

Name _____	# _____	
K - 19	A - 17	C - 2

Knowledge and Understanding (K)

- Given:

$f = \{(-4, 6), (-2, 4), (0, 2), (4, -5)\}$
 $g(x) = 2x + 5 \xrightarrow{\text{green arrow}} (-4, -3), (-2, 1), (0, 5), (4, 13)$
 $h = \{(-4, -1), (-3, 1), (-2, 4), (0, 7), (4, 10)\}$
 $h^{-1}(x) = \{(-1, -4), (1, -3), (4, -2), (7, 0), (10, 4)\}$

 Determine each of the following: [2 marks each]

- $(f + g) = \{(-4, 3), (-2, 5), (0, 7), (4, 8)\}$
 - $\frac{f}{g} = \{(-4, -\frac{2}{3}), (-2, \frac{1}{2}), (0, \frac{5}{5}), (4, -\frac{5}{13})\}$
 - $f \circ h = \{(-2, -5)\}$
 - $f \times h = \{(-4, -6), (-2, 16), (0, 14), (4, -50)\}$
 - $f \circ h^{-1} = \{(-1, 6), (4, 4), (7, 2), (10, -5)\}$

- Given: [9 marks total]

$f(x) = 3 \sin(x)$
 $g(x) = 6 \cos(x)$
 $h(x) = (x - 4)(x + 2)(x + 5)$
 $k(x) = x - 2$
 $m(x) = \frac{1}{x}$
 $n(x) = x(x + 5)$

 Write the simplified function for each of the following

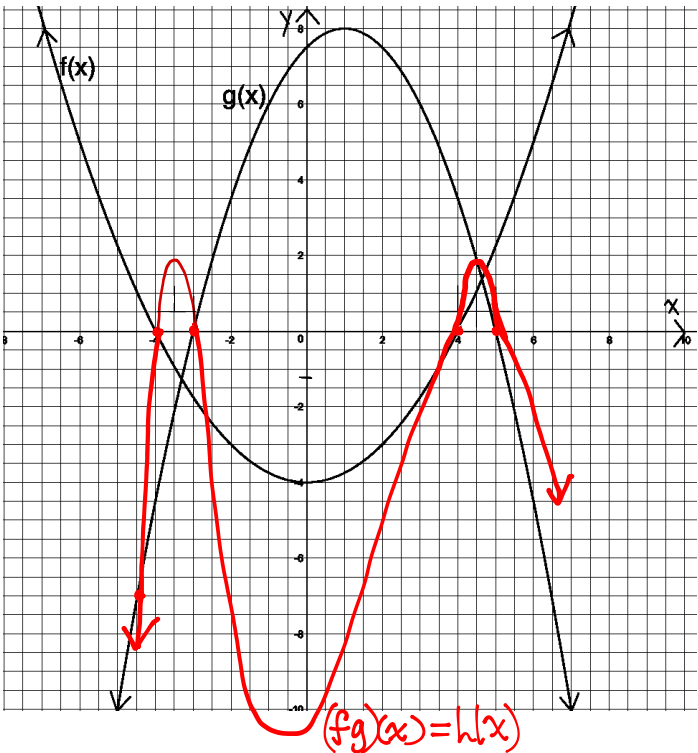
- $(f + g)(x) = 3 \sin(x) + 6 \cos(x)$
 - $(fg)(x) = 3 \sin(x) 6 \cos(x)$
 $= 18 \sin(x) \cos(x)$
 $= 9 \sin(2x)$
 - $\left(\frac{n}{m}\right)(x) = \frac{x(x+5)}{\frac{1}{x}}$
 $= x^2(x+5), x \neq 0$
 - $(h \circ k)(x) = h(x-2)$
 $= ((x-2)-4)((x-2)+2)((x-2)+5)$
 $= (x-6)(x)(x+3)$
 $= x(x-6)(x+3)$
 - $\left(\frac{h}{n}\right)(x) = \frac{(x-4)(x+2)(x+5)}{x(x+5)}$
 $= \frac{(x-4)(x+2)}{x}, x \neq -5, 0$

Application

- Given the graphs of the following functions, sketch a graph of the requested new function. (Be sure to label or mention any asymptotes)
 - Sketch $(fg)(x)$ [4 marks]

$f(x): a(x-4)(x+4)$ vertex $(0, -4)$
 $-4 = a(-4)(4)$
 $\frac{-4}{-4(4)} = a$
 $a = \frac{1}{4}$
 $\therefore f(x) = \frac{1}{4}(x^2 - 16)$

$g(x): a(x+3)(x-5)$ vertex $(1, 8)$
 $8 = a(1+3)(1-5)$
 $8 = a(4)(-4)$
 $\frac{8}{-16} = a$
 $a = -\frac{1}{2}$
 $g(x) = -\frac{1}{2}(x+3)(x-5)$



b) Sketch $\frac{f}{g}(x)$ [6 marks]

$$h(x) = \frac{0.25(x-4)(x+4)}{-0.5(x+3)(x-5)}$$

$$x\text{-int} = 4, -4$$

$$y\text{-int} = \frac{8}{15}$$

$$V.A. \ x = -3, \ x = 5$$

$$H.A. \ y = \frac{0.25}{-0.5} = -\frac{1}{2}$$

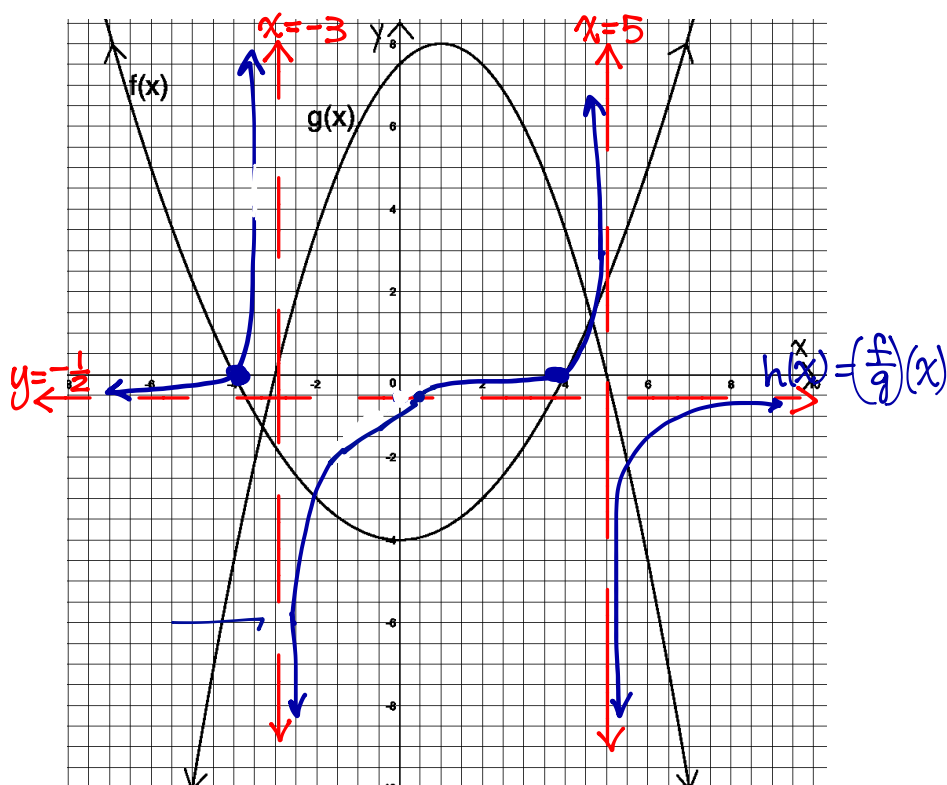
Cross test:

$$\frac{1}{2} = \frac{(x+4)(x-4)}{2(x+3)(x-5)}$$

$$x^2 - 16 = x^2 - 2x - 15$$

$$-1 = -2x$$

$$\frac{1}{2} = x$$



c) Sketch $(f + g)(x)$ [3 marks]

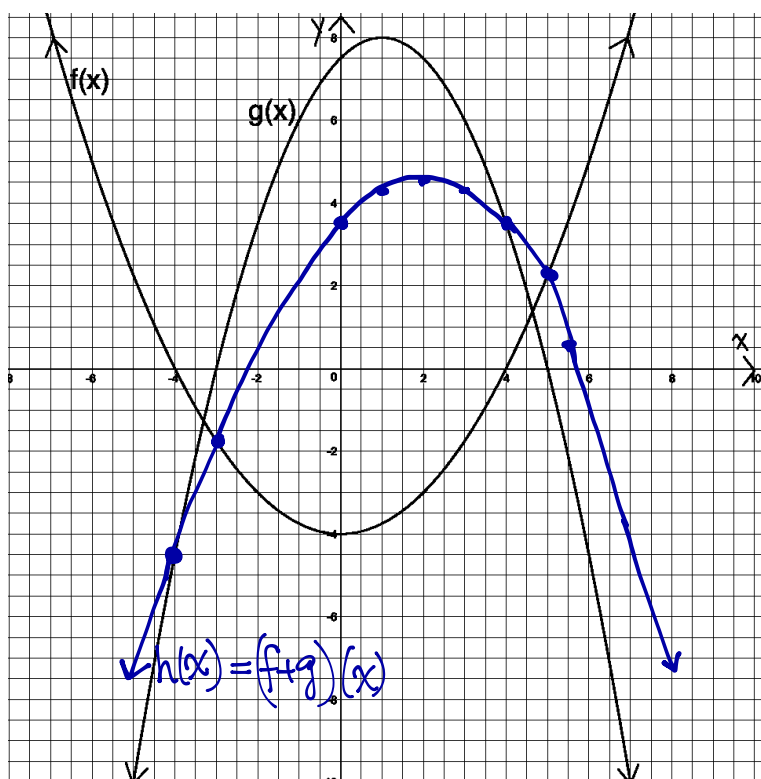
$$h(x) = (f+g)(x)$$

$$= \frac{1}{4}(x^2 - 16) - \frac{1}{2}(x+3)(x-5)$$

$$= \frac{1}{4}x^2 - 4 - \frac{1}{2}(x^2 - 2x - 15)$$

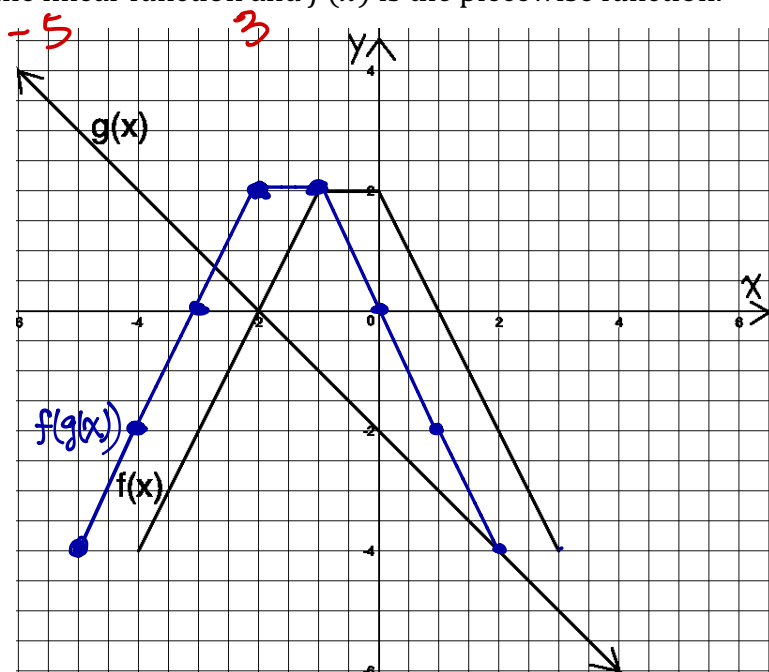
$$= \frac{1}{4}x^2 - 4 - \frac{1}{2}x^2 + x + \frac{15}{2}$$

$$= -\frac{1}{4}x^2 + x + \frac{15}{2}$$



d) Sketch $(f \circ g)(x)$ given that $g(x)$ is the linear function and $f(x)$ is the piecewise function. [4 marks]

$g(x)$					$f(x)$	
x	y	$f(g(x))$	x	y	x	y
-5	3	-4	-5	-4	-5	-4
-4	2	-2	-4	-2	-4	-2
-3	1	0	-3	0	-3	0
-2	0	2	-2	2	-2	2
-1	-1	2	-1	2	-1	2
0	-2	-2	0	-2	0	-2
1	-3	-4	1	-4	1	-4
2	-4	DNE	2	-4	2	-4
3	-5	DNE	3	-5	3	-5



Two marks will be awarded for proper mathematical forms throughout the assessment.

[2 marks]