

Calculus

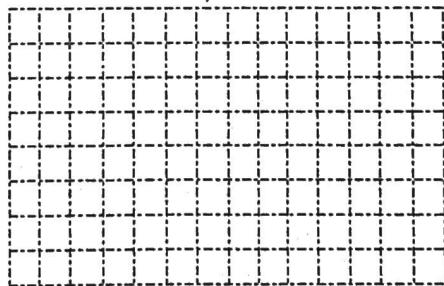
- Used to compare quantities which vary in a non-linear way
- 2 main branches:
 - Differentiation-helps us to find a rate of change of one quantity compared to another
 - Integration-(reverse of differentiation) given a rate of change, find the original relationship between the 2 quantities
- The idea of zooming in to get a better approximation of the slope of the curve was the breakthrough that led to the development of differentiation
- Methods: ① Numerical approach to finding slopes (limits & slope of a tangent)
② Algebraic approach to finding slopes (differentiation from first principles & derivatives)

1-1 Exploring the concept of a limit



Recall: A sequence is a list of numbers written in a definite order: $t_1, t_2, t_3, t_4, \dots, t_n, \dots$, where t_1 is the first term and in general t_n is the nth term.

Ex 1: List the first 5 terms of the sequence defined by $t_n = \frac{n}{n+1}$ and draw the graph of the sequence.



In general, we say that the sequence with general term t_n has the limit L , and we write:

$$\lim_{n \rightarrow \infty} t_n = L$$

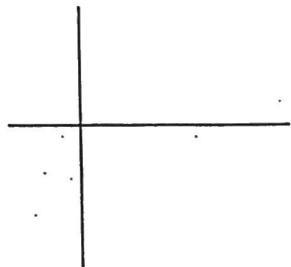
Ex 2: Find $\lim_{n \rightarrow \infty} \frac{1}{n}$

Ex 3: Find $\lim_{n \rightarrow \infty} (-1)^n$

Introduction to Limits

Example 1: Consider the graph of the function $f(x) = 3x - 5$.

As the value of x "approaches" 2 (on the graph), what does the value of the function approach?



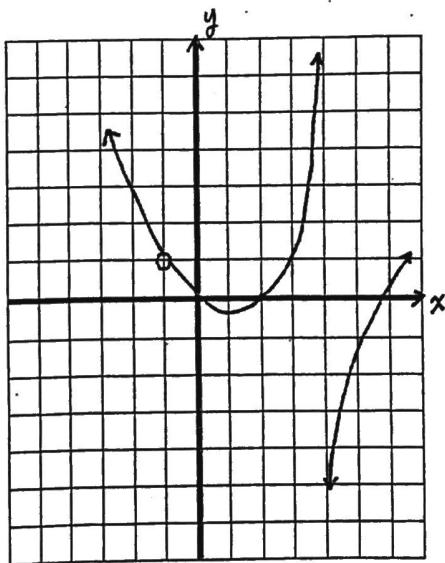
We say that "the limit of $3x-5$ as x approaches 2 equals 1," and we write

$$\lim_{x \rightarrow 2} (3x - 5) = 1$$

Notes:

- In this case $f(2) = 1$ and $\therefore \lim_{x \rightarrow 2} f(x) = f(2)$.
- In retrospect it follows that $\lim_{x \rightarrow 2} f(x)$ could have been evaluated simply by substituting $x = 2$ into the formula for $f(x)$.
- However, NOT all limits can be evaluated by substitution.

Example 2: Use the given graph of f to state the value of the limit, if it exists.



a. $\lim_{x \rightarrow 3} f(x)$

b. $\lim_{x \rightarrow 2} f(x)$

c. $\lim_{x \rightarrow -1} f(x)$

d. $\lim_{x \rightarrow 4} f(x)$

Example 3: Evaluate the following limits:

a. $\lim_{x \rightarrow 3} x^2$

b. $\lim_{x \rightarrow 2} \frac{1}{x}$

c. $\lim_{x \rightarrow 1} \sqrt{x+3}$

Notes: For the following types of functions

- polynomial functions
- rational functions
- algebraic functions

you can evaluate $\lim_{x \rightarrow a} f(x)$ by substitution ($\lim_{x \rightarrow a} f(x) = f(a)$), provided that $f(a)$ is defined.

Example 4: Evaluate the following limit: $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

Example 5: Evaluate the following limits:

$$\lim_{x \rightarrow 2} \frac{(x-3)^2 - 1}{x-2}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{4}{2+h} - 2}{h}$$