Unit 1: Polynomial Functions
1.8 Solving Polynomial Equations

If a polynomial equation, P(x)=0, is factorable, the roots of the equation are determined by factoring the polynomial, setting each factor to zero, and then solving each of these equations individually.

- **An n^{th} degree polynomial equation has at most n distinct roots.**
- **The solutions to a polynomial equation P(x)=0 are the zeros of the corresponding** polynomial function y=P(x).

The x-intercepts of the graph of y=P(x) are the real zeros of the polynomial function.

- **If a polynomial equation of degree 3 or greater cannot be factored, the roots of the equation must be determined using technology or higher-level mathematical procedures.**

Steps to Solve Polynomial Equations:

Example1:

$$f(x) = 2x^3 + 3x^2 - 8x + 3$$

1^{st} Find all the factors of the constant and of the leading coefficient.

2^{nd} Form all rational numbers by making fractions with the numerator a factor of the constant and the denominator a factor of the leading coefficient

3^{rd} Using synthetic substitution, find one solution to the problem

4^{th} Rewrite the function as two factors.

5^{th} Repeat this process as needed to get all your factors of the polynomial.

6^{th} Set each factor equal to zero and solve the resulting equations.

Example 2: Solve for x, $x \lfloor \ \Sigma$

a. $x^3 - x = 0$

b. $8x^3 + 26x^2 + 17x - 6 = 0$

c. $x^3 - 3x^2 - 4x + 12 = 0$

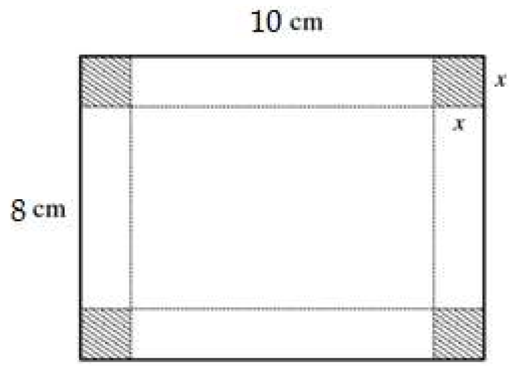
d. $3x^4 - 10x^3 = 24x^2 + 6x - 5$

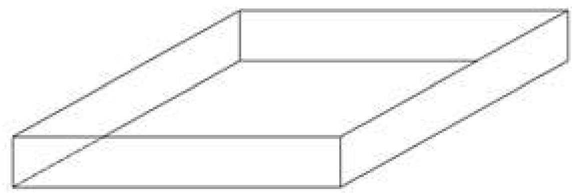
e. $6x^3 + x^2 - 5x - 2 = 0$

f. $x^4 - 15x^2 - 16 = 0$

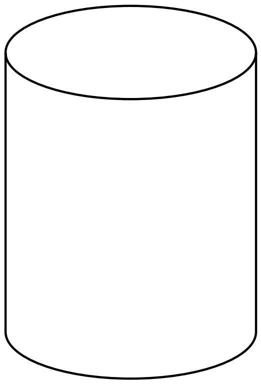
g. $(x^2 - 5x - 5)(x^2 - 5x + 3) = 9$

Example 3: A rectangular piece of cardboard measuring 10 cm by 8 cm is made into an open box by cutting squares from the corners and turning up the sides. If the box is to hold a volume of 48 cm^3 , what size of square must be removed?





Example 4: A cylindrical chemical storage tank must have a height 4 meters greater than radius of the top of the tank. Determine the radius of the top and the height of the tank if the tank must have a volume of 5π cubic meters.



Example 5: The length, width and height of a small box are three consecutive **odd** integers, where the width is the least and the length is the greatest integer. If the width is double and the length and height are increased by 2 cm each, then the volume is increased by 273 cm^3. Find the dimensions of the original box

Practice

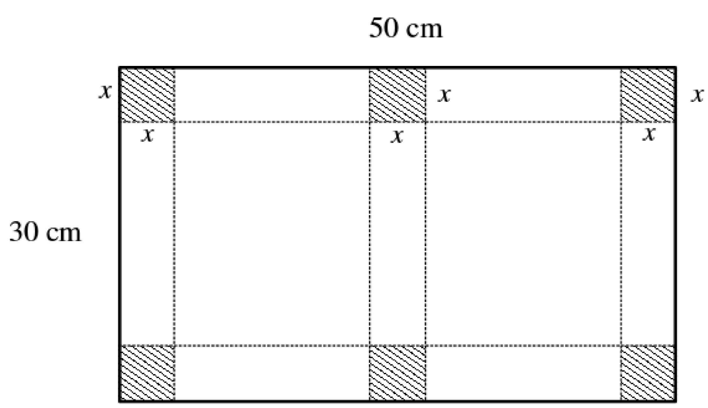
1. Solve the following polynomial equations by factoring where $x \in \mathbb{R}$.
   a) $x^3 - 5x^2 - 4x + 20 = 0$
   b) $2x^3 + 3x^2 = 11x + 6$
   c) $4x^2 = x^3 + 2x + 3$
   d) $x^4 - 7x^2 + 12 = 0$
   e) $2x^3 + 15 = 6x^2 + 5x$
   f) $2x(x^3 + 1) = x^2(4x + 1)$
   g) $2x^4 + 8x + 12 = 3x^2(x + 3)$

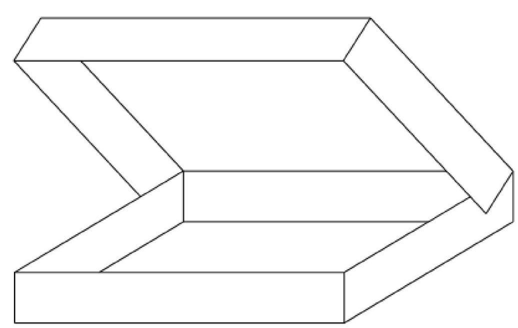
2. Explain why
   a) $15x^5 + 4x^4 + 9x^2 + 7x + 380 = 0$ has at least one real root.
   b) $5x^6 + 3x^4 + 8x^2 + 120 = 0$ has no real roots.

3. A rectangular holding tank is x metres deep, (6x–8) metres long, and (6x–16) metres wide. Find the dimensions of the tank with a volume of 512 m^3.

4. The product of the squares of two consecutive integers is 1764. Find all possible values for the integers.

5. A box with a lid is to be created from a 50 cm by 30 cm piece of cardboard by cutting x by x squares from the four corners of the cardboard, and at the centre of the two sides, as shown in the diagram. Determine the function that represents the volume of the box in terms of x, and state the restrictions on x. If the box is to have a volume of 1750 cm^3, determine the side length of the squares that need to be cut





6*. Solve $x^2\left(x^2 + 6\right) = 5x^3 - x + 1, x \in \mathbb{R}$.