4.7 Applications Involving Exponential Functions

1) Exponential Growth / Appreciation

Increasing Function

$y = \underbrace{a \cdot b^x}$, $b > 1$

for growth/appreciation $b = 1 + \text{growth rate}$

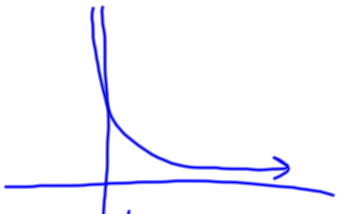
$1 + 0.08$

1.08

2) Exponential Decay / Depreciation

Decreasing Function

$y = \underbrace{a \cdot b^x}$, $0 < b < 1$



for decay/depreciation $b = 1 - \text{decay rate}$

$1 - 0.2 = 0.80$

The Exponential function:

$y = a \cdot b^x$

y → final amount

a → initial amount

x → number of growth/decay periods

b → growth rate = 1 + growth rate; decay rate = 1 − decay rate

Function	Exponential Growth or Decay?	Initial Value	Growth or Decay Rate
$V(t) = 20(\cancel{1.02})^t$	Growth	20	0.02 = 2%
$P(n) = (\cancel{0.8})^n$	Decay	1	1 − decay = 0.8 ∴ decay = 20%
$A(x) = 0.5(\underline{3})^x$	Growth	0.5	200%
$Q(w) = 6\underline{00}\left(\frac{5}{8}\right)^w$	Decay	600	1 − decay rate = $\frac{5}{8}$

$1 - \frac{5}{8} = \text{decay rate}$

$\frac{8}{8} - \frac{5}{8} = \text{decay}$

$\frac{3}{8} = \text{decay rate.}$

EX1: When the Sens enter the playoffs, their fan base increases by approximately 4% daily. If the Sens had 600,000 fans prior to the start of the playoffs, **how many fans will the Sens have after 6 playoff days?**

$y = a \cdot b^x$

growth

$b = 1 + \text{growth rate}$

$a = 600{,}000$

$= 1 + 0.04$

$= 1.04$

$y = 600(1.04)^x$ where y is in thousands

$x = 6$ days

$y = 600(1.04)^6$

$y = 759.191 \Rightarrow 759{,}191$ fans

EX2: The municipality of Wood Buffalo, Alberta has experienced a large population increase in recent years due to the discovery of one of the world's largest oil deposits. Its population, of 35,000 in 1996, has grown at an annual rate of approximately 8%

How long will it take for the population to double?

growth function
$b = 1 + \text{growth rate}$
$= 1 + 0.08 = 1.08$

$y = a \cdot (1.08)^x$

Assume 1996 is $x = 0$

$y = 35(1.08)^x$ y is given in thousands

$\frac{70}{35} = \frac{35(1.08)^x}{35}$

$2 = (1.08)^x$

$\boxed{x = \frac{\log(2)}{\log(1.08)}}$

$x = 1, \ y = 1.08$
$x = 1.5, \ y = (1.08)^{1.5} = 1.12$
$x = 5, \ y = (1.08)^5 = 1.46$
$x = 10, \ y = (1.08)^{10} = 2.15$
$x = 9, \ y = (1.08)^9 = 1.99905$

$\rightarrow x = 9$ years

$\therefore$ It takes 9 years for the pop. to double.

ex: $2^x = 5$
$x = \frac{\log 5}{\log 2}$

$x = 0$ at 1996
$x = 9$ at 2005

EX3: A 200g sample of radioactive substance has a *half-life* of 138 days. This means that every 138 days, the amount of substance left in the sample is half of the original amount. The mass of the substance, in grams, that remains after *t*-days can be modelled by the function:

$$M(t)=200\boxed{\left(\frac{1}{2}\right)^{\frac{t}{138}}}$$

a) Determine the mass that remains after 5 years

b) How long does it take for this 200g sample to decay to 110g?

5 years = 1825 days

$5(365 \text{ days/yr})$

a) $M(t)=200\left(\frac{1}{2}\right)^{\frac{t}{138}}$

$M(t)=200\left(\frac{1}{2}\right)^{\frac{1825}{138}}$

$=200\left(\frac{1}{2}\right)^{13.22}$

$M(t)=200(0.0001045)$

$1.045E^{-4}$

$.0001045 \times 10^{-4}$

$=0.021g$

b) $y=110g$

$\frac{110}{200}=\frac{200\boxed{\left(\frac{1}{2}\right)^{\frac{t}{138}}}}{200}$

$0.55=\left(\frac{1}{2}\right)^{\frac{t}{138}}$

$\frac{t}{138}=\frac{\log(0.55)}{\log(0.5)}$

$\cancel{138}\left(\frac{t}{\cancel{138}}\right)=(0.8625)138$

$t=119.02$ days

RECALL

$2^x=5$

$x=\frac{\log 5}{\log 2}$

EX4: A biologist tracks the population of a new species of frog over several years. From the table of values below, **determine an equation that models the frog's population growth, and determine the number of years before the population triples.**

Year	0	1	2	3	4	5
Population	400	480	576	691	829	995

1st diff. 80 96 115 138 166

×1.2 1.2 1.2 1.2 ⇒ b=1.2

a) Find an equation

b) find # years before pop. triples

a) $y=a\cdot b^x$

$a=400$ $\quad b=1.2$ $\quad y=400(1.2)^x$

b) final population: $400\times 3=1200$

$$\frac{1200}{400}=\frac{400(1.2)^x}{400}$$

$$3=(1.2)^x$$

$$x=\frac{\log(3)}{\log(1.2)}$$

$$x=6.026$$

6.03 years

EX5: A new car costs $24,000. Each year, the value of the car depreciates by approximately 18%.

Determine the value of the car after 30 months.

decreasing function

$b = 1 - \text{decay rate}$ (depreciation rate)

$b = 1 - 0.18$

$b = 0.82$

$a = 24{,}000$

$Y = 24{,}000(0.82)^x$

$\$100$

$8\% = 1 + 0.08$

$100(1.08)^x$

$30 \text{ months}\left(\frac{1 \text{ year}}{12 \text{ months}}\right) = 2.5 \text{ years}.$

$Y = 24{,}000(0.82)^{2.5}$

$= 24{,}000(0.6089) = \$14{,}613$

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