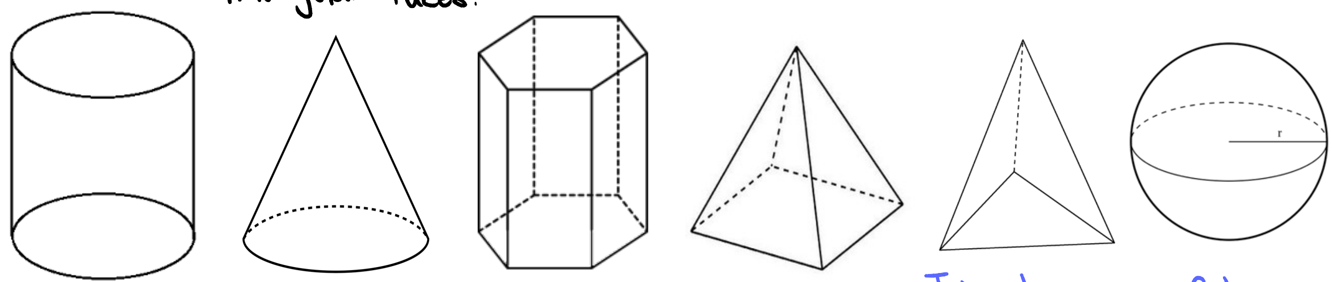
Prism vs Pyramid

Prism: A polyhedron with two identical polygonal bases connected by rectangular faces.

Pyramid: A polyhedron with a polygonal base connected to an apex by triangular faces.



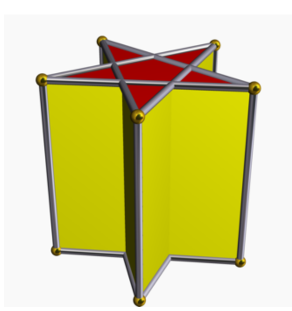
Cylinder | Cone | Hexagonal-based prism | Square-based pyramid | Triangular-based pyramid | Sphere

Now, we are going to look at a relationship between the volume of a prism and pyramid with the same base and the same height. Any guesses?

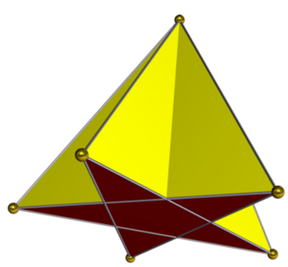
from video

Relationship: Volume of prism is 3 times the volume of related pyramid.

If the prism shown below has the same base area and height as the pyramid shown below, find the volume of the pyramid.



Volume: $300\ cm^3$



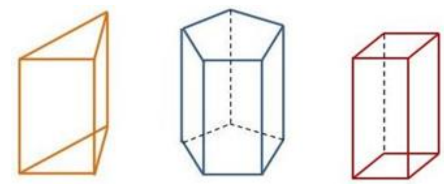
Volume: $\frac{300}{3} = 100\ cm^3$

Volume of Prisms and Pyramids

Recall:

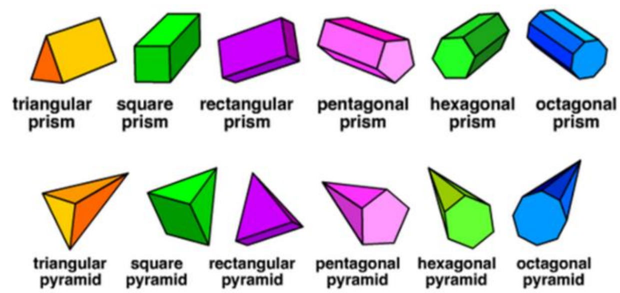
How can we find the volume of any prism given the area of the base and the height?

$$V_{prism} = (A_{base})(height)$$



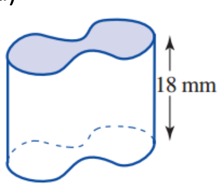
What is the relationship between the volume of a pyramid and the volume of a prism with same base area and height?

$$V_{pyramid} = \frac{1}{3}(V_{prism})$$



Example 1: Determine the volume of the following solids.

a)

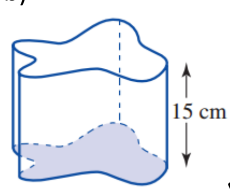


[Base area: 25 mm²]

$$V_{prism} = (A_{base})(height)$$
$$= (25)(18)$$
$$= 450 \text{ mm}^3$$

volume units are cubed

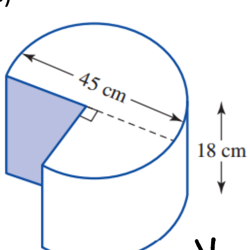
b)



[Base area: 24 cm²]

$$V_{prism} = (A_{base})(height)$$
$$= (24)(15)$$
$$= 360 \text{ cm}^3$$

c)



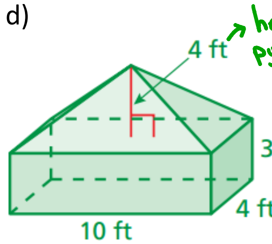
$A_{circle} = \pi r^2$

$A_{base} = \frac{3}{4}$ of circle

$r = \frac{45}{2}$ cm

$$V_{prism} = (A_{base})(height)$$
$$= \left(\tfrac{3}{4}\pi r^2\right)(h)$$
$$= \tfrac{3}{4}\pi\left(\tfrac{45}{2}\right)^2(18)$$
$$\doteq 21470.82 \text{ cm}^3$$

d)



height of pyramid

height of prism

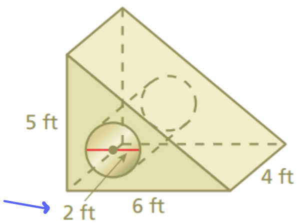
$A_{base} = lw$
$= (10)(4)$
$= 40 \text{ ft}^2$

$$V_{total} = V_{prism} + V_{pyramid}$$
$$= (40)(3) + \tfrac{1}{3}(40)(4)$$
$$= 120 + \tfrac{160}{3}$$
$$\doteq 173.33 \text{ ft}^3$$

Example 2: Determine the volume of the following solids.

a)

$r = \frac{2}{2} = 1\text{ ft}$

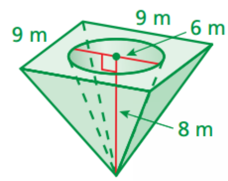


$$V = V_{prism} - V_{cylinder}$$
$$= \tfrac{1}{2}(lwh) - (\pi r^2)(h)$$
$$= \tfrac{1}{2}(6)(4)(5) - \pi(1)^2(4)$$
$$= 60 - 4\pi$$
$$\doteq 47.43\text{ ft}^3$$

$\therefore$ The volume is about 47.43 ft^3.

b)

$r = \frac{6}{2} = 3\text{m}$

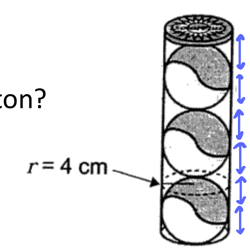


$$V = V_{pyramid} - V_{cone}$$
$$= \tfrac{1}{3}s^2h - \tfrac{1}{3}\pi r^2 h$$
$$= \tfrac{1}{3}(9)^2(8) - \tfrac{1}{3}\pi(3)^2(8)$$
$$= 216 - 24\pi$$
$$\doteq 140.6\text{ m}^3$$

$\therefore$ The volume is about 140.6 m^3.

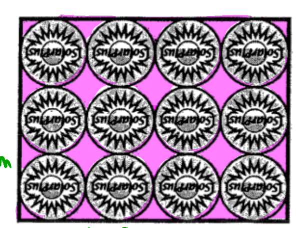
Example 3: Three tennis balls that have a radius of 4 cm are stacked in a cylindrical can.
A rectangular carton holds 12 cylindrical cans that each contains 3 tennis balls.
How much empty space is in the carton once the 12 cans are placed in the carton?

$h = 6r$
$= 6(4)$
$= 24\text{ cm}$



$$V_{empty\ space} = V_{carton} - 12V_{can}$$
$$= lwh - 12(\pi r^2 h)$$
$$= (32)(24)(24) - 12[\pi(4)^2(24)]$$
$$= 18432 - 4608\pi$$
$$\doteq 3955.54\text{ cm}^3$$

$w = 6r$
$= 6(4)$
$= 24\text{cm}$



$l = 8r$
$= 8(4)$
$= 32\text{ cm}$

$\therefore$ There is about 3955.54 cm^3 of empty space.