Unit 1: Polynomial Functions
1.1 Power Functions

POLYNOMIAL EXPRESSIONS

A polynomial expression is one or more terms where each term is the product of a constant and a variable raised to a non-negative integral exponent only.

Example 1: Which of the following is a polynomial expression? Explain.

	Yes/No	Reason
$5x$		
$2x^{-3} + 3\sqrt{x} - 4$		
$t^2 + 3.5t$		
$3xy + 4x^2$		
$\frac{1}{2}x^5 - 3x^2 - x + 1$		

POLYNOMIAL FUNCTIONS

A polynomial function is a function defined by a polynomial in one variable written in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$.

To be a polynomial function, the following conditions must be met:

1. $a_n \neq 0$ (This means the lead coefficient cannot equal zero) if n is the highest term
2. The coefficients (a_n, a_{n-1}, ...a_0) are all real numbers
3. The exponents are all whole numbers
   - The degree of a polynomial in one variable *x* is the highest power of *x*.
   - The leading coefficient of a polynomial function is the constant belonging to the power with the highest exponent.
   - The domain of a polynomial function is all real numbers.
   - There are n+1 terms in a polynomial function of degree n.

Types of Polynomial Functions

Type	Degree	Standard Form a,b,c,d,e are Real numbers	Example
Constant	0	$f(x) = a$	
Linear	1	$f(x) = ax+b$, $a\neq 0$	
Quadratic	2	$f(x) = ax^2+bx+c$, $a\neq 0$	
Cubic	3	$f(x) = ax^3+bx^2+cx+d$, $a\neq 0$	
Quartic	4	$f(x) = ax^4+bx^3+cx^2+dx+e$, $a\neq 0$	
Quintic	5	$f(x) = ax^5+bx^4+cx^3+dx^2+ex+f$, $a\neq 0$	

Example 2: Complete the table below:

	Type	Degree	Leading Coefficient
$f(x) = 10 + 7x$			
$f(x) = 3.7x^4 - 2x^2 + 7.4$			
$g(x) = 2.5$			
$g(x) = -\frac{1}{2}x^2 + \sqrt{2}$			
$s(t) = t^3 - 3t$			
$f(x) = x(x+1)(x-2)$			
$h(x) = x(2x+1)^2(2-x)^3$			

Example 3: Explain why each of the following are not polynomial functions.

a) $h(x) = 3x^2 + 2x - 5x^{-1}$

b) $f(x) = \frac{1}{x}$

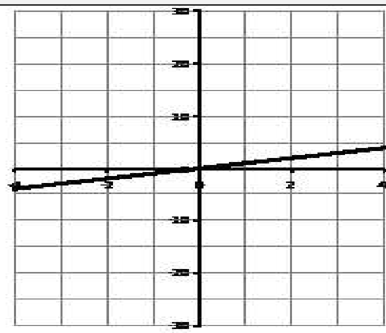
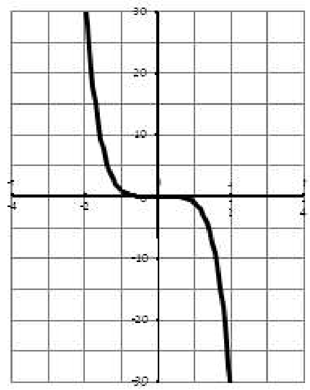
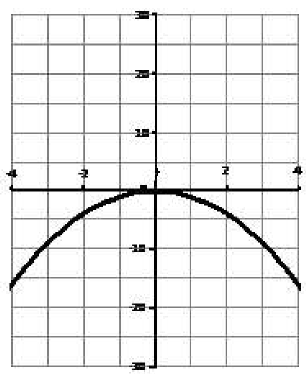
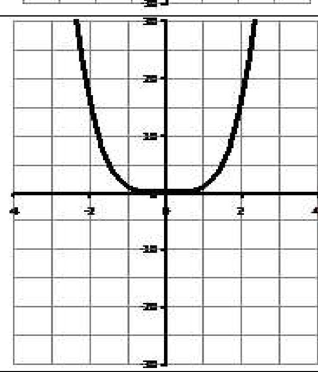
c) $g(x) = 2^x$

POWER FUNCTIONS

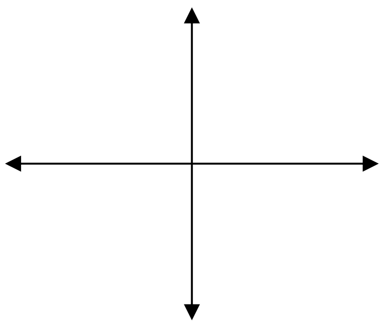
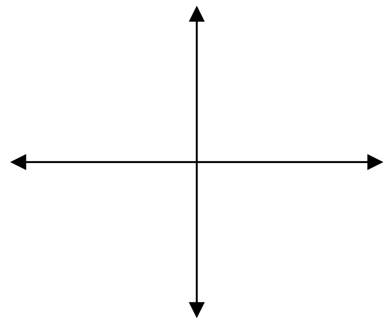
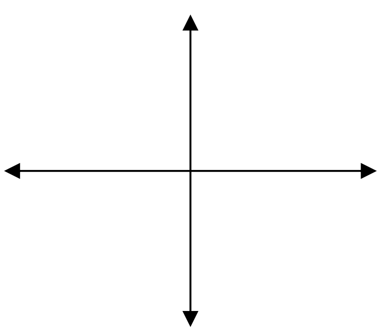
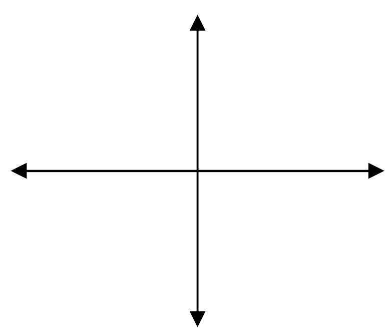
A power function is a polynomial of the form $f(x) = ax^n$, where n is a whole number.
Power functions have similar characteristics depending on whether their degree is even or odd.

____
____
____

____
____
____

Function	Degree	Graph	Sign of Leading coefficient	End behavior
$y = \frac{1}{3}x$				As $x \to -\infty$, $y \to$ ______ As $x \to \infty$, $y \to$ ______
$y = -2x^3$				As $x \to -\infty$, $y \to$ ______ As $x \to \infty$, $y \to$ ______
$y = -x^2$				As $x \to -\infty$, $y \to$ ______ As $x \to \infty$, $y \to$ ______
$y = x^4$				As $x \to -\infty$, $y \to$ ______ As $x \to \infty$, $y \to$ ______

Summary

If a polynomial function has an ***odd degree and its lead coefficient is positive***, then, the function extends from the _______ quadrant to the _______ quadrant. Therefore: *as* $x \to -\infty$, ____________ *as* $x \to \infty$, ____________ . 	If a polynomial function has an ***odd degree and its lead coefficient is negative***, then, the function extends from the _____ quadrant to the _______ quadrant. Therefore: *as* $x \to -\infty$, ____________ *as* $x \to \infty$, ____________ . 
If a polynomial function has an ***even degree and its lead coefficient is positive***, then, the function extends from the _______ quadrant to the _______ quadrant. Therefore: *as* $x \to -\infty$, ____________ *as* $x \to \infty$, ____________ . 	If a polynomial function has an ***even degree and its lead coefficient is negative***, then, the function extends from the _____ quadrant to the _______ quadrant. Therefore: *as* $x \to -\infty$, ____________ *as* $x \to \infty$, ____________ . 

Polynomial Identities

An equation shows that two mathematical expressions are equal.

$$2x-5=6x-1$$

It can be solved to give the ***roots*** or ***solutions*** of the equation. An *identity* is an equation that is true for **all values of *x*.**

If an equation is an identity, the symbol $"="$ in the equation can be replaced by $"\equiv"$ which means "*is identical to*".

For example: $(x+1)^3 \equiv x^3+3x^2+3x+1$ is an **identity** since the equation is satisfied for all values of *x*.

Ex#1.Given that $x^3-2x^2+4x+3 \equiv (x-1)(x^2-x+a)+b$, find the value of *a* and *b*.

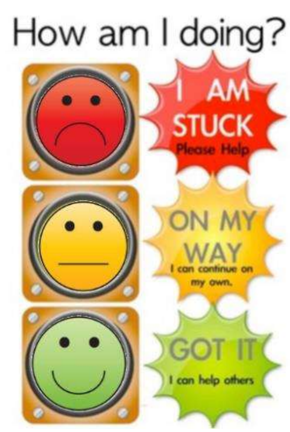
Ex#2. Given that $15x^3+Cx^2-x+2 \equiv (3x+1)(Ax+B)(x-1)$, find the values of A,B and C.

Ex#3.Given $2x^2+x+C \equiv A(x+1)^2+B(x-1)+4$ for all values of x, find the values of A,B and C.

Exit Card!

Complete the chart below.

Function	**Type**	**degree**	**leading coefficient**	**End behavior**
$y = x^2 - 2x$				As $x \to -\infty$, $y \to$ ________ As $x \to \infty$, $y \to$ ________
$y = -(x-1)(1-x)^2$				As $x \to -\infty$, $y \to$ ________ As $x \to \infty$, $y \to$ ________
$y = -2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$				As $x \to -\infty$, $y \to$ ________ As $x \to \infty$, $y \to$ ________
$y = -(1-2x)^3(x+1)^2$				As $x \to -\infty$, $y \to$ ________ As $x \to \infty$, $y \to$ ________
$y = -3x^2(2-x)^3(2x-1)^2$				As $x \to -\infty$, $y \to$ ________ As $x \to \infty$, $y \to$ ________



Warm Up

1. Which of the following relations are polynomial functions? For each polynomial function name the type of polynomial, its degree, the leading coefficient and end behaviour. If a relation is not a polynomial function, provide at least one reason why it is not.

a. $y = 3x^5 - 2x + 17$	b. $y = 3x^4 + \sqrt{5x}$
Type of polynomial: __________ Degree : __________ Leading coefficient:__________ $x \to -\infty, y \to$ $x \to \infty, y \to$	Reason:
c. $y = -\frac{1}{2}x(x-4)^2(x+4)^2$	d. $y = (x-2)^2(4-2x)(x+5)$
Type of polynomial: __________ Degree : __________ Leading coefficient:__________ $x \to -\infty, y \to$ $x \to \infty, y \to$	Type of polynomial: __________ Degree : __________ Leading coefficient:__________ $x \to -\infty, y \to$ $x \to \infty, y \to$
e. $y = 4$	f. $y = \left(-(x-4)\right)^3 + 1$
Type of polynomial: __________ Degree : __________ Leading coefficient:__________ $x \to -\infty, y \to$ $x \to \infty, y \to$	Type of polynomial: __________ Degree : __________ Leading coefficient:__________ $x \to -\infty, y \to$ $x \to \infty, y \to$

2. From the graphs given, select all graphs that represent power functions of the form $y=ax^n$.

a)	**b)**	**c)**	**d)**