Example 6: Describe the transformations for each of the following functions compared to y = x^4 in two different ways.

a) $y = -\left(\frac{1}{2}x - 5\right)^4$

- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________

- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________

b) $y = -7(-x+6)^4 + 9$

- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________

- ______________________________
- ______________________________
- ______________________________
- ______________________________
- ______________________________

1.4 Part B : EVEN AND ODD FUNCTIONS

EVEN FUNCTIONS

- A function is an even function if it shows symmetry in the y-axis.
- This means the left half of the function looks like a mirror image of the right half of the function on either side of the y-axis.
- To test if a function is even, we check to see if f(x) = f(-x).
- Alternately, if (x, y) maps onto (-x,y), the function is even.

ODD FUNCTIONS

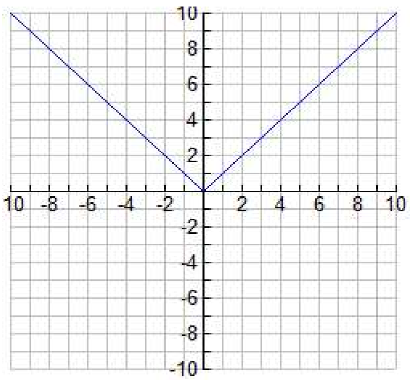
- A function is an odd function if it shows symmetry in the origin.
- This means that any line drawn across the plane through the origin that crosses the curve will cross pairs of points on the graph that are the same distance from the origin.
- To test if a function is odd, we check to see if f(-x) = -f(x).
- Alternately, if (x, y) maps onto (-x, -y), the function is odd.

NOTE:

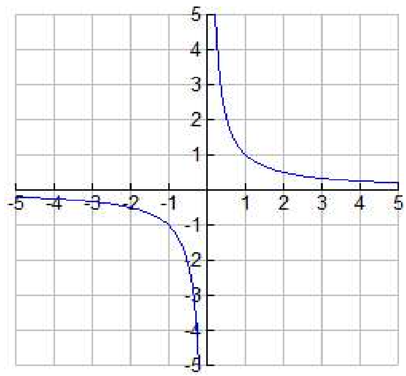
- Not all functions can be classified as even or odd. Some functions are neither.

EXAMPLE 1: Determine if the following functions are even, odd, or neither. Justify your reasoning.

1.



2.



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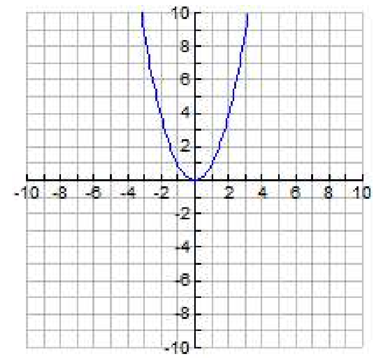

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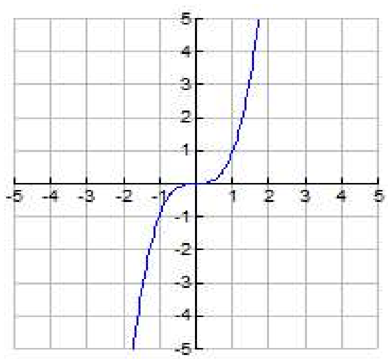

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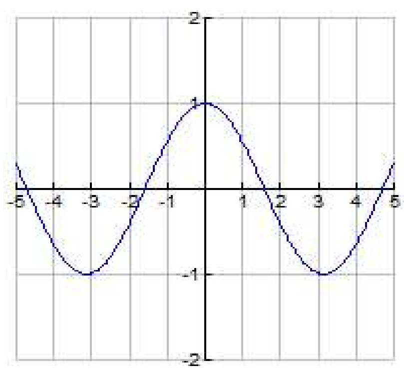

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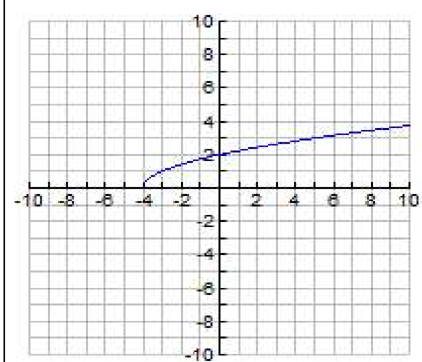


EXAMPLE 2: Determine if the following functions are even, odd or neither. Justify your reasoning algebraically.

a) $f(x) = 3x^5 - 2x^3 + x$

b) $f(x) = \dfrac{6x^4 + x^2}{(x^3 - 2x)^2}$

c) $h(x) = \dfrac{-5x^3 + x}{(3x - 5)^2}$

Practice:

1. Review your notes and the examples below. Then, for each of the practice questions prove algebraically that the function is even, odd, or neither. Use good form!

Function	*Algebraic Proof:*	Odd or even
$f(x) = x^2$	$f(x) = x^2$ $f(-x) = (-x)^2$ $= x^2$ $= f(x)$ *Therefore f(x) is even.*	Even since *f(x)* is symmetrical about the y-axis.
$f(x) = x^3$	*If f(x) is odd then f(-x)=-f(x)* $f(-x) = (-x^3)$ $= -x^3$ $= -f(x)$ $\therefore f(x)$ *is odd*	Odd since *f(x)* is symmetrical through the origin.
$f(x) = x\sqrt{(5-x)}$	$f(x) = x\sqrt{(5-x)}$: $f(-x) = (-x)\sqrt{(5-(-x)}$ $= -x\sqrt{(5+x)}$ $\neq f(x)$ $\neq -f(x)$ *f(x) is neither even nor odd* *Hint: Always check if the function is even first. Then, check if it is odd.*	Neither even nor odd since the function is not symmetrical through the origin or about the y-axis.
$f(x) = x^2 - x$		
$f(x) = (x + x^3)^5$		

$f(x)=2^x$		
$f(x)=\dfrac{1}{x^4+1}$		
$f(x)=\dfrac{x-1}{x+1}$		
$f(x)=\dfrac{x}{x+2}$		

2 Determine an equation, in factored form, for an **even** polynomial function with a turning point at (2,0). Show algebraically that the function is even.

3. Determine an equation, in factored form, for an **odd** polynomial function with one of its zeros at x=2. Show algebraically that the function is odd.

4. Sketch the graph of a polynomial function with the following characteristics:
   i. the function is an even function
   ii. as $x \to \infty, y \to -\infty$
   iii. the function has exactly 3 x-intercepts

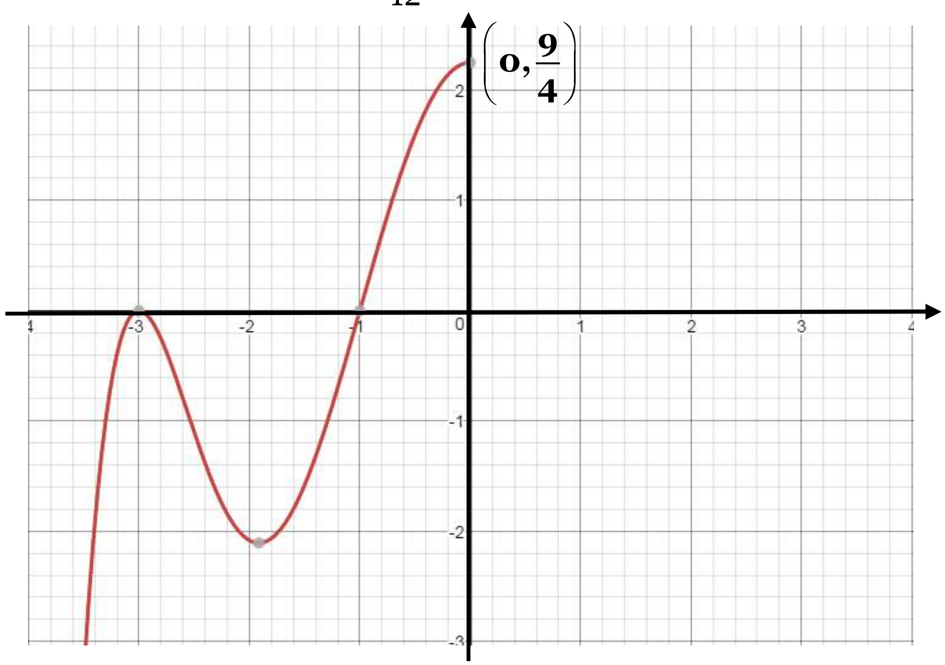
5. Sketch the graph of two polynomial functions of different degree with the following characteristics:
   i. the function is an odd function
   ii. as $x \to \infty, y \to \infty$
   iii. the function has exactly 3 x-intercepts

6. Determine, algebraically, if $f(x) = -2x(x-4)^2(x+4)^2$ is even, odd, or neither.

7. Determine, algebraically, if $f(x) = (4x+8)(x-2)^2(2+x)$ is even, odd, or neither.

8. A quintic polynomial function that is classified as an **odd** function passes through the points $(2,0), (3,210), (4,0)$. Determine the equation of $f(x)$ in factored form.

9. Given the function, $f(x)$, below is classified as an **even** polynomial function. Determine the exact value of $f(2)$. [Answer: $\frac{-25}{12}$]



Warm- up

1. Determine algebraically if the function is an even function, odd function, or neither.
   $f(x) = x^2(2x^5 - 7x^3 + 9x)$.

2. Given the graph of $f(x)$, determine the equation of $f(x)$.