# Unit 6: Combinations of Functions

# 6.1 Sum and Difference of functions

Throughout grade 11 and 12, we have expressed functions using function notation.

Ex) $f(x) = 2x + 1$ $\qquad$ $g(x) = 2x^2 - 7x - 4$ $\qquad$ $h(x) = 2\sin(3x)\cos(2x)$

Functions are made of terms separated by **arithmetic operators** $(+, -, \times, \div)$. Because terms can be added, subtracted, multiplied, and divided and functions are made of terms, functions may be added, subtracted, multiplied, and divided.

a) Whenever arithmetic operations are applied to functions, a new function may be created.
   - The new function is called a **combined function**.

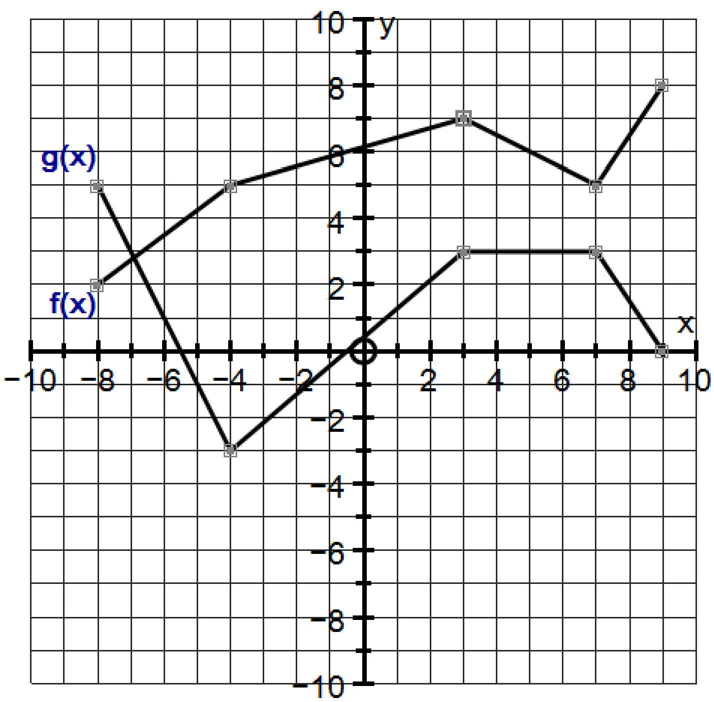
Function Notation: $f(x) + g(x)$ is often represented as $(f+g)(x)$
$f(x) - g(x)$ is often represented as $(f-g)(x)$

$$\boxed{D_{f\pm g} = \qquad\qquad}$$

## The Superposition Principal

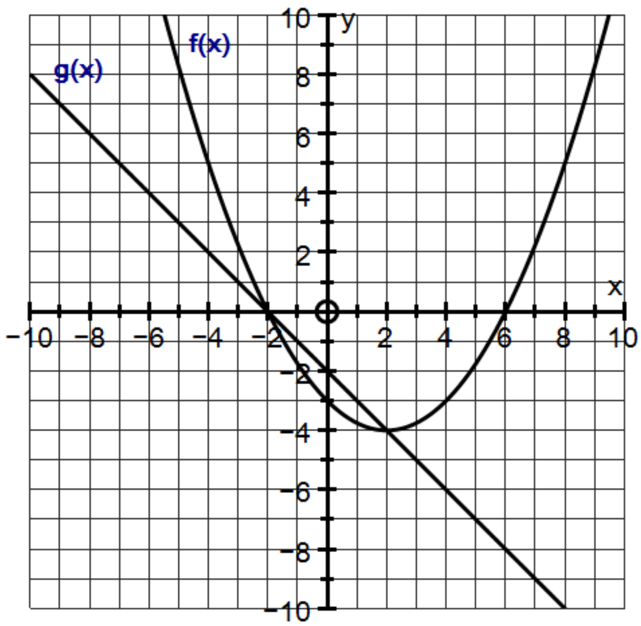
If $f(x)$ and $g(x)$ are functions then the sum of the two functions $\boldsymbol{h}(\boldsymbol{x}) = f(x) + g(x)$ can be found by adding the y-coordinates at each point along the x-axis. The difference of the two functions $(f-g)(x)$ can be found by subtracting the y-coordinates at each point along the x-axis.

Ex 1) Given the following graphs, find the graph of $(f+g)(x)$ and $(f-g)(x)$.




| x | $f(x)$ | $g(x)$ | $(f+g)(x)$ | $(f-g)(x)$ |
|---|---|---|---|---|
| -8 | | | | |
| -7 | | | | |
| -4 | | | | |
| 0 | | | | |
| 3 | | | | |
| 7 | | | | |
| 9 | | | | |

Ex 2) Given the following graphs, find the graph of $h(x) = f(x) + g(x)$




| x | $f(x)$ | $g(x)$ | $(f+g)(x)$ |
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Ex 3) Given $f = \{(-2,6),(-1,8),(0,5),(1,0),(2,-2)\}$ *and* $g = \{(-2,2),(-1,4),(0,-6)\}$

a) State the domain of $f$

b) State the domain of $g$

c) State the domain of $f + g$

d) State the domain of $f - g$

e) List $f + g$

f) List $f - g$

Ex 4) If $f(x) = 2x^2 - 12x + 8$ and $g(x) = x^2 - 5x + 12$

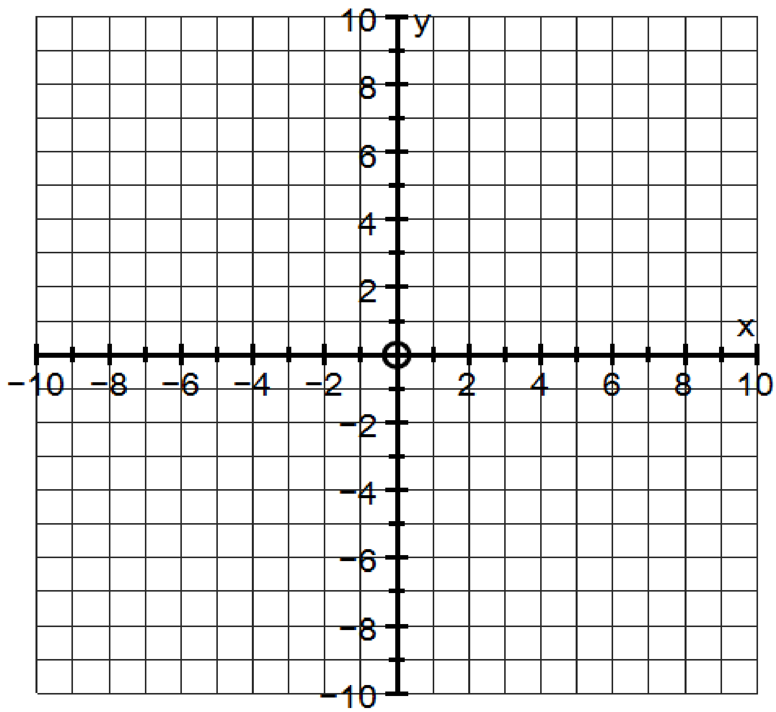
a) find the value(s) of x for which $(f + g)(x) = 0$

b) Find the optimal value of $(f - g)(x)$

Ex 5) If $f(x) = \frac{x}{x-1}$ and $g(x) = \frac{3}{x^2-1}$, find the value of x for which $(f + g)(x) = 1$

# Practice: Addition and Subtraction of Functions of the Same Family

1. Given $f(x) = \frac{1}{4}x^2 - 4$ and $g(x) = -\frac{1}{2}(x-3)^2 + 8$.
   a) Graph the functions on the same set of axes with a graphing calculator and sketch the functions on the given axes with a different colour.
   b) Complete the table below
   c) Determine $h(x) = f(x) + g(x)$
   d) Determine $k(x) = f(x) - g(x)$




| $x$ | $f(x) = \frac{1}{4}x^2 - 4$ | $g(x) = -\frac{1}{2}(x-3)^2 + 8$ | $h(x) = f(x) + g(x)$ | $k(x) = f(x) - g(x)$ |
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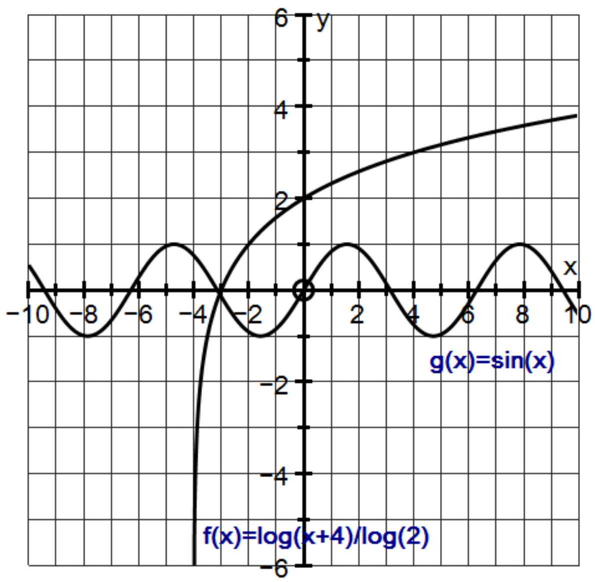
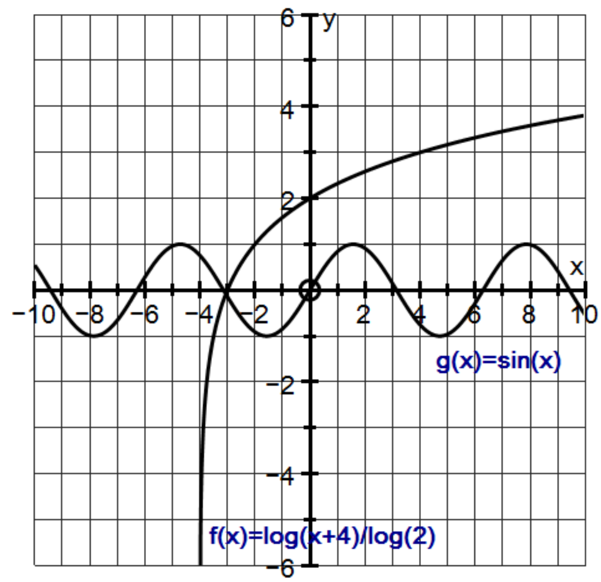
2. Given $f(x) = \log_2(x+4)$ and $g(x) = \sin(x)$.

a) Complete the table below
b) Sketch $h(x) = f(x) + g(x)$ on the grid to the left
c) Sketch $k(x) = f(x) - g(x)$ on the grid to the right




| $x$ | $f(x) = \log_2(x-4)$ | $g(x) = \sin(x)$ | $h(x) = f(x) + g(x)$ | $k(x) = f(x) - g(x)$ |
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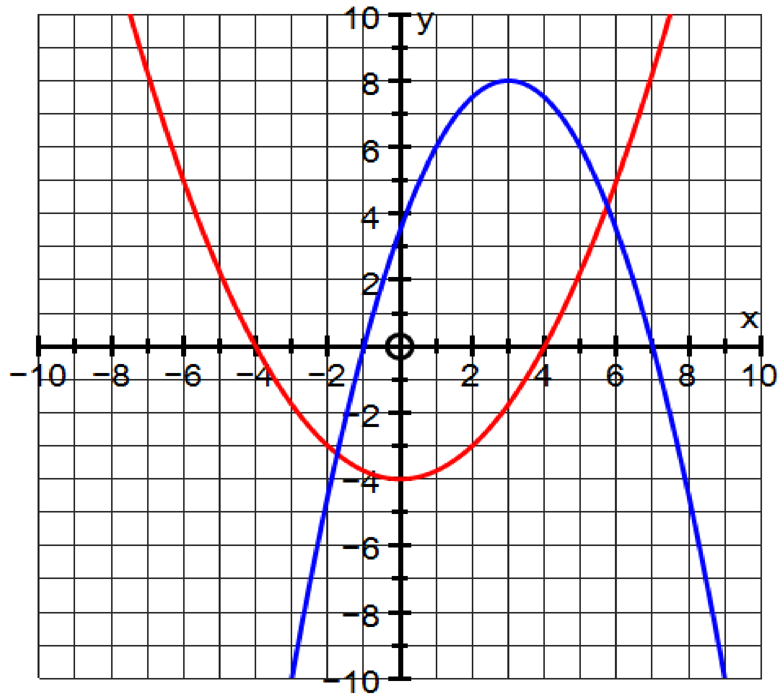
3. Given $f(x) = \frac{1}{4}x^2 - 4$ and $g(x) = -\frac{1}{2}(x-3)^2 + 8$.

a) Complete the table below

b) Determine $h(x) = f(x) + g(x)$

c) Determine $k(x) = f(x) - g(x)$




| $x$ | $f(x) = \frac{1}{4}x^2 - 4$ | $g(x) = -\frac{1}{2}(x-3)^2 + 8$ | $h(x) = f(x) + g(x)$ | $k(x) = f(x) - g(x)$ |
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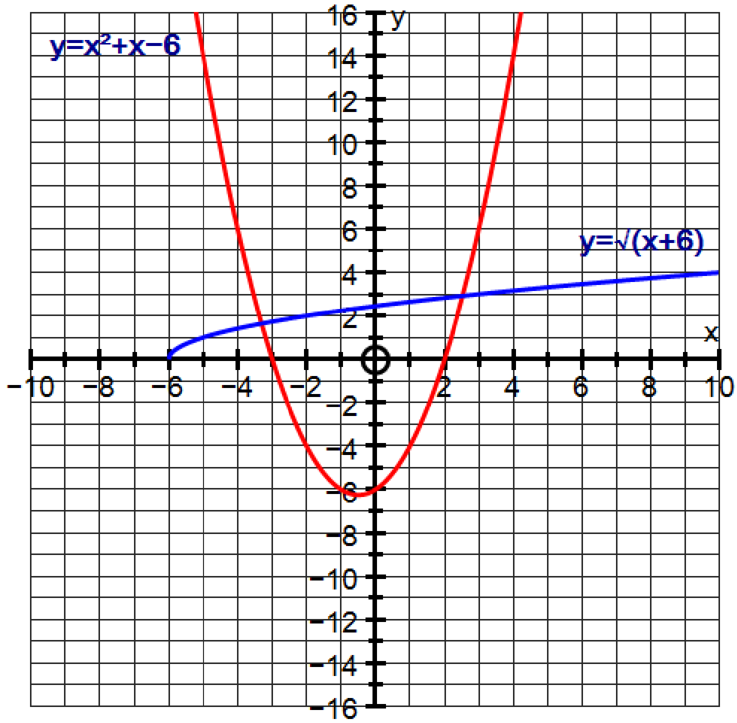
# Warm Up

1. Determine the graph of $h(x) = f(x) + g(x)$, given the graphs of $f(x) = x^2 + x - 6$ and $g(x) = \sqrt{x+6}$ by creating a table of values

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
|---|---|---|---|
| $-6$ | | | |
| $-5$ | | | |
| $-4$ | | | |
| $-3$ | | | |
| $-2$ | | | |
| $-1$ | | | |
| $0$ | | | |
| $1$ | | | |
| $2$ | | | |
| $3$ | | | |




$h(x)$ in un-simplified form:________________________ Domain of $h(x)$:____________________

2. Let $f(x) = mx^2 + 2x + 5$ and $g(x) = 2x^2 - nx - 2$. The functions are combined to form the new function $h(x) = (f - g)(x)$. Points $(1, 10)$ and $(-1,16)$ satisfy the new function. Determine the values of m and n. [m=8, n=-5]

# 6.2 Product and Quotient of two functions

If you are given the functions $f(x)$ and $g(x)$

- the **product of two functions** is defined as $f(x)g(x)$ or $(fg)(x)$
- the **quotient of two functions** is defined as $\frac{f(x)}{g(x)}$ or $\left(\frac{f}{g}\right)(x)$

Similarly, the product of two functions can be found by multiplying the y-coordinates with the same x-coordinates. I.e.)$\left(x, f(x)g(x)\right)$

- **The Domain of $fg$=**

To find the quotient of two functions, we need to divide the y-coordinates with the same x-coordinates.

For the functions $f$ and $g$, $\frac{f}{g}$ is defined by $\frac{f}{g} = \left\{\left(x, \frac{f(x)}{g(x)}\right) \middle| \frac{f(x)}{g(x)} \; is \; defined\right\}$

Note: The quotient of two functions may not be a function due to possible Holes and V.A.(s)

- **The Domain of $\frac{f}{g}$ =**

Note: The product of two functions will remain a function!

Ex 1) Given $f = \{(-3,7), (-2,5), (-1,-4), (2,8)\}$ and $g = \{(-4,6), (-2,3), (-1,0), (1,5), (2,-4)\}$
Find

a) $f + g$

b) $f - g$

c) $fg$

d) $\frac{f}{g}$

e) $\frac{g}{f}$

Ex 2) Let $f(x) = \sqrt{x+6}$ and $g(x) = x^2 + x - 6$

a) Determine the equation of $h(x) = f(x)g(x)$

b) Determine the equation of $k(x) = \frac{g(x)}{f(x)}$

c) Sketch the graphs of the combined functions

d) State the domain and range of $h(x)$ and $k(x)$.

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ | $k(x)$ |
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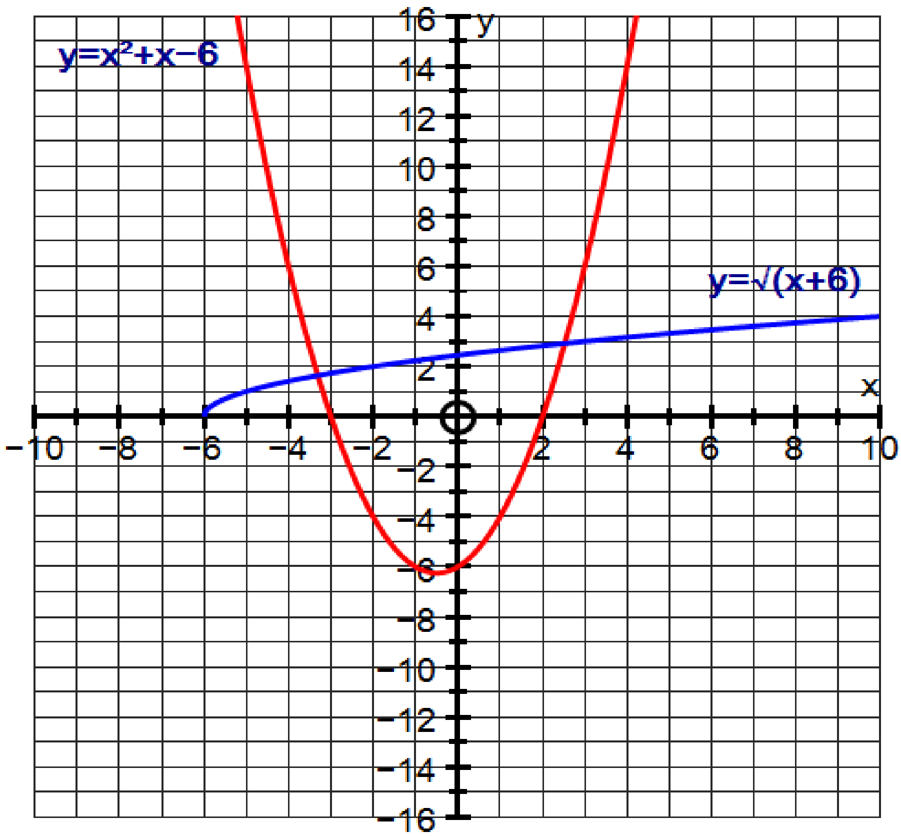


Ex 3) Consider the following table illustrating the product of functions $f(x)$ and $g(x)$. Let the product be defined as $h(x) = f(x)g(x)$ and the quotient be defined as $k(x) = \frac{f(x)}{g(x)}$. Determine if $h(x)$ and $k(x)$ are even or odd in the following scenarios.

| $f(x)$ | $g(x)$ | $f(x)g(x)$ | $\frac{f(x)}{g(x)}$ |
|---|---|---|---|
| Even | Even | | |
| Even | Odd | | |
| Odd | Even | | |
| Odd | Odd | | |

Ex 4) For each of the following pairs of functions, write the equation of $h(x) = \left(\frac{f}{g}\right)(x)$ and state the domain of $h(x)$

1. $f(x) = 10, g(x) = x$
2. $f(x) = 4x, g(x) = 3x - 2$
3. $f(x) = 4x^2, g(x) = x^2 - 4$
4. $f(x) = x^2 - x - 12, g(x) = x^2 + 2x - 3$

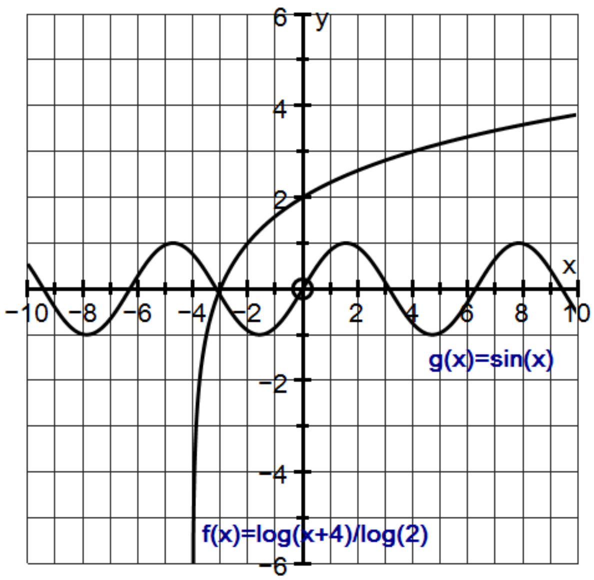
# Practice: Product and Quotient of Functions of Different Families

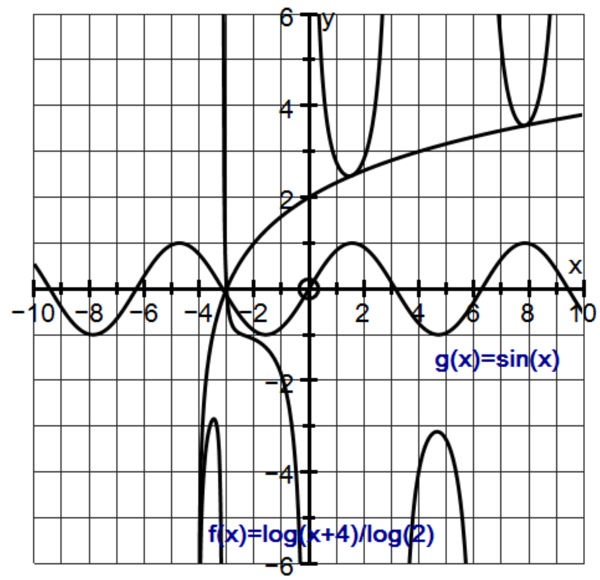
Given $f(x) = \log_2(x+4)$ and $g(x) = \sin(x)$.

a) Complete the table below

b) Sketch $h(x) = f(x)g(x)$ on the grid to the left

Note: The grid to the right illustrates $k(x) = \frac{f(x)}{g(x)}$




| $x$ | $f(x) = \log_2(x+4)$ | $g(x) = \sin(x)$ | $h(x) = f(x)g(x)$ | $k(x) = \frac{f(x)}{g(x)}$ |
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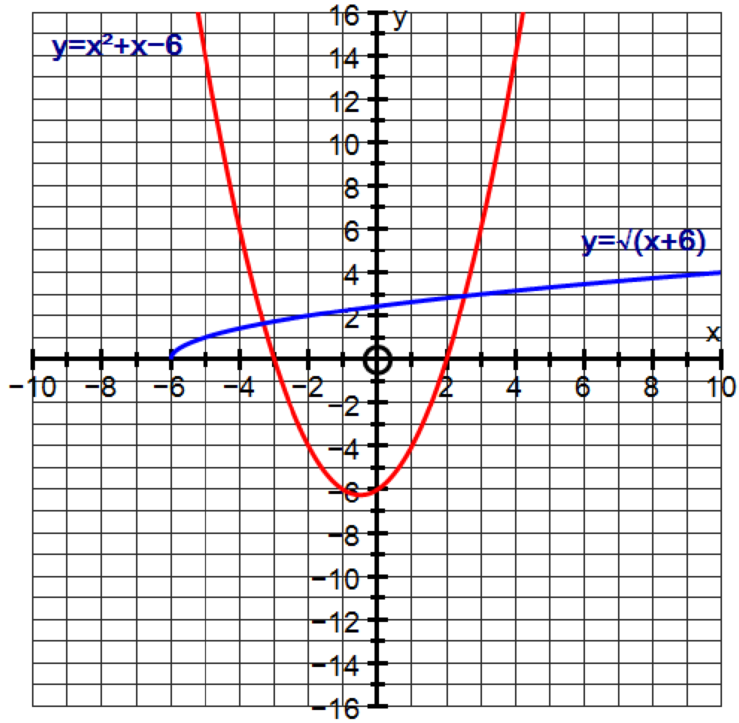
# Warm Up

3. Determine the graph of $h(x) = f(x)g(x)$, given the graphs of $f(x) = x^2 + x - 6$ and $g(x) = \sqrt{x+6}$ by creating a table of values

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
|---|---|---|---|
| $-6$ | | | |
| $-5$ | | | |
| $-4$ | | | |
| $-3$ | | | |
| $-2$ | | | |
| $-1$ | | | |
| $0$ | | | |
| $1$ | | | |
| $2$ | | | |
| $3$ | | | |




$h(x)$ in un-simplified form:________________________ Domain of $h(x)$:___________________

4. Let $f(x) = mx^2 + 2x + 5$ and $g(x) = 2x^2 - nx - 2$. The functions are combined to form the new function $h(x) = (fg)(x)$. Points $(1, -40)$ and $(-1, 24)$ satisfy the new function. Determine the values of m and n. [m=3, n=4]

# 6.3 More Practice with the Quotient of Two Functions

To find the quotient of two functions, we need to divide the y-coordinates with the same x-coordinates.

For the functions $f$ and $g$, $\frac{f}{g}$ is defined by $\frac{f}{g} = \left\{\left(x, \frac{f(x)}{g(x)}\right) \middle| \frac{f(x)}{g(x)} \text{ is defined}\right\}$

Note: The quotient of two functions may not be a function due to possible Holes and V.A.(s)

- **The Domain of $\frac{f}{g}$ = Domain of f ∩ Domain of g, for which $g(x) \neq 0$**
  **= $\{x \epsilon R | x \epsilon D_f \cap D_g, g(x) \neq 0\}$**

Ex 1) Given $f = \{(-5,1), (-3,1), (0,1), (1,1), (2,1), (4,1)\}$ and
$g = \{(-5,6), (-4,5), (-3,4), (0,1), (1,0), (3,-2), (4,-3)\}$

a) Find the domain of $f + g$

b) Find the domain of $f - g$

c) Find the domain of $fg$

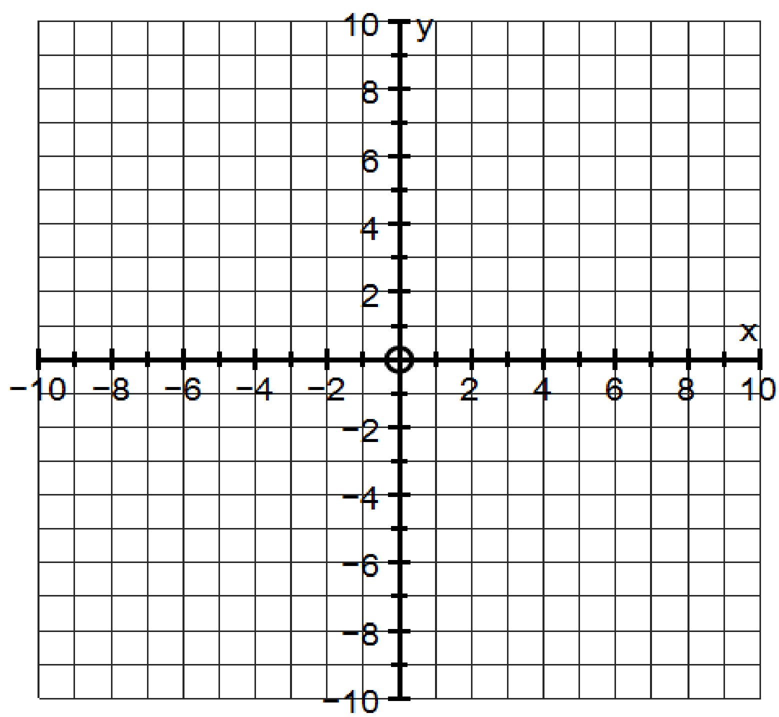
d) Find the domain of $\frac{f}{g}$

e) List $\frac{f}{g}$

Ex 2) Let $f(x) = x + 2$ and $g(x) = x^2 - 3x - 10$.

a) Determine the equation of $h(x) = \frac{f(x)}{g(x)}$

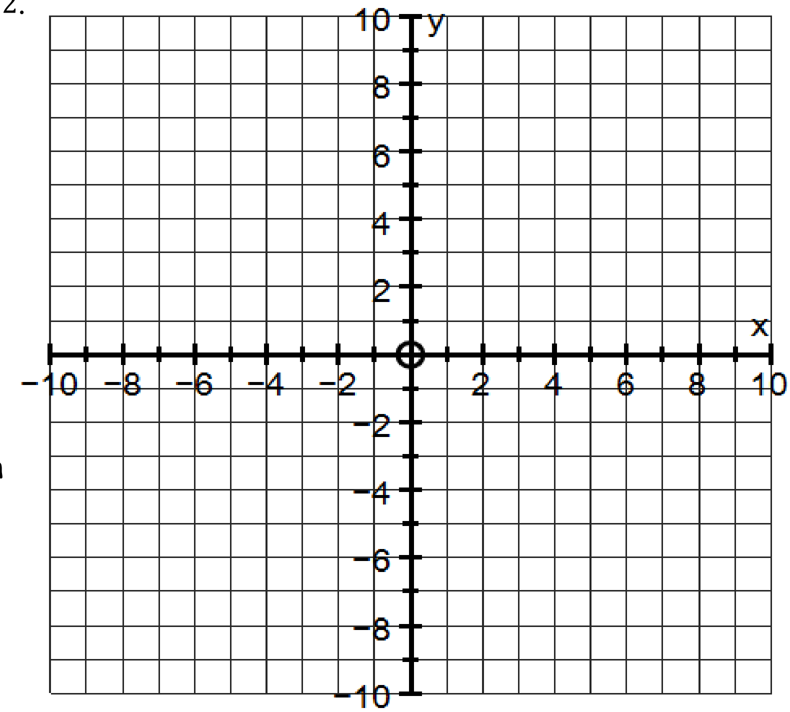
b) Sketch a graph of the combined function




c) State the domain and range of $h(x)$

Ex 3) Let $f(x) = x^3 - 7x - 6$ and $g(x) = x^2 + 3x + 2$.

a) Determine the quotient function $\left(\frac{f}{g}\right)(x)$



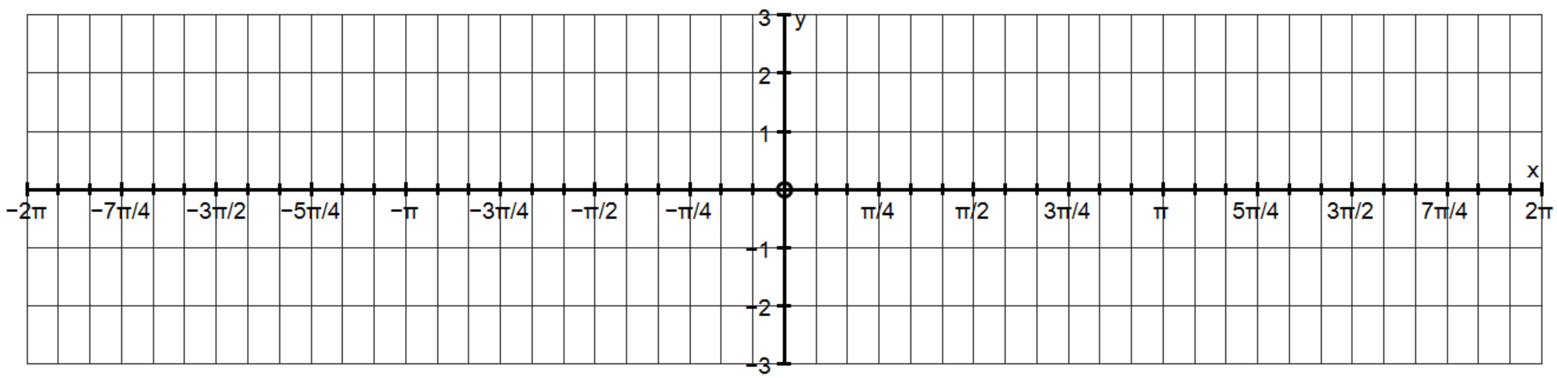
b) Determine the domain of the quotient function

c) Make a sketch of the quotient function

## Practice: Quotient of Functions of Different Families

Given $f(x) = \sin(x)$ and $g(x) = \cos(x)$.

a) Complete the table below
b) Graph the functions on the same set of axes. Be as accurate as possible!
c) Sketch $h(x) = \frac{f(x)}{g(x)}$ on the grid below. Be as accurate as possible!




| $x$ | $f(x) = \sin(x)$ | $g(x) = \cos(x)$ | $h(x) = \frac{f(x)}{g(x)}$ |
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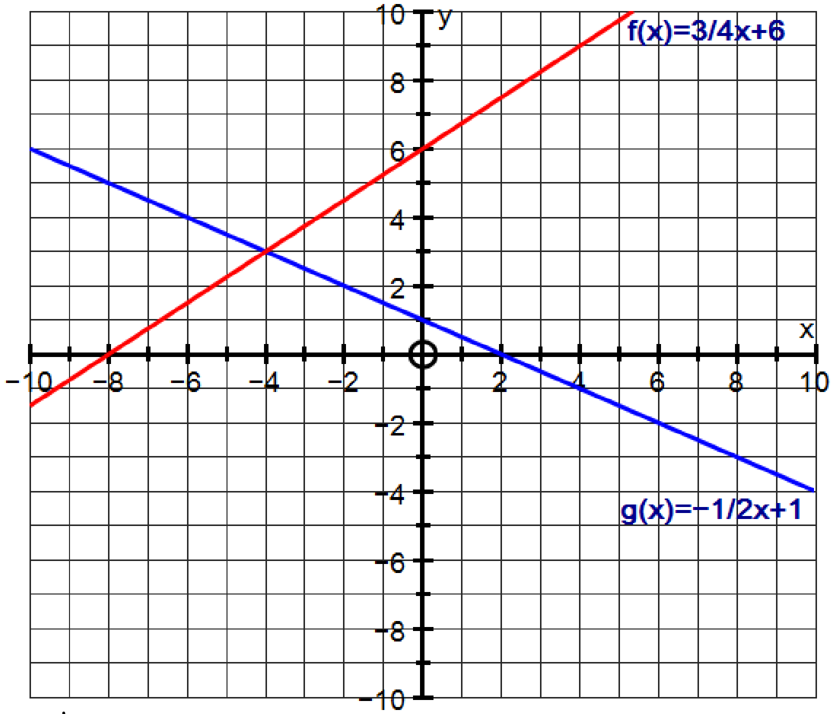
# Warm Up

1. Given the graphs of $f$ and $g$, graph $fg$ and $\frac{f}{g}$ using different colours on the grid to the right.
   Label any V.A. if needed.

| $x$ | $fg(x)$ | $\frac{f}{g}(x)$ |
|---|---|---|
| $-10$ | | |
| $-8$ | | |
| $-6$ | | |
| $-4$ | | |
| $-2$ | | |
| $0$ | | |
| $2$ | | |
| $4$ | | |
| $6$ | | |
| $8$ | | |
| $10$ | | |




2. Using the signs (+ or -) of $f$ and $g$, determine when:
   a. $(fg)(x) > 0$

   b. $(fg)(x) < 0$

   c. $(\frac{f}{g})(x) > 0$

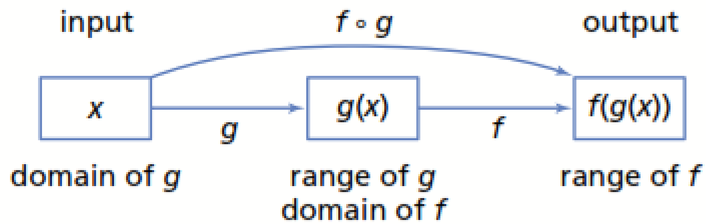
   d. $(\frac{f}{g})(x) < 0$

# 6.4 Compositions of Functions

Let $f(x)$ and $g(x)$ represent two functions. We have learned how to

- **Add** two functions $(f+g)(x)$
- **Subtract** two functions $(f-g)(x)$
- **Multiply** two functions $(fg)(x)$
- **Divide** two functions $\frac{f}{g}(x)$
- Find the I**nverse** of a function $f^{-1}(x)$

Now, we need to learn how to put a function into another function. Say we want to put $g(x)$ into $f(x)$.
This is written as $f\big(g(x)\big)$ or $(f \circ g)(x)$.
$f\big(g(x)\big)$ is an example of a **Composite Function**




**Always remember to put the inner function in the outer function.**

- If we want to put $f(x)$ into $g(x)$ the composition function is written as ___________

**Example.** Let $f(x) = 4 + \sqrt{x^2 + 4x}$ and $g(x) = x - 1$. Determine the equation for each composition Function.

a) $f(g(x))$

b) $g\big(f(x)\big)$

c) $g\big(g(x)\big)$

Find $g^{-1}(x)$

d) $g\big(g^{-1}(x)\big)$

e) $g^{-1}\big(g(x)\big)$

**Method for obtaining inverses**

This method utilizes the fact that

$$f\left(f^{-1}(x)\right)=x\ ,\quad x\in D_{f^{-1}}$$
$$f^{-1}\left(f(x)\right)=x\ ,\quad x\in D_{f}$$

This may seem confusing, but it is a result of the most basic principles of function inverses. Think of a function as some sort of process that we put x through, and it outputs some term. A functions inverse is simply the reverse process. So if we put x through a process, f, then put it through the reverse process, $f^{-1}$, we end up with just x again.

The methodical approach can be summarized in the following steps:

### 1. Replace x with $f^{-1}(x)$ on both sides of the equation.

At this point, you should have a $f(f^{-1}(x))$on one side of your equation and then a function of $f^{-1}(x)$ on the other.

**2. Substitute in $f(f^{-1}(x)) = x$.**

If we take the inverse of a function on the output of the function, f(x), we are left with the input. Therefore, if we take $f(f^{-1}(x))$, we are left with the original input, which is x. Therefore we know $f(f^{-1}(x)) = x$. Remember that, since the two expressions are equal, we can just replace $f(f^{-1}(x))$ with x.

**3. Solve for** $f^{-1}(x)$ **in terms of x.**

At this point, we have an x on one side of the equation, and then a function of $f^{-1}(x)$ on the other side. Try to solve for $f^{-1}(x)$ by getting it by itself on one side of the equation. This will tell you what the inverse function is.

**Examples:** Find the inverses for the following functions.

a) $f(x) = x + 4$

b) $f(x) = (x + 4)^2 - 5$

**Example.** If $(f \circ g)(x)=8x^2-10x+15$ and $f(x)-2x+9$ find $g(x)$.

**Example.** Use the graphs of y = f(x) and y = g(x) to find each of the following compositions.

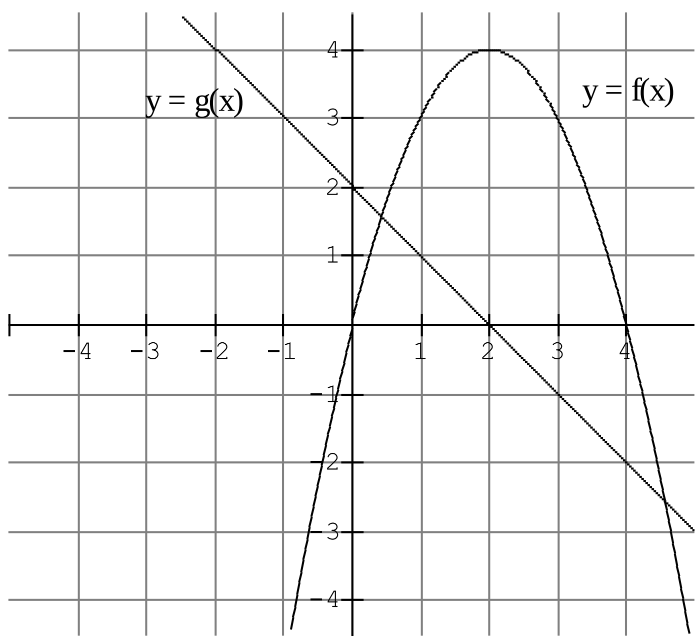
a. $f(g(3))$

b. $g(f(3))$

c. $f(g(0))$

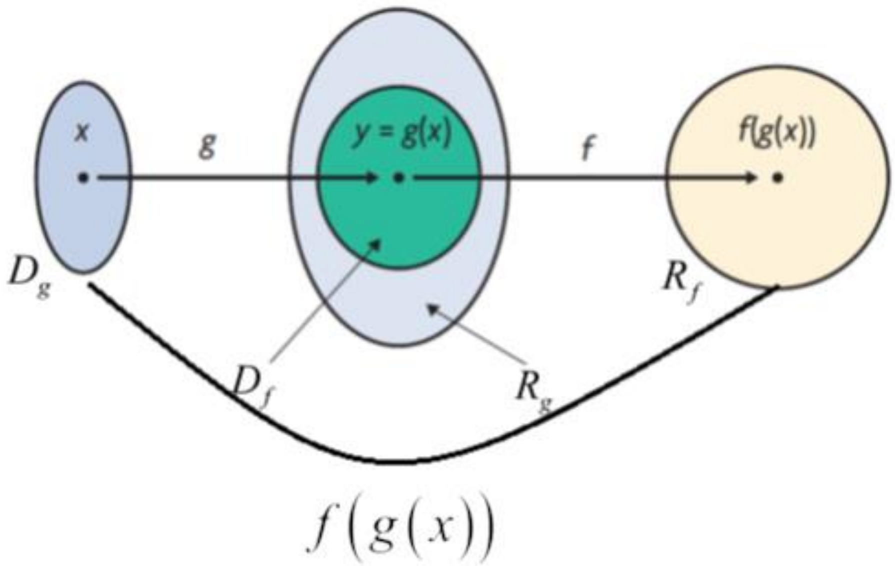
d. $g(f(0))$

e. $g(g(3))$



**Domain of Composite Function Diagram**

Let $(a, b) \epsilon g$ and $(b, c) \epsilon f$. Determine $f(g(a))$




NOTE: To define $(fg)(x)$ ,

$$\boxed{D_f \cap R_g \neq \varnothing}$$

Then

$$\boxed{D_{fog} = \{x \in D_g, g(x) \in D_f\}}$$

**Example** . Given that $f = \{(0,1), (1,3), (2,5), (3,7), (4,9)\}$ and $g = \{(0,0), (1,2), (2,4), (3,6), (4,8)\}$. Determine

a) $(f \circ g)(x)$

b) $(f \circ g)^{-1}(x)$

c) $g(f(x))$

d) $f^{-1}(x)$

e) $(g \circ f^{-1})(x)$

**Example.** Given the following graph of $f(x)$ and $g(x)$. Graph $(f \circ g)(x)$.

| $x$ | $g(x)$ | $f(g(x))$ |
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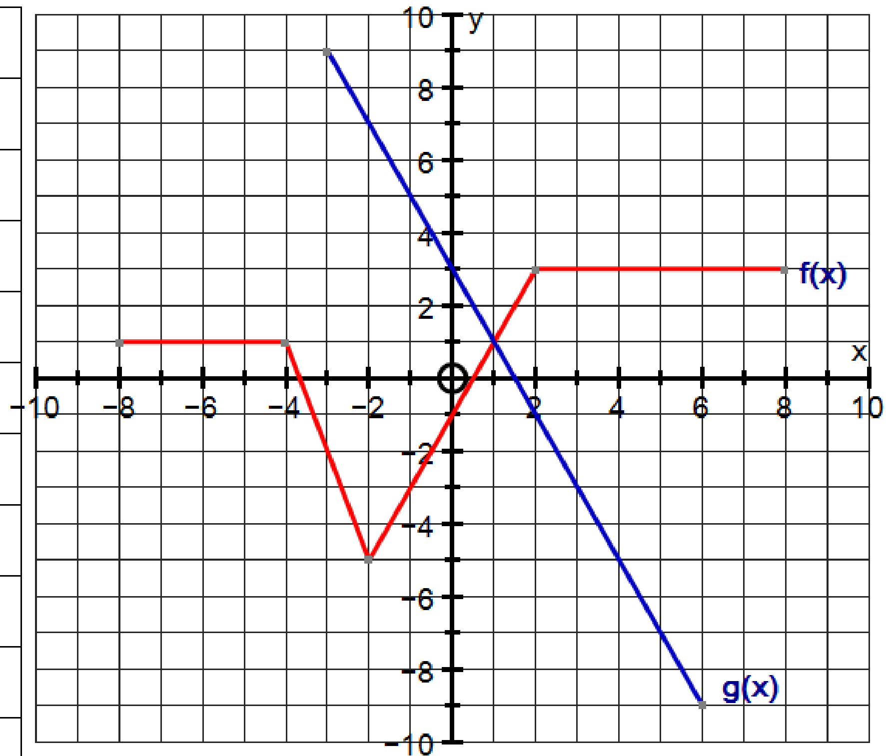
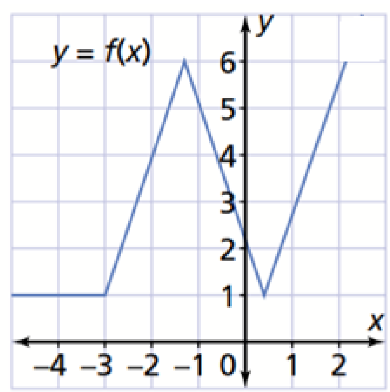



**Example**. Given the graph of y = f (x) on the right and the functions g(x) = 2x +1, h(x) =-x+3, and $k(x) = (g \circ f \circ h)(x)$,




(a) evaluate k(2)

(b) graph $(g \circ h \circ f)(x)$.

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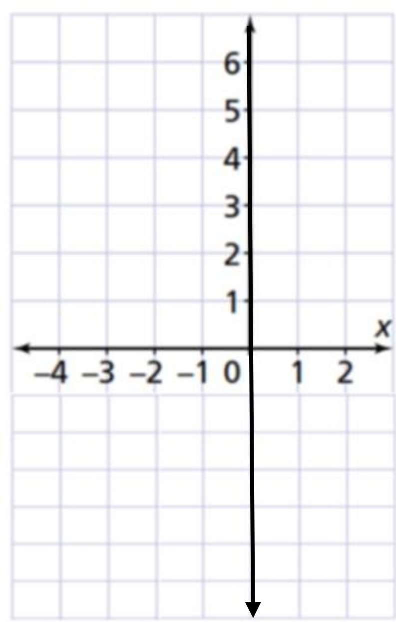


**Example.** Express h as the composition of two functions f and g, such that $h(x) = f(g(x))$.

a. $h(x) = 3^{2x^2-1}$

b. $h(x) = x^4 + 5x^2 + 6$

c. $h(x) = \dfrac{2x^2 - 1}{x^2}$

d. $h(x) = \dfrac{1}{x-4}$

## Composite Functions

# Exercise 6.4

### Part A

1. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find the following:

   a. $f(g(1))$ b. $g(f(1))$ c. $g(f(0))$

   d. $f(g(-4))$ e. $f(g(x))$ f. $g(f(x))$

2. For each of the following pairs of functions, find the composite functions $f \circ g$ and $g \circ f$. What is the domain of each composite function? Are the composite functions equal?

   a. $f(x) = x^2$, $g(x) = \sqrt{x}$

   b. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 1$

   c. $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x + 2}$

### Part B

3. Use the functions $f(x) = 3x + 1$, $g(x) = x^3$, $h(x) = \frac{1}{x + 1}$, and $u(x) = \sqrt{x}$ to find expressions for the indicated composite function.

   a. $f \circ u$ b. $u \circ h$ c. $g \circ f$

   d. $u \circ g$ e. $h \circ u$ f. $f \circ g$

   g. $h \circ (f \circ u)$ h. $(f \circ g) \circ u$ i. $g \circ (h \circ u)$

4. Express $h$ as the composition of two functions $f$ and $g$, such that $h(x) = f(g(x))$.

   a. $h(x) = (2x^2 - 1)^4$ b. $h(x) = \sqrt{5x - 1}$

   c. $h(x) = \frac{1}{x - 4}$ d. $h(x) = (2 - 3x)^{\frac{5}{2}}$

   e. $h(x) = x^4 + 5x^2 + 6$ f. $h(x) = (x + 1)^2 - 9(x + 1)$

5. If $f(x) = \sqrt{2 - x}$ and $f(g(x)) = \sqrt{2 - x^3}$, then what is $g(x)$?

6. If $g(x) = \sqrt{x}$ and $f(g(x)) = (\sqrt{x} + 7)^2$, then what is $f(x)$?

7. Let $g(x) = x - 3$. Find a function $f$ so that $f(g(x)) = x^2$.

8. Let $f(x) = x^2$. Find a function $g$ so that $f(g(x)) = x^2 + 8x + 16$.

9. Let $f(x) = x + 4$ and $g(x) = (x - 2)^2$. Find a function $u$ so that $f(g(u(x))) = 4x^2 - 8x + 8$.

10. If $f(x) = \frac{1}{1 - x}$ and $g(x) = 1 - x$, determine

    a. $g(f(x))$ b. $f(g(x))$

11. If $f(x) = 3x + 5$ and $g(x) = x^2 + 2x - 3$, determine $x$ such that $f(g(x)) = g(f(x))$.

12. If $f(x) = 2x - 7$ and $g(x) = 5 - 2x$,

    a. determine $f \circ f^{-1}$ and $f^{-1} \circ f$.

    b. show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

**Exercise**

**1. a.** $0$ **b.** $0$ **c.** $-1$ **d.** $\sqrt{15}$ **e.** $\sqrt{x^2 - 1}$ **f.** $x - 1$

**2. a.** $f(g(x)) = x, x \geq 0$; $g(f(x)) = |x|, x \in R$; $f \circ g \neq g \circ f$

**b.** $f(g(x)) = \frac{1}{x^2 + 1}, x \in R$; $g(f(x)) = \frac{1}{x^2} + 1, x \neq 0$; $f \circ g \neq g \circ f$

**c.** $f(g(x)) = \frac{1}{\sqrt{x + 2}}, x > -2$; $g(f(x)) = \sqrt{\frac{1 + 2x}{x}}, x < -\frac{1}{2}$ or $x > 0$; $f \circ g \neq g \circ f$

**3. a.** $3\sqrt{x} + 1$ **b.** $\frac{1}{\sqrt{x + 1}}$ **c.** $(3x + 1)^3$

**d.** $\sqrt{x^3}$ **e.** $\frac{1}{\sqrt{x} + 1}$ **f.** $3x^3 + 1$ **g.** $\frac{1}{3\sqrt{x} + 2}$ **h.** $3x\sqrt{x} + 1$

**i.** $\frac{1}{(\sqrt{x} + 1)^3}$

**4. a.** $f(x) = x^4$, $g(x) = 2x^2 - 1$ **b.** $f(x) = \sqrt{x}$, $g(x) = 5x - 1$

**c.** $f(x) = \frac{1}{x}$, $g(x) = x - 4$ **d.** $f(x) = x^{\frac{5}{2}}$, $g(x) = 2 - 3x$

**e.** $f(x) = x(x + 1)$, $g(x) = x^2 + 2$

**f.** $f(x) = x^2 - 9x$, $g(x) = x + 1$

**5.** $g(x) = x^3$

**6.** $f(x) = (x + 7)^2$

**7.** $f(x) = (x + 3)^2$

**8.** $g(x) = x + 4$ or $g(x) = -x - 4$

**9.** $u(x) = 2x$ or $u(x) = -2x + 4$

**10. a.** $\frac{x}{x - 1}$ **b.** $\frac{1}{x}$ **11.** $-2, -3$ **12. a.** $x$

1. Given the following functions, determine the following in simplest **exact** form.

$f(x)=\{(1,2),(2,3),(3,5),(5,7)\}$ $\quad$ $g(x)=\{(1,4),(3,7),(5,9),(9,2)\}$

$h(x)=-2\cos(2x+\pi)+1$ $\quad$ $m(x)=x^2+1$

a. $\left(f^{-1}+g\right)(x)$

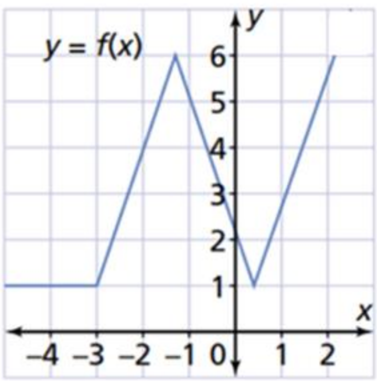
b. $(f\times g)(3)$

c. $(g\circ h)(\pi)$

d. $\left(\frac{f}{m}\right)(x)$

2. Given the graph of y = f (x) on the right and the functions $\boldsymbol{g(x) = 2x+1}$, $\boldsymbol{h(x) = -x+3}$, and $\boldsymbol{k(x) = (g\circ f\circ h)(x)}$

(a) evaluate $k(2)$




(b) graph $(g\circ h\circ f)(x)$.

| $x$ | $(g\circ h\circ f)(x)$ |
|---|---|
| | |
| | |
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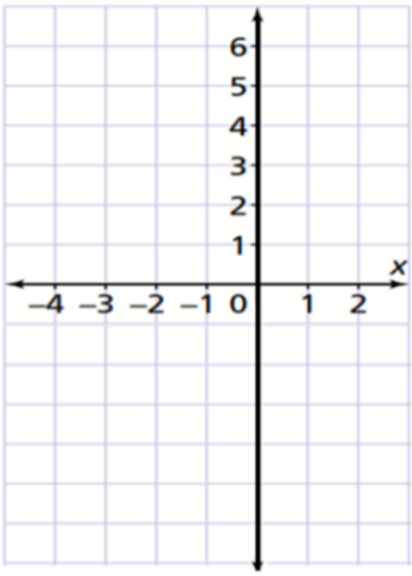
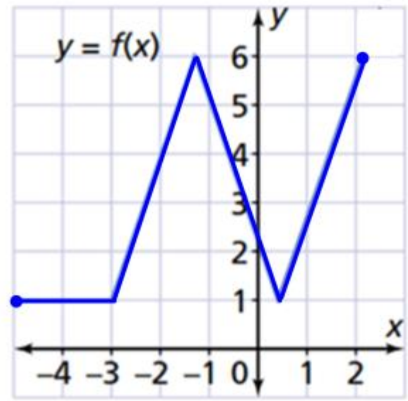


3. Given the graph $y = k(x)$ below and the functions $g(x) = 2^{x-1}$, $h(x) = \log_2(x^2+1)$, $m(x) = \sqrt{16-x^2}$ ,graph of $y = f(x)$ below, $k(x) = \dfrac{(g \circ f \circ h)(x)}{h(x)}$ and $n(x) = (f \circ m \circ f)(x)$ determine the following:




a. $k(2)$

b. $n(-2)$

# 6.5 Solving Inequalities of Combined Functions

Recall: An **inequality** is a mathematical statement that contains one of the following symbols:

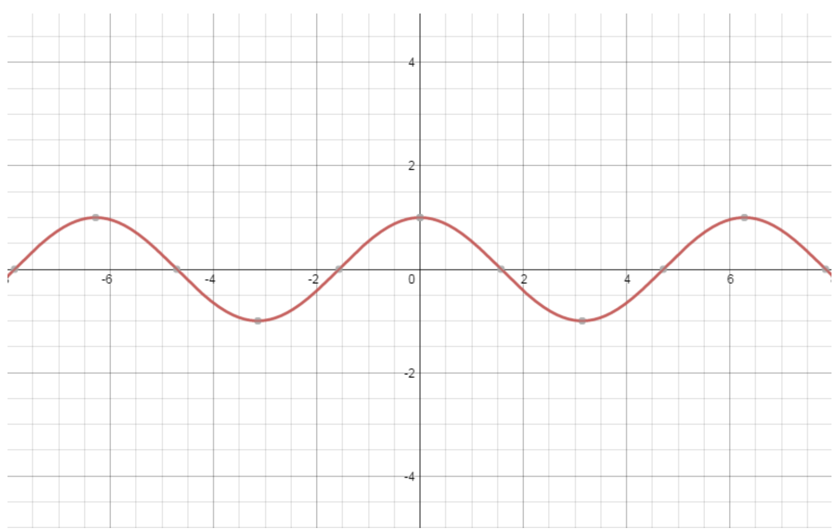
$$>, \geq, <, \leq, \neq$$

There are two main strategies used to solve an inequality involving $f(x)$ and $g(x)$

1. Inequality related to **<u>Zeros</u>**
   - The **chart method**
     - Used whenever you can factor $f(x) - g(x)$ or $g(x) - f(x)$
2. Inequality related to **<u>Points of Intersection</u>**
   - The **graphing method**
     - Used whenever you cannot factor the above but you can still graph $f(x)$ and $g(x)$

Ex.1: Given $f(x) = x^3 - 2x^2 + 5x + 20$ and $g(x) = 2x^2 + 14x - 16$. Solve $f(x) \geq g(x)$ .

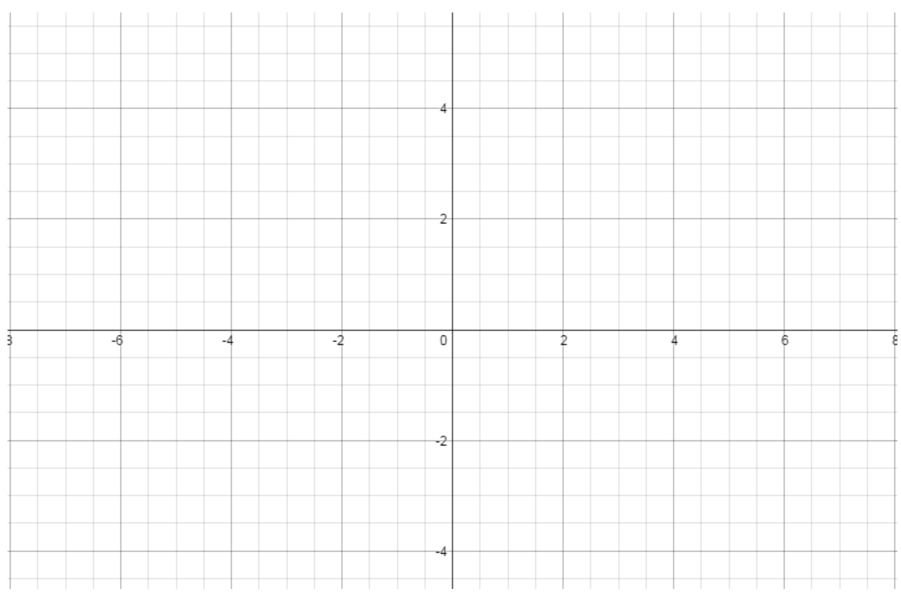
Ex.2: Given $f(x) = \cos(x)$ and $g(x) = x$. Determine all values of x such that $f(x) > g(x)$ .



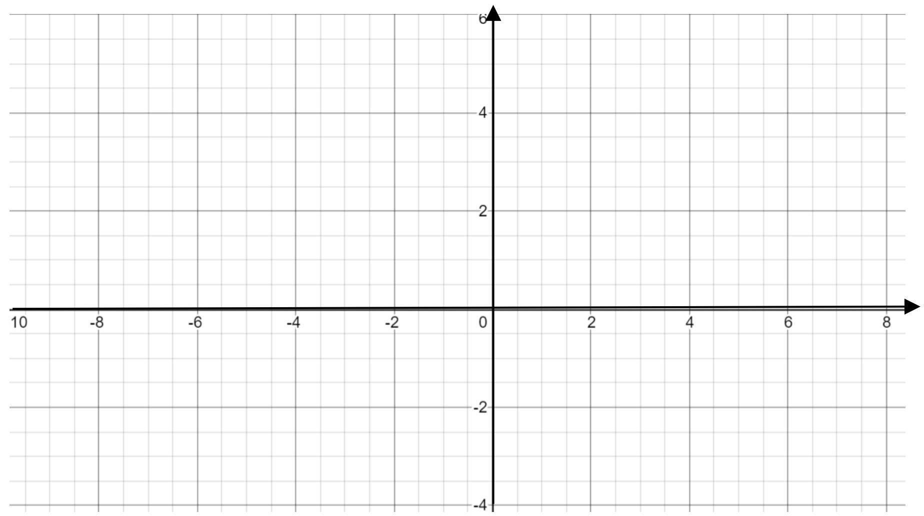
Ex.3: Given $f(x)=\dfrac{3x}{x+8}$ and $g(x)=\dfrac{x}{x-6}$ , determine all values of x such that $f(x) < g(x)$.

Ex.4: Given $f(x) = x^2 + 4x - 5$ and $g(x) = x + 1$. Determine all values of x such that $f(x) > g(x)$ .

Ex.5: Given $f(x) = \log_2(x^2)$ and $g(x)=\dfrac{1}{x^2}$ determine all values of x such that $f(x) > g(x)$.



Ex.6: Given $f(x) = \sqrt{2 - x}$ and $g(x) = 2\sqrt{x} - 1$, sketch the graphs of f and g on the same set of axes. Using the graphs of these functions, solve $f(x) > g(x)$.



# Group work

**Multiple Choice**: Write the CAPITAL LETTER corresponding to the most correct answer on the line provided.

1. If $\left(\frac{m}{n}\right)(x)=\frac{x+3}{2x+1}$ and $n(x)=2x^2-5x-3$, the function that defines $m(x)$ is: _______

   A) $m(x)=x^2+6x-9$ B) $m(x)=x+3$ C) $m(x)=x^2-9$
   D) $m(x)=x-3$ E) None of the above

2. If $D_f=\{x\,|\,x\in\mathbb{R}\}$ and $D_g=\{x\,|\,x\geq 0, x\in\mathbb{R}\}$, then which of the following is correct? _______

   A) $D_{f+g}=\{x\,|\,x\in\mathbb{R}\}$ B) $D_{f+g}=\{x\,|\,x\geq 0, x\in\mathbb{R}\}$ C) $D_{fg}=\{x\,|\,x\geq 0, x\in\mathbb{R}\}$ D) $D_{f-g}=\{x\,|\,x\in\mathbb{R}\}$ E) Both B and C

3. If $f(x)$=2$x$-2 , $g(x)$ = $x$+4 and $h(x)$ = $f(x)$-$g(x)$ then which of the following statements is true? _______

   A) The y-intercept of $h(x)$ is 2 B) $f(x)$, $g(x)$ and $h(x)$ all intersect at the same point.

   C) $h(x)=f\left(f^{-1}(x)\right)-g(-10)$ D) $h(3)=-3$ E) Both C and D

4. Given $f(x)=3x^2$ and $g(x)=3x^4-x^2-5$, which of the following statements are true? _______

   A) $\left(\frac{g}{f}\right)(x)$ is quadratic B) $(f - g)(x)$ is quadratic C) $(f + g)(x)$ is cubic
   D) $(fog)(x)$ is an even function E) $(f - g)(x)$ is an odd function

**Short Answer**

5. Given $f(x)=11x^5$ and $g(x)=22x^2\log(x)$ determine :

   a) the domain of $\left(\frac{f}{g}\right)(x)$ b) equation of $\left(\frac{f}{g}\right)(x)$

6. Given the following functions, determine the following in simplest form:
   $f(x)=\{(1,2),(2,3),(3,5),(9,7)\}$ $g(x)=\{(1,4),(3,7),(5,9),(9,2)\}$
   $h(x)=\sqrt{x-4}$ $m(x)=\sqrt{x-3}$

   a) $(f+g)(x)$ b) $(fg)(3)$

   c) $D_{h+m}$ d) $D_{\frac{h}{m}}$

7. Given the functions $f(x)=2x^2-4x+3$ and $g(x)=3x^2-6x-4$ then determine the range of $(f-g)(x)$ .

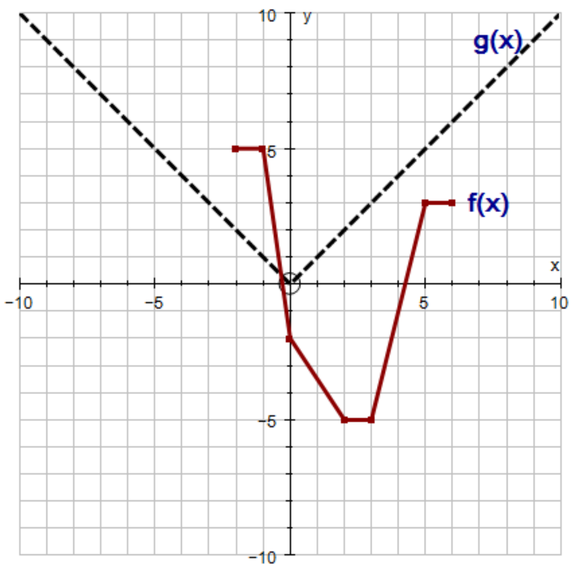
8. Determine two functions, f and g, whose product would result in $(fg)(x)=8\sin^3 x+27$ .

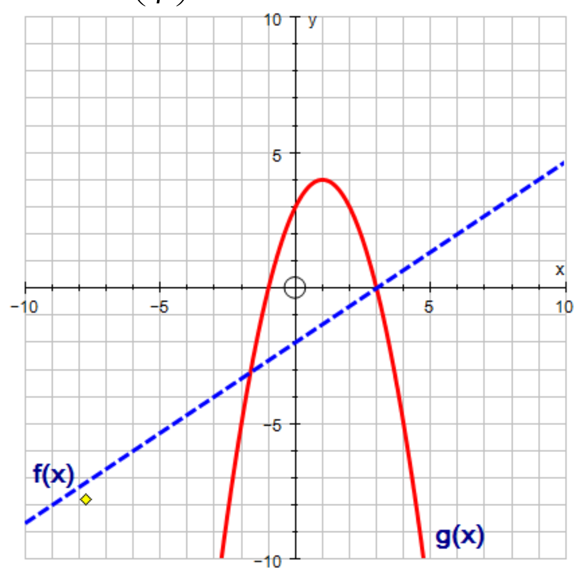
9. Given $f(x)=9x^4+6x^3+270$ and $g(x)=9x^4+51x^2+57x$ solve $f(x)\geq g(x)$.

10. Given the graphs below, sketch each of the following on the same set of axes provided.

a. $(f+g)(x)$

b. $\left(\frac{g}{f}\right)(x)$




11. If $f(x)=2x+3$ and $k(x)=2x^2+6x+9$, find $g(x)$ so that $(f\circ g)(x)=k(x)$.

12. Complete the table below.

| Point on f(x) | Point on g(x) | Point on $(f\circ g)$(x) |
|---|---|---|
| | (3,4) | (3,1) |
| (1,5) | (2,1) | |
| (0,-1) | (1,__) | |
| (__,5) | (__,1) | (0,__) |
| (3,__) | | (-1,3) |

13. Using the functions listed below sketch the graph of $R(x)=\left(\frac{f}{g}\cdot h\cdot P\right)(x)$.

$f(x)=x^2+3x-40$ $\quad$ $g(x)=-3x^2+12x+15$

$h(x)=3x^3+3x^2$ $\quad$ $K(x)=2x$

$M(x)=4x+4$ $\quad$ $P(x)=(M\circ K)(x)$

14. Given the information below determine the range of $(f+g)(x)$ .

$f(x)=2(x+7)^2-4$ $\quad$ $D_f=\{x|x\leq -9, x\in\mathbb{R}\}$

$g(x)=x+5$ $\quad$ $D_g=\{x|x\geq -12, x\in\mathbb{Z}\}$

# Unit 6 Review

1. Given that

   $f(x) = x^2 + 2x - 8$ $\qquad$ $g(x) = x^2 - 3x + 2$ $\qquad$ $h(x) = x^3 + 3x^2 - x - 3$

   $l(x) = \{(-5,-3), (-2,-4), (-1,2), (0,4), (3,5)\}$ $\qquad$ $m(x) = \{(-3,5), (-2,-1), (-1,3), (5,0)\}$

Find

a. $f(x) + h(x)$
b. $f(x) - g(x)$
c. $f^{-1}(x)$
d. $\frac{h(x)}{g(x)}$
e. $(f \circ g)(x)$
f. $(lm)(x)$
g. $D_{\frac{l}{m}}$
h. $(l \circ m)^{-1}(x)$
i. $l^{-1}\big(m(x)\big)$

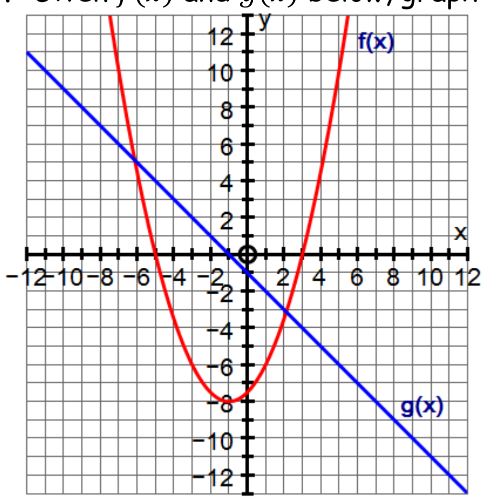
2. Given $f(x) = 2x - 7$, $h(x) = \{(-1,3), (0,4), (1,7), (2,5), (3,-2)\}$, and $k(x) = \{(-4,1), (-3,5), (-2,3), (1,6)\}]$

Find

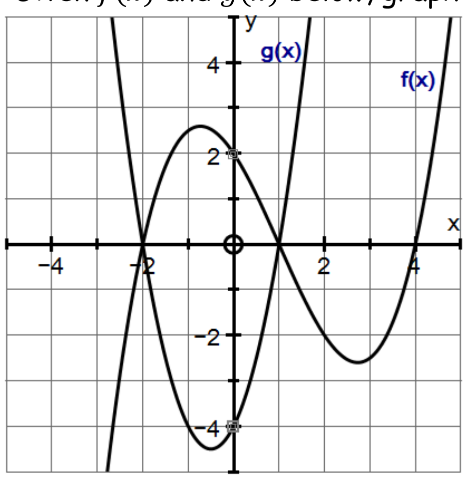
a. $(h \circ k)(x)$ $\qquad$ b. $(k \circ h)(x)$ $\qquad$ c. $(f \circ h)(x)$ $\qquad$ d. $(k \circ f)(x)$

3. If $D_f$: $x\epsilon(-\infty,-3) \cup (2,5) \cup (7,\infty), x\epsilon R$ and $D_g$: $x\epsilon(-8,-2) \cup [0,3] \cup [5,15], x\epsilon R$ then determine $D_{f+g}$.

4. Given $f(x)$ and $g(x)$ below, graph $(fg)(x)$ using points. Also, determine the equation of $(fg)(x)$.




5. Given $f(x)$ and $g(x)$ below, graph $\frac{f(x)}{g(x)}$ by determining its equation.




Using the graph

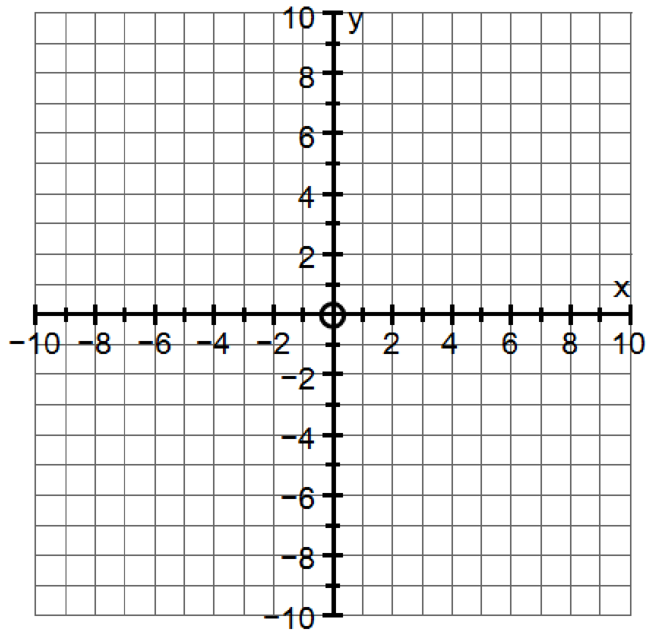
a) How would you determine the location of the VA(s) of $\frac{f(x)}{g(x)}$.

b) How would you determine the location of the Hole(s) of $\frac{f(x)}{g(x)}$.

6. Solve $\frac{x+1}{x-3} \geq \frac{4x+9}{x^2-2x-3}$ by method of zeros (by completing a chart). $[x\epsilon(-\infty,-2] \cup (-1,3) \cup [4,\infty), x\epsilon R]$

7. Solve $\frac{2}{x-4} \geq x-3$ by method of points of intersection (by graphing). $[x\epsilon(-\infty,2] \cup (4,5], x\epsilon R]$

**Points of Intersection Work:**




8. Find the domain of $f(g(x))$ if $f(x) = \frac{3x-4}{2x-6}$ and $g(x) = x^2 - 6x + 8$. $[\{x\epsilon R | x \neq 1, 5\}]$

9. Find $g(x)$ if $f(g(x)) = h(x)$ where $f(x) = x^2 - 6x + 2$ and $h(x) = 9x^2 - 24x + 9$. $[g(x) = 3x - 1 \; or \; g(x) = -3x + 7]$