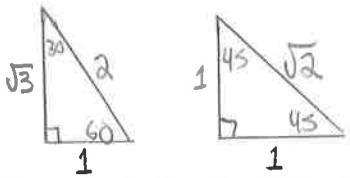
Exam Review Part 5 – Trig Geometry

MCR3U

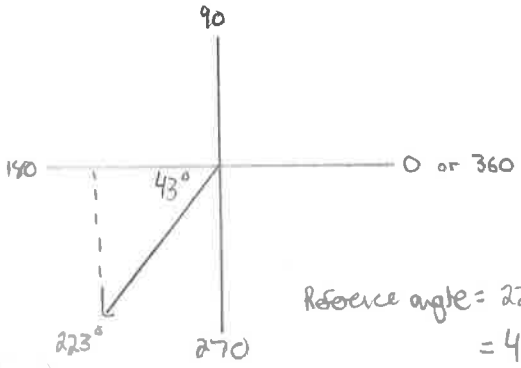
SOLUTIONS

1) Draw both special triangles learned in this unit. Make sure to label all angles and side lengths.



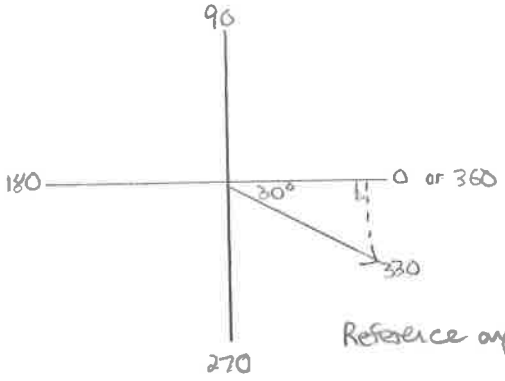
2) Find a reference angle for the following obtuse angles

a) $\theta = -137^\circ = 223$



Reference angle = 223 − 180
= 43°

b) $\theta = 330^\circ$



Reference angle = 360 − 330
= 30°

3) Determine the exact sine value and cosine value of each angle.

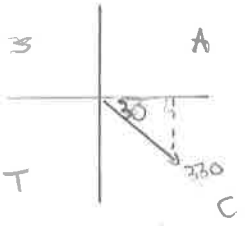
a) $\theta = 330^\circ$

$\sin 330 = -\sin 30$

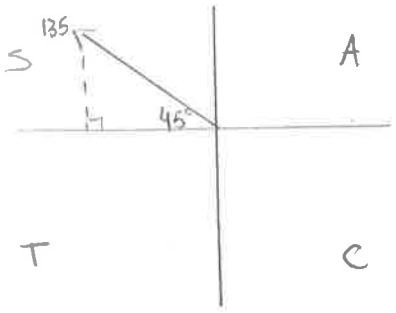
$= -\frac{1}{2}$

$\cos 330 = \cos 30$

$= \frac{\sqrt{3}}{2}$



b) $\theta = 135^\circ$



$\sin 135 = \sin 45$

$= \frac{1}{\sqrt{2}}$

$\cos 135 = -\cos 45$

$= -\frac{1}{\sqrt{2}}$

4) Determine two angles that are coterminal with angle $\theta = 97^\circ$.

$\theta_1 = 97 + 360 = 457^\circ$

$\theta_2 = 457 + 360 = 817^\circ$

5) Determine ***TWO*** angles between 0° and 360° that have the following trigonometric function value. Write each angle to one decimal place.

a) $\tan\theta = -0.32$

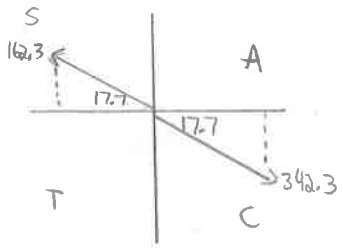
$\theta_1 = \tan^{-1}(-0.32)$

$= -17.7°$

$= 342.3°$

$\theta_2 = 180 - 17.7$

$= 162.3$



$\tan 342.3 = \tan 162.3 = -0.32$

b) $\sin\theta = -0.46$

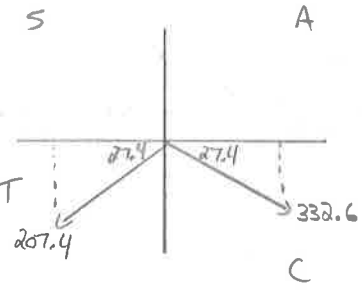
$\theta_1 = \sin^{-1}(-0.46)$

$= -27.4$

$= 332.6°$

$\theta_2 = 180 + 27.4$

$= 207.4$



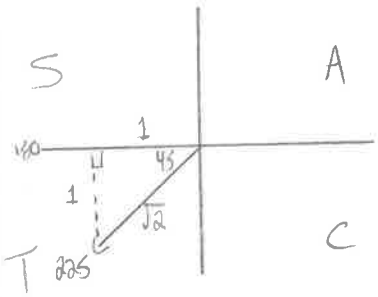
$\sin 332.6 = \sin 207.4 = -0.46$

6) State each exact value using special triangles.

a) $\sin 225°$

$\sin 225 = -\sin 45$

$= -\frac{1}{\sqrt{2}}$

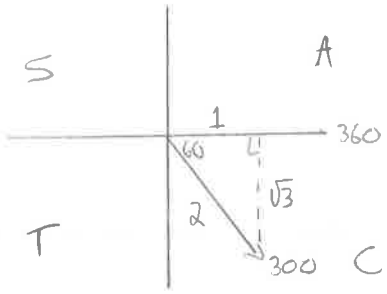


b) $\cot 300°$

$\cot 300 = \frac{1}{\tan 300}$

$= -\frac{1}{\tan 60}$

$= -\frac{1}{\sqrt{3}}$



7) Find the exact value of the following trigonometric ratios using special triangles and the definition of the inverse functions.

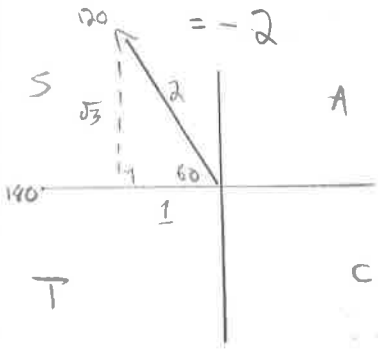
a) $\sec 120°$

$= \frac{1}{\cos 120}$

$= -\frac{1}{\cos 60}$

$= -\frac{1}{(\frac{1}{2})}$

$= -2$

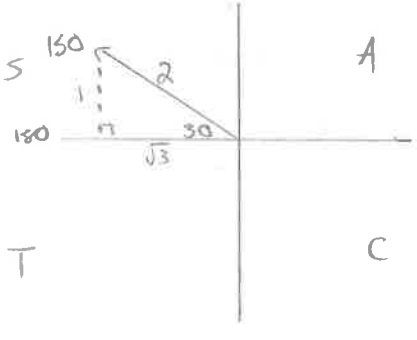


b) $\csc 150°$ $= \frac{1}{\sin 150}$

$= \frac{1}{\sin 30}$

$= \frac{1}{(\frac{1}{2})}$

$= 2$

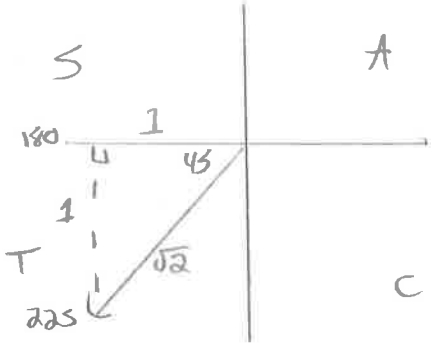


c) $\cot 225°$ $= \frac{1}{\tan 225}$

$= \frac{1}{\tan 45}$

$= \frac{1}{1}$

$= 1$

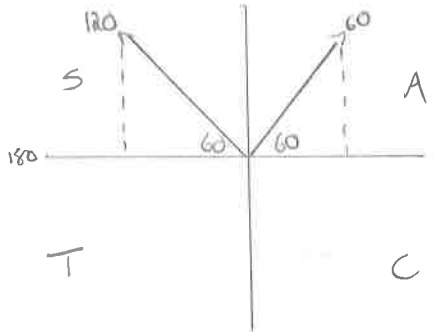


8) Determine two angles between 0 and 360° that have the following trigonometric function value.

a) $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta_1 = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$ $\quad \theta_2 = 180-60 = 120°$

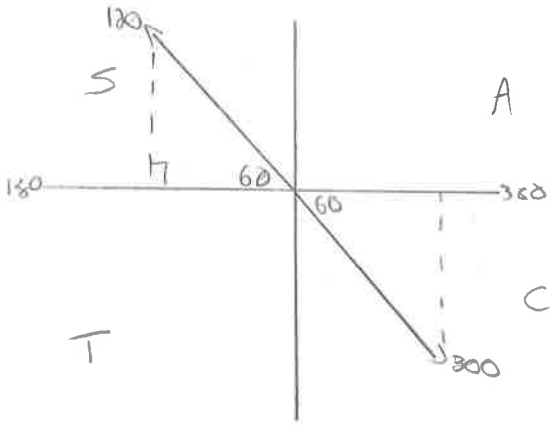
$\sin 60 = \sin 120 = \frac{\sqrt{3}}{2}$



b) $\tan\vartheta = -\sqrt{3}$

$\theta_1 = \tan^{-1}(-\sqrt{3}) = -60 = 300°$ $\quad \theta_2 = 180-60 = 120°$

$\tan 300 = \tan 120 = -\sqrt{3}$

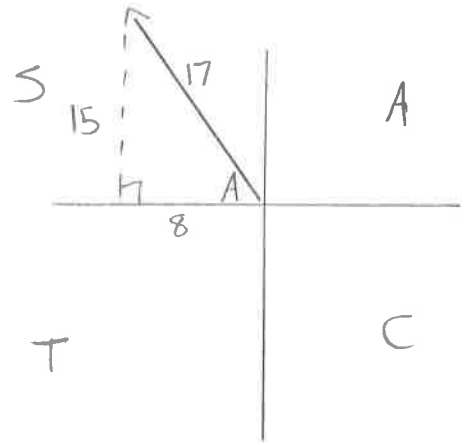


) One of the primary trig ratios of an angle is given, as well as the quadrant in which the terminal arm lies. Find the other two primary trig ratios.

a) $\cos A = -\frac{8}{17}$, second quadrant

$\sin A = \frac{15}{17}$

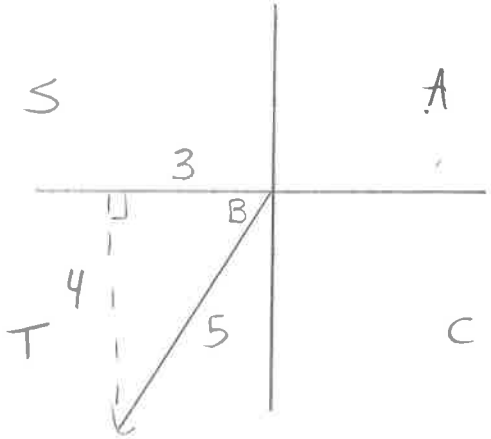
$\tan A = -\frac{15}{8}$



b) $\sin B = -\frac{4}{5}$, third quadrant

$\cos B = -\frac{3}{5}$

$\tan B = \frac{4}{3}$



10) Each point lies on the terminal arm of an angle in standard position. Determine exact expressions for the six trigonometric ratios for the angle.

a) P(-3, 4)

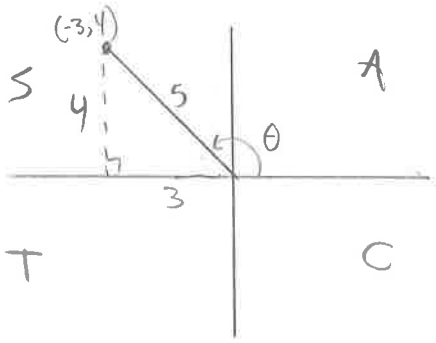
$\sin\theta = \frac{4}{5}$ $\cos\theta = -\frac{3}{5}$ $\tan\theta = -\frac{4}{3}$

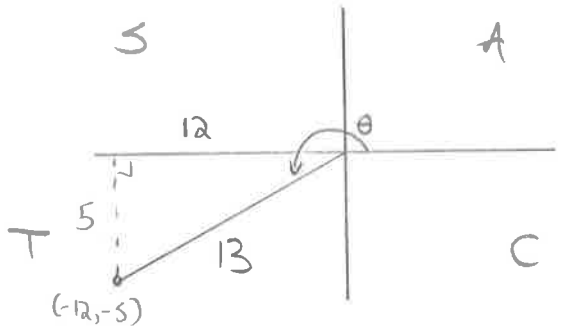
$\csc\theta = \frac{5}{4}$ $\sec\theta = -\frac{5}{3}$ $\cot\theta = -\frac{3}{4}$

b) Q(-12, -5)

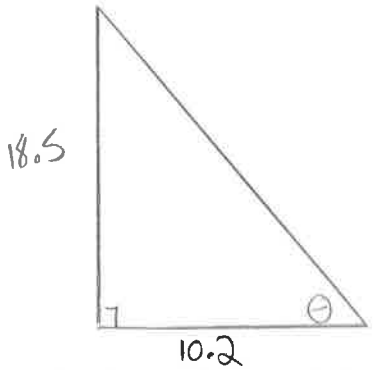
$\sin\theta = -\frac{5}{13}$ $\cos\theta = -\frac{12}{13}$ $\tan\theta = \frac{5}{12}$

$\csc\theta = -\frac{13}{5}$ $\sec\theta = -\frac{13}{12}$ $\cot\theta = \frac{12}{5}$





11) The shadow of a tree that is 18.5 m tall measures 10.2 m in length. Determine the angle of elevation of the sun.



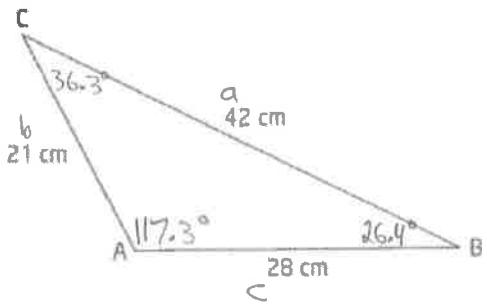
$\tan\theta = \frac{18.5}{10.2}$

$\theta = \tan^{-1}\left(\frac{18.5}{10.2}\right)$

$\theta = 61.1^\circ$

12) Solve each of the following triangles.

a)



$\cos A = \frac{42^2 - 21^2 - 28^2}{-2(21)(28)}$

$\cos A = -0.458333$

$\angle A = 117.3^\circ$

$\cos B = \frac{21^2 - 42^2 - 28^2}{-2(42)(28)}$

$\cos B = 0.8958333$

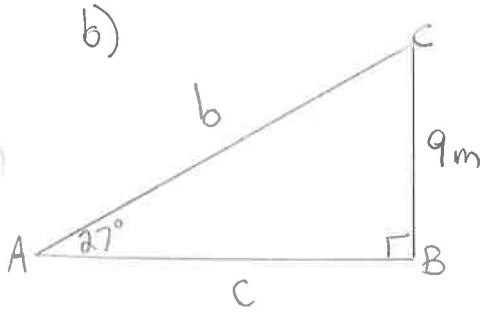
$\angle B = 26.4^\circ$

$\angle C = 180 - 117.3 - 26.4$

$\angle C = 36.3^\circ$

~~b)~~

b)



$\sin 27 = \frac{9}{b}$

$b = \frac{9}{\sin 27}$

$b = 19.8\text{m}$

$\tan 27 = \frac{9}{c}$

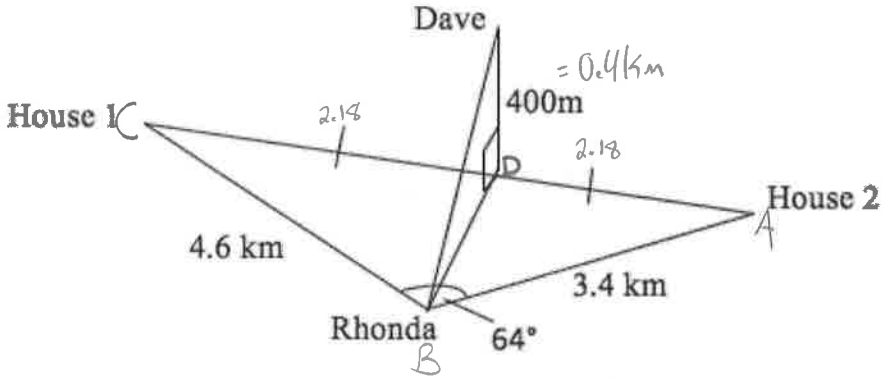
$c = \frac{9}{\tan 27}$

$c = 17.7\text{m}$

$\angle C = 180 - 27 - 90$

$= 63°$

13) Dave is in a hot air balloon 400m in the air exactly halfway between two houses on the ground. His wife, Rhonda, is at her friend's house which is 4.6km from the first house and 3.4km from the second house. The angle of the two houses, from Rhonda's point of view, is 64°. Find the angle of elevation if Rhonda looks up at Dave.



$b^2 = 4.6^2 + 3.4^2 - 2(4.6)(3.4)(\cos 64)$

$b = 4.36$

$\frac{4.36}{\sin 64} = \frac{4.6}{\sin A}$

$\sin A = \frac{4.6(\sin 64)}{4.36}$

$\angle A = 71.5°$

$BD^2 = 3.4^2 + 2.18^2 - 2(3.4)(2.18)(\cos 71.5)$

$BD = 3.4$

$\tan\theta = \frac{0.4}{3.4}$

$\theta = 6.7°$

14) Prove the following trigonometric identities.

14) Prove the Following trig identities.

a) $\sec\theta\cos\theta + \sec\theta\sin\theta = 1 + \tan\theta$

LS

$= \frac{1}{\cos\theta}(\cos\theta) + \frac{1}{\cos\theta}(\sin\theta)$

$= 1 + \frac{\sin\theta}{\cos\theta}$

$= 1 + \tan\theta$

RS

$= 1 + \tan\theta$

LS = RS

b) $tan^2x + cos^2x + sin^2x = \frac{1}{cos^2x}$

LS

$= \tan^2x + 1$

$= \frac{\sin^2x}{\cos^2x} + \frac{\cos^2x}{\cos^2x}$

$= \frac{1}{\cos^2x}$

RS

$= \frac{1}{\cos^2x}$

LS = RS

c) $csc^4x + cot^4x = 1 + 2csc^2xcot^2x$

LS

$= \frac{1}{\sin^4x} + \frac{\cos^4x}{\sin^4x}$

$= \frac{1 + \cos^2x(\cos^2x)}{\sin^4x}$

$= \frac{1 + (1 - \sin^2x)(\cos^2x)}{\sin^4x}$

$= \frac{\sin^2x + \cos^2x + \cos^2x - \cos^2x\sin^2x}{\sin^4x}$

$= \frac{\sin^2x - \cos^2x\sin^2x + 2\cos^2x}{\sin^4x}$

$= \frac{\sin^2x(1 - \cos^2x) + 2\cos^2x}{\sin^4x}$

$= \frac{\sin^2x(\sin^2x) + 2\cos^2x}{\sin^4x}$

$= \frac{\sin^4x + 2\cos^2x}{\sin^4x}$

RS

$= 1 + 2\left(\frac{1}{\sin^2x}\right)\left(\frac{\cos^2x}{\sin^2x}\right)$

$= 1 + \frac{2\cos^2x}{\sin^4x}$

$= \frac{\sin^4x}{\sin^4x} + \frac{2\cos^2x}{\sin^4x}$

$= \frac{\sin^4x + 2\cos^2x}{\sin^4x}$

LS = RS

d) $\frac{cotx-tanx}{csc^2x-sec^2x} = \frac{1}{cscxsecx}$

LS

$= \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}}$

$= \frac{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x}}$

$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \cdot \frac{\sin^2 x \cos^2 x}{\cos^2 x - \sin^2 x}$

$= \sin x \cos x$

RS

$= \frac{1}{\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)}$

$= 1 \cdot \frac{\sin x \cos x}{1}$

$= \sin x \cos x$

LS = RS

e) $\frac{cotx-tanx}{sinxcosx} = csc^2x - sec^2x$

LS

$= \frac{\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right)}{\sin x \cos x}$

$= \frac{\left(\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}\right)}{\sin x \cos x}$

$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \cdot \frac{1}{\sin x \cos x}$

$= \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x}$

RS

$= \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}$

$= \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x}$

LS = RS

~~Answers~~

1)

2) a) $43°$ **b)** $30°$

3) a) $\sin 330 = -\frac{1}{2}$; $\cos 330 = \frac{\sqrt{3}}{2}$ **b)** $\sin 135 = \frac{1}{\sqrt{2}}$; $\cos 135 = -\frac{1}{\sqrt{2}}$

4) $457°$ and $817°$

5) a) $342.3°$ and $162.3°$ **b)** $332.6°$ and $207.4°$

6) a) $-\frac{1}{\sqrt{2}}$ **b)** $-\frac{1}{\sqrt{3}}$

7) a) $\sec 120 = -2$ **b)** $\csc 150 = 2$ **c)** $\cot 225 = 1$

8) a) $60°$ and $120°$ **b)** $300°$ and $120°$

9) a) $\sin A = \frac{15}{17}, \tan A = -\frac{15}{18}$ **b)** $\cos B = -\frac{3}{5}, \tan B = \frac{4}{3}$

10) a) $\sin\theta = \frac{4}{5}, \csc\theta = \frac{5}{4}, \cos\theta = -\frac{3}{5}, \sec\theta = -\frac{5}{3}, \tan\theta = -\frac{4}{3}, \cot\theta = -\frac{3}{4}$

b) $\sin\theta = -\frac{5}{13}, \csc\theta = -\frac{13}{5}, \cos\theta = -\frac{12}{13}, \sec\theta = -\frac{13}{12}, \tan\theta = \frac{5}{12}, \cot\theta = \frac{12}{5}$

11) $61.1°$

12) a) $\angle A = 117.3°, \angle B = 26.4°, \angle C = 36.3°$ **b)** $b = 19.8, c = 17.7, \angle C = 63°$

13) $6.65°$

14) See posted solutions