The Derivative

Definition: The derivative is the slope of the tangent line to the curve at any point x. It can also be defined as the rate of change at any point x.

The function must be defined at the point x.

Examples of where the function is **not** defined include: Vertical Asymptotes (V. A.) and holes.

Examples of where the function is **not** differentiable are sharp corners, endpoints, vertical tangents.

1st Principles Definition of the Derivative

Lagrange Notation

read as "y prime"

$$y' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

read as "f prime of x"

or

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Leibniz Notation

read as "dy by dx"

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Example 1: Determine y' if $y=\sqrt{x+1}$.

$$y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$y' = \lim_{h\to 0} \frac{(\sqrt{x+h+1}-\sqrt{x+1})}{h}\frac{(\sqrt{x+h+1}+\sqrt{x+1})}{(\sqrt{x+h+1}+\sqrt{x+1})}$$

$$y' = \lim_{h\to 0} \frac{\cancel{x}+h+\cancel{1}-(\cancel{x}+\cancel{1})}{\cancel{h}(\sqrt{x+h+1}+\sqrt{x+1})}$$

$$y' = \lim_{h\to 0} \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

$$y' = \frac{1}{2\sqrt{x+1}}$$

Example 2: Determine $\frac{dy}{dx}$ at $x=3$ if $y=2x^2+6x+11$. $\left.\frac{dy}{dx}\right|_{x=3}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{2(x+h)^2+6(x+h)+11-(2x^2+6x+11)}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+6x+6h+11-2x^2-6x-11}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+\cancel{6x}+6h+\cancel{11}-\cancel{2x^2}-\cancel{6x}-\cancel{11}}{h}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{4x\cancel{h}+2h^{\cancel{2}}+6\cancel{h}}{\cancel{h}} \qquad \therefore \left.\frac{dy}{dx}\right|_{x=3} = 18$$

$$\frac{dy}{dx} = 4x+6$$