3.3 Discover the Exponent Laws

DO IT NOW

I have a bag containing 24 coloured marbles. The colours are red, green and blue. There are twice as many red marbles as green marbles, and one more red marble than blue marble. How many of each colour marble are there?

Red: 10

Green: 5

Blue: 9

Part 1: Exponent Laws Investigation

Product Rule: Complete the following table

Product	Expanded Form	Single Power
$3^2 \cdot 3^4$	(3 x 3) x (3 x 3 x 3 x 3) = 3 x 3 x 3 x 3 x 3 x 3	3^6
$4^3 \cdot 4^3$	$(4\times4\times4)\times(4\times4\times4)$ $=4\times4\times4\times4\times4\times4$	4^6
$2^3 \cdot 2^4 \cdot 2^2$	$(2\times2\times2)\times(2\times2\times2\times2)\times(2\times2)$ $=2\times2\times2\times2\times2\times2\times2\times2\times2$	2^9
$k^3 \cdot k^5$	$(k\times k\times k)\times(k\times k\times k\times k\times k)$ $=k\times k\times k\times k\times k\times k\times k\times k$	k^8
create your own example		

Describe any trends you see:

When multiplying powers with the same base, keep the base the same and add the exponents.

Quotient Rule: Complete the following table

Quotient	Expanded Form	Single Power
$5^5 \div 5^3$	$\frac{5\times5\times\cancel{5}\times\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times\cancel{5}}$	5^2
$7^4 \div 7^1$	$\frac{7\times7\times7\times\cancel{7}}{\cancel{7}}$	7^3
$10^6 \div 10^4$	$\frac{10\times10\times\cancel{10}\times\cancel{10}\times10\times10}{\cancel{10}\times\cancel{10}\times10\times10}$	10^2
$x^8 \div x^5$	$\frac{(x)(x)(x)(x)(x)\cancel{(x)}\cancel{(x)}\cancel{(x)}}{\cancel{(x)}\cancel{(x)}\cancel{(x)}(x)(x)}$	x^3
create your own example		

Describe any trends you see:

When dividing powers with the same base, keep the base the same and subtract the exponents.

Power of a Power Rule: Complete the following table

Power of a Power	Expanded Form	Single Power
$(2^2)^3$	$(2^2) \times (2^2) \times (2^2)$ $= (2 \times 2) \times (2 \times 2) \times (2 \times 2)$ $= 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6
$(5^3)^4$	$(5^3)\times(5^3)\times(5^3)\times(5^3)$ $=(5\times5\times5)\times(5\times5\times5)\times(5\times5\times5)\times(5\times5\times5)$ $=5\times5\times5\times5\times5\times5\times5\times5\times5\times5\times5\times5$	5^{12}
$(10^4)^2$	$(10^4)\times(10^4)$ $=(10\times10\times10\times10)\times(10\times10\times10\times10)$ $=10\times10\times10\times10\times10\times10\times10\times10$	10^8
Create your own example		

Describe any trends you see:

A power of a power can be written as a single power by keeping the base the same and multiplying the exponents.

Summary of Exponent Laws:

Product Rule	$x^a \cdot x^b = x^{a+b}$
Quoutient Rule	$x^a \div x^b = x^{a-b}$
Power of a Power Rule	$(x^a)^b = x^{a \times b}$
Zero Exponent Rule	$x^0 = 1$

Part 2: Summary of Exponent Laws

Product of Powers Rule

When multiplying powers with the **same base,** keep the same BASE and ADD the exponents.

General Rule:

$$x^a \cdot x^b = x^{a+b}$$

Quotient of Powers Rule

When dividing powers with the **same base,** keep the same BASE and SUBTRACT the exponents.

General Rule:

$$x^a \div x^b = x^{a-b}$$

Power of a Power Rule

A power of a power can be written as a single power by MULTIPLYING the exponents.

General Rule:

$$(x^a)^b = x^{a \times b}$$

Power of a Quotient

When you have a single power with a rational base, you can evaluate it by applying the exponent to the NUMERATOR and the DENOMINATOR.

Rule:

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

Power of a Product

When you have a single power with a base that is a product, the exponent gets put on to each FACTORS in the brackets. Please notice that this only works when inside the brackets is a single term (no + or – signs separating terms)

Rule:

$$(ab)^x = a^x \cdot b^x$$

Part 3: Apply the Product Rule

$$x^a \cdot x^b = x^{a+b}$$

Write each product as a single power. Then, evaluate the power where possible.

1) $3^2 \times 3^3$

$= 3^{2+3}$

$= 3^5$

$= 243$

2) $5^2 \times 5 \times 5^2$

$= 5^{2+1+2}$

$= 5^5$

$= 3125$

3) $(x^2)(x^7)$

$= x^{2+7}$

$= x^9$

4) $(a^4)(a^4)(a^5)$

$= a^{4+4+5}$

$= a^{13}$

5) $(-2)^4 \times (-2)^3$

$= (-2)^{4+3}$

$= (-2)^7$

$= -128$

6) $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

$= \left(\frac{1}{2}\right)^{3+2}$

$= \left(\frac{1}{2}\right)^5$

$= \frac{1^5}{2^5}$

$= \frac{1}{32}$

Part 4: Apply the Quotient Rule

$x^a \div x^b = x^{a-b}$

Write each quotient as a single power. Then, evaluate the power where possible.

7) $8^7 \div 8^5$

$= 8^{7-5}$

$= 8^2$

$= 64$

8) $4^7 \div 4 \div 4^3$

$= 4^{7-1-3}$

$= 4^3$

$= 64$

9) $x^{70} \div x^{40} \div x^{29}$

$= x^{70-40-29}$

$= x^1$

$= x$

10) $\frac{x^7}{x^3}$

$= x^{7-3}$

$= x^4$

11) $\frac{(-0.5)^6}{(-0.5)^3}$

$= (-0.5)^{6-3}$

$= (-0.5)^3$

$= -0.125$

12) $\frac{\left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^2}{\left(\frac{3}{4}\right)^5}$

$= \frac{\left(\frac{3}{4}\right)^5}{\left(\frac{3}{4}\right)^5}$

$= \left(\frac{3}{4}\right)^{5-5}$

$= \left(\frac{3}{4}\right)^0$

$= 1$

13) $\frac{a^5 a^2}{a^6 a^1}$

$= \frac{a^{5+2}}{a^{6+1}}$

$= \frac{a^7}{a^7}$

$= a^{7-7}$

$= a^0$

$= 1$

Note: An exponent of zero always gives the answer of 1

Part 5: Apply the Power of a Power Rule

$(x^a)^b = x^{a \times b}$

Write each power of a power as a single power. Then, evaluate the power where possible.

14) $(3^2)^4$

$= 3^{2 \times 4}$

$= 3^8$

$= 6561$

15) $[(-2)^3]^4$

$= (-2)^{3 \times 4}$

$= (-2)^{12}$

$= 4096$

16) $\left[\left(\frac{2}{3}\right)^2\right]^2$

$= \left(\frac{2}{3}\right)^{2 \times 2}$

$= \left(\frac{2}{3}\right)^4$

$= \frac{2^4}{3^4}$

$= \frac{16}{81}$

17) $(3ab^7)^2$

$= (3)^2(a)^2(b^7)^2$

$= 9a^2b^{7 \times 2}$

$= 9a^2b^{14}$

Note: *for #16 you will need the power of a quotient rule and #17 you will need the power of a product rule.*

Summary of Exponent Laws

Product of Powers Rule	$x^a \cdot x^b = x^{a+b}$
Quotient of Powers Rule	$x^a \div x^b = x^{a-b}$
Power of a Power Rule	$(x^a)^b = x^{a \times b}$
Power of a Quotient	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Power of a Product	$(ab)^x = a^x \cdot b^x$
Zero Exponent Rule	$x^0 = 1$
Negative Exponent Rule	$x^{-a} = \frac{1}{x^a}$