Bayview Secondary School
Math Department – Course Code: **MHF4U**
Assessment As Learning For **Unit THREE – Trigonometry**

Name ________________ #_____

K – 12	T – 12	A – 17	C - 9

Knowledge and Understanding (K)

1. Covert the following to **exact** radian measures:

a) 132° $\frac{11\pi}{15}$ [1 mark]

b) $-292.5°$ $\frac{-13\pi}{8}$ [1 mark]

2. Convert the following to **degree** measures:
(Round to the nearest degree)

a) $\frac{5\pi}{7}$ 129° [1 mark]

b) $\frac{-9\pi}{4}$ $-405°$ [1 mark]

3. State one positive and one negative co-terminal angles for $\frac{13\pi}{2}$

Positive: $\frac{17\pi}{2}$ [1 mark]

Negative: $\frac{-3\pi}{2}$ [1 mark]

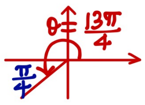
4. Write the following in terms of its co-function identity: [4 marks]

a) $-sin\left(\frac{13\pi}{4}\right)$ $\left(\sin\left(\frac{3\pi}{2}-x\right)=-\cos x\right)$

$= -\sin\left[\frac{3\pi}{2}-\left(-\frac{7\pi}{4}\right)\right]$

$= \cos\left(-\frac{7\pi}{4}\right)$

OR $= \cos\left(\frac{7\pi}{4}\right)$



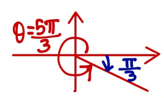
$\frac{13\pi}{4}=\frac{3\pi}{2}-\theta$

$\theta=\frac{-7\pi}{4}$

b) $cos\left(\frac{5\pi}{3}\right)$

$= \cos\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{6}\right)$



$\frac{3\pi}{2}+\theta=\frac{5\pi}{3}$

$\theta=\frac{\pi}{6}$

5. Express as a single sine or cosine function.

a) $15\sin(3x)\cos(3x)$ [1 mark]

$=\frac{15}{2}\sin(6x)$ $\theta=3x$, $2\theta=6x$

$\left(\sin 2\theta=2\sin\theta\cos\theta \Rightarrow \sin\theta\cos\theta=\frac{1}{2}\sin 2\theta\right)$

b) $2cos^2(10\theta)-1$ [1 mark]

$=\cos(20\theta)$ $A=10\theta$, $2A=20\theta$

$\left(\cos 2A=2\cos^2 A-1\right)$

THINKING

1. If $A=\frac{2sin^2(\theta)-[sin^2(\theta)+cos^2(\theta)]}{cos\left(\frac{3\pi}{2}+\theta\right)\cos(-\theta)+sin\left(\frac{\pi}{2}-\theta\right)\sin(\pi-\theta)}$, determine the exact value of $csc^2(2\theta)-A^2$. [4 marks]

$$=\frac{2\sin^2\theta-1}{\sin\theta\cos\theta+\cos\theta\sin\theta}$$

$$=\frac{-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$

$$=\frac{-\cos 2\theta}{\sin 2\theta}$$

$$=-\cot(2\theta)$$

$\csc^2(2\theta)-A^2$

$=\csc^2(2\theta)-(-\cot(2\theta))^2$

$=\csc^2(2\theta)-\cot^2(2\theta)$

$=1$

Pythagorean Identity:
$1+\cot^2 x=\csc^2 x$

2. Prove the following identities:

a) $\sin(4\theta)=4\sin(\theta)\cos(\theta)-8sin^3(\theta)\cos(\theta)$ [4 marks]

R.S.
$4\sin\theta\cos\theta-8\sin^3\theta\cos\theta$

$=4\sin\theta\cos\theta\left(1-2\sin^2\theta\right)$

$=2(\sin 2\theta)(\cos 2\theta)$ θ'

$=\sin(4\theta)$ $2\theta'$

∴ LS=RS, the identity is true.

b) $\frac{\cos(2x)}{1+\sin(2x)}=\frac{\cot(x)-1}{\cot(x)+1}$ [4 marks]

L.S. $\frac{\cos 2x}{1+\sin 2x}$

$=\frac{\cos^2 x-\sin^2 x}{\sin^2 x+\cos^2 x+2\sin x\cos x}$

$=\frac{(\cos x-\sin x)\cancel{(\cos x+\sin x)}}{(\sin x+\cos x)^{\cancel{2}}}$

$=\frac{\frac{\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\sin x}}$

$=\frac{\cot x-1}{\cot x+1}$

OR

R.S. $\frac{\frac{\cos x}{\sin x}-1}{\frac{\cos x}{\sin x}+1}$

$=\frac{\frac{\cos x-\sin x}{\sin x}}{\frac{\cos x+\sin x}{\sin x}}$

$=\frac{\cos x-\sin x}{\cos x+\sin x}\left(\frac{\cos x+\sin x}{\cos x+\sin x}\right)$

$=\frac{\cos^2 x-\sin^2 x}{2\cos x\sin x+\sin^2 x+\cos^2 x}$

$=\frac{\cos(2x)}{\sin(2x)+1}$

∴ LS=RS, the identity is true.

APPLICATION

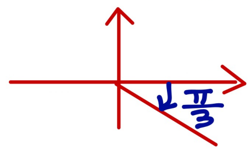
1. Find the exact values of the following. Draw a diagram and show work to support your answer.

a) $sin\left(\frac{-\pi}{3}\right)$ [3 marks]

$= -\sin\left(\frac{\pi}{3}\right)$

$= \frac{-\sqrt{3}}{2}$

RAA $= \frac{\pi}{3}$

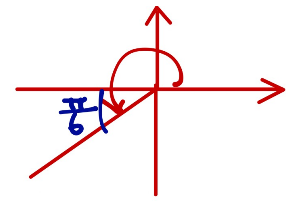


b) $cot\left(\frac{7\pi}{6}\right)$ [3 marks]

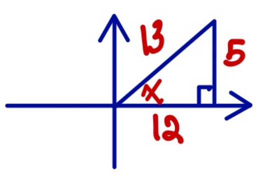
$= \cot\left(\frac{\pi}{6}\right)$

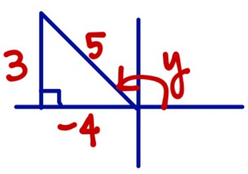
$= \sqrt{3}$

RAA $= \frac{\pi}{6}$



2. Determine the exact value of $\sin[2(x-y)]$, if $\sin(x) = \frac{5}{13}$; $0 \leq x \leq \frac{\pi}{2}$, $\cos(y) = -\frac{4}{5}$; $\frac{\pi}{2} \leq y \leq \pi$. [4 marks]





$\sin 2(x-y)$

$= 2\sin(x-y)\cos(x-y)$

$= 2(\sin x\cos y - \cos x\sin y)(\cos x\cos y + \sin x\sin y)$

$= 2\left[\left(\frac{5}{13}\right)\left(\frac{-4}{5}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)\right]\left[\left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)\right]$

$= \frac{3696}{4225}$

OR

$\sin 2(x-y)$

$= \sin(2x-2y)$

Then do it ☺

3. Determine a formula for $\cos(3\theta)$ in terms of $\cos(\theta)$. [3 marks]

$\cos(2\theta+\theta) = \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$

$= \cos\theta(\cos^2\theta - \sin^2\theta) - \sin\theta(2\sin\theta\cos\theta)$

$= \cos^3\theta - \cos\theta\sin^2\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$

$= \cos^3\theta - 3(1-\cos^2\theta)\cos\theta$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

COMMUNICATION

1. Explain how to convert from radians to degrees and from degrees to radians. [3 marks]

To convert from degrees to radians, take the number of the degrees and divide by 180° then multiply by π.
To convert from radians to degrees, take the number of radians and divide by π then multiply by 180°.

2. If you know the values of $\sin\left(\frac{x}{2}\right)$ and $\cos\left(\frac{x}{2}\right)$, how can you determine the value of $\sin(x)$ without calculating x? [4 marks]

Since we know that the double angle formula for sine is $\sin 2x = 2\sin x\cos x$, the formula for $\sin x$ would equal $2\sin\frac{x}{2}\cos\frac{x}{2}$. You then substitute the known values of $\sin\frac{x}{2}$ and $\cos\frac{x}{2}$ and calculate the value of $\sin x$.

Two marks will be awarded for proper mathematical forms throughout the assessment. [2 marks]