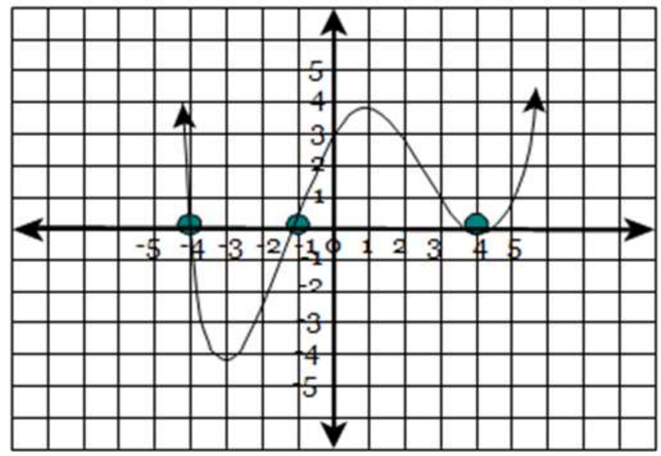
Unit 1: Polynomial Functions
1.10 Solve Inequalities

PART A: Solve Factorable Polynomial Inequalities Graphically
The solutions to polynomial inequalities are intervals or sets of numbers that are subset of the domain of the corresponding function.

Examples:

1. For the function on the right state when
   i) $f(x) > 0$



   ii) $f(x) < 0$

2. Use a **graphing calculator** to solve each of the following polynomial inequalities.

   a) $f(x) = x^3 - x^2 + 3x - 9$, solve $f(x) \geq 0$ ____________________

   b) $f(x) = x^3 + 5x^2 + 3x - 9$, solve $f(x) < 0$ ____________________

   $f(x) > 0$ ____________________

   c) $f(x) = x^4 - 1$, solve $f(x) \leq 0$ ____________________

   d) $f(x) = x(x+2)^2(x-3)$, solve $f(x) > 0$ ____________________

   $f(x) \geq 0$ ____________________

PART B: Solve Factorable Polynomial Inequalities Algebraically

Method:

1. Rearrange inequality so that the right side is 0.
2. Find the zeros(or x-intercepts) of the polynomial.
3. Draw a number line representing the x-axis and label the zero(s) (or x-intercepts).

4. Pick a test value(your choice!) between the zero(s) to determine if the interval or region is positive(+) or negative(-) .
   - positive(+) : y-values are positive for all the x in the interval (Graph is above x-axis)
   - negative(-) : y-values are negative for all the x in the interval (Graph is above x-axis)

NOTE: May sketch function to determine positive or negative intervals.

5. State the solution to the inequality given.
   If $f(x) > 0$, positive interval(s) are required only
   If $f(x) \geq 0$, positive(including zeros) interval(s) are required only
   If $f(x) < 0$, negative interval(s) are required only
   If $f(x) \leq 0$, negative(including zeros) interval(s) are required only

Example#1: Solve each of the following, $x \in R$

a) $x^2 - 3x > 10$

b) $x^3 + 4x^2 + x - 6 < 0$

c) $125 - 8x^3 \leqslant 0$

d) $(3x-1)^5(x+5)^7 - (3x-1)^4(x+5)^8 > 0$

e) $(4-x^2)(x^2-3x+2)<0$

f) $x^2+1>0$

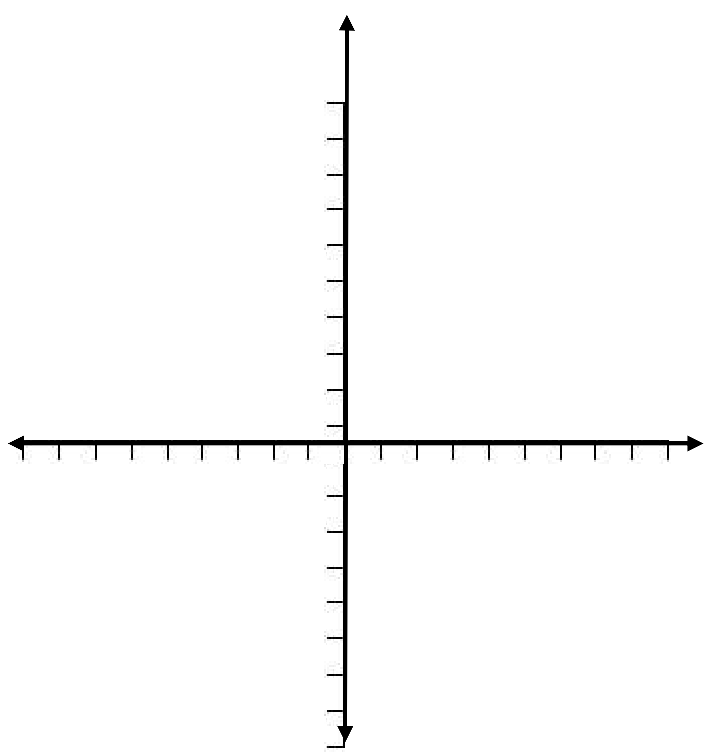
g) $x^4+2x^3-4x^2>8x$

h) $x^3+3x^2+x+3\leq 0$

Example #2: Laurie and Dave play on an Ultimate Frisbee team. On a windy day, and throwing against the wind, the height, in metres, of the Frisbee, t seconds after it leaves Laurie's hand, is determined by the function $h(t) = -t^3 + 2t^2 + t - 2$. How many seconds after it is thrown must Dave catch the Frisbee to ensure that it does not hit the ground?

Example#3 : Determine the equation of a quartic function $f(x)$ that satisfies the following conditions:

- $f(x) \geq 0$ when $x \in (-2, 5)$
- $f(x) < 0$ when $x \in (-\infty, -2) \cup (5, \infty)$
- $f(x)$ has a root of order 2 at x= 2
- $f(x)$ has a maximum point at (4 ,10).



Practice

1. Solve the following polynomial inequalities, $x \in \mathbb{R}$.

   a) $x^2 - 4x + 3 < 0$
   b) $x^3 - 3x - 2 \geqslant 0$
   c) $x^4 - 1 \geqslant 0$
   d) $-x^2 + 3x + 1 < 0$
   e) $-2x^4 - 2x^3 + 16x^2 + 24x < 0$
   f) $2(x+3)(x-1)^2(x-5) \leqslant 0$
   g) $-3(x+4)(x-3)^3 > 0$
   h) $x^4 < 22x^2 + 75$
   i) $2x^2 - 2x \geqslant 2 - x$

2. Let $f(x) = -2x + 1, g(x) = x^2 - 2x + 1$ and $h(x) = x^3 - 1$. Determine all values of x such that $f(x) < g(x) < h(x)$ and illustrate the situation graphically.

3. The number *n* (**in hundreds**), of mosquitoes in a camping area after *t* weeks can be modelled by the equation $n(t) = 2t^4 - 5t^3 - 16t^2 + 45t$ .According to this model, when will the population of mosquitoes be greater than 1800?

4. A zoo wishes to construct an aquarium in the shape of a rectangular prism such that the length is twice the width and 5 m greater than the height. If the aquarium must have a volume strictly between 1125 m^3 and 3000 m^3, determine the restrictions on the length of the aquarium.

5. Determine the equation of a quintic function $f(x)$ that satisfies the following conditions:

   - $f(-3) = f(0) = f(4) = 0$
   - $f(1) = -9$
   - $f(x) > 0$ when $x < -3$ or $-3 < x < 0$
   - $f(x) < 0$ when $0 < x < 4$ or $x > 4$

   Illustrate the situation graphically.

6. The solution to $x^2 + bx + 24 < 0$ is the set of all values of x such that $k < x < k + 2$ for some real value of k. Determine all possible values of b, **b**$\in \mathbb{R}$. Justify your answer.

7. A quartic function has turning points at $(-3,0)$,$(1,0)$, and $(-1,-16)$. Determine all values of x such that $-9 < f(x) < 0$.

Warm up

1. Solve the inequality $x^3 - x^2 - 4x + 4 > 0$ graphically.

2. Solve the inequality $-x^3 + 7x^2 - 48 \le 0$ algebraically.