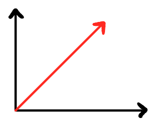
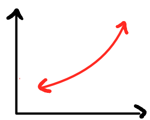
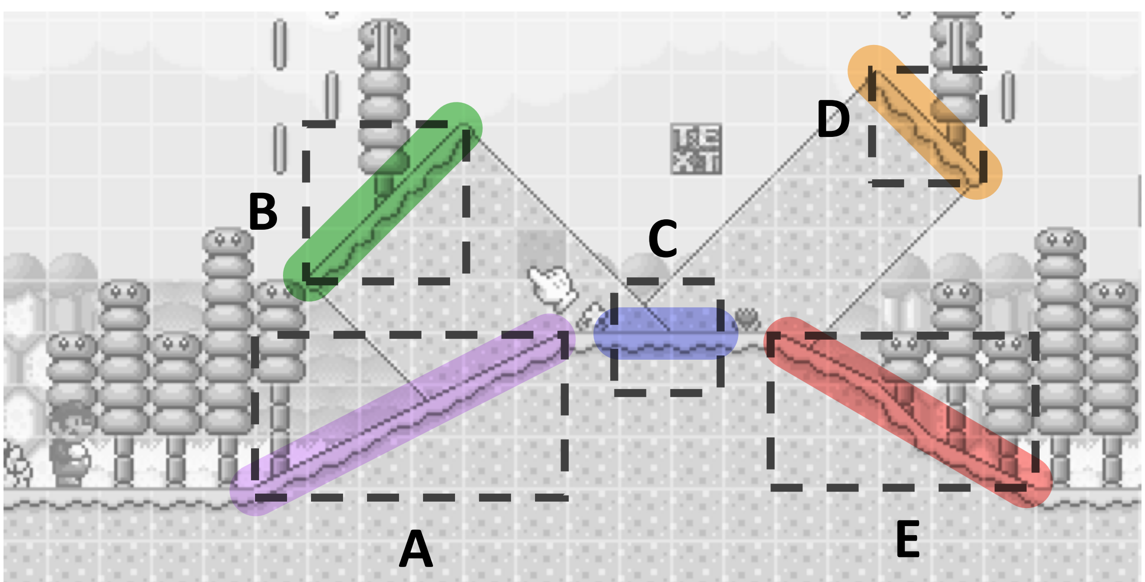
Determining Slopes (Graphically and Algebraically)

Recall: How can we tell if a relation is linear or non-linear based on various representations?

	Linear	Non-Linear
Graph	- straight line - continuous 	- curves - different directions 
Table of Values	- first differences are constant $(y_2 - y_1)$	- first differences are not constant
Equation (more on this later)	y= polynomial of degree one e.g. $y = 5x^1 + 1$	degree not one e.g. $y = 3x^2$

Now that we know how to tell linear relations from non-linear relations, let's look more closely at **linear relations**! How can we tell one linear relation from another?

Let's examine the possible paths that Mario can take, how is each of the section different from another?

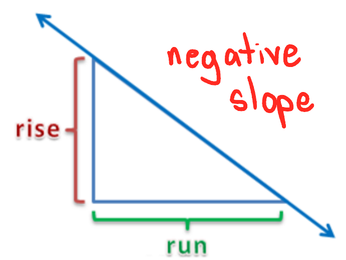


A and B
- incline
- B more steep

C
- flat

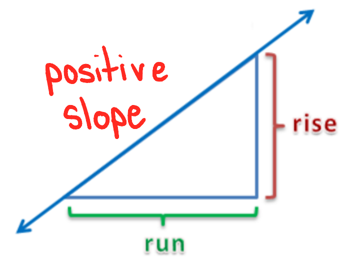
D and E
- decline
- D more steep

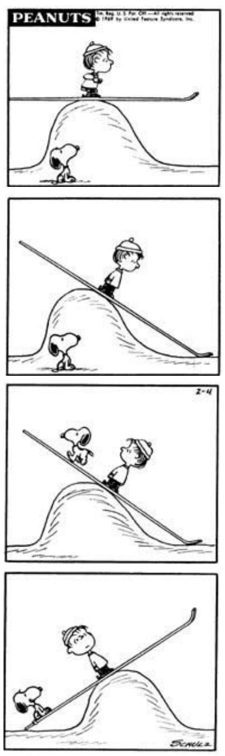
One of the features that is unique to linear relations is the **slope of the line**!



What is the slope of a line?

> The slope of a line is the ___ratio___ between the ___rise___ and the ___run___ between any two given points on that line





How can we describe the slope of a line?

- amount of slope (steepness)
- direction of slope

Amount of slope

moderate	Makes an angle of 45° with the horizontal
gentle	Makes an angle between 0° and 45° with the horizontal
steep	Makes an angle between 45° and 90° with the horizontal
flat	Makes an angle between 0° with the horizontal

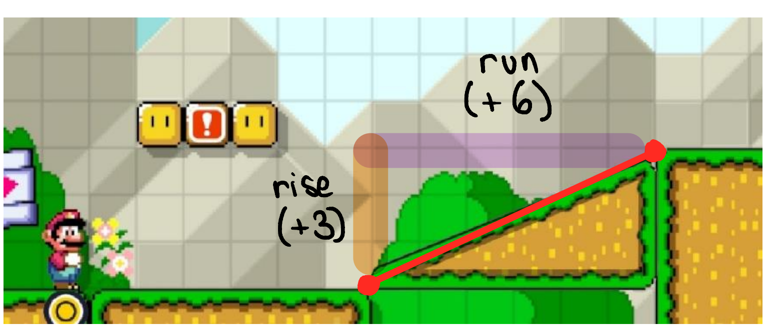
Direction of slope

positive	Ascending, rises as we move from left to right
negative	Descending, falls as we move from left to right

Describing slope using words is not bad, but we can do better!

We can find a numerical value to describe the slope!!

Example 1: Find the slope to the ramp below.



Therefore, the ramp has a slope of…

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$$

Step 1: Find two **CLEAR** grid points

Step 2: Use a ruler and make a right triangle <u>starting with the point on the left</u> by moving to the right, then move up or down

Step 3: Label run and rise

Step 4: Count the "run"

*it can ONLY move to the right (+)

Step 5: Count the "rise"

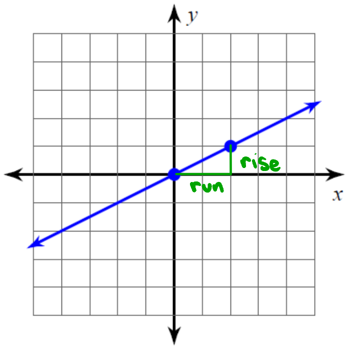
*it can move up (+) or down (−)

Step 6: Write the slope ratio (rise/run)

*simplify/reduce if necessary

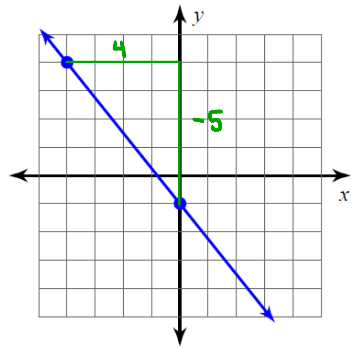
Example 2: Find the slope for each of the lines below. Remember to always move from LEFT to RIGHT on the line.

a)



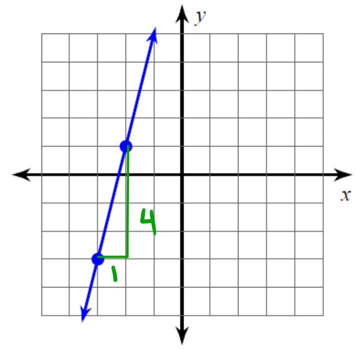
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$

b)



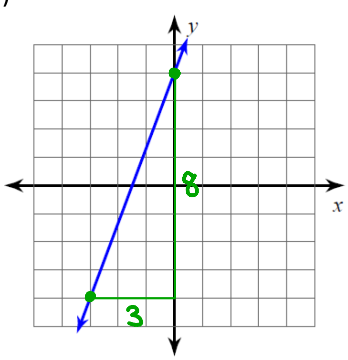
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-5}{4}$

c)



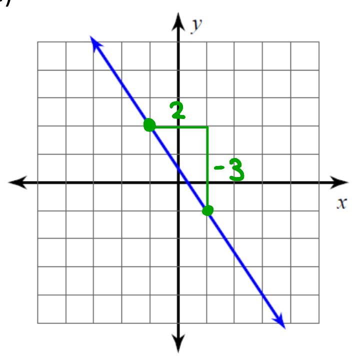
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{1} = 4$

d)



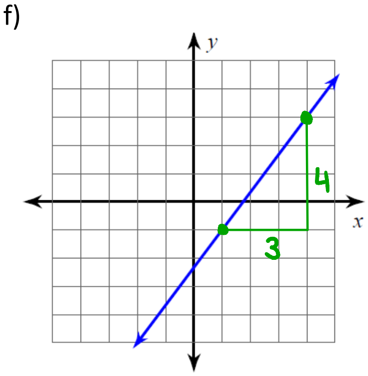
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{8}{3}$

e)



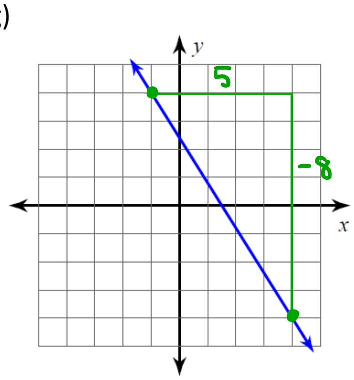
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2}$

f)



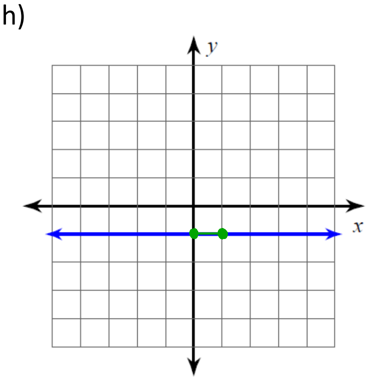
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{3}$

g)



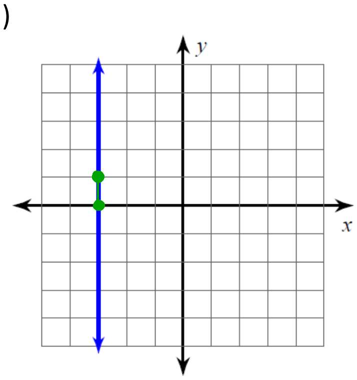
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-8}{5}$

h)



$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{0}{1} = 0$

i)



$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{0} =$ undefined

cannot divide by 0

Some Interesting Observations!

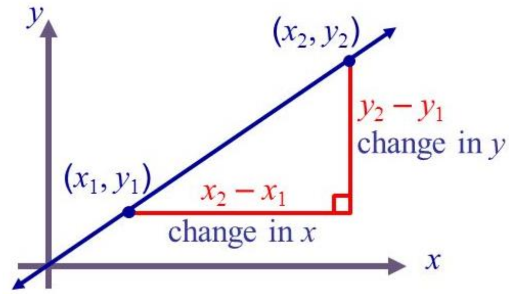
1) The value for uphill slopes are always positive
2) The value for downhill slopes are always negative
3) Horizontal slopes always have a value of 0
4) Vertical slopes are undefined

Calculating Slope Algebraically

We can also calculate the slope of a line as long as we have two points **on** that line!

Given two points on a line $A(x_1, y_1)$ and $B(x_2, y_2)$, we can calculate the slope by using the formula

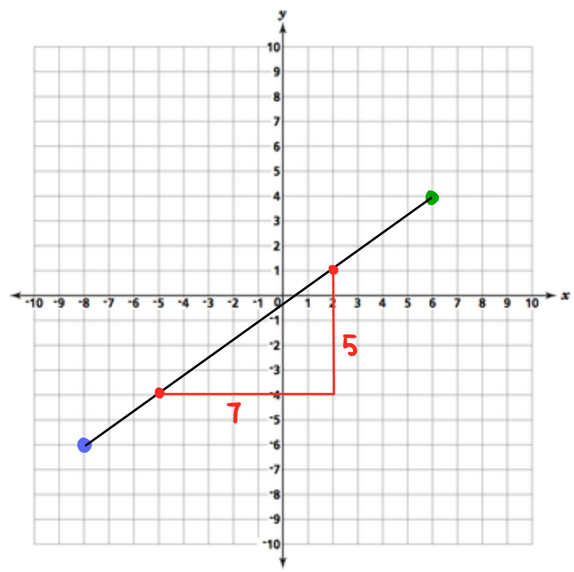
$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\overset{\text{delta } y \text{ (change in } y)}{\Delta y}}{\underset{\text{delta } x \text{ (change in } x)}{\Delta x}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example 3: Determine the slope formed by the two points algebraically. Verify your answer graphically.

a) $\overset{1}{(-8,-6)}$ and $\overset{2}{(6,4)}$

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-(-6)}{6-(-8)}$$
$$= \frac{10}{14}$$
$$= \frac{5}{7}$$



Example 4: Determine the slope given the following pairs of points.

a) $(-5,-3)$ and $(-8,6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6-(-3)}{-8-(-5)}$$
$$= \frac{9}{-3}$$
$$= -3$$

b) $(6,-5)$ and $(6,3)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-(-5)}{6-6}$$
$$= \frac{8}{0}$$
$$= \text{undefined}$$

vertical line