Unit 1: Polynomial Functions
1.9 Families of Polynomial Functions

Families of Polynomial Functions

A **family of** n^{th} **degree polynomial functions** that share the same x-intercepts and differ only in vertical scale factors can be defined by $f(x)=k(x-a_1)(x-a_2)\cdots(x-a_n)$ where k is the leading coefficient, $k\in\mathbb{R}$, $k\neq 0$ and $a_1,a_2,a_3,\ldots,a_n$ are the zeros of the function.

- **General Forms in factored form:**
  quadratic: $f(x) = k(x-s)(x-u)$ s and u are zeros, $k\in R$
  cubic: $f(x) = k(x-s)(x-t)(x-u)$ s, u, and t are zeros ,$k\in R$ e.g. $f(x) = 3(x-1)(x+2)(x+5)$
  $f(x) = -5(x-1)(x+2)(x+5)$
  quartic: $f(x) = k(x-s)(x-t)(x-u)(x-v)$ s,u,t, and v are zeros, $k\in R$

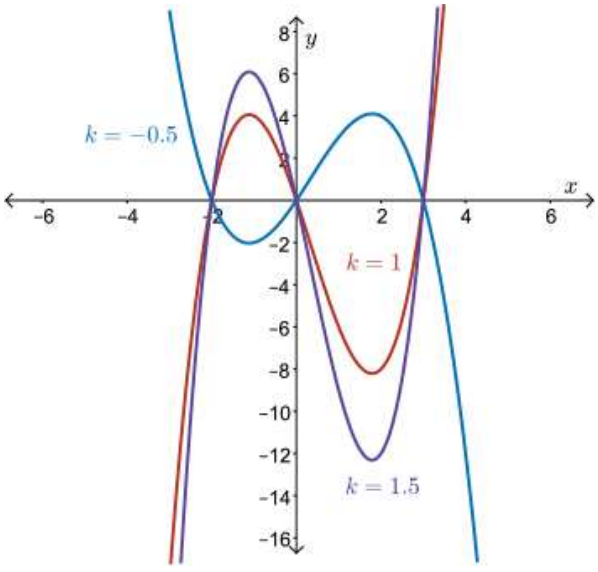
Example 1: The function $f(x) = -3x^3 + kx^2 - 5x + k$ has a zero/x-intercept at $x = 1$. Determine the value of 'k' and the equation of the polynomial function.

Example 2:

a) Find the family of cubic functions whose x-intercepts are -2,0, and 3.

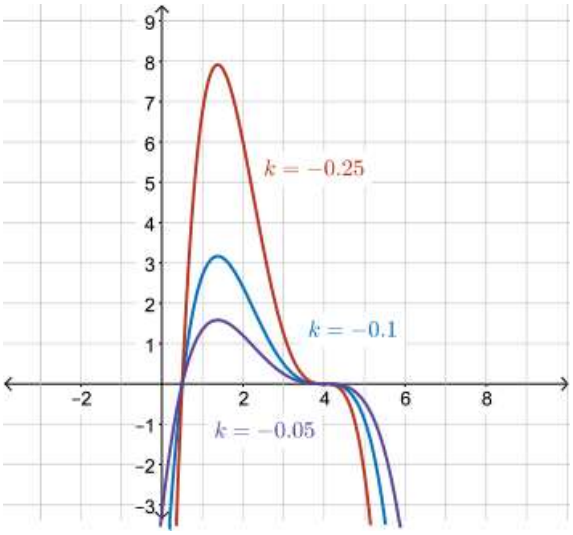
Factors: _____

Family:_____



b) Find the specific member of this family that has remainder of 12 when it's divided by $x+1$.

Example 3:
Determine the general equation of a quartic function with end behavior f(x)→−∞ as x→±∞, a zero at $x = \frac{1}{2}$, and a point of inflection at $x = 4$.



Example 4:
a) Give an example of polynomial function that has single roots at $2 \pm \sqrt{3}$ and a double root at 4.

b) How many other relations share the same zeros? How do you know?

c) Find specific member of family that has y-intercept of -8.

Practice

1. Match each graph with the corresponding equation.

(a) $y = -x(x-2)^2$

(b) $y = -(x-2)(x^2+2x+3)$

(c) $y = x(x-2)(x+2)$

(d) $y = 12x^3(x-2)$

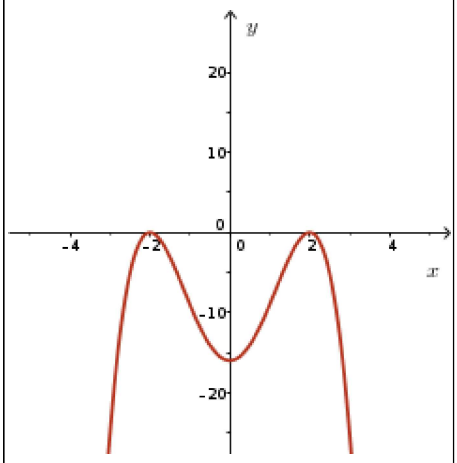
(e) $y = x(x+2)^2$

(f) $y = -2x(x+2)(x-2)$

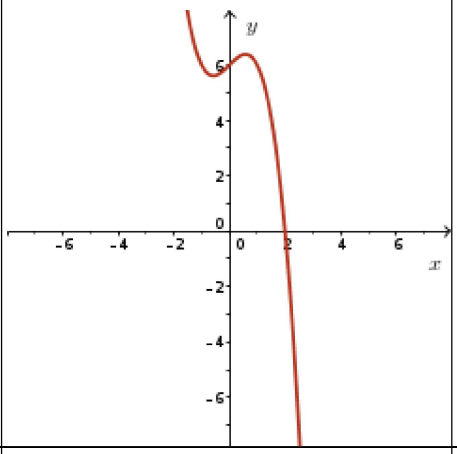
(g) $y = -(x-2)^2(x+2)^2$

(h) $y = -(x-2)(x^2+2x-5)$

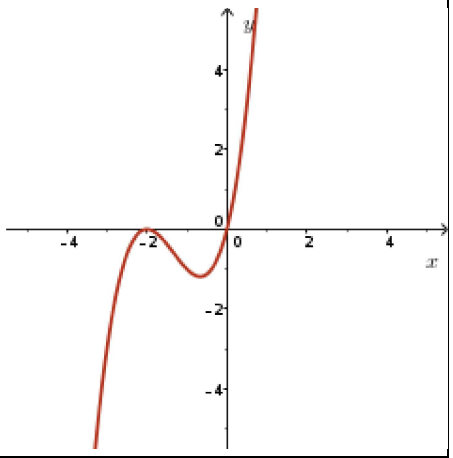
I.



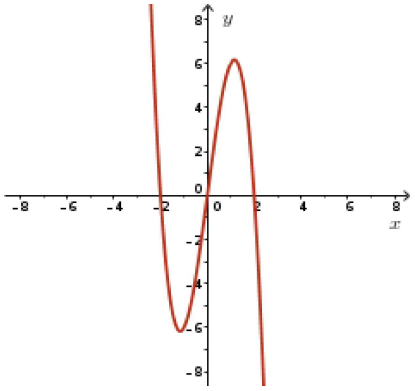
II.



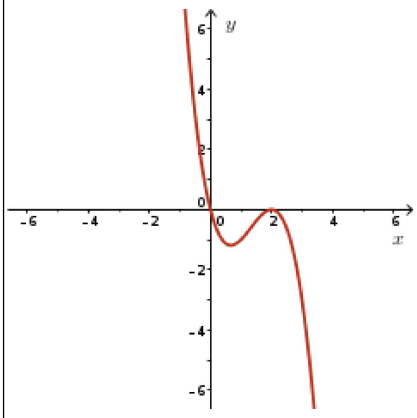
III.



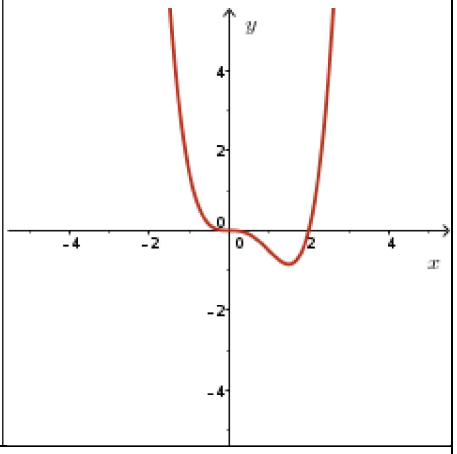
IV.



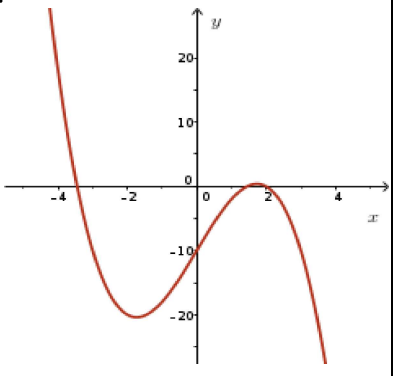
V.



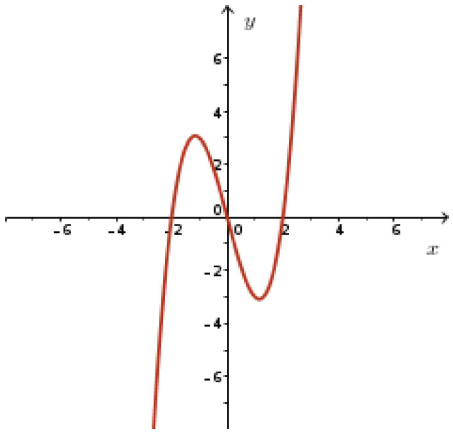
VI.



VII.

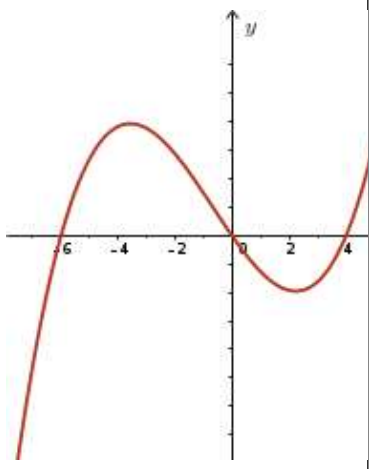


VIII.

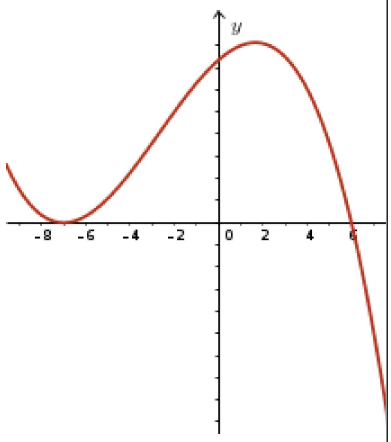


2. Given the graph of y=f(x), determine **a general equation** for a family of polynomials with the same end behavior and zeros of f(x) (note: all zeros are integer in value).

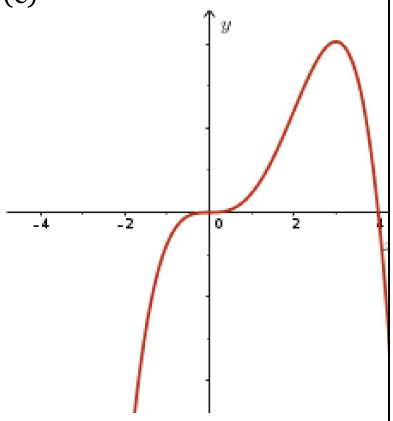
(a)



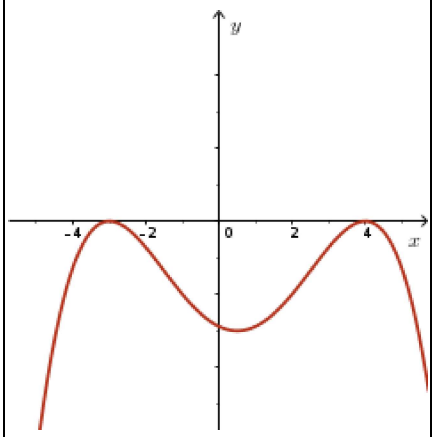
(b)



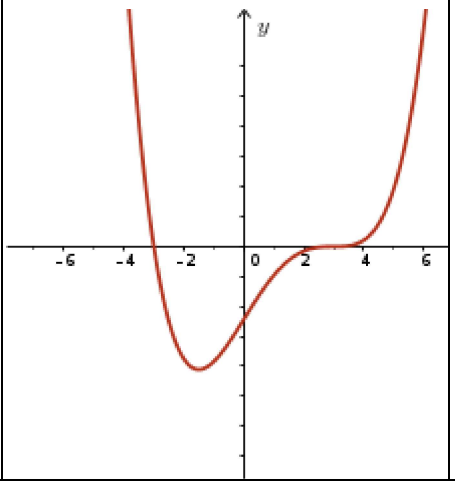
(c)



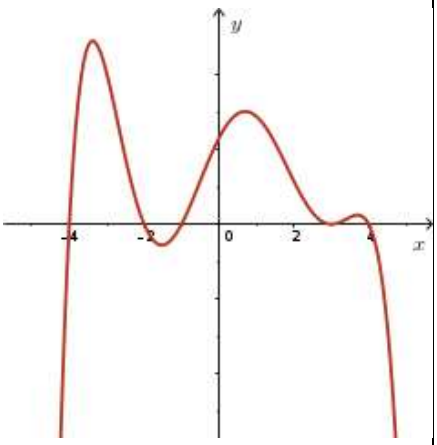
(d)



(e)

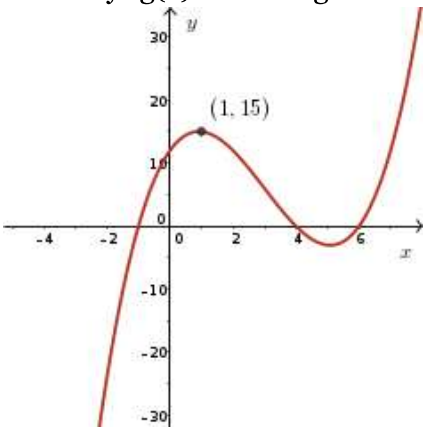


(f)



3. State the equation of the quartic function with zeros $x = \frac{-1}{2}$ and 5 (both of multiplicity 1) and x=2 (multiplicity 2), having a y-intercept of 4.

4. Determine the equation given the graph of the polynomial function y=g(x) with integer zeros.

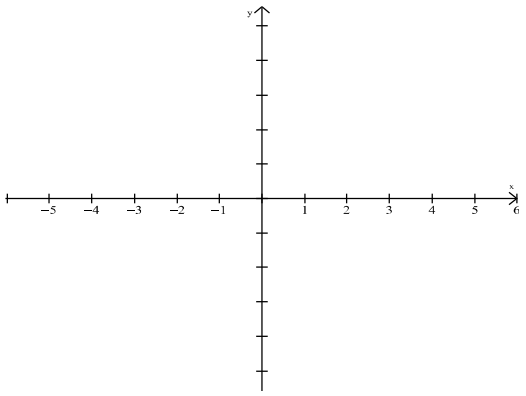


5. Determine the equation of the
    a. quadratic functions with zeros $-3 \pm \sqrt{5}$,passing through (-1,2).
    b. cubic functions with zeros 0 and $1 \pm 2\sqrt{3}$, passing through (2,22).
    c. quartic functions with zeros –2,1, and $-1 \pm \sqrt{2}$, and y-intercept -36.
    d. Cubic function with zeros –1,2/3, and 3, passing through (4,5).
6. Determine the equation of the quartic function with rational coefficients, zeros $4 - \sqrt{2}$ and $-3 + \sqrt{6}$ and a y-intercept of –21.

Warm Up

1. Sketch the graph of the following functions using in the properties of functions discussed in class.

a) $f(x) = -(2-x)(x^2 - 4)$



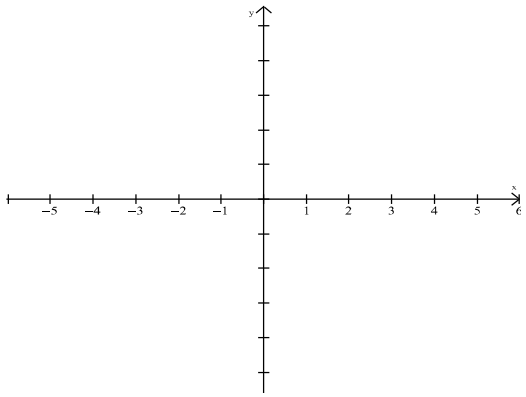
Degree of the function: _____

End behaviour:

$x \to \infty$,

$x \to -\infty$,

b) $f(x) = -(x - 2)^2(x + 1)^3(x+3)$



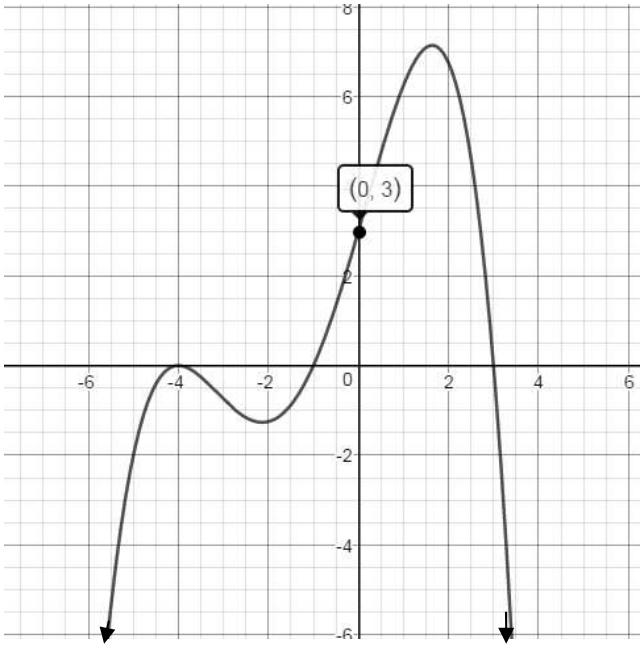
Degree of the function: _____

End behaviour:

$x \to \infty$,

$x \to -\infty$,

2. Use the graph of the polynomial function to answer the following questions.



a. The least possible degree of the function is _____.

b. The sign of the leading coefficient is _____.

c. The x-intercepts of the function are _____.

d. The intervals where the function is increasing are _____.

e. The intervals where the function is negative are _____.

f. Determine an equation in factored form.