1.5 Dividing Polynomials

Dividing a polynomial by another polynomial is similar to performing a division of numbers using long division. For example, divide the polynomial $x^3 + 13x^2 + 39x + 46$ by $x + 9$

Solution:

1)
$$\begin{array}{r|l} & x^2 \\ \hline x+9 & x^3+13x^2+39x+46 \end{array}$$

first divide x into x^3 to get x^2

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2)
$$\begin{array}{r|l} & x^2 \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \\ & \quad\;\; 4x^2 \end{array}$$

now multiply x^2 by $x + 9$ to get $x^3 + 9x^2$
then subtract $x^3 + 9x^2$ from $x^3 + 13x^2$ to get $4x^2$

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3)
$$\begin{array}{r|l} & x^2+4x \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \;\; \downarrow \\ & \quad\;\; 4x^2+39x \end{array}$$

bring down the + 39x
divide $4x^2$ by x to get 4x

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4)
$$\begin{array}{r|l} & x^2+4x \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \;\; \downarrow \\ & \quad\;\; 4x^2+39x \\ & \quad\;\; \underline{4x^2+36x} \\ & \qquad\qquad 3x \end{array}$$

now multiply 4x by $x + 9$ to get $4x^2 + 36x$
then subtract $4x^2 + 36x$ from $4x^2 + 39x$ to get 3x

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5)
$$\begin{array}{r|l} & x^2+4x+3 \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \;\; \downarrow \quad \downarrow \\ & \quad\;\; 4x^2+39x \quad \downarrow \\ & \quad\;\; \underline{4x^2+36x} \quad \downarrow \\ & \qquad\qquad 3x+46 \\ & \qquad\qquad \underline{3x+27} \\ & \qquad\qquad\qquad 19 \end{array}$$

bring down the + 46
divide 3x by x to get 3

multiply 3 by $x + 9$ to get $3x + 27$
then subtract $3x + 27$ from $3x + 46$ to get 19

Since the remainder has a lower degree than the divisor, the division is now complete. The result can be written as:

$$x^3 + 13x^2 + 39x + 46 = \underbrace{(x+9)}_{\text{divisor}}\underbrace{(x^2+4x+3)}_{\text{quotient}} + \underbrace{19}_{\text{remainder}}$$

The result of the division of a polynomial $P(x)$ by a binomial of the form $x - b$ is $\frac{P(x)}{x-b} = Q(x) + \frac{R}{x-b}$, where $Q(x)$ is the quotient and R is the remainder. The corresponding statement that can be used to check the division, is $P(x) = (x - b)Q(x) + R$.

Dividing Polynomials

Using the previous example, complete the polynomial division questions below:

1. $x^3 - 5x^2 - x - 10$ by $x - 2$

2. $y^4 + 2y^2 - 28y - 36$ by $2y^2 + 4y - 2$

Remainder Theorem

Remainder Theorem:
When a polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$.

Proof of the Remainder Theorem:

Ex. 1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

(a) $x-1$

(b) $x-2$

(c) $x-3$

(d) $x+1$

Similarly, when a polynomial $f(x)$ is divided by $ax+b$, its remainder is given by $f\left(-\frac{b}{a}\right)$.

Ex. 2. When $8x^3+4kx^2-2x+3$ is divided by $2x+1$ the remainder is 6, find *k*.

Ex. 3. When $f(x)=2x^3-px^2+qx-1$ is divided by $x+2$ the remainder is -3; $f(x)$ is divisible by $x-1$. Find the values of p and q.

Ex. 4. Polynomial $f(x)$ has a remainder of 3 when divided by $x-2$ and a remainder of -5 when it is divided by $x+2$. Determine the remainder when the polynomial is divided by x^2-4.

Practice

1. Without using long division, find each remainder:
   (a) $(2x^2+6x+8)\div(x+1)$ (b) $(x^2+4x+12)\div(3x-1)$
2. When the polynomial x^n+x-8 is divided by $x-2$ the remainder is 10. What is the value of n ?
3. Given that $g(x)=(x+2)(3x^2+4)+5$ *and* $h(x)=(6x+1)(3x^3-2x^2+x)+8$. Find the remainder when $g(x)+h(x)$ is divided by $x+1$.
4. The remainders when a polynomial is divided by $x-1$ and $x+3$ are 2 and -6 respectively. Find the remainder when the same polynomial is divided by $(x-1)(x+3)$.
5. Find the remainder when $x^{2012}+x-1$ is divided by $x+1$.
6. Find the value of k for which $x^2+(k-1)x+k^2-16$ is exactly divisible by x-3 but not divisible by x+4.
7. Given that the expression $2x^3+px^2-8x+q$ is exactly divisible by $2x^2-7x+6$,evaluate p and q.
8. The polynomial $2x^3-3ax^2+ax+b$ has a factor x-1 and a remainder of -10 when divided by x+1.Find the values of a and b.
9. Find each quotient and remainder:

(a) $(2x^2+6x+15)\div(x+3)$ (b) $(x^2-4x+13)\div(2x-1)$
(c) $(x^2-x+3)\div(x+2)$ (d) $(2x^4+x^3-24x^2-3x+2)\div(x^2+x-4)$

10. When a certain polynomial is divided by $x+3$, the quotient is x^2-3x+5 and the remainder is 6. What is the polynomial?

Warm- up

1. Divide $8x^4 - 30x^2 + 6x - 3$ by $1 + x + 2x^2$ using long **division** and write the division statement.

2. Consider the function $f(x) = ax^3 + 3x + b + 5$, where a and b are constants and $a \neq 0$ & $b \neq 0$. $f(x)$ has a remainder of 2a when divided by x and a remainder of 2b when divided by $x - 1$. Determine the values of a and b.