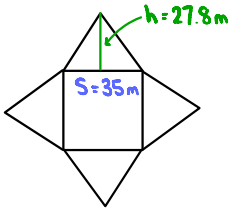
Surface Area of Prisms and Pyramids

Let's revisit some important terms!

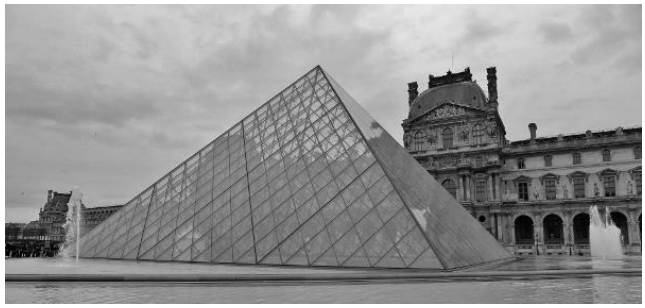
perimeter	The distance around the outside of a 2D figure
area	The amount of space occupied by a 2D figure, measured in square units
surface area	The amount needed to cover the surface of a 3D object, measured in square units
volume	The amount of space that an 3D object occupies, measured in cubic units
prism	A polyhedron whose top and base is the same polygon and whose faces are rectangles connecting the top and the base
pyramid	A polyhedron whose base is a polygon and whose faces are triangles that meet at a common vertex (apex)
lateral faces	Faces of a prism or pyramid that are not bases

Example 1: A modern example of a pyramid can be found at the Louvre in Paris, France. The glass square-based pyramid was built as an entrance to this famous museum. Given that the base is 35 m long and the slant height is 27.8 m, calculate the surface area of the pyramid, including the base area.



$$SA = A_{base} + 4A_{lateral}$$
$$= s^2 + 4\left(\tfrac{1}{2}bh\right)$$
$$= (35)^2 + 4\left(\tfrac{1}{2}\right)(35)(27.8)$$
$$= 1225 + 1946$$
$$= 3171\ m^2$$

∴ The surface area is $3171\ m^2$.



Example 2: Chocolate is sometimes packaged in a box that is shaped like a triangular prism. Calculate the amount of material required to make this box, to the nearest square centimeter, if each side of the triangle is 3 cm and the length of the chocolate bar is 12 cm.

1) Find h (Pythagorean Thm)

$a^2 + b^2 = c^2$

$\left(\frac{3}{2}\right)^2 + h^2 = 3^2$

$2.25 + h^2 = 9$

$h^2 = 9 - 2.25$

$h^2 = 6.75$

$h = \sqrt{6.75}$

$h \doteq 2.6$ cm

2) $SA = 2A_{base} + 3A_{lateral}$

$= 2\left(\frac{bh}{2}\right) + 3(\ell w)$

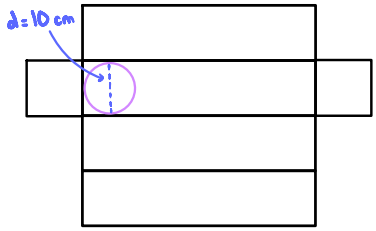
$\doteq 2\left(\frac{1}{2}\right)(3)(2.6) + 3(12)(3)$

$\doteq 7.8 + 108$

$\doteq 115.8$ cm^2

$\therefore$ The surface area is about 115.8 cm^2.

Example 3: 5 tennis balls are packed in a rectangular prism package. The diameter of one ball is 10 cm. What is the minimum amount of material needed to make the box?



$SA = 2A_{base} + 4A_{lateral}$

$= 2(s^2) + 4(\ell w)$

$= 2(d^2) + 4(5d)(d)$

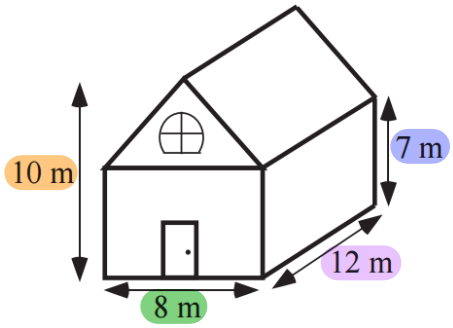
$= 2(10^2) + 4(5)(10)(10)$

$= 200 + 2000$

$= 2200$ cm^2

$\therefore$ The minimum amount of material needed is 2200 cm^2.

Example 4: Calculate the surface area of the house below (not including the floor) given the roof is the same on both sides.



$SA_{House} = 2(8)(7) + 2(7)(12)$

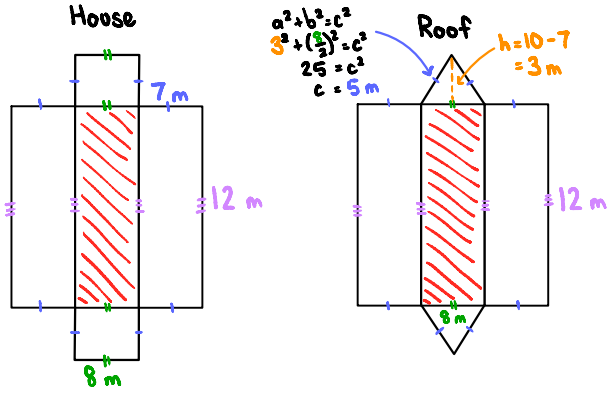
$= 112 + 168$

$= 280$ m^2

$SA_{Roof} = 2\left(\frac{1}{2}\right)(8)(3) + 2(12)(5)$

$= 24 + 120$

$= 144$ m^2



$SA_{Total} = 280 + 144$

$= 424$ m^2

$\therefore$ The surface area is 424 m^2.