Mr. Flynn Key Mr. Flynn June 2006 2008

**Part A: Completion**

*Answers must be placed in the space provided. It is not necessary to show work. Each question is worth one mark.*

1. If $f(x) = 2x - 7$, find $f(-3) + 1$. — 1. $-12$

2. State the domain for the following function: $y = \dfrac{2}{(x-4)}$. — 2. $\{x \in R \mid x \neq 4\}$

3. State the range of the following: y = 5. — 3. $\{y = 5\}$

4. For $p(t) = 5t + 4$, determine $p^{-1}(-2)$. $x = 5y + 4$; $\frac{x-4}{5} = y$; $\frac{-2-4}{5}$ — 4. $-\frac{6}{5}$

5. Given the point $(1,-3)$ on $y = f(x)$, what would be the image of the point under the following transformation: $y = f(\frac{1}{2}x)$ — 5. $(2,-3)$

6. How many zeroes does the following function have? $y = -3(x-2)^2 - 4$ — 6. 0 NONE

7. Simplify the following expression: $\dfrac{m^2 + m - 12}{m^2 + 5m + 4}$. $= \dfrac{(m-3)(m+4)}{(m+1)(m+4)}$ — 7. $\dfrac{m-3}{m+1}$

8. Simplify: $4x[3(2x-8) + 11x]$. $4x(6x - 24 + 11x)$; $4x(17x - 24)$ — 8. $68x^2 - 96x$ or $4x(17x-24)$

9. Express $\sqrt{80}$ as a mixed radical. $\sqrt{16 \times 5}$ — 9. $4\sqrt{5}$

10. Simplify: $\sqrt{32} - \sqrt{8}$. $4\sqrt{2} - 2\sqrt{2}$ — 10. $2\sqrt{2}$

11. Evaluate: $32^{\frac{-2}{5}}$ (No decimals) $\dfrac{1}{(\sqrt[5]{32})^2} = \dfrac{1}{4}$ — 11. $\frac{1}{4}$

12. Express $5\sqrt{6}$ as an entire radical. — 12. $\sqrt{150}$

13. What is the horizontal asymptote of the function $y = 2^x - 1$? — 13. $y = -1$

14. What is the y-intercept of the function $y = -2(3)^x + 1$? — 14. $(0,-1)$

15. Find the next two terms of the sequence: 3, 5, 8, 13, 21, ___, ___. — 15. 34, 55 or 33, 50

16. Identify the type of sequence represented by: $\frac{3}{7}, \frac{2}{7}, \frac{12}{63}, \frac{24}{189}, \ldots$ $r = 0.\bar{6}$ $(\frac{2}{3})$ — 16. Geometric

17. Write the recursive sequence for $2, 10, 50, 250, \ldots$ — 17. $t_n = t_{n-1} \times 5$, $t_1 = 2$, $n > 1$

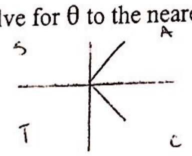
18. What is the interest rate per period of an investment at 4.4%/a, compounded semi-annually? $\frac{0.044}{2}$ — 18. 0.022 or 2.2%

19. What is the principal angle of -127°? — 19. 233°

20. What is the related acute angle for -327°? — 20. 33°

21. State the period of the function $y = 2\sin 4\theta - 5$ in degrees. — 21. 90°

22. Solve for θ to the nearest degree: $\cos\theta = 0.2218$, $0° \le \theta \le 360°$. — 22. 77° and 283°



$\theta = 77$

$\overline{\phantom{00}}$ 22

What is the range of the function $y = -4\sin x + 3$ 8 6 7 - 1

23. $\{y \in R \mid -1 \le y \le 7\}$

24. What is the equation of the axis for $y = 2\cos(\theta - 30°) - 4$

24. $y = -4$

25. What is the exact value of $\sec 330°$ ? $= \frac{1}{\cos 330} = \frac{1}{\frac{\sqrt{3}}{2}}$

25. $\frac{2}{\sqrt{3}}$

**Part B: Full Solutions**

*Answer the following questions in the space provided. Complete solutions are required and all answers should be expressed in their simplest form.*

1. For the geometric sequence with terms $t_8 = 210$ and $t_9 = 630$, find *a* and *r*.

[3]

$\underline{t_n = ar^{n-1}}$

$t_8 = ar^{8-1}$

$210 = ar^7$

$t_9 = ar^{9-1}$

$630 = ar^8$

$\underline{210 = ar^7}$

$\boxed{3 = r}$

(or) $r = 630 \div 210$

$= 3$

$\therefore 210 = a(3)^7$

$\frac{210}{2187} = a$

$\boxed{a = \frac{70}{729}}$

$\therefore r = 3$

$a = \frac{70}{729}$

or $= 0.096$

2. Determine the number of terms in the following arithmetic sequence: $-109, -95, -81, -67, \ldots, 101$. (Show appropriate work)

[4]

$t_n = a + (n-1)d$

$101 = -109 + (n-1)14$

$\frac{210}{14} = n-1$

$15 = n-1$

$n = 16$

$\therefore n = 16$

16 terms

3. Determine the sum of the series $5 - 10 + 20 - 40 + \ldots + 1280$ by using the appropriate formulas.

[4]

Sol: $S_n = \frac{a(r^n - 1)}{r-1}$

Find n: $t_n = ar^{n-1}$

$1280 = 5(-2)^{n-1}$

$256 = (-2)^{n-1}$

$(-2)^8 = (-2)^{n-1}$

$8 = n-1$

$9 = n$

$S_9 = \frac{5((-2)^9 - 1)}{-2-1}$

$= 855$

(or) $S_n = \frac{t_{n+1} - t_1}{r-1}$

$S_n = \frac{-2560 - 5}{-2-1}$

$= \frac{-2565}{-3}$

$\boxed{= 855}$

$r = -2$

$t_1 = 5$

$t_{n+1} = 1280 \times -2$

$= -2560$

Barry purchases a new truck worth $45000. It depreciates in value by 12% each year. How much is the truck worth after 7 years?

[2] $V(t) = 45000(0.88)^7$

$\doteq 18390.40$

$\boxed{\therefore \$18390.40}$

5. A 200 gram sample of radioactive plutonium has a half-life of 138 days. The mass of plutonium, in grams, that remains after $t$ days can be modeled by $M = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$.

a) Determine the mass that remains after 5 years. (1825d) [2] round to two dp.

$M = 200\left(\frac{1}{2}\right)^{\frac{1825}{138}}$

$\doteq 0.02089\ldots$ $\therefore$ 0.02 grams

b) How long does it take for this 200 gram sample to decay to 110 grams? to nearest day. [3]

$110 = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$

$0.55 = \left(\frac{1}{2}\right)^{\frac{t}{138}}$

(or) $\frac{\log 0.55}{\log 0.5} = 0.86$

$0.86 = \frac{t}{138}$

$\boxed{t \doteq 119 \text{ days}}$

Trial & Error

$0.5^{0.85} = 0.554$

$0.5^{0.86} = 0.551$

$\therefore \frac{t}{138} = 0.86$

$t \doteq 119$ days

6. Determine the equation of the quadratic function in standard form that has roots $(3+\sqrt{5})$ and $(3-\sqrt{5})$ and passes through the point $(4,2)$.

[4]

$f(x) = a[x-(3+\sqrt{5})][x-(3-\sqrt{5})]$

$f(4) = a[4-3-\sqrt{5}][4-3+\sqrt{5}]$

$2 = a(1-\sqrt{5})(1+\sqrt{5})$

$2 = a(1-5)$

$\frac{2}{-4} = a$

$a = -\frac{1}{2}$

$f(x) = -\frac{1}{2}[x-(3+\sqrt{5})][x-(3-\sqrt{5})]$

$= -\frac{1}{2}[x^2 - x(3-\sqrt{5}) - x(3+\sqrt{5}) + (3+\sqrt{5})(3$

$= -\frac{1}{2}[x^2 - 3x + x\sqrt{5} - 3x - x\sqrt{5} \ldots$

$= -\frac{1}{2}[x^2 - 6x + 9 - 5]$

$\boxed{\therefore f(x) = -\frac{1}{2}x^2 + 3x - 2}$

11

Determine the coordinates of the vertex of the following parabola by completing the square.

$f(x) = -3x^2 + 5x - 7$ Use fractions.

[3]

$= -3(x^2 - \frac{5}{3}x) - 7$

$= -3(x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}) - 7$

$= -3(x - \frac{5}{6})^2 + \frac{75}{36} - 7$

$= -3(x - \frac{5}{6})^2 - \frac{59}{12}$

$\therefore$ vertex $(\frac{5}{6}, -\frac{59}{12})$

Rough

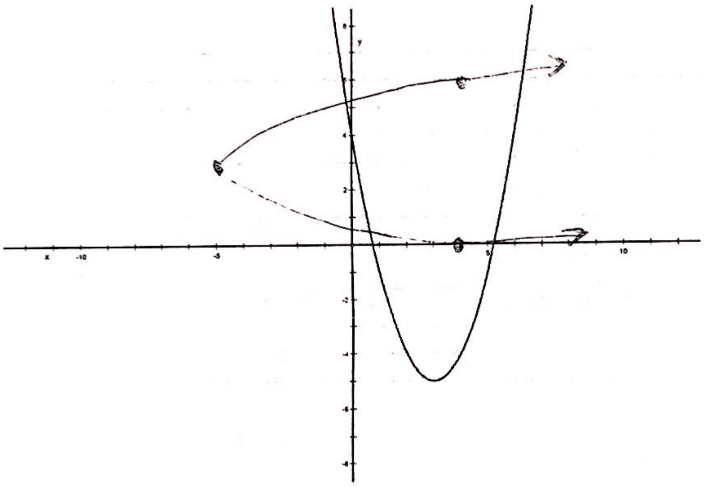
$\frac{5}{3} \times \frac{1}{2} = \frac{5}{6}$

$(\frac{5}{6})^2 = \frac{25}{36}$

$\frac{25}{12} - \frac{84}{12}$

$= \frac{-59}{12}$

8. The graph of $y = (x-3)^2 - 5$ is given below.



| Points | Inverse |
|---|---|
| (3,-5) | (-5,3) |
| (6,4) | (4,6) |
| (0,4) | (4,0) |

a) On the same set of axes, draw the inverse of the above graph. [1]

b) What is the equation of the inverse? [3]

$x = (y-3)^2 - 5$

$\sqrt{x+5} + 3 = y$

$\therefore y = \pm\sqrt{x+5} + 3$

c) State a restriction on the original function, so that the inverse is also a function. [1]

D: $\{x \in R \mid x \geq 3\}$ or $\{x \in R \mid x \leq 3\}$

8

Expand and simplify the first three terms of $(3x+5y)^6$ [4]

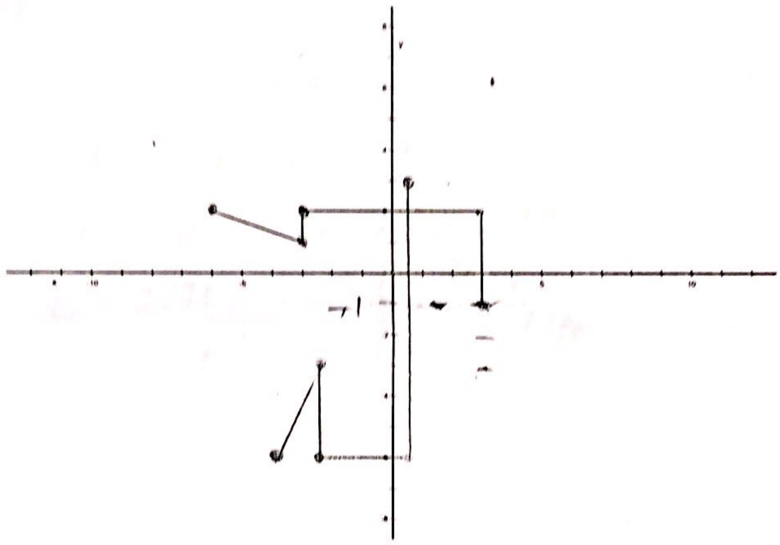
$= 1(3x)^6(5y)^0 + 6(3x)^5(5y)^1 + 15(3x)^4(5y)^2$

$= 729x^6 + 7290x^5y + 30375x^4y^2$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6

6th row.

10. Given the graph $y = f(x)$, sketch $y = -3f(2(x+1))$. State all of the transformations. [4]




- left one
- reflection in $x$ axis
- vertical stretch of 3
- Horizontal compression of $\frac{1}{2}$

11. Simplify the following: $\dfrac{x^2+x-12}{6x^2+7x-5} \div \dfrac{x^2-x-20}{9x^2+30x+25} \times \dfrac{1}{x-3}$

[5]

$= \dfrac{(x+4)(x-3)}{(2x-1)(3x+5)} \div \dfrac{(x+4)(x-5)}{(3x+5)(3x+5)} \times \dfrac{1}{x-3}$

$x \neq -\frac{5}{3}, \frac{1}{2}, 3$

$= \dfrac{x+4}{(2x-1)(3x+5)} \times \dfrac{(3x+5)(3x+5)}{(x+4)(x-5)}$

$x \neq -4, 5$

$= \dfrac{3x+5}{(2x-1)(x-5)}$

13

The average monthly temperature, T, in degree Celsius for Ottawa can be modelled by the function

$T(t) = -20\cos(30t)° + 10$ , where $t$ represents the number of months. January is represented by $t=0$, February by $t = 1$ and so on.

a) What is the period? Explain the period in relation to this problem.

[1] $P = \frac{360}{30}$ There are 12 months in the year

$P = 12$

b) What is the minimum temperature?

[1] $10 - 20 = -10°C$

c) In what months does the temperature reach 7°C ? Show this algebraically.

[4] $7 = -20\cos(30t)° + 10$

$-3 = -20\cos(30t)°$

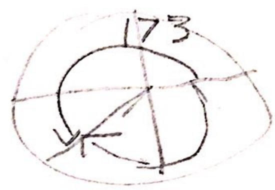
$0.15 = \cos(30t)°$

$(30t)° = \cos^{-1}(0.15)$

$30t = 81.37$ or $30t = 278.63$

$t = 2.71$ $t = 9.29$

late March Early october



14. Find the intersection of $x^2 + y^2 = 10$ and $y - 2x = 7$ algebraically.

[4]

$y = 2x+7$

$\therefore x^2 + (2x+7)^2 = 10$

$x^2 + 4x^2 + 28x + 49 = 10$

$5x^2 + 28x + 39 = 0$

$(5x+13)(x+3) = 0$

$5x+13 = 0$ $x+3=0$

$x = -\frac{13}{5}$ $x = -3$

$y = 2\left(-\frac{13}{5}\right) + 7$ $y = 2(-3)+7$

$= -\frac{26}{5} + 7$ $y = -6+7$

$= -\frac{26}{5} + \frac{35}{5} = \frac{9}{5}$ $y = 1$

$\therefore \left(-\frac{13}{5}, \frac{9}{5}\right)$ & $(-3, 1)$

10

Prove the following trigonometric identity:

a) $\cos x + \sin x \tan x = \dfrac{1}{\cos x}$

[3]

$\cos x + \sin x \cdot \dfrac{\sin}{\cos x}$

$\dfrac{\cos x}{1} + \dfrac{\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos x}$

$\dfrac{1}{\cos x} = R.S.$

June 200?

$\cos\theta(1+\sec\theta)(\cos\theta - 1) = -\sin^2\theta$

$\cos\theta\left(1+\frac{1}{\cos\theta}\right)(\cos\theta - 1)$

$(\cos\theta + 1)(\cos\theta - 1)$

$\cos^2\theta - 1$

$-(1-\cos^2\theta)$

$-\sin^2\theta$

$= R.S.$

16. A river flows at 4 km/h. Kelly takes 3 hours to row 15 km up the river and 15 km back. How fast can Kelly row in still water?

[6]

Let $x$ = Kelly's Speed in Still water

| | Distance | Speed | Time |
|---|---|---|---|
| UP | 15 | $x-4$ | $15/x-4$ |
| DOWN | 15 | $x+4$ | $15/x+4$ |

$\left[\dfrac{15}{x-4} + \dfrac{15}{x+4} = 3\right](x-4)(x+4)$

$15(x+4) + 15(x-4) = 3(x-4)(x+4)$

$15x + 60 + 15x - 60 = 3(x^2 - 16)$

$30x = 3x^2 - 48$

$3x^2 - 30x - 48 = 0$

$x^2 - 10x - 16 = 0$

$x = \dfrac{10 \pm \sqrt{(-10)^2 - 4(1)(-16)}}{2(1)}$

$x = \dfrac{10 \pm \sqrt{100 + 64}}{2}$

$x = \dfrac{10 \pm 12.8}{2}$

$x = 11.4$ & $x = -1.4$

$\therefore$ Kelly can row 11.4 Km/h in still water.

9

Peter has a part time job at McDonald's and is saving money for a school trip to France in two years. He needs $3200 for the trip. He wants to deposit equal amounts at the end of every month for two years in a savings account that pays 3.6%/a, compounded monthly. How much money does Peter need to deposit at the end of every month in order to save the $3200?

[3]

$i = \frac{3.6\%}{12}$

$i = 0.3\%$

$i = 0.003$

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$3200 = \frac{R\left[(1.003)^{24} - 1\right]}{0.003}$$

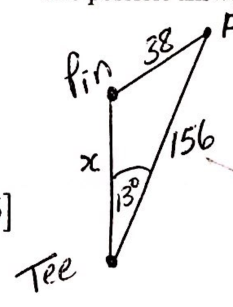
$$9.6 = 0.074539591\, R$$

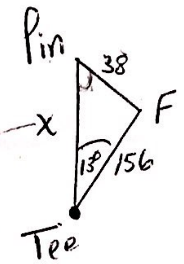
$$R = 128.79$$

$\therefore$ monthly deposit is \$128.79

18. On a particular par 3 hole, Mike Weir's first shot was 156 yards but sliced 13° to the right. He estimated that he was still 38 yards from the pin. Find the straight line distance from the tee to the pin. (There are two possible answers)

[6]




$\frac{\sin 13^\circ}{38} = \frac{\sin P}{156}$

$\angle P = 67.4^\circ$

$\angle F = 99.6^\circ$

$\frac{\sin 13^\circ}{38} = \frac{\sin 99.6}{x}$

$x = 166.6$ yards

$\frac{\sin 13^\circ}{38} = \frac{\sin P}{156}$

$\angle P = 180 - 67.4$

$\angle P = 112.6$

$\angle F = 54.4^\circ$

$\frac{\sin 13^\circ}{38} = \frac{\sin 54.4}{x}$

$x = 137.5$ yards