## 1.5 Dividing Polynomials

Dividing a polynomial by another polynomial is similar to performing a division of numbers using long division. For example, divide the polynomial  $x^3 + 13x^2 + 39x + 46$  by x + 9

#### **Solution:**

5)

1) 
$$x+9\sqrt{x^3+13x^2+39x+46}$$
 first divide x into  $x^3$  to get  $x^2$ 

2)  $x+9\sqrt{x^3+13x^2+39x+46}$  now multiply  $x^2$  by  $x+9$  to get  $x^3+9x^2$  then subtract  $x^3+9x^2$  from  $x^3+13x^2$  to get  $4x^2$ 

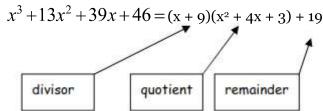
3)  $x+9\sqrt{x^3+13x^2+39x+46}$   $x^3+9x^2$  bring down the  $x=3$ 0 divide  $x=3$ 1 bring down the  $x=3$ 2 divide  $x=3$ 2 bring down the  $x=3$ 3 divide  $x=3$ 3 bring down the  $x=3$ 3 from  $x=3$ 4 divide  $x=3$ 4 from  $x=3$ 5 from  $x=3$ 4 divide  $x=3$ 5 from  $x=3$ 

Since the remainder has a lower degree than the divisor, the division is now complete. The result can be written as:

multiply 3 by x + 9 to get 3x + 27

then subtract 3x + 27 from 3x + 46 to get 19

divide 3x by x to get 3



 $4x^2+36x \qquad \downarrow$ 

The result of the division of a polynomial P(x) by a binomial of the form x - b is  $\frac{P(x)}{x - b} = Q(x) + \frac{R}{x - b}$ , where Q(x) is the quotient and R is the remainder. The corresponding statement that can be used to check the division, is P(x) = (x - b)Q(x) + R.

### **Dividing Polynomials**

Using the previous example, complete the polynomial division questions below:

1. 
$$X^3 - 5X^2 - X - 10$$
 by  $X - 2$ 

**2.** 
$$y^4 + 2y^2 - 28y - 36$$
 by  $2y^2 + 4y - 2$ 

# **Remainder Theorem**

#### **Remainder Theorem:**

When a polynomial f(x) is divided by x-a, the remainder is f(a).

Proof of the Remainder Theorem:

Ex. 1. Find the remainder when  $2x^3+3x^2-17x-30$  is divided by each of the following:

(a) 
$$x-1$$

(b) 
$$x-2$$

(c) 
$$x-3$$

(d) 
$$x+1$$

Similarly, when a polynomial f(x) is divided by ax + b, its remainder is given by  $f\left(-\frac{b}{a}\right)$ .

**Ex. 2.** When  $8x^3+4kx^2-2x+3$  is divided by 2x+1 the remainder is 6, find k.

**Ex. 3.** When  $f(x) = 2x^3 - px^2 + qx - 1$  is divided by x + 2 the remainder is -3; f(x) is divisible by x - 1. Find the values of p and q.

**Ex. 4.** Polynomial f(x) has a remainder of 3 when divided by x-2 and a remainder of -5 when it is divided by x+2. Determine the remainder when the polynomial is divided by  $x^2-4$ .

### **Practice**

1. Without using long division, find each remainder:

(a) 
$$(2x^2+6x+8)\div(x+1)$$

(b) 
$$(x^2+4x+12)\div(3x-1)$$

- 2. When the polynomial  $x^n+x-8$  is divided by x-2 the remainder is 10. What is the value of n?
- 3. Given that  $g(x) = (x+2)(3x^2+4)+5$  and  $h(x) = (6x+1)(3x^3-2x^2+x)+8$ . Find the remainder when g(x)+h(x) is divided by x+1.
- 4. The remainders when a polynomial is divided by x-1 and x+3 are 2 and -6 respectively. Find the remainder when the same polynomial is divided by (x-1)(x+3).
- 5. Find the remainder when  $x^{2012} + x 1$  is divided by x + 1.
- 6. Find the value of k for which  $x^2+(k-1)x+k^2-16$  is exactly divisible by x-3 but not divisible by x+4.
- 7. Given that the expression  $2x^3+px^2-8x+q$  is exactly divisible by  $2x^2-7x+6$ , evaluate p and q.
- 8. The polynomial  $2x^3-3ax^2+ax+b$  has a factor x-1 and a remainder of -10 when divided by x+1. Find the values of a and b.
- 9. Find each quotient and remainder:

(a) 
$$(2x^2+6x+15)\div(x+3)$$

(b) 
$$(x^2-4x+13)\div(2x-1)$$

(c) 
$$(x^2-x+3)\div(x+2)$$

(d) 
$$(2x^4+x^3-24x^2-3x+2)\div(x^2+x-4)$$

10. When a certain polynomial is divided by x+3, the quotient is  $x^2-3x+5$  and the remainder is 6. What is the polynomial?

#### Warm-up

1. Divide  $8x^4 - 30x^2 + 6x - 3$  by  $1 + x + 2x^2$  using long **division** and write the division statement.

2. Consider the function  $f(x) = ax^3 + 3x + b + 5$ , where a and b are constants and  $a \ne 0 \& b \ne 0$ . f(x) has a remainder of 2a when divided by x and a remainder of 2b when divided by x-1. Determine the values of a and b.