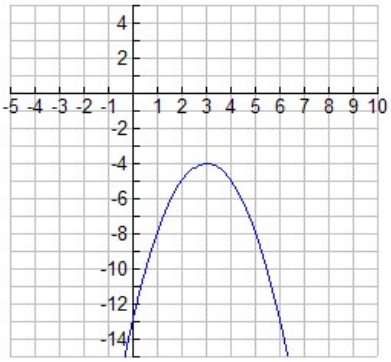
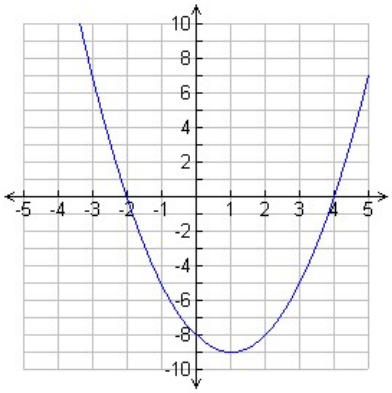
1.2 PROPERTIES OF POLYNOMIAL FUNCTIONS (Part 2)

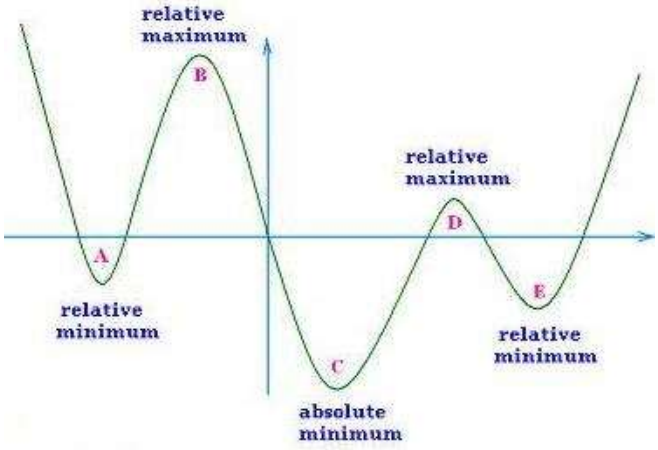
ABSOLUTE (GLOBAL) MAXIMUM AND MINIMUM VALUES:
An **absolute maximum** is **the highest y-value** on the graph.
An **absolute minimum** is **the lowest y-value** on the graph.



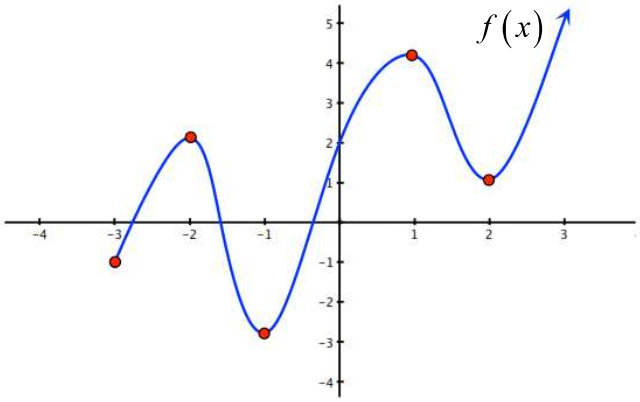


RELATIVE (LOCAL) MAXIMUM AND MINIMUM VALUES:

A **relative maximum** is the greatest value of a *function in* ***its neighborhood***.
A **relative minimum** is the least value of a *function in* ***its neighborhood***.



Ex1. Graph of function f(x) is given .Identify global and local max/min values.



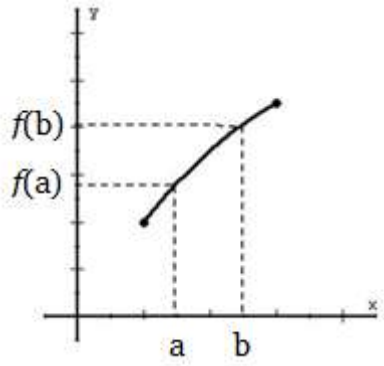
Local max:_____
Local min:_____
Absolute max: _____
Absolute min: _____

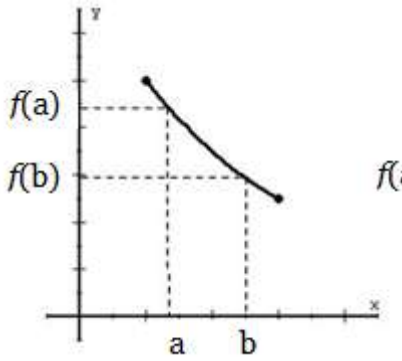
INTERVALS OF INCREASE/DECREASE

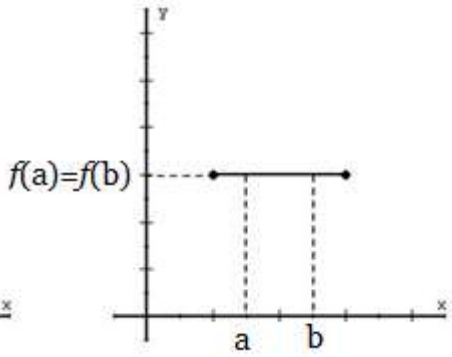
Suppose S is an interval in the domain of $f(x)$,so $f(x)$ is defined for all x in S.

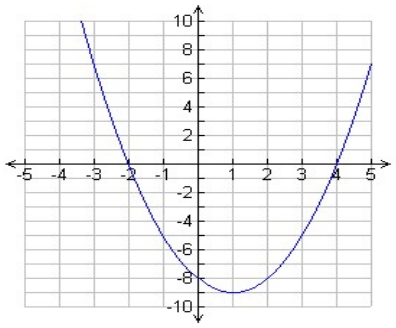
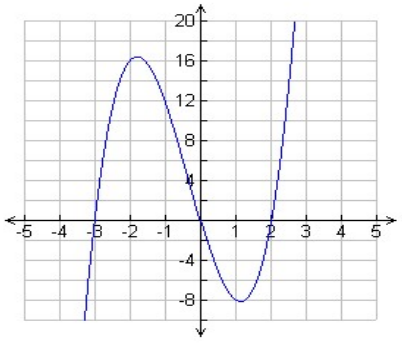
$f(x)$ is **increasing** on S , $\Leftrightarrow f(a) < f(b)$ for all $a,b \in S$ such that $a < b$

$f(x)$ is **decreasing** on S $\Leftrightarrow f(a) > f(b)$ for all $a,b \in S$ such that $a < b$





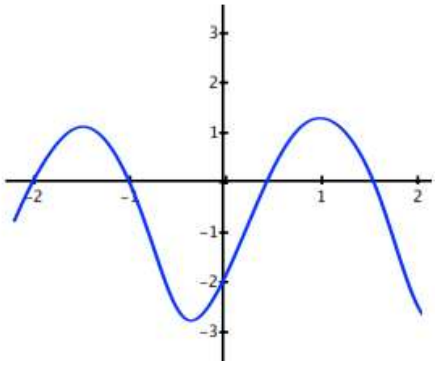


FUNCTION GRAPH	INTERVALS OF INCREASE/DECREASE	TYPE OF MAX OR MIN
		
		

Conclusion:

ZEROS OF A FUNCTION:

The **zeros** of a function are known by two other names: ***x-intercepts*** and **roots**.
A zero/x-intercept/root of a function is the value of the x-coordinate where the function cuts or just touches the x-axis. The x-intercept is the **value** of the x-coordinate from the point (x, 0).
The Y-INTERCEPT is the value of the y-coordinate of the point where the graph crosses the y axis. It is the value of y coordinate of the point (0, y).



x-intercepts :_____
y-intercept :_____

Investigation of the Properties of Polynomial Functions:

Using a graphing calculator or Desmos[1], fill-in the following charts and draw appropriate conclusions:

a) Quadratic Functions:

Function	Degree	Number of zeroes/ x-intercepts/ roots	Number of Turning Points (MAX or Min)
$y = x^2$			
$y = x^2 + 1$			
$y = 3x^2 - 4x - 1$			

Conclusions:

- Quadratic functions have a degree of _____.
- The maximum number of roots that a quadratic function can have is _____
- The least number of roots that a quadratic function can have is _____
- The maximum number of turning points (max.min) a quadratic function can have is _____

b) Cubic Functions:

Function	Degree	Number of zeroes/ x-intercepts/ roots	Number of Turning Points (MAX or Min)
$y = x^3$			
$y = x^3 + 2x^2 - x - 2$			
$y = -4x^3 + 16x^2 - 13x + 3$			

Conclusions:

- Cubic functions have a degree of _____.
- The maximum number of roots that a cubic function can have is _____
- The least number of roots that a cubic function can have is _____
- The maximum number of turning points (max/min) a cubic function can have is _____

(1) https://www.desmos.com/calculator/kxbhcq6bix

c) Quartic Functions:

Function	Degree	Number of zeroes/ x-intercepts/ roots	Number of Turning Points (MAX or Min)
$y = x^4$			
$y = -x^4 - 5$			
$y = x^4 + 3x^3 + x^2 - 3x - 2$			
$y = -x^4 + 5x^2 - 4$			

Conclusions:

- Quartic functions have a degree of _____.
- The maximum number of roots that a quartic function can have is _____
- The least number of roots that a quartic function can have is _____
- The maximum number of turning points a quartic function can have is _____

d) Quintic Functions:

Function	Degree	Number of zeroes/ x-intercepts/ roots	Number of Turning Points (MAX or Min)
$y = x^5 + 7$			
$y = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$			
$y = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$			

Conclusions:

- Quintic functions have a degree of _____.
- The maximum number of roots/x-intercepts that a quintic function can have is _____
- The least number of roots/x-intercepts that a quintic function can have is _____
- The maximum number of turning points a quintic function can have is _____

e) 6th Degree Functions:

Function	Degree	Number of zeroes/ x-intercepts/ roots	Number of Turning Points (MAX or Min)
$y = x^6$			
$y = 2x^6 - 12x^4 + 18x^2 + x - 5$			
$y = -x^6 - 3$			

Conclusions:

- The maximum number of roots/x-intercepts that a 6th degree function can have is _____
- The least number of roots/x-intercepts that a 6th degree function can have is _____
- The maximum number of turning points a 6th degree function can have is _____

Overall Conclusions

Number of Zeros:

The maximum number of zeros/x-intercepts that a polynomial function can have is the _____as its _____.

The minimum number of zeros/x-intercepts that an ***odd degree*** polynomial can have is _____.

However, an ***even degree*** polynomial function can have _____ zeros/x-ints at all.

Turning Points:

The maximum number of turning points that a polynomial function can have is _____.

An ***even degree*** function must have at least _____ turning point.

An ***odd degree*** function could have _____ turning points at all.

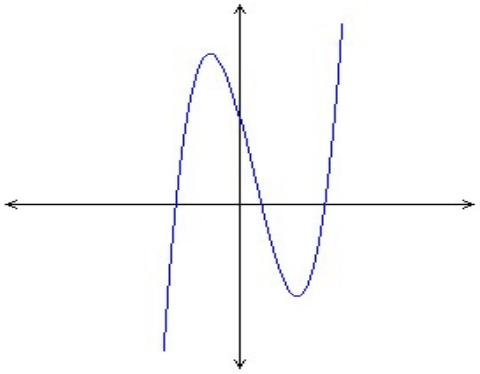
Complete the chart:

Type of Polynomial	Degree	Maximum Number of zeros / x-intercepts / roots	Minimum Number of Zeros/ x-intercepts/roots	Maximum Number of Turning Points
Linear				
Quadratic				
Cubic				
Quartic				
Quintic				
	6			
	7			
	n		If n is even:_____ If n is odd:_____	

Exit Card!

Identify number of zeros/x-intercepts/roots, the sign of the leading coefficient and describe the end behaviour. Using this information, decide if each function is cubic or quartic.

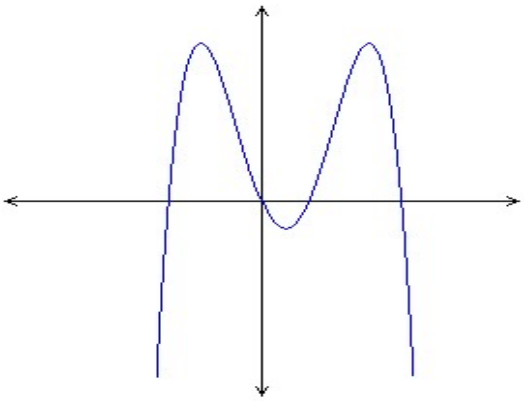
a)



# of zeros/x-int/roots: _____
Sign of Leading Coefficient: _____
End Behaviour:

Type of function:

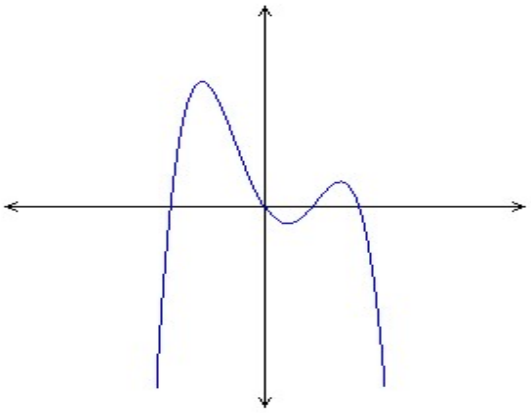
b)



# of zeros/x-int/roots: _____
Sign of Leading Coefficient: _____
End Behaviour:

Type of function:

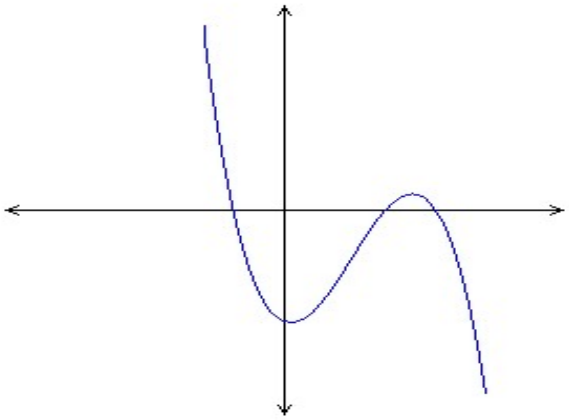
c)



# of zeros/x-int/roots: _____
Sign of Leading Coefficient: _____
End Behaviour:

Type of function:

d)



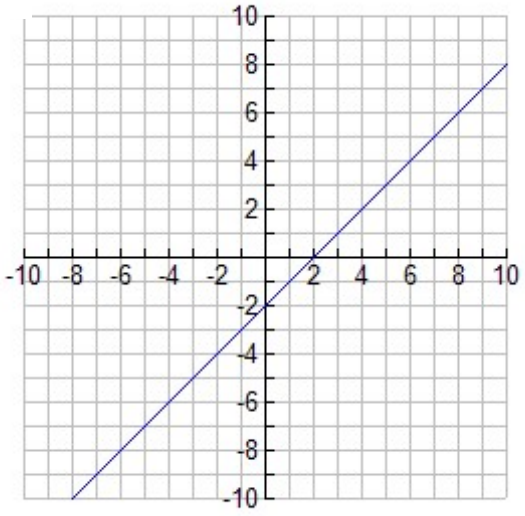
# of zeros/x-int/roots: _____
Sign of Leading Coefficient: _____
End Behaviour:

Type of function:

Practice:

Take a look at the following graphs and answer the questions.

a)



Domain:

Range:

Number of roots:

Roots:

End Behaviour:

Degree:

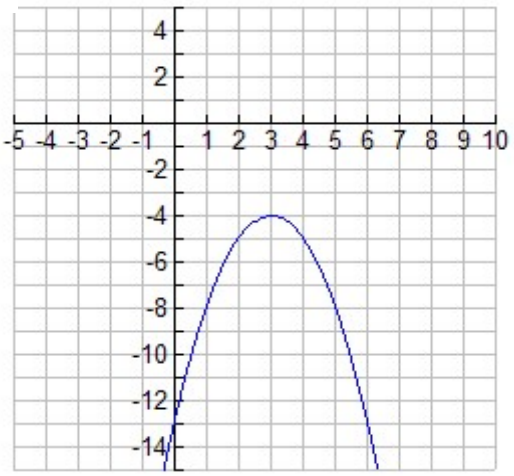
Intervals of positive:

Intervals of negative:

Intervals of increasing:

Intervals of decreasing:

b)



Domain:

Range:

Number of roots:

Roots:

End Behaviour:

Degree:

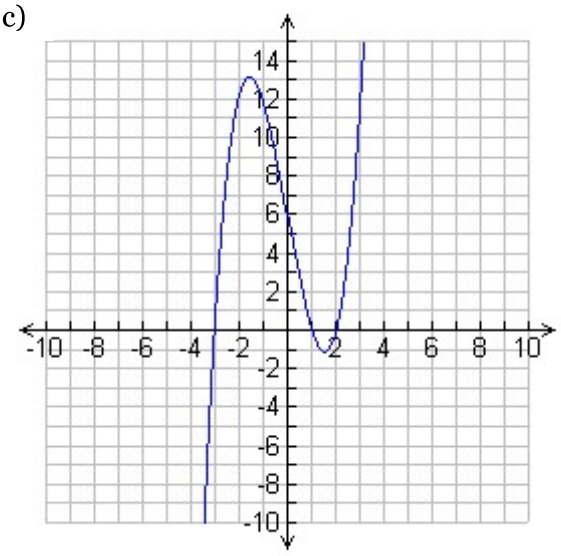
Intervals of positive:

Intervals of negative:

Intervals of increasing:

Intervals of decreasing

c)



Domain:

Range:

Number of roots:

Roots:

End Behaviour:

Degree: Name:

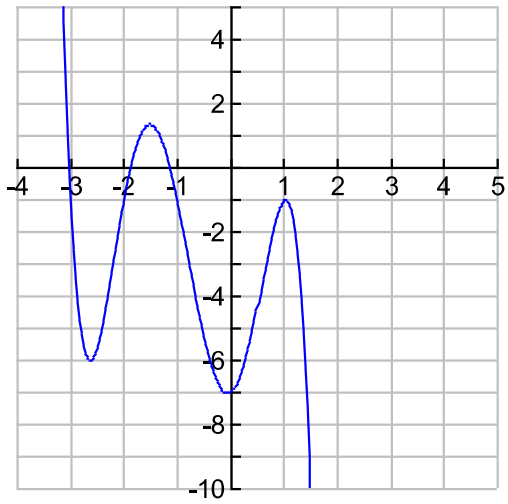
Intervals of positive:

Intervals of negative:

Intervals of increasing:

Intervals of decreasing

d)



Domain:

Range:

Number of roots:

Roots:

End Behaviour:

Degree: Name:

Intervals of positive:

Intervals of negative:

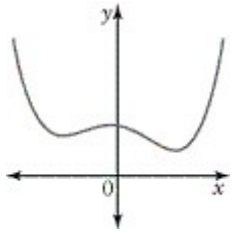
Intervals of increasing:

Intervals of decreasing

Warm up

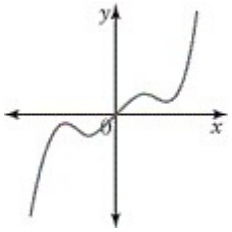
Part I. Multiple Choice

1. The least possible degree of the polynomial function represented by the graph shown is



a) 2 c) 4
b) 3 d) 5

2. The least possible degree of the polynomial function represented by the graph shown is



a) 3 c) 5
b) 4 d) 7

3. If y = f(x) is a quartic function with a constant difference of -48, then the following statement is **false**:

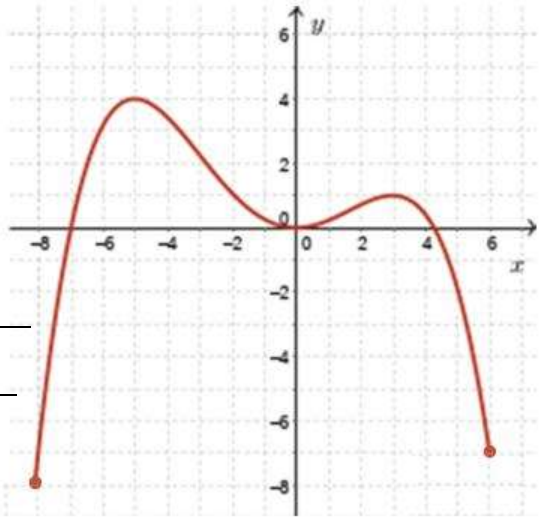
a) the function starts in Q3 and ends in Q4
b) the sign of the leading coefficient is negative
c) the function might not have any roots
d) the sign of the leading coefficient is positive

Part II. True/False

a) The function $y = -3x^4 + 1$ extends from quadrant 3 to quadrant 4. _____
b) Odd-degree polynomials have at least one x-intercept. _____
c) Even-degree polynomial functions always begin and end on the same side of the x-axis. _____
d) The graph of a quartic function cannot have exactly three x-intercepts. _____
e) The function $y = x^4 + 2x^2 + 1$ never crosses the x-axis. _____
f) All quartic polynomial equations have at least one real solution. _____

Part III. Short Answers

The following is the graph of $y = f(x)$. Answer the following questions.



a) absolute max:_____

b) absolute min: _____

c) local max:_____

d) local min.: _____

e) interval(s) of increasing : _____

f) interval(s) of decreasing : _____