Final Review For MHF4U Advanced Functions

Study Tips

- The best way to study for a math exam is to PRACTICE!
- Look at your old assessments and determine what you remember and what you forget
  - MAKE NOTES!
- Do not spend a lot of time practicing material that you already know! You should focus on mastering the skills you have not acquired.
- Look at old Unit Review Sheets. They are an excellent source of review.
- Work on this review
  - Note: This review alone is NOT a sufficient amount of review that will guarantee you an amazing mark (whatever that may be), but it's a good start ☺

Unit 1 Questions

1. Factor Fully
   a. $x^3 + 3x^2 - 25x - 75$ $[(x-5)(x+5)(x+3)]$
   b. $64x^3 + 27y^3$ $[(4x+3y)(16x^2 - 12xy + 9y^2)]$
   c. $2x^4 + 3x^3 - 11x^2 - 6x$ $[x(x-2)(2x+1)(x+3)]$

2. Graph $y = (x-3)^2(x^2 - 4x - 7)$

3. Solve the following
   a. $x^3 - 5x = 5x^2 - 1$ $\left[-1, 3 \pm 2\sqrt{2}\right]$
   b. $x(x+1)(x-2)(x-4) > 0$ $[x\epsilon(-\infty, -1) \cup (0, 2) \cup (4, \infty), x\epsilon R]$
   c. $(x+7)^2(x-3)^3 < 0$ $[x\epsilon(-\infty, -7) \cup (-7, 3), x\epsilon R]$
   d. $2x^3 + 3x^2 - 11x \geq 6$ $\left[x\epsilon\left[-3, -\frac{1}{2}\right] \cup [2, \infty), x\epsilon R\right]$

4. Determine if each of the following functions are even, odd, or neither
   a. $f(x) = \frac{1}{x^3+1}$ [N] b. $f(x) = 2x^4 + 3x^2$ [E] c. $f(x) = \left(\frac{1}{x^3+x}\right)^5$ [O]

5. When $x^4 - 4x^3 + ax^2 + bx + 1$ is divided by $(x-1)$, the remainder is 7. When it is divided by $(x+1)$, the remainder is 3. Determine the values of a and b. $[a = 3, b = 6]$

6. An open box, no more than 5 cm in height, is to be formed by cutting four identical squares from the corners of a sheet of metal 25 cm by 32 cm, and folding up the metal to form sides. The volume of the box must be 1650 cm^3. What is the side length of the squares removed? $\left[\frac{1}{4}\left(47 - \sqrt{889}\right) cm\right]$

7. The population of a town is modelled by $p(t) = 6t^2 + 110t + 3000$, where P is the population and t is the number of years since 1990. Find the average rate of change in population between 1995 and 2005. $\left[230 \frac{people}{year}\right]$

8. Determine the instantaneous rate of change of $f(x) = x^3 + x^2 \; at \; x = 2$ [16]

9. The graph of $f(x) = x^4$ is horizontally stretched by a factor of 2, reflected in the y-axis, and shifted up 5 units. Find the equation of the transformed function. $\left[f(x) = \left(-\frac{1}{2}x\right)^4 + 5\right]$

10. What other set of transformations applied to $f(x) = x^4$ would yield the same graph as that in #9?

11. A polynomial of degree 5 has a negative leading coefficient
    a. How many turning points // local max/mins could the polynomial have? $[0, 2, or\ 4]$
    b. How many zeros could the function have? $[at\ most\ 5]$
    c. Describe the end behaviour. $[Q2\ to\ Q4]$
    d. Sketch two possible graphs, each passing through the point $(1, -2)$

12. A polynomial has a bounce at $x = -1$ and two x-intercepts of order 1 at $x = 2$ and $x = 4$.
    a. Determine the equation of the polynomial if it goes through the point $(1, -24)$. $[a = -2]$
    b. Solve for p if $(3, p)$ is a point on the graph of the function $[p = 32]$

Unit 2 Questions

13. Sketch the graph of $f(x) = \frac{2x^2+x-3}{x^2-4x+3}$

14. Sketch the graph of $f(x) = \frac{x^2+4}{x^2-4}$

15. Solve the following using any method you want.
    a. $x + \frac{1}{x-4} = 0$ $\quad [2 \pm \sqrt{3}]$
    b. $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}$ $\quad \left[-\frac{1}{2}\right]$
    c. $\frac{2x}{x-1} + \frac{1}{x-3} \geq \frac{2}{x^2-4x+3}$ $\quad \left[\left\{x\epsilon R \middle| x \leq -\frac{1}{2}, x > 1, x \neq 3\right\}\right]$
    d. $\frac{5}{x+3} < -\frac{3}{x-1}$ $\quad \left[x\epsilon(-\infty, -3) \cup \left(-\frac{1}{2}, 1\right)\right]$

Unit 3 and 4 Questions

16. Convert the following
    a. $\frac{11\pi}{15}$ to degrees [132°]
    b. 420° to radians $\left[\frac{7\pi}{3}\right]$

17. The point P(-5, 4) is on the terminal arm of an angle measure of θ in standard position.
    a. Sketch the principal angle
    b. Determine the exact value of $\sin(\theta)$ $\left[\frac{4}{\sqrt{41}}\right]$
    c. Determine the exact value of $\cos\left(\theta - \frac{\pi}{6}\right)$ $\left[\frac{-5\sqrt{3}+4}{2\sqrt{41}}\right]$
    d. Determine the value of θ in degrees and radians. $[\theta = 141.34° \text{ or } 2.4669]$

18. Find the exact value of
    a. $\cos\left(\frac{3\pi}{4}\right)$ $\left[-\frac{1}{\sqrt{2}}\right]$
    b. $\csc\left(-\frac{3\pi}{2}\right)$ $[1]$
    c. $\sin\left(\frac{7\pi}{12}\right)$ $\left[\frac{\sqrt{6}+\sqrt{2}}{4}\right]$
    d. $\sec\left(\frac{5\pi}{6}\right)\cos\left(\frac{7\pi}{4}\right) - \cot\left(-\frac{\pi}{3}\right)$ $\left[\frac{\sqrt{3}-\sqrt{6}}{3}\right]$
    e. $\cos\left(\frac{\pi}{8}\right)$ $\left[\frac{\sqrt{2+\sqrt{2}}}{2}\right]$
    f. $\csc\left(\frac{7\pi}{12}\right)$

19. Given $\sin(A) = -\frac{6}{7}$ where $\frac{3\pi}{2} \leq A \leq 2\pi$ and $\tan(B) = \frac{2}{3}$ where $\pi \leq B \leq \frac{3\pi}{2}$. Determine the exact value of

a. $Sin(A+B)$ $\left[\frac{18\sqrt{13}-26}{91}\right]$

b. $Cos(2B)$ $\left[\frac{5}{13}\right]$

20. Solve the following

a. $\cos(2x) = \cos(x), 0 \leq x \leq 2\pi$ $\left[0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi\right]$

b. $\sqrt{2}\tan(x)\cos(x) = \tan(x), 0 \leq x \leq 2\pi$ $\left[0, \frac{\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi\right]$

c. $2\cos(2x) = 1, 0 \leq x \leq 2\pi$ $\left[\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right]$

d. $\cos(2x) = -0.9541$ $[x = 1.4187 + k\pi, k\epsilon I, and\ x = 1.7229 + k\pi, k\epsilon I]$

e. $6\sin^2(x) - 5\cos(x) - 2 = 0, 0 \leq x \leq 2\pi$ $\left[x = \frac{\pi}{3}, \frac{5\pi}{3}\right]$

21. Prove that $f(x) = \tan(x)$ is an odd function.

22. Prove the following

a. $\cos(x) + \sin(x) = \frac{1+\tan(x)}{\sec(x)}$

b. $\frac{1}{1-\sec(x)} + \frac{1}{1+\sec(x)} = -2\cot^2(x)$

c. $\cos^2(2\theta) - \cos^2(\theta) = \sin^2(\theta) - \sin^2(2\theta)$

d. $\cos(x+y)\cos(x-y) = \cos^2(x) + \cos^2(y) - 1$

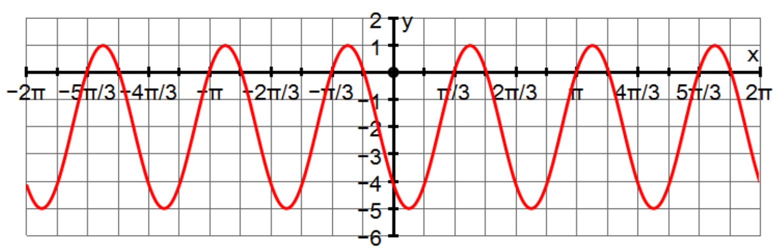
e. $\frac{\cos(x)}{1+\sin(x)} + \frac{\cos(x)}{1-\sin(x)} = \frac{2}{\cos(x)}$

f. $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) = -\cot(x)$

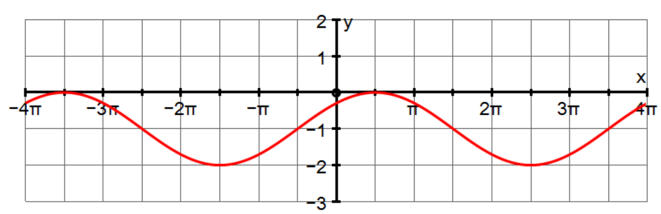
g. $\frac{\sin(\pi-x)\cos(\pi+x)\tan(2\pi-x)}{\sec\left(\frac{\pi}{2}+x\right)\csc\left(\frac{3\pi}{2}-x\right)\cot\left(\frac{3\pi}{2}+x\right)} = \sin^4(x) - \sin^2(x)$

23. How does $y = -2\cos\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right] + 1$ compare to $f(x) = \cos(x)$?

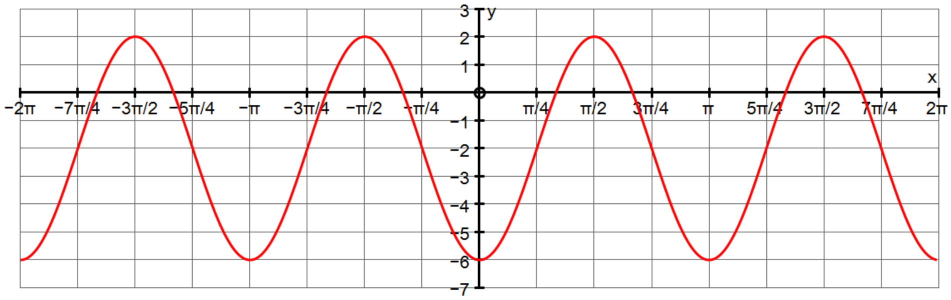
24. Graph $y = -3\cos\left(3x - \frac{\pi}{4}\right) - 2$



25. Graph $y = \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right) - 1$



26. Determine the equation of $f(x)$ given the following graph
   a. in terms of sine
   b. in terms of cosine



Unit 5 Questions

27. Write as a single logarithm
   a. $alog_5(x-7)-\frac{2}{3}\log_5(w)+2$ $\left[\log_5\left(\frac{25(x-7)^a}{w^{\frac{2}{3}}}\right)\right]$
   b. $2\log_3\left(\frac{x^2-25}{x-5}\right)+\frac{7}{\log_{x+5}(9)}$ $\left[\log_3\left((x+5)^{\frac{11}{2}}\right)\right]$

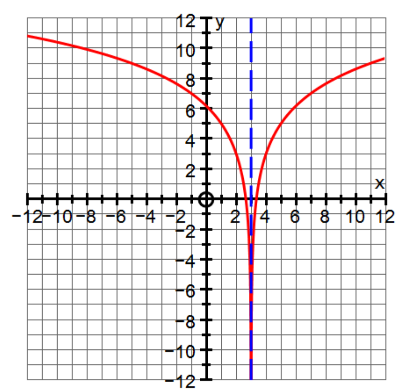
28. Determine the exact value of
   a. $\log_8(6)-\log_8(3)+\log_8(4)$ $[1]$
   b. $\log_9(3^7\sqrt[5]{81})$ $\left[\frac{39}{10}\right]$

29. Solve the following
   a. $\log_4(x+3)=2$ $[13]$
   b. $\log_7(x+2)=1-\log_7(x-4)$ $[5]$
   c. $\log_9(x-5)+\log_9(x+3)=1$ $[6]$
   d. $\log_5(x+1)+\log_5(2)-\log_5(x+3)=\log_5(x-1)$ $[\sqrt{5}]$
   e. $5\cdot 8^{x+2}=5^{7x}$ $[approx\ 0.627]$
   f. $(4^2)(2^{2x-3})=(16^{x-2})\left(\frac{1}{\sqrt{2}}\right)$ $\left[\frac{19}{4}\right]$
   g. $3^{2x}-2(3^x)-15=0$ $\left[\frac{log5}{log3}\right]$

30. If $\log_b(a)=\frac{1}{x}$ and $\log_a(\sqrt{b})=3x^2$, show that $x=\frac{1}{6}$.

31. Graph $\log_2(8(x-3)^2)$.

Q31) Answer:



32. In the year 1980, both towns had an earthquake. Springfield's earthquake measured 7.5 on the Richter Scale. The magnitude of the earthquake was 7.5. The earthquake in Shelbyville measured 6.4. How many times as intense was Springfield's earthquake when compared to Shelbyville's earthquake? $[12.59\ times\ as\ intense]$

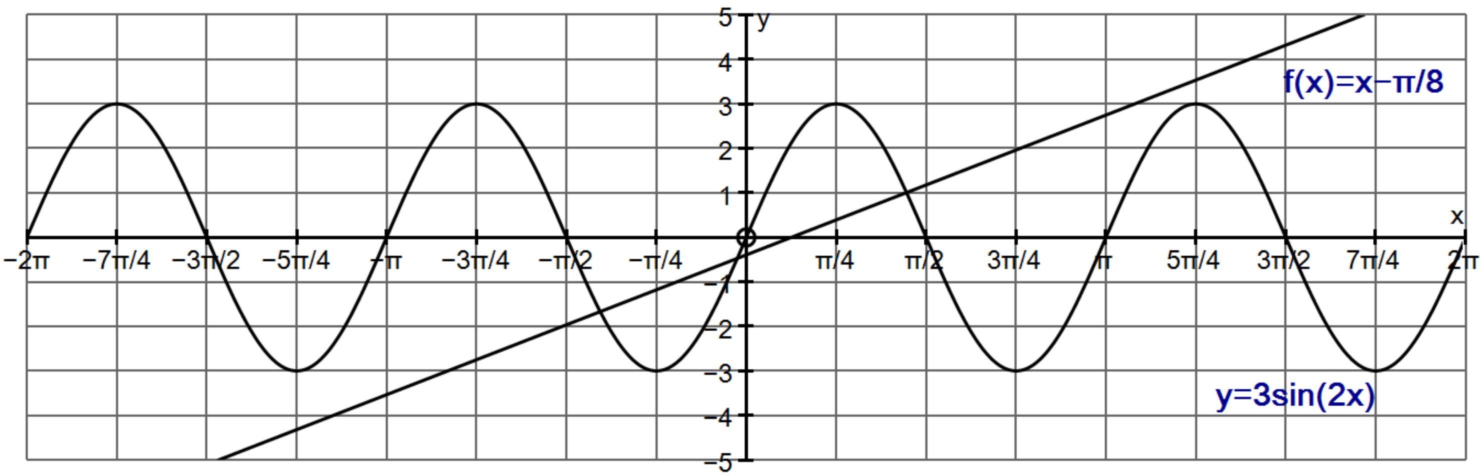
Unit 6 Questions

33. Let $f(x) = \{(3,2),(5,1),(7,4),(9,3),(11,5)\}$ and $g(x) = \{(1,3),(2,5),(3,7),(4,9),(5,11)\}$. Determine
   a. $f(g(3))$ [4] b. $g(f(9))$ [7] c. $(f-g)(x)$ $[\{(3,-5),(5,-10)\}]$

34. Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2+8$. Find
   a. $(f-g)(x)$ b. $g(g(x))$ c. $f^{-1}(x)$ d. $f(g(x))$ e. $f(f^{-1}(x))$
   f. $g(f(x))$ g. $f(g(5))$ h. $g(f(5))$

35. Prove whether each of the following are Even, Odd, or Neither.
   a. $f(x) = 2^x + 2^{-x}$ [E] b. $f(x) = \frac{\sin(x)}{x^2-4}$ [O] c. $f(x) = \log(x^x)$ $[N]$

36. The graphs of $f(x)$ and $g(x)$ are given below. On the same grid (in different colours), sketch
   a. $l(x) = f(g(x))$ b. $j(x) = g(f(x))$



37. What are the zeros of $f(x) = (x+2)(x+3)(x-4)\log(x)$?

38. Determine the domain of $f(g(x))$ if $f(x) = \frac{1}{x-5}$ and $g(x) = \frac{2}{(x+3)(x-4)}$ $\left[\left\{x \epsilon R \middle| x \neq -3, 4, \frac{5 \pm \sqrt{1265}}{10}\right\}\right]$