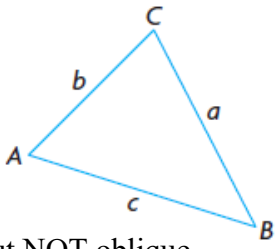
Lesson 5: The Ambiguous Case of the Sine Law

To solve a triangle means to determine the length of all sides and the measure of all angles.



The primary trig ratios are useful for solving right triangles, but NOT oblique triangles (triangles that are either acute or obtuse)

Sine Law $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Sine law can be used if you know two angles and any side (AAS or ASA), or two sides and one angle opposite a given side (SSA – a.k.a "The Donkey Case")

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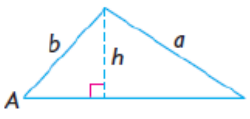
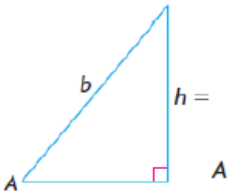
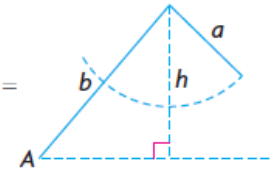
SSA: Given angle A is acute. There are 4 different possible outcomes as depicted in the chart. In one case, two triangles are actually possible based on the information provided. In this case the information is "ambiguous" and thus, you must solve for **both possibilities**.

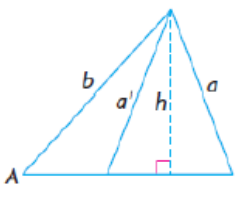
- Determine the vertical height of the triangle (using primary trig ratios)
  - ***Always draw the height so it doesn't cut into any given angles or sides.***
- Compare the height to the given sides. **Given ∠A & sides *a* & *b*:**

If ∠A is ACUTE			
$a \geq b$	$a < b$		
	Check the height		
	$h = a$	$h > a$	$h < a$
1 triangle	1 triangle (right angle)	No triangles	2 triangles









CAREFUL! These outcomes are based on given angle A and given sides a & b. You will need to adapt if different letters are used!**

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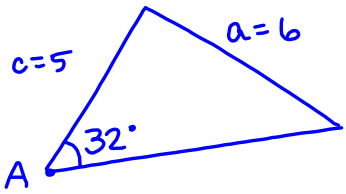
Ex 1) In ΔABC, ∠A = 32°, a = 6 cm, c = 5 cm. Draw and label a diagram then determine the number of solutions. Do not solve.



because $a > c$; 1 triangle that exist

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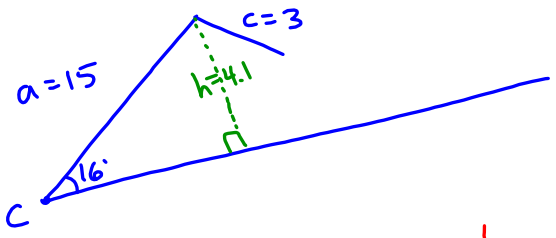
Ex 2) In ΔABC, ∠C is 16°, c = 3 m, a = 15 m. Draw and label a diagram then determine the number of solutions. Do not solve.



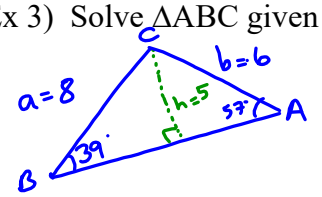
① because $c < a$, find h:

② $\sin 16° = \frac{h}{15}$

$h = 4.1$

∴ because $h > c$, No triangles exist.

Ex 3) Solve ΔABC given $\angle B = 39°$, $a = 8$ cm, $b = 6$ cm.



① because $b < a$, find height:

$\sin 39° = \frac{h}{8}$

$h = 5$

① $\frac{\sin 39°}{6} = \frac{\sin A}{8}$

$\angle A_1 = 57°$

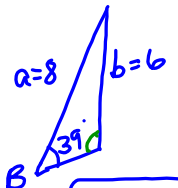
$\angle C_1 = 84°$

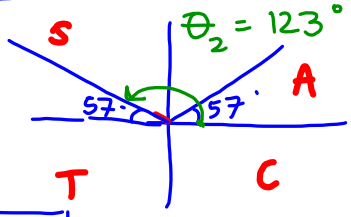
$180° - 39° - 57°$

$c_1 = 9.5$

$\frac{\sin 39°}{6} = \frac{\sin 84°}{c}$

$c = 9.5$





$\angle A_2 = 123°$

$\angle C_2 = 18°$

$180° - 123° - 39°$

$c_2 = 2.9$ cm

$\frac{\sin 39°}{6} = \frac{\sin 18°}{c}$

$c = 2.9$ cm

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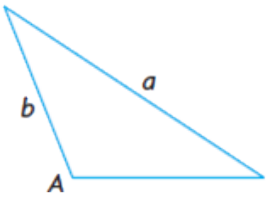
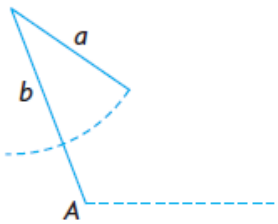
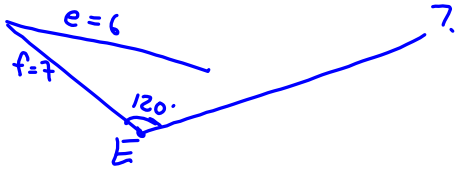
SSA: Given angle A is obtuse. There are 2 different possible outcomes as depicted in the chart. You do NOT need to determine the height. **Given $\angle A$ & sides a & b:**



If $a > b$	One triangle exists
If $a \leq b$	No triangle exists



Ex 4) In ΔDEF, $\angle E = 120°$, $e = 6$ cm and $f = 7$ cm. Draw and label a diagram then determine the number of solutions. Do not solve.



Because $e < f$; No triangles exist.

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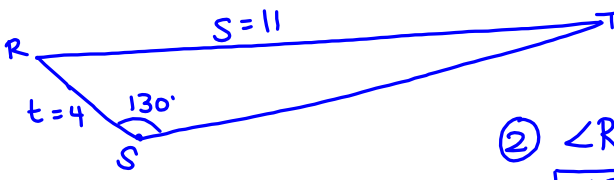
Ex 5) Solve ΔRST given ∠S = 130°, s = 11 m and t = 4 m

because $s > t$, 1 triangle exists.



① $\frac{\sin 130^\circ}{11} = \frac{\sin T}{4}$

$\angle T = 16^\circ$

② $\angle R = 180^\circ - 16^\circ - 130^\circ$

$\angle R = 34^\circ$

③ $\frac{\sin 130^\circ}{11} = \frac{\sin 34^\circ}{r}$

$r = 8$ m

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HW U3L5:

1. Handout
2. p. 318 #1b, 2, 4, 5 (# of triangles, don't solve), 13, 14

   *#4b s/b 68 degrees or 112 degrees
3. sign and correct trig quizzes