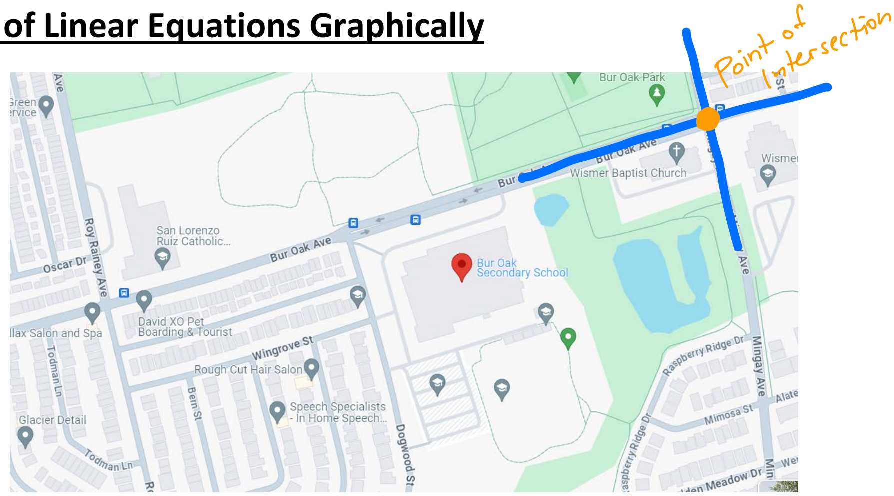
Solving a System of Linear Equations Graphically

Warm up: Here's a Google map of the area near our school. Can you identify an intersection on the map?

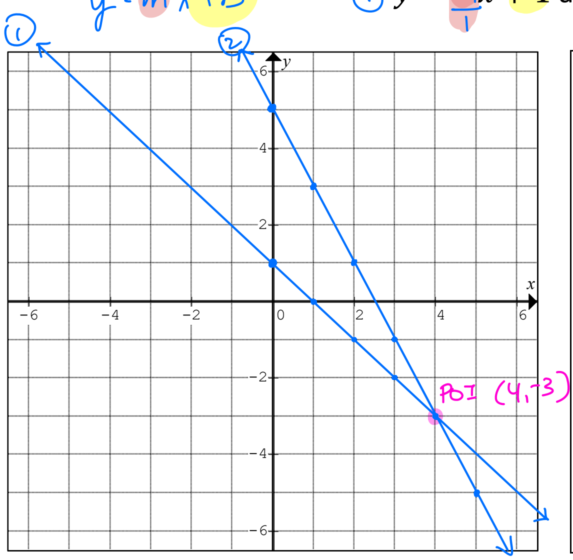


Similarly, a point of intersection in math is a point where 2 or more lines cross on the graph. Today, we are going to solve graphically for a point of intersection of a system of linear equations!

A **system** of equations is a collection of equations. Hence, a system of linear equations is a collection of linear equations. When we have a system of equations, we can often solve for the point(s) of intersection, also known as the solution(s) / root(s) to the system of equations.

Example 1: Plot the two linear relations on the same plane:

$y = mx + b$ ① $y = -x + 1$ and ② $y = -2x + 5$



Observations:

The two lines intersect (meet) at the point (4, -3).
This point lies on BOTH straight lines, $y = -x + 1$ and $y = -2x + 5$. Thus, (4, -3) is said to satisfy both linear equations.

Furthermore, (4, -3) is said to be the SOLUTION / root to the **linear system**

$$\begin{cases} y = -x + 1 \\ y = -2x + 5 \end{cases}$$

Because (4, -3) satisfies both linear equations

What can we do to make sure that our solution is indeed correct? LS/RS check

① sub in (4, -3)

For $y = -x + 1$	
LS	RS
y $= -3$	$-x+1$ $= -(4)+1$ $= -3$

LS = RS ☺

② sub in (4, -3)

For $y = -2x + 5$	
LS	RS
y $= -3$	$-2x+5$ $= -2(4)+5$ $= -3$

LS = RS ☺

since LS = RS for both lines, (4, -3) is the POI!

Example 2: Verify whether or not $(7, -3)$ is the solution (POI) to the linear system $\begin{cases} y = 2x - 17 \\ y = -3x - 2 \end{cases}$

$y = 2x - 17$

LS	RS
y $= -3$	$2x - 17$ $= 2(7) - 17$ $= -3$

LS = RS ☺

$y = -3x - 2$

LS	RS
y $= -3$	$-3x - 2$ $= -3(7) - 2$ $= -23$

LS ≠ RS ☹

Since LS ≠ RS for one equation, (7, -3) is NOT the POI!

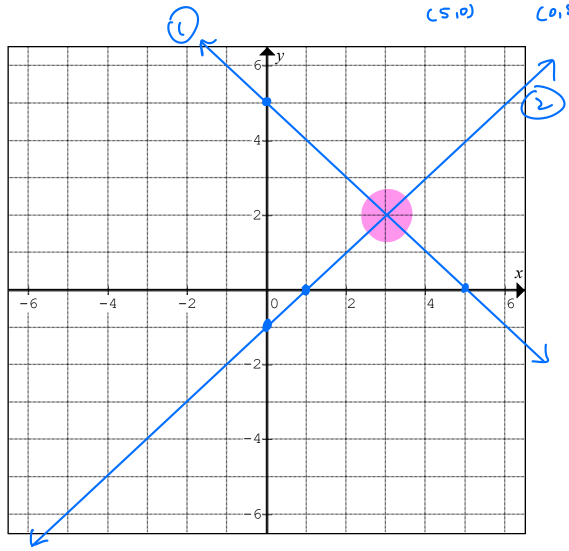
Example 3: Find the solution to the linear system $\begin{cases} ① \; x + y = 5 \\ ② \; x - y = 1 \end{cases}$

For ① $x + y = 5$

x-int	y-int
set $y=0$	set $x=0$
$x + 0 = 5$	$0 + y = 5$
$x = 5$	$y = 5$
$(5,0)$	$(0,5)$

For ② $x - y = 1$

x-int	y-int
set $y=0$	set $x=0$
$x - 0 = 1$	$0 - y = 1$
$x = 1$	$-y = 1$
$(1,0)$	$y = -1$
	$(0,-1)$



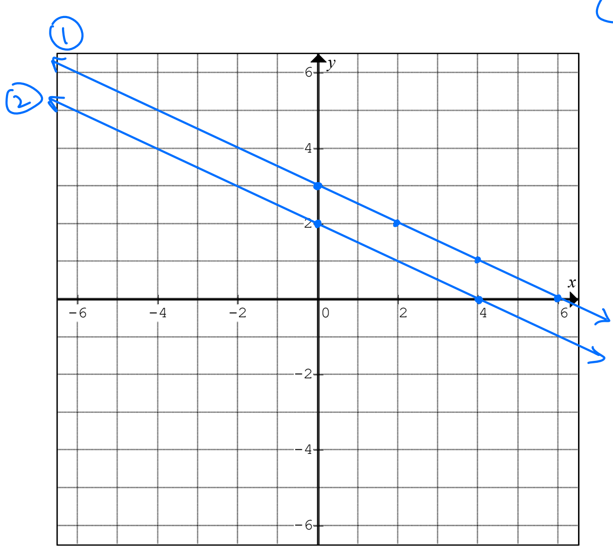
Observations:

POI is $(3,2)$

Do you think there will always be a point of intersection for a system of linear equations?

Example 4: Find the solution to this linear system $\begin{cases} ① \; y = -\frac{1}{2}x + 3 \\ ② \; 2x + 4y = 8 \end{cases}$

x-int	y-int
set $y=0$	set $x=0$
$2x + \cancel{4(0)} = 8$	$\cancel{2(0)} + 4y = 8$
$2x = 8$	$4y = 8$
$x = 4$	$y = 2$
$(4,0)$	$(0,2)$



Observations:

Parallel and distinct lines!

∴ no POI!

So how can we know without graphing that a linear system will have a point of intersection or not? $y = mx + b$

- Infinite solutions : same slopes + same y-int's (identical lines)
- No solutions : same slopes but diff y-int's (parallel and distinct)
- 1 solution: diff slopes (y-int doesn't matter)