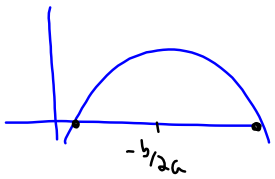
3.2 Max and Min of Quadratics

Mar 11

Methods for finding the vertex, $y = ax^2 + bx + c$

1. FACTOR, FIND ZEROS, MIDPOINT OF ZEROS, SUB INTO FUNCTION
   x_{symmetry}
2. COMPLETING THE SQUARE $y = ax^2 + bx + c \longrightarrow y = a(x-h)^2 + k$
3. QUADRATIC FORMULA, FIND ZEROS, MIDPOINT, SUB...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \underbrace{\frac{-b}{2a}}_{x = -b/2a \text{ VERTEX}} \pm \frac{\sqrt{b^2-4ac}}{2a}$$



Determine the vertex of the function using each of the methods listed above. $f(x) = -5x^2 + 40x + 100$

1) FACTORING

$f(x) = -5(x^2 - 8x - 20)$

$0 = -5(x+2)(x-10)$

$x = -2, 10$

SYMMETRY $x = \frac{-2+10}{2} = \frac{8}{2} = 4$

$\longrightarrow f(4) = -5(4)^2 + 40(4) + 100$

$= 180$

$\therefore$ VERTEX $(4, 180)$

2) COMPLETE THE SQUARE.

$f(x) = -5x^2 + 40x + 100$

$f(x) = -5(x^2 - 8x) + 100$

$= -5(\underline{x^2 - 8x + 16} - 16) + 100$

$f(x) = -5(\underline{x-4})^2 + 80 + 100$

$f(x) = -5(x-4)^2 + 180$

$\therefore$ VERTEX $(4, 180)$

c) QUADRATIC FORMULA $\longrightarrow -\frac{b}{2a}$

$f(x) = -5x^2 + 40x + 100$

$x = -\frac{b}{2a} = \frac{-40}{2(-5)} = \frac{-40}{-10} = 4$

$f(4) = -5(4)^2 + 40(4) + 100 = 180$

$\therefore$ Vertex $(4, 180)$

The cost, $c(x)$, in dollars per hour of running a certain steamboat is modelled by the quadratic function $c(x) = 1.8x^2 - 14.4x + 156.5$, where x is the speed in kilometres per hour. At what speed should the boat travel to achieve the minimum cost?

$$c(x) = 1.8x^2 - 14.4x + 156.5$$
$$= 1.8(x^2 - 8x\ ) + 156.5$$
$$= 1.8(x^2 - 8x + 16 - 16) + 156.5$$
$$= 1.8(x-4)^2 - 28.8 + 156.5$$
$$= 1.8(x-4)^2 + 127.7$$

$$x = -\frac{b}{2a}$$
$$= \frac{-(-14.4)}{2(1.8)}$$
$$= \frac{14.4}{3.6}$$
$$= 4$$

∴ MIN COST WHEN SPEED IS 4 km/h

The demand function for a new magazine is $p(x) = -6x + 40$, where $p(x)$ represents the selling price, in thousands of dollars, of the magazine and x is the number sold, in thousands. The cost function is $C(x) = 4x + 48$. Calculate the maximum profit and the number of magazines sold that will produce the maximum profit.

PROFIT = REVENUE − COST

$$P(x) = R(x) - C(x)$$

REVENUE = PRICE/ITEM × # ITEMS

$$R(x) = p(x) \cdot x$$

$$P(x) = x\,p(x) - C(x)$$
$$= x(-6x+40) - (4x+48)$$
$$= -6x^2 + 40x - 4x - 48$$
$$= -6x^2 + 36x - 48$$
$$= -6(x^2 - 6x + 8)$$
$$0 = -6(x-4)(x-2)$$
$$x = 2, 4$$

symmetry $x = 3 \longrightarrow P(3) = -6(3)^2 + 36(3) - 48 = 6$

∴ MAX PROFIT WHEN 3000 MAGAZINES ARE SOLD AT A PROFIT OF \$6000.

Homework p. 153 #4,5,7-9,11c