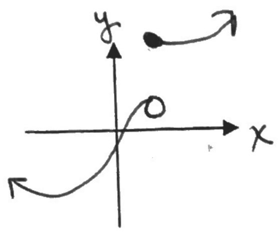
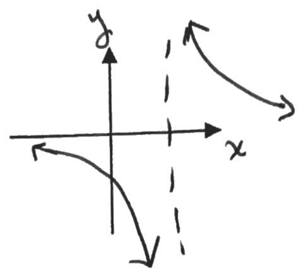
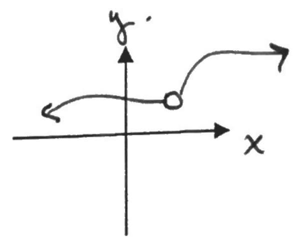
Functions can have 3 types of discontinuities:

1. Jump discontinuity
2. Infinite discontinuity
3. Removable discontinuity

1. 

2. 

3. 

Ex. 1 Consider the function:

$$f(x)=\begin{cases} x^2-3 & if \quad x\le -1 \\ x-1 & if \quad x>-1 \end{cases}$$

By evaluating one-sided limits, determine whether $\lim\limits_{x\to -1} f(x)$ exists. Sketch the graph of $f(x)$.

$\lim\limits_{x\to -1^-} f(x)$

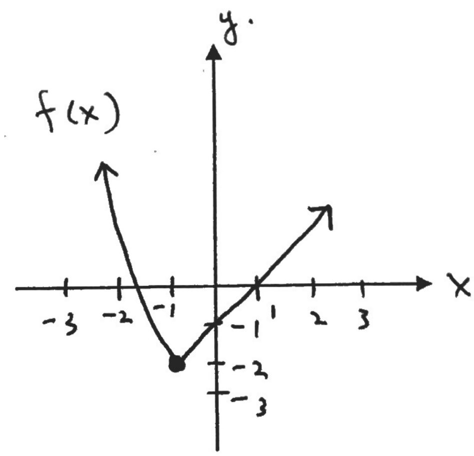
$= \lim\limits_{x\to -1^-} x^2-3$

$= -2$

$\lim\limits_{x\to -1^+} f(x)$

$= \lim\limits_{x\to -1^+} x-1$

$= -2$

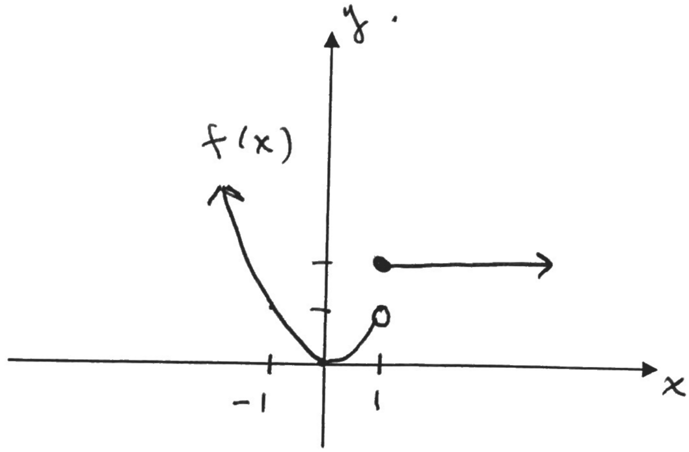


$\therefore \lim\limits_{x\to -1} f(x) = -2$

- Note that $f(-1)=-2$ and hence $\lim\limits_{x\to -1} f(x)=f(-1)$
- This equality describes the fact that the separate parts of the graph meet at $x=-1$ without a break, so that $f(x)$ is continuous at $x=-1$.

Ex. 2 By sketching the graph of the function, find all x values at which the function is discontinuous.

$$f(x)=\begin{cases} x^2 & \text{if } x<1 \\ 2 & \text{if } x\geq 1 \end{cases}$$



From the graph:

Discontinuous at $x=1$

Algebraically: $\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1^+} f(x) = 2$ $\Rightarrow \lim_{x\to 1} f(x)$ DNE

$\therefore$ Discontinuous at $x=1$.

Ex. 3 Let $f(x)=\begin{cases} \frac{1}{x} & \text{if } x\leq -1 \\ \frac{x-1}{2} & \text{if } -1<x<+1 \\ \sqrt{x} & \text{if } x\geq 1 \end{cases}$

a) Find the following limits.

i) $\lim_{x\to -1^-} f(x)$ $= -1$

ii) $\lim_{x\to -1^+} f(x)$ $= -1$

iii) $\lim_{x\to 1^-} f(x)$ $= 0$

iv) $\lim_{x\to 1^+} f(x)$ $= 1$

b) Sketch the graph of $f(x)$.

c) Where is $f(x)$ discontinuous? @ $x=1$.

Date: ______________ Name: ______________________

Intuitive Approach to Limits

1. Based on the graph below, determine:

a) $\lim_{x \to -12^-} f(x) = $ 0

b) $\lim_{x \to -9^+} f(x) = $ 0

c) $\lim_{x \to 5^-} f(x) = $ 4

d) $\lim_{x \to -9^-} f(x) = $ −4

e) $\lim_{x \to -6^+} f(x) = $ 5

f) $\lim_{x \to 0^+} f(x) = $ 2

g) $\lim_{x \to -6^-} f(x) = $ 5

h) $\lim_{x \to -3^+} f(x) = $ 1

i) $\lim_{x \to 2^+} f(x) = $ 3

j) $\lim_{x \to -3^-} f(x) = $ 1

k) $\lim_{x \to 0^-} f(x) = $ 1

l) $\lim_{x \to 5^+} f(x) = $ $-\infty$

m) $\lim_{x \to -12^+} f(x) = $ 0

n) $\lim_{x \to 2^-} f(x) = $ 3

State whether or not the following limits exist. State the values of those limits that do exist.

a) $\lim_{x \to -12} f(x) = $ 0

b) $\lim_{x \to 5} f(x) = $ DNE

c) $\lim_{x \to -3} f(x) = $ 1

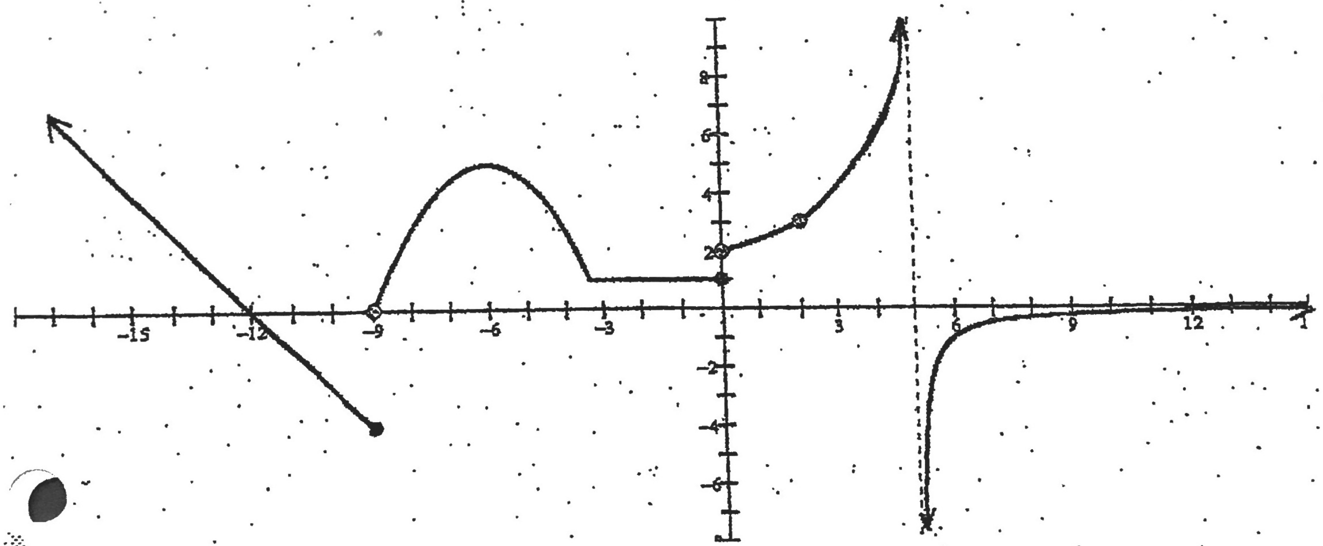
d) $\lim_{x \to -6} f(x) = $ 5

e) $\lim_{x \to -9} f(x) = $ DNE

f) $\lim_{x \to 2} f(x) = $ 3

g) $\lim_{x \to 0} f(x) = $ DNE

h) $\lim_{x \to \infty} f(x) = $ 0



2. a) Given the function defined by:

$$f(x) = \begin{cases} x \text{ if } x \geq 1 \\ x^2 \text{ if } x < 0 \end{cases}$$

explain why:

$$\lim_{x \to 1} f(x) \neq 1$$

The $\lim_{x \to 1^-} f(x)$ DNE

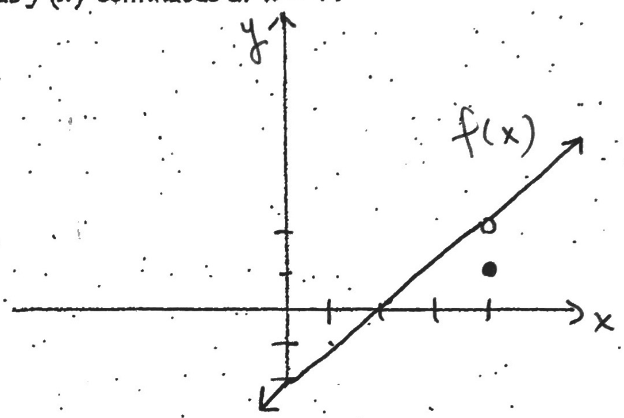
b) Let the function f be defined by:

$$f(x) = \begin{cases} x-2 \text{ if } x \neq 4 \\ 1 \text{ if } x = 4 \end{cases}$$

i) Draw the graph of this function.

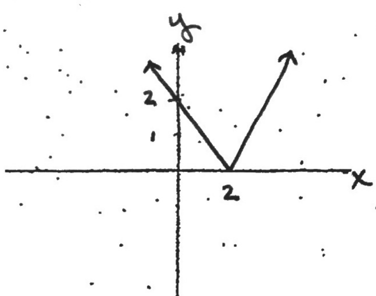
ii) $\lim_{x \to 4} f(x) = ?$

iii) Is $f(x)$ continuous at $x = 4$?

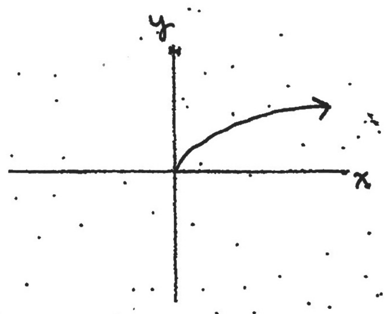


By drawing the graph for each of the following, find the limits:

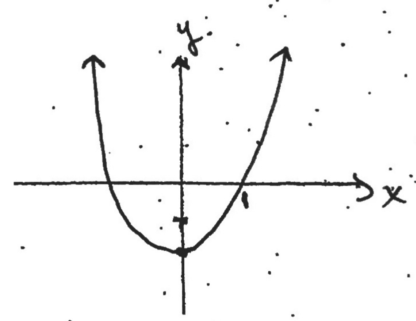
a) $\lim_{x \to 0^+} |x-2| =$ 2



b) $\lim_{x \to 0^+} \sqrt{x} =$ 0



c) $\lim_{x \to 1} (x^2 - 2) =$ -1



1. Find the slope of the secant line to the curve $f(x) = 3x^2 + 4x - 10$ when $x_1 = -2$ and $x_2 = 3$.

$$m_{sec} = \frac{f(3) - f(-2)}{3 - (-2)}$$

$$= \frac{29 + 6}{5} = 7$$

2. Find the slope of the tangent line to the curve $f(x) = 3x^2 + 4x - 10$ when $x = 2$.

$$m_t = \lim_{h \to 0} \frac{3(x+h)^2 + 4(x+h) - 10 - (3x^2 + 4x - 10)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + \cancel{4x} + 4h \cancel{-10} \cancel{-3x^2} - \cancel{4x} \cancel{+10}}{h}$$

$$= \lim_{h \to 0} \frac{6x\cancel{h} + 3h^{\cancel{2}} + 4\cancel{h}}{\cancel{h}} \qquad \therefore m_t|_{x=2} = 16$$

$$= 6x + 4$$

3. Find the equation of the tangent line to the curve $f(x) = 3x^2 + 4x - 10$ when $x = 2$.
   - Hint: use the point on the curve that the tangent line touches to solve $y = mx + b$

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 16(x - 2)$$

4. Find the slope of the tangent line to the curve $f(x) = \frac{2}{x+1}$ when $x = 3$.

$$m_t = \lim_{h \to 0} \frac{\frac{2}{x+h+1} - \frac{2}{x+1}}{h} \qquad \therefore m_t|_{x=3} = \frac{-2}{16}$$

$$= \lim_{h \to 0} \frac{2(x+1) - 2(x+h+1)}{h(x+h+1)(x+1)} \qquad = \frac{-1}{8}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} \cancel{+2} \cancel{-2x} - 2\cancel{h} \cancel{-2}}{\cancel{h}(x+h+1)(x+1)}$$

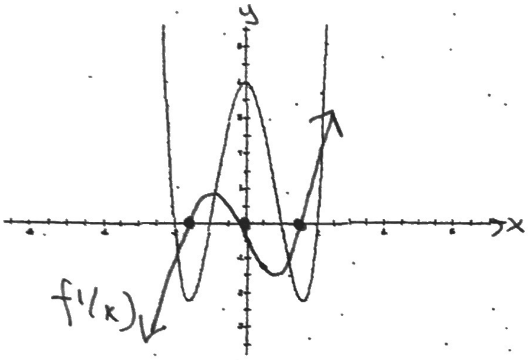
$$= \frac{-2}{(x+1)^2}$$

Name________________

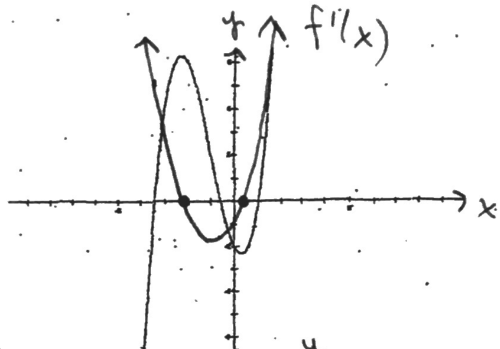
GRAPH OF THE DERIVATIVE FUNCTION

Given graph of function, sketch graph of derivative on the same grid:

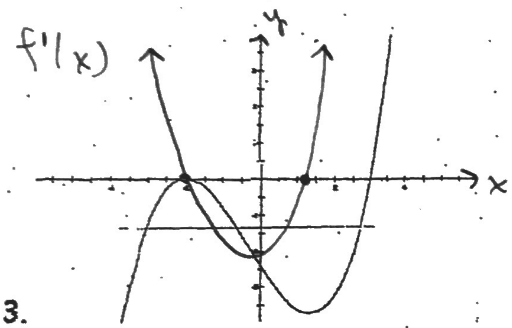
1\.



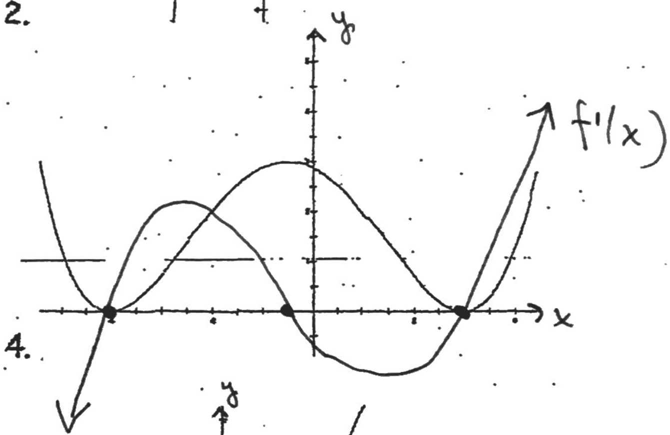
2\.



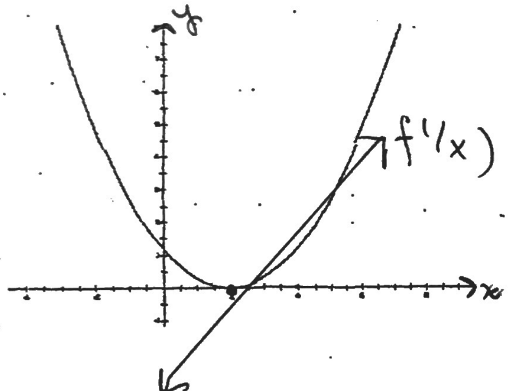
3\.



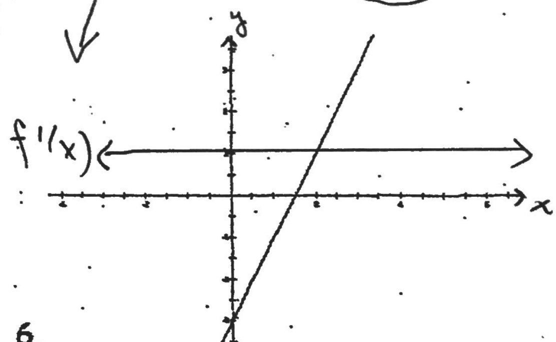
4\.



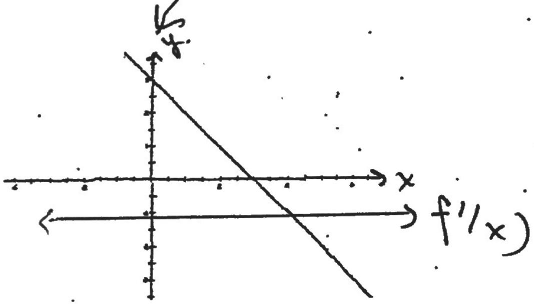
5\.



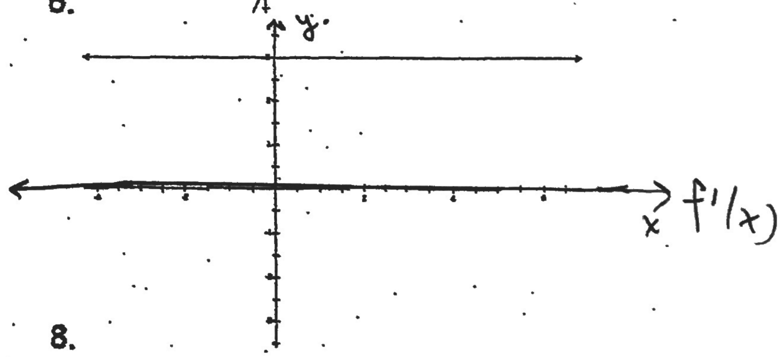
6\.



7\.



8\.



Quiz: Limits Name: ______________

ind the following limits.

a) $\lim_{x\to 0} \pi$ ✓

$= \pi$

b) $\lim_{x\to -1} 2x^2 - 5x + 3$ ✓

$= 10$

c) $\lim_{x\to 4} \frac{x^2+2x-6}{x-4}$ ✓✓

$=$ DNE (V.A.)

d) $\lim_{x\to 4} \frac{x^2-16}{3x-12}$ ✓✓

$= \lim_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{3\cancel{(x-4)}}$

$= \frac{8}{3}$

e) $\lim_{h\to 0} \frac{h}{\sqrt{3+h}-\sqrt{3}}$ ✓✓✓

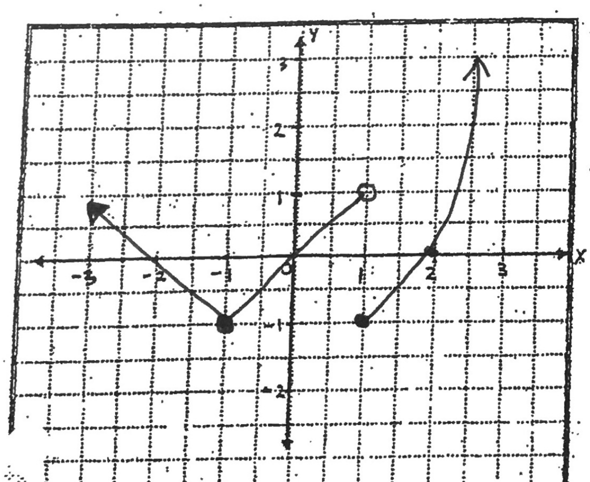
$= \lim_{h\to 0} \frac{h}{(\sqrt{3+h}-\sqrt{3})} \frac{(\sqrt{3+h}+\sqrt{3})}{(\sqrt{3+h}+\sqrt{3})}$

$= \lim_{h\to 0} \frac{\cancel{h}(\sqrt{3+h}+\sqrt{3})}{\cancel{3}+\cancel{h}-\cancel{3}}$

$= 2\sqrt{3}$

2. Sketch the graph of the function $f(x) = \begin{cases} -x-2, & \text{if } x \le -1 \\ x, & \text{if } -1 < x < 1 \\ x^2 - 2x, & \text{if } x \ge 1 \end{cases}$. ✓✓✓

Determine $\lim_{x\to -1} f(x)$ and $\lim_{x\to 1} f(x)$. ✓✓✓✓



* $\lim_{x\to -1^-} f(x) = -1$

* $\lim_{x\to 1} f(x) =$ DNE

30