Lesson 4: Annuities: Future & Present Value

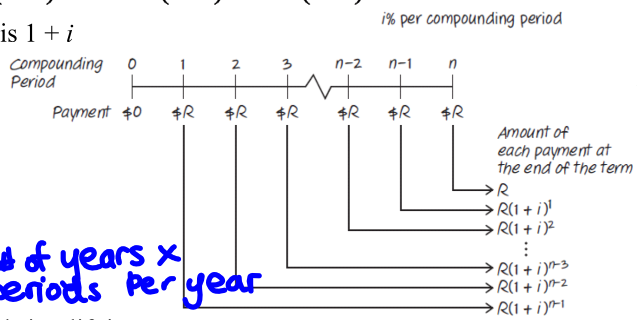
An **annuity** is a series of payments at regular intervals. It can be a loan or an investment.

The **future value** of an annuity is the final amount at the end of the term of the annuity. The future value includes all of the periodic payments and the compound interest. The future value is the geometric series:

$$FV = R + R(1+i) + R(1+i)^2 + \ldots + R(1+i)^{n-2} + R(1+i)^{n-1}$$

Note that the common ratio is $1 + i$

We use the formula for the sum of a geometric series,



with: $S_n = \frac{a(r^n - 1)}{r - 1}$

$a = R$ (the regular payment)

$r = 1 + i$

# of years × periods per year

Substituting these values and simplifying, we get:

$$FV = \frac{R\left((1+i)^n - 1\right)}{i}$$

annual rate / # of periods per year

where R is the ***regular payment***

i is the interest rate ***per conversion period***

n is the # ***of payments***

Jan 31-9:35 PM

Ex 1) \$750 is deposited at the end of every 3 months for 5 years at 3.25%/a, compounded quarterly. Find the value of the annuity on the date of the last payment.

→ 0.0325

$R = 750$

$i = \frac{0.0325}{4}$

$i = 0.008125$

$n = 5 \times 4$

$n = 20$

$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$

$FV = \frac{750\left[(1+0.008125)^{20} - 1\right]}{0.008125}$

$\boxed{FV = 16216.26\ \$}$

Jun 2-1:30 PM

Ex 2) Jenni wants to save up money to buy a house. She knows she will need \$20,000 for a down payment. How much should she put aside each month, at 4.5%/a interest, compounded monthly, if she has 5 years to save up?

$i = \frac{0.045}{12}$

$i = 0.00375$

$n = 5 \times 12$

$n = 60$

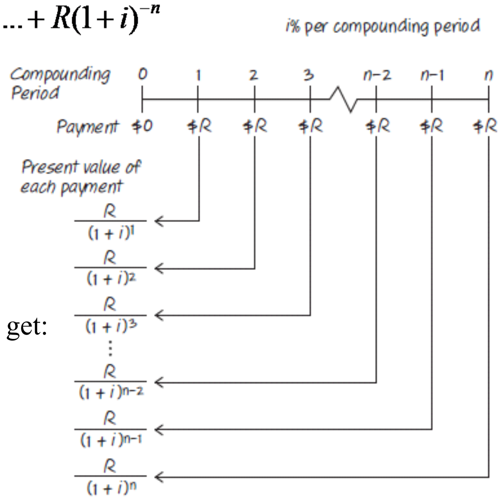
$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$20\,000 = \frac{R\left[(1+0.00375)^{60} - 1\right]}{0.00375}$$

$$\boxed{\$297.86 = R}$$

The **present value** of an annuity is the value of the annuity at the beginning of the term. It is the sum of all present values of the payments and can be written as the geometric series

$$PV = R(1+i)^{-1} + R(1+i)^{-2} + R(1+i)^{-3} + \ldots + R(1+i)^{-n}$$



Note that the common ratio is $(1+i)^{-1}$

We use the formula for the sum of a geometric series, $S_n = \frac{a(r^n - 1)}{r - 1}$

with:

$a = R\ (1+i)^{-1}$ *and* $r = (1+i)^{-1}$

Substituting these values & simplifying, we get:

$$PV = \frac{R(1+i)^{-1}\left((1+i)^{-n} - 1\right)}{(1+i)^{-1} - 1} \times \frac{1+i}{1+i}$$

$$PV = \frac{R\left((1+i)^{-n} - 1\right)}{1 - (1+i)}$$

$$PV = \frac{R\left(1 - (1+i)^{-n}\right)}{i}$$

where R is the *regular payment*

i is the interest rate ***per conversion period***

n is the # ***of payments***

Jun 2-1:31 PM

Ex 3) Victor is looking to buy a car. He determines that he can afford payments of at most $400 per month, and he is hoping to pay off the car in 4 years. He can get a loan from his bank for 6.75%, compounded monthly. What should he set as his budget for car shopping?

$R = 400$

$i = \frac{0.0675}{12}$

$i = 0.005625$

$n = 4 \times 12$

$n = 48$

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$PV = \frac{400\left[1-(1+0.005625)^{-48}\right]}{0.005625}$$

$$\boxed{PV = 16\,785.26\ \$}$$

Jun 2-1:38 PM

Ex 4) Jenni manages to save up her $20,000 for her down payment. But she decides to buy a new boat instead. She finds a small cabin cruiser advertised at $105,000. The bank approves her loan at an interest rate of 5.25%, compounded monthly. If she wants to pay off her boat in 10 years, what should her monthly payments be? How much interest will she pay over the life of the loan?

$PV = 85\,000 \leftarrow (105\,000 - 20\,000)$

$R = ?$

$i = \frac{0.0525}{12}$

$i = 0.004375$

$n = 120$

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$85\,000 = \frac{R\left[1-(1+0.004375)^{-120}\right]}{0.004375}$$

$$\boxed{R = 911.98\ \$}$$

$911.98 \times 120 = 109\,437.60\ \$$

$109\,437.60 - 85\,000 = 24\,437.60\ \$$

HW U6L4:

1. p.511 # 5ac, 6, 7, 9
2. p.520 # 3bd, 4, 6, 7, 9
3. study for unit test on Wednesday