Unit-6: Introduction to Vectors

Day	Date	Topic	Textbook Reference Student Work
1		6.1 Introduction to Vectors 6.2 Vector Addition	p. 279, #1, 4, 8ac, 9, 10, 11ab p. 290, #2, 5, 6, 7, 9, 11, 13, 14, 15, 16
2		6.3 Multiplication of a Vector by a Scalar; and 6.4 Properties of Vectors	p. 298, #1, 3, 7, 13, 14, 15, 17, 21 p. 306, #6, 7, 9, 10, 11
3		6.5 Vectors in R^2 and R^3	p. 316, #3, 6, 7, 12, 13, 14, 17, 19
4		6.6 Operations with Vectors in R^2	p. 324, #3, 4, 6ac, 8ac, 9, 10, 12, 13a, 14, 16
5		6.7 Operations with Vectors in R^3; and	p. 332, #1, 4, 6ac, 11, 12, 13, 14
6		6.8 Linear Combinations and Spanning Sets	p. 340 #2, 4, 6, 8, 9, 11, 13
7		Review	p. 344, #1, 3, 9, 11a, 12, 15, 16a, 17, 23, 24
8		Test	

6.1 Introduction to Vectors
6.2 Vector Addition

Learning Goals / Success Criteria:
Represent vectors as directed line segments / p.280 #6.
Recognize a vector as a quantity with both magnitude and direction / p.281 #10.
Perform mathematical operations on geometric vectors / p.292 #12.

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In mathematics and science, you often encounter different types of numbers which can be separated into two groups or types of numbers:

A scalar is ____

A vector is ____

1. Identify of each of the following is a scalar or a vector.

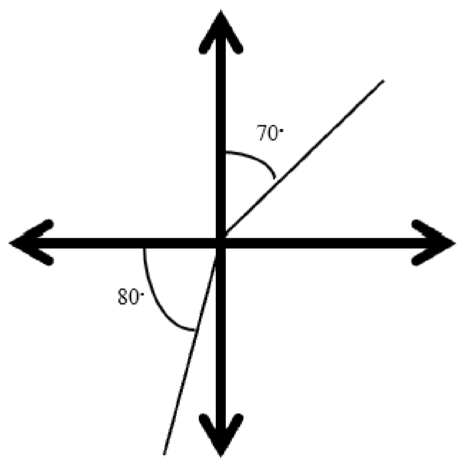
Height	S	Temperature	S
Weight A scale uses the downward acceleration of gravity to determine weight; weight is sometimes used as a synonym for force; $weight\ (in\ Newtons) = mass\ (in\ kg) \times 9.8\ m/s^2$	V	Mass	S
Area	S	Volume	S
Distance	S	Displacement	V
Speed	S	Force	V
Velocity	V	Friction	V
A ball is rolling along a floor with a speed of 2.5 m/s		A dog walks up a ramp inclined at an angle of 3° to the horizontal at 4km/h	
Jack is pushing a crate with a force of 250 N toward the west.		A box has a weight of 84 N	
A sound has an intensity of 50 dB		A cat has mass 12 kg	
A train travels northwest at 115 km/h		The high temperature yesterday was 12°	
A bird flies 500 m north		A car drives east for 4 hours	

A vector can be represented in many ways:

Words	Geometric Vector	Symbols using endpoints	Symbols with a single letter

Directions can be given with a "true bearing" (degrees clockwise from north) or with a "quadrant bearing" (direction-angle-direction).

2. (a) Find the true bearing and quadrant bearing of the two lines in the diagram.



(i) For the line in QI.

(ii) For the line in QIII.

(b) Find the true bearing of $S12°W$.

(c) Find the quadrant bearing of $100°$.

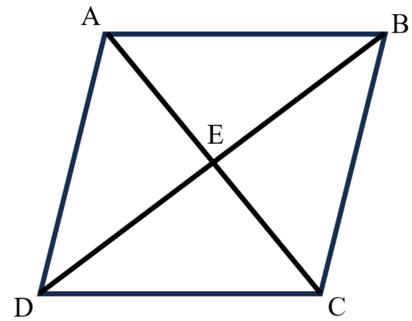
3. Draw a geometric vector for each of the following.

(a) $\vec{v} = 2$ km/h at a bearing of 020°

(b) $\vec{v} = 40$ km/h at a quadrant bearing of S50°E

3. Given rhombus $ABCD$ with diagonals as shown, identify the following.

(a) All vectors parallel to $\overrightarrow{AB}$.



(b) All vectors equal to $\overrightarrow{DC}$.

(c) All vectors opposite to $\overrightarrow{CA}$.

(d) All vectors with the same magnitude as $\overrightarrow{BC}$.

(e) All vectors equal to $\overrightarrow{DE}$.

The **magnitude** of a vector $\overrightarrow{AB}$ is denoted by $|\overrightarrow{AB}|$.

Also, **opposite vectors** are: Vectors that have the same magnitude but point in opposite directions.
Therefore, if $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are opposite then $|\overrightarrow{AB}| = |\overrightarrow{BA}|$ and $\overrightarrow{AB} = -\overrightarrow{BA}$

And **equal vectors** are: Vectors that have the same direction and have the same magnitude.
Therefore $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal (or equivalent) if and only if:
$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel
and the direction from A to B is the same as from C to D
and $|\overrightarrow{AB}| = |\overrightarrow{CD}|$

To **add vectors $\vec{a} + \vec{b}$**, we draw them head-to-tail and form a triangle by connecting the tail of $\vec{a}$ to the head of $\vec{b}$. The new vector is called the **resultant**. (Note: another method to add vectors is to draw them tail-to-tail and form a parallelogram).

4. A hiker starts at point A, travels 8 km north to point B and then 6 km east to point C. Find the resulting displacement.

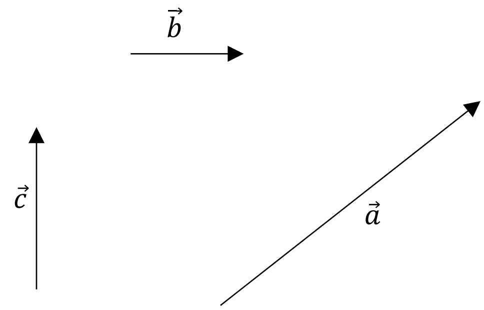
When we state, "the angle formed by two vectors," we are referring to the **angle formed with the vectors placed tail to tail**.

5. (a) Vectors $\vec{a}$ and $\vec{b}$ form a $60°$ angle. Given $|\vec{a}| = 3, |\vec{b}| = 2$, determine $|\vec{a} + \vec{b}|$.

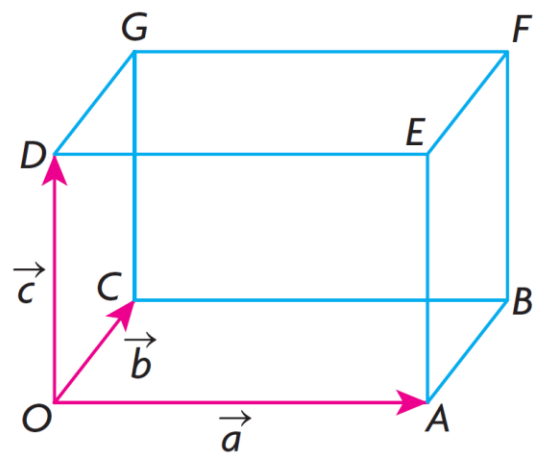
To **subtract vectors $\vec{a} - \vec{b}$**, we either: (1) add the opposite of $\vec{b}$ to $\vec{a}$; or (2) draw the vectors tail-to-tail and find the vector that must be added to $\vec{b}$ to get to $\vec{a}$.
The new vector is called the resultant.

5. (b) Determine $|\vec{a} - \vec{b}|$.

6. Given vectors $\vec{a}, \vec{b}, \vec{c}$, draw a diagram that find the resultant $\vec{a} - \vec{b} + \vec{c}$



7. In the rectangular box shown, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, $\overrightarrow{OD} = \vec{c}$.
Express each of the following in terms of $\vec{a}, \vec{b}, \vec{c}$:



(a) $\overrightarrow{BC}$

(b) $\overrightarrow{EF}$

(c) $\overrightarrow{OB}$

(d) $\overrightarrow{AC}$

(e) $\overrightarrow{BG}$

(f) $\overrightarrow{OF}$

8. An airplane heads due south at a speed of 300 km/h and meets a wind from the west at 100 km/h. Find the resultant velocity of the plane (relative to the ground).

6.3 Multiplication of a Vector by a Scalar

Learning Goals / Success Criteria:
Perform mathematical operations on geometric vectors / p.300 #12.
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1. An airplane is heading north at 1000 km/h. The airplane's velocity is represented by $\vec{v}$. Draw the vectors, and state what each vector represents:

$\vec{v}$ $\frac{1}{2}\vec{v}$ $-\vec{v}$ $-2\vec{v}$

When a vector $\vec{v}$ is multiplied by a scalar, k, then

the magnitude ____________________, and the direction ______________________.

If $k\vec{v} = \vec{u}$, then $\vec{v}$ and $\vec{u}$ are: ___________________.

A **unit vector** is a vector one unit long.

2. Vectors $\vec{x}$ and $\vec{y}$ are unit vectors that form a $30°$ angle with each other.

(a) Find $|2\vec{x} - \vec{y}|$

(b) Find the direction of $2\vec{x} - \vec{y}$

What happens to a vector $\vec{x}$ when it is multiplied by $\frac{1}{|\vec{x}|}$?

3. Given $|\vec{u}| = 4, |\vec{v}| = 5$ and the angle between $\vec{u}$ and $\vec{v}$ is $120°$, determine a unit vector in the same direction as $\vec{u} + \vec{v}$.

6.4 Properties of Vectors

Learning Goals / Success Criteria:
Determine some properties of the operations performed on vectors / p.306 #7.
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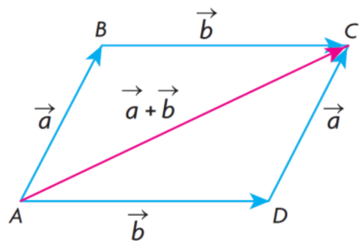
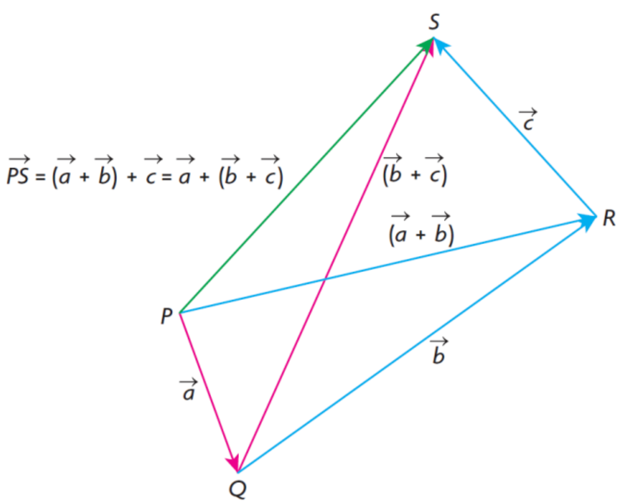
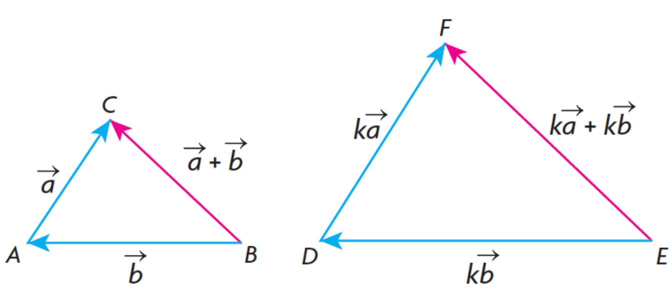
Is $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ for all vectors? If so, we define vector addition to be commutative. Use a diagram to illustrate.



Is $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$? If so, then vector addition is associative.



Is $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$? If so, then vector addition is distributive.



1. Simplify $3\left(2\vec{a} + \vec{b} + \vec{c}\right) - (\vec{a} + 3\vec{b} - 2\,\overrightarrow{c)}$

2. If $\vec{x} = 3\vec{\imath} - 4\vec{\jmath} + \vec{k}$, $\vec{y} = \vec{\jmath} - 5\vec{k}$ and $\vec{z} = -\vec{\imath} - \vec{\jmath} + 4\vec{k}$, then determine each of the following in terms of $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$.

(a) $\vec{x} + \vec{y}$

(b) $\vec{x} - \vec{y}$

c) $\vec{x} - 2\vec{y} + 3\vec{z}$

6.5 Vectors in R^2 and R^3

Learning Goals / Success Criteria:
Determine the Cartesian representation of a vector in two- and three-dimensional space / p.318 #15.

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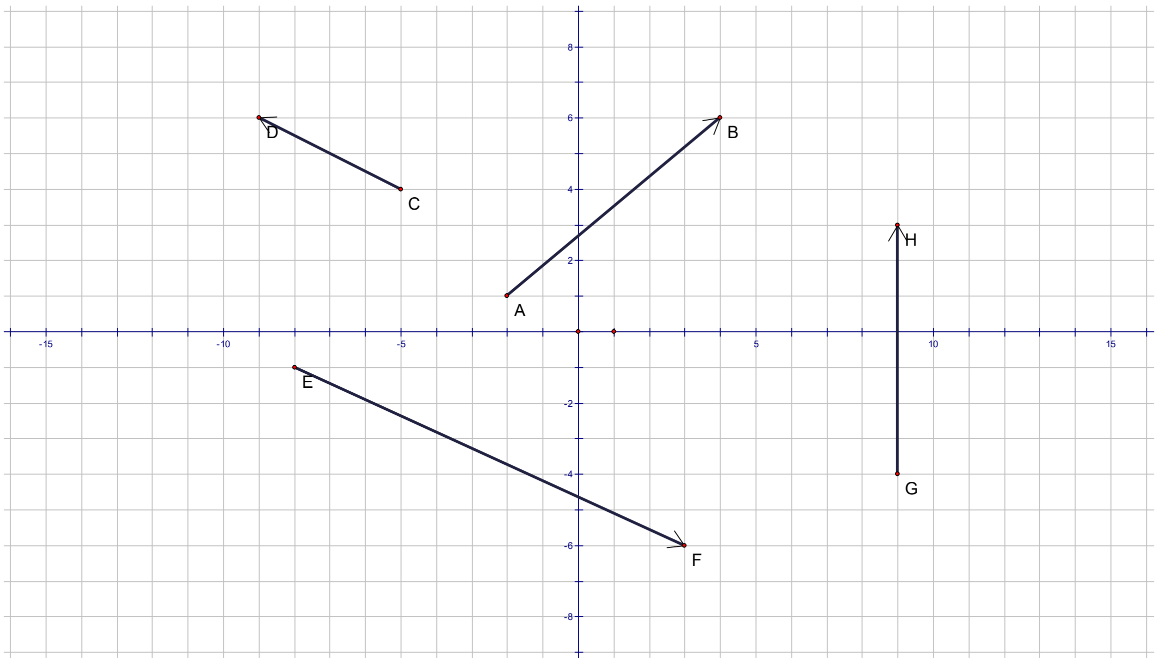
Vectors can be written in two-dimensional space R^2 or in three-dimensional space R^3 as co-ordinates.

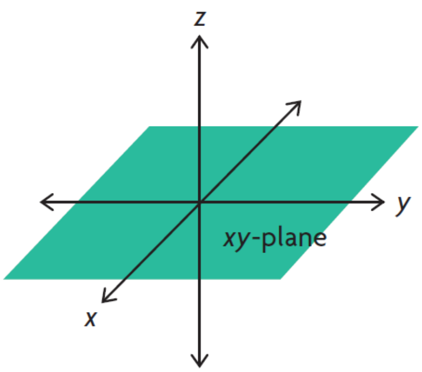
1. Write each of these vectors in R^2 as a set of co-ordinates:



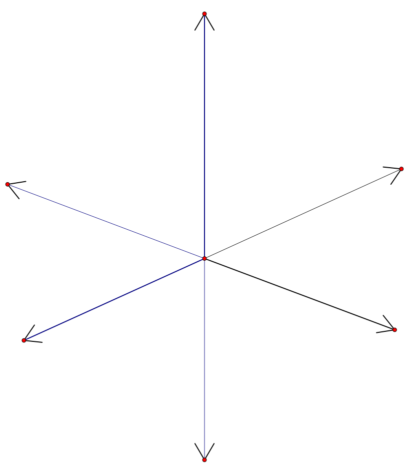
A vector that starts at (0,0) is called a **position vector**.

2. On the gride above, draw the vector $\overrightarrow{OP} = (6,5)$ starting at the origin. At what point does it end? Why is this called a position vector?

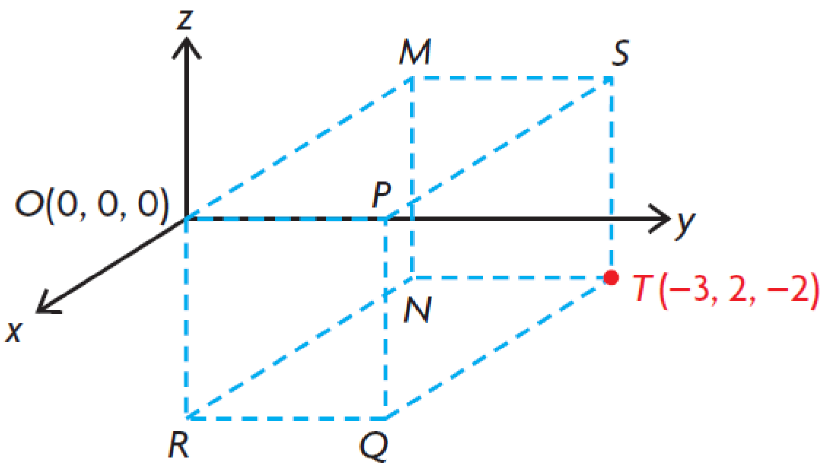
In three dimensions, we need a different set of axes to represent where points are found.



3. Draw the point (6,2,4).



4. In the following diagram, point T is located at $(-3, 2, -2)$.



(a) Find the co-ordinates of points P, Q, R, M, N and S.

(b) Determine whether each point is on an axis. For the points not on axes, determine the <u>one</u> plane on which the point lies (the xy-plane, the xz-plane, or the yz-plane).

(c) Write an equation for the plane containing the points $O, P, R,$ and Q.

(d) Write an equation for the plane containing the points $P, Q, S,$ and T.

6.6 Operations with Vectors in R^2

Learning Goals / Success Criteria:
Determine the Cartesian representation of a vector in two- and three-dimensional space / p.325 #9.
Perform mathematical operations on algebraic vectors in two- and three-dimensional space / p.326 #10.
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Unit vectors $\vec{\imath} = (1,0)$ and $\vec{\jmath} = (0,1)$ can be used to write vectors in two dimensions.

1. Rewrite the following vectors in terms of $\vec{\imath}$ and $\vec{\jmath}$.

a) $\overrightarrow{OP} = (1,2)$ (b) $\overrightarrow{OQ} = (-3,0)$ (c) $\overrightarrow{OR} = (-4,-1)$

2. Rewrite the following vectors as co-ordinates:

(a) $\overrightarrow{OA} = -\vec{\imath}$ (b) $\overrightarrow{OB} = \vec{\imath} + 5\vec{\jmath}$ (c) $\overrightarrow{OD} = \sqrt{2}\vec{\imath} - 4\vec{\jmath}$

3. Given $\vec{a} = (1,3)$ and $\vec{b} = (4,-2)$, find

(a) $\vec{a} + \vec{b}$ (b) $\vec{a} - \vec{b}$

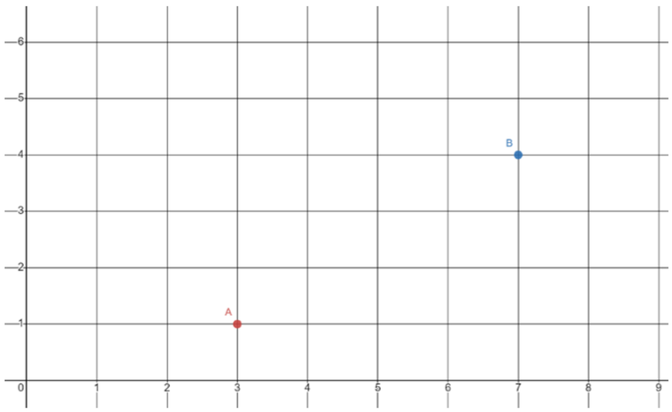
Adding **two vectors**, subtracting **two vectors**, scalar multiplication of a vector.

If $\vec{a} = (x_1, y_1)$ and $\vec{b} = (x_2, y_2)$ then $\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2)$

$\vec{a} - \vec{b} = (x_1 - x_2, y_1 - y_2)$

$k\vec{a} = (kx_1, ky_1)$

4. Given the **points** $A(3,1)$ and $B(7,4)$, how do we find $\overrightarrow{AB}$? How do we find $|\overrightarrow{AB}|$?



This if for two points using position vectors!

If $\overrightarrow{OA} = (x_1, y_1)$ and $\overrightarrow{OB} = (x_2, y_2)$, then $\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

5. Given points $A(-3,7), B(5,22)$ and $C(8,18)$, determine each of the following.

(a) $|\overrightarrow{AB}| + |\overrightarrow{BC}|$

(b) $|\overrightarrow{AB} + \overrightarrow{BC}|$

6. Given $\vec{x} = 2\vec{\imath} - 3\vec{\jmath},\ \vec{y} = -4\vec{\imath} - 3\vec{\jmath}$, find $|\vec{x} + \vec{y}|$

6.7 Operations with Vectors in R^3

Learning Goals / Success Criteria:
Determine the Cartesian representation of a vector in two- and three-dimensional space / p.332 #4.
Perform mathematical operations on algebraic vectors in two- and three-dimensional space / p.332 #10.
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The **standard unit vectors** are $\vec{\imath} = (1,0,0)$ and $\vec{\jmath} = (0,1,0)$ and $\vec{k} = (0,0,1)$ can be used to write vectors in three dimensions.

1. Rewrite each of the following using component form of the standard unit vectors $\vec{\imath}, \vec{\jmath}$, and $\vec{k}$.

(a) $\overrightarrow{OP} = (2,1,-3)$

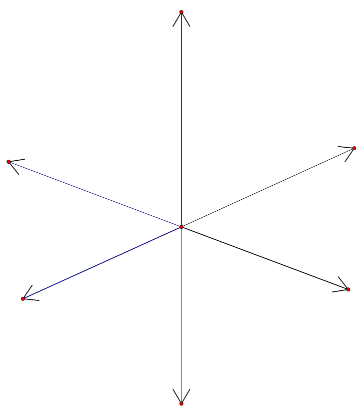
(b) $\overrightarrow{OM} = 2\vec{\imath} - 6\vec{k}$

2. Given $\vec{a} = -\vec{\imath} + 2\vec{\jmath} + \vec{k}$, $\vec{b} = 2\vec{\jmath} - 3\vec{k}$, $\vec{c} = \vec{\imath} - 3\vec{\jmath} + 2\vec{k}$, find $2\vec{a} - \vec{b} + \vec{c}$.

(a) Use component form.

(b) Use the standard unit vectors

3. Find the magnitude of $\overrightarrow{OP} = (3,-4,12)$ by first finding the magnitude of $\overrightarrow{OR} = (3,-4,0)$



If $\vec{a} = (x_1, y_1, z_1)$ and $\vec{b} = (x_2, y_2, z_2)$ then

$$\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

$$\vec{a} - \vec{b} = (x_1 - x_2, y_1 - y_2, z_1 - z_2)$$

$$k\vec{a} = (kx_1, ky_1, kz_1)$$

If $\overrightarrow{OA} = (x_1, y_1, z_1)$ and $\overrightarrow{OB} = (x_2, y_2, z_2)$, then

$$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

4. Given points $A\ (3, 1, 4)$ and $B(7,6,4)$, how do you find $\overrightarrow{AB}$?

5. If $A(7, -11, 13)$ and $B(4, -7, 25)$ are points, determine:

(a) $\left|\overrightarrow{OA}\right|$

(b) $\left|\overrightarrow{OB}\right|$

(c) $\overrightarrow{AB}$

(d) $\left|\overrightarrow{AB}\right|$

6.8 Linear combinations and spanning sets:

Learning Goals / Success Criteria:
Determine the Cartesian representation of a vector in two- and three-dimensional space / p.341 #7.
Perform mathematical operations on algebraic vectors in two- and three-dimensional space / p.341 #11.
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	Linear combinations	Spanning sets
	For noncolinear vectors $\vec{u}$ and $\vec{v}$, a linear combination of these vectors is given by the vector: $a\vec{u} + b\vec{v}$ where a and b are scalars (real numbers)	A set of vectors is a spanning set if every vector can be written as a linear combination of those vectors.
R^2	Every vector in R^2 can be written as a linear combination of the unit vectors $\vec{\imath}$ and $\vec{\jmath}$, where: $\vec{\imath} = (1,0)$ and $\vec{\jmath} = (0,1)$	Any pair of nonzero, noncolinear vectors will span R^2. This means that every vector in R^2 can be expressed as a linear combination of this pair of vectors.
R^3	Every vector in R^3 can be written as a linear combination of the unit vectors $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$, where: $\vec{\imath} = (1,0,0)$, $\vec{\jmath} = (0,1,0)$, and $\vec{k} = (0,0,1)$	Any pair of nonzero, noncolinear vectors will span a plane in R^3. This means that every vector in the plane can be expressed as a linear combination of this pair of vectors. --- Any set of three nonzero, non-coplanar vectors will span R^3. This means that every vector in R^3 can be expressed as a linear combination of these three vectors.

1. Consider the two geometric vectors $\vec{a}$ and $\vec{b}$ shown below and any two points on the page, M and S. Show that we can use a linear combination of vectors $\vec{a}$ and $\vec{b}$ to get from M to S and therefore we can state the set $\{\vec{a}, \vec{b}\}$ is a spanning set for R^2.

2. Show that $\vec{x} = (4,23)$ can be written as a linear combination of $\vec{a} = (-1,4)$ and $\vec{b} = (2,5)$.

3. Show that the set of vectors $\{(2,3), (4,6)\}$ does not span R^2.

4. Show that the $\vec{a} = (2,3,-5)$ can be written as a linear combination of $\vec{i}, \vec{j}$, and $\vec{k}$.

5. Given vectors $\vec{a} = (-1, -2, 1)$ and $\vec{b} = (3, -1, 1)$, does the vector $\vec{c} = (-9, -4, 1)$ lie on the plane determined by $\vec{a}$ and $\vec{b}$? Explain.

6. Prove that the set of vectors $\{(2,1), (-3,-1)\}$ is a spanning set for R^2.