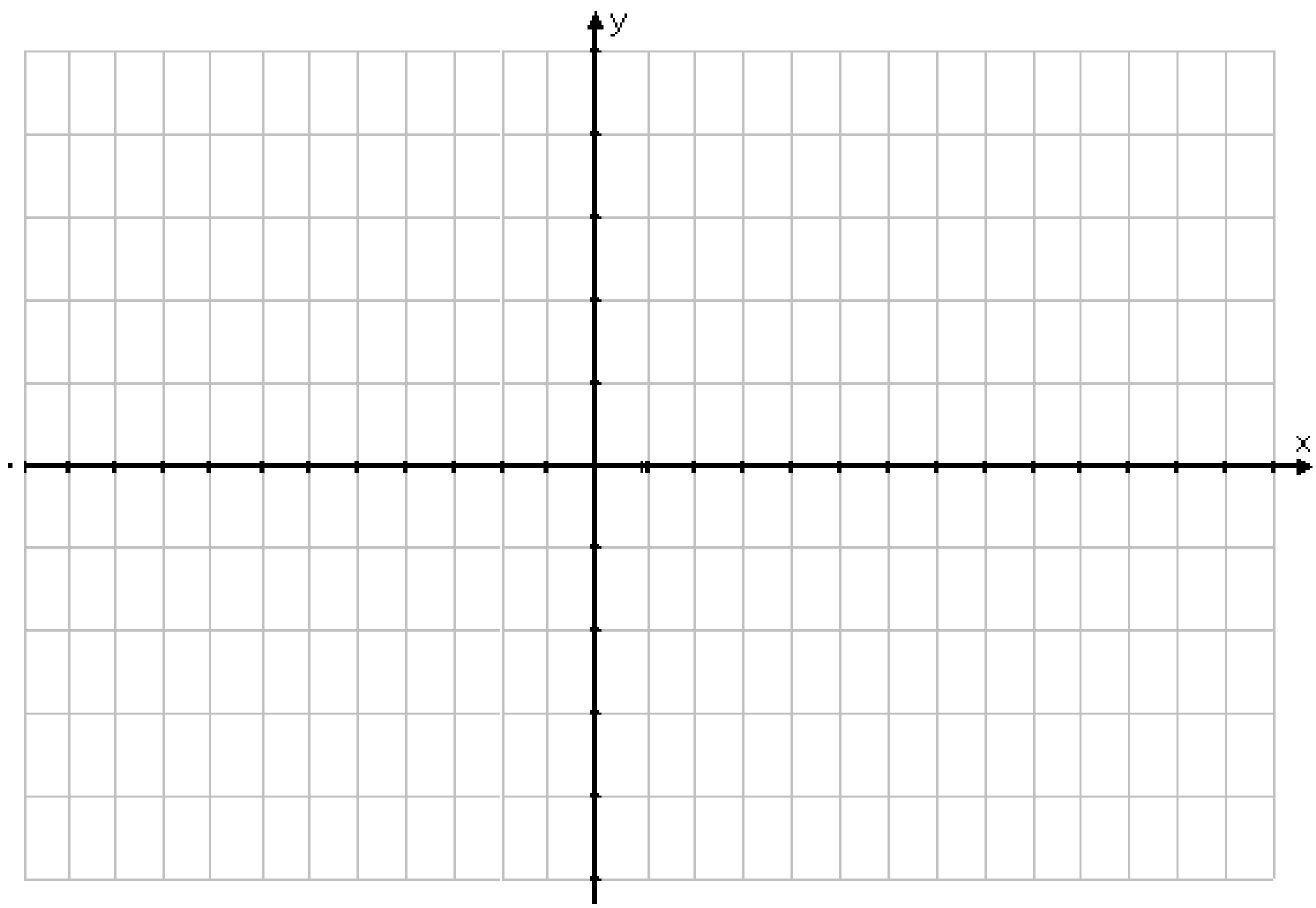
Unit 4: Trigonometric Functions(part II)

4.1 GRAPHING THE SINE FUNCTION

Complete the table of values for $y = \sin(\theta)$.

$\theta°$	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
Radian Measure	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y									

Let 6 spaces represent π along the θ-axis. Let 2 spaces along the y-axis represent 1 unit. Scale the axes. Then, plot each (θ , y) point. Then, join the points in a smooth curve. Using your calculator, determine the points in Quadrants II and III.
Then, extend your graph to cover the domain $-2\pi \le \theta \le 2\pi$.

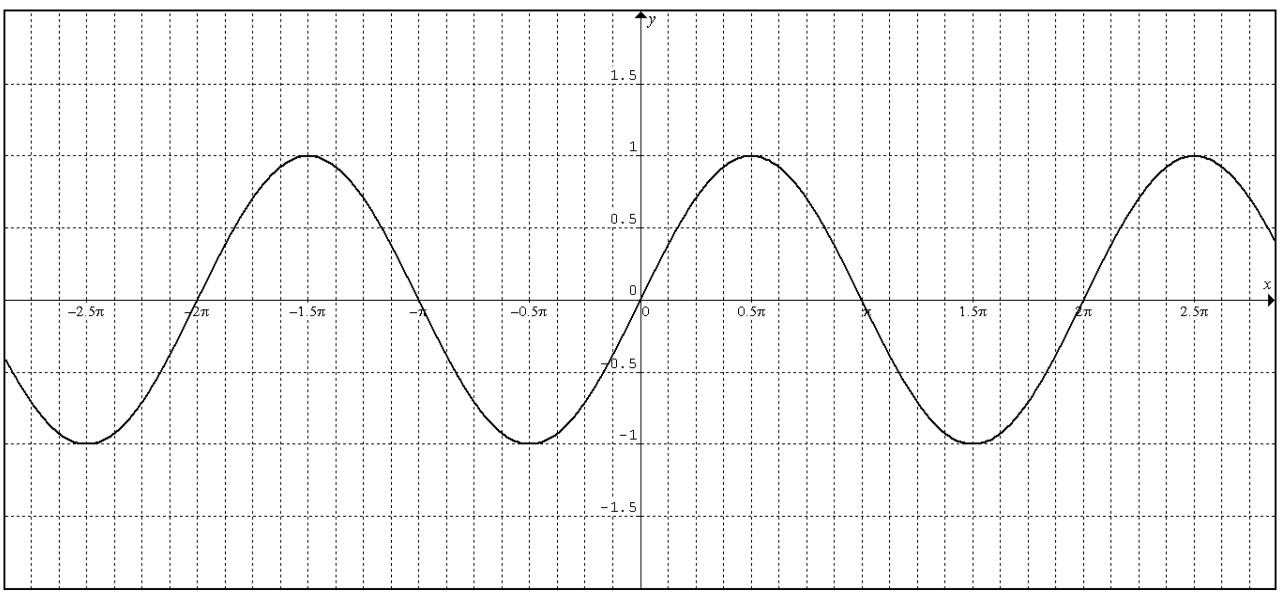


GRAPHING THE COSECANT FUNCTION

Complete the table of values for $y = \csc(\theta)$. Remember, $\csc(\theta) = \dfrac{1}{\sin(\theta)}$. To find the y-values for the graph $y = \csc\theta$, first evaluate $y = \sin(\theta)$. Then, take the reciprocal. (Your calculator has a 1/x button or an x^{-1} button that will give the reciprocal of an input.)

$\theta°$	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
Radian Measure	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y									

Notice if $\sin(\theta) = 0$, then $\csc(\theta)$ is undefined. An undefined value for y implies that there is a vertical asymptote at the corresponding θ value, so draw a vertical dotted line at these locations. Plot the values of $y = \csc(\theta)$ and draw smooth curves through the points for each defined region. Notice that the graph for $y = \sin(\theta)$ has been penciled in so you can make comparisons between it and the graph of $y = \sec(\theta)$.

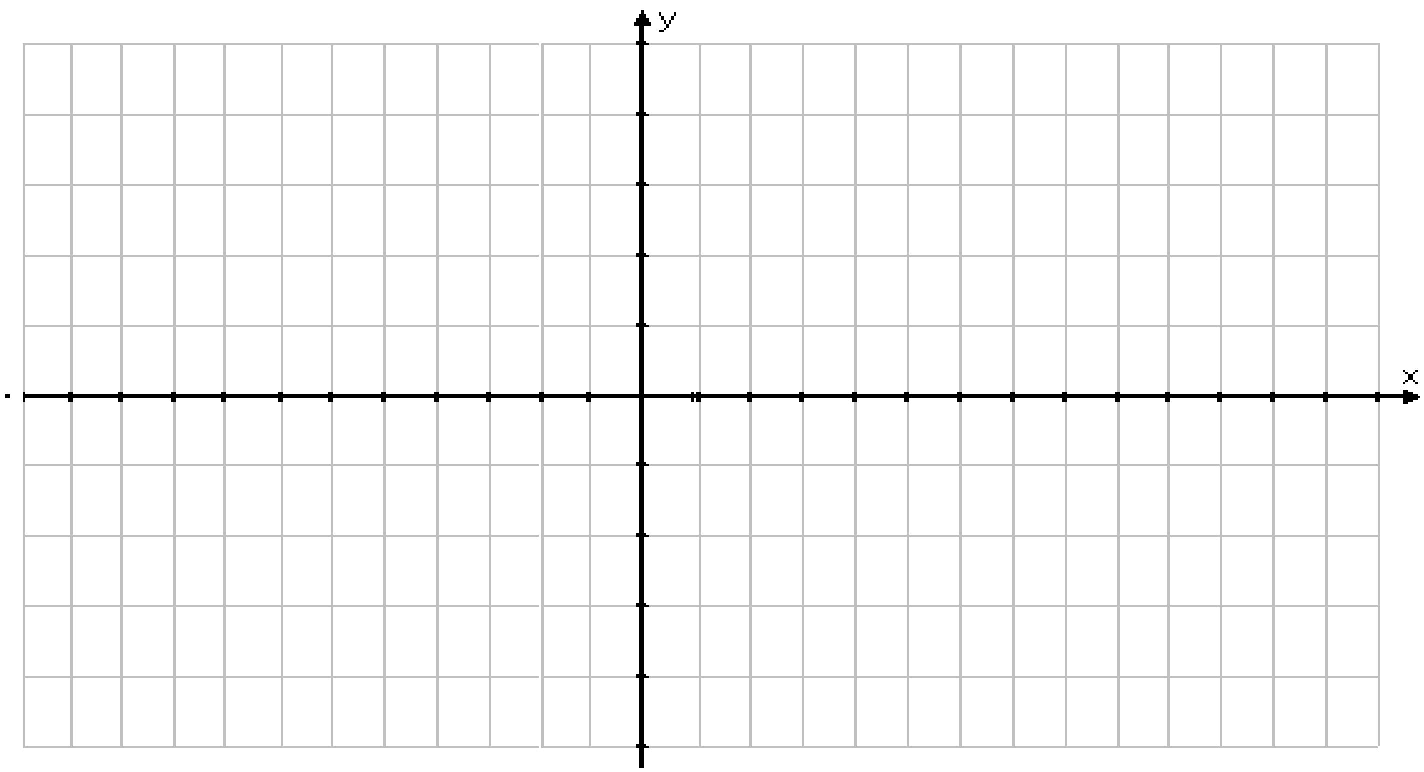


GRAPHING THE COSINE FUNCTION

Complete the table of values for $y = \cos(\theta)$.

$\theta°$	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
Radian Measure	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y									

Let 6 spaces represent π along the θ-axis. Let 2 spaces along the y-axis represent 1 unit. Scale the axes. Then, plot each (θ , y) point. Then, join the points in a smooth curve. Using your calculator, determine the points in Quadrants II and III. Then, extend your graph to cover the domain $-2\pi \le \theta \le 2\pi$.

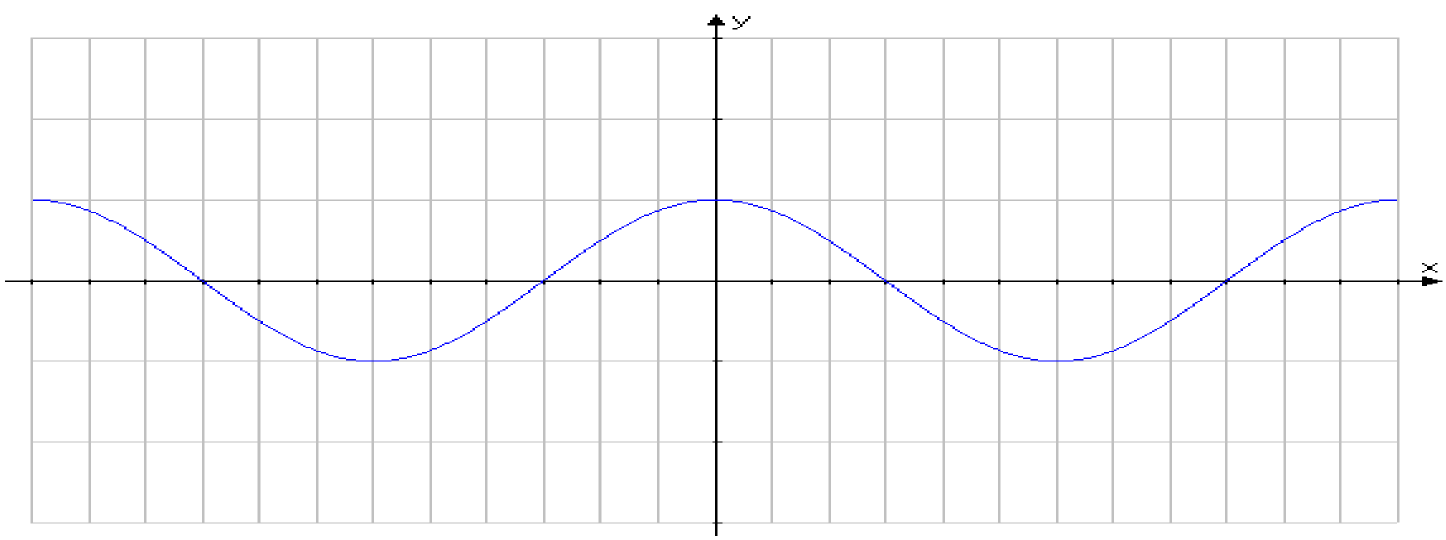


GRAPHING THE SECANT FUNCTION

Complete the table of values for $y = \sec(\theta)$. Remember, $\sec(\theta) = \dfrac{1}{\cos(\theta)}$. To find the y-values for the graph $y = \sec(\theta)$, first evaluate $y = \cos(\theta)$. Then, take the reciprocal. (Your calculator has a 1/x button or an x^{-1} button that will give the reciprocal of an input.)

$\theta°$	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
Radian Measure	-2π	$-\dfrac{3\pi}{2}$	$-\pi$	$-\dfrac{\pi}{2}$	0	$\dfrac{\pi}{2}$	π	$\dfrac{3\pi}{2}$	2π
y									

Notice if $\cos(\theta) = 0$, then $\sec(\theta)$ is undefined. An undefined value for y implies that there is a vertical asymptote at the corresponding θ value, so draw a vertical dotted line at these locations. Plot the values of $y = \sec(\theta)$ and draw smooth curves through the points for each defined region. Notice that the graph for $y = \cos(\theta)$ has been penciled in so you can make comparisons between it and the graph of $y = \sec(\theta)$.



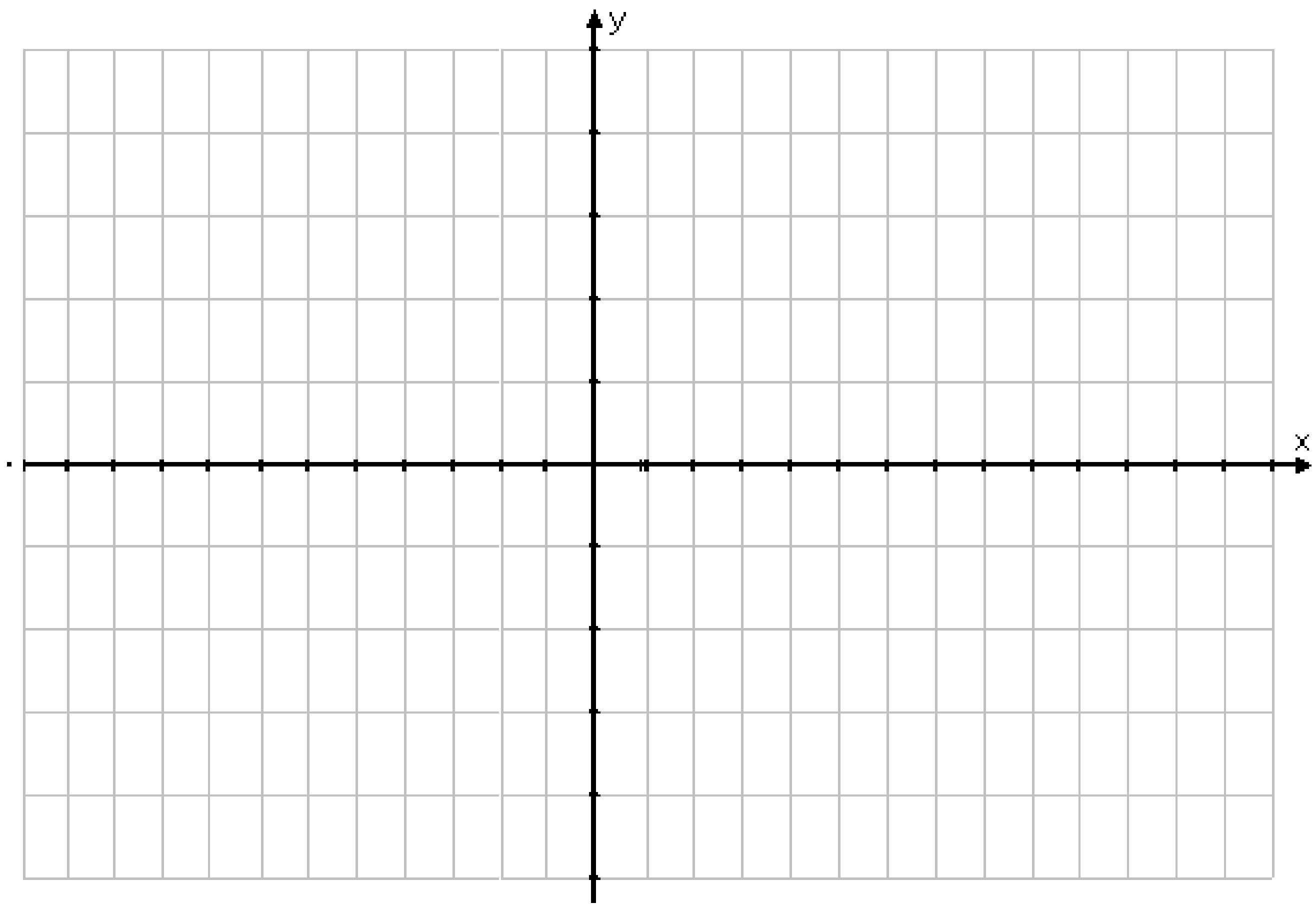
GRAPHING THE TANGENT FUNCTION

Complete the table of values for $y = \tan(\theta)$. Then graph the function.

$\theta°$	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
Radian Measure	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y									

Notice if $y = \tan(\theta)$ is undefined, then there is a vertical asymptote at this value for θ.

Draw a dotted line parallel to the y-axis. Your curve should approach these asymptotes closely, but not touch or cross them. Plot the (θ, y) points and draw smooth curves through the points for each defined region.

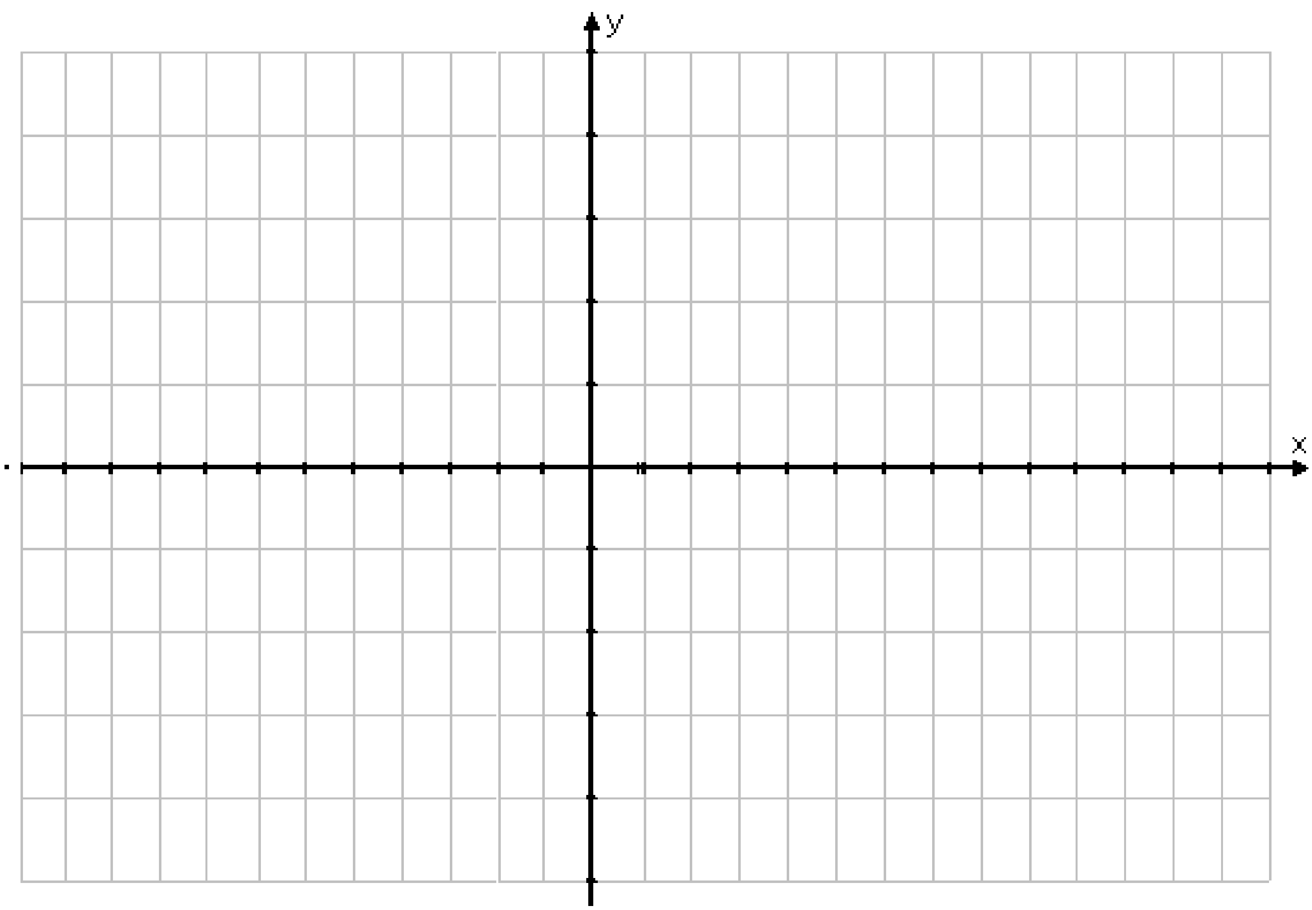


GRAPHING THE COTANGENT FUNCTION

Complete the table of values for $y = \cot(\theta)$. Remember, $\cot(\theta) = \dfrac{1}{\tan(\theta)}$. To find the y-values for the graph $y = \cot(\theta)$ first evaluate $y = \tan(\theta)$. Then, take the reciprocal. (Your calculator has a 1/x button or an x^{-1} button.)

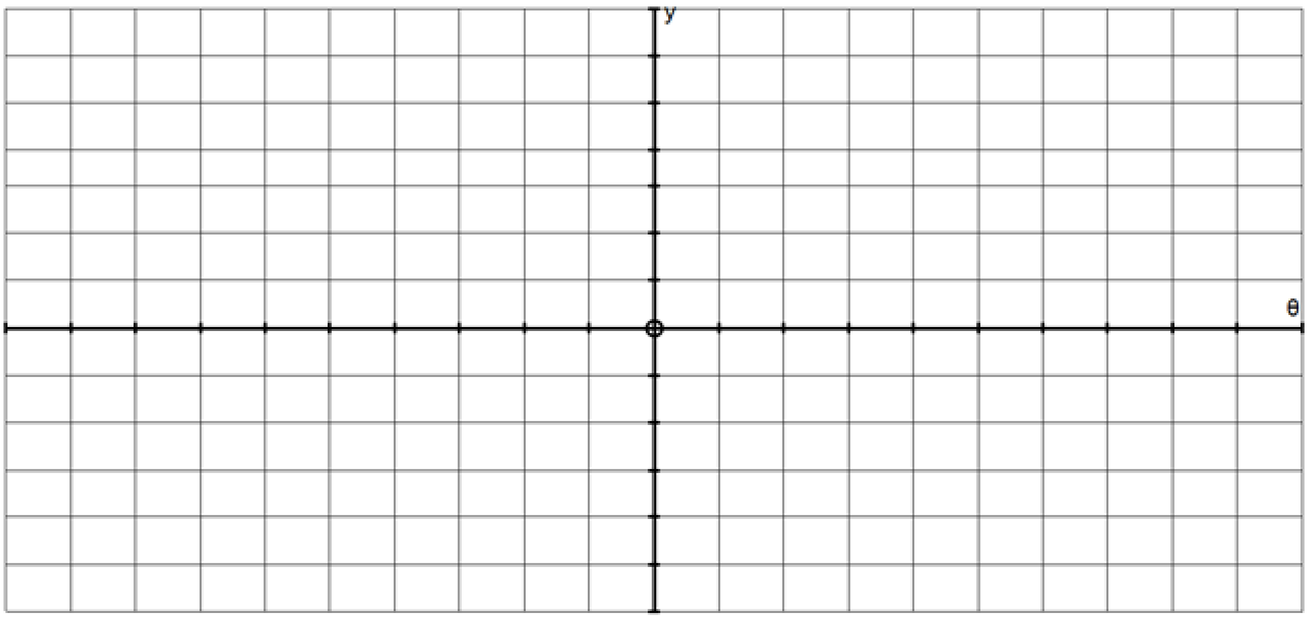
$\theta°$	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
Radian Measure	-2π	$-\dfrac{3\pi}{2}$	$-\pi$	$-\dfrac{\pi}{2}$	0	$\dfrac{\pi}{2}$	π	$\dfrac{3\pi}{2}$	2π
y									

Notice if $\tan(\theta) = 0$, then $\cot(\theta)$ is undefined. An undefined value for y implies that there is a vertical asymptote at the corresponding θ value, so draw a vertical dotted line at these locations. Also, notice that if $\tan(\theta)$ is undefined, then $\cot(\theta) = 0$ .Plot the values of $y = \cot(\theta)$ and draw smooth curves through the points for each defined region.

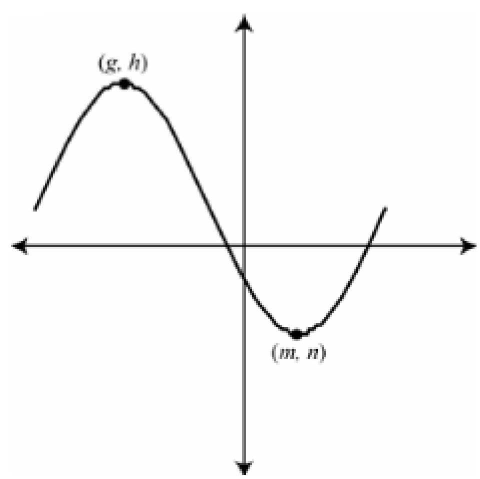


Warm up

1. Graph one complete cycle of $y = -3\cos\frac{1}{4}\left(\theta + \frac{\pi}{3}\right) + 1$.



2. The graph of $y = 6\cos\left(x + \frac{3\pi}{4}\right) + 1$ is illustrated below. Determine the exact values of $\boldsymbol{g}$,$\boldsymbol{h}$,$\boldsymbol{m}$ and **n**.



4.2 Properties of Sinusoidal Functions

y = a sin (k(x -d)) + c and y = a cos (k(x -d)) + c

Amplitude – Half the distance between the maximum and minimum values of a periodic function.

$$\boxed{\text{amplitude} = \frac{\text{max - min}}{2}}$$

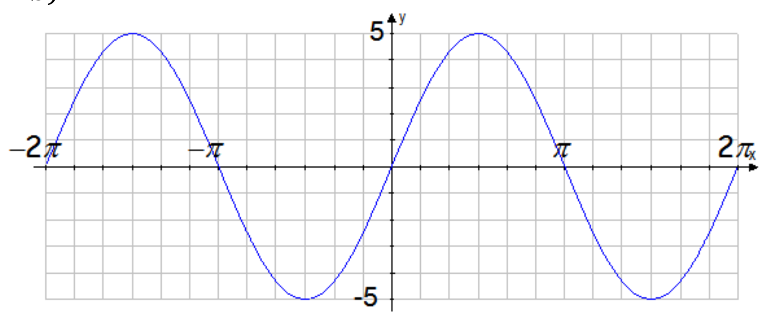
For $y = a\sin(\theta)$ or $y = a\cos(\theta)$

$$\boxed{\text{Amplitude} = |a|}$$

Q1. Find the amplitude for the following:

a) $y = -2\cos(\theta)$

b)

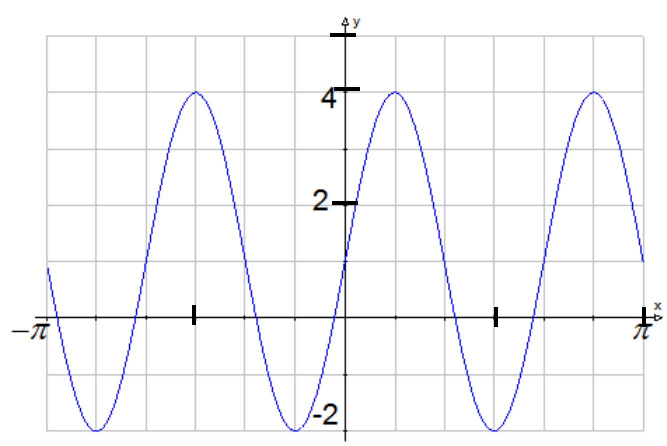


Axis of the curve – The horizontal line that is half way between maximum and minimum of values of a periodic curve is called the axis of the curve

The equation of the axis of the curve is:

$$\boxed{y = \frac{\text{maximum value} + \text{minimum value}}{2}}.$$

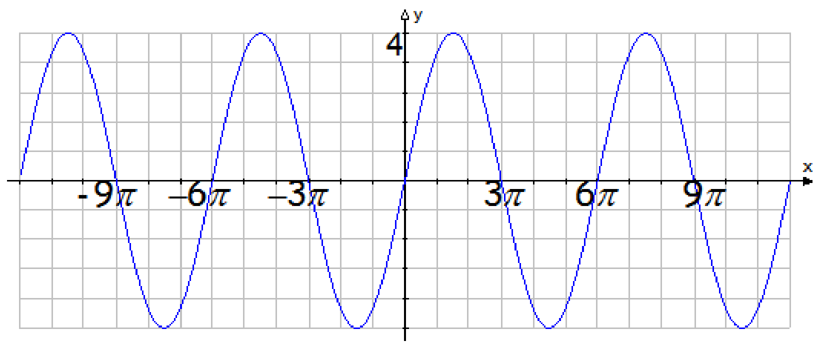
Q2. Find the equation of axis of the curve for the following graph :



Period – The horizontal length of one cycle. (The horizontal distance between two consecutive maximum or two consecutive minimum)

Recall: Pick a point on the curve. Follow the graph until it starts repeating. That is one **cycle**.

Q3. Indicate one cycle on the graph, then, state the period of the function shown:



For $y = \sin(k\theta)$ or $y = \cos(k\theta)$

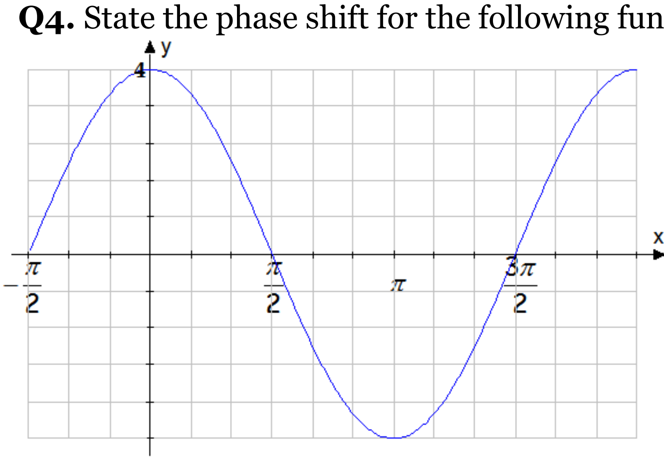
$$\boxed{\text{Period} = \frac{2\pi}{|k|}}$$

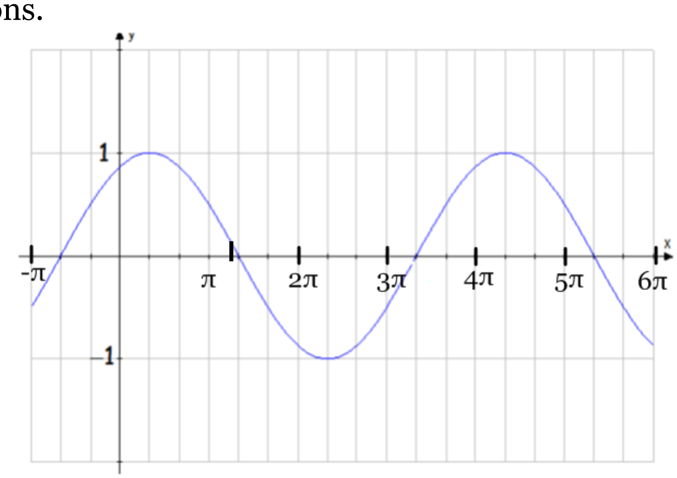
Phase Shift – The horizontal translation of a trigonometric function.

For $y = \sin\left[k(x-d)\right]+c$ or $y = \cos\left[k(x-d)\right]+c$

$$\boxed{\text{Phase shift} = x_{max} - \frac{P}{4}}$$

Q4. State the phase shift for the following functions.



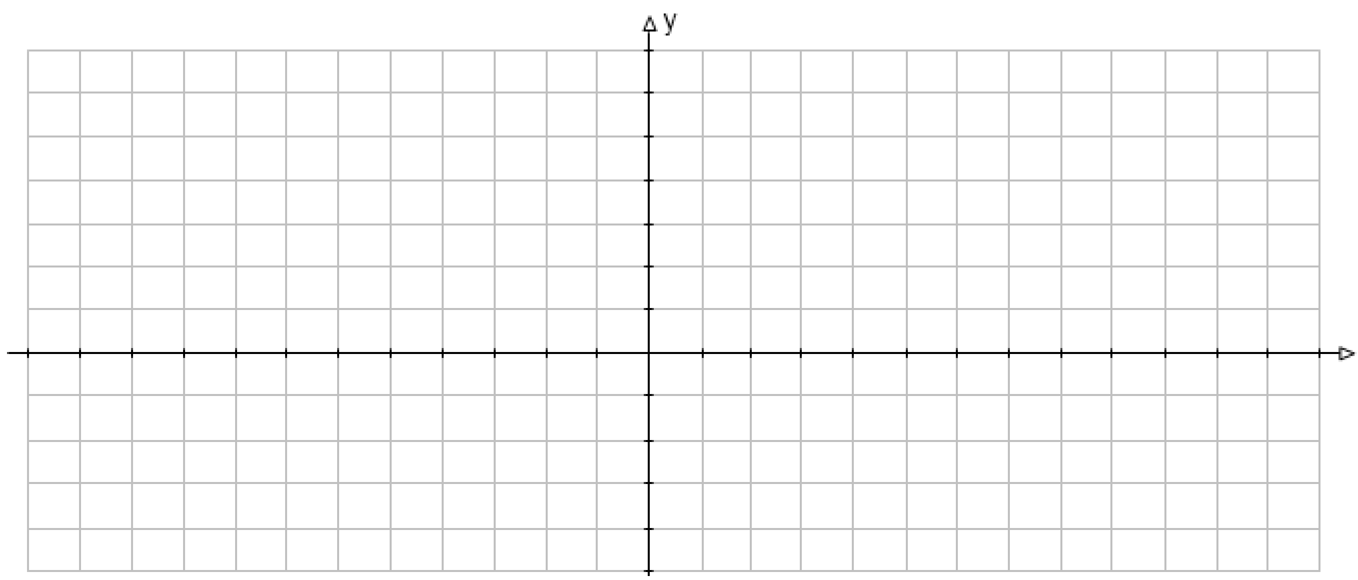


EXAMPLE 1: Complete the following table and then sketch the graph of the function using mapping rule.

a) $f(\theta) = 2\sin(2\theta - \frac{\pi}{3})$.

$(\theta, y) \rightarrow$

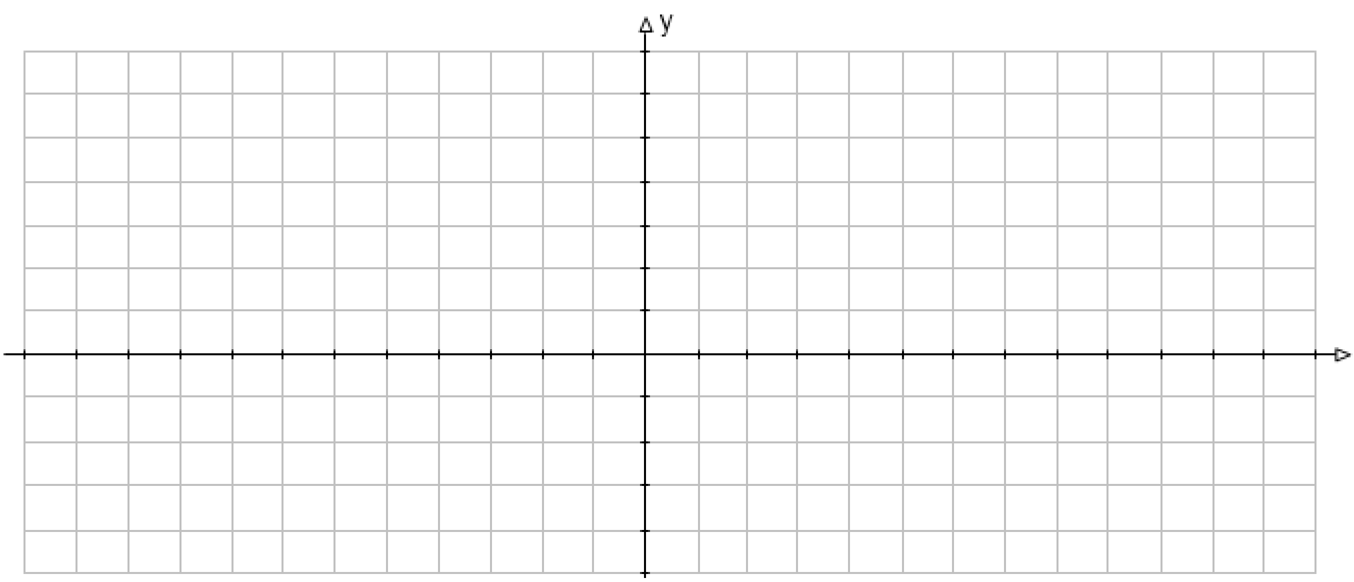
Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



b) $f(\theta) = -3\cos\frac{1}{2}(-\theta + \frac{\pi}{4})$.

$(\theta, y) \rightarrow$

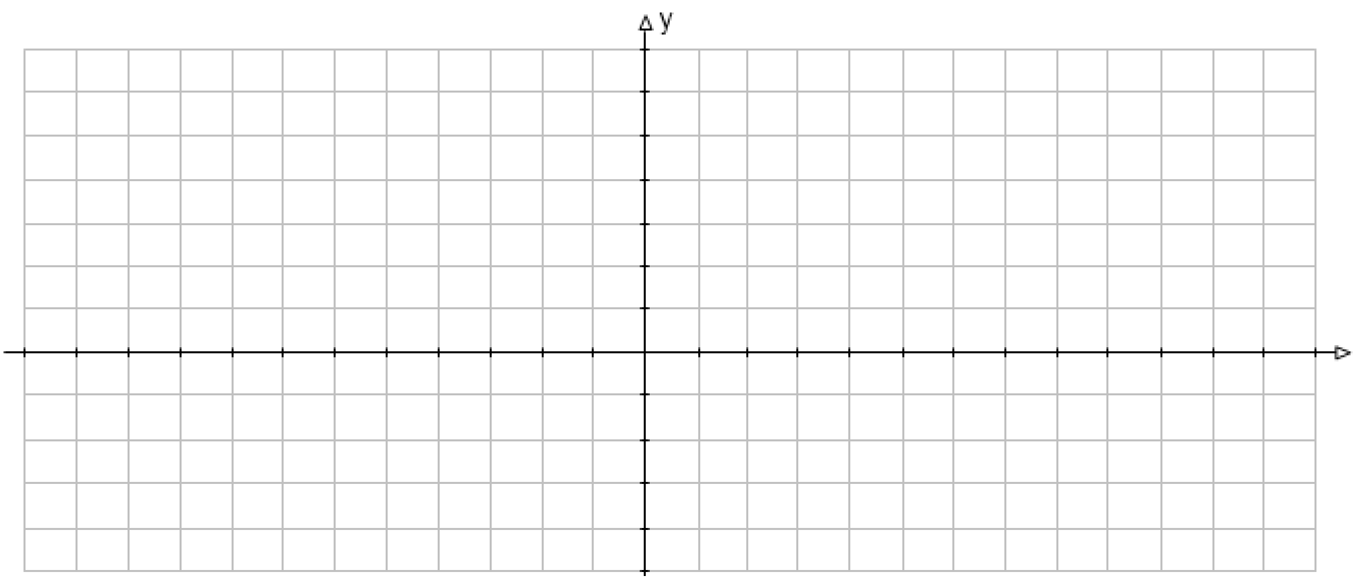
Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



c) $f(\theta)=\frac{1}{2}\cos\left(3\theta-\frac{\pi}{2}\right)+3$.

$(\theta, y)\rightarrow$

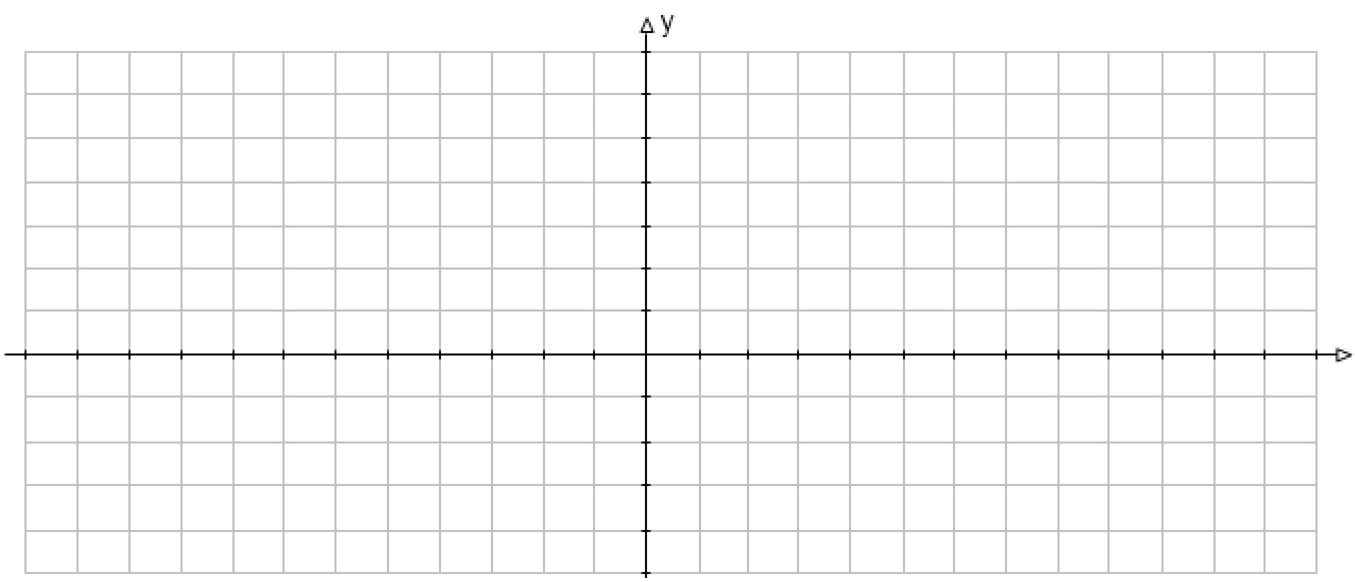
Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



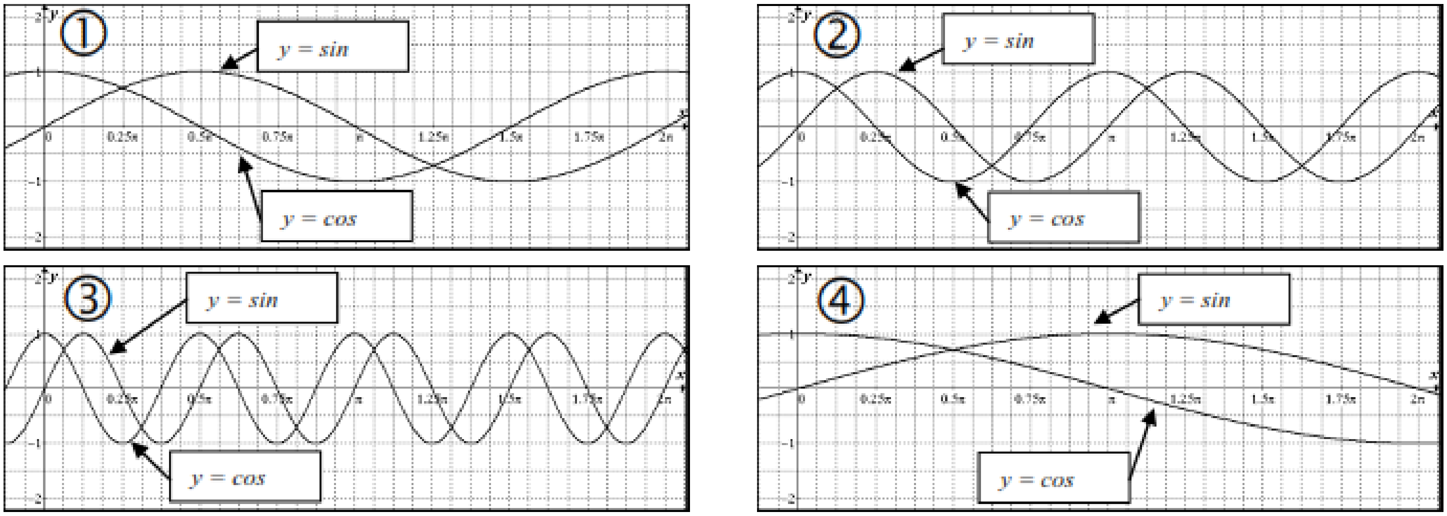
d) $f(\theta) = -2\sin\left(\frac{\theta}{2} - \frac{\pi}{3}\right) + 1.$

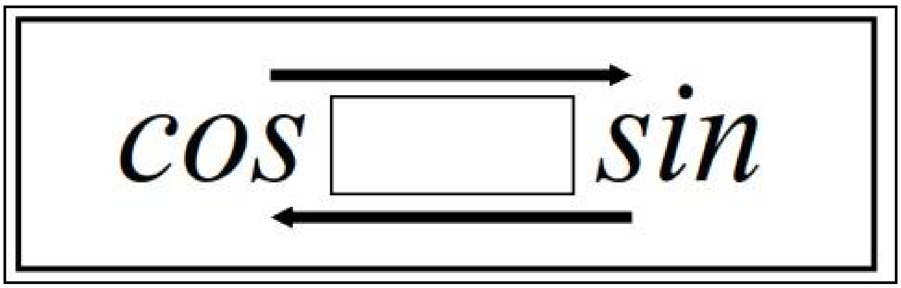
$(\theta, y) \rightarrow$

Amplitude	
Equation of axis of curve	
Period	
maximum	
minimum	
Phase Shift	



4.2 Converting between Sine and Cosine





Q1. Convert sin to cos or cos to sin in each of the following:

#	a	P	d	c	Equation with *sin*	Same curve using *cos*
a)	1	2π	$\frac{\pi}{4}$ left	0		
b)	2	4π	$\frac{3\pi}{2}$ left	2 up		
c)	7	π	$\frac{\pi}{4}$ left	1 down		
d)	$\frac{1}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{8}$ right	$\frac{3}{5}$ up		
e)	3	8π	$\frac{5\pi}{6}$ right	$\frac{1}{8}$ down		

f) $y = \sin\left(\frac{1}{8}x\right)$ ____________________________________

g) $y = 2\cos\left(\frac{3}{4}x\right)$ ____________________________________

h) $y = \sin 4\left(x - \frac{\pi}{8}\right)$ ______________________________

i) $y = 2\sin\frac{1}{3}\left(x - \frac{7\pi}{3}\right)$ ______________________________

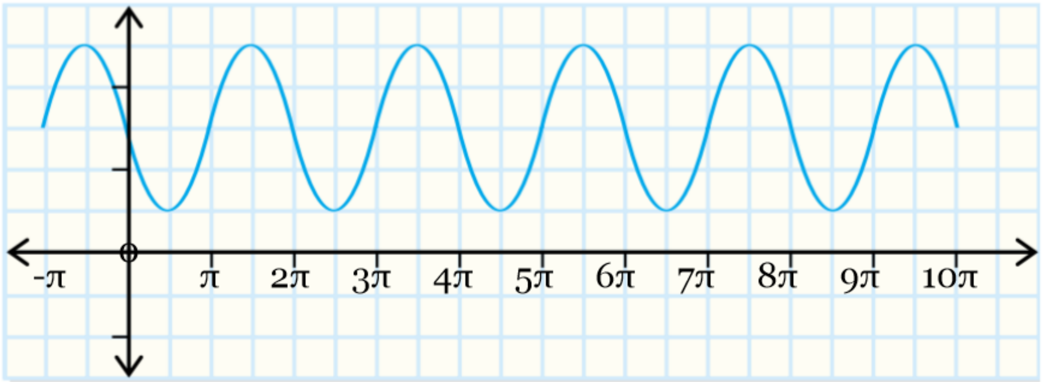
j) $y = 3\cos 3\left(x + \frac{\pi}{4}\right)$ ______________________________

Q2. Write an equation to represent a i) Sine function, ii) Cosine function with the following properties. Maximum: 23 Minimum: 11 Period 5 Phase shift: 9

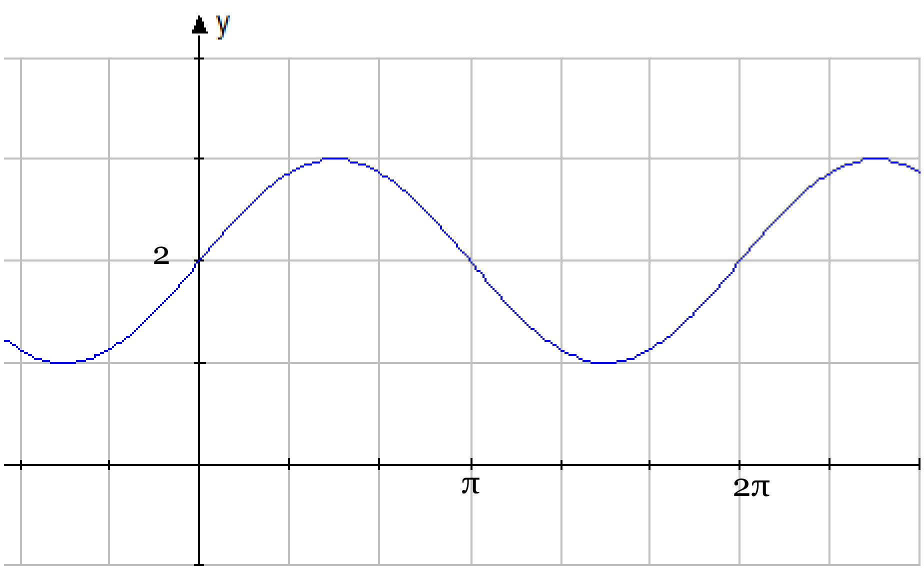
Q3. Find an equation of a function of the form $y = \cos(\theta) + q$ whose graph passes through the point $A\left(\frac{\pi}{3}, \frac{-1}{2}\right)$.

Q.4: State the amplitude and period, phase shift, and vertical shift of each function. Then, write the equation of the function as :

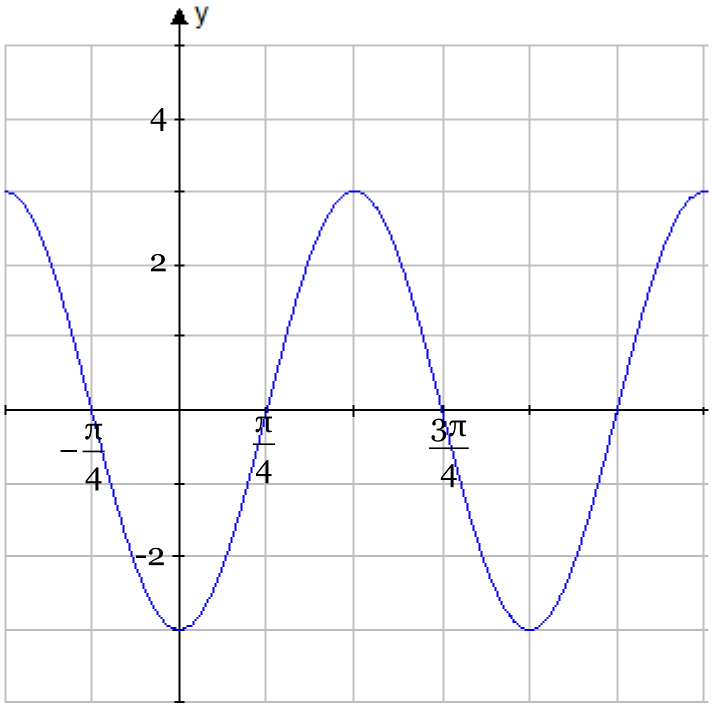
a) a sine function



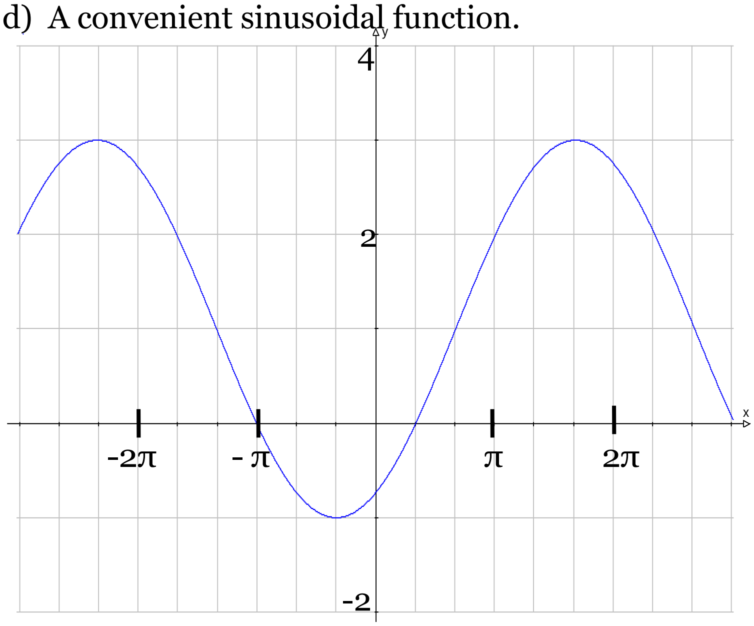
b) a cosine function:



c) A cosine function.

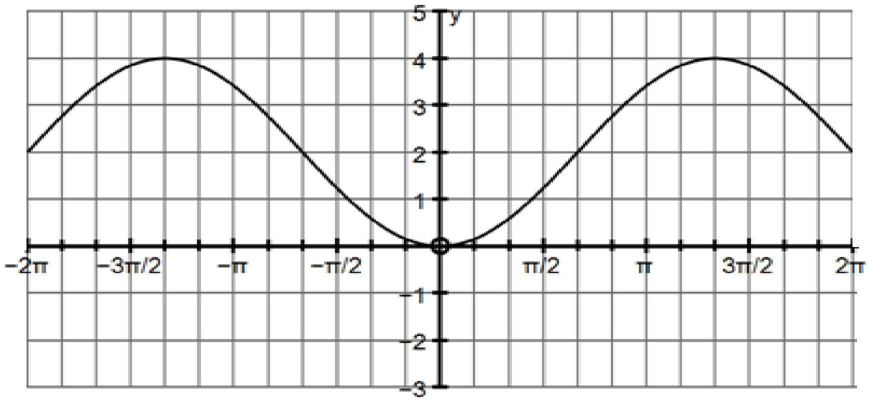


d) A convenient sinusoidal function.



Warm up

1) Write the equation of the following function as both sine and cosine function.



Sine Equation: ____

Cosine Equation: ____

2) An equation that relates the height of the tide with respect to time is given by the equation $h(t) = 1.8\sin\left[\dfrac{2\pi(t-4)}{12.4}\right] + 3.1$. Time is given in hours using a 24 hour clock and the height is given in meters above sea level.

(a) What is the minimum height of the tide?_______________________

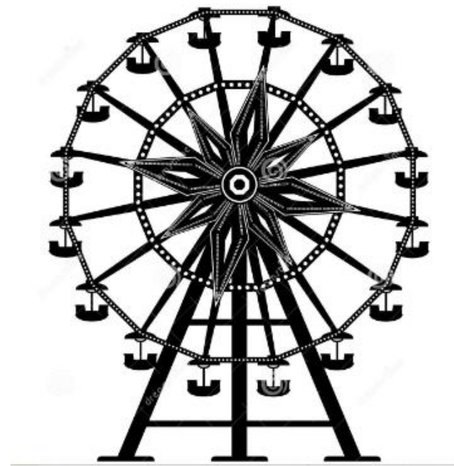
(b) Calculate the depth of the water at 5 A.M. ______________________

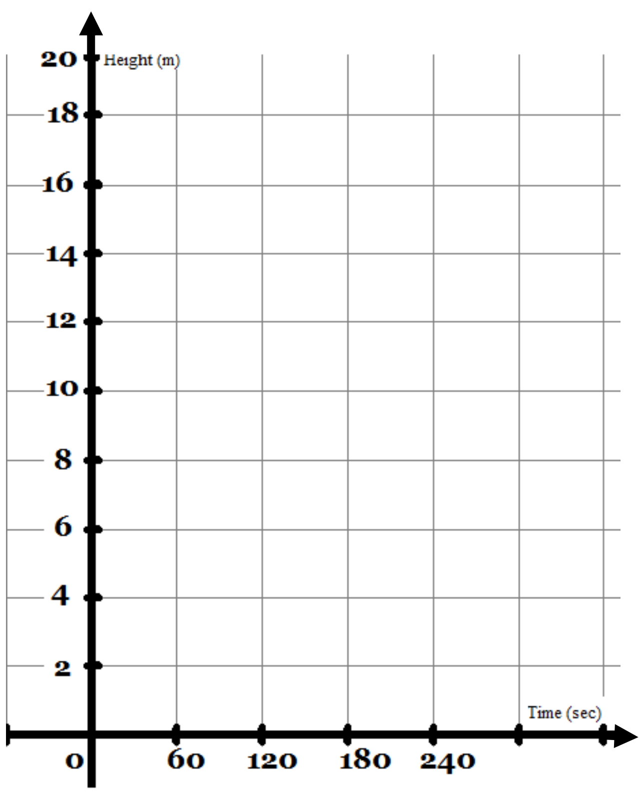
(c) At **what time** is the first occurrence of the maximum height? ________________________

4.3 Modeling Sinusoidal Functions

1. At a county fair, the Ferris wheel has a diameter of 16 meters, and its lowest point is 2 meters above the ground. The wheel completes one complete revolution every 4 minutes. Riders begin a ride at the lowest position on the wheel.
   (a) Draw a sketch of the trip.
   (b) Determine the equation of this periodic function using both sine and cosine function.
   (c) Find the height of passengers at 2 minutes and 35 seconds.
   (d) Find the time(s) when the passengers are at a height of 7 m.

Time (sec)	Height (m)
0 sec	
60 sec	
120 sec	
180 sec	
240 sec	

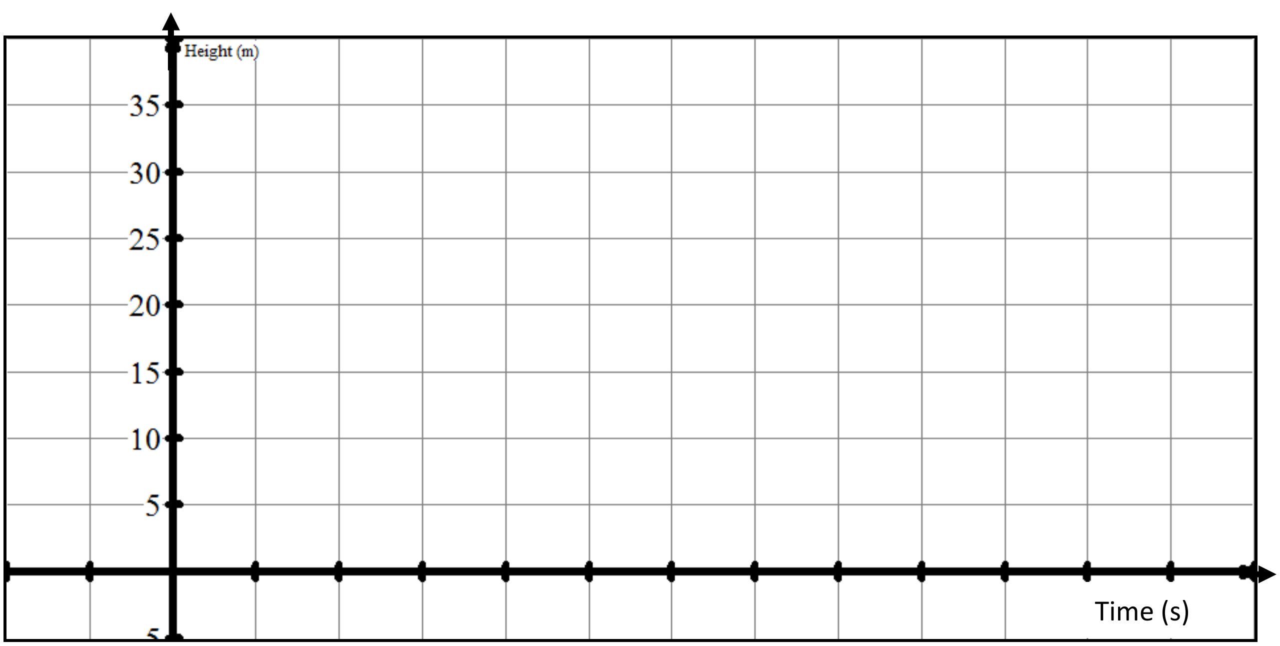




2. The SkyWheel is a Ferris wheel in Niagara Falls. It has a diameter of 30 metres and the ride lasts for 12 minutes for a total of 6 revolutions. It has a total of 42 gondolas that can each hold 6 passengers. Assuming that the height of the gondola follows a sinusoidal model, if you enter the gondola at a height of 2 m above the ground, what is your altitude at 11 minutes and 21 seconds? At what time(s) is your height 21 m high?

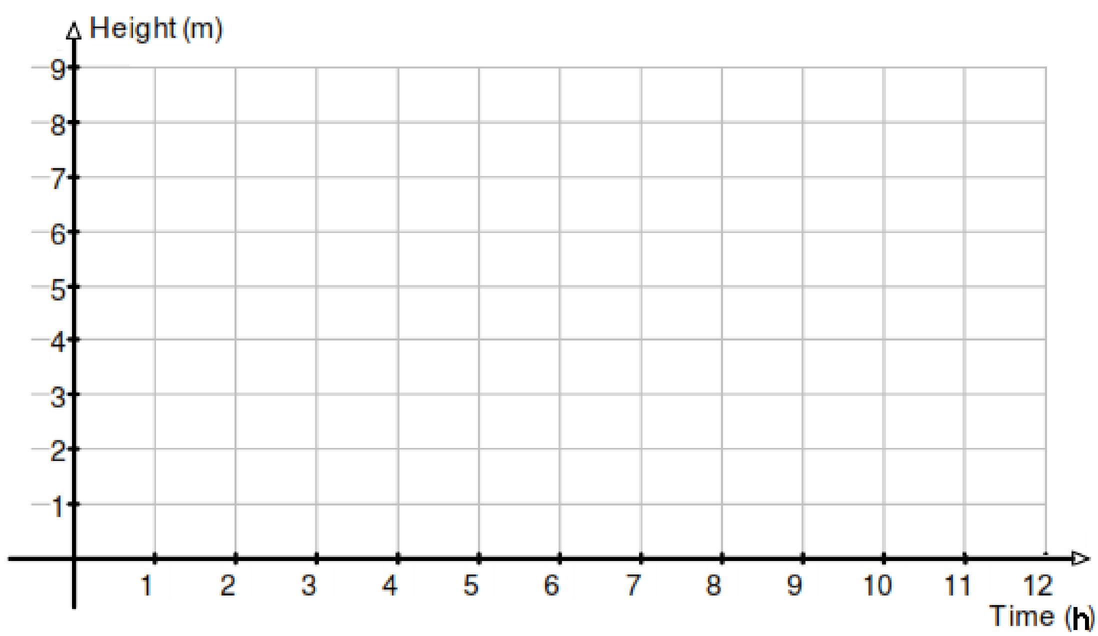
   (a) Draw a sketch of the trip.
   (b) Determine the equation of this periodic function using both sine and cosine function.
   (c) Find the height of your gondola at 11 minutes and 21 seconds.
   (d) Find the time(s) when your gondola is at a height of 21 m.

Time (sec)	Height (m)



3. The water at a local beach has an average depth of 1 meter at low tide. The average depth of the water at high tide is 8 m. If one cycle takes 12 hours:
   (a) Determine the equation of this periodic function using cosine as the base function where 0 time is the beginning of high tide.
   (b) What is the depth of the water at 2 am?
   (c) Many people dive into the beach from the nearby dock. If the water must be at least 3 m deep to dive safely, between what daylight hours should people dive?

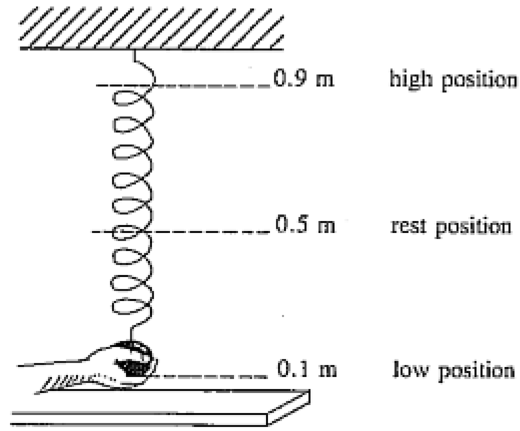
Time (h)	Height (m)

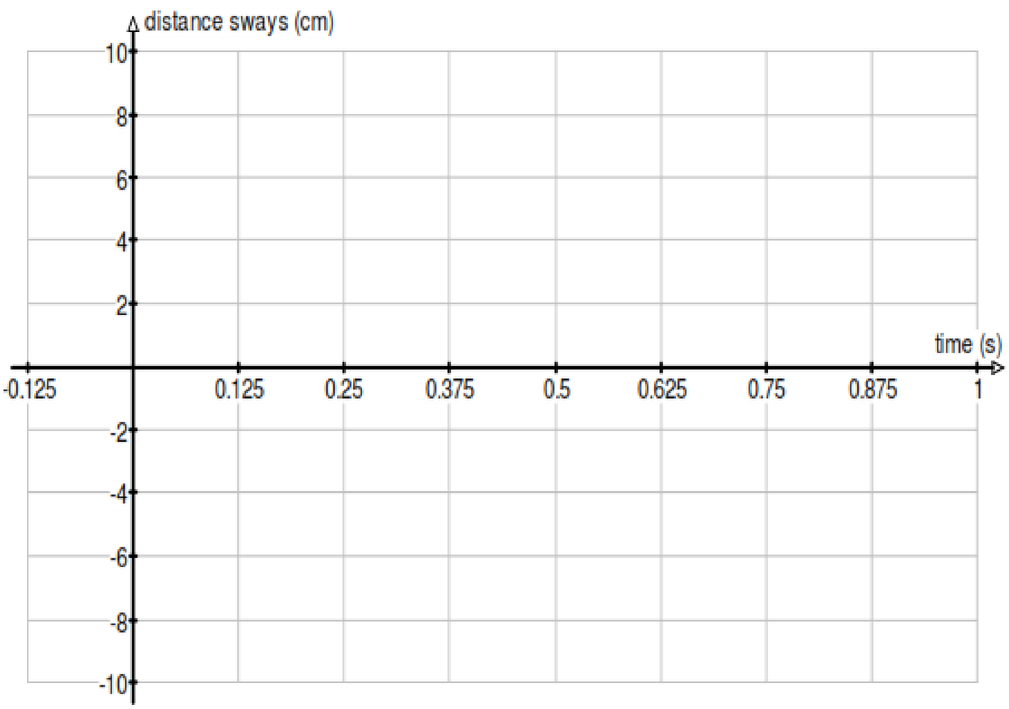


4. A certain mass is supported by a spring so that it is at rest 0.5 m above a table top. The mass is pulled down 0.4 m to its lowest position and released at time t=0, creating a periodic up and down motion, called simple harmonic motion. It takes 1.2 s for the mass to reach the highest position of 0.9 m and return to the lowest position each time.

a) Draw a graphing showing the height of the mass above the table top as a function for the first 2.0s.

b) Write an equation for this function.



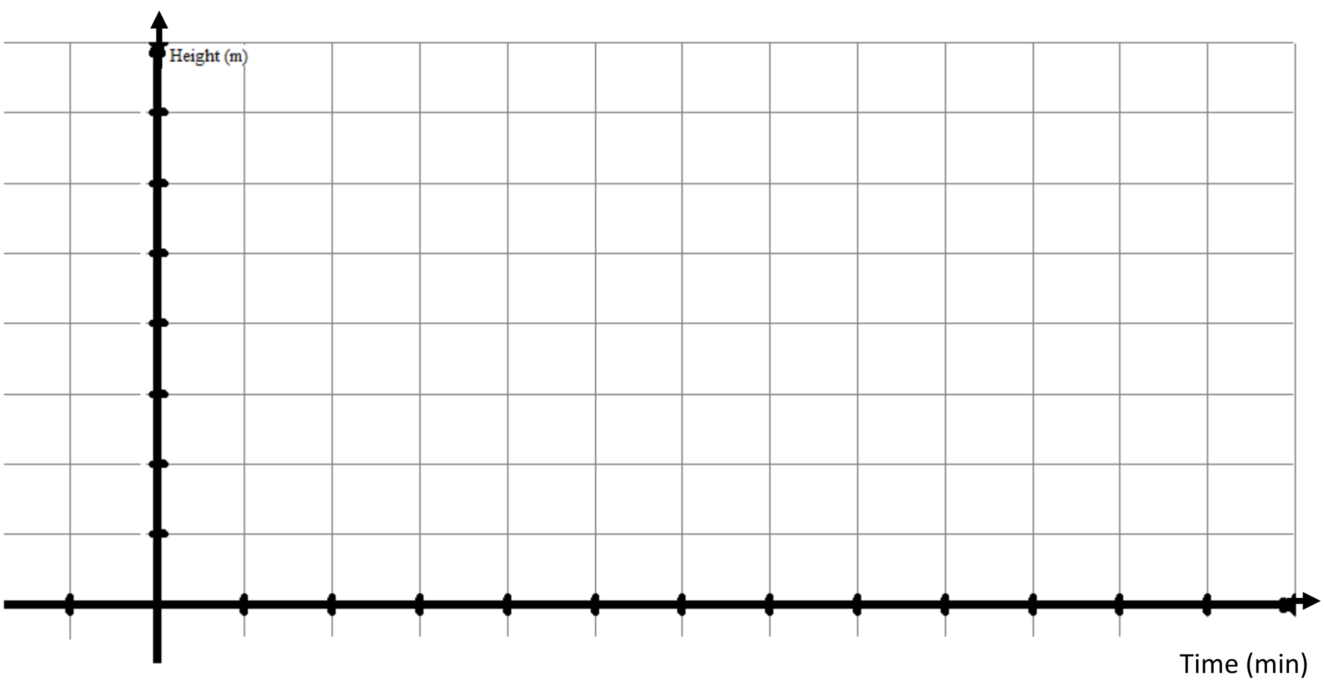


EXIT CARD

A Ferris wheel with a radius of 9.5 m rotates once every 10 min. The bottom of the wheel is 1.2 m above the ground. Draw a graph to show how a person's height above the ground varies with time for two complete revolutions, starting when the person gets onto the Ferris wheel at its lowest point.

(a) Determine an equation for the graph based on a sine function.
(b) Determine an equation for the graph based on a cosine function.
(c) Find the time(s) when the passengers are at a height of 8.6 m.

Time (min)	Height (m)



4.3 Applications of Sinusoidal Functions-HW

1. Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time $t = 0$. A minimum number, 200 foxes, occurred when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t = 5.1$ years.

a) Sketch a graph of this sinusoid.
b) Write an equation, using the sine function, to express the number of foxes as a function of time, t.
c) Predict the population when $t = 7$.
d) Foxes are declared to be an endangered species when their population drops below 300. Between what two non-negative values of t were foxes first endangered?

2. A Ferris wheel ride reaches a maximum height of 30m and a minimum height of 2m above the ground. The Ferris wheel rotates at a speed of 0.8π m/s. You start the ride at the lowest point of the wheel and travel counterclockwise. [$\text{Hint}: k = \dfrac{speed}{radius}$]

a) Sketch a graph of this sinusoid.
b) Write an equation, using the cosine function, to express your height above the ground as a function of time, t.
c) How high will you be after 30s on the ride?

3. A group of students decided to study the sinusoidal nature of tides. Values for the depth of the water level were recorded at various times. At t=2 hours low tide was recorded at a depth of 1.8 m. At t=8 hours, high tide was recorded at a depth of 3.6 m.

a) Sketch the graph of this function
b) Write an equation expressing distance in terms of time
c) Give the depth of water at t=21 hours.

4. A city averages 14 hours of daylight in June, 10 in December, and 12 in both March and September. Assume that the number of hours of daylight varies sinusoidally over a period of one year.

a) Sketch the graph of this relationship.
b) Write an equation for n, the number of hours of daylight, as a cosine function of t.
c) (Let t be in months and t = 0 correspond to the month of January)
d) How many hours of daylight are there in the month of July?

5. At a certain ocean bay, the maximum height of the water is 4 m above mean sea level at 8:00 a.m. The height is at a maximum again at 8:24 p.m. Assuming that the relationship between the height, h, in meters, and the time, t, in hours, is sinusoidal, determine the height of the water above mean sea level at 10:00 a.m.

ANSWERS

1. b) $p(t) = 300\sin[\frac{5\pi}{11}(t-4)]+500$ or $p(t) = -300\cos[\frac{5\pi}{11}(t-2.9)]+500$

   c) 227 foxes

   d) 2.31years, 3.49years

2. b) $h(t) = 14\cos[\frac{2\pi}{35}\ (t-17.5)]+16$ or $h(t) = -14\cos\frac{2\pi}{35}t +16$

   c) 7.27m

3. b) $h(t) = 0.4\sin[\frac{5\pi}{3}(t-0.6)]+0.5$ or $h(t) = -0.4\cos\left(\frac{5\pi}{3}t\right)+0.5$

4. b) $n(t) = 2\cos[\frac{\pi}{6}(t-5)]+12$

   c) 13.7h

5. a) $h(t) = 4\sin\frac{5\pi}{31}(t+3.1)$ or $h(t) = 4\cos\left(\frac{5\pi}{31}t\right)$

   b) $h(2) = 2.1\text{m}$

Warm up

Oceanic tides also display periodic behavior, where the high tide corresponds to the peak and the low tide corresponds to the trough of a sinusoidal curve. On a certain day, the depth of water at Sydney Harbour Bridge at high tide was 16m. After 6 hours (at low tide) the depth was 6m.

(a) How long is the tidal cycle.

(b) Write an equation for the depth of the water (in meters), in terms of time (in hours).

(c) Using your equation, find the depth of the water 4 hours and 15 minutes *after the low tide*.

(d) When will the tide first reach a height of 14 m?

4.4 Solving Trigonometric Equations

What is a trig equation?

- A trig equation is an equation that contains one or more trigonometric functions.

How is it similar to a trig identity?

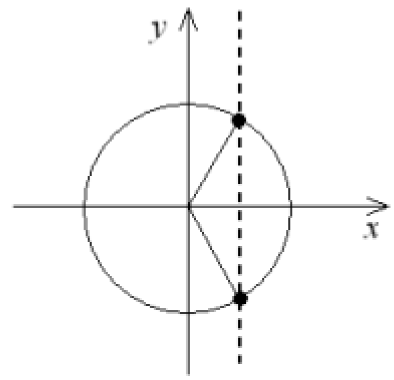
- A trig equation *can* be but *does not have to* be a trig identity
    - A trig identity is an equation that is true for ***all values*** of the variable for which expressions of both sides of the equation are defined
- A trig equation that is not an identity is only true for ***certain values*** of the variable.

What does it mean to **SOLVE** a trig equation?

- Much like solving a linear equation, we are looking for all of the values of the variable that makes the equation true.

General Solutions

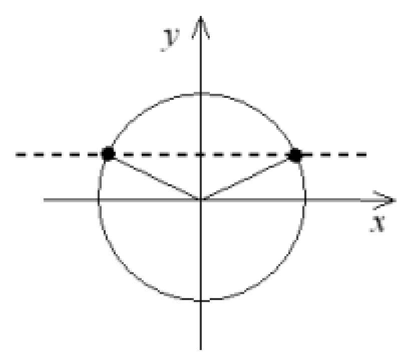
i) If $\cos(x)=\cos(\alpha)$, then $x = 2\pi k \pm \alpha$, $k \in Z$



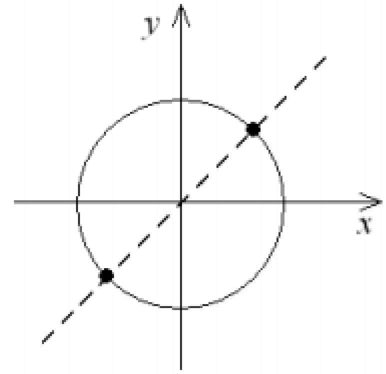
Ex.1) Find the exact solutions for $2\cos(x)+\sqrt{3}=0$ for $x \in [0, 2\pi]$.

ii) If $\sin(x)=\sin(\alpha)$, then $x = 2\pi k + \alpha$ or $x = 2\pi k + (\pi-\alpha)$, $k \in Z$

Ex.2) Find the exact solutions for $2\sin(2x)=\sqrt{3}$ for $x \in [0, 2\pi]$.



iii) If $\tan(x)=\tan(\alpha)$, then $x = k\pi + \alpha$, $k\epsilon Z$



Ex.3) Find the exact solutions for $3\tan^2(\theta) = 1$ for $\theta \in [0, 2\pi]$.

Ex.4) Find the exact solutions for $\tan^2(\theta) + 3\sec(\theta) + 3 = 0$, $0 \le \theta \le 2\pi$

Ex.5) Find the solutions for $3\,cos^2(\theta) - \sin(\theta) - 1 = 0\,, 0 \le \theta \le 2\pi$

Ex.6) Solve the equation $\sin(4x) - \cos(2x) = 0\ , x \in [0, 2\pi]$

Ex.7) Solve $\sin(2x) - \cos(2x) = 0, \quad 0 \le x \le 2\pi$

Ex.8) Solve $\tan(\theta)(\csc(\theta) + 2) = 0 \; , 0 \le \theta \le 2\pi$

Exit Card!

Solve $4\sin^4(\theta)+3\sin^2(\theta)-1=0$, $0\leq\theta\leq 2\pi$

Practice

1. Solve for x on the interval $[0,2\pi]$.

a) $6\sin^2(x)-\sin(x)-1=0$

b) $\cot(x)\cos^2(x)=\cot(x)$

c) $4\tan(x)-\sec^2(x)=0$

d) $2\sin^2(x)-\sin(x)-1=0$

e) $4\sin^3(x)+2\sin^2(x)-2\sin(x)-1=0$

f) $\sin(x)+\sqrt{2}=-\sin(x)$

g) $2\cos(3x-1)=0$

h) $\sin(2x)\cos(x)-\cos(2x)\sin(x)=0$

i) $\sec^2(x)-2\tan(x)=4$

j) $3\tan\left(\frac{x}{2}\right)+3=0$

k) $-6\sin(2x)\cos(x)+8\cos(2x)+3\sin(x)+4=0$

l) $\cot(x)\cos^2(x)=2\cot(x)$

m) $\frac{1+\sin(x)}{\cos(x)}+\frac{\cos(x)}{1+\sin(x)}=4$

n) $2\sin^2(x)+3\cos(x)-3=0$

o) $2\sin(x)\tan(x)-\tan(x)-2\sin(x)+1=0$

p) $\cos(x)\tan(x)-1+\tan(x)-\cos(x)=0$

2. A weight hanging from a spring is set in motion moving up and down. Its distance, d, (in cm) above or below its "rest" position is described by $d(t)=5(\sin(6t)-4\cos(6t))$. At what times during the first 2 seconds is the weight at the rest position (d=0).

Warm up

Solve the following equations for $x \in [0, 2\pi]$.

a) $3\sec^2(x) - 7 = -1$

b) $1 + \sin(x) = \sqrt{3}\cos(x)$

c) $\cos^2(2x) = 3\sin^2(x) - 2$

d) $\dfrac{\cos(2x)}{\cos(x) + \sin(x)} = 1$

4.5 Rates of Change in Trigonometric Functions

(1) Average Rate of Change

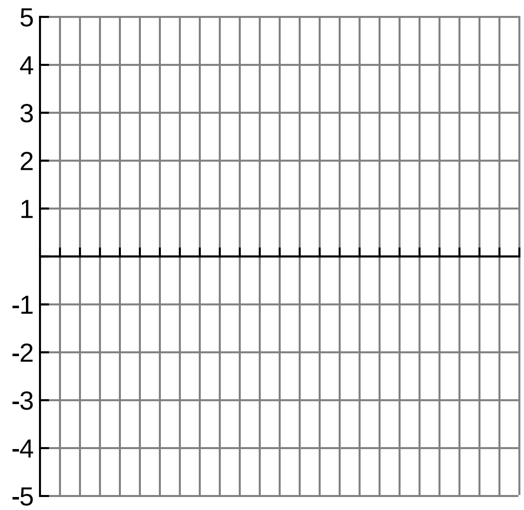
Average Rate of Change = $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

(2) Instantaneous Rate of Change

Instantaneous Rate of Change= $\dfrac{f(a+h)-f(a)}{h}, h \to 0$

<u>Example #1:</u> Consider the trigonometric function $y = -3\sin\left[\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right]$.

(a) Sketch one cycle of the function.



(b) Determine an interval where the avg. rate of change is:
(i) positive (ii) negative (iii) zero

(c) Determine a point where the inst. rate of change is:
(i) positive (ii) negative (iii) zero

(d) Calculate the average rate of change for $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$.

(e) Describe how the instantaneous rate of change varies over the interval $[0, 4\pi]$.

Example #2: The position of a particle as it moves horizontally is described by the equation $s(t) = 12\sin\left(\frac{\pi t}{90}\right) + 15$, where *s* is the displacement, in metres, and *t* is the time, in seconds.

(a) Calculate the average rate of change of *s(t)* for the following intervals:

(i) 5 s to 10 s

(ii) 9 s to 10 s

(b) Estimate the instantaneous rate of change of *s(t)* at *t* = 10s.

(c) What physical quantity does this instantaneous rate of change represent?

Unit 4- Review

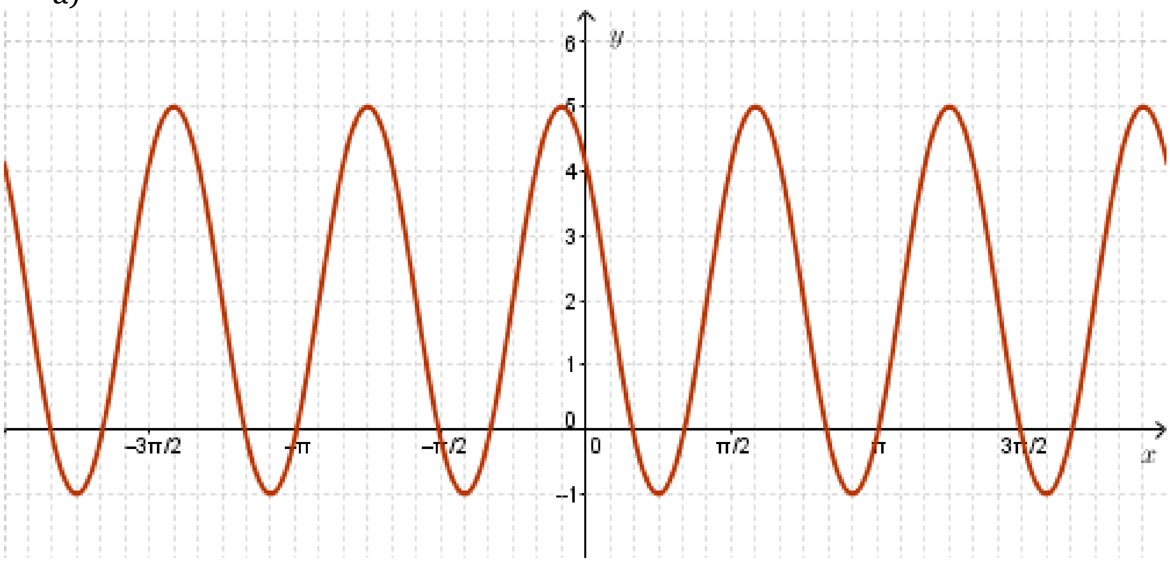
1. State the intervals where the graph of $y = \sec(x), -2\pi \le x \le 2\pi$ is increasing.
2. Sketch the graphs of $y = \tan(x)$ and $y = \cot(x), -\frac{\pi}{2} \le x \le \pi$. Using the letters A,B and C, label the three intersections points of the two functions .Determine the area and perimeter of ΔABC.
3. Identify the amplitude, period, phase shift, and vertical displacement for each of the following:
   a) $y = 6\cos\left[12\left(x - 30^\circ\right)\right] + 3$
   b) $y = -2 + 3\sin\left(x + \frac{\pi}{4}\right)$
   c) $y = -4\cos\left(2x - \frac{\pi}{3}\right) - 2$
4. Sketch $y = 5\sin\left[\frac{3}{2}\left(x - 30^\circ\right)\right], -120^\circ \le x \le 120^\circ$.
5. Sketch one period of the function $f(x) = -\cos\left[\frac{1}{3}\left(x + \frac{5\pi}{6}\right)\right] - 2$.
6. Sketch one period of the function $f(x) = 3\cos\left(2x - 60^\circ\right) + 1$.
7. Sketch the graph of $y = -2\sin\left(\frac{x}{2} + \frac{\pi}{4}\right) + 3$.
8. a) Determine a sine function that is defined for all x≥0 and has its first minimum at (π/3,3) and its first maximum at (4π/3,9).

   b) State an equivalent cosine function for part a).
9. a) Determine a sinusoidal function f(x) that
   - has a maximum of 100;
   - has a minimum of 20;
   - a period of 30;
   - has the point (15,60) on its curve; and
   - for x≥0, reaches its first maximum before its first minimum.

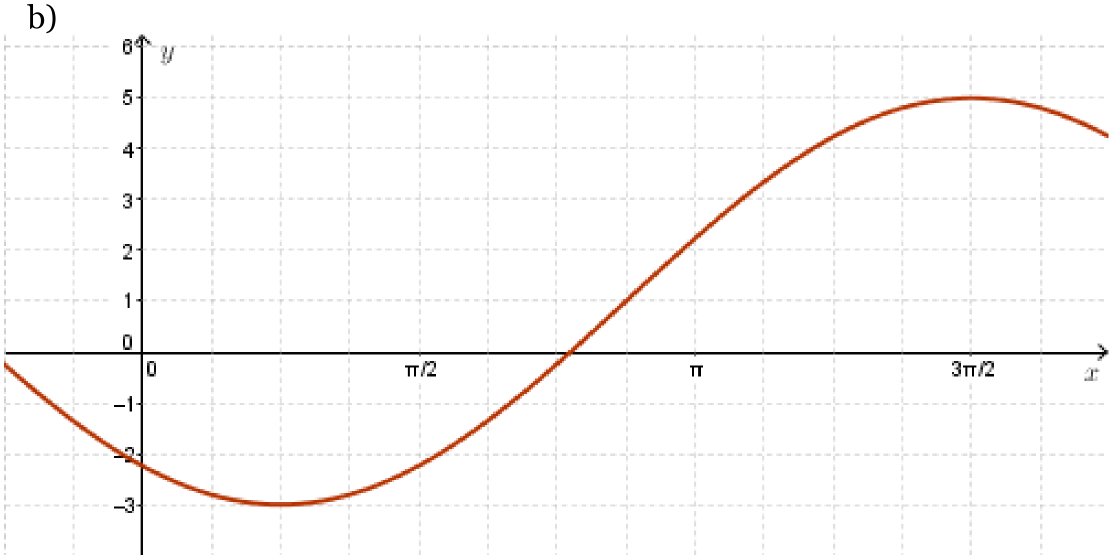
   b) Use your function from part a) to determine the first value of x,x≥0 such that f(x)=80.

10. Determine a sinusoidal function that could represent the graph drawn below.

a)



b)



11. For the function $y = -3\cos\left(2x + \frac{\pi}{2}\right) + 6$:

a) Graph four periods of the function for $x \geq 0$.

b) On your graph, sketch the line $y = \frac{15}{2}$.

c) Determine all solutions to $-3\cos\left(2x + \frac{\pi}{2}\right) + 6 = \frac{15}{2}$, in the interval $0 \leq x \leq 4\pi$.

d) Using the graph, verify that all of the solutions in the interval $0 \leq x \leq 4\pi$ have been determined in part c).

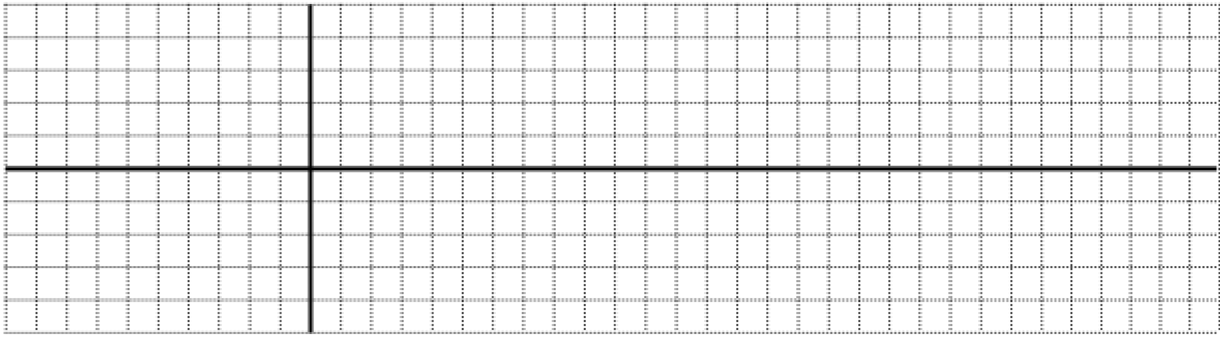
12. Ashley is riding a Ferris wheel that has a diameter of 40 metres. The wheel revolves at a rate of 1.5 revolutions per minute. If Ashley's height above the ground is shown by h after t minutes
   a) Find the equation of h(t).
   b) What is Ashley's maximum height above the ground while she is riding the Ferris wheel?
   c) If Ashley gets on the ride at its lowest point, how high above the ground will she be to start the ride?
   d) How many times does Ashley go around the Ferris wheel in four minutes?
   e) Draw a sketch of the rider's height above the ground at any time during the first four minutes.
   f) How long after she starts riding will her height be 31 metres above the ground?
   g) In the four minutes that she spends riding the Ferris wheel, what is the total amount of time that Ashley's height is above 31 metres?

13. The minimum depth, d (in metres), of water in a harbour, t hours after midnight, can be approximated by the function $d(t)=5\cos(0.5t)+12$, where $0\leq t\leq 24$.
   a) Determine the maximum and minimum depths of water in the harbour.
   b) Determine the period of the depth function.
   c) What is the depth of water, to the nearest tenth of a metre, at 2:00 AM?
   d) A ship, which requires a minimum depth of 8.5 metres, is docked at midnight. By what time, to the nearest minute, must it leave in order to prevent being grounded?
   e) What is the next time, to the nearest minute, that the ship can return to the harbour?

14. Determine the period and equation of vertical asymptotes of $y=-2\tan(50x)$.

15. The height of a rung on a hamster wheel can be modeled by $h(t)=-25\cos\left[2\pi\left(\frac{t-4}{12}\right)\right]+27$, where ***h(t)*** represents the height of the rung above the bottom of the cage in centimeters and ***t*** is the time in seconds after the wheel starts moving. Show all your work for these questions.
   a) Determine the height of the rung at the start of the ride.
   b) Determine the maximum height of the rung during one rotation.
   c) How long will it take for the wheel to complete one full rotation?
   d) How long will it take for the wheel to reach its maximum height?
   e) Determine the average rate of change in the height of the rung between 2 and 3 seconds.

16. Graph one complete cycle of $y = 4\csc\left[\frac{\pi(t-1)}{2}\right]$.



17. Solve for x, $4\cos(x) - 3\sin(x) = 2 \quad (0 \le x \le 2\pi)$ (2 **DECIMAL PLACES**)

18. Solve for θ, $-10\cos^2(x) - 3\sin(x) + 9 = 0$, $(0 \le x \le 2\pi)$ (2 **DECIMAL PLACES**)

19. Find an **equation** of a function in the form $y = a\sin k(x-d) + c$ whose graph has a maximum at the point $A\left(\frac{\pi}{4}, 1\right)$ and a minimum at $B\left(\frac{5\pi}{4}, -1\right)$.

20. Graph $y = -2\sin\left(\frac{1}{2}\theta - \frac{\pi}{4}\right) + 3$, $-\pi \le \theta \le \pi$