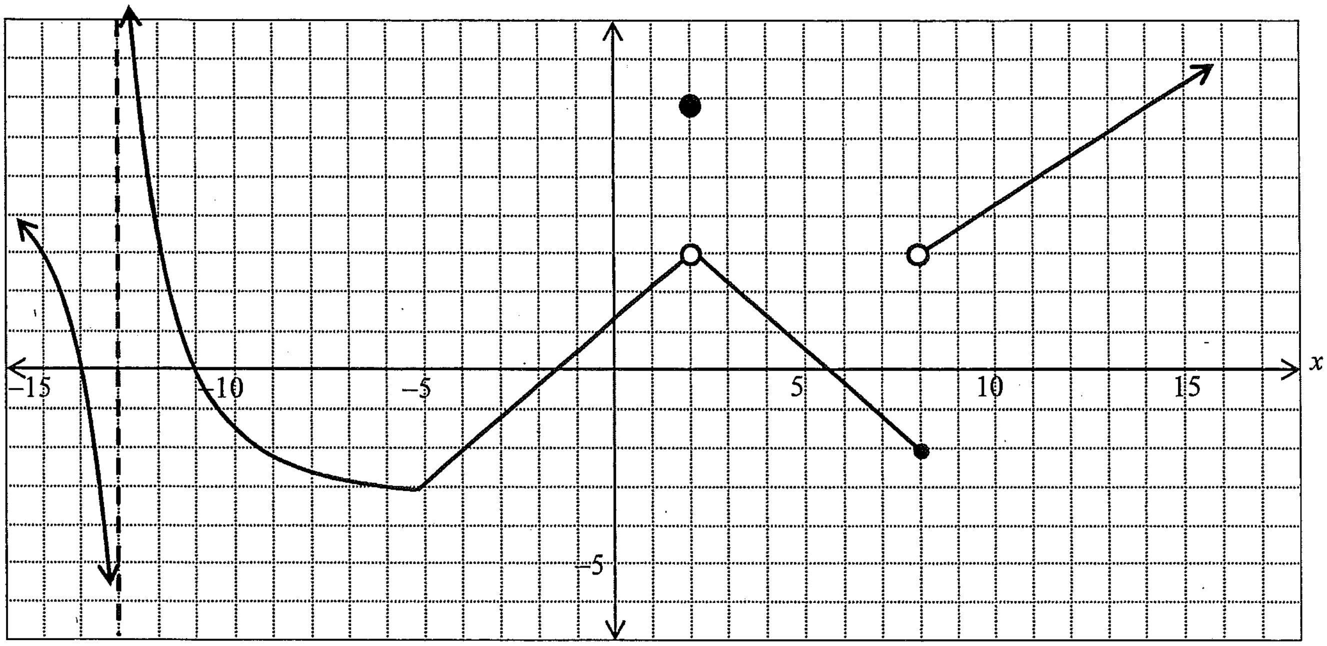
LIMITS AND CONTINUITY WORKSHEET

1. Consider the following graph of the function, $f(x)$.



a) Evaluate the following, if it exists.

i) $\lim_{x\to 2^+} f(x) = 3$ **ii)** $\lim_{x\to 2} f(x) = 3$ **iii)** $f(2) = 7$ **iv)** $f(-5) = -3$

v) $\lim_{x\to 8^-} f(x) = -2$ **vi)** $\lim_{x\to 8} f(x)$ DNE **vii)** $\lim_{x\to -13^-} f(x)$ DNE OR $-\infty$ **viii)** $f(-13)$ undefined

b) Identify where $f(x)$ is discontinuous and whether it is a removable, jump or infinite discontinuity?

@ $x = -13$ (infinite discontinuity)

@ $x = 2$ (removable discontinuity)

@ $x = 8$ (jump discontinuity)

2. Evaluate the following limits.

a) $\lim_{x\to -3} \frac{x+3}{x^2-6x-27}$

$= \lim_{x\to -3} \frac{\cancel{x+3}}{(x-9)\cancel{(x+3)}}$

$= \frac{-1}{12}$

b) $\lim_{h\to 0} \frac{\sqrt{25+h}-5}{h}$

$= \lim_{h\to 0} \left(\frac{\sqrt{25+h}-5}{h}\right)\left(\frac{\sqrt{25+h}+5}{\sqrt{25+h}+5}\right)$

$= \lim_{h\to 0} \frac{\cancel{25}+\cancel{h}-\cancel{25}}{\cancel{h}(\sqrt{25+h}+5)}$

$= \frac{1}{10}$

3. Given the function

$$f(x)=\begin{cases} 2x & x\in(-\infty,-1] \\ x^2 & x\in(-1,2) \\ 0.5x+3 & x\in[2,\infty) \end{cases}$$

a) Determine if and where the function is discontinuous. If discontinuous explain why.

b) Determine each limit, if it exists.

i) $\lim_{x\to 2^-} f(x) = 4$ ii) $\lim_{x\to 2^+} f(x) = 4$ iii) $\lim_{x\to 2} f(x) = 4$

iv) $\lim_{x\to -1^-} = -2$ v) $\lim_{x\to -1^+} f(x) = 1$ vi) $\lim_{x\to -1} f(x)$ DNE

Discontinuous @ $x = -1$

Reason: $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x) \Rightarrow \lim_{x\to -1} f(x)$ DNE

Condition 2 fails!

4. Evaluate the following limits:

a. $\lim_{x\to 3} \frac{x^2+2}{x-2}$

$= 11$

b. $\lim_{x\to 0} \frac{x^2-2x}{x}$

$= \lim_{x\to 0} \frac{\cancel{x}(x-2)}{\cancel{x}}$

$= -2$

c. $\lim_{x\to 125} \frac{125-x}{x^{\frac{1}{3}}-5}$

Let $t = x^{\frac{1}{3}}$

$t^3 = x$

$\lim_{t\to 5} \frac{125-t^3}{t-5}$

$= \lim_{t\to 5} \frac{\cancel{(5-t)}(25+5t+t^2)}{-\cancel{(t-5)}}$

$= -75$

d. $\lim_{x\to 1} \frac{3-\sqrt{8+x}}{1-x}$

$= \lim_{x\to 1} \frac{(3-\sqrt{8+x})}{(1-x)} \frac{(3+\sqrt{8+x})}{(3+\sqrt{8+x})}$

$= \lim_{x\to 1} \frac{9-(8+x)}{(1-x)(3+\sqrt{8+x})}$

$= \lim_{x\to 1} \frac{\cancel{1-x}}{\cancel{(1-x)}(3+\sqrt{8+x})}$

$= \frac{1}{6}$

5. Consider the function f(x) = $\begin{cases} x^2, \text{ if } x \le 0 \\ 1+x, \text{ if } x > 0 \end{cases}$. Does $\lim_{x\to 0} f(x)$ exist? Explain.

$\lim_{x\to 0^-} f(x)$

$= \lim_{x\to 0^-} x^2$

$= 0$

$\lim_{x\to 0^+} f(x)$

$= \lim_{x\to 0^+} 1+x$

$= 1$

since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$ then $\lim_{x\to 0} f(x)$ DNE

6. Let $f(x) = \begin{cases} -x^2 &, x<0 \\ ax+b &, 0 \le x < 1 \\ \sqrt{x+3} &, x \ge 1 \end{cases}$. Determine the values of a and b that make the function continuous.

$\lim_{x\to 0^-} f(x)$

$= \lim_{x\to 0^-} -x^2$

$= 0$

$\lim_{x\to 0^+} f(x)$

$= \lim_{x\to 0^+} ax+b$

$= b$

$\therefore \boxed{b=0}$ in order for $\lim_{x\to 0} f(x)$ to exist

$\lim_{x\to 1^-} f(x)$

$= \lim_{x\to 1^-} ax+b$

$= a+b$

$\lim_{x\to 1^+} f(x)$

$= \lim_{x\to 1^+} \sqrt{x+3}$

$= 2$

$\therefore a+b=2$ but $b=0 \Rightarrow \boxed{a=2}$

7. Use 1st **Principles** to determine the derivative of the following functions.

a. $f(x) = \dfrac{1}{x-1}$

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0} \dfrac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$

$f'(x) = \lim_{h\to 0} \dfrac{\cancel{x}\cancel{-1} - (\cancel{x}+h\cancel{-1})}{h(x+h-1)(x-1)}$

$f'(x) = \lim_{h\to 0} \dfrac{-1}{(x+h-1)(x-1)}$

$f'(x) = \dfrac{-1}{(x-1)^2}$

b. $y = x^3 + 2x^2 + 4$

$y' = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$y' = \lim_{h\to 0} \dfrac{(x+h)^3 + 2(x+h)^2 + 4 - (x^3+2x^2+4)}{h}$

$y' = \lim_{h\to 0} \dfrac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{2x^2} + 4xh + 2h^2 \cancel{+4} \cancel{-x^3} \cancel{-2x^2} \cancel{-4}}{h}$

$y' = \lim_{h\to 0} \dfrac{\cancel{h}(3x^2 + 3xh + h^2 + 4x + 2h)}{\cancel{h}}$

$y' = 3x^2 + 4x$

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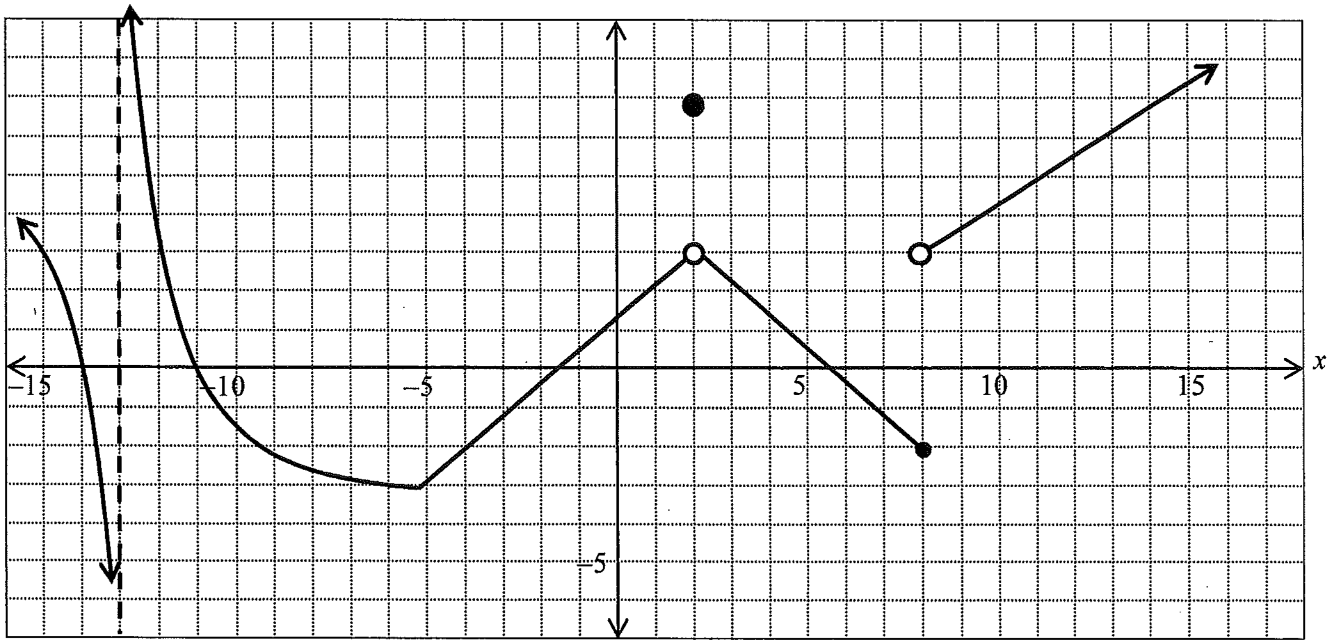

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$$= \lim_{h\to 0} \frac{(\sqrt{25+h}-5)}{h} \frac{(\sqrt{25+h}+5)}{(\sqrt{25+h}+5)}$$

$$= \lim_{h\to 0} \frac{\cancel{25}+\cancel{h}-\cancel{25}}{\cancel{h}(\sqrt{25+h}+5)}$$

$$= \frac{1}{10}$$

5. Consider the function $f(x) = \begin{cases} x^2, & \text{if } x \le 0 \\ 1 + x, & \text{if } x > 0 \end{cases}$. Does $\lim_{x \to 0} f(x)$ exist? Explain.

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since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

then $\lim_{x \to 0} f(x)$ DNE

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$\lim_{x \to 1^-} f(x)$

$= \lim_{x \to 1^-} ax + b$

$= a + b$

$\lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1^+} \sqrt{x+3}$

$= 2$

$\therefore\ a + b = 2$ but $b = 0 \Rightarrow \boxed{a = 2}$

$\therefore\ \boxed{b = 0}$ in order for $\lim_{x \to 0} f(x)$ to exist

7. Use **1st Principles** to determine the derivative of the following functions.

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$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \dfrac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$

$f'(x) = \lim_{h \to 0} \dfrac{\cancel{x-1} - (\cancel{x} + \cancel{h} - \cancel{1})}{h(x+h-1)(x-1)}$

$f'(x) = \lim_{h \to 0} \dfrac{-1}{(x+h-1)(x-1)}$

$f'(x) = \dfrac{-1}{(x-1)^2}$

b. $y = x^3 + 2x^2 + 4$

$y' = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

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$y' = \lim_{h \to 0} \dfrac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{2x^2} + 4xh + 2h^2 \cancel{+4} \cancel{-x^3} \cancel{-2x^2} \cancel{-4}}{h}$

$y' = \lim_{h \to 0} \dfrac{\cancel{h}(3x^2 + 3xh + h^2 + 4x + 2h)}{\cancel{h}}$

$y' = 3x^2 + 4x$