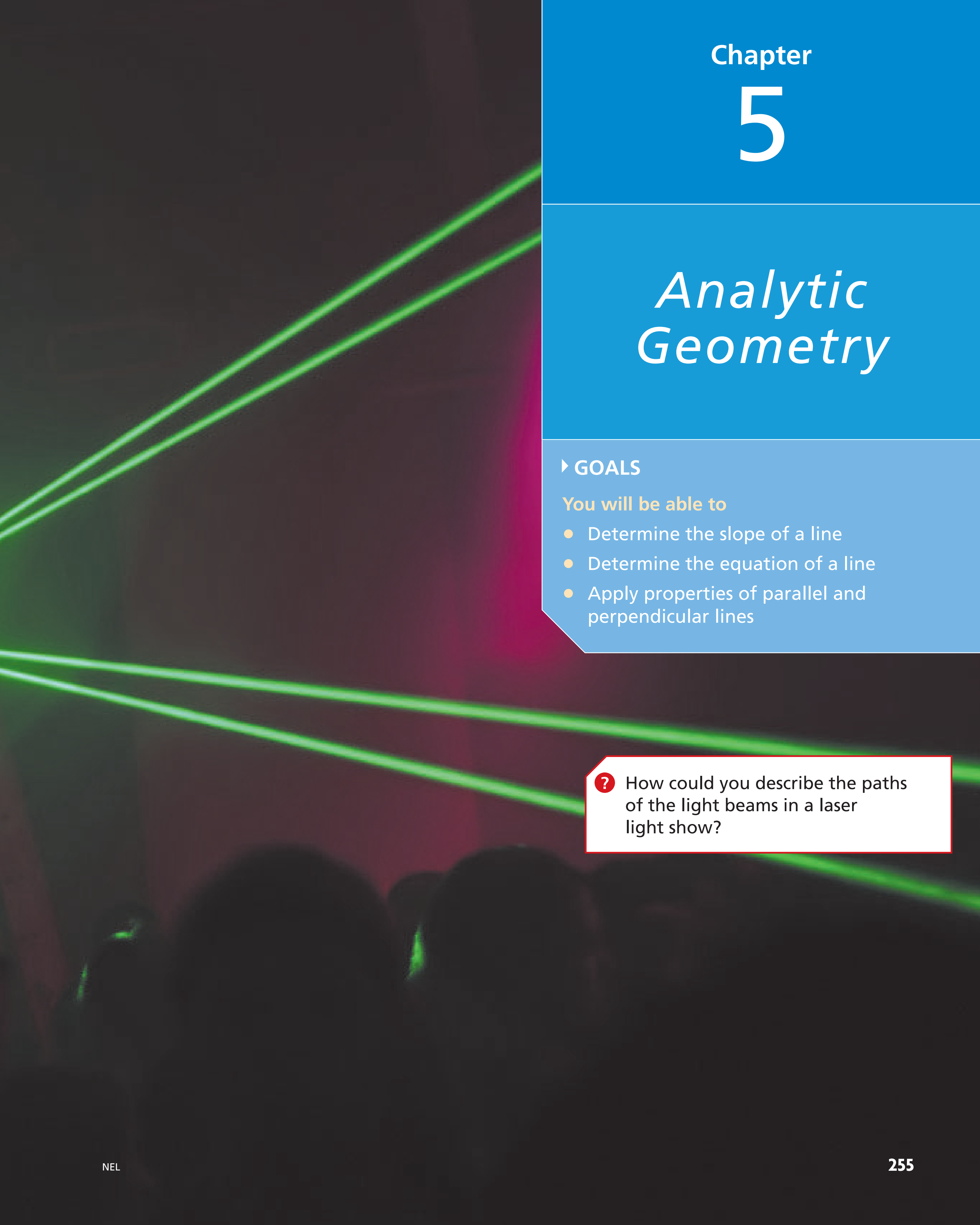


Chapter
5

Analytic Geometry

GOALS

You will be able to

- Determine the slope of a line
- Determine the equation of a line
- Apply properties of parallel and perpendicular lines

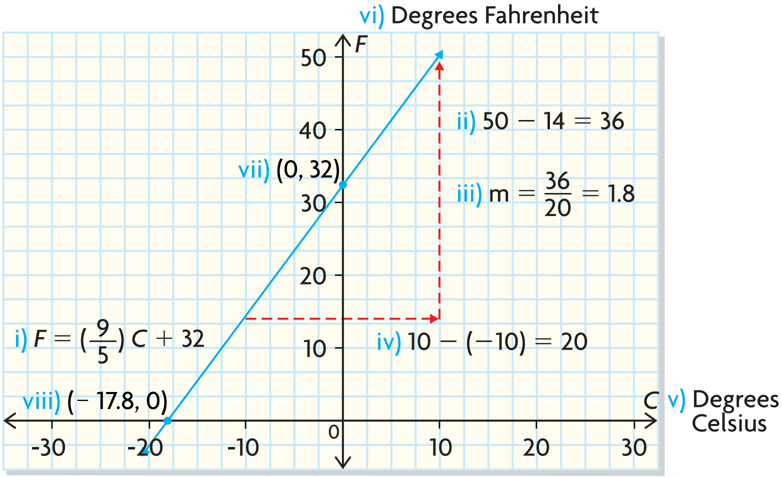
? How could you describe the paths of the light beams in a laser light show?

5 Getting Started

WORDS YOU NEED to Know

1. The graph shows the relationship between temperatures recorded using the Celsius scale and those recorded using the Fahrenheit scale. Match each word with the labelled part of the graph below that most closely represents it.

- **a)** x-intercept
- **b)** y-intercept
- **c)** equation of a line
- **d)** slope of a line
- **e)** rise
- **f)** run
- **g)** independent variable
- **h)** dependent variable



SKILLS AND CONCEPTS You Need

Determining the Reciprocal of a Rational Number

The product of a rational number and its reciprocal is 1.

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

EXAMPLE

Determine the reciprocal of $-1\frac{1}{2}$.

Solution

$$-1\frac{1}{2} = \frac{-3}{2}$$

The reciprocal of $\frac{-3}{2}$ is $\frac{2}{-3} = \frac{-2}{3}$.

2. Determine the reciprocal of each rational number.

a) $\frac{-3}{5}$ **b)** $2\frac{3}{4}$ **c)** $-3\frac{1}{3}$ **d)** 1.5

Using Inverse Operations to Isolate a Variable

You can solve a relation or formula for one variable in terms of the other(s) by using inverse operations to isolate that variable.

EXAMPLE

Solve for m.

$$4m - 3n + 2 = 4$$

Solution

$$\begin{aligned} 4m - 3n &= 4 - 2 \\ 4m - 3n + 3n &= 4 - 2 + 3n \\ 4m &= 2 + 3n \\ \frac{4m}{4} &= \frac{2 + 3n}{4} \\ m &= \frac{2 + 3n}{4} \\ m &= \frac{3n}{4} + \frac{2}{4} \\ m &= \frac{3n}{4} + \frac{1}{2} \end{aligned}$$

3. Isolate or solve for the indicated variable.

a) $V = bh; b$

b) $3x + 5y = 15; y$

c) $P = 2l + 2w; w$

d) $v = \frac{d}{t}; d$

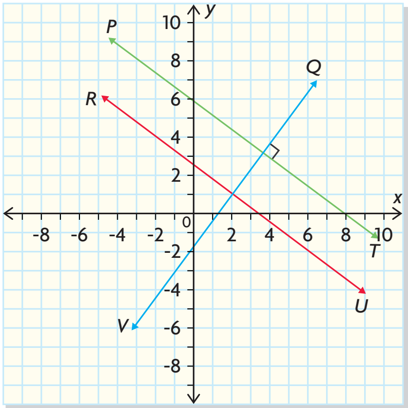
e) $I = Prt; r$

f) $5x + 2y - 20 = 0; x$

Recognizing Parallel and Perpendicular Lines Geometrically

Parallel lines do not intersect and perpendicular lines intersect at 90°.

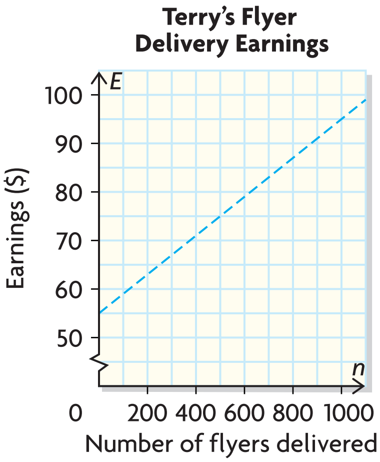
4. From the following graph, name pairs of lines that are parallel or perpendicular.



Study | *Aid*

- For help, see the following examples.

Question	Examples
5	Lesson 4.2 Examples 1–3 and Lesson 4.3 Examples 1 and 3.
6	Lesson 3.5 Examples 1 and 2.
8 and 9	Lesson 3.4 Examples 2 and 3.



PRACTICE

5. Solve each equation.

a) $4x - 15 = 13$ **c)** $12 - 5p = 3p + 8 - 7p$

b) $-4 = \frac{2}{3}m + 6$ **d)** $6t + 3(2t - 5) = 4 - 7t$

6. Tell why each of the following does or does not represent a linear relationship.

a) $y = 7.5x$ **b)** $y = 2x^2 - 8$ **c)** $4x + 2y - 14 = 0$

7. Create a table of values, then use it to graph each relation.

a) $y = 4x + 1$ **c)** $y = \frac{-1}{2}x - 3$

8. Determine the x- and y-intercepts for each of these linear equations.

a) $y = -5x + 4$ **b)** $2x + 7y + 14 = 0$

9. Terry delivers the local paper. He earns a fixed amount plus an additional amount based on the number of flyers he delivers with the paper as shown in the graph to the left.

a) What is the fixed amount Terry earns?

b) How much does Terry earn for each flyer delivered?

c) How does your answer in part b) relate to the slope of the line in the graph?

d) Express the slope of the graph as a rate of change value.

10. Keisha double-booked herself for babysitting Saturday night and now she must make a choice. The Herteises will pay her $10 plus $2/hour and the Farids will pay her $6 plus $4/hour.

a) Write an equation for each family to represent the relationship between the numbers of hours worked and Keisha's earnings.

b) Graph both equations.

c) For which family should Keisha choose to babysit? Explain.

11. Complete the following Anticipation Guide in your notes. Decide whether you agree or disagree with each statement. Justify your thinking.

	Statement	Agree	Disagree
a)	The slope of a line can only be found by graphing the line.		
b)	A line will always pass through at least two quadrants on a grid.		
c)	All points on a vertical line have the same *x*-coordinate.		
d)	When two lines on a Cartesian grid intersect at a 45° angle, one line has a slope greater than 1 and the other has a slope less than 1.		

APPLYING What You Know

Counting Coins

Meaghan has decided to cash in her piggy bank to buy her mom a birthday present. She has 59 coins, which are all quarters and dimes.

YOU WILL NEED

- grid paper

? How much money might Meaghan have?

A. If there were 10 quarters in Meaghan's piggy bank, how much money would she have?

Number of Quarters	Number of Dimes	Value of Quarters	Value of Dimes	Total Value
10				

B. Copy the table above and calculate at least five possible coin combinations using all 59 coins in her piggy bank.

C. What is the least amount of money that Meaghan could have in her piggy bank? How do you know?

D. What is the greatest amount of money that Meaghan could have in her piggy bank? How do you know?

E. Create a graph showing the total value versus number of quarters.

F. Is this a linear relationship? How do you know?

G. Use your graph to determine how many of each coin Meaghan has if the total value of her coins is $11.

5.1

Exploring the Equation of a Line

YOU WILL NEED

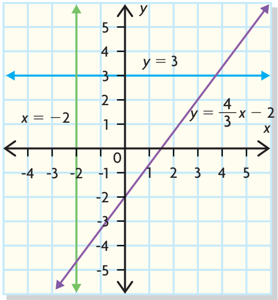
- dynamic geometry software

GOAL

Determine the significance of the values of m and b in the equation $y = mx + b$.

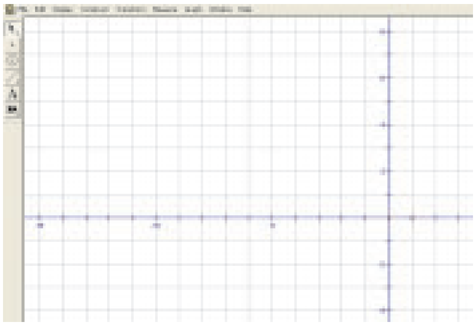
EXPLORE the Math

The graph on the left shows three lines and their equations.



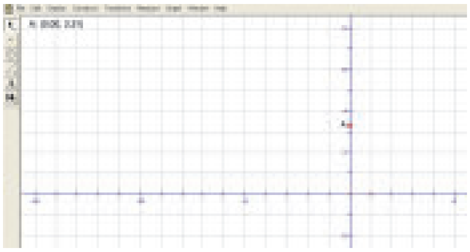
? How is the graph of a line related to its equation?

A. Use *The Geometer's Sketchpad* (GSP) to create a Cartesian grid with *x*- and *y*-axes.



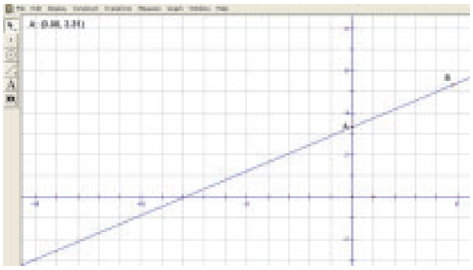
B. Plot a point on the *y*-axis and label it A.

Use MEASURE to determine A's coordinates.



C. Plot a point *not* on the *y*-axis and label it B.

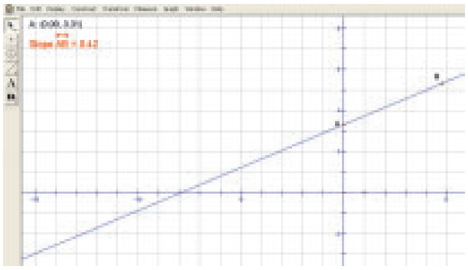
Construct a line to join A and B.



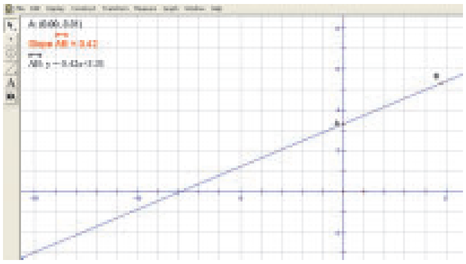
Tech | *Support*

For information on how to use *The Geometer's Sketchpad*, to plot points, and construct a line and determine its slope and equation, see Appendices B13 to B19.

D. Use MEASURE to determine the slope of the line.



E. Use MEASURE to determine the equation of the line.



F. Move the line around the grid and observe what happens.

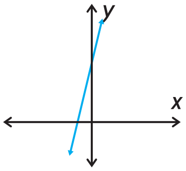
Copy and complete the table, then record five to ten different observations.

Slope	*y*-intercept	Linear Equation

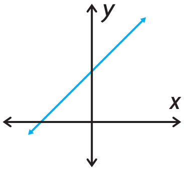
G. Select point B and move the line around on the grid to create each of the types of lines shown below. Each time, observe the changes to the line's slope and equation.

Lines Rising to the Right

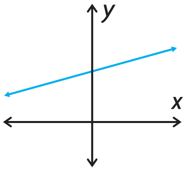
STEEP



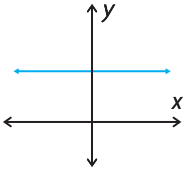
ABOUT 45°



GENTLE

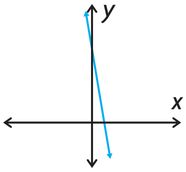


Horizontal Line

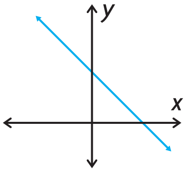


Lines Falling to the Right

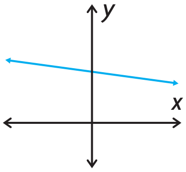
STEEP



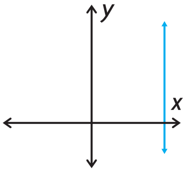
ABOUT 45°



GENTLE



Vertical Line



Record your observations in a copy of the following table.

Description and Picture of Line	Possible Value(s) of Slope	Sample Equations

Reflecting

H. How does the slope value influence the steepness and direction of the line?

I. Given the equation of a line, how could you tell if the line was horizontal or vertical?

J. In the linear relation $y = mx + b$, how do the position and slope of the line change as m and b change?

K. Given the equation of the line in the form $y = mx + b$, how could you determine the line's slope and y-intercept? Use an example in your explanation.

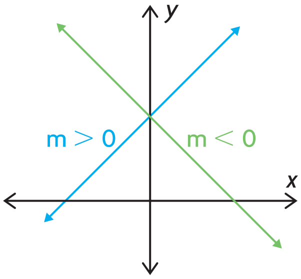
In Summary

Key Idea

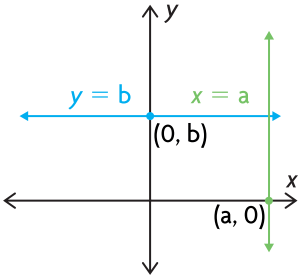
- $y = mx + b$ is the equation of a line where m represents the slope of the line and b represents the y-intercept of the line.

Need to Know

- The value of the slope determines the steepness and direction of the line. The greater the **magnitude** of the m-value, the steeper the line.
- The value of the y-intercept is a distance from the origin, where the graph crosses the y-axis.
- A line rising to the right has a positive slope.
- A line falling to the right has a negative slope.



- A horizontal line has a slope of 0 and its equation has the form $y = b$, where b is the value of the y-intercept.
- A vertical line has an undefined slope and its equation has the form $x = a$, where a is the value of the x-intercept.



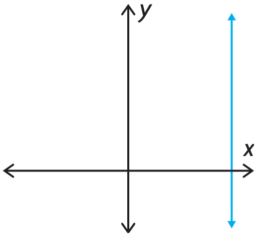
FURTHER Your Understanding

1. Match each linear equation with the graph that best represents it.

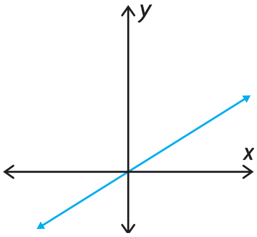
a) $y = -3x + 5$ **c)** $y = \frac{5}{8}x$ **e)** $x = 5$

b) $y = 7x - 4$ **d)** $y = -\frac{1}{4}x - 4$ **f)** $y = 3$

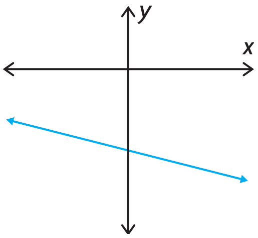
i)



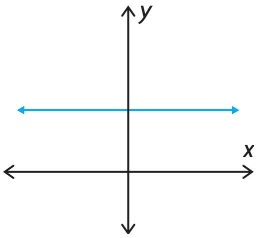
iv)



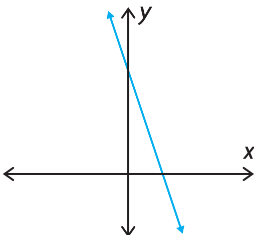
ii)



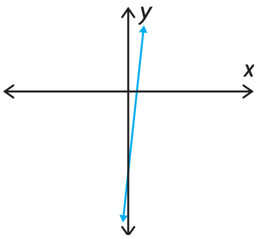
v)



iii)



vi)



2. Consider the lines formed by each of the following equations.

a) $y = -2x + 8$

$y = -\frac{15}{2}x + 3$

$y = -\frac{1}{2}x - 7$

b) $y = \frac{1}{3}x + 1$

$y = 3x - 9$

$y = x + 5$

i) Identify the steepest and the least steep line in each of parts a) and b).

ii) Use the slope and y-intercept to sketch the graphs to verify your answers in i).

3. a) What linear equation represents the x-axis?

b) What linear equation represents the y-axis?

5.2

Different Forms of the Equation of a Line

YOU WILL NEED

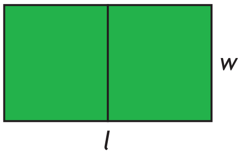
- graphing calculator

GOAL

Given an equation in the form $Ax + By + C = 0$ or $Ax + By = D$, express the equation in the form $y = mx + b$.

LEARN ABOUT the Math

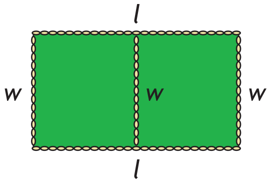
David is a dog breeder and needs to construct two identical, adjacent rectangular pens to contain the male and female puppies. He has 24 m of fencing material available.



? What are some possible values for the length and width of the pens?

EXAMPLE 1 Using an algebraic strategy

Determine possible lengths and widths for the pens.



Pietr's Solution: Isolating *l* and using the slope and *y*-intercept to sketch a graph

l represents the length of the pens and *w* represents the width.

I chose variables to represent each value.

$$2l + 3w = 24$$

I wrote an equation using two lengths and three widths to represent the total amount of fencing.

$$\begin{aligned} 2l + 3w - 3w &= 24 - 3w \\ 2l &= -3w + 24 \\ \frac{2l}{2} &= \frac{-3w + 24}{2} \\ \frac{2l}{2} &= \frac{-3w}{2} + \frac{24}{2} \\ l &= -\frac{3}{2}w + 12 \\ y &= -\frac{3}{2}x + 12 \end{aligned}$$

I used inverse operations to isolate the length variable *l*, so that I could calculate a length to go with any width I chose.

I replaced *l* with *y* since it is the dependent variable and *w* with *x* since it is the independent variable.

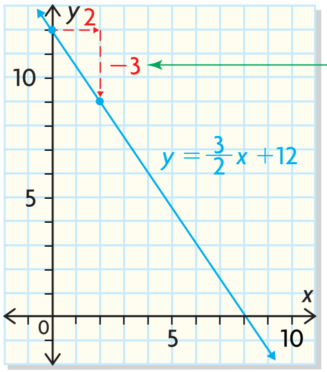
$$y = -\frac{3}{2}x + 12$$

The equation is in the form $y = mx + b$.

I know that m tells the slope and b tells the y-intercept.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = -\frac{3}{2}$$

$$y\text{-intercept} = 12$$



I plotted the y-intercept first.

Starting at the y-intercept, I used the run to move 2 units right and the rise to move 3 units down to determine a second point on the line.

Then, I drew the line joining the two points.

Some possible dimensions for the pens are:

$w = 2$ m and $l = 9$ m

$w = 3$ m and $l = 7.5$ m

$w = 4$ m and $l = 6$ m

$w = 6$ m and $l = 3$ m

I used the graph to locate other points that were on the line. Each ordered pair (x, y) or (w, l) represents the dimensions of a pen with a perimeter of 24 m.

Hanxiang's Solution: Isolating a variable to graph the relation with a graphing calculator

l represents the length of the pens and w represents the width.

I chose variables to represent each value.

$$2l + 3w = 24$$

The diagram has 2 horizontal and 3 vertical sides, so I wrote a sum equal to the amount of fencing.

$$2l + 3w - 2l = 24 - 2l$$

$$3w = -2l + 24$$

$$\frac{3w}{3} = \frac{-2l + 24}{3}$$

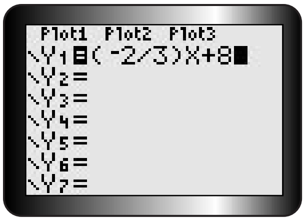
$$\frac{3w}{3} = \frac{-2l}{3} + \frac{24}{3}$$

$$w = -\frac{2}{3}l + 8$$

My graphing calculator requires linear relations to be entered in the form $y = mx + b$. I decided to isolate w so that l would be the independent variable.

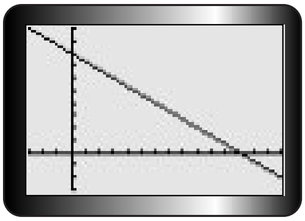
Tech | ***Support***

For help with graphing a linear equation using a graphing calculator, see Appendix B-3.



```
Plot1 Plot2 Plot3
\Y1=(-2/3)X+8
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

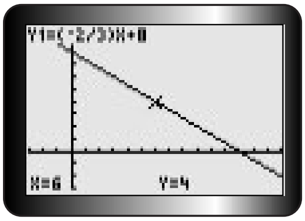
I entered the equation into the graphing calculator. I replaced the independent variable *l* with *x*, and the dependent variable *w* with *y*.



Since I knew that both the width and the length had to be positive, I changed my window settings so that I could focus on the graph's values in quadrant 1.

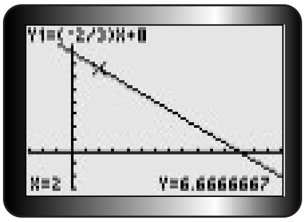
Tech | ***Support***

For help determining values of a relation, see Appendix B-4.

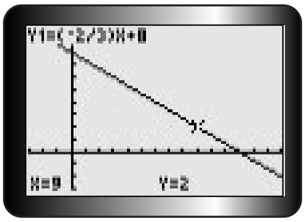


$l = 6$ and $w = 4$

I used the value operation to get some possible values for *l* and *w*.



$l = 2$ and $w = 6.\overline{66}$



$l = 9$ and $w = 2$

Some possible dimensions are:
$l = 6$ m and $w = 4$ m
$l = 9$ m and $w = 2$ m
$l = 2$ m and $w \doteq 6.7$ m

This used a little more than 24 m of fencing, so I knew that the answer wasn't exact.

Reflecting

A. Why do Pietr's and Hanxiang's forms of the equation give some of the same values for l and w?

B. How did isolating a variable help each student to solve the problem?

APPLY the Math

EXAMPLE 2 Using an algebraic strategy to determine the slope and the *y*-intercept

Determine the slope and the y-intercept of the line $3x + 4y + 8 = 0$.

Sara's Solution

$$\begin{aligned} 3x + 4y + 8 &= 0 \\ 3x - 3x + 4y + 8 - 8 &= 0 - 3x - 8 \\ 4y &= -3x - 8 \\ \frac{4y}{4} &= \frac{-3x - 8}{4} \\ y &= \frac{-3}{4}x - \frac{8}{4} \\ y &= \frac{-3}{4}x - 2 \end{aligned}$$

I wanted the equation in the form $y = mx + b$, to determine the values of m and b.

I used inverse operations to isolate y.

I knew that if the equation was in the form $y = mx + b$, m would give the slope, and b would give the y-intercept.

The slope is $-\frac{3}{4}$ and the y-intercept is -2.

EXAMPLE 3 Using an equation to represent and solve a problem

Sam has two part-time jobs. At the grocery store he earns \$8/h and at the library he earns \$10/h. Before going on vacation, he would like to save \$280. Determine the fewest number of hours he needs to work to achieve his goal.

Aaron's Solution

G is the number of hours worked at the grocery store. L is the number of hours worked at the library.

I chose variables for the two unknowns.

$$\begin{aligned}
8G + 10L &= 280 \\
8G - 8G + 10L &= -8G + 280 \\
10L &= -8G + 280 \\
\frac{10L}{10} &= \frac{-8G + 280}{10} \\
\frac{10L}{10} &= \frac{-8G}{10} + \frac{280}{10} \\
L &= \frac{-4}{5}G + 28
\end{aligned}$$

I had to multiply each hourly rate by the number of hours to get the total earnings.

I used inverse operations to isolate *L*.

G	**L**	**Total Hours Worked**
0	$-\frac{4}{5}(0) + 28$ $= 28$	28
5	$-\frac{4}{5}(5) + 28$ $= 24$	29
7.5	$-\frac{4}{5}(7.5) + 28$ $= 22$	29.5
20	$-\frac{4}{5}(20) + 28$ $= 12$	32
45	$-\frac{4}{5}(45) + 28$ $= -8$	?

I calculated some possible solutions for the problem by choosing a value for *G* and substituting it into my equation.

My last choice meant that Sam worked a negative number of hours at the library, which is impossible. On the other hand, if Sam worked 45 hours at the grocery store he would earn \$360, which is more than the \$280 he wants to save.

If Sam worked 0 h at the grocery store and 28 h at the library, he would earn enough money for his vacation.

I chose the answer that showed the fewest total hours Sam had to work to earn \$280.

I knew that this made sense because (28 h)(\$10/h) = \$280.

In Summary

Key Idea

- You can take an equation that is in the form $Ax + By + C = 0$ or $Ax + By = D$ and rewrite it into the form $y = mx + b$ by using inverse operations to solve for y.
- You can locate two points on most lines by plotting the y-intercept and locating a second point using the rise and run of the slope. Joining these points with a straight line gives you a sketch of the relation.

Need to Know

- Equations in the form $Ax + By + C = 0$, $Ax + By = D$, and $y = mx + b$ represent linear relations.
- To enter equations into a graphing calculator, write linear equations in the form $y = mx + b$.

CHECK Your Understanding

1. Express the equation $5x + 6y + 15 = 0$ in the form $y = mx + b$.

2. A room contains three-legged stools and four-legged chairs. There are 48 legs altogether.

a) Write an equation to represent the relationship between the number of stools, the number of chairs, and the total number of legs.

b) How many stools could there be?

PRACTISING

3. Express each of the following equations in the form $y = mx + b$.
K Then, state the slope and y-intercept of each line.

a) $4x - 3y = 24$

b) $2x + 5y = 15$

c) $3x - 6y - 14 = 0$

d) $8x + 5y = 0$

e) $4x + 7y - 11 = 0$

f) $2.4x + 1.5y = -3$

4. Use the slope and y-intercept to sketch the graphs of each of the linear relations in question 3.

5. a) Without graphing, predict whether each of the following lines will rise or fall to the right. How do you know?

i) $2x + 3y = 5$

ii) $x - 4y + 10 = 0$

iii) $3x + 5y - 8 = 0$

iv) $2x + 5y = 15$

v) $2.5x - 15y = 20$

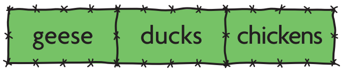
vi) $\frac{x}{2} - 3y = 6$

b) Check your predictions by graphing each line.

6. The dependent variable is d in each of the following equations. Isolate d to determine the d-intercept and the slope of each line.

a) $4t + 3d = 9$
b) $8d - 2h + 16 = 0$
c) $15 + 5k - 6d = 0$

7. A farmer wants to build new enclosures for geese, ducks, and chickens.
A He has 40 m of fencing to build the three identical, adjacent enclosures.



a) Write an equation to represent the amount of fencing required.
b) Rearrange your equation to isolate one of the variables.
c) Graph the relationship.
d) Suggest three possible sets of dimensions for the farmer's enclosures.

8. Evan spent a total of \$18 on gourmet jellybeans and chocolate-covered almonds. The jellybeans cost \$12/kg. The almonds cost \$21/kg.

a) Write an equation to represent Evan's purchases.
b) Isolate the variable for the quantity of jellybeans in your equation.
c) If Evan bought 250 g of almonds, how many grams of jellybeans did he buy?
d) If Evan bought 100 g of almonds, how many grams of jellybeans did he buy?

9. Orenda has a total of 41 loonies and toonies in her piggy bank. Their
T total value is \$59.

a) Write one equation for the total number of coins and a second equation for the total value.
b) Graph both lines.
c) Determine the coordinates of the point of intersection of the lines.
d) How do you know that the coordinates of the point of intersection are the only possible combination of loonies and toonies that meets the conditions of this situation?

10. Amanda plans to make chocolate-chip cookies and oatmeal cookies for a bake sale. The chocolate-chip cookies use three eggs per batch. The oatmeal cookies use two eggs per batch. How many batches of each recipe can she make using two dozen eggs?

11. Textbooks have an average mass of 0.9 kg and notebooks have an average mass of 0.6 kg. To avoid straining his back, Stephen never puts more than 6 kg of books in his backpack.

- **a)** Write an equation to represent the relationship between the number of each type of book and the total maximum mass.
- **b)** Isolate one of the variables in your equation from part a).
- **c)** Determine all possible combinations of textbooks and notebooks that would have a total mass of 6 kg.

12. **a)** Show that $3x - 8y + 5 = 0$ and $y = \frac{3}{8}x + \frac{5}{8}$ represent the same line.

- **b)** Do $y = \frac{2}{3}x + \frac{1}{3}$ and $2x + 3y + 1 = 0$ represent the same line? How do you know?

13. **C** Punitha really only understands how to graph a line if it is in the form $y = \mathrm{m}x + \mathrm{b}$.

- **a)** As her tutor, how would you ensure that Punitha is able to graph lines expressed in any form?
- **b)** What can you tell Punitha about the similarities between all the linear equation forms?

Extending

14. Show that $y = \frac{2}{3}x + \frac{7}{3}$ and $x = \frac{3}{2}y - \frac{7}{2}$ represent the same line.

15. **a)** Determine the slope and y-intercept for each linear equation.

- **i)** $3x + 4y - 8 = 0$
- **ii)** $2x + 5y - 9 = 0$
- **iii)** $4x - 3y = -12$

b) An equation is given in the form $\mathrm{A}x + \mathrm{B}y + \mathrm{C} = 0$.

- **i)** What is the slope of this line?
- **ii)** What is the y-intercept of this line?

5.3 Slope of a Line

YOU WILL NEED

- grid paper
- ruler

GOAL

Determine the slope of a line.

INVESTIGATE the Math

Julien's parents are hiring a caterer for his brother's graduation party.

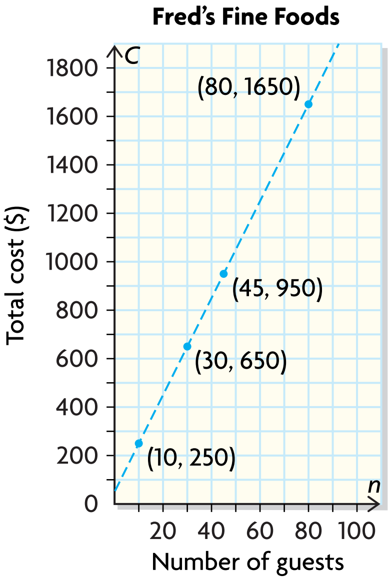
They found the advertisement to the right in the newspaper.

They wonder what the cost per person would be if they used Fred's catering service.

Fred's Fine Foods

Number of Guests	Total Cost ($)
10	250
30	650
45	950
80	1650

? How can Julien's parents determine the cost per person?



Fred's Fine Foods

A. Copy the graph into your notebook.

B. Select two of the four points given on the graph.

C. Draw a right triangle to illustrate the rise and run between the points.

D. Calculate the slope and express it in lowest terms.

E. Record your results in the following table.

First Point	Second Point	Rise	Run	Slope

F. Repeat parts B to E with a second and a third pair of points.

G. How could you have calculated the rise and run without drawing the graph?

H. What is the cost per person if Julien's parents hire Fred's Fine Foods?

Reflecting

I. Why is the slope the same for any pair of points that you choose on the line?

J. If (x_1, y_1) and (x_2, y_2) are points on a line, what formula could you write to calculate the slope of the line?

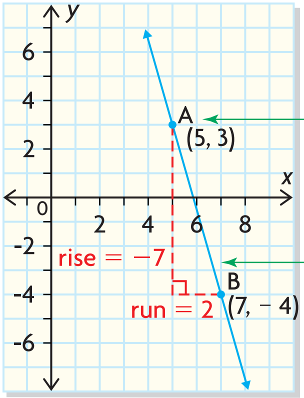
K. Explain why it does not matter which point is (x_1, y_1) and which is (x_2, y_2) when calculating the slope of a line.

APPLY the Math

EXAMPLE 1 Selecting a strategy to determine slope

Calculate the slope of the line passing through points A(5, 3) and B(7, −4).

Rory's Solution: Reasoning by using a graph



I plotted A and B on a grid and drew a line through them. Then, I drew a right triangle to determine the rise and run.

The line sloped down to the right, so I knew that the slope had to be negative.

The slope of the line is $m = -\frac{7}{2}$.

My answer seemed reasonable because the line was decreasing and steep.

Communication | *Tip*

AB often is used to indicate the line segment joining points A and B.

m_{AB} often is used to represent the slope of the line or line segment through points A and B.

Chong Sun's Solution: Calculating using a formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

I decided to use the slope formula.

$$A(\underset{\underset{x_1}{\uparrow}}{5}, \underset{\underset{y_1}{\uparrow}}{3}) \qquad B(\underset{\underset{x_2}{\uparrow}}{7}, \underset{\underset{y_2}{\uparrow}}{-4})$$

I chose point A(5, 3) to be (x_1, y_1) and point B(7, −4) to be (x_2, y_2).

$$m_{AB} = \frac{(-4) - (3)}{(7) - (5)}$$

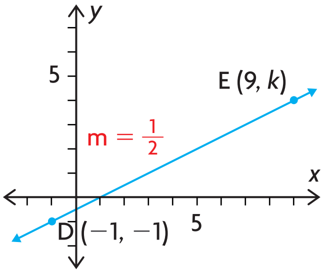
I substituted the values into the slope formula.

$$= \frac{-7}{2}$$

The slope of the line is $m = -\frac{7}{2}$.

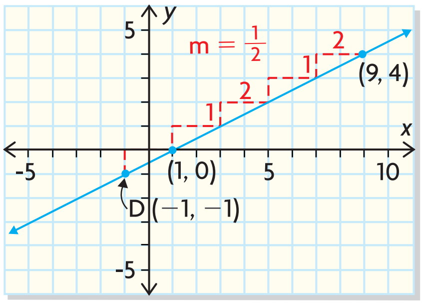
Since the *y*-value decreased a lot and the *x*-value increased only a little, I knew that my answer was reasonable.

EXAMPLE 2 Selecting a strategy to determine an unknown coordinate



Determine the value of *k* in point E using the graph to the left.

Galen's Solution: Reasoning by using a graph



I started at point D. Since the slope was $\frac{1}{2}$, the rise was 1 and the run was 2.

I went up 1 unit from D and to the right 2 units and I got to (1, 0). I continued to go up 1 and right 2 until I got to point E where *x* was 9.

The value of *k* is 4.

The point $(x, y) = (9, 4)$ was on the line with a slope of $\frac{1}{2}$ through points D and E.

Pierce's Solution: Calculating using a formula

The slope of segment DE is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{2} = \frac{(k) - (-1)}{(9) - (-1)}$$

I substituted $\frac{1}{2}$ for m because it was given.

I chose point D as (x_1, y_1) and point E as (x_2, y_2).

I substituted the *x*- and *y*-values into the formula.

$$\frac{1}{2} = \frac{k + 1}{9 + 1}$$

$$\frac{1}{2} = \frac{k + 1}{10}$$

I simplified the equation.

$$10\left(\frac{1}{2}\right) = 10\left(\frac{k + 1}{10}\right)$$

I multiplied both sides of the equation by 10 so that I could work with integers instead of fractions.

$$5 = k + 1$$
$$5 - 1 = k + 1 - 1$$
$$4 = k$$

I solved the equation using inverse operations.

The value of k is 4.

Point E must have the coordinates (9, 4).

EXAMPLE 3 Using the slope formula to calculate a rate of change

A bathtub is filling with water at a constant rate. After 3 min the water is 7.5 cm deep, and after 8 min the water is 15 cm deep. At what rate is the depth of water increasing?

Quinn's Solution

Time (min)	3	8
Depth (cm)	7.5	15

I organized the given information in table form.

$$m = \frac{15 - 7.5}{8 - 3}$$
$$= \frac{7.5}{5}$$
$$= 1.5$$

I knew that the slope of a line gave the rate of change of the relation, so I calculated the slope using the given values.

The water depth is increasing 1.5 cm/min.

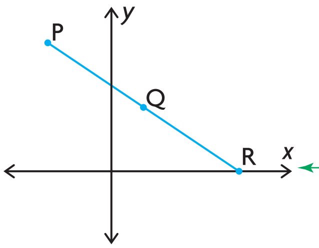
collinear

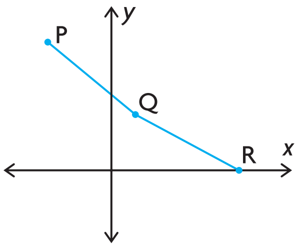
three or more points are collinear if they lie on the same line

EXAMPLE 4 Using reasoning involving slope to determine collinearity

Determine if the points P(−6, 12), Q(3, 6), and R(12, 0) are **collinear**.

Angus's Solution





I started with a rough sketch of the three points, but I couldn't tell if they were on a straight line or not.

I knew that if the points were on a straight line then the graph would look like my first sketch. Otherwise, the graph would look like my second sketch.

Need to determine:
slope of $\overline{PQ}$ and slope of $\overline{QR}$

I noticed that in the second sketch, the slopes of $\overline{PQ}$ and $\overline{QR}$ were different. So, I wanted to calculate the slopes of those two line segments.

$$
\begin{aligned}
m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{12 - 6}{(-6) - 3} \\
&= \frac{6}{-9}
\end{aligned}
$$

I calculated the slope of line segment $\overline{PQ}$.

$Q(\underset{x_1}{3}, \underset{y_1}{6})$ $P(\underset{x_2}{-6}, \underset{y_2}{12})$

The slope of segment $\overline{PQ}$ is $-\frac{2}{3}$.

$$
\begin{aligned}
m_{QR} &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{0 - 6}{12 - 3} \\
&= \frac{-6}{9}
\end{aligned}
$$

I calculated the slope of line segment $\overline{QR}$.

$Q(\underset{x_1}{3}, \underset{y_1}{6})$ $R(\underset{x_2}{12}, \underset{y_2}{0})$

The slope of segment $\overline{QR}$ is $-\frac{2}{3}$.

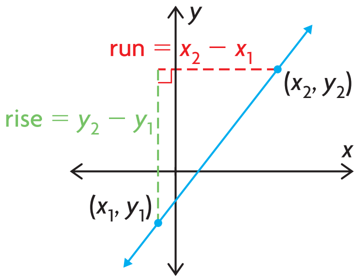
Since $m_{PQ} = m_{QR}$, P, Q, and R are collinear.

Since the slopes were the same for $\overline{PQ}$ and $\overline{QR}$, I knew that an accurate graph would look like my first rough sketch. My points had to be collinear.

In Summary

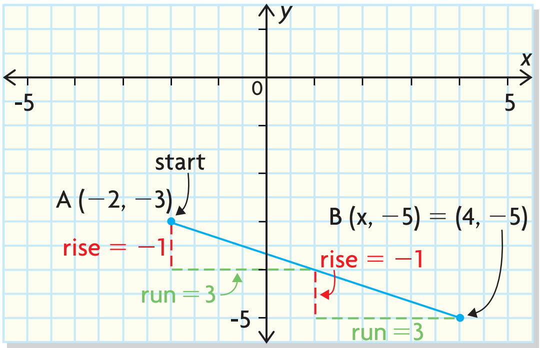
Key Idea

- You can use the formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope of a line if you know the coordinates of two points on the line.



Need to Know

- When you use the coordinates of two points to calculate the slope of a line, either point can be (x_1, y_1).
- Points A, B, and C are collinear if the slopes of any pair of line segments (e.g., $\overline{AB}$, $\overline{BC}$, $\overline{AC}$) are equal.
- If you are given the slope and one point on a line, you can determine an unknown coordinate of another point on the line in two ways:



  - Draw an accurate sketch by starting at the known point, and then use the rise and run repeatedly until you reach the required point. This method only works easily if the coordinates are all integers. For example: Determine the value of x if the slope of $\overline{AB}$ is $-\frac{1}{3}$ with A$(-2, -3)$ and B$(x, -5)$. The value of x is 4.
  - Substitute both points and the given slope into the slope formula and use inverse operations to solve for the unknown coordinate.

CHECK Your Understanding

1. Calculate the slope of the line through each pair of points.

a) A(3, 8) and B(10, 15) **b)** C(9, -2) and D(8, 4)

2. The point $(-2, -3)$ lies on a line with slope $\frac{2}{3}$. Determine the y-coordinate of the point on the line with x-coordinate 13.

PRACTISING

3. Calculate the slope of the line through each pair of points.

K **a)** A(-2, 5) and B(4, -8) **d)** G(-7, 8) and H(4, 8)

b) C(0, 5) and D(-2, 3) **e)** I(3.5, 4.8) and J(1.4, 6.2)

c) E(5, 10) and F(5, -4) **f)** K(32, 630) and L(58, 1020)

4. Write the coordinates of one other point that would be on the line passing through the point A(2, 5) with each of the following slopes.

a) $-\frac{1}{4}$ **b)** $\frac{8}{3}$ **c)** -4 **d)** 0

5. For the points J, K, and L, the slope of segment $\overline{JK}$ is -4 and the slope of segment $\overline{KL}$ is -2. Explain how you know that J, K, and L are not collinear.

6. Determine whether the given points are collinear.

a) A(-8, 0), B(-6, 1), and C(4, 6)
b) D(-5, 17), E(-12, 40), and F(-42, 128)
c) G(-30, -70), H(-15, -38), and I(17, 26)
d) J(-9, 1), K(-12, 3), and L(6, -9)

7. a) Plot the points $(-3, 8)$ and (5, 8) and draw the line that passes through them.
b) Calculate the slope of the line using the slope formula.
c) What can you conclude about the slope of a horizontal line?

8. a) Plot the points (4, 10) and $(4, -1)$ and draw the line that passes through them.
b) Calculate the slope of the line using the slope formula.
c) What can you conclude about the slope of a vertical line?

9. a) Is the rise equal to zero for a vertical line or a horizontal line? Explain.
b) Is the run equal to zero for a horizontal line or a vertical line? Explain.

10. How can you tell from the coordinates of two points if the line passing through them is horizontal, vertical, or slanted?

11. **C** Estimate the slope of the red line in the “No cell phones” sign. (Assume the grid is square.) Explain how you determined your estimate.

12. **A** Nolen was cycling toward his home. After 2 h of cycling he was 55 km from home, and after 4.5 h of cycling he was 17.5 km from home. Assuming he was cycling at a constant rate, how fast was he cycling?

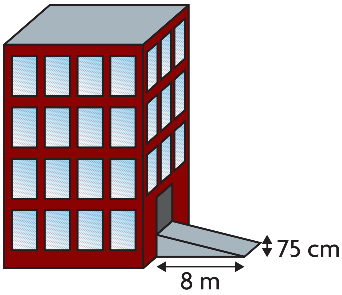
13. Manpreet works for Vision Optical where she earns an hourly rate and receives a fixed amount each week to cover her expenses. This table shows her earnings for various hours worked.

Hours Worked	20	25	30	40	50
Weekly Earnings ($)	390	450	510	630	750

a) Draw a graph of weekly earnings vs. hours worked.
b) Calculate her rate of pay per hour.
c) How long would Manpreet have to work in order to earn $900?
d) Do you think it is likely that Manpreet will earn $900 in a single week? Explain.

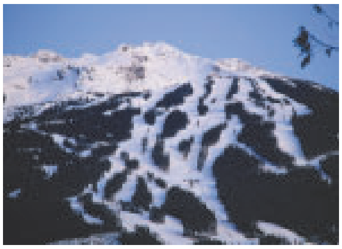
14. A house worth $150 000 in 1999 increased by a constant rate to its value of $255 000 in 2007. Calculate the home’s annual rate of increase in value.

15. **T** A wheelchair ramp should have a slope of $\frac{1}{12}$ or less.



a) Does this wheelchair ramp meet the requirements?
b) If a second ramp is to be built with a rise of 90 cm, what is the shortest length that will still meet the building code?

16. Information about the three most popular runs at a ski resort is shown in the following table.



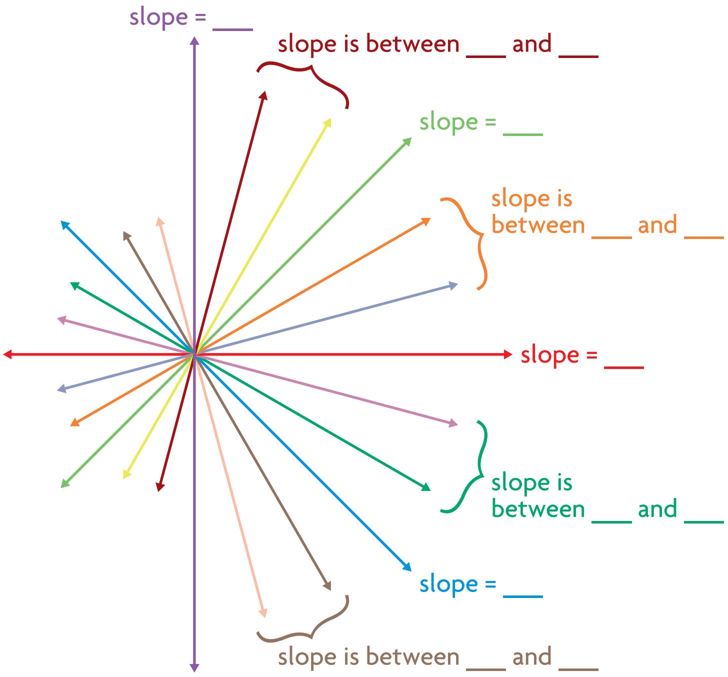
Name of Run	**Vertical Drop (feet)**	**Length of Slope (feet)**
Snowbowl	256	890
Bear Claw	480	4824
The Vortex	510	3438

a) Use the Pythagorean theorem to determine the horizontal “run” for each ski slope. Assume each ski slope is perfectly straight (no dips or moguls).
b) Use slopes to rank each ski run from easiest to most difficult.

17. If you hire Daminga's Delicious Dinners to cater a party, it will cost \$450 for 20 guests and \$675 for 35 guests. If the company charged a fixed rate per guest, calculate the cost per person.

18. Determine the value of k if the points X(3, 2), Y(k, 8), and Z($k + 7$, 29) are collinear.

19. Complete the following picture in your notebook to summarize what you know about slopes of lines.



Extending

20. Consider the points A(7, k), B(11, 4), and C(13, $1 - 3k$).

a) If A, B, and C are collinear, determine the value of k.

b) If A, B, and C are collinear, determine the coordinates of A and C.

c) Determine a possible value for k for which the points would not be collinear.

21. For each situation, write an equation of the line in the form $y = mx + b$.

a) The slope is 5 and the y-intercept is 2.

b) The slope is -4 and the point (4, -3) is on the line.

c) The slope is $\frac{2}{3}$ and the point (6, 4) is on the line.

d) The line is vertical and passes through the point (2, 5).

e) The line is horizontal and passes through (-1, -2).

5 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: In the equation $y = mx + b$, how are the values of m and b related to the graph of the line?

A: The value of b identifies the y-intercept of the line. Changing the value of b will cause a change in where the line crosses the y-axis.

The value of m identifies the slope of the line, which indicates the steepness and direction of the line. The greater the **magnitude** of the m-value, the steeper the line. If m is negative, the line falls to the right; if m is positive, the line rises to the right.

The slope of a vertical line is undefined and its equation looks like $x = a$.

The slope of a horizontal line is 0 and its equation looks like $y = b$.

Study *Aid*

- See Lesson 5.1.
- Try Mid-Chapter Review Questions 1 and 5.

Q: If the equation of a line is in the form $y = mx + b$, how can the values of m and b be used to graph the line?

A: To graph a line in the form $y = mx + b$, start by plotting the y-intercept. From the y-intercept, follow the rise up if the slope is positive (or down, if the slope is negative), and then the run to the right. Plot this position as a new point. Draw the line through the y-intercept and the new point, and then label the line with its equation.

Study *Aid*

- See Lesson 5.1.
- Try Mid-Chapter Review Questions 1 and 5.

EXAMPLE

Graph the line $y = -\frac{1}{4}x + 4$.

Solution

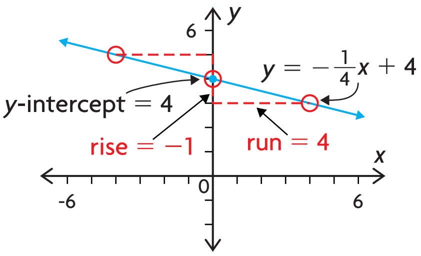
1) Plot the y-intercept, which is 4.

2) Slope $= \frac{-1}{4}\left(= \frac{\text{rise}}{\text{run}}\right)$.

From the y-intercept, go down 1 unit and to the right 4 units and plot the position.

or

From the y-intercept, go up 1 unit and to the left 4 units and plot the position.



Study | *Aid*

- See lesson 5.2, Example 2.
- Try Mid-Chapter Review Question 6.

Q: How can you convert an equation from the form $Ax + By + C = 0$ to the form $y = mx + b$?

A: You use inverse operations to isolate y.

EXAMPLE

Write the equation $4x - 3y + 10 = 0$ in the form $y = mx + b$.

Solution

$$\begin{aligned}
4x - 3y + 10 &= 0 \\
4x - 4x - 3y + 10 - 10 &= 0 - 4x - 10 \\
-3y &= -4x - 10 \\
\frac{-3y}{-3} &= \frac{-4x - 10}{-3} \\
y &= \frac{-4x}{-3} - \frac{10}{-3} \\
y &= \frac{4}{3}x + \frac{10}{3}
\end{aligned}$$

Study | *Aid*

- See Lesson 5.3, Example 1.
- Try Mid-Chapter Review Questions 9 and 10.

Q: How can you use the coordinates of points on a line to determine the slope of the line?

A: You can use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two points on the line.

EXAMPLE

Determine the slope of the line through the points $A(4, -3)$ and $B(-2, 5)$.

Solution

Let $A(4, -3) = (x_1, y_1)$ and $B(-2, 5) = (x_2, y_2)$.

$$\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{(5) - (-3)}{(-2) - (4)} \\
&= \frac{8}{-6} \\
&= -\frac{4}{3}
\end{aligned}$$

Let $A(4, -3) = (x_2, y_2)$ and $B(-2, 5) = (x_1, y_1)$.

$$\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{(-3) - (5)}{(4) - (-2)} \\
&= \frac{-8}{6} \\
&= -\frac{4}{3}
\end{aligned}$$

PRACTICE Questions

Lesson 5.1

1. Identify the slope and y-intercept of each line.

a) $y = 4x - 5$

b) $y = -2x + 3$

c) $y = \frac{3}{7}x - \frac{2}{3}$

2. Describe each line using these words: *horizontal*, *vertical*, *rising to the right*, or *falling to the right*.

a) $y = -3x + 5$

b) $y = -2$

c) $y = \frac{2}{3}$

d) $x = 4.5$

e) $y = 4x - 1$

f) $y = \frac{3}{4}x + \frac{1}{3}$

3. Order each of the following sets of lines based on slope, from closest to horizontal to closest to vertical.

a) $y = x$
$y = 7$
$x = 2$

b) $y = \frac{2}{3}x - 7$
$y = 2.5x - 3.7$
$y = \frac{9}{2}x + 4$

c) $y = -\frac{1}{5}x + 8$
$y = -6x - \frac{5}{8}$
$y = -2x + 4$

4. Suppose each equation in question 3 represents a ski hill.

a) Which two equations could not possibly represent ski hills? Why?

b) Organize the rest of the equations in question 3 into three categories: Bunny Hills (most gentle), Intermediate Hills (moderately sloped), and Double Black Diamond Hills (steepest).

5. Sketch a graph of each of the following equations without using the slope and y-intercept.

a) $y = 2x - 4$

b) $y = -\frac{1}{4}x + 3$

c) $y = -\frac{7}{6}x$

Lesson 5.2

6. Rewrite each of the following equations into the form $y = mx + b$.

a) $6x - 3y - 15 = 0$

b) $3x + 6y + 12 = 0$

c) $2x - 8y = 10$

d) $y - 10 = 0$

e) $4x + y - 9 = 0$

f) $2x - 3y = -1$

7. Movie tickets are \$8 each and concert tickets are \$12 each. Andrew spent a total of \$100 on movie and concert tickets.

a) Write an equation to represent the total cost for movie and concert tickets.

b) Rewrite the equation in the form $y = mx + b$.

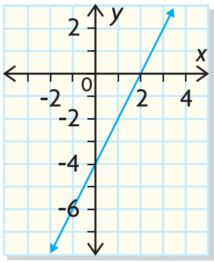
c) Graph the relation.

d) Determine possible combinations of movie and concert tickets that Andrew might have bought.

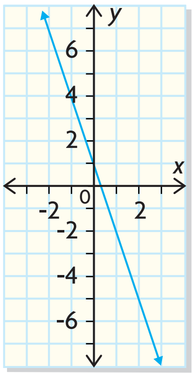
Lesson 5.3

8. Determine the slope of each of the following lines.

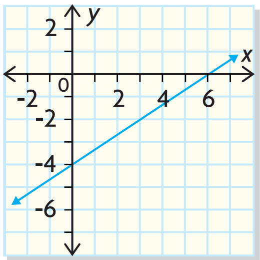
a)



b)



c)



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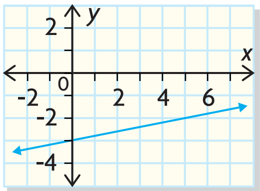

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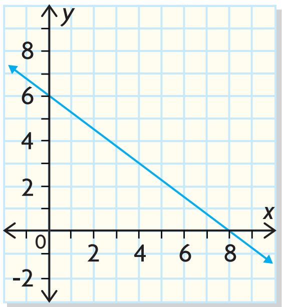

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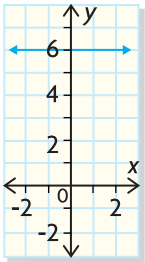


9. Calculate the slope of the line passing through each pair of points.

a) A(3, 8) and B(5, 7)
b) C(8, 9) and D(−2, −2)
c) E(−8, 4) and F(4, −8)
d) I(0, 0) and J(−3, −5)
e) M(0, 4) and N(−3, 4)
f) P(−2, −1) and Q(−2, −9)

10. Determine if the points in each part are collinear.

a) A(−3, −2), B(−2, 1), and C(2, 10)
b) D(7, −1), E(6, 5), and F(2, 1)
c) G(−7, −5), H(−2, 10), and I(−9, −11)
d) J(8, 9), K(−6, 7), and L(24, 11)

11. Point A has coordinates A(3, *k*), and the slope of $\overline{\text{AB}}$ is $\frac{2}{5}$. Determine the value of *k* for each point B.

a) B(7, −2)
b) B(13, 5)
c) B(−2, 2)
d) B(9, 10)

12. A catering company charges $550 for 20 guests and $775 for 35 guests. What is the cost per person?

13. At the end of July, the Robillard family headed home after a vacation. The Robillards were 750 km from home when they started out, but 4 h later they were only 394 km from home. They didn't stop and they maintained a constant speed. How fast were they driving?

5.4 Using Points to Determine the Equation of a Line

GOAL

Determine the equation of a line given information about related points.

YOU WILL NEED

- graphing calculator

LEARN ABOUT the Math

Ken's Kanine Kennel provides suites that dogs in the same family can share. Ken's charges a room fee for the family plus an additional amount for each dog. One day's stay costs \$71 for 2 dogs and \$113 for 5 dogs. Julie wants to know the daily cost to board her 3 dogs.

? How can you determine the equation of this relationship?

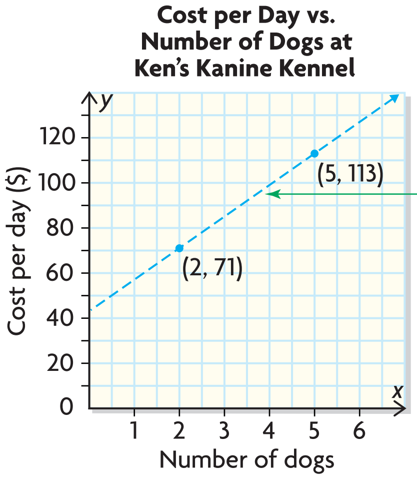
EXAMPLE 1 Using a strategy involving the slope formula and equation solving

Determine the equation that describes the relationship between the number of dogs and the daily boarding cost.

Katerina's Solution

Let x represent the number of dogs and y represent the daily cost.

I chose x to represent the number of dogs because it was the independent variable, and y to represent the cost because the cost depended on the number of dogs.



I sketched the given information so that I could get an idea of what was happening.

$y = mx + b$

I knew that the equation of a line can have the form $y = mx + b$. I needed to determine the slope (m) and the y-intercept (b).

Determine the slope:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{113 - 71}{5 - 2} \\ &= \frac{42}{3} \\ &= 14 \end{aligned}$$

I used the slope formula and the coordinates of the two points.

$(\underset{x_1}{2}, \underset{y_1}{71}) \qquad (\underset{x_2}{5}, \underset{y_2}{113})$

$$y = 14x + b$$

I substituted the value of the slope into my equation.

Determine the *y*-intercept:

$$\begin{aligned} 71 &= 14(2) + b \\ 71 &= 28 + b \\ 71 - 28 &= 28 - 28 + b \\ 43 &= b \end{aligned}$$

From my sketch, I could tell the *y*-intercept was between 40 and 50. I also knew that the point (2, 71) was on the line, so if I substituted 2 for *x* in the equation, then the *y*-value had to be 71. That gave me enough information to solve for b.

$$y = 14x + 43$$

Since I knew the values for both m and b, I could write the equation.

Check:

Left Side	Right Side
	$14x + 43$
113	$= 14(5) + 43$
	$= 70 + 43$
	$= 113$

I checked to see if boarding 5 dogs would cost $113 using my equation. Since both sides were equal, I knew that my equation was correct.

The relationship between the daily boarding cost and the number of dogs is $y = 14x + 43$. This means that Ken's charges a room fee of $43, plus an additional $14 per dog per day.

Reflecting

A. How was Katerina able to determine the slope and the exact value of the *y*-intercept of the line?

B. Explain why Katerina could have used the point (5, 113) to determine a value for b.

C. Katerina determined the value of m before she determined the value for b. Could she have determined b before m? Explain.

APPLY the Math

EXAMPLE 2 Using the slope and one point to determine the equation of a line

Determine the equation of the line that has a slope of 4 and passes through the point (2, 6).

Sherif's Solution

$$y = mx + b$$
$$y = 4x + b$$

I was given a value for the slope, so I substituted 4 for m.

Determine the y-intercept:

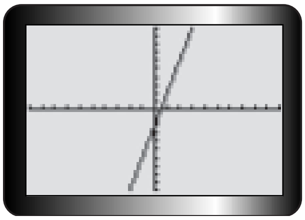
$$6 = 4(2) + b$$

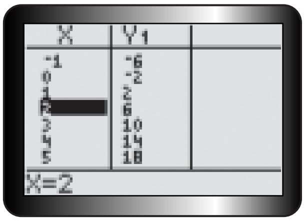
(2, 6) was a point on the line, so I substituted its coordinates for x and y.

$$6 = 8 + b$$
$$6 - 8 = 8 - 8 + b$$
$$-2 = b$$

I solved the equation for b.

The equation of the line is $y = 4x - 2$.





X	Y_1
-1	-6
0	-2
1	2
2	6
3	10
4	14
5	18

X=2

I graphed my equation and used the table to verify that my line passed through the point (2, 6).

EXAMPLE 3 Reasoning from properties of a line to determine its equation

Determine the equation of the line that has a slope of $-\frac{1}{3}$ and the same y-intercept as the line $2x - 4y + 7 = 0$.

Anayis's Solution

Determine the slope:

$$y = mx + b$$
$$y = -\frac{1}{3}x + b$$

I substituted the given value of $-\frac{1}{3}$ for the slope, m.

Determine the y-intercept:

$2x - 4y + 7 = 0$

I needed to determine the y-intercept of $2x - 4y + 7 = 0$ because the lines had the same y-intercept.

$2(0) - 4y + 7 = 0$

I knew that the x-coordinate was 0 for the point where the line crossed the y-axis.

$$\begin{aligned} -4y + 7 &= 0 \\ -4y + 7 - 7 &= 0 - 7 \\ -4y &= -7 \\ \frac{-4y}{-4} &= \frac{-7}{-4} \\ y &= \frac{7}{4} \end{aligned}$$

I used inverse operations to solve the equation for y.

$$b = \frac{7}{4}$$

I knew that the b in the equation represented the y-intercept.

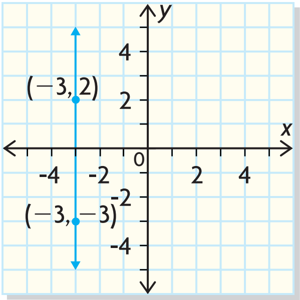
The equation of the line is

$y = -\frac{1}{3}x + \frac{7}{4}$.

EXAMPLE 4 Using reasoning to determine the equation of a line through two points with the same x-coordinate

Determine the equation of the line that passes through the points $(-3, -3)$ and $(-3, 2)$.

Omar's Solution



This line is vertical. The equation of the line is $x = -3$.

From the sketch of the graph, I could see that the line was vertical and that every point on the line had −3 for its x-coordinate.

EXAMPLE 5 Using the equation of a line to solve a problem

The speed of sound in air can be calculated based on temperature using a linear relation. At 10 °C the speed of sound is 337.4 m/s, and at 21.5 °C the speed is 344.3 m/s. What is the speed of sound when the temperature is 32.3 °C?

Suhaila's Solution

Let T represent the temperature and S represent the speed of sound.

I chose variables to represent each unknown quantity.

$$S = mT + b$$

I wanted to find an equation for the relationship first. Using $y = mx + b$, I replaced y with S since sound is the dependent variable and x with T since temperature is the independent variable.

Determine the slope:

$$\begin{aligned} m &= \frac{344.3 - 337.4}{21.5 - 10} \\ &= \frac{6.9}{11.5} \\ &= 0.6 \end{aligned}$$

I substituted the points (10, 337.4) and (21.5, 344.3) into the slope formula.

This means the speed increases 0.6 m/s for each Celsius degree increase in temperature.

Determine the *S*-intercept:

$$S = 0.6T + b$$

I substituted the slope value 0.6 into my equation.

$$\begin{aligned} 337.4 &= 0.6(10) + b \\ 337.4 &= 6 + b \\ 337.4 - 6 &= 6 - 6 + b \\ 331.4 &= b \end{aligned}$$

I used the point $(T, S) = (10, 337.4)$ in my equation to solve for b.

This means that when the temperature is 0 °C the speed of sound is 331.4 m/s. $S = 0.6T + 331.4$ represents the speed of sound S at temperature T.

$$S = 0.6(32.3) + 331.4$$
$$S \doteq 350.8$$

I substituted the given temperature 32.3 °C into my equation to solve the equation.

At 32.3 °C, the speed of sound is 350.8 m/s.

In Summary

Key Ideas

- You can determine the equation of a line in the form $y = mx + b$ if you know two points on the line or one point and the slope.

Need to Know

- You can determine the equation of a line as follows:
  - If the slope is not given, and you know two points on the line, use the coordinates of the points to calculate the slope.
  - Substitute the value of the slope for m and the coordinates (x, y) of a point on the line into $y = mx + b$ and solve for b.
  - Use the values of m and b to write the equation of the line.

CHECK Your Understanding

1. Complete the table on the right by determining the missing values.

	Slope	*y*-intercept	Equation
a)	3	5	
b)			$y = 5x + 1$

2. Match each equation to its corresponding graph.

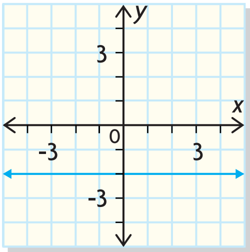
a) $x = -2$

b) $x - 2y = 4$

c) $y = -2$

d) $y = -\frac{4}{3}x + 3$

i)



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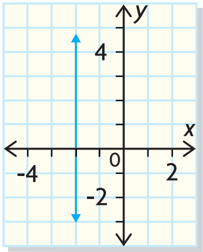

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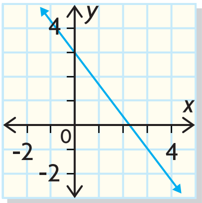

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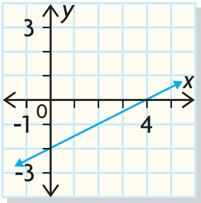


3. Determine the equation of the line with the following characteristics.

a) has a slope of -2 and passes through the point A(5, 2)

b) passes through the points B(4, 6) and C(1, -3)

PRACTISING

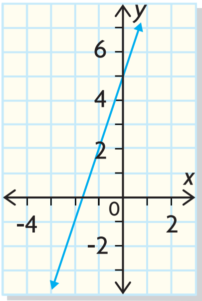
4. Complete the table shown below by determining the missing values.

K

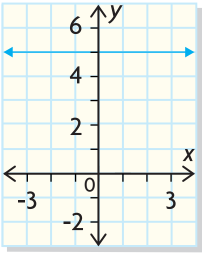
	Slope	y-intercept	Equation
a)	-5	3	
b)			$y = \frac{4}{3}x - 2$
c)	0	2	
d)			$y = \frac{1}{2}x$

5. Determine the equation of each line shown below.

a)



b)



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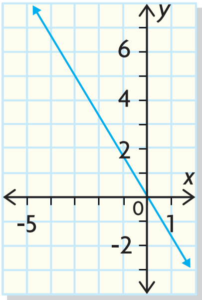
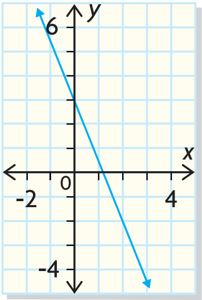


d)



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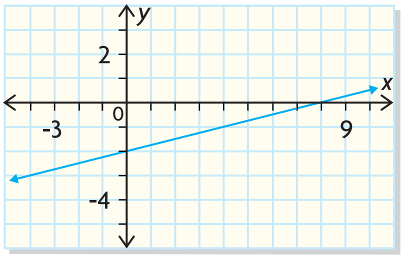

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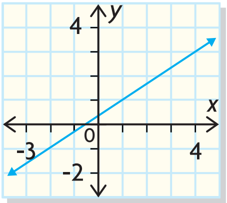


6. If the equation $y = 3x + \text{b}$ represents a line that passes through the given point, determine the value of the y-intercept, b.

a) (4, 1) **b)** $(-3, 2)$ **c)** $(1, -3)$

7. If the equation $y = \text{m}x + 3$ represents a line that passes through the given point, determine the slope value, m.

a) (2, 4) **b)** $(-3, 7)$ **c)** (8, 2)

8. Determine the equation of each line described below.

a) passing through the point A(0, 4), with a slope of $-\frac{8}{9}$

b) passing through the point A(3, -5), with a slope of $\frac{1}{5}$

c) has an x-intercept of 4 and a y-intercept of -3

d) has an x-intercept of 6 and passes through the point (6, 4)

9. Determine the equation of the line passing through each pair of points.

a) A(1, 9) and B(1, -7)

b) C(-8, -3) and D(8, 27)

c) E(-12, 7) and F(4, 7)

d) G(6, 18) and H(-12, 3)

e) I(0, 5) and J(0, 12)

f) K(-5, -1) and L(15, 1)

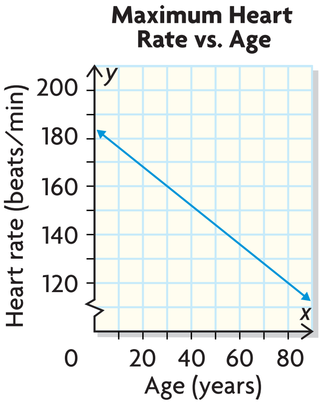
10. Determine the equation of the line that has the same x-intercept as the line described by $x - 5y + 10 = 0$, and the same y-intercept as the line $3x + 2y - 6 = 0$.

11. The LeBlanc family is driving home. The LeBlancs are using cruise control so their speed is constant. After 3 h, they are 350 km from home. After 5 h, they are 130 km from home.

a) Write an equation to represent this distance–time relationship.

b) What do the slope and y-intercept of your equation mean in this situation?

12. A stress test evaluates the health of a patient's heart. While riding on a stationary bike or running on a treadmill, a patient has his or her heart rate measured by a technician and compared with a safe maximum heart rate. This safe heart rate is based on the patient's age as shown in the graph.



a) What does the y-intercept represent in this situation?

b) What does the slope of the graph represent?

c) Determine the equation for the line.

d) Ellen is 14 years old. Using your equation, determine her maximum safe heart rate.

13. The local fall fair charges a flat fee for admission plus an additional cost for ride tickets. Last year, Kelsey purchased 15 tickets and spent a total of \$19.50. His brother Quinn purchased 36 tickets and spent a total of \$30.00 at the fair.

T

a) Determine an equation to represent the relationship between the total amount of money spent and the number of tickets purchased.

b) A ride pass, which gives a person entrance to the park and unlimited use of the rides, costs \$21. Write the equation for the relationship between the total amount spent on a ride pass and the number of rides it can be used for.

c) Last year, Erin used 25 tickets at the fall fair. Should Erin purchase tickets again this year, or buy a ride pass? Explain.

d) Heather only likes the fun house, which requires one ticket. She went on this ride 10 times last year. How much money would Heather save by purchasing tickets instead of a ride pass?

14. Lori downloads music from the Music Genie site, which charges a monthly membership fee plus an amount for each song downloaded. A three-month record of her activity on the site is shown.

A

Month	Number of Songs Downloaded	Monthly Bill (\$)
January	54	26.90
February	38	25.30
March	21	23.60

a) Use two points from the table to determine the equation of the relationship between numbers of downloads and her monthly bill.

b) Verify that the third point from the table also satisfies your equation.

c) Lori's brother thinks she should change to Web Waves, which doesn't have a membership fee and charges \$0.95 per song. Based on your calculations, do you think Lori should change music companies? Explain.

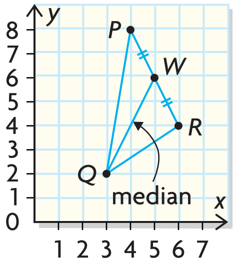
d) Digital Beats charges \$25 for a monthly membership, with unlimited free downloads. Would you recommend Lori change to Digital Beats? Explain.

15. Shawn says he can only figure out the equation of a line if he is given the y-intercept and the slope of the line. Barb says that she can figure out the equation using the coordinates of any two points on the line. With whom do you agree? Why?

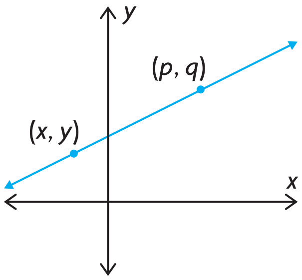
C

Extending

16. Determine the equation of the **median** from Q to the midpoint of PR, in triangle PQR, with P(4, 8), Q(3, 2), and R(6, 4).



17. Given any two points on a line, the equation of the line can be determined from the point-slope form of the equation of the line: $y = \mathrm{m}(x - p) + q$.



a) Show how the formula can be developed using the slope formula, $\mathrm{m} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

b) Use the point-slope form of the equation of a line to determine the equation of a line that has a slope of 3 and passes through the point $(\underset{\overset{\uparrow}{p}}{1},\ \underset{\overset{\uparrow}{q}}{2})$.

c) Determine the equation of the line in part b) using $y = \mathrm{m}x + \mathrm{b}$ to verify that the new formula works.

d) Use the point-slope form of the equation of a line to determine the equation for each of the following lines.

- **i)** passing through the points $(4, -6)$ and $(5, -1)$
- **ii)** passing through the points $(3, -1)$ and $(9, 3)$
- **iii)** passing through the points $(4, 5)$ and $(3, 9)$

5.5 Parallel and Perpendicular Lines

GOAL

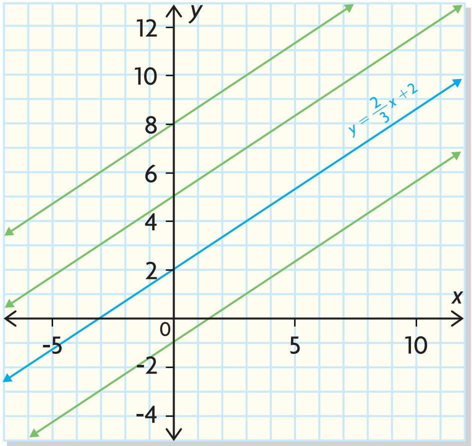
Determine and apply properties and equations of parallel and perpendicular lines.

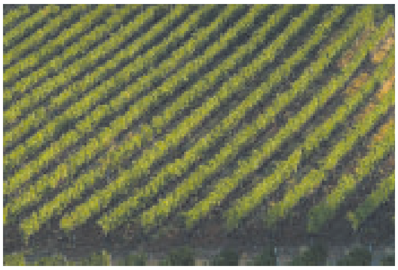
YOU WILL NEED

- grid paper
- graphing calculator
- protractor (optional)

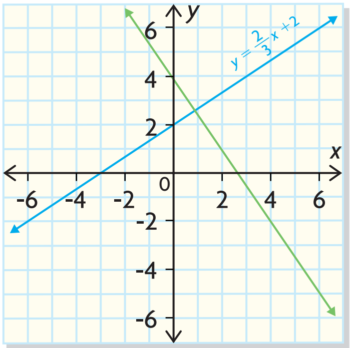
INVESTIGATE the Math

The graph below could represent the rows of vines in a Niagara vineyard.





This graph could represent two jet-plane trails that intersect at right angles.



? How can you tell from the equation of a line whether it is parallel or perpendicular to a given line?

A. Graph and label each of the following lines on separate grids.

$$y = \frac{2}{3}x - 4 \qquad y = -\frac{1}{5}x + 3$$

B. Carefully draw two lines parallel to each of the original lines, and then complete the following table.

	Rise	Run	Slope	y-intercept	Equation
First Original Line					$y = \frac{2}{3}x - 4$
Parallel line 1					
Parallel line 2					
Second Original Line					$y = -\frac{1}{5}x + 3$
Parallel line 1					
Parallel line 2					

C. For each of the lines in part A, write two linear equations that represent lines parallel to the original line.

D. Draw two lines that are perpendicular to each of the original lines, and then complete the following table.

	Rise	Run	Slope	y-intercept	Equation
First Original Line					$y = \frac{2}{3}x - 4$
Perpendicular line 1					
Perpendicular line 2					
Second Original Line					$y = -\frac{1}{5}x + 3$
Perpendicular line 1					
Perpendicular line 2					

E. For each of the lines in part A, write two more linear equations that represent lines perpendicular to the original line.

Reflecting

F. How were the equations of lines parallel to $y = \frac{2}{3}x - 4$ and $y = -\frac{1}{5}x + 3$ like the original equations? How were they different?

G. How were the equations of the lines perpendicular to $y = \frac{2}{3}x - 4$ and $y = -\frac{1}{5}x + 3$ related to the equations of the original lines?

APPLY the Math

EXAMPLE 1 Reasoning about slope to determine whether lines are parallel or perpendicular

Determine which of the following lines are parallel and which are perpendicular to the line defined by $x - 2y = 4$.

a) $y = -2x + 8$ **b)** $y = \frac{2}{4}x + 2$ **c)** $y = \frac{4}{5}x + 2$

Kimmy's Solution

Original line:

$$x - 2y = 4$$
$$-2y = -x + 4$$
$$\frac{-2y}{-2} = \frac{-x + 4}{-2}$$
$$y = \frac{-x}{-2} + \frac{4}{-2}$$
$$y = \frac{1}{2}x - 2$$

I rearranged the original line into the form $y = mx + b$ so that I could determine its slope.

So, $m = \frac{1}{2}$

I determined the slope of the line in part a).

a) $y = -2x + 8$

$m = -2$

$y = -2x + 8$ is perpendicular to $y = \frac{1}{2}x + 4$

I knew that this line was perpendicular to the original line because their slopes were negative reciprocals of each other.

negative reciprocals

numbers that multiply to produce -1 are negative reciprocals of each other (e.g., $\frac{3}{4}$ and $-\frac{4}{3}$; $-\frac{1}{2}$ and 2)

b) $y = \frac{2}{4}x + 2$

I determined the slope of the line in part b).

$$m = \frac{2}{4}$$
$$= \frac{1}{2}$$

$y = \frac{2}{4}x + 2$ is parallel to

$y = \frac{1}{2}x - 2$

I knew that the two lines were parallel because their slopes were equal.

c) $$y = \frac{4}{5}x + 2$$

$$m = \frac{4}{5}$$

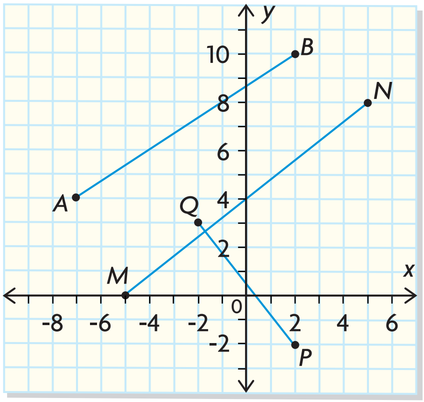
This line is neither parallel nor perpendicular to $y = \frac{1}{2}x - 2$.

I determined the slope of the line in part c).

Since the slope of this line was neither equal to nor the negative reciprocal of the slope of the original line, I knew that the lines could not be described as parallel or perpendicular.

EXAMPLE 2 Identifying perpendicularity by reasoning

Which line segments in the following diagram are perpendicular?



Liz's Solution

$$A(\underset{\substack{\uparrow\\ x_1}}{-7}, \underset{\substack{\uparrow\\ y_1}}{4}) \qquad B(\underset{\substack{\uparrow\\ x_2}}{2}, \underset{\substack{\uparrow\\ y_2}}{10})$$

$$\begin{aligned} m_{AB} &= \frac{10 - 4}{2 - (-7)} \\ &= \frac{6}{9} \\ &= \frac{2}{3} \end{aligned}$$

I calculated the slope of each line segment using the slope formula,

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$M(\underset{\substack{\uparrow\\ x_1}}{-5}, \underset{\substack{\uparrow\\ y_1}}{0}) \qquad N(\underset{\substack{\uparrow\\ x_2}}{5}, \underset{\substack{\uparrow\\ y_2}}{8})$$

$$\begin{aligned} m_{MN} &= \frac{8 - 0}{5 - (-5)} \\ &= \frac{8}{10} \\ &= \frac{4}{5} \end{aligned}$$

$$P(\underset{\substack{\uparrow\\ x_1}}{2}, \underset{\substack{\uparrow\\ y_1}}{-2}) \qquad Q(\underset{\substack{\uparrow\\ x_2}}{-2}, \underset{\substack{\uparrow\\ y_2}}{3})$$

$$\begin{aligned} m_{PQ} &= \frac{3 - (-2)}{-2 - 2} \\ &= \frac{5}{-4} \\ &= -\frac{5}{4} \end{aligned}$$

$$\begin{aligned} m_{AB} \times m_{MN} &= \frac{2}{3} \times \frac{4}{5} \\ &= \frac{8}{15} \end{aligned}$$

I multiplied the slopes to see if any were negative reciprocals.

$$\begin{aligned} m_{AB} \times m_{PQ} &= \frac{2}{3} \times \left(-\frac{5}{4}\right) \\ &= -\frac{10}{12} \\ &= -\frac{5}{6} \end{aligned}$$

$$\begin{aligned} m_{MN} \times m_{PQ} &= \frac{4}{5} \times \left(-\frac{5}{4}\right) \\ &= -\frac{20}{20} \\ &= -1 \end{aligned}$$

I knew that the line segments $\overline{MN}$ and $\overline{PQ}$ were perpendicular because the product of their slopes was -1.

The line segments $\overline{MN}$ and $\overline{PQ}$ are perpendicular.

EXAMPLE 3 Reasoning about slope to determine the equation of a line that is parallel to another line

Determine the equation of the line that is parallel to $y = -\frac{2}{7}x + 3$ and passes through the point (14, 9).

Rahim's Solution

$$y = mx + b$$

I started with the general equation of a line.

Determine the slope:

$$m = -\frac{2}{7}$$

Since the new line is parallel to $y = -\frac{2}{7}x + 3$, I knew that it had the same slope.

$$y = -\frac{2}{7}x + b$$

I substituted the slope into my equation.

Determine the y-intercept:

$$9 = -\frac{2}{7}(14) + b$$

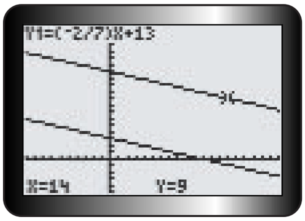
Since the line passed through the point (14, 9), I used $x = 14$ and $y = 9$ in the equation of the line to determine b.

$$9 = -4 + b$$
$$9 + 4 = -4 + 4 + b$$
$$13 = b$$

I simplified, and then used inverse operations to solve for b.

Therefore, $y = -\frac{2}{7}x + 13$.

I substituted my value for b to complete the equation.



```
Y1=(-2/7)X+13
X=14        Y=9
```

I used a graphing calculator to check that the equation I found was parallel to the original equation and passed through (14, 9).

EXAMPLE 4 Selecting a strategy to determine the equation of a line that is perpendicular to another line

Determine the equation of the line that is perpendicular to $y = 3x + 1$ and has the same y-intercept.

Priya's Solution: Using a strategy based on slope and *y*-intercept

$$y = mx + b$$

I knew that if I could determine the line's slope and y-intercept, I could write its equation in the form $y = mx + b$.

Determine the slope:

For $y = 3x + 1$:

$$m = 3$$

The slope of the given line was 3 because the line was in the form $y = mx + b$.

For the new perpendicular line:

$$m = \frac{-1}{3}$$
$$= -\frac{1}{3}$$

The given line had a slope of 3, but I needed to find its negative reciprocal. I rewrote 3 as $\frac{3}{1}$. This made it easier for me to get the negative reciprocal.

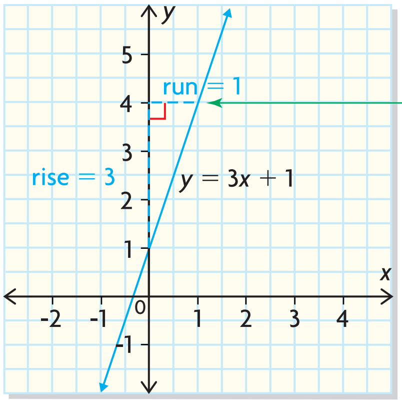
Determine the *y*-intercept:

$$b = 1$$

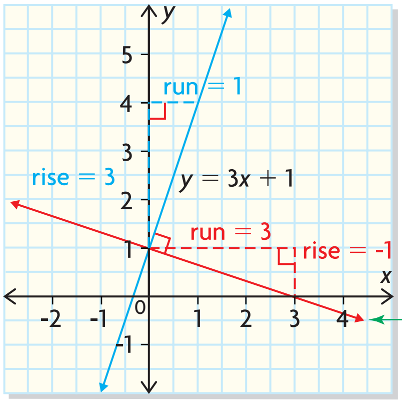
I knew that the *y*-intercept of the new line had to be the same as the *y*-intercept of the line $y = 3x + 1$.

The equation is $y = -\frac{1}{3}x + 1$.

Jacob's Solution: Reasoning from a graph



The slope was $\frac{3}{1}$ and the *y*-intercept was 1, so I used this to draw a graph of the original line.



I rotated the original line 90° to get my new line.

$$\text{new slope} = \frac{1}{-3} = -\frac{1}{3}$$

y-intercept $= 1$

The equation of the new line is

$y = -\frac{1}{3}x + 1.$

I noticed that my rise and run also rotated, so the new rise was 1 and the new run was −3. I used those values to calculate the slope. The y-intercept was the same.

In Summary

Key Ideas

- The slopes of parallel lines are equal.
- The slopes of perpendicular lines are negative reciprocals.

Need to Know

- Two numbers are negative reciprocals if they have opposite signs and their denominators and numerators are exchanged.

  For example, $\frac{-2}{3}$ and $\frac{3}{2}$ are negative reciprocals.

  So are 3 and $\frac{-1}{3}$.
- The product of two negative reciprocals is always −1.

CHECK Your Understanding

1. a) State an equation of a line parallel to $y = -\frac{3}{2}x + 9$.

b) State an equation of a line perpendicular to $y = -\frac{3}{2}x + 9$.

2. Determine which of the following lines are parallel and which are perpendicular to each other.

a) $y = -\frac{1}{3}x + 2$

b) $y = -3x + 2$

c) $y = \frac{7}{2}x - 4$

d) $y = \frac{2}{7}x - 3$

e) $y = \frac{1}{3}x + 1$

f) $y = \frac{1}{-3}x - 8$

g) $y = \frac{-3}{9}x$

h) $y = \frac{-2}{7}x - 9$

PRACTISING

3. For each pair of equations, state whether the lines are parallel,
K perpendicular, or neither.

a) $y = 2x + 5$

$y = -\frac{1}{2}x - 4$

b) $y = \frac{2}{3}x - 2$

$y = -1.5x - 6$

c) $y = \frac{3}{7}x - 4$

$y = -\frac{3}{7}x - 4$

d) $x - 4y = 2$

$2x - 8y = 3$

e) $y = -0.2x - 1$

$y = -\frac{1}{5}x + 3$

f) $x - 5y + 8 = 0$

$5x - y = 0$

4. The following sets of points define the endpoints of line segments. Determine which line segments are parallel and which line segments are perpendicular.
A(6, 5) and B(12, 3)
P(−3, −4) and Q(5, 20)
G(0, −4) and H(6, −2)
U(−5, 9) and V(−6, 12)
K(2, 4) and L(6, 16)

5. Are the lines defined by the equations $y = 4$ and $x = 3$ parallel, perpendicular, or neither? Explain.

6. a) Write the equation of a line parallel to the x-axis that passes through the point (1, 4).
b) Write the equation of a line parallel to the x-axis that passes through the point (3, −8).
c) In general, what is true about the equation of any line parallel to the x-axis?

7. a) Write the equation of a line parallel to the y-axis that passes through the point (−9, 3).
b) Write the equation of a line parallel to the y-axis that passes through the point (6, 2).
c) In general, what is true about the equation of any line parallel to the y-axis?

8. Use the given information to write the equation of each line.

A **a)** a line parallel to the line defined by $y = 3x + 5$ and passing through the point $(3, -5)$

b) a line perpendicular to the line defined by $y = 3x + 5$ and passing through the point $(3, -5)$

c) a line parallel to the line defined by $3x + 2y = 7$ with y-intercept $= 3$

d) a line perpendicular to the line defined by $2x - 3y + 18 = 0$ with the same y-intercept

9. Determine the equation of a line perpendicular to $4x - 3y - 2 = 0$ with the same y-intercept as the line defined by $3x + 4y = -12$.

10. Determine the equation of a line perpendicular to $2x - 5y = 6$ with the same x-intercept as the line defined by $3x + 8y - 15 = 0$.

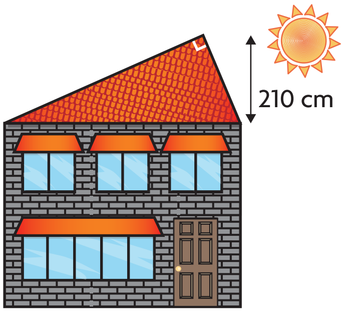
11. For the given vertices, determine whether or not ΔABC is a right triangle.

a) A$(13, 3)$, B$(3, 5)$, and C$(-2, -20)$

b) A$(5, 4)$, B$(-1, 2)$, and C$(2, -1)$

12. Show algebraically that the points A$(-4, 7)$, B$(6.5, 1)$, C$(-8, 0)$, and

T D$(2.5, -6)$ form a rectangle.

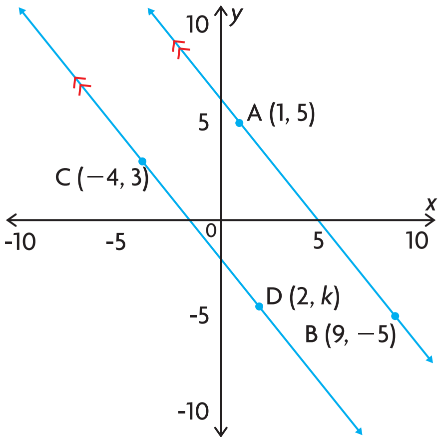


13. Show algebraically that the points E$(-9, 4)$, F$(-5, -3)$, G$(1, -4)$, and H$(-3, 3)$ form a parallelogram, but not a rectangle.

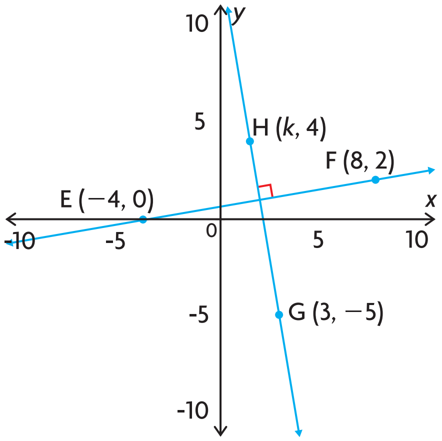
14. Mr. Rite wants his roof to be 90° at its peak and have a slope of $-\dfrac{7}{2}$ on the sunny side of the house. If the height of his roof must be 210 cm, how wide is his house?

15. Determine the value of k in each graph.

a)



b)



16. Explain why a line cannot be perpendicular to $y = \frac{3}{4}x + 2$ and also be parallel to $y = \frac{4}{5}x - 8$.

C

Extending

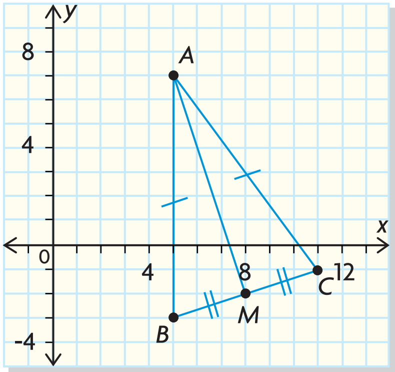
17. A line segment has endpoints A(1, −5) and B(4, 1).

a) Determine the coordinates of two points, C and D, that would make ABCD a parallelogram.

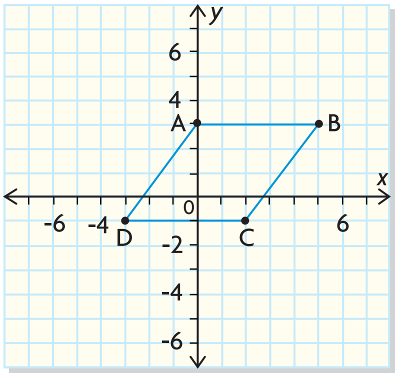
b) Determine the coordinates of two points, C and D, that would make ABCD a rectangle.

c) Determine the coordinates of two points, C and D, that would make ABCD a square.

18. $\overline{AM}$ is a median. Show that $\overline{AM}$ is perpendicular to $\overline{BC}$.



19. ABCD is a rhombus. Show that the diagonals of the rhombus are perpendicular to each other.



YOU WILL NEED

- grid paper
- ruler

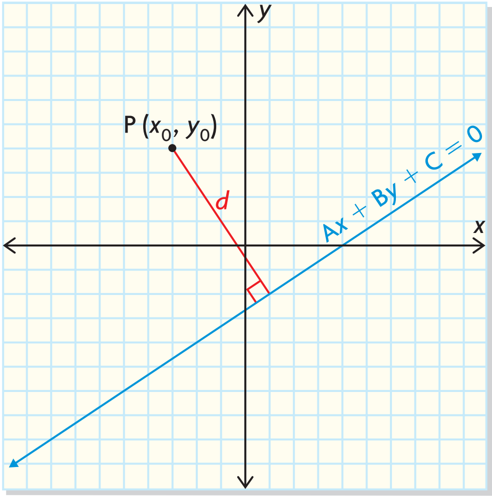
Curious Math

Distance from a Point to a Line

Calculating the shortest distance from a point to a line can be quite complicated. However, if the equation of the line is written in the form $Ax + By + C = 0$, then finding the distance is as easy as substituting into a formula.

The shortest distance d from a point $P(x_0, y_0)$ to a line $Ax + By + C = 0$ is $d = \dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$

Note: The two lines surrounding the numerator are called absolute value bars. The **absolute value** of a number is its distance from zero, and distance is always positive. For example: $|-3| = 3$ and $|3| = 3$.



1. On grid paper, plot the point $A(-5, 4)$ and the line with equation $2x - 3y - 4 = 0$.
2. Determine the shortest distance from point $A(-5, 4)$ to the line in step 1, using the Pythagorean theorem.
3. Determine the shortest distance from point $A(-5, 4)$ to the line $2x - 3y - 4 = 0$ using the distance formula $d = \dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$.
4. Determine the distance from the given point to the given line for each of the following. Round to one decimal place as necessary.
   a) $A(5, -5)$ and $2x - 3y - 4 = 0$
   b) $A(1, 4)$ and $5x + 2y - 1 = 0$
5. Identify the coordinates of any point you wish. Create the equation of a line in the form $Ax + By + C = 0$. Use the distance formula above to determine the distance from your point to your line.
6. Determine the distance from the point $A(2, 1)$ to the line $4x - 9y + 1 = 0$. What does your answer tell you about the point and the line?

FREQUENTLY ASKED *Questions*

Q: How can you determine the equation of a line?

A1: If you know the slope and y-intercept of the line, you can write the equation in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Study *Aid*

- See Lesson 5.4, Example 1.
- Try Chapter Review Questions 10 and 11.

EXAMPLE

To determine the equation of the line that has a slope of $\frac{4}{3}$ and a y-intercept of -3, substitute the values directly into the formula.

Solution

$$y = mx + b$$

$$m = \frac{4}{3}$$

$$b = -3$$

The equation is $y = \frac{4}{3}x - 3$.

A2: If you know two points on the line, determine the slope and then use either point to calculate the y-intercept.

EXAMPLE

To determine the equation of the line that passes through points A$(-3, 17)$ and B$(4, -11)$, first determine the slope.

$$\begin{aligned} m &= \frac{(-11) - 17}{4 - (-3)} \\ &= \frac{-28}{7} \\ &= -4 \end{aligned}$$

Next, substitute the slope into the equation.

$$y = -4x + b$$

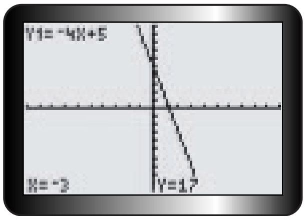
Then, use this equation and substitute the coordinates of the point $(-3, 17)$ for x and y.

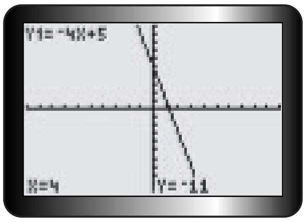
$$17 = -4(-3) + b$$

Then, solve for b.

$$\begin{aligned} 17 &= 12 + b \\ b &= 5 \end{aligned}$$

Substitute the values for m and b to create the equation.
The equation is $y = -4x + 5$.
To verify your work, use a graphing calculator.





Study ***Aid***

- See Lesson 5.5, Example 1.
- Try Chapter Review Question 13.

Q: How can you tell if two linear equations represent lines that are parallel or perpendicular without graphing them?

A1: Two lines are parallel if they have the same slope.

EXAMPLE

Determine the slope of $3x + 2y + 24 = 0$.

Solution

$$\begin{aligned} 3x + 2y + 24 &= 0 \\ 3x - 3x + 2y + 24 - 24 &= -3x - 24 \\ 2y &= -3x - 24 \\ \frac{2y}{2} &= \frac{-3x - 24}{2} \\ y &= \frac{-3x}{2} - \frac{24}{2} \\ y &= -\frac{3}{2}x - 12 \end{aligned}$$

Slope $= -\frac{3}{2}$

Determine the slope of $y = -\frac{3}{2}x + 2$.

Slope $= -\frac{3}{2}$

The lines $3x + 2y + 24 = 0$ and $y = -\frac{3}{2}x + 2$ are parallel because their slopes are equal.

Study ***Aid***

- See Lesson 5.5, Example 2.
- Try Chapter Review Question 13.

A2: Two lines are perpendicular if the product of their slopes equals -1.

EXAMPLE

Given the lines $y = 2.5x - 3.2$ and $y = -0.4x + 8.1$.

$m_{\text{line 1}} = 2.5$ $\qquad$ $m_{\text{line 2}} = -0.4$

These lines are perpendicular because $m_{\text{line 1}} \times m_{\text{line 2}} = 2.5 \times -0.4$
$= -1$

PRACTICE Questions

Lesson 5.1

1. Identify the slope and y-intercept for each line.

a) $y = 3x + 4$ **c)** $y = -1.11 + 9.7x$

b) $y = -\frac{2}{5}x - 6.8$ **d)** $y = 3$

2. Order each set of lines from closest to horizontal to closest to vertical.

a) $y = 2x - 4$
$y = x + 8$
$y = \frac{1}{3}x - 2$

b) $y = -\frac{1}{3}x + 5$
$y = -8x - 2$
$y = -\frac{5}{2}x + 3$

3. Copy and complete the table to identify whether the lines will rise or fall to the right.

	Equation	Rises to the Right	Falls to the Right
a)	$y = 4x + 5$		
b)	$y = -\frac{2}{3}x - 8$		
c)	$y = -2.8x + 4$		
d)	$y = \frac{21}{8}x$		
e)	$y = 1.5x + 4.5$		

Lesson 5.2

4. Determine the slope and y-intercept for each of these lines.

a) $3x - 4y + 9 = 0$ **c)** $2x + 6y = 32$

b) $5x - y = 12$ **d)** $8x + 2y - 4 = 0$

5. Evan and his sister Sarah shovel driveways during the winter. They charge $10 for a double driveway and $5 for a single driveway. This past winter, Evan earned $255 and Sarah earned $230.

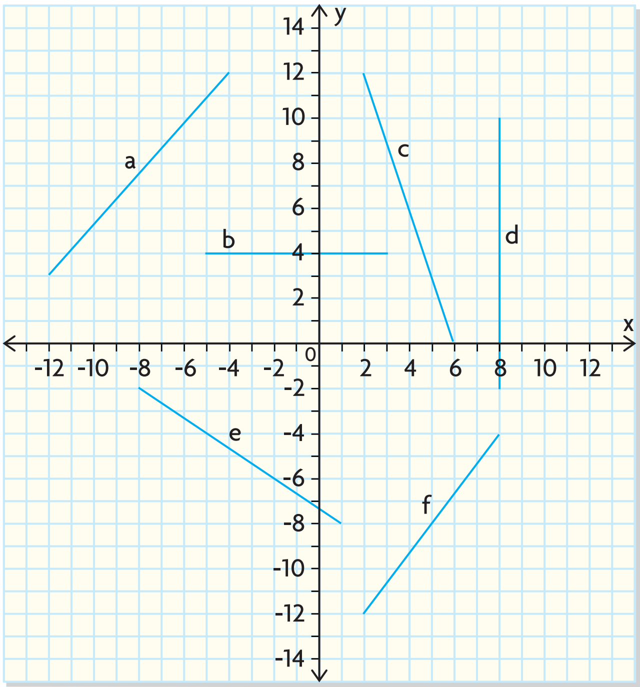
a) Write equations for both Evan and Sarah to represent the relationship between the amounts earned shovelling single and double driveways.

b) Isolate the variable used for single driveways in both equations.

c) If they both shovelled 10 double driveways, how many single driveways did each shovel?

Lesson 5.3

6. Calculate the slopes of the line segments shown below.



7. Calculate the slopes of the lines that pass through each of the following pairs of points.

a) A(8, 2) and B(1, 9)

b) E(−1, 5) and F(3, 2)

c) C(−1, 2) and D(3, −8)

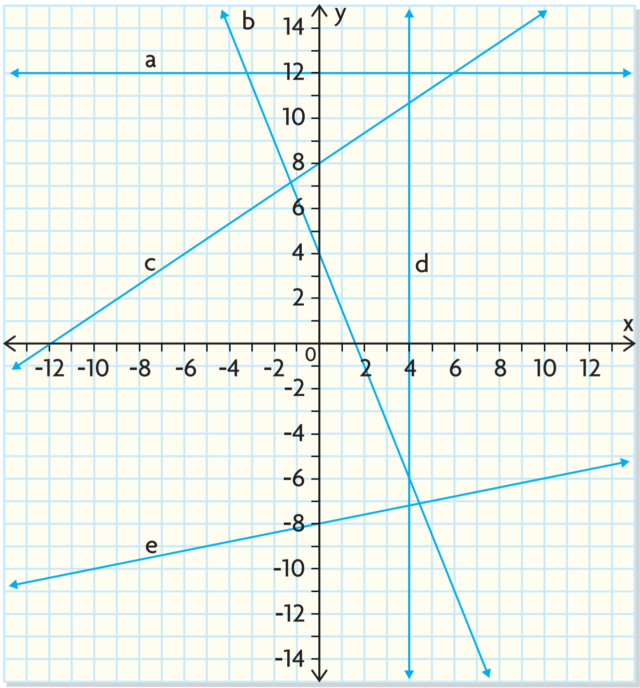
d) G(−3, 2) and H(−9, −11)

8. The points (−6, −3), (k, 1), and (8, 4) are collinear. Determine the value of k.

9. Three hours after beginning her long-distance bicycle trip, Cathy was 98 km from home. After seven hours, she was 182 km from home. Assuming she maintained the same speed throughout the trip, how fast was she cycling?

Lesson 5.4

10. Determine the equation of each line.



11. Determine the equations of the lines described below.

a) passing through the point M(6, 9) with slope $= -\frac{3}{4}$

b) passing through the points P(3, -11) and Q(0, 5)

c) passing through the points D(2, 9) and E(1, 13)

d) passing through the points A(5, 2) and B(5, -3)

e) passing through the points X(8, 5) and Y(2, 3)

12. Determine whether the points A(2, -6) and B(-3, 10) lie on the line $y = -4x + 2$.

Lesson 5.5

13. For each pair of linear equations, determine if the lines are parallel, perpendicular, or neither. Justify your answers.

a) $y = 3x - 5$
$y = -3x - 5$

b) $y = 0.25x - 2$
$y = \frac{1}{4}x - 9$

c) $y = \frac{1}{2}x + 4$
$y = -2x - 8$

d) $2x - 4y = 9$
$x + 2y + 7 = 0$

e) $y = 0.625x - 2$
$y = -1.6x - 9$

f) $3x - 5y - 10 = 0$
$5x + 3y + 2 = 0$

14. Determine the equation for each line.

a) passing through the point W(2, 9) and parallel to $y = \frac{7}{2}x + 3$

b) passing through the point V(1, 6) and perpendicular to $y = -\frac{1}{4}x + 11$

c) passing through the *y*-intercept of the line defined by $2x + 3y - 18 = 0$ and perpendicular to $4x - 9y = 27$

15. a) Do you think that the diagonals of a square are perpendicular to each other?

b) Test your conjecture by plotting 4 points on grid paper that form a square. Draw the sides and diagonals of the square.

c) Calculate the slopes of the diagonals. Does this support your conjecture? Explain.

d) Repeat parts b) and c) using 4 different points. Is your result the same?

5 Chapter Self-Test

1. Which choice best describes the line defined by the equation $y = -4x + 27$?

A. rising to the right **C.** horizontal

B. falling to the right **D.** vertical

2. Which of the following statements is true about the line defined by the equation $y = \frac{1}{3}x + 2$?

A. It is steeper than the line defined by $y = \frac{1}{6}x - 4$.

B. It has the same y-intercept as the line defined by the equation $y = \frac{1}{5}x + 2$.

C. It is less steep than the line defined by $y = 5x - 6$.

D. all of the above

3. Which of the following equations represents the same line as described by $12x - 3y + 21 = 0$?

A. $y = \frac{1}{4}x - 7$ **C.** $y = 4x + 7$

B. $y = -4x + 21$ **D.** $y = \frac{1}{4}x + 63$

4. What can be said about the lines given by the equations $3x + 7y = 28$ and $y = \frac{7}{3}x - 2$?

A. they are perpendicular

B. they are parallel

C. they are the same

D. none of the above

5. A line passes through the point $(1, -4)$ and has a slope of $\frac{5}{2}$. Which of the following points would also be on this line?

A. $(6, -2)$ **C.** $(-1, 1)$

B. $(3, 1)$ **D.** $(3, -9)$

6. Sketch the graph of $y = \frac{-4}{5}x + 3$ using the slope and y-intercept.

7. Are the points A$(-10, -4)$, B$(-3, 7)$, and C$(2, 14)$ collinear? Explain how you know.

8. Points M$(14, 6)$ and N$(-7, k)$ lie on a line that has a slope of $\frac{3}{7}$. Determine the value of k.

9. Tickets for this year's major drama production cost \$8 for adults and \$6 for students. Last night's show earned \$2200.

a) Write an equation to represent the relationship between the number of adult tickets sold and the number of student tickets sold.

b) Rearrange your relationship to isolate the variable representing the number of adult tickets.

c) If no students purchased tickets, how many adult tickets were sold?

d) If 148 students purchased tickets, how many adult tickets were sold?

10. Determine the equation of the line that passes through the points $(-5, 7)$ and $(5, 15)$.

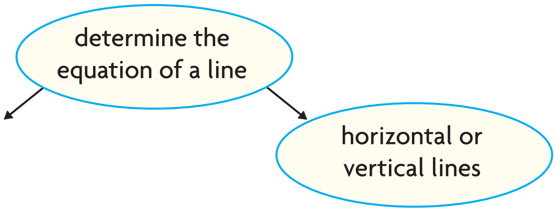
11. Carrie-Lynn has just recorded her first CD and would like to create 400 CDs for her upcoming CD release party. The company CD-Clone charges \$159 for 100 CDs and \$297 for 250 CDs.

a) Write the equation for the relationship between the numbers of CDs created and the total cost.

b) Use your equation to calculate the cost for 400 CDs.

12. Determine the equation of the line that is perpendicular to the line $6x + 10y - 1 = 0$ and has the same y-intercept as $3x + y = 1$.

13. Create a mind map to organize what you know about determining the equation of a line. Include the following words in your mind map: *horizontal*, *vertical*, *slope*, *y-intercept*, *point(s)*, *parallel*, and *perpendicular*. Start your mind map like this:



5 Chapter Task

Urban Planning

Many downtown centres—such as Montreal and Ottawa—were laid out in a grid pattern by early urban planners. This image of Toronto illustrates the grid pattern. The coordinate axes have the origin at the CN Tower.

? Are the streets of downtown Toronto parallel and/or perpendicular to each other?

Using a scale of 1 unit = 100 m and dynamic graphing software, Macy determined the approximate coordinates of several major intersections and recorded them in a table.

Intersection	Label	Coordinates
Jarvis and Bloor	A	(13.9, 20.5)
Jarvis and Adelaide	B	(14.0, 1.1)
Yonge and Bloor	C	(6.3, 20.5)
Yonge and College	D	(6.4, 11.2)
Yonge and Dundas	E	(6.5, 5.7)
University and Adelaide	F	(0.6, 0.6)
Spadina and College	G	(−6.9, 11.4)
Spadina and Dundas	H	(−6.8, 5.7)
Bathurst and Bloor	I	(−13.1, 20.5)
Bathurst and College	J	(−12.9, 11.2)

A. Plot and label the given coordinates on a graph.

B. Assuming the streets are straight, determine equations to represent the following streets: Jarvis, Yonge, Spadina, Bathurst, Bloor, Adelaide, College, and Dundas. Express the slopes and y-intercepts to two decimal places.

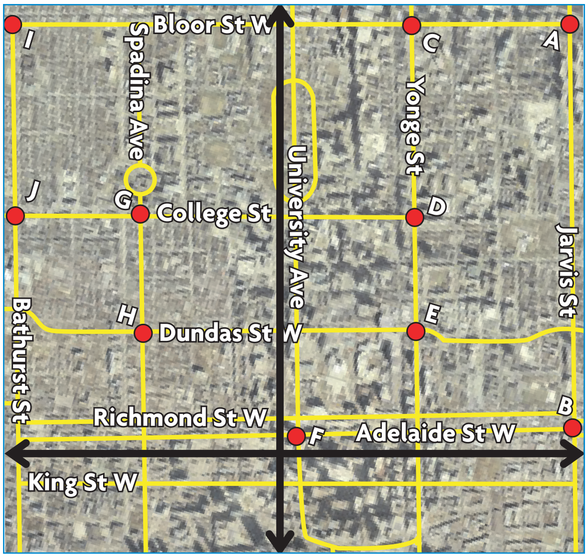
C. Are any of the streets in part B parallel? Justify your answer.

D. Are any of the streets in part B perpendicular? Justify your answer.

E. Use the information you have to discuss whether the streets of downtown Toronto really do form a grid.

YOU WILL NEED

- grid paper



Task Checklist

- ✔ Did you check to make sure that your equations match your graph?
- ✔ Did you remember to label your graph?
- ✔ Did you show all your work for your calculations?
- ✔ Did you check your calculations?
- ✔ Did you explain your thinking clearly?
- ✔ Did you justify your answers mathematically?