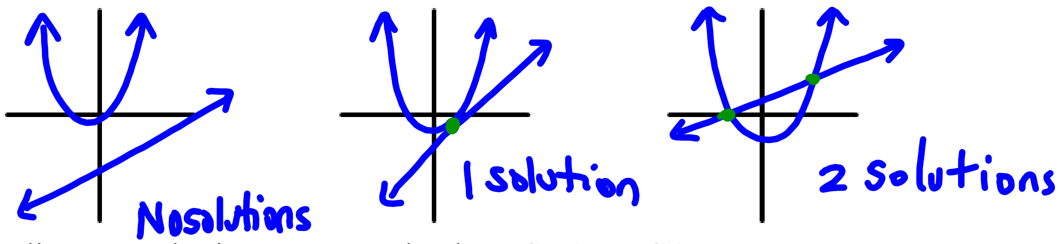
Lesson 7: Linear-Quadratic Systems

Recall: A **linear system** involves 2 linear functions with the same independent and dependent variables. The solution of the linear system is the point of intersection (POI) of the 2 lines. Linear systems can be solved graphically or algebraically (substitution or elimination).

A **linear-quadratic system** involves one **linear function**, and one **quadratic function**. The solution of the system is the point(s) of intersection of the 2 functions. There may be 0, 1 or 2 solutions



Solving a linear-quadratic system can be done GRAPHICALLY or ALGEBRAICALLY

GRAPHICALLY – graph each function and identify the **point(s) of intersection**.
ALGEBRAICALLY – solve the system using **substitution**

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$y = mx + b$

Example: Given $p(x) = x^2 - 4x$ and $q(x) = 2x - 5$, graph to find the point(s) of intersection.

① $0 = x^2 - 4x$

$0 = x(x - 4)$

zeros : 0 & 4

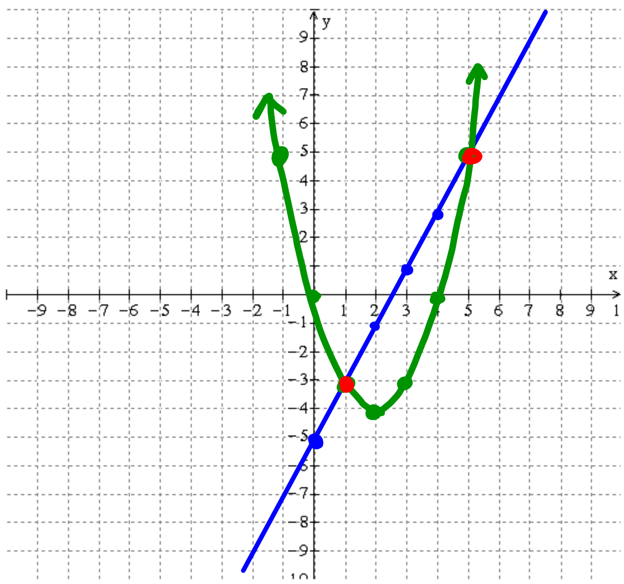
② AOS = 2

③ plug $x = 2$ in :

$p(2) = (2)^2 - 4(2)$

$p(2) = 4 - 8$

$p(2) = -4$



∴ The solutions (POI) are : $(1, -3)$, $(5, 5)$.

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Example: Given $g(x) = 2x - 2$ and $f(x) = x^2 - 3x + 2$, determine the point(s) of intersection algebraically.

1. Isolate y in the linear equation
2. Sub. into the quadratic equation
3. Solve the quadratic (factor or quadratic formula)
4. Sub.each x-value back into the line to get y

① $y = 2x - 2$

② $y = x^2 - 3x + 2$

set ① equal to ② :

$$2x - 2 = x^2 - 3x + 2$$

$$0 = x^2 - 5x + 4$$

$$0 = (x - 1)(x - 4)$$

$$x = 1 \qquad x = 4$$

plug x=1 into solve for y:

$g(1) = 2(1) - 2$

$g(1) = 0$ $(1, 0)$

plug x=4 into solve for y:

$g(4) = 2(4) - 2$

$g(4) = 6$ $(4, 6)$

∴ the solutions (PoI) are $(1,0)$ & $(4,6)$.

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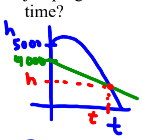
We can also solve problems involving linear and quadratic functions.

Example: A skydiver jumped from an airplane and fell freely for several seconds before releasing her parachute. Her height in metres, above the ground t seconds after jumping out is given by $\boldsymbol{h_1(t) = -4.9t^2 + 5000}$ before she released her parachute, and $\boldsymbol{h_2(t) = -4t + 4000}$ after she released the parachute. How long after jumping did she release her parachute? How high was she above the ground at that time?



$$h_1(t) = h_2(t)$$

$$-4.9t^2 + 5000 = -4t + 4000$$

$$0 = 4.9t^2 - 4t - 1000$$

Quadratic Formula:

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \mp \sqrt{(-4)^2 - 4(4.9)(-1000)}}{2(4.9)}$$

$$x = \frac{4 \mp \sqrt{19616}}{9.8}$$

$$x = \frac{4 + \sqrt{19616}}{9.8} \qquad x = \frac{4 - \sqrt{19616}}{9.8}$$

$$\boxed{x = 14.70} \qquad x = \cancel{-13.88}$$ outside the domain

Now plug x=14.70 in to solve for the height:

$h_2(14.70) = -4(14.70) + 4000$

$h_2(14.70) = 3941.2$

∴ she pulled the parachute at 14.70 seconds and the height was 3941.2 m.

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Example: Determine the equations of the lines that have a slope of 2 and that intersects the quadratic function $f(x) = x(6 - x)$ once; twice; never.

$g(x) = 2x + b \qquad f(x) = -x^2 + 6x$

$2x + b = -x^2 + 6x$

$x^2 - 4x + b = 0$

ONE ROOT

$0 = b^2 - 4ac$

$0 = (-4)^2 - 4(1)(b)$

$0 = 16 - 4b$

$-16 = -4b$

$\frac{-16}{-4} = b$

$\boxed{4 = b}$

2 ROOTS

$0 < b^2 - 4ac$

$0 < 16 - 4b$

$-16 < -4b$

$\frac{-16}{-4} > b$ Flip it!

$\boxed{4 > b}$

NO ROOTS

$0 > b^2 - 4ac$

$0 > 16 - 4b$

$-16 > -4b$

$\frac{-16}{-4} < b$ flip it!

$\boxed{4 < b}$

1 root

4

$\boxed{g(x) = 2x + 4}$

$g(x) = 2x + 3$
$g(x) = 2x + 2$
...

$g(x) = 2x + 5$
$g(x) = 2x + 6$
...

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HW U1L7:

1. p. 198 #1b, 3, 4ac, 8, 10-12

Hint for #6 – Breakeven point for a business will happen when the revenue is equal to the cost.