

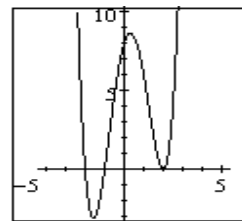
Formative Quiz – Polynomial Functions

Multiple Choice

/5-K

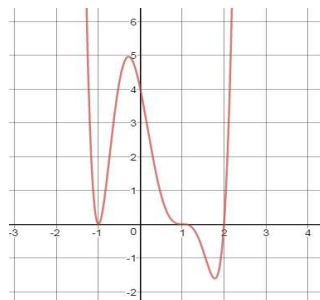
Identify the letter of the choice that best completes the statement or answers the question.

- ___ **B** ___ 1. The equation of $f(x)$ **to the right**, is:
 a. $f(x) = -(x-1)(x-2)(x+2)^2$ c. $f(x) = (x+1)(x+2)(x-2)$
 b. $f(x) = (x+1)(x+2)(x-2)^2$ d. $f(x) = -(x-1)(x-2)(x+2)$
- ___ **C** ___ 2. $x-2$ is **not** a factor of:
 a. $x^3 - 5x^2 + 6x$ c. $x^3 - 3x^2 - x + 3$
 b. $x^3 - x^2 - 4x + 4$ d. $x^4 - 2x^3 - x^2 + 2x$
- ___ **C** ___ 3. $f(x) = -2x(x-1)(x-2)(x+2)$ and $g(x) = -2x(x-1)(x+2)^2$ have the same:
 a. number of zeros c. fourth finite difference
 b. intervals of increase and decrease d. maximum value
- ___ **A** ___ 4. $f\left(\frac{1}{2}\right) = 0$ and $f(-1) = 6$ indicates that:
 a. there is no remainder when $f(x)$ is divided by $2x-1$
 b. there is no remainder when $f(x)$ is divided by $x+1$
 c. $(2x-1)$ and $(x+1)$ are factors of $f(x)$
 d. $f(x) = (2x-1)(x+1) + 6$
- ___ **D** ___ 5. For the function $f(x) = x^3 + 2x$, determine the average rate of change of $f(x)$ over the interval $1 \leq x \leq 3$.
 a. -30 b. -2 c. 2 d. 15



Short Answer

1. For the function: $f(x) = -2(x-1)^3(2-x)(x+1)^2$
 a) State
 ▪ the degree of the function **6**
 ▪ the leading coefficient **2**
 ▪ the end behaviour of this function,
 as $x \rightarrow -\infty$, **$y \rightarrow \infty$**
 as $x \rightarrow \infty$, **$y \rightarrow \infty$**
 b) Sketch the function using the above information:



/4-K

2. Determine the equation of a quintic function with zeros at $1 \pm 2\sqrt{3}$ and a point of inflection at $x = -2$, whose graph passes through $(2, -16)$. /3-K

$$\begin{aligned}
 x &= 1 \pm 2\sqrt{3} \\
 x-1 &= \pm 2\sqrt{3} \\
 (x-1)^2 &= (\pm 2\sqrt{3})^2 \\
 x^2 - 2x + 1 &= 12 \\
 x^2 - 2x - 11 &= 0 \\
 f(x) &= k(x^2 - 2x - 11)(x+2)^3
 \end{aligned}$$

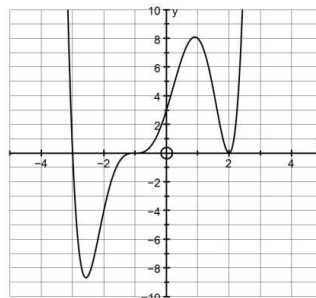
$$\begin{aligned}
 -16 &= k(4 - 4 - 11)(4)^3 \\
 -16 &= k(-11)(64) \\
 k &= \frac{1}{44} \\
 \therefore f(x) &= \frac{1}{44}(x^2 - 2x - 11)(x+2)^3
 \end{aligned}$$

3. Factor and simplify completely: $3(x-3)^3 + 24(x-1)^3$. /3-K

$$\begin{aligned}
 &= 3[(x-3)^3 + 8(x-1)^3] \\
 &= 3[(x-3) + 2(x-1)][(x-3)^2 - 2(x-1)(x-3) + 4(x-1)^2] \\
 &= 3(3x-5)(x^2 - 6x + 9 - 2x^2 + 8x - 6 + 4x^2 - 8x + 4) \\
 &= 3(3x-5)(3x^2 - 6x + 7)
 \end{aligned}$$

4. Determine the specific equation of the following function. /3-K

$$\begin{aligned}
 f(x) &= k(x+3)(x-2)^2(x+1)^3, f(0) = 3 \\
 3 &= k(3)(4)(1) \\
 k &= \frac{1}{4} \\
 \therefore f(x) &= \frac{1}{4}(x+3)(x-2)^2(x+1)^3
 \end{aligned}$$



Full Solution

1. Determine the equation of the polynomial function represented by the following table of values. /4-A

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-2	-23			
-1	3	26		
0	7	4	-22	
1	1	-6	-10	12
2	-3	-4	2	12
3	7	10	14	12
4	43	36	26	12

$$\Delta^3 y = a(3!)$$

$$12 = 6a$$

$$a = 2$$

$$f(x) = 2x^3 + bx^2 + cx + d$$

$$f(-1) = 3 : -2 + b - c + 7 = 3 \rightarrow b - c = -2 \quad (1)$$

$$f(0) = 7 : d = 7$$

$$f(1) = 1 : 2 + b + c + 7 = 1 \rightarrow b + c = -8 \quad (2)$$

$$\begin{cases} b - c = -2 & (1) \\ b + c = -8 & (2) \end{cases}$$

$$\begin{cases} b - c = -2 & (1) \\ b + c = -8 & (2) \end{cases}$$

$$(1) + (2) : 2b = -10$$

$$b = -5 \quad \& \quad c = -3$$

$$\therefore f(x) = 2x^3 - 5x^2 - 3x + 7$$

2. Divide $(2x^3 + 5x^2 - 4x - 5) \div (x^2 + 1)$ using long division. Write the division statement. /4-K

$$\begin{array}{r}
 2x + 5 \\
 x^2 + 0x + 1 \overline{) 2x^3 + 5x^2 - 4x - 5} \\
 \underline{-2x^3 - 0x^2 - 2x} \\
 5x^2 - 6x - 5 \\
 \underline{-5x^2 + 0x - 5} \\
 -6x - 10
 \end{array}$$

$$2x^3 + 5x^2 - 4x - 5 = (x^2 + 1)(2x + 5) + (-6x - 10)$$

OR

$$\frac{2x^3 + 5x^2 - 4x - 5}{x^2 + 1} = (2x + 5) - \frac{2(3x + 5)}{x^2 + 1}$$

3. Solve the following algebraically.

a) $x^4 + x^3 + 2x^2 = -4x + 8$ /3-A

$$x^4 + x^3 + 2x^2 + 4x - 8 = 0$$

$$\text{Possible zeros: } \pm\{1, 2, 4, 8\}$$

$$\text{Let } f(x) = x^4 + x^3 + 2x^2 + 4x - 8$$

$$f(1) = 0$$

$$\begin{array}{r|rrrrr}
 1 & 1 & 1 & 2 & 4 & -8 \\
 \downarrow & & 1 & 2 & 4 & 8 \\
 \hline
 & 1 & 2 & 4 & 8 & 0
 \end{array}$$

$$f(x) = (x - 1)(x^3 + 2x^2 + 4x + 8)$$

$$f(-2) = 0$$

$$\begin{array}{r|rrrr}
 -2 & 1 & 2 & 4 & 8 \\
 \downarrow & & -2 & 0 & -8 \\
 \hline
 & 1 & 0 & 4 & 0
 \end{array}$$

$$f(x) = (x - 1)(x + 2)(x^2 + 4)$$

$$f(x) = 0 : x = 1, -2$$

b) $x^3 + 3x^2 - 4 \geq x^2 + 4x + 4$ /3-A

$$x^3 + 3x^2 - 4 - x^2 - 4x - 4 \geq 0$$

$$x^3 + 2x^2 - 4x - 8 \geq 0$$

$$(x - 2)(x^2 + 4x + 4) \geq 0$$

$$(x - 2)(x + 2)^2 \geq 0$$

$$- \quad \downarrow \quad - \quad +$$

$$\begin{array}{c}
 \leftarrow \quad | \quad | \quad \rightarrow \\
 -\infty \quad -2 \quad 2 \quad \infty
 \end{array}$$

$$x \in [2, \infty) \text{ and } \{x = -2\}$$

4. The polynomial $2x^3 - 5x^2 + kx + 6$ has the same remainder when divided by $x + 1$ and $x - 2$. Find value of k . /3-A

$$f(2) = f(-1)$$

$$16 - 20 + 2k + 6 = -2 - 5 - k + 6$$

$$3k = -3$$

$$k = -1$$

5. A medical researcher establishes that a patient's reaction time, r , in minutes, to a dose of a particular drug is $r(d) = -0.8d^3 + d^2$, where d is the amount of the drug, in millilitres, that is absorbed into the patient's blood. Determine the instantaneous rate of change of the time reaction with respect to the amount of the drug at 1 millilitre.

/3-A

$$r(1) = 0.2$$

$$r(1.001) = 0.1995$$

$$\text{IROC} = \frac{r(1.001) - r(1)}{0.001}$$

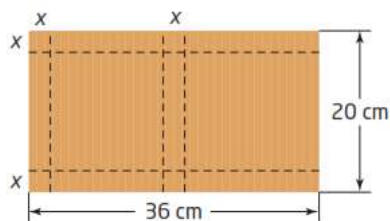
$$= \frac{0.1995 - 0.2}{0.001}$$

$$= -0.4 \text{ min / mL}$$

the instantaneous rate of change of the time reaction with respect to the amount of the drug at 1 millilitre decreases at a rate of 0.4 min per mL.

6. Boxes for candies are to be constructed from cardboard sheets that measure 36 cm by 20 cm. Each box is formed by folding a sheet along the dotted lines, as shown. What are the possible whole number dimensions of the box if the volume is to be 512 cm³?

/5-T



$$\begin{aligned} \text{The volume is given by } V(x) &= x(20 - 2x)\left(\frac{36 - 2x}{2}\right) \\ &= x(20 - 2x)(18 - x), 0 < x < 10. \end{aligned}$$

$$x(20 - 2x)(18 - x) = 512$$

$$2x^3 - 56x^2 + 360x - 512 = 0$$

$$2(x - 2)(x^2 - 26x + 128) = 0$$

$$\boxed{x = 2} \text{ or } x = 13 - \sqrt{41} \text{ or } x = 13 + \sqrt{41}$$

inadmissible extraneous

7. Explain how the sign of the leading coefficient of an even degree polynomial function impacts the functions end behaviours.

/3-C

The sign of the leading coefficient is sufficient to predict the end behavior of the function. For the even degree polynomial function if the sign of lead coefficient is positive, the graph of function extends from quadrant 2 to quadrant 1. If the sign of lead coefficient is negative, the graph of the function extends from quadrant 3 to quadrant 4.

Two marks are given throughout the assessment for mathematical form.

/2-C

Knowledge & Understanding	Application	Communication	Thinking
/22	/16	/5	/5