

Application of Average and Instantaneous Rate of Change

Example: Chevonne kicks a soccer ball off the top of a building that follows the path given by the function $h(t) = -4.9t^2 + 500$, where $h(t)$ is the height, in metres, above the ground after t seconds.

- a. Determine the average speed of the soccer ball between 1 second and 5 seconds.

$$\begin{aligned} \text{AROC} &= \frac{h(5) - h(1)}{5 - 1} \\ &= \frac{-122.5 + \cancel{500} - (-4.9 + \cancel{500})}{4} \\ &= \frac{-117.6}{4} \\ &= -29.4 \text{ m/s} \end{aligned}$$

- b. Determine the speed of the soccer ball at $t = 4$ seconds.

$$\begin{aligned} \text{IROC} &= \lim_{h \rightarrow 0} \frac{h(t+h) - h(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(4+h) - h(4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-4.9(4+h)^2 + 500 - (-4.9(4)^2 + 500)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-4.9(16 + 8h + h^2) + 4.9(16)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-39.2h - 4.9h^2}{h} \\ &= \lim_{h \rightarrow 0} -39.2 - 4.9h \\ &= -39.2 \text{ m/s} \end{aligned}$$