1.2 Characteristics of Polynomial Functions (Part 1)
INVESTIGATING FINITE DIFFERENCES

A **first difference** or **finite difference** is the difference between **consecutive y-coordinates** for **evenly spaced integral x-coordinates**. To calculate a first difference, **subtract consecutive y-values**.

1. Complete the following table for the relation $y = 2x + 1$:

x	f(x)	$\Delta f(x)$
-2		
-1		
0		
1		
2		

Observations

How can you identify a linear relation?

In general, function in the form
$y = mx + b$, is a ________________.

For a linear relation, all of the first differences are ______________________.

The constant differences = _____× _____

2. Complete the following table for the relation $y = 3x^2 - 1$:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$
-2			
-1			
0			
1			
2			

Observations
In general, function in the form
$f(x) = ax^2 + bx + c$ is a _________________.

Its first differences form an arithmetic sequence.

Its second differences are _______________.

The constant differences = _____× ____

3. Complete the following table for the relation $y = 2x^3 + 1$:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
-2				
-1				
0				
1				
2				

Observations
In general, a function in the form $f(x) = ax^3 + bx^2 + cx + d$ is a _________________.
Its third differences are _______________.
The constant differences = _____× _____

Summary and Extension:

1. If the first differences are equal:
   - ➔ The function is a ______ degree function;
   - ➔ The function is called a _________________;
   - ➔ The constant differences = _____ × _____
2. If the second differences are equal:
   - ➔ The function is a ______ degree function;
   - ➔ The function is called a _________________;
   - ➔ The constant differences = _____ × _____
3. If the third differences are equal:
   - ➔ The function is a ______ degree function;
   - ➔ The function is called a _________________;
   - ➔ The constant differences = _____ × _____

GENERAL RULE:

- Finite differences can be used to determine the degree of a polynomial function.
- For example, the fourth differences of a quartic function are constant.
- The constant finite differences have the same sign as the leading coefficient.
- The constant finite differences are equal to **a[n x (n – 1) x ... x 2 x 1]=an!** where **a** is the value of the leading coefficient. (**The constant differences= an!**)

Example 1:
The table of values represents a polynomial function.

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-2	-54				
-1	-8				
0	0				
1	6				
2	22				
3	36				
4	12				
5	-110				

Use finite differences to determine:

(a) The degree of the polynomial function. ___________________

(b) The sign of the leading coefficient ______________________

(c) The value of the leading coefficient ______________________

Example 2: The points (1,-4), (2,0), (3,30), (4, 98) (5,216) (6,396) are on a function. Find the equation of the function.

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	-4				
2	0				
3	30				
4	98				
5	216				
6	396				

<u>Method:</u>

Step 1: Complete a table of values to determine the type of function and its general equation.
Step 2: Find the equations for consecutive values of y.
Step 3: Create a system of equations and solve for the variables ‘a’, ‘b’, and ‘c’.
Step 4: Write the equation.

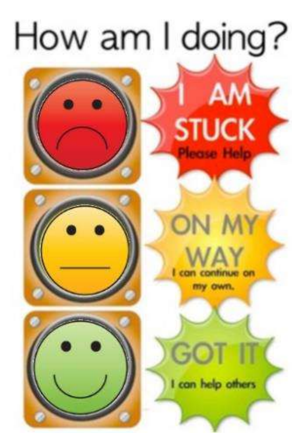
Example 3: The points (1,0), (2,-2), (3,-2), (4, 0) (5,4) (6,10) are on a function. Find the equation of the function.

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1					
2					
3					
4					
5					
6					

Exit Card!

1. For the polynomial function $f(x)=(9x^3-9x^2-9x+9)(x^2-x+2)$. State:
   (a) the degree of the function:_________________
   (b) the leading coefficient:________________________
   (c) the value of the constant finite differences :__________________

2. A polynomial function has a constant fourth difference of -132. Determine
   (a) the type of the function: _____________
   (b) the degree of the function _____________
   (c) the value of the leading coefficient: ______________



Practice

1. In each of the following, the points given lie on the graph of a polynomial f unction. Determine the equation of the function using the algebraic method developed in class:

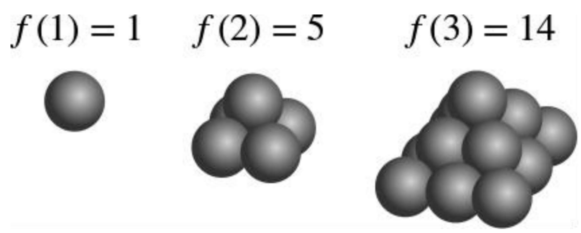
   a) (1,4), (2,15), (3,30), (4,49), (5,72), (6,99)

   b) (1, -34), (2,-42), (3,-38), (4,-16), (5,30), (6,106)

   c) (1,12), (2,-10), (3,-18), (4,0), (5,56), (6,162)

   d) (1,-2), (2,-4), (3,-6), (4,-8), (5,14), (6,108), (7,346)

2. The first three square pyramidal numbers are 1,5, and 14, as shown in the diagram. Find the next three pyramidal numbers and determine the equation of a polynomial function that gives the x^{th} square pyramidal number.



Warm-Up

Determine the equation of the function that has the following points on its curve:

(-2,-24), (-1, -7), (0, -2), (1, -3), (2, -4), (3, 1) .

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-2	-24				
-1	-7				
0	-2				
1	-3				
2	-4				
3	1				