Investigation: Rational Exponents

Consider the following pattern:

A. Fill in the blanks based off of the examples. Then answer the questions to the right.

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$2^3 = \underline{2} \cdot \underline{2} \cdot \underline{2} = \underline{8}$

$2^2 = \underline{2} \cdot \underline{2} = \underline{4}$

$2^1 = \underline{2}$

$2^{\frac{1}{2}}$ →

$2^{\frac{1}{3}}$ ↗

$2^0 = \underline{1}$

$2^{-1} = \underline{\frac{1}{2}}$

$2^{-2} = \frac{1}{2^2} = \frac{1}{2} \cdot \frac{1}{2} = \underline{\frac{1}{4}}$

$2^{-3} = \frac{1}{2^3} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

$2^{-4} = \frac{1}{2^4} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

1. With the number system in mind, what type of exponents is being used?

integer (whole number)

2. What is the most specific number classification for the results? (Final products)

rational (fraction)

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B. Now consider $2^{\frac{1}{2}}$. Where would this fit in the pattern above? Draw an arrow where you think $2^{\frac{1}{2}}$ should be placed. What do you think the value will be?

My estimated value of $2^{\frac{1}{2}}$: 1.5

- Now enter $2^{\frac{1}{2}}$ in your calculator. Record the value below. What is the most specific number classification for the result?

1.41421... irrational

- What is another key sequence on your calculator to find $2^{\frac{1}{2}}$? [redacted] $\sqrt{2}$

C. Now consider $2^{\frac{1}{3}}$. Where would this fit in the pattern above? Draw an arrow where you think $2^{\frac{1}{3}}$ should be placed. What do you think the value will be?

My estimated value of $2^{\frac{1}{3}}$: 1.2 Calculator value: 1.25992... Number classification: irrational

- What is another key sequence on your calculator to find $2^{\frac{1}{3}}$? [redacted] $\sqrt[3]{2}$

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D. Evaluate the following using your calculator:

$36^{\frac{1}{2}} = 6$ $81^{\frac{1}{2}} = 9$

$64^{\frac{1}{2}} = 8$ $144^{\frac{1}{2}} = 12$

$25^{\frac{1}{2}} = 5$

index, radical sign, radicand: $\sqrt[n]{a}$

Write a statement about what the exponent $\frac{1}{2}$ represents.

Try to write this symbolically in *radical form*: $a^{\frac{1}{2}} = \sqrt{a}$

E. Based on your observations from part D, try to evaluate the following **without** your calculator.

$8^{\frac{1}{3}} = 2$ $(2)(2)(2) = 8$

$27^{\frac{1}{3}} = 3$

$1000^{\frac{1}{3}} = 10$

$125^{\frac{1}{3}} = 5$

Write a statement about what the exponent $\frac{1}{3}$ represents?

Try to write this symbolically in *radical form*: $a^{\frac{1}{3}} = \sqrt[3]{a}$

F. Look back at parts D and E to complete the following symbolic rule in *radical form*:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

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G. Another way of understanding this rule:

Evaluate $\left(4^{\frac{1}{2}}\right)\left(4^{\frac{1}{2}}\right)$ using product rule

$4^{\frac{1}{2}+\frac{1}{2}} = 4^1$

Evaluate $\left(\sqrt{4}\right)\left(\sqrt{4}\right)$

$(2)(2) = 4$

Evaluate $\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)$ using product rule

$8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8^1$

Evaluate $\left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right)$

$(2)(2)(2) = 8$

What do you notice?

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G. Another way of understanding this rule:

Evaluate $\left(4^{\frac{1}{2}}\right)\left(4^{\frac{1}{2}}\right)$ using product rule

$4^{\frac{1}{2}+\frac{1}{2}} = 4^1$

$= 4$

Evaluate $\left(\sqrt{4}\right)\left(\sqrt{4}\right) = (2)(2)$

$= 4$

Evaluate $\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)$ using product rule

$8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8^1$

$= 8$

Evaluate $\left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right)\left(\sqrt[3]{8}\right) = (2)(2)(2)$

$= 8$

What do you notice?

$4^{\frac{1}{2}} = \sqrt{4}$

$8^{\frac{1}{3}} = \sqrt[3]{8}$

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Lesson 3: Working with Rational Exponents

Rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$, means the n^{th} root of x

Ex 1) Evaluate

a) $49^{\frac{1}{2}}$

$= \sqrt{49}$

$= 7$

b) $(-64)^{\frac{1}{3}}$

$= \sqrt[3]{(-64)}$

$= -4$

c) $8^{-\frac{1}{3}}$

$= \left(\frac{1}{8}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{1}{8}}$

$= \frac{1}{2}$

or

$= \frac{1}{\sqrt[3]{8}}$

$= \frac{1}{2}$

d) $\left(\frac{1}{36}\right)^{\frac{1}{2}}$

$= \frac{1}{\sqrt{36}}$

$= \frac{1}{6}$

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Rule: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$, means the n^{th} root of the m^{th} power of x

Ex 2) Evaluate. Write in radical form first.

a) $8^{\frac{2}{3}}$

$= \sqrt[3]{8}^2$

$= 2^2$

$= 4$

b) $-25^{\frac{5}{2}}$

$= -\sqrt{25}^5$

$= -5^5$

$= -3125$

c) $81^{-\frac{3}{4}}$

$= \left(\frac{1}{81}\right)^{\frac{3}{4}}$

$= \sqrt[4]{\frac{1}{81}}^3$

$= \left(\frac{1}{3}\right)^3$

$= \frac{1}{27}$

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Ex 2) Evaluate. Write in radical form first.

d) $16^{0.75}$

$= 16^{\frac{3}{4}}$

$= \sqrt[4]{16}^3$

$= 2^3$

$= 8$

e) $\left(-\frac{1}{64}\right)^{\frac{2}{3}}$

$= \sqrt[3]{-\frac{1}{64}}^2$

$= \left(-\frac{1}{4}\right)^2$

$= \frac{1}{16}$

f) $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

$= \sqrt{\frac{4}{9}}^3$

$= \left(\frac{2}{3}\right)^3$

$= \frac{8}{27}$

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Ex 3) Evaluate, no decimals:

$128^{-\frac{5}{7}} - 16^{0.75}$

$= \left(\frac{1}{128}\right)^{\frac{5}{7}} - 16^{\frac{3}{4}}$

$= \sqrt[7]{\left(\frac{1}{128}\right)^5} - \sqrt[4]{16^3}$

$= \left(\frac{1}{2}\right)^5 - 2^3$

$= \frac{1}{32} - 8$

$= \frac{1}{32} - \frac{256}{32}$

$= \frac{-255}{32}$

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Ex 4) **Simplify**, then evaluate to 4 decimal places.

$\left(3^{-\frac{4}{5}}\right)\left(3^{\frac{1}{15}}\right) \div \left(3^{\frac{2}{3}}\right)$

$= 3^{-\frac{4}{5} + \frac{1}{15} - \frac{2}{3}}$

$= 3^{-\frac{12}{15} + \frac{1}{15} - \frac{10}{15}}$

$= 3^{-\frac{21}{15}}$

$= 3^{-\frac{7}{5}}$

$= 0.21479\ldots$

$= 0.2148$

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Ex 5) Express as a single, positive power, then evaluate.

$\left(\sqrt[3]{27}\right)\left(\sqrt[4]{81}\right)^3$

$= (3^1)(3)^3$

$= 3^4$

$= 81$

Do ALL homework questions without a calculator, unless specified otherwise.

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HW U2L3:

1. p. 229 #4-6ace,8-11,12ace,14,15a

(calculator permitted for #8,9,12

2. study for quiz (Day 2/3/4/5)

↳ Thursday

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