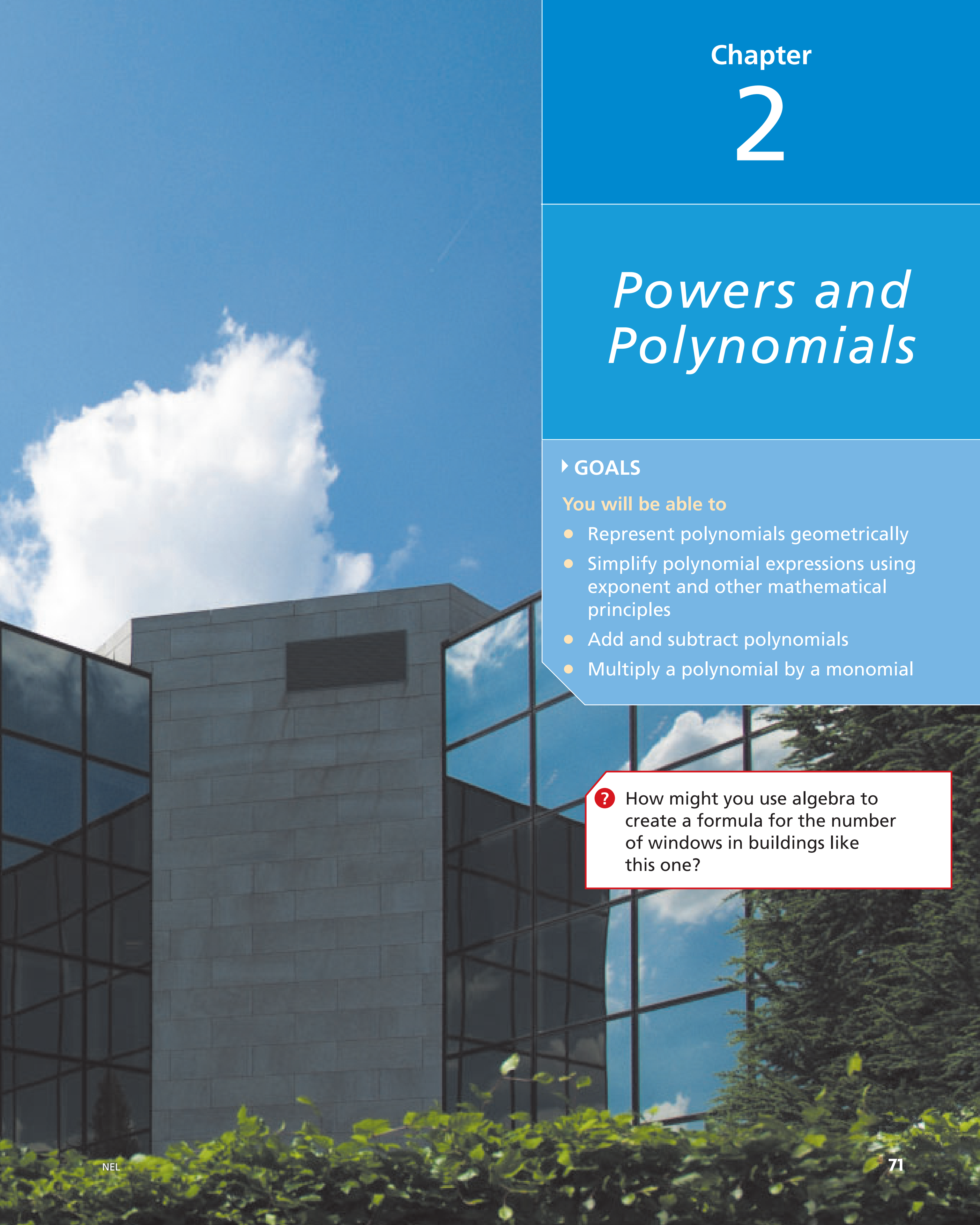


Chapter

2

Powers and Polynomials

GOALS

You will be able to

- Represent polynomials geometrically
- Simplify polynomial expressions using exponent and other mathematical principles
- Add and subtract polynomials
- Multiply a polynomial by a monomial

? How might you use algebra to create a formula for the number of windows in buildings like this one?

2 Getting Started

WORDS YOU NEED to Know

1. Match each mathematical term to the highlighted example it most closely describes.

a) variable
b) constant
c) power
d) base
e) exponent
f) algebraic expression

i) $5x^2 - 3x + 2$
ii) $5x^2 - 3x + 2$
iii) 2^3
iv) 3^4
v) $5x^2 - 3x + 2$
vi) $5x^2 - 3x + 2$

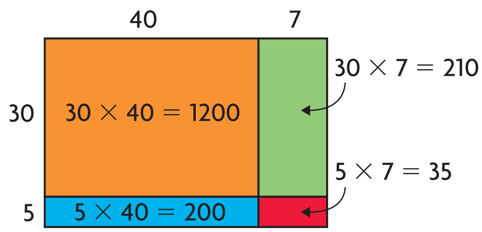
SKILLS AND CONCEPTS You Need

Using an Area Model to Calculate a Product

You can use an area model to multiply multi-digit numbers. You can visualize how this works by representing the product of two numbers as the area of a rectangle. The final product is the sum of the areas of the smaller rectangles that make up the larger one.

EXAMPLE

To multiply 35×47, visualize this area model.



$$\begin{aligned} \text{So, } 35 \times 47 &= (30 + 5) \times (40 + 7) \\ &= 30 \times 40 + 30 \times 7 + 5 \times 40 + 5 \times 7 \\ &= 1200 + 210 + 200 + 35 \\ &= 1645 \end{aligned}$$

2. Use an area model to evaluate each product.

a) 24×52

b) 12×46

c) $2\frac{1}{2} \times 3\frac{3}{4}$

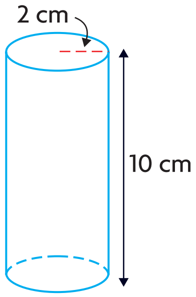
d) $24 \times 1\frac{1}{4}$

Evaluating Algebraic Expressions

It's a good idea to place the values you substitute for the variables in brackets before you begin evaluating an expression. Then, follow the order of operations represented by the memory aid BEDMAS.

EXAMPLE

Determine the volume of the cylinder.



Solution

The formula for the volume of a cylinder is $V = \pi r^2 h$, where h is the height of the cylinder and r is the radius of its base.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi (2)^2(10) \\ &= \pi (4)(10) \\ &= \pi (40) \\ &\doteq 125.6 \text{ cm}^3 \end{aligned}$$

3. Evaluate these algebraic expressions if $a = 0$, $b = 1$, $c = -1$, and $d = 2$.

a) $b + 3c$

b) $3b + 2c - d$

c) $2a^2 + b - d$

d) $3(2b - 3c)$

e) $-4(a + b + c)$

f) $(5c - 6b)^2$

4. Use each formula and the given values to calculate the value of the indicated variable.

a) $P = 2(l + w)$; determine P when $l = 3$ cm and $w = 5$ cm

b) $A = \dfrac{b \times h}{2}$; determine A when $b = 5.5$ m and $h = 4$ m

c) $V = s^3$; determine V when $s = 12$ cm

d) $c = \sqrt{a^2 + b^2}$; determine c when $a = 5$ m and $b = 12$ m

Study Aid

- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
5 and 6	A-1
7 and 8	A-10
10 and 11	A-11

PRACTICE

5. Represent each repeated multiplication as a power.
   a) $13 \times 13 \times 13 \times 13$
   b) $(-8)(-8)(-8)(-8)(-8)(-8)$
   c) $7 \times 6 \times 7 \times 6$

6. Represent each power using repeated multiplication.
   a) 7^4 **b)** $(-7)^4$ **c)** -7^4

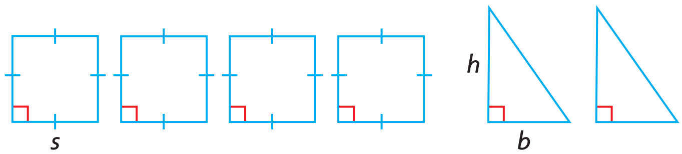
7. Why does the expanded form $2 \times 10^3 + 6 \times 10^2 + 7 \times 10^1 + 3$ describe 2673?

8. Represent 1254 in expanded form.

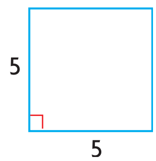
9. Which of the following numbers are perfect squares: 16, 35, 100, 25, 1? Explain how you know.

10. Create an algebraic expression to describe the total cost for a group to go skating. Use A to represent the number of adults and C the number of children in the group.

11. Use variables and numbers to represent the total area of the congruent squares and congruent triangles.



12. A word association chart is shown for the mathematical term "perfect square." Make another chart for one of these terms: base of a power, algebraic expression, or variable.

Math term	Drawing or description
perfect square	
My definition	**Reminds me of**
A perfect square is a number that is found by multiplying an integer by itself; 25 is a perfect square.	

APPLYING What You Know

Paper Folding

YOU WILL NEED

- sheet of paper

Toni folds a piece of paper in half. She does this many times. When she unfolds the paper, there are a lot of sections created by the fold lines.

? If Toni could fold the paper in half 12 times, how many sections would be created?

A. Fold the paper in half. Now open it up. How many sections does the fold create? Refold the paper.

B. Fold the paper in half again. How many sections are there now when you open it up? Refold the paper.

C. Continue to repeat part B for as long as you can. After each new fold, open the paper up and record the total number of sections in a table like this one before refolding.

Number of Folds	Number of Sections
0	1
1	
2	
3	

D. Write an algebraic expression to describe the pattern that relates the number of folds to the number of sections.

E. Use this expression to predict the number of sections if the paper could be folded 12 times.

2.1 Representing Powers Up to Degree 3

YOU WILL NEED

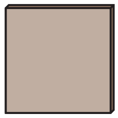
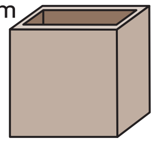
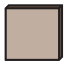
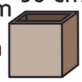
- straws
- scissors
- modelling clay

GOAL

Create geometric representations for powers and square roots.

LEARN ABOUT the Math

Anne's landscaping company sells large and small rectangular trim tiles, interior square tiles, and cube planters. She usually fills orders on an order form like the one below. But this time, Anne's supplier wants a one-line order from her.

	Trim Tiles	Interior Tiles	Planters
Large	60 cm 	60 cm, 60 cm 	60 cm, 60 cm, 60 cm 
Small	30 cm 	30 cm, 30 cm 	30 cm, 30 cm, 30 cm 

Order Form						
Order Number	**Trim Tiles**		**Interior Tiles**		**Planters**	
	Small	**Large**	**Small**	**Large**	**Small**	**Large**
1		20		25		4
2	16		30		2	
3		15		50		1
4	40		100		6	
Total	56	35	130	75	8	5

? How can Anne represent the total numbers of each type of tile and planter on a one-line order?

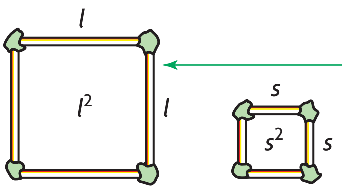
EXAMPLE 1 Representing variables with models

Anne's Solution

I used straw lengths and **algebraic terms** of **degree** 1 to represent the lengths of the trim tiles.

I used straws to model the trim tiles. I used the variables l and s to represent the lengths of the large and small trim tiles.

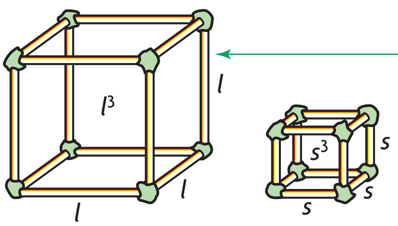
I used algebraic terms of degree 2 to represent the areas of the square tiles.



The side lengths of the interior tiles and planters are the same as the trim lengths. I used modelling clay and straws to build models of the square tiles.

I labelled the sides of the tiles and used their areas to name the tiles. The larger tile's area is l^2. The smaller tile's area is s^2.

I used algebraic terms of degree 3 to represent the volumes of the cubes.



I built models of the planters and labelled their sides.

I used the volume of each of the cubes to name the planters. The larger planter's volume is l^3. The smaller planter's volume is s^3.

The order will ask for $56s$, $35l$, $130s^2$, $75l^2$, $8s^3$, and $5l^3$.

I wrote an algebraic expression to represent the number of each type of tile and planter. The **coefficient** of each term told how many of each I needed to order.

algebraic term

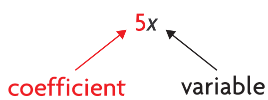
part of an algebraic expression; often separated from the rest of the expression by an addition or subtraction symbol (e.g., the expression $2x^2 + 3x + 4$ has three terms: $2x^2$, $3x$, and 4)

degree of a term

for a power with one variable, the degree is the variable's exponent; when there is more than one variable, the degree is the sum of the exponents of the powers of the variables (e.g., x^4, x^3y, and x^2y^2 all have degree 4)

coefficient

the factor by which a variable is multiplied (e.g., in the term $5x$, the coefficient is 5)



Reflecting

A. Why did Anne need only two variables to describe all of the items in her order?

B. How did Anne relate each algebraic expression to the shape it described?

APPLY the Math

Communication *Tip*

A product often is written with the factors side by side without a multiplication symbol. For example, $2ab$ represents the product $2 \times a \times b$.

Brackets often are used to clearly show the factors. For example, $(3x)(2y)$ represents $(3x) \times (2y)$ and $(2x)^3$ represents $(2x) \times (2x) \times (2x)$.

EXAMPLE 2 Representing algebraic terms geometrically

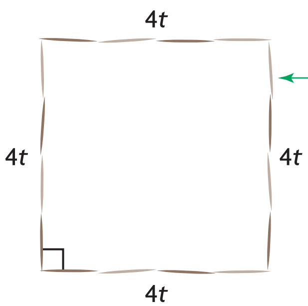
Joe is having difficulty understanding the difference between $4t$, $(4t)^2$, and $(4t)^3$. How might his friend Caleb model these expressions to help him see the difference?

Caleb's Solution

I used 4 toothpicks in a row to model $4t$.

I thought that if each toothpick represented a length of t, 4 in a row would represent a length of $4t$.

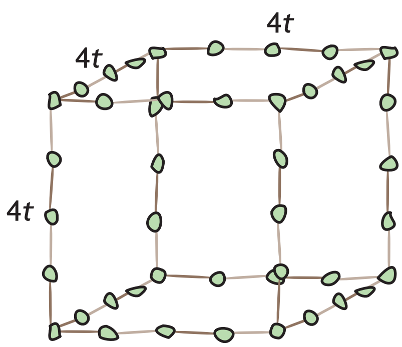
I used a square with side lengths of $4t$ to model $(4t)^2$.



I know that the area of a square is the square of its side length. I represented that square using toothpicks. A square has 2 dimensions, so the exponent of 2 makes sense.

I used a cube with side lengths of $4t$ to model $(4t)^3$.

I know that the volume of a cube is the cube of its side length. A cube has 3 dimensions, so the exponent of 3 makes sense.



EXAMPLE 3 Connecting squares and square roots

A square wall tile has an area of 116 cm². Determine the length of its side to two decimal places.

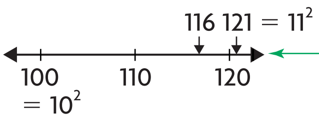
Jesselina's Solution

$$Area = 116\text{ cm}^2$$
$$l^2 = 116$$

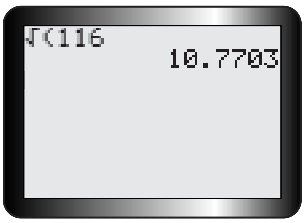
I knew that the area of a square is calculated by multiplying its side length *l* by itself.

$$10^2 = 10 \times 10$$
$$= 100$$
$$11^2 = 11 \times 11$$
$$= 121$$

I knew that 10 squared is 100 and 11 squared is 121. So, I knew that the length was between 10 and 11.



116 is quite a bit closer to 121 than to 100, so I thought that the answer was between 10.5 and 11.



To find the side length, I used my calculator to determine the square root of 116, since this is the inverse operation of squaring.

$$\sqrt{116} \doteq 10.77$$

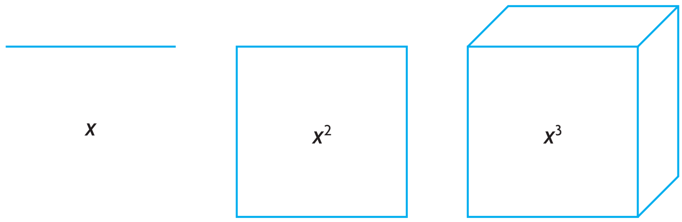
The side length of the tile is about 10.77 cm.

I rounded the result to two decimal places.

In Summary

Key Idea

- You can represent single variable terms with powers of 1, 2, or 3 using concrete materials or drawings. The degree of each term corresponds to the number of dimensions in the model.



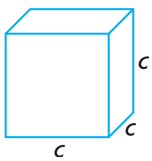
1 dimension 2 dimensions 3 dimensions

Need to Know

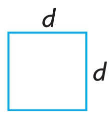
- You can think of $\sqrt{x^2} = x$ as the length of the side of a square with area x^2.
- Expressions involving terms of degree 1, 2, or 3 can be represented using combinations of the models shown above.

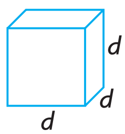
CHECK Your Understanding

1. Copy and label each object, then represent it with one of the following algebraic expressions: c, d, c^2, d^2, d^3 c^3.

a) 

b) 

c) 

d) 

2. a) Calculate the side length of a square with an area of 49 cm^2.

b) A square has an area of 110 cm^2. Without using a calculator, state the two whole numbers between which its side length is located.

PRACTISING

3. Sketch models to represent each of the following algebraic expressions.
K The variables x and y do not represent the same number.

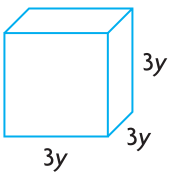
a) x^2
b) x^3
c) y
d) $(2y)^2$

4. The areas of some squares are shown. Determine or estimate the length of the sides of each square. Use a calculator to check your answers.

a) 144 km^2
b) 75 cm^2
c) 0.01 m^2

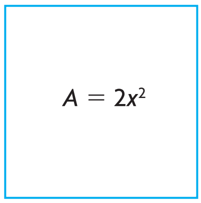
5. Choose the expression that represents the indicated quantity:
$\sqrt{(2x^2)}$, $2x$, $(2x)^2$, $(3x)^3$, $3y$, $(3y)^2$, $(3y)^3$.

a) the volume of the cube

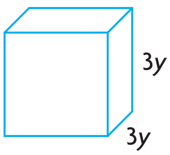


b) the length of the line

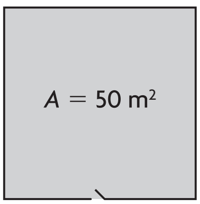
c) the side length



d) the area of the square face

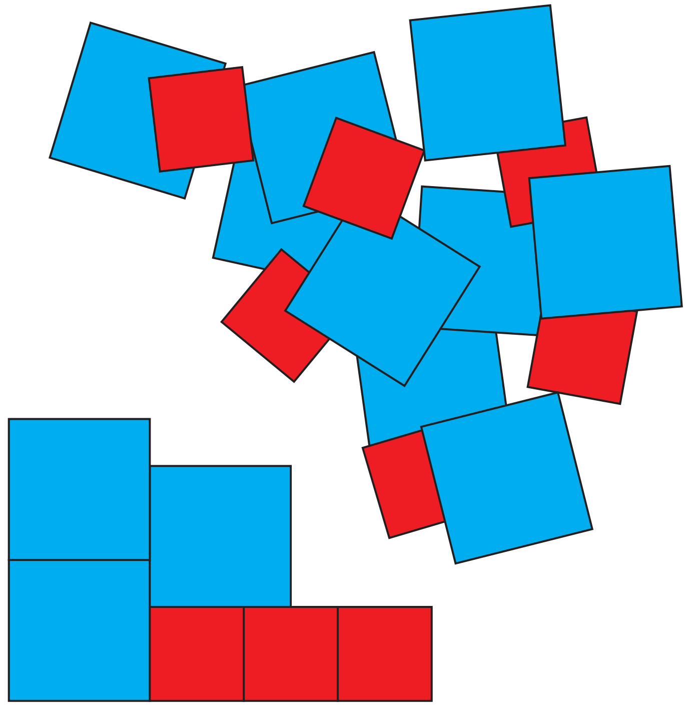


6. If the footprint of a square building has an area of 50 m^2, which is a
A better estimate for the length of the front of the building: 7.2 m or 7.7 m? Explain how you can answer this without using a calculator.



7. If the length of the side of a square game board is $\sqrt{1000}$ cm, what is the area of the game board? How can you check your answer?

8. A mosaic design is created to form a square from nine 2 cm × 2 cm red ceramic tiles and twelve 3 cm × 3 cm blue ceramic tiles.
T
   a) Sketch how this would look.
   b) What is the length of the side of the completed design?
   c) Explain how you know.



9. We read 4^2 as "4 squared" and 4^3 as "4 cubed." Sketch or build models that show why this makes sense.
C

Extending

10. Draw or build a model of the algebraic expression *ab* where *a* is not equal to *b*. Explain how this model is the same as and how it is different from a model of the algebraic expression a^2.

11. Draw or build a model of the algebraic expression *abc* where *a*, *b*, and *c* are not equal to each other. Compare this to a model of the algebraic expression ab^2.

12. The symbol $\sqrt[3]{\ }$ represents the cube root. For example, $4 \times 4 \times 4 = 64$, $\sqrt[3]{64} = 4$. Create a geometric model to represent $\sqrt[3]{64}$. Explain why the model makes sense.

2.2 Multiplying and Dividing Powers

GOAL

Develop and apply exponent principles to multiply and divide powers.

INVESTIGATE the Math

Amir thought there was a way to simplify $\frac{(3^6)(3^9)}{3^{12}}$ without using a calculator.

? How could Amir simplify this expression without a calculator?

A. Draw a table like the one shown. Use it to record the products of powers that have the same base but different exponents.

Extend the table until you see a pattern. Use the pattern to determine how to quickly multiply powers with the same base.

Multiplication	Expanded Form	Product Expressed as a Single Power
$(3^1)(3^2)$	$(3)(3 \times 3)$	3^3
$(3^2)(3^2)$	$(3 \times 3)(3 \times 3)$	3^4
$(3^3)(3^2)$		

B. How is the exponent of the product related to the exponents of the factors?

C. Test your answer to part B using these expressions. Check using a calculator.

a) $(5^2)(5^4)$ **b)** $(2^2)(2^3)(2^4)$

D. Draw a new table like the one shown. Use it to record quotients with the same base and different positive integers for exponents.

Continue to add rows to the table until you see a pattern. Use the pattern to determine how to quickly divide powers with the same base.

Division	Expanded Form	Quotient Simplified and Expressed as a Single Power
$3^2 \div 3^1$	$\frac{(3 \times 3)}{3}$	$\frac{(3 \times \cancel{3})}{\cancel{3}} = 3^1$
$3^4 \div 3^2$	$\frac{(3 \times 3 \times 3 \times 3)}{(3 \times 3)}$	$\frac{(3 \times 3 \times \cancel{3 \times 3})}{(\cancel{3 \times 3})} = 3^2$

E. How is the exponent of the quotient related to the exponents of the terms in the division statement?

F. Use your results in parts B and E to simplify $\frac{(3^6)(3^9)}{3^{12}}$ as a single power with base 3. Check your answer by calculating the value of the original and simplified expressions.

Reflecting

G. How can you determine the exponent of the product of powers with the same base?

H. How can you determine the exponent of the quotient of powers with the same base?

principle
a basic truth or rule about the way something works

I. Why do the **principles** in parts G and H only apply to products and quotients of powers with the same base?

APPLY the Math

EXAMPLE 1 Representing an expression involving powers

Simplify $\frac{(x^7)(x^3)}{x^6}$.

Tony's Solution: Making a conjecture based on numeric examples

$$\frac{(2^7)(2^3)}{(2^6)}$$

I substituted 2 for *x* and calculated the value of the expression.

$$= \frac{(128)(8)}{(64)}$$
$$= \frac{1024}{64}$$
$$= 16$$
$$= 2^4$$

16 is 2 × 2 × 2 × 2, which equals 2^4.

$$\frac{(3^7)(3^3)}{(3^6)}$$

Then, I substituted 3 for *x* and calculated the value of the expression.

$$= \frac{2187 \times 27}{729}$$
$$= \frac{59049}{729}$$
$$= 81$$
$$= 3^4$$

81 is 3 × 3 × 3 × 3, which equals 3^4.

I think that $\frac{(x^7)(x^3)}{x^6} = x^4$

In both cases, the final power had an exponent of 4.
I thought the same pattern would work for any base.

Danny's Solution: Reasoning using the definition of a power

$$\frac{(x^7)(x^3)}{x^6}$$
$$= \frac{(xxxxxxx)(xxx)}{x^6}$$

I looked at the numerator and wrote out the powers in expanded form. This showed me that the numerator was x^{10}.

$$\frac{(\cancel{xxxxxx}^{\,1}x)(xxx)}{\cancel{xxxxxx}_{\,1}}$$

I wrote out the denominator in expanded form. I simplified the fraction by dividing 6 of the xs in the numerator and denominator.

$$xxxx = x^4$$

This left 4 xs multiplied together, which I wrote as x^4.

So, $\frac{(x^7)(x^3)}{x^6} = x^4$.

Patty's Solution: Reasoning using exponent principles

$$\left(\frac{x^7}{x^6}\right)x^3$$
$$= (x)(x^3)$$

I divided x^7 by x^6 using the exponent principle for division. Subtracting 6 from 7, I got 1. I didn't need to write the exponent because x^1 is the same as x.

$$(x)(x^3) = x^4$$

I used the exponent principle for multiplication. Since 1 + 3 is 4, my final answer was x^4.

$$\frac{(x^7)(x^3)}{x^6}$$
$$= x^{7+3-6}$$
$$= x^4$$

I saw that I could have reached the same answer in one step by just adding the exponents in the numerator and subtracting the exponent in the denominator.

Communication *Tip*

Any variable without a visible exponent is understood to have an exponent of 1. For example, $x = x^1$, $2y = 2y^1$, and $-4c = -4c^1$.

EXAMPLE 2 Selecting a strategy to evaluate an expression with two variables

Simplify $\dfrac{(x^4y^3)(x^3y^5)}{x^5y^5}$ and evaluate when $x = 3$ and $y = -2$.

Kathryn's Solution: Using an algebraic strategy to first simplify the expression

$$\frac{(x^4y^3)(x^3y^5)}{x^5y^5}$$
$$= \frac{(x^4x^3)(y^3y^5)}{x^5y^5}$$
$$= \frac{x^7y^8}{x^5y^5}$$

Everything in the numerator was multiplied together. I rewrote it so that powers of the same base were side by side. Then, I simplified by adding the exponents of the powers having the same base.

$$= x^{7-5}y^{8-5}$$
$$= x^2y^3$$

I divided powers having the same base. I did this by subtracting the exponent of the denominator from the exponent of the numerator.

$$= 3^2(-2)^3$$
$$= 9(-8)$$
$$= -72$$

I substituted 3 for *x* and -2 for *y*, and evaluated. The result was -72.

Jeremy's Solution: Using a substitution strategy

$$\frac{(x^4y^3)(x^3y^5)}{x^5y^5}$$
$$= \frac{[3^4(-2)^3][3^3(-2)^5]}{3^5(-2)^5}$$

I substituted 3 for *x* and -2 for *y* into the question.

$$\frac{[81(-8)][27(-32)]}{243(-32)}$$

I calculated the expressions inside each set of brackets.

$$= \frac{[-648][-864]}{-7776}$$
$$= \frac{559\,872}{-7776}$$
$$= -72$$

I did the final multiplication and division.

EXAMPLE 3 Applying exponent principles to simplify expressions where the base has multiple factors

Simplify $\dfrac{\left(-\frac{2}{5}xy\right)^5}{\left(-\frac{2}{5}\right)^3 xy}$.

Miranda's Solution: Connecting exponent principles to fraction operations

$$\frac{\left(-\frac{2}{5}xy\right)^5}{\left(-\frac{2}{5}\right)^3 xy}$$

$$= \frac{\left(-\frac{2}{5}\right)^5 x^5y^5}{\left(-\frac{2}{5}\right)^3 xy}$$

I expressed the numerator as three separate powers because I knew that the exponent 5 outside of the brackets referred to each of the factors inside.

$$= \left(-\frac{2}{5}\right)^{(5-3)} x^{(5-1)}y^{(5-1)}$$

Then, I divided powers having the same base by subtracting their exponents. I knew that x and y in the denominator each had an exponent of 1.

$$= \left(-\frac{2}{5}\right)^2 x^4y^4$$

$$= \frac{4}{25}x^4y^4$$

I evaluated $\left(-\frac{2}{5}\right)^2$ by multiplying.

$\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) = \frac{4}{25}$.

If each rational number in an algebraic expression can be expressed as a terminating decimal, you can write each number as such and then simplify the equivalent expression.

Communication *Tip*

You can rewrite a product or quotient raised to an exponent by applying the exponent to each of the terms. For example, $(xy)^2 = x^2y^2$ and $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$.

Lee's Solution: Connecting exponent principles to decimal operations

$$\frac{(-0.4xy)^5}{(-0.4)^3xy}$$

$$= \frac{(-0.4)^5x^5y^5}{(-0.4)^3xy}$$

I converted each fraction to its decimal equivalent. Then, I expressed the numerator as three separate powers because I knew that the exponent 5 outside the brackets referred to each of the factors inside.

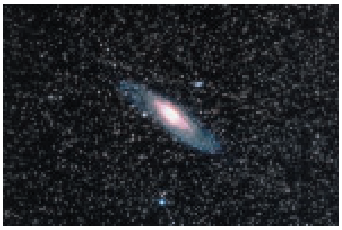
$$= (-0.4)^{(5-3)}x^{(5-1)}y^{(5-1)}$$

I divided powers having the same base by subtracting their exponents. The x and y in the denominator each had an exponent of 1.

$$= (-0.4)^2x^4y^4$$
$$= 0.16x^4y^4$$

I evaluated $(-0.4)^2$. The final result was $0.16x^4y^4$.

EXAMPLE 4 Using exponent principles to solve a problem involving large numbers



The M31 galaxy in the constellation of Andromeda is about 2.4×10^{19} km away. Light travels at about 9.5×10^{12} km/year. Estimate how long it would take light to reach Earth from M31.

Kyle's Solution

I used the formula $t = \frac{d}{s}$.

I needed to divide the distance to the M31 galaxy by the speed of light to find the time in years.

$d = 2.4 \times 10^{19}$ km

$s = 9.5 \times 10^{12}$ km/year

$$t = \frac{2.4 \times 10^{19}}{9.5 \times 10^{12}}$$

I estimated this to be about $\frac{2 \times 10^{19}}{10 \times 10^{12}}$.

2.4×10^{19} is close to 2×10^{19}, 9.5×10^{12} is close to 10×10^{12}, and 10×10^{12} is 10^{13}.

$$= \frac{2 \times 10^{19}}{10^{13}}$$

I used the exponent principle for quotients to simplify the expression.

$$= 2 \times 10^{19-13}$$
$$= 2 \times 10^6$$

$$\frac{2.4 \times 10^{19}}{9.5 \times 10^{12}} \doteq 2\ 000\ 000$$

$10^6 = 1\ 000\ 000$, so $2 \times 10^6 = 2\ 000\ 000$.

The light from M31 takes about 2 000 000 years to reach Earth!

In Summary

Key Idea

- When two powers have the same base, these principles can be used to simplify their product or quotient:

Exponent Principle for Products	Exponent Principle for Quotients
$(a^m)(a^n) = a^{m+n}$	$(a^m) \div (a^n) = a^{m-n}\ (a \neq 0)$

For example, $(2^2)(2^3) = 2^{2+3} = 2^5$ and $3^4 \div 3^2 = 3^{4-2} = 3^2$.

Need to Know

- These exponent principles only work when the powers involved have the same base.
- It is more efficient to simplify an algebraic expression involving powers before substituting to evaluate it.
- An exponent applied to a product or quotient can be written by applying the exponent to each of its terms. In general $(ab)^m = a^m b^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$.

CHECK Your Understanding

1. Simplify.

a) $(2^2)(2^3)$ **b)** $(x^4)(x^3)$

2. Simplify.

a) $\frac{2^5}{2^2}$ **b)** $\frac{y^6}{y^3}$

3. Simplify if possible, and then evaluate.

a) $\frac{(2^7)}{(2^5)}$ **b)** $\frac{(5^5)(3)(3^4)}{(3^3)(5^4)}$

4. Simplify, and then evaluate for $x = 2$ and $y = 5$.

a) $\frac{(x^4)(x^3)}{x^6}$ **b)** $\frac{y^6x^4}{x^3y^3}$

PRACTISING

5. Simplify.

a) $(5^2)(5^8)$

b) $(m^4)(m^2)$

c) $(7^3)(7)(x^4)(x^2)$

d) $\left(\frac{2}{5}\right)^3\left(\frac{2}{5}\right)^2\left(\frac{2}{5}\right)^4$

6. Simplify.

a) $(n^5)(w^6)(n^3)(w^7)$

b) $(m^3)(m^4)(r^8)(m^2)(r^2)$

c) $2^5(p^3)2^2(p^2)(p^8)$

d) $(b^5)3^4(b^3)3^2(b^7)$

e) $(x^4)(-2)(x^5)(-2)^3$

f) $(a^5)(3^2)(a^4)(a)(3)$

7. Why do you get the same result for each of these expressions?

a) $(5^7)(5^4)$

b) $(5^6)(5^5)$

c) $(5^4)(5^2)(5^5)$

d) $(5^3)(5)(5^5)(5^2)$

8. Simplify.

a) $\frac{5^7}{5^2}$

b) $\frac{m^4}{m^2}$

c) $\frac{(2^5)(x^3)}{(2^4)(x^2)}$

d) $\frac{(-5)^3y^{10}}{(-5)(y^6)(y^3)}$

9. Simplify.

a) $\frac{(7^6)(a^3)(7^2)}{(7^3)a}$

b) $\frac{(10^{10})x^4y^5}{(10^8)xy}$

c) $\frac{(xy)^5}{x^4y^3}$

d) $\frac{x^2y^4}{x^3y}$

10. Create four different expressions involving exponents that simplify to 7^8.

11. Simplify if possible, and then evaluate.

K

a) $\frac{2^8}{2^5}$

b) $\frac{(4^5)(4^6)}{4^7}$

c) $\frac{(7^3)(3^2)(3^4)(7)}{(3^3)(7^2)}$

d) $\frac{(4.2^3)(4.2^5)}{4.2^7}$

e) $\frac{\left(\frac{2}{7}\right)^4}{\left(\frac{2}{7}\right)^2}$

f) $\frac{\left(\frac{4}{5}\right)^5\left(\frac{4}{5}\right)^4}{\left(\frac{4}{5}\right)^6}$

12. Simplify, and then evaluate for $x = 2$ and $y = 5$.

a) $\frac{(x^5)(x^4)}{x^8}$

b) $\frac{(y^6)(y^4)}{(y^8)(y)}$

c) $\frac{(y^6)(x^4)}{(x^3)(y^3)}$

d) $\frac{250y^6}{125y^3}$

e) $\frac{6(x^4)(y^6)}{3(x^3)y^3}$

f) $\frac{\left(\frac{3}{4}xy\right)^3}{\left(\frac{3}{4}\right)^2xy^2}$

13. If you know that the product of two powers is 7^{10} and that the quotient is 7^2, what could the two powers be? How could you verify your answer?
A

14. Scientists estimate that there are 50×10^{12} cells in the average human. There are approximately 6×10^9 humans in the world. Approximately how many cells do all the humans on Earth have? Write your answer using a power with base 10.
T

15. Explain why it is necessary for the bases to be the same in order to apply the multiplication and division principles for exponents.
C

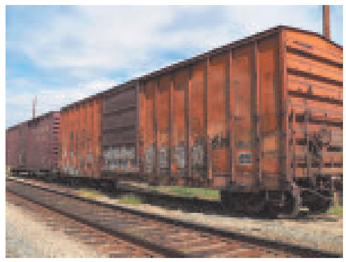
Extending

16. a) Complete the table to show the relationship between the metric units of length. Express each relationship as a power with base 10.

	Millimetres	Centimetres	Metres	Kilometres
Millimetres				
Centimetres				
Metres				
Kilometres				

b) Determine the number of centimetres in 5 km.
c) Determine the number of millimetres in 4 m.

17. A piece of steel plate is used to make a railway car. The plate is 2.5 m wide, 3.2 m long, and 0.5 cm thick. Determine the volume of steel in cubic centimetres.



18. The annual worldwide production of all grains is about 9×10^{12} kg. How much grain is produced per person if there are approximately 6×10^9 people in the world?

19. a) Evaluate $\frac{3^5}{3^5}$.

b) Simplify $\frac{3^5}{3^5}$ using the exponent principle for quotients.

c) Use the meaning of the powers in $\frac{3^5}{3^5}$ to simplify the expression.

d) Discuss what 3^0 might mean.

e) Discuss whether a^0 would have a similar meaning for any value of a.

2.3 Power of a Power

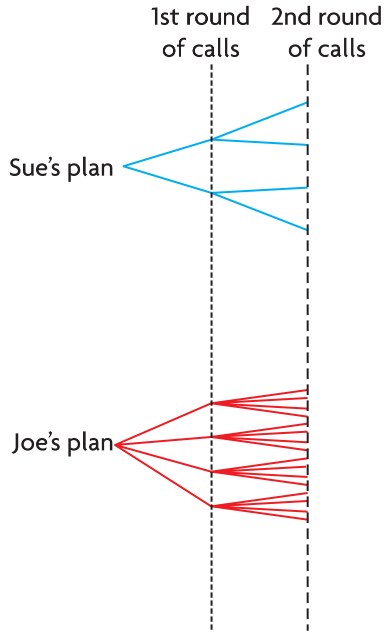
GOAL

Simplify expressions involving a power of a power.

LEARN ABOUT the Math

Sue and Joe want to spread the news about school picture day. Sue suggests that she call 2 people and ask each person called to call 2 more people, and so on. Joe suggests that he call 4 people and ask each person called to call 4 more people, and so on.

Joe says that with his plan, the same number of people would be called on the 4th round of calls as on the 8th round of calls with Sue's plan.



? Is Joe right?

EXAMPLE 1 Representing a power as an equivalent power

Determine and compare the number of people called on the 8th round of calls using Sue's plan and on the 4th round of calls using Joe's plan.

Jaan's Solution: Representing 2^8 as a power with base 4

$$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

I started by looking at 2^8, since that is the number of people who would be called in the 8th round with Sue's plan.

$$\begin{aligned} 2^8 &= (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \\ &= (2^2)(2^2)(2^2)(2^2) \\ &= (2^2)^4 \\ &= 4^4 \end{aligned}$$

I grouped the 2s in pairs. Each pair could be written as 2^2 and was equal to 4. I now had four 4s multiplied together.

4^4 is how many people would be called after the 4th round with Joe's plan.

2^8 is the same as 4^4. So, Joe is right.

I used my calculator to evaluate the powers. $2^8 = 256$ and $4^4 = 256$, so I knew Joe was right.

Marie's Solution: Representing 4^4 as a power with base 2

$4^4 = 4 \times 4 \times 4 \times 4$

I started by looking at 4^4 because that is the number called in round 4 of Joe's plan.

$4 \times 4 \times 4 \times 4$
$= (2^2)(2^2)(2^2)(2^2)$
$= (2^2)^4$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^8$

I wrote each 4 as 2^2.

4^4 is the same as 2^8. So, Joe is right.

I used my calculator to evaluate the powers. $2^8 = 256$ and $4^4 = 256$, so I knew Joe was right.

Reflecting

A. How did Jaan and Marie use different representations to show that Joe was correct?

B. Why could 2^8 also have been written as $(2^4)^2$?

C. How could you use Jaan's approach to write 8^4 as a single power of 2? How does this strategy demonstrate a principle for calculating a power of a power?

Communication *Tip*

When expressing a power of a power, use brackets to indicate the base to which the outermost exponent applies. For example, $(2^3)^4$ means $(2^3)(2^3)(2^3)(2^3)$.

APPLY the Math

EXAMPLE 2 Selecting a strategy to simplify a power of a power

Simplify $(x^5)^3$.

Jordi's Solution: Reasoning using products of powers with the same base

$(x^5)^3$
$= (x^5)(x^5)(x^5)$
$= x^{5+5+5}$
$= x^{15}$

I used the principle for multiplying powers with the same base.

$(x^5)^3$
$= x^{15}$

So, *x* to the power of 5 all to the power of 3 is the same as *x* to the power of 15.

Parm's Solution: Reasoning using the power-of-a-power principle

$$(x^5)^3$$
$$= x^{5\times 3}$$
$$= x^{15}$$

When I use numbers I can multiply or divide exponents if the bases are the same, so I assumed that this would also be true with variables.

I applied the power-of-a-power principle by multiplying the exponents 5 and 3 together.

$$(x^5)^3$$
$$= x^{15}$$

So, *x* to the power of 5 all to the power of 3 is *x* to the power of 15.

EXAMPLE 3 Simplifying an expression when the base is a term with more than one variable

Simplify $\dfrac{(2x^2y^3)^3}{(2xy^2)^2}$.

Teresa's Solution: Reasoning using the exponent principle for products

$$\frac{(2x^2y^3)^3}{(2xy^2)^2}$$
$$= \frac{(2x^2y^3)(2x^2y^3)(2x^2y^3)}{(2xy^2)(2xy^2)}$$

I started by writing the numerator and denominator using repeated multiplication.

$$= \frac{(2)(2)(2)(x^2)(x^2)(x^2)(y^3)(y^3)(y^3)}{(2)(2)(x)(x)(y^2)(y^2)}$$

Then I rearranged the numerator and denominator by putting the factors with the same base side-by-side.

$$= \frac{(2^3)(x^6)(y^9)}{(2^2)(x^2)(y^4)}$$

I rewrote each expression by using the exponent principle for multiplication, adding the exponents where the base was the same.

$= (2x^4y^5)$

I simplified the expression by using the exponent principle for division, subtracting the exponents where the base was the same.

Marty's Solution: Reasoning using the power-of-a-power principle

$$\frac{(2x^2y^3)^3}{(2xy^2)^2}$$

$$= \frac{(2)^3(x^2)^3(y^3)^3}{(2)^2(x)^2(y^2)^2}$$

The exponents outside the brackets in the numerator and the denominator apply to all of the factors inside the brackets. This means that I could write the expression as a product of separate factors.

$$= \frac{(2^3)(x^6)(y^9)}{(2^2)(x^2)(y^4)}$$

To simplify a power of a power, I multiplied the exponents.

$$= 2x^4y^5$$

To divide when the bases are the same, I subtracted the exponent in the denominator from the exponent in the numerator.

In Summary

Key Idea

- When a power is raised to another exponent the following principle can be used to simplify the power.

Exponent Principle for Power of a Power
$(a^m)^n = a^{mn}$

For example, $(a^4)^3 = a^{4\times 3} = a^{12}$.

Need to Know

- If you have the power of a product, the outer exponent refers to each factor inside the brackets. For example:
  $(a^mb^n)^p = (a^m)^p \times (b^n)^p = a^{mp}b^{np}$.
- If you have the power of a quotient, the outer exponent refers to each term inside the brackets. For example:
  $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}} (b \neq 0)$.

CHECK Your Understanding

1. Express each of the following as a power with a single exponent.

a) $(7^3)^5$ **b)** $(x^4)^6$ **c)** $(c^3)^2$

2. Express each of the following as a power with a different base.

a) 16 **b)** 4^3 **c)** 9^4

PRACTISING

3. Express each of the following as a power with a single exponent.

a) $(3^4)^2$ **c)** $(2^5)^3$ **e)** $(x^2)^3$

b) $(9^4)^3$ **d)** $(10^6)^6$ **f)** $(5^2)^4$

4. Express each of the following as a power with the base indicated.

a) 16^2 with a base of 4

b) 16^2 with a base of 2

c) 25^3 with a base of 5

d) 27^3 with a base of 3

5. Explain each principle, and then give a numerical example.

C **a)** $(a^m)^n = a^{mn}$

b) $(a^m b^n)^p = a^{mp} b^{np}$

c) $\left(\dfrac{a^m}{b^n}\right)^p = \dfrac{a^{mp}}{b^{np}}$

6. Simplify.

K **a)** $(3^4)^2(3^5)$ **c)** $\dfrac{(2^3)^3}{2^4}$ **e)** $\dfrac{(5^5 \times 5^2)^2}{(5^4 \times 5)^2}$

b) $(5^4)^3(5^4)^3$ **d)** $\dfrac{(10^4)^2}{(10^2)^3}$ **f)** $\left(\dfrac{3^5}{3^3}\right)^2$

7. Simplify.

a) $(y^3)^4$ **b)** $(m^2)^3$ **c)** $(c^3)^3$ **d)** $(n^3)^4$

8. Simplify.

a) $(v^2)^2(v)$ **c)** $\dfrac{(k^5)^3}{k^2}$ **e)** $\dfrac{(x^2x^3)^4}{(x^5x)^3}$

b) $(n^4)^3(n^2)^3$ **d)** $\dfrac{(j^8)^2}{(j^5)^2}$ **f)** $\left(\dfrac{y^6}{y^4}\right)^3$

9. Simplify.

a) $(3a^2)^3$ **c)** $(-2m^2)^4$ **e)** $(5a^2 \times 2b^3)^2$

b) $(5x^5)^2$ **d)** $(4^3p^4)^2$ **f)** $(3x^4y^2)^3$

10. Simplify.

a) $(4^3 \times 3^2)^2(4^5 \times 3^2)^3$

b) $(2x^3)^4(2x^2)^5$

c) $\dfrac{(2^5 \times 5^2)^2}{(2^4 \times 5)^2}$

d) $\dfrac{(5a^3)^5}{(5a^5b^2)^2}$

11. Simplify.

a) $(2y^3)^4$

b) $(3x^5)^2$

c) $(3a^3)^2(3^3a^5b^2)^2$

d) $\dfrac{(5^3a^4)^5}{(5^4a^3)^2}$

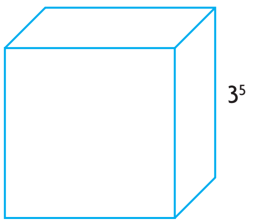
12. Without actually computing the values, explain how you know that each expression below is equal to 0.

T

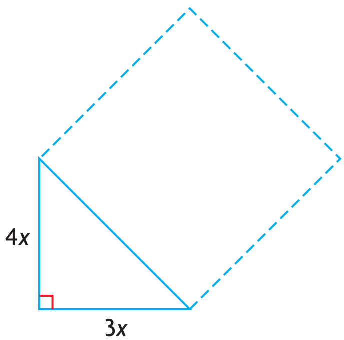
a) $(3^2)^6 - (3^3)^4$ b) $(10^2)^8 - (10^4)^4$ c) $(-2^3)^2 - (-2^2)^3$

13. The length of the side of a cube is 3^5. Express its surface area (SA) and volume (V) using powers and simplify.

A



14. Determine an expression for the area of the square drawn on the **hypotenuse**.



15. Evaluate.

a) $\dfrac{(2^3)^4}{(2^2)^5}$

b) $\dfrac{(5^3)^6}{(5^3)^5}$

c) $\dfrac{(6)(2^3)^3}{(2^2)^4}$

d) $\dfrac{(5^2)^3(7^3)^4}{(7^{11})(5^5)}$

e) $\dfrac{(5^2)(6^6)}{(5^1)^4(6^2)^3}$

f) $\dfrac{[(2^4)^2]^3}{[(2^2)^3]^2}$

16. Simplify and evaluate each. Use $a = 2$, $b = -1$, and $c = 4$.

a) $\dfrac{a^5}{a^2}$

b) $(b^3)^2$

c) $\dfrac{(c^2)^3}{c^5}$

d) $\dfrac{a^3b^3}{ab}$

17. Show that 3^{10} is the same as 9^5 using your understanding of exponents.

18. Simplify and evaluate each.

a) $\dfrac{(x^5)^2(x^7)^3}{(x^4)^6}$ when $x = 2$

b) $\dfrac{(m)^{11}}{(m^5)^2} + \dfrac{n^7}{(n^2)^3}$ when $m = 3$ and $n = 4$

c) Explain how using exponent principles helped you to solve these problems.

19. Determine the value of the exponent that makes each statement true.

a) $4^3 = 2^{\blacksquare}$

b) $6^9 = 216^{\blacksquare}$

c) $625^2 = 25^{\blacksquare}$

d) $27^4 = 3^{\blacksquare}$

20. Write each power in simplified form.

a) 4^5 as a power of 2

b) 9^6 as a power of 3

c) 27^4 as a power of 3

d) $(-125)^7$ as a power of (-5)

21. Knowing that 2^3 is 8 and 3^2 is 9, how do you know that $2^{30} < 3^{20}$?

22. Describe the relationship between the power-of-a-power principle and the other exponent principles you know.

Extending

23. Jody's calculator will only input one-digit numbers. The exponent key and the display are working fine. Explain how she can use her calculator to evaluate each of the following.

a) 25^4

b) 16^2

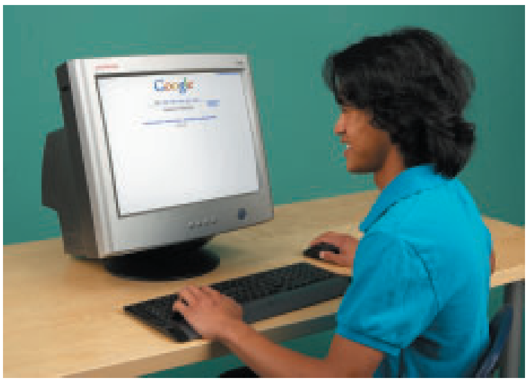
24. Explain why you can write 2^8 as a power having a base of 4 and an integer as an exponent, but cannot do this for 2^7.

Curious | Math

Google This!

In 1920, mathematician Edward Kassner asked his nine-year-old nephew Milton Sirotta what name he should give to the number 10^{100}. “A googol,” came the boy’s reply, and the name stuck. How large do you think a googol is?

1. What does 10^{100} mean?
2. If you were to write 10^{100} out in long hand, how many zeros would there be after the 1?
3. The number 10^{googol} is called a googolplex. Describe what the number 10^{googol} would look like.
4. Express 10^{googol} another way.
5. How long might it take you to write all the digits of one million (10^6)? What about one billion (10^9)?
6. Suppose you could keep writing zeros without taking a break. About how long would it take you to write out the whole number equivalent of a googol? What about a googolplex?
7. Can you think of a number greater than a googolplex?
8. Explain why the founders of Google™ might have chosen this name as the name of their search engine.



2 Mid-Chapter Review

FREQUENTLY ASKED Questions

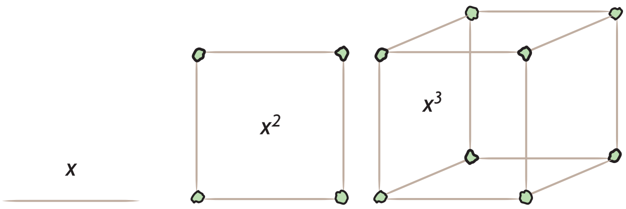
Study Aid

- See Lesson 2.1, Examples 1 and 2.
- Try Mid-Chapter Review Question 1.

Q: How can you use models to represent powers and square roots?

A1: You can draw a line segment to represent the variable x, for example. You can then use it to represent x^2 and x^3.

EXAMPLE



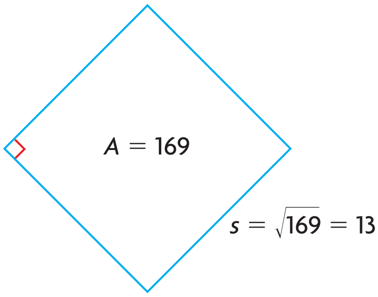
Study Aid

- See Lesson 2.1, Example 3.
- Try Mid-Chapter Review Question 2.

A2: If x^2 represents the area of a square, then $\sqrt{x^2}$ represents the side length of the square. They are inverse operations of each other.

EXAMPLE

$13^2 = 169$; $\sqrt{169} = 13$.



Study Aid

- See Lesson 2.2, Example 1 and Lesson 2.3, Examples 2 and 3.
- Try Mid-Chapter Review Questions 3, 4, 5, and 6.

Q: How can you simplify an expression involving powers?

A: You can apply one or all of the following principles.

	Product Principle for Powers with the Same Base	**Quotient Principle for Powers with the Same Base**	**Power-of-a-Power Principle**
Statement of Principle	$(a^m)(a^n) = a^{m+n}$	$(a^m) \div (a^n) = a^{m-n}$ if $a \neq 0$	$(a^m)^n = a^{mn}$
Example	$3^2 \times 3^3 = 3^{2+3}$ $= 3^5$	$5^{10} \div 5^6 = 5^{10-6}$ $= 5^4$	$(4^3)^5 = 4^{3\times5}$ $= 4^{15}$

PRACTICE Questions

Lesson 2.1

1. Suppose you created models of a, b, a^2, b^2, a^3, and b^3 and that a and b are different. How would the models of each pair be alike and how would they be different?

a) a and b **c)** a^3 and b^3

b) a^2 and b^2 **d)** a^2 and a^3

2. Draw a diagram or make a model to represent each of the following:

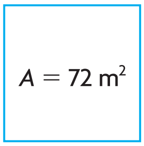
a) $(3y)^2$ **b)** $3y$ **c)** y^3

3. The area of this square is 72 m^2.

a) Estimate the length of the side of the square.

b) Represent the exact side length using the square root sign.

c) Use a calculator to determine the square's side length to two decimal places.



Lesson 2.2

4. Simplify.

a) $(5^3)(5^6)$ **c)** $\dfrac{(5^6)}{(5^2)}$ **e)** $(7^3)(2^6)(7^2)(2^5)$

b) $(-2)^3(-2^7)$ **d)** $\dfrac{(7^3)(7^6)}{7}$ **f)** $\dfrac{(7^3)(2^6)}{(7^2)(2^5)}$

5. Simplify.

a) $\left(\dfrac{5}{7}\right)^3\left(\dfrac{5}{7}\right)^6$ **c)** $(3.1)^8(3.1)^2$

b) $\dfrac{\left(\dfrac{-2}{5}\right)^{13}}{\left(\dfrac{-2}{5}\right)^{7}}$ **d)** $\dfrac{(0.012)^3}{(0.012)}$

6. Simplify.

a) $(x^2)(x^6)$ **c)** $\dfrac{(m^6)}{(m^5)}$ **e)** $(ab^7)(a^3)(b^4)$

b) $(y^3)(y^6)$ **d)** $\dfrac{(n^3)(n^2)}{n}$ **f)** $\dfrac{(x^5)(y^6)}{(x^3)(y)}$

7. Simplify.

a) $(2^3)(x^2)(2^2)(x^5)$ **c)** $\dfrac{(4^4)(m^6)}{(4m)}$

b) $(5^3)(y^3)(5^3)(y^6)$ **d)** $\dfrac{(6^3)(q^5)(q^2)}{6q^4}$

8. Explain why the bases must be the same to apply the exponent principle when multiplying powers.

9. The diameter of the Earth is about 1.3×10^4 km. The diameter of the Sun is about 1.4×10^6 km. About how many "Earths" could you line up along the Sun's diameter?

Lesson 2.3

10. Simplify:

a) $(5^3)^5$ **c)** $(2x^3)^7$

b) $(x^3)^4$ **d)** $\left[\left(\dfrac{8}{5}\right)^3\right]^4$

11. Express each of the following numbers with the base indicated:

a) 32^3 with a base of 2

b) 81^2 with a base of 9

c) 81^2 with a base of 3

d) 100^{15} with a base of 10

12. Simplify.

a) $(5^3)^5(5^4)^4$ **c)** $(m^3)^4(2x^3)^7(m^3)^4(x^3)^7$

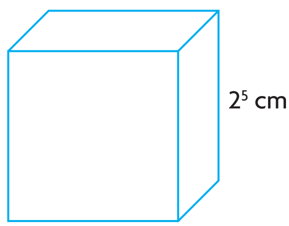
b) $(x^3)^4(x^5)^4$ **d)** $\dfrac{(m^3)^4(2x^2)^7}{2^5(m^3)^3(x^3)^2}$

13. Explain why each of the following simplifies to x^{36}:

a) $(x^2)^{18}$ **c)** $(x^3)^{12}$

b) $(x^6)^6$ **d)** $(x^9)^4$

14. Determine a simplified expression for the surface area and volume of this cube. The length of each side is 2^5 cm.



2.4 Adding and Subtracting Polynomials

YOU WILL NEED

- algebra tiles

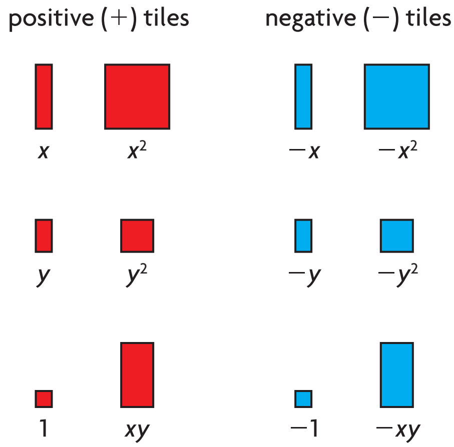
GOAL

Add and subtract like terms.

LEARN ABOUT the Math

A class is playing a game with algebra tiles. The game has the following rules:

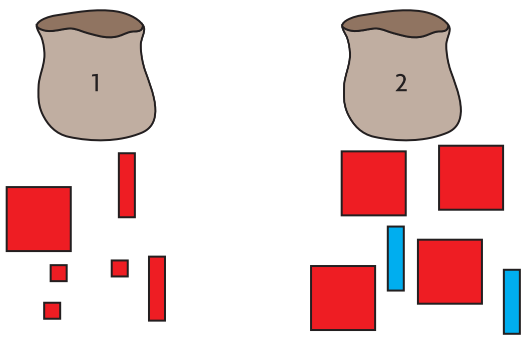
- A player gets two pouches. Each contains six randomly selected algebra tiles.
- A player can use the **zero principle** to add the tiles in the two pouches or subtract the tiles in the second pouch from those in the first.
- The goal is to end up with the fewest tiles.



? How can you decide if you should add or subtract?

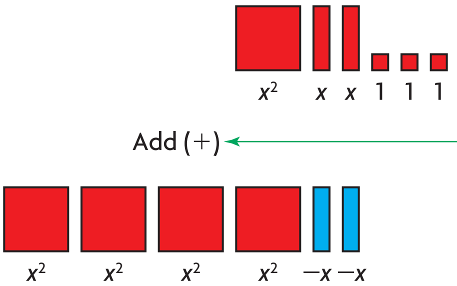
EXAMPLE 1 Using a concrete model to represent an operation

Farell and Peter received the following tiles in their pouches.



Should they add or subtract to get a result that uses the fewest number of tiles?

Farell's Solution: Representing and simplifying a sum using algebra tiles

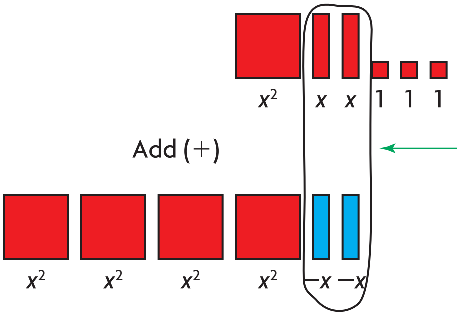


I arranged the algebra tiles from the two pouches and decided to add them by combining **like terms**.

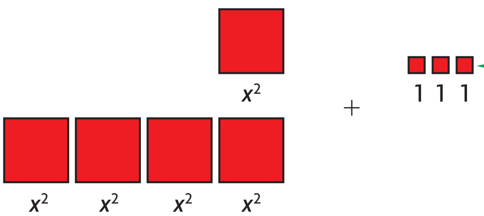
like terms

algebraic terms that have the same variables and exponents apart from their numerical coefficients (e.g., $2x^2$ and $-3x$)

I need to calculate
$(x^2 + 2x + 3) + (4x^2 - 2x)$



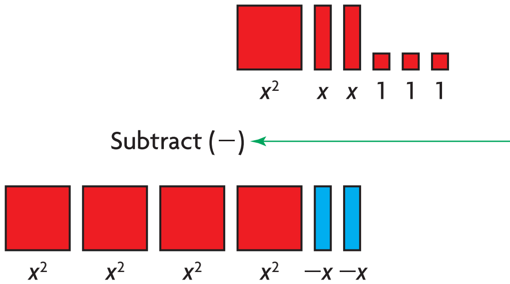
I figured positive- and negative-x tiles could be combined to makes zeros using the zero principle, just as with opposite integers. So, I paired the two positive-x tiles with the two negative-x tiles and removed them.



There were 5 x^2 tiles and 3 unit tiles left. I couldn't combine the x^2 tiles and the unit tiles since they were different things.

The expression I get by adding the two polynomials is $5x^2 + 3$ and that uses 8 tiles.

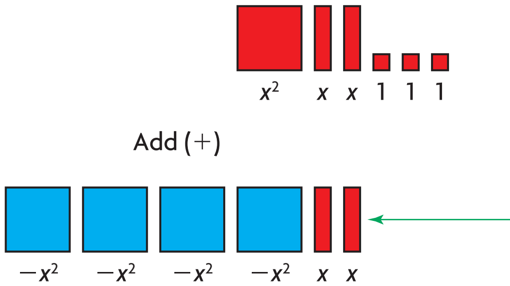
Peter's Solution: Representing and simplifying a difference using algebra tiles



I decided to subtract the tiles of the second pouch from those in the first pouch.

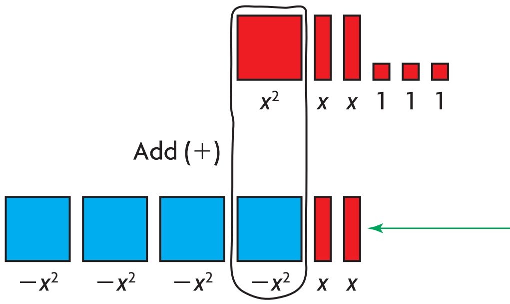
I needed to figure out the difference and how many tiles it would take to represent that difference.

I need to calculate
$(x^2 + 2x + 3) - (4x^2 - 2x)$.



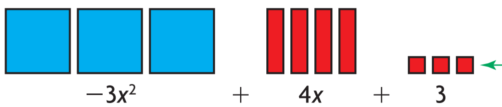
To subtract, I added the opposite of each tile in the second pouch, just like I would do with integers. I replaced red tiles with blue tiles and blue tiles with red tiles, and then added.

The opposite of $4x^2$ is $-4x^2$.
The opposite of $(-2x)$ is $2x$.



I saw that I could eliminate an x^2 tile and a negative-x^2 tile using the zero principle, just as with opposite integers. Then, I gathered together congruent tiles of the same colour.

Since $1 + (-4) = (-3)$,
$x^2 + (-4x^2) = -3x^2$.
Since $2 + 2 = 4$,
$2x + 2x = 4x$.



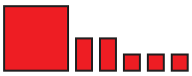
I saw and counted 3 negative-x^2 tiles, 4 x tiles and 3 unit tiles.

The expression I get by subtracting is $-3x^2 + 4x + 3$ and that uses 10 tiles. Subtraction resulted in more tiles than addition, so I should use addition.

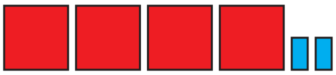
EXAMPLE 2 Reasoning about expressions algebraically

Jay and Sierra got two new pouches of tiles. The first pouch contained 1 x^2 tile, 2 y tiles, and 3 unit tiles. The second pouch had 4 x^2 tiles and 2 $-y$ tiles. They represented their contents using algebraic expressions. Jay added and Sierra subtracted the expressions. Which expression results in the fewest tiles?

Jay's Solution: Representing and simplifying a sum algebraically



Add (+)



I represented the tiles using algebraic expressions. I expressed the contents of the first pouch as three **monomials** and the contents of the second pouch as two monomials.

$$x^2 + 2y + 3 + 4x^2 + (-2y)$$

I wrote an algebraic expression that represented the sum.

$$= \underline{x^2} + \underline{\underline{2y}} + 3 + \underline{4x^2} + \underline{\underline{(-2y)}}$$

I identified the algebraic terms that could be combined by looking at the variable part of each term. If the variable parts and their exponents were identical, the coefficients of the terms could be added using integer arithmetic. The variable part stayed the same.

$$= \underline{x^2} + \underline{\underline{2y}} + 3 + \underline{4x^2} \underline{\underline{- 2y}}$$
$$= 5x^2 + 3$$

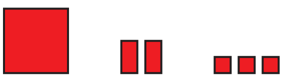
I ended up with $5x^2 + 3$.

This would use 8 tiles.

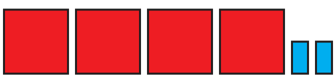
monomial

an algebraic expression with one term; for example, $5x^2$, $4xy$

Sierra's Solution: Representing and simplifying a difference algebraically



Subtract (−)



I represented the tiles using algebraic expressions. I represented the contents of the first pouch as a **trinomial** and the contents of the second pouch as a **binomial**.

trinomial

an algebraic expression containing three terms; for example, $2x^2 - 6xy + 7$

binomial

an algebraic expression containing two terms; for example, $3x + 2$

polynomial
an expression that comprises a sum and/or difference of monomials

$$(x^2 + 2y + 3) - (4x^2 - 2y)$$

I wrote this as a difference of two polynomials.

$$= x^2 + 2y + 3 + (-4x^2) + 2y$$

To subtract the second polynomial I added the opposite of each term.

$$= \underline{x^2} + \underline{\underline{2y}} + 3 \underline{- 4x^2} + \underline{\underline{2y}}$$

I simplified my expression by combining like terms.

$$= -3x^2 + 4y + 3$$

The result was $-3x^2 + 4y + 3$.

This would use 10 tiles.

Communication *Tip*

- Simplifying an algebraic expression means representing the expression using as few terms as possible.

Reflecting

A. How did using algebra tiles help Farell and Peter know which terms could be added or subtracted?

B. How did the appearance of algebraic terms help Jay and Sierra know which terms could be added or subtracted?

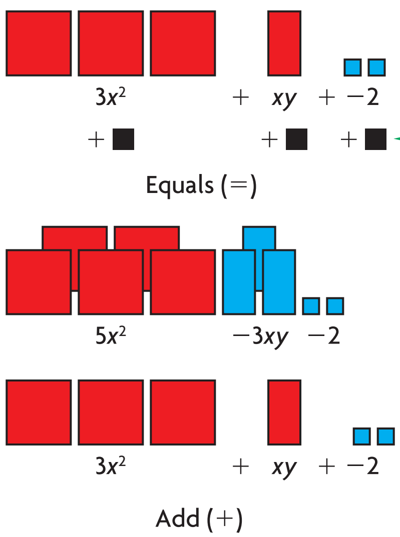
C. How did an understanding of integer operations and the zero principle help each student simplify his or her polynomials?

APPLY the Math

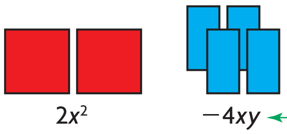
EXAMPLE 3 Reasoning about like terms to determine missing terms in a polynomial

What polynomial must be added to $3x^2 + xy - 2$ to give the result $5x^2 - 3xy - 2$?

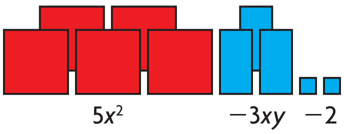
Barry's Solution



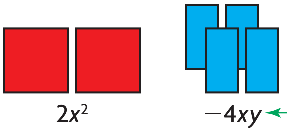
I used algebra tiles to show $3x^2 + xy - 2$ and $5x^2 - 3xy - 2$. I thought about what tiles needed to be added to the ones in the first line to get the ones in the last line.



Equals (=)



I used algebra tiles and integer arithmetic to determine how many and what colour of tiles were needed. I saw that I needed 2 x^2 tiles, 4 negative-xy tiles, and no unit tiles.



When I decided on the algebra tiles I needed, I wrote the algebraic representation.

$3x^2 + 1xy - 2 + 2x^2 - 4xy$
$= 5x^2 - 3xy - 2$

I checked my work by writing my new expression algebraically and combining like terms.

I have to add $2x^2 - 4xy$.

EXAMPLE 4 Using algebraic reasoning to simplify a difference of polynomials

Simplify $(5x^2y + 4xy) - (2x^2y - xy)$.

Raman's Solution

$(5x^2y + 4xy) - (2x^2y - xy)$
$= 5x^2y + 4xy + (-2x^2y + 1xy)$

I knew that subtracting a polynomial is the same as adding its opposite terms.

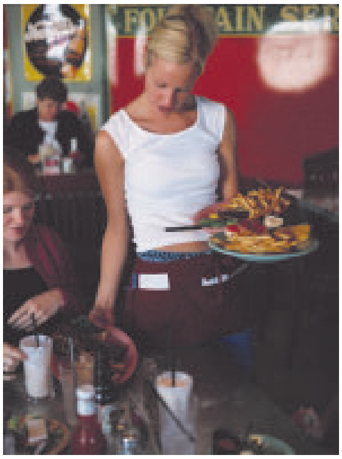
$= 5x^2y + 4xy - 2x^2y + 1xy$
$= 5x^2y - 2x^2y + 4xy + 1xy$

I identified the like terms and grouped them together.

$= (5 - 2)x^2y + (4 + 1)xy$
$= 3x^2y + 5xy$

I combined the coefficients of the like terms by adding or subtracting them.

EXAMPLE 5 Using polynomials to represent and solve a problem



Joan and Chris both have jobs. They both work the same number of hours per week. Their pay rates and expenses are shown.

	Pay Rate	Weekly Expenses
Joan	\$15.50/h	\$40 uniform rental
Chris	\$14/h	\$35 cafeteria charge

Write an algebraic expression in simplest form to describe Joan and Chris's combined take-home pay each week.

Use this polynomial to determine their combined income if they both work 38 hours in a week.

Susie's Solution

Joan and Chris each work h hours in a week.

I used h for the number of hours per week that Joan and Chris each worked.

Joan's income for a week: $15.5h - 40$
Chris's income for a week: $14h - 35$

To represent Joan's weekly income, I multiplied her hourly rate by h and subtracted \$40 for her uniform.

To represent Chris's weekly income, I multiplied her hourly rate by h and subtracted \$35 for her meals.

combined income

$$= (15.5h - 40) + (14h - 35)$$
$$= 15.5h + 14h - 40 - 35$$
$$= 29.5h - 75$$

I added the two expressions by combining like terms.

combined income for 38 hours

$$= 29.5(38) - 75$$
$$= 1121 - 75$$
$$= 1046$$

To determine their combined weekly income, I substituted 38 for h and evaluated.

For 38 hours, their combined income was \$1046.

In Summary

Key Idea

- You can simplify a sum or a difference of polynomials by adding or subtracting the coefficients of like terms.

  For example: $(2y + 3x^2) + (8y - 5x^2)$
  $= (3x^2 - 5x^2) + (2y + 8y)$
  $= -2x^2 + 10y$

Need to Know

- Like terms can be combined by adding or subtracting their numerical coefficients.
- The sum or difference of the coefficients of like terms can be calculated using the principles for adding and subtracting rational numbers.
- It often is easier to subtract two polynomials by using the same strategy you use with integers: adding the opposite.

  For example: $(2y - 2x^2) - (3y + 4x^2)$
  $= 2y - 2x^2 + (-3y - 4x^2)$
  $= 2y - 2x^2 - 3y - 4x^2$
  $= -6x^2 - 1y$

CHECK Your Understanding

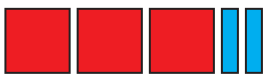
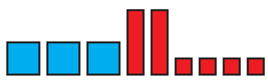
1. Draw an algebra tile representation of each polynomial.

a) $2x^2 - x$ **b)** $x^2 + 3$ **c)** $2y - 2x + 2$

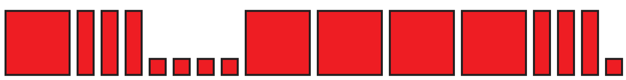
2. Copy each question. Identify the like terms in each and circle their coefficients.

a) $3x, 4y, -2x$ **b)** $6m, -1.5m, 4n, 3m^2$

3. Write an algebraic expression for each algebra tile representation.

a)  **b)** 

4. Simplify the following algebra tile representation. State your result as a polynomial.



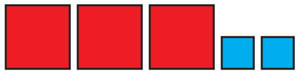
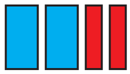
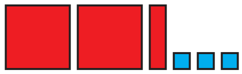
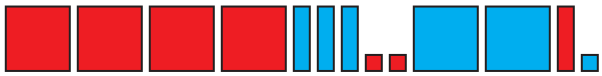
5. Simplify the following.

a) $2x + 3x$
b) $3y^2 - 2y^2 + 4y^2$
c) $3x - 2y + 4x$
d) $(2x + 3) + (5x - 4)$
e) $(3x - 5) + (-2x + 6)$
f) $(3x + 2) - (5x + 2)$

PRACTISING

6. Draw an algebra tile representation of each polynomial.
   a) $x^2 + 3x$
   b) $2x^2 - y^2$
   c) $xy + 4x$
   d) $2x^2 - 3x - 4$

7. Copy each question. Identify the like terms in each and circle their coefficients.
   a) $-2g, 3f, -5g$
   b) $-\frac{1}{2}y, -4x, 2\frac{1}{2}y, 6x^2$
   c) $5x, -2.1y^3, -0.8y^3, 2y$
   d) $-3.75rs, 3.3r, -5.1s, 4.25rs$

8. Write a simplified algebraic expression for each algebra tile representation.
   a) 
   b) 
   c) 
   d) 
   e) 

9. Simplify the following.
   K
   a) $3h + 1 + 2h + 5$
   b) $7y - 3y - x^2 + 4x^2$
   c) $\frac{3}{4}w^2 - \frac{2}{3}w^2 + \frac{1}{4}w^2 - \frac{4}{3}w^2$
   d) $\frac{3}{4}a - \frac{1}{5}b - \frac{1}{4}a + \frac{2}{5}b$

10. Simplify the following.
    a) $(2x - 3y) + (3x + y)$
    b) $(2y^2 - 3y + 4) + (-5y^2 + 5y - 3)$
    c) $(3x^2 - 4xy + 6y^2) + (6x^2 - 8xy - 3y^2)$

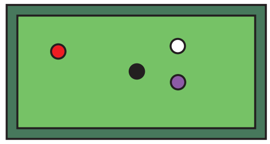
11. Simplify the following.
    a) $(5x - 4y) - (3x + 2y)$
    b) $(3y^2 - 2y + 1) - (-5y^2 + 2y - 3)$
    c) $(3x^2 - 4xy + 6y^2) - (6x^2 - 8xy - 3y^2)$

12. Simplify each of the following. Check your answer using a different tool or strategy.
    a) $2y - 3y - x^2 + 3x^2$
    b) $(y^2 - 2y + 2) - (3y^2 - 2y + 3)$

13. Determine the polynomials that need to be added to each row of the table.

	Initial Polynomial	Polynomial To Be Added	Final Polynomial
a)	$x^2 + 3x$		$-x^2 + 5x$
b)	$2x^2y^2 - 4y^2$		$5x^2y^2 - 3y^2$
c)	$-7xy + 4x$		$-7xy + 3x - 2$
d)	$2x^2 - 3x - 4$		$-2x^2 + 3x - 6$

14. A A pool table is always twice as long as it is wide. The Cue Ball Company makes pool tables in many different sizes. Each table top must have rubber bumpers around the outside edge and a felt top. The rubber bumpers cost \$2.25/m and the felt material for the top costs \$28/m^2. Determine an algebraic expression that represents the total cost for felt and rubber for the table top. Use this to determine the cost of the materials for a top that has a width of 1.5 m.



15. Jan is a plumber. She charges \$35 to visit a job site. Her hourly rate is \$43.50. Fred repairs furnaces. He charges \$41 for a service call plus \$38.75/h. Let x represent the number of hours they work.

a) Represent Jan's bill as a polynomial.
b) Represent Fred's bill as a polynomial.
c) Write a new polynomial that represents Jan's and Fred's combined charge, assuming that they both work x hours at a site.
d) Calculate their combined charge if they both work 8 h at the same complex.

16. Elizabeth and Dragan serve food at different restaurants on a cruise ship. Their earnings are based on tips, as shown, from which they have to pay for room and board.

	Elizabeth	Dragan
Average Weekly Tips	\$220/table	\$160/table
Room and Board	\$160/week	\$125/week

a) Write a polynomial to represent Elizabeth's weekly earnings after she pays for room and board.
b) Write a polynomial to represent Dragan's weekly earnings after he pays for room and board.
c) Dragan and Elizabeth work the same number of tables. Write a single polynomial that combines Dragan's and Elizabeth's earnings.

d) Evaluate the earnings for five tables.
e) Suppose Dragan works seven tables and Elizabeth works five tables. Can the single polynomial in part c) be used to calculate their joint earnings? Explain.

17. T In a TV game show, each player begins with $1000. For each question answered correctly, a player receives $125. For each one answered incorrectly, a player must pay $250.

a) Express the total winnings for a player using an algebraic expression.
b) Use the expression from part a) to find the total winnings for the three players if:
   - player 1 answered 12 questions correctly and 8 incorrectly
   - player 2 answered 10 questions correctly and 2 incorrectly, and
   - player 3 answered 15 questions correctly and 5 incorrectly.

18. Create two 3-term polynomials such that:

a) When the polynomials are combined there are 5 terms.
b) When the polynomials are combined there are 3 terms.
c) When the polynomials are combined there is 1 term.

19. C Describe how your knowledge of the zero principle and of adding and subtracting rational numbers helps simplify a sum or difference of polynomials.

Extending

20. Simplify.

a) $2x + 3y + 4z - 4x + 3y - z$
b) $-4abc - 3ab - 6abc - 4ab$
c) $3xy + 5yz - 2xyz + 6xy - xyz$

21. Simplify.

a) $(2x + 3y) + (5x - 4y) + (2x - y)$
b) $-(4ab - 3a) - (6ab - 4a) + (2ab + 6a)$
c) $(3xy + 5y^2) - (3xy + 5y^2) + (3xy + 5y^2)$

22. Two polynomials are added and the sum is $3x^2 - y + 4$. For each statement, state whether it is always true, sometimes true, or never true. Explain or provide a counter-example to justify your answer.

a) Both are monomials.
b) Both include a y-term.
c) If there is an x-term in one polynomial then there must be an x-term in the other.
d) Both are binomials.

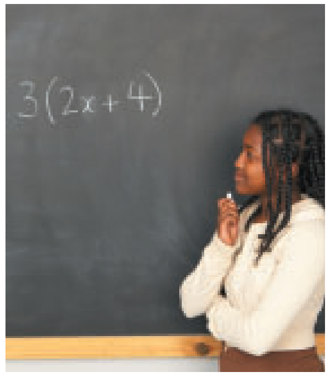
2.5 Multiplying a Polynomial by a Monomial

GOAL

Apply the distributive property to polynomials.

YOU WILL NEED

- algebra tiles
- algebra tile frame



LEARN ABOUT the Math

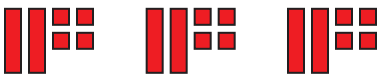
Judy has been asked to determine the product $3(2x + 4)$.

? How might Judy think about this operation in order to determine the product?

EXAMPLE 1 Multiplying a monomial by a polynomial

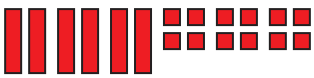
Determine the product $3(2x + 4)$.

Judy's Solution: Representing the product using algebra tiles



I knew that multiplying a number by 3 is the same as adding 3 copies of that number. I decided to show the same repeated addition strategy using algebra tiles.

I gathered enough algebra tiles to show 3 sets of $2x + 4$.



There were 3 sets of 2 x tiles and 3 sets of 4 unit tiles. That meant altogether there were 6 x tiles and 12 unit tiles.

$3(2x + 4) = 6x + 12$

Tamara's Solution: Representing the product using an area model

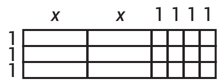
I knew that the area of a rectangle is the product of its length and width. I used algebra tiles to represent the length $2x + 4$ and the width of the rectangle.

I filled in the rectangle with algebra tiles that matched each section of the area inside the rectangle.

$$3(2x + 4)$$
$$= 3(2x) + 3(4)$$
$$= 6x + 12$$

The inside of the rectangle had 3 rows of tiles. Each row contained 2 x tiles and 4 unit tiles. Altogether the area was made up of 6 x tiles and 12 unit tiles.

Sue's Solution: Representing the product using a diagram



I imagined that the factors were the length and width of a rectangle that was divided into sections. I calculated the area of each section separately and added them together to get the total area. The total area was $6x + 12$.

$$3(2x + 4)$$
$$= 3(2x) + 3(4)$$
$$= 6x + 12$$

I noticed that there were 3 rows of sections and each row had 2 sections with area of x and 4 with area of 1.

Shania's Solution: Comparing the product to a product of numbers

$$\begin{aligned} 20 \times 23 &= 20 \times (20 + 3) \\ &= 20 \times 20 + 20 \times 3 \\ &= 400 + 60 \\ &= 460 \end{aligned}$$

Sometimes, when I have to calculate a product, I split one of the factors into parts and use the **distributive property**.

$$\begin{aligned} 3(2x + 4) &= 3(2x) + 3(4) \\ &= 6x + 12 \end{aligned}$$

I decided to use the same strategy to multiply with a polynomial.

distributive property or law

the property that states that when a sum is multiplied by a number, each value in the sum is multiplied by the number separately and the products are then added; for example, $4 \times (7 + 8) = (4 \times 7) + (4 \times 8)$

Reflecting

A. Which student's approach would you use? Why?

B. How does each student's way of thinking about the problem involve an application of the distributive property?

APPLY the Math

EXAMPLE 2 Connecting the distributive property to products of polynomials of degrees greater than 1

Multiply $(2x^3 + 4x - 5)3x^2$.

Peng Bo's Solution

$$(2x^3 + 4x - 5)3x^2$$

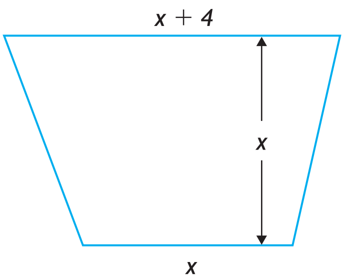
I used the distributive property to multiply each term in the trinomial factor by the factor $3x^2$.

$$= (2x^3)3x^2 + (4x)3x^2 - (5)3x^2$$
$$= 6x^5 + 12x^3 - 15x^2$$

I multiplied the coefficients of the terms to get the coefficients of the product. I added the exponents to determine the exponents of the variable terms.

EXAMPLE 3 Using reasoning and the distributive property to expand a product

Parm is an artist who works in metal and ceramics. He is building a sculpture in the shape of a large trapezoid. The sculpture's proportions are shown on the diagram. To order materials, he needs to determine the trapezoid's area.



Barry's Solution

$$A = \frac{1}{2}h(a + b)$$
$$= \frac{1}{2}x[x + (x + 4)]$$
$$= \frac{1}{2}x(2x + 4)$$

I remembered the formula for the area of a trapezoid. I used x for the height, x for the length of one parallel side, and $x + 4$ for the length of the other side.

$$A = \frac{1}{2}x(2x + 4)$$
$$= \frac{1}{2}x(2x) + \frac{1}{2}x(4)$$
$$= x^2 + 2x$$

I knew that with numbers, I could use the distributive property to multiply each number inside the brackets by the number outside the brackets. I decided to simplify the formula further by multiplying each term inside the brackets by the factor $\frac{1}{2}x$.

Communication *Tip*

The words "expand," "use the distributive property," and "remove brackets" all mean to multiply a polynomial by a factor.

EXAMPLE 4 Using reasoning and the distributive property to determine a missing factor

Complete and verify this equation: $\blacksquare(2x^2 - 5) = 8x^3 - 20x$.

Mohab's Solution

$\blacksquare(2x^2 - 5) = 8x^3 - 20x$ ← Using the distributive property, I knew that whatever factor was represented by the blank box would be multiplied by $2x^2$ and result in $8x^3$.

$4x(2x^2) = 8x^3$ ← $4x$ multiplied by $2x^2$ gives me $8x^3$.

$4x(2x^2 - 5) = 8x^3 - 20x$ ← To check, I replaced the box with the factor $4x$ and multiplied to make sure that the other term was $-20x$.

In Summary

Key Idea

- You can determine the product of a monomial and a polynomial by using the distributive property to expand it.

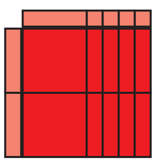
  For example: $2x(3x^2 + 5) = 2x(3x^2) + 2x(5)$
  $= 6x^3 + 10x$

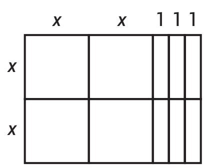
Need to Know

- You can sometimes determine the product of a monomial and a polynomial using the area model. This can be represented with concrete materials or diagrams.

CHECK Your Understanding

1. State the factors and product represented in each model as an algebraic equation.

a) 

b) 

2. Expand.
a) $2a^3(4a^2 - a)$ **b)** $-2(y^2 - y - 1)$

3. Expand using the tool or strategy of your choice. Verify your answer using a different tool or strategy.
a) $2(3x + 4)$ **b)** $3x(x + 2)$

PRACTISING

4. Expand using a different tool or strategy for each.

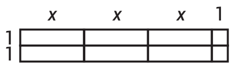
a) $x(2x + 1)$

b) $4(5 + x)$

c) $(3x + 5)(2x)$

5. What multiplication equation does each model represent?

a)



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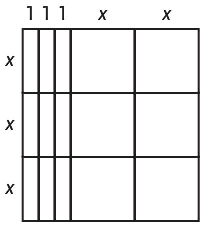

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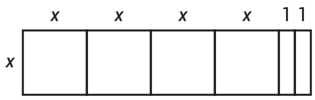


6. Expand.

a) $2(-y^2 - y - 1)$

b) $b^2(2b^3 - 4b + 1)$

c) $3m^3(5m^2 + 6m - 4)$

d) $-(x^2 - 3x + 7)$

e) $-4x(x^2 - 3x)$

f) $-2n^2(3n - 5 + 4n^3)$

7. Determine the missing factor and verify.

a) $\blacksquare(2x - 10) = 6x - 30$

b) $\blacksquare(x^3 - 5x - 4) = 2x^5 - 10x^3 - 8x^2$

c) $-4a^3(\blacksquare + \blacksquare + \blacksquare) = -12a^7 + 4a^4 - 8a^3$

8. Evaluate each statement for $p = 5$ once before you expand it and once after you expand it.

a) $4(3p + 2)$

b) $3p(6 - p)$

c) $2p^2(4p + 3)$

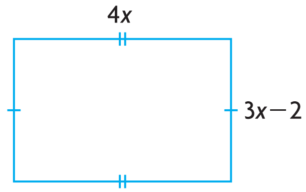
d) $p(3p^2 - 4p + 4)$

e) Explain how you know that you should get the same result both before you expand each statement and after you expand it.

f) Was it always easier to evaluate after you expanded? Explain.

9. Write simplified algebraic expressions for the perimeter, P, and area, A,
A of the following figures.

a)



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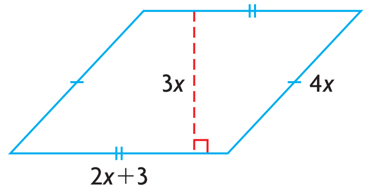
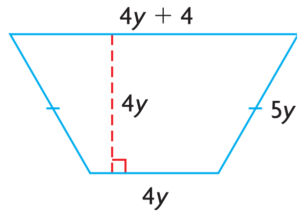


c)



d) Evaluate the perimeter and the area of the rectangle in part a) if $x = 4$ cm.

10. You could evaluate 20×47 by doing the calculation
C $20 \times 40 + 20 \times 7$. How is this like using the distributive property to simplify $x(2x + 7)$?

Extending

11. Expand.

a) $2x(y - 3z)$

b) $-3x(xy + yz)$

12. Fill in the missing information to make the statements true.

a) $20x + 15 = 5(\blacksquare + \blacksquare)$

b) $5x^2 + 25x = 5\blacksquare(\blacksquare + \blacksquare)$

c) $4x^5 + 8x^3 - 2x^2 = \blacksquare x^2(\blacksquare + \blacksquare + \blacksquare)$

d) How could you check your answers for parts a), b), and c) to see if they are correct?

13. These polynomials were expanded using the distributive principle. Restate them as a product of two factors.

a) $12x^2 - 6x$

b) $21y^3 + 7y^2 - 14y$

c) $5x^2 - 10xy + 30x$

2.6 Simplifying Polynomial Expressions

GOAL

Expand and simplify polynomial expressions in one variable.

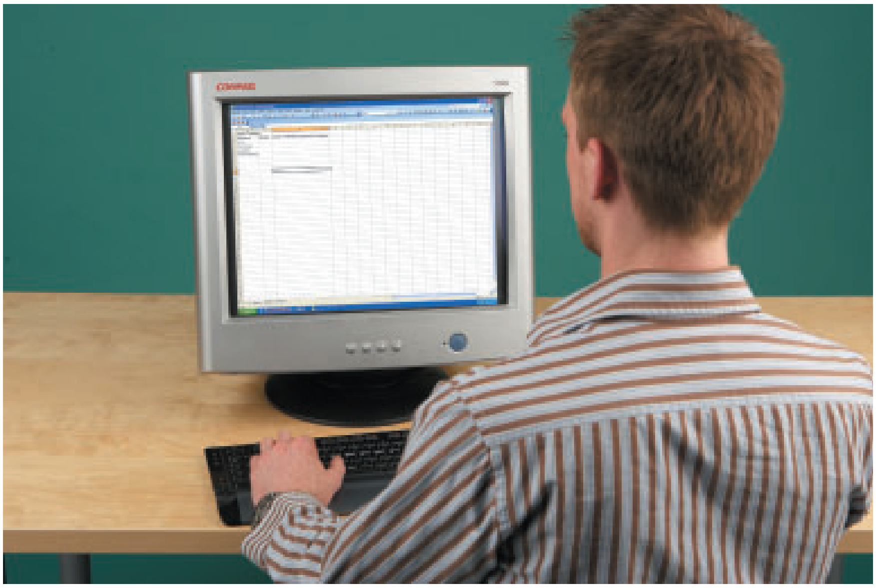
YOU WILL NEED

- algebra tiles

LEARN ABOUT the Math

Todd has a landscaping business. He employs 4 truck drivers, 3 assistants, and 15 student labourers. The pay structure for the business is shown below.

Employees	Number	Hourly and Weekly Payment per Employee
Owner	1	\$800/week
Truck drivers	4	\$17/h plus \$150 for gas
Assistants	3	\$12/h plus \$50 for expenses
Student labourers	15	\$10/h



Todd wants to use a spreadsheet to determine his weekly payroll. He assumes that in any one week all of the employees will work the same number of hours, but also that the number of hours they work from week to week will vary.

? What formula can Todd use to represent his total weekly payroll in terms of number of hours worked?

EXAMPLE 1 Using algebraic reasoning to simplify a polynomial expression

Todd's Solution

Let h represent the number of hours each employee works in a week.

Employees work the same number of hours each week, so I can use the same variable to represent hours worked.

Truck drivers are paid $17h + 150$.
Assistants are paid $12h + 50$.
Student labourers are paid $10h$.

I wrote an algebraic expression to represent each type of worker's weekly pay.

Pay for truck drivers: $4(17h + 150)$
Pay for assistants: $3(12h + 50)$
Pay for labourers: $15(10h)$
Pay for owner, Todd: 800/week

I multiplied the representation for each position's weekly pay by the number of people employed in that position.

Total weekly payroll
$$P = 800 + 4(17h + 150) + 3(12h + 50) + 15(10h)$$

I added the weekly payrolls for all of the positions to describe the total weekly payroll.

$$P = 800 + 4(17h + 150) + 3(12h + 50) + 15(10h)$$
$$= 800 + 68h + 600 + 36h + 150 + 150h$$
$$P = 254h + 1550$$

I simplified this using the distributive property, and then collected like terms.

I can use the formula $P = 254h + 1550$ to represent the total payroll per week.

Reflecting

A. What mathematical principles did Todd use to simplify his formula?

B. What did the only variable in Todd's polynomial expression represent? How did using only one variable help him to expand and simplify the expression to determine the simplified payroll formula?

Communication Tip

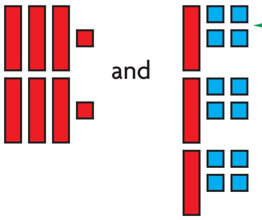
Computer spreadsheets use "formulas" differently than mathematics does. For example, in mathematics the formula $P = 2l + 2w$ is used to determine the perimeter of a rectangle. When a spreadsheet is used, only $= 2l + 2w$ is entered as the formula in a spreadsheet cell.

APPLY the Math

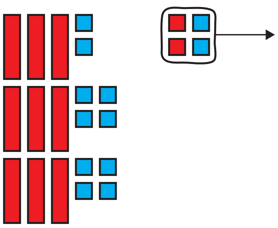
EXAMPLE 2 Simplifying a sum of products of polynomials

Simplify $2(3x + 1) + 3(x - 4)$.

Jaspal's Solution: Reasoning from an algebra tile representation



I used algebra tiles to represent each product.



I combined like tiles and used the zero principle to simplify the polynomial.

$$2(3x + 1) + 3(x - 4) = 9x - 10$$

I wrote a summary statement by counting the remaining tiles in my representation.

Katerina's Solution: Reasoning using the distributive property

$$\begin{aligned}&2(3x + 1) + 3(x - 4)\\&= 6x + 2 + 3x - 12\\&= 9x - 10\end{aligned}$$

I used the distributive property to expand each product.

I collected like terms.

$$\begin{aligned}&2(3x + 1) + 3(x - 4)\\&= 9x - 10\end{aligned}$$

EXAMPLE 3 Simplifying polynomial expressions

Determine the missing factor:

$$(4x^2 - 3x + 2) - (x^2 - 9x - 1) = \blacksquare(x^2 + 2x + 1)$$

Jordan's Solution: Reasoning using the distributive property

$$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$$

$$(4x^2 - 3x + 2) + (-1)(x^2 - 9x - 1)$$

I simplified the polynomials on the left side of the equal sign. The negative sign in front of the second bracket represents (-1), so I used the distributive property to expand, and then collected like terms.

$$= 4x^2 - 3x + 2 - x^2 + 9x + 1$$
$$= 3x^2 + 6x + 3$$

$$\blacksquare(x^2 + 2x + 1) = 3x^2 + 6x + 3$$

Then, I looked at the original expression on the right side of the equal sign. I thought about what factor outside the brackets would give me $3x^2 + 6x + 3$ when I used the distributive property.

$$\blacksquare x^2 = 3x^2$$
$$\blacksquare = 3$$

I started with the first term, $3x^2$. To get this, I would have to multiply x^2 by 3, so I chose the number 3 to fill the box.

$$3(x^2 + 2x + 1) = 3x^2 + 6x + 3$$

I checked by using the distributive property, and it worked.

$$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$$
$$= 3(x^2 + 2x + 1)$$

Danika's Solution: Connecting to adding opposites

$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$

$= 4x^2 - 3x + 2 + (-x^2) + 9x + 1$
$= 3x^2 + 6x + 3$

I simplified the polynomials on the left side of the equal sign. The negative sign in front of the second bracket meant I had to add the opposite of each term inside the bracket. I rewrote the problem and collected like terms.

$3x^2 + 6x + 3 = \blacksquare(x^2 + 2x + 1)$

I looked at the simplified expression. I thought about what factor was common to each of the terms.

$\frac{3x^2}{3} = x^2,$

$\frac{6x}{3} = 2x$, and

$\frac{3}{3} = 1$

$\blacksquare = 3$

Each term had a factor of 3. When I divided each term by 3, the results were the terms of the trinomial on the right hand side.

$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$
$= 3(x^2 + 2x + 1)$

EXAMPLE 4 Connecting exponent principles to simplifying a polynomial expression

Expand and simplify $3x^2(4x^3 - 2x^2 + 6x) - (x^5 + 5x^4 - 4x^3) + 7x^5$.

George's Solution

$3x^2(4x^3 - 2x^2 + 6x) - (x^5 + 5x^4 - 4x^3) + 7x^5$
$= 12x^5 - 6x^4 + 18x^3 - x^5 - 5x^4 + 4x^3 + 7x^5$

I used the distributive property to expand the product for each bracketed expression. To simplify the first set of brackets, I had to apply the exponent principles for multiplying powers. To simplify the second set of brackets, I multiplied each term by the factor (-1). The last term wasn't multiplied by anything.

$= 12x^5 - x^5 + 7x^5 - 6x^4 - 5x^4 + 18x^3 + 4x^3$
$= 18x^5 - 11x^4 + 22x^3$

I simplified the expression by grouping then collecting like terms.

EXAMPLE 5 Solving a problem by simplifying a polynomial expression

A sporting goods company provides skis and snowboards to instructors at ski resorts.

	Number Provided	Original Value	Yearly Drop in Value
Skis	200	\$600	\$50
Snowboards	300	\$800	\$60

Determine an expression that represents the combined value of the equipment after y years of use. Use this value to determine the combined value of all the equipment after 2 years of use.

Mark's Solution

After y years, the values of the skis and snowboards will be:

- skis: $600 - 50y$
- snowboards: $800 - 60y$

I wrote algebraic expressions to represent the future value of each item.

$$V = 200(600 - 50y) + 300(800 - 60y)$$

I multiplied each expression by the number of skis or snowboards. This gave me an equation for the combined value V after y years.

I used the distributive property, and then simplified the expressions.

$$= 120\ 000 - 10\ 000y + 240\ 000 - 18\ 000y$$
$$= 360\ 000 - 28\ 000y$$

$$V = 360\ 000 - 28\ 000(2)$$
$$= 360\ 000 - 56\ 000$$
$$= 304\ 000$$

I substituted 2 for y to determine their combined value after 2 years.

After 2 years, the combined value will be \$304 000.

In Summary

Key Idea

- You can use the distributive property and collect like terms to simplify a sum or difference of products of polynomials.

Need to Know

- Use the order of operations to determine the sequence in which operations must be performed.

CHECK Your Understanding

1. Simplify using the tool or strategy of your choice. Verify using a different tool or strategy.

a) $3(x - 1) + 2(2x + 2)$

b) $2(y^2 - 3) - 2(y^2 - 1)$

2. Simplify.

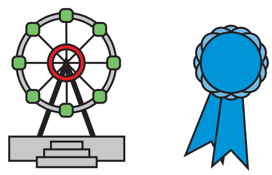
a) $3x^2(x^3 - 1) + 2x^3(2x^2 + 2)$

b) $2(y^2 - 3y^5) - 3(y^2 - y^5)$

3. On average, the following numbers of adults and children pay to enter the fall fair.

	Adults	Children
At 9 a.m.	20	25
Each Hour After 9 a.m. Until Closing	95	120

FALL FAIR
9 a.m.–10 p.m.
Children: $8.00
Adults: $12.00



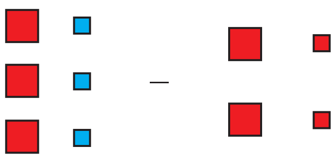
Determine an expression for the total entrance fees collected h hours after opening.

PRACTISING

4. C Write an algebraic representation that corresponds to the algebra tile models shown. Simplify the expression using the strategy of your choice.

a)

b)



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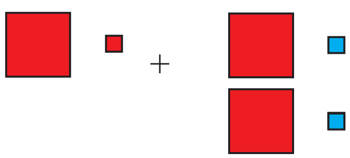


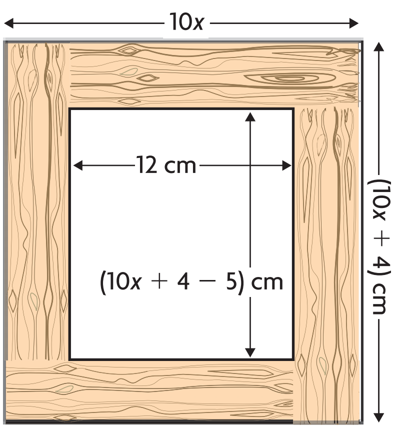
5. Simplify the following expressions using the tool or strategy of your **K** choice. Verify using a different tool or strategy.
   a) $6(8 + 3c) + 4(10 + 2c)$
   b) $5(2x - 3) - 4(3x + 6)$
   c) $2(x^2 - 3x + 6) - 3(2x^2 - 4x - 1)$
   d) $-y(y^2 + 5y + 4) + 3y(2y^2 - y + 6)$
   e) $2(3y^2 + 4y) - 3(2y^2 - y)$
   f) $2(4x^3 - 3x + 6) + 3(2x^5 + x^3 - 4x)$

6. Simplify each expression, and then evaluate for $a = 3$.
   a) $6(2a + 4) - 3a$
   b) $15 - 2(a - 5)$
   c) $-10a - 2(a^2 + 7)$
   d) $-(2a - a^3) - a^2$

7. Simplify the following polynomial expressions.
   a) $3(2x^2 - 1) + 6(2x - 3) - (2x^2 - 5x)$
   b) $6(x + 5) - 2(x + 4) + 3(x - 5)$
   c) $3(2y^2 - 1) + 6(2y - 3) - (2y^2 - 5y)$
   d) $3(4p^2 - 2p + 6) + 6(4p - 2) - (7p^2 + 5p + 1)$

8. Simplify.
   a) $4x^2(x^3 - 2x^2) + 2x^3(2x^2 + 2x)$
   b) $2y^2(y^3 - 3y^5) - 3(y^2 - y^5)$
   c) $3m^3(2m^2 - 5m + 3) - 4m(m^4 + 2m^3 - m^2)$
   d) $-5x(x^3 - 2x^2) + 2x^2(3x^2 - 5x) - 4x^3(x - 2)$

9. Expand and simplify.
   a) $\frac{3}{5}\left(2\frac{1}{3}a - 2\frac{1}{2}\right) - \frac{1}{2}\left(2\frac{1}{5}a + 3\frac{2}{3}\right)$
   b) $\frac{1}{6}\left(3\frac{1}{5}a + \frac{2}{3}b\right) + \frac{1}{3}\left(\frac{1}{2}a - \frac{1}{2}\right)$
   c) $-1.25(3.1m + 2.2) - 2.15(1.2m - 3.2)$



10. Mary is making rectangular picture frames to the proportions shown.
   a) Determine a simplified expression for the outside perimeter of the frame.
   b) Determine the outside perimeter when $x = 5$ cm.
   c) Determine a simplified expression for the area of one picture frame.
   d) Determine the area of one frame when $x = 5$ cm.
   e) Determine a simplified expression for the number of square centimetres of wood needed to make 20 frames the same size. Assume there is no waste.

11. A company purchased two kinds of cars for its sales force. The
A following expressions give the value of each vehicle after it has lost value for x years.

- sedans: $V = -2400x + 19\,600$
- sport utility vehicles (SUVs): $V = -3100x + 24\,500$

a) The company has 12 sales representatives who drive sedans and 3 executives who drive SUVs. Write an expression that represents the combined value of the company's automobile fleet after x years.

b) Create a spreadsheet that will determine the cost of each vehicle type and the combined value of all the cars each year for 0 to 6 years.

c) What did the company pay for the SUVs? the sedans?

d) Which vehicle type is losing value at a faster rate?

12. Simplify.

a) $15 - 10(x - 4) - (3x + 3)$

b) In part a), explain why the 10 is not subtracted from the 15 before you expand.

13. a) Is the following statement always true, sometimes true, or never
T true? "Algebra tiles can be used to represent and simplify algebraic expressions that require the distributive property."

b) If you chose "always true" or "never true," explain how you know. If you chose "sometimes true," provide an example that shows when it is true, and another that shows when it isn't true.

Extending

14. Simplify the following.

a) $5(2x - 3y) - 4(3x + 6y)$

b) $x(x + y) + 2x(x - y)$

c) $2(x^2 - 3xy + 6y) - 3(2x^2 - 4xy - y)$

15. Apply the distributive property to simplify the following.

a) $(x + 3)(2x + 4)$

b) $(y + 2)(y + 1)$

c) $(2x + y)(x + y)$

16. Simplify.

a) $(x - 3)(x + 4) + 3(x^2 - x + 2)$

b) $(2x + y)(x - y) - (x^2 + y^2)$

c) $(3x + 5)(2x - 4) + (x + 1)(2x + 5)$

2 Chapter Review

FREQUENTLY ASKED Questions

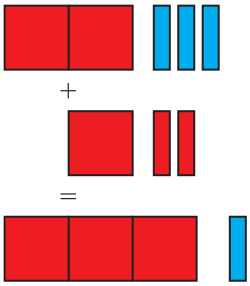
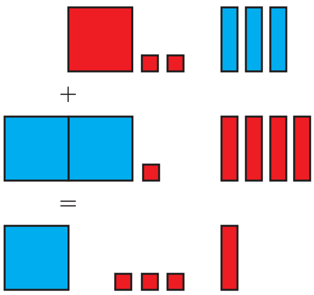
Study *Aid*

- See Lesson 2.4, Example 3.
- Try Chapter Review Questions 11 and 12.

Q: What tools and strategies can you use to simplify an algebraic expression?

A: Sometimes you can recognize like terms by using an algebra tile model to represent the expression. You can always use the variables and the degree to identify like terms. You use rational number arithmetic to add or subtract the coefficients of like terms.

EXAMPLE

Expression	Algebra Tile Model	Identifying Like Terms
$2x^2 - 3x + x^2 + 2x$ $= 3x^2 - x$		$\underline{2x^2} - \underline{\underline{3x}} + \underline{x^2} + \underline{\underline{2x}}$ $= 3x^2 - x$
$x^2 + 2y^2 - 3xy - 2x^2 + y^2 + 4xy$ $= -x^2 + 3y^2 + xy$		$\underline{x^2} + \underline{\underline{2y^2}} - \underline{\underline{\underline{3xy}}} - \underline{2x^2} + \underline{\underline{y^2}} + \underline{\underline{\underline{4xy}}}$ $= -x^2 + 3y^2 + xy$
$2x^4y - 3xy - 5x^4y - 4xy$ $= -3x^4y - 7xy$	This cannot be modelled with algebra tiles.	$\underline{2x^4y} - \underline{\underline{3xy}} - \underline{5x^4y} - \underline{\underline{4xy}}$ $= -3x^4y - 7xy$

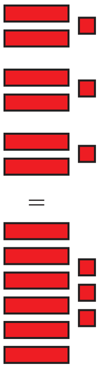
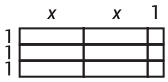
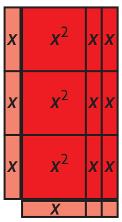
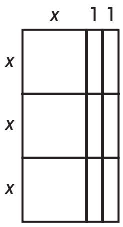
Q: **What strategies can you use to multiply a polynomial by a monomial?**

A: You can use the distributive property to multiply a polynomial by a monomial. Sometimes you can use algebra tiles and an understanding of multiplication as repeated addition. Sometimes you can use an area model.

Study Aid

- See Lesson 2.5, Example 2.
- Try Chapter Review Questions 14 and 15.

EXAMPLE

Expression	Algebra Tile Model	Area Model	Distributive Property
$3(2x + 1)$ $= 6x + 3$		 The area is $6x + 3$.	$3(2x + 1)$ $= 6x + 3$
$3x(x + 2)$ $= 3x^2 + 6x$		 The area is $3x^2 + 6x$	$3x(x + 2)$ $= 3x^2 + 6x$

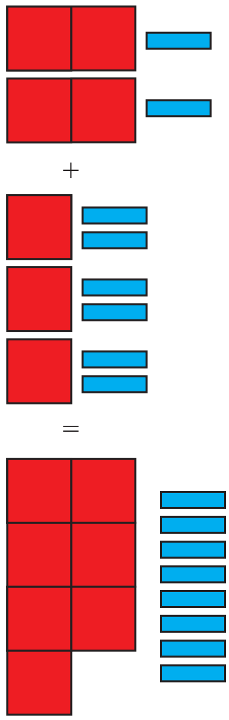
Study Aid

- See Lesson 2.6, Example 2.
- Try Chapter Review Questions 17 and 19.

Q: How can you simplify a sum or difference involving products of polynomials?

A: You can multiply any products using the distributive property to expand, and then combine like terms. Sometimes you can use algebra tiles to model and simplify the expression.

EXAMPLE

Expression	Algebra Tile Model	Distributive Property and Algebraic Skills
$2(2x^2 - x) + 3(x^2 - 2x)$ $= 7x^2 - 8x$		$2(2x^2 - x) + 3(x^2 - 2x)$ $= 4x^2 - 2x + 3x^2 - 6x$ $= 7x^2 - 8x$
$2x^2(2x^2 - 3x) + x^2(2x - 1)$ $= 4x^4 - 4x^3 - x^2$	This cannot be modelled with algebra tiles.	$2x^2(2x^2 - 3x) + x^2(2x - 1)$ $= 4x^4 - 6x^3 + 2x^3 - x^2$ $= 4x^4 - 4x^3 - x^2$

PRACTICE Questions

Lesson 2.1

1. Sketch models to represent each of the following algebraic expressions. The variables x and y are not equal.

a) y^2
b) y^3
c) $2x$
d) $(2x)^2$

2. Rob is finishing a floor with square tiles. Each tile has an area of 412 cm^2. Estimate the length of the side of each tile. Use a calculator to check your answer.

Lesson 2.2

3. Why do you get the same result for each of the following expressions?

a) $\dfrac{(5^7)}{(5^4)}$

b) $\dfrac{(5^6)}{(5^3)}$

c) $\dfrac{(5^4)(5^5)}{(5^6)}$

d) $\dfrac{(5^4)(5)}{(5)(5)}$

4. Simplify, and then evaluate for $x = -2$ and $y = 3$.

a) $\dfrac{(x^3)(x^4)}{x^6}$ **b)** $\dfrac{(y^6)(x^4)}{(x^2)(y^4)}$ **c)** $\dfrac{-32x^6}{16x^3}$

5. If you know that the product of two numbers is 9^6 and the quotient is 9^2, what could the two numbers be? How do you know?

6. About how long does it take for light to travel from one end of our galaxy to the other?

- It is about 9.5×10^{16} km from one end of our galaxy to the other.
- Light travels at about 1.1×10^9 km/h.

Lesson 2.3

7. Simplify.

a) $(a^3)^2$ **b)** $(4x^3)^4$ **c)** $\dfrac{(2^3y^4)^3}{(2^4y^3)^2}$

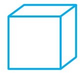
8. Without doing the calculations, how do you know that each result will be zero?

a) $(10^3)^5 - (10^5)^3$ **b)** $(9^2)^2 - (3^4)^2$

9. Express each of the following as a power with a lesser base.

a) 8^3 **b)** 25^4 **c)** 9^3

10. The length of the side of a cube is 5^3. Express its surface area (SA) and volume (V) using powers and simplify each expression.



Lesson 2.4

11. Simplify.

a) $5y - 4y$
b) $3xy^2 + 3xy^2$
c) $2x^2 - 5x + 5x^2 - x$
d) $y^2 + 5xy + y^2 - xy$

12. Simplify.

a) $\dfrac{4}{5}a - \dfrac{1}{5}a$

b) $2\dfrac{1}{2}a + \dfrac{2}{3}b + \dfrac{1}{2}a - \dfrac{1}{3}b$

c) $-1.75m + 2.7 - 2.25m + 2.3$

13. Manuel and Kim have a picture framing business. Manuel cuts the wooden frames. He charges \$25 for each one plus \$10/h for his labour. Kim cuts the picture mats and assembles the product. She charges \$8 for each mat plus \$9/h for her labour.

a) Represent Manuel's bill as a polynomial.
b) Represent Kim's bill as a polynomial.
c) Write a new polynomial that represents their total charges to frame a picture. Assume that they both work h hours on the frame.
d) Calculate the cost for a frame if they both work 5 h on it.

Lesson 2.5

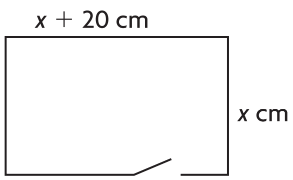
14. Expand. Check one of your answers using a different tool or strategy.

a) $3(y - 2)$ **d)** $-3x(x^2 - x)$
b) $x(2x + 4)$ **e)** $2y^3(y^3 + 3y^2 - y)$
c) $5m(3m^3 + 2n)$ **f)** $-a^2(2a - 5a^2 + 4a^3)$

15. Expand.

a) $\frac{1}{3}(3x + 12)$
b) $\frac{2}{5}\left(\frac{5}{8}a + 10b\right)$
c) $-1.5m(2.8m + 2.2)$

16. Rick runs a pet store and is building rectangular pens for the animals. The length of the pens is always 20 cm longer than the width.



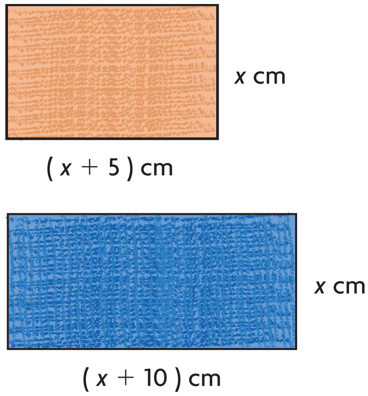
a) One way of determining the perimeter is to use $P = 2(l + w)$. Use this formula to create an expression for the perimeter in terms of x.
b) Simplify your formula in part a).
c) Find the perimeter of a pen with a width of 45 cm.
d) Suppose you use the formula $P = 2l + 2w$ instead. Use this formula to create another expression for the perimeter in terms of x.
e) Why should the expressions in parts a) and d) be equivalent?

Lesson 2.6

17. Expand and simplify. Check one of your answers using a different tool or strategy.

a) $2(x - 3) + 3(x + 2)$
b) $3(y^2 + y - 2) - (y^2 + 2y + 4)$
c) $2x(3x - 2) + x^2 + 2(x^2 + 3)$
d) $3x(4x^2 - 5x) + x^3 - x^2$

18. Ms. Smith needs fabric pieces for an art project for her students. The pieces will be cut to two rectangular sizes, as shown.



a) Determine a simplified expression for the area of fabric needed if 14 students choose the larger size and 12 choose the smaller size.
b) The class decides that the width of each piece of fabric will be 20 cm. Use your answer from part a) to determine how much material will be needed.

19. Expand and simplify.

a) $\frac{1}{4}(8x - 12) - \frac{1}{2}(6 - 14x)$
b) $\frac{5}{6}(6x - 18y) + \frac{2}{3}(21x - 6y)$
c) $-5.2x(2.5x + 6.6) - 0.25x(1.6x - 2.4)$

20. Mickey has a baseball card collection. He is wondering about the future value of his rookie and big star cards.

Card Type	Number of Cards	Value Today	Increase in Value per Year
Rookie cards	22	$15	$3
Big star cards	18	$12	$2

a) Write an expression that represents the combined value of these cards in y years.
b) Use your answer from part a) to determine the combined value of the cards in 6 years.

2 Chapter Self-Test

1. Sketch and label a representation for each of the following algebraic terms. Explain how your sketches are both alike and different.

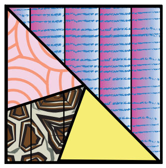
a) y^2 **b)** m **c)** m^3

2. Jacqueline is making a quilt using finished squares like the one shown. Each square has an area of 310 cm^2.

a) Write an expression that represents the length of the side of the square in terms of its area.

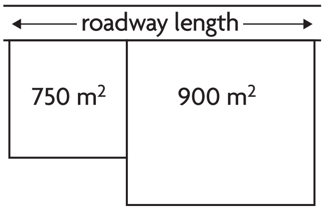
b) Estimate the length. Describe the process you used.

c) Use a calculator to determine the correct length to two decimal places.



$A = 310$ cm^2

3. Determine the approximate length of the roadway bordering the adjacent square properties.



4. Simplify.

a) $(3^4)(3^6)$

b) $\dfrac{\left(\dfrac{4}{5}\right)^6}{\left(\dfrac{4}{5}\right)^5}$

c) $\dfrac{(x^3)(x^6)}{x}$

d) $(6.1)(6.1^6)(6.1^5)(5^2)$

5. Which expression below is equivalent to $(-7)^4(-7^6)$?

A. -49^{10}

B. 49^{10}

C. $(-7)^{10}$

D. $(-7)^{24}$

E. none of the above

6. Use an example to explain how you know that, to simplify a power of a power, you multiply the exponents.

7. Simplify.

a) $(6^3)^2$

b) $(x^2)^5$

c) $(5x^4)^5$

d) $\left[\left(\dfrac{2}{5}\right)^4\right]^5$

e) $(y^3)^2(y^5)^2$

f) $\left(\dfrac{5}{x^2}\right)^2$

8. What is meant by "like terms"? Explain how algebra tiles or diagrams can help you to identify like terms.

9. Simplify. Check one of your answers using a different tool or strategy.

a) $5x - 2x$

b) $7xy + 2xy$

c) $(3x^2 - 5a) + (4x^2 - a)$

10. Which expression below is equivalent to $(y^2 + 5y) - (3y^2 - y)$?

A. $-2y^2 + 6y$

B. $4y^2 + 4y$

C. $2y^2 + 6y$

D. $-2y^2 + 4y$

E. none of the above

11. Expand. Check one of your answers using a different tool or strategy.

a) $2(x - 3)$

b) $3x(4x^2 - 5x)$

c) $y^5(5y^4 + 3y^3 - y^2)$

12. Expand and simplify. Check one of your answers using a different tool or strategy.

a) $2(x - 3) + 3(x + 2)$

b) $3(y^2 + y - 2) - (y^2 + 2y + 4)$

c) $2x(3x - 2) + x^2 + 0.2(x^2 + 3)$

13. Mary and Bill are setting up a summer concession stand at the park. They need these jobs filled:

Position	Number of Positions	Hourly Pay Rate and Weekly Bonus
Manager	3	\$14/h plus \$50 bonus per week for working at least 30 h per week
Server	8	\$9/h plus \$35 bonus per week for working at least 30 h per week

a) Determine a simplified algebraic expression to represent the weekly payroll for all 11 employees. Assume that they will always work at least 30 h per week.

b) Use your expression to determine the weekly payroll if each employee works 32 h per week.

14. Determine an expression that simplifies to $3x^2$ if the expression contains:

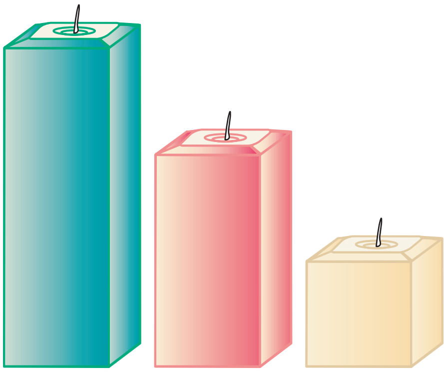
- exactly two binomials
- the distributive property used once

2 Chapter Task

Candle Fundraiser to Brighten Lives

Your class is going to raise money for charity by ordering and selling candle sets. Each set will contain three different-sized candles. The candles are rectangular prisms with congruent square bases. The other specifications for the candles are as follows:

- The length of the side of the base for each set is between 6 cm and 10 cm.
- The smallest candle's height is equal to the length of the side of the base.
- The middle candle's height is double the length of the side of the base.
- The tallest candle's height is triple the length of the side of the base.
- The wick extends 2 cm past the top of the candle for easy lighting.



The cost of the wick is \$0.004/cm. The candle wax costs \$0.02/cm^3. Your class sets the price of each candle as the cost of the materials plus a 40% profit.

? What expression can you use to calculate the selling price of any candle set?

A. Determine an expression to represent the volume of a candle set.

B. Determine an expression to represent the wick length needed for a candle set.

C. Determine an expression to represent the purchase price of a candle set.

D. Use your expressions to calculate the purchase price for a candle set when the area of the base is 55 cm^2.

Task Checklist

- ✔ Did you choose an appropriate variable to represent the length of the side of the base of the candles?
- ✔ Did you draw and label a diagram accurately?
- ✔ Did you show all of your steps?
- ✔ Did you explain your reasoning clearly?
- ✔ Did you consider all of the information provided?