First Differences

How can we determine if a relation is linear or not given a table of values?

We can determine whether a relation is linear or not based on the value of the **first differences**. First differences are differences between ___Consecutive___ y-values with EVENLY spaced x-values. When calculating first differences, we are ___subtracting___ the current y-value by the previous y-value ($y_2 - y_1$).

Example 1: Let's practice calculating the first differences of the following relations.

a)

evenly spaced

x	y	First differences
0	7	———
1	3	$3-7=-4$
2	-1	$-1-3=-4$
3	-5	$-5-(-1)=-4$
4	-9	$-9-(-5)=-4$

+1, +1, +1, +1

= linear

b)

x	y	First differences
-4	-8	———
-2	-2	$-2-(-8)=6$
0	0	$0-(-2)=2$
2	-2	$-2-0=-2$
4	-8	$-8-(-2)=-6$

= non-linear

Remember to check that the x-values have the same increments!

Now let's put our knowledge of calculating first differences to the test...
Let's complete the Modeling Linear and Non-Linear Relations lesson and draw some conclusions!

Now let's compare the graphs and their first differences. Here's the million dollar question... what is the relationship between the linear relations and their first differences?

A relation is linear if the values of the ___first___ ___differences___ are ___constant___

Modelling Linear and Non-Linear Relations

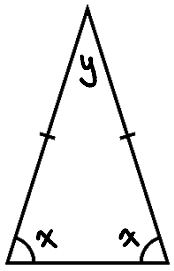
Think: When we say a relation is a linear relation, what is formed when the two variables are graphed?

straight continuous line (constant rate of change)

Let's spend some time today and explore some relationships!

Example 1: How are the angles in an isosceles triangle related?

a) Make a table of values to show the measures of the equal angles and the third angle in an isosceles triangle.

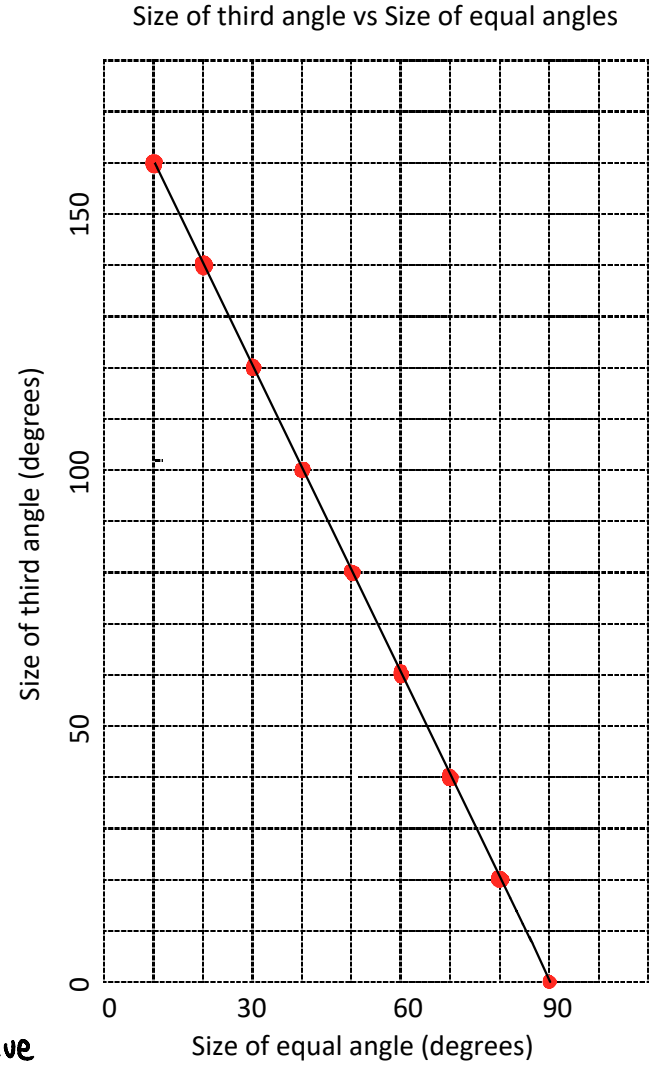


x	y
Size of Equal Angle (degrees)	Size of Third Angle (degrees)
10	180 - 2(10) = 160
20	180 - 2(20) = 140
30	120
40	100
50	80
60	60
70	40
80	20
90	0

b) What trend do you see in the data?

as size of equal angles increases by 10°, the size of third angle decreases by 20°.

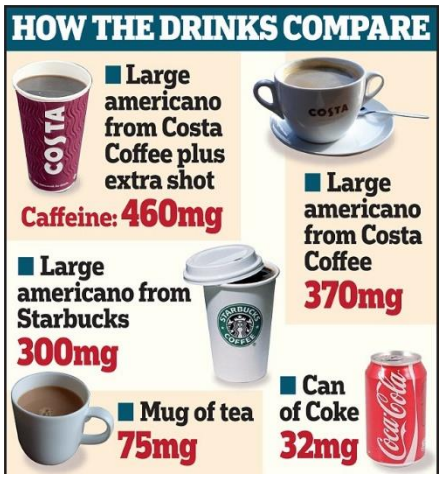
c) Graph the data on the graph on the right



d) Is the relationship linear? Justify your answer.

Yes, the graph forms a straight line and the y-value decreases by a constant rate.

Example 2: A typical North American adult consumes about 200 mg of caffeine a day. Caffeine has a half-life of about 6 hours. This means that about 6 hours after consumption, half the caffeine remains in a person's body.



a) Complete the table of values to show how much caffeine is left in a person's body over time.

Time (hours)	Mass of Caffeine (mg)
0	200
6	$\frac{200}{2} = 100$
12	$\frac{100}{2} = 50$
18	25
24	12.5

+6

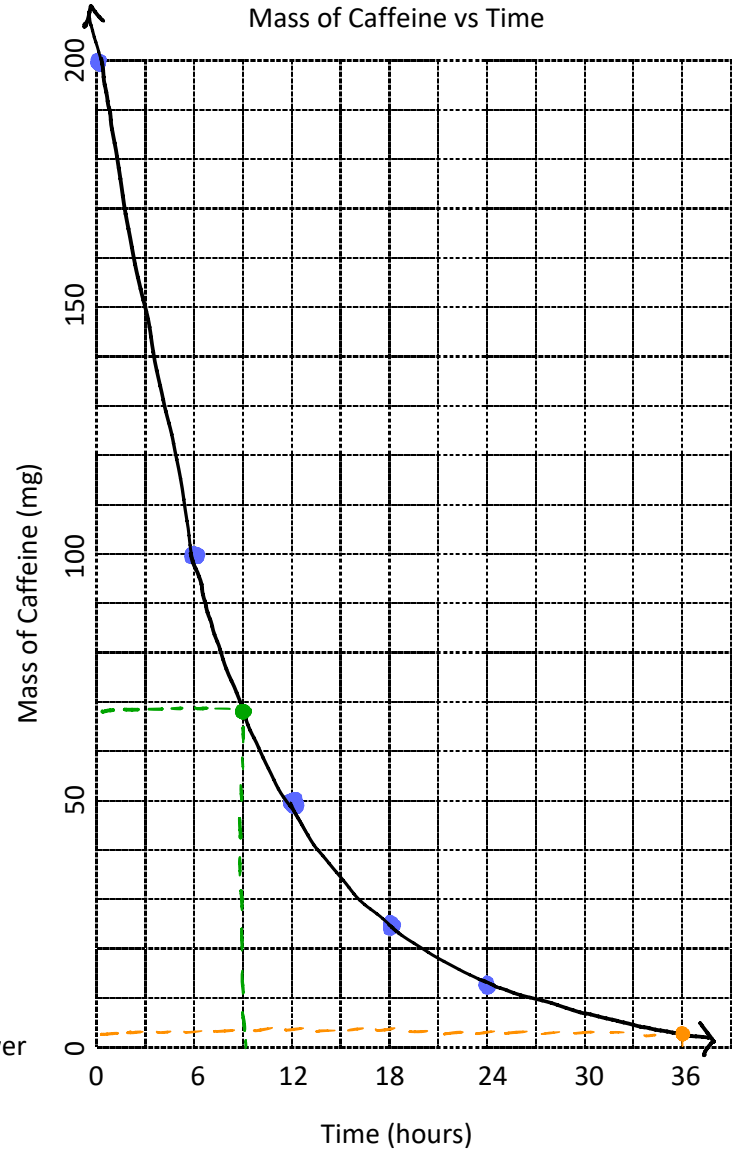
b) What trends do you see in the data?

Every 6 hours, the mass of caffeine decreases by one-half.

c) What do you think the graph will look like?

curved decline

d) Graph the data on the graph on the right.



e) About how much caffeine will remain after 9 hours?

about 70 mg

f) About how much caffeine will remain after 36 hours?

about 3 mg

g) Is the relationship linear? Justify your answer

No, rate of change is not constant and first differences are not equal.

h) What assumptions did you make?

→ No additional caffeine intake

→ Rate is the same for anybody