

Knowledge and Understanding (K)

1. Identify the following given the function:  $f(\theta) = -3 \sin \left[ 2 \left( \theta - \frac{\pi}{6} \right) \right] - 4$ . [8 marks]

Amplitude: 3	Domain: $\{ \theta \mid \theta \in \mathbb{R} \}$
Maximum value: -1	Range: $\{ f(\theta) \mid -7 \leq f(\theta) \leq -1, f(\theta) \in \mathbb{R} \}$
Minimum value: -7	Phase shift: $\frac{\pi}{6}$ right
Period: $\pi$	Vertical displacement: 4 down

2. Write an equation for the function with the given characteristics:

a) A sine graph that has an amplitude of 43, a period of $\frac{\pi}{2}$ , a phase shift of 9 units to the left and a vertical displacement of 104 units down. [3 marks] $y = 43 \sin[4(x+9)] - 104$	b) A cosine graph that has a reflection in the x-axis, a maximum value of 14, a minimum value of 8, a period of $10\pi$ , a phase shift of 27 units to the right. [4 marks] $y = -3 \cos\left[\frac{1}{5}(x-27)\right] + 11$
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3. State:

a) The domain of $y = \sec(x)$ , $[0, 2\pi]$	$\{ x \mid 0 \leq x < \frac{\pi}{2}, \frac{\pi}{2} < x < \frac{3\pi}{2}, \frac{3\pi}{2} < x \leq 2\pi, x \in \mathbb{R} \}$
b) The range of $y = \csc(x)$	$\{ y \mid y \leq -1 \text{ or } y \geq 1, y \in \mathbb{R} \}$
c) The y-intercept of $y = \tan(x)$	0

Application

4. Solve for  $x$ . Give the exact value, if possible, otherwise round to the nearest tenth of a radian:

a) $2 \sin(x) + \sqrt{3} = 0$ , $[0, 2\pi]$ [3 marks] $\sin x = -\frac{\sqrt{3}}{2}$ $a = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $a = \frac{\pi}{3}$ $x_1 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ $x_2 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ $\therefore$ the solutions are $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$	b) $4 \cos^2(x) + 5 \cos(x) - 6 = 0$ , $[0, 2\pi]$ [4 marks] $(4 \cos x - 3)(\cos x + 2) = 0$ $4 \cos x = 3 \quad \cos x = -2$ $\cos x = \frac{3}{4} \quad \downarrow$ $\text{inadmissible}$ $a = \cos^{-1}\left(\frac{3}{4}\right) \approx 0.723$ $x_1 = 0.7$ or $x_2 = 2\pi - 0.723 = 5.6$ $\therefore$ the solutions are 0.7 or 5.6
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5. The formula  $y = 10 \cos(0.507t)$ , where  $t$  is measured in hours after midnight, gives the height,  $y$ , in metres of the tide at a spot on the shore of Vancouver Island. Help the fisherman determine what times the tide will be 8 metres high for the interval  $0 \leq t \leq 24$ . [4 marks]

$8 = 10 \cos(0.507t)$   
 $\frac{4}{5} = \cos(0.507t)$   
Let  $A = 0.507t$   
 $A = \cos^{-1}\left(\frac{4}{5}\right) \approx 0.6435$   
 $A_1 \approx 0.6435$  or  $A_2 = 2\pi - 0.6435 = 5.6397$

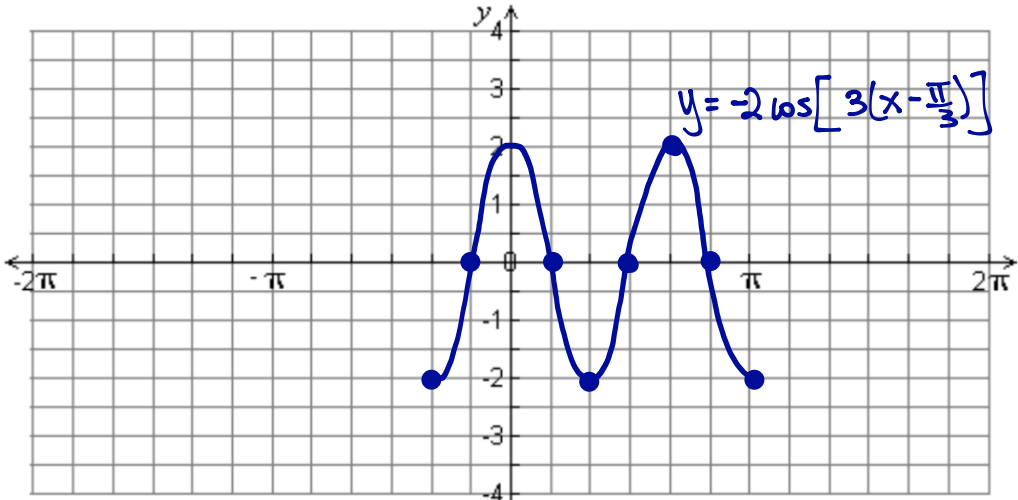
$\text{If } A_1 = 0.6435$   
 $0.507t = 0.6435$   
 $t_1 = 1.27 \text{ h} = 1:16$   
 $t_3 \approx 1.27 + 12.39 = 13.66 = 13:40$

$\text{If } A_2 = 5.6397$   
 $0.507t = 5.6397$   
 $t_2 = 11.12 \text{ h} = 11:07$   
 $(\text{Period} = \frac{2\pi}{0.507} = 12.3929)$   
 $t_4 \approx 11.12 + 12.39 = 23.51 = 23:31$

$\therefore$  The tide will be 8m high at the following times: 1:16, 11:07, 13:40 and 23:31

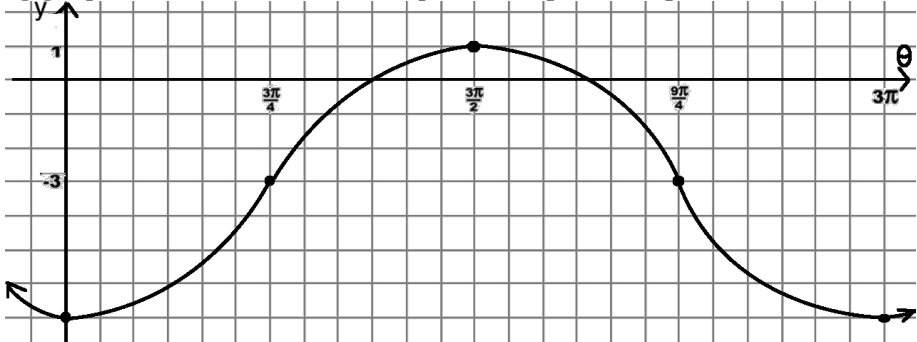
6. Sketch the following graph:  $y = -2 \cos(3x - \pi)$  over 2 cycles. State the amplitude, period and domain and range. [7 marks]

a) Amplitude: 2	b) Domain: $\{x   -\frac{\pi}{3} \leq x \leq \pi, x \in \mathbb{R}\}$
c) Period: $\frac{2\pi}{3}$	d) Range: $\{y   -2 \leq y \leq 2, y \in \mathbb{R}\}$



**Thinking**

7. Given the following graph, state the indicated equations: [3 marks]



The equation in terms of

a) Sine:[2 marks]	$y = 4 \sin[\frac{2}{3}(\theta - \frac{3\pi}{4})] - 3$
b) Cosine: [1 mark]	$y = 4 \cos[\frac{2}{3}(\theta - \frac{3\pi}{2})] - 3$

8. An object attached to the end of a spring is oscillating up and down. The displacement of the object,  $y$ , in centimetres, is a function of the time,  $t$ , in seconds, and is given by  $y = 2.4 \cos\left(12t + \frac{\pi}{6}\right)$ .

a. What is the maximum distance through which the object oscillates? [1 mark]  4.8 cm	b. What is the period of the function? [1 marks]  $\frac{\pi}{6}$
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- c. When is the object displaced 1.0 cm above the mean position during the first cycle? [4 marks] (Round all final answers to 3 decimal places)

$$1 = 2.4 \cos\left(12t + \frac{\pi}{6}\right)$$

$$\text{Let } A = 12t + \frac{\pi}{6}$$

$$\cos A = \frac{1}{2.4}$$

$$RAA = \cos^{-1}\left(\frac{1}{2.4}\right)$$

$$\doteq 1.141$$

$$A_1 = 1.141$$

$$A_2 = 2\pi - 1.141 = 5.142$$

$$\text{If } A_1 = 1.141$$

$$12t + \frac{\pi}{6} = 1.141$$

$$12t = 1.141 - \frac{\pi}{6}$$

$$t \doteq 0.052 \text{ s}$$

$$\text{If } A_2 = 5.142$$

$$12t + \frac{\pi}{6} = 5.142$$

$$t = 0.385 \text{ s}$$

$\therefore$  The object is displaced 1 cm above the mean position at 0.052 s and 0.385 s

**Communication**

9. List the transformations applied to  $y = \sin(x)$  to obtain the following graph:

$$y = -57 \sin\left(\frac{5}{3}\left(x + \frac{\pi}{4}\right)\right) + 2.3. \text{ [4 marks]}$$

- reflected in the x-axis
- vertically stretched by a factor of 57
- horizontally compressed by a factor of  $\frac{3}{5}$
- horizontally translated  $\frac{\pi}{4}$  units left
- vertically translated by 2.3 units up

10. Compare the graphs of  $f(x) = -\sin(8x) + 6$  and  $g(x) = -\sin\left(\frac{1}{8}(x + 6)\right)$ . [4 marks]

Both graphs are sine functions reflected in the x-axis.

However  $f(x)$  is horizontally compressed by a factor of  $\frac{1}{8}$

whereas  $g(x)$  is horizontally expanded by a factor of 8.

Two marks will be awarded for proper mathematical forms throughout the assessment.

[2 marks]