Unit 5 – Applications of Functions

Day	Lesson	Practice Questions	Struggles?
1	U5L1 – Solving Problems with Quadratic Functions	**Read:** Pg. 148 – 153, 172 – 177 **Do:** Pg. 153 # 5bd, 7ac, 12 Pg. 178 # 9 – 12, 14	
2	U5L2 – Solving Problems with Exponential Functions (Growth & Decay)	**Read:** Pg. 254 – 260 **Do:** Pg. 261 # 2, 3/4a-c, 6, 9*(use a graph for c)*, 10, 11 a-d, 12, 15	
3	U5L3 – Solving Problems with Exponential Functions (Compound Interest)	**Read:** 483 – 489, 493-497 **Do:** Pg. 490 # 4bde, 5, 7, 10, 14 Pg. 498 # 3b, 4 – 6, 8, 9	
4	U5L4 – Interpreting Sinusoidal Functions	**Read:** Pg. 346 – 351, Pg. 365 – 369 **Do:** Pg. 355 # 8efg, 11 Pg. 370 # 1, 4, 6, 8, 13 ***1e) Yes. It's 14m away, 6d) graph is wrong, 8a) y-axis is wrong (should be min = 0, max = 52)*	
5	U5L5 – Solving Problems with Sinusoidal Functions	**Read:** Pg. 394 – 398 **Do:** Pg. 398 # 1, 3, 4, 6, 8, 9 *1d) d=0.5sin(180t)+1.5, 9a) -30cos(1.43x)+40*	
6	U5L6 – Solving Problems with Functions	**Do:** **Handout**	
7-9	U5L7 – Modelling Data with Functions	**Read:** Pg. 389-390 **Do:** Pg. 392 #5, 8, 9, 10 **Handouts**	
10	Review	**Read:** Pg. 374-375 **Do:** Pg. 170 # 8 Pg. 203 # 14 Pg. 269 # 13 – 15, 16ac, 17 Pg. 404 # 4, 6, 11, 13	
11	TEST		

Unit 5, Lesson 1: Solving Problems with Quadratic Functions

When solving problems involving quadratic functions, we are either interested in finding the optimum value (maximum or minimum) for a situation, or finding the value of the independent variable that produces a particular output (solving an equation). You must pay careful attention to which type of question is being asked.

- Is the question asking for a minimum, maximum, smallest or largest possible value?
  - **Find the vertex** by completing the square, or factoring to determine the axis of symmetry
- Does the question give you enough information to create one equation with a single variable, or two equations with 2 variables (*linear/quadratic systems*)?
  - **Solve the equation(s)** by factoring, quadratic formula, or inverse operations (if possible!)
  - Quadratic equations often have 2 roots, consider if you need both in your solution.

You will need to ***create equations*** to represent the situation.

- Pay careful attention to how you define your variables!
- Make sure you write a therefore statement to answer the question being asked.

Example 1: Phil and Shelly are playing a number game. Shelly says, "I am thinking of two numbers that add to 17 and the sum of their squares is 185." What are the two numbers?

When defining your variable, always consider if you can successfully create an equation with that variable.

Example 2: Research for a given orchard has shown that if 100 pear trees are planted, then the annual profit is $91.50 per tree. If more trees are planted they have less room to grow and generate fewer pears per tree. As a result, the annual profit per tree is reduced by $0.75 for each additional tree planted. How many pear trees should be planted to maximize the profit from the orchard for one year?

For questions involving "Uniform width", always ***draw a picture*** and define your independent variable to be the uniform width.

Example 3: A rectangular pool measures 10m by 5m. A deck, of uniform width, is to be built all the way around the pool such that the total area of the pool and deck will be 126 m^2. Algebraically determine the width of the deck.

Many economic problems are based on the **demand function**, and how it relates to **revenue** and **profits**.

Demand Function	$p(x)$	**Price** of a commodity as a function of the number of items sold (the # of items people are willing to purchase at the price)
Revenue Function	$R(x) = x \cdot p(x)$	**Income** as a function of the number of items sold
Cost Function	$C(x)$	**Expenses** incurred as a function of the number of items
Profit Function	$P(x) = R(x) - C(x)$	Difference between the **revenue** and **costs**

Example 4: The demand function for a new product is $p(x) = -5x + 39$, where p represents the selling price of the product and x is the number sold in thousands.
The cost function is $C(x) = 4x + 30$. How many items must be sold to maximize profit?

Unit 5, Lesson 2: Solving Problems with Exponential Functions (Growth/Decay)

Exponential functions can be used to model situations involving growth or decay.

Consider the function $f(x) = aB^x$

If $B > 1$, the function models growth **If $0 < B < 1$, the function models decay**

***f*(*x*)** is the final amount
a is the starting or initial amount
B is the change factor
for exponential **growth**, B = **1 + growth rate**
for exponential **decay**, B = **1 – decay rate**
x is the number of growth or decay periods

Example 1: Determine the change factor (B) for each situation

a) money is growing at 5%/year?

b) a car is depreciating in value by 20%/year?

c) bacteria is doubling every 8 minutes?

d) a chemical is decaying by half every 10 hours?

The growth or decay rate can be found by evaluating $B - 1$

Example 2: Determine the growth or decay rate for each exponential function

a) $f(x) = 3(0.75)^x$

b) $f(x) = \frac{1}{2}(1.75)^x$

c) $f(x) = 3^{0.5x}$

Example 3: The following is an excerpt from the Stittsville Village Association website

Stittsville remained a small farming community serving the local area for many years. By 1899, the population was only 205 and by the mid-1950s, had only reached about 500 people. ... By 1993, the population had increased to nearly 10,000. ... The pace and distribution of current development indicates our population figures will continue to rise.

a) Determine the average annual growth rate, as a percent, from 1899-1993.

b) Create an equation to model the population of Stittsville as a function of years since 1899. Use your function to estimate Stittville's current population.

Example 4: The half-life of a radioactive element is the time taken for the element to decay to one half of its initial amount. The half-life of Iodine 131 is 8 days. The function that models the mass, in g, of a 320g sample of Iodine, $m(t)$ after t days is given by $m(t) = 320\left(\frac{1}{2}\right)^{\frac{t}{8}}$

a) Determine the mass of Iodine after 10 days

b) How long does it take for the mass of iodine to decay to 10g?

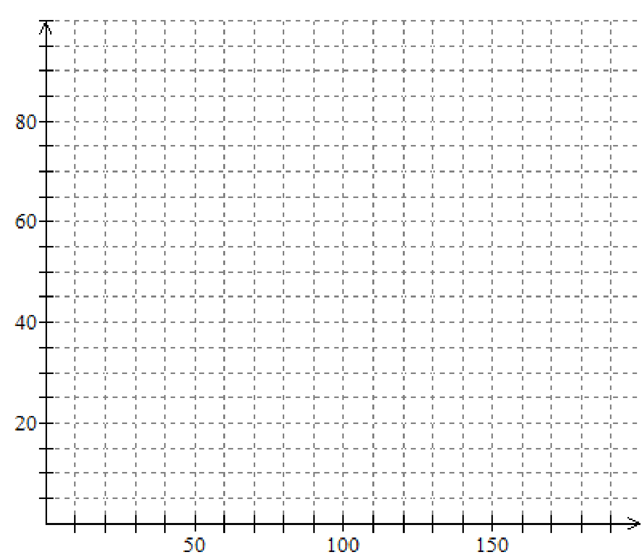
Example 5: Mrs. McKinnell's coffee cools at a rate defined by the function $T(t) = 65\left(\frac{1}{2}\right)^{\frac{t}{75}} + 23$, where T is temperature in degrees Celsius and t is time in minutes.

a) What is the initial temperature of the coffee?

b) What is the ambient temperature in the room? How do you know?

c) Mrs. McKinnell pours her coffee every morning at 7:30 am. All first and second period her students are in need of her attention so she doesn't have a chance to sip her coffee. When she finally drinks it at the start of lunch, what temperature will her coffee be?

d) Sketch a graph of the function and use the graph to estimate when Mrs. McKinnell should drink her coffee, if her preferred temperature is 63°.



Unit 5, Lesson 3: Solving Problems with Exponential Functions (Compound Interest)

When you **deposit** money in a bank account, you are actually lending money to the bank. In return, the bank pays out money to you in the form of interest. When you **borrow** money from the bank, you must pay them a fee, in the form of interest, for the convenience of borrowing.

Most banks use **compound interest,** which is interest that is added to the principal *before* new interest earned is calculated. This is done at regular intervals, called **compounding periods**.

The equation for compound interest is given by $A = P(1+i)^n$, where:

A	is the future value (the total value of an investment or loan)
P	is the principal (a sum of money that is borrowed or invested)
r	is the interest rate for the period (expressed as a decimal)
n	is the number of compounding periods (see chart)

Compounding frequency	*n*
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Bi-Weekly	26
Weekly	52
Daily	365

To calculate the total interest, ***I***, use the formula $\boldsymbol{I = A - P}$

Example 1: Complete the chart

Term	Annual Interest Rate, *r* (%)	Compounding Frequency	Principal, *P*	Future Value, *A* ($)	Compound Interest Earned, *I* ($)
3 years	4%	Semi-annually	$5100		
2 years	5.2%	weekly	$550		
21 months	10.55%	quarterly	$2000		

Using the compound interest formula, we can solve for the principal or annual interest of an investment or loan by using inverse operations.

Example 2: Jordan's brother borrows some money from him and he is charged 20%/a compounded semi-annually. He repays Jordan $512 after 2 years. How much did he borrow?

Example 3: Chris has $2500 to invest right now. He hopes to have $3500 in 3 years. At what annual interest rate, compounded monthly, must he invest his principal?

Example 4: Jackie is looking to borrow $4000. She is offered 2 options

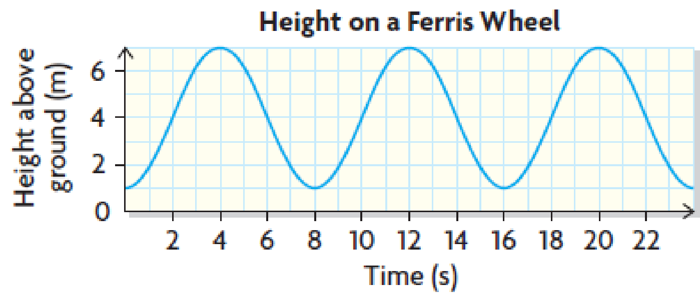
- An annual interest rate of 5.5%, compounded semi-annually, or
- An annual interest rate of 5.45%, compounded weekly.

Which has a lower cost of borrowing?

Unit 5, Lesson 4: Interpreting Sinusoidal Functions

Sinusoidal functions can be used model periodic phenomena. Each characteristic or property of the graph is related to the situation that it is modelling

Example 1: The graph shows Raymond's height above the ground as a function of time as he rides a Ferris wheel.



a) What is the diameter of the Ferris wheel?

b) What is Raymond's initial height above the ground?

c) How high above the ground is the axle on the wheel?

d) How long does it take for Raymond to complete one revolution of the Ferris wheel?

e) What is the speed of the Ferris wheel? What assumption(s) are you making about the Ferris wheel when you calculate its speed?

f) Determine his height above the ground 61s after the ride began.

Example 2: Annette's shop teacher was discussing table saws.
The teacher produced two different graphs for two different types of saw. In each case, the graphs show the height of one tooth on the circular blade relative to the cutting surface of the saw in terms of time.

Table Saw A

Table Saw B

Table Saw A

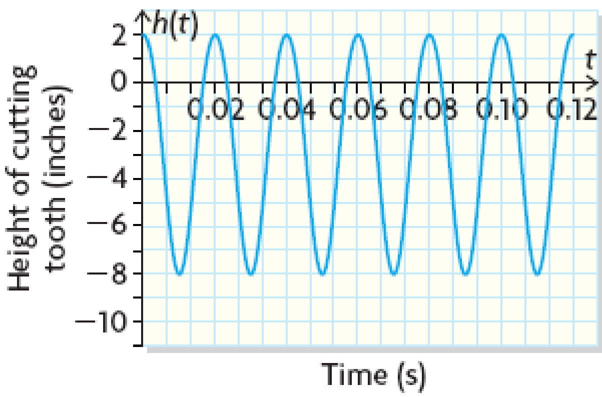
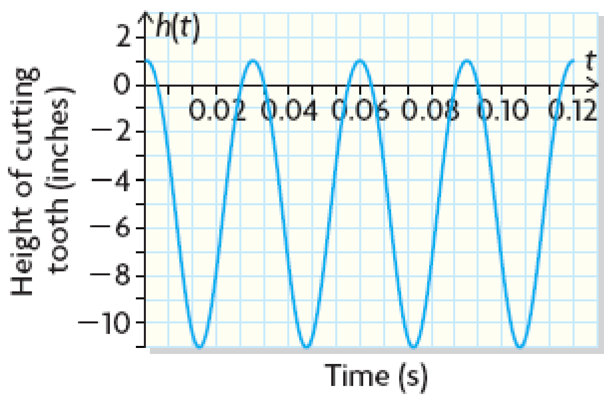


Table Saw B



a) How high above the cutting surface is each blade set?

b) What is the radius of each blade?

c) Where is the axle of the blade?

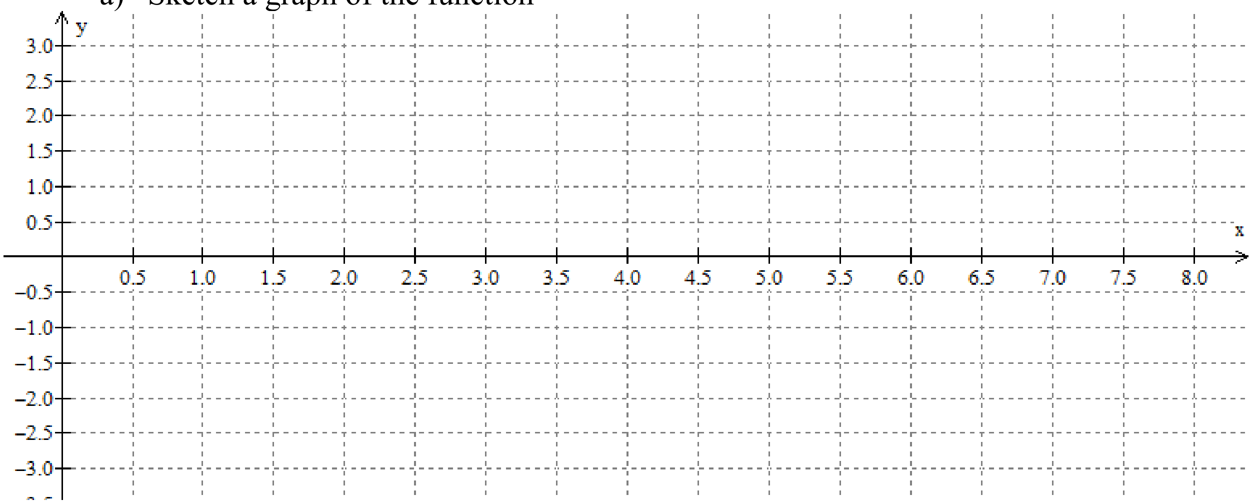
d) How long does it take for each blade to complete one revolution?

Unit 5, Lesson 5: Solving Problems with Sinusoidal Functions

Many periodic situations can be modeled by a sinusoidal function. If this is the case, then developing an equation for that function can help make predictions.

Example 1: Greg is swinging on a swing at a steady rate. He swings forward a distance of 2.5 meters and back a distance of 2.5 meters from the original resting position during each swing. At $t = 0$, he is at resting position and then swings forward. He is able to complete 15 swings forward and back in 1 minute.

a) Sketch a graph of the function



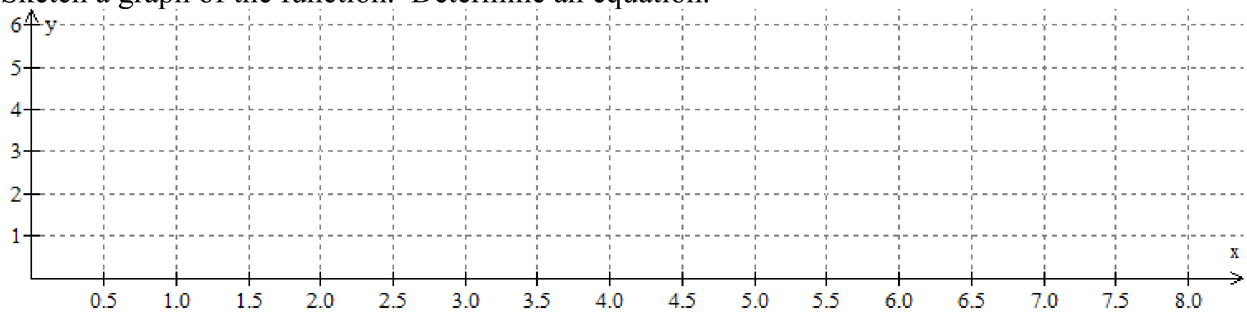
b) Determine the equation of the sinusoidal function which describes Greg's distance (in meters) to the resting position at each time, t (in seconds).

c) What is Greg's distance from the starting point at 12.5s?

d) At what time(s) will Greg be the farthest away from his resting position?

Example 2: Joe's mom, Helen, is pushing him on a swing. At $t = 0$ seconds she gives him a big push. At $t = 1.5$ seconds Joe is the furthest away from his mom at 6m. Determine an equation that models Joe's horizontal distance from his mother in terms of time seconds/degrees.

Sketch a graph of the function. Determine an equation.



Equation:

a) What is Joe's distance from his mom after 13 seconds?

b) How long does it take him to be 1m away from his mom on his 3rd swing?

c) How long does it take him to be 1m away from his mom **again** on his 3rd swing?

Unit 5, Lesson 6: Solving Problems with Functions

So far we have seen 4 types of functions that model real-world problems

- Linear Functions $f(x) = mx + b$
- Quadratic Functions $f(x) = ax^2 + bx + c$
- Exponential Functions $f(x) = aB^{kx} + c$
- Sinusoidal Functions $f(x) = a\sin k(x-d) + c$ or $f(x) = a\cos k(x-d) + c$

Deciding which model to use for a given situation is an important skill that takes practice!

Some hints:

- Most Revenue & Profit functions are quadratic in nature. Demand functions are linear in this course, but can be non-linear as well.
- If a problem involves area, it will likely require a quadratic function.
- If a situation models growth or decay, with a rate given as a %, this will be an exponential function, and the base will depend on the growth or decay rate.
- Any situation involving half-life (medication in bloodstream, chemical isotopes) is exponential decay, with $B = \frac{1}{2}$.
- If a situation is periodic in some way (repeating motion, seasonal sales, etc), use a sinusoidal function.

Example 1: The half life of caffeine in the bloodstream for an adult is 5.5 hours.
If Mrs. McKinnell drinks a Starbucks grande brewed coffee at 8:00 am, and then another one at 11:00 am, how much caffeine is still in her bloodstream at 9pm?

Example 2: Sales of ice cream are seasonal. Daily sales at an ice cream parlour peak in July at \$4800 per day, while in January they are at their lowest at \$1000 per day. During which month(s) are sales **at least** \$3500 per day?

Example 3: You want to sell your handmade jewellery at the Carp Farmer's market. Some market research has shown that you will sell 300 necklaces per month when the price is \$10 per necklace, and will sell 250 necklaces per month when the price is \$15 per necklace. Each necklace costs you \$4.00 in materials to make, and the monthly rental on your market booth is \$800. Determine the price you should set for your necklaces to ensure the maximum profit.

Solve each problem using techniques taught in this course.

1. Find 3 consecutive odd integers such that the sum of the squares of the first two is 15 less than the square of the third.

2. The demand function for a new product is $p(x) = -x + 24$, where p represents the selling price of the product and x is the total number sold in thousands. The cost function is $C(x) = 2x + 28$. What price should the company set for the item to yield a maximum profit?

3. Strontium-90 has a half-life of 28 years. How much remains of an initial sample of 40 g, after 21 years?

4. The maximum height of a Ferris wheel is 35 m. The wheel takes 2 min to make one revolution. The last passengers boards the Ferris wheel at 2 m above the ground at the bottom of the rotation. How high is the passenger after 45 s?

5. A family plans to surround their pool with a patio of uniform width. The pool has an area of 150 square feet. The dimensions of the pool with the patio will be 15 feet by 20 feet. Find the dimensions of the pool.

6. In 1947 an investor bought Vincent Van Gogh's painting Irises for $84,000. In 1987 she sold it for $49 million. What was her annual rate of interest to the nearest tenth based on the growth in the value of the painting?

7. A gaming company has been selling 1200 computer game discs per week at $18 each. Data indicates that for each $0.50 price increase, there will be a loss of 40 sales per week. If it costs $10 to produce each disc, what should the selling price be in order to maximize the profit?

8. Houses tend to sell more during certain times of year. For a particular neighbourhood, sales in June are at their highest at 120 houses sold. In December, sales are at their lowest with only 10 houses sold. During which months are at least 100 houses sold?

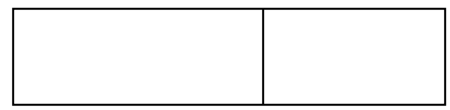
9. A store is to be built on a rectangular lot that measures 70 m by 45 m. A lawn of uniform width, equal to the area of the store, must surround it. How wide is the strip of lawn?

10. A car was purchased for $32 500. The car depreciates at 15% per year. How much should the car be sold for, in 5 years, to not lose money?

11. A skyscraper sways 55 cm back and forth from “the vertical” during high winds. At $t = 5$s, the building is 55cm to the right of vertical. The building sways back to the vertical and, at $t = 35$s, the building sways 55cm to the left of the vertical. During which time(s) within the first 60 seconds is the building within 10 cm of the vertical?

12. Cindy drinks an extra large coffee that contains approximately 320mg of caffeine. How many hours does it take for the amount of caffeine in her bloodstream to be reduced to 10mg? Assume that the half life of caffeine in the bloodstream is 5.5 hours.

13. A farmer has 450m of fencing to enclose a rectangular area and divide it into two sections as shown. Determine the dimensions that gives the maximum area.



14. The increase in the consumer price index (CPI) for the year 2012 was 1.5%. This means that prices increased an average of 1.5% from the previous year. A pair of jeans cost $89.99 in 2012. Assuming the CPI increases at the same rate in each of the next 4 years, determine the cost of the jeans in 2016.

15. The diameter of a car’s tire is 50 cm. While the car is being driven, the tire picks up a nail. When the nail reaches a height of 10cm above the ground for the sixth time, how far has the car traveled?

1) 1,3,5 or 3,5,7 2) $13 3) 23.8 g 4) 30.2 m 5) 10ft by 15ft 6) 17.4% 7) $21.50 8) April to August 9) 7.95 m 10) $14 420.42 11) 18.3-21.7s and 48.3-51.7s 12) 27.5 hours 13) 112.5m by 75m 14) $95.51 15) 4.5 m

Unit 5, Lesson 7: Modelling Data with Functions

Regression is the process of determining the equation of a curve of best fit for a set of data. They are useful for modelling data that has been gathered to predict future behaviour.

To perform a regression by hand:

- Graph the data & draw a curve of best fit
  - pay attention to your scale; make it consistent and fitting to the data
- Determine the parent function
  - look at the shape of the curve
  - consider past and future behaviour
- Decide which transformations you need
  - the fewer, the better!
  - some transformations you can determine looking at the curve, others need algebra
- Choose 1 (or more) point(s) to substitute into your equation to solve for the parameters needed.
  - in general, the more parameters you are solving for, the more points you will need – you may end up with a system of equations.

Example 1: The data for sales of a seasonal product are collected. Results are shown in the table below. Graph the data and determine the model that gives the sales as a function of time in months.

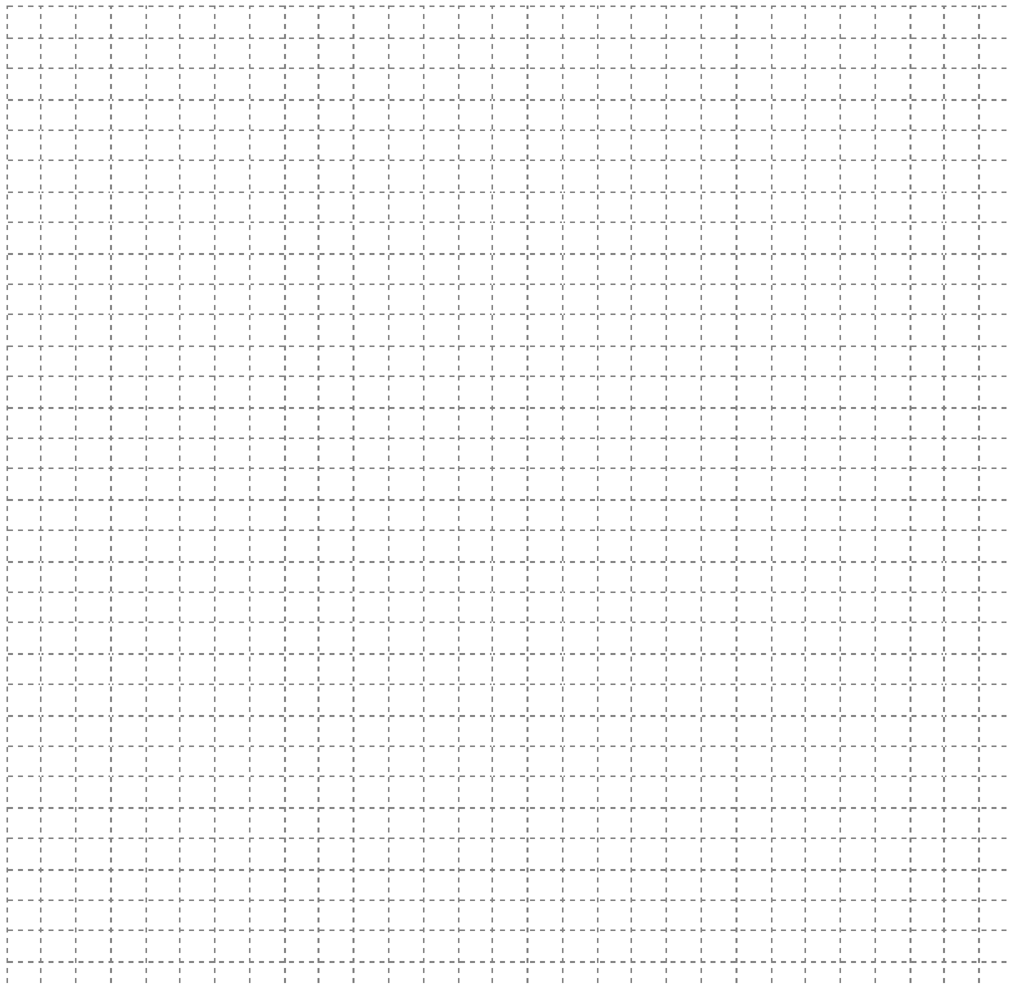
Month	**Sales ($1000)**
Feb	101
May	150
Aug	98
Nov	50
Feb	100
May	147

For exponential regression, you need to determine both the base of the parent function, as well as any transformations you need. Determining the base should be your first step!

Example 2: The Consumer Price Index is a measure of the cost of living. It is found by tracking the average family's typical living expenses. An upward trend in CPI is called inflation. The table gives the CPI for Canadians over a 7-year period.

a) Graph the data and determine the model that gives the CPI as a function of time in years.

Year	CPI ($)
2002	100
2003	102.9
2004	105.9
2005	109
2006	112.1
2007	115.4
2008	118.8



b) What would be the CPI for the year

i) 2016?

ii) 1990?

Part A - For each table, **graph the data** and draw a **curve of best fit**. Then define your variables and determine an algebraic model for each situation. If necessary, restrict the domain of your function so it makes sense for the situation.

1. The following table shows the population of a small town from 2000 – 2005. Determine an algebraic model for the population as a function of time.

Year	Population
2000	2000
2001	2150
2002	2300
2003	2500
2004	2700
2005	2950

2. The following table shows the length of time needed to drive a fixed distance at varying speeds. Determine an algebraic model for time as a function of speed.

Speed (km/h)	Time (h)
10	12.0
15	8.0
20	6.0
25	4.8
30	4.0
40	3.0
50	2.4
80	1.5

3. The following table shows the maximum height of a bouncing ball after it bounces. Determine an algebraic model for the maximum height as a function of the number of bounces.

# of bounces	Height (cm)
0	100
1	76
2	57
3	43
4	32
5	24

4. The following table shows the lengths and periods for different pendulums. Determine an algebraic model for the period as a function of the length.

Length (m)	Period (s)
0.1	0.64
0.2	0.90
0.3	1.10
0.4	1.27
0.5	1.49
0.6	1.55
0.7	1.70
0.8	1.76
0.9	1.90
1.0	2.01

5. The following table shows the stopping distance for cars driving at various speeds. Determine an algebraic model for the stopping distance as a function of speed

Speed (km/h)	Stopping Distance (m)
10	3.7
20	7.6
30	12.0
40	17.1
50	22.9
60	29.8
70	37.9
80	47.5
90	58.6
100	71.6
110	86.5
120	103.5
130	122.8
140	144.7

6. The following table shows the speed of a tsunami at different depths of water. Determine an algebraic model for the speed of a tsunami as a function of water depth.

Water Depth (m)	Speed of Tsunami (m/s)
0	0
10	9.90
100	31.30
500	70.00
1000	100.00
1500	121.24
2000	140.00

7. The following table shows the number of hours spent studying and the grade received as a percent for 12 students. Determine an algebraic model for grade as a function of hours spent studying.

Hours Spent Studying	Grade (%)
6	53
7	60
6.5	56
8	79
6.6	58
8.1	85
6.9	76
6.5	65
7.3	74
8.6	84
7.6	76
7.4	78
8.2	79
6.8	65

8. The following table shows the temperature of a small cup of coffee as time passes. Determine an algebraic model for temperature as a function of time.

Time (s)	Temperature (˚C)
30	75.5
120	73.63
210	72.27
300	71.01
390	69.78
480	68.59
570	67.64
660	66.5
750	65.59
840	64.57
930	63.76
1020	62.58
1110	61.56
1200	60.73
1290	59.92
1380	59.12
1470	58.33
1560	57.56
1650	56.62
1740	55.86

9. The following table show the average daily high temperature for one year. Determine an algebraic model for the temperature as a function of time.

Month	Daily High Temperature
Jan	-4
Feb	-5
Mar	1
Apr	10
May	19
Jun	24
Jul	26
Aug	25
Sep	20
Oct	13
Nov	5
Dec	-4

Part B – Determine an algebraic model for each table of values. You may wish to graph the data first to help you.

x	y
-2	$\frac{5}{9}$
-1	$\frac{5}{3}$
0	5
1	15
2	45
3	135
4	405

x	Y
-3	$5\frac{1}{54}$
-2	$5\frac{1}{18}$
-1	$5\frac{1}{6}$
0	5.5
1	6.5
2	9.5
3	18.5

x	y
-2	100
-1	20
0	4
1	0.8
2	0.16
3	0.032
4	0.0064

x	y
-3	-0.000064
-2	-0.0016
-1	-0.04
0	-1
1	-25
2	-625
3	-15625

x	y
30.0	-10.0
37.5	-12.0
45.0	-10.0
52.5	-8.0
60.0	-10.0
67.5	-12.0
75.0	-10.0

x	y
10	8
20	18
30	28
40	18
50	8
60	18
70	28

x	y
0.0	12.5
1.5	17.5
3.0	12.5
4.5	7.5
6.0	12.5
7.5	17.5
9.0	12.5

x	y
-50	50
-45	40
-40	30
-35	40
-30	50
-25	40
-20	30