Unit 2-Review SOLUTIONS

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided.

1. Given the function $f(x)=\dfrac{x^a+k}{x^b+m}$, a linear oblique asymptote will occur when: C

A) $a \geq b$ B) $b > a$ C) $a-b=1$ D) $a-b=2$ E) none of the above

2. Which of the following statements is **true** if $f(x)$ is the reciprocal of a quadratic with x intercepts at $x=\pm 4$ and a vertex of $(0,8)$? A

A) $f(x)$ has two vertical asymptotes B) $\frac{1}{f(x)}$ has a local maximum at $\left(0,\frac{1}{8}\right)$

C) $\frac{1}{f(x)}$ has a local minimum at $\left(0,\frac{-1}{8}\right)$ D) $\frac{1}{f(x)} > 0$ when $x \in (-\infty,\infty)$

3. Given $f(x)=\dfrac{(6-2x)}{(x^2-4)(x-3)}$ which of the following is **true**? B

$= \dfrac{-2\cancel{(x-3)}}{(x^2-4)\cancel{(x-3)}} = \dfrac{-2}{x^2-4}$

A) $f(x)$ crosses at least one of its asymptotes B) $f(x)$ has a hole at $\left(3,\frac{-2}{5}\right)$

C) $f(x)$ has a horizontal asymptote at $y=-2$ D) $f(x)$ has 3 vertical asymptotes

4. Which of the following functions has an asymptote that passes through the origin? D

A) $f(x)=\dfrac{x(x-4)(x-9)}{(x-8)(x-3)^2}$ B) $f(x)=\dfrac{(x^2-4)(2x-9)}{(x)(2x-3)}$, $x \neq -1$

C) $f(x)=\dfrac{(x-6)(x+4)}{x^3-8}$ D) Both B and C

5. Which of the following function(s) cross at least one of their asymptotes? C

A) $f(x)=\dfrac{(x+1)(x+5)(2x-9)}{2x^2-7x-9} = \dfrac{\cancel{(x+1)}(x+5)\cancel{(2x-9)}}{\cancel{(2x-9)}\cancel{(x+1)}}$ B) $f(x)=\dfrac{1}{x^2+16}$

C) $f(x)=\dfrac{x(x-4)(x-9)}{x^3-14x^2+57x-72}$ D) All of the above

6. Complete the table below given the following function $f(x)=\dfrac{-x(x-3)(x-4)}{(2x-8)(x+2)(x+5)} = \dfrac{-x(x-3)\cancel{(x-4)}}{2\cancel{(x-4)}(x+2)(x+5)}$

x-intercept(s), if any.	0 and 3
y-intercept, if any.	0
Equation of vertical asymptote(s), if any.	$x=-2$ and $x=-5$
Equation of horizontal or oblique asymptote, if any.	$y=-\frac{1}{2}$

7. Determine the equation of the oblique asymptote given $f(x) = \frac{2x^2+9x-12}{x+4}$

$$\begin{array}{r|rrr} -4 & 2 & 9 & -12 \\ & & -8 & -4 \\ \hline & 2 & 1 & -16 \end{array}$$

$f(x) = 2x+1 - \frac{16}{x+4}$

$\therefore$ O.A is $y = 2x+1$

8. Create the equation of a function $g(x)$ with the following properties:

B) x-intercept of $\frac{1}{4}$, y-intercept of $\frac{-1}{2}$, vertical asymptote of $x = \frac{-2}{3}$ and horizontal asymptote of $y = \frac{4}{3}$.

$$g(x) = \frac{\frac{4}{3}\left(x-\frac{1}{4}\right)}{b\left(x+\frac{2}{3}\right)}$$

$$= \frac{4\left(x-\frac{1}{4}\right)}{3\left(x+\frac{2}{3}\right)}$$

$$= \frac{4x-1}{3x+2}$$

$$\frac{\frac{4}{3}\left(0-\frac{1}{4}\right)}{b\left(0+\frac{2}{3}\right)} = -\frac{1}{2}$$

$$\frac{-\frac{1}{3}}{\frac{2}{3}b} = -\frac{1}{2}$$

$$-\frac{1}{3} = -\frac{1}{3}b$$

$$1 = b$$

9. For the function, $g(x) = \frac{mx-3}{4-nx}$, find the values of m and n such that $g(x)$ has a vertical asymptote when $x = 6$ and a horizontal asymptote at $y = -3$.

V.A at $x=6$: $4-n(6) = 0$

$4 = 6n$

$\frac{2}{3} = n$

H.A at $y=-3$: $\frac{m}{-n} = -3$

$\frac{m}{-\frac{2}{3}} = -3$

$m = 2$

10. Sketch the graph of $f(x) = \frac{x^3-x^2-4x+4}{x^2+x-20}$

$f(x) = \frac{x^2(x-1)-4(x-1)}{(x+5)(x-4)} = \frac{(x+2)(x-2)(x-1)}{(x+5)(x-4)}$

x-inter: $-2, 2, 1$ y-inter: $-\frac{1}{5}$

V.A: $x=-5 \Rightarrow$ As $x \to -5^-$, $f(x) \to -\infty$

As $x \to -5^+$, $f(x) \to \infty$

$x = 4 \Rightarrow$ As $x \to 4^-$, $f(x) \to -\infty$

As $x \to 4^+$, $f(x) \to \infty$

O.A:

$$\begin{array}{r|l} & \quad x - 2 \\ \hline x^2+x-20 & x^3 - x^2 - 4x + 4 \\ & -(x^3 + x^2 - 20x) \\ & \quad -2x^2 + 16x + 4 \\ & \quad -(-2x^2 - 2x + 40) \\ & \qquad 18x - 36 \end{array}$$

$f(x) = x-2 + \frac{18x-36}{x^2+x-20}$

Sub. $x=100$: $\frac{1800-36}{10000+100-20} > 0$ (above)

Sub. $x=-100$: $\frac{-1800-36}{10000-100-20} < 0$ (below)

$\therefore$ As $x \to \infty$, $f(x) \to x-2$ (above)

As $x \to -\infty$, $f(x) \to x-2$ (below)

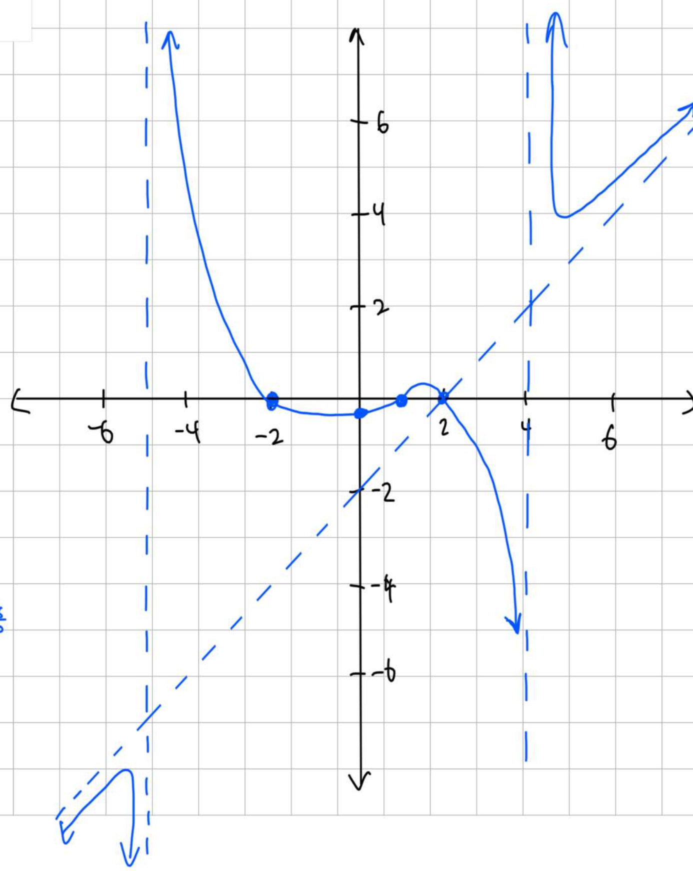
cross-over: $x-2 = x-2 + \frac{18x-36}{x^2+x-20}$

$0 = 18x-36$

$2 = x$

Sub $x=2$: $y = 2-2 = 0$

$\therefore$ Cross-over at $(2, 0)$



11. Determine any points of intersection for the function $f(x) = -x^2 - 5x - 6$ and its reciprocal function. Leave answer(s) in exact form where necessary.

P.O.I. at $y=1$ and $y=-1$

Sub. $y=1$: $1 = -x^2 - 5x - 6$

$0 = -x^2 - 5x - 7$

$0 = x^2 + 5x + 7$

No roots

Sub. $y=-1$: $-1 = -x^2 - 5x - 6$

$0 = -x^2 - 5x - 5$

$0 = x^2 + 5x + 5$

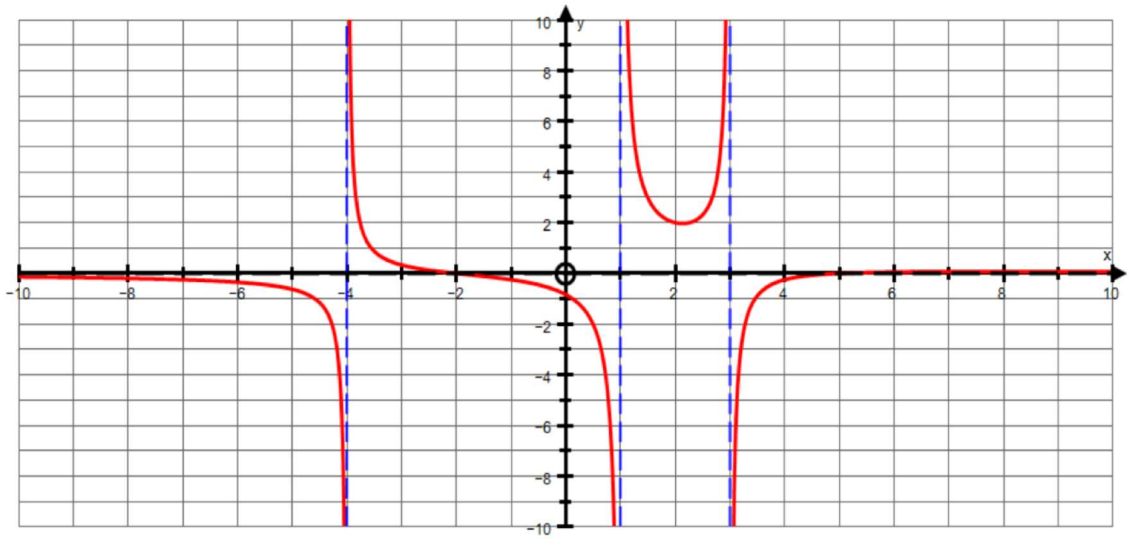
$$x = \frac{-5 \pm \sqrt{25 - 4(1)(5)}}{2} = \frac{-5 \pm \sqrt{5}}{2}$$

$\therefore$ P.O.I are $\left(\frac{-5+\sqrt{5}}{2}, -1\right)$ and $\left(\frac{-5-\sqrt{5}}{2}, -1\right)$

12. Given the graph of the rational function $f(x)$ below solve $\frac{4x^2-20x}{f(x)} \geq 0$

Please note two things regarding $f(x)$:

a) The y intercept is $\left(0, \frac{-5}{6}\right)$
b) $f(x)$ crosses its horizontal asymptote at $x = -2$ and $x = 5$
c) The degree of the numerator of $f(x)$ is 2 and the degree of the denominator of $f(x)$ is 3.



$$f(x) = \frac{k(x+2)(x-5)}{(x+4)(x-1)(x-3)}$$

Sub. $\left(0, -\frac{5}{6}\right)$: $-\frac{5}{6} = \frac{k(2)(-5)}{(4)(-1)(-3)}$

$-\frac{5}{6} = \frac{-10k}{12}$

$-60 = -60k$

$1 = k$

$$\therefore f(x) = \frac{(x+2)(x-5)}{(x+4)(x-1)(x-3)}$$

$$\frac{4x^2-20x}{\frac{(x+2)(x-5)}{(x+4)(x-1)(x-3)}} \geq 0$$

$$\frac{4x\cancel{(x-5)}(x+4)(x-1)(x-3)}{(x+2)\cancel{(x-5)}} \geq 0$$

Hole at $x=5$ V.A: $x=-2$

x-inter: $0, -4, 1, 3$

(−) (+) (−) (+) (−) (+) (+)

−4 −2 0 1 3 5

$\therefore x \in [-4, -2) \cup [0, 1] \cup [3, 5) \cup (5, \infty)$

13. Solve

a) $\dfrac{x^3+7x^2+12x}{x^2+9x+20} \geq \dfrac{x+1}{x^2-3x-4} - \dfrac{2x+5}{2x^2-3x-20}$

b) $\dfrac{x^3+2x^2-46x-125}{(x^2+5x-6)(x+3)} \geq \dfrac{-1}{(x+6)(x+3)}$

$\dfrac{x(\cancel{x+4})(x+3)}{(x+5)(\cancel{x+4})} - \dfrac{\cancel{x+1}}{(x-4)(\cancel{x+1})} + \dfrac{\cancel{2x+5}}{(\cancel{2x+5})(x-4)} \geq 0$

$\dfrac{x(x+3)}{x+5} - \dfrac{1}{x-4} + \dfrac{1}{x-4} \geq 0$

$\dfrac{x(x+3)}{x+5} \geq 0$

Hole @ $x=-\frac{5}{2}$, $x=-4$, $x=-1$ and $x=4$

Roots: 0 and -3

V.A at $x=-5$

(−) (+) (+) (−) (−) (−) (+) (+)

-5 -4 -3 $-\frac{5}{2}$ -1 0 4

$\therefore x \in (-5,4) \cup (-4,-3] \cup [0,4) \cup (4,\infty)$

$\dfrac{x^3+2x^2-46x-125}{(x+6)(x-1)(x+3)} + \dfrac{1}{(x+6)(x+3)} \geq 0$

$\dfrac{x^3+2x^2-46x-125+(x-1)}{(x+6)(x-1)(x+3)} \geq 0$

$\dfrac{x^3+2x^2-45x-126}{(x+6)(x-1)(x+3)} \geq 0$

$\begin{array}{r|rrrr} -3 & 1 & 2 & -45 & -126 \\ & & -3 & 3 & 126 \\ \hline & 1 & -1 & -42 & 0 \end{array}$

$\dfrac{(x+3)(x^2-x-42)}{(x+6)(x-1)(x+3)} \geq 0$

$\dfrac{(\cancel{x+3})(x-7)(\cancel{x+6})}{(\cancel{x+6})(x-1)(\cancel{x+3})} \geq 0$

$\dfrac{x-7}{x-1} \geq 0$

Hole @ $x=-3$ and $x=-6$

Root: 7 V.A: $x=1$

(+) (+) (+) (−) (+)

-6 -3 1 7

$\therefore x \in (-\infty,-6) \cup (-6,-3) \cup (-3,1) \cup [7,\infty)$

14. Determine the perimeter of the quadrilateral, rounded to the nearest tenth of a unit, created by the intersections of $h(x)$ and its reciprocal graph $f(x)$. Given: $h(x) = -2(x+4)(x-6)$ and

$f(x) = \dfrac{1}{-2x^2+4x+48}$.

P.O.I when $y=1$ and $y=-1$

Sub. $y=1$: $1 = -2x^2+4x+48$

$0 = -2x^2+4x+47$

$0 = 2x^2-4x-47$

$x = \dfrac{4 \pm \sqrt{16-4(2)(-47)}}{4}$

$= \dfrac{4 \pm \sqrt{392}}{4}$

$= \dfrac{4 \pm 14\sqrt{2}}{4}$

$= \dfrac{2 \pm 7\sqrt{2}}{2}$

$\doteq 5.95$ or -3.95

Sub. $y=-1$: $-1 = -2x^2+4x+48$

$0 = -2x^2+4x+49$

$0 = 2x^2-4x-49$

$x = \dfrac{4 \pm \sqrt{16-4(2)(-49)}}{4}$

$= \dfrac{4 \pm \sqrt{408}}{4}$

$= \dfrac{4 \pm 2\sqrt{102}}{4}$

$= \dfrac{2 \pm \sqrt{102}}{2}$

$= 6.05$ OR -4.05

Points on quadrilateral: A(-3.95, 1), B(5.95, 1), C(-4.05, -1), D(6.05, -1)

$AB = \sqrt{(5.95+3.95)^2+(1-1)^2} \doteq 9.9$

$BD = \sqrt{(5.95-6.05)^2+(1+1)^2} \doteq 2$

$CD = \sqrt{(6.05+4.05)^2+(-1+1)^2} \doteq 10.1$

$CA = \sqrt{(-4.05+3.95)^2+(-1-1)^2} \doteq 2$

$\therefore P = 9.9 + 2 + 10.1 + 2$

$= 24$ units

15. Describe what is known about the equation of a rational function with vertical asymptotes at $x = 5$ and $x = -3$ and a horizontal asymptote of $y = 0$.

It's a rational function with a numerator that is degree 0 or 1 and a denominator of degree 2.

$$f(x) = \frac{k}{(x-5)(x+3)} \quad \text{OR} \quad f(x) = \frac{k(x-a)}{(x-5)(x+3)}$$

16. The concentration of a toxic chemical in a spring-fed lake is given by the equation $C(x) = \dfrac{60x}{x^2+3x+6}$, where C is given in grams per litre and x is the time in days. Find the instantaneous rate of change at 4 days.

$$C(4.001) = \frac{60(4.001)}{(4.001)^2+3(4.001)+6}$$

$$\doteq 7.058304459$$

$$C(4) = \frac{60(4)}{(4)^2+3(4)+6} \doteq 7.058823529$$

$$\text{I.R.O.C} = \frac{C(4.001)-C(4)}{0.001}$$

$$\doteq \frac{7.058304459-7.058823529}{0.001}$$

$\doteq -0.52$ grams in litre/day

17. Sketch the graph of $y = \dfrac{x^3-6x^2+32}{x^3-x^2-4x+4}$.

-2	1	-6	0	32
		-2	16	-32
	1	-8	16	0

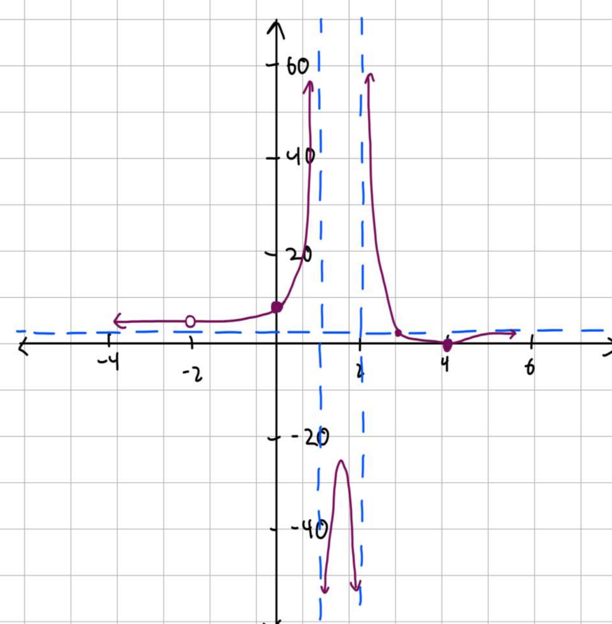
$$y = \frac{(x+2)(x^2-8x+16)}{x^2(x-1)-4(x-1)}$$

$$= \frac{\cancel{(x+2)}(x-4)^2}{(x-2)\cancel{(x+2)}(x-1)}$$

$$= \frac{(x-4)^2}{(x-2)(x-1)}$$

Hole @ $(-2, f(-2)) = (-2, 3)$

Roots: 4 y-intercept: 8



V.A: $x = 2$ and $x = 1$

As $x \to 2^-$, $f(x) \to -\infty$

As $x \to 2^+$, $f(x) \to +\infty$

As $x \to 1^-$, $f(x) \to +\infty$

As $x \to 1^+$, $f(x) \to -\infty$

H.A: $y = 1$

As $x \to \infty$, $y \to 1$ (below)

As $x \to -\infty$, $y \to 1$ (above)

$f(100) = \frac{9216}{9702} < 1$

$f(-100) = \frac{10816}{10302} > 1$

Crossovers: $1 = \dfrac{(x-4)^2}{(x-2)(x-1)}$

$$x^2 - 3x + 2 = x^2 - 8x + 16$$

$$5x = 14$$

$$x = 14/5$$

cross-overs @ $(14/5, 1)$

18. Use the information below to sketch the function.

- There is a horizontal asymptote at $f(x) = -2$.
- The Domain of the function is $D:\{x \neq -3, x \neq 4, x \in \Re\}$ V.A: $x = -3$
- The range of the function is $R:\{y > -8, y \in \Re\}$
- $f(0) = \frac{28}{81}$
- There is a hole at $(4, \frac{9}{40})$
- One of the factors of the numerator is $(x^2 + 5x - 6) = (x+6)(x-1)$ Roots: -6, 1
- $f(x) > 0$ *when* $-6 < x < -3,\ -3 < x < 1,\ 1 < x < 4,\ 4 < x < 7$
- $f(7) = 0$ Root: 7