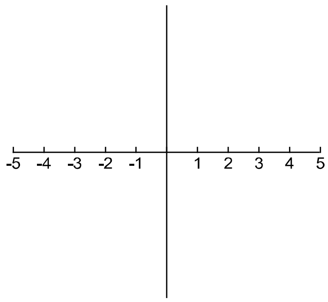
1.3 Equations and Graphs of Polynomial Functions

Use your graphing calculator to complete the following charts:

<u>Cubic Functions:</u>

1. $y = x^3$



Degree:

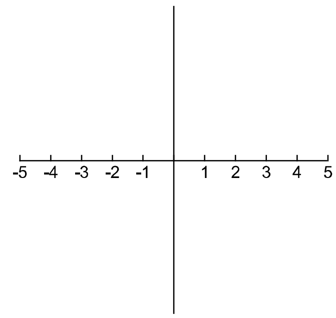
Sign of the lead coefficient:

Quadrants:

Roots/x-intercepts:

y-intercept:

2. $y = -x^3$



Degree:

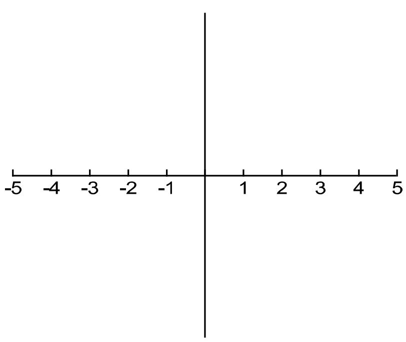
Sign of the lead coefficient:

Quadrants:

Roots/x-intercepts:

y-intercept:

3. $y = x(x + 2)(x-1)$



Degree:

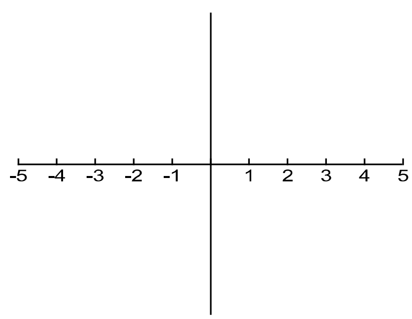
Sign of the lead coefficient:

Quadrants:

Roots/x-intercepts:

y-intercept:

4. $y = -(x - 3)(x + 2)(x - 1)$



Degree:

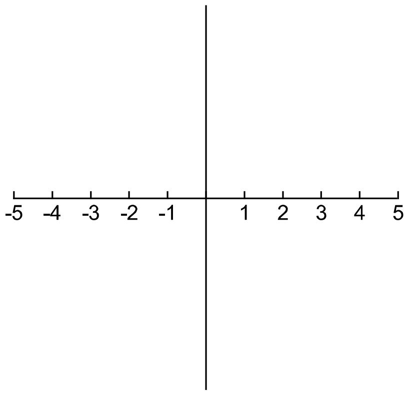
Sign of the lead coefficient:

Quadrants:

Roots/x-intercepts:

y-intercept:

5. $y = (x + 2)(x-4)^2$



Degree:

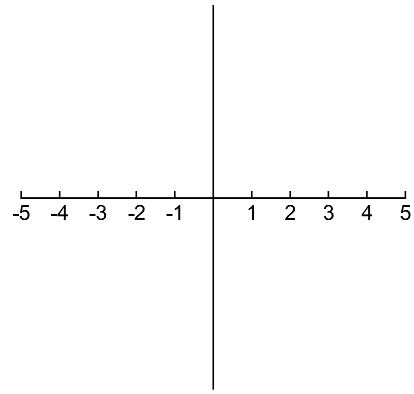
Sign of the lead coefficient:

Quadrants:

Roots/x-intercepts:

y-intercept:

6. $y = -(x + 3)^3$



Degree:

Sign of the lead coefficient:

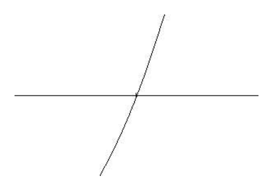
Quadrants :

Roots/x-intercepts:

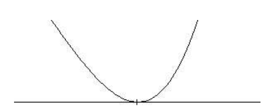
y-intercept:

Cubic functions

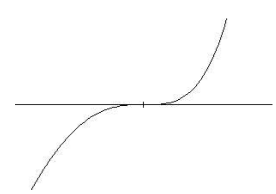
- Have a hill and a valley in the middle.
- Their degree is ________________
- The maximum number of zeros or x-intercepts is ________
- If the lead coefficient is positive, the graph starts in the ______ quadrant and ends in the _______ quadrant.
- If the lead coefficient is negative, the graph starts in the ______quadrant and ends in the ________ quadrant.



- If the exponent on the variable or bracket is 1 [i.e. x or (x+2)], the curve _____________________________ the x-intercept.

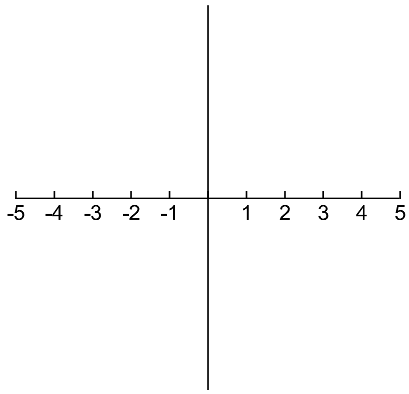


- If the exponent on the variable or bracket is squared [i.e. x^2 or $(x+2)^2$], the curve will______________________ the x-intercept.
- If the exponent on the variable or bracket is cubed [i.e. x^3 or $(x+2)^3$], the curve will appear flatter and have a "**point of inflection**" as it passes through the x-intercept.



Quartic Functions

1. $y = x^4$



Degree:

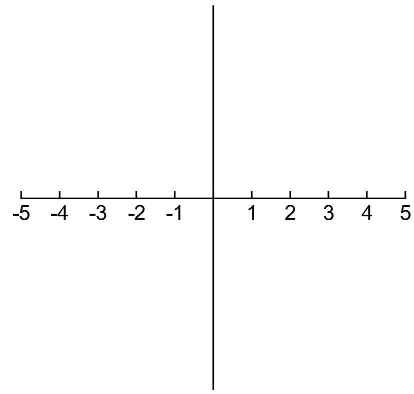
Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept:

2. $y = -x^4$



Degree:

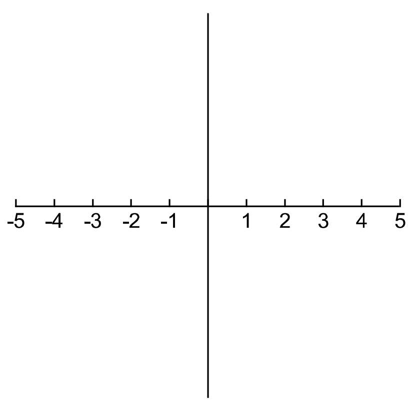
Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept:

3. $y = (x-2)(x-1)(x+3)(x-3)$



Degree:

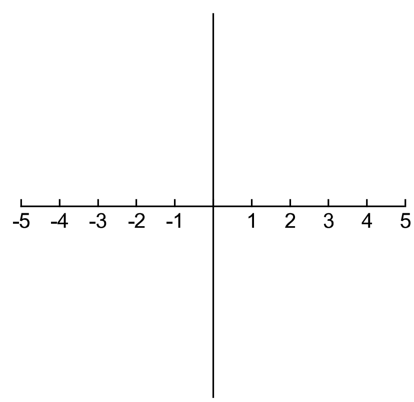
Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept:

4. $y = -(x + 2)^2(x-3)^2$



Degree:

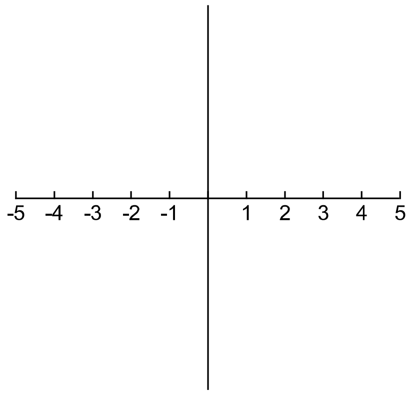
Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept :

5. $y = (x+2)^3(x-1)$



Degree:

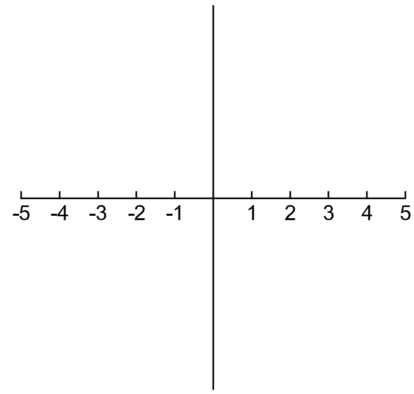
Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept

6. $y = (x+1)^2(x-2)(x+3)$



Degree:

Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept

Quartic Functions

There degree is ______Their shape is similar to a ____________________, or U-shaped. The maximum number of x-intercepts is ________.
If the lead coefficient /value in front of the bracket is positive, the curve opens __________ and it begins and ends in quadrants ________ ←→_______.
If the lead coefficient /value in front of the bracket is negative, the curve opens ___________. and it begins and ends in quadrants ________ ←→______.

- Note that if the variable or bracket has an exponent of 1, the curve__________________.
- If the variable or bracket has an even exponent, the curve_____________________ at the x-intercept.
- If the variable or bracket has an odd exponent greater that 1, the curve passes through the x-intercept with a slight____.

<u>Example 1</u>: Do not use a graphing calculator. Sketch the following graphs**:**

a) $y = (x^2 - 4)(x+2)$ (BE SURE TO FACTOR FULLY BEFORE GRAPHING!)

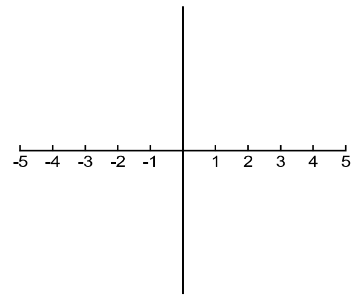
Degree:

Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept



b) $y = - x^3 + x$

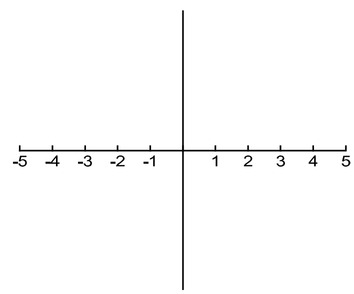
Degree:

Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept



c) $y = x(x - 3)^3$

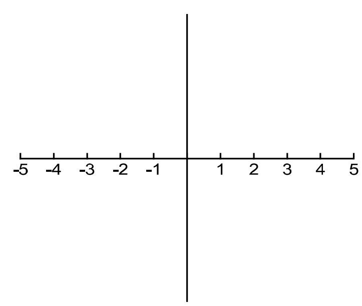
Degree:

Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept



d) $y = x(x-2)^2(x+3)^3$

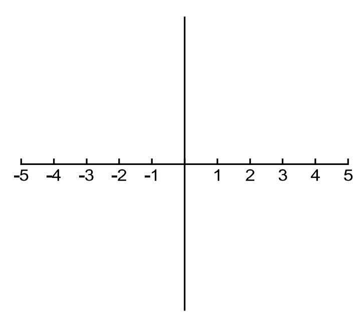
Degree:

Sign of the lead coefficient:

Quadrants :

Roots/x-intercepts:

y-intercept :



Sketching Polynomial Functions Summary

To sketch a polynomial function,

- Factor the polynomial fully, if it is not in factored form.
- Identify the degree. This will indicate the general shape of the curve.
- Look to see if the lead coefficient is positive or negative. This will help peg down the shape and quadrants.
- Find the x-intercepts. Let y = 0 solve for x.
- Find the y-intercept. Let x = 0 solve for y.
- Plot the intercepts and use the shape to sketch the curve.
- **Remember that if the variable or bracket has an even exponent, the curve "bounces" off the intercept**.
  However, if the variable or bracket has an odd exponent, the curve passes through the intercept in one of two ways:
  - If the odd exponent is 1, then the curve passes straight through the curve.
  - If the odd exponent is greater than 1, (i.e. 3, 5, 7. . .) then, the curve bends creating a slight shelf(saddle) at the x-intercept.

HOMEWORK:

By writing down the degree, the sign of the lead coefficient, quadrants/shape, the roots/x-intercepts, and the y-intercepts, Sketch each of the following in separate sheet of paper:

a) $y = x(x-2)(x+3)$

b) $y = -(x-1)(x+3)^2$

c) $y = -x(x-3)(x+2)(x+4)$

d) $y = (x+2)^3(x-3)$

e) $y = x(x+2)^2(x-2)$

f) $y = x^2(x+2)^3$

g) $y = (x-1)^2(x+1)^3(x-2)(x+2)$

h) $y = x^3 - x$

i) $y = -x^3 + 4x^2 - 3x$

j) $y = -(x^2 - 9)$

Exit card!

Sketch the graph of the following function: $y = (x-1)^2(x+1)^3(x-2)(x+2)$

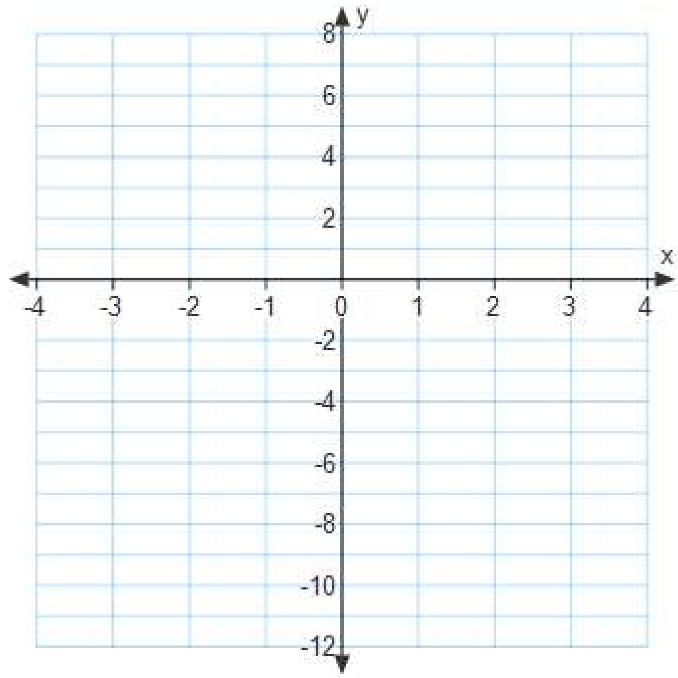
Degree:

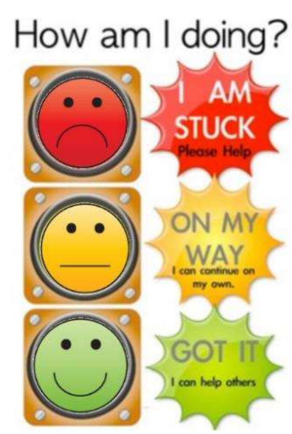
Sign of the lead coefficient:

Quadrants:

Roots/x-intercepts:

y-intercept :





Group Activity

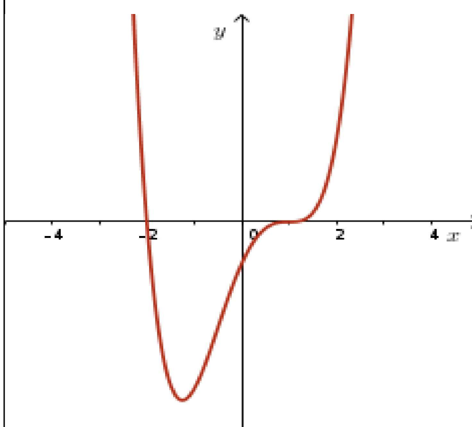
Names: ____

1. Match each equation to the most appropriate graph (There are more graphs than equations)

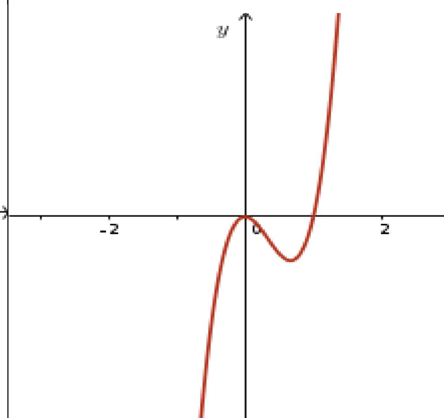
a. $f(x) = 2x(x-1)^2$ _____ b. $y = -(x-1)^2(x+2)^2$ _____ c. $y = -x^3(x+1)$ _____

d. $g(x) = (x+2)(x-1)^3$ _____ e. $g(x) = -2(x+1)^2(x-2)^3$ _____

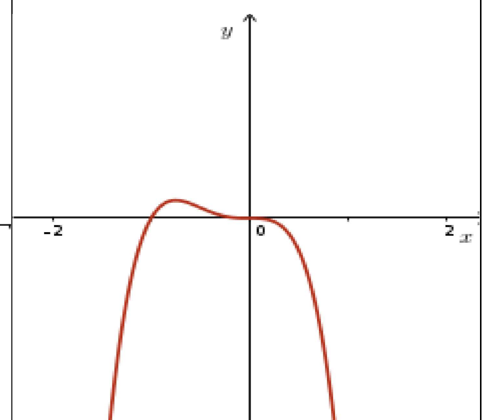
I.



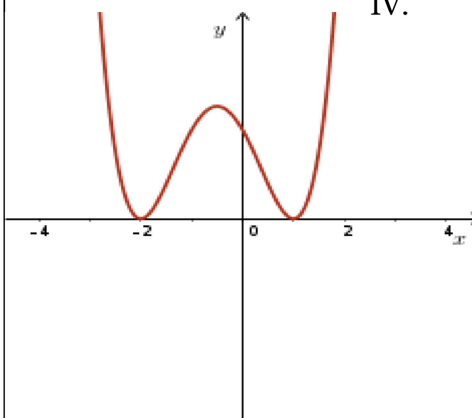
II.

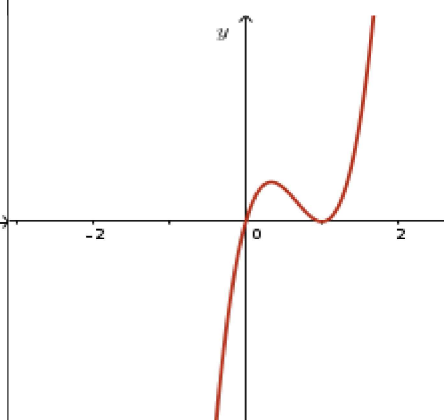


III.

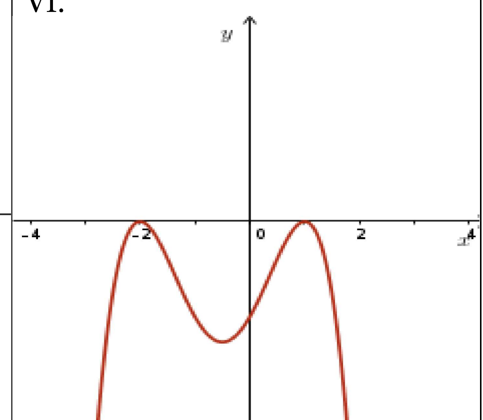


IV.

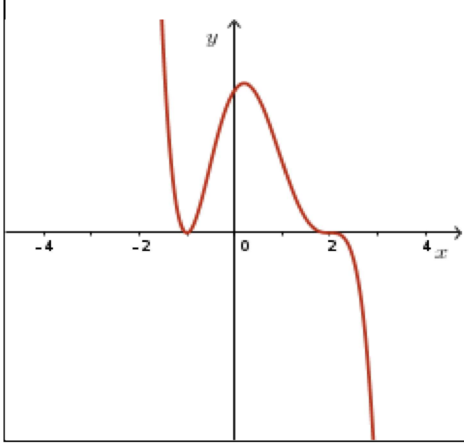




VI.

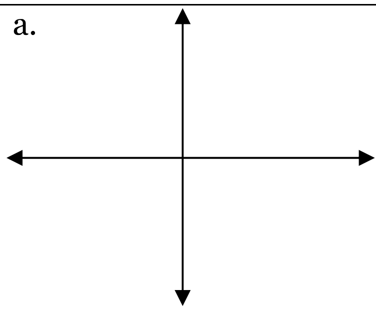
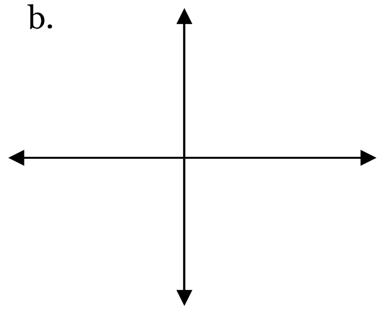
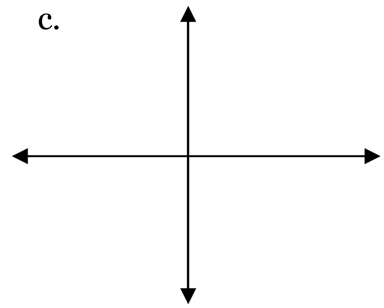


VII.



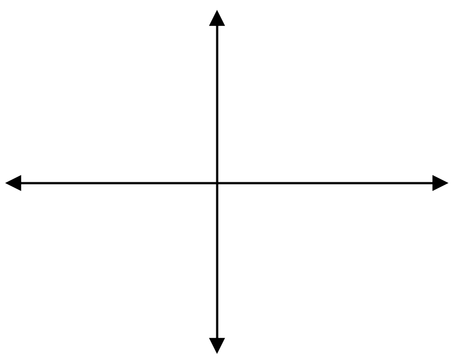
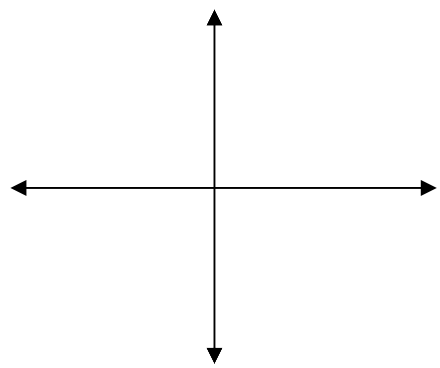
2. Sketch a possible graph of a polynomial function that satisfies the following conditions.

   (a) A quadratic function with a negative leading coefficient and a zero at $x=-5$ of order 2.
   (b) A 5th degree function with a positive leading coefficient, a zero at the origin of order 2, and a zero at $x=3$ of order 3.
   (c) A quartic function with a negative leading coefficient and two real zeros, $x=0$ and $x=3$ of order 2.

a.	b.	c.
		

3. Sketch a possible graph for each of the following functions.

   (a) $y=-0.5(3-x)(x+1)^3$ (b) $f(x)=-x(2x+3)(x-2)^2$

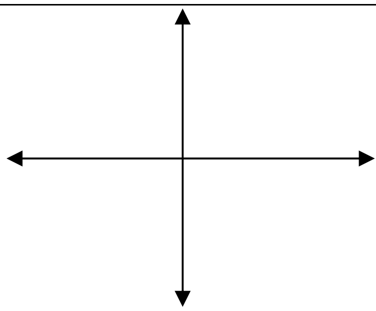
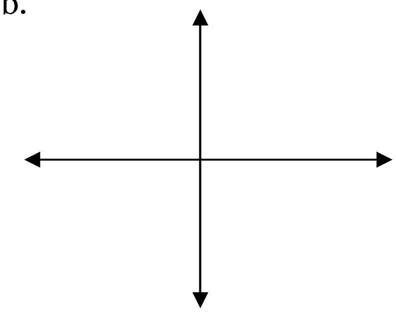
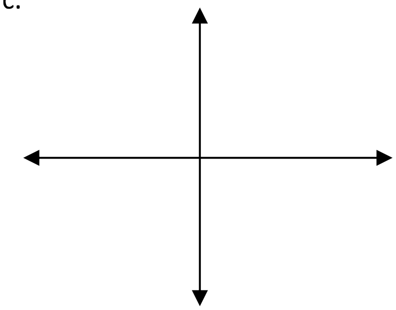
a.	b.
	

4. State the x-intercepts of each function and identify at which zeros the value of the function, f(x), changes sign.

   (a) $f(x)=-2(x-1)^3(x+4)^2$ (b) $f(x)=-2(x+3)(x-4)^2$

5. Identify the intervals in which the following polynomial functions are positive and the intervals in which they are negative. (Hint: sketch the graph of each function)

(a) $f(x) = (x-2)(x+1)(x+4)$ (b) $f(x) = -2x(x-4)(x+3)^2$ (c) $f(x) = -x^2(x-3)^3$

a. 	b. 	c. 

6. Determine a possible equation for the polynomial function y=f(x) shown below.