

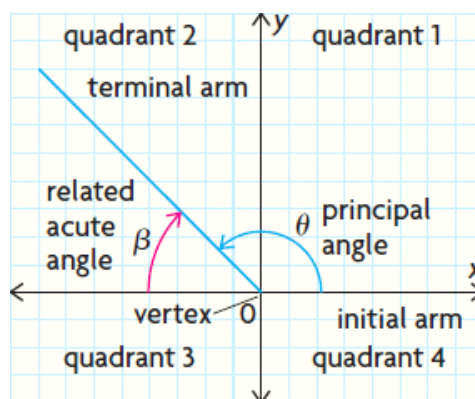
## Lesson 3: Trig Ratios for Angles between $0^\circ$ and $360^\circ$

### Important Terminology

An angle has 3 parts: **Initial arm**, **vertex**, **terminal arm**

For an angle to be in **standard position**, it must meet the following criteria:

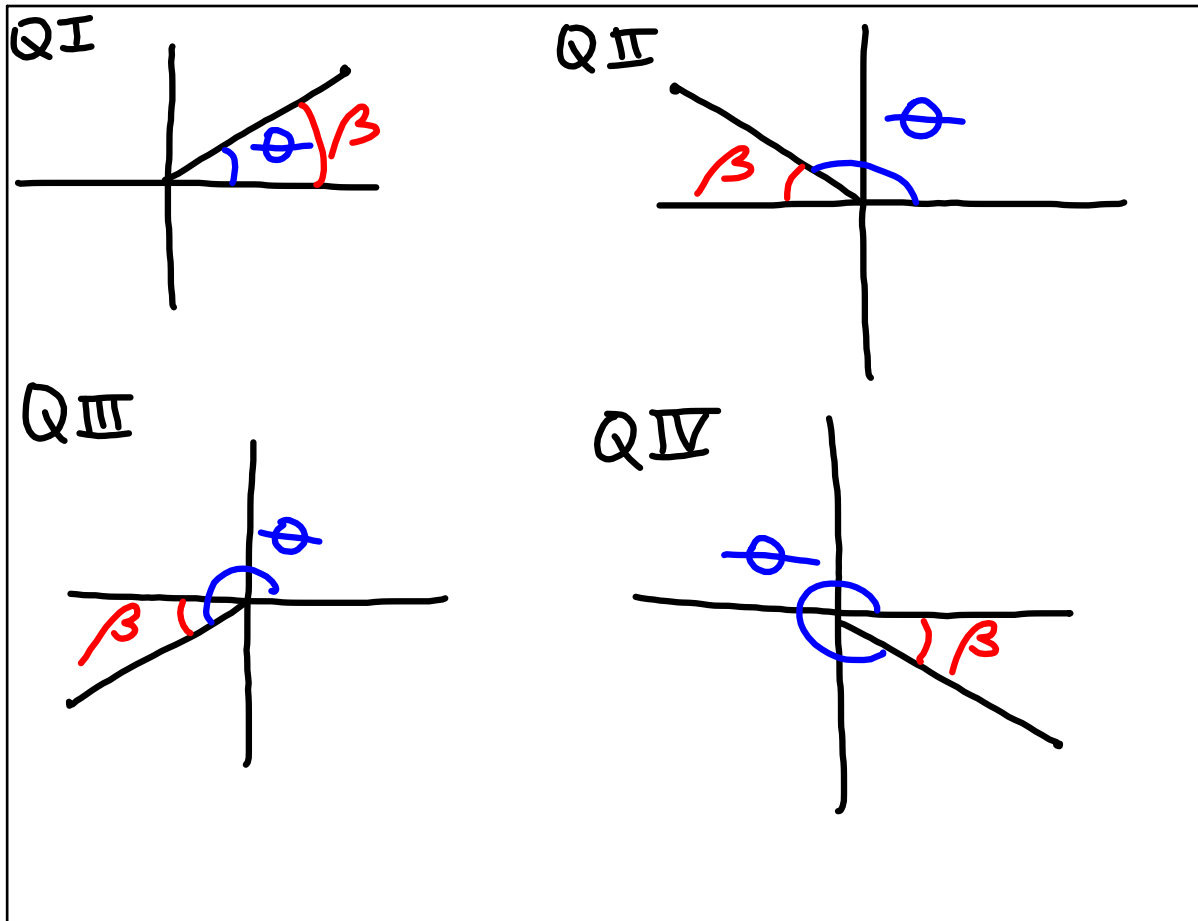
- Vertex must be at the origin
- Initial arm must be on the positive  $x$ -axis
- Angle is measured from initial arm to terminal arm



The **principal angle ( $\theta$ )** is the counter clockwise angle between the initial arm and the terminal arm of an angle in standard position. Its value is between  $0^\circ$  and  $360^\circ$ .

The **related acute angle ( $\beta$ )** is the acute angle between the terminal arm of an angle in standard position and the  $x$ -axis when the terminal arm lies in quadrants 2, 3 or 4.

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**Exploration using GSP**

**A.**  $P(4, 3)$  lies on the terminal arm of an angle in standard position. Determine the primary trigonometric ratios for BOTH  $\theta$  and  $\beta$ . Record these ratios in the first 2 lines of the chart below.

**B.** Reflect point  $P$  in the  $y$ -axis; it should now be in the *second quadrant*. Record the values of the primary trig ratios for the principal angle  $\theta$  and the related acute angle  $\beta$  in the 3<sup>rd</sup> and 4<sup>th</sup> row of the table.

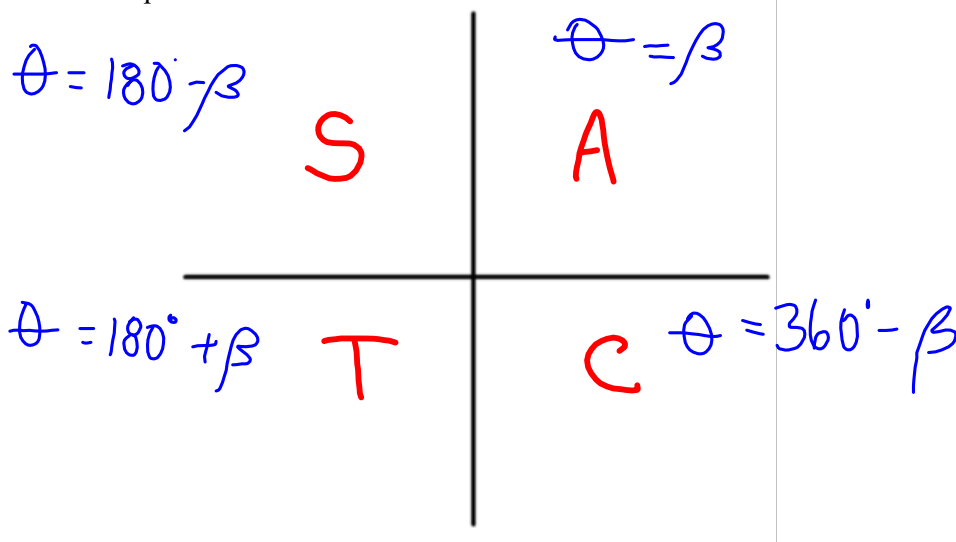
**C.** Reflect point  $P$  in the  $x$ -axis; it should now be in the *third quadrant*. Record the values of the primary trig ratios for the principal angle  $\theta$  and the related acute angle  $\beta$  in the 5<sup>th</sup> and 6<sup>th</sup> row of the table.

**D.** Reflect point  $P$  in the  $y$ -axis; it should now be in the *fourth quadrant*. Record the values of the primary trig ratios for the principal angle  $\theta$  and the related acute angle  $\beta$  in the 7<sup>th</sup> and 8<sup>th</sup> row of the table.

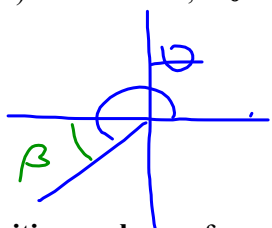
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**Summary**

1. In which quadrants were each of the primary trig ratios for  $\theta$  positive? <sup>1<sup>st</sup></sup>
2. What do you notice about the sign of the primary trig ratios for  $\beta$  for all quadrants? *all positive*
3. What do you notice about the *absolute value* of the ratios for the principal angle compared to the ratios for the related acute angle? *all the same*
4. For each quadrant, determine an expression for  $\theta$  in terms of  $\beta$ .



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Ex 1) Determine  $\beta$ , if  $\theta = 220^\circ$ 

$$\theta = 220^\circ$$

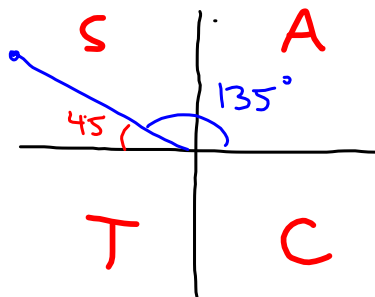
$$\beta = 220^\circ - 180^\circ$$

$$\beta = 40^\circ$$

**Positive angles** are formed by a **counter clockwise** rotation of the terminal arm.

**Negative angles** are formed by a **clockwise** rotation of the terminal arm.

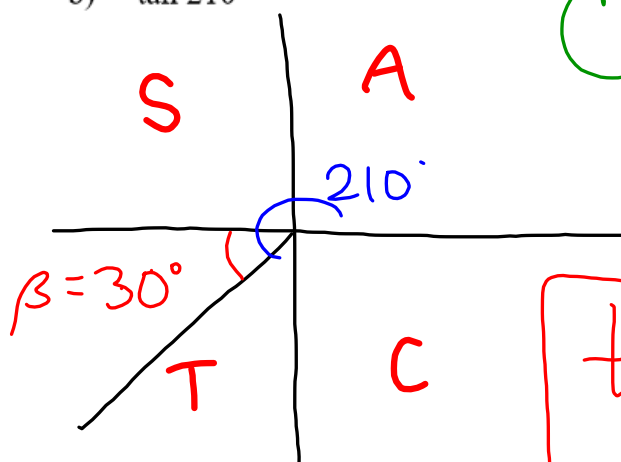
Ex 2) Evaluate without the use of a calculator (special triangles!)

a)  $\cos 135^\circ$ 

recall:  $\cos 45^\circ = \frac{\sqrt{2}}{2}$

$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

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b)  $\tan 210^\circ$ 

recall:  $\tan 30^\circ = \frac{\sqrt{3}}{3}$

$$\tan 210^\circ = \frac{\sqrt{3}}{3}$$

type  $\sin 37^\circ$  into your calculator ...

Angles	Quadrant	Sine Ratio	Cosine Ratio	Tangent Ratio
Principal angle $\theta = 37^\circ$	1	0.60	0.80	0.75
Related acute angle $\beta = 37^\circ$		0.60	0.80	0.75
Principal angle $\theta = 143^\circ$	2	0.60	-0.80	-0.75
Related acute angle $\beta = 37^\circ$		0.60	0.80	0.75
Principal angle $\theta = 217^\circ$	3	-0.60	-0.80	0.75
Related acute angle $\beta = 37^\circ$		0.60	0.80	0.75
Principal angle $\theta = 323^\circ$	4	-0.60	0.80	-0.75
Related acute angle $\beta = 37^\circ$		0.60	0.80	0.75

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c)  $\csc 300^\circ$

recall:  $\sin 60^\circ = \frac{\sqrt{3}}{2}$   
 $\therefore \csc 60^\circ = \frac{2}{\sqrt{3}}$

$\csc 300^\circ = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\csc 300^\circ = -\frac{2\sqrt{3}}{3}$

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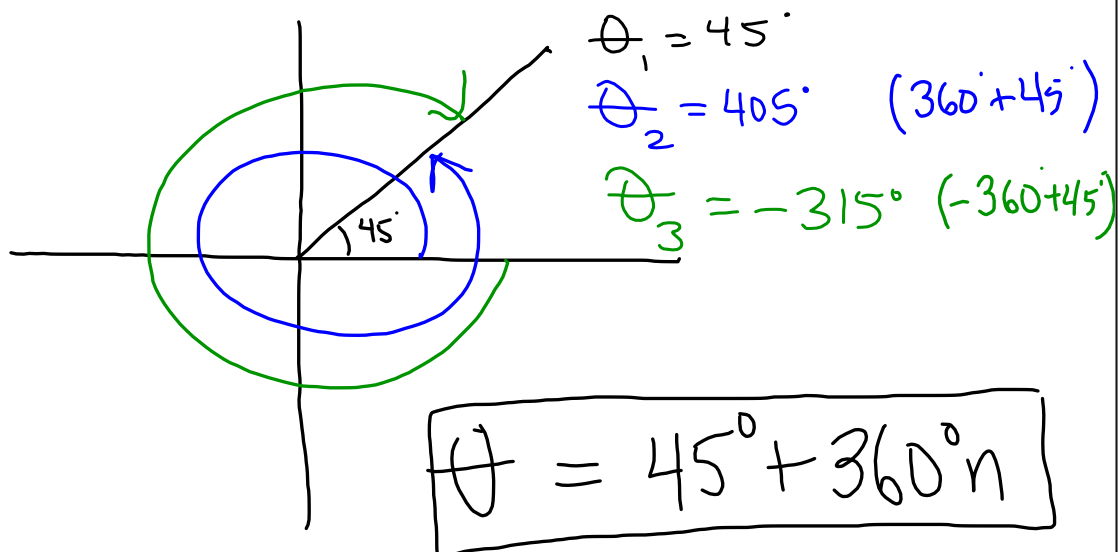
d)  $\sec(-120^\circ)$

Recall:  $\cos 60^\circ = \frac{1}{2}$   
 $\therefore \sec 60^\circ = 2$

$\sec(-120^\circ) = -2$

**Co-terminal angles** have the same initial arm, the same terminal arm but have different angle measurements.

Ex 3) Determine 3 angles that are co-terminal to  $45^\circ$



HW U3L3:

1. p. 292 #4
2. p. 299 #1bc, 12ab
3. handout