Math Skills

Scientific Notation

It is difficult to work with very large or very small numbers when they are written in common decimal notation. Usually it is possible to accommodate such numbers by changing the SI prefix so that the number falls between 0.1 and 1000; for example, 237 000 000 mm can be expressed as 237 km and 0.000 000 895 kg can be expressed as 0.895 mg. However, this prefix change is not always possible, either because an appropriate prefix does not exist or because it is essential to use a particular unit of measurement. In these cases, the best method of dealing with very large and very small numbers is to write them using scientific notation. Scientific notation expresses a number by writing it in the form $a \times 10^n$, where $1 < |a| < 10$ and the digits in the coefficient a are all significant. **Table 1** shows situations where scientific notation would be used.

Table 1: Examples of Scientific Notation

Expression	Common decimal notation	Scientific notation
124.5 million kilometres	124 500 000 km	1.245×10^8 km
154 thousand picometres	154 000 pm	1.54×10^{-5} pm
602 sextillion /mol	602 000 000 000 000 000 000 000 /mol	6.02×10^{23}/mol

To multiply numbers in scientific notation, multiply the coefficients and add the exponents; the answer is expressed in scientific notation. Note that when writing a number in scientific notation, the coefficient should be between 1 and 10 and should be rounded to the same certainty (number of significant digits) as the measurement with the least certainty (fewest number of significant digits). Look at the following examples:

$$(4.73 \times 10^5 \text{ m})(5.82 \times 10^7 \text{ m}) = 27.5 \times 10^{12} \text{ m}^2 = 2.75 \times 10^{13} \text{ m}^2$$

$$(3.9 \times 10^4 \text{ N})(5.3 \times 10^{-3} \text{ m}) = 0.74 \times 10^7 \text{ N·m} = 7.4 \times 10^6 \text{ N·m}$$

On many calculators, scientific notation is entered using a special key, labelled EXP or EE. This key includes "× 10" from the scientific notation; you need to enter only the exponent. For example, to enter

7.5×10^4	press	7.5 EXP 4
3.6×10^{-3}	press	3.6 EXP +/–3

Uncertainty in Measurements

There are two types of quantities that are used in science: exact values and measurements. Exact values include defined quantities (1 m = 100 cm) and counted values (5 cars in a parking lot). Measurements, however, are not exact because there is some uncertainty or error associated with every measurement.

There are two types of measurement error. **Random error** results when an estimate is made to obtain the last significant figure for any measurement. The size of the random error is determined by the precision of the measuring instrument. For example, when measuring length, it is necessary to estimate between the marks on the measuring tape. If these marks are 1 cm apart, the random error will be greater and the precision will be less than if the marks are 1 mm apart.

Systematic error is associated with an inherent problem with the measuring system, such as the presence of an interfering substance, incorrect calibration, or room conditions. For example, if the balance is not zeroed at the beginning, all measurements will have a systematic error; if using a metre stick that has been worn slightly, all measurements will contain an error.

The precision of measurements depends upon the gradations of the measuring device. **Precision** is the place value of the last measurable digit. For example, a measurement of 12.74 cm is more precise than a measurement of 127.4 cm because the first value was measured to hundredths of a centimetre whereas the latter was measured to tenths of a centimetre.

When adding or subtracting measurements of different precision, the answer is rounded to the same precision as the least precise measurement. For example, using a calculator, add

$$11.7 \text{ cm} + 3.29 \text{ cm} + 0.542 \text{ cm} = 15.532 \text{ cm}$$

The answer must be rounded to 15.5 cm because the first measurement limits the precision to a tenth of a centimetre.

No matter how precise a measurement is, it still may not be accurate. Accuracy refers to how close a value is to its true value. The comparison of the two values can be expressed as a percentage difference. The percentage difference is calculated as:

$$\% \text{ difference} = \frac{|\text{experimental value} - \text{predicted value}|}{\text{predicted value}} \times 100$$
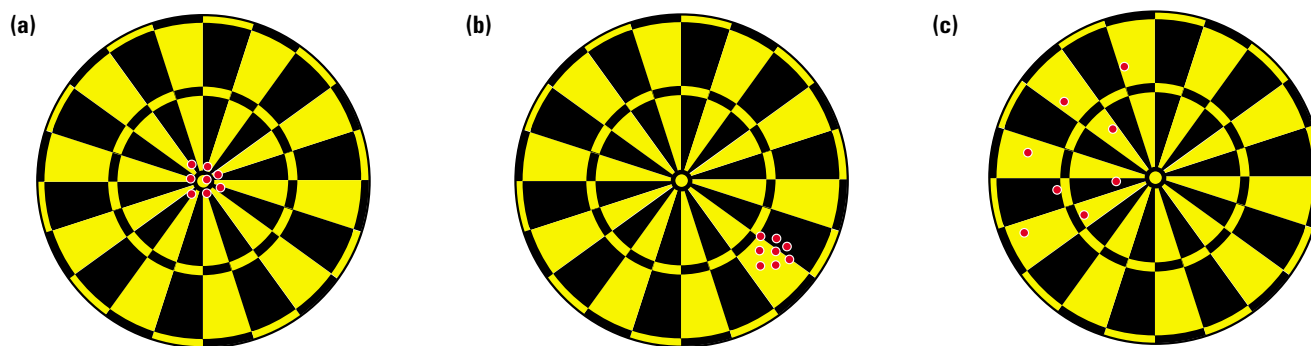
Figure 1
The positions of the darts in each of these figures are analogous to measured or calculated results in a laboratory setting.
The results in **(a)** are precise and accurate, in **(b)** they are precise but not accurate, and in **(c)** they are neither precise nor accurate.

Figure 1 shows an analogy between precision and accuracy, and the positions of darts thrown at a dartboard.

How certain you are about a measurement depends on two factors: the precision of the instrument used and the size of the measured quantity. More precise instruments give more certain values. For example, a mass measurement of 13 g is less precise than a measurement of 12.76 g; you are more certain about the second measurement than the first. Certainty also depends on the measurement. For example, consider the measurements 0.4 cm and 15.9 cm; both have the same precision. However, if the measuring instrument is precise to ± 0.1 cm, the first measurement is 0.4 ± 0.1 cm (0.3 cm or 0.5 cm) or an error of 25%, whereas the second measurement could be 15.9 ± 0.1 cm (15.8 cm or 16.0 cm) for an error of 0.6%. For both factors—the precision of the instrument used and the value of the measured quantity—the more digits there are in a measurement, the more certain you are about the measurement.

Significant Digits

The certainty of any measurement is communicated by the number of significant digits in the measurement. In a measured or calculated value, significant digits are the digits that are certain plus one estimated (uncertain) digit. Significant digits include all digits correctly reported from a measurement.

Follow these rules to decide if a digit is significant:

1. If a decimal point is present, zeros to the left of the first non-zero digit (leading zeros) are not significant.

2. If a decimal point is not present, zeros to the right of the last non-zero digit (trailing zeros) are not significant.

3. All other digits are significant.

4. When a measurement is written in scientific notation, all digits in the coefficient are significant.

5. Counted and defined values have infinite significant digits.

Table 2 shows some examples of significant digits.

An answer obtained by multiplying and/or dividing measurements is rounded to the same number of significant digits as the measurement with the fewest number of signifi-

Table 2: Certainty in Significant Digits

Measurement	Number of significant digits
32.07 m	4
0.0041 g	2
5×10^5 kg	1
6400 s	2
204.0 cm	4
10.0 kJ	3
100 people (counted)	infinite

cant digits. For example, if we use a calculator to solve the following equation:

$$77.8 \text{ km/h} \times 0.8967 \text{ h} = 69.76326 \text{ km}$$

However, the certainty of the answer is limited to three significant digits, so the answer is rounded up to 69.8 km.

Rounding Off

The following rules should be used when rounding answers to calculations.

1. When the first digit discarded is less than five, the last digit retained should not be changed.
   3.141 326 rounded to 4 digits is 3.141

2. When the first digit discarded is greater than five, or if it is a five followed by at least one digit other than zero, the last digit retained is increased by 1 unit.
   2.221 672 rounded to 4 digits is 2.222
   4.168 501 rounded to 4 digits is 4.169

3. When the first digit discarded is five followed by only zeros, the last digit retained is increased by 1 if it is odd, but not changed if it is even.
   2.35 rounded to 2 digits is 2.4
   2.45 rounded to 2 digits is 2.4
   –6.35 rounded to 2 digits is –6.4

Measuring and Estimating

Many people believe that all measurements are *reliable* (consistent over many trials), *precise* (to as many decimal places as possible), and *accurate* (representing the actual value). But there are many things that can go wrong when measuring.

- There may be limitations that make the instrument or its use unreliable (inconsistent).
- The investigator may make a mistake or fail to follow the correct techniques when reading the measurement to the available precision (number of decimal places).
- The instrument may be faulty or inaccurate; a similar instrument may give different readings.

For example, when measuring the temperature of a liquid, it is important to keep the thermometer at the correct depth and the bulb of the thermometer away from the bottom and sides of the container. If you set a thermometer with its bulb on the bottom of a liquid-filled container, you will be measuring the temperature of the bottom of the container, and not the temperature of the liquid. There are similar concerns with other measurements.

To be sure that you have measured correctly, you should repeat your measurements at least three times. If your measurements appear to be reliable, calculate the mean and use that value. To be more certain about the accuracy, repeat the measurements with a different instrument.

Logarithms

Any positive number N can be expressed as a power of some base b where $b > 1$. Some obvious examples are:

$16 = 2^4$ base 2, exponent 4

$25 = 5^2$ base 5, exponent 2

$27 = 3^3$ base 3, exponent 3

$0.001 = 10^{-3}$ base 10, exponent −3

In each of these examples, the exponent is an integer; however, exponents may be any real number, not just an integer. If you use the x^y button on your calculator, you can experiment to obtain a better understanding of this concept.

The most common base is base 10. Some examples for base 10 are

$10^{0.5} = 3.162$

$10^{1.3} = 19.95$

$10^{-2.7} = 0.001995$

By definition, the exponent to which a base b must be raised to produce a given number N is called the **logarithm** of N to base b (abbreviated as $\log_b$). When the value of the base is not written, it is assumed to be base 10. Logarithms to base 10 are called **common logarithms**. We can express the previous examples as logarithms:

$\log 3.162 = 0.5$

$\log 19.95 = 1.3$

$\log 0.001995 = -2.7$

Most measurement scales you have encountered are linear in nature. For example, a speed of 80 km/h is twice as fast as a speed of 40 km/h and four times as fast as a speed of 20 km/h. However, there are several examples in science where the range of values of the variable being measured is so great that it is more convenient to use a logarithmic scale to base 10. One example of this is the scale for measuring the acidity of a solution (the pH scale). For example, a solution with a pH of 3 is 10 times more acidic than a solution with a pH of 4 and 100 times (10^2) more acidic than a solution with a pH of 5. Other situations that use logarithmic scales are sound intensity (the dB scale) and the intensity of earthquakes (the Richter scale).