MHF4U1

TRIGONOMETRY FORMATIVE QUIZ

NAME: SOLUTIONS

1. Find two angles co-terminal with $\frac{4\pi}{3}$. …/1-K

Answers may vary

$\theta_1 = \frac{4\pi}{3} + 2\pi = \frac{10\pi}{3}$

$\theta_2 = \frac{4\pi}{3} - 2\pi = -\frac{2\pi}{3}$

2. Find the exact value for the $2\sin^2\left(\frac{3\pi}{8}\right) - 1$: …/2-K

$2\sin^2\left(\frac{3\pi}{8}\right) - 1 = -\left[1 - 2\sin^2\left(\frac{3\pi}{8}\right)\right]$

$= -\cos\left[2\left(\frac{3\pi}{8}\right)\right]$

$= -\cos\left(\frac{3\pi}{4}\right)$

$= -\cos\left[\pi - \frac{\pi}{4}\right]$

$= -\left(-\cos\frac{\pi}{4}\right)$

$= \sqrt{2}/2$

3. Convert $\frac{5\pi}{24}$ to degrees …/1-K

$\frac{5\cancel{\pi}}{24} \cdot \frac{180^\circ}{\cancel{\pi}} = 37.5^\circ$

4. Convert -55° to radians …/1-K

$-55^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{11\pi}{36}$

Multiple Choice: Circle one answer that best answers each question #5-6

5. A circle has a radius of 20 cm . The **exact** length of arc that subtends by a central angle of 135° is:…/1-K

↳ $135\left(\frac{\pi}{180}\right) = \frac{3\pi}{4}$ rads

a) 2700 cm
b) 50 cm
c) 15 cm
d) 15πcm

$a = 20\left(\frac{3\pi}{4}\right) = 15\pi$

6. $2\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$ is equivalent to: …/1-K

↳ $\sin\left[2\left(\frac{\pi}{4}\right)\right] = \sin\left(\frac{\pi}{2}\right)$

a) $\sin\left(\frac{\pi}{2}\right)$
b) $\cos\left(\frac{\pi}{2}\right)$
c) $\sin\left(\frac{\pi}{8}\right)$
d) $\cos\left(\frac{\pi}{8}\right)$

7. Completely simplify $\frac{\sin\left(\frac{\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right)}{\cos(x+\pi)} - \frac{\csc\left(\frac{3\pi}{2} - x\right)}{\sec(2\pi - x)}$. Show all steps. …/4-K

$= \frac{\cancel{\cos(x)}\cdot[-\tan(x)]}{-\cancel{\cos(x)}} - \frac{-\cancel{\sec(x)}}{\cancel{\sec(x)}}$

$= \tan(x) + 1$

8. Fin the **exact** value for the following. Show all steps.

$\sin\left(\frac{3\pi}{8}\right)$. …/4-A

$= \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$

$= \cos\left(\frac{\pi}{8}\right)$

$\cos\left(\frac{\pi}{4}\right) = 2\cos^2\left(\frac{\pi}{8}\right) - 1$

$\frac{\sqrt{2}}{2} = 2\cos^2\left(\frac{\pi}{8}\right) - 1$

$\frac{\sqrt{2}}{2} + 1 = 2\cos^2\left(\frac{\pi}{8}\right)$

$\frac{\sqrt{2}+2}{2} = 2\cos^2\left(\frac{\pi}{8}\right)$

$\sqrt{\frac{\sqrt{2}+2}{4}} = \sqrt{\cos^2\left(\frac{\pi}{8}\right)}$

$\therefore \frac{\sqrt{\sqrt{2}+2}}{2} = \cos\left(\frac{\pi}{8}\right)$

$\sin\left(\frac{7\pi}{12}\right)$ …/3-A

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

9. Prove $\frac{\sin(2x)}{1+\cos(2x)} = \frac{\tan(x)\sin(2x)}{2\sin(x)\cos(x)}$. …/4-A

$LS = \frac{2\sin(x)\cos(x)}{1 + [2\cos^2(x) - 1]}$

$= \frac{\cancel{2}\sin(x)\cancel{\cos(x)}}{\cancel{2}\cos^{\cancel{2}}(x)}$

$= \frac{\sin(x)}{\cos(x)}$

$= \tan(x) \cdot \frac{\sin(2x)}{\sin(2x)}$

$= \frac{\tan(x)\sin(2x)}{\sin(2x)}$

$= \frac{\tan(x)\sin(2x)}{2\sin(x)\cos(x)}$

$= RS$

TRIGONOMETRY
FORMATIVE QUIZ

NAME:____________________

10. Determine the **exact** value of $\dfrac{\csc\left(\frac{5\pi}{3}\right)+\tan\left(\frac{\pi}{6}\right)}{\sin\left(\frac{5\pi}{4}\right)\cot\left(\frac{\pi}{3}\right)}$ (rationalize if necessary). …/4-A

$= \dfrac{\csc\left(2\pi-\frac{\pi}{3}\right)+\frac{1}{\sqrt{3}}}{\sin\left(\pi+\frac{\pi}{4}\right)\cdot\frac{1}{\sqrt{3}}}$

$= \dfrac{-\csc\left(\frac{\pi}{3}\right)+\sqrt{3}/3}{-\sin\left(\frac{\pi}{4}\right)\cdot\frac{1}{\sqrt{3}}}$

$= \dfrac{-\frac{2}{\sqrt{3}}+\frac{1}{\sqrt{3}}}{-\frac{\sqrt{2}}{2}\cdot\frac{1}{\sqrt{3}}}$

$= \dfrac{-1/\sqrt{3}}{-\sqrt{2}/2\sqrt{3}}$

$= \dfrac{1}{\cancel{\sqrt{3}}}\cdot\dfrac{2\cancel{\sqrt{3}}}{\sqrt{2}}$

$= \dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$= \sqrt{2}$

11. If $\tan(x) = -\frac{3}{4}$, and $\frac{3\pi}{2} < x < 2\pi$, then determine the value of $\tan\left(\frac{x}{2}\right)$. …/5-T

$\tan(x) = \dfrac{2\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}$

$-\dfrac{3}{4} = \dfrac{2\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}$

Let $A = \tan\left(\frac{x}{2}\right)$:

$-\frac{3}{4} = \frac{2A}{1-A^2}$

$-3(1-A^2) = 8A$

$-3+3A^2 = 8A$

$3A^2-8A-3=0$

$(3A+1)(A-3)=0$

$3A+1=0$ $\quad$ $A-3=0$

$A=-\frac{1}{3}$ $\quad$ $A=3$

$\tan\left(\frac{x}{2}\right)=-\frac{1}{3}$ $\quad$ $\tan\left(\frac{x}{2}\right)=3$

↳ inadmissible since $\frac{3\pi}{2}<x<2\pi$

since the tangent is negative in Q4

$\therefore \tan\left(\frac{x}{2}\right) = -\frac{1}{3}$

12. If $\cos(\theta)+\sin(\theta) = \frac{1+\sqrt{3}}{2}$ — ① and $\cos(\theta)-\sin(\theta)=\frac{1-\sqrt{3}}{2}$ — ② find the value of $\sin(2\theta)$. …/5-T

① + ②: $\cos\theta + \sin\theta = \frac{1+\sqrt{3}}{2}$

$+)\ \cos\theta - \sin\theta = \frac{1-\sqrt{3}}{2}$

$2\cos\theta = \frac{2}{2}$

$\cos\theta = \frac{1}{2}$

Sub. $\cos\theta = \frac{1}{2}$ into ①: $\frac{1}{2}+\sin(\theta) = \frac{1+\sqrt{3}}{2}$

$\sin(\theta) = \frac{\sqrt{3}}{2}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2}$

$\therefore \sin(2\theta) = \frac{\sqrt{3}}{2}$

KNOWLEDGE	APPLICATION	THINKING
___/11	____/15	____/10