Solutions

Graph the below function. Clearly indicate at least 4 points for the graph, any asymptotes and intercepts.

$f(x) = -2\log_4(4x-12)+3$

$(x,y) \to \left(\frac{x}{4}+3, -2y+3\right)$

x	y
$\frac{1}{4}$	-1
1	0
4	1
16	2

x_{new}	y_{new}
3.1	5
3.25	3
4	1
7	-1

x-int: $y=0$

$0 = -2\log_4(4x-12)+3$

$2\log_4(4x-12) = 3$

$\log_4(4x-12) = \frac{3}{2}$

$4^{\frac{3}{2}} = 4x-12$

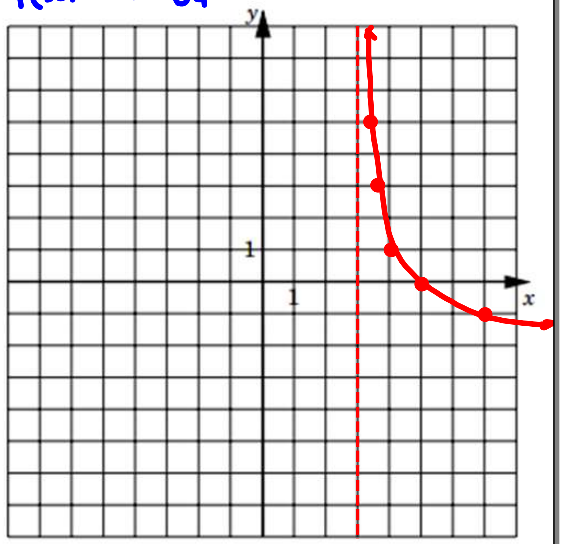
$\left(\sqrt{4}\right)^{3} = 4x-12$

$8 = 4x-12$

$4x = 20$

$\boxed{x=5}$

$f(x) = -2\log_4 4(x-3)+3$



y-int: $x=0$
None

V.A: $x=3$

If $\log_a(b) = \frac{1}{x}$ and $\log_b(a^2) = 12x^2$, solve for x.

$x \neq 0$

$2\log_b(a) = 12x^2$

$\log_b(a) = 6x^2$

$\frac{1}{\log_a(b)} = 6x^2$

$\frac{1}{\frac{1}{x}} = 6x^2$

$x = 6x^2$

$0 = 6x^2 - x$

$0 = x(\cancel{6-x})$ (6x-1)

$x=0$ inadmissible

$\boxed{\cancel{x=6}}$ x=1/6

Solutions

Solve for x without a calculator.

a) $3^{2x} - 4(\underbrace{3^{x+1}}) + 27 = 0$

$3^{2x} - 4(3^x \cdot 3^1) + 27 = 0$

$3^{2x} - 12(3^x) + 27 = 0$

Let $3^x = A$

$A^2 - 12A + 27 = 0$

$(A-9)(A-3) = 0$

$A = 9$, $A = 3$

$3^x = 3^2$, $3^x = 3$

$\boxed{x = 2}$; $\boxed{x = 1}$

b) $3\log_x \sqrt[3]{x^4 - 15} = \log_5(1)$

Res. $x^4 - 15 > 0$ ①

$x > \sqrt[4]{15} \approx 1.96$

$x > 0, x \neq 1$ ②

① ∩ ② : $x > \sqrt[4]{15}$

$3\log_x (x^4 - 15)^{\frac{1}{3}} = 0$

$3(\cancel{\tfrac{1}{3}}) \log_x (x^4 - 15) = 0$

$x^0 = x^4 - 15$

$1 = x^4 - 15$

$x^4 = 16$

$x = \pm 2$

inadmissible

$\boxed{x = 2}$

c)★ $x\log 5 + \log 6 = x + \log(1 + 2^x)$

$\log 5^x + \log 6 = x + \log(1 + 2^x)$

$\log 6(5^x) - \log(1 + 2^x) = x$

$\log \frac{6(5^x)}{1 + 2^x} = x$

$10^x = \frac{6(5^x)}{1 + 2^x}$ $\div 5^x$

$\frac{10^x}{5^x} = \left(\frac{10}{5}\right)^x$

$2^x = \frac{6}{1 + 2^x}$

$2^x(1 + 2^x) = 6$

$2^x + 2^{2x} = 6$

$2^{2x} + 2^x - 6 = 0$

Let $2^x = A$

$A^2 + A - 6 = 0$

$(A+3)(A-2) = 0$

$A = -3$ inadmissible

$A = 2$

$2^x = 2$

$\boxed{x = 1}$

Solutions

Simplify. State any restrictions on the variables.

$$\log_3\left(\frac{\sqrt{x}}{x^3}\right)+\log_3\left(\sqrt{x}\right)^3-\log_3\left(\sqrt[3]{x^2}\right)$$

Res. $x>0$

$= \log_3\left(x^{\frac{1}{2}-3}\right)+\log_3 x^{\frac{3}{2}}-\log_3 x^{\frac{2}{3}}$

$= \log_3 \dfrac{x^{-\frac{5}{2}}\cdot x^{\frac{3}{2}}}{x^{\frac{2}{3}}}$

$= \log_3 x^{-1-\frac{2}{3}}$

$= \log_3 x^{-\frac{5}{3}}$

Determine the inverse of $g(x)=-2\log_5(x-4)-6$.

$x=-2\log_5(y-4)-6$

$\dfrac{x+6}{-2}=\log_5(y-4)$

$5^{\frac{x+6}{-2}}=y-4$

$4+5^{-\frac{1}{2}(x+6)}=y$

$\therefore g^{-1}(x)=5^{-\frac{1}{2}(x+6)}+4$

Determine the domain of $f(x) = \log_{2022}\left(x^3 - 4x^2 - 15x + 18\right)$.

$$x^3 - 4x^2 - 15x + 18 > 0$$

$$(x-1)(x^2-3x-18) > 0$$

$$(x-1)(x-6)(x+3) > 0$$

$$\begin{array}{c|cccc} 1 & 1 & -4 & -15 & 18 \\ & \downarrow & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

$$\begin{array}{ccccccccc} & - & & + & & - & & + & \\ -\infty & & -3 & & 1 & & 6 & & \infty \end{array}$$

$$D_f = (-3, 1) \cup (6, \infty)$$

The 1906 earthquake in San Francisco had a magnitude of 8.3 on the Richter scale. At the same time in Japan there was an earthquake of magnitude 4.8 which caused only minor damage. How many times more intense was the San Francisco earthquake than the Japan. earthquake?

$$M_{SF} - M_J = \log\left(\frac{I_{SF}}{I_J}\right)$$

$$8.3 - 4.8 = \log\left(\frac{I_{SF}}{I_J}\right)$$

$$3.5 = \log\left(\frac{I_{SF}}{I_J}\right)$$

$$10^{3.5} = \frac{I_{SF}}{I_J}$$

$$3162.3 = \frac{I_{SF}}{I_J}$$

Solutions

A baby cries at a loudness of 70dB. When the baby screams, the resulting sound is 20 times louder. Calculate the loudness of the scream in decibels to the nearest whole number.

$$L_s - L_c = 10 \log\left(\frac{I_s}{I_c}\right)$$

$$L_s - 70 = 10 \log(20)$$

$$L_s \doteq 70 + 13$$

$$L_s \doteq 83 \text{ dB}$$