Name: SOLUTIONS

Knowledge/ Understanding	Application
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UNIT 6 FORMATIVE QUIZ
COMBINING FUNCTIONS

KNOWLEDGE

Multiple Choice: Place the CAPITAL letter of the correct answer on the provided line.

C **1)** Given the functions $f(x)=x^2-x$ and $g(x)=x-1$, determine an equation for the composite function $y=f(g(x))$ $= (x-1)^2-(x-1) = x^2-2x+1-x+1 = x^2-3x+2$

A. $y=x^2-3x+1$ C. $y=x^2-3x+2$

B. $y=x^2-x-1$ D. $y=x^2-x+1$

D **2)** Given the functions $f(x)=\dfrac{x+4}{2}$ and $g(x)=x^2-5$ determine the value of $g(f(4))$.

$f(4)=4$

$g(4)=16-5=11$

A. 3.5 C. 7

B. 7.25 D. 11

A **3)** Given the functions $f(x)=\sqrt{x+3}$ and $g(x)=x^2+2x-8$, determine the domain of the combined function $f(x)-g(x)$. $D_{f-g}=D_f\cap D_g=\{x\in\mathbb{R}\mid x\geq -3\}$

A. $\{x\in\mathbb{R}\mid x\geq -3\}$ C. $\{x\in\mathbb{R}\mid 4\leq x\leq 2\}$

B. $\{x\in\mathbb{R}\mid x\geq -8\}$ D. cannot be determined

A **4)** What is the range of the function $f(x)=\sin\left(2^{x^2+1}\right)$?

A. $\{y\in\mathbb{R}\mid -1\leq y\leq 1\}$ C. $\{y\in\mathbb{R}\mid -2\leq y\leq 0\}$

B. $\{y\in\mathbb{R}\mid -2\leq y\leq 2\}$ D. cannot be determined

C **5)** Given the functions $f(x)=2^x$ and $g(x)=\log(1-x)$, determine domain of the combined function $y=f(x)g(x)$. $D_{f\cdot g}=D_f\cap D_g=\{x\in\mathbb{R}\mid x<1\}$

A. $\{x\in\mathbb{R}\mid x>0\}$ C. $\{x\in\mathbb{R}\mid x<1\}$

B. $\{x\in\mathbb{R}\mid x\geq 0\}$ D. $\{x\in\mathbb{R}\mid x\leq 1\}$

A **6)** Which combination is always an odd function?

A. the sum of two odd functions

B. the difference of an odd function and an even function

C. the sum of an odd function and an even function

D. the difference of two even functions

B **7)** If $f(x)=\sqrt{3-x}$ and $g(x)=\dfrac{1}{3^x}$, what is the domain of $(f\times g)(x)$? $D_{f\cdot g}=D_f\cap D_g=\{x\in\mathbb{R}\mid x\leq 3\}$

A. $\{x\in\mathbb{R}\mid -3\leq x\leq 3\}$ C. $\{x\in\mathbb{R}\mid -1\leq x\leq 1\}$

B. $\{x\in\mathbb{R}\mid x\leq 3\}$ D. $\{x\in\mathbb{R}\mid x\geq 0\}$

8) Let $f(x)=\{(9,8),(5,7),(6,12),(7,1),(2,-13)\}$, $g(x)=\{(12,5),(2,11),(-8,9),(7,-3),(5,0)\}$, $h(x)=2-x$, and $k(x)=-\sin\left[2\left(\dfrac{\pi}{4}+x\right)\right]-1$, determine the following.

a) $(g+f)(x)$ [2]

$=\{(5,7),(7,-2),(2,-2)\}$

b) $(f\circ g)(-8)-2(g\circ f)(6)$ [3]

$=f(9)-2g(12)$

$=8-2(5)$

$=-2$

c) $(f\circ g^{-1})(-3)$ [2]

$=f(7)$

$=1$

d) $h(5)+k\left(\dfrac{\pi}{3}\right)$ [2]

$=(2-5)+\left(-\sin\frac{7\pi}{6}-1\right)$

$=-3+\left(\frac{1}{2}-1\right)$

$=-3-\frac{1}{2}$

$=-7/2$

9) Given $f(x)=x^3-x^2-9x+12$ and $g(x)=x^2+6x+12$, determine where $(f-g)(x)\leq 0$. [4]

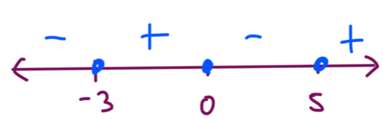
$x^3-x^2-9x+12-(x^2+6x+12)\leq 0$

$x^3-2x^2-15x\leq 0$

$x(x^2-2x-15)\leq 0$

$x(x-5)(x+3)\leq 0$

Roots: 0, 5, -3



$\therefore x\in(-\infty,-3]\cup[0,5]$

APPLICATION

1) Let $f(x)=x^2+3x$ and $(f\circ g)(x)=x^2+7x+10$. Determine $g(x)$. [3]

$f(g(x)) = x^2+7x+10$

$[g(x)]^2+3g(x) = x^2+7x+10$

$\underbrace{1}_{a}[g(x)]^2+\underbrace{3}_{b}g(x)\underbrace{-x^2-7x-10}_{c}=0$

$g(x)=\dfrac{-3\pm\sqrt{9-4(1)(-x^2-7x-10)}}{2}$

$=\dfrac{-3\pm\sqrt{9+4x^2+28x+40}}{2}$

$=\dfrac{-3\pm\sqrt{4x^2+28x+49}}{2}$

$=\dfrac{-3\pm\sqrt{(2x+7)^2}}{2}$

$=\dfrac{-3\pm(2x+7)}{2}$

$\therefore g(x)=\dfrac{-3+(2x+7)}{2}=\dfrac{2x+4}{2}=x+2$

OR

$g(x)=\dfrac{-3-(2x+7)}{2}=\dfrac{-2x-10}{2}=-x-5$

2) Determine two functions $f(x)$ and $g(x)$ such that $(f+g)(x)=2x^2-2x+8$ and $(f-g)(x)=8x-4$. [4]

.

$f(x)+g(x)=2x^2-2x+8$ —— ①

$f(x)-g(x)=8x-4$ —— ②

①+②: $f(x)+g(x)=2x^2-2x+8$

$+)\ f(x)-g(x)=8x-4$

$2f(x)=2x^2+6x+4$

$f(x)=x^2+3x+2$

Sub. $f(x)=x^2+3x+2$ into ②:

$x^2+3x+2-g(x)=8x-4$

$x^2-5x+6=g(x)$

$\therefore f(x)=x^2+3x+2$

$g(x)=x^2-5x+6$

3) If $f(x)=\cos^2(x)-\sin^2(x)$ and $g(x)=x+\pi$, find the exact value of $f\left(g\left(\frac{\pi}{8}\right)\right)$. [3]

$g\left(\frac{\pi}{8}\right)=\frac{\pi}{8}+\pi=\frac{9\pi}{8}$

$f\left(\frac{9\pi}{8}\right)=\cos^2\left(\frac{9\pi}{8}\right)-\sin^2\left(\frac{9\pi}{8}\right)$ double angle

$=\cos\left(2\cdot\frac{9\pi}{8}\right)$

$=\cos\left(\frac{9\pi}{4}\right)$

$=\cos\left(2\pi+\frac{\pi}{4}\right)$

$=\cos\left(\frac{\pi}{4}\right)$

$=\frac{\sqrt{2}}{2}$

4) Given: $f(x)=\log_2 x$ and $g(x)=16^x$. Find:

[2] a) $(g\circ f)(x)-f(g(x))$

$=16^{\log_2 x}-\log_2(16^x)$

$=2^{4\log_2 x}-\log_2(2^{4x})$

$=2^{\log_2 x^4}-4x$

$=x^4-4x$

b) $f(x)\div g^{-1}(x)$ [2]

$g^{-1}(x)=\log_{16}(x)$

$f(x)\div g^{-1}(x)=\log_2 x\div\log_{16}(x)$

$=\log_2 x\div\log_{2^4}(x)$

$=\log_2 x\div\frac{1}{4}\log_2(x)$

$=\dfrac{\cancel{\log_2 x}}{\frac{1}{4}\cancel{\log_2 x}}$

$=4$

5) Given: $f(x)=\left(\frac{2}{3}\right)^{x^2}-5$ and $g(x)=x-x^3$ State if $\dfrac{f(x)}{g(x)}$ is even, odd, or neither. Justify your answer. [2]

$f(-x)=\left(\frac{2}{3}\right)^{x^2}-5$

$=f(x)$

$\therefore f(x)$ is even

$g(-x)=-x-(-x)^3$

$=-x+x^3$

$=-(x-x^3)$

$=-g(x)$

$\therefore g(x)$ is odd.

$\dfrac{f(-x)}{g(-x)}=\dfrac{f(x)}{-g(x)}$

$\therefore \dfrac{f(x)}{g(x)}$ is odd

6) If $f(x)=3x+k$ and $g(x)=\dfrac{x-4}{3}$ for what value of k is $f(g(x))=g(f(x))$? [4]

$f\left(\frac{x-4}{3}\right)=g(3x+k)$

$3\left(\frac{x-4}{3}\right)+k=\dfrac{3x+k-4}{3}$

$x-4+k=\dfrac{3x+k-4}{3}$

$3x-12+3k=3x+k-4$

$2k=8$

$k=4$