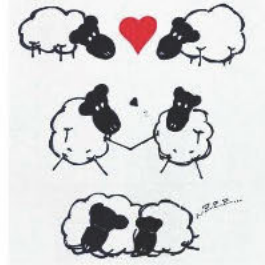


## Modelling Linear Relations

## RelationSheep

**Recall:** What are the four different ways to represent a relationship in math?

- 1) using words
- 2) using an equation
- 3) using a table of values
- 4) using a graph



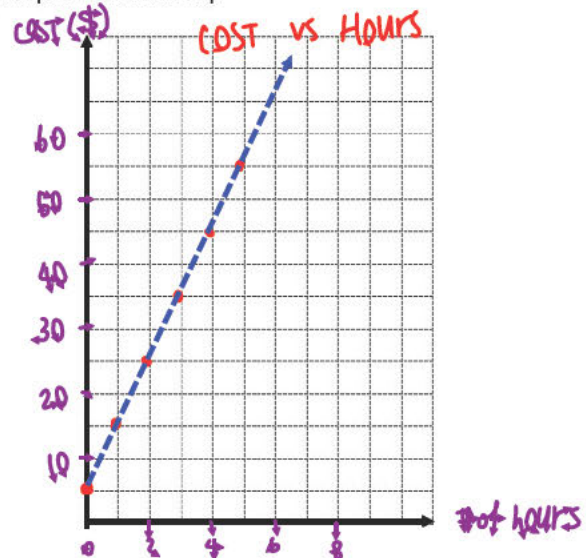
There are many connections between these representations! Let us explore using some examples!

**Example 1:** The cost to have a student cut your lawn includes a flat fee of \$5 for booking the student, plus \$10 per hour.

a) Complete the following table of values

Number of Hours	Cost	First differences
0	5	
1	15	$15 - 5 = 10$
2	25	$25 - 15 = 10$
3	35	$35 - 25 = 10$
4	45	$45 - 35 = 10$
5	55	$55 - 45 = 10$

b) Graph the relationship



c) Is this a linear relation? Explain using 2 reasons.

Yes  
 ① graphs a line  
 ② First differences are the same

d) Determine an equation that models the relationship between the number of hours and cost.

Let  $C$  be the cost

Let  $n$  be the number of hours

$$C = 10n + 5$$

e) Find two points on the graph and calculate the slope. What is the relationship between the value of the slope and the first difference? Do you see this value in the equation?

$(0, 5)$   
 $(1, 15)$   
 $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 5}{1 - 0} = \frac{10}{1} = 10$   
 $\rightarrow$  slope is equal to the 1st difference  
 $\rightarrow$  slope is the coefficient of  $n$

f) What is the initial cost of the relation? Do you see this value in the equation and the graph?

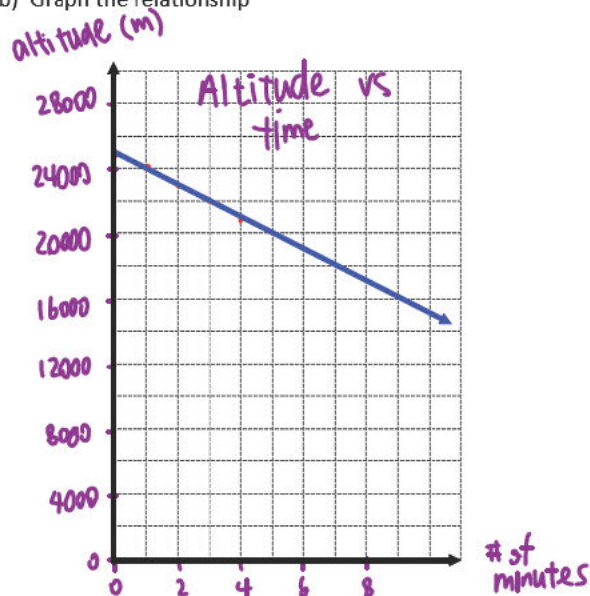
initial cost = \$5  
 $\rightarrow$  initial cost is the constant  
 $\rightarrow$  initial cost is the "y-intercept"

**Example 2:** An airplane is flying at an altitude of 25000 m, and is descending at a rate of 1000 m per minute.

a) Complete the following table of values

Number of Minutes	Altitude	First differences
0	25000	
1	24000	-1000
2	23000	-1000
3	22000	-1000
4	21000	-1000
5	20000	-1000

b) Graph the relationship



c) Is this a linear relation? Explain using 2 reasons.

**Yes!** 1) graphs a line  
2) the first differences are constant

d) Determine an equation that models the relationship between the number of minutes and altitude.

Let A be the altitude of the plane  
Let n be the number of minutes

$$A = 25000 - 1000n \quad \text{or} \\ A = -1000n + 25000$$

e) Find two points on the graph and calculate the slope. What is the relationship between the value of the slope and the first difference? Do you see this value in the equation?

$(0, 25000)$   
 $(5, 20000)$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20000 - 25000}{5 - 0} = \frac{-5000}{5} = -1000 \frac{\text{m}}{\text{min}} //$$

→ slope is the 1st difference

f) What is the initial altitude of the relation? Do you see this value in the equation and the graph?

initial = 25000 m **Yes! & yes!**

h) What is the altitude of the airplane after 14 minutes?

$$A = ? \quad A = -1000n + 25000$$

$$n = 14 \quad A = -1000(14) + 25000$$

$$A = -14000 + 25000$$

$$A = 11000 \text{ m} //$$

i) How long does it take for the plane to reach the ground?

$$A = ? \quad A = -1000n + 25000$$

$$A = 0 \quad 0 = -1000n + 25000$$

$$\underline{1000n = 25000}$$

$$\underline{1000} \quad \underline{1000}$$

$$n = 25 \text{ min} //$$



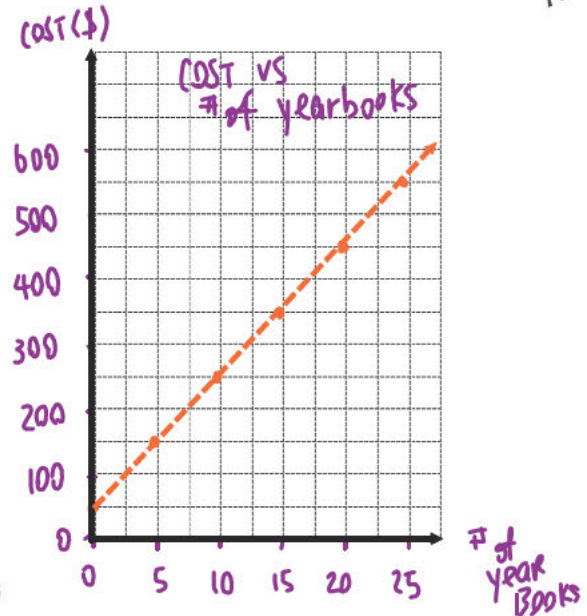
**Example 3:** A yearbook company charges an initial fee of \$50 to produce the layout of the book, and then \$20 per book printed.



a) Complete the following table of values

Number of yearbooks	Cost	First differences
0	50	
5	150	100
10	250	100
15	350	100
20	450	100
25	550	100

b) Graph the relationship



c) Is this a linear relation? Explain using 2 reasons.

**YES!** 1) constant first differences  
2) graphs a line

d) Determine an equation that models the relationship between the number of yearbooks and cost.

Let  $C$  be the cost

Let  $n$  be the number of year books

$$C = 20n + 50$$

e) How much will it cost if 38 yearbooks need to be printed?

$$\begin{aligned} C &= ? & C &= 20n + 50 & \rightarrow C &= 810 // \\ n &= 38 & C &= 20(38) + 50 \end{aligned}$$

f) How many yearbooks are printed if the total cost is \$450?

$$\begin{aligned} C &= 450 & C &= 20n + 50 & \rightarrow 400 &= 20n \\ n &= ? & 450 &= 20n + 50 & n &= 20 // \end{aligned}$$

g) Find two points on the graph and calculate the slope. Is the slope equal to the first difference in this example? Why do you think that is?

$$\begin{aligned} &(5, 150) \\ &(10, 250) \end{aligned} \quad \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{250 - 150}{10 - 5} = \frac{100}{5} = 20 \text{ /book}$$

$\rightarrow$  not equal to 1st difference because the increment of "n" is 5

h) Do you see where the value of the slope is in your equation? What can you conclude about the slope and the rate of change?

The coefficient for  $n$ , slope is rate of change

i) What is the initial value of cost of this relation? Do you see this value in your equation and graph?

initial = 50, equation  $\Rightarrow$  constant  
graph  $\Rightarrow$  "y"-intercept

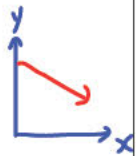


Let's summarize!

slope



x	y
0	
1	
2	
3	



$$y = ax + b$$

Model	Rate of Change	Initial/Fixed Value
Table of Values (Numeric Model)	- first differences if the x-values have an increment of 1	- when x is equal to zero
Graph (Graphical Model)	- slope $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$	- y-intercept (where line touches the y-axis)
Equation (Algebraic Model)	- coefficient of x "a" value	- constant "b" value

**Practice:** Write an equation to represent the following relations. Don't forget the "let" statements!

- a) Angela is a mechanic. She charges a fee of \$40 to examine the car, and then \$55 per hour that it takes her to fix the car.

Let C be the cost

$$C = 55n + 40$$

Let n be the number of hours

- b) The cost of a large pizza with sauce and cheese is \$14.00. Additional toppings costs \$2.00 each.

Let C be the cost

$$C = 2n + 14$$

Let n be the number of toppings

- c) Dana has a part time job at The Gap. She earns \$11.50 per hour she works.

Let P be her pay

$$P = 11.50n$$

Let n be the number of hours she works

- d) A taxi costs \$3.50 to hire the taxi, and \$0.75 per kilometer travelled.

Let C be cost

$$C = 0.75n + 3.5$$

Let n be the number of kilometers travelled

- e) A parking lot charges \$3.00 per hour for parking.

Let C be cost

$$C = 3n$$

Let n be the number of hours



encourage mint