

Limits at Infinity

Given $\lim_{x \rightarrow \infty} f(x)$ there are four (4) possible values for this limit.

$$\lim_{x \rightarrow \infty} f(x) = \begin{cases} 0 \\ +\infty \\ -\infty \\ \frac{a}{b}, \quad a \text{ & } b \text{ constants} \end{cases}$$

Let $f(x) = \frac{P_m(x)}{Q_n(x)}$, where m and n are the degrees of the polynomials $P(x)$ and $Q(x)$.

Note: To determine the value of $\lim_{x \rightarrow \infty} f(x)$, divide the numerator and the denominator by the highest degree of the denominator.

CASE 1: $m < n$

ex: $\lim_{x \rightarrow \infty} \frac{x+1}{x^2+4}$

Summary:

CASE 2: $m > n$

ex: $\lim_{x \rightarrow \infty} \frac{x^3 + x^2 + 4}{3x^2 + 1}$

ex: $\lim_{x \rightarrow -\infty} \frac{x^2 + 2}{x - 1}$

Summary:

CASE 3: $m = n$

ex: $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 3}{4x^3 + x^2 + x + 1}$

Summary: