

Formative Quiz – Exponential and Logarithmic Functions

Part A - Multiple Choice : Place the CAPITAL letter of the correct answer on the line provided. /10-K

1. State the domain and range of $f(x) = \frac{2}{3}\log_2(-4x+12)+3$

A) $D = \{x \in \mathbb{R} \mid x > 3\}$
 $R = \{y \in \mathbb{R}\}$

C) $D = \{x \in \mathbb{R} \mid x > 3\}$
 $R = \{y \in \mathbb{R} \mid y > 3\}$

B) $D = \{x \in \mathbb{R} \mid x < 3\}$
 $R = \{y \in \mathbb{R}\}$

D) $D = \{x \in \mathbb{R} \mid x < 3\}$
 $R = \{y \in \mathbb{R} \mid y > 3\}$

2. The x -intercept of the graph of $y = -\log_2(x+4)+1$ is at:

A) $x = -2$

C) $x = 2$

B) $x = -1$

D) $x = -4$

3. The graph of $y = -\frac{2}{3}(730)^{12x+28} - 4$ has a horizontal asymptote at:

A) $x = \frac{7}{3}$

C) $y = \frac{7}{3}$

B) $x = -3$

D) $y = -4$

4. $\sin^2\left(\frac{\pi}{2}-4^{\log_4(x)}\right), 0 \leq x \leq \frac{\pi}{2}$ is equal to:

A) $\cos(x)$

C) $2 \sin(x) \cos(x)$

B) $1 - \cos^2(x)$

D) $\cos^2(x)$

5. The graph shown has the equation:

A) $y = 4^{x-1}$

B) $y = 4^x - 1$

C) $y = 4^{1-x}$

D) $y = 4^{-x} - 1$

6. The hydronium ion concentration of a solution with a pH of 4.04 is approximately

A) 0.606 mol/L

C) 8.63×10^{-7} mol/L

B) 9.12×10^{-5} mol/L

D) 2.47×10^{-6} mol/L

7. $\log_8\left(\frac{\sqrt{2}}{4}\right)$ is equal to:

A) $\frac{3}{6}$

B) $-\frac{8}{6}$

C) $-\frac{1}{2}$

D) $-\frac{3}{5}$

8. The graph of $y = -\frac{2}{3}\log_3(3x-9)-4$ has a vertical asymptote at:

A) $x = 3$

C) $y = 3$

B) $x = 9$

D) $y = -4$

9. The ratio of intensities of two sounds is 647. If the louder sound has a decibel level of 97 dB, what is the decibel level of the other sound?

A) 57.7 dB

C) 125.1 dB

B) 28.1 dB

D) 68.9 dB

10. The half-life of a radioactive substance is 97 years. If 10 mg are produced after a reaction, approximately how long will it take for only 1 mg to remain?

A) 29 years

C) 322 years

B) 180 years

D) 462 years

Page 1 of 5

Part B - Short Answer

1. Write the following as a single logarithm and simplify as much as possible.

$$\begin{aligned} \text{a) } \log_4 \left(64 \times \sqrt{64^4} \right) + \log_4 \left(4^6 \div \frac{4^3}{\sqrt{256}} \right) \\ = \log_4 (4^3 \times 4^6) + \log_4 (4^{10}) \\ = \log_4 (4^{19}) \\ = 19 \end{aligned}$$

$$\begin{aligned} \text{b) } \log(\sqrt{x})^3 + \frac{1}{2} \log x^4 - \log \sqrt{x} \\ = \log x^{\frac{3}{2}} + \log x^2 - \log x^{\frac{1}{2}} \\ = \log \frac{x^{\frac{3}{2}} \cdot x^2}{x^{\frac{1}{2}}} \\ = \log x^3 = 3 \log x \end{aligned}$$

/5

2. Without a calculator, showing all steps, solve for x in each of the following (EXACT ANSWERS ONLY):

$$\text{a) } 32(8^{x-2}) = 1$$

$$\begin{aligned} 8^{x-2} &= \frac{1}{32} \\ 3(x-2) &= -5 \end{aligned}$$

$$\begin{aligned} 3x - 6 &= -5 \\ 3x &= 1 \rightarrow x = \frac{1}{3} \end{aligned}$$

/3

$$\text{b) } \log_2 [\log_x (16)] = 1$$

Res. $x > 0, x \neq 1$

$$\log_x 16 = 2$$

$$x^2 = 16$$

$$x = \pm 4 \rightarrow x = 4$$

/2

$$\text{c) } \log_3 (x-1) = \log_3 (x^2) - \log_3 (x+3)$$

/3-A

$$\log_3 (x-1) = \log_3 \left(\frac{x^2}{x+3} \right) \quad \text{Res: } x > 1$$

$$x-1 = \frac{x^2}{x+3}$$

$$(x-1)(x+3) = x^2$$

$$x^2 + 2x - 3 = x^2$$

$$2x - 3 = 0$$

$$x = \frac{3}{2}$$

$$\text{d) } \frac{1}{\log_9 x} - \frac{1}{\log_3 x} = 2$$

Res. $x > 0, x \neq 1$

$$\log_x 9 - \log_x 3 = 2$$

$$2 \log_x 3 - \log_x 3 = 2$$

$$\log_x 3 = 2$$

$$x^2 = 3 \rightarrow x = \sqrt{3}$$

/4

3. Solve the following equations. Show all steps. Round answers to two decimal places if necessary.

$$\text{a) } 12^{2x+3} = 10(4)^{5x-2}$$

$$(3 \times 4)^{2x+3} = 10(4)^{5x-2}$$

$$\begin{aligned} 2x+3 &= 10 \frac{4}{4} \\ 2x+3 &= 10 \frac{4}{4} \end{aligned}$$

$$\begin{aligned} 2x+3 &= 32-5 \\ 3 &= 10(4) \end{aligned}$$

$$(2x+3) \log 3 = 1 + (3x-5) \log 4$$

$$3 \log 3 - 1 + 5 \log 4 = x(3 \log 4 - 2 \log 3)$$

$$x = \frac{3 \log 3 + 5 \log 4 - 1}{3 \log 4 - 2 \log 3}$$

/3

$$x \approx 4.04$$

$$\text{b) } \log_2 (x) + \log_{\sqrt{2}} (x-7) = 3$$

$$\log_2 (x) + 2 \log_2 (x-7) = 3 \quad \text{Res: } x > 7$$

$$\log_2 (x) + \log_2 (x-7)^2 = 3$$

$$\log_2 (x(x-7)^2) = 3$$

$$x(x-7)^2 = 2^3$$

$$x(x^2 - 14x + 49) = 8$$

$$x^3 - 14x^2 + 49x = 8$$

$$x^3 - 14x^2 + 49x - 8 = 0$$

$$(x-8)(x^2 - 6x + 1) = 0$$

$$x = 8, x = 3 \pm 2\sqrt{2}$$

inadmissible

5. Determine the equation of the inverse of $f(x) = 4 - 5\log_3(x - 10)$.

/3-A

$$x = 4 - 5\log_3(y - 10)$$
$$\frac{x - 4}{-5} = \log_3(y - 10)$$
$$3^{\frac{-(x - 4)}{-5}} + 10 = y$$
$$f^{-1}(x) = (3)^{\frac{4 - x}{5}} + 10$$

6. Sketch the graph of function $f(x) = -\log_2 \sqrt{9 - 3x} + 1$. Clearly state the equation of vertical asymptote and identify the x and y- intercepts.

/4-A

$$f(x) = -\frac{1}{2}\log_2(-3(x - 3)) + 1$$
$$(x, y) \rightarrow \left(-\frac{1}{3}x + 3, -\frac{1}{2}y + 1\right)$$

x - int: $y = 0$

$$-\frac{1}{2}\log_2(-3(x - 3)) + 1 = 0$$
$$-\frac{1}{2}\log_2(-3(x - 3)) = -1$$
$$\log_2(-3(x - 3)) = 2$$
$$-3(x - 3) = 4$$
$$x = \frac{5}{3} \approx 1.66$$

y - int: $x = 0$

$$-\frac{1}{2}\log_2(-3(0 - 3)) + 1 \approx -0.58$$

x	y = log ₂ (x)
1/2	-1
1	0
2	1
4	2

➡

x	y = -1/2 log ₂ (-3(x-3)) + 1
2.8	1.5
2.7	1
2.3	0.5
1.7	0

7. If \$25 000 is invested in a hedge fund that pays 2.75% interest compounded weekly, how long will it take for that investment to be worth \$75 000?

/3-A

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$75000 = 25000\left(1 + \frac{0.0275}{52}\right)^{52t}$$
$$3 = (1.00053)^{52t}$$
$$\log(3) = 52t\log(1.00053)$$
$$t = \frac{\log(3)}{52\log(1.00053)} \approx \cancel{70.78 \text{ weeks}} \quad 39.94$$

write statement

8. The half-life of radium is 1620 years. If a laboratory has 12 grams of radium, how long will it take before it has 8 grams of radium left?

$$A = A_o \left(\frac{1}{2} \right)^{\frac{t}{h}}$$

$$8 = 12 \left(\frac{1}{2} \right)^{\frac{t}{1620}}$$

$$\frac{2}{3} = \left(\frac{1}{2} \right)^{\frac{t}{1620}}$$

$$\log \left(\frac{2}{3} \right) = \frac{t}{1620} \log \left(\frac{1}{2} \right)$$

$$t = \frac{1620 \log \left(\frac{2}{3} \right)}{\log \left(\frac{1}{2} \right)} \approx 947.64 \text{ years}$$

/3-A

Thinking

1. Solve for the exact value of x : $\log_5(x) + \log_{x^2}(5) + \log_x(25) = -\frac{7}{2}$

$$\log_5(x) + \frac{1}{\log_5(x^2)} + \frac{1}{\log_{25}(x)} = -\frac{7}{2} \quad \text{Res.: } x > 0, x \neq 1$$

$$\log_5(x) + \frac{1}{2\log_5(x)} + \frac{2}{\log_5(x)} = -\frac{7}{2}$$

Let $\log_5(x) = A$, we get :

$$A + \frac{1}{2A} + \frac{2}{A} = -\frac{7}{2}$$

$$2A^2 + 1 + 4 = -7A$$

$$2A^2 + 7A + 5 = 0$$

$$(2A + 5)(A + 1) = 0$$

$$A = \frac{-5}{2} \text{ or } A = -1$$

$$\log_5(x) = \frac{-5}{2} \quad \text{or} \quad \log_5(x) = -1$$

$$5^{\frac{-5}{2}} = x$$

$$5^{-1} = x$$

$$x = \frac{\sqrt{5}}{125}$$

$$x = \frac{1}{5}$$

/4-T

2. Solve the following system of equations algebraically.

$$y = \log_2(2x - 2)$$

$$y = 5 - \log_2(x - 1)$$

$$5 - \log_2(x - 1) = \log_2(2x - 2) \quad \text{Res: } x > 1$$

$$5 = \log_2(2x - 2) + \log_2(x - 1)$$

$$5 = \log_2 2(x - 1)(x - 1)$$

$$5 = \log_2 2 + \log_2 (x - 1)^2$$

$$5 = 1 + 2\log_2(x - 1)$$

$$4 = 2\log_2(x - 1)$$

$$2 = \log_2(x - 1)$$

$$4 = x - 1$$

$$\boxed{x = 5} \text{ \& } y = \log_2(2(5) - 2)$$

$$y = \log_2(8)$$

$$\boxed{y = 3}$$

\therefore P.O.I is (5,3)

/4-T

Knowledge $\overline{21}$

Application $\overline{26}$

Thinking $\overline{8}$