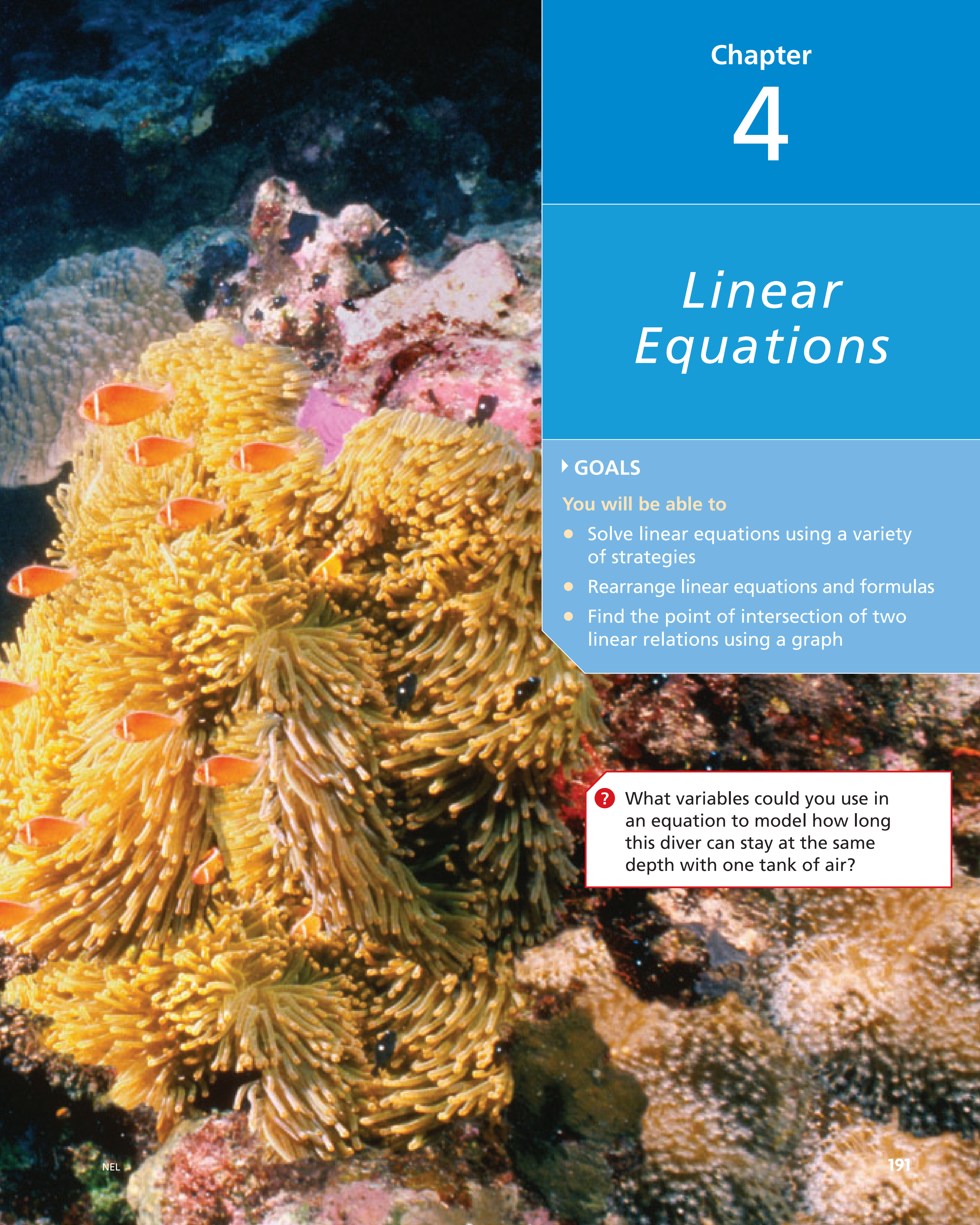


Chapter

4

Linear Equations

GOALS

You will be able to

- Solve linear equations using a variety of strategies
- Rearrange linear equations and formulas
- Find the point of intersection of two linear relations using a graph

? What variables could you use in an equation to model how long this diver can stay at the same depth with one tank of air?

4 *Getting* Started

WORDS YOU NEED to Know

1. Match each term with the highlighted example that best represents it.

a) equation **b)** solution to an equation **c)** variable **d)** algebraic expression **e)** constant **f)** coefficient

i) $3x + 4 = 10$
ii) $3x + 4$
iii) $3x + 4$
iv) $3x + 4$
v) $3x + 4$
vi) $3x + 4 = 10$
$x = 2$

SKILLS AND CONCEPTS You Need

Study *Aid*

- For more help and practice, see Appendix A-13.

Equation Solving Strategies

You can use different strategies to solve an equation:

- systematic trial
- drawing a graph
- inspection and logical reasoning
- balancing

EXAMPLE

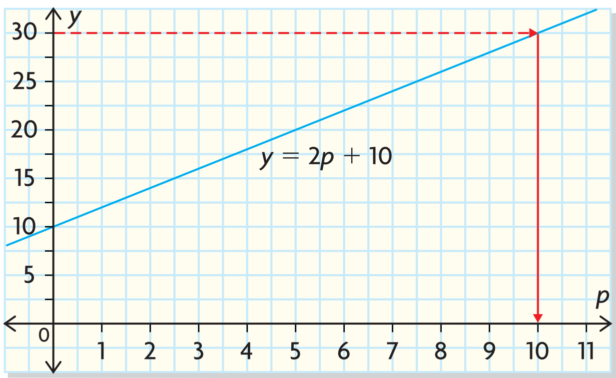
Solve: $2p + 10 = 30$.

Solution

Systematic Trial

p	$2p + 10$	Comparison to 30
8	$16 + 10 = 26$	too low
11	$22 + 10 = 32$	too high
10	$20 + 10 = 30$	correct

Drawing a Graph



Inspection and Logical Reasoning

$2p + 10 = 30$

That means $2p$ must be 20.

If $2p = 20$, then p must be 10.

Balancing

$$2p + 10 = 30$$
$$2p + 10 - 10 = 30 - 10$$
$$2p = 20$$
$$2p \div 2 = 20 \div 2$$
$$p = 10$$

2. Use a graphing strategy to solve the following equations.
  a) $3n - 1 = 11$ **b)** $2x + 5 = 21$

3. Solve the following equations using the strategy of your choice.
  a) $6a + 12 = 18$ **b)** $10 = 4m + 2$ **c)** $3x - 5 = 10$

Solving an Equation to Solve a Problem

You can use equations to solve word problems. To do so, you need to define the variable for the unknown quantity, and then, create an equation using the variable and the information provided.

EXAMPLE

Rishi ordered 3 pizzas. What was the cost of each pizza if the delivery charge was \$1.50 and the total bill was \$27.00?

Solution

Let n represent the cost of each pizza.

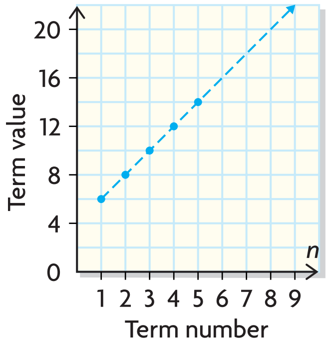
$$\begin{aligned} 3n + 1.50 &= 27.00 \\ 3n + 1.50 - 1.50 &= 27.00 - 1.50 \\ 3n &= 25.50 \\ 3n \div 3 &= 25.50 \div 3 \\ n &= 8.50 \end{aligned}$$

Check:
If each pizza cost \$8.50, 3 pizzas would cost $3 \times \$8.50 = \$25.50$.
The \$1.50 delivery charge would bring the total to \$27.00.

Each pizza costs \$8.50.

4. Use an equation to solve each of the following.
  a) Jillian bought 5 DVD's and the cost was \$99.75. Determine the cost of each DVD.
  b) Bill's age 3 years ago was 18. How old is he now?
  c) Nazir has \$14.50 in her pocket. Of this amount, \$10 is in bills and the rest is quarters. How many quarters are in her pocket?
  d) Bart rented a truck for the day. His bill was \$177. The rental company charged him a flat fee of \$45 and a fee of \$0.55/km for the distance he drove. How many kilometres did he drive?

Number Sequence (Rule: $2n + 4$)



Study Aid

- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
5, 6, 7, and 12	A-13
8	A-6
11	A-11

PRACTICE

5. Use the Number Sequence graph on the left to solve the equation $2n + 4 = 20$.

6. Solve for x.

a) $\frac{x}{3} = \frac{6}{27}$ **b)** $\frac{2}{x} = \frac{10}{40}$ **c)** $\frac{3}{7} = \frac{x}{18}$

7. Solve each equation.

a) $78 = 6x$

b) $45 = 4m + 11$

c) $25n - 8.5 = 101.5$

8. Calculate.

a) $-8 + 1\frac{3}{5}$ **b)** $-3\frac{1}{5} - \left(-6\frac{3}{8}\right)$

9. Simplify.

a) $-3x - 2(4x + 7)$ **b)** $2x - 5.5 - 4.5x + 9$

10. **a)** Identify the initial value and the rate of change for $y = -3x + 13$.

b) How would you sketch the graph of $y = -3x + 13$?

11. Ethel is a junior programmer for EDUCAT Software. She is paid by the hours worked. She is also paid a fixed amount for her expenses every week. The table shows how much she will be paid for hours worked in a week, including expenses.

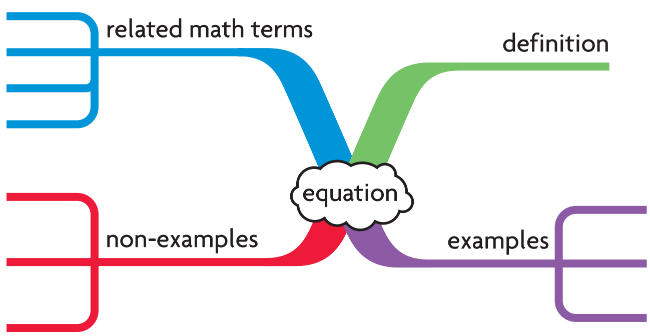
Hours Worked	20	25	30	35	40
Earnings ($)	700	825	950	1075	1200

a) What fixed amount is Ethel paid for her expenses?

b) Write an equation for the relationship.

c) How much would Ethel be paid if she worked 32 h? 53 h?

12. Use what you know about equations to complete the vocabulary organizer below. Add branches as needed.



APPLYING What You Know

YOU WILL NEED

- interlocking cubes
- grid paper

Painted Cubes

Toni made a large cube out of wooden blocks. She painted the outside of the large cube red. She let the paint dry, and then, took the cube apart. She noticed that

- some blocks had no paint on them
- some blocks had one face painted
- some blocks had two faces painted
- some blocks had three faces painted

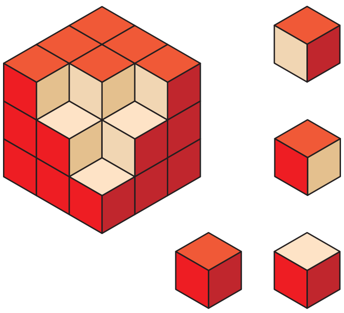
For a 10 × 10 × 10 cube, how can you predict the number of blocks that have paint on 3, 2, 1, or 0 faces?

A. Build several different cubes from centimetre blocks.

B. Imagine painting each of your cubes.
Visualize how many faces of each centimetre block will be painted.

C. Complete a table of values showing the relationship between the number of centimetre blocks that are painted on 3 faces and the length of the side of the big cube.

D. Graph the relationship you found in part C.
Write a rule that describes the relationship.



E. Repeat parts C and D for the number of centimetre blocks that are painted on 2 faces, on 1 face, and on 0 faces.

F. Which rules from parts C to E represent linear relationships?

G. Predict the number of small blocks painted on 3, 2, 1, and 0 faces for a $10 \times 10 \times 10$ cube. Explain how you made your prediction.

4.1 Interpreting the Solution of a Linear Equation

YOU WILL NEED

- grid paper

GOAL

Relate equations to tables of values and graphs.

LEARN ABOUT the Math

Mariane wants U-Host to host her website.

U-Host's Charges per Month
\$19.00 monthly charge
\$1.15/megabyte (MB) of storage used

? How many megabytes of storage can Mariane purchase for \$60 per month?

EXAMPLE 1 Solving a problem using different representations of the situation

Determine the amount of storage Mariane can afford each month.

Jordan's Solution: Using a table of values to estimate a solution

MB	Total Cost (\$)
0	$19 + 1.15 \times 0 = 19.00$
10	$19 + 1.15 \times 10 = 30.50$
20	$19 + 1.15 \times 20 = 42.00$
30	$19 + 1.15 \times 30 = 53.50$
40	$19 + 1.15 \times 40 = 65.00$

I created a table of values to estimate the amount of storage that Mariane can purchase for \$60. I used multiples of 10 for the numbers of megabytes to make the calculations easier.

I estimate that Mariane can afford about 35 MB of storage each month.

Since \$60 is between \$53.50 and \$65.00, the answer must be between 30 MB and 40 MB of storage.

Check:

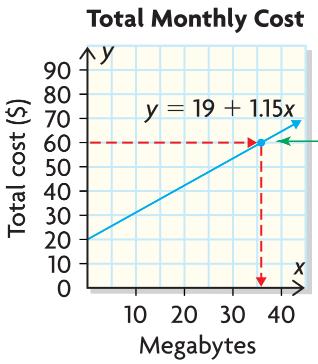
$19 + 1.15 \times 35 = 59.25$

So, 35 MB will cost Mariane just under \$60 per month.

I substituted my solution into the cost expression to see if my answer was reasonable.

Xavier's Solution: Using a graph to estimate the solution

total cost = monthly charge + cost per MB × number of MB



I graphed the relation $y = 19 + 1.15x$, where x represents the number of megabytes of storage, y represents the cost in dollars, 19 is the monthly charge, and 1.15 is the cost per MB.

I knew that the x-coordinate of the point that has a y-coordinate of 60 would be the solution.

From the graph, I saw that $60 would pay for about 36 MB.

Based on the graph, Mariane could purchase about 36 MB of storage for $60 per month.

Check:

$y = 19 + 1.15x$

$y = 19 + 1.15 \times 36$

$y = 60.40$

I substituted my solution into the cost expression to see if my answer was reasonable.

Jordan and Xavier's strategies provided estimates. You can use an equation to calculate an exact solution.

Eva's Solution: Using an equation to get an exact solution

total cost = monthly charge + cost per MB × number of MB

I used x to represent the amount of storage (in MB) Mariane could purchase. I wrote a word equation and a **linear equation**.

$$60 = 19 + 1.15x$$

I used a balancing strategy to solve the equation.

$$60 - 19 = 19 + 1.15x - 19$$
$$41 = 1.15x$$
$$\frac{41}{1.15} = \frac{1.15x}{1.15}$$
$$35.65 \doteq x$$

Mariane can purchase about 35.65 MB of storage for $60.

linear equation

an equation in the form $ax + b = 0$, or an equation that can be rewritten in this form; the algebraic expression involved is a polynomial of degree 1 (e.g., $2x + 3 = 6$ or $y = 3x - 5$)

Check:

$19 + 1.15x$
$= 19 + 1.15(35.65)$
$= 59.9975$

After rounding to the nearest cent, 35.65 MB will cost Mariane $60 per month.

I checked to see if 35.65 actually solved the equation.

Reflecting

A. Why does the algebraic solution from an equation provide the most accurate answer for Mariane?

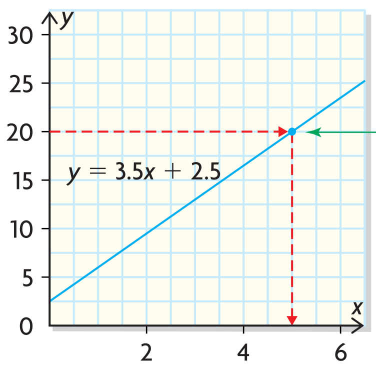
B. Why can the table of values, the graph, and the equation all be used to find possible solutions to Mariane's problem?

APPLY the Math

EXAMPLE 2 Using a graphical representation of a relation to solve an equation

Solve the equation $3.5x + 2.5 = 20$.

Kevin's Solution



To solve the equation, I graphed the linear relation $y = 3.5x + 2.5$.

Since $y = 20$ in the equation I drew a horizontal line from 20 on the y-axis to find the point that lies on the linear relation.

When $y = 20$, I estimate that $x = 5$.

To estimate the x-value of this point I drew a vertical line from this point to the x-axis.

Check:

$3.5x + 2.5$
$= 3.5(5) + 2.5$
$= 20$

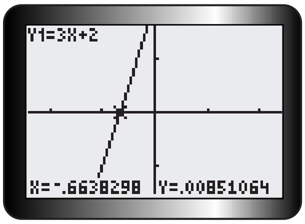
I substituted my solution into the equation to see if it solved the equation.

So, $x = 5$ solves the equation.

EXAMPLE 3 Determining the x-intercept for a linear relation

Determine the x-intercept of the graph of the relation $y = 3x + 2$.

Shayla's Solution: Using graphing technology



I graphed the linear relation on a graphing calculator. Since the x-intercept occurs when $y = 0$, I traced along the line until I found the point with a y-coordinate close to 0.

$x \doteq -0.66$

The x-intercept is approximately -0.66.

The line passes through $(-0.66, 0)$ on the x-axis.

Shayla's strategy provided an estimate of the location of the x-intercept. You can use algebra and the equation of the linear relation to determine its exact location.

Mike's Solution: Using algebra

$$y = 3x + 2$$

I substituted $y = 0$ because the y-coordinate of the x-intercept is 0.

$$0 = 3x + 2$$
$$0 - 2 = 3x + 2 - 2$$
$$-2 = 3x$$
$$\frac{-2}{3} = \frac{3x}{3}$$
$$-\frac{2}{3} = x$$

I used a balancing strategy to solve the equation.

So, the x-intercept is $-\frac{2}{3}$.

The line passes through $\left(-\frac{2}{3}, 0\right)$ on the x-axis.

Tech Support

For help using the TRACE function on your calculator, see Appendix B-4.

EXAMPLE 4 Solving a problem represented by a linear equation

A cell-phone company is offering this plan:

- \$9.95 per month
- 50 minutes free
- \$0.09 per minute after the first 50 minutes

Each month, after 50 minutes of air time, the company uses the exact air time to calculate the monthly bill.

About how many minutes can be purchased each month for \$40?

Gail's Solution

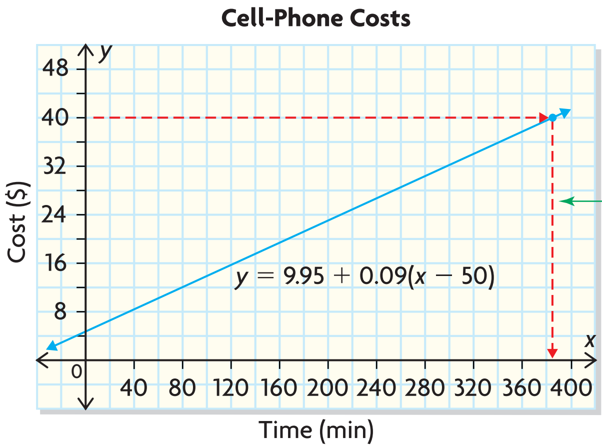
Let x represent the number of minutes used each month and y the total cost for the month.

I used a linear equation to represent the problem.

This equation shows how the monthly charge is calculated.

$y = 9.95 + 0.09(x - 50)$

Since the first 50 minutes are free, I subtracted 50 from the number of minutes before I multiplied by the per-minute cost. I added the fixed cost of \$9.95 to get the total cost.



To estimate the solution, I graphed the relation.

I located $y = 40$ on the graph of the relation, and then, drew horizontal and vertical lines to estimate the x-value for $y = 40$. The solution is about $x = 385$.

So, about 385 minutes can be purchased each month for \$40.

In Summary

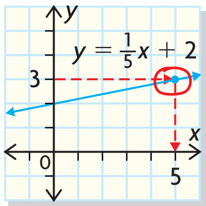
Key Ideas

- You can use a table or a graph to estimate the solution to an equation.

  For example, solve $3 = \frac{1}{5}x + 2$.

$y = \frac{1}{5}x + 2$

x	y
0	2
5	3
10	4



- The exact solution to an equation can be determined using a balancing strategy and algebra.

  For example,

$$3 = \frac{1}{5}x + 2$$

$$3 - 2 = \frac{1}{5}x + 2 - 2$$

$$1 = \frac{1}{5}x$$

$$1 \times 5 = \frac{1}{5}x \times 5$$

$$5 = x$$

Need to Know

- The solution to a linear equation is the x- or y-coordinate of a point on the graph of its corresponding linear relation. If you know one of the coordinates, you can read or estimate the other coordinate from the graph.
- To check a solution, substitute the value of the solution into the equation and calculate both sides. If both sides work out to the same number, the solution is correct.

CHECK Your Understanding

1. Estimate the solution to $4.25x - 3 = 9.5$ using a table of values. Verify your solution.

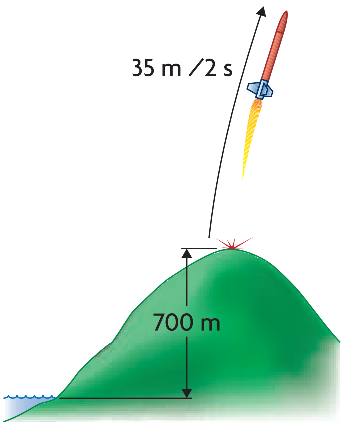
2. Estimate the solution to $-2 = 5 - \frac{1}{4}x$ using a graph. Verify your solution.

3. Solve the equations in questions 1 and 2 using algebra.

4. Justin is purchasing chain link fence for his yard. It costs \$5.25 per linear foot of fencing. How many feet of fencing can he buy if his budget is \$600?
   a) Estimate the amount of fencing using a table of values or a graph.
   b) Determine the exact amount of fencing using algebra.
   c) Verify your solution.

PRACTISING

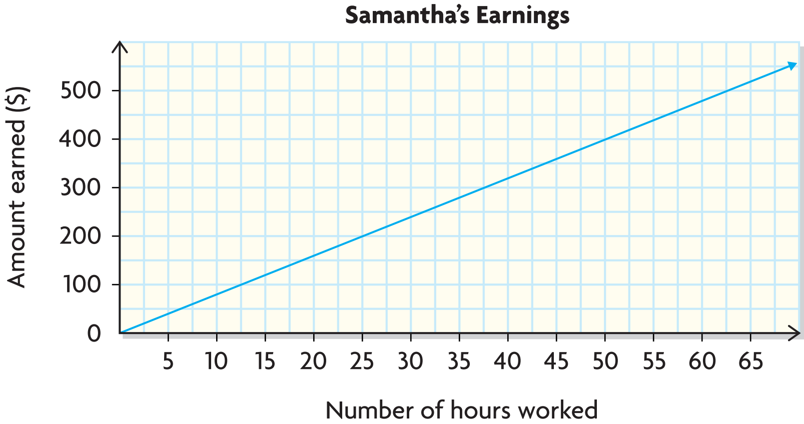
5. Estimate the solutions to the following equations using a graph.
   a) $-3x - 11 = 7$ b) $2x + 9 = 4$ c) $35 - 2t = 13$
6. Solve the equations in question 5 using algebra.
7. Estimate the solutions to the following equations using a table of values of the corresponding linear relation.
   a) $2x - 8 = -9$ b) $7 - 3x = 16$ c) $2.75x + 3.8 = 3.8$
8. Solve the equations in question 7 using algebra.



9. A rocket is launched from a hill that is 700 m high. The rocket's altitude increases by 35 m every 2 s.
   a) Create the linear relation that models the rocket's upward path.
   b) Graph the linear relation.
   c) Write the equations you would solve to determine the height of the rocket at 50 s and 100 s. Estimate the solution to these equations using the graph.
   d) Write the equation you would use to determine when the rocket reaches a height of 1000 m. Use the graph to estimate the solution to this equation.
10. Party Planners is catering a party. Its services cost \$25 per person with a minimum of 50 guests, but it does not charge for the first 10 people.
    a) Create the linear relation that models the catering costs in terms of the number of people attending the party.
    b) Graph the linear relation.
    c) Write the equations you would solve to determine the costs for 50 people and 75 people. Solve these equations using the graph.
    d) Write the equations you would use to determine how many people could attend for a total cost of \$1500. Estimate the solution using the graph.
11. K Estimate the solution to each equation using either a graph or a table of values. Verify your solutions.
    a) $2x + 3 = 11$ b) $-17 = 3 - 4n$ c) $-2 + \frac{1}{2}n = -5$
12. Solve the equations in question 11 using algebra. Verify your solution.

13. This graph shows Samantha's earnings against hours worked.

A **a)** Write the linear relation that models this graph.



b) Write the equation you would solve to determine the number of hours worked if Samantha's earnings were \$500.

c) Write the equation you would solve to determine her earnings for 40 hours worked.

d) How are the equations in parts b) and c) similar? How are they different?

e) Estimate the solution to each of the equations using the graph. Verify your solutions.

f) Solve the equations using algebra.

14. Use algebra to determine the x-intercept for each of the following:

a) $y = 4x + 8$ **b)** $y = \frac{1}{2}x - 5$ **c)** $y = x$ **d)** $y = -1$

15. A large water tank holds 100 L of water. It is leaking at a rate of 5 L/min.

a) Write the linear relation that models the amount of water remaining in the tank in terms of the number of minutes since the tank started leaking.

b) Write the equation you would solve to determine the amount of water remaining in the tank 13 minutes after the water started leaking.

c) Write the equation you would solve to determine when the tank would be half full.

d) How are the equations in parts b) and c) similar? How are they different?

e) Solve each equation using a graph.

f) Solve each equation using algebra. Verify your solution using the equation.

16. A rectangular field is 100 m long. It is fully enclosed by 500 m of fencing.

a) Explain why the equation $500 = 2(100) - 2w$ can be used to determine w, the width of the field.

b) Estimate the solution to the equation. Verify your solution.

c) Solve the equation using algebra.

d) How wide is the field?

17. T **a)** Write an equation for the relationship between the figure number and number of counters in each figure.

b) Use the equation to determine the figure that can be made with exactly 60 counters.

c) Explain why there is no figure in this pattern that can be made with exactly 100 counters.

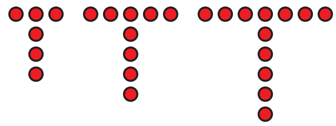


figure 1 figure 2 figure 3

18. C Create a linear equation of the form $ax + b = c$. Draw a flow chart that shows the steps needed to estimate the solution using a graph.

Extending

19. Use a table of values or a graph to estimate solutions to the following equations. Verify your solution.

a) $\dfrac{(x + 2)}{5} - 3x = 7$

b) $\dfrac{1}{2}(x + 2) - \dfrac{1}{3}(x - 1) = 4$

c) $x^2 + 7 = 16$

d) $2x^2 - 3 = 11$

e) $3x^3 - 9 = 72$

20. Determine an exact solution to each equation in question 19 using any method you can.

21. Given the linear relation $y = mx + b$, what equation would you solve to determine the x-intercept? Justify your answer.

4.2 Solving Linear Equations Using Inverse Operations

GOAL

Solve equations by working backward.

LEARN ABOUT the Math

Michelle delivers paper waste to a recycling centre. She had net earnings of \$23.90 on her first trip.

Michelle's Costs	Michelle's Earnings
\$8.00/trip for gas	\$72.50/tonne of paper

Communication *Tip*

Using inverse operations is the same as balancing.

? How much paper did Michelle deliver?

EXAMPLE 1 Using inverse operations as a strategy to solve an equation

Determine the amount of paper waste Michelle delivered on her first trip to the recycling centre.

Michelle's Solution

\$72.50 × (# of tonnes) − \$8.00 = \$23.90

I created a word equation to represent the situation.

Let p represent the number of tonnes of paper I delivered.

$72.50p - 8.00 = 23.90$

I wrote a linear equation that used a variable instead of words to represent the situation.

Try $p = 1$.

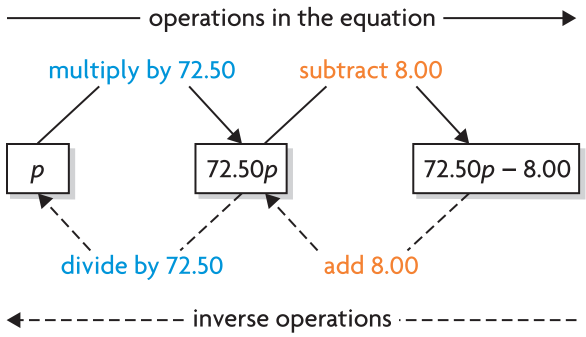
I used guess-and-check to get an idea of the amount of paper.

$72.50(1) - 8.00$

$= 72.50 - 8.00$

$= 64.50$

64.50 is much greater than 23.90, so my guess was too high. There must have been less than 1 tonne of paper.



I used a diagram to show the operations in the equation and the **inverse operations** I needed to undo them.

inverse operations
operations that undo, or reverse, each other, for example: addition is the inverse of subtraction; multiplication is the inverse of division

$$72.50p - 8.00 + 8.00 = 23.90 + 8.00$$
$$72.50p = 31.90$$

I undid the last step by adding 8.00. This **isolated the term** 72.50*p*. Since I got the new equation by doing the same operation to both sides of the original, I knew they were **equivalent equations**.

isolating a term or a variable
performing math operations (e.g., addition, subtraction, multiplication, division) to get a term or a variable by itself on one side of an equation

equivalent equations
equations that have the same solution

$$72.50p \div 72.50 = 31.90 \div 72.50$$
$$p = 0.44$$

I isolated *p* by dividing both sides of the equation by 72.50 and solved the equation.

Check:

Left Side	Right Side
$72.50(0.44) - 8.00$	23.90
$= 23.90$	

When I checked my answer, it gave me the correct amount.

I delivered 0.44 tonnes of paper.

Reflecting

A. How did Michelle's estimate of 1 tonne help her determine the inverse operations needed to solve the equation?

B. Why is "isolating the term or variable" a good name for the process used to solve the equation?

C. How would doing the inverse operations in a different order affect Michelle's solution?

APPLY the Math

EXAMPLE 2 Using an inverse operation strategy to solve an equation

Solve $-2x + 4 = 14$.

Chelsea's Solution

The calculation steps are
- multiply by -2
- add 4

I listed the operations on x in the equation following order of operations.

The reverse steps to solve are
- subtract 4
- divide by -2

Then, I listed the inverse operations that would undo each operation in the equation.

$$-2x + 4 - 4 = 14 - 4$$
$$-2x = 10$$

I isolated the term $-2x$ by subtracting 4 from each side to undo $+ 4$.

$$-2x \div (-2) = 10 \div (-2)$$
$$x = -5$$

Then, I isolated the variable x by undoing multiplication by -2.

Check:

Left Side	Right Side
$-2(-5) + 4$	14
$= 10 + 4$	
$= 14$	

I substituted my solution into the equation to make sure that it worked.

$x = -5$ is the correct solution.

EXAMPLE 3 Using an inverse operations strategy to solve an equation with fractional coefficients

Solve $\frac{w}{3} - 13 = 7$.

Drake's Solution

$$\frac{w}{3} - 13 = 7$$

$$\frac{w}{3} - 13 + 13 = 7 + 13$$

I performed the inverse operations. The inverse operation of subtracting 13 is adding 13.

$$\frac{w}{3} = 20$$

$$3 \times \frac{w}{3} = 3 \times 20$$

$$w = 60$$

$\frac{w}{3} = w \div 3$

The inverse operation of dividing by 3 is multiplying by 3.

$w = 60$ is the correct solution.

I used mental math to check that $\frac{60}{3} - 13$, or $20 - 3$, equals 7.

EXAMPLE 4 Solving a problem represented by linear equation

A photographer charges a sitting fee of \$100. The first four prints are free. Each additional print costs \$5.25. How many prints can you buy with \$257.50?

Asad's Solution

$$\$100 + \$5.25 \times (\text{\# of prints} - 4 \text{ free prints}) = \$257.50$$

$$100 + 5.25(P - 4) = 257.50$$

I wrote a word equation, and then, an algebraic equation to describe the situation. I used P to represent the number of prints ordered.

$$100 - 100 + 5.25(P - 4) = 257.50 - 100$$

$$5.25(P - 4) = 157.50$$

I used inverse operations to isolate the term with the variable in it.

$$5.25(P - 4) \div 5.25 = 157.50 \div 5.25$$

$$1(P - 4) = 30$$

Then, I used other inverse operations to isolate the term in brackets.

$$P - 4 + 4 = 30 + 4$$

$$P = 34$$

I used inverse operations and added 4.

Check:

$100 + 5.25(34 - 4)$

$= 100 + 157.50$

$= 257.50$

I checked to see if 34 prints would actually cost \$257.50.

You can order 34 prints.

In Summary

Key Ideas

- You can write an equation that is equivalent to a given equation by applying the same operation to both sides.
- You can use inverse operations to isolate individual terms or variables.

Need To Know

- You can substitute a value for the variable to get a sense of the inverse operations to use to solve an equation.
  For example, for the equation $3x - 17 = 34$, you might try $x = 10$. Since $3 \times 10 - 17 = 13$, is too low, you know the solution is greater than 10. You also know that the operations you have to undo are subtracting 17 and multiplying by 3.
- An equation in which the variable appears on only one side can be solved by
  - listing the operations that can be used to evaluate the expression in the order in which they would be used
  - performing the inverses of the operations one at a time, in their opposite order, until the variable is isolated

  For example:

$$\begin{aligned} 3x - 4 &= 2 \\ 3x - 4 + 4 &= 2 + 4 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

- You can check your solution by substituting it into the original equation. The solution is correct if both sides evaluate to the same number.
- There is sometimes more than one way to solve an equation using inverse operations. As long as you perform the same operations on both sides of the equation, the solution will be correct.

Communication *Tip*

It's common practice to line up equal signs when solving equations. This makes it easier to check the results of each step. Only one equal sign appears in each line of a solution and it separates the left side from the right side.

CHECK Your Understanding

1. List the inverse operations and the order in which you would apply them to isolate the variable in each equation.

a) $-3x + 2 = 15$

b) $12.4x - 3.2 = 21.5$

c) $\frac{x}{2} + 5 = 11$

2. Solve each equation in question 1. Show all steps.

3. An author is paid \$5000. In addition, he receives a royalty of \$1.25 for every book sold.

a) Write an equation to represent the number of books that have to be sold for the author to earn \$10 000.

b) Solve the equation using inverse operations. Show all steps.

c) Verify your solution.

PRACTISING

4. List the operations you would use to isolate the variable in each equation.

K **a)** $6b - 10 = -2$

b) $2.5c + 1.0 = 1.5$

c) $3f - 4 = 10$

d) $6 - 2d = 4$

e) $6 - 2e = 6$

f) $-3 - h = -2$

5. Solve each equation in question 4. Show all steps and verify each solution.

6. The relation $C = 8.00 + 0.50T$ represents the cost of a pizza in dollars. T represents the number of toppings ordered.

a) Write an equation that represents a \$10 order.

b) Solve the equation in a) to determine the number of toppings. Show all steps.

7. A submarine is currently submerged at a depth of 600 m. It rises at a rate of 4 m/s.

a) Write a linear relation that shows the relationship between the depth of the submarine and the number of seconds it has been rising.

b) Write the equation you must solve to determine when the submarine will reach a depth of 486 m.

c) List the inverse operations you need to use to isolate the variable and solve the equation.

d) Solve the equation. Show all the steps.

e) Verify your solution.

8. Caroline told Marc that using balancing and solving linear equations
C was different from using inverse operations to isolate the variable. Was Caroline right? Explain.

9. A hot-air balloon is at a height of 500 m. It develops a steady leak and begins to descend at a rate of 60 m/min. Write and solve an equation to determine how long it takes for the balloon to reach a height of 20 m.

10. Solve each equation.

a) $\frac{x}{4} + 1 = 3$

b) $\frac{x}{2} - 10 = 3$

c) $5 - \frac{y}{3} = 3$

d) $10 + \frac{b}{5} = -1$

e) $\frac{w}{3} + 5 = 1$

f) $3 - \frac{d}{6} = -1$

11. Liz was testing Jane on solving with equations. She gave Jane the following problem:

"I am a number such that when you divide me by 7, and then, add 13 you get 32. What number am I?"

Write and solve an equation to determine Liz's number.

12. Solve each equation.

a) $3(x + 1) = 12$

b) $2(x - 4) = 4$

c) $\frac{(w + 3)}{4} = 2$

d) $\frac{(y - 5)}{3} = 6$

e) $\frac{(2a + 3)}{3} = 5$

f) $-2 = \frac{-2c}{5} + 1$

13. T Jack's Restaurant charges $22.95 for brunch but allows one person per table to eat free. To figure out how many people attended the Sunday brunch, Jack collected the information in this table.

Table Number	Bill Total ($)
1	137.70
2	68.85
3	160.65
4	91.80
5	91.80

a) Why is it reasonable that Jack used the equation $22.95(x - 1) = T$ to determine the number of people at each table? What do the variables x and T represent?

b) Create and solve the equation for each table number.

c) How many people in total sat at the five tables?

14. The relationship between Celsius and Fahrenheit is represented by

A $C = \frac{5}{9}(F - 32)$.

- **a)** Determine the Celsius temperature that is equivalent to 58 °F.
- **b)** List the operations you used to calculate the Celsius temperature.
- **c)** List the inverse operations you would use to isolate *F*.
- **d)** Determine the Fahrenheit temperature that is equivalent to 25 °C.

15. When you use an inverse operation to isolate a variable, why can you say that the equation you get at each step is equivalent to the original one?

Extending

16. Solve each equation.

a) $2x + 3 = 4(1 - x) + 5$ **b)** $\frac{1}{2}x - \frac{1}{4} = \frac{x}{3} + \frac{1}{6}$

17. The intercepts of a graph are the points at which the line crosses the x-axis and the y-axis. Consider the relation $3x + 5y = 15$.

- **a)** List the inverse operations you would use to express the relation in the form $y = \text{m}x + \text{b}$.
- **b)** Use your answer from part a) to solve the relation for y.
- **c)** List the inverse operations you would use to express and isolate the x-variable.
- **d)** Use your answer from part c) to solve the relation for x.
- **e)** Explain how you can use your answers to parts b) and d) to quickly graph this relation.

18. The members of a scout troop held a car wash for charity. They washed 49 vehicles. They charged \$4 per car and \$6 per truck and earned a total of \$230. How many of each type of vehicle did they wash?

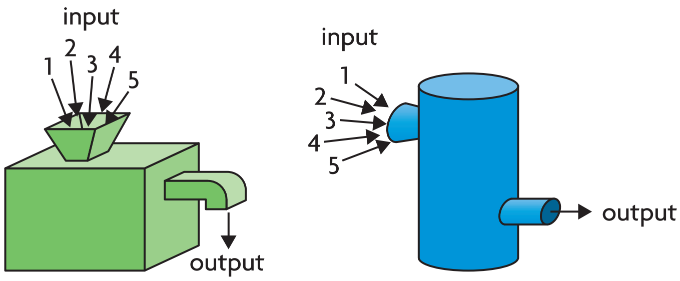
- **a)** Write an algebraic relation that models the number of vehicles washed.
- **b)** Write an algebraic relation that models the total money earned.
- **c)** Graph the relations and use the graphs to solve the problem.

4.3 Equation-Solving Strategies

GOAL

Solve equations where the variable appears on both sides of the equality.

LEARN ABOUT the Math



machine 1: $\frac{1}{2}x + \frac{3}{4}$ machine 2: $x - \frac{1}{2}$

Atish and Sara are playing a game on their computers. The goal of the game is to be the first to figure out which input will make the two machines generate the same output.

They have input several values. Here are their results:

	Machine 1	**Machine 2**
Input	Output	Output
0	0.75	−0.5
2	1.75	1.5
4	2.75	3.5
6	3.75	5.5

? What strategies can they use to win the game quickly?

EXAMPLE 1 Selecting a strategy to solve an equation

Determine the input value that will result in the same output value for both machines.

Klint's Solution: Using a guess-and-test strategy

Input Value	Output Value	
x	Machine 1 $\frac{1}{2}x + \frac{3}{4}$	Machine 2 $x - \frac{1}{2}$
0	0.75	−0.5
2	1.75	1.5
4	2.75	3.5

I noticed that when the input is 2, the output value of Machine 1 is greater than the output value of Machine 2. But, when the input is 4, the reverse is true.

$x = 3$

I thought the answer must be between 2 and 4, so I tried 3.

Input Value	Output Value	
x	Machine 1 $\frac{1}{2}x + \frac{3}{4}$	Machine 2 $x - \frac{1}{2}$
0	0.75	−0.5
2	1.75	1.5
3	2.25	2.5

The output value of Machine 1 is still greater when $x = 2$ and less when $x = 3$. The solution must be between 2 and 3.

$x = 2\frac{1}{2}$

I chose an input value of $2\frac{1}{2}$, which is between 2 and 3. I tested this value in each machine.

$x = \frac{5}{2}$

I rewrote the fraction as an improper fraction, and then, calculated the outputs.

Input Value	Output Value	
x	Machine 1 $\frac{1}{2}x + \frac{3}{4}$	Machine 2 $x - \frac{1}{2}$
0	0.75	−0.5
2	1.75	1.5
2.5	2.00	2.0

When the input is $2\frac{1}{2}$, or 2.5, both machines have an output value of 2.

An input of $2\frac{1}{2}$ will produce equal outputs on both machines.

When an equation contains fractions, it is often easier to solve if you can rewrite it as an equivalent equation that does not contain fractions. This can be done using a common denominator. You can then use inverse operations to solve the resulting equation.

Dion's Solution: Using a common denominator and inverse operations to isolate the variable

$$\frac{1}{2}x + \frac{3}{4} = x - \frac{1}{2}$$

I created an equation to figure out what value of *x* would give the same output values

$$4\left(\frac{1}{2}x + \frac{3}{4}\right) = 4\left(x - \frac{1}{2}\right)$$

I tried to write an equivalent equation that didn't contain fractions. I knew that if I performed the same operation on both sides of the equation, the result is an equivalent equation. I decided to multiply both sides by 4, since both denominators will divide into this number, eliminating the fractions. This is also the lowest common denominator between 2 and 4.

$$4\left(\frac{1}{2}x\right) + 4\left(\frac{3}{4}\right) = 4(x) - 4\left(\frac{1}{2}\right)$$

$$\frac{4}{2}x + \frac{12}{4}x = 4x - \frac{4}{2}$$

$$2x + 3 = 4x - 2$$

I used the distributive property to expand, and then, simplified by multiplying.

$$2x + 3 - 3 = 4x - 2 - 3$$

$$2x = 4x - 5$$

I used inverse operations to solve the equation. I decided to undo + 3 by subtracting 3 from both sides.

$$2x + 5 = 4x - 5 + 5$$

$$2x + 5 = 4x$$

To undo − 5, I added 5 to both sides.

$$2x + 5 - 2x = 4x - 2x$$

$$5 = 2x$$

To isolate the variable term I had to undo + 2*x*. I subtracted 2x from both sides

$$\frac{5}{2} = \frac{2x}{2}$$

$$\frac{5}{2} = x$$

To solve for *x*, I used the inverse operation of multiply by 2 and divided both sides by 2.

$$2\frac{1}{2} = x$$

I rewrote the fraction as a mixed number.

The input that will produce the same output on both machines is $2\frac{1}{2}$.

Check:

When $x = \frac{5}{2}$,

Left Side	Right Side
$\frac{1}{2}x + \frac{3}{4}$	$x - \frac{1}{2}$
$= \frac{1}{2}\left(\frac{5}{2}\right) + \frac{3}{4}$	$= \frac{5}{2} - \frac{1}{2}$
$= \frac{5}{4} + \frac{3}{4}$	$= \frac{4}{2}$
$= \frac{8}{4}$	$= 2$
$= 2$	

I checked the solution by entering the input value into the left hand and right hand sides separately.

Since the left side and right side both resulted in the same output, I knew my solution was correct.

Reflecting

A. How does solving the equation $\frac{1}{2}x + \frac{3}{4} = x - \frac{1}{2}$ show that there is only one value of x for which the machines have the same output?

B. Why does Klint's strategy only provide an estimate of the solution, while Dion's provides an exact solution?

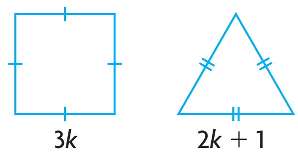
C. How did Dion decide what number to multiply both sides of the equation by to eliminate the fractions?

D. How did Dion know which inverse operations to use in order to group the variables on one side of the equation and the constant terms on the other side?

APPLY the Math

EXAMPLE 2 Using an inverse operations strategy to solve a problem represented by an equation

The square and equilateral triangle shown have the same perimeters. What are the dimensions of each figure?



Kayla's Solution

$$4(3k) = 3(2k + 1)$$

I used the expression $4 \times 3k$ to calculate the perimeter of the square and $3 \times (2k + 1)$to calculate the perimeter of the triangle.
I made these two expressions equal because the perimeters are the same.

$$12k = 6k + 3$$

I used the distributive property on the right side of the equation to simplify it.

$$12k - 6k = 6k + 3 - 6k$$
$$6k = 3$$
$$\frac{6k}{6} = \frac{3}{6}$$
$$k = \frac{1}{2}$$

I used inverse operations to group the variables on the left side of the equation. I chose the left because $12k$ is larger than $6k$ and my variable would have a positive coefficient.

When $k = \frac{1}{2}$,
Perimeter of the square:
$4(3k)$
$$= 4\left(3 \times \frac{1}{2}\right)$$
$$= 6$$

I checked my solution by finding the perimeter of the square and the triangle when $k = \frac{1}{2}$.

Since the perimeters of both shapes are 6 units when $k = \frac{1}{2}$, I knew my solution was correct.

Perimeter of the triangle:
$3(2k + 1)$
$$= 3\left(2 \times \frac{1}{2} + 1\right)$$
$$= 6$$

When $k = \frac{1}{2}$,

Dimensions of the square:

$3k$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

To find the dimensions, I substituted $k = \frac{1}{2}$ into the expression for the side lengths of the triangle and of the square.

Dimensions of the triangle:

$2k + 1$

$= 2\left(\frac{1}{2}\right) + 1$

$= 1 + 1$

$= 2$

The perimeters are the same when the square has a side length of $1\frac{1}{2}$ and the triangle has a side length of 2.

EXAMPLE 3 Using an equation and an inverse operations strategy to solve a problem

It takes Ryan 2 h to mow the lawn and water the garden. It takes Maria 3 h to do the same. How long would it take them if they worked together?

Abby's Solution

Let x represent the amount of time needed to complete the tasks if Ryan and Maria work together.

I used a variable to represent the time needed to complete the job if they worked together.

Fraction of work done by Ryan: $\frac{x}{2}$

Fraction of work done by Maria: $\frac{x}{3}$

I wrote an expression to represent the fraction of work done by each person:

fraction of work done $= \frac{\text{time actually spent}}{\text{time to do the whole job alone}}$

$$\frac{x}{2} + \frac{x}{3} = 1$$

Since both people are working together, the total of the fractions of work done must be 1. I used this to write an equation.

$$6\left(\frac{x}{2}\right) + 6\left(\frac{x}{3}\right) = 6(1)$$

I wanted to create an equivalent equation without fractions. I knew that if I multiplied both sides of the equation by 2, the $\frac{x}{2}$ would become $1x$, or just x. But I would still have the thirds on this side of the equation. So, I decided to multiply both sides by 2, and then, by 3. That's the same as multiplying by 6. I had to remember to multiply every term and not just the fraction terms or the new equation would not be equivalent.

$$\frac{6x}{2} + \frac{6x}{3} = 6$$

$$3x + 2x = 6$$

$$5x = 6$$

I multiplied, and then, simplified.

$$\frac{5x}{5} = \frac{6}{5}$$

I divided both sides by 5 to isolate the variable.

$$x = 1.2$$

It would take Ryan and Maria 1.2 h to mow the lawn and water the garden if they worked together.

Check:

Left Side

Right Side

1

I used my calculator to check my solution. Since the left side equals the right side when $x = 1.2$, I know my solution is correct.

In Summary

Key Ideas

- When you solve an equation in which the variable appears on both sides of the equal sign ($ax + b = cx + d$), you can use inverse operations to group the variable terms on one side of the equation.
- If an equation has fraction coefficients and constants, you can use a common denominator to write an equivalent equation with integer coefficients. To do this, multiply each term of the equation by the common denominator. Using the lowest common denominator keeps the numbers in the new equivalent equation as small as possible.

Need To Know

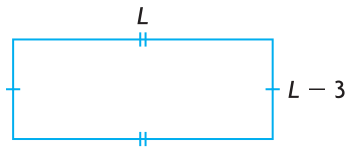
- Sometimes you have to use the distributive property to expand expressions involving brackets before you can collect like terms on each side of the equation before solving for the variable.
- You can check your solution to an equation of the form $ax + b = cx + d$ by substituting the value in each side of the equation and calculating the result. If you get the same value on both sides, then your result is correct.

CHECK Your Understanding

1. Use inverse operations to solve $2x + 4 = 4x - 2$.

2. To determine the dimensions of the rectangle with
   - perimeter 44 cm and
   - width 3 cm less than the length,

   Florence drew a diagram:



   a) Why is it reasonable for Florence to label the width "$L - 3$"?
   b) Create and solve the equation to determine the length of one side.
   c) What are the dimensions of the rectangle?

PRACTISING

3. Given each solved equation below, explain the mathematical reasoning for each step.

   a)
$$
\begin{aligned}
2x + 8 &= 4x - 18 \\
2x + 8 - 2x &= 4x - 18 - 2x && \text{Step A} \\
8 &= 2x - 18 \\
8 + 18 &= 2x - 18 + 18 && \text{Step B} \\
26 &= 2x \\
\frac{26}{2} &= \frac{2x}{2} && \text{Step C} \\
13 &= x
\end{aligned}
$$

   b)
$$
\begin{aligned}
\frac{1}{2}x + \frac{2}{3} &= 5 \\
6 \times \left(\frac{1}{2}x + \frac{2}{3}\right) &= 5 \times 6 && \text{Step A} \\
3x + 4 &= 30 && \text{Step B} \\
3x + 4 - 4 &= 30 - 4 && \text{Step C} \\
3x &= 26 \\
\frac{3x}{3} &= \frac{26}{3} && \text{Step D} \\
x &= 8\frac{2}{3} && \text{Step E}
\end{aligned}
$$

4. Explain why the equations in each group are equivalent equations.

a) $5x + 8 = 2(2x - 3)$, $5x + 8 = 4x - 6$, and $5x - 4x = -6 - 8$

b) $\frac{x}{4} + 5 = \frac{1}{3}$, $\frac{3x}{12} + \frac{60}{12} = \frac{4}{12}$, and $3x + 60 = 4$

c) $5x - 8 = 12$, $\frac{5x}{6} - \frac{4}{3} = 2$, and $\frac{5x}{6} - \frac{8}{6} = \frac{12}{6}$

5. Solve each equation. Verify each solution.

K **a)** $5x + 24 = 2x$ **c)** $-4x - 1 = -3x + 5$ **e)** $3b - 4 - 5b = -3b - 2$

b) $2k = 4k - 15$ **d)** $2x - 3x + 6 = 7 - x + 2$ **f)** $a + 2a + 3a - 6 = 7a - 6$

6. Solve $n + (n + 1) + (n + 2) = 54$.

7. Solve each equation. Verify each solution.

a) $3(x - 5) = 6$ **c)** $-3(5 - 6m) = 39$ **e)** $3(c + 5) = 4(1 - 2c)$

b) $-5 = 5(3 + 2d)$ **d)** $2(x - 2) = 3x - 14$ **f)** $4(x - 2) = -3(2x + 6)$

8. A number, n, decreased by 5, is equal to 3 times the number plus 1. Determine the number.

9. The perimeter of a rectangle is 36 cm. The width is 5 cm less than the length. Determine the dimensions of the rectangle.

10. George is three times as old as Sam. Five years from now, the sum of their ages will be 46.

a) Create an equation that represents the relationship between George's and Sam's ages five years from now.

b) Use your equation to determine their current ages.

11. Fill in the missing column for each equation.

	Equation	Common Denominator of All Terms	Equation with Denominators Eliminated
a)	$\frac{3x}{4} + \frac{2}{3} = 2$		
b)	$\frac{1}{2} - \frac{x}{3} = \frac{1}{3}$		
c)	$\frac{2}{3} = 5 + x$		
d)	$\frac{x - 5}{4} + 1 = \frac{1}{2}$		
e)	$-16 = \frac{x}{5} + \frac{x}{3}$		
f)	$\frac{-2}{5}(x - 8) = 4$		
g)	$\frac{y + 2}{3} = \frac{1}{5}(2y + 3)$		

12. Solve each equation. Verify each solution.

a) $\frac{x}{3} = 2$

b) $\frac{d}{4} + 3 = 2$

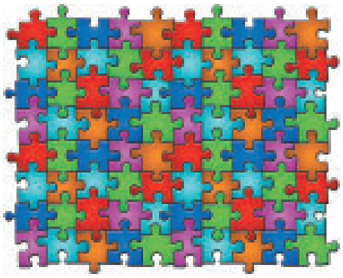
c) $\frac{x}{2} + \frac{x}{3} = 10$

d) $\frac{c}{3} - \frac{c}{4} = 3$

e) $\frac{3k}{5} - 6 = \frac{k}{3}$

f) $\frac{2x + 1}{3} = 5$

13. The sum of one-half of a number, q, and three-fifths is two-thirds the number q. Determine the number.



14. For each of the following, create and solve an equation.

A **a)** It takes Eli 4 hours to paint a room. It takes Mia 3 hours to paint a room. How long would it take them to paint the room together?

b) Amir can put together a puzzle in 30 minutes. Bob takes double the amount of time. How long will it take them to do it together?

c) A jet left Toronto for Vancouver, travelling at a speed of 600 km/h. At the same time, a jet left Vancouver for Toronto, travelling at a speed of 800 km/h. If the distance between Toronto and Vancouver is 3500 km, when will the jets pass each other?

15. A square has sides of length $2k - 1$ units. An equilateral triangle has sides of length $k + 2$ units. The square and the triangle have the same perimeter. What is the value of k?

16. Show that the equation $2x - 3 = 4 + 2x$ has no solution. Why do
T you think this happens?

17. Show that the equation $\frac{10 - 6x}{2} = 5 - 3x$ has an infinite number of solutions. Why do you think this happens?

18. Jennifer solved the equation $\frac{4x - 1}{4} + \frac{2x - 1}{5} = 2$ below. Explain
C the mathematical operation she used in each step.

$$\frac{4x - 1}{4} + \frac{2x - 1}{5} = 2$$

$$\begin{aligned}
20\left(\frac{4x - 1}{4}\right) + 20\left(\frac{2x - 1}{5}\right) &= 20(2) && \text{Step A} \\
5(4x - 1) + 4(2x - 1) &= 40 && \text{Step B} \\
20x - 5 + 8x - 4 &= 40 && \text{Step C} \\
28x - 9 &= 40 && \text{Step D} \\
28x - 9 + 9 &= 40 + 9 && \text{Step E} \\
28x &= 49 && \text{Step F} \\
\frac{28x}{28} &= \frac{49}{28} && \text{Step G} \\
x &= \frac{49}{28} && \\
x &= \frac{7}{4} \text{ or } 1\frac{3}{4} && \text{Step H}
\end{aligned}$$

19. Samir thinks that solving an equation with x on both sides is like solving an equation where x is only on one side. Do you agree or disagree with Samir? Use an example to justify your answer.

Extending

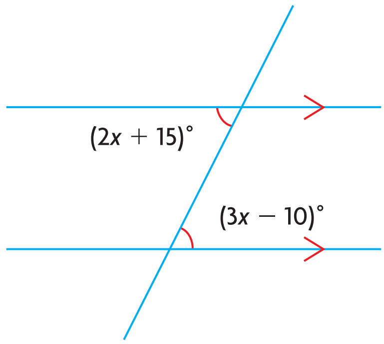
20. David has 16 dimes and quarters. Colin has twice as many dimes and $\frac{1}{3}$ as many quarters as David.

They both have the same amount of money. What coins does each boy have?

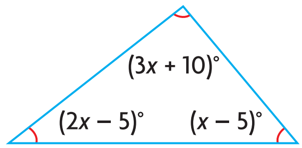
	Number of Quarters	Number of Dimes	Value of Quarters (¢)	Value of Dimes (¢)
David	q	$16 - q$	$25q$	
Colin	$\frac{q}{3}$			

21. Determine the value of x in each diagram.

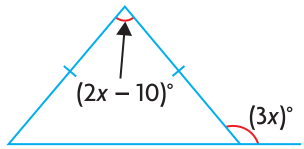
a)



b)



c)



22. Solve the equations:

a) $3x^2 - 2 = 25$

b) $2(x + 1)^2 - 1 = 71$

23. Chiaki is organizing a candy hunt for the children in her neighbourhood. She spent $102 to buy 500 large candies and 400 small candies. The ratio of the price of a large candy to the price of a small candy is 7:4. Find the prices of one large and one small candy.

4 Mid-Chapter Review

FREQUENTLY ASKED Questions

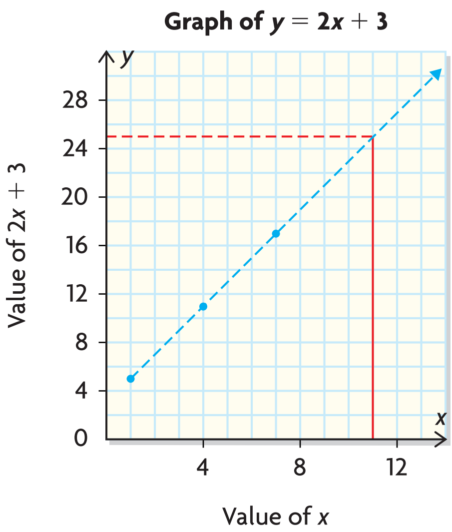
Study *Aid*
- See Lesson 4.1, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1 and 2.

Q: **What is the connection between the solution of a linear equation and the graph of a linear relation?**

A: The solution to an equation is the x- or y-coordinate of a point on its graph.

EXAMPLE

Consider the graph of the linear relation represented by $y = 2x + 3$.



Solving for y in $y = 2x + 3$ when $x = 11$ using the graph:
$y = 2(11) + 3$
- Locate 11 on the x-axis.
- Locate the point on the graph with that x-coordinate.
- Determine the y-coordinate of that point.

The solution is $y = 25$.

Solving for x in $y = 2x + 3$ when $y = 25$ using the graph:
$25 = 2x + 3$
- Locate 25 on the y-axis.
- Locate the point on the graph with that y-coordinate.
- Determine the x-coordinate of that point.

The solution is $x = 11$.

Q: How can you use inverse operations to solve a linear equation?

A: First, identify the operations acting on the variable. Remember to follow the order of operations. Then, isolate the variable by undoing each operation in the reverse order. Keep the equations equivalent by performing the same operation on both sides of the equation.

Study Aid

- See Lesson 4.2, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1 and 2.

EXAMPLE

Solve $-3x + 2 = 14$.

Solution

$-3x + 2 = 14$

Operations acting on x:
Multiply by -3.
Add 2.

$-3x + 2 - 2 = 14 - 2$

$-3x = 12$

Inverse operations to isolate x:
Subtract 2.

$\frac{-3x}{-3} = \frac{12}{-3}$

Divide by -3.

$x = -4$

Q: How can you verify a solution to a linear equation?

A: Substitute the solution into each side of the equation. If this number produces the same value on both sides of the equation, the solution is correct.

Study Aid

- See Lesson 4.2, Examples 1 and 2 and Lesson 4.3, Examples 1 and 3.
- Try Mid-Chapter Review Question 5.

EXAMPLE

To verify that $x = -4$ is the solution to $-3x + 2 = 14$:

Left Side	Right Side
$-3x + 2$ when $x = -4$	14
$= -3(-4) + 2$	
$= 12 + 2$	
$= 14$	

The left side equals the right side, so $x = -4$ is the correct solution.

Q: How can you solve an equation with variables on both sides?

A: Inverse operations can be used to collect variables and numbers on one side of an equation, regardless of how complicated the equation looks. As long as the inverse operations are performed on both sides of the equation in the reverse order, the equation remains equivalent.

Study Aid

- See Lesson 4.3, Examples 1 and 4.
- Try Mid-Chapter Review Question 6.

EXAMPLE

Solve $3(2x - 1) - 3(x - 2) = x + 5$.

Solution

$$\begin{aligned} 3(2x - 1) - 3(x - 2) &= x + 5 \\ 6x - 3 - 3x + 6 &= x + 5 \end{aligned}$$

Expand using the distributive property.

$$3x + 3 = x + 5$$

Collect like terms.

$$3x + 3 - 3 = x + 5 - 3$$

Subtract 3 from both sides.

$$\begin{aligned} 3x &= x + 2 \\ 3x - x &= x + 2 - x \end{aligned}$$

Subtract x from both sides.

$$\begin{aligned} 2x &= 2 \\ 2x \div 2 &= 2 \div 2 \\ x &= 1 \end{aligned}$$

Divide both sides by 2.

Study Aid

- See Lesson 4.3, Examples 1, 2, and 4.
- Try Mid-Chapter Review Questions 5 and 6.

Q: How can you solve an equation that has rational coefficients and constants?

A1: The most efficient way is to create an equivalent equation that contains only integers by eliminating the fractions. You can do this by multiplying both sides of the equation by the lowest common denominator (LCD) of the fractions. Then, use inverse operations to solve for the variable.

EXAMPLE

Solve $6 \times \left(-\frac{1}{6}x + \frac{1}{2}\right) = 7 \times 6$.

Solution

$$6 \times \left(-\frac{1}{6}x + \frac{1}{2}\right) = 7 \times 6$$

Multiply all terms on both sides by the LCD of 6.

$$\begin{aligned} -\frac{6}{6}x + \frac{6}{2} &= 42 \\ -x + 3 &= 42 \\ -x + 3 - 3 &= 42 - 3 \end{aligned}$$

Subtract 3 from both sides.

$$\begin{aligned} -x &= 39 \\ \frac{-1x}{-1} &= \frac{39}{-1} \end{aligned}$$

Divide both sides by -1.

$$x = -39$$

A2: You could also choose to keep the rational numbers, use inverse operations, and solve for the variable.

EXAMPLE

Solve $-\frac{1}{6}x + \frac{1}{2} = 7$.

Solution

$$-\frac{1}{6}x + \frac{1}{2} = 7$$

$$-\frac{1}{6}x + \frac{1}{2} - \frac{1}{2} = 7 - \frac{1}{2}$$

Subtract $\frac{1}{2}$ from both sides.

$$-\frac{1}{6}x = 6\frac{1}{2}$$

$$-\frac{1}{6}x \div \left(-\frac{1}{6}\right) = 6\frac{1}{2} \div \left(-\frac{1}{6}\right)$$

Divide both sides by $-\frac{1}{6}$.

$$x = \frac{13}{2} \times -\frac{6}{1}$$

$$x = -39$$

A3: You could write an equivalent equation by using the same denominator on both sides.

EXAMPLE

Solve $-\frac{1}{6}x + \frac{1}{2} = 7$.

Solution

$$-\frac{1}{6}x + \frac{1}{2} = 7$$

Express both sides of the equation using an LCD of 6.

$$-\frac{1}{6}x + \frac{1 \times 3}{2 \times 3} = \frac{7 \times 6}{1 \times 6}$$

$$-\frac{1}{6}x + \frac{3}{6} = \frac{42}{6}$$

Write the left side using a single denominator.

$$\frac{-1x + 3}{6} = \frac{42}{6}$$

Equate the numerators.
Solve for x using inverse operations.

$$-1x + 3 = 42$$

$$-1x + 3 - 3 = 42 - 3$$

$$-1x = 39$$

$$\frac{-1x}{-1} = \frac{39}{-1}$$

$$x = -39$$

PRACTICE Questions

Lesson 4.1

1. Use a graph to determine the solution to each equation.

a) $x + 2 = 7$

b) $4x - 4 = 8$

c) $-2x - 9 = -5$

d) $-7x - 14 = -14$

2. A submarine starts at sea level and descends 50 m every 5 min.

a) Make a table of values of the submarine's depth. Use intervals of 5 min, up to 30 min.

b) Graph the submarine's depth at 30 min.

c) What patterns do you see in the table and the graph?

d) If the submarine started at a depth of 219 m, what relation would model the submarine's location in relation to time?

e) Create an equation to show how long it would take the submarine to reach a depth of 428 m. Use the graph to solve this equation.

Lesson 4.2

3. Solve using inverse operations.

a) $2x - 5 = 7$

b) $3x + 4 = 10$

c) $-6 = 3 + 3x$

d) $-2.1k + 5.6 = 20.2$

e) $-8.75z + 12.5 = 12.5$

f) $-a + 5 = 0$

4. Amit joins a book club. The first six books are free, but after that he pays $8.98 per book.

a) Write an expression for the cost of *b* books.

b) How much would he pay for eight books?

c) Amit receives his first shipment of books with a bill for $53.88. Create and solve an equation to determine how many books he ordered.

Lesson 4.3

5. Solve. Verify your solution.

a) $-x + 6 = 2x - 12$

b) $\frac{2}{3}x - 2 = 4x + \frac{4}{3}$

c) $4(x - 8) = -2(x - 5)$

d) $\frac{2}{3}x - \frac{1}{2} = -\frac{1}{2} + \frac{1}{4}x$

e) $\frac{1}{5}(a + 1) = \frac{1}{3}(2a - 3)$

f) $\frac{(4a - 2)}{5} + \frac{1}{2} = \frac{(3a + 7)}{2} - 1$

6. For each equation, write an equivalent equation you could use to help solve it. Then, solve each equation.

a) $\frac{k}{3} + 1 = 4$

b) $\frac{x}{2} - 3 = 1\frac{1}{6}$

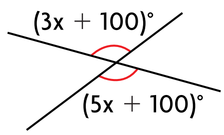
c) $-1\frac{2}{3}g + \frac{7}{9} = 0$

d) $\frac{2}{3}h + \frac{1}{4} = 3\frac{1}{2}$

7. Create and solve an equation to answer each problem below.

a) The perimeter of a rectangle is 210 m. The length is 7 m longer than the width. Determine the dimensions of the rectangle.

b) Recall that opposite angles are equal. Determine the value of each angle measure:



c) Tom has 117 quarters and dimes worth a total of $15.75. How many of each coin does he have?

d) Jim can paint a house in 10 h and Mario can paint the same house in 12 h. How long will it take if they work together?

Curious Math

Crossing the T

Jeff told Erica that if she shaded a letter T on a number grid and told him the sum of all the numbers, he could tell her where the T is located.

Erica: "My sum is 320."
Jeff: "Your numbers are 57, 58, 59, 68, and 78!"

How did Jeff know what numbers Erica chose?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. What expression can you use to describe the sum of the numbers in a T?
2. Try this trick with a partner.
3. Create another trick that would predict the numbers that form a letter E on the hundred chart if you know the sum.

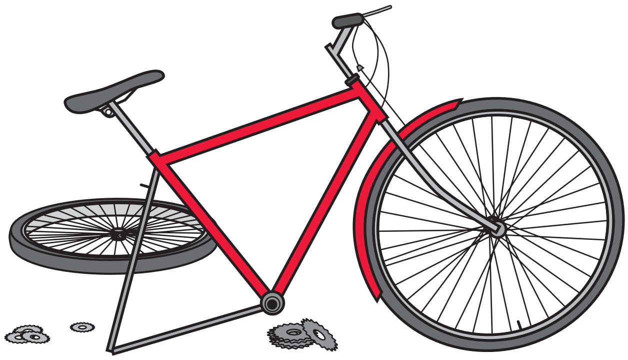
4.4 Solving for a Variable in a Linear Relation

GOAL

Use inverse operations to solve for a variable in a linear relation.

LEARN ABOUT the Math

Ralph and Bill work part time repairing bikes. They are paid \$2 to install a tire and \$5 to install gears. Their boss will pay a maximum of \$100 per week.



? Which combinations of tire and gear installations will earn the boys exactly \$100?

EXAMPLE 1 Strategies for determining ordered pairs in a linear relation

The boys developed the linear relation $2T + 5G = 100$ to represent the situation. T represents the number of tire installations and G represents the number of gear installations.
Determine the combinations of tire and gear installations that will earn them \$100.

Ralph's Solution: Using guess-and-check to solve for one variable after substituting a value for the other

Let $T = 5$.
So, $2T + 5G = 100$.

I chose a value for T. I knew the value of G should depend on the value of T that I chose.

$$\begin{aligned} 2(5) + 5G &= 100 \\ 10 + 5G &= 100 \\ 10 + 5G - 10 &= 100 - 10 \\ 5G &= 90 \\ \frac{5G}{5} &= \frac{90}{5} \\ G &= 18 \end{aligned}$$

I substituted the value for *T* into the equation and solve for *G*.

Let $T = 6$:

I chose a different value for *T*.

$$\begin{aligned} 2T + 5G &= 100 \\ 2(6) + 5G &= 100 \end{aligned}$$

I substituted the value into the equation.

$$12 + 5G - 12 = 100 - 12$$

I used inverse operations to solve for *G*.

$$\begin{aligned} 5G &= 88 \\ \frac{5G}{5} &= \frac{88}{5} \\ G &= 17.6 \end{aligned}$$

It is impossible to do 17.6 gear installations, so $T = 6$ is not a reasonable value to choose.

Each time I chose a value, I ended up with an equation like $5G = \blacksquare$. I realized that whatever I subtract from 100 must be a multiple of 5 in order to divide and get a whole number. *T* must be a multiple of 5.

I tried $T = 10$ as a value.

Let $T = 10$:

$$\begin{aligned} 2T + 5G &= 100 \\ 2(10) + 5G &= 100 \\ 20 + 5G - 20 &= 100 - 20 \\ 5G &= 80 \\ \frac{5G}{5} &= \frac{80}{5} \\ G &= 16 \end{aligned}$$

5 tire installations and 18 gear installations will earn exactly $100. So will 10 tire installations and 16 gear installations.

Bill's Solution: Using algebra to rearrange the relation to solve for one variable in terms of the other

$$2T + 5G - 5G = 100 - 5G$$
$$2T = 100 - 5G$$

I thought I could save time if I wrote an equivalent relation that showed how to calculate one of the variables in terms of the other.

I decided to write an equation that **solved for *T* in terms of *G***. That meant I first had to undo the + 5*G* on the left.

solve for a variable in terms of other variables

the process of using inverse operations to express one variable in terms of the other variable(s)

$$\frac{2T}{2} = \frac{100 - 5G}{2}$$
$$T = \frac{100 - 5G}{2}$$

I used inverse operations to isolate the variable *T*.

Let $G = 10$:

$$T = \frac{100 - 5G}{2}$$
$$T = \frac{100 - 5(10)}{2}$$
$$T = \frac{50}{2}$$
$$T = 25$$

I chose a value for *G* and substituted the value into the equation.

10 gear installations and 25 tire installations will earn exactly \$100.

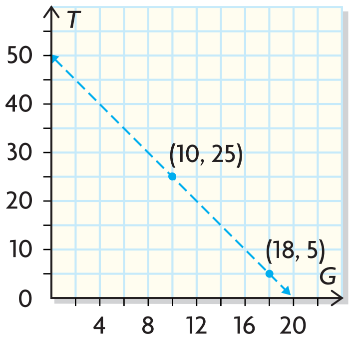
Let $G = 11$:

$$T = \frac{100 - 5G}{2}$$
$$T = \frac{100 - 5(11)}{2}$$
$$T = \frac{45}{2}$$
$$T = 22.5$$

I chose a different value for *G* and substituted the value into the equation.

It is impossible to do 22.5 tire installations, so $G = 11$ is not a reasonable value to choose.

$T = \frac{100 - 5G}{2}$



I also found that 18 gear and 15 tire installations would work, so I plotted the points (10, 25) and (18, 5) and joined these with a dashed line since that data is discrete. This gave me the graph of the relation

$$T = \frac{100 - 5G}{2}.$$

Points outside the first quadrant are not possible because they would mean a negative number of tire or gear installations.

I can see from the graph that there are several combinations that would earn exactly \$100. Some possible combinations are (0, 50), (2, 45), (4, 40), (6, 35), (8, 30).

I noticed that each increase of 2 tire installations resulted in a decrease of 5 gear installations.

Any point that lies on the line that has whole-number coordinates represents a combination that will earn \$100.

Reflecting

A. How did Ralph and Bill know which inverse operations to use and in what order to apply them?

B. How would Bill's solution, equation, and graph be different if he chose to solve for G instead of T?

C. How is using inverse operations to solve a linear equation similar to using inverse operations to solve for one variable in terms of another? How is it different?

APPLY the Math

EXAMPLE 2 Using an inverse operations strategy to solve a relation for one variable in terms of the other

Solve for y in terms of x for the line $\frac{2}{3}x + \frac{1}{5}y = 2$.

Agatha's Solution

$$\frac{2}{3}x + \frac{1}{5}y - \frac{2}{3}x = 2 - \frac{2}{3}x$$

$$\frac{1}{5}y = 2 - \frac{2}{3}x$$

I used an inverse operation to remove the $\frac{2}{3}x$ term and isolate the $\frac{1}{5}y$ term on the left.

$$5\left(\frac{1}{5}y\right) = 5\left(2 - \frac{2}{3}x\right)$$

Then, I multiplied both sides by 5 to solve for y. I added brackets to remind me to use the distributive property on the right side of the equation.

$$y = 10 - \frac{10}{3}x$$

I simplified the right side of the equation.

$$y = -\frac{10}{3}x + 10$$

I reordered the terms on the left.

The equation $\frac{2}{3}x + \frac{1}{5}y = 2$ is the same as $y = -\frac{10}{3}x + 10$.

EXAMPLE 3 Rearranging an equation before solving it

Vicki puts \$1500 in an investment account that earns 7.5% simple interest per year. If she wants to earn \$200 in interest, how long must she leave the money in the investment?

Latifa's Solution

The simple interest formula is: $I = prt$.

- I represents the interest earned;
- p represents the principal amount invested (the initial amount put into the bank account);
- r represents the interest rate per year (expressed as a decimal);
- t represents the time (in years) that the money was invested.

$$I = prt$$

$$\frac{I}{pr} = \frac{prt}{pr}$$

$$\frac{I}{pr} = t$$

Since I wanted to calculate the time needed for the investment, I decided to solve for T. I had to undo the multiplication by p and by r. The inverse of multiplying by p and r is to divide by p and r.

$$I = \$200$$
$$r = 0.075$$
$$p = \$1500$$
$$\frac{200}{1500 \times 0.075} = t$$

$r = 7.5\% = \frac{7.5}{100} = 0.075$

I substituted the values for *I*, *p*, and *r* into the equation for *T*.

$$1.8 = t$$

1.8 years is about 1 year and 80% of the second year.

$$80\% \text{ of } 12 \text{ months}$$
$$= \frac{80}{100} \times 12$$
$$= 0.80 \times 12$$
$$\doteq 10 \text{ months}$$

I calculated the number of months.

Vicki must leave her money in the investment for about 1 year and 10 months to earn $200 in interest.

In Summary

Key Idea

- You can use inverse operations to isolate any variable in a relation. This has the effect of solving for that variable in terms of the other variable(s) in the relation.

Need To Know

- The following strategy can be used to solve a relation for any variable:
  - Imagine that each of the other variables has been replaced by a number.
  - List the inverse operations needed to solve for the target variable.
  - Perform the inverse operations, one at a time, on the original relation, using the original variables until the target variable is isolated.

CHECK Your Understanding

1. Solve for the variable indicated.
   a) $3x + y = 5$; solve for x
   b) $2x + 5y = -10$; solve for y

2. Henri sharpens skates at a local arena. He charges \$3.00 to sharpen a pair of figure skates and \$2.50 for a pair of hockey skates. Last Saturday he earned a total of \$240.

a) To model this situation, John wrote: \$3(number of pairs of figure skates) + \$2.5(number of pairs of hockey skates) = \$240. Write the relation using variables.

b) Write the equation that expresses the number of hockey skates sharpened in terms of the number of figure skates.

c) Write the equation that expresses the number of figure skates sharpened in terms of the number of hockey skates.

Practising

3. In each set of equations, identify the equation that is *not* equivalent to the others.

a) $2a - b = 4$; $2a = b + 4$; $a = \frac{b}{2} + 2$; and $b = 2a + 4$

b) $x + 2y = -6$; $y = \frac{x}{2} + 3$; $x = 2y - 6$; and $x - 2y + 6 = 0$

c) $4m - 3n + 2 = 4$; $3n = 4m + 2$; $4m = 3n - 2$; and $3n - 4m = 2$

4. Solve for y in terms of x.

K a) $2y = 8 - 4x$

b) $-2x - 3y = 12$

c) $2.8x + 1.1y - 5.3 = 0$

d) $\frac{7}{5}y + \frac{2}{3}x = \frac{11}{13}$

e) $\frac{4}{5} = \frac{2}{3}x + 1\frac{1}{2}y$

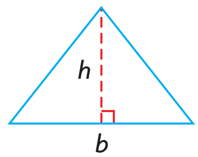
f) $3(y - 2) + 2x = 8$

5. A cell-phone company offers a plan of \$25 per month and \$0.10 per minute of talk. The cost, C, in dollars, is given by the relation $C = 25 + 0.10n$, where n is the number of minutes used per month. Each month the company uses the exact air time to calculate the monthly bill.

a) Solve the relation for n in terms of C.
b) Create a table of values for this new relation.
c) Graph this relation.
d) What is the independent variable? What is the dependent variable?
e) Why might someone want to rearrange this relation and express it in terms of the cost?

6. Start with the relation $2x - 5y = 20$.

a) Solve for y in terms of x.
b) Graph this relation using x as the independent variable.
c) State the slope and the intercepts of the graph.
d) Solve for x in terms of y.
e) Graph the relation using y as the independent variable.
f) State the slope and intercepts of the graph.
g) Compare the slope of the two graphs. Justify your comparison.



7. Solve the relation or formula for the variable indicated:

a) $2a - 5b = 12$; solve for a
b) $0.35m + 2.4n = 9$; solve for n
c) $\frac{1}{2}p - \frac{2}{3}q = \frac{1}{4}$; solve for p
d) $I = prt$; solve for r
e) $P = 2L + 2W$; solve for L
f) $C = 2\pi r$; solve for r

8. Look at the diagram.

T **a)** Write a formula for h in terms of the base, b, and the area, A.
b) Determine the height of the triangle if the area is 55 cm^2 and the base is 4 cm.

9. Ben has \$42.50 in quarters and dimes.

a) Write a linear relation expressing the total amount of money in terms of the number of quarters and dimes.
b) Write an equation to express the number of quarters in terms of the number of dimes.
c) Write an equation to express the number of dimes in terms of the number of quarters.
d) Use one of your equations to determine the possible combinations of quarters and dimes Ben could have.

10. A candy store is making a mixture of chocolate-coated almonds and chocolate-coated raisins. The almonds cost \$30/kg and the raisins cost \$8/kg. The total cost of the mixture is to be \$150.

A

a) Write a linear relation expressing the total cost in terms of the mass of almonds and the mass of raisins purchased.

b) Write an equation to express the mass of almonds in terms of the mass of raisins.

c) Write an equation to express the mass of raisins in terms of the mass of almonds.

d) Which combinations of almonds and raisins will cost exactly \$150?

11. The Alltime Watch Company makes and sells two kinds of watches. The profit on digital watches is \$15 per watch. The profit on analog watches is \$20 per watch. The watch factory can only produce watches in the ratio of 3 digital : 2 analog because of the machines it uses. Given this ratio, how many watches of each type must be produced to meet the company's profit target of at least \$20 000 per week?

C

12. When you multiply a number, x, by k, add n, and then divide by r, the answer is w.

a) Write the relation that models this situation.

b) List the inverse operations that you would use, in the correct order, to isolate x.

c) Solve the relation for x.

d) How is rearranging a relation or formula for a particular variable similar to isolating a variable in a linear equation? How is it different?

Extending

13. Solve for x.

a) $\frac{5}{x} + 2y = 9$

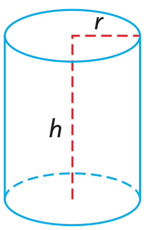
b) $3x^2 + 50 = 197$

c) $(x - 4)^2 - 12 = 24$

d) $\frac{(3 + y)}{x} = -4$

e) $\sqrt{x + 1} = 9$

f) $2 - 8x^3 = 3$



14. The formula for determining the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

a) Solve for h in terms of SA and r.

b) Determine the height of a cylinder with radius 5 cm and surface area 300 cm^2.

c) Solve for r in terms of the other variables.

4.5 Solving a Linear System Graphically

YOU WILL NEED

- grid paper

GOAL

Use a graph to solve a problem modelled by two linear relations.

Sari wants to join a website that allows its users to share music files. SHAREIT charges a $5 membership fee, plus $2.50 for each downloaded song. FILES 'R' US charges $3.00 per downloaded song.

? How can Sari determine which website she should join?

EXAMPLE 1 Solving a problem modelled by a system of linear equations

Determine the website that Sari should join.

Nick's Solution: Using a hand-drawn graph to solve a system of linear equations

I chose x to represent the number of music files downloaded and y to represent the costs for each company.

SHAREIT: $y = 5 + 2.5x$

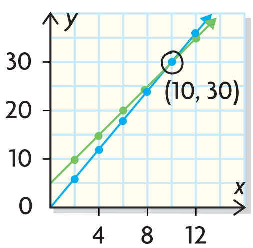
FILES 'R' US: $y = 3x$

I created a **system of linear equations** to model this situation. The total cost, y, in each case depends on the number of music files, x, that are downloaded. I used each equation to create a table of values.

system of linear equations

a set of equations (at least two) that represent linear relations between the same two variables

Number of Songs, x	2	4	6	8	10	12
SHAREIT Cost, $y = 5 + 2.5x$	10	15	20	25	30	35
FILES'R'US Cost, $y = 3x$	6	12	18	24	30	36



I drew both linear relations on the same graph.

Since the coordinate (10, 30) lies on both lines, this is the **point of intersection** and the **solution to the system of equations**.

If Sari plans to download exactly 10 songs, it doesn't matter which site she purchases from. They will both charge $30.

point of intersection

the point in common between two lines

solution to a system of linear equations

a point that satisfies both relations in a system of linear equations; the point of intersection represents an ordered pair that solves the system of linear equations

Check:

SHAREIT

$y = 5 + 2.5x$

$y = 5 + 2.5(10)$

$y = 5 + 25$

$y = 30$

FILES 'R' US

$y = 3x$

$y = 3(10)$

$y = 30$

I checked that the point (10, 30) works in both relations.

Sari should choose FILES 'R' US if she plans to download fewer than 10 songs. If she is going to download more than 10 songs, she should choose SHAREIT.

From 1 to 9 songs downloaded, it costs less to purchase from FILES "R" US. I can tell because the graph is below the graph for SHAREIT. This switches when more than 10 songs are purchased.

Ben decided to use a graphing calculator to solve the problem.

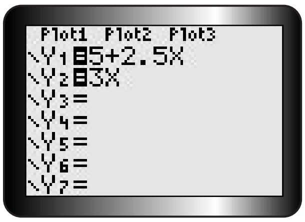
Ben's Solution: Using technology to solve a system of linear equations

I chose x to represent the number of music files downloaded and y to represent the costs for each company.

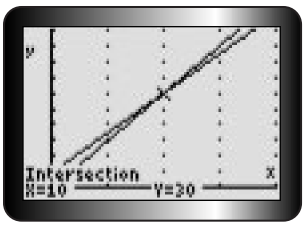
SHAREIT: $y = 5 + 2.5x$

FILES 'R' US: $y = 3x$

I created a **system of linear equations** to model this situation.



I used my calculator to graph both relations and determine the point of intersection.



The point of intersection is (10, 30).

Sari should choose FILES 'R' US if she plans to download fewer than 10 songs. If she is going to download more than 10 songs, she should choose SHAREIT.

Tech Support

For help with using the graphing calculator to determine the point of intersection, see Appendix B-8.

Reflecting

A. Why is it reasonable for the point of intersection to be called the solution to a system of linear equations?

B. Which student's strategy would you prefer to use? Why?

APPLY the Math

EXAMPLE 2 Determining the point of intersection of two lines

What is the point of intersection of the graphs of

$3x + 2y = 12$ and $y = -\frac{3}{2}x + 2$?

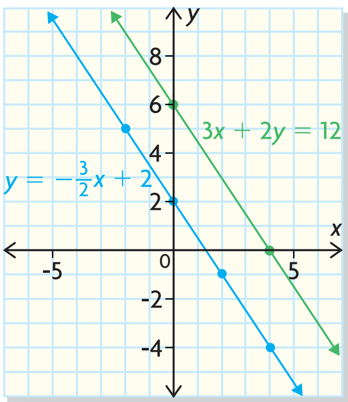
Julie's Solution

$3x + 2y = 12$	
x	y
4	0
0	6

$y = \frac{-3}{2}x + 2$	
x	y
−2	5
0	2
2	−1
4	−4

To graph $3x + 2y = 12$, I calculated the intercepts and connected the points to see the relation.

To graph $y = -\frac{3}{2}x + 2$, I used a table of values. I chose numbers for x that were divisible by 2 because the coefficient of x has 2 in its denominator. I substituted these into the equation to find the values for y. I connected the points to see the relation. The lines are parallel, so they don't have a point in common.



There is no point of intersection.

EXAMPLE 3 Using a graphing strategy to estimate a break-even point

Jean just opened a new company that makes MP3 players.
He uses two equations to compare cost and revenue. In order for the company to break even, the cost must equal the revenue.

Cost equation:

- The company paid $5750 to set up the manufacturing line.
- The materials and labour cost for each machine is $50.
- The cost equation is $y = 50x + 5750$, where x is the number of MP3 players produced and y is the cost to produce x number of players.

Revenue equation:

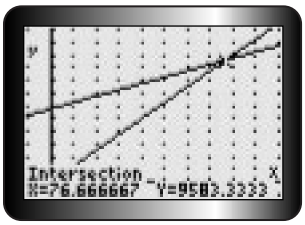
- The company sells each player for $125.
- The revenue equation is $y = 125x$, where x is the number of MP3 players sold and y is the revenue made for x number of MP3 players.

How many MP3 players must the company sell to break even?

Mason's Solution

Number of MP3 Players	Cost ($)	Revenue ($)
50	8 250	6 250
100	10 750	12 500
150	13 250	18 750

I created a table of values to find the break-even point. This is the point where the cost and the revenue are the same.
The cost is **more than** revenue when 50 MP3 players are sold.
The cost is **less than** the revenue when 100 MP3 players are sold.
This means that the break-even point must occur when more than 50 but fewer than 100 players are sold.



I used graphing technology to get a more accurate solution. I entered the equation $y = 50x + 5750$ in Y1 and $y = 125x$ in Y2. Then, I determined the point of intersection.

Intersection at approximately (76.67, 9583.33).

The break-even point is at (76.67, 9583.33).

Since you cannot sell part of an MP3 player, the company needs to sell about 77 MP3 players to break even.

If the company sells 77 MP3 players, its revenue and costs will be about $9600.

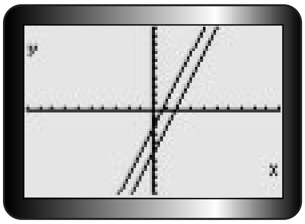
EXAMPLE 4 Determining the number of solutions to a system of linear equations

Determine the solution to each of the following:

a) $3x - y = 5$ and $y = 3x - 2$ **b)** $x - y = 2$ and $y = \dfrac{(2x - 4)}{2}$

Brent's Solution: Using graphing technology to solve the system of linear equations

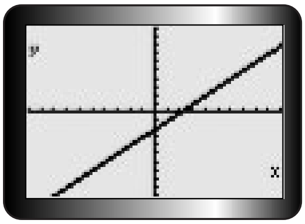
a)



There is no solution to this system of linear equations because the lines are parallel.

I used graphing technology to graph each linear relation. I could not find the point of intersection. I rewrote $3x - y = 5$ in its equivalent $y = mx + b$ form: $y = 3x - 5$. It appears that the slopes of the two lines are the same because the lines look parallel. This means that they do not intersect at any point.

b)



When I graphed both relations, the lines were identical.

$$x - y = 2$$
$$2 \times (x - y) = 2 \times 2$$
$$2x - 2y = 4$$

I realized that the equations were equivalent to each other. When I multiplied all the terms in the first equation by 2, I got the second equation.

There is an infinite number of solutions for this system of linear equations.

Since the equations are equivalent, all values that lie on one line also lie on the other line.

EXAMPLE 5 Using technology to solve a system of linear equations

Find two numbers where:

- The sum of both numbers divided by 4 is 3.
- Two times the difference of the two numbers is –36.

Donna's Solution

Let x represent one number and y represent the other.

$$\frac{(x + y)}{4} = 3 \qquad 2(x - y) = -36$$

I created a system of linear equations to model the two statements.

$$4 \times \frac{(x + y)}{4} = 3 \times 4 \qquad \frac{2(x - y)}{2} = \frac{-36}{2}$$

$$(x + y) = 12 \qquad x - y = -18$$

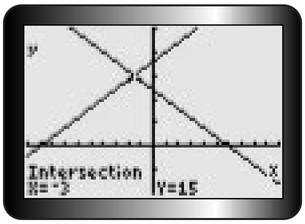
$$x + y - x = 12 - x \qquad x - y + y = -18 + y$$

$$y = 12 - x \qquad x = -18 + y$$

$$y = -x + 12 \qquad x + 18 = -18 + y + 18$$

$$x + 18 = y$$

I used inverse operations to rearrange the relations into the form $y = mx + b$, so that I could graph both relations on a graphing calculator.



I entered both relations into the equation editor and graphed the relations. I found the point of intersection using the graphing calculator. The point of intersection is $(-3, 15)$.

The numbers are -3 and 15.

Check:

Since the point of intersection satisfies both equations, the numbers are correct.

$$\frac{(x + y)}{4} = 3$$

Left Side	Right Side
$\frac{(-3 + 15)}{4}$	3
$= \frac{12}{4}$	
$= 3$	

$$2(x - y) = -36$$

Left Side	Right Side
$2(-3 - 15)$	-36
$= 2(-18)$	
$= -36$	

In Summary

Key Ideas

- When solving a system of linear relations, the point of intersection of their graphs is the solution to that system of linear equations.

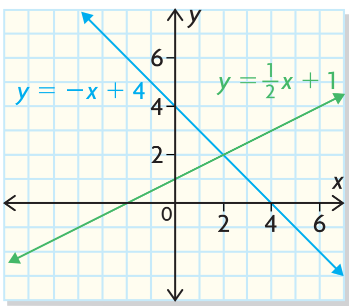
Need To Know

- The coordinates of the point of intersection can be estimated by graphing the relations by hand.
- Graphing technology helps determine the point of intersection with greater accuracy than is possible with a hand-drawn graph.
- A system of linear equations can have one point of intersection, zero points of intersection (if the graphs are parallel), or infinite points of intersection (if they are equivalent equations).

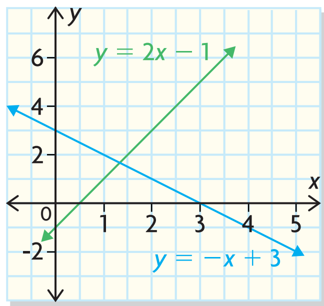
CHECK Your Understanding

1. Determine the point of intersection for each system of linear equations shown below.

a) $y = \frac{1}{2}x + 1$ and $y = -x + 4$



c) $y = 2x - 1$ and
$y = -x + 3$



b) $y = x + 1$ and $y = 4x - 5$

d) $y = x$ and $y = -x$

2. Bill wants to earn extra money selling lemonade in front of his house. It costs \$1.20 to start his business and each glass of lemonade costs \$0.06 to make. He plans to sell the lemonade for \$0.10 a glass.

a) Write an equation that represents his cost.

b) Write an equation that represents his revenue.

c) Graph both equations on the same set of axes.

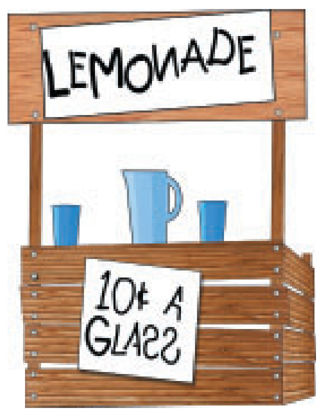
d) What does the point of intersection mean in this case?

e) Does Bill make a profit or lose money for

i) 20 glasses sold?

ii) 35 glasses sold?

iii) 50 glasses sold?



PRACTISING

3. Determine the point of intersection of each pair of lines.
   a) $y = -3x - 2$ and $2x + 3y = 5$
   b) $2x + 4y = 7$ and $-x + 0.75y = 5$
   c) $0.25x - 0.5y = 1$ and $3.25x + 4y = 22.5$
   d) $y = 3x + 6$ and $1 = 3x - y$

4. The sum of two integers is 42. The difference of the two numbers is 17.
   a) Create a system of linear equations to model each statement above.
   b) Determine the integers using a graph.

5. Mike has \$9.85 in dimes and quarters. If there are 58 coins altogether, how many dimes and how many quarters does Mike have?

6. Does each pair of lines intersect at the given point?
   a) $(2, 3)$: $y = x + 1$, $y = 4x - 5$
   b) $(1, -1)$: $y = 5x - 4$, $y = 2x - 3$
   c) $(0, 2)$: $y = 3x + 2$, $y = 5x - 1$
   d) $(-1, -3)$: $y = 4x + 1$, $y = x - 5$

7. Given the lines $y = 2$ and $y = 4x + 9$,
   a) Determine the point of intersection using a graph.
   b) Create the linear equation that you would solve to determine the x-value of the point of intersection.
   c) Solve the linear equation in part b) to verify your solution from part a).

8. Determine the point of intersection of each pair of lines:
K
   a) $y - x = 9$ and $x - \frac{1}{6}y = -\frac{2}{3}$
   b) $y = 2$ and $y = 5$
   c) $2x - y = 0$ and $y = 5 + 2x$
   d) $y = -4$ and $x = 1$

9. Marie charges \$3 for every 4 bottles of water purchased from her store. She pays her supplier \$0.25 per bottle, plus \$250 for shelving and water delivery.
   a) Create a system of two linear equations to model this situation.
   b) How many bottles of water does she need to sell to break even?

10. Mr. Smith is trying to decide which Internet service provider (ISP) to use for his home computer. UPLINK offers a flat fee of \$19 per month; BLUELINE offers a fee of \$10 per month, but charges \$0.59 per hour after the first 30 hours.
A
   a) Write the linear relation that models the cost in relation to the number of hours used for each plan.
   b) Estimate the point at which the costs for both companies would be the same.

c) What equation would you set up and solve to determine the exact point at which the costs would be the same? Why is this equation reasonable?
d) What advice would you give to Mr. Smith about which ISP to choose?

11. Mrs. Smith was trying to help her husband decide which ISP to use and she investigated two other companies on her own:
DOWNLINK offers a plan of \$5 per month plus \$1.15 per hour after the first 20 hours.
REDLINE offers a plan of \$2.50 per month plus \$1.80 per hour after the first 10 hours.
Should the Smiths consider either of these two companies in their decision? Why or why not?

12. Determine the point of intersection of each pair of lines:
a) $5x + 8y - 12 = 0$ and $-5x + 16y - 12 = 0$
b) $4x + y - 2 = 0$ and $8x + 2y - 4 = 0$
c) $\frac{1}{3}x - \frac{2}{5}y + \frac{1}{4} = 0$ and $2x - \frac{1}{7}y + \frac{1}{2} = 0$
d) $5x - 2.5y = 10$ and $3.1x + 4y = 6.2$

13. Given the relation $x + y = 5$, determine a second relation that:
T a) intersects $x + y = 5$ at $(2, 3)$
b) *does not* intersect $x + y = 5$

14. Movies to Go rents DVDs for \$2.50 and has no membership fee. Films
C 'R' Us rents videos for \$2 but has a \$10 membership fee. What advice would you give to someone who is deciding which video store to use?

15. Why does a system of two linear equations usually have only one solution for each of the two variables?

Extending

16. To determine the point of intersection of $y = 2x + 5$ and $y = 4x - 3$, Elena wrote $2x + 5 = 4x - 3$ and solved the equation. Why is this a reasonable strategy for determining the point of intersection of the two lines?

17. Compare the strategies of solving $3x + 4 = 5x + 3$ by using inverse operations and by graphing the two relations.

18. a) Determine the point(s) of intersection of $y = 2x^2$ and $y = 8$ using a graph.
b) Create and solve the equation that you would use to determine where the point of intersection lies.
c) Are your solutions from parts a) and b) the same? Explain.

4 Chapter Review

FREQUENTLY ASKED Questions

Study Aid

- See Lesson 4.4, Examples 2 and 3.
- Try Chapter Review Questions 5 and 6.

Q: How does solving a linear relation for a variable compare to solving a linear equation?

A: The processes are the same, but the solution for a linear equation is a number. When solving for a variable, you get an equation equivalent to the original one. To isolate the variable, you must undo each operation in the reverse order. The operation must be done to both sides of the equation to keep the equivalence of the relation or formula.

EXAMPLE

Rearrange the equation $90x + 45y = 360$ to solve for y in terms of x.

Solution

$$\begin{aligned} 90x + 45y &= 360 \\ 90x + 45y - 90x &= 360 - 90x \\ 45y &= 360 - 90x \\ \frac{45y}{45} &= \frac{360 - 90x}{45} \\ y &= \frac{360 - 90x}{45} \\ y &= 8 - 2x \end{aligned}$$

Study Aid

- See Lesson 4.5, Example 1.
- Try Chapter Review Questions 7 and 8.

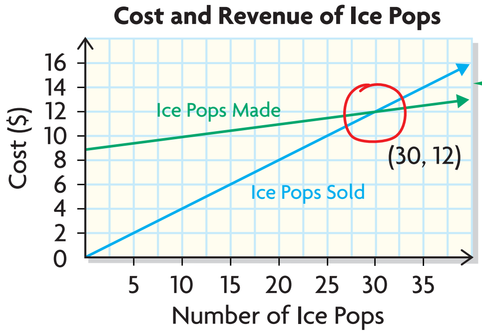
Q: How do you solve a problem that can be modelled by a system of linear relations?

A1: Use a hand-drawn graph to estimate the point of intersection.

EXAMPLE

The cost to make ice pops is \$0.10 per ice pop, plus \$9.00 in supplies. Each ice pop sells for \$0.40. How many do you need to sell to break even?

Solution



Create a table of values for each equation, then graph both equations on the same axes. Find the point of intersection. Then, interpret the point of intersection.

The break-even point is located at (30, 12).

You need to sell 30 ice pops to break even.

A2: Use graphing technology to determine more accurate coordinates of the point of intersection.

EXAMPLE

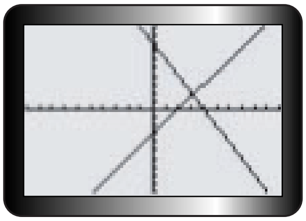
Determine the point of intersection of the graphs of $2x + y = 8$ and $3x - 2y = 6$.

Solution

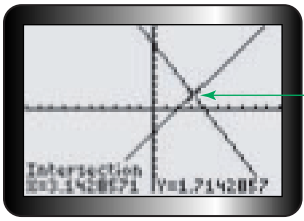
$$\begin{aligned} 2x + y &= 8 \\ 2x + y - 2x &= 8 - 2x \\ y &= -2x + 8 \end{aligned}$$

$$\begin{aligned} 3x - 2y &= 6 \\ 3x - 2y - 3x &= 6 - 3x \\ -2y &= -3x + 6 \\ \frac{-2y}{-2} &= -3x + \frac{6}{-2} \\ y &= \frac{3}{2}x - 3 \end{aligned}$$

Solve for y in terms of x in both equations, so that they can be graphed on a graphing calculator.



Enter both equations into the equation editor of a graphing calculator and graph.



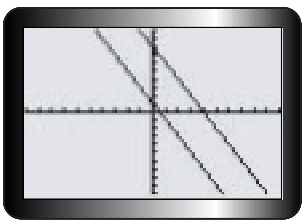
Determine the point of intersection using the Intersect operation.

The point of intersection is about (3.14, 1.71).

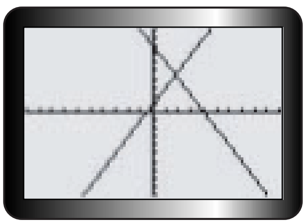
Q: In what ways can two lines intersect?

A: Two lines can intersect in one of three different ways.

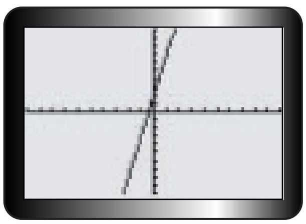
The lines could be parallel resulting in 0 points of intersection.



The lines could intersect at exactly one point of intersection.



The lines could be identical resulting in an infinite number of points of intersection.



PRACTICE Questions

Lesson 4.1

1. Write the linear relation that corresponds to each equation. Estimate the solution graphically.

a) $4x - 5 = 3$ **c)** $-2(x - 3) = -4$

b) $\frac{1}{2}x + 3 = 5$ **d)** $\frac{1}{4}\left(x + \frac{2}{5}\right) = 0$

2. Solve each equation using algebra.

a) $3x + 6 = 12$

b) $5 - 2x = 11$

c) $4x - 8 = 12$

d) $-6x + 8 = -10$

3. Determine the x-intercept of each of the following.

a) $y = -5x + 20$ **b)** $2x + y = 10$

4. A promoter is holding a video dance. Tickets cost \$15 per person, and he has given away 10 free tickets to radio stations.

a) Create the linear relation that models the money the promoter will earn in ticket sales in terms of the number of people attending the dance.

b) Graph the linear relation.

c) Write the equation you would solve to determine the money from tickets sales if 100 people attend. Solve the equation using the graph.

d) Write the equation you would use to determine how many people attended if ticket sales were only \$600. Estimate the solution using the graph.

Lesson 4.2

5. Solve each equation in question 1 using inverse operations.

6. Erynn joins a CD club. The first 10 CDs are free, but after that she pays \$15.95 for each CD she orders.

a) Write an expression for the cost of x CDs.

b) How much would she pay for 15 CDs?

c) Erynn receives her first order of CDs with a bill for \$31.90. Create and solve an equation to determine how many she ordered.

Lesson 4.3

7. Solve and verify each equation.

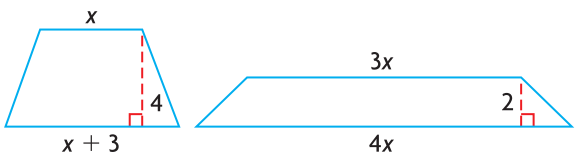
a) $9x + 2 = 11x - 10$

b) $-\frac{4}{5}x + \frac{2}{3} = 1\frac{3}{4}x + 2$

c) $-3(x + 1) - 2 = 4x - 5(x - 3)$

d) $\frac{(4 + x)}{3} + 4 = \frac{(x - 6)}{2} - 6$

8. To calculate the area of a trapezoid, you would use the expression: $\frac{h}{2}(b_1 + b_2)$. Determine the length of each base for the trapezoids below if they have the same area.



9. Is $x = 3$ the solution to $5(3x - 2) = 4 - 10(x + 1)$? Explain how you know.

Lesson 4.4

10. Solve each equation for the variable indicated.

a) $P = 2l + 2w$; l **c)** $V = \pi r^2 h$; h

b) $A = P + Prt$; t **d)** $Ax + By = C$; y

11. The formula $C = \frac{5}{9}(F - 32)$ is used to convert Fahrenheit temperatures to Celsius.

a) Determine the Celsius temperature when $F = 90$.

b) Solve for F in terms of C.

c) Determine the Fahrenheit temperature when $C = 25$.

12. Solve for y in terms of x.

a) $8x - 4y = 12$

b) $5x = 10y - 20$

c) $3x - 3y - 9 = 0$

d) $\frac{x}{4} + \frac{y}{8} = 2$

13. Josh has \$32.00 in loonies and toonies.

a) Write a linear relation expressing the total amount of money in terms of the number of loonies and toonies.

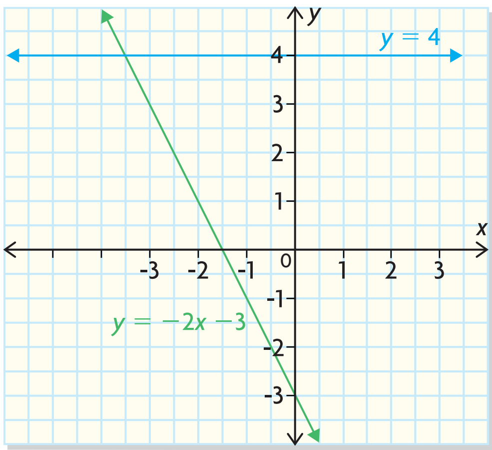
b) Write an equation to express the number of toonies in terms of the number of loonies.

c) Use your equation to determine the 4 different possible combinations of coins Josh could have.

d) Is it possible that Josh has 13 toonies and 5 loonies? Explain.

Lesson 4.5

14. John said, "To solve the equation $4 = -2x - 3$, I graphed $y = -2x - 3$ and $y = 4$. The x-value where the two lines intersect is the solution."



a) What is the solution to this equation based on the graph?

b) Verify the solution using algebra.

c) Why is John's strategy reasonable?

d) How could you use this strategy to solve $3x - 4 = 2x + 3$?

15. Solve each of the following systems of equations using a graph.

a) $y = 3x - 1$ and $y = -x + 5$

b) $y = -0.5x$ and $y = 6.5x + 3$

c) $3x - 4y = -12$ and $2x - 3y = 6$

16. Faster Fitness has a monthly membership fee of \$90. Members pay \$5 to take an aerobics class. At Drop-in Fitness, there is no membership fee, but clients pay \$10 per class.

a) Write a linear relation for the yearly cost in terms of the number of aerobics classes.

b) Graph the equations on the same set of axes.

c) State the point of intersection.

d) What does the point of intersection mean in this case?

e) How would you advise someone who is trying to choose between the two fitness clubs?

17. The Video Vault rents DVDs for \$3.00 each and has no membership fee. Videorenters rents DVDs for \$2 each but has a \$15 membership fee.

a) Write an equation for each situation.

b) Graph both equations on the same set of axes. Find the point of intersection.

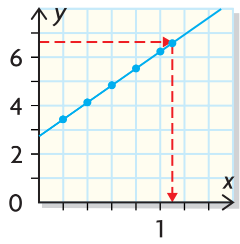
c) What does the point of intersection mean in this case?

d) What advice would you give to someone who is deciding which video store to use?

4 Chapter Self-Test

1. a) Explain how each of the following illustrates a valid approach to solving the equation $3.50x + 2.70 = 6.55$.

b) Which method leads to an exact solution?

<table>
<tr><th>Table of Values</th><th>Graph</th><th>Solve an Equation</th></tr>
<tr>
<td>
<table>
<tr><th>x</th><th>$3.50x + 2.70$</th></tr>
<tr><td>0.0</td><td>2.70</td></tr>
<tr><td>0.2</td><td>3.40</td></tr>
<tr><td>0.4</td><td>4.10</td></tr>
<tr><td>0.6</td><td>4.80</td></tr>
<tr><td>0.8</td><td>5.50</td></tr>
<tr><td>1.0</td><td>6.20</td></tr>
<tr><td>1.2</td><td>6.90</td></tr>
<tr><td>1.4</td><td>7.60</td></tr>
<tr><td>1.6</td><td>8.30</td></tr>
<tr><td>1.8</td><td>9.00</td></tr>
<tr><td>2.0</td><td>9.70</td></tr>
</table>
The solution is between 1.0 and 1.2.
</td>
<td>
$y = 3.50x + 2.70$


The solution is $x \doteq 1.1$.
</td>
<td>
$3.50x + 2.70 = 6.55$
$3.50x + 2.70 - 2.70 = 6.55 - 2.70$
$3.5x = 3.85$
$3.5x \div 3.5 = 3.85x \div 3.5$
$x = 1.1$
The solution is $x = 1.1$.
</td>
</tr>
</table>

2. Solve and verify each equation.

a) $-2a + 5 = 3$

b) $4 - 2x = 8$

c) $\frac{5}{6}x - \frac{3}{4} = \frac{1}{4}$

d) $3(x - 1) + 2(3x + 1) = 2$

3. Rearrange each equation to solve for the variable indicated.

a) $-3x + 2y = 6$; solve for y.

b) $y = 3x + 2$; solve for x.

4. David has two part-time jobs. He earns \$14/h at one and \$11/h at the other. David wants to know how many hours it will take him to earn \$1000.

a) Find two combinations of the numbers of hours David could work at each job to earn \$1000.

b) Graph the relation.

5. Determine the point of intersection for each system of linear equations.

a) $y = 7x - 9$ and $y = -x - 1$

b) $-x - 2y = -3$ and $3x + y = -2$

6. Justin charges \$21 per linear foot to install a wood fence. It costs him \$19 per linear foot plus \$4000 to purchase materials and hire installers each month. How many linear feet of fencing would he need to install each month to break even?

4 Chapter Task

Planning the Burn

Carol enjoys high-impact aerobics and mountain biking. Steve prefers to swim and jog. Each of them works out for 45 minutes in a session.

Carol wants to burn 480 calories during her exercise sessions. Steve's target is 560 calories. The table shows the rate at which they burn calories in each type of activity.

	Calories Burned per Hour	
Activity	**Carol**	**Steve**
high-impact aerobics	444	620
jogging	636	880
mountain biking	540	752
swimming	508	708

? How should Carol and Steve plan their activities to meet their calorie targets?

A. Use algebraic expressions to represent each of Carol's and Steve's exercise plans.

B. Graph a relation for each.

C. Decide the amount of time Carol and Steve should spend on their preferred activities. Justify your choice.

D. Carol and Steve would like to choose two activities from the four to do together. Determine if they can do the same two activities for 45 minutes and meet their calorie targets.

Task *Checklist*

- ✔ Did you draw and label your graphs?
- ✔ Did you explain your solutions?
- ✔ Did your solutions answer the questions?
- ✔ Did you use appropriate math language?