Unit 4: Trigonometric Functions Part II –Formative Quiz

Name: SOLUTIONS , Period: ________

Knowledge

Multiple Choice: Write the **CAPITAL letter** corresponding to the correct answer on the line provided

1. A solution to $2\sin(x)=1$ is C

   A) $\frac{\pi}{4}$ B) $-\frac{\pi}{6}$ C) $\frac{13\pi}{6}$ D) $\frac{\pi}{3}$

2. The graph of $y=\frac{1}{2}\tan\left(\frac{1}{4}x-\frac{\pi}{2}\right)+4$ has a period of B

   $y=\frac{1}{2}\tan\left[\frac{1}{4}(x-2\pi)\right]+4$

   $P: \pi/\frac{1}{4} = 4\pi$

   A) $\frac{\pi}{2}$ B) 4π C) $\frac{\pi}{4}$ D) 6π

3. If $4\sin^2(x)-3=0$,where $0 \le x \le 3\pi$ then C

   A) 2 solutions exist B) 4 solutions exist C) 6 solutions exist D) 8 solutions exist

4. A Ferris wheel starts spinning at $t = 0$ s and stops at $t = 12$ s. If the Ferris wheel made 5 loops during that time, what is its period, k? A

   $2\pi/\frac{12}{5} = \frac{10\pi}{12} = \frac{5\pi}{6}$

   A) $\frac{5\pi}{6}$ B) $\frac{2\pi}{5}$ C) $\frac{2\pi}{12}$ D) 2π

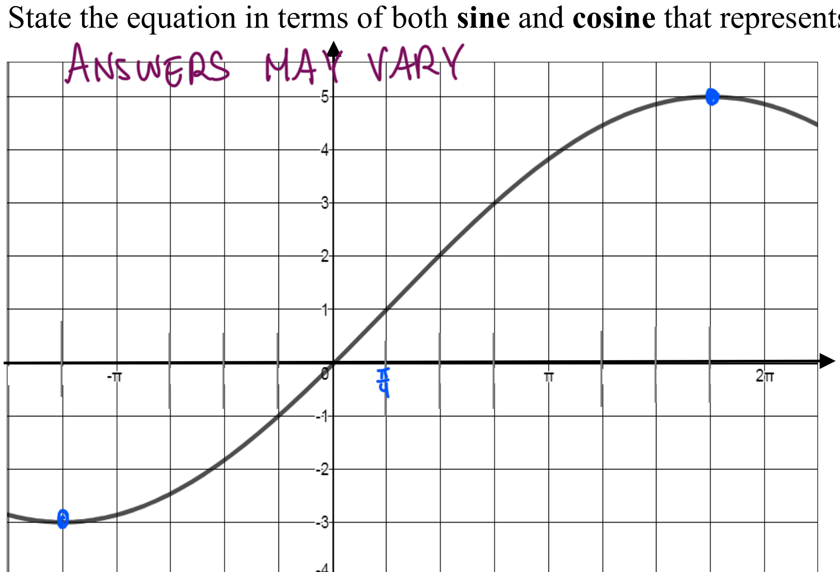
5. $y = \cot(x)$ has x-intercepts that occur at C

   A) $k\pi, k\epsilon Z$ B) $\frac{k\pi}{2}, k\epsilon Z$ C) $k\pi-\frac{\pi}{2}, k\epsilon Z$ D) $2k\pi, k\epsilon Z$

Full Solutions

6. State the equation in terms of both **sine** and **cosine** that represents the graph below.

ANSWERS MAY VARY



$a = \frac{max-min}{2} = \frac{5-(-3)}{2} = 4$

$period = \left(\frac{7\pi}{4}-\left(-\frac{5\pi}{4}\right)\right)\times 2 = (3\pi)\times 2 = 6\pi$

$k = \frac{2\pi}{k} = \frac{2\pi}{6\pi} = \frac{1}{3}$

$d = \frac{\pi}{4}$ (for sine) $d = \frac{7\pi}{4}$ (for cosine)

$c = \frac{max+min}{2} = \frac{5+(-3)}{2} = 1$

Sine equation: $y = 4\sin\left[\frac{1}{3}\left(x-\frac{\pi}{4}\right)\right]+1$

Cosine equation: $y = 4\cos\left[\frac{1}{3}\left(x-\frac{7\pi}{4}\right)\right]+1$

7. Solve the following.

a) $\tan^4(x)-4\sec^2(x)+7=0$, $-2\pi \le x \le 2\pi$

$\tan^4(x)-4\left[1+\tan^2(x)\right]+7=0$

$\tan^4(x)-4-4\tan^2(x)+7=0$

$\tan^4(x)-4\tan^2(x)+3=0$

factor

$[\tan^2(x)-3][\tan^2(x)-1]=0$

$\tan^2(x)-3=0$

$\tan(x)=\pm\sqrt{3}$

R.A.A $=\frac{\pi}{3}$

$x_1=\frac{\pi}{3}$

$x_2=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

$x_3=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$

$x_4=2\pi-\frac{\pi}{3}=\frac{5\pi}{3}$

$\tan^2(x)-1=0$

$\tan(x)=\pm 1$

R.A.A $=\frac{\pi}{4}$

$x_5=\frac{\pi}{4}$

$x_6=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$

$x_7=\pi+\frac{\pi}{4}=\frac{5\pi}{4}$

$x_8=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$

$\therefore x \in \left\{\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \pm\frac{5\pi}{3}, \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}\right\}$

b) $[\sin(x)]^2=[1+\sqrt{3}\cos(x)]^2$, $0 \le x \le 2\pi$

$\sin^2(x) = 1+2\sqrt{3}\cos(x)+3\cos^2(x)$

$1-\cos^2(x) = 1+2\sqrt{3}\cos(x)+3\cos^2(x)$

$0 = 4\cos^2(x)+2\sqrt{3}\cos(x)$

$0 = 2\cos(x)[2\cos(x)+\sqrt{3}]$

$2\cos(x)=0$

$\cos(x)=0$

$x_1=\frac{\pi}{2}$

$x_2=\frac{3\pi}{2} \rightarrow$ extraneous

$2\cos(x)+\sqrt{3}=0$

$\cos(x)=-\sqrt{3}/2$

R.A.A $=\pi/6$

$x_2=\pi-\frac{\pi}{6}=\frac{5\pi}{6} \rightarrow$ extraneous

$x_3=\pi+\frac{\pi}{6}=\frac{7\pi}{6}$

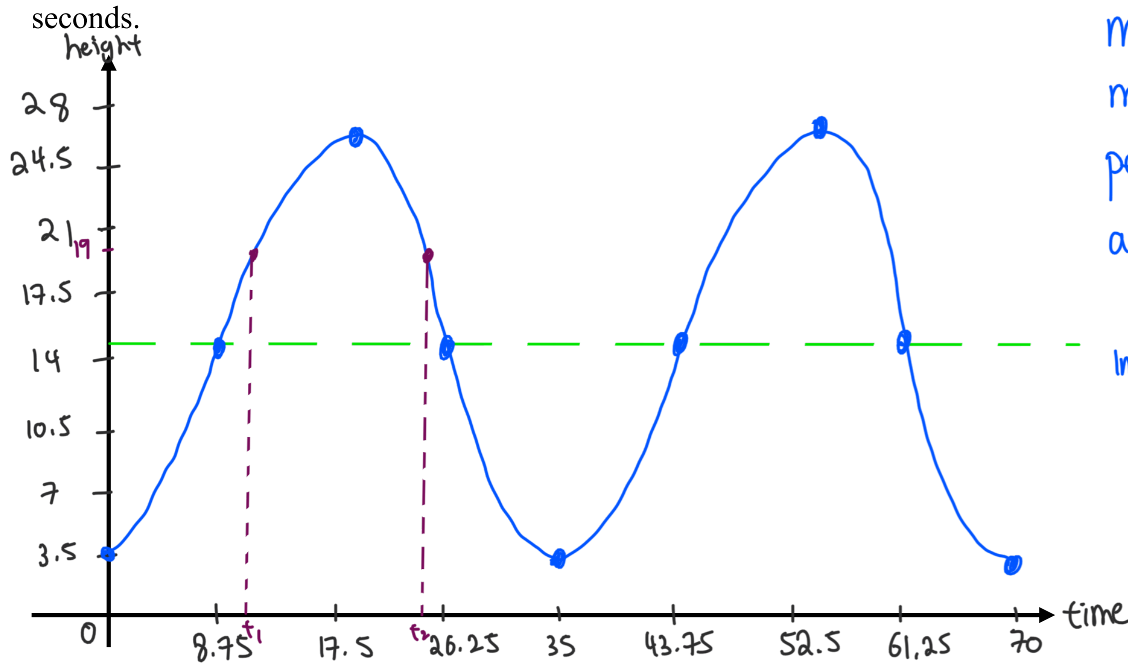
$\therefore x \in \left\{\frac{\pi}{2}, \frac{7\pi}{6}\right\}$

APPLICATION

8. A Ferris wheel ride is to be built next to a 19 metre high building. Riders get on at the lowest point of the Ferris wheel which is 3.5 m above the ground. The ride lasts 7 minutes and the Ferris wheel completing 12 revolutions per ride. The centre of the wheel is at 14.5 metres above the ground.

7 mins = 420 s

a) Draw a fully labelled graph of the function for only the **first two revolutions** of the ride using time in seconds.



max = 25.5

min = 3.5

period = 420 s / 12 = 35 seconds

axis of the curve: $y = \frac{25.5+3.5}{2}$

$y = 14.5$

Interval = $\frac{35}{4} = 8.75$

t	h
0	3.5
8.75	14.5
17.5	25.5
26.25	14.5
35	3.5

b) Determine a cosine equation that models the height h, in metres with respect to time t, in seconds.

$a = \frac{25.5-3.5}{2} = 11$

$k = \frac{2\pi}{35}$

$d = 0$

$c = 14.5$

$\therefore h(t) = -11\cos\left(\frac{2\pi}{35}t\right) + 14.5$

c) Determine how long a rider is below the height of the building for each 7 minute ride.

$19 = -11\cos\left(\frac{2\pi}{35}t\right) + 14.5$

$-\frac{9}{22} = \cos\left(\frac{2\pi}{35}t\right)$

R.A.A = $\cos^{-1}\left(\frac{9}{22}\right) \doteq 1.149$

$\frac{2\pi}{35}t_1 = \pi - 1.149$

$\frac{2\pi}{35}t_1 \doteq 1.99$

$t_1 \doteq 11.1$

$\frac{2\pi}{35}t_2 = \pi + 1.149$

$\frac{2\pi}{35}t_2 \doteq 4.29$

$t_2 \doteq 23.9$

Time below 19 m in first cycle = 11.1 + (35 - 23.9)

= 22.2 seconds

7 mins = 12 revolutions

total time = 22.2 × 12 = 266.4 seconds

∴ The rider is below the height of the building for a total of 266.4 seconds or 4.44 mins.

9. Solve $\cos^2(2x) = 3\sin^2(x) - 2$ for $x \epsilon [0, 2\pi]$.

double angle

$[1 - 2\sin^2(x)]^2 = 3\sin^2(x) - 2$

$1 - 4\sin^2(x) + 4\sin^4(x) = 3\sin^2(x) - 2$

$4\sin^4(x) - 7\sin^2(x) + 3 = 0$

$[4\sin^2(x) - 3][\sin^2(x) - 1] = 0$

$4\sin^2(x) = 3$

$\sin(x) = \pm\frac{\sqrt{3}}{2}$

R.A.A = $\frac{\pi}{3}$

$x_1 = \frac{\pi}{3}$

$x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$x_3 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$x_4 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\sin^2(x) - 1 = 0$

$\sin(x) = \pm 1$

$x_5 = \frac{\pi}{2}$

$x_6 = \frac{3\pi}{2}$

$\therefore x \epsilon \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

10. For the following function $y=-2\cos\left[\frac{2}{3}(\theta-\pi)\right]+1$. Fill in the blanks below, and sketch a labeled graph over two complete cycle.

Sub $\theta=0$: $y=-2\cos\left[\frac{2}{3}(-\pi)\right]+1$

$y=2$

Period: $\frac{2\pi}{2/3}=3\pi$ Phase Shift: π y-int : $(0,2)$

Domain: $\{x\in\mathbb{R}\}$ Range: $\{y\in\mathbb{R}\mid -1\le y\le 3\}$

$max = 1+2 = 3$

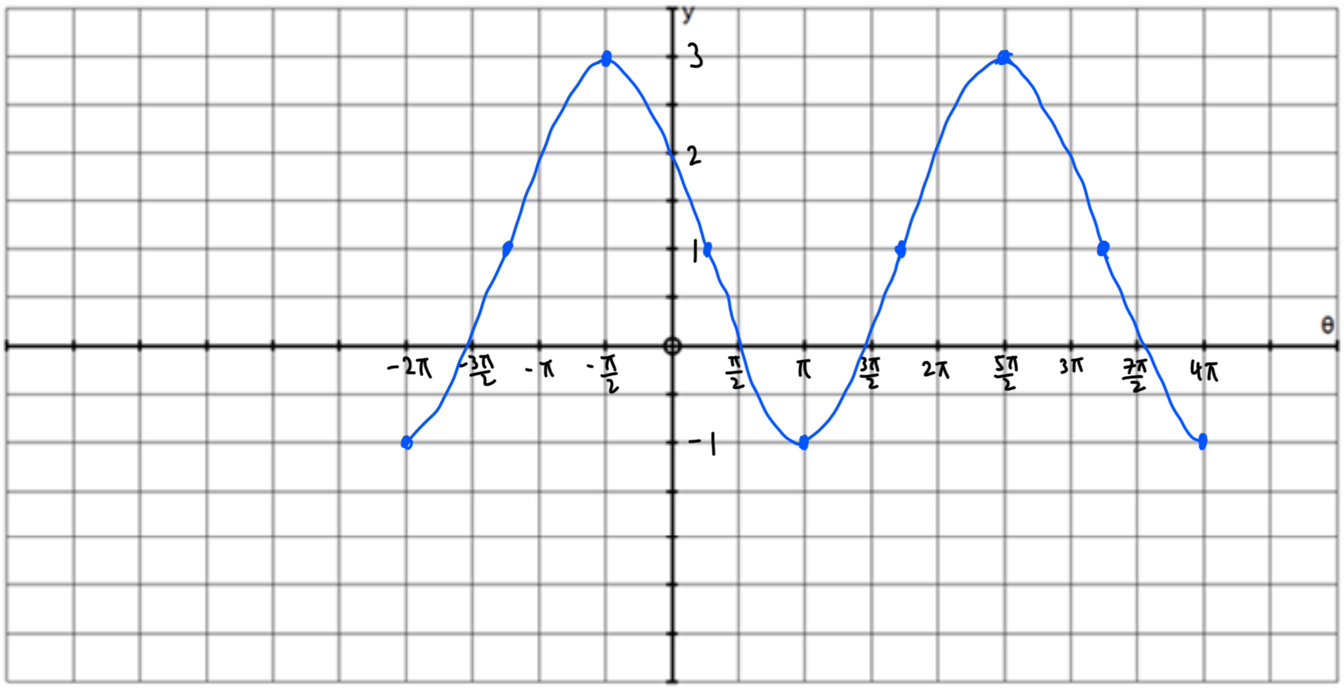
$min = 1-2 = -1$

$(x,y)\to\left(\frac{3}{2}\theta+\pi,\ -2y+1\right)$

$(0,1)\to(\pi,-1)$ $\left(\frac{3\pi}{2},0\right)\to\left(\frac{13\pi}{4},1\right)$

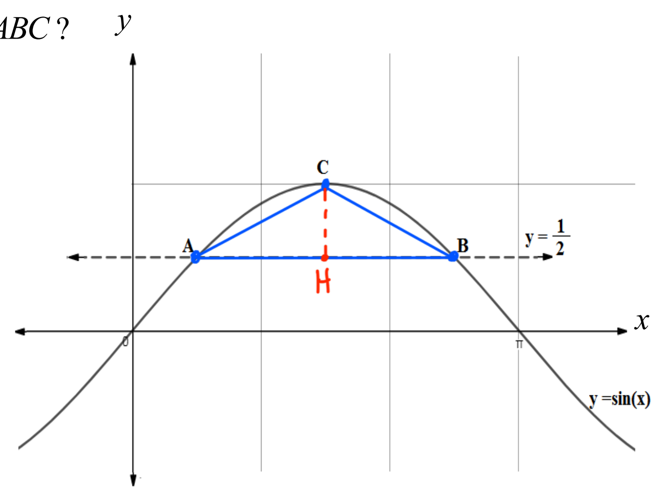
$\left(\frac{\pi}{2},0\right)\to\left(\frac{7\pi}{4},1\right)$ $(2\pi,1)\to(4\pi,-1)$

$(\pi,-1)\to\left(\frac{5\pi}{2},3\right)$



THINKING

11. In the diagram, the line $y=\frac{1}{2}$ intersects the graph of $y=\sin(x)$ at points A and B. Point C is the maximum point on the graph. What is the **exact** value of the area of ΔABC?



$C=\left(\frac{\pi}{2},1\right)$

$\frac{1}{2}=\sin(x)$

R.A.A $=\frac{\pi}{6}$

$x_1=\frac{\pi}{6} \longrightarrow A=\left(\frac{\pi}{6},\frac{1}{2}\right)$

$x_2=\pi-\frac{\pi}{6}=\frac{5\pi}{6}\longrightarrow B\left(\frac{5\pi}{6},\frac{1}{2}\right)$

$d_{AB}=\frac{5\pi}{6}-\frac{\pi}{6}=\frac{2\pi}{3}$

$d_{CH}=\frac{1}{2}$

$\therefore A=\frac{1}{2}(\text{base})(\text{height})$

$=\frac{1}{2}\left(\frac{2\pi}{3}\right)\left(\frac{1}{2}\right)$

$=\frac{\pi}{6}$ units^2

12. Solve: $\sec^2(x)-2\sec(x)\le 0$ for $x\in[0,2\pi]$.

$\frac{1}{\cos^2(x)}-\frac{2}{\cos(x)}\le 0$

$\frac{1-2\cos(x)}{\cos^2(x)}\le 0$

V.A : $\cos^2(x)=0$

$x=\frac{\pi}{2}$, $x=\frac{3\pi}{2}$

Roots: $1-2\cos(x)=0$

$\cos(x)=\frac{1}{2}$

R.A.A $=\frac{\pi}{3}$

$x_1=\frac{\pi}{3}$, $x_2=\frac{5\pi}{3}$

0		$\frac{\pi}{3}$		$\frac{\pi}{2}$		$\frac{3\pi}{2}$		$\frac{5\pi}{3}$		2π
	$-$		$+$		$+$		$+$		$-$	

$\therefore x\in\left[0,\frac{\pi}{3}\right]\cup\left[\frac{5\pi}{3},2\pi\right]$