

## Section 5.1 – Direct Variation

MPM1D

### Part 1: DO IT NOW!!! “What is speed?”



You and a friend decide to have a friendly bicycle race on Saturday. How far ahead will you be after one hour, given that you travel at the following speeds?

- a) Heading North      your speed is 12km/h, your friend's speed is 10 km/h
- b) Heading South      you travel 5 km in 30 minutes, your friend travels 5 km in 20 minutes

**Calculations:**

a)

b)

Conclusion: Speed is an example of \_\_\_\_\_

## Part 2: Investigate Direct Variation using Tables



**Example 1:** Use a table to organize the information in the following problem.

**a)** How long does it take to fill a 3 000 litre hot tub if a water truck supplies water at a rate of 250 litres per hour and the tub is initially empty?

Time ( $h$ )	Water ( $L$ )

**b)** What if the water truck was able to double the flow rate?

Time ( $h$ )	Water ( $L$ )

Which variable is dependent? (If a variable is dependent its value cannot be known without knowing the other variable's value). **To determine the dependent variable ask yourself the following key questions:**

Does the amount of water depend upon time ?

**OR**

Does time depend upon the amount of water ?

$$\textbf{Rate of Change} = \frac{\Delta \text{dependent variable}}{\Delta \text{independent variable}} = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{water}}{\Delta \text{time}}$$

**a)** Calculate the rate of change of the amount of water in the hot tub between the following times, using the first table:

**i)** Between 0 hours and 1 hour: rate = \_\_\_\_\_ =

**ii)** Between 0 hours and 2 hour: rate = \_\_\_\_\_ =

**iii)** Between 2 hours and 5 hour: rate = \_\_\_\_\_ =

What do you notice about these rates?

When the rate is ***constant***, it is often referred to as the ***constant of variation***.

A **direct variation** is a relationship between two variables in which one variable is a constant multiple of the other. The rate of change is constant (constant of variation). A direct variation is a situation in which two quantities, such as hours and pay, increase or decrease at the same rate. That is, as one quantity doubles, the other quantity also doubles. The variables are said to be directly proportional.

**For example:** if you get paid hourly, when you work twice as many hours, you will make twice as much money. How much you make is a constant multiple of how many hours you work.

**Example 2:** Identify the dependent and independent variables in the following rates of change.

	Dependent variable ( $y$ )	Independent variable ( $x$ )
The car travelled at 75 km/h.		
In November the temperature drops 1.2 degrees Celsius per day.		

**Example 3:** Determine the constant of variation (rates of change) given the data in the following tables.

**Recall:** Rate of change (constant of variation) =  $\frac{\Delta \text{dependent}}{\Delta \text{independent}}$

a)

Hours worked (h)	Money made (\$)
0	0
1	35
2	70
3	105

b)

Mass of grain (kg)	Cost (\$)
0	0
20	125
40	250
60	375

Rate of change ( $m$ ) =

Rate of change ( $m$ ) =

i) What is the same in the relationships given in parts a) and b)?

Both tables represent **direct variations**, the independent variable directly affects the dependent variable.

**Direct variation will always have ( 0 , 0 ) as the initial value!**

ii) Write an equation to model the data in the tables given in Example 3. Remember that 'y' varies directly with 'x';  $y = mx$

a)

b)

iii) Which is the independent variable ( $x$ )?

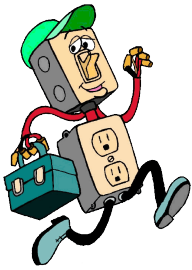
Which is the dependent variable ( $y$ )?

In general, the equation of a **direct variation** has the following properties:

- we say that the equation of a direct variation is always in the form  $y = mx$ , where  $m$  is the constant of variation

## Part 2: Investigate Direct Variations using Graphs

**Example 4:** The cost to do electrical work varies directly with time. Electric company charges \$25 per hour to do electrical work while AC-DC electrical charges \$50 per hour.



a) Write equations to model each relationship.

Electric company:

AC-DC electrical:

b) Use the equations to complete the tables for 0 to 4 hours.

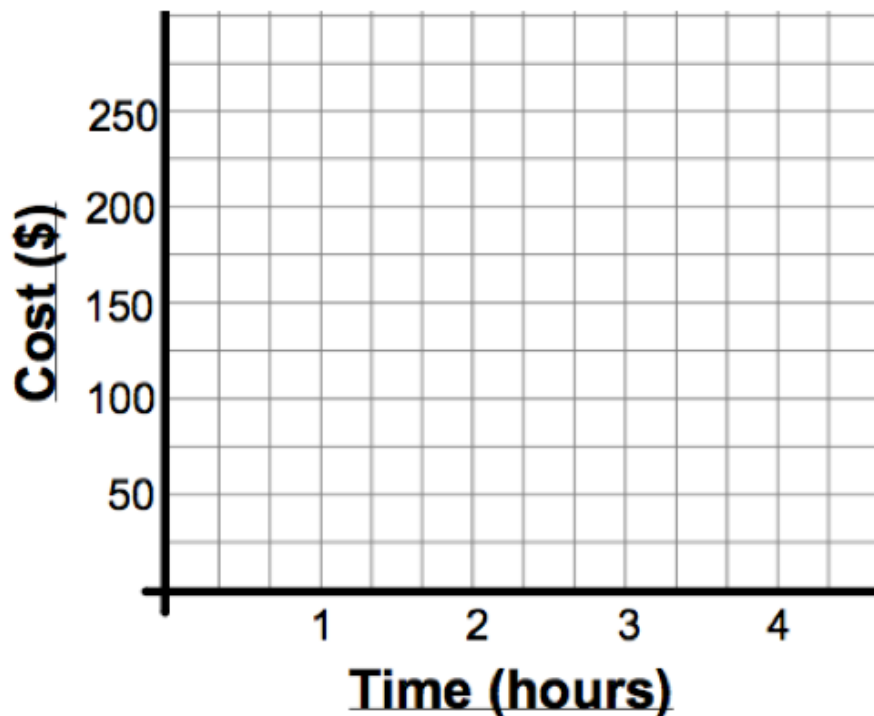
Electric company:

AC-DC electrical:

Hours ( $h$ )	Cost (\$)
0	
1	
2	
3	
4	

Hours ( $h$ )	Cost (\$)
0	
1	
2	
3	
4	

c) Graph the data for both companies on the same Cartesian coordinate grid.



**d)** Looking at the **graph or the table**, which company should I choose if I have a job that requires 3 hours of electrical work?

**e)** How can we show the rate of change on the graph? How does the steepness of the line relate to the rate of change?

**f)** What is the same for both graphs?

In general, the graph of a **direct variation** has the following properties:

- it is a straight line which always passes through the origin

### **Part 3: Interpreting Direct Variation Word Problems**

#### **Example 5:**

The distance travelled by Mr. Jensen when he drives varies directly with time. His car travels 630 km in 3 hours.

- What is the constant of variation?
- Write an equation to represent the relationship between the variables
- Which variable is the dependent/independent variable?

**Example 6:**

The total cost varies directly with the number of MP3's downloaded. 13 MP3 downloads costs \$12.87

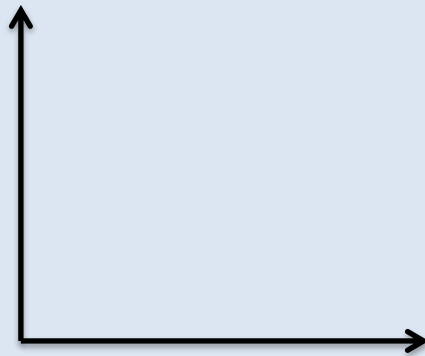
- i) What is the constant of variation?
- ii) Write an equation to represent the relationship between the variables
- iii) Which variable is the dependent/independent variable?

**Consolidate:**

Direct variation occurs when the dependent variable \_\_\_\_\_ of the independent variable.

Direct variation can be defined algebraically as \_\_\_\_\_ where  $m$  is the constant of variation.

The graph of a direct variation is a straight line that passes through the origin (0, 0).

**Remember:**

$$\text{Rate of change (constant of variation)} = m = \frac{\Delta \text{dependant variable}}{\Delta \text{independent variable}}$$