Review - Course Pack - Page 31

1a) $\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4}$

$= \lim_{x \to -4} \frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-1)}$

$= \frac{3}{5}$

b) $\lim_{x \to 3} \frac{2x^2-x-15}{3x^2-13x+12}$

$= \lim_{x \to 3} \frac{(2x+5)\cancel{(x-3)}}{(3x-4)\cancel{(x-3)}}$

$= \lim_{x \to 3} \frac{2x+5}{3x-4}$

$= \frac{11}{5}$

c) $\lim_{x \to -4} \frac{\sqrt{x^2+9}-5}{x+4}$

$= \lim_{x \to -4} \frac{(\sqrt{x^2+9}-5)(\sqrt{x^2+9}+5)}{(x+4)(\sqrt{x^2+9}+5)}$

$= \lim_{x \to -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)}$

$= \lim_{x \to -4} \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(\sqrt{x^2+9}+5)}$

$= \frac{-8}{10}$

$= \frac{-4}{5}$

d) $\lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2}$

$= \lim_{x \to 16} \frac{(4-\sqrt{x})}{16x-x^2} \frac{(4+\sqrt{x})}{(4+\sqrt{x})}$

$= \lim_{x \to 16} \frac{\cancel{16-x}}{x\cancel{(16-x)}(4+\sqrt{x})}$

$= \frac{1}{128}$

(2)

1e) $$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$$

$$= \lim_{x\to -4} \frac{\cancel{x+4}}{4x\cancel{(4+x)}}$$

$$= \frac{-1}{16}$$

f) $$\lim_{x\to 4} \frac{x^3-64}{x-4}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(x^2+4x+16)}{\cancel{x-4}}$$

$$= 48$$

g) $$\lim_{x\to -\infty} \frac{-2x^3+5x-7}{x^2+5x+4}$$

$$= \frac{\lim\limits_{x\to -\infty} \frac{-2x^3}{x^2} + \lim\limits_{x\to -\infty} \frac{5x}{x^2} - \lim\limits_{x\to -\infty} \frac{7}{x^2}}{\lim\limits_{x\to -\infty} \frac{x^2}{x^2} + 5\lim\limits_{x\to -\infty} \frac{x}{x^2} + \lim\limits_{x\to -\infty} \frac{4}{x^2}}$$

$$= \frac{\lim\limits_{x\to -\infty} -2x + \lim\limits_{x\to -\infty} \frac{5}{x} - \lim\limits_{x\to -\infty} \frac{7}{x^2}}{\lim\limits_{x\to -\infty} 1 + 5\lim\limits_{x\to -\infty} \frac{1}{x} + \lim\limits_{x\to -\infty} \frac{4}{x^2}} = \frac{\infty+0-0}{1+0+0} = \infty$$

h) $$\lim_{x\to \infty} \frac{2x^3-1}{5x^3+2x^2-3x}$$

$$= \frac{\lim\limits_{x\to \infty} \frac{2x^3}{x^3} - \lim\limits_{x\to \infty} \frac{1}{x^3}}{5\lim\limits_{x\to \infty} \frac{x^3}{x^3} + 2\lim\limits_{x\to \infty} \frac{x^2}{x^3} - 3\lim\limits_{x\to \infty} \frac{x}{x^3}}$$

$$= \frac{\lim\limits_{x\to \infty} 2 - \lim\limits_{x\to \infty} \frac{1}{x^3}}{5\lim\limits_{x\to \infty} 1 + 2\lim\limits_{x\to \infty} \frac{1}{x} - 3\lim\limits_{x\to \infty} \frac{1}{x}}$$

$$= \frac{2-0}{5+0-0} = \frac{2}{5}$$

1e) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

$= \lim_{x\to -4} \dfrac{\cancel{x+4}}{4x\cancel{(4+x)}}$

$= \dfrac{-1}{16}$

f) $\lim_{x\to 4} \dfrac{x^3-64}{x-4}$

$= \lim_{x\to 4} \dfrac{\cancel{(x-4)}(x^2+4x+16)}{\cancel{x-4}}$

$= 48$

g) $\lim_{x\to -\infty} \dfrac{-2x^3+5x-7}{x^2+5x+4}$

$= \dfrac{\lim_{x\to -\infty} \frac{-2x^3}{x^2} + \lim_{x\to -\infty} \frac{5x}{x^2} - \lim_{x\to -\infty} \frac{7}{x^2}}{\lim_{x\to -\infty} \frac{x^2}{x^2} + 5\lim_{x\to -\infty} \frac{x}{x^2} + \lim_{x\to -\infty} \frac{4}{x^2}}$

$= \dfrac{\lim_{x\to -\infty} -2x + \lim_{x\to -\infty} \frac{5}{x} - \lim_{x\to -\infty} \frac{7}{x^2}}{\lim_{x\to -\infty} 1 + 5\lim_{x\to -\infty} \frac{1}{x} + \lim_{x\to -\infty} \frac{4}{x^2}} = \dfrac{\infty + 0 - 0}{1+0+0} = \infty$

h) $\lim_{x\to \infty} \dfrac{2x^3-1}{5x^3+2x^2-3x}$

$= \dfrac{\lim_{x\to \infty} \frac{2x^3}{x^3} - \lim_{x\to \infty} \frac{1}{x^3}}{5\lim_{x\to \infty} \frac{x^3}{x^3} + 2\lim_{x\to \infty} \frac{x^2}{x^3} - 3\lim_{x\to \infty} \frac{x}{x^3}}$

$= \dfrac{\lim_{x\to \infty} 2 - \lim_{x\to \infty} \frac{1}{x^3}}{5\lim_{x\to \infty} 1 + 2\lim_{x\to \infty} \frac{1}{x} - 3\lim_{x\to \infty} \frac{1}{x}}$

$= \dfrac{2-0}{5+0-0} = \dfrac{2}{5}$

2 a) $f(x) = 2x^2 - 3x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}} - 3\cancel{h}}{\cancel{h}}$$

$$= 4x - 3$$

b) $f(x) = \dfrac{1}{3x+2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3(x+h)+2} - \frac{1}{3x+2}}{h}$$

$$= \lim_{h \to 0} \frac{3x+2 - (3(x+h)+2)}{h(3(x+h)+2)(3x+2)}$$

$$= \lim_{h \to 0} \frac{\cancel{3x+2} - \cancel{3x} - 3\cancel{h} \cancel{- 2}}{\cancel{h}(3x+3h+2)(3x+2)}$$

$$= \frac{-3}{(3x+2)^2}$$

3. $y = x^2 - 6x + 9$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 9 - (x^2 - 6x + 9)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h + \cancel{9} - \cancel{x^2} + \cancel{6x} - \cancel{9}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h}$$

$$= 2x - 6$$

$\therefore m_t|_{x=2} = -2 \Rightarrow y - 1 = -2(x-2)$

$\therefore m_\perp|_{x=2} = \frac{1}{2} \Rightarrow y - 1 = \frac{1}{2}(x-2)$

4. $y = x^2 - 5x + 4$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 5(x+h) + 4 - (x^2 - 5x + 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{5x} - 5h \cancel{+4} - \cancel{x^2} + \cancel{5x} \cancel{-4}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}$$

$\perp$ to $y = -\frac{1}{3}x + 5$

$\Rightarrow m_t = 3$

$$= 2x - 5$$

$\therefore 2x - 5 = 3$

$x = 4$

$\therefore y = 3(x-4)$

5. a) $\text{AROC} = \dfrac{h(t_2) - h(t_1)}{t_2 - t_1}$

$= \dfrac{\cancel{300} - 5(3)^2 - (\cancel{300} - 5(1)^2)}{3-1}$

$= \dfrac{-45+5}{2}$

$= -20$ m/s.

b) Evaluate $\lim\limits_{a \to 2} \dfrac{h(t+a) - h(t)}{a}$.

Answer is -20 m/s.

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6. $f(x) = \begin{cases} -2x-8 & x \leq -2 \\ \frac{3}{2}x - 1 & -2 < x \leq 2 \\ (x-4)^2 - 6 & x > 2 \end{cases}$

At $x = -2$

① $f(-2) = -4$

② $\lim\limits_{x \to -2^-} f(x)$

$= \lim\limits_{x \to -2^-} -2x-8$

$= -4$

$\lim\limits_{x \to -2^+} f(x)$

$= \lim\limits_{x \to -2^+} \frac{3}{2}x - 1$

$= -4$

$\Rightarrow \lim\limits_{x \to -2} f(x) = -4$

③ $f(-2) = \lim\limits_{x \to -2} f(x)$

$*$ $\therefore$ cont. @ $x = -2$

At $x = 2$

① $f(2) = 1$

② $\lim\limits_{x \to 2^-} f(x)$

$= \lim\limits_{x \to 2^-} \frac{3}{2}x - 1$

$= 1$

$\lim\limits_{x \to 2^+} f(x)$

$= \lim\limits_{x \to 2^+} (x-4)^2 - 6$

$= -2$

$\Rightarrow \lim\limits_{x \to 2} f(x)$ DNE.

$*$ $\therefore$ Not cont. @ $x = 2$