Unit 1 - Algebra

Chapter 4 – Equations

MPM1D

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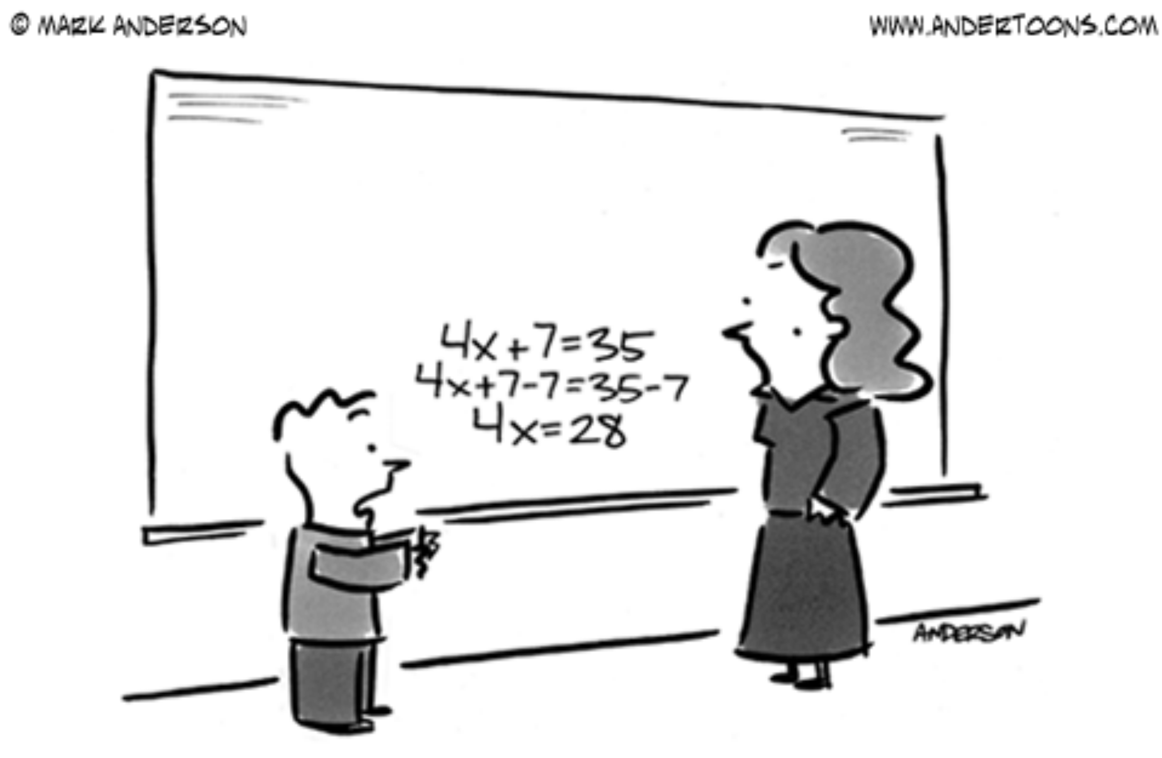


"You *knew* X was 7 the *whole time* and you *never said anything*?!"

Chapter 4 - Equations

4.1 Solving Simple Equations

Part 1: Do It Now

Byron spent a total of $11 on two magazines. The cost of one magazine is $5. You can use an equation to find the cost of the other magazine.

a) Write an equation to represent this situation

$x+5=11$

b) What value of the variable makes the equation true? Describe the math operations you used to find the value?

$x=6$ makes the equation true.

You can calculate this value by subtracting 5 from 11.

Part 2: Keeping Equations Balanced

An equation is still true if you apply identical operations to both sides

$$5 = 5$$

$$5 + 1 = 5 + 1$$

If I add 1 to each side; both sides are still equal

$$5 \times 2 = 5 \times 2$$

If I multiply both sides by 2; both sides are still equal

Keeping Equations Balanced

Solve for x (what value of x makes the equation true?)

when solving an equation, the goal is to isolate the variable

$$x + 4 = 12$$

$$x + 4 - 4 = 12 - 4$$

Subtract 4 from the left because you will be left with just x by itself because 4 - 4 = 0. That means you will have to subtract 4 from the right as well to keep the equation equivalent

$$x = 12 - 4$$

$$x = 8$$

Part 3: Solving Simple Equations Examples

1) $x - 2 = 8$

$x - 2 + 2 = 8 + 2$

$x = 8 + 2$

$x = 10$

2) $x + 7 = 5$

$x + 7 - 7 = 5 - 7$

$x = 5 - 7$

$x = -2$

3) $-4 + x = -1$

$-4 + 4 + x = -1 + 4$

$x = -1 + 4$

$x = 3$

4) $10 + x = 5$

$10 - 10 + x = 5 - 10$

$x = 5 - 10$

$x = -5$

Now You Try!

5) $x - 7 = 8$

$x - 7 + 7 = 8 + 7$

$x = 8 + 7$

$x = 15$

6) $x + 5 = 5$

$x + 5 - 5 = 5 - 5$

$x = 5 - 5$

$x = 0$

Hopefully you are starting to notice that the trick to isolating a variable is to move numbers away from the variable by applying the opposite operation!

7) $3x = 18$

The opposite of multiplication is: DIVISION

$\frac{\cancel{3}x}{\cancel{3}} = \frac{18}{3}$

$x = \frac{18}{3}$

$x = 6$

8) $\frac{x}{4} = 3$

The opposite of division is: MULTIPLICATION

$\cancel{4}\left(\frac{x}{\cancel{4}}\right) = 4(3)$

$x = 4(3)$

$x = 12$

Now You Try!

9) $-1x = 9$

$\frac{\cancel{-1}x}{\cancel{-1}} = \frac{9}{-1}$

$x = \frac{9}{-1}$

$x = -9$

10) $5x = 30$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{30}{5}$

$x = \frac{30}{5}$

$x = 6$

11) $\frac{x}{7} = 3$

$\cancel{7}\left(\frac{x}{\cancel{7}}\right) = 7(3)$

$x = 7(3)$

$x = 21$

Part 4: Two Step Equations

$$5x + 25 = 500$$

Isolate variable term first. (you will perform BEDMAS in reverse when isolating variables)

$$5x+25-25=500-25$$

Step 1: Subtract 25 from both sides

$$\frac{\cancel{5}x}{\cancel{5}}=\frac{475}{5}$$

Step 2: Divide both sides by 5

$$x=95$$

Remember: isolate variable term first!

12) $2x - 7 = 9 + 7$

$$2x=9+7$$

$$\frac{\cancel{2}x}{\cancel{2}}=\frac{16}{2}$$

$$x=\frac{16}{2}$$

$$x=8$$

13) $\frac{x}{2} + 4 = 20 - 4$

$$\frac{x}{2}=20-4$$

$$\cancel{2}\left(\frac{x}{\cancel{2}}\right)=2(16)$$

$$x=2(16)$$

$$x=32$$

14) $16x + 3 = 15 - 3$

remember to always put fraction in lowest terms!

$16x = 15 - 3$

$\frac{\cancel{16}x}{\cancel{16}} = \frac{12}{16}$

$x = \frac{12}{16}$

$x = \frac{3}{4}$

Before Moving On…

Solve the following equation:

$\frac{2x}{3} + 7 = 15 - 7$

$\frac{2x}{3} = 15 - 7$

$\cancel{3}\left(\frac{2x}{\cancel{3}}\right) = 3(8)$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{24}{2}$

$x = \frac{24}{2}$

$x = 12$

Summary of Key Concepts

- To solve an equation means to find the value of the variable that makes the statement true.
- To solve a one step equation, isolate the variable by performing the opposite operation.
- In a two-step equation, isolate the variable term first, then isolate the variable.
- You can check a solution by substituting the root back in to the equation.

4.2 Solve Multi-Step Equations

Part 1: Do it Now!

Solve the following equation:

$-9x + 8 = 23$

$-9x = 23 - 8$

$\frac{\cancel{-9}x}{\cancel{-9}} = \frac{15}{-9}$

$x = -\frac{15}{9}$

$x = -\frac{5}{3}$

Part 2: Solving Multi-Step Equations

To solve an equation involving multiple terms:

1) Get rid of any brackets by expanding

2) Collect variable terms on one side of the equation and constant terms on the other.

3) Collect like terms

4) Isolate the variable

Don't forget you can check your solutions!

1) $3x + 2 = 2x - 4$

$3x - 2x + 2 = -4$

$x + 2 = -4$

$x = -4 - 2$

$x = -6$

Start by moving all the variable terms to one side. Collect the variable terms on the left by subtracting $2x$ from both sides.

Now isolate the variable terms by moving the constant terms to the right. Do this by subtracting 2 from each side.

2) $7 - 2x = 8 - 5x$

Remember: start by collecting variable terms on to the left side, then move the constant terms to the right. (you can do this in one step with more practise)

$-2x + 5x = 8 - 7$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{1}{3}$

$x = \frac{1}{3}$

Try on Your Own

3) $5 - 3m = -2 - 2m$

$5 + 2 = -2m + 3m$

$7 = m$

$m = 7$

4) $5(x-3)-1(x-2)=19$

Start by expanding using the distributive property to get rid of the brackets.

$5x-15-1x+2=19$

$5x-1x=19+15-2$

$\frac{\cancel{4}x}{4}=\frac{32}{4}$

$x=8$

Complete the question just like the first three examples. Collect the variable terms on the left and the constant terms on the right.

Check your answer......

LS	RS
$=5(x-3)-1(x-2)$	$=19$
$=5(8-3)-1(8-2)$	
$=5(5)-1(6)$	
$=25-6$	
$=19$	

$LS=RS$

5) $2(x - 3) = -3(x + 5) - 6$

$2x - 6 = -3x - 15 - 6$

$2x + 3x = -15 - 6 + 6$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{-15}{5}$

$x = -3$

Try on Your Own

6) $5(5x – 13) = 23x – 13$

$25x - 65 = 23x - 13$

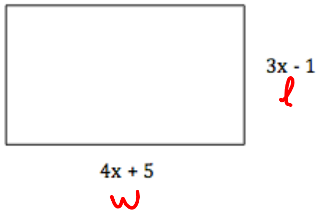
$25x - 23x = -13 + 65$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{52}{2}$

$x = 26$

Part 3: Application

7) The perimeter of the given rectangle is 50cm. Determine the length of each side of the rectangle.



$$P = 2(l) + 2(w)$$

$$50 = 2(3x-1) + 2(4x+5)$$

$$50 = 6x - 2 + 8x + 10$$

$$50 + 2 - 10 = 6x + 8x$$

$$\frac{42}{14} = \frac{\cancel{14}x}{\cancel{14}}$$

$$x = 3$$

Length $= l = 3x - 1 = 3(3) - 1 = 8\text{cm}$

Width $= w = 4x + 5 = 4(3) + 5 = 17\text{cm}$

Practice Practice Practice!!!!!

4.3a Solve Equations Involving Fractions

Part 1: Do it Now

Solve the following equation:

$$\frac{x}{4} = 7$$

Hint: You can simplify equations involving one fraction by multiplying both sides by the denominator of the fraction.

$\cancel{4}\left(\frac{x}{\cancel{4}}\right) = 4(7)$

$x = 4(7)$

$x = 28$

Part 2: Solve Equations With One Fraction

You can simplify equations involving one fraction by multiplying both sides by the denominator of the fraction.

Don't distribute the fraction; multiply both sides by 3 to get rid of the fraction.

1) $6 = \frac{1}{3}(8 + x)$

$3(6) = \cancel{3}(\frac{1}{\cancel{3}})(8+x)$

$18 = 1(8+x)$

$18 = 8 + x$

$18 - 8 = x$

$10 = x$

$x = 10$

2) $\frac{7(x-5)}{4} = 7$

$\cancel{4}\left[\frac{7(x-5)}{\cancel{4}}\right] = 4(7)$

$7(x-5) = 4(7)$

$7x - 35 = 28$

$7x = 28 + 35$

$\frac{\cancel{7}x}{\cancel{7}} = \frac{63}{7}$

$x = 9$

3) $\frac{1}{6}(2x+4)=5$

$\cancel{6}\left(\frac{1}{\cancel{6}}\right)(2x+4)=6(5)$

$1(2x+4)=6(5)$

$2x+4=30$

$2x=30-4$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{26}{2}$

$x=13$

Part 3: Solve Equations With More Than 1 Fraction

When eliminating more than 1 fraction, find the lowest common denominator of all the fractions and then multiply both sides of the equation by this value to eliminate the fractions.

4) $\frac{1}{3}(2x-5)=\frac{3}{4}(x-2)$

Find the lowest common denominator:

3, 6, 9, 12

4, 8, 12

${}^{4}\cancel{12}\left(\frac{1}{\cancel{3}}\right)(2x-5)={}^{3}\cancel{12}\left(\frac{3}{\cancel{4}}\right)(x-2)$

$4(1)(2x-5)=3(3)(x-2)$

$4(2x-5)=9(x-2)$

$8x-20=9x-18$

$-20+18=9x-8x$

$-2=x$

$x=-2$

5) $\frac{1}{5}(7x-3)=\frac{1}{10}$

Find the lowest common denominator:

5, 10, 15, 20

10, 20, 30

$\overset{2}{\cancel{10}}\left(\frac{1}{\cancel{5}}\right)(7x-3)=\overset{1}{\cancel{10}}\left(\frac{1}{\cancel{10}}\right)$

$2(1)(7x-3)=1(1)$

$2(7x-3)=1$

$14x-6=1$

$14x=1+6$

$\frac{\cancel{14}x}{\cancel{14}}=\frac{7}{14}$

$x=\frac{1}{2}$

6) $-\frac{3}{4}(d+3)=\frac{4}{5}(3d-2)$

Find the lowest common denominator:

4, 8, 12, 16, 20

5, 10, 15, 20

$\overset{5}{\cancel{20}}\left(\frac{-3}{\cancel{4}}\right)(d+3)=\overset{4}{\cancel{20}}\left(\frac{4}{\cancel{5}}\right)(3d-2)$

$5(-3)(d+3)=4(4)(3d-2)$

$-15(d+3)=16(3d-2)$

$-15d-45=48d-32$

$-45+32=48d+15d$

$\frac{-13}{63}=\frac{\cancel{63}d}{\cancel{63}}$

$d=\frac{-13}{63}$

7a) $$\frac{k+2}{3}=\frac{k-4}{5}$$

Find the lowest common denominator:

3, 6, 9, 12, 15

5, 10, 15

$$\overset{5}{\cancel{15}}\left(\frac{k+2}{\cancel{3}}\right)=\overset{3}{\cancel{15}}\left(\frac{k-4}{\cancel{5}}\right)$$

$$5(k+2)=3(k-4)$$

$$5k+10=3k-12$$

$$5k-3k=-12-10$$

$$\frac{\cancel{2}k}{\cancel{2}}=\frac{-22}{2}$$

$$k=-11$$

Part 4: Cross Multiplication

Method:

1) Multiply the numerator of the left fraction with the denominator of the right fraction. Put the product on either side of the equation.

2) Multiply the numerator of the right fraction with the denominator of the left fraction. Put the product on the other side of the equation.

3) Solve for the variable

Note: Cross-multiplication can only be used if you have two rational expressions equal to each other. If you have more than two expressions, you must clear denominators using the lowest common denominator.

You could have used the method of cross multiplication for #7

Try solving using cross-multiplication:

7b) $$\frac{k+2}{3} = \frac{k-4}{5}$$

$5(k+2) = 3(k-4)$

$5k+10 = 3k-12$

$5k-3k = -12-10$

$\frac{\cancel{2}k}{\cancel{2}} = \frac{-22}{2}$

$k = -11$

Can you use cross multiplication for this question?......

$$\frac{(k+2)}{3} - 7 = \frac{(k-4)}{5}$$

NO!

8) $$\frac{5-2x}{3}=\frac{15x-7}{2}$$

$$3(15x-7)=2(5-2x)$$

$$45x-21=10-4x$$

$$45x+4x=10+21$$

$$\frac{\cancel{49}x}{\cancel{49}}=\frac{31}{49}$$

$$x=\frac{31}{49}$$

9) $$\frac{1}{4}x+3=2$$

Make sure each term on both sides are multiplied by the LCD

$$4\left(\frac{1}{4}x+3\right)=4(2)$$

$$\cancel{4}\left(\frac{1}{\cancel{4}}x\right)+4(3)=4(2)$$

$$1x+12=8$$

$$x=8-12$$

$$x=-4$$

10) $\frac{1}{5}m + \frac{2}{3} - 2 = m - \frac{2}{5}$

Make sure each term on both sides are multiplied by the LCD

$\overset{3}{\cancel{15}}\left(\frac{1}{\cancel{5}}m\right) + \overset{5}{\cancel{15}}\left(\frac{2}{\cancel{3}}\right) - 15(2) = 15(m) - \overset{3}{\cancel{15}}\left(\frac{2}{\cancel{5}}\right)$

$3m + 10 - 30 = 15m - 6$

$3m - 20 = 15m - 6$

$-20 + 6 = 15m - 3m$

$\frac{-14}{12} = \frac{\cancel{12}m}{\cancel{12}}$

$-\frac{7}{6} = m$

11) $\frac{3}{2}x + \frac{x-4}{2} = \frac{x+14}{3}$

Make sure each term on both sides are multiplied by the LCD

$\overset{3}{\cancel{6}}\left(\frac{3}{\cancel{2}}x\right) + \overset{3}{\cancel{6}}\left(\frac{x-4}{\cancel{2}}\right) = \overset{2}{\cancel{6}}\left(\frac{x+14}{\cancel{3}}\right)$

$3(3x) + 3(x-4) = 2(x+14)$

$9x + 3x - 12 = 2x + 28$

$9x + 3x - 2x = 28 + 12$

$\frac{\cancel{10}x}{\cancel{10}} = \frac{40}{10}$

$x = 4$

Before homework, make sure you can solve:

a) $\frac{3}{4}(x+3)=9$

$4\left(\frac{3}{\cancel{4}}\right)(x+3)=4(9)$

$3(x+3)=4(9)$

$3x+9=36$

$3x=36-9$

$\frac{\cancel{3}x}{\cancel{3}}=\frac{27}{3}$

$x=9$

x = 9

b) $\frac{x-5}{3}=\frac{x+10}{6}$

$6(x-5)=3(x+10)$

$6x-30=3x+30$

$6x-3x=30+30$

$\frac{\cancel{3}x}{\cancel{3}}=\frac{60}{3}$

$x=20$

x = 20

4.3b Solve Equations Involving Fractions

You can simplify equations involving one fraction by multiplying both sides of the equation by the denominator of the fraction.

When eliminating more than 1 fraction, find the lowest common denominator of all the fractions and then multiply both sides of the equation by this value to eliminate the fractions.

Cross Multiplication:

1) Multiply the numerator of the left fraction with the denominator of the right fraction. Put the product on either side of the equation.

2) Multiply the numerator of the right fraction with the denominator of the left fraction. Put the product on the other side of the equation.

3) Solve for the variable

Note: Cross-multiplication can only be used if you have two rational expressions equal to each other. If you have more than two expressions, you must clear denominators using the lowest common denominator.

1) $\frac{1}{3}(x-2)=5$

$\cancel{3}\left(\frac{1}{\cancel{3}}\right)(x-2)=3(5)$

$1(x-2)=3(5)$

$x-2=15$

$x=15+2$

$x=17$

2) $-14=\frac{2(x-3)}{5}$

$5(-14)=\cancel{5}\left[\frac{2(x-3)}{\cancel{5}}\right]$

$-70=2(x-3)$

$-70=2x-6$

$-70+6=2x$

$\frac{-64}{2}=\frac{\cancel{2}x}{\cancel{2}}$

$x=-32$

3) $3 = \frac{2}{5}(n + 7)$

$5(3) = \cancel{5}\left(\frac{2}{\cancel{5}}\right)(n+7)$

$15 = 2(n+7)$

$15 = 2n+14$

$15-14 = 2n$

$\frac{1}{2} = \frac{\cancel{2}n}{\cancel{2}}$

$n = \frac{1}{2}$

LCD is 12

4) $\frac{1}{4}(x - 3) = \frac{1}{3}(x - 2)$

$\overset{3}{\cancel{12}}\left(\frac{1}{\cancel{4}}\right)(x-3) = \overset{4}{\cancel{12}}\left(\frac{1}{\cancel{3}}\right)(x-2)$

$3(x-3) = 4(x-2)$

$3x-9 = 4x-8$

$-9+8 = 4x-3x$

$-1 = x$

$x = -1$

4) $\frac{1}{4}(x-3)=\frac{1}{3}(x-2)$

$$\frac{x-3}{4}=\frac{x-2}{3}$$

$$3(x-3)=4(x-2)$$

$$3x-9=4x-8$$

$$-9+8=4x-3x$$

$$-1=x$$

$$x=-1$$

5) $\frac{3x-2}{5}=\frac{2x-1}{3}$

$$3(3x-2)=5(2x-1)$$

$$9x-6=10x-5$$

$$-6+5=10x-9x$$

$$-1=x$$

$$x=-1$$

LCD is 15

6) $\frac{2}{3}(5x-1) = -\frac{3}{5}(x+2)$

$$\overset{5}{\cancel{15}}\left(\frac{2}{\cancel{3}}\right)(5x-1) = \overset{3}{\cancel{15}}\left(\frac{-3}{\cancel{5}}\right)(x+2)$$

$$5(2)(5x-1) = 3(-3)(x+2)$$

$$10(5x-1) = -9(x+2)$$

$$50x-10 = -9x-18$$

$$50x+9x = -18+10$$

$$\frac{\cancel{59}x}{\cancel{59}} = \frac{-8}{59}$$

$$x = \frac{-8}{59}$$

6) $\frac{2}{3}(5x-1) = -\frac{3}{5}(x+2)$

$$\frac{2(5x-1)}{3} = \frac{-3(x+2)}{5}$$

$$5(2)(5x-1) = 3(-3)(x+2)$$

$$10(5x-1) = -9(x+2)$$

$$50x-10 = -9x-18$$

$$50x+9x = -18+10$$

$$\frac{\cancel{59}x}{\cancel{59}} = \frac{-8}{59}$$

$$x = \frac{-8}{59}$$

LCD is 12

7) $\frac{p}{2}=\frac{3}{4}+\frac{p}{3}$

$12\left(\frac{p}{2}\right)=12\left(\frac{3}{4}+\frac{p}{3}\right)$

$\overset{6}{\cancel{12}}\left(\frac{p}{\cancel{2}}\right)=\overset{3}{\cancel{12}}\left(\frac{3}{\cancel{4}}\right)+\overset{4}{\cancel{12}}\left(\frac{p}{\cancel{3}}\right)$

$6p=3(3)+4p$

$6p-4p=9$

$\frac{\cancel{2}p}{\cancel{2}}=\frac{9}{2}$

$p=\frac{9}{2}$

LCD is 12

8) $\frac{3x}{4}+\frac{x-5}{3}=\frac{1}{2}$

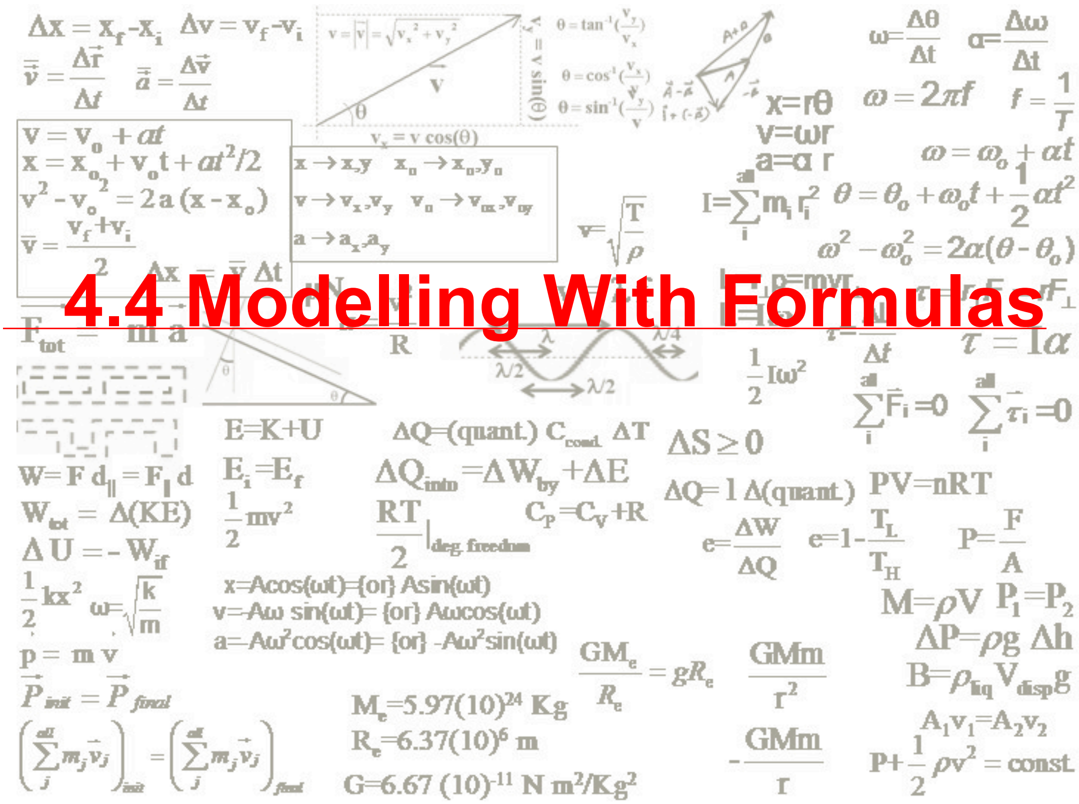
$\overset{3}{\cancel{12}}\left(\frac{3x}{\cancel{4}}\right)+\overset{4}{\cancel{12}}\left(\frac{x-5}{\cancel{3}}\right)=\overset{6}{\cancel{12}}\left(\frac{1}{\cancel{2}}\right)$

$3(3x)+4(x-5)=6(1)$

$9x+4x-20=6$

$\frac{\cancel{13}x}{\cancel{13}}=\frac{26}{13}$

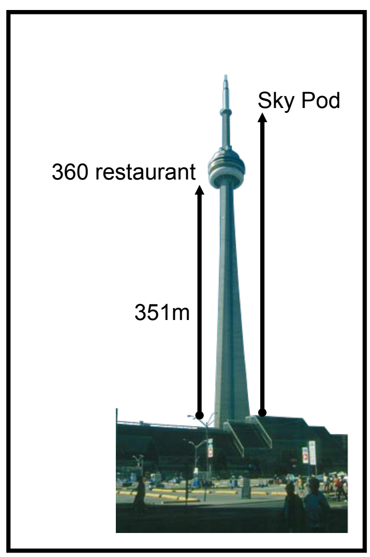
$x=2$



4.4 Modelling With Formulas

Part 1: Do It Now

The relationship between how high you are and how far you can see is given by the formula $d = 2\sqrt{3.2h}$, where h is your height in metres above the ground, and d is your distance in kilometres to the horizon.



Using this information,

a) How far can you see from the 360 restaurant?

b) from the Sky Pod the horizon is 75.64 km away. How high up is the Sky Pod to the nearest meter?

a) $d = 2\sqrt{3.2h}$

$d = 2\sqrt{3.2(351)}$

$d = 2\sqrt{1123.2}$

$d \simeq 67.03$ km

The horizon is about 67 km away.

$d = 2\sqrt{3.2h}$

b) $d = 2\sqrt{3.2h}$

$\frac{75.64}{2} = \frac{\cancel{2}\sqrt{3.2h}}{\cancel{2}}$

$37.82 = \sqrt{3.2h}$

$(37.82)^2 = 3.2h$

$\frac{1430.3524}{3.2} = \frac{\cancel{3.2}h}{\cancel{3.2}}$

$h \simeq 446.99$ m

The Sky Pod is about 447 m tall.

$d = 2\sqrt{3.2h}$

c) Which was easier to solve? Why?

a) was easier because the unknown variable was already isolated.

Learning Goals: In this lesson you will learn how rearrange formulas to isolate different variables.

When working with a formula

The known values for variables can be substituted first and then this equation can be solved for the required variable.

OR

The formula can be rearranged to isolate the required variable and then the known values can be substituted into the formula.

Part 2: Rearranging Formulas

To rearrange a formula, isolating one variable, think of the other variables as constants and isolate the variable in the same way that you would solve an equation.

For each of the following examples, rearrange each formula to isolate the variable indicated.

1) $d = a + b$ isolate for a

$d - b = a$

$a = d - b$

2) $C = 2\pi r$ isolate for r

$\frac{C}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$\frac{C}{2\pi} = r$

$r = \frac{C}{2\pi}$

3) $y = x^2$ isolate for x

$\sqrt{y} = \sqrt{x^2}$

$\sqrt{y} = x$

$x = \sqrt{y}$

4) $y = mx + b$ isolate for x

First isolate the **term** that contains the variable we want to isolate, then isolate the variable.

$\frac{y-b}{m} = \frac{\cancel{m}x}{\cancel{m}}$

$\frac{y-b}{m} = x$

$x = \frac{y-b}{m}$

5) $V = \frac{W}{q}$ isolate for W

$$q(V) = \cancel{q}\left(\frac{W}{\cancel{q}}\right)$$

$$qV = W$$

$$W = qV$$

6) $k = \frac{1}{2}mv^2$ isolate for v

$$2k = \cancel{2}\left(\frac{1}{\cancel{2}}\right)mv^2$$

$$\frac{2k}{m} = \frac{\cancel{m}v^2}{\cancel{m}}$$

$$\frac{2k}{m} = v^2$$

$$\sqrt{\frac{2k}{m}} = \sqrt{v^2}$$

$$v = \sqrt{\frac{2k}{m}}$$

7) $C = \frac{5}{9}(F - 32)$ isolate for f

$9(C) = \cancel{9}\left(\frac{5}{\cancel{9}}\right)(F-32)$

$\frac{9C}{5} = \frac{\cancel{5}(F-32)}{\cancel{5}}$

$\frac{9C}{5} = F - 32$

$\frac{9C}{5} + 32 = F$

$F = \frac{9C}{5} + 32$

Part 3: Application

The area, A, of a square is related to its side length, l, by the formula $A = l^2$. Find the length, to the nearest tenth of a centimeter, of a side of a square with an area of 32 cm^2.

$A = l^2$

$\sqrt{A} = l$

$l = \sqrt{A}$

$l = \sqrt{32}$

$l = 5.66\,cm$

4.5 Modelling With Algebra

Part 1: English to Algebra

Example 1: Write an algebraic expression for each English phrase.

a) the sum of 5 and y $5+y$

b) the product of 4 and x $4x$

c) the product of 4 and m, then increase the result by 7 $4m+7$

d) the sum of 4 and d, then multiply the result by 2 $2(4+d)$

e) add 4 to d, then double the result $2(d+4)$

f) three consecutive numbers $x, x+1, x+2$

Example 2: Write an algebraic expression for each English phrase.

a) 7 more than twice a number $2x+7$

b) one-quarter of a number increased by 3 $\frac{x}{4}+3$

c) double the sum of a number and 5 $2(x+5)$

d) triple a number $3x$

e) 6 less than one-half of a number $\frac{x}{2}-6$

f) the quotient of a number and 4 $\frac{x}{4}$

Example 3: Write an equation for each English statement.

a) Five more than a number is twenty-seven. $x+5=27$

b) Seven less than a number is 4. $x-7=4$

c) Double a number less eleven is sixteen. $2x-11=16$

d) The sum of 4 consecutive integers is fifty. $x+(x+1)+(x+2)+(x+3)=50$

e) Six times a number is 42. $6x=42$

Example 4: Write an equation for each sentence.

a) A number increased by six is twenty $x+6=20$

b) A number multiplied by four is sixteen $4x=16$

c) Seven less than a number is fifteen $x-7=15$

d) One fifth of a number is six $\frac{x}{5}=6$

e) A number divided by six is seven. $\frac{x}{6}=7$

f) Two more than triple a number is 14 $3x+2=14$

Part 2: Word Problems

When solving word problems,

- define the unknowns.
- write an equation to model the situation.
- solve the equation.
- answer the question asked in the problem.

Example 5: Mr. Jensen operates a variety store with his two best friends, Sidney and Evgeni. Sidney makes twice as much as Evgeni. Mr. Jensen makes $200 a week more than Sidney. The total weekly payroll is $1450. How much does each friend make?

Step 1: Let's define our variables:

Worker	Expression
Evgeni	x
Sidney	$2x$
Mr. Jensen	$2x+200$
Total	1450

Step 2: Write an equation that relates these expressions to the total payroll

$$x+2x+2x+200=1450$$

Step 3: Solve the equation

$$5x+200=1450$$

$$5x=1450-200$$

$$\frac{\cancel{5}x}{\cancel{5}}=\frac{1250}{5}$$

$$x=250$$

Step 4: Answer the question in context.

Each friend makes the following amount per week:

Eugeni $= x = \$250$

Sidney $= 2x = \$500$

Mr. Jensen $= 2x + 200 = \$700$

Example 6: Curtis works at a ballpark selling peanuts. He is paid $6/h plus a 50 cent commission for every bag of peanuts he sells.

a) Find Curtis' earnings if he sells 42 bags of peanuts during a 4 hour shift.

E = earnings
h = hours
p = peanuts

$E = 6h + 0.50p$

$E = 6(4) + 0.5(42)$

$E = 24 + 21$

$E = \$45$

He would earn $45.

b) How many bags of peanuts must he sell to earn \$100 in 7 hours?

$E = 6h + 0.5p$

$100 = 6(7) + 0.5p$

$100 = 42 + 0.5p$

$100 - 42 = 0.5p$

$\frac{58}{0.5} = \frac{\cancel{0.5}p}{\cancel{0.5}}$

$116 = p$

He must sell 116 bags.

Example 7: The length of a rectangle is 7m more than its width. The perimeter of the rectangle is 60m. What are the dimensions?

Length $= x + 7$

Width $= x$

$P = 2(\text{length}) + 2(\text{width})$

$60 = 2(x+7) + 2(x)$

$60 = 2x + 14 + 2x$

$60 - 14 = 4x$

$\frac{46}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$x = 11.5$

Length $= x + 7 = 18.5\text{m}$

Width $= x = 11.5\text{m}$

4.5 Model With Algebra (Day 2)

Do It Now!

Write an equation for each phrase:

a) triple a number is 18 $3x = 18$

b) 7 more than a number is 11 $x + 7 = 11$

c) half a number is 10 $\frac{x}{2} = 10$

d) double a number, less 3 is 7 $2x - 3 = 7$

e) 5 less than one third a number is 1 $\frac{x}{3} - 5 = 1$

f) 2 more than triple a number is 14 $3x + 2 = 14$

Part 2: Word Problems

When solving word problems,

- define the unknowns.
- write an equation to model the situation.
- solve the equation.
- answer the question asked in the problem.

1) The length of a rectangle is triple its width. The perimeter of the rectangle is 40 cm. What are the length and width?

Length $= 3x$

Width $= x$

$P = 2(\text{length}) + 2(\text{width})$

$40 = 2(3x) + 2(x)$

$40 = 6x + 2x$

$\frac{40}{8} = \frac{\cancel{8}x}{\cancel{8}}$

$x = 5$

Length $= 3x = 15\text{cm}$

Width $= x = 5\text{cm}$

2) Three consecutive integers have a sum of 75. What are the three integers?

1st integer $= x$

2nd integer $= x+1$

3rd integer $= x+2$

$$x+(x+1)+(x+2)=75$$
$$x+x+1+x+2=75$$
$$3x+3=75$$
$$3x=75-3$$
$$\frac{\cancel{3}x}{\cancel{3}}=\frac{72}{3}$$
$$x=24$$

1st integer $= x = 24$

2nd integer $= x+1 = 25$

3rd integer $= x+2 = 26$

3) Three consecutive even integers have a sum of 102. What are the three integers?

1st integer $= 2x$

2nd integer $= 2x+2$

3rd integer $= 2x+4$

$$2x+(2x+2)+(2x+4)=102$$
$$2x+2x+2+2x+4=102$$
$$6x+6=102$$
$$6x=102-6$$
$$\frac{\cancel{6}x}{\cancel{6}}=\frac{96}{6}$$
$$x=16$$

1st integer $= 2x = 32$

2nd integer $= 2x+2 = 34$

3rd integer $= 2x+4 = 36$

4) Katherine is 2 years older than Christine. The sum of their ages is 16. How old is each girl?

Katherine $= x+2$

Christine $= x$

$$x+(x+2)=16$$

$$x+x+2=16$$

$$2x+2=16$$

$$2x=16-2$$

$$\frac{\cancel{2}x}{\cancel{2}}=\frac{14}{2}$$

$$x=7$$

Katherine $= x+2 = 9$ years old

Christine $= x = 7$ years old.