Chapter

8

Measurement

GOALS

You will be able to

- Develop and apply formulas to calculate the surface areas and volumes of pyramids, cones, and spheres
- Determine the best perimeter and area measurements for rectangles for given situations
- Investigate the effects of varying dimensions on surface areas and volumes of prisms and cylinders

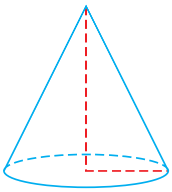
These pyramid-shaped greenhouses are part of the Muttart Conservatory in Edmonton, Alberta.
What would you estimate the volume of each greenhouse to be?

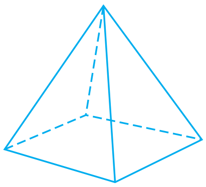
8 Getting Started

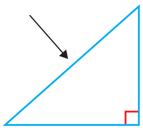
WORDS YOU NEED to Know

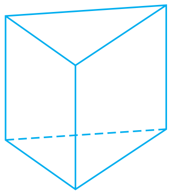
1. Match each term with the most appropriate picture.

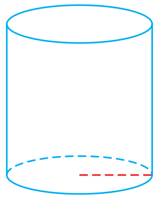
a) hypotenuse
b) prism
c) net
d) pyramid
e) surface area
f) cylinder
g) volume
h) sphere
i) composite shape
j) cone

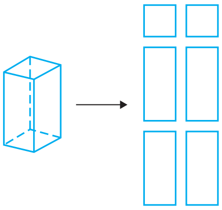
i) 

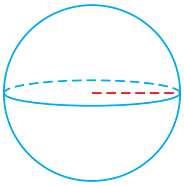
iii) 

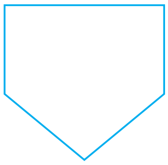
v) 

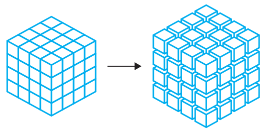
vii) 

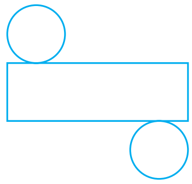
ix) 

ii) 

iv) 

vi) 

viii) 

x) 

SKILLS AND CONCEPTS You Need

Calculating Volume and Surface Area

Study Aid

- For more help and practice, see Appendix A-20.

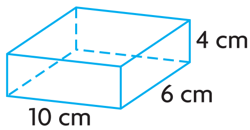
EXAMPLE

Area of base

$B =$ area of top
$= 6 \times 10$
$= 60 \text{ cm}^2$

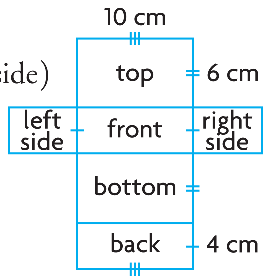
Volume

$V = Bh$
$= 60 \times 4$
$= 240 \text{ cm}^3$



Surface area

$SA =$ surface area of faces
$SA = 2(\text{area of top}) + 2(\text{area of front}) + 2(\text{area of side})$
$= 2(10 \times 6) + 2(10 \times 4) + 2(4 \times 6)$
$= 248 \text{ cm}^2$



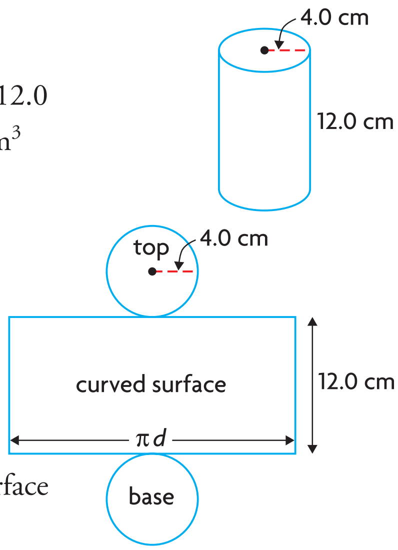
2. Determine the surface area of each figure.

a) a cube with a side length of 10 cm
b) a rectangular prism that is 3 cm by 5 cm by 8 cm
c) a rectangular prism that is 10 cm by 20 cm by 5 cm

EXAMPLE

Area of base
$B =$ area of top
$= \pi r^2$
$= \pi(4.0)^2$
$= 16\pi \text{ cm}^2$

Volume
$V = Bh$
$= 16\pi \times 12.0$
$\doteq 603.2 \text{ cm}^3$



Length of curved surface
$=$ circumference of base
$= \pi d$
$= \pi\,(8.0)$
$= 8\pi$ cm

Surface area
$SA = 2(\text{area of base}) + \text{area of curved surface}$
$= 2\pi r^2 + \pi dh$
$= 2\pi(4.0)^2 + \pi(8.0)(12.0)$
$\doteq 402.1 \text{ cm}^2$

3. Determine the surface area of each cylinder.
- **a)** a cylinder with a height of 10 cm and a radius of 5 cm
- **b)** a cylinder with a height of 8 cm and a diameter of 8 cm
- **c)** a cylinder with a diameter of 12 cm and a height of 9 cm

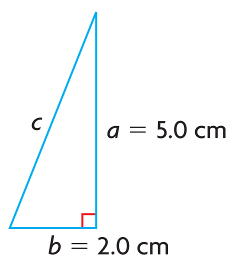
Communication *Tip*

When calculating volume and surface area of 3-D figures, it is common practice to round off your answers to the same number of decimal places present in the given measurements.

Calculating the Hypotenuse Using the Pythagorean Theorem

EXAMPLE

By the Pythagorean theorem, $c^2 = a^2 + b^2$.
$c^2 = 5.0^2 + 2.0^2$
$c^2 = 25.0 + 4.0$
$c^2 = 29.0$
$c = \sqrt{29.0}$
$\doteq 5.4$ cm



4. Determine the hypotenuse of each right triangle.
- **a)** a right triangle with legs of 4 cm and 8 cm
- **b)** a right triangle with both legs of 10 cm
- **c)** a right triangle with legs of 12 cm and 5 cm

Study *Aid*

- For more help and practice, see Appendix A-19.

Study Aid

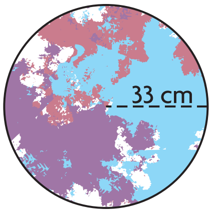
- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
5	A-17 and A-18
6, 7, and 10	A-20
8	A-19

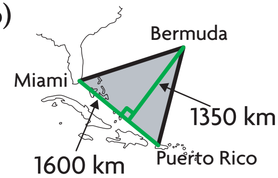
PRACTICE

5. Calculate the area of each shape.

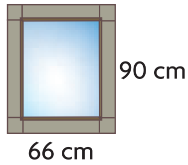
a)



b)

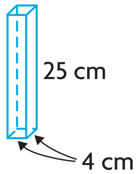


c)

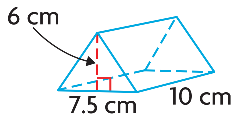


6. Calculate the total surface area and volume of each figure.

a)



b)



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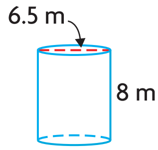


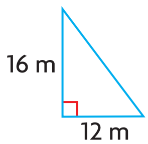
7. A company sells mints in three types of boxes: a cube, a triangular prism, and a cylinder. Each box has a capacity of 64 mL and they all have the same height.

a) Draw a sketch that shows possible dimensions to the nearest tenth, for each type of box.

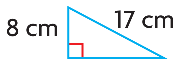
b) Calculate the surface area of each box you sketched.

8. Calculate the length of the missing side.

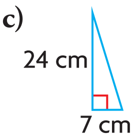
a)



b)



c)



9. Do you agree or disagree?

a) Rectangles with different side lengths can have the same area.

b) The number of square units in the surface area of a cube is usually less than the number of cubic units in its volume.

c) The longest that a rectangle with a perimeter of 50 cm can be is 24 cm.

d) The area of a circle with diameter d is always less than d^2.

10. Copy and complete the table.

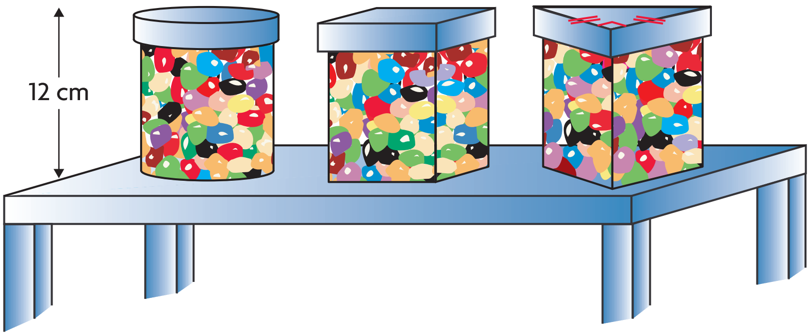
	Prism	Pyramid	Cone
Definition			
Properties			
Diagram			

APPLYING What You Know

Judging Jars

The student council is holding a jellybean guessing contest for a fundraiser. They must use one of these jars. They know the following:

- Each jar is 12 cm high.
- Each jar holds 2400 jellybeans.
- Each jellybean has a volume of about 1 cm^3.
- 1 cm^3 = 1 mL



What are possible dimensions for the base of each jar?

A. Copy the table.

Jar	Capacity Estimate (mL)	Base Area Estimate (cm^2)	Shape of Base	Base Dimensions (Estimates in cm)
cylinder				diameter = ■
rectangular prism				side length = ■
triangular prism				base = ■, height = ■

B. Estimate the capacity of each jar.

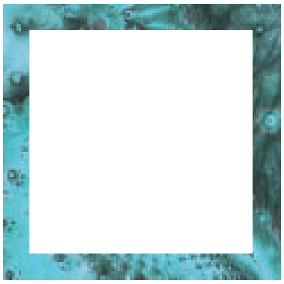
C. Estimate the area of each base. Use the formula $V_{\text{prism or cylinder}} = A_{\text{base}} \times$ height to help you.

D. Estimate the measurements of the base of each jar.

8.1 Determining Optimum Area and Perimeter

YOU WILL NEED

- 1 cm grid paper
- 24 cm piece of string
- geoboard and elastic band



6 cm

6 cm

Julie's Note Card

GOAL

Solve problems involving the dimensions of rectangles.

INVESTIGATE the Math

Julie showed Wyatt and Nick a rectangular note card she had made.

- Wyatt wanted to make a card with the greatest possible area and the same perimeter as Julie's.
- Nick wanted to make a card with the same area as Julie's and the least possible perimeter.

	Julie's Card	Wyatt's Card	Nick's Card
Area	36 cm^2	greatest possible	36 cm^2
Perimeter	24 cm	24 cm	least possible

? What are the dimensions for Wyatt's and Nick's cards?

A. Outline rectangles on 1 cm grid paper using a 24 cm piece of string. Include some rectangles with sides that are not whole numbers.

B. Draw each rectangle on the grid paper.

C. Determine the area of each rectangle.

D. What are the dimensions of the rectangle with the greatest area?

E. Form rectangles with an area of 36 cm^2 on a geoboard using an elastic band.

F. Draw each rectangle on grid paper.

G. Determine the perimeter of each rectangle.

H. What are the dimensions of the rectangle with the least perimeter?

I. Describe how you know that the dimensions you discovered are **optimum** measures.

optimum

the most desirable of a number of possible choices

Reflecting

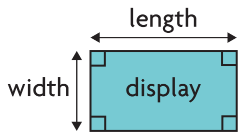
J. What was special about the rectangle with the greatest area for a given perimeter?

K. What was special about the rectangle with the least perimeter for a given area?

APPLY the Math

EXAMPLE 1 Using a graphing strategy to determine minimum perimeter

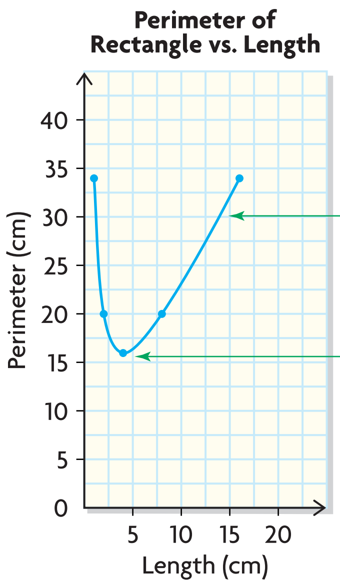
At the local craft fair, the rectangular displays must have an area of 16 m^2. Marek wants to use the least amount of border for his entry. What dimensions can he use?



Marek's Solution

	Factors of 16				
Length (m)	1	2	4	8	16
Width (m)	16	8	4	2	1
Perimeter (m)	34	20	16	20	34

Since the area must be 16 m^2, I compared rectangles with lengths and widths whose products were 16.



I created a scatter plot of perimeter vs. length. The data were continuous, so I connected the points with a smooth curve.

The graph shows that the least perimeter occurs when the length is 4 m.

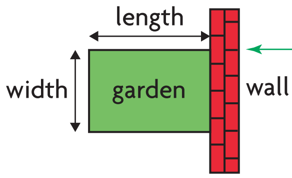
The display has to be a 4 m by 4 m square.

EXAMPLE 2 Using graphing technology to determine maximum area

Sunia's horticulture club is exhibiting at the city garden show. Each garden must be bordered by 18.0 m of wood against a brick display wall. What dimensions will maximize the area of the garden?

Sunia's Solution

The total length of the three sides is 18.0 m.



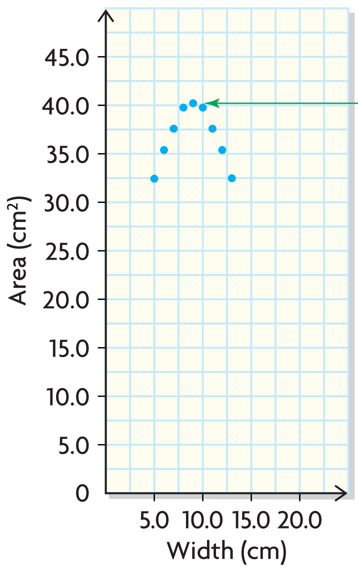
I sketched how the garden might look.

	A	B	C
1	Width, w (m)	Length, l $l = (18.0 - w) \div 2$ (m)	Area: $l \times w$ (m^2)
2	5.0	6.5	32.5
3	6.0	6.0	36.0
4	7.0	5.5	38.5
5	8.0	5.0	40.0
6	9.0	4.5	40.5
7	10.0	4.0	40.0
8	11.0	3.5	38.5
9	12.0	3.0	36.0

I entered possible dimensions for the garden in a table of values on a spreadsheet.

I chose the width, and then, calculated the length and area for that width.

Area of Garden vs. Width



I graphed area versus width using the spreadsheet program.

I noticed the area was greatest when the width was double the length: 9.0 m and 4.5 m.

This makes sense. If the border was on all four sides, a square with dimensions 4.5 m by 4.5 m would give the greatest area.

I think the club should plan for a 9.0 m by 4.5 m garden exhibit.

Tech *Support*

For information on selecting columns from a table of values to form a scatter plot using a spreadsheet, see Appendix B-30.

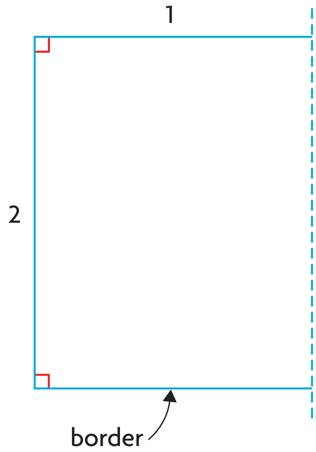
In Summary

Key Ideas

- Rectangles with the same perimeter can have different areas, and the rectangle with a maximum area for a given perimeter is a square.
- Rectangles with different areas can have the same perimeter, and the rectangle with a minimum perimeter for a given area is a square.
- Collecting and plotting possible solution data helps to estimate optimal values.

Need to Know

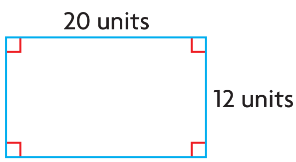
- A rectangle with a border on three sides has a maximum area and a minimum border length when the side without a border and its opposite side are twice the length of the other two sides.

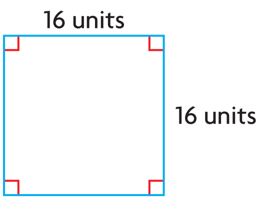


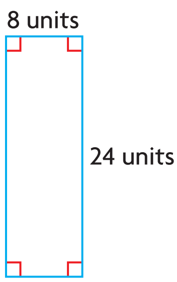
CHECK Your Understanding

1. Each rectangle has a perimeter of 64 units. Which one has the greatest area?

A.


C.


B.


D.


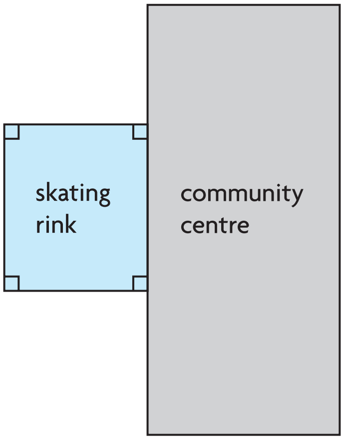
2. Draw rectangles with areas of 36 square units on grid paper. Which one has the least perimeter?

PRACTISING

3. a) Determine the maximum area of a rectangle with each perimeter, to one decimal place.
K
   i) 100 cm ii) 72 m iii) 169 km iv) 143 mm

   b) Determine the minimum length of wood needed to build a rectangular frame for an art sketch of each area, to one decimal place.
   i) 1 m^2 ii) 70 cm^2 iii) 15.4 cm^2 iv) 28 cm^2

4. Sarah is fencing a vegetable garden to keep rabbits out. The hardware
A
store sells fencing for $25.50/m. Her family has $165 to spend. What dimensions should Sarah use to build a garden with the greatest area?

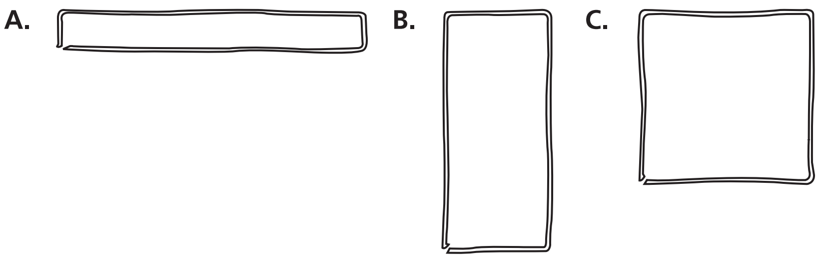


5. An outdoor rectangular skating rink with an area of 126 m^2 will be built with one of its side lengths next to the community centre. To enclose the rink, 3 sides of fencing are needed.
   a) Create a table of values that compares width, length, and perimeter of the rink for various widths of fencing shown.

Width (m)	1	2	3	4	6	9	12	14	21	42
Length (m)										
Perimeter (m)										

   b) Create a scatter plot of perimeter versus width.
   c) Use your scatter plot to estimate the dimensions of the rink that will use the least amount of fencing needed to enclose the 3 sides.

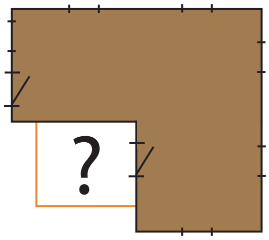
6. The same piece of string was used to create these three rectangles.
C
Which one has the maximum area? Explain your thinking.



7. A rectangular, indoor fish pond is being added to the lobby of a hotel. The budget allows for a stone border of 36 m around the pond. What dimensions will create a pond with the greatest area? How do you know?

8. Determine the dimensions of a rectangle with a perimeter of 40 cm and the greatest possible area.

9. What is the largest rectangular area that can be built with a 20 m fence in the corner of a building?

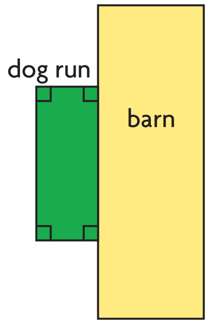


a) Create a table of values showing possible perimeters, widths, lengths, and areas.

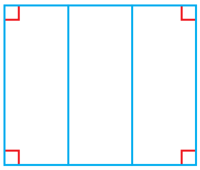
b) Construct a scatter plot of the area versus the width.

c) Explain your solution.

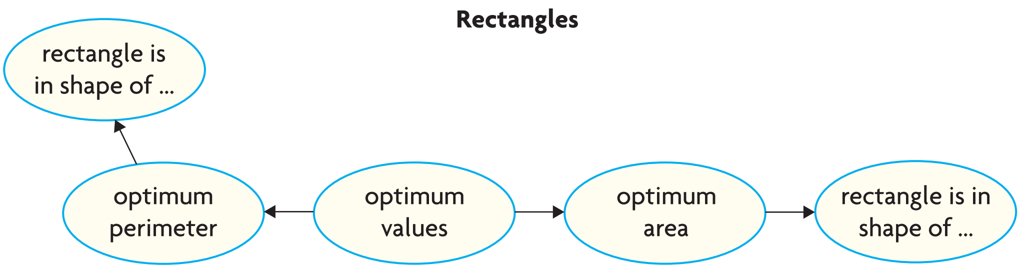
10. **T** Randy is building a rectangular, fenced dog run beside his barn. He has 120 m of fencing and plans to use the side of the barn as one side of the fenced area. What are the dimensions of a dog run that maximizes the area Randy can enclose?



11. A farmer has $3600 to spend on fencing for three adjoining rectangular pastures, as shown. The pastures all have the same dimensions. A local contracting company charges $6.25/m for fencing. What is the largest area that the farmer can enclose?



12. Complete and extend the mind map for rectangles.



Extending

13. A 60 cm length of rope is to be cut into two pieces to form the perimeters of two separate squares. The total area of the two squares is to be a maximum. Calculate the dimensions of the squares to the nearest hundredth.

14. Diane is building a kennel with two stalls. She has 80 m of wood panelling for the outer walls and an inside wall to separate the two stalls. What dimensions would give each stall the maximum equal area?

8.2 Problems Involving Composite Shapes

YOU WILL NEED

- grid paper
- spreadsheet software (optional)

GOAL

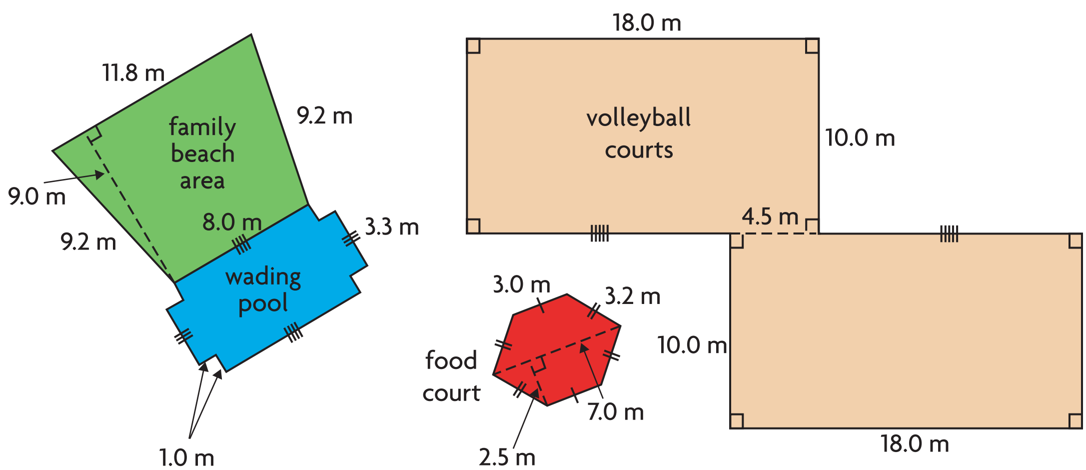
Solve problems involving the area and perimeter of composite 2-D shapes.

LEARN ABOUT the Math

The town of Maple Beach is accepting proposals to create a new beach play area. All proposals must give the area and perimeter of each part of the design, the total area and perimeter, and the cost of the materials. Students from the local high school are submitting a proposal.

Beach Play Area Measurements Table

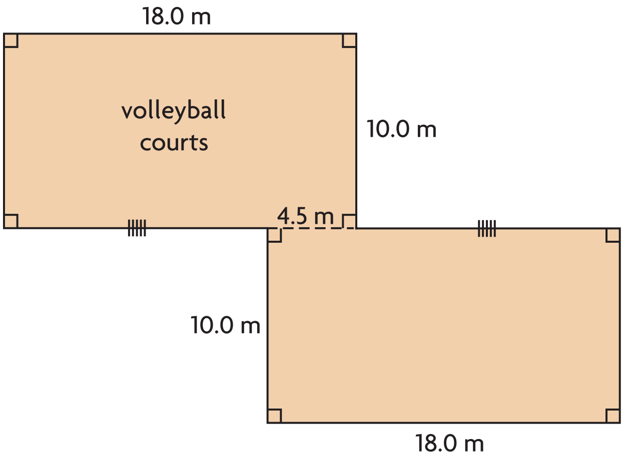
Area	Material	Area (m^2)	Perimeter (m)
volleyball courts	sand and chalk trim		
food court	tinted cement and stone trim		
wading pool	pool tiles and fencing		
family beach area	lawn sod and plastic lawn trim		
Total Beach Play Area Measurements:			



How can the students determine the areas and perimeters of the figures to complete their proposal?

EXAMPLE 1 Decomposing shapes to solve an area and perimeter problem

Jamie's Solution



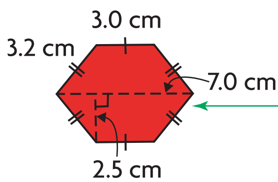
$A_{\text{rectangle}}$
$= l \times w$
$= 10.0 \times 18.0$
$= 180.0 \text{ m}^2$
$A_{\text{volleyball courts}} = 2 \times 180.0$
$= 360.0 \text{ m}^2$

I started with the two volleyball courts. I calculated the area of one court, and then, doubled it.

Missing side length
$= 18.0 - 4.5$
$= 13.5 \text{ m}$
$P_{\text{volleyball courts}} = 2 \times 18.0 + 4 \times 10.0 + 2 \times 13.5$
$= 103.0 \text{ m}$

The perimeter was the sum of the side lengths. Two sides were 18.0 m long and four sides were 10.0 m long. To determine the length of the last two sides, I subtracted the 4.5 m section from 18.0 m to get 13.5 m.

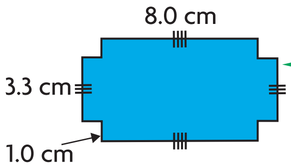
$$A_{\text{trapezoid}} = \frac{(b_1 + b_2) \times h}{2}$$
$$= \frac{(3.0 + 7.0) \times 2.5}{2}$$
$$= \frac{10.0 \times 2.5}{2}$$
$$= \frac{25.0}{2}$$
$$= 12.5 \text{ m}^2$$
$$A_{\text{food court}} = 2 \times 12.5$$
$$= 25.0 \text{ m}^2$$



I divided the food court into two trapezoids. I calculated the area of one trapezoid and doubled it.

$$P_{\text{food court}} = 2 \times 3.0 + 4 \times 3.2$$
$$= 6.0 + 12.8$$
$$= 18.8 \text{ m}$$

I calculated the perimeter of the food court.



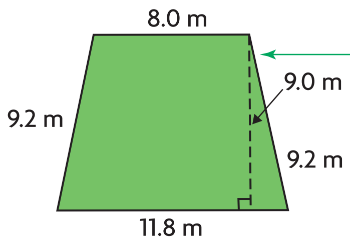
I noticed the wading pool was a rectangle with four corners removed.

$$\begin{aligned} A_{\text{rectangle}} &= 10.0 \times 5.3 \\ &= 53.0 \text{ m}^2 \\ A_{\text{wading pool}} &= 53.0 - 4(1.0 \times 1.0) \\ &= 53.0 - 4.0 \\ &= 49.0 \text{ m}^2 \end{aligned}$$

I calculated the area of the rectangle, and then, subtracted the area of the four corners.

$$\begin{aligned} P_{\text{wading pool}} &= 2(8.0) + 2(3.3) + 8(1.0) \\ &= 16.0 + 6.6 + 8.0 \\ &= 30.6 \text{ m} \end{aligned}$$

I calculated the perimeter of the wading pool.



The family beach area is like a trapezoid.

$$\begin{aligned} A_{\text{beach area}} &= \frac{(11.8 + 8.0) \times 9.0}{2} \\ &= \frac{19.8 \times 9.0}{2} \\ &= 89.1 \text{ m}^2 \end{aligned}$$

I calculated its area with the same formula as the food court.

$$\begin{aligned} P_{\text{beach area}} &= 11.8 + 8.0 + 2(9.2) \\ &= 38.2 \text{ m} \end{aligned}$$

I calculated the perimeter of the beach area.

$$\begin{aligned} A_{\text{total}} &= 360.0 + 25.0 + 49.0 + 89.1 \\ &= 523.1 \text{ m}^2 \end{aligned}$$

I calculated the total area.

$$\begin{aligned} P_{\text{total}} &= 103.0 + 18.8 + 30.6 + 38.2 \\ &= 190.6 \text{ m} \end{aligned}$$

I calculated the total perimeter.

Reflecting

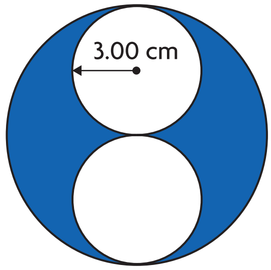
A. How did looking at simple shapes help Jamie determine the areas and perimeters of the composite shapes?

B. How else could Jamie have decomposed the shapes?

APPLY the Math

EXAMPLE 2 Using a subtraction strategy to calculate area

Matti is designing a logo in his graphic arts class. How can Matti calculate the area of the blue section?

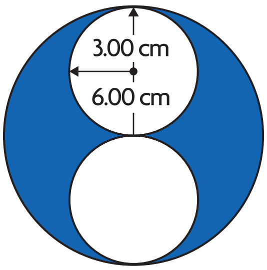


Matti's Solution

$$A_{\text{blue circle}} = \pi r^2$$
$$\doteq 3.14 \times 6.00^2$$
$$= 3.14 \times 36.00$$
$$= 113.04 \text{ cm}^2$$

I decided to calculate the area of the blue circle, and then, subtract the area of the white circles.

The radius of the blue circle was the same as the diameter of a white circle, or 6.00 cm.



$$A_{\text{white circle}} = \pi r^2$$
$$A_{\text{both white circles}} = 2 \times (\pi r^2)$$
$$\doteq 2 \times (3.14 \times 3.00^2)$$
$$= 2 \times (3.14 \times 9.00)$$
$$= 2 \times (28.26)$$
$$= 56.52 \text{ cm}^2$$

Both white circles have a radius of 3 cm, so I calculated the area of one, and then, doubled it.

$$A_{\text{blue circle}} \doteq 113.04 \text{ cm}^2$$
$$A_{\text{white circle}} \doteq 56.52 \text{ cm}^2$$
$$A_{\text{blue section}} \doteq 113.04 - 56.52$$
$$\doteq 56.52 \text{ cm}^2$$

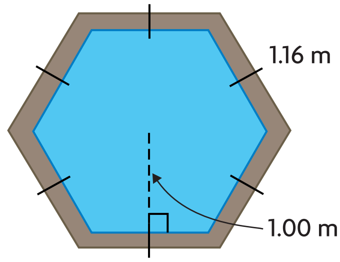
I subtracted to determine the area of the blue part.

The area of the blue section is about 56.52 cm^2.

I answered to two decimal places, because that is how the dimensions were given.

EXAMPLE 3 Solving a problem using a right triangle

Rani is replacing a regular hexagonal window. The side length is 1.16 m and the distance from the centre to the middle of each side is 1.00 m. How can Rani calculate the length of the wooden framing and the area of the glass?

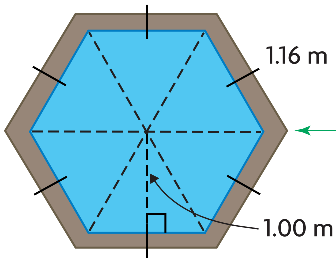


Rani's Solution

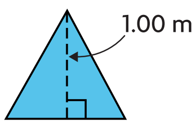
$$\begin{aligned} P_{\text{hexagon}} &= n \times s \\ &= 6 \times 1.16 \\ &= 6.96 \text{ m} \end{aligned}$$

The perimeter of the hexagon is 6.96 m.

Each side of the frame is the same length, so I multiplied one side's length by 6 to get the perimeter.



To determine the area, I divided the hexagon into 6 identical triangles.



$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 1.16 \times 1.00 \\ &= \frac{1}{2} \times 1.16 \\ &= 0.58 \text{ m}^2 \end{aligned}$$

I used the distance from the centre as the height and the side of the hexagon as the base.

$$\begin{aligned} A_{\text{hexagon}} &= \text{number of sides} \times A_{\text{triangle}} \\ &= 6 \times 0.58 \\ &= 3.48 \text{ m}^2 \end{aligned}$$

I calculated the area of the hexagon by multiplying the area of one triangle by 6.

The area of the hexagon is 3.48 m^2.

I realized my formula would work for any regular polygon because you can always divide it into identical triangles.

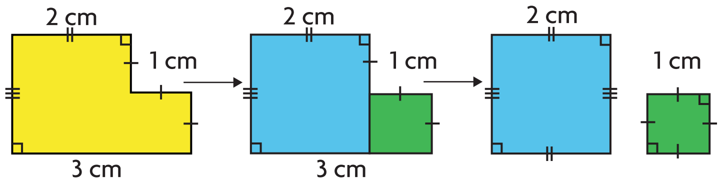
In Summary

Key Idea

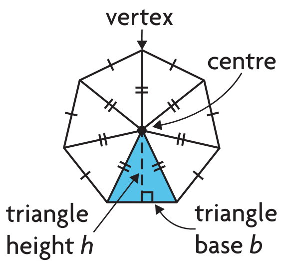
- You can determine the area or perimeter of a geometric shape by decomposing it into simpler shapes whose formulas you know.

Need to Know

- The area of a shape created by joining smaller shapes is equal to the combined area of the smaller shapes. For example, the area of the yellow shape is equal to the sum of the areas of the blue and green shapes.



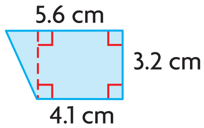
- When one shape is removed from another, the area of the remaining shape is equal to the area of the original shape minus the area of the shape that was removed.
- To calculate the perimeter of a new shape created from other shapes, determine whether some of the original shapes' sides are either duplicated or no longer part of the new perimeter. The perimeter of the yellow shape is not the same as the sum of the perimeters of the blue and green shapes.
- The formula for the perimeter of a regular polygon is $P = n \times s$, where n is the number of sides and s is the length of each side.
- To calculate the area of a regular polygon, divide it into triangles, and then, add their areas. Form the triangles by drawing a line from the centre to each vertex. The polygon side length is the base of each triangle, and the distance from the centre to the middle of each side is the height.

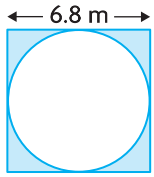


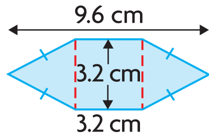
CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

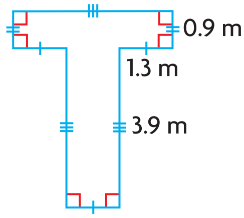
1. Calculate the shaded area of each figure.

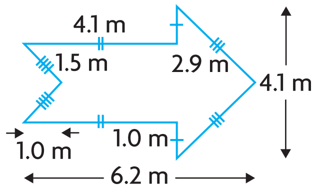
   a) 

   b) 

   c) 

2. Calculate the perimeter and area of each shape.

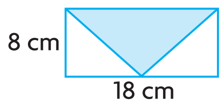
   a) 

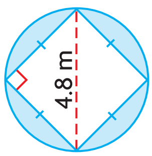
   b) 

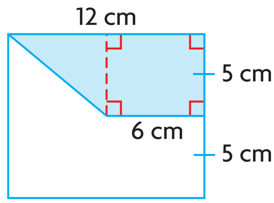
PRACTISING

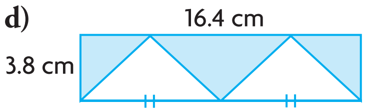
3. Calculate the shaded area of each figure.

K

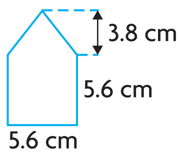
   a) 

   b) 

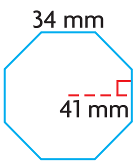
   c) 

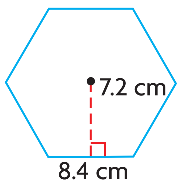
   d) 

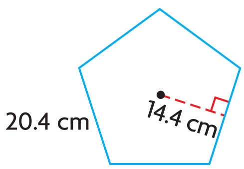
4. Calculate the area and perimeter of this shape.



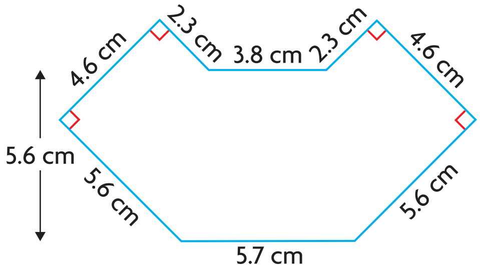
5. Calculate the area and perimeter of each regular polygon.

   a) 

   b) 

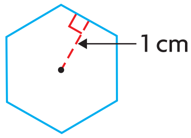
   c) 

6. **a)** Copy this shape and divide it into simpler polygons.
A
b) Calculate the area of the shape. Explain your thinking.



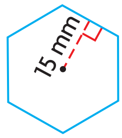
7. Calculate the length of one side of each regular polygon.

a)



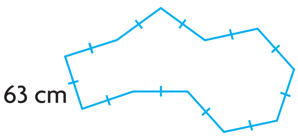
A = 5.2 cm^2

b)



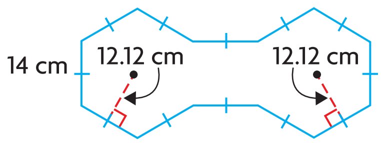
A = 758 mm^2

8. **a)** Explain how you can calculate the area of this shape. Include what measurements you would need to know to calculate the area.
T



b) Calculate the perimeter of the shape in part a).

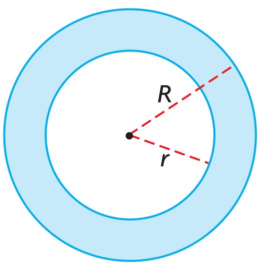
9. Calculate the area and perimeter of this shape. Explain what you did.
C



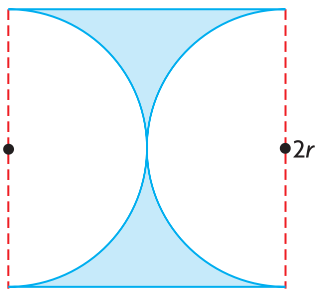
Extending

10. Determine an expression for the shaded area of each figure.

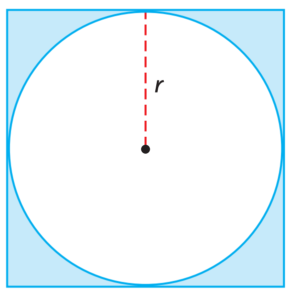
a)



b)



c)

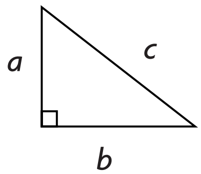


11. Show how to calculate the area of a regular polygon using only its perimeter and the distance from its centre to the midpoint of each side.

8.3 The Pythagorean Theorem

YOU WILL NEED

- grid paper
- scissors



GOAL

Solve problems using the Pythagorean theorem.

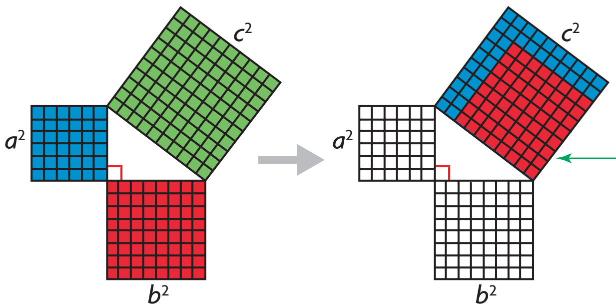
LEARN ABOUT the Math

Julie is tutoring her friend, Annie, on using the Pythagorean theorem to solve problems. To help Annie understand, Julie creates a geometric representation of the theorem using a picture.

? What geometric model can Julie use to represent the Pythagorean theorem?

EXAMPLE 1 Representing the Pythagorean theorem geometrically

Julie's Solution

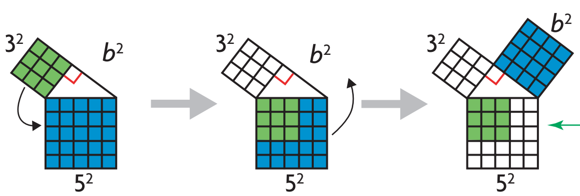


I used grid paper. I drew a right triangle with legs of 6 cm and 8 cm. I cut out a square to fit on each side of the triangle. I coloured the squares blue, red, and green.

I rearranged the blue and red squares on top of the green square on the long side or **hypotenuse**. The hypotenuse square had the same area as the sum of the two other squares.

square of a + square of b = square of c

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2 \text{ or } c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$\sqrt{c^2} = \sqrt{100}$$
$$c = 10 \text{ cm}$$



I tried another triangle. I discovered that the square on the side with a length of 3 had an area of 9 and the square on the side with a length of 5 had an area of 25. I subtracted 9 from 25 to get 16, which is 4 squared. I decided that side b had a length of 4.

Reflecting

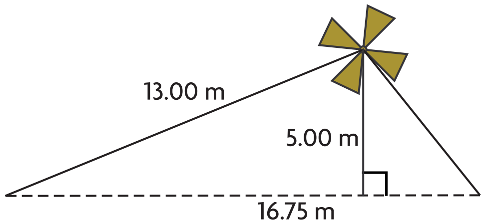
A. How did a geometric model help to represent the Pythagorean theorem?

B. How can you use known sides of a right triangle to calculate an unknown side?

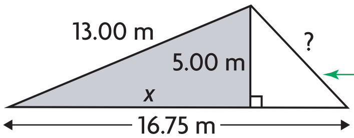
APPLY the Math

EXAMPLE 2 Applying the Pythagorean theorem to calculate a length

Anil is constructing a 5.00 m tall windmill supported by wires. One wire must be 13.00 m long and the distance between the wires must be 16.75 m. Anil wanted to know what length to cut for the other wire.



Anil's Solution

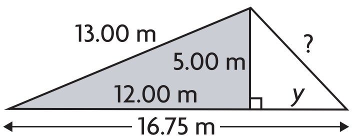


I divided the triangle into two right triangles. I started with the left-hand triangle since I knew two of its dimensions.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
x^2 + 5.00^2 &= 13.00^2 \\
x^2 + 25.00 &= 169.00 \\
x^2 &= 169.00 - 25.00 \\
x^2 &= 144.00 \\
x &= \sqrt{144.00} \\
x &= 12.00 \text{ m}
\end{aligned}
$$

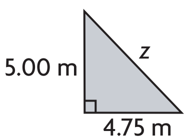
I determined the distance, x, from the windmill to the base of the left-hand wire. I substituted the sides I knew into the Pythagorean theorem and solved for x.

The distance was 12.00 m.



$$
\begin{aligned}
y &= 16.75 - 12.00 \\
&= 4.75 \text{ m}
\end{aligned}
$$

I calculated the distance, y, from the windmill to the right-hand wire's base.



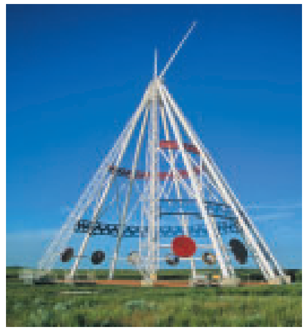
$$
\begin{aligned}
z^2 &= 5.00^2 + 4.75^2 \\
z^2 &= 47.56 \\
z &= 6.90 \text{ m}
\end{aligned}
$$

The hypotenuse, z, is the length of wire needed. I calculated the hypotenuse using the legs.

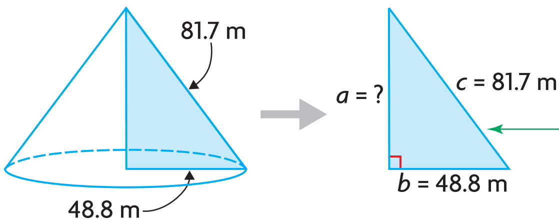
The other wire should be 6.90 m long.

EXAMPLE 3	Solving a problem modelled by a right triangle

The Saamis Teepee in Medicine Hat, Alberta, is the tallest teepee in the world. In 2007, a windstorm damaged the teepee, reducing its height. Each beam originally was 81.7 m long and touched the ground 48.8 m from the centre of the base. What was the original height of the teepee?



Dave's Solution



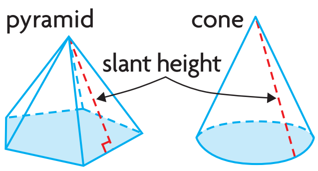
I assumed that the Teepee was a cone and I visualized a right triangle inside it.

The slant height of the cone is 81.7 m, and the radius of the base is 48.8 m.

I used the slant height, *c*, for the hypotenuse and the base radius for the horizontal leg, *b*.

slant height

the distance from the top to the base, at a right angle, along a slanted side of a **pyramid** or **cone;** it is measured to the midpoint of the base side for a pyramid



Height of cone:

$a^2 + b^2 = c^2$

So,

$a^2 = c^2 - b^2$

$a^2 = 81.7^2 - 48.8^2$

$a^2 = 6674.89 - 2381.44$

$a^2 = 4293.45$

$a = \sqrt{4293.45}$

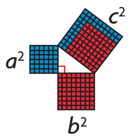
$a = 65.5$ m

I calculated the original height of the teepee, *a*, using the Pythagorean theorem. It is 65.5 m, to one decimal place.

In Summary

Key Idea

- The Pythagorean theorem describes both a numerical and a geometric relationship between the three sides of a right triangle.

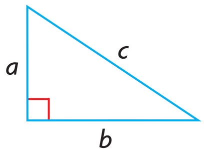


geometric relationship

$a^2 + b^2 = c^2$
numerical relationship

Need to Know

- The formula for the hypotenuse of a right triangle is $c = \sqrt{a^2 + b^2}$, where a and b are the lengths of the legs.

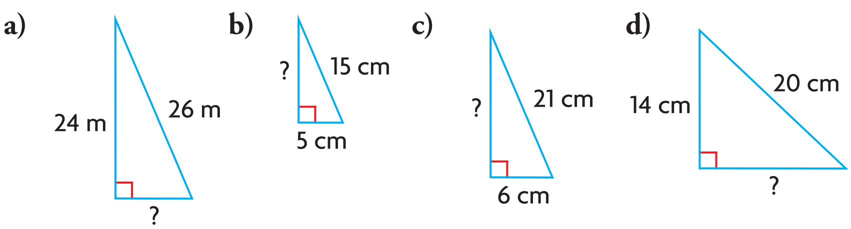


- The formula for the length of a leg of a right triangle is $a = \sqrt{c^2 - b^2}$, where c is the length of the hypotenuse and b is the length of the other leg.

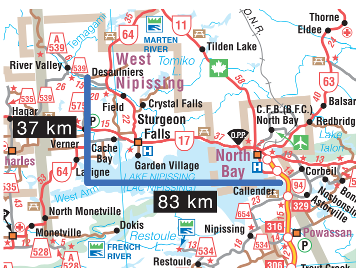
CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Determine the missing length.

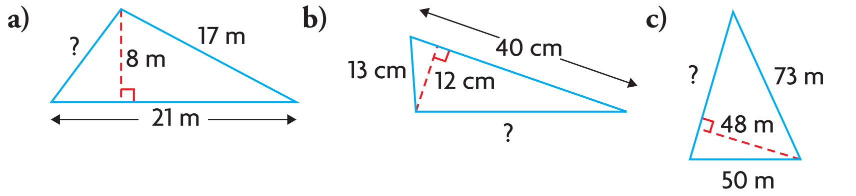


2. What is the length of the direct flight path from Desaulniers to Callander?



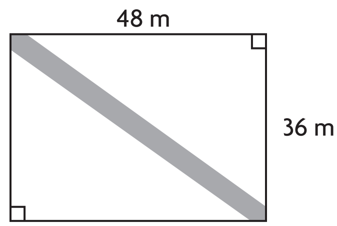
PRACTISING

3. Calculate the missing length.

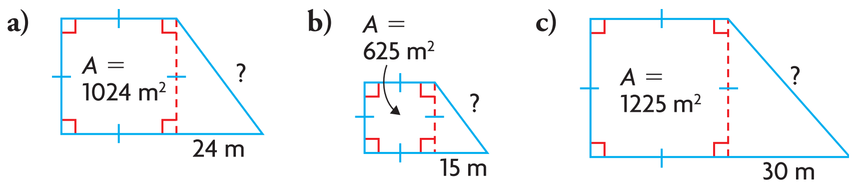


4. A path is being constructed between the corners of the school playground, as shown. Determine the length of the path.

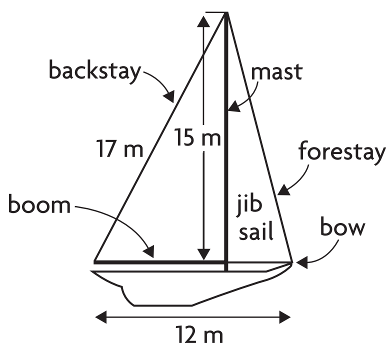
K



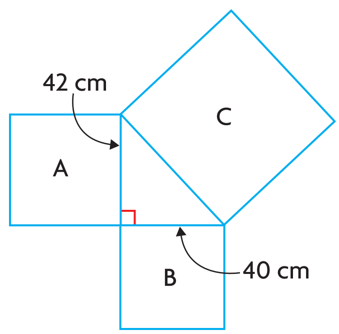
5. Determine the length of the hypotenuse.



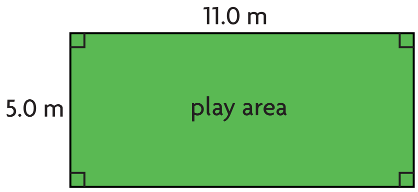
6. Determine the lengths of the boom and the forestay to one decimal place.



7. Determine the area of each square.



8. **A** The outside play area of a daycare centre is shown. Show how you can use the Pythagorean theorem to ensure that the fence corners are at right angles.



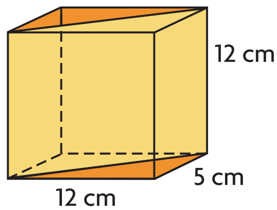
9. **T** A Pythagorean triple is a group of three whole numbers that can represent the lengths of the sides of a right triangle. The smallest Pythagorean triple is 3, 4, 5. Which of the following are Pythagorean triples?

a) 7, 24, 25 **b)** 3, 6, 8 **c)** 9, 21, 23 **d)** 31, 35, 38

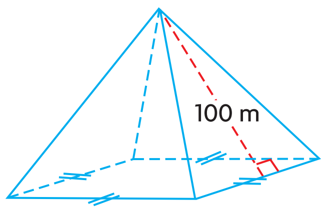
10. **C** Create a geometric problem that you would have to solve using the Pythagorean theorem. Write the problem and its solution, with diagrams.

Extending

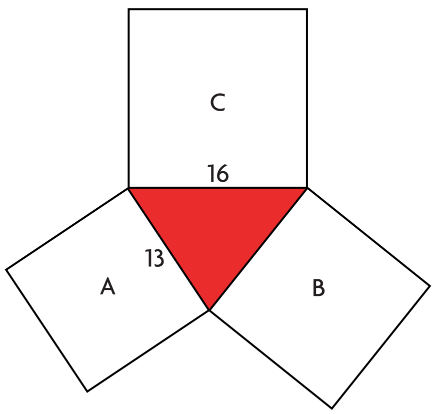
11. A box is 12 cm long, 5 cm wide, and 12 cm high. A cardboard rectangle is inserted along the diagonal to divide the box vertically into two equal spaces. Determine the dimensions of the cardboard rectangle.



12. A square-based pyramid has a slant height of 100 m. Determine two possible sets of dimensions for the height and side length of the pyramid.



13. The red triangle shown is not right-angled. Explain how you know that the combined area of squares A and B does not equal the area of square C.



8.4 Surface Area of Right Pyramids and Cones

YOU WILL NEED

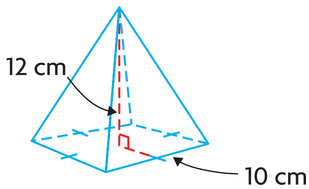
- grid paper

GOAL

Determine the surface area of a pyramid and a cone using a variety of strategies.

LEARN ABOUT the Math

Yvonne is printing slogans on the side of this **right pyramid**. She wants to calculate its surface area.



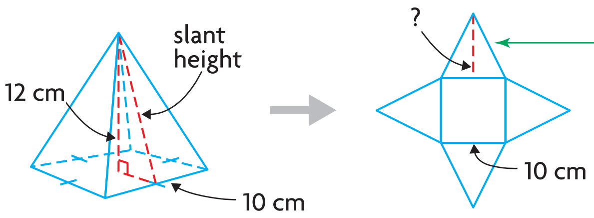
right pyramid

a **pyramid** whose base is a regular polygon and whose top vertex is directly above the centre of the base

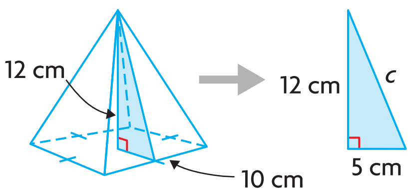
? How can Yvonne determine the area for slogans?

EXAMPLE 1 Calculating surface area using a net and slant height

Yvonne's Solution



I visualized the box's net. It had four identical triangles for the sides and a square base.



$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 12^2 + 5^2 \\ c^2 &= 144 + 25 \\ c^2 &= 169 \\ c &= \sqrt{169} \\ c &= 13 \end{aligned}$$

I calculated the slant height.

The height of the pyramid, *a*, was 12 cm. The distance, *b*, from the centre of the base to the side was half of 10 cm, or 5 cm. I visualized the 12 cm and 5 cm lengths as legs of a right triangle. The slant height, *c*, was the hypotenuse.

$$\begin{aligned} A_{\text{base}} &= s^2 \\ &= 10 \times 10 \\ &= 100 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 10 \times 13 \\ &= 65 \text{ cm}^2 \end{aligned}$$

I calculated the area of the square base and of each triangular face.

$$\begin{aligned} SA_{\text{pyramid}} &= A_{4\text{ triangles}} + A_{\text{base}} \\ &= 4 \times 65 + 100 \\ &= 360 \text{ cm}^2 \end{aligned}$$

I calculated the total surface area.

Reflecting

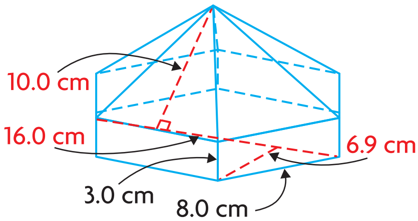
A. How did Yvonne use what she already knew about the area of 2-D shapes to determine the area for slogans on the box?

B. How would you explain to a friend how to calculate the surface area of a pyramid?

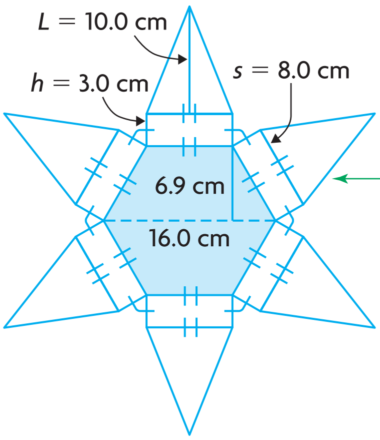
APPLY the Math

EXAMPLE 2 Solving a surface area problem using nets

Judy found a new box. It is a pyramid with six triangular faces on top of a hexagonal prism. What is its surface area?



Judy's Solution



I used a net to see all the faces.

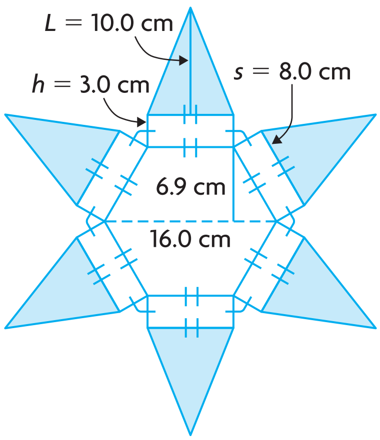
There were six identical triangular faces and six identical rectangular faces. The base was a hexagon, so I divided it into two trapezoids.

$$A_{\text{trapezoid}} = \frac{1}{2}(\text{base}_1 + \text{base}_2) \times 6.9$$
$$= \frac{1}{2}(8.0 + 16.0) \times 6.9$$
$$= 82.8\ \text{cm}^2$$

I calculated the area of one trapezoid.

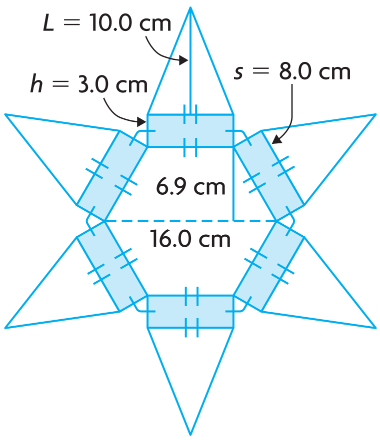
$$A_{\text{hexagon}} = 2 \times A_{\text{trapezoid}}$$
$$= 2 \times 82.8$$
$$= 165.6\ \text{cm}^2$$

I doubled that area to calculate the area of the base.



$$A_{6\text{ triangles}} = 6 \times \frac{sL}{2}$$
$$= 6 \times \frac{8.0 \times 10.0}{2}$$
$$= 240.0\text{ cm}^2$$

I calculated the surface area of the six triangular faces using the base side length *s* and slant height *L*.



$$A_{6\text{ rectangles}} = 6 \times sh$$
$$= 6 \times 8.0 \times 3.0$$
$$= 144.0\text{ cm}^2$$

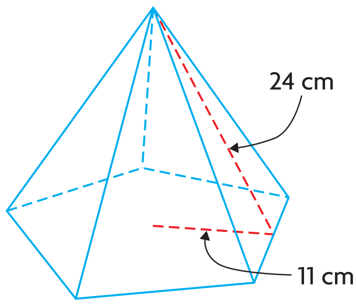
I calculated the surface area of the six rectangular faces using the base side length *s* and rectangle height *h*.

$$SA = A_{\text{base}} + A_{\text{sides}}$$
$$= A_{\text{hexagon}} + A_{6\text{ triangles}} + A_{6\text{ rectangles}}$$
$$= 165.6 + 240.0 + 144.0$$
$$= 549.6\text{ cm}^2$$

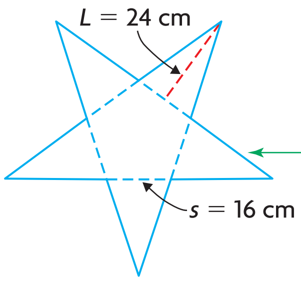
I calculated the total surface area of the box.

EXAMPLE 3 Using reasoning to develop a formula for surface area of a pyramid

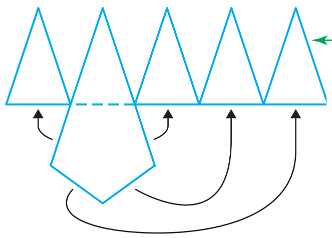
Sarah wants to calculate the surface area of this pyramid. The perimeter of its base is 80 cm.



Sarah's Solution



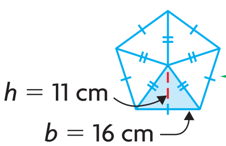
I created a net. I labelled it with the dimensions that I needed. The base side length, s, was $\frac{1}{5}$ of 80 cm, or 16 cm, and the slant height, L, was 24 cm.



There were five triangles, one for each side of the base.

$$A_{5\text{ triangular faces}} = 5 \times \frac{sL}{2}$$
$$= 5 \times \frac{16 \times 24}{2}$$
$$= 960\text{ cm}^2$$

I multiplied the area of one triangle by the number of sides on the base.



I divided the base into five congruent triangles. Each triangle had a base length of 16 cm and a height of 11 cm.

$$A_{\text{triangle}} = \frac{bh}{2}$$
$$= \frac{11 \times 16}{2}$$
$$= 88\text{ cm}^2$$

I calculated the area of one triangle.

$$A_{base} = 5 \times 88$$

There were five triangles, so I multiplied the area by 5.

$$= 440 \text{ cm}^2$$

The area of the base was 440 cm^2.

$$SA = A_{\text{triangular sides}} + A_{base}$$
$$= 960 + 440$$
$$= 1400 \text{ cm}^2$$

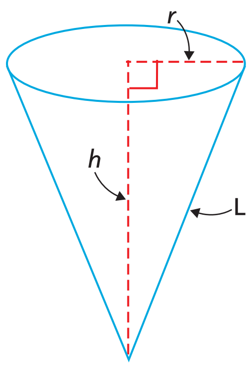
I calculated the total surface area.

The pyramid has a surface area of 1400 cm^2.

EXAMPLE 4 Using reasoning to develop surface area of a cone

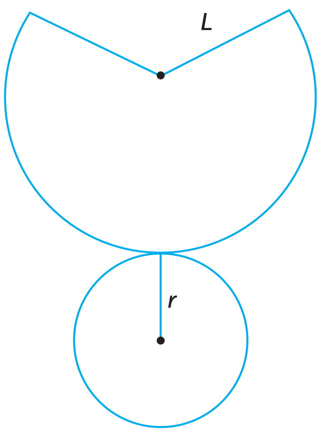
a) Develop a formula for calculating the surface area of any cone with radius r, height h, and slant height L.

b) Use the formula to calculate the surface area of a cone with a radius of 3 cm and a height of 7 cm.



Melinda's Solution

a)



I drew a net for any cone. It is made up of two shapes. The base is a circle with a radius of r. The curved surface opens up to form a sector of the circle with a radius of L, the slant height of the cone.

The surface area of a cone is the sum of the areas of these two shapes.

$$A_{base} = \pi r^2$$

I used the formula for area of a circle to represent the area of the base of the cone.

$$\frac{\textit{Area of curved surface}}{\textit{Area of circle}(\textit{radius} = L)} = \frac{\textit{Circumference of cone}}{\textit{Circumference of circle}(\textit{radius} = L)}$$

For the curved surface, I reasoned that its arc length must be equal to the circumference of the circular base. I used proportional reasoning to write two equal ratios that compare areas to circumferences.

$$\frac{\textit{Area of curved surface}}{\pi L^2} = \frac{2\pi r}{2\pi L}$$

$$\pi L^2 \times \frac{\textit{Area of curved surface}}{\pi L^2} = \pi L^2 \times \frac{2\pi r}{2\pi L}$$

$$\overset{1}{\cancel{\pi L^2}} \times \frac{\textit{Area of curved surface}}{\underset{1}{\cancel{\pi L^2}}} = \overset{1\ L}{\cancel{\pi L^2}} \times \frac{\overset{1}{\cancel{2}}\pi r}{\underset{1\,1\,1}{\cancel{2}\cancel{\pi}\cancel{L}}}$$

$$\textit{Area of curved surface} = \pi r L$$

I want to find the *Area of curved surface*, so I multiplied both sides of the equation by πL^2. Then, I simplified.

$$\begin{aligned}\textit{Surface area of a cone} &= \textit{area base} + \textit{area of curved surface}\\ &= \pi r^2 + \pi r L\end{aligned}$$

I added the two areas that make up the surfaces. This gave me the formula where r = radius of the circular base and L = the slant height of the cone.

b) $r = 3$ cm
$h = 7$ cm
$L = ?$

I know the radius of the cone and its height but I need to find the slant height L to calculate the surface area.

$$\begin{aligned}r^2 + h^2 &= L^2\\ 3^2 + 7^2 &= L^2\\ 9 + 49 &= L^2\\ 58 &= L^2\\ \sqrt{58} &= L\end{aligned}$$

r, h, and L are sides in a right triangle, so I used the Pythagorean Theorem to calculate L.

$$\begin{aligned}\textit{Surface area of a cone} &= \pi r^2 + \pi r L\\ &= (3.14)(3)^2 + (3.14)(3)(\sqrt{58})\\ &= 28.26 + 9.42(\sqrt{58})\\ &\doteq 100\ \text{cm}^2\end{aligned}$$

I substituted the values into the formula, and then, calculated the answer.

In Summary

Key Idea

- To calculate the surface area of a right pyramid, add the area of the base and the area of the faces.
- To calculate the surface area of a cone, add the area of the circular base and the area of the curved surface.

Need to Know

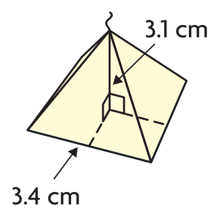
- The slant height of a right pyramid is the height of the triangular faces.
- To calculate the slant height of a right pyramid, use its height, the side length of the base, and the Pythagorean theorem.
- To calculate the area of the base of a right pyramid, divide it into isosceles triangles by drawing lines from the centre of the base to each vertex.
- The surface area of a 3-D figure is the combined area of the 2-D shapes in its net.
- The formula for the surface area of a square-based prism is $SA = A_{4\text{triangles}} + A_{\text{base}}$ or $2bL + b^2$, where b is the base side length and L is the slant height.
- The height of a cone is the distance from the top of the cone to the centre of its circular base.
- To calculate the slant height of a cone, use its radius and height and the Pythagorean theorem.
- The formula for the surface area of a cone is $SA = \pi r^2 + \pi rL$, where r is the radius of the circular base and L is the slant height.

CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Calculate the surface area of each type of candle.

a)



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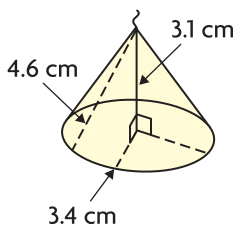
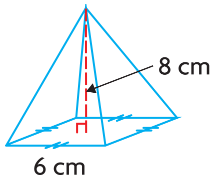


2. Calculate the surface area of each shape.

a)



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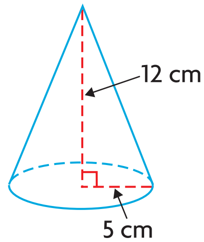
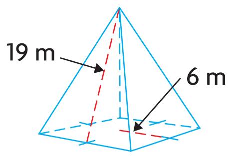


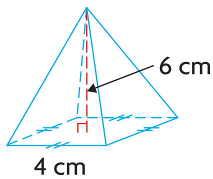
PRACTISING

3. Calculate the surface area.

K **a)**



b)



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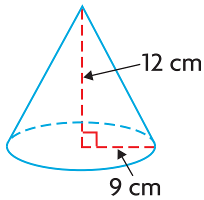
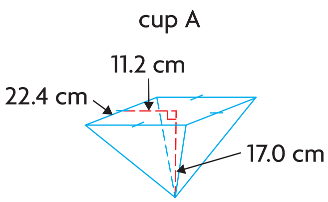


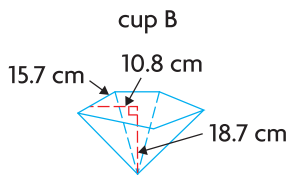
4. Determine the surface area of a square pyramid with a height of 11.0 cm and a base area of 36.0 cm^2.

5. a) Determine the slant height of a cone with a height of 8 cm and a radius of 4 cm.

b) Calculate the cone's surface area.

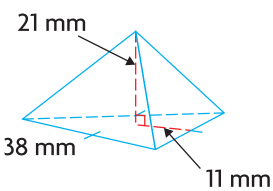
6. There are two shapes of snow-cone cups at the Fall Fair. Which cup uses less material? Assume that the bases are regular polygons.



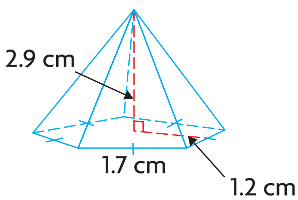


7. Calculate the surface area of each regular pyramid.

a)



b)



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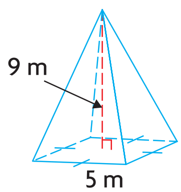
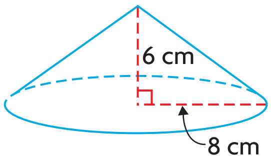


8. Calculate the surface area of each cone.

a)



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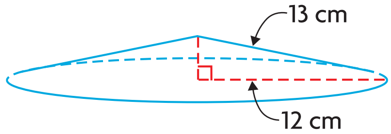

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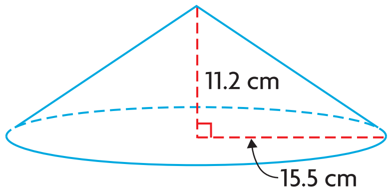
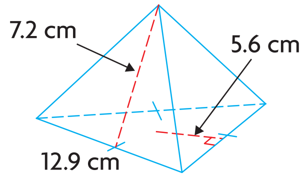


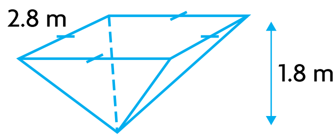
9. A The local party store sells pyramid-shaped gift boxes. They have either a square base with a side length of 10 cm or a regular octagon base with a distance of 6 cm from the centre of the base to the midpoint of each side. Both boxes have a base perimeter of 40 cm. Each box has a height of 8 cm. Which box requires more wrapping paper? Explain your solution.

10. Calculate the surface area of this pyramid.

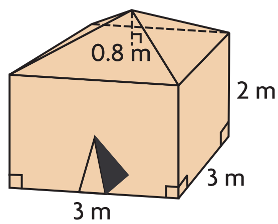


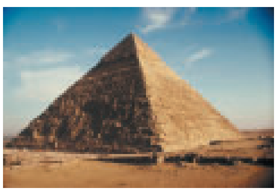
11. Dennis bought a paperweight shaped like a regular hexagonal pyramid for his sister's birthday. It has a measure of 2.6 cm from the centre of its base to the midpoint of each side, a base perimeter of 18 cm, and a height of 4 cm. He wants to know if he has enough wrapping paper for it. Determine the pyramid's surface area.

12. Salt is stored in a bin shaped like an inverted square-based pyramid. The sides of the base are 2.8 m long. The bin is 1.8 m high. Determine the surface area of the bin, including the square base.



13. Determine the surface area of the tent. Include the floor in your calculation.





14. The Great Pyramid of Cheops was originally 147 m high. Its square base had a side length of 230.4 m.

a) Calculate the surface area of the Great Pyramid, including its base.

b) The outside surface of each block in the Great Pyramid is 2.3 m by 1.8 m. Estimate the number of blocks that make up the outside facing of the Great Pyramid.

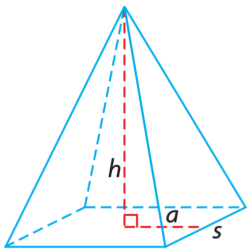
15. Two regular octagonal pyramids are 8 cm high. Pyramid A has a
T surface area of 318.08 cm^2 and a measure of 6 cm from the centre of its base to the midpoint of each side. Pyramid B has a measure of 15 cm from the centre of the base to the midpoint of each side. What is the surface area of pyramid B?

16. Sketch a pyramid and label its dimensions. Show how to calculate its
C surface area in at least two different ways.

Extending

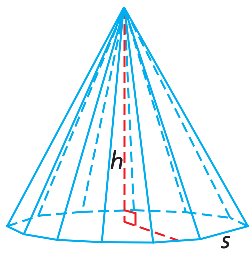
17. Each of these regular pyramids is 10 cm high and measures 4 cm from the centre of the base to the midpoint of each side. Which pyramid do you think has the greatest surface area? Explain.

A.



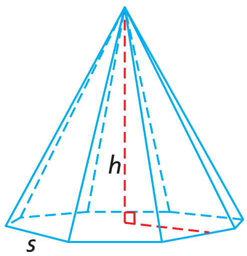
4-sided

C.



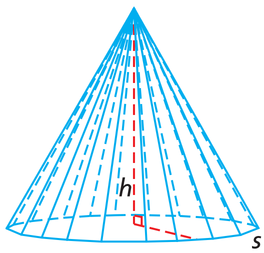
12-sided

B.



8-sided

D.



20-sided

18. a) This shape is composed of two identical regular pyramids. They each have a height of 5 cm and a base side length of 7 cm. Determine the surface area.

b) Another identical pyramid is joined to the shape on one of its triangular faces, as shown. Determine the new surface area.

c) Write a formula for the surface area of a shape with n pyramids joined in this way.

8 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study | *Aid*

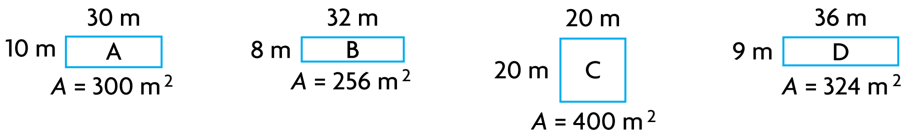
- See Lesson 8.1, Example 1.
- Try Mid-Chapter Review Questions 1 and 3.

Q: If many rectangles have the same perimeter, how can you determine which one has the greatest area?

A1: The rectangle that is most like a square will have the greatest area.

EXAMPLE

These rectangles all have a perimeter of 80 m, but C has the greatest area.

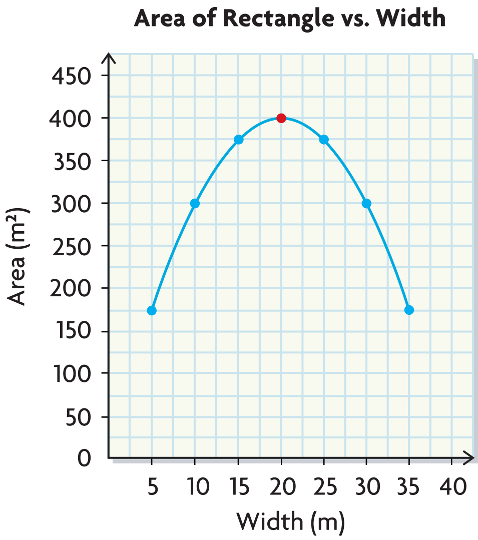


A2: You can use a table of values and a graph.

EXAMPLE

Here is a table of values for rectangles with perimeters of 80 m. The graph shows that the rectangle with the greatest area is a square.

Length (m)	Width (m)	Area (m^2)
5	35	175
10	30	300
15	25	375
20	20	400
25	15	375
30	10	300
35	5	175



Study | *Aid*

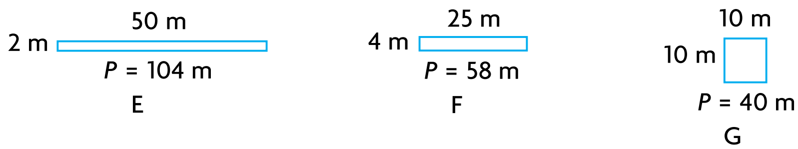
- See Lesson 8.1, Example 2.
- Try Mid-Chapter Review Questions 2 and 4.

Q: If several rectangles have the same area, how can you determine which one has the least perimeter?

A: The rectangle that is most like a square will have the least perimeter.

EXAMPLE

These rectangles all have an area of 100 m^2, but G has the least perimeter.



Q: How can you determine the area or perimeter of a composite 2-D shape?

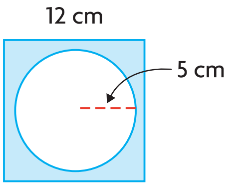
A1: Separate the shape into simpler shapes and calculate their areas. Then, add the areas. To calculate its perimeter, add the length of each side.

A2: Subtract the area of a smaller shape from a larger shape to calculate the area left over. Add the length of each side on the border to calculate the perimeter.

Study Aid
- See Lesson 8.2, Examples 1, 2, and 3.
- Try Mid-Chapter Review Question 5.

EXAMPLE

The area of this square is 144 m^2 and the area of the circle is πr^2 or about 79 cm^2, so the blue area is about $144 - 79 = 65$ cm^2.



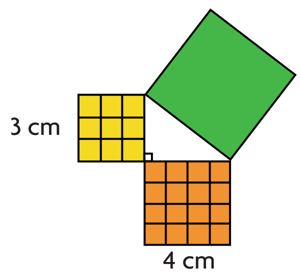
Q: When is the geometric relationship of the Pythagorean theorem useful as a part of a problem-solving strategy?

A: You can use the Pythagorean theorem to determine the third side of a right triangle or the area of a square on the third side. The hypotenuse is $c = \sqrt{a^2 + b^2}$ and the length of a leg is $a = \sqrt{c^2 - b^2}$.

Study Aid
- See Lesson 8.3, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 6 and 7.

EXAMPLE

The length of one side of the green square is $\sqrt{3^2 + 4^2} = 5$ cm and its area is 5^2 or 25 cm^2.



Q: How can you calculate the surface area of a right pyramid or cone?

A: For a right pyramid, you can use the formula $SA = A_{\text{base}} + A_{\text{triangular faces}}$

For a cone, use the formula $SA = \pi r^2 + \pi rL$, where r is the radius of the circular base and L is the slant height.

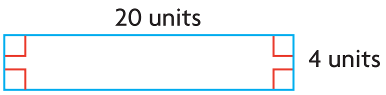
Study Aid
- See Lesson 8.4, Examples 1, 2, and 4.
- Try Mid-Chapter Review Questions 8 and 9.

PRACTICE Questions

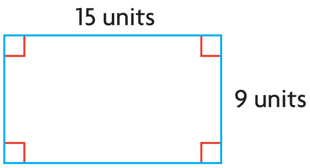
Lesson 8.1

1. Each rectangle has a perimeter of 48 units. Predict which has the greatest area. Explain.

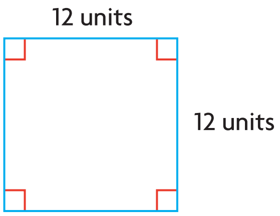
A.



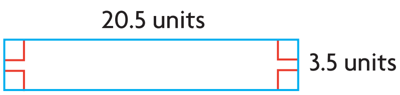
B.



C.



D.



2. Draw rectangles with areas of 72 square units on grid paper. Determine which rectangle has the least perimeter, and then, calculate its perimeter.

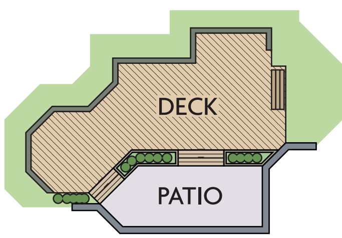
3. Calculate the maximum area for a rectangle with each perimeter.

a) 100 cm **b)** 20 m **c)** 24 km

4. Josie is building a rectangular garden centre with an area of 98 m^2 attached to the side of her store. Determine the minimum length of wood needed for a fence on the three open sides.

Lesson 8.2

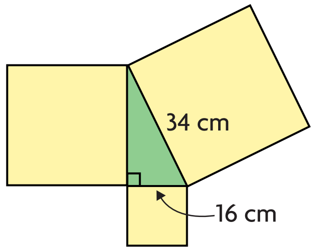
5. Sketch the deck plan. Divide the deck into polygons to show how to determine the area of the wooden section. List the dimensions you need to calculate the perimeter and area of the deck.



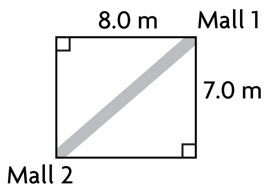
DECK PLAN

Lesson 8.3

6. Arshad is creating this tile pattern. He wants to use a right triangle tile and several square tiles around it. What is the area of each tile?



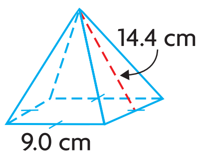
7. A new covered walkway is being constructed to connect two malls. The rectangular space between the two malls is 8.0 m by 7.0 m. The walkway will connect the malls' opposite corners. How long is the reference chalk line drawn between the corners?



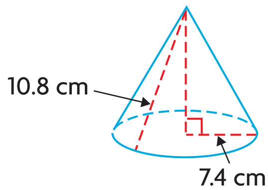
Lesson 8.4

8. Determine the surface area of each shape.

a)



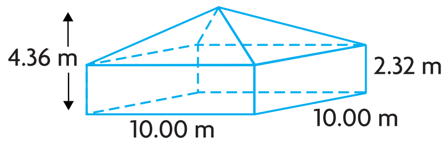
b)



9. Janice needs to re-shingle the roof of her house. One bundle of shingles costs $35.99 and covers 2.25 m^2.

a) How many bundles of shingles does she need for the roof?

b) What is the total cost of re-shingling the roof?



8.5 Volumes of Pyramids and Cones

GOAL

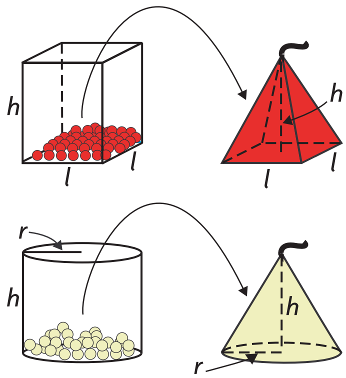
Investigate formulas for the volume of pyramids and cones.

YOU WILL NEED

- 3-D containers: cylinders, pyramids, cones, prisms
- sand or other fine filler

INVESTIGATE the Math

The student art club recycles used candles. The students store yellow wax in cylinders, and red wax in prisms, as shown. They will pour the red wax into pyramids and the yellow wax into cones. Then, they will sell them with wicks as candle kits to raise money for field trips. The pyramids and prisms have the same height and base area. Likewise, the cylinders and cones have the same height and base area.



? How many pyramidal candles can be made from one prism, and how many conical candles can be made from one cylinder?

A. Estimate the pyramid's volume as a fraction of the prism's volume.

B. Fill the prism with sand.

C. How many times can you fill the pyramid with sand from the prism?

D. Estimate the cone's volume as a fraction of the cylinder's volume.

E. Fill the cylinder with sand.

F. How many times can you fill the cone with sand from the cylinder?

Reflecting

G. How is calculating the volume of the pyramid, from the volume of its corresponding prism, like calculating the volume of the cone from the volume of its corresponding cylinder?

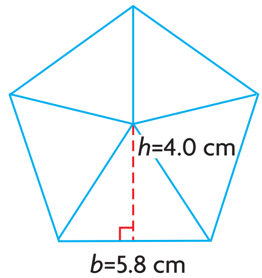
H. What formula describes how to calculate the volume of a square-based pyramid with base length l and height h?

I. What formula describes how to calculate the volume of a cone with radius r and height h?

APPLY the Math

EXAMPLE 1 Calculating the volume of a pyramid

John and Lisa bought an oil lamp with a reservoir in the shape of a pyramid with a regular pentagonal base. A diagram of the base is shown. The reservoir has a height of 8.1 cm. The oil comes in 750 mL bottles. How many times can they fill the lamp completely with one bottle of oil?



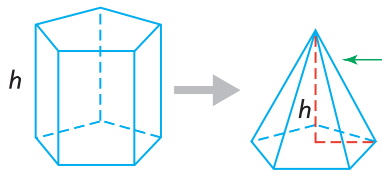
Henri's Solution

$$\begin{aligned} A_{\text{pentagon}} &= 5 \times \frac{bh}{2} \\ &= 5 \times \frac{5.8 \times 4.0}{2} \\ &= 5 \times \frac{23.2}{2} \\ &= 5 \times 11.6 \\ &= 58.0 \text{ cm}^2 \end{aligned}$$

I divided the base into five triangles.

I multiplied by 5 to determine the total area.

The area of the base was 58.0 cm^2.



I drew the prism that corresponds to the pyramid. Its volume is Ah. The volume of the pyramid is $\frac{1}{3}$ that of the prism.

$$\begin{aligned} V_{\text{pyramid}} &= \frac{1}{3}Ah \\ &= \frac{1}{3} \times 58 \times 8.1 \\ &= \frac{1}{3} \times 469.8 \\ &= 156.6 \text{ cm}^3 \end{aligned}$$

I calculated the volume of the pyramid.

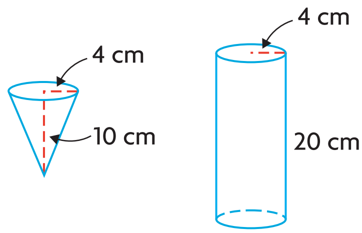
The volume was 156.6 cm^3.
The capacity was 156.6 mL.

$750 \div 156.6 = 4.8$
You can fill the lamp completely four times with one bottle of oil.

I determined how many times you can fill the lamp.

EXAMPLE 2 Selecting a strategy to calculate volume

A conical paper cup has a radius of 4 cm and a height of 10 cm. A cylindrical glass has a radius of 4 cm and a height of 20 cm. How many times do you need to fill the paper cup and pour it into the glass to fill the glass?



Marcy's Solution: Determining volume using a formula

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

I calculated the volume of the paper cup.

$$V_{\text{cylinder}} = \pi r^2 h$$

I calculated the volume of the glass.

$$V_{\text{cylinder}} \div V_{\text{cone}}$$

I divided the volume of the glass by the volume of the cup.

You have to fill the paper cup 6 times to fill the glass.

Tech Support

You can store answers from several calculations using the STO→ and ALPHA keys on a graphing calculator. Select a different letter for each answer. To perform calculations using these answers, use the ALPHA key and each letter in place of the numbers in your expression.

Wyatt decided he could solve the problem without any calculations.

Wyatt's Solution: volume using reasoning

If the glass were the same height as the paper cup, I would need to fill the paper cup 3 times. But, the glass is 20 cm high, so 2 cylinders with a height of 10 cm each will fit inside it. This means that I need to fill the paper cup 2 × 3 or 6 times.

The glass has the same radius as the cup, and twice the height of the cup.

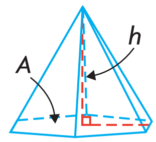
In Summary

Key Ideas

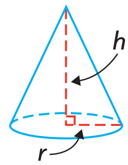
- The volume of a pyramid is $\frac{1}{3}$ the volume of a prism with an identical base and height.
- The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with an identical base and height.

Need to Know

- The formula for the volume of a pyramid is $V = \frac{1}{3}Ah$ where A is the area of its base and h is its height.



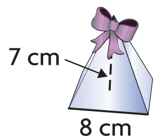
- The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$ where r is the radius of its base and h is its height.



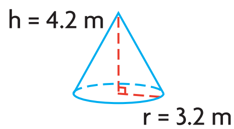
CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Calculate the volume of the gift box.



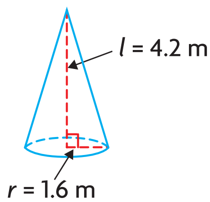
2. Calculate the volume of the cone.



PRACTISING

3. Determine the volume of sand that would fill a cone with a base radius of 6.5 cm and a height of 12.0 cm.
4. Sammy has a regular octagonal-based pyramidal paperweight filled with coloured liquid. It has a distance of 4.2 cm from the centre of its base to the midpoint of each side, a base perimeter of 19.0 cm, and a height of 6.0 cm. Determine the volume of the pyramid.
5. Calculate the volume of the cone.

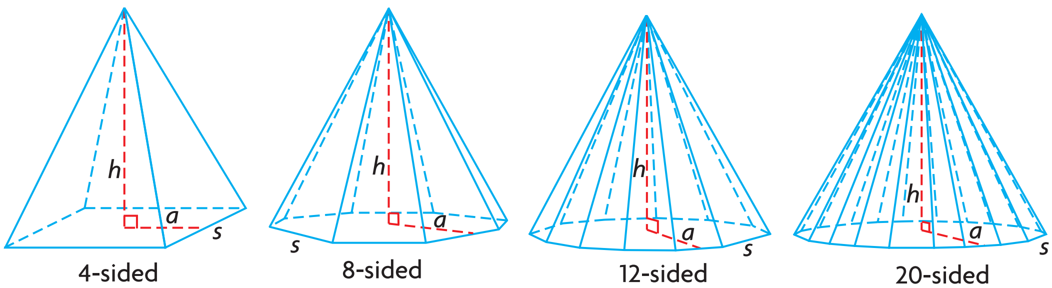
K



6. Sand for icy roads is stored in a conical pile 14.2 m high with a base diameter of 34.4 m.
   - a) Calculate the volume of the pile.
   - b) One sander can take 6.9 m^3 of sand. How many sanders can be filled from the pile?
7. A square-based pyramid has a volume of 100 cm^3 and a base area of 40 cm^2. What is its height?
8. Candles in the shape of square-based pyramids are sold in three volumes: 75 cm^3, 150 cm^3, and 175 cm^3. The base side length of each candle is 5 cm. What are the heights of the candles?
9. **T** A pyramid and a prism with the same height both have a base area of 64 cm^2. How do their volumes compare?
10. **A** Each conical paper cup for a water fountain has a height of 9 cm and a radius of 3 cm. An average of 45 cups of water is drunk each day. What volume of water is drunk each week?
11. **C** Describe the problem-solving process you would use to compare the volume of a square-based pyramid and a cone with the same height.

Extending

12. For each right pyramid, the base is a regular polygon with $a = 4$ cm and $h = 10$ cm.



   - a) Develop a formula to compute the volume of each pyramid in terms of a and h.
   - b) Use your result to state a formula for the volume of a cone with a height of 10 cm and a radius of a.

8.6 Volume and Surface Area of a Sphere

YOU WILL NEED

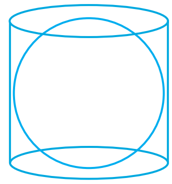
- orange
- scissors and tape
- paper
- sand
- paper plates

GOAL

Develop formulas for the volume and surface area of a sphere.

INVESTIGATE the Math

Exercise balls are **spheres** filled with liquid for weight training. They are sold in cylindrical packages. The manufacturer wants to calculate how much water will fill an exercise ball with a radius of 18 cm, and how much material is needed to make the ball.



? How can you determine the volume and surface area of a spherical shape like the exercise ball?

A. Use an orange to represent the exercise ball.
Construct a paper tube to represent the cylindrical package.
It should be the same height as the orange and have the same circumference as the equator of the orange.

B. Calculate the volume of the paper tube in millilitres using the formula $V = \pi r^2 h$ (1 mL = 1 cm^3).

C. Place the tube on a paper plate.
Put the orange in the tube.
Pour the sand into the tube, filling the regions above and below the orange.

D. Remove the tube, leaving the sand and orange on the plate.
Pour the same sand back into the tube again, using a second plate.

E. Compare the volume of the sand left in the tube with the volume of the tube.

F. Trace the base of the paper tube several times on paper.

G. Calculate the area of the circles, using the formula $A = \pi r^2$.

H. Peel the orange and place the pieces of peel over the circles that you traced using the base of the paper tube.

I. Estimate the area of the orange.

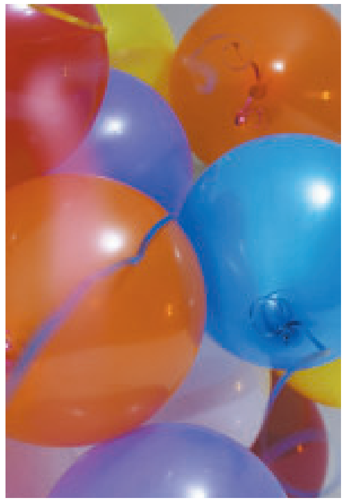
J. Compare the surface area of the peel (sphere) to the area of the base of the tube.

Reflecting

K. About what fraction of the cylinder did the orange fill?
The cylinder's height was twice its radius.
Use this fact and your result to create a formula to describe the volume of a sphere in terms of its radius.

L. About how many copies of the base of the cylinder did you cover with the orange peel?
How might you use your results to create a formula for the surface area of a sphere?

APPLY the Math



EXAMPLE 1 Using a formula to calculate volume

Dylan must buy 100 spherical balloons for \$0.08 each and enough helium to inflate them. Helium costs \$0.024/L. Each balloon will inflate to a surface area of 900.00 cm^2. How much will it cost to buy and inflate them?

Dylan's Solution

$$SA_{\text{sphere}} = 4\pi r^2$$
$$4(3.14 \times r^2) \doteq 900.00$$

I used the surface area to determine the radius.

$$3.14 \times r^2 = \frac{900.00}{4}$$
$$3.14 \times r^2 = 225.00$$
$$r^2 = \frac{225.00}{3.14}$$
$$r^2 \doteq 71.66$$
$$r \doteq 8.47 \text{ cm}$$

I took the square root of 71.66 to calculate *r*.

$$V = \frac{4}{3}\pi r^3$$
$$\doteq \frac{4}{3}(3.14) \times (8.47)^3$$
$$\doteq 2544 \text{ mL or } 2.544 \text{ L}$$

I calculated the volume of one balloon.

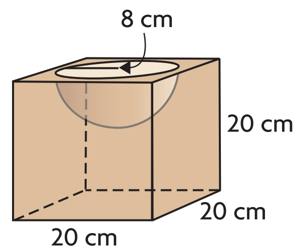
Cost of helium for one balloon
= cost of helium × volume of balloon
= 0.024 × 2.544
= \$0.061

Cost of 100 balloons with helium
100 × (0.08 + 0.061)
= 100 × 0.141
= \$14.10

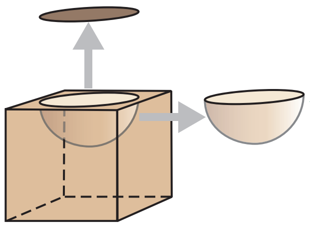
The total cost will be \$14.10.

EXAMPLE 2 Using a visualization strategy to understand and solve a problem

Zuri wanted to make a bowl in shop class. She decided to hollow out a half-sphere from a cube. She needed to know the surface area to varnish the bowl. She also wanted to know the final volume of wood used.



Zuri's Solution



I visualized the surface area as a half-sphere plus a cube. But the cube was missing the area of the circle where the half-sphere was cut.

$$SA_{bowl} = SA_{1\text{ half-sphere}} + SA_{6\text{ squares}} - SA_{1\text{ circle}}$$

I counted the half-sphere and six square sides minus the circle.

$$SA_{sphere} = 4\pi r^2$$

$$\begin{aligned} SA_{half\text{-}sphere} &= \frac{1}{2} \times 4\pi r^2 \\ &\doteq \frac{1}{2} \times 4 \times 3.14 \times 8^2 \\ &= 2 \times 3.14 \times 64 \\ &= 2 \times 200.96 \\ &\doteq 402\text{ cm}^2 \end{aligned}$$

The surface area of the half-sphere is about 402 cm^2.

I calculated the surface area of the half-sphere.

$$SA_{square} = s^2$$

$$SA_{circle} = \pi r^2$$

$$\begin{aligned} SA_{6\text{ squares}} - SA_{1\text{ circle}} &= 6 \times s^2 - \pi r^2 \\ &\doteq 6 \times 20^2 - 3.14 \times 8^2 \\ &= 6 \times 400 - 3.14 \times 64 \\ &= 2400 - 200.96 \\ &= 2199.04 \\ &\doteq 2199\text{ cm}^3 \end{aligned}$$

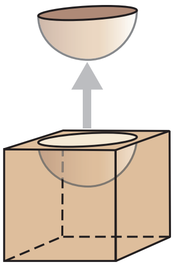
I calculated the surface area of the square sides minus the circle.

The surface area is about 2199 cm^3.

$$SA_{\text{bowl}} = 402 + 2199$$
$$= 2601 \text{ cm}^3$$

The surface area of the bowl is 2601 cm^3.

I calculated the total surface area of the bowl.



$$V_{\text{wood}} = V_{\text{cube}} - V_{\text{half-sphere}}$$

I determined the volume of wood. I visualized the volume as a cube minus a half-sphere.

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$V_{\text{half-sphere}} = \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$\doteq \frac{1}{2} \times \frac{4}{3} \times 3.14 \times 8^3$$
$$= \frac{2}{3} \times 3.14 \times 512$$
$$= \frac{2}{3} \times 1607.68$$
$$\doteq 1071.79 \text{ cm}^3$$

The volume of the half-sphere is about 1071.79 cm^3.

I calculated the volume of the half-sphere.

$$V_{\text{cube}} = s^3$$
$$= 20^3$$
$$= 8000 \text{ cm}^3$$

The volume of the cube is 8000 cm^3.

I calculated the volume of the cube.

$$V_{\text{wood}} = 8000 - 1071.79$$
$$= 6928.21 \text{ cm}^3$$

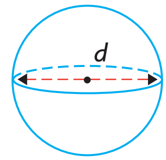
The volume of wood used for the bowl is 6928.21 cm^3.

I calculated the total volume of the wood used.

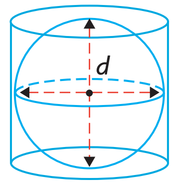
In Summary

Key Ideas

- The surface area of a sphere is four times the area of the circular cross-section that goes through its diameter.



- The volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same radius and height.



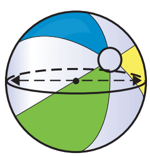
Need to Know

- The formula for the surface area of a sphere with radius r is $SA = 4\pi r^2$.
- The formula for the volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.
- The surface area of a 3-D figure composed of other 3-D figures is the sum of the exposed surface areas of the other figures.
- The volume of a 3-D figure composed of other figures is the combined volume of the other figures.
- When one 3-D figure is removed from another, the volume of the remaining figure is the volume of the original figure minus the volume of the figure that was removed.

CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Calculate the surface area of a tennis ball with a radius of 3.0 cm.
2. Calculate the volume of the beach ball.



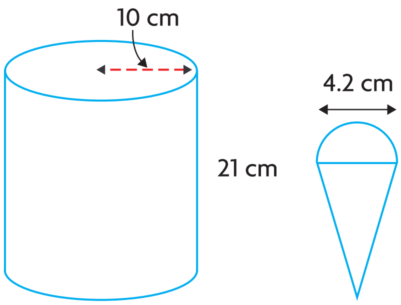
PRACTISING

3. Calculate the surface area of a soccer ball with a radius of 12 cm. Explain what you did.
4. Calculate how much water you would need to fill a round water balloon with a radius of 5 cm.

K

5. Jim runs a company that makes ball bearings. The bearings are shipped in boxes that are then loaded onto trucks. Each bearing has a diameter of 0.96 cm.
   a) Each box can hold 8000 cm^3 of ball bearings. How many ball bearings can a box hold?
   b) Each ball bearing has a mass of 0.95 g. Determine the mass of each box.
   c) The maximum mass a truck can carry is 11 000 kg. What is the maximum number of boxes that can be loaded into a truck?
   d) Besides the ball bearings' mass, what else must Jim consider when loading a truck?

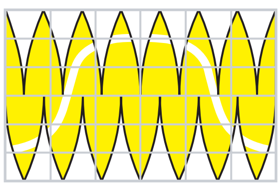
6. Ice cream is sold to stores in cylindrical containers as shown. Each scoop of ice cream in a cone is a sphere with a diameter of 4.2 cm.
   a) How many scoops of ice cream are in each container?
   b) An ice cream cone with one scoop sells for 86¢. How much money will the ice cream store charge for each full cylinder of ice cream that it sells in cones?



7. a) Earth has a circumference of about 40 000 km. Estimate its radius to the nearest tenth of a kilometre and use the radius to calculate the surface area to the nearest hundred square kilometres.
   b) Mars has a surface area of about 144 800 000 km^2. Determine the circumference of Mars to the nearest hundred kilometres.

8. a) Frederic has a sphere of clay with a radius of 10 cm. What additional volume of clay does he need to enlarge his sphere to one with a radius of 20 cm?
   b) How much foil would be needed to wrap the larger sphere?

9. a) A tennis ball has a radius of 3.4 cm. What volume of this cylinder is empty?
T
   b) This pattern is used to create the surface of one tennis ball. How much material will be left over?



10. A baseball has an inner core covered with string. The ball's circumference is between 23 cm and 23.5 cm. Between what values must the surface area fall?

11. A cylinder just fits inside a 10 cm by 10 cm by 10 cm cubic box. Which shape has the smaller surface area? Verify your answer by determining the surface area of each shape.

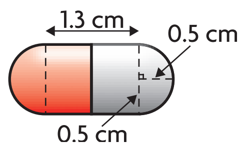
12. **a)** Complete the table.

Shape	Surface Area (cm^2)	Dimensions (cm)	Volume (cm^3)
square-based prism	1000	$s = 10$, $h = \blacksquare$	
cylinder	1000	$r = 10$, $h = \blacksquare$	
sphere	1000	$r \doteq \blacksquare$	

b) Which shape has the greatest volume?

13. Determine the surface area of a ball bearing with a volume of 6.75 cm^3.

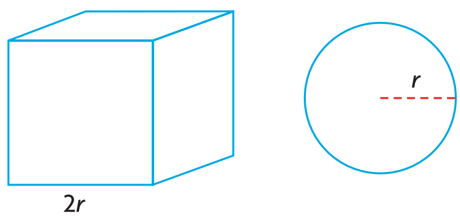
14. **A** A pharmaceutical company creates a capsule for medication in the shape of a cylinder with hemispherical ends as shown. How much medication will the capsule hold?



15. **C** How can you calculate the volume and surface area of a sphere if you know its radius? Create a diagram and dimensions for a sphere from your experience to support your explanation.

Extending

16. Which has a larger volume: a sphere with a radius of r or a cube with a side length of $2r$? Which has a larger surface area?



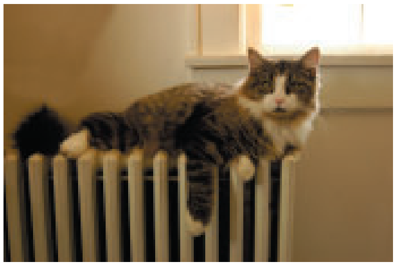
17. A balloon is inflated to a radius of 10 cm. By how much will the radius increase if you add 1 L of air to the balloon?

Curious Math

Surprising Surface Area

The greater an object's surface area, the faster it will give off heat. That is why radiators have a large surface area.

Radiators are coiled, so that they do not take up much space. This way, they give off a large amount of heat without taking up much room.



The same principle applies to living things. For instance, a penguin's body has the smallest possible surface area. This way, it will not lose much body heat in cold weather.

1. Compare the ears of the jackrabbit and the Arctic hare. Why do you think they are so different in size?

jackrabbit in desert

Arctic hare

2. Investigate how surface area plays a role in the bodies of other animals, such as in the ears of an African elephant.

3. Investigate whether your lungs or your classroom floor covers a greater area.

8.7 Exploring Optimum Volume and Surface Area

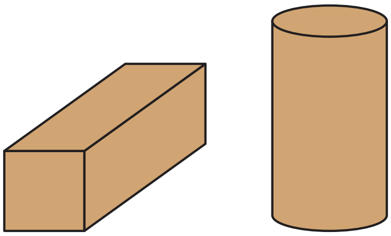
YOU WILL NEED

- grid paper
- graphing calculator or spreadsheet software (optional)

GOAL

Explore to determine optimum measures.

EXPLORE the Math



Diane showed Michael and Katie two packages, one a square-based prism and one a cylinder. Each package had a volume of 1331 cm^3 and a surface area of 728 cm^2.

- Michael wanted one of each type with the same volume and least possible surface area.
- Katie wanted one of each type with the same surface area and greatest possible volume.

	Diane's Packages		Michael's Packages		Katie's Packages	
	Cylinder	**Square-Based Prism**	**Cylinder**	**Square-Based Prism**	**Cylinder**	**Square-Based Prism**
Volume	1331 cm^3	1331 cm^3	1331 cm^3	1331 cm^3	greatest volume	greatest volume
Surface Area	728 cm^2	728 cm^2	least surface area	least surface area	728 cm^2	728 cm^2

? How can Michael and Katie determine the optimum dimensions for their packages?

A. Complete the table, showing possible dimensions for five to ten square-based prisms with a volume of 1331 cm^3.

Prism	Volume (cm^3)	Base Side Length (cm)	Height (cm)	Surface Area (cm^2)
1	1331			
2	1331			

B. Graph the relationship between base side length and surface area. Use the base side length as the independent variable.

C. Repeat parts A and B for cylinders. This time, graph the relationship between the surface area and radius of the base using the radius as an independant variable.

D. Use strategies like those in parts A and C to investigate what happens when the surface area remains at 728 cm^2 and the volume changes.

Reflecting

E. What were the dimensions of the figures with the optimum surface area? What were the dimensions of the figures with the optimum volume?

F. How did using a graph help you to determine the optimum surface area and the optimum volume?

In Summary

Key Idea

- When the volume or surface area of a square-based prism or cylinder is given, you can determine the shape with the least surface area or greatest volume in these ways:
  - List possible dimensions for various figures.
  - Locate the corresponding points on the graph of surface area or volume versus one of the dimensions.

Need to Know

- Graphing a table of values will often help you to recognize relationships, patterns, and/or trends.
- Changing one of the dimensions of a 3-D figure will affect the surface area and volume of the figure.

FURTHER Your Understanding

1. What strategy did you use to select dimensions to investigate?

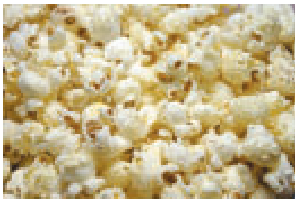
2. a) Create a problem requiring the minimum surface area for a fixed volume.

b) Graph possible dimensions to determine the dimensions that best solve the problem.

8.8 Optimum Volume and Surface Area

YOU WILL NEED

- grid paper
- graphing calculator or spreadsheet software



GOAL

Determine and apply optimum measures to solve problems.

LEARN ABOUT the Math

The student council sells popcorn in square-based prisms and cylinders. Both packages are made from 600.0 cm^2 of card stock. Meredith wants to ensure it is priced fairly.

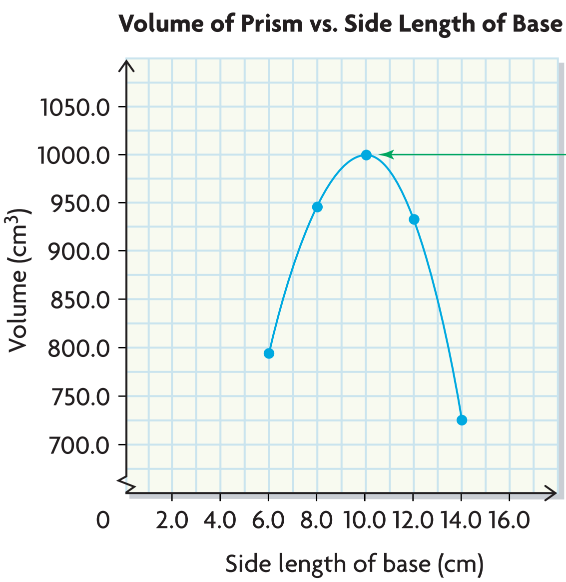
? How can Meredith determine the maximum volume of each package?

EXAMPLE 1 Using a graphing strategy to determine maximum volume

Meredith's Solution

Side Length (cm) s	Height (cm) $h = (600 - 2s^2) \div 4s$	Volume (cm^3) $V = s^2h$
6.0	22.0	792.0
8.0	14.8	947.2
10.0	10.0	1000.0
12.0	6.5	936.0
14.0	3.7	725.2

I created a table for square-based prisms with a surface area of 600.0 cm^2. I chose a side length, s, and then, determined the height and volume for that length.



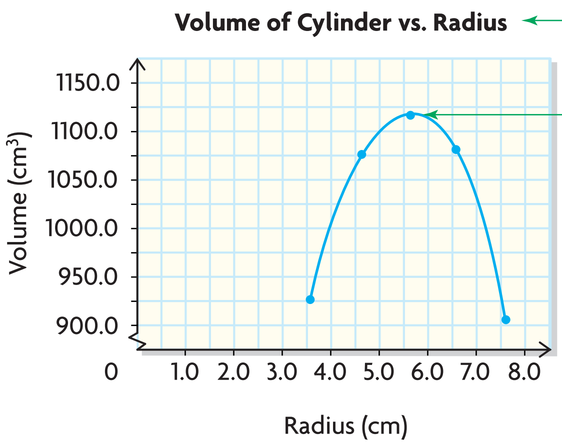
I graphed the relation between volume and side length.

The highest point was at a volume of 1000.0 cm^3.

The maximum volume of a square-based prism with a surface area of 600 cm^2 is 1000.0 cm^3.

Radius (cm) r	Height (cm) $h = \frac{(600 - 2\pi r^2)}{2\pi r}$	Volume (cm^3) $V = \pi r^2 h$
3.6	22.9	932.0
4.6	16.2	1077.0
5.6	11.4	1123.0
6.6	7.9	1081.0
7.6	5.0	907.0

I created a table for cylinders with a surface area of 600.0 cm^2. I chose a radius, r, and then, determined the height and volume for that radius.



I graphed the relation between volume and radius.

The highest point was at a volume of about 1123.0 cm^3.

The maximum volume of a cylinder with a surface area of 600.0 cm^2 is about 1123.0 cm^3.

Reflecting

A. When either the volume or surface area was kept the same, how did changing the value of one dimension affect the values of the other dimensions of the shapes?

B. For the prism and the cylinder that hold the optimum volume of popcorn, what is the relation between the height and the base dimensions?

APPLY the Math

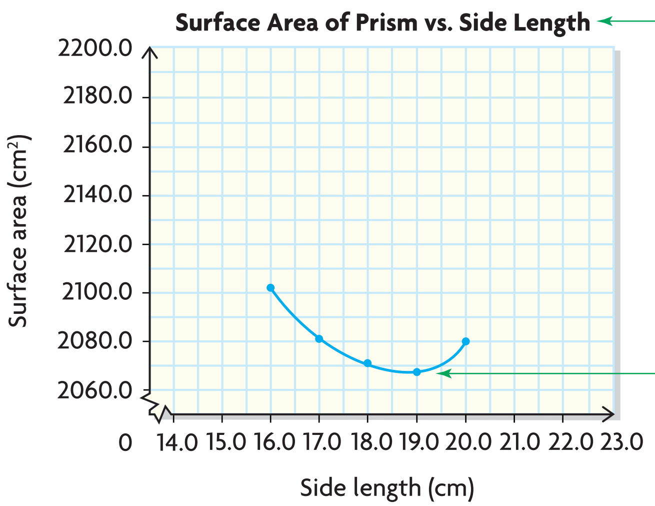
EXAMPLE 2 Using graphing technology to solve a problem

Sasha will use 6400.0 cm^3 of ice to make an ice sculpture. It will be either a prism with a square base or a cylinder. The less surface area the sculpture has, the more slowly it will melt. Which shape should Sasha make and what dimensions should it have?

Sasha's Solution

	A	B	C
1	**Side Length (cm)** **s**	**Height (cm)** **h = 6400 ÷ s²**	**Surface Area (cm²)** **SA = 2s² + 4sh**
2	16.0	25.0	2112.0
3	17.0	22.1	2080.8
4	18.0	19.8	2073.6
5	19.0	17.8	2067.2
6	20.0	16.0	2080.0

I created a table using a spreadsheet for square-based prisms with a volume of 6400.0 cm^3. I chose a side length, *s*, and then, determined the height and surface area for that side length.

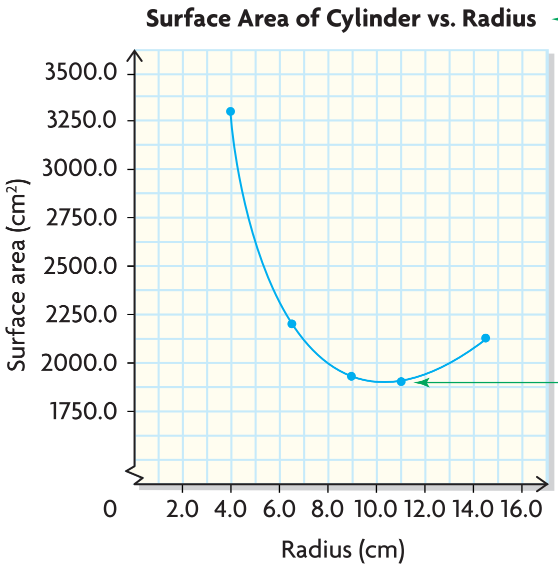


I graphed the relation between surface area and side length.

The lowest point was for a prism with a side length of 19 cm and a surface area of 2067.2 cm^2.

	A	B	C
1	**Radius (cm)** **r**	**Height (cm)** **h = 6400 ÷ r²**	**Surface Area (cm²)** **SA = 2πr² + 2πrh**
2	4.0	127.4	3300.5
3	6.5	48.2	2234.6
4	9.0	25.2	1930.9
5	11.0	16.8	1923.5
6	14.5	9.7	2203.1

I created a table using a spreadsheet for cylinders with a volume of 6400.0 cm^3. I chose a radius, *r*, and then, determined the height and volume for that radius.



I graphed the relation between surface area and radius.

The lowest point occurred for a cylinder with a radius of 10.0 cm and a surface area of about 1909.0 cm^2.

The cylinder has less surface area because 1909.0 cm^2 are less than 2067.2 cm^2. I should create a cylinder with a radius of 10.0 cm and a height of 20.4 cm.

In Summary

Key Ideas

- If you multiply one dimension of a prism or cylinder, you must divide another dimension by a proportional amount to keep the surface area or volume constant.
- A cube is the square-based prism with the least surface area for a given volume, and a cylinder with equal height and diameter is the cylinder with the least surface area for a given volume.
- A cube is the square-based prism with the greatest volume for a given surface area, and a cylinder with equal height and diameter is the cylinder with the greatest volume for a given surface area.

Need to Know

- You can use tables, graphs, graphing calculators, or spreadsheets to determine the effect of varying a dimension on the volume or surface area of a prism or cylinder.
- When you graph a changing dimension of a prism or cylinder versus surface area or volume, the graph has one of these shapes: ∪ or ∩. The optimum value for the dimension is at either the lowest or the highest point.

CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. a) Determine the maximum possible volume of a square-based prism with a surface area of 325 cm^2.
   b) Determine the maximum possible volume of a cylinder with a surface area of 325 cm^2.

2. In a bulk food store, rice is kept in cardboard boxes shaped like square-based prisms. Each box has a volume of 28 000 cm^3.
   a) Determine the dimensions of the box that will use the least amount of cardboard.
   b) Customers use scoops to take the rice from a container. Each scoop holds 1275 cm^3 of rice. How many scoopfuls are in a full box of rice?

PRACTISING

3. Determine the dimensions of the square-based prism with the least
K possible surface area for each volume.
   a) 125 m^2
   b) 3375 cm^3
   c) 21.952 cm^3
   d) 3112.136 cm^3

4. Sugar is sometimes packaged as cubes. Each cube of sugar must have a volume of 3.376 cm^3. Determine the following.
   a) the dimensions of a cube, to one decimal place
   b) the volume of 64 cubes of sugar, to the nearest cubic centimetre
   c) the dimensions of a box in the shape of a square-based prism, made from the least possible amount of material, that will hold 64 cubes

5. The student council is testing new shapes for popcorn boxes. One box will be a cylinder and one will be a square-based prism. They will be made from 900.0 cm^2 of card stock.
   a) What is the maximum volume the cylinder can be?
   b) What is the maximum volume the prism can be?

6. Parnehoi is making an ice sculpture with 8200 cm^3 of ice. It will either be a prism with a square base or a cylinder. Which shape will have the least surface area and what dimensions will it have?

7. Each area below is the surface area of a square-based prism with the greatest possible volume. Determine the height, the side length of the base, and the volume of each prism.
   a) 150 m^2 b) 864 cm^2 c) 541.5 cm^2 d) 4873.5 cm^2

8. Complete this table for cylinders that hold 1000 mL. Which cylinder uses the least amount of material?

Radius (cm)	Height (cm)	Surface Area (cm^2)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

9. Determine the dimensions of a square-based, open-topped prism with a volume of 24 cm^3 and a minimum surface area.

10. **A** An office supply company is producing an open-topped cylindrical pen-holder with a volume of 314.0 cm^3. Determine the base radius and height that will use the minimum amount of material. What is the optimal surface area?

11. A cylinder has a radius of 12 cm and a height of 29 cm. What are the dimensions of a square-based prism with the same volume as the cylinder?

12. **T** What is the greatest volume for an open-topped cylinder with a surface area of 25 cm^2?

13. **C** Create a set of cue cards that a newscaster could read to explain to an audience how to calculate optimum surface area and volume of a cylinder and square-based pyramid. Include any visuals that might appear behind the newscaster as the cue cards were read.

Extending

14. A rectangular cardboard box must be designed to package 12 cans of peas. Each can has a radius of 5 cm and a height of 10 cm.

a) Determine the dimensions of the box that would require the least amount of cardboard.

b) If you had to ship 144 cans, would packaging 12 cans per box be the most economical use of cardboard? Explain.

8 Chapter Review

Study Aid

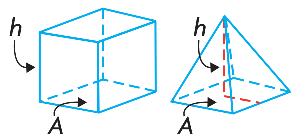
- See Lesson 8.5, Example 1.
- Try Chapter Review Questions 9 a) and 10.

FREQUENTLY ASKED Questions

Q: How can you calculate the volume of a pyramid?

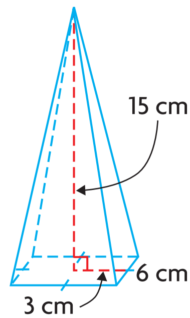
A1: Determine the volume of the prism with the same base and height as the pyramid. Multiply the area of the base by the height. Then, multiply by $\frac{1}{3}$.

A2: Use the formula $V = \frac{1}{3}Ah$ where A is the area of the base and h is the height of the pyramid.



EXAMPLE

Calculate the volume of this pyramid.



Solution

The base is a square with side length 6 cm.
The height of the pyramid is 15 cm.

$$
\begin{aligned}
V &= \frac{1}{3}Ah \\
&= \frac{1}{3}(6^2)(15) \\
&= \frac{1}{3}(36)(15) \\
&= 180 \text{ cm}^3
\end{aligned}
$$

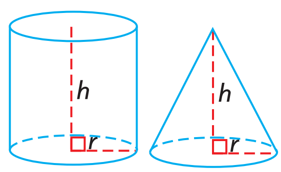
Study Aid

- See Lesson 8.5, Example 2.
- Try Chapter Review Question 9 b).

Q: How can you calculate the volume of a cone?

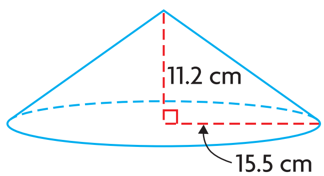
A1: Determine the volume of the cylinder with the same base and height as the cone. Multiply the area of the base, A, by the height, h. Then, multiply by $\frac{1}{3}$.

A2: Use the formula $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the base and h is the height of the cone.



EXAMPLE

Calculate the volume of this cone.



Solution

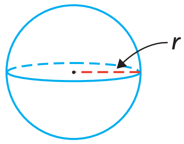
The radius of the base is 15.5 cm.
The height of the cone is 11.2 cm.

$$V = \frac{1}{3}\pi r^2 h$$
$$\doteq \frac{1}{3} \times 3.14 \times 15.5^2 \times 11.2$$
$$= \frac{1}{3} \times 3.14 \times 240.25 \times 11.2$$
$$\doteq 2816.4 \text{ cm}^3$$

Q: How can you calculate the surface area and volume of a sphere?

A1: Use the formula $SA = 4\pi r^2$ where r is the radius.

A2: Use the formula $V = \frac{4}{3}\pi r^3$ where r is the radius.

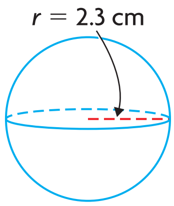


Study Aid

- See Lesson 8.6, Examples 1 and 2.
- Try Chapter Review Questions 11 and 12.

EXAMPLE

Calculate the volume and surface area of this sphere.



Solution

The radius is 2.3 cm.

$$SA = 4\pi r^2$$
$$= 4\pi(2.3)^2$$
$$= 4 \times 3.14 \times 5.29$$
$$\doteq 66.4 \text{ cm}^2$$

$$V = \frac{4}{3}\pi r^3$$
$$\doteq \frac{4}{3} \times 3.14 \times 2.3^3$$
$$= \frac{4}{3} \times 3.14 \times 12.167$$
$$\doteq 50.1 \text{ cm}^3$$

Q: Which square-based prisms and cylinders optimize volume and surface area?

A: This table shows the square-based prisms and cylinders that optimize volume and surface area:

Study Aid

- See Lesson 8.8, Examples 1 and 2.
- Try Chapter Review Question 14.

Figure	Optimizes Surface Area	Optimizes Volume
Square-Based Prism	A cube has the minimum surface area for a given volume.	A cube has the greatest volume for a given surface area.
Cylinder	The cylinder whose height equals its diameter has the minimum surface area for a given volume.	The cylinder whose height equals its diameter has the maximum volume for a given surface area.

PRACTICE Questions

Lesson 8.1

1. Arianna is creating a rectangular outdoor space for her pet rabbit. Fencing material costs \$15.25/m. She has \$145. What dimensions give the greatest area, to the nearest tenth of a metre?

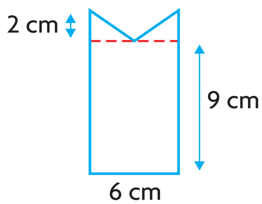
2. What is the minimum perimeter possible for a rectangle with an area of 500 cm^2?

3. Sarah has 20 m of garden edging. What are the dimensions of the rectangular garden with the greatest area can she enclose with the edging?

4. Denzel wants to rope off a 800 m^2 rectangular swimming area using the beach as one of the sides. What should the dimensions of the rectangle be in order to use the minimum amount of rope?

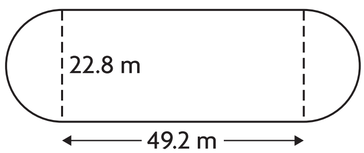
Lesson 8.2

5. Calculate the area of the figure.



6. Michelle created an octagonal quilt piece for a quilt-making class project. It was cut from a square with a side length of 10.0 cm. To make the piece, Michelle cut off the four corners of the square, by measuring 2.9 cm from each corner, and then, cutting the diagonal. What are the area and perimeter of the octagonal quilt piece?

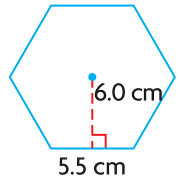
7. A school field has the dimensions shown.



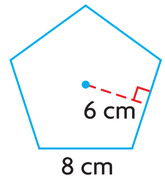
a) Calculate the length of one lap of the track.
b) If Amanda ran 625 m, how many laps did she run?
c) Calculate the area of the field.

8. Calculate the area and perimeter of each regular polygon.

a)



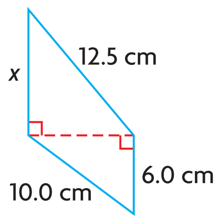
b)



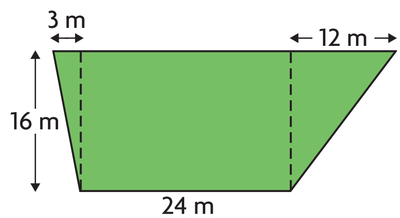
Lesson 8.3

9. A baseball diamond is a square. The distance between the bases is 27.4 m. Calculate the direct distance from first base to third base.

10. Find the length of x accurate to the nearest tenth.



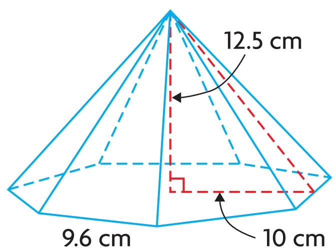
11. Determine the length of the fence around the playground.



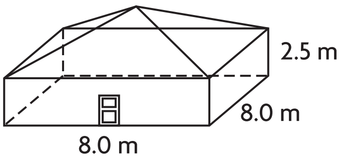
12. A right triangle's legs are 20 cm and 48 cm. What is the area of the square whose side length is equal to the hypotenuse?

Lesson 8.4

13. Calculate the surface area of the regular pyramid.



14. Janice and Wilson have bought a new house. They decide to paint the exterior of the house, including the door, and re-shingle the roof. One 4-L can of paint covers 35 m^2. One bundle of shingles covers 2.25 m^2.



a) How many bundles of shingles do they need for the roof? (Hint: Find the slant height of the roof first.)
b) How many cans of paint do they need?
c) One can of paint is $29.95 and one bundle of shingles is $35.99. Find the total cost of the job.

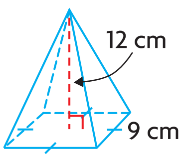
15. Determine the surface area of a square-based pyramidal candle with a base side length of 8 cm and a slant height of 10 cm.

16. Determine the height of a square-based pyramid with a base side length of 8.0 cm and a surface area of 440.0 cm^2.

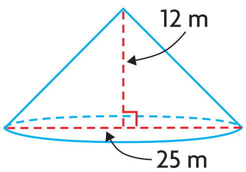
Lesson 8.5

17. Calculate the volume and surface area of each figure.

a)

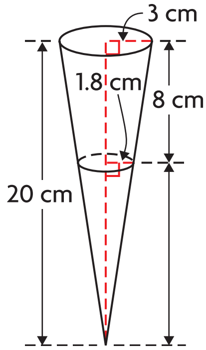


b)



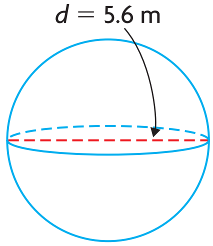
18. Gum is packaged in a square-based pyramid-shaped box with a distance of 6 cm from the centre of the base to the sides and a height of 12 cm.
a) How much material was used to create the box?
b) What is the volume of the box?

19. A solid figure is said to be truncated when a portion of the bottom is cut and removed. The cut line must be parallel to the base. Many paper cups, such as the one shown here, are truncated cones. Calculate the volume of this paper cup.



Lesson 8.6

20. Calculate the volume and surface area of this sphere.



21. A spherical bar of soap just fits inside its package, which is a cube with a side length of 8 cm.
a) What is the volume of the bar of soap?
b) Calculate the amount of empty space in the box.

22. A toy company makes rubber balls with a diameter of 20 cm. How much rubber would be saved per ball if the balls had a diameter of 15 cm?

Lesson 8.7

23. A square-based pyramid has a base side length of 13 cm and a height of 16 cm. What are the dimensions for a cylinder having the same volume as the pyramid?

Lesson 8.8

24. Determine, to one decimal place, the dimensions of the rectangular square-based prism that would have the greatest volume for each surface area. Show your solution.
a) 210 cm^2
b) 490 cm^2

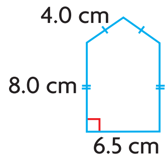
25. What is the greatest volume for an open-topped rectangular prism with a surface area of 101.25 cm^2?

8 Chapter Self-Test

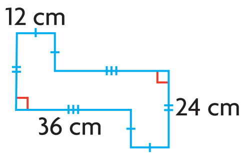
1. Jamal wants to install a rectangular 2025 cm^2 window in his garden shed. That area of window comes in several length/width combinations, and he wants to minimize the perimeter to prevent drafts. What is the least perimeter possible for the window?

2. Calculate the area and perimeter of each shape.

   a)

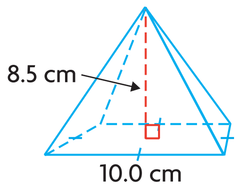


   b)



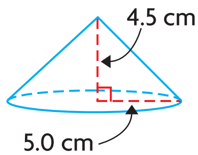
3. Determine the volume of a square-based pyramid with base length 8.0 cm and slant height 10.0 cm. Sketch and label the pyramid in your solution. Explain your thinking.

4. Choose the surface area of the square-based pyramid.
   A. 85.5 cm^2
   B. 100.0 cm^2
   C. 297.2 cm^2
   D. not possible to calculate

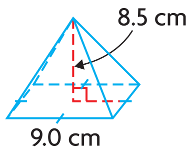


5. Calculate the volume and surface area of each figure.

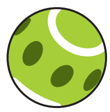
   a)



   b)



6. The diameter of an inflatable dog toy is 6.5 cm.
   a) What amount of material was required to produce it?
   b) What is the volume of air inside it?



7. A cylinder is being designed to hold rice pudding. It will hold 1078.0 mL of pudding. Which radius minimizes the surface area?
   A. 5.6 cm
   B. 7 cm
   C. 8.5 cm
   D. 10 cm

8. Suppliers sell centimetre cubes to schools in packages shaped like square-based prisms. Determine the dimensions of the package that would require the least material to hold 1200 cubes.

Storage Capacity of a Silo

Tony is building a new silo to store corn as animal feed. It will be a cylinder topped with a half-sphere, and must store 21 000 t of corn. The entire silo can be filled with corn. Tony wants to minimize the surface area of the silo to reduce materials and paint costs. He has the following information:

- 1 m^3 of corn has a mass of 700 kg.
- Building costs are \$8/m^2, taxes included.
- Paint comes in 3.8 L cans. Each can covers 40 m^2 and costs \$35, taxes included.
- Corn costs \$140 per tonne (\$140/t), taxes included. Recall that 1 t = 1000 kg.

? What is the total cost to build, paint, and fill a silo with the least surface area?

A. Sketch the silo. Label any measurements you will need.

B. Calculate the volume of the silo using the mass of feed it must hold.

C. Create a table listing possible dimensions for the silo.

D. Graph the surface area versus base radius.

E. Determine the minimum surface area.

F. Calculate the silo's building cost (before painting).

G. Calculate the silo's paint cost.

H. Calculate the cost to fill the silo with corn.

I. Determine the total cost.

J. Prepare a written report that shows your calculations and explains your thinking.

Task Checklist

- ✔ Did you label all your table values and calculate entries correctly?
- ✔ Did you draw your sketch and label your graph accurately?
- ✔ Did you support your choice of surface area?
- ✔ Did you explain your thinking clearly?

Chapters 7–8 Cumulative Review

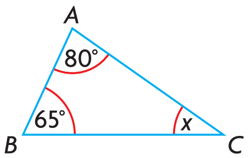
Multiple Choice

1. Melissa cut two vertices off a triangle. What is the maximum amount the sum of the interior angles of the resulting shape will increase by?

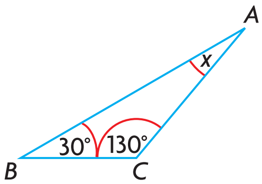
A. 180° **C.** 360°
B. 540° **D.** 90°

2. In which diagram is $x = 150°$?

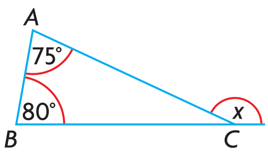
A.



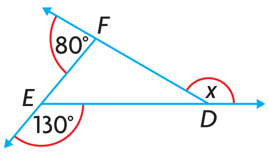
B.



C.



D.



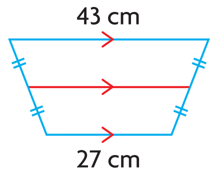
3. Which statement is not always true?

A. If the two pairs of opposite sides of a quadrilateral are congruent, the figure must be a parallelogram.
B. The diagonals of a rhombus are perpendicular.
C. The diagonals of a square are perpendicular bisectors.
D. The diagonals of a parallelogram are always congruent.

4. Which statement is not always true?

A. The midsegments of a rhombus form a rectangle.
B. The midsegments of a square form a square.
C. The midsegments of a parallelogram form a parallelogram.
D. The midsegments of a rectangle form a rectangle.

5. Determine the length of the red line segment.



A. 60 cm **C.** 30 cm
B. 35 cm **D.** 16 cm

6. The police seal off accident scenes with yellow tape. Determine the dimensions of the maximum rectangular area that can be surrounded with 300 m of tape.

A. 100 m by 50 m **C.** 75 m by 75 m
B. 150 m by 2 m **D.** 125 m by 25 m

7. A stop sign shaped like a regular octagon is 120 cm from side to side and each side is 50 cm long. Estimate the area of the sign.

A. 24 000 cm^2 **C.** 6000 cm^2
B. 12 000 cm^2 **D.** 3000 cm^2

8. Determine the radius of a sphere with a volume of 117.00 cm^3.

A. 3.03 cm **C.** 6.04 cm
B. 1.02 cm **D.** 58.50 cm

9. A sugar sculpture is a triangular pyramid 18.0 cm high. The base is an equilateral triangle with 3.0 m sides. Determine the volume of the sculpture.

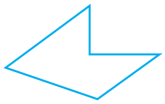
A. 23.4 cm^3 **C.** 54.0 cm^3
B. 27.0 cm^3 **D.** 70.1 cm^3

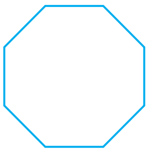
10. The sum of the interior angles in a polygon is 1800°. How many sides does it have?

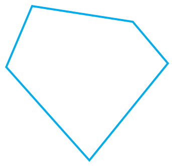
A. 9 **C.** 11
B. 10 **D.** 12

11. Which of the following is not a convex polygon?

A. 

C. 

B. 

D. 

12. What is the measure of each exterior angle in a regular 12-gon?

A. 30° **C.** 20°
B. 45° **D.** 35°

13. In any polygon what is the sum of any interior angle and its corresponding exterior angle?

A. 360° **C.** 90°
B. 180° **D.** 270°

14. If the diagonals of a quadrilateral are perpendicular, equal in length, and bisect each other, then the shape is a:

A. rectangle **C.** kite
B. rhombus **D.** square

15. How many counterexamples are needed to disprove a conjecture?

A. 1 **C.** 5
B. 2 **D.** 10

16. In which of the following quadrilaterals do the midsegments form a parallelogram?

A. rhombus **C.** rectangle
B. trapezoid **D.** all of the above

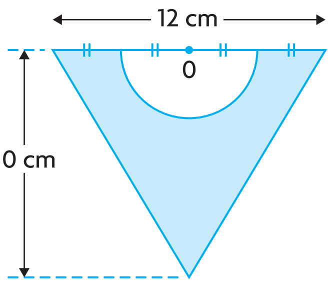
17. What lines can be used to locate the centroid of any quadrilateral?

A. diagonals **C.** midsegments
B. bimedians **D.** medians

18. What is the greatest rectangular area that can be enclosed with a 100 m roll of fencing?

A. $100\ m^2$ **C.** $625\ m^2$
B. $250\ m^2$ **D.** $825\ m^2$

19. Determine the area of the shaded region.



A. $60\ cm^2$ **C.** $106\ cm^2$
B. $46\ cm^2$ **D.** $75\ cm^2$

20. A sphere has a diameter of 10 cm. Both the diameter and the height of a cone are 10 cm. A cube has a side length of 10 cm. Both the side length and the height of a square-based pyramid are 10 cm. Which shape has the least volume?

A. sphere **C.** cone
B. cube **D.** pyramid

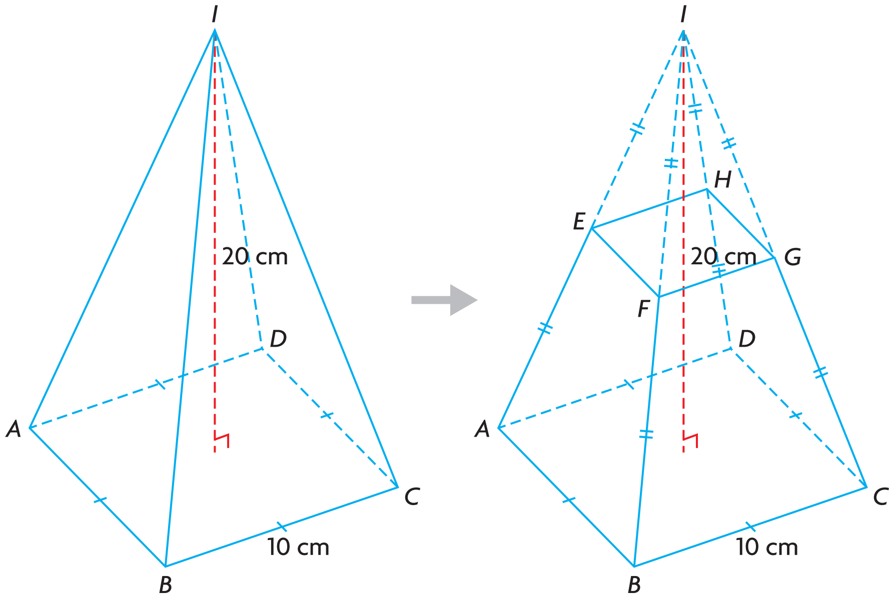
21. For a given volume, the cylinder with the least surface area occurs when:

A. radius = height
B. radius = height ÷ 2
C. diameter = height
D. 2(diameter) = height

Investigation

Mystery of the Pyramids

22. Jeremy is creating a piece of art for an exhibit. He starts with a square-based right pyramid, as shown. He makes a cut parallel to the base through the midpoints of the lateral edges. Then, he removes the top of the pyramid.



a) Determine the volume of the original pyramid.

b) Determine what volume of the pyramid was removed.

c) In terms of volume, what fraction of the original pyramid was removed?

d) Investigate whether this fraction would be the same if the original pyramid had a rectangular base.