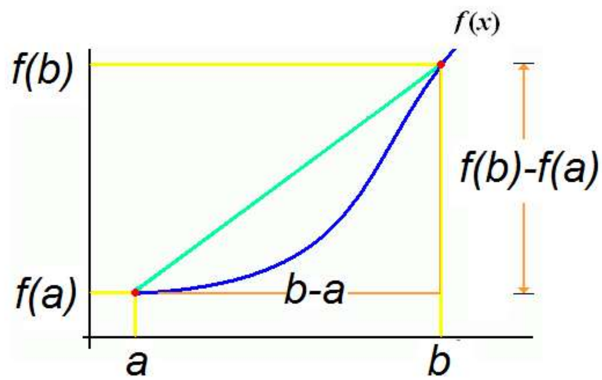
1.11 Average Rate of Change & Instantaneous Rate of Change

The average rate of change of f(x) on the interval [a,b] is defined as the slope of the secant drawn to the graph over the interval [a,b].

$$A.R.O.C = m_s = \frac{f(b) - f(a)}{b - a}$$

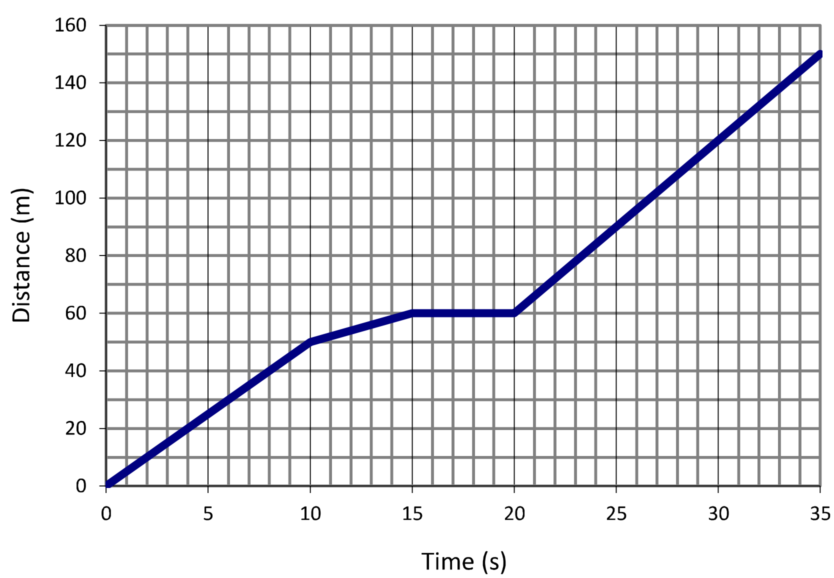
Recall: A secant line is a straight line that joins two points on the function.



Example#1: Calculate the average rate of change

i. between 10s and 15 s.

ii. between 4s and 30s.

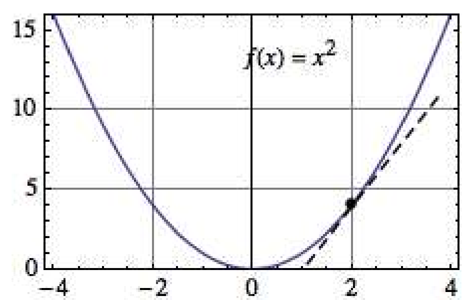


Example#2 Determine the average rate of change of the secant line on the graph of $g(x) = x^2 - 4x$ in interval [1,5].

Instantaneous Rate of Change

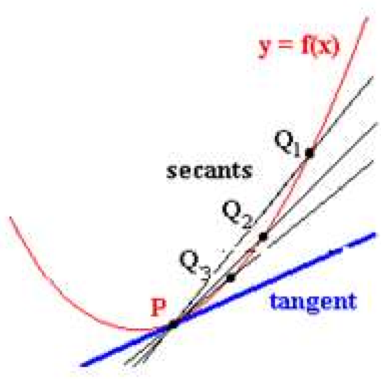
The exact rate of change of a function $y = f(x)$ at a specific value of the independent variable $x = a$

- ***Instantaneous rate of change*** = slope of tangent
  Recall: **Tangent Line** is a line which only touches the curve at one point



Example #3: Estimate the instantaneous rate of change of $f(x) = x^2$ at the point $x = 2$. We cannot find the slope of the tangent because we only know one point on the tangent line, P.

We can estimate the slope of the tangent at P (i.e. the instantaneous rate of change at P) by finding the slopes of secants for smaller and smaller intervals around P.As the point Q on the function approaches the point P, the slope of the secant approaches the slope of the tangent.



Mathematically:

(1) select points closer and closer to the point P

(2) calculate the slope of each secant

(3) the slopes of the secants will approach a value, which is the slope of the tangent

Method 1

Choose a point whose x-value is very close to the one given and calculate the slope using a table of values until a pattern emerges

Point 1	
X1	Y1
2	4
2	4
2	4
2	4

Point 2		$Slope = \frac{y_2 - y_1}{x_2 - x_1}$
X2	Y2	
2.1	4.41	4.1
2.01		
2.001		
2.0001		

The slope of tangent is ______.

Method 2

If we want to know the instantaneous rate of change of *f(x)* at $x = a$, consider the intervals between $x = a$ and $x = a \pm h$, where *h* is a really small number.(We can always consider *h*=0.001)

$$\textbf{Instantaneous Rate of Change} = \frac{f(a+h) - f(a)}{(a+h) - a}$$

$$\boxed{I.R.O.C = \frac{f(a+h) - f(a)}{h}, as\ h \to 0}$$

(This is known as the **difference quotient**.)

$$I.R.O.C = \frac{f(2+0.001) - f(2)}{0.001}$$

$$= \underline{\hspace{4cm}}$$

Example #4: An emergency flare is shot into the air. Its height, in metres, above the ground at various times in its flight is given by the following table.

Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	2.0	15.75	27.0	37.1	42.0	46.8	47.0	45.75	42.0

Estimate the instantaneous rate of change in height at exactly t = 2.0 s.

Example #5: The height of a soccer ball above the ground at time t after it is kicked into the air, is given by the formula $h(t) = -4.9t^2 + 3.5t + 1$ where h is the height in metres, t is the time in seconds.

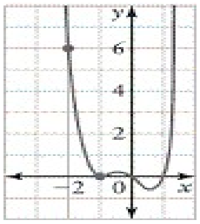
(a) Calculate the average velocity of the ball between t=0.1s and t =0. 3s

(b) Calculate the instantaneous velocity at t=0.6 s.

Practice

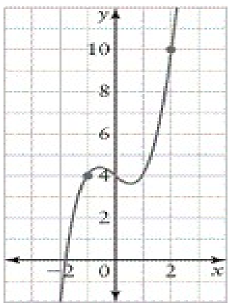
Part A –*Identify the choice that best completes the statement or answers the question.*

____ 1. A secant drawn through the points shown on the graph has a slope of



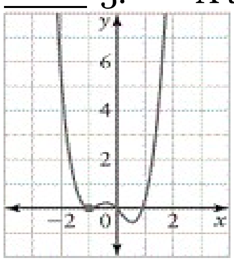
a. −1
b. −2
c. −3
d. −6

____ 2. A secant drawn through the points shown on the graph has a slope of



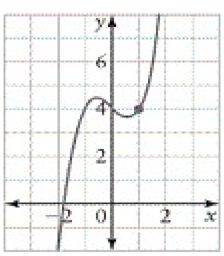
a. $\frac{1}{2}$
b. 2
c. 5
d. $\frac{10}{3}$

____ 3. A tangent to the graph of the function at the point shown has a slope of



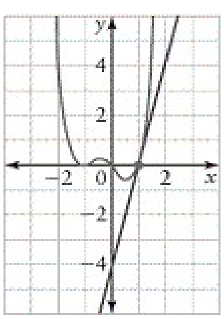
a. −2
b. −1
c. 0
d. 1

____ 4. A tangent to the graph of the function at the point shown has a slope of approximately



a. $\frac{1}{2}$
b. 2
c. −2
d. 0

____ 5. The slope of the tangent at the point indicated on the graph is



a. $\frac{1}{2}$
b. 1
c. 2
d. 4

_____ 6. The area, A, in square metres, of a circular oil spill after t min is given by $A = kt^2$. The average rate of change of the area of the oil spill over the first 5 min is approximately 15 m^2/min, the value of k is :

a. 3
b. 6
c. 2
d. 5

Part B

1. Suppose the revenue, R, in dollars, from the sales of x (in hundred) MP3 players is given by $R(x) = x(350 - 0.1x^2)$. Find the average rate of change of revenue from selling from 100 to 200 MP3 players.

2. A medical researcher establishes that a patient's reaction time, r, in minutes, to a dose of a particular drug is $r(d) = -0.8d^3 + d^2$, where d is the amount of the drug, in millilitres, that is absorbed into the patient's blood. Determine the instantaneous rate of change of the time reaction with respect to the amount of the drug at 1 millilitre.

3. The following table represents the growth of a bacteria population over a 4.5 h period.

Time (h)	0	0.75	1.5	2.25	3	3.75
Number of bacteria	850	1122	1481	1954	2577	3400

Estimate the instantaneous rate of change in number of bacteria at exactly t = 3.

4. For the function $f(x) = -2x^3 + 3x - 1$, determine the slope of a tangent at x = 1
5. Find the equation of tangent line to the curve $f(x) = -x^4 + 1$ at x = -1 on the curve.

Unit 1 Review – Polynomial Functions

1. Fill in the blanks.

a) State the remainder when $-4x^3+3x^2+2x-1$ is divided by $x-2$. _______________

b) State the roots and the order of each root of $g(x)=2x^2(2x+3)^3$. _______________

c) When a function is divided by $2x-1$, the remainder is -2; Determine the remainder when the same function is divided by $x-\frac{1}{2}$. _____________

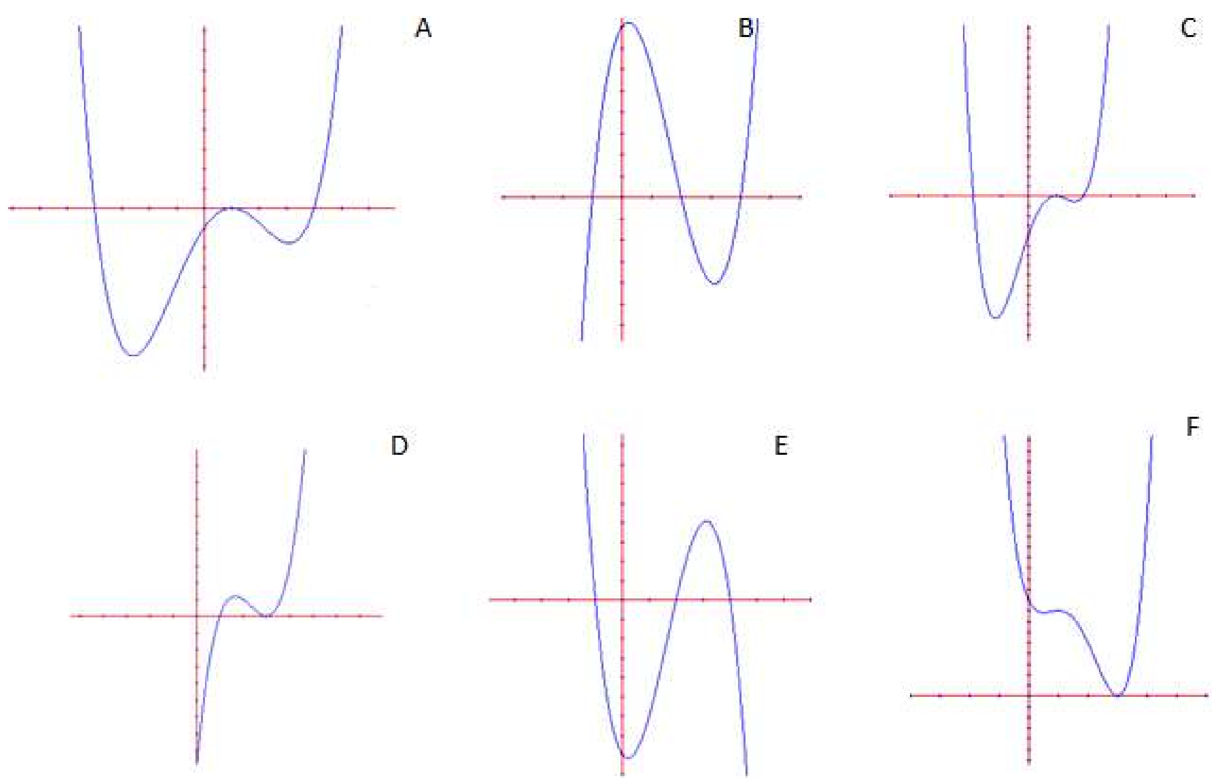
d) Values that could be zeros for the polynomial $f(x)=4x^3+2x^2-7x-8$ are___________

e) State if $y=-2x^4+3x^2+1$ is odd, even or neither. ______________________________

f) Beside each equation below, put the letter of the graph that best describes the equation:

i) $y=(x^2-16)(x-1)^2$ ______ ______

ii) $y=(2-x)(x-4)(x+1)$ ______ ______



2. Write the equation in factored form of any quartic function with following characteristics. Sketch the graph of function:

- $f(0)=0$
- $f(x)<0$, when $x<-2$
- $f(x)\geq 0$, when $-2\leq x\leq 3$
- $f(x)<0$, when $x>3$

3. Fully factor

a) $2x^5 - 2x^4 - 4x^3 + 4x^2 + 2x - 2$ b) $64y^6x^3 - 125$

c) $6(x+2)^{-5} + 2x^2(x+2)^{-4}$ d) $4x^4 - 13x^3 - 13x^2 + 28x - 6 = 0$

4. Divide $8x^4 - 30x^2 + 6x - 3$ by $1 + x + 2x^2$ using **long division** and write the division statement.

5. When $f(x) = x^4 - 4x^3 + mx^2 + nx + 1$ is divided by $x-1$, the remainder is 7. When it is divided by $x+1$, the remainder is 3. Determine the values of m and n.

6. Solve each of the following, $x \in R$.

a) $x(x-1)(3-x)(x+3) < 0$ b) $x^3 - x^2 < 5x + 3$

7. Find the value of a and b and the remaining factor if the expression $ax^3 - 11x^2 + bx + 3$ is divisible by $x^2 - 4x + 3$.

8. Graph the following function $f(x) = (x^2+x+1)(2x+5)^2(x-3)^3$.Show all your work.

9. The passenger section of a train has a width $2x - 7$, length $2x + 3$, and height $x - 2$, with all dimensions in metres. Solve a polynomial equation to determine the dimensions of the section of the train if the volume is $117m^3$.

10. Determine algebraically, whether each function is even, odd, or neither.

a) $f(x) = 4x^3$ b) $f(x) = 2x^4 - x^2$ c) $g(x) = \sqrt[3]{2x^2 + 1}$

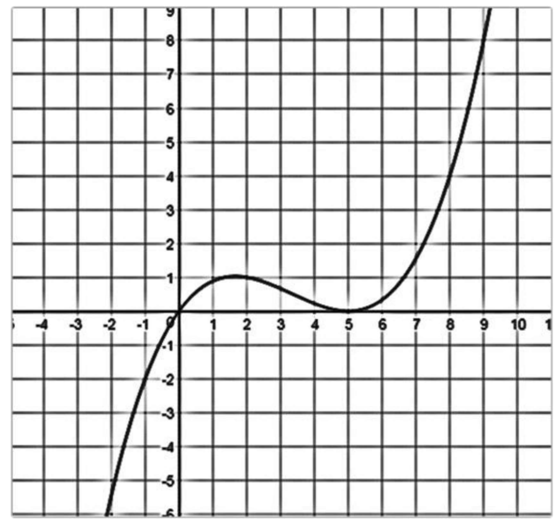
d) $h(x) = \dfrac{-x^3}{(3x^3 - 9x)^2}$ e) $f(x) = x + |x|$ f) $g(x) = \dfrac{2x}{|x|}$

11. The table of values below represents a polynomial function. Determine the equation of this function.

x	y			
-2	-19			
-1	-3			
0	1			
1	-1			
2	-3			
3	1			

12. Find the general equation of quartic functions that has negative leading coefficient, two equal roots at 2, and roots at $3 \pm 2\sqrt{2}$.

13. Given the graph of a polynomial function g(x), answer the following:



a) Is the function even-degree or odd-degree?

b) Is the function even or odd or neither?

c) State the zeroes and the lowest possible order of each zero ___________________

d) State the interval where the function is positive

____

e) State the interval where the function is negative

____

f) Determine the value of the remainder when f(x) is divided by x+2. ______________

14. Water is draining from a container. The height, in millimeters, of the water as a function of time, in seconds, can be modeled by the function

$$h(t) = 0.00185(250 - t)^2.$$

a) Calculate the average rate of change of height with respect to time from 50s to 100s.

b) Calculate the instantaneous rate of change of height with respect to time at t=60s.

c) Create a sketch of the function indicating the secant line and tangent line from part a.

15. When polynomial x^3-**a**x+21 is divided by x +**b**, the quotient is $x^2 - 3x + 5$ and the remainder is 6. Determine values of **a** and **b**.

16. Is x+b a factor of $x^9+5b^2x^7+5bx^8- b^9$?