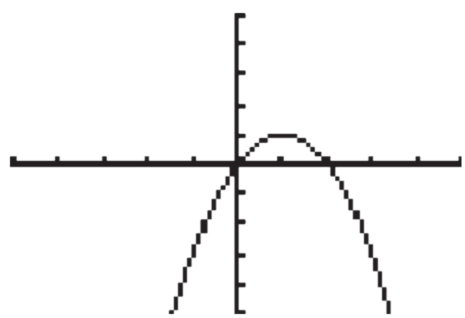
E $x^2 + y^2 = 36$

F $y = -3x^2 + 1$

G



H



- **Function Notation**

Functions can be described using function notation. The linear equation $y = -3x + 6$ is a function.
In function notation: $f(x) = -3x + 6$.

output *read "f of x" or "f at x"*

x is the input of the function
f(x) is the output of the function (**it does not mean *f* times *x***)
f is the name of the function

Example: Let $f(x) = -3x + 6$. Determine the following:

a) $f(0)$

b) $f(-4)$

c) $f(a - 1)$

d) $f(2) - f(1)$

e) $3f(5)$

Example: Let $g(x) = -2x^2 + 2x - 6$. Determine the following

a) $g(2)$

b) $g(2) + g(-1)$

c) $g(a + 5)$

Example: Using the table of values provided, determine the following:

x	$h(x)$
-5	8
-3	2
0	-1
2	9
8	4
9	4
20	0

a) $h(20)$

b) $h(-1)$

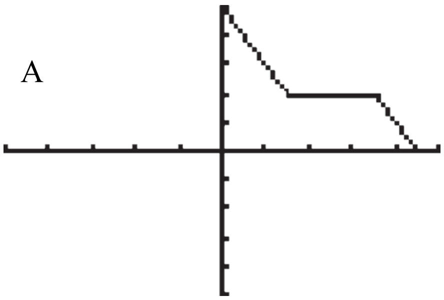
c) $h(8)$

d) value(s) for x such that $h(x) = 9$

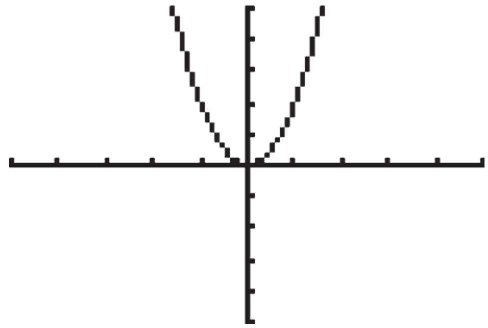
Example: Let $f(x) = x^2 + 5x - 14$. Determine value(s) for x such that $f(x) = -20$

1. Classify each as a function or a non-function.

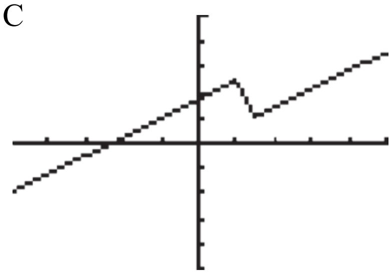
A



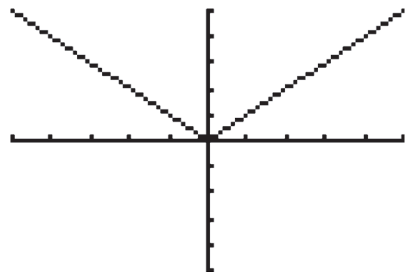
B



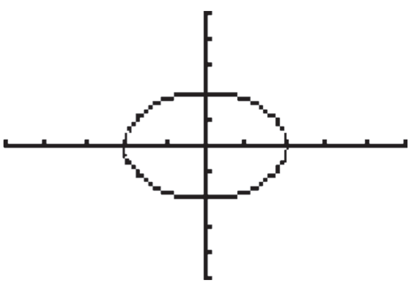
C



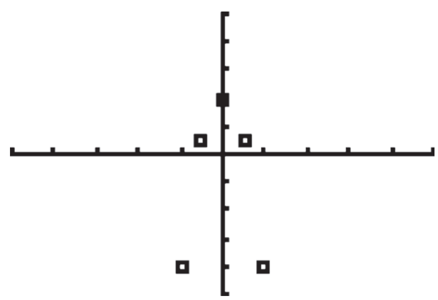
D



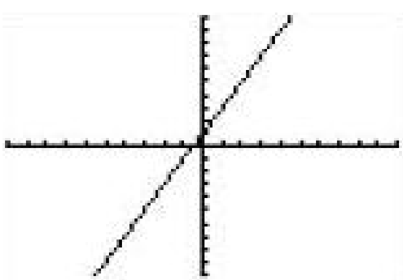
E



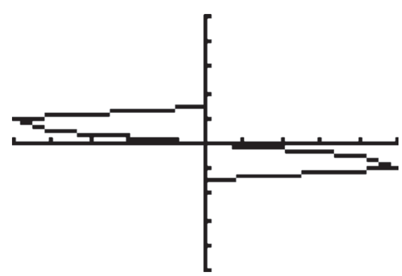
F



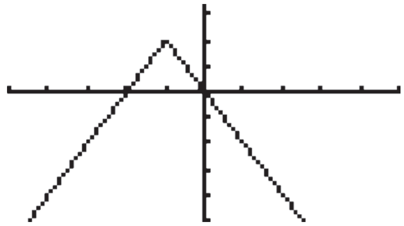
G



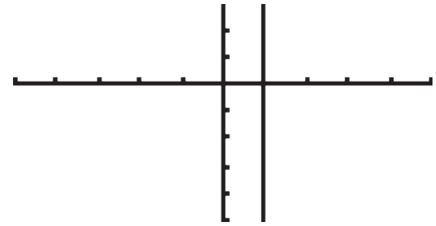
H



I



J



K $\{(0,0), (2,5), (6,10)\}$

L $\{(2,1), (1,5), (2,6)\}$

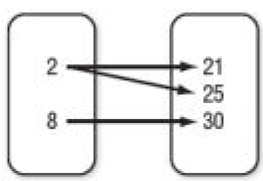
M

x	y
−3	0
−1	−1
0	0
2	−2
3	4

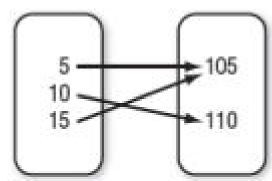
N

x	y
−2	−1
−2	1
−1	0
1	0
2	1

O



P



Q $y = 5(x-2)^2 + 8$

R $4x^2 - 3y^2 = 12$

2. Refer to yesterday's HW Handout (Domain & Range). For each graph, classify as a function or non-function

Skill Builder: Factoring MCR3U

- Factoring involves changing a polynomial relation from standard form (i.e. ____________________) to factored form (i.e. __________________________)
- Remember! **ALWAYS LOOK FOR COMMON FACTORS FIRST!**
- <u>THEN</u>, if a quadratic remains, FACTOR WHATEVER IS INSIDE THE BRACKETS, IF POSSIBLE.

1) $x^2 - 14x + 45$

2) $y^2 + 2y - 15$

3) $3x^2 - 11x - 4$

4) $9x^2 - 25$

5) $16 - t^2$

6) $4x^2 - 16$

7) $4a^2 - 12a + 9$

8) $x^2 + 4x + 4$

9) $18x^2 - 32$

10) $6x^2 + x - 12$

11) $6x^2 + 2x - 4$

Factoring by Grouping (*should have 4 terms in polynomial*)

- Separate the four terms into two groups of two or three terms (you may have to rearrange the terms)
    - For groups of two terms
        - Factor each group so that each has the same common factor remaining
        - Factor this common factor into one bracket and the remaining terms into another bracket
    - For groups of three terms
        - Factor a trinomial into two identical brackets which can then be written as $(\quad)^2$.
        - This may result in a difference of squares which can then be factored into two large brackets.
        - Simplify the two large brackets as much as possible, eliminating any brackets within.

Example – factor each expression by grouping

a) $xy + 6x + 5y + 30$

b) $2ab + 2a - 3b - 3$

c) $x^3 + x^2 + x + 1$

d) $y^2 - 4y + 4 - 16x^2$

e) $8x^4 - 18y^2 - 60y - 50$

f) $2m^2 + 10m + 10n - 2n^2$

Unit 1, Lesson 3: Max and Min of Quadratic Functions

Different forms of a quadratic function

FORM	GIVES US
Standard form: $f(x) = ax^2 + bx + c$ (Pretty but almost useless)	*Direction of opening* *Vertical stretch* *y-intercept*
Vertex form: $f(x) = a(x - h)^2 + k$ Found by completing the square	*Direction of opening* *Vertical stretch* *Vertex*
Factored form: $f(x) = a(x - r)(x - s)$ Found by factoring	*Direction of opening* *Vertical stretch* *Zeros or x-intercepts*

The **maximum** or **minimum (optimal) value** of a quadratic function is the ***y*-coordinate of the vertex**. There are a variety of strategies to determine the vertex of a quadratic function.

Method 1: Factoring to determine the zeroes & use to determine vertex

Example: $f(x) = -3x^2 - 12x + 15$

Method 2: Partial Factoring to determine the axis of symmetry (x-coordinate of the vertex), then substitute.

Example: $f(x) = 4x^2 + 10x + 3$

Method 3: Completing the square & read vertex (h, k) from equation in vertex form.

Example: $f(x) = 7x^2 - 9x - 2$

When reading word problems, pay close attention to how variables are defined.

Example: The cost function in a computer manufacturing plant is $C(x) = 0.28x^2 - 0.7x + 1$, where *C(x)* is the cost per hour in millions of dollars and x is the number of items produced per hour in thousands.
Determine the number of items that will produce the minimum cost and give the minimum production cost.

U1L3 Completing the Square Handout MCR3U

Determine the **optimal value** for each function by **completing the square** and indicate whether it is a **maximum** or **minimum**.

1. $f(x) = -0.2x^2 - 3.1x + 7.3$

Optimal Value : __________ Max / Min

2. $g(x) = -5x^2 + 4x - 6$

Optimal Value : __________ Max / Min

3. $P(x) = 2x^2 - 9x$

Optimal Value : __________ Max / Min

4. $h(t) = -t^2 + \frac{3}{4}t - \frac{1}{2}$

Optimal Value : __________ Max / Min

5. $k(x) = -\frac{2}{3}x^2 + \frac{4}{5}x - 1$

Optimal Value : __________ Max / Min

6. $R(x) = 50x^2 - 120x + 3$

Optimal Value : __________ Max / Min

1). 19.3, max 2) $-\frac{26}{5}$, max 3) $-\frac{81}{8}$, min 4) $-\frac{23}{64}$, max 5) $-\frac{19}{25}$, max 6) – 69, min

Investigation: Radicals MCR3U

Part A) Evaluate without the use of a calculator:

$\sqrt{4\times 9}=$ $\sqrt{4}\times\sqrt{9}=$

$\sqrt{4\times 16}=$ $\sqrt{4}\times\sqrt{16}=$

$\sqrt{9\times 16}=$ $\sqrt{9}\times\sqrt{16}=$

$\sqrt{4\times 25}=$ $\sqrt{4}\times\sqrt{25}=$

What do you notice? Write a rule to explain your observations.

Part B) Evaluate (round to 4 decimal places) with a calculator:

$\sqrt{5\times 5}=$ $\sqrt{5}\times\sqrt{5}=$

$\sqrt{9\times 3}=$ $\sqrt{9}\times\sqrt{3}=$

$\sqrt{2\times 10}=$ $\sqrt{2}\times\sqrt{10}=$

$\sqrt{6\times 3}=$ $\sqrt{6}\times\sqrt{3}=$

Does the rule you created in part A still work?

Part C) Without a calculator, write each radical in a different way. (Notice the similarity to the numbers from part B)

$\sqrt{27}$

$\sqrt{20}$

$\sqrt{18}$

Skill Builder: Radicals MCR3U

The **product property** of radicals states:

$$\text{For } a \geq 0, b \geq 0; \sqrt{a}\sqrt{b} = \sqrt{ab}$$

We can use this property to simplify & perform operations with radicals

Simplifying Radicals

1. Find **2** factors, one of which is a perfect square (highest perfect square possible).
2. Rewrite as two radicals. (First radical must be the perfect square.)
3. Evaluate the perfect square.

*** If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further. ***

Practice: Simplify

a) $\sqrt{12}$ b) $2\sqrt{24}$ c) $\sqrt{32}$ d) $-\sqrt{8}$

Multiplying Radicals

- Outside times outside, stays outside.
- Inside times inside, stays inside.
- Simplify radical

a) $2\sqrt{2} \times 3\sqrt{7}$ b) $5\sqrt{6} \times \sqrt{5}$ c) $-8\sqrt{10} \times 2\sqrt{2}$

Dividing Radicals

- Outside divided by outside, stays outside.
- Inside divided by inside, stays inside.
- Simplify radical.

a) $\dfrac{2\sqrt{15}}{\sqrt{3}}$ b) $\dfrac{\sqrt{24}}{\sqrt{2}}$

Adding/Subtracting Radicals

- You can only add or subtract like radicals (think algebra: like terms)
- Reduce if needed, then collect like radicals

Simplify any individual radical terms first.

Example: $3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$

Practice: Simplify

a) $3\sqrt{11} + 2\sqrt{11}$

b) $5\sqrt{8} - 3\sqrt{18}$

Multiplying Radical Expressions

To multiply radical expressions use the distributive law and simplify where possible.

Example: $3\sqrt{3}\left(4 - 2\sqrt{8}\right)$

Example: $\left(2 + 3\sqrt{5}\right)\left(3 - 2\sqrt{6}\right)$

Example: $\left(7 + 2\sqrt{6}\right)\left(6 - \sqrt{6}\right)$

Unit 1, Lesson 4: Solving Quadratic Equations

Quadratic Equation: An equation of the form $ax^2 + bx + c = 0$, where a, b, c ε $\Re$ and $a \neq 0$.

The **solution** to a quadratic equation is also called the **roots** of the equation.

There are 3 methods to solve a quadratic equation.

Method 1: Inverse Operations

Use this method when there is a single *x*-term (vertex form)

- Use inverse operations to isolate x
- When you take the square root, recall that there should be 2 answers.
- Leave answer in simplified radical form, unless specified otherwise.

Example: Solve $2(x-9)^2 - 19 = 5$

Method 2: Factoring

Try this method before resorting to method #3

- Rearrange equation so it is in standard form $ax^2 + bx + c = 0$
- Factor, if possible.
- Set each factor to 0 and solve each linear equation.

Example: Determine the roots of $x(4x-5) = 6$

Method 3: Quadratic Formula

Use this method when the equation is not factorable.

- Rearrange equation so it is in standard form $ax^2 + bx + c = 0$
- Substitute *a*, *b*, *c* into the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Leave answer in simplified radical form, unless specified otherwise.

Example: Solve $2x(2x+3)+6=5$

Example: The Profit function for a business is modelled by the equation $P(x) = -0.5x^2 + 10x - 16$, where x is the number of items sold in thousands, and $P(x)$ is the profit in thousands of dollars. Determine the number of items the company must sell in order to break even.

Unit 1, Lesson 5: Zeros of a Quadratic Function

How to Determine the Number of Zeros (*x*-intercepts)

Factored form: $f(x) = a(x-r)(x-s)$

- The number of zeros will be equivalent to the number of **unique factors**.
- If there are no zeros, the equation cannot be written in factored form.

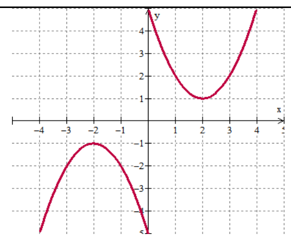
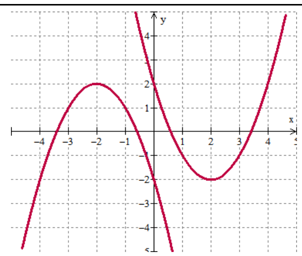
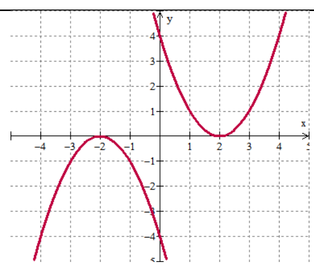
Example 1: Without drawing the graph, find the number of zeros of the following functions.

a) $f(x) = 0.4(x-1)(x+2)$

b) $g(x) = 3(x+5)^2$

c) $h(x) = 4x(x-2)$

Vertex form: $f(x) = a(x-h)^2 + k$

	if a & k have the **same sign** $a > 0$ & $k > 0$ or $a < 0$ & $k < 0$ **no real zeros**
	if a & k have **opposite signs** $a > 0$ & $k < 0$ or $a < 0$ & $k > 0$ **two real zeros**
	if $\boldsymbol{k = 0}$ **one real zero**

Example 2: Without drawing the graph, find the number of zeros of the following functions.

a) $f(x) = 1.3(x-4)^2 + 2.2$

b) $g(x) = 1.7(x+2)^2 - 4.5$

c) $h(x) = 3(x-2.4)^2$

Standard form: $f(x) = ax^2 + bx + c$

- Calculate the **Discriminant** → $\mathbf{b^2 - 4ac}$ (from the quadratic formula)
  - if $b^2 - 4ac > 0 \rightarrow$ 2 real zeros
  - if $b^2 - 4ac = 0 \rightarrow$ 1 real zero
  - if $b^2 - 4ac < 0 \rightarrow$ no real zeros (2 complex zeros)

Example 3: Determine the **number** of zeros.

a) $f(x) = x^2 - 8x + 16$ **b)** $g(x) = 3x^2 + 2x + 4$

Example 4: For what values of k does the equation $2x^2 + kx + 8 = 0$ have

a) two distinct, real roots?

b) one real root?

c) no real roots?

Try: Determine the value(s) of k such that the function $f(x) = 3x^2 + kx - 3 + k$ has exactly one zero.

Skill Builder: Operations with Polynomials MCR3U

- **Polynomial** – numerical coefficients are real numbers, exponents are non-negative integers
  - Monomial – one term
  - Binomial – two terms
  - Trinomial – three terms

- **Degree** of a polynomial is the value of the highest exponent
  - Polynomial of degree 0 is called a constant
  - Polynomial of degree 1 is called a linear expression
  - Polynomial of degree 2 is called a quadratic expression
  - Polynomial of degree 3 is called a cubic expression
  - Polynomial of degree 4 is called a quartic expression

Adding and Subtracting Polynomials

- To **add or subtract** polynomials, combine like terms.
- Remember that if you are subtracting a polynomial, you must subtract **each term** of the polynomial.

Ex 1) Simplify $(-2x^2 + 5x - 3) + (x^2 - 6x + 1) - (-3x^2 - 2x - 4)$

Multiplying Polynomials

- To **multiply** (or **expand**) polynomials, use the **distributive property –** multiply each term inside the bracket by the number/term outside of the brackets.
  - When a polynomial is multiplied by another polynomial, this means that **every term** in the first polynomial is multiplied by **every term** in the second polynomial.

$$(2x + 3y + 4z)^2 = (2x + 3y + 4z)(2x + 3y + 4z)$$

- After applying the distributive property don't forget to **collect like terms**!

Ex 2) **Expand** $(2y-5)(3y^2+4y-6)$

Ex 3) **Expand** $(2x^2-3x+1)(4x^2+5x-6)$

Ex 4) **Expand** $(2x-5)^3$

Unit 1, Lesson 6: Equation of a Quadratic Function

Investigate:

1. Using graphing software, graph each of the following quadratic functions. How are the graphs the same? How are they different?

$f(x) = x^2 - 3x - 10$
$g(x) = -2x^2 + 6x + 20$
$h(x) = 4x^2 - 12x - 40$
$k(x) = -0.5x^2 + 1.5x + 5$

2. Write each function in factored form. What do you notice?

3. This group of functions forms a **family of quadratic functions**. What is the **common characteristic** of this family?

A **Family of Parabolas** is a group of parabolas that share a common characteristic

Vertex Form: Where **'a'** is varied, this results in a family of parabolas with the same vertex and axis of symmetry

Factored Form: Where **'a'** is varied, this results in a family of parabolas with the same *x*-intercepts and axis of symmetry

Standard Form: Where **'a'** and **'b'** are varied, this results in a family of parabolas with the same *y*-intercept

Example: Write the equation (in standard form) of the quadratic function that passes through the point (2, -9), if the roots of the corresponding quadratic equation are 5 and -7.

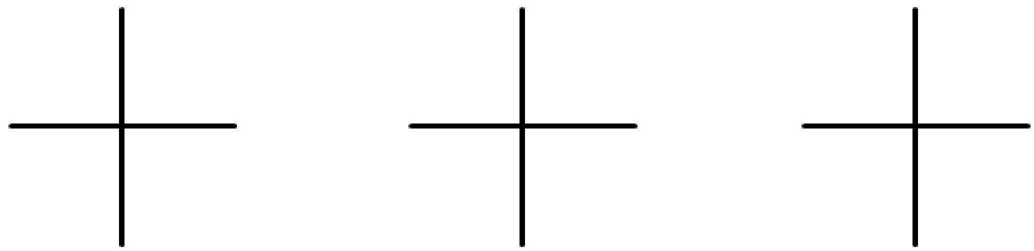
Recall: Expand $(5-6\sqrt{3})(5+6\sqrt{3})$

Example: Write the equation (in standard form) of the quadratic function that passes through the point (-1, 3), if the roots of the corresponding quadratic equation are $5 \pm 3\sqrt{2}$.

Unit 1, Lesson 7: Linear/Quadratic Systems

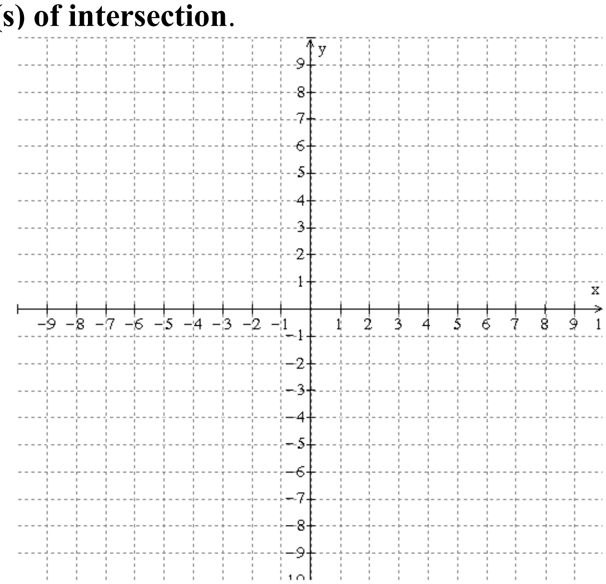
Recall: A **linear system** involves 2 linear functions with the same independent and dependent variables. The solution of the linear system is the point of intersection (POI) of the 2 lines. Linear systems can be solved graphically or algebraically (substitution or elimination).

A **linear-quadratic system** involves one **linear function**, and one **quadratic function**. The solution of the system is the point(s) of intersection of the 2 functions. There may be 0, 1 or 2 solutions.



Solving a linear-quadratic system can be done GRAPHICALLY or ALGEBRAICALLY
GRAPHICALLY – graph each function and identify the **point(s) of intersection**.
ALGEBRAICALLY – solve the system using **substitution**

Example: Given $p(x) = x^2 - 4x$ and $q(x) = 2x - 5$, graph to find the point(s) of intersection.



Example: Given $g(x) = 2x - 2$ and $f(x) = x^2 - 3x + 2$, determine the point(s) of intersection algebraically.

1. Isolate y in the linear equation
2. Sub. into the quadratic equation
3. Solve the quadratic (factor or quadratic formula)
4. Sub. **each** x-value back into the line to get y

We can also solve problems involving linear and quadratic functions.

Example: A skydiver jumped from an airplane and fell freely for several seconds before releasing her parachute. Her height in metres, above the ground t seconds after jumping out is given by $\boldsymbol{h_1(t) = -4.9t^2 + 5000}$ before she released her parachute, and $\boldsymbol{h_2(t) = -4t + 4000}$ after she released the parachute. How long after jumping did she release her parachute? How high was she above the ground at that time?

Example: Determine the equations of the lines that have a slope of 2 and that intersect the quadratic function $f(x) = x(6 - x)$ once; twice; never.

Hint for #6 – Breakeven point for a business will happen when the revenue is equal to the cost.