MHF 4U1 Name: SOLUTIONS

Formative Quiz – Rational Functions

Part A - Multiple Choice: *Clearly circle the correct answer.* /6-K

1. What are the zeros of $f(x)=\dfrac{(x-3)(x+2)}{x(x+5)(x-1)}$?

a. $x=3,-2,-5,1,0$ c. $x=3,-2$
b. $x=-5,1$ d. $x=0,-5,1$

2. Which of the following rational functions has a graph that is linear?

a. $y=\dfrac{x+3}{x^2-5}$ c. $y=\dfrac{x^2-25}{x-5}$
b. $y=\dfrac{x}{(x-3)^2}$ d. $y=-\dfrac{2}{x}$

3. Which of the following functions has an oblique asymptote?

a. $y=\dfrac{x^2-3x-4}{x^2-4}$ c. $y=\dfrac{x+3}{x^2-5x-14}$
b. $y=\dfrac{x^2-4}{x+3}$ d. $y=\dfrac{(x-3)^2}{(x-3)(x+4)}$

4. Which of the following functions has a vertical asymptote at x =2 and a horizontal asymptote of y=1 ?

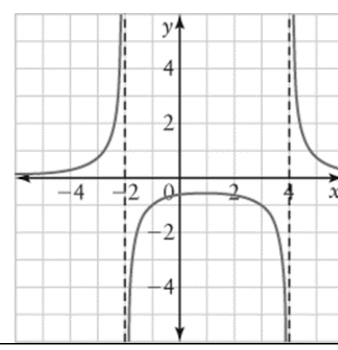
a. $y=\dfrac{x^2-6x+9}{x^2+3x+2}$ c. $y=\dfrac{x+3}{x^2-4}$
b. $y=\dfrac{3}{x-2}$ d. $y=\dfrac{x^2-9}{x^2-4x+4}$

5. Which of the following statements is false for the function $f(x)=\dfrac{g(x)}{h(x)}$ where both g and h are polynomial functions?

a. It is possible to have both a vertical asymptote and a horizontal asymptote.
b. It is possible to have both an oblique asymptote and a horizontal asymptote.
c. It is possible to have both a vertical asymptote and an oblique asymptote.
d. It is possible to have no vertical asymptote

6. Over what interval(s) is the graph of the rational function decreasing?

a. $x\in(-2,4)$
b. $x\in(1,4)\cup(4,\infty)$
c. $x\in(-\infty,-2)\cup(-2,1)$
d. $x\in(-\infty,-2)\cup(4,\infty)$



Part B – Full Solution

1. Given the following function : $f(x)=\dfrac{x^2-5x+3}{x-2}$

a) Does it have a vertical asymptote? yes / 1-K
b) Does it have an oblique or horizontal asymptote? Oblique / 1-K
c) Find the equation of the oblique/horizontal /vertical asymptote. / 2-K

$$\begin{array}{c|ccc} 2 & 1 & -5 & 3 \\ & & 2 & -6 \\ \hline & 1 & -3 & -3 \end{array}$$

$f(x)=x-3+\left(\dfrac{-3}{x-2}\right)$

$\therefore$ O.A is $y=x-3$

d) Analyze the end behavior of the function near the asymptotes. /3-K

As $x\to\infty$, $[f(x)-(x-3)]\to 0$ (below)
As $x\to-\infty$, $[f(x)-(x-3)]\to 0$ (above)
As $x\to 2^+$, $f(x)\to-\infty$
As $x\to 2^-$, $f(x)\to\infty$

2. Solve the following: $\dfrac{x+4}{x^2+5x-24}\geq\dfrac{2}{x-3}$.

$$\frac{x+4}{(x+8)(x-3)}-\frac{2}{x-3}\geq 0$$

$$\frac{x+4-2(x+8)}{(x+8)(x-3)}\geq 0$$

$$\frac{x+4-2x-16}{(x+8)(x-3)}\geq 0$$

$$\frac{-x-12}{(x+8)(x-3)}\geq 0$$

$$\frac{-(x+12)}{(x+8)(x-3)}\geq 0$$

Root : -12
V.A: x=-8, x=3

+ − + −
-12 -8 3

$\therefore x\in(-\infty,-12]\cup(-8,3)$ /4-K

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3. Sketch the following function using the domain, intercepts, vertical, horizontal asymptotes.

$$f(x)=\frac{x^3-3x^2-13x+15}{x^3-12x-16}\begin{matrix}\} g(x)\\ \} h(x)\end{matrix}$$

1	1	-3	-13	15
		1	-2	-15
	1	-2	-15	0

$g(x)=(x-1)(x^2-2x-15)$
$=(x-1)(x-5)(x+3)$

-2	1	0	-12	-16
		-2	4	16
	1	-2	-8	0

$h(x)=(x+2)(x^2-2x-8)$
$=(x+2)(x-4)(x+2)$
$=(x+2)^2(x-4)$

$$f(x)=\frac{(x-1)(x-5)(x+3)}{(x+2)^2(x-4)}$$

$\boxed{\text{V.A:}}$ $x=-2$
↳ As $x\to-2^+$, $f(x)\to-\infty$
As $x\to-2^-$, $f(x)\to-\infty$

$x=4$
↳ As $x\to4^+$, $f(x)\to-\infty$
As $x\to4^-$, $f(x)\to\infty$

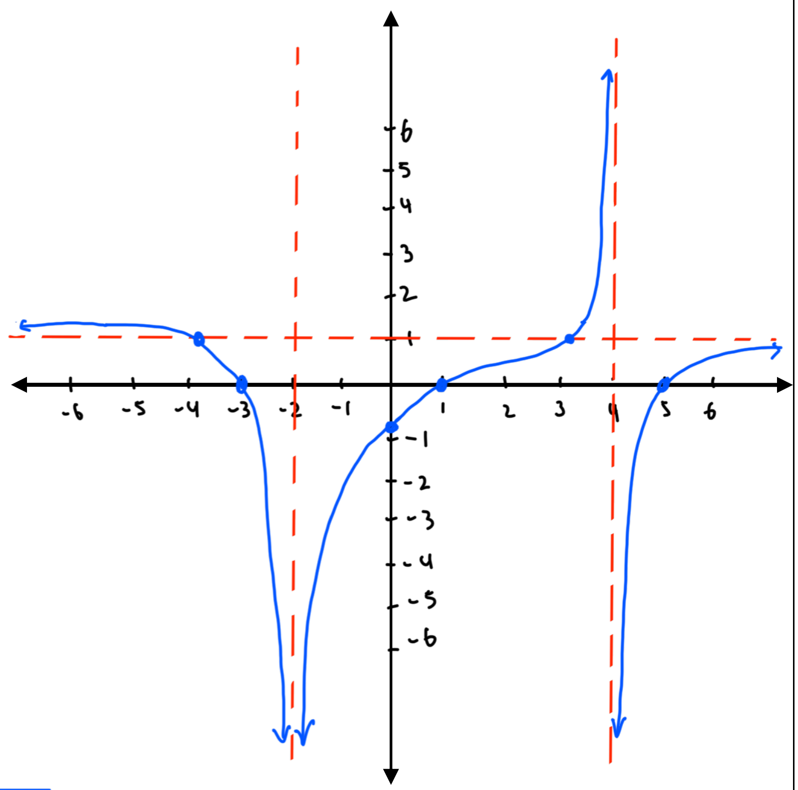
$\boxed{\text{H.A:}}$ $y=1$
↳ As $x\to\infty$, $f(x)\to1$ (below)
As $x\to-\infty$, $f(x)\to1$ (above)

Cross-over: $f(x)=1$

$\frac{x^3-3x^2+13x+15}{x^3-12x-16}=1$

$x^3-3x^2-13x+15=x^3-12x-16$
$-3x^2-x+31=0$
$3x^2+x-31=0$
$x=\frac{-1\pm\sqrt{1-4(3)(-31)}}{2(3)}$
$=\frac{-1\pm\sqrt{373}}{6}$

∴ Cross-overs at $(-3.39, 1)$ and $(3.05, 1)$



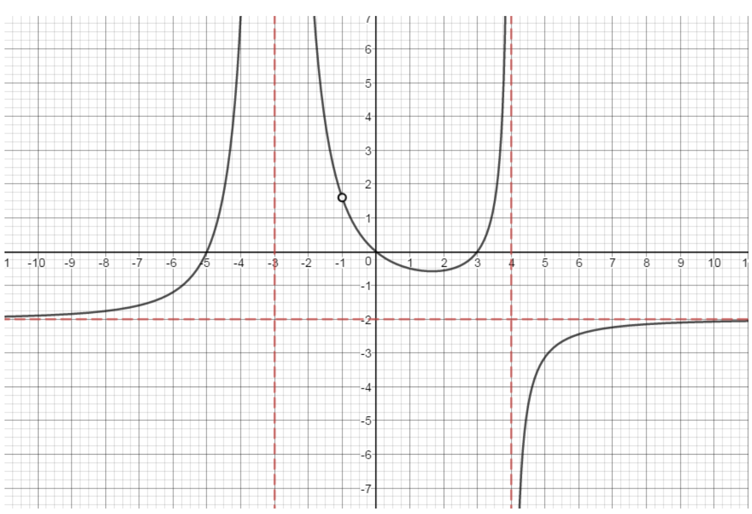
$\boxed{\text{x-inter:}}$ $(1,0)$, $(5,0)$, $(-3,0)$

$\boxed{\text{y-inter:}}$ $y=\frac{(-1)(-5)(3)}{(2)^2(-4)}=\frac{15}{-16}$

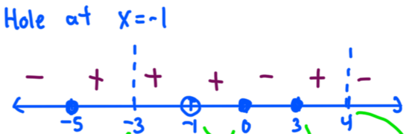
$(0,-\frac{15}{16})$

/10-A

4. Write an equation for the function graphed below.



V.A: $x=-3$, $x=4$
H.A: $y=-2$
Roots: $-5, 0, 3$
Hole at $x=-1$



$$f(x)=\frac{-2x(x+5)(x+1)(x-3)}{(x+3)^2(x+1)(x-4)}$$

/5-A

5. Sketch the graph of $f(x)=3x^2+8x-3$ and its reciprocal on the same axis.

$f(x)=(3x-1)(x+3)$

Roots: $x=\frac{1}{3}, -3$

Vertex: $h=\frac{\frac{1}{3}-3}{2}=\frac{-8/3}{2}=-\frac{4}{3}$

$f(-\frac{4}{3})=(-4-1)(\frac{5}{3})=-\frac{25}{3}$

∴ Vertex at $(-\frac{4}{3},-\frac{25}{3})$

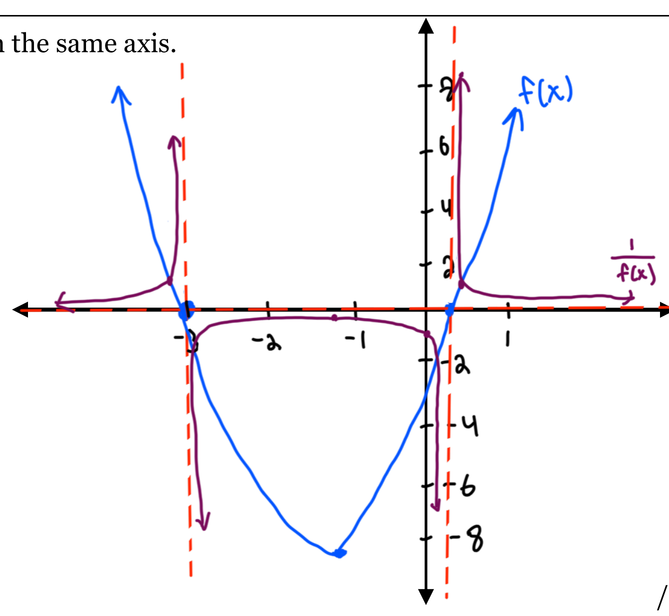
y-inter: $y=(-1)(3)=-3$

$\frac{1}{f(x)}$: V.A at $x=\frac{1}{3}$ and $x=-3$

H.A at $y=0$

Max at $(-4/3, -\frac{3}{25})$

y-Inter: $y=-\frac{1}{3}$



/4-K

6. If the graph of function $f(x)=\frac{(\mathbf{1-ax})^3}{x^2}$ crosses its oblique asymptote at $x=\frac{1}{3}$ determine the value of **a**. What is the equation of the oblique asymptote?

$$f(x)=\frac{(1-ax)(1-2ax+a^2x^2)}{x^2}=\frac{1-2ax+a^2x^2-ax+2a^2x^2-a^3x^3}{x^2}=\frac{-a^3x^3+3a^2x^2-3ax+1}{x^2}=\underbrace{-a^3x+3a^2}_{O.A}+\frac{\overbrace{-3ax+1}^{R(x)}}{x^2}$$

$R(x)=-3ax+1$

Cross-over at $x=\frac{1}{3}\Rightarrow R\left(\frac{1}{3}\right)=0$

$$-3a\left(\tfrac{1}{3}\right)+1=0$$
$$-a+1=0$$
$$1=a$$

$\therefore$ O.A: $y=-(1)^3x+3(1)^2$

$y=-x+3$

/5-T

7. The rational function $y=\frac{ax^2}{bx^2+(2b+1)x+b}$ has one vertical asymptote and passes through point (2,-3). Determine the values of a and b.

One V.A $\Rightarrow bx^2+(2b+1)x+b$ has only 1 root

$\Rightarrow$ discriminent $=0$

$$(2b+1)^2-4(b)(b)=0$$
$$4b^2+4b+1-4b^2=0$$
$$4b=-1$$
$$b=-\frac{1}{4}$$

Passes through $(2,-3)\Rightarrow -3=\frac{a(2)^2}{-\frac{1}{4}(2)^2+\left(-\frac{1}{2}+1\right)(2)-\frac{1}{4}}$

$$-3=\frac{4a}{-1+1-1/4}$$
$$-3=\frac{4a}{-1/4}$$
$$\frac{3}{4}=4a$$
$$\frac{3}{16}=a$$

/5-T

8. Explain why it is important to know how to use long division (not just synthetic division) when determining the equation of an oblique asymptote for *some* rational functions.

For a rational function $\frac{P(x)}{Q(x)}$, an Oblique Asymptote exists if and only if the degree of P(x) is one degree larger than Q(x). To find the O.A, we divide the polynomial P(x) by Q(x). If Q(x) is degree 2 or higher, then synthetic division would be challenging or impossible.

Hence, knowing long division is important to know for such situations.

/4-C

Knowledge	Application	Communication	Thinking
/21	/15	/4	/10