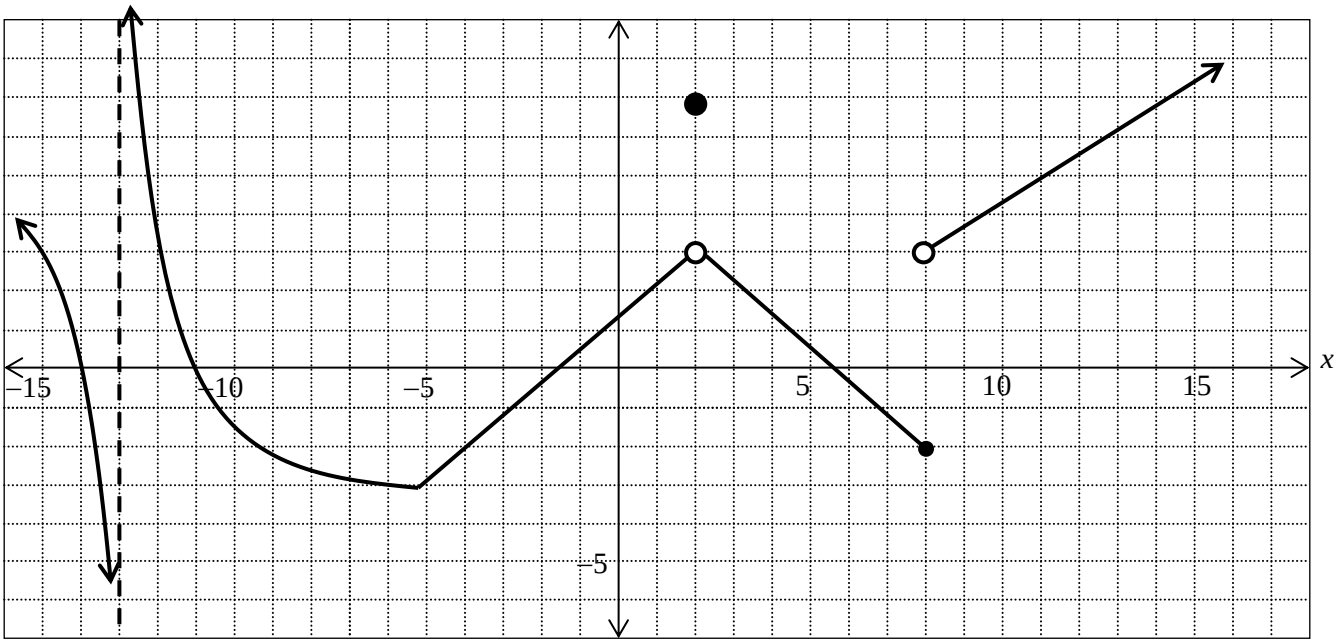
LIMITS AND CONTINUITY WORKSHEET

1. Consider the following graph of the function, $f(x)$.



a) Evaluate the following, if it exists.

i) $\lim\limits_{x\to 2^+} f(x)$ **ii)** $\lim\limits_{x\to 2} f(x)$ **iii)** $f(2)$ **iv)** $f(-5)$

v) $\lim\limits_{x\to 8^-} f(x)$ **vi)** $\lim\limits_{x\to 8} f(x)$ **vii)** $\lim\limits_{x\to -13^-} f(x)$ **viii)** $f(-13)$

b) Identify where $f(x)$ is discontinuous and whether it is a removable, jump or infinite discontinuity?

2. Evaluate the following limits.

a) $\lim\limits_{x\to -3} \dfrac{x+3}{x^2-6x-27}$

b) $\lim\limits_{h\to 0} \dfrac{\sqrt{25+h}-5}{h}$

3. Given the function

$$f(x)=\begin{cases}2x & x\in(-\infty,-1]\\ x^2 & x\in(-1,2)\\ 0.5x+3 & x\in[2,\infty)\end{cases}$$

a) Determine if and where the function is discontinuous. If discontinuous explain why.

b) Determine each limit, if it exists.

i) $\lim_{x\to 2^-} f(x)$ **ii)** $\lim_{x\to 2^+} f(x)$ **iii)** $\lim_{x\to 2} f(x)$

iv) $\lim_{x\to -1^-}$ **v)** $\lim_{x\to -1^+} f(x)$ **vi)** $\lim_{x\to -1} f(x)$

4. Evaluate the following limits:

a. $\lim_{x\to 3}\dfrac{x^2+2}{x-2}$

b. $\lim_{x\to 0}\dfrac{x^2-2x}{x}$

c. $\lim_{x\to 125}\dfrac{125-x}{x^{\frac{1}{3}}-5}$

d. $\lim_{x\to 1}\dfrac{3-\sqrt{8+x}}{1-x}$

5. Consider the function f(x) = $\begin{cases} x^2, \text{ if } x \le 0 \\ 1 + x, \text{ if } x > 0 \end{cases}$. Does $\lim_{x \to 0} f(x)$ exist? Explain.

6. Let $f(x) = \begin{cases} -x^2 & , x < 0 \\ ax + b & , 0 \le x < 1 \\ \sqrt{x+3} & , x \ge 1 \end{cases}$. Determine the values of *a* and *b* that make the function continuous.

7. Use **1st Principles** to determine the derivative of the following functions.

a. $f(x) = \dfrac{1}{x-1}$

b. $y = x^3 + 2x^2 + 4$